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Network Control in Realistic Settings with Heterogeneous Vehicles

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Vehicle scheduling in roadway conflict areas is a common yet challenging problem due to the large vehicle population and heterogeneity, including the significant presence of transit vehicles. Traditional methods are time-consuming and fail to meet real-time operation needs. This study proposes a platooning strategy for connected and automated vehicles in conflict areas. Vehicles are clustered into platoons and treated as scheduling units, reducing problem complexity. Theoretical analyses show a bounded solution gap between single-vehicle and platoon-based strategies of vehicle delay. Numerical experiments confirm the superiority of the proposed strategy in reducing problem size without significant loss of optimality. This platooning approach has the potential to optimize vehicle scheduling, including transit vehicles, and enhance network control in heterogeneous vehicle environments where specific groups of vehicles such as transit are prioritized.</td></tr>
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Executive Summary

Vehicle scheduling in roadway conflict areas is a pervasive and complex challenge. The sheer volume of vehicles exacerbates the time-consuming nature of solving the scheduling problem, rendering it inadequate for real-time operational needs. To address this issue effectively, this study introduces a platooning strategy for connected automated vehicles when traversing through general conflict areas. The proposed strategy aggregates vehicles into platoons, treating each platoon as a scheduling unit and thereby reducing the problem's complexity. The heterogeneity of the vehicle is simultaneously transformed into the characteristics of the platoon. Theoretical analyses demonstrate that the disparity in optimal solutions (in terms of vehicle delay) between the single-vehicle–based strategy and the proposed platoon-based strategy is bounded. Numerical experiments further validate the effectiveness of the proposed strategy, showing its superiority in reducing the scheduling problem size without significant compromise to solution optimality. These findings contribute to the domain of network control in realistic settings with heterogeneous vehicles, addressing the challenges associated with coordinating diverse vehicle types efficiently. By leveraging platooning and treating platoons as scheduling units, the proposed strategy improves scalability and enhances the potential for real-time network control in heterogeneous vehicle environments.

Chapter 1. Introduction

Conflict areas commonly exist in roadway systems, such as intersections, work zones, and ramps. Conflict areas are usually the bottlenecks of the road network, which cause serious delays. Vehicle scheduling decides the appropriate passing sequence for vehicles from different approaches. Preferable vehicle scheduling enhances the efficiency of the conflict area (e.g., reducing vehicle delay and elevating throughput).

In the existing literature, vehicle scheduling is usually formulated into a Mixed-Integer Linear Programming (MILP) problem. These studies include vehicle scheduling at intersections (Li & Wang, 2006; Long et al., 2020; Xu, Zhang, Li, & Li, 2019; Yu et al., 2018; Yu et al., 2019), roundabouts (Debada, Makarem, & Gillet, 2016; Long et al., 2022; Martin-gasulla & Elefteriadou, 2021), and ramps (Xu, Feng, Zhang, & Li, 2019). The difficulty of solving the problem increases exponentially with the number of vehicles (Chen & Englund, 2016). Efforts have been made to facilitate MILP solving with various methods proposed in the past decades. Existing methods are generally categorized into two groups. The first group is constructing efficient heuristics to approximate the MILP optimal solution (Debada, Makarem, & Gillet, 2016; Long et al., 2022; Martin-gasulla & Elefteriadou, 2021; Rios-Torres & Malikopoulos, 2016; Zhang, Liu, & Waller, 2019). The second group is developing MILP-solving algorithms to expedite the optimal solution (Li & Wang, 2006; Xu, Zhang, Li, & Li, 2019).

Recently, some pioneer studies have approached the vehicle scheduling problem from another perspective by reducing the problem size (Bashiri et al., 2017; Xu, Feng, Zhang, & Li, 2019). Specifically, vehicles are clustered into platoons within which they closely follow each other. Given the tightness, multiple vehicles in a platoon can be viewed and scheduled as one unit. Once the number of scheduling units decreases, the dimension of the MILP is much reduced. As a result, the corresponding solution is facilitated (Xu, Feng, Zhang, & Li, 2019).

The existing literature numerically demonstrates the effectiveness of the platoon-based method in helping address vehicle scheduling problems (Li & Li, 2022; Xu, Feng, Zhang, & Li, 2019). However, no theoretical insights have been revealed about the platoon-based method performance. The impact and usage scenarios of the platoon-based method have not been studied. This limits our understanding of the vehicle scheduling problem and impedes platoon parameter selections.

This study is motivated to analytically investigate the performance of the platoon-based method in approaching vehicle scheduling problems. Theoretical analyses show that the optimal solution gap (in terms of vehicle delay) between the traditional vehicle-based scheduling method and the platoon-based scheduling method is bounded, and the upper bound is analytically solved. Further, the number of platoons is analytically derived. These theoretical insights provide the basis for constructing an appropriate platooning method that reduces the number of scheduling units while guaranteeing scheduling performance. Experiments are carried out to verify the effectiveness of the proposed platoon-based method in solving vehicle scheduling problems by comparing it with a benchmark.

This paper is organized as follows. Chapter 2 presents the investigated vehicle scheduling problem. Chapter 3 discusses the theoretical analyses and develops a platooning method. Chapter 4 conducts numerical experiments to test the performance of the proposed platoon-based scheduling method by comparing it with a benchmark. Chapter 5 concludes this paper and briefly discusses future research directions.

Chapter 2. Problem Statement

Vehicle scheduling is to find the optimal sequence of vehicles from different approaches to pass a conflict area (e.g., an intersection or a merging point) to minimize the average vehicle delay.

Figure 1 illustrates this problem with two conflicting approaches. We consider n_l vehicles following one another from Approach $l \in \mathcal{L} \coloneqq \{1,2\}$. We index vehicle $i \in \{1, \cdots, n_l\}$ from approach $l \in \mathcal{L}$ as a tuple (l, i). Let $\mathcal{I}_l \coloneqq \{(l, 1), \cdots, (l, n_l)\}$ denote the sets of vehicle indexes from Approach $l \in \mathcal{L}$. Define $n: = \sum_{l \in \mathcal{L}} n_l$ as the total number of vehicles in the system. And let $\mathcal{I}: = \cup_{l \in \mathcal{L}} \mathcal{I}_l$ denote the set of vehicle indexes from all approaches for the convenience of the notation. Let $\underline{t}_{li}$ denote the earliest time for Vehicle $(l, i) \in \mathcal{I}$ to pass the conflict area if it keeps proceeding at the free-flow speed without impedance from downstream or conflicting-approach vehicles.

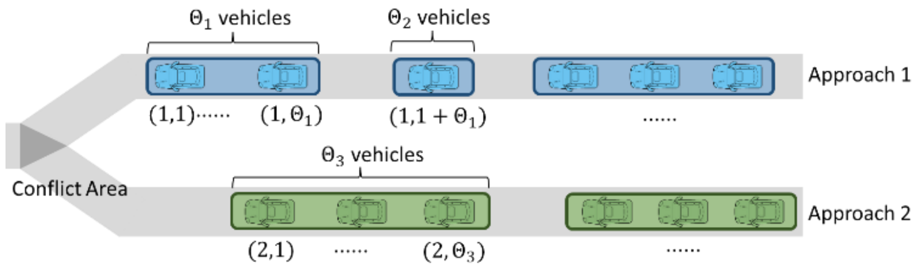

Figure 1. An illustration of the problem.

A passing sequence is denoted by the vector of the corresponding vehicle indexes according to their order of passing the conflict area; that is, $s \coloneqq [(l_{s1}, i_{s1}), (l_{s2}, i_{s2}), \cdots, (l_{sn}, i_{sn})]$ where Vehicle (l_{sk}, i_{sk}) (or Vehicle sk for short) denotes the k^{th} vehicle passing the conflict area $\forall k \in \{1, \cdots, n\}$ and $\cup_{k \in \{1,\cdots,n\}} (l_{sk}, i_{sk}) = \mathcal{I}$. Let $\mathcal{S}$ denote the set of all possible passing sequences. For a given passing sequence $s \in \mathcal{S}$, let t_{sk} denote the actual time when Vehicle (l_{sk}, i_{sk}) passes the conflict area, and $h_{sk} \coloneqq t_{sk} - t_{s(k-1)}$ denote the time headway from Vehicle sk to Vehicle $s(k-1)$ to pass the conflict area. We assume the headway of two consecutive vehicles from the same approach needs to be always no less than the minimum following headway denoted as h^{F}. And the headway of two vehicles from different approaches consecutively passing the conflict area is no less than the minimum switching headway h^{S}; that is, $h_{sk} \geq h^{\text{F}}$ if $l_{sk} = l_{s(k-1)}$ or $h_{sk} \geq h^{\text{S}}$ if $l_{sk} \neq l_{s(k-1)}, \forall k \in \{2, \cdots, n\}$. Since it usually takes a long time to switch from one approach to another (e.g., due to clearance time), we assume that $h^{\text{F}} < h^{\text{S}}$ without loss of generality. With this, the actual passing time set $\{t_{sk}\}$ can be solved with the following iterative equation:

$$t_{s1} = \underline{t}_{i_{s1} l_{s1}}, \tag{1}$$

$$t_{sk} = \max\left\{\underline{t}_{i_{sk} l_{sk}}, \underline{t}_{i_{s(k-1)} l_{s(k-1)}} + \underline{h}_{sk}\right\} \forall k \in \{2, \cdots, n\} \tag{2}$$

where $\underline{t}_{il}$ is the earliest time for Vehicle (i, l) to pass the conflict area if it keeps proceeding at the free-flow speed without impedance from downstream or conflicting-approach vehicles, $\underline{h}_{sk}$ is the minimum headway between Vehicle sk and Vehicle $s(k-1)$ with respect to a given Sequence s.

$$\underline{h}_{sk} = \begin{cases} h^{\text{F}}, & \text{if } l_{sk} = l_{s(k-1)}; \\ h^{\text{S}}, & \text{otherwise}, \end{cases} \tag{3}$$

where h^{F} is the minimum following headway between two consecutive vehicles from the same approach, h^{S} is the minimum switching headway between two vehicles from different approaches consecutively passing the conflict area.

Equation (1) indicates that the first vehicle will always pass the conflict area at its earliest passing time without downstream impedance. Equation (2) indicates that each following vehicle sk will pass the conflict area at its earliest passing time unless it is held back by the previous Vehicle $s(k-1)$ by the following or switching headway.

Define the average vehicle delay with respect to a given sequence s as follows:

$$\overline{D}(s) \coloneqq \frac{\sum_{k=1}^{n} t_{sk} - \underline{t}_{l_{sk} i_{sk}}}{n}, \forall s \in \mathcal{S}. \tag{4}$$

Define the optimal solution (OS) as the set of optimal sequences s^* that minimizes the average vehicle delay:

$$S^* \coloneqq \left\{ s^* | s^* = \arg\min_{s \in \mathcal{S}} \overline{D}(s) \right\} \tag{5}$$

Based on the investigated problem, we propose a platoon method. In this method, adjacent vehicles at the same approach are platooned according to a rule. Platooned vehicles cross the conflict area continuously without the insertion of vehicles from other approaches. We index platoon $j \in \left\{1, \cdots, n_l^{\mathrm{P}}\right\}$ from approach $l \in \mathcal{L}$ as a tuple (l, j). Platoon (l, j) is defined by a vector of vehicles $\left[\left(l, i_j\right), \left(l, i_j + 1\right), \cdots, \left(l, i_j + \Theta_j\right)\right]$ where $\underline{t}_{l, i_j + \theta} - \underline{t}_{l, i_j + \theta - 1} \leq \delta, \forall \theta \in \{2, \cdots, \Theta_j\}$. Vehicle $\left(l, i_j\right)$ is the first vehicle in Platoon (l, j), Θ_j is the number of vehicles in Platoon (l, j), and $\Theta_j \geq 1$. Define $n^{\mathrm{P}} := \sum_{l \in \mathcal{L}} n_l^{\mathrm{P}}$ as the total number of platoons in the system. Let $\mathcal{J}_l^{\mathrm{P}} \coloneqq \{(l, 1), \cdots (l, n_l^{\mathrm{P}})\}$ denote the sets of platoon indexes from Approach $l \in \mathcal{L}$. And let $\mathcal{J}^{\mathrm{P}} := \cup_{l \in \mathcal{L}} \mathcal{J}_l^{\mathrm{P}}$ denote the set of platoon indexes from all approaches.

A platoon passing sequence is denoted by the vector of the platoon indexes according to their order of passing the conflict area; that is, $s^{\mathrm{P}} \coloneqq [(l_{s1}, j_{s1}), (l_{s2}, j_{s2}), \cdots, (l_{sn}, j_{sn})]$. Let $\mathcal{S}^{\mathrm{P}}$ denote the set of all possible passing sequences when the platoon rule is applied, $\mathcal{S}^{\mathrm{P}} \subseteq \mathcal{S}$.

Define the platooned optimal solution (POS) as the set of optimal sequences that minimizes the average vehicle delay when the platoon rule is applied:

$$S^{\mathrm{P}*} \coloneqq \left\{ s^{\mathrm{P}*} | s^{\mathrm{P}*} = \arg\min_{s^{\mathrm{P}} \in \mathcal{S}^{\mathrm{P}}} \overline{D}\left(s^{\mathrm{P}}\right) \right\} \tag{6}$$

$$\overline{D}\left(s^{\mathrm{P}}\right) \coloneqq \frac{\sum_{k=1}^{n} t_{sk} - \underline{t}_{l_{sk}, i_{sk}}}{n}, \forall s^{\mathrm{P}} \in \mathcal{S}^{\mathrm{P}}. \tag{7}$$

$s^{\mathrm{P}*}$ cannot be a better solution than s^* since the solution space of the platoon-considered problem $\mathcal{S}^{\mathrm{P}}$ is smaller than that of the original problem $\mathcal{S}$. Define $G = \min_{s^{\mathrm{P}} \in \mathcal{S}^{\mathrm{P}}} \overline{D}\left(s^{\mathrm{P}}\right) - \min_{s \in \mathcal{S}} \overline{D}(s)$ as the optimal solution gap between the minimized average vehicle delay and the minimized average vehicle delay when the platoon rule is applied.

Chapter 3. Methodology

This section first conducts theoretical analyses on platoon-based vehicle scheduling problems. The yielding insights are used to develop a specific platooning method that is expected to facilitate vehicle scheduling problem solving by reducing the number of scheduling units without much loss of the solution optimality.

Theoretical Analyses

In this subsection, the optimal gap between the vehicle-based scheduling method and the platoon-based scheduling method is analyzed. The upper limit is analytically solved. Next, the number of scheduling units (platoons) after platooning is theoretically studied.

The Upper Limit of the Optimal Solution Gap

We first proposed a proposition that describes the feature of situations with a non-zero G. Then this can help us bound the upper limit of G.

For convenience, we define breaking headway as $h_{sk}^{\mathrm{B}} \coloneqq \underline{t}_{sk} - \underline{t}_{s(k-1)}, l_{sk} = l_{s(k-1)}, \forall k \in \{2, \cdots, n\}$. For example, given the passing sequence of vehicles shown in Figure 1, which can be denoted as $[(1,1), \cdots, (1, \Theta_1), (2,1), \cdots, (2, \Theta_3), (1,1+\Theta_1), \cdots]$, the time gap between the earliest arrival time of Vehicle $(1, i_j + \Theta_1 - 1)$ and Vehicle $(1, i_j + \Theta_1)$ is breaking headway.

Proposition. In an OS, every breaking headway should be greater than h^{F}.

Prof: The contrary proposition of Proposition 1 is there existed one instance, and the optimal solution to this instance has at least one headway smaller than h^{F}. Therefore, proving Proposition 1 can be achieved by proving the incorrectness of the contrary proposition of Proposition 1.

We assume a general form of the optimal solution, defined as s_1 shown in Figure 2. The breaking headway between Vehicle $\left(l_{s_1,1}, i_{s_1,1}\right)$ (or Vehicle $s_1, 1$ for short) and Vehicle $\left(l_{s_1,2+\Theta_2}, i_{s_1,2+\Theta_2}\right)$ (or Vehicle $s_1, 2+\Theta_2$ for short) is smaller than h^{F}. Another solution s_2 is formed by switching the passing sequence: Vehicle $s_1, 2+\Theta_2$ passes the conflict area earlier than the platoon from Vehicle $s_1, 2$ to Vehicle $s_1, 1+\Theta_2$. Suppose the passing sequence of other vehicles keeps the same. Let $t_{s_1,1}$ denote the actual time when Vehicle $s_1, 1$ passes the conflict area, $t_{s_1,1} = t_{s_2,1}$. Therefore, we can get the actual time of each vehicle passes the conflict area in Solution 1:

$$\begin{cases} t_{s_1,2} = t_{s_1,1} + h^{\mathrm{S}} \\ \vdots \\ t_{s_1,1+\Theta_2} = t_{s_1,1} + h^{\mathrm{S}} + (\Theta_2 - 1) \cdot h^{\mathrm{F}} \\ t_{s_1,2+\Theta_2} = t_{s_1,1} + 2h^{\mathrm{S}} + (\Theta_2 - 1) \cdot h^{\mathrm{F}} \\ \vdots \\ t_{s_1,1+\Theta_1+\Theta_2} = t_{s_1,1} + 2h^{\mathrm{S}} + (\Theta_1 + \Theta_2 - 2) \cdot h^{\mathrm{F}} \\ t_{s_1,2+\Theta_1+\Theta_2} = t_{s_1,1} + 3h^{\mathrm{S}} + (\Theta_1 + \Theta_2 - 2) \cdot h^{\mathrm{F}} \end{cases} \tag{8}$$

Similarly, we can get the actual time each vehicle passes the conflict area in Solution 2:

$$\begin{cases} t_{s_2,2} = t_{s_2,1} + h^{\mathrm{F}} \\ \vdots \\ t_{s_2,1+\Theta_1} = t_{s_2,1} + \Theta_1 \cdot h^{\mathrm{F}} \\ t_{s_2,2+\Theta_1} = t_{s_2,1} + h^{\mathrm{S}} + \Theta_1 \cdot h^{\mathrm{F}} \\ \vdots \\ t_{s_2,1+\Theta_1+\Theta_2} = t_{s_2,1} + h^{\mathrm{S}} + (\Theta_1 + \Theta_2 - 1) \cdot h^{\mathrm{F}} \\ t_{s_2,2+\Theta_1+\Theta_2} = t_{s_2,1} + h^{\mathrm{S}} + (\Theta_1 + \Theta_2) \cdot h^{\mathrm{F}} \end{cases} \tag{9}$$

The total travel time of the $3 + \Theta_1 + \Theta_2$ vehicles can get, and we have:

$$\sum_{k=1}^{3+\Theta_1+\Theta_2} t_{s_1,k} = (3 + \Theta_1 + \Theta_2) t_{s_1,1} + (2\Theta_1 + \Theta_2 + 3) h^{\mathrm{S}} + \frac{{\Theta_1}^2 + {\Theta_2}^2 + 2\Theta_1\Theta_2 - \Theta_1 + \Theta_2 - 4}{2} h^{\mathrm{F}} \tag{10}$$

$$\sum_{k=1}^{3+\Theta_1+\Theta_2} t_{s_2,k} = (3 + \Theta_1 + \Theta_2) t_{s_2,1} + (\Theta_2 + 1) h^{\mathrm{S}} + \frac{{\Theta_1}^2 + {\Theta_2}^2 + 2\Theta_1\Theta_2 + 3\Theta_1 + \Theta_2}{2} h^{\mathrm{F}} \tag{11}$$

$$\bar{D}(s^1) - \bar{D}(s^2) = \frac{\sum_{k=1}^{3+\Theta_1+\Theta_2}\left(t_{s_1,k} - \underline{t}_k\right) - \sum_{k=1}^{3+\Theta_1+\Theta_2}\left(t_{s_2,k} - \underline{t}_k\right)}{3 + \Theta_1 + \Theta_2} = \frac{2\Theta_1 + 2}{3 + \Theta_1 + \Theta_2}\left(h^{\mathrm{S}} - h^{\mathrm{F}}\right) \tag{12}$$

$\bar{D}(s^1) - \bar{D}(s^2) > 0$ given the fact that $\Theta_1 \geq 1$ and $h^{\mathrm{S}} > h^{\mathrm{F}}$. This indicates that the total system delay of Solution 2 is smaller than that of Solution 1. Solution 1 is not the optimal solution. This proves the correctness of Proposition 1.

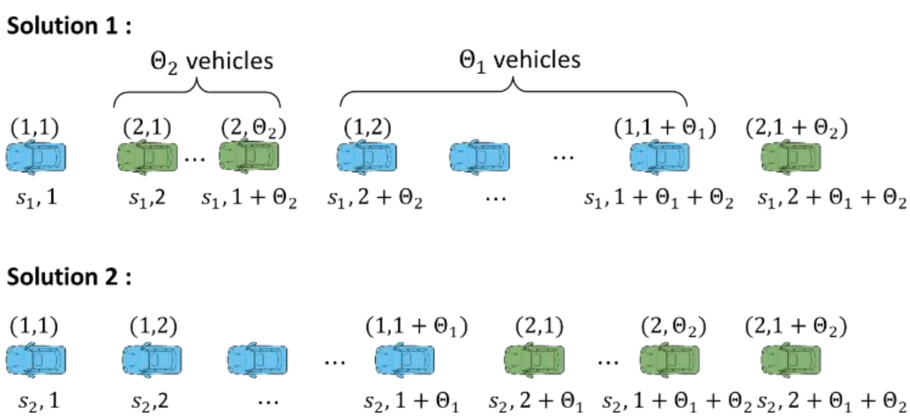

Figure 2. An illustration of Proposition 1.

This proposition indicates that vehicles with headways not greater than h^{F} can be platooned without sacrificing the solution optimality. That is to say, when $\delta = h^{\mathrm{F}}$, POS achieves the same total system delay as OS, $G = 0$. When $\delta > h^{\mathrm{F}}$, G would be possible to be greater than 0. Based on this result, we assume a general form of OS, defined as s^* shown in Figure 3. Then we calculate the upper limit of G in this situation. The headway between Vehicle $(1, k)$ and Vehicle $(1, k + \Theta - 1)$ is smaller than δ. Other two solutions s_1^{P} and s_2^{P} are formed by switching the passing sequence. The POS can only be s_1^P or s_2^P. Therefore, G^+ can be calculated as:

$$G^+ = \frac{\min\{D(s_1^{\mathrm{P}}) - D(s^*), D(s_2^{\mathrm{P}}) - D(s^*)\}}{\Theta + 1} \tag{13}$$

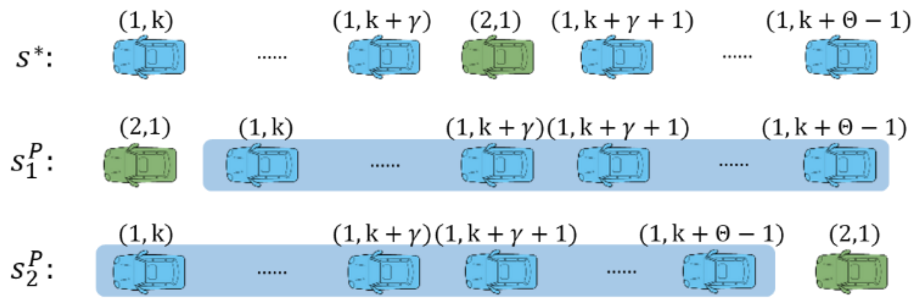

Figure 3. An illustration of the OS and two PS.

$D(s_1^{\mathrm{P}})$ and $D(s_2^{\mathrm{P}})$ can be calculated as:

$$D(s_1^{\mathrm{P}}) = \Theta \underline{t}_{2,1} + \Theta h^{\mathrm{S}} + \frac{\Theta^2 - \Theta}{2} h^{\mathrm{F}} - \sum_{i=k}^{k+\Theta-1} \underline{t}_{1,i} - \underline{t}_{2,1} \tag{14}$$

$$D(s_2^{\mathrm{P}}) = \underline{t}_{1,k+\Theta-1} + h^{\mathrm{S}} - \underline{t}_{2,1} \tag{15}$$

The actual time when the last vehicle passes the conflict area with respect to three sequences is calculated as:

$$t_{s^*,1,k+\Theta-1} = \max\{\underline{t}_{1,k+\gamma} + 2h^{\mathrm{S}} + (\Theta - \gamma - 1)h^{\mathrm{F}}, \underline{t}_{2,1} + h^{\mathrm{S}} + (\Theta - \gamma - 2)h^{\mathrm{F}}, \underline{t}_{1,k+\Theta-1}\} \tag{16}$$

The calculation of $D(s^*)$ has three cases, depending on which value $t_{s^*,1,k+c}$ takes in Equation (16). Suppose $t_{s^*,1,k+\Theta-1} = \underline{t}_{1,k+\gamma} + 2h^{\mathrm{S}} + (\Theta - \gamma - 1)h^{\mathrm{F}}$. In this case, the total system delay $D(s^*)_1 = (\Theta - \gamma)\underline{t}_{1,k+\gamma} + (2\Theta - 2\gamma - 1)h^{\mathrm{S}} + \frac{(\Theta-\gamma-1)^2}{2} h^{\mathrm{F}} - \underline{t}_{2,1} - \sum_{i=k+\gamma+1}^{k+\Theta-1} \underline{t}_{1,i}$. Therefore, the upper limit of $D(s_1^{\mathrm{P}}) - D(s^*)$ and $D(s_2^{\mathrm{P}}) - D(s^*)$ can be calculated in sequence, then combine to get G^+.

$$D(s_1^{\mathrm{P}}) - D(s^*)_1 < \frac{\Theta^2 - \Theta - 1}{2} h^{\mathrm{F}} + (2\Theta - 2)h^{\mathrm{S}} + \frac{\Theta^2 - 3\Theta + 2}{2} \delta \tag{17}$$

$$D(s_2^{\mathrm{P}}) - D(s^*)_1 < -\frac{1}{2} h^{\mathrm{F}} - 2h^{\mathrm{S}} + 2\delta \tag{18}$$

Therefore, in this situation G^+ can be calculated as:

$$G_1^+ = \frac{\min\left\{\frac{\Theta^2 - \Theta - 1}{2} h^{\mathrm{F}} + (2\Theta - 2)h^{\mathrm{S}} + \frac{\Theta^2 - 3\Theta + 2}{2}\delta, -\frac{1}{2} h^{\mathrm{F}} - 2h^{\mathrm{S}} + 2\delta\right\}}{\Theta + 1} \tag{19}$$

Similarly, the upper limit of the optimal solution gap G^+ when $t_{s^*,1,k+\Theta-1}$ choose other two values can be calculated as:

$$G_2^+ = \frac{\min\left\{\frac{n^2-n-1}{2}t_1 + \frac{n^2-n}{2}\delta, \frac{-2n^2+4n-3}{2}t_1 + 2t_2\right\}}{\Theta+1} \tag{20}$$

$$G_3^+ = \frac{\min\left\{\frac{n^2-n}{2}t_1 + \frac{n^2-n}{2}\delta, (n-1)t_1 + 2t_2\right\}}{\Theta+1} \tag{21}$$

Summary of three situations, $G^+ = \max\{G_1^+, G_2^+, G_3^+\}$. In this example, we set $h^{\mathrm{F}} = 2s$, $h^{\mathrm{S}} = 3s$. How G^+ is influenced by the platoon size Θ and the platoon threshold δ is shown in Figure 4. When the platoon threshold δ is fixed and greater than 10, G^+ increases steeply when the platoon size n increases from 2 to 3. When n is greater than 3, G^+ decreases gradually and finally tends to be stable. When δ is less than 10, G^+ decreases slowly with the increase of the platoon size. When the platoon size n is less than 12, G^+ increases with the increase of δ. When n is greater than 12, G^+ no longer increases with the increase of δ.

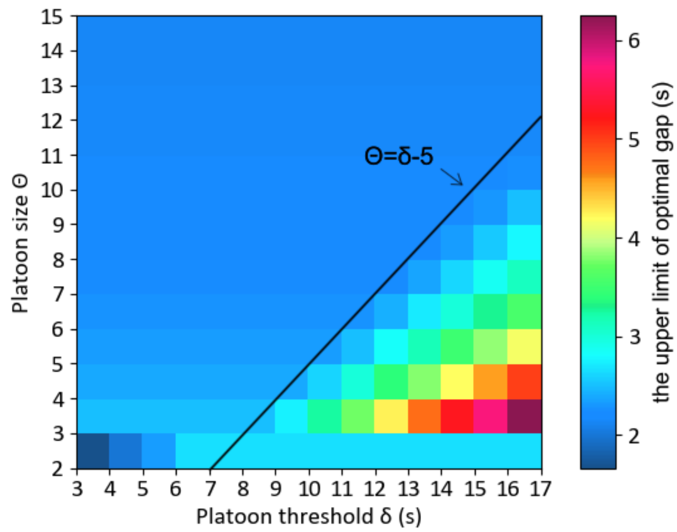

Figure 4. Numerical analysis of $\overline{G}$.

These results indicate that the case should be avoided that the platoon size n is small and the platoon threshold δ is large, which leads to a great G^+. This can be simplified as a guide to vehicle platooning:

$$\delta \leq \Theta + 5 \tag{22}$$

Probability of Optimality Sacrificing

This section finds the probability of optimality sacrificing, which means the average vehicle delay of POS is smaller than that of OS, $G > 0$. We try to calculate the $P(G > 0)$. However, although we obtain the sufficient necessary condition for $G > 0$, the computation is too complicated and contains 3 situations. Therefore, we choose to use the necessary and insufficient condition of $G > 0$. It has been proven that the necessary condition of $G > 0$ can be expressed as:

$$
\begin{cases}
\underline{t}_{1,k+\gamma} - \underline{t}_{2,1} < h^{\mathrm{S}} \\
-h^{\mathrm{S}} < \underline{t}_{1,k+\Theta-1} - \underline{t}_{2,1} < (\Theta - \gamma - 1)h^{\mathrm{F}} + h^{\mathrm{S}} \\
\underline{t}_{1,k+\Theta-1} - \underline{t}_{1,k+\gamma} > (\Theta - \gamma - 1)h^{\mathrm{F}} \\
h^{\mathrm{F}} \leq \delta
\end{cases} \tag{23}
$$

Therefore, the probability that Equation (23) is satisfied P' is larger than $P(G > 0)$. We can use P' to fit the upper limit of $P(G > 0)$. Suppose the Vehicle arrivals follow the Poisson distribution and the headway of each approach h_l obeys a negative exponential distribution $h_l \sim \mathrm{Exp}(\lambda_l), \forall l \in \mathcal{L} \coloneqq \{1,2\}$.

$$
P(h_l \leq z) = 1 - e^{-\lambda_l z} \tag{24}
$$

Therefore, the probability density function of the virtual headway of vehicles of two-approach h^{V} is:

$$
h^{\mathrm{V}} = \underline{t}_{1,i} - \underline{t}_{2,j}
$$

$$
f(h^{\mathrm{V}}) = \frac{\lambda_1 \cdot \lambda_2}{\lambda_1 + \lambda_2} \begin{cases} e^{-\lambda_1 h^{\mathrm{V}}} & if\ h^{\mathrm{V}} > 0 \\ e^{\lambda_2 h^{\mathrm{V}}} & if\ h^{\mathrm{V}} < 0 \end{cases} \tag{25}
$$

$$
P(h^{\mathrm{V}} \leq z) = 1 - \frac{\lambda_2}{\lambda_1 + \lambda_2} e^{-\lambda_1 z} \tag{26}
$$

Considering that $(\Theta - \gamma - 2)h^{\mathrm{F}} < \underline{t}_{1,k+\Theta-1} - \underline{t}_{1,k+\gamma+1} < (\Theta - \gamma - 2)\delta$, Equation (23) can be transferred into:

$$
\begin{cases}
\underline{t}_{1,k+\gamma} - \underline{t}_{2,1} < h^{\mathrm{S}} \\
-h^{\mathrm{S}} - (\Theta - \gamma - 2)\delta < \underline{t}_{1,k+\gamma+1} - \underline{t}_{2,1} < h^{\mathrm{F}} + h^{\mathrm{S}} \\
\underline{t}_{1,k+\Theta-1} - \underline{t}_{1,k+\gamma} > (\Theta - \gamma - 1)h^{\mathrm{F}} \\
h^{\mathrm{F}} \leq \delta
\end{cases} \tag{27}
$$

Thus, the probability of satisfying all the constraints in Equation (27) is:

$$
P' = P_1 \cdot P_2 \cdot P_3 \tag{28}
$$

$$
P_1 = P(\underline{t}_{1,k+\gamma} - \underline{t}_{2,1} < h^{\mathrm{S}}) = 1 - \frac{\lambda_2}{\lambda_1 + \lambda_2} e^{-\lambda_1 h^{\mathrm{S}}} \tag{29}
$$

$$
P_2 = P(-h^{\mathrm{S}} < \underline{t}_{1,k+\gamma+1} - \underline{t}_{2,1} < h^{\mathrm{F}} + h^{\mathrm{S}}) = \frac{\lambda_2}{\lambda_1 + \lambda_2} e^{\lambda_1 h^{\mathrm{S}}} \cdot \left(1 - \frac{\lambda_2}{\lambda_1 + \lambda_2} e^{-\lambda_1 (h^{\mathrm{F}} + h^{\mathrm{S}})}\right) \tag{30}
$$

$$
P_3 = P(\underline{t}_{1,k+\gamma+1} - \underline{t}_{1,k+\gamma} > h^{\mathrm{F}}) = e^{-\lambda_1 h^{\mathrm{F}}} \tag{31}
$$

When both λ_1 and λ_2 equals to 1, $P' < 0.1$. As λ_1 and λ_2 are smaller than 1, P' becomes even smaller. This demonstrates that the case we estimated in the preceding section is extremely unlikely to occur. In most cases, the losses are less than the highest numbers calculated previously.

Number of Platoons

This subsection investigates the number of scheduling units after platooning.

The average headway is calculated as:

$$h = \frac{1}{v\rho} \tag{32}$$

As suggested before, we assume that the h follows a negative exponential distribution,

$$\mathrm{P}_1 = \mathrm{P}(h > \delta) = e^{-\lambda\delta} \tag{33}$$

where n is the number of vehicles from all approaches. The probability that n^{P} platoon is formed can be calculated as:

$$\mathrm{P}\left(n^{\mathrm{P}} = y|\mathrm{P}_1\right) = C_{n-n^{\mathrm{A}}}^{y-n^{\mathrm{A}}} \cdot {\mathrm{P}_1}^{y-n^{\mathrm{A}}} \cdot (1-\mathrm{P}_1)^{n-y} \tag{34}$$

where n^{A} is the number of entrance lanes. $C_{n-n^{\mathrm{A}}}^{y-n^{\mathrm{A}}}$ is a combinatorial number. The expected value of n^{P} given P_1 is:

$$\mathrm{E}\left(n^{\mathrm{P}}|\mathrm{P}_1\right) = \left(n - n^{\mathrm{A}}\right) \cdot \mathrm{P}_1 + n^{\mathrm{A}} \tag{35}$$

Then we set a simple case with $n^{\mathrm{A}} = 2$, $n = 60$. We vary the platoon threshold δ from 2 to 16 seconds and the vehicle density ρ of two lanes from 0 to 36 vehicles/km. The corresponding headway distribution can be estimated by Equation (33). Then the expected value of n^{P} under different platoon threshold δ and vehicle density ρ can be plotted in Figure 5.

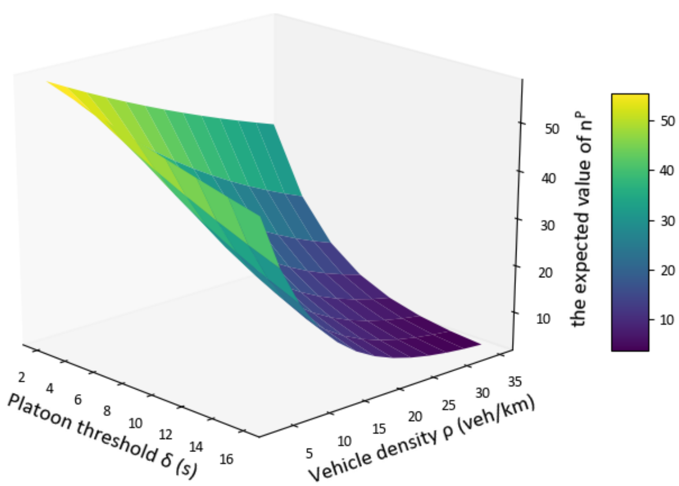

Figure 5. The expected value of the number of platoons n^{P} under different platoon threshold δ and vehicle density ρ.

The distribution n^{P} under different platoon threshold δ and vehicle arrival rate λ can be plotted in Figure 6 and Figure 7. Larger δ leads to averagely less platoons, and the upper limit of the number of platoons also decreases. Larger λ means higher vehicle arrival rate, which means more vehicles in the system. This leads to more platoons on average, and the fluctuation of the number of formations also becomes larger.

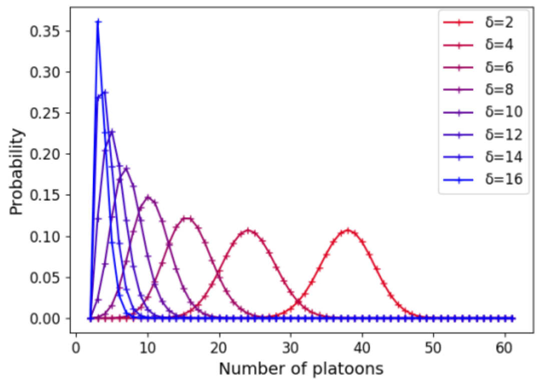

Figure 6. Distribution of the number of platoons n^{P} under different platoon threshold δ.

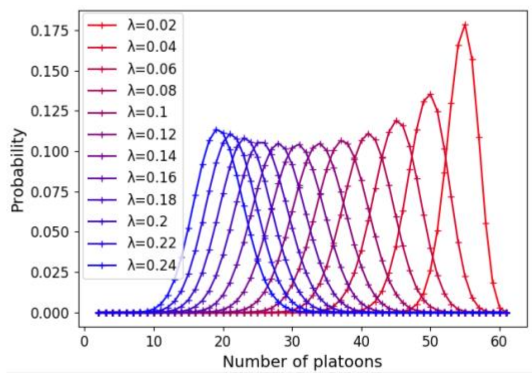

Figure 7. Distribution of the number of platoons n^{P} under different vehicle arrival rate λ.

Platooning Method

Based on the above theoretical analysis, a platooning method is proposed to guarantee all formed platoons satisfy the following criteria:

Criterion 1: Headway between vehicles < δ;

Criterion 2: Cumulate headway of one platoon < $k\delta$;

Criterion 3: The number of vehicles in one platoon < δ−5.

For all vehicles inside a certain area, the proposed platooning method is an inflating method to transfer n vehicles into n^{P} controlled units. This platooning method can be formally stated as the pseudocode in **Algorithm 1**.

Table 1. Pseudocode of Algorithm 1

Algorithm 1. Platooning method of vehicles.

Input: $\mathcal{I}_l, \forall l \in \mathcal{L};\ \underline{t}_{il}, i \in \{1, \cdots, n_l\}, \forall l \in \mathcal{L};\ h^{\mathrm{F}};\ h^{\mathrm{S}}; \delta; k;$
For $l \in \mathcal{L}$ **do**
    $q \leftarrow 0$
    $n \leftarrow 0$
    $platoon\_num \leftarrow 0$
    cumulate_h$\leftarrow 0$
    While $(q < n_l)$ **do**
        $h^{\mathrm{new}} \leftarrow \underline{t}_{q+1,l} - \underline{t}_{q,l}$
        If ($h^{\mathrm{new}} \leq \delta$) **and** (cumulate_h+$h^{\mathrm{new}} \leq k\delta$) **do**
            $P_{l,j} \leftarrow [P_{l,j}, (l, n+1)]$
            q$\leftarrow q + 1$
            n$\leftarrow \mathrm{n} + 1$
            $\mathrm{cumulate\_h} \leftarrow \mathrm{cumulate\_h} + h^{\mathrm{new}}$
        Else do
            $platoon_{num} \leftarrow platoon_{num} + 1$
            $\mathcal{I}_l^{\mathrm{P}} \leftarrow [\mathcal{I}_l^{\mathrm{P}}, [(l, n+1)]]$
            $q \leftarrow q + 1$
            n$\leftarrow \mathrm{n} + 1$
            $\mathrm{cumulate\_h} \leftarrow 0$
    End while
End for
Output: $\mathcal{I}_l^{\mathrm{P}}, \forall l \in \mathcal{L}; P_{l,j}, \Theta_j, \forall l \in \mathcal{L}, \forall j \in \{1, \cdots, n_l^{\mathrm{P}}\}$

Chapter 4. Numerical Experiments

Experimental Design

This section verifies the effectiveness of the proposed platooning method in expediting the vehicle scheduling problem-solving. After formulation, the scheduling MILP is solved using Gurobi 9.0 (Gurobi Optimization Inc., 2019). All experiments were conducted on a computer with Intel®CoreTMi5-1.80GHz, and the proposed methods were implemented with Python.

Three methods are tested:

NP: No platooning method is applied as the baseline.

CHP: Critical headway platooning is the widely applied platooning strategy based on vehicle kinematics. In this method, vehicles with a headway smaller than a critical vehicle headway are platooned (Bashiri et al., 2017; Jiang, Li, & Shamo, 2006; Xu, Feng, Zhang, & Li, 2019). It satisfies criterion 1.

PP: Proposed platooning method satisfies criteria 1, 2, and 3.

All experiments consider an isolated conflict area consisting of two one-lane approaches, shown in Figure 1. The headway between every two adjacent connected automated vehicles (CAVs) at the same approach is randomly generated following a negative exponential distribution. For the convenience of the reader, Table 2 summarizes the parameter values. We fix the total number of vehicles and vary the vehicle density.

Table 2. Parameter Settings of CAVs

Parameter	Value
Platoon threshold δ (s)	4
Vehicle density ρ (veh/km)	4-34
Number of vehicles at each approach n_l	30

Results and Discussion

We first compare the solution efficiency. Figure 6 plots the number of scheduling units (either platoons or vehicles) as the vehicle density increases. It is observed that both PP and CHP decrease the number of scheduling units. The reduction magnitude of CHP is even more significant than PP because PP is imposed with more criteria. Figure 7 shows the MILP solution time of Gurobi. Because of the decreased problem size, both CHP and PP have a significantly smaller solution time than NP. Despite the greater reduction in the number of scheduling units achieved by CHP, the improvement of its solution time is marginal compared to PP.

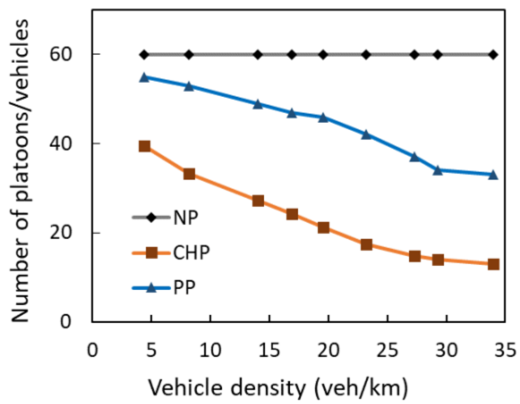

Figure 8. Number of platoons or vehicles under different vehicle densities.

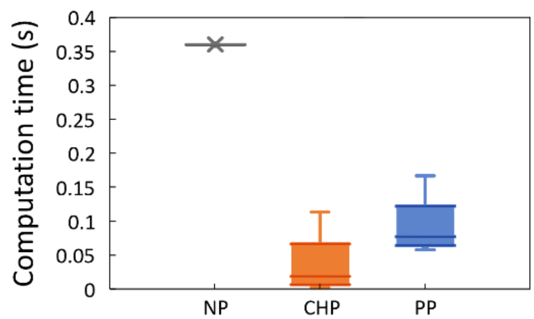

Figure 9. Computation time of Gurobi when three methods are applied.

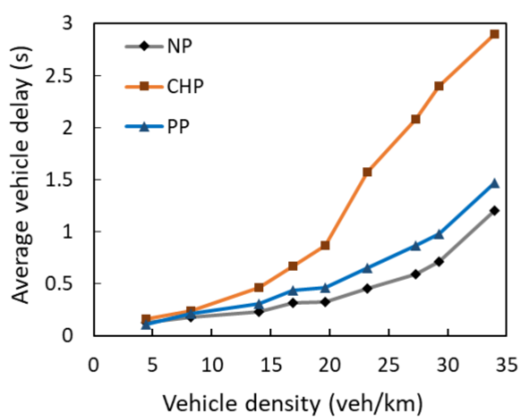

Figure 10. Average vehicle delay under different vehicle densities.

The solution quality was also compared. In Figure 8, the optimal solution (average vehicle delay) of CHP greatly increases with vehicle density compared to NP. The average vehicle delay of PP is also greater than NP. Yet, the increment is relatively marginal. Specifically, PP leads to an average of 30.3% increase in vehicle delay compared with NP, while CHP leads to an average of 158.1% increase in delay compared with NP. This demonstrates the superiority of PP in helping solve vehicle scheduling problems compared with the existing benchmark because of the insights generated from theoretical analyses.

Figure 11 presents the details about the effectiveness of the platooning method. The results without platooning are solved, and the critical headway platooning (CHP) method, which is most common in previous studies, is solved. Figure 11 plots out the average vehicle delay under different platoon thresholds. With the growth of δ, the vehicle delay increases, but PP grows more slowly than the CHP method.

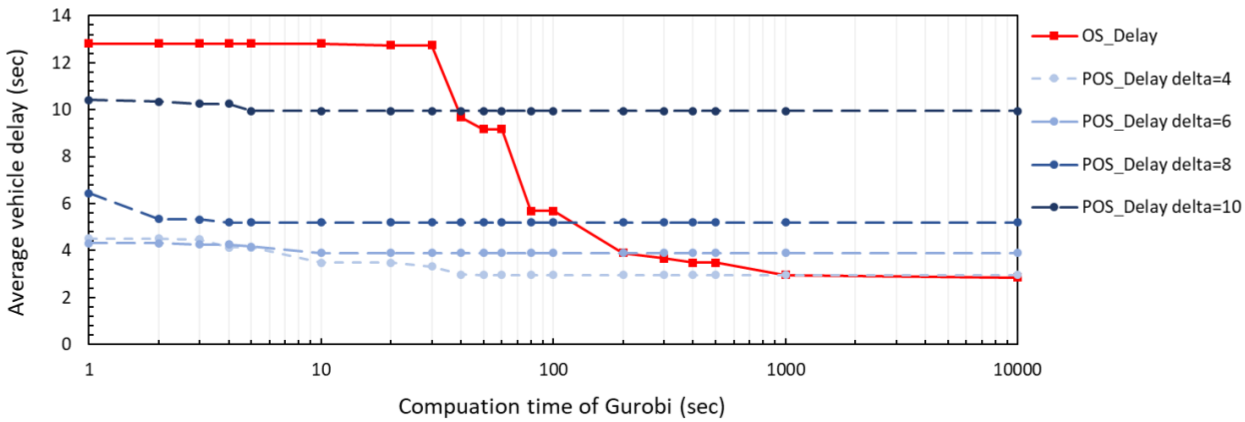

Figure 11. Average vehicle delay under different platoon thresholds δ.

To summarize, experiment results demonstrate that the proposed platooning method (PP) is an effective approach to help expedite solving vehicle scheduling problems without much loss of the solution optimality.

Chapter 5. Conclusion

In conclusion, this study proposes a platoon-based strategy for heterogeneous vehicle scheduling in roadway conflict areas. By clustering vehicles into platoons and treating each platoon as a scheduling unit, the proposed strategy effectively reduces the problem dimension, offers a promising solution to the challenges posed by the large vehicle population, and acknowledges the diverse capabilities and characteristics of the vehicles involved. Theoretical analyses demonstrate that the platoon-based strategy yields a bounded optimal solution gap in terms of vehicle delay when compared to traditional vehicle-based scheduling methods. The upper bound of this gap is analytically derived, providing valuable insights into the potential benefits of the platoon-based approach. This approach not only reduces the problem dimension, but also enables efficient coordination and collaboration within each platoon, taking into account the variations in vehicle types and functionalities.

Furthermore, the expected number of platoons is analytically determined, offering a quantitative understanding of the system's behavior and facilitating practical implementation. The analytical findings serve as a basis for constructing an appropriate platooning method that reduces the number of scheduling units while guaranteeing scheduling performance. The experimental results validate the effectiveness of the proposed platoon-based strategy, showing significant improvements in solving the vehicle scheduling problem without considerable loss of solution optimality compared to a benchmark approach.

Overall, the platoon-based strategy presents a viable and efficient approach to vehicle scheduling in heterogeneous and congested environments. By leveraging platooning and treating platoons as scheduling units, this strategy offers the potential to enhance the scalability and real-time operation capabilities in roadway conflict areas. The theoretical analyses and experimental validations provided in this study contribute to the body of knowledge on network control in realistic settings with heterogeneous vehicles and pave the way for more effective and efficient solutions in the field of vehicle scheduling. Future research can further explore the implementation and deployment of the platoon-based strategy in real-world scenarios, considering factors such as communication protocols, safety measures, and integration with emerging technologies.

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