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Smartphone-Based Incentive Framework for Dynamic Network-Level Traffic Congestion Management

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16. Abstract - This study proposes to develop smartphone-based frameworks to develop/utilize real-time incentives (monetary, value-based, travel-related credits, information etc.) to influence drivers' en route routing decisions to manage network-level system performance in congested dynamic traffic networks. The framework consists of: (i) analytical model and algorithm, (ii) driving simulator-based experiments to analyze drivers' responses to the incentives, and (iii) a smartphone-based app. The analytical model and the algorithm determine the characteristics of the specific incentives to provide or utilize in real-time, including how, where, when, type and amount. The driving simulator-based experiments elicit contextual driver responses to the specific incentives provided in real time, which are used to understand driver behavior in this context, and to finetune the analytical model to be consistent with driver behavior/responses. The smartphone-based app is developed to populate incentives in real-time and identify incentives available en route to the specific driver using the app during his/her origin-destination trip. Accordingly, this study composes of two tasks. Task 1 of this study investigates the role of demand management techniques in generating system level benefits such as reduction in congestion or pollution. Task 2 of this study aims to alleviate traffic congestion by exploiting a novel information provision strategy. Specifically, it takes advantage of the information gaps between individuals and the central planner (CP) and developed a correlated equilibrium routing mechanism (CeRM), which suggests priorities to individual vehicles' route choices and drives their route choices to an equilibrium with a systematically optimal traffic condition while still satisfying individuals' selfish nature. Overall, the output of the two research tasks together will help understand how different types of incentives can be used to alleviate traffic congestion using smart phones/on-board smart devices. The completion of this study will help develop more efficient traffic congestion management tools.

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ABSTRACT

In recent years, dynamic traffic data from multiple entities (public transportation agencies, Google, transportation network companies, etc.) and sensor types is available. This study proposes to develop smartphone-based frameworks to develop/utilize real-time incentives (monetary, value-based, travel-related credits, information etc.) to influence drivers' en route routing decisions to manage network-level system performance in congested dynamic traffic networks. The framework consists of: (i) analytical model and algorithm, (ii) driving simulator-based experiments to analyze drivers' responses to the incentives, and (iii) a smartphone-based app. The analytical model and the algorithm determine the characteristics of the specific incentives to provide or utilize in real-time, including how, where, when, type and amount. The driving simulator-based experiments elicit contextual driver responses to the specific incentives provided in real time, which are used to understand driver behavior in this context, and to finetune the analytical model to be consistent with driver behavior/responses. The smartphone-based app is developed to populate incentives in real-time and identify incentives available en route to the specific driver using the app during his/her origin-destination trip.

Accordingly, this study composes of two tasks. Task 1 of this study investigates the role of demand management techniques in generating system level benefits such as reduction in congestion or pollution. This study explored two such techniques, namely tangible incentives, and nudges. Both incentives and nudges were modeled in the context of network-level traffic congestion to be behavior consistent, real-time, and market-based. A reinforcement learning-based approach is employed to design and generate the incentives. A conceptual smartphone-based framework is illustrated to disseminate these techniques in the real world.

Task 2 of this study aims to alleviate traffic congestion by exploiting a novel information provision strategy. Specifically, it takes advantage of the information gaps between individuals and the central planner (CP) and developed a correlated equilibrium routing mechanism (CeRM), which suggests priorities to individual vehicles' route choices and drives their route choices to an equilibrium with a systematically optimal traffic condition while still satisfying individuals' selfish nature. A distributed Augmented Lagrangian algorithm (D-AL) is developed to efficiently solve the CeRM to provide online real-time navigation services, taking advantage of the smart phones and/or on-board computation resources of individual vehicles. The simulation experiments show that the CeRM results in better system performance (have less system cost) compared with the existing Independent Routing (IR) mechanism and user-oriented Equilibrium Routing (uoER) mechanism. Overall, the output of the two research tasks together will help understand how different types of incentives can be used to alleviate traffic congestion using smart phones/on-board smart devices. The completion of this study will help develop more efficient traffic congestion management tools.

Keywords (up to 5):
Congestion mitigation; Driver behavior; Travel demand; Real-time incentive; Big data analysis

EXECUTIVE SUMMARY

By leveraging advances in smartphone-based personalization, big data availability for traffic, network-level integration through information-based connectivity, this study proposes to manage congestion in real-time in traffic networks, especially during peak period commutes and under debilitating incidents.

Task 1 investigates two demand management techniques, i.e., tangible incentives and nudges, in generating system level benefits such as reduction in congestion or pollution. Both incentives and nudges are designed and generated dynamically according to system-level congestion. Task 1 employs a reinforcement learning-based approach to design and generate the incentives and illustrates a ubiquitous smartphone-based framework to present the incentives to the users. Such a solution is practical in its real-world solution as the models can be trained offline and later implemented online.

Task 2 investigates the use of information incentives and designs a correlated routing mechanism that calculates and provides online routing guidance for vehicles with smart phones and/or onboard computing and communication devices. By exploiting information discrepancies between individual vehicles and the Central Planner (CP), the proposed mechanism drives the snapshot equilibrium route choice of a group of vehicles toward a more systematic optimal condition while still preserving the individual's selfish nature. The simulation experiments show that the proposed routing mechanism in Task 2 can significantly reduce traffic congestion and system travel time by 55% and 3.6% compared to the existing Independent Routing Mechanism and User-oriented Equilibrium Routing Mechanism. Furthermore, this study proposes the D-AL, an effective distributed algorithm that could quickly solve the routing problem for an online real-time navigation service with the help of smart phones and/or individual vehicles' on-board smart devices.

The results and insights provided by this study can be used by state/local transportation agencies as new complementary tools in their portfolio to dynamically manage traffic congestion at the network level. It may help transit agencies and planners in understanding the potential of using smart-phones and different types of incentives to alleviate traffic congestions. In the future, Task 1 could be conducted on real-world networks such as the city of Atlanta to build and test the smartphone-based framework with human subjects to understand the effect of the behavioral change solutions. For task 2, this study assumes drivers make decisions purely based on information provided by the CP without using their prior knowledge. However, in reality, drivers' ex-ante knowledge may affect their compliance to the routing guidance. Therefore, a possible future work is to incorporate individual drivers' ex-ante beliefs into the correlated routing game.

1.0 Task 1: Smart-Phone Based Real-Time Incentive Framework for Travel Behavior Change

1.1 INTRODUCTION

1.1.1 Background

Traffic congestion and pollution are some of the major transportation related problems faced by the Urban areas. Traffic congestion further leads to loss in productivity and has economic cost to it. INRIX (*1*) study showed that the Average American spends almost 100 hours in congestion and leads to $1350 in economic cost per American. Majority of urban travel in the United States is through single occupancy personal vehicles. Such travel patterns severely contributed to the pollution in the urban areas. Study conducted by the US EPA (*2*) study shows that the transportation sector alone contributes to 40% of greenhouse gas emissions, highest among all the sectors. Rapid urbanization in the past few years and the decades to come is expected to increase the demand in the transportation network. Such an increase in demand would lead to further increase in congestion to the current transportation system.

To tackle urban congestion, both supply and demand side solutions were explored. Supply side solutions include infrastructure developments and investments, traffic management devices, ITS solutions and increasing the capacity of existing roadways. These solutions are not sustainable, expensive and time consuming.

Demand side solutions uses tools to target and influence individuals' travel behavior. These tools included tolls, congestion pricing, incentives, and tradeable credit schemes. Incentives are intended to influence their travel decisions subtly. Incentives based solutions are more acceptable and equitable than tolls. Incentive based solutions fall under a broader research of behavioral change strategies within behavioral psychology.

1.1.2 Objectives

The main objective of this study is two-fold. The first objective is to list, characterize and classify behavioral change strategies that can be used to influence travel related decisions. The second objective is to model the behavioral changes strategies in the context of a dynamic traffic network.

1.1.3 Scope

The scope of this research is limited to studying the effect of behavioral change strategies on route choice behavior. Specifically, the research addresses the effect of behavioral change strategies on initial route choice and subsequent en-route choices. While the behavioral change strategies can be applied to the various aspects of individual trip making such as choice of mode, route, departure time, etc., this study does not model its effects on mode choice and departure time choice.

1.1.4 Report organization

This report includes five sections. Section 2 covers literature review. Section 3 classifies and characterizes the incentives. Section 4 presents the problem formulation and solution methods. Section 5 presents the real-time solution deployment framework. Section 6 presents the numerical studies.

1.2 LITERATURE REVIEW

A review of published literature and practices on the use of incentives and nudges in the context of travel behavior is presented below.

1.2.1 Research studies using behavioral intervention strategies

Behavioral change strategies operate on the motivation behind user actions. Often the research studies target the users economic, health or environmental values to influence their decisions. For example, by providing environmental incentives the users that are concerned with global warming, or their carbon footprint would be motivated to change behavior. Among the past travel-based studies, most studies target mode choice behavior and encourage users to shift to public transit. The behavioral intervention tools utilized in the literature can be classified into benefits such as tangible or in-tangible incentives or gamification techniques such as points, badges, etc. Benefits are further classified into value-based incentives, monetary incentives, and in-tangible incentives such as nudges. While monetary incentives are direct dollar amounts, the value-based incentives are points that can be traded for goods or services. Gamification techniques on the other hand are used to induce competition among users of the system. The table below provides an overview of various behavioral change strategies formulated in the real-world

TABLE 1 DIFFERENT TYPE OF INCENTIVES EXPLORED IN LITERATURE

	Economic	Health	Environmental	Route	Mode	Value-based	Monetary	In-tangible	Badge	Points	Leaderboard
Ubigreen (*1*)	✓	✓	✓		✓			✓	✓		
PEIR (*2*)			✓		✓			✓		✓	
i-Tour (*3*)	✓		✓		✓	✓		✓			
Trip-zoom	✓	✓	✓		✓	✓	✓	✓	✓	✓	

SUPER-HUB (*3*)	✓	✓	✓		✓	✓		✓	✓	✓	✓
Matka-Hupi (*4*)			✓		✓	✓		✓			✓
Peacox (*5*)			✓		✓	✓		✓			
QT	✓	✓	✓		✓	✓		✓			
IPET (*6*)	✓	✓	✓		✓	✓		✓	✓	✓	
Viaggia (*7*)	✓	✓	✓		✓	✓		✓	✓	✓	✓
trafficO2 (*8*)	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓
Metropia (*9*)	✓			✓		✓	✓	✓			
IAM	✓				✓	✓		✓			
MM		✓	✓			✓		✓	✓	✓	✓
MUV	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓
Tripod	✓			✓	✓	✓	✓	✓		✓	
RMTP	✓	✓	✓		✓	✓	✓	✓		✓	✓
Roider	✓	✓	✓		✓	✓	✓	✓		✓	
Commuter Connections (*10*)				✓		✓	✓				
Commutifi (*11*)				✓	✓	✓	✓			✓	
Hytch (*12*)	✓		✓	✓		✓	✓			✓	

Spitsmijden (*13*)	✓			✓		✓	✓				
INSTANT (*14*)	✓			✓		✓				✓	✓
INSINC (*15*)	✓				✓	✓				✓	✓
CAPRI (*16*)			✓		✓	✓					
Flex-Pass (*17*)											
Steptacular (*18*)		✓			✓	✓					

1.3 Behavior Change Strategies and their Characteristics

In this study, two behavioral change strategies have been considered. First is the tangible incentive such as monetary or value-based rewards. Second is the in-tangible incentives or the nudges.

1.3.1 Characteristics of tangible incentives

Tangible incentives are one of the most intuitive forms of behavioral change strategies. They can be either monetary or value based. Value-based incentives usually comprise of point systems that can be exchanged for real goods or services. The incentives are positive quantities as opposed to tolls and can be present on all of the edges in a road network. They are updated in real-time and are dependent on the system congestion levels making them dynamic in nature. They can also be based on time of the day and differ from off-peak periods to peak periods.

1.3.2 Characteristics of nudges

Nudges are an application of the Nudge theory proposed by *Thaler, R. et al. 2008*. They are a design mechanism on choice architecture. Nudges in this study are implemented as en-route prompts that encourage the user to change their route during the trip. Nudges do not have a tangible value and cannot be quantified. These prompts are provided to the user based on the availability of incentives and update of routes. These prompts are personalized to each user based on their travel preferences such as value of time or value of incentive. The table below summarizes the incentive and nudge characteristics.

TABLE 2 INCENTIVE CHARACTERISTICS

	Tangible incentives	Nudges
Time-based	Yes	No
Dynamic	Yes	No
System level	Yes	No
Personalized	No	Yes
Market-based	Yes	No
Geographical	Yes	No

1.4 Problem Formulation

1.4.1 Overview of the reinforcement learning problem

Reinforcement learning is a type of machine learning paradigm that uses rewards and punishments to train the model. The reinforcement learning is modeled as a Markov decision process with an agent and an environment. At every step, the agent receives the system current state and reward from the environment. The agent performs an action to maximize its reward. The environment simulates the affect of the action and generates the state value and the reward value to pass to the agent.

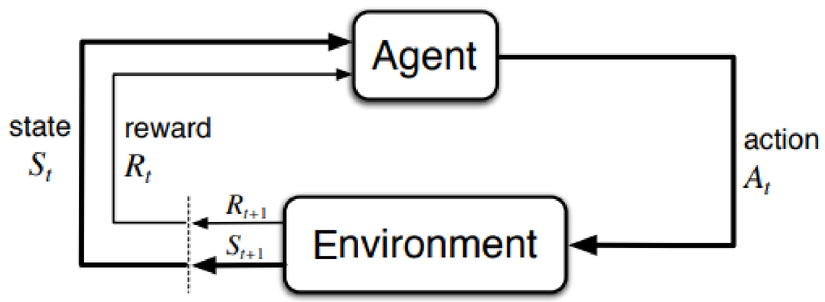

FIGURE 1 OVERVIEW OF THE REINFORCEMENT LEARNING PROBLEM

1.4.2 Problem formulation

The tangible incentives are generated in real-time based on the current system state. The incentives are generated as a response of the current system conditions and congestion. Within each trip, every user makes multiple micro-travel decisions such as changing routes within a single trip. The incentives are generated to influence such en-route decisions. The generation of incentives is sequential in nature and is a response to the system congestion levels or travel time. The travel time and incentives are analogous to the state-action pairs present in the reinforcement learning framework. Like the action values within an RL framework, the incentives affect the transition of system from one state to the other. The transition between one traffic state to another is hard to model and computationally intensive.

The tangible incentives and intangible nudges will be provided to the user through a mobile app. The generation of incentives themselves will be in real-time as a response to the travel time. Often the travel time is the only real-time value available to model incentives. Because of all the above reasons, the above problem is modeled as a Markov decision process and trained using a reinforcement learning problem. For the remainder of this section, we will formulate the incentive generation problem as a reinforcement learning problem.

Consider a dynamic road network $G(N, E)$ with N nodes and E links. The time horizon of interest T is broken down into multiple time intervals. Each time interval t corresponds to a step in the RL/Markov decision process. Let $tt_{e,t}$ be the travel time on link e at the end of time interval t. Let $i_{e,t}$ be the number of incentives present on the link e in the beginning of time interval t.

Within a reinforcement learning framework, the state needs to be a sufficient statistic of the history. Within real-world deployments it is realistic to assume that the travel-time on every link is available for all links of a traffic network. Keeping this in mind, state S_t is defined as an array of travel times on every link at the end of time interval t.

$$S_t = \{\dots, tt_{e,t}, \dots\} \; \forall \; e \in E, t \in T$$

The action value taken by the agent affects the transition of the traffic environment from one state to another. In this scenario, the action A_t is defined as an array of incentives present on every link at the beginning of time interval t. This definition allows incentives to present on all links of the network based on congestion. This is contrary to tolls where they are only present on select roads. The incentives are non-negative values. This indicates that while there can be incentives on an edge, there could be a few edges without incentives. The values of the incentives are purely dependent on the congestion levels of the network.

$$A_t = \{\dots, i_{e,t}, \dots\} \; \forall \; e \in E, t \in T$$

The RL agent maximizes the expected reward in the future steps. Since the objective of this study is to generate behavioral change strategies to reduce congestion using tangible (often monetary) incentives, the total system travel time is incorporated into the reward function. The reward value R_t is defined as a linear combination of the total system travel time and the total incentives generated in the time interval t.

$$R_t = -\left(\sum_{e \in E} tt_{e,t}\right) - \beta * \left(\sum_{e \in E} i_{e,t}\right) \; \forall \; e \in E, t \in T$$

The value β is the scaling/weight factor used. This can also be interpreted as the value of time for the network operator.

1.4.3 Incentive usage and route generation

The incentives generated to influence the users on the traffic network to change their routes. Since the incentives are updated at every time step, the routes and users' perception of the routes could also be affected. In each time interval t, the routes of all users is updated. The route updating contains two stages. In the first stage of route generation, a personalized route is computed based on the user preferences and the incentives available during that time step. This stage ensures that the incentives provided to the user are personalized by including user preferences such as value of time and value of incentives. The incentives are limited in quantity and consumed through the time interval. The users reaching a particular link first could gain incentives and the users reaching the same link later might not gain any incentives.

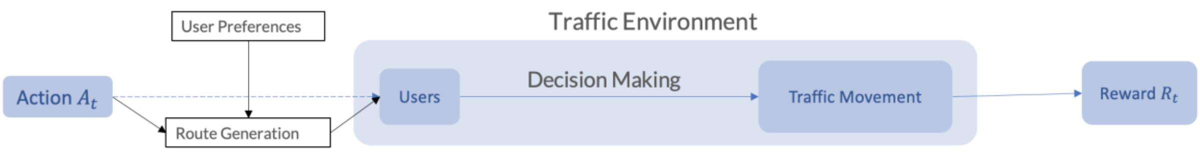

FIGURE 2 ROUTE GENERATION IN THE PRESENCE OF INCENTIVES

A personalized route is generated based on the utility function of a trip for each user. This utility function incorporates the user preferences such as the value of time and the value of incentive. Let $\beta_{tt,u}$ and $\beta_{i,u}$ be the value of time and value of incentive for user u. Let tt_μ and i_μ be the travel time and incentives on route μ. The utility function for user u for a route μ is defined as a linear combination of travel time and incentives on that particular route.

$$c_\mu = \beta_{tt,u} * tt_\mu + \beta_{i,u} * i_\mu$$

Each user has a unique cost function as it incorporates the user preferences. The user preferences can be estimated through the mobile app usage.

The second stage is the decision-making stage. The updated route is provided as a prompt to the user. When prompted to change the route, a user can choose to change to the new updated route or choose to remain on the current route.

The decision-making among users can be different based on their affinity towards incentives and nudges. The users can be classified into two categories. Habitual users that are reluctant to change the routes even when the cost of the new route is lower that their current routes. Nudged users who shift routes when prompted to do so even when the cost of the new route is higher. During the simulation, the users are classified into either nudged users or habitual users to model their corresponding behavior.

To avoid the cost function from reaching negative values, the upper limits are implemented on the incentives. These upper limits are implemented such that the cost of the users remains positive.

1.4.4 Traffic Environment

The traffic environment is responsible for computing the transition between one state to another based on the provided incentives. Analytical models that handle the transition between different traffic states can be complex and computationally intensive. In this study, the traffic environment is simulated using the Simulation of Urban Mobility (SUMO) simulation platform. This platform is responsible for simulating the trajectories of vehicles and generate the resulting system state and reward.

1.4.5 Training Algorithm

A training algorithm is used to learn the optimal actions of the agent that maximizes reward. Within the RL literature, the algorithms can be classified into value-based methods or policy-based methods. The value-based methods learn actions based on the reward functions and expected value of action. The policy-based methods involve the use of a policy function that assess the value of an action. The proposed algorithm utilized in this study is the Deep Deterministic Policy Gradient (DDPG) developed by (*20*). Most RL algorithms deal with discrete state-action values, while the above formulation has both continuous state and action values. The DDPG algorithm is designed to allow continuous state and action pairs. This algorithm has an actor and a critic modeled using neural networks. The actor generates actions while the critic evaluates the actions and updates the policy.

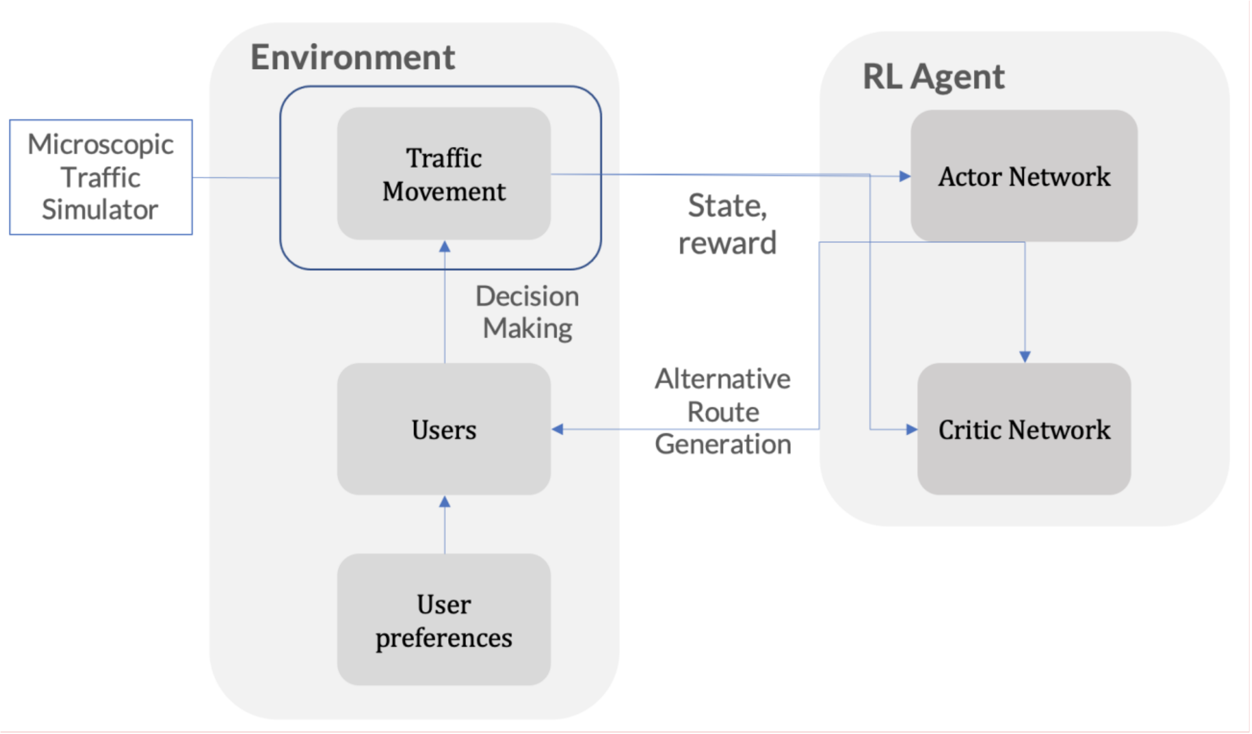

FIGURE 3 TRAINING ALGORITHM

1.5 Real-time Deployment Solutions Framework

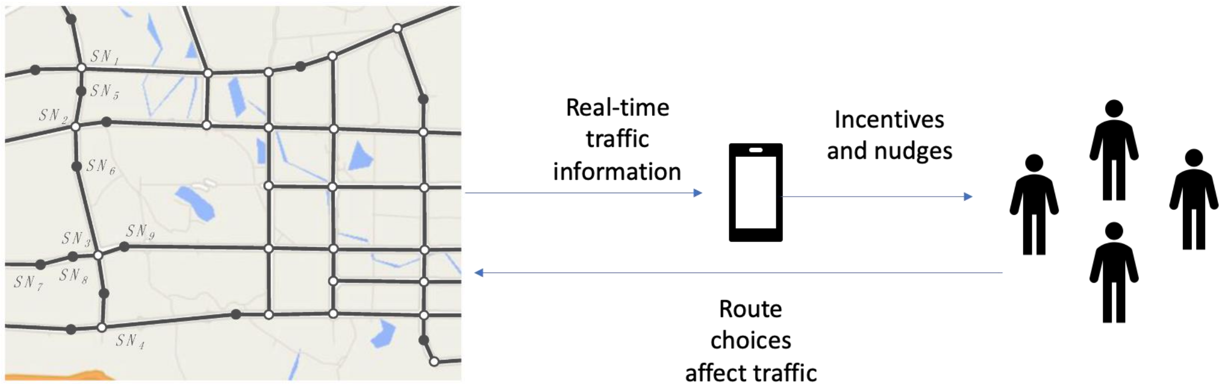

FIGURE 4 REAL-TIME INCENTIVE DEPLOYMENT FRAMEWORK

The figure 5 describes the real-world incentive deployment framework. This conceptual framework employs a mobile app to present the incentives and nudges to the users. Such a mobile app can also be used to track the user's travel choices and interpret the user preferences such as their value of time or value of incentive.

1.6 Numerical Experiments

The Simulation of Urban Mobility (SUMO) simulation platform is used to simulate the traffic state transitions. To test the effectiveness of the incentives, the following the RL agent is trained in six scenarios. Each scenario corresponds to different mobile app penetration levels. App penetration levels is a direct indication of the number of users that have access to the incentives on the network.

For all the six scenarios Peak hour simulated between the 4 PM and 5 PM on Hannover South City network. Table 3 shows the parameters used by the DDPG algorithm to train on various scenarios.

TABLE 3 DEEP DETERMINISTIC POLICY GRADIENT ALGORITHM PARAMETERS

Parameter	**Value**
Optimizer	Adam
Actor network learning rate	10^{-4}
Critic network learning rate	10^{-3}
Discount factor	0.99
Tau	0.001
NN Layers	ReLU
Batch size	64
Replay buffer size	100000
Number of episodes	1000
Actor network	2 hidden layers (400 X 300)

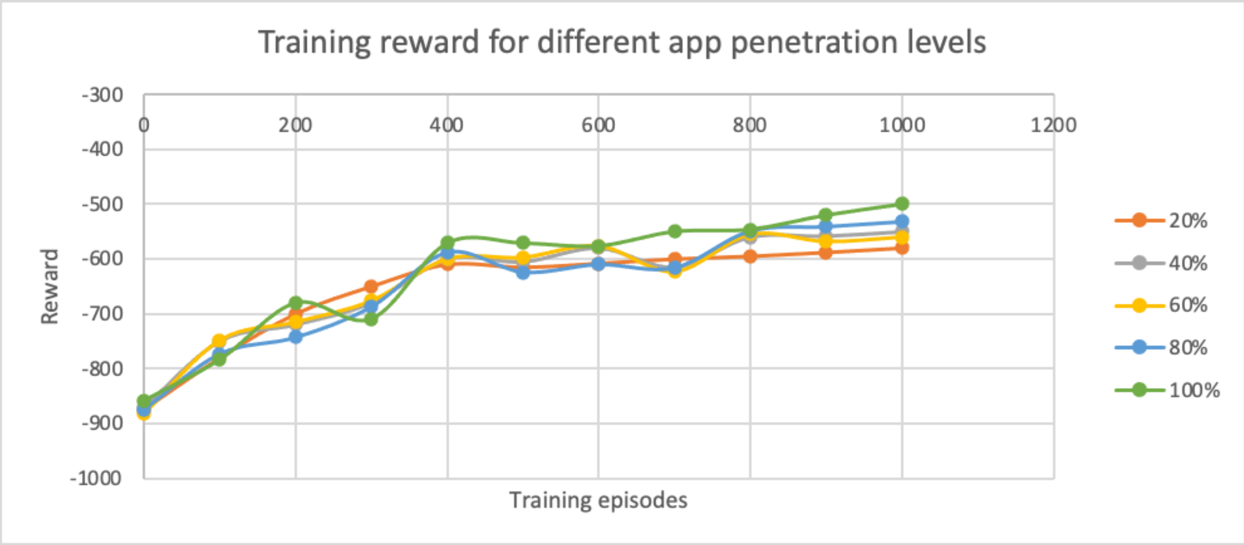

FIGURE 5 REWARD VALUE FOR DIFFERENT APP PENETRATION LEVELS

Figure 5 shows the trend in the reward value over number of episodes. The different lines show the reward values for different app penetration levels. For example, a 40% app penetration scenario has 40% of the vehicles equipped with the incentive provision app and can change routes, the rest 60% do not change their initial routes. Rewards under all scenarios appear to be following similar trajectory. The reward value is the highest under the 100% app penetration scenario. Although unrealistic, this shows that the system congestion level is improving with increasing app penetration levels. The reward value is the lowest under the 20% app penetration scenario. The scenarios yielding similar results could indicate that the neural network might need additional training. For the training process, the basic actor and critic neural network architecture from the DDPG algorithm was adopted. The dimensionality of action values and state values are high and perhaps require a customized neural network architecture. Future studies could also explore multi-agent DDPG algorithm to train on the same data.

1.7 REFERENCE LIST

This list appears at the end of your report. References should appear as a numbered list. Below are suggestions for acceptable reference citations mostly borrowed from TRB. For in-text citations provide (last name of author, year) or (last name et al. Year)

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2.0 Task 2: An Equilibrium Routing Mechanism for Traffic Congestion Mitigation Built upon Mixed Strategy Correlated Game and Distributed Optimization

2.1 INTRODUCTION

Traffic congestion has long been a major issue in urban areas, which happens when the demand is higher than road capacity. With the development of mobile intelligent devices and wireless communication technologies, drivers nowadays rely heavily on traveler information systems (TIS) such as Google Maps, Apple Maps, and Waze to get real-time traffic information and make their best response to avoid congestion. This study refers to this widely used routing mechanism as Independent Routing (IR). However, if too many vehicles simultaneously choose similar routes/links according to the same provided information, these recommended routes/links would become congested later. This phenomenon resulting from the collective snapshot routing decisions of drivers relying on IR is known as the flash crowd effect [13], which may cause severe traffic congestion and incur high system costs for all drivers. On the contrary to IR, System Optimum routing (SOR) seeks to minimize the total travel cost of all drivers, which is the ideal traffic state that a system wants to achieve. However, SOR will sacrifice some users' interest (experience more travel cost) in exchange for better system performance, which conflicts with individuals' selfish nature. In addition, the computation load to implement real-time SOR is prohibitive.

Transportation researchers have long been devoted to developing navigation systems for reducing traffic congestion and pushing the routing pattern of IR to be more systematically efficient. One of the most widely studied approaches is congestion pricing. Along with its variants like credit- or permit-based regulations, congestion pricing influences the behavior of individuals by changing their perceived cost physically/monetarily. Interested readers can refer to [8][16] for a comprehensive review. While theoretically, congestion pricing could push the traffic state close to SO and is undoubtedly a powerful tool to improve traffic conditions in practice, the study of [8] summarized seven main complications when implementing congestion pricing. Along with social equity, fairness, legislature, and political issues, congestion pricing still has obstacles to address in practice, and researchers are thus motivated to explore alternative options [15][21][22][25].

Several recent studies (i.e., [10][11][13]) proposed a different routing approach based on snapshot equilibrium routing (ER) to mitigate traffic congestion and improve traffic conditions. Mainly originating from game theory in economics, ER aided by real-time traffic information is a class of emerging routing mechanisms that try to manipulate individual drivers' real-time route choices to approach a snapshot equilibrium by coordination or information provision technologies (discussed in detail in the literature review). To be noted, the collective snapshot routing decisions relying on IR do not lead to such snapshot route choice equilibrium. Therefore, ER seeks to reduce traffic congestion by mitigating the flash crowd effect resulting from the over-competition on the provided shortest paths, often observed in IR. In addition, unlike compulsive law enforcement or congestion pricing, ER influences individuals' perceptions and decisions without using external regulation and incentives, thus avoiding many impedances faced in typical road pricing schemes. However, though ER could reduce system costs compared with IR, most existing ER mechanisms are still away from SOR since they are still user-oriented, and system interest (system optimality) is only implicitly considered by the routing mechanism design. We categorize this type of ER as user-oriented ER (uoER).

Motivated by the above views, this study seeks to go one step further and design an equilibrium routing mechanism toward a better system performance than uoER. Specifically, this study develops a Correlated equilibrium online routing mechanism (CeRM) based on the correlated game and distributed solution algorithm. The CeRM explores a snapshot equilibrium route choice decision among all drivers opting in the routing service to reduce traffic congestion and improve system performance from IR and uoER. In addition, the CeRM does not violate individuals' selfish nature, and thus drivers would be willing to follow the proposed routing decision.

The idea of developing such an ER using the correlated game is invoked by exploiting the information discrepancies between individual drivers and the Central Planner (CP) in existing route navigation systems. Specifically, in a navigation scenario, each driver only knows their own trip information but has no idea of others, e.g., how many travelers are there on the road, where they are going, their personal choice characteristics, etc. The lack of panoramic information at the individual level makes drivers unable to know/predict traffic conditions in the future and often make the best response to the real-time traffic information provided by the navigation service. On the other hand, the CP is omniscient and could better predict the traffic condition in the near future since it can get comprehensive information by collecting travelers' trip plans when they require navigation services. Such an information gap allows the CP to act as a trusted agent, which can strategically design and provide individual drivers with route choices that no one would want to deviate from. Correspondingly, this study develops the CeRM, which calculates and suggests each traveler's routing preferences according to their trip features (mainly origins, destinations, and personal choice characteristics), aiming to achieve the desired system optimal performance while guaranteeing that an individual driver would not be better off if they unilaterally derivate from the suggested routing preferences (i.e., correlated equilibrium).

Moreover, to satisfy the computation need of online navigation services, this study develops a distributed solution algorithm (D-AL). It distributes the computation load of searching a correlated equilibrium route decision to individual vehicles' smartphones and or/ onboard communication and computing devices.

Overall, this study has four distinguished characteristics and methodology contributions:

(i) We discovered the usually overlooked information discrepancies between the users and CP in a navigation service and developed a new information provision strategy for traffic congestion mitigation.

(ii) Taking advantage of the information discrepancies, we developed a Correlated equilibrium Routing Mechanism (CeRM) built upon an atomic mixed strategy correlated game to coordinate individual travelers' real-time routing decisions, which push traffic conditions toward a desired system optimal performance without introducing external regulation and incentives.

(iii) To serve a large-scale of travelers' online navigation requests, we developed a problem-specific distributed solution algorithm (D-AL) by taking advantage of the model's unique structure features. The D-AL could solve the CeRM problem efficiently by distributing the computation load to individual vehicles' smart phones and/or onboard computing devices.

(iv) The numerical experiments validate our solution algorithms' computation performance and convergence properties and demonstrate that the CeRM could reduce traffic congestion and system cost compared with existing IR and uoER. More exactly, our experiments show that the CeRM can significantly reduce traffic congestion and system travel time by 55% and 3.6% compared to the existing IR and uoER mechanisms. The D-AL could handle a scenario with more than a thousand vehicles by a leading time smaller than 22 seconds.

The rest of the paper is organized as follows. Section 2.2 reviews relevant works in the existing literature and identifies the research gaps this study addresses. Following that, we introduce related notations and concepts in section 2.3, and then design the correlated equilibrium routing scheme in section 2.4. Section 2.5 develops an effective distributed solution algorithm. Last, section 2.6 presents numerical experiments and section 2.7 concludes the task. We will use "vehicle" and "driver" interchangeably in the following context for better illustration.

2.1.1 OBJECTIVE

This task has two main objectives. The first objective is to design an equilibrium routing mechanism to reduce traffic congestion and achieve better system performance than IR and uoER while still satisfy individual's selfish nature. The second objective is to design a distributed solution algorithm with the help of smart phones and/or on-board computing and communication devices to solve our problem efficiently to satisfy the fast computation need of online navigation services.

2.1.2 SCOPE

To the best of our knowledge, this is the first research to design a correlated equilibrium routing mechanism with an efficient distributed solution algorithm using individuals' smart phones and/or on-board smart devices to mitigate traffic congestion and reduce system cost. It significantly contributes to the methodology development and practice for the field of traffic congestion mitigation.

2.2 LITERATURE REVIEW

This study aims to develop a correlated routing mechanism by exploiting the information gap between drivers and the CP, which could reduce congestion at the system's level while still maintaining individuals' selfish nature. In literature, this research sits in the field of flash crowd effect, equilibrium routing, correlated game, and information design. This section will briefly review some of the most relevant works to our study and identify their research gaps.

We first recognized the studies that improve system performance by congestion pricing and its variants like credit- or permit-based regulations, which has been briefly introduced in the Introduction section. Considering these studies mainly use a different line of approaches (i.e., imposing physical externalities) than our study (i.e., using information provision to form ER), the following survey does not provide detailed reviews for them. Interested readers can refer to [8] and [16] for a comprehensive review.

In literature, the flash crowd effect [13], also known as overreaction [2], occurs in the traffic when a large number of drivers receive similar traffic information and make routing decisions based on it selfishly and independently. Different ER mechanisms have been proposed to mitigate such adverse phenomenon in either distributed or centralized ways. In [10], the author developed an online coordinated routing mechanism based on an atomic mixed strategy congestion game. By iteratively sharing and updating the routing preference for each vehicle, the coordinated routing mechanism guarantees to converge to an equilibrium routing decision which leads to better system cost than the IR mechanism. On the other hand, [13] proposed to perform route selections centrally. Vehicles are assigned to suggested routes with probabilities calculated by the central server. The probability is inversely proportional to the estimated travel time, and the resulting routing decision avoids the situation that a large number of vehicles choose the same route. There are many other approaches to derive an ER, and most of them share similar thoughts with [13] and [10]. For example, [11] conducted the coordinated routing under pure strategy setting, [37] proposed to solve the mixed strategy coordinated routing problem similar to [10] using reinforcement learning method, and [20] proposed an anticipatory navigation service that predicts and disseminates the near future traffic condition based on real-time data. While simulation results show that all these approaches could reduce the system cost compared with IR, their performance is still away from SOR. This is because the routing decision in uoER can be considered as a spontaneous equilibrium resulting from drivers' selfish reactions to perfect traffic information. Though certain machinimas design of uoER could mitigate overreaction of drivers' routing decisions, the resulting system cost is still sub-optimal compared with SOR since there are conflicts between user performance and system performance. The CeRM proposed in this study falls into the category of ER but differs from existing uoER in that it explicitly incorporates system cost minimization in the routing mechanism design.

Another research area that shares similar thoughts to our study in literature is system optimal traffic assignment with users' constraints. They conduct system optimum traffic assignment under the consideration of user fairness and cooperation willingness. The approaches used in this field can be divided into two branches. One incorporates users' fairness constraints in the system optimum to stabilize the resulting system optimum flow. The other relaxed the user equilibrium condition to improve the system performance of user equilibrium flows. Interested readers can refer to [23] for a detailed review. Even though these studies achieve further system cost reduction without using externalities in road pricing schemes, they only reveal the aggregated traffic flow on each route/link, but do not provide specific routing decisions for drivers, thus cannot be used for navigation services. On the contrary, our study works on the atomic game, which yields detailed routing decisions for each driver and could be used in a navigation service.

Recently, [9] and [29] proposed to improve the performance of uoER by providing a perturbed travel time to drivers in the navigation service. Mainly, this study seeks to manipulate individual drivers' real-time route choices toward a better system performance by strategically involving bias into travel time provision. While this approach could reduce system cost and rational drivers are likely to comply, it may induce fairness issues. Specifically, some drivers responding to perturbed traffic information may experience sacrifice in travel time compared with responding to unperturbed traffic information. In comparison, in our work, we propose to reach a systematically efficient equilibrium routing decision based on the naturally existing information gap between drivers and the central planner. The proposed CeRM guarantees that every driver would be better off (at least not worse off) given their limited individual information and thus precludes the issue of individual fairness.

The method in our study is built upon the correlated equilibrium (CE) in game theory, in which a trusted agent assigns strategy to players according to a probability distribution [1], and no player could unilaterally deviate from the assigned strategy to increase their expected utility. By designing the informational environment, the agent can manipulate players' behavior and direct the resulting equilibrium to serve its own interests. CE has a great potential to improve the system performance without using externalities, but it is still an emerging area in the transportation field. Existing studies using CE to reduce traffic congestion focus on exploring the effects of CE in simplified scenarios such as small networks with simple one-link or parallel-link routes [12][17][7][13][18][30]. Three recent studies proposed CE models for routing games on general transportation networks and discussed their impact theoretically [19][33][34] by adopting linear travel cost functions, which do not well capture traffic flow features in reality. Moreover, to the authors' best knowledge, no existing work provides efficient solution algorithms to solve the CE models for online navigation services over a city network.

To conclude, state of the art indicates two major research gaps. 1) Most existing ER mechanisms do not completely address the inefficiencies brought by the conflicts between individual performance and system performance or user compliance issues. 2) Existing CE research in routing games can be further improved by involving realistic transportation network modeling and practical solution algorithms. This study seeks to partially fill such research gaps by involving these enhanced features and addressing the new research challenges. Briefly, this study develops a CE-based ER mechanism (i.e., CeRM), which reduces system cost compared with uoER and satisfies individuals' selfish nature using a different line of approach from congestion pricing. Briefly, the CeRM is built upon a transportation network with multi origin-destinations, multi-link non-parallel routes, and well-accepted link cost functions. To adapt this CeRM to the online application, we develop a distributed solution algorithm and prove it to be efficient for real-world scenarios with thousands of vehicles opting in the services. The following sections introduce the technical details for developing the CeRM.

2.3 Preliminary

This section will first introduce some mathematic notations and the concept of correlated equilibrium and then propose the correlated equilibrium routing mechanism.

2.3.1 Mathematic notations

Denote $G = (N, L)$ to be the directed graph of a transportation network, where N is the set of nodes and L is the set of arcs (links). Let $v = 1, \ldots, m$ be the qualified vehicle on roads. Each vehicle v has a specific origin-destination (OD) pair $(o_v, d_v) \in N \times N$ and a set of k_v possible routes. Denote r_v^i as the ith possible route of vehicle v, where $i = 1, \ldots, k^v$. In a mixed strategy setting, every player places a probability distribution (i.e., preference) on their set of available choices. In the routing game, each vehicle is a player, and their possible paths are potential alternatives. Denote $p^{v,i}$ as the probability that vehicle v places on the route r_v^i, Then clearly, we have

$$\sum_{i=1}^{k_v} p^{v,i} = 1\ , \quad \forall v = 1, \ldots, m. \tag{1}$$

The probability $p^{v,i}$ can also be viewed as the expected volume generated by vehicle v on route r_v^i. Thus, we could form the expected flow on link l as

$$f^l = \sum_{v=1}^{m} \sum_{i=1}^{k^v} p^{v,i} \, \delta_{v,i}^l \,, \tag{2}$$

where the link-route incidence indicator $\delta_{v,i}^l = 1$ if link l is used by route r_v^i, and 0 otherwise. Associated with each link is a link travel cost $c_l(f_l)$. Then, for each route i of vehicle v, a generalized travel cost C_v^i could be defined.

$$C_v^i(\boldsymbol{P}) = \sum_{l \in r^{v,i}} c_l(f^l) \,, \tag{3}$$

where $\boldsymbol{P}$ is the set of all route choice preferences, i.e., $\boldsymbol{P} = \{p^{\boldsymbol{v},\boldsymbol{i}}\}, v = 1, \dots m, i = 1, \dots k^v$. Denote the current (initial) traffic information on route i of vehicle v as $C_{v,o}^i$. Then according to $C_{v,o}^i$, individual vehicle's selfish routing choice preference could then be calculated by a multinomial logit (MNL) choice model:

$$p_o^{v,i} = \frac{e^{-V_{v,i}}}{\sum_{i=1}^{k_v} e^{-V_{v,i}}} \,, \tag{4}$$

Where

$$V_{v,i} = \alpha^v + \beta^v C_{v,o}^i \,, \tag{5}$$

is the measured utility of route r_v^i for vehicle v and α^v, β^v are vehicle-specific constant scalars representing the characteristics of each individual.

2.3.2 Correlated Equilibrium (CE)

In this subsection, we briefly introduce the concept of correlated equilibrium (CE) used in the proposed routing mechanism. We consider a N-player strategic game (N, A_i, u_i) which is characterized by an action set A_i and utility function u_i for each player i. Let S denotes the strategy set given by a trusted CP, let $s \in S$ be the single strategy and s_i be the action allocated to player i under strategy s. In a correlated game, a trusted agent assigns a strategy s to every player according to a probability distribution $p(s)$, if no player wants to deviate from the suggested action s_i, then a correlated equilibrium (CE) is reached, i.e.,

$$\sum_{s_{-i}} p(s_i, s_{-i}) \big(u_i(s_i, s_{-i}) - u_i(s_i', s_{-i}) \big) \geq 0 \,, \tag{6}$$

where s_i' is an action of player i different from s_i, and s_{-i} represents the action sets of all other players except i.

In our routing problem, the CE is used to measure individual vehicles' selfish rationality. A rational individual will not want to deviate from the routing guidance if it satisfies the CE condition. Namely, the CE condition ensures that no player can be better off by unilaterally derivate from the suggested strategy.

There is always more than one solution satisfying the CE condition [14]. However, not all of them will lead to a better system-level performance than IR or uoER. Thus, this study is interested in finding an optimal CE that minimizes the expected system cost in Eq. (7).

$$System\ Cost\colon \sum_{s\in S}\sum_{i=1}^{N} p(s)\, u_i(s_i, s_{-i}) \tag{7}$$

2.4 Correlated equilibrium Routing Mechanism (CeRM)

Given the research gaps mentioned in the literature review, this study seeks to design a correlated equilibrium routing mechanism (CeRM) that could drive the traffic condition from an inefficient IR to a more systematically optimal one that outperforms existing uoER. By doing that, we consider a traffic scenario where there are a large number of qualified vehicles en route - vehicles equipped with on-board computing and communication devices - trying to make routing decisions at a given short-time period. The CeRM will ensure that every rational vehicle would be better off compared to the now widely adopted IR and would thus follow the scheme voluntarily.

More exactly, at the beginning of the CeRM navigation service, each participating vehicle provides their OD pairs and receives the current traffic information (current route travel cost) $C_{v,o}^{i}$ as they usually do with regular navigation services like Google or Apple map. According to the received traffic information, each vehicle v will calculate their initial route preference $p_o^{v,i}$, $i = 1, \dots, k^v$ (as done in IR by equations (4)-(5)), and then proposed to the CP. The collective information from all vehicles opting in the services is denoted as $\boldsymbol{P_o} = \{p_o^{v,i}\}, v = 1, \dots m, i = 1, \dots, k^v$. Built upon the collected information, the CP will generate the suggested CE route choice preferences $\boldsymbol{P_s} = \{p_s^{v,i}\}, v = 1, \dots m, i = 1, \dots, k^v$, in which $p_s^{v,i}, i = 1, \dots, k^v$ represents the suggested route preference for a driver v. $\boldsymbol{P_s}$ seeks to minimizes the system cost while guarantees every driver would not be better off by deviating from the suggestion. The whole process is conducted automatically in the navigation apps/electronic devices, where the drivers only need to provide their OD and personal choice parameters (α^v, β^v in Eq. (5)) and wait for the CP to calculate and display the suggested route preferences.

Note that our solution algorithm designed in Section 5 ensures that the CeRM takes no more than half a minute to generate the optimal CE routing guidance. Considering the traffic condition in such a short time period is not likely to change dramatically, we assume that the initial traffic conditions, i.e., $\{C_{v,o}^{i}\}, v = 1, \dots m, i = 1, \dots, k^v$ won't change during the decision process of the CeRM. Travelers departing at different times may be treated as different coordination groups. If traffic condition changes or travelers change their routes en route for unexpected reasons within the coordination groups, they may rejoin the CeRM again as new travelers. For example, the apps on the individual vehicles can periodically reconduct the CeRM to obtain route suggestions whenever they are approaching traffic intersections and have the opportunity to re-route their trips.The calculation will be solved distributedly with the help of vehicles' on-board computation resources, and the suggested route preference will be disseminated to individual vehicles. Each vehicle would then pick a route based on the preferences (probabilities).

2.4.1 Modeling CeRM

There are a few assumptions we make before introducing the mathematical model for the CeRM:

Assumption 1: Every participating driver is rational and would assume all others are rational.

Assumption 2: Drivers don't know other drivers' choices.

Assumption 3: Drivers either follow the guidance or stick to their initial route preference calculated by given real-time travel time.

Assumption 4: The link cost function c_l is assumed to be continuously differentiable, strictly increasing, and convex with respect to link flow f_l.

Assumption 1 and 4 are common assumptions used in the transportation field, while assumption 2 and 3 captures the properties of most popular navigation services such as Google or Waze in reality.

The CP suggests individual vehicles the optimal route choice preference by solving the following mathematical programming (MP) problem:

$$\min Z = \sum_{l \in L} f_s^l c_l(f_s^l) \tag{8.1}$$

$s.t$

$$\sum_{i=1}^{k_v} p_s^{v,i}\left(-C_v^i(\boldsymbol{P}_s) - \frac{1}{\beta_v}\ln\left(p_s^{v,i}\right)\right) \geq \sum_{i=1}^{k_v} p_o^{v,i}\left(-C_v^i(\boldsymbol{P}_o^v) - \frac{1}{\beta_v}\ln\left(p_o^{v,i}\right)\right), \ \ \forall v = 1, \dots, m \tag{8.2}$$

$$\sum_{i=1}^{k_v} p_s^{v,i} = 1, \ \ \forall v = 1, \dots, m \tag{8.3}$$

$$p_s^{v,i} \geq \epsilon, \ \ \forall v = 1, \dots, m, \forall\, i = 1, \dots, k^v \tag{8.4}$$

$$f_s^l = \sum_{v=1}^{m} \sum_{i=1}^{k^v} p_s^{v,i}\, \delta_{v,i}^L, \quad \forall l \in L \tag{8.5}$$

Where, $\boldsymbol{P}_o^v = \left\{p_s^{v_1,i_1}, \dots, p_s^{v-1,i_{k^{v-1}}}, p_o^{v,i_1}, \dots, p_o^{v,i_{k^v}}, p_s^{v+1,i_1}, \dots, p_s^{v_m,i_{k^{v_m}}}\right\}$ is the set of route choice preferences in which only vehicle v sticks to its initial preferences $p_o^{v,i}$ and all others follow the CeRM guidance. Accordingly, the perceived link flow for vehicle j on link l can be calculated by $f_{v_j}^{l} = \sum_{v=1}^{m\backslash v_j} \sum_{i=1}^{k^v} p_S^{v,i}\, \delta_{v,i}^{l} + \sum_{i=1}^{k^{v_j}} p_O^{v_j,i} \delta_{v_j,i}^{l}$.

The MP aims to find an optimal solution that minimizes total system cost (8.1) while satisfying the correlated equilibrium condition (8.2) and other related feasibility constraints. Note that f_S^l is the expected flow on link l if everyone follows the CeRM routing guidance $\boldsymbol{P_s}$. Thus, the objective function (8.1) here is the expected total system travel time incurred by all vehicles in the network. The objective function can also incorporate other performance measurements such as emissions. It won't affect the applicability of our model and the solution approach as long as its gradient satisfies the Lipschitz continuous condition. The left-hand side of Equation (8.2) is the driver's expected net economic benefit if they follow the guidance, and the right-hand side is the expected net economic benefit if he unilaterally deviates from the guidance (sticks to their original routing preference $\boldsymbol{P}_o^v$). We refer to constraint (8.2) as the rationality constraint, since it represents the decision process a rational driver would consider (Assumption 2 and 3). Below gives a further justification about using net economic benefit to build this constraint.

It has been well known that under discrete behavior choice models, the perceived utility $U_{v,i}$ of a route i for vehicle v considers not only the exact measured travel time $V_{v,i}$, but also a random term ϵ that represents the influence of unobserved attributes or measurement errors [26], i.e., $U_{v,i} = V_{v,i} + \epsilon$. Under the commonly adopted multinomial logit choice model, the error term ϵ follows an i.i.d Gumbel distribution. The welfare/consumer surplus an individual vehicle v receives if it chooses a particular route i among other candidate routes could then be expressed as $-\frac{1}{\beta_v}\ln\left(p^{v,i}\right)$ [28]. Then, the net economic benefit of an individual vehicle could be expressed as the welfare it receives minus the actual transportation cost it experiences [35], i.e., $\left(-\frac{1}{\beta_v}\ln\left(p^{v,i}\right) - C_v^i(\boldsymbol{P})\right)$. It is worth noting that under the CeRM, the suggested routing preference $p_S^{v,i}$ of each individual vehicle are calculated from the MP rather than determined by logit choice model. But as a rational driver possesses consistent behavior patterns throughout a decision-making process to determine whether to follow the independent routing preference $\boldsymbol{p}_o^v$ or the suggested routing preference $\boldsymbol{p}_s^v$, it is reasonable to use the consistent measurement to measure the welfare of the choice from an individual vehicle's view. Namely, a rational driver would choose the routing decision with the largest net economic benefit. To ensure the compliance of the equilibrium routing guidance, the rationality constraints (8.2) guarantee that each driver would not be better off (receive more net economic benefit) if they choose not to follow the proposed guidance. In other words, if constraints (8.2) are satisfied, each driver would have no incentive to deviate from the suggested routing guidance.

Constraints (8.3) – (8.4) ensure the conservation and positiveness of probability variables. It's worth pointing out that the route preference (probability) under the logit choice model is strictly positive. Namely, each route's probability could not be 0 because of uncertainties and user heterogeneities. To align the same route choice behavior pattern of a rational driver, constraints (8.4) ensure that the decision variables of route choice preference are positive. Here, we set ϵ as a sufficiently small positive constant, e.g., $\epsilon = 1 \times 10^{-6}$. According to the theory of bounded rationalities [27][31], a rational driver cannot sense the travel time difference within a certain threshold. Thus, the introduction of ϵ to constraints (8.4) would not cause any noticeable difference to the routing decision of an individual driver in practice. Constraints (8.5) are the flow conservation constraints.

From a mathematical point of view, the MP of (8.1) – (8.5) has a convex objective function but nonconvex constraint sets, and the detailed proof is shown in the Appendix A. Moreover, the MP is always feasible. When no routing guidance is given to individual vehicles, i.e., $\boldsymbol{P}_s = \boldsymbol{P}_o$, all the constraints are satisfied, which means that there always exists a feasible solution for our problem. For simplicity issue, we reform the rationality constraint as:

$$r_v(\boldsymbol{P}_s) = \sum_{i=1}^{k_v} p_s^{v,i}\left(C_v^i(\boldsymbol{P}_s) + \frac{1}{\beta_v}\ln\left(p_s^{v,i}\right)\right) - \sum_{i=1}^{k_v} p_o^{v,i}\left(C_v^i(\boldsymbol{P}_o^v) + \frac{1}{\beta_v}\ln\left(p_o^{v,i}\right)\right) \le 0, \quad \forall v = 1, \dots, m \tag{9}$$

2.5 Distributed Augmented Lagrangian (D-AL) algorithm

The CeRM seeks to provide online routing guidance for every participating vehicle to mitigate traffic congestion. It requires us to solve the large-scale, highly coupled, and nonlinear nonconvex MP in (8.1) – (8.5) promptly (i.e., less than 30 seconds) since it is not likely that a driver en route would wait several minutes or even longer to get the route guidance. Even though there exist many methods to cope with nonconvex optimization problems, such as interior-point methods [4], SQP (sequential quadratic programming) [4], and problem-specific heuristic algorithms [19], etc., state of the art shows that none of them could satisfy the computation needs for such online service involving a large scale of vehicles in a large transportation network (i.e., a large number of decision variables). On the other hand, thanks to recent developments in vehicular on-board computing devices and wireless communication technologies, distributed computation is becoming a possible solution to be implemented in practice.

Motivated by this view, this study develops a distributed solution algorithm, i.e., distributed Augmented Lagrangian (D-AL), to solve the proposed MP for the CeRM problem by taking advantage of the problem's unique structure features. Mainly, the D-AL will efficiently solve the problem by distributing a large portion of the computation loads to individual CVs smart phones and/or on-board smart devices. Figure 1 illustrates the framework of the D-AL implemented between the CP and individual vehicles. Specifically, it includes the four essential procedures.

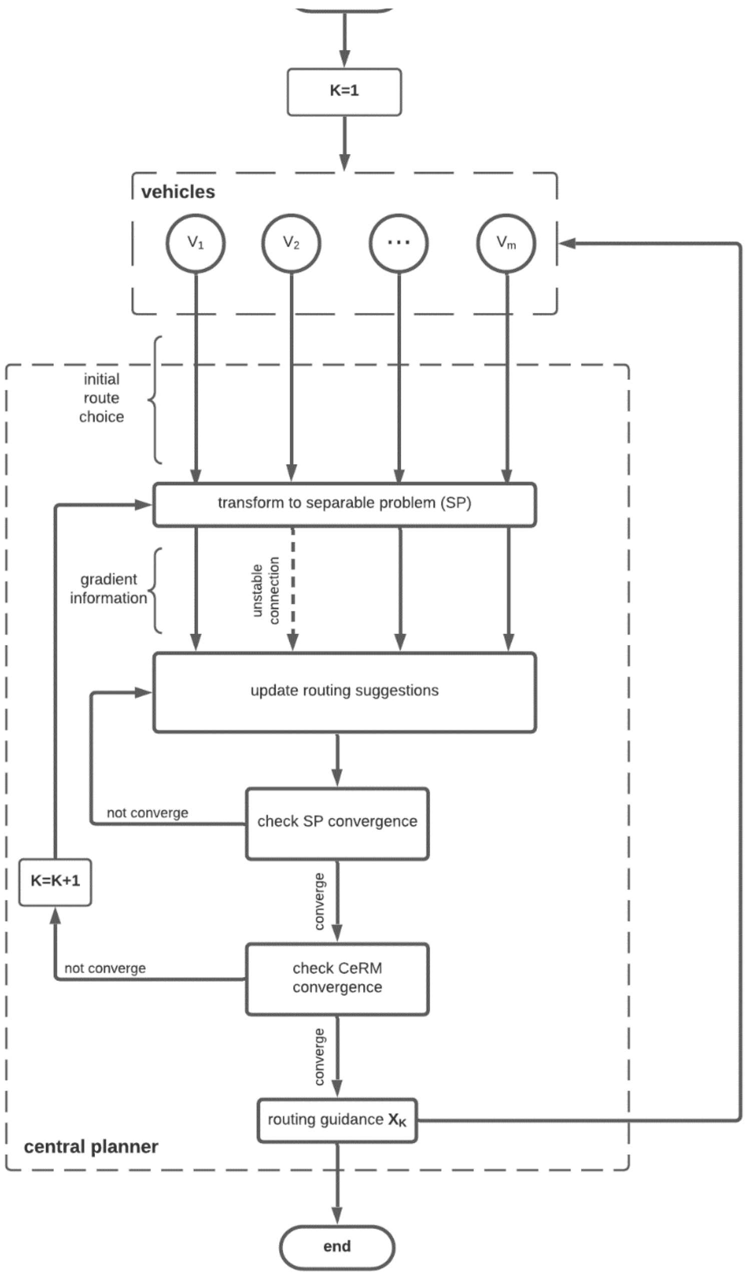
Step (i): Individual vehicles first locally evaluate traffic conditions and propose their routing preferences to the CP;

Step (ii): Upon receiving the information, the CP forms the MP for the CeRM problem, transforms it into a separable problem ($MP - S$), and then separated into individual problems ($MP - I$) and dispatch to each vehicle;

Step (iii): Each vehicle iteratively calculates the assigned computation tasks and proposes the result to the CP. The CP synchronizes individuals' responses and updates the solution until the $MP - S$ converges;

Step (iv): If the outcome of Step (iii) does not satisfy the convergence criteria of the MP for the CeRM model, the $MP - S$ is updated and the algorithm returns to step (ii).

FIGURE 6 THE FLOW CHART OF THE D-AL SOLUTION ALGORITHM



The proposed D-AL is guaranteed to converge to a local solution to the MP of our CeRM problem. The subsections below introduce the technical details for developing such a solution algorithm, including model transformation, distribution scheme, and a customized projection algorithm. For simplicity issues, we denote the solution (routing guidance) of our problem $\boldsymbol{P}_s = \{p_s^{v,i}\} = \{x^{v,i}\} = \boldsymbol{X} \in \mathbb{R}^{\sum_v k^v}$ hereafter.

2.5.1 The Augmented Lagrangian Transformation

To develop the D-AL, we first notice that the MP's constraint set (8.2) – (8.5) presents a unique feature. It involves complicated nonconvex and highly coupled rationality constraints (8.2) and a relatively simple and separable ϵ-probability simplex constraint set (8.3) – (8.4) regarding each vehicle's route choice decision variables (preference). Invoked by these features, we consider transforming the MP for the CeRM problem into the Augmented Lagrangian form (10) by changing the inequality constraints (9) (an equivalent transformation of rationality constraints (8.2)) into equality constraints and penalizing them into the objective function (8.1).

$$\mathcal{L}(\boldsymbol{X}, \boldsymbol{\lambda_K}, c_K) = \sum_{\underset{Z}{\underbrace{L=1}}}^{n} f_s^L c_L(f_s^L) + \frac{1}{2c_K}\sum_{v=1}^{m}\left(max^2\{0, \lambda_K^v + c_K r_v(\boldsymbol{X})\} - \lambda^{v\,2}\right), \tag{10}$$

where $\boldsymbol{\lambda_K} = \{\lambda_K^v\}, v = 1, \dots, m$ is the set of Lagrangian multipliers, and c is the penalty parameter. Then we have the first transformations of the MP for the CeRM problem given below.

$$\boldsymbol{MP-AL}: \quad \begin{matrix} \min_{\boldsymbol{X}} \mathcal{L}(\boldsymbol{X}, \boldsymbol{\lambda}, c) \\ s.t.\ (8.3) - (8.5) \end{matrix} \tag{11}$$

The merits of this transformation lie in eliminating the complex constraints (8.2) in the MP and transforming them into more manageable sub-problems. Accordingly, existing studies [3] show that iteratively solving and updating the transformed problem in (11) and associated parameters by Augmented Lagrangian method (AL), we can find a local solution of the MP developed for the CeRM. Below we first briefly introduce the procedure of the AL algorithm to update the Lagrangian multipliers $\boldsymbol{\lambda}_K$ and penalty parameter c_K with the given solution of (11):

Parameter updating scheme

1: **if** $\mathcal{Y}_{K+1} \le \gamma_m \mathcal{Y}_K$ (rationality constraint violation has been decreased):

2: set $c_{K+1} = c_K$;

3: **else**:

4: set $c_{K+1} = \gamma_c c_K$;

5: **end if**;

6: **for** $v \in V$:

7: set $\lambda_{K+1}^v = \max\{0, \lambda_{K+1}^v + c_K r_v(\boldsymbol{X}_{K+1})\}$;

Where $\mathcal{Y} = \max_{v \in V} r_v(\boldsymbol{X})$ is the maximum violation of all rationality constraints, $\gamma_c > 1$ is a constant scalar for updating c_K, γ_m is a positive constant to compare the change in constraint violations, and $\boldsymbol{X}_K$ is the solution of (11) corresponding to $(\boldsymbol{\lambda_K}, c_K)$. The AL stops until the convergence criterion of the MP for the CeRM problem is satisfied. The updating scheme of the Lagrangian multipliers takes the merit of Augmented Lagrangian algorithms and the updating scheme of the penalty parameter c_K has shown to be efficient for our problem in numerical experiments.

On the other hand, with each given parameters $(\boldsymbol{\lambda}, c)$, the AL algorithm solves the transformed problem in (11) by iteratively updating the solution using the gradient projection method as follows.

$$\boldsymbol{X}_{k+1} = [\boldsymbol{X}_k - \alpha^k \nabla \mathcal{L}]^{\mathcal{H}} \ , \tag{12}$$

where $[\cdot]^{\mathcal{H}}$ stands for the projection onto the constraint set (8.3) – (8.4) denoted by $\mathcal{H}$ and α^k is the step size. Note that we use upper case K to denote each iteration of updating Lagrangian multipliers and penalty parameter $(\boldsymbol{\lambda}_K, c_K)$, and lower case k to denote the iteration of updating $\boldsymbol{X}_k$ for a given transformed problem (11). However, the standard gradient projection method along with the AL algorithm is not efficient enough to satisfy the computation need of our online navigation service. This study further develops a distribution scheme and a customized $\boldsymbol{\epsilon}$-probability simplex projection algorithm to expedite the computation of the solution algorithm.

2.5.2 Distribution scheme

It is noticed that the link-based objective function (8.1) (the first item Z in (10)) is not user separable, but the second item in (10) and the constraint set (8.3) – (8.4) are. Thus, to accommodate a distribution scheme, we transform the objective function Z into the equivalent path-based and user separable form Z_u:

$$Z_u = \sum_{v=1}^{m} \sum_{i=1}^{k_v} p_s^{v,i} \, C_v^i(\boldsymbol{P}_s) \tag{13}$$

After this second transformation, the objective function (10) can be rewritten as the summation of individuals' augmented objective functions in (14) (i.e., $MP - AL$ of (11) is then transformed to $MP - S$ of (14)), which could then be separated among individual vehicles.

$$\boldsymbol{MP} - \boldsymbol{S}: \min_{\boldsymbol{X}} \mathcal{L}(\boldsymbol{X}, \boldsymbol{\lambda}, c) = Z_u \ + \frac{1}{2c} \sum_{v=1}^{m} \left(max^2\{0, \lambda^v + c r_v(\boldsymbol{X})\} - {\lambda^v}^2 \right) \ = \sum_{v=1}^{m} \mathscr{f}_v \tag{14}$$
$$s.t. \ \ (8.3) - (8.5)$$

Where $\mathscr{f}_v = \sum_{i=1}^{k_v} p_s^{v,i} \, C_v^i(\boldsymbol{P}_s) + \frac{1}{2c} \sum_{v=1}^{m} \left(max^2\{0, \lambda^v + c r_v(\boldsymbol{X})\} - {\lambda^v}^2 \right)$ is the individual vehicle's augmented objective function. The first part of $\mathscr{f}_v$ is the expected travel cost of vehicle v and the second part is related to the violation of vehicle v's rationality. From the individual's perspective, a vehicle v only cares about its own augmented objective

$\mathcal{g}_v$, and tries to minimize it subject to constraints (8.3) – (8.5). We refer to the following model for each vehicle v as the individual problem $(MP - I)$.

$$\boldsymbol{MP} - \boldsymbol{I}: \min_{\boldsymbol{X}^v} \mathcal{g}_v(\boldsymbol{X}^v, \boldsymbol{\lambda}, c) = \sum_{i=1}^{k_v} p_s^{v,i}\, C_v^i(\boldsymbol{P}_s) + \frac{1}{2c}\left(max^2\{0, \lambda^v + cr_v(\boldsymbol{X}^v)\}\right. \quad (15)$$
$$s.t.\ (8.3) - (8.5)$$

Note that an $MP - I$ is a separation of $MP - S$ regarding vehicles, not the decision variables. Namely, each $MP - I$ holds the same decision variables as the $MP - S$. It can be seen as an individual vehicle v trying to design routing preferences for all vehicles that minimize its own augmented objective. An $MP - I$ can also be solved using the gradient projection method by iteratively performing the updating process shown in (16):

$$\boldsymbol{X}_{k+1}^v = [\boldsymbol{X}_k^v - \alpha^k \nabla \mathcal{g}_v]^{\mathcal{H}} \quad (16)$$

Where, $\boldsymbol{X}^v \in \mathbb{R}^{\sum_v k^v}$ is the solution of vehicle v's $MP - I$. A solution $\boldsymbol{X}^v$ from vehicle v can be seen as the solution mostly favorable to vehicle v's interest. However, for two vehicles, most likely we will have $\boldsymbol{X}^{v'} \neq \boldsymbol{X}^v$. Consequently, $\boldsymbol{X}^v$ is not in accordance with the overall objective in (14). To balance individual's will and produce a consensus solution that converges to the $MP - S$, we design a customized distribution scheme (c-DS) that only requires individuals to propose their interest-related gradients $\nabla \mathcal{g}_v = \{\frac{\partial \mathcal{g}_v}{\partial p_s^{v_1,i_1}}, \dots, \frac{\partial \mathcal{g}_v}{\partial p_s^{v_m,i_{k_{v_m}}}}\}$. By synchronizing individuals' gradients, the CP can obtain $\nabla \mathcal{L} = \sum_{v=1}^{m} \nabla g_v$, and then perform the update through (17) to solve the $MP - S$ of (14).

$$\boldsymbol{X}_{k+1} = \left[\boldsymbol{X}_k - \alpha^k \sum_{v=1}^{m} \nabla \mathcal{g}_v\right]^{\mathcal{H}} \quad (17)$$

Here, the step size α^k is determined by a centralized line search along the projection arc [3]. Clearly, $\sum_{v=1}^{m} \nabla \mathcal{g}_v = \nabla \mathcal{L}$ and the solution updating process (17) is equivalent to performing the gradient projection algorithm (12) on the $MP - AL$ (11), but using a distributed way to conduct this computation.

This study also noticed another more general and straightforward way to distribute the calculation load of $\nabla \mathcal{L}$ in (12) by letting each vehicle compute the partial derivatives related to their own decision variables. We label this naive approach as n-DS: vehicle v computes $\frac{\partial \mathcal{L}}{\partial p_s^{v,i}}, i = 1, \dots, k_v$. However, there are two drawbacks in n-DS. First, unlike in c-DS, individuals have no direct interest in the computation task under n-DS, making them less willing to contribute their computing power and propose the needed information. Second, the naive approach n-DS is less efficient than our problem-specific c-DS regarding the computation workload. We prove this merit in Theorem 1 below.

Theorem 1: Assume there are m vehicles each with k possible candidate routes, then each vehicle undertakes $\frac{1-\frac{1}{m}}{k+1}$ less workload in c-DS than in n-DS.

The proof of Theorem 1 is shown in Appendix B. It is worth noting that each vehicle usually faces 2 to 4 possible routes. Then the c-DS developed in this study can reduce the computation load by nearly 20% to 33%, which is quite considerable in practice given that the main computation burden of the D-AL lies in this part.

2.5.3 Projection onto the $\boldsymbol{\epsilon}$-probability simplex

It should be noted that the updating process (17) involves a projection process $[\cdot]^{\mathcal{H}}$, which is not easy to perform in general procedures. It usually involves solving a quite computationally costly optimization problem: $\min_{x \in \mathcal{H}} \|x - y\|$. However, after conducting the Augmented Lagrangian transformation and further transforming the problem to $MP - S$ in (14), the remaining constraints are of ϵ-probability simplex form for each vehicle. Several studies, i.e., [5][32] have developed projection algorithms with the probability simplex ($x_i \geq 0, \sum_i x_i = 1$). To be noted, our study works on the projection onto the ϵ-probability simplex space. Thus, we cannot directly use their algorithms. This subsection thus develops a ϵ-simplex projection algorithm that could conduct the projection efficiently to the ϵ-probability simplex space without solving the extra optimization problem. We first give the projection algorithm and then prove its correctness.

Algorithm 1 ϵ-Simplex Projection

1: input $Y = (y_1, \dots, y_n) \in \mathbb{R}^n$;

2: sort Y in descending order such that $y_{(1)} \geq y_{(2)} \geq \cdots \geq y_{(n)}$;

3: find the largest index $k \in [1, n]$, such that

$$y_{(k)} - \frac{\sum_{i=1}^{k} y_{(i)} + (n-k)\epsilon - 1}{k} > \epsilon$$

4: set $\lambda = \frac{\sum_{i=1}^{k} y_{(i)} + (n-k)\epsilon - 1}{k}$

5: **return** $x_i = \max\{y_i - \lambda, \epsilon\}, i = 1 \dots, n$.

The main complexity of the algorithm lies in sorting the elements of Y into descending order, which has a worst-case time complexity of $O(n \log n)$ [6]. Theorem 2 below proves the algorithm correctly projects a vector into the ϵ-probability simplex space.

Theorem 2: the ϵ-Simplex Projection algorithm returns a vector X that satisfies $X = \arg\min_{X \in \mathcal{H}} \|X - Y\|^2$, where $\mathcal{H}$ is the ϵ-probability simplex space.

Proof:

Projecting a vector $Y = (y_1, \dots, y_n)$ into the ϵ-probability simplex space equals to solving the optimization problem of:

$$\begin{gathered}\min_X \frac{1}{2}\|X - Y\|^2 \\ s.t. X^T\mathbf{1} = 1 \\ \epsilon \le x_1, \dots, x_n\end{gathered} \tag{18}$$

KKT conditions of the problem are:

$$\nabla_{x_i}\mathcal{L}(X^*, \lambda^*, \mu^*) = x_i - y_i + \lambda - \mu_i = 0, \quad i = 1, \dots, n \tag{19.1}$$

$$\nabla_{\lambda}\mathcal{L}(X^*, \lambda^*, \mu^*) = \sum_{i=1}^{n} x_i - 1 = 0 \tag{19.2}$$

$$\mu_i(\epsilon - x_i) = 0, \qquad i = 1, \dots, n \tag{19.3}$$

$$\epsilon \le x_i, \qquad i = 1, \dots, n \tag{19.4}$$

$$\mu_i \ge 0, \qquad i = 1, \dots, n \tag{19.5}$$

From above, we know that

$$\begin{cases}\text{if } x_i > \epsilon, \text{then } \mu_i = 0 \text{ and } y_i - \lambda = x_i > \epsilon \\ \text{if } x_i = \epsilon, \text{then } \mu_i \ge 0 \text{ and } y_i - \lambda = x_i - \mu_i = \epsilon - \mu_i \le \epsilon\end{cases} \tag{20}$$

(20) indicates that if $y_i \ge y_j$, then $x_i \ge x_j$. Without loss of generality, assume that Y has been sorted in descending order, and X is arranged using the same index, i.e.,

$$\begin{gathered}y_1 \ge y_2 \ge \cdots \ge y_n \\ x_1 \ge x_2 \ge \cdots \ge x_k > x_{k+1} = \cdots = x_n = \epsilon\end{gathered} \tag{21}$$

Replacing (20) and (21) to (19.2), we have:

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{k} x_i + (n-k)\varepsilon = \sum_{i=1}^{k}(y_i - \lambda) + (n-k)\epsilon = 1 \tag{22}$$

Then,

$$\lambda = \frac{\sum_{i=1}^{k} y_i + (n-k)\epsilon - 1}{k} \tag{23}$$

To this end, if we find the boundary index k, we could determine the value of λ and calculate

the projected vector by:

$$x_i = \max\{y_i - \lambda, \epsilon\} \tag{24}$$

By substituting (24) to (19.1) – (19.5), It's easy to find that it would satisfy all the KKT conditions and would thus be the optimal solution to the problem (18).

Next, we show that step 3 of Algorithm 1 will successfully find the boundary index k, i.e., if k is the index found in step 3 of Algorithm 1, then

$$\begin{cases} y_j - \frac{\sum_{i=1}^{j} y_i + (n-j)\epsilon - 1}{j} > \epsilon, & \forall j \le k \\ y_j - \frac{\sum_{i=1}^{j} y_i + (n-j)\epsilon - 1}{j} \le \epsilon, & \forall j > k \end{cases}.$$

Suppose that now we have found the largest index k, such that $y_k - \frac{\sum_{i=1}^{k} y_i + (n-k)\epsilon - 1}{k} > \epsilon$. Let $\lambda = \frac{\sum_{i=1}^{k} y_i + (n-k)\epsilon - 1}{k}$. Then,

1) for index $j \le k$,

$$\begin{aligned} y_j - \frac{\sum_{i=1}^{j} y_i + (n-j)\epsilon - 1}{j} &= \frac{j y_j - \sum_{i=1}^{j} y_i - (n-j)\epsilon + 1}{j} \\ &= \frac{j y_j + \sum_{i=j+1}^{k} y_i - \sum_{i=1}^{k} y_i - (n-j)\epsilon + 1}{j} \end{aligned} \tag{25}$$

From (22), we have

$$\sum_{i=1}^{k} y_i = 1 + k\lambda - (n-k)\epsilon \tag{26}$$

Insert (26) into (25), we have

$$\begin{aligned} y_j - \frac{\sum_{i=1}^{j} y_i + (n-j)\epsilon - 1}{j} &= \frac{j y_j + \sum_{i=j+1}^{k} y_i - k\lambda + (j-k)\epsilon}{j} \\ &= \frac{j(y_j - \lambda) + \sum_{i=j+1}^{k}(y_i - \lambda) + (j-k)\epsilon}{j} \\ &> \frac{j\epsilon + (k-j)\epsilon + (j-k)\epsilon}{j} = \epsilon \end{aligned} \tag{27}$$

Note that the inequality in (27) results from that y is sorted in descending order, thus for $i \le k$, $y_i - \lambda \ge y_k - \lambda > \epsilon$.

2) for index $j > k$, incorporate (26), we have

$$
\begin{aligned}
y_j - \frac{\sum_{i=1}^{j} y_i + (n-j)\epsilon - 1}{j} &= \frac{j y_j - \sum_{i=1}^{k} y_i - \sum_{i=k+1}^{j} y_i - (n-j)\epsilon + 1}{j} \\
&= \frac{j y_j - \sum_{i=k+1}^{j} y_i - k\lambda + (j-k)\epsilon}{j} \\
&= \frac{k(y_j - \lambda) + \sum_{i=k+1}^{j} (y_j - y_i) + (j-k)\epsilon}{j} \\
&\leq \frac{k\epsilon + (j-k)\epsilon}{j} = \epsilon
\end{aligned} \tag{28}
$$

Note that the inequality in (28) results from that y is sorted in descending order, thus for $j > k$, $y_j - \lambda \leq \epsilon$ and for $i < j$, $y_j - y_i \leq 0$.

Combining (27) and (28), we conclude that if we find the largest index k such that $y_k - \frac{\sum_{i=1}^{k} y_i + (n-k)\epsilon - 1}{k} > \epsilon$, then for index $j < k$, $y_j - \frac{\sum_{i=1}^{j} y_i + (n-j)\epsilon - 1}{j} > \epsilon$, for index $j > k$, $y_j - \frac{\sum_{i=1}^{j} y_i + (n-j)\epsilon - 1}{j} \leq \epsilon$, and thus k would be the boundary index we need.

Q.E.D

With the help of our customized distribution scheme (sec 5.1.2) and ϵ-Simplex Projection algorithm, a complete description of Step (iii) in D-AL can then be given as follows: After an $MP - S$ is formed in Step (ii), the CP distributes individual-specific objective functions along with the current step solution to each vehicle. Upon receiving the information, individual vehicles calculate the gradients related to their own functions and propose them to the CP. The CP then aggregates all the information and performs process (17) to update a new solution. This process keeps iterating until the $MP - S$ is converged. The solution updating process of (17) takes the merit of the gradient projection algorithm and is guaranteed to converge to a local solution of $MP - S$ [3].

To this end, combining subsections 5.1.1 – 5.1.3, we provide the steps of the D-AL as follows:

Algorithm 2 D-AL solution algorithm

1: **Initialization**: initial route choice probabilities $p_v^i, i = 1, \dots, k_v, v = 1, \dots, m$, Lagrangian multipliers $\boldsymbol{\lambda_1}$ and penalty parameter c_1;

2: **For** $K = 1,2, \dots,$:

3: **If** the convergence criterion of the MP for the CeRM problem is satisfied:

4: **break**;

Else:

transform to / update $(\boldsymbol{\lambda}_K, c_K)$ for the $MP-S$ according to Eq. (11), (14) and the Parameter updating scheme;

For $k = 1,2, \dots,$:

If the convergence criterion of the $MP-S$ is satisfied:

break;

Else:

distribute computation task ∇f_v to each vehicle v;

collects the result from each vehicle and update the route choice probabilities according to Eq. (17) with the help of Algorithm 1;

End

End

The D-AL algorithm developed here relies on the distributed computation of the gradient information, i.e., $\sum_{v=1}^{m} \nabla f_v$ in (17). Specifically, when the computation results aggregated from individual vehicles are exactly the gradient of the $MP-S$ $(\nabla\mathcal{L})$, the solving process (17) of $MP-S$ takes the merits of the gradient projection methods and guarantees to converge [3]. Then, as $MP-S$ is iteratively updated according to (11) and the Parameter updating scheme, the convergence of the D-AL resembles that of the Augmented Lagrangian algorithm (see sec 5.1.1), which has been proved in literature to converge to a local solution [3]. In other words, the convergence of D-AL is guaranteed when every vehicle is well-connected throughout the navigation process.

2.6 Numerical Experiment

This study conducts numerical experiments to demonstrate the efficiency of the D-AL and the efficacy of the CeRM. Specifically, our experiments investigate three aspects: (1) the computation efficiency and convergence pattern of the D-AL algorithm; (2) the system cost reduction brought by the CeRM compared with benchmarks (IR, uoER, SOR) routing mechanisms.

2.6.1 Experiment Settings

The experiments are conducted upon the topology of the Sioux Falls city network, as shown in Figure 2. The middle-sized network has 24 nodes and 76 links [35] and has been widely used as a testbed in the transportation field.

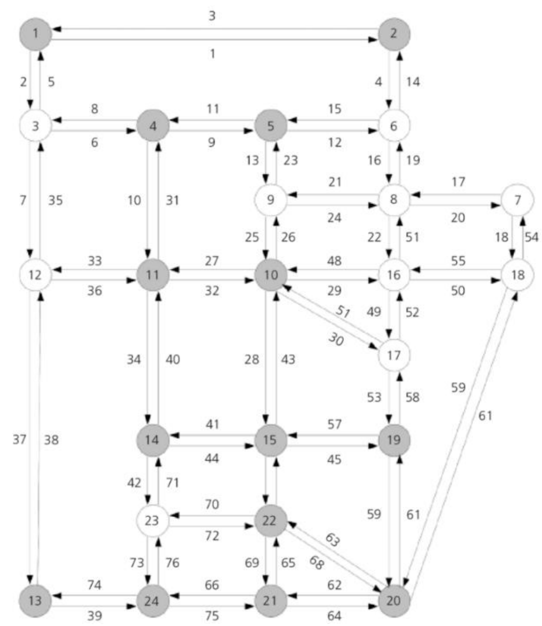


FIGURE 7 SIOUX FALLS CITY NETWORK

The standard BPR function is adopted to capture link travel time with the given flow, i.e.,

$$c_l(f_l) = t_0^l\left(1 + 0.15\left(\frac{f_l}{k_l}\right)^4\right)$$

where t_0^l and k_l is the free-flow travel time and capacity of link l separately. Vehicles represented by a three tuple (OD, α, β) are generated randomly with OD denoting the origin-destination pair and $\alpha, \beta \in [0,1]$ being personal parameters used in the multinomial logit choice model as defined in (5). Each vehicle v has two possible routes found by the k-shortest paths algorithm [36] under current (initial) traffic conditions. Detailed parameters used in the D-AL algorithm are shown in Table 4.

TABLE 4 EXPERIMENT PARAMETERS

Parameter use	notation	value
penalty updating parameter	γ_c	10
Constraint feasibility comparison	γ_m	0.7
first order optimality tolerance	ε_m	0.01
feasibility tolerance	ε_c	0.01

Initial Lagrangian multiplier	λ_0	1
Initial penalty	p_0	1
initial step size	$\bar{\alpha}$	1
Armijo parameter	β, σ	0.5, 0.01

To measure the algorithm's efficiency, we implement the proposed D-AL algorithm in MATLAB R2020a and compare it with the SQP-based MATLAB solver since Sequential Quadratic Programming (SQP) is commonly used to solve nonconvex large-scale optimization problems. In addition, it has been used by recent works such as [24] to solve nonconvex problems in routing games. To measure the routing mechanism's efficacy, we compare the system cost of the CeRM with that of (i) IR, by which each vehicle conducts one snapshot best response to real-time traffic information, (ii) uoER, by which the route choices of the vehicles are coordinated to a snapshot equilibrium resolution, and (iii) SOR, by which vehicles' route choices are systematically manipulated toward the minimum system cost. The experiments are conducted on the laptop with processor: Intel® Core™ i5-8300H CPU @ 2.30GHz.

2.6.2 Computation performance of D-AL

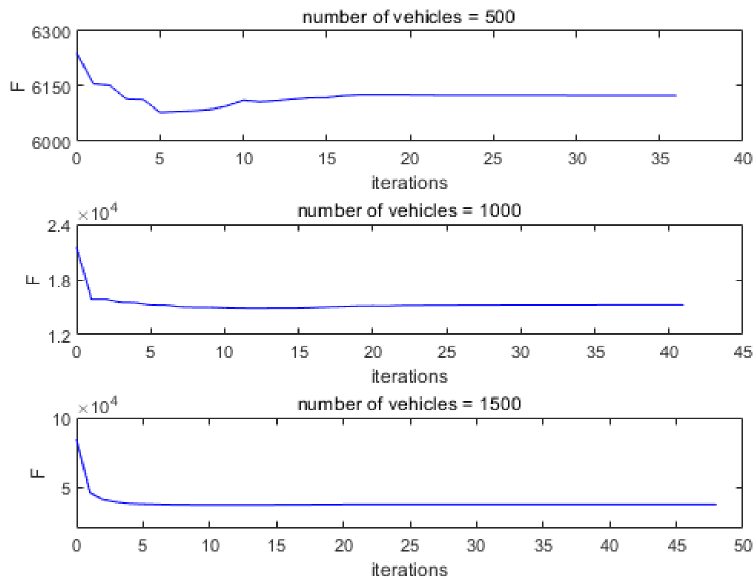


FIGURE 8 CONVERGENCE PATTERN UNDER DIFFERENT NUMBER OF VEHICLES

To demonstrate the proposed algorithm's computation performance, we run 14 traffic scenarios with the number of qualified participating vehicles increasing from 200 to

1500 by an increment of 100. Each scenario was run ten times and then we took the average performance to reduce contingency and randomness.

Figure 3 displays the convergence pattern under the cases of 500, 1000, and 1500 vehicles. It indicates that the objective function drops quickly in the early period (when the penalty is small), fluctuates a little bit, then enters a flat district and stays stable (as penalty increases), which coincides with the typical convergence pattern of the Augmented Lagrangian algorithm.

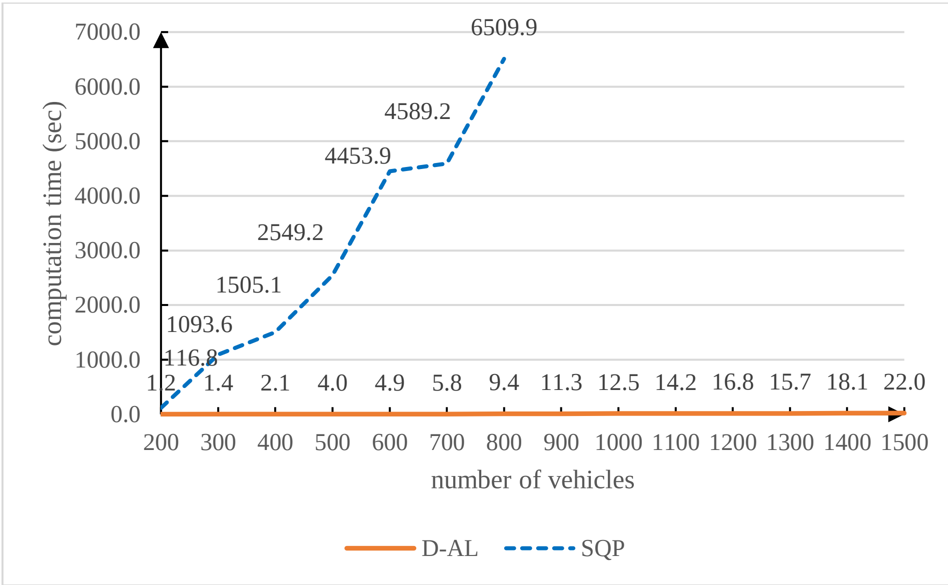


FIGURE 9 COMPUTATION TIME UNDER DIFFERENT NUMBER OF VEHICLES

Figure 4 shows the average computation time for D-AL and SQP as the number of vehicles increases. It demonstrates that the SQP solver becomes computationally intractable (i.e., computation time is larger than 6000 seconds, which cannot adapt to online routing requirements) when the number of vehicles exceeds 800. The coefficient of variation for the computation time of the D-AL ranges between 0.19 to 0.43, with a maximum computation time of 28.7 sec happened in the scenario with 1500 vehicles. The D-AL dramatically outperforms SQP in all scenarios and we conclude that it could satisfy the fast computation need for an online navigation service (i.e., handle a scenario with more than a thousand vehicles by an average leading time smaller than 22 seconds). It's worth noting that it is not proper to compare the computation time of the CeRM to that of the IR mechanism applications such as Google Map or Waze, since the formal one seeks to coordinate the routing decision of a group of vehicles, which is highly complicated and time-consuming, while the latter one only determines individual vehicles' route choices independently without coordination. To conclude, the D-AL algorithm shows stable convergence quality and is far more efficient than the traditional SQP algorithm regarding convergence speed. The computation performance of the D-AL can satisfy the need for a realistic online navigation service with a large number of vehicles.

2.6.3 System Performance of the CeRM

This section further investigates the effectiveness of the CeRM in mitigating traffic congestion, while sustaining individual vehicles'' trip interest. The proposed D-AL is used to calculate the routing guidance under the CeRM. The system cost resulting from the collective route choices under the CeRM is compared with three benchmarks: IR. uoER, and SOR. The detailed formulations of these benchmarks are shown in Appendix C. Fourteen scenarios of experiments are conducted, in which the number of vehicles increased from 200 to 1500 with an increment of 100. Similar to above, every scenario is run ten times to reduce the effect of randomness in the presented results. The coefficient of variation for different scenarios ranges between 0.018 to 0.027, which is very small and shows that the CeRM has stable system performance.

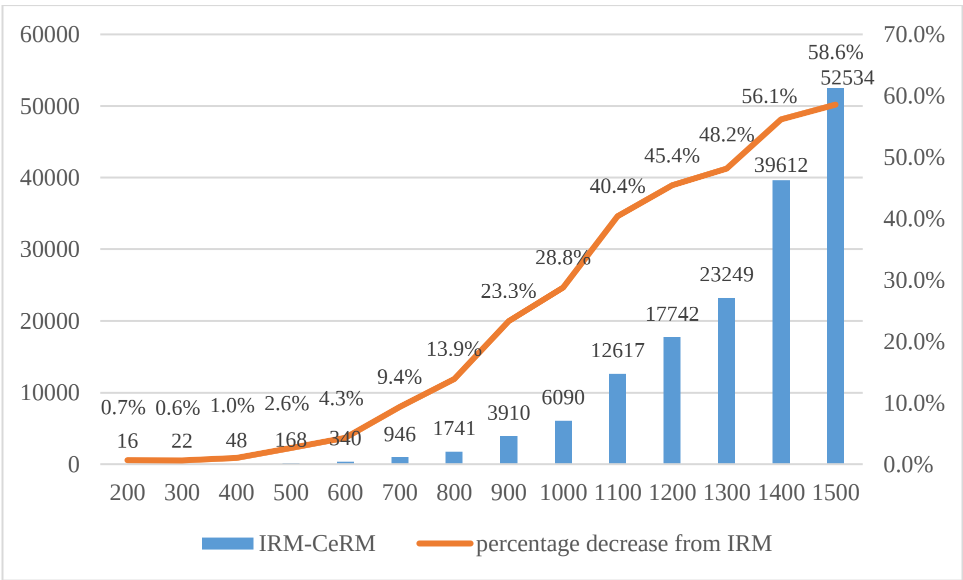


FIGURE 10 SYSTEM COST COMPARISON BETWEEN IR AND CERM

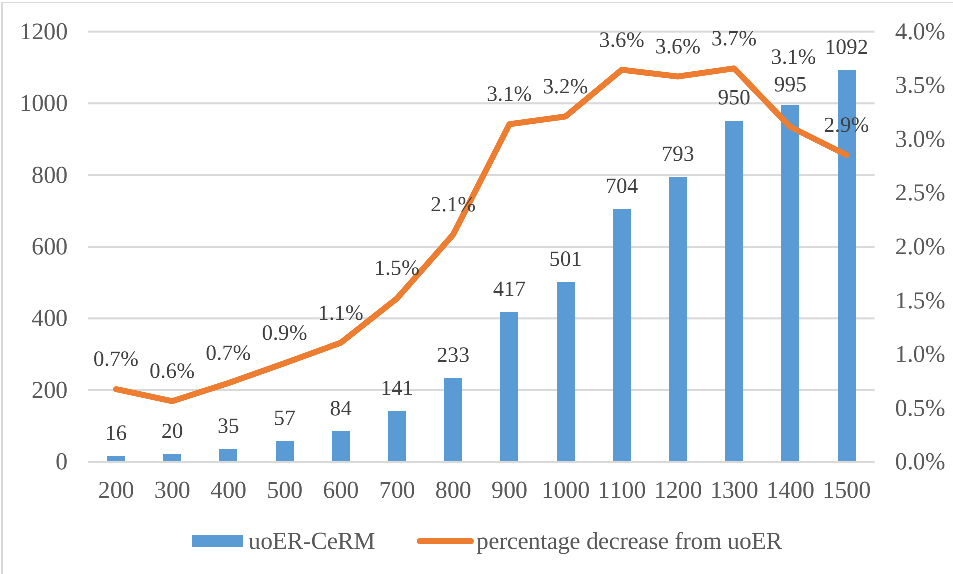


FIGURE 11 SYSTEM COST COMPARISON BETWEEN UOER AND CERM

The experiment results are shown in Figure 5 and Figure 6. Figure 5 shows the system cost comparison between IR and the CeRM, and Figure 6 shows the same comparison between the uoER and CeRM. Define $Z_{CeRM}, Z_{IR}, Z_{uoER}, Z_{SOR}$ to be the system cost under the CeRM, IR, uoER, and SOR, respectively. The blue bars show the amount of the system cost reduction by the CeRM from the benchmark mechanisms, and the orange line shows the percentage of the system cost reduction as comparing the CeRM with IR or uoER, i.e., $\frac{Z_{IR}-Z_{CeRM}}{Z_{IR}}$ and $\frac{Z_{uoER}-Z_{CeRM}}{Z_{uoER}}$. It can be seen that the system cost of the CeRM is always lower than that under IR and uoER. As the number of vehicles increases, the system cost reduction by the CeRM increases. In congested scenarios, the system cost could be reduced by around 55% compared with IR and approximately 3.6% compared with uoER.

We also compare the system performance of the CeRM, IR, and uoER by measuring how much their induced system costs are higher than the System Optimum (SOR) cost, i.e., compare $\Delta_{CeRM} = Z_{CeRM} - Z_{SOR}$, $\Delta_{IR} = Z_{IR} - Z_{SOR}$ and $\Delta_{uoER} = Z_{uoER} - Z_{SOR}$. Given SOR represents the best system performance, the smaller Δ is, the better the resulting system performance it represents. Table 1 clearly shows that the CeRM approaches the system optimum cost closely and it outperforms IR and uoER under all scenarios. To conclude, the CeRM pushes the snapshot traffic resulting from widely used IR to a more systematically efficient state. It proves to be efficient in reducing traffic congestion at a systematic level.

Number of Vehicles	Δ_{CeRM}	Δ_{uoER}	Δ_{IR}

200	72.74	88.40	88.41
300	110.34	130.51	132.34
400	155.29	190.40	203.74
500	242.11	299.35	410.44
600	305.83	389.87	645.76
700	441.50	582.75	1387.15
800	526.70	759.24	2268.00
900	589.15	1006.36	4498.82
1000	651.79	1152.41	6742.16
1100	663.79	1367.56	13280.52
1200	571.55	1364.15	18313.46
1300	575.57	1526.05	23824.54
1400	440.08	1435.10	40052.14
1500	443.80	1535.47	52978.12

TABLE 5 SYSTEM PERFORMANCE UNDER DIFFERENT NUMBER OF VEHICLES

2.7 Conclusion

This study designs a correlated routing mechanism that calculates and provides online routing guidance for vehicles with onboard computing and communication devices. By exploiting information discrepancies between individual vehicles and the CP, the proposed mechanism drives the snapshot equilibrium route choice of a group of vehicles toward a more systematic optimal condition while still preserving the individual's selfish nature. By following the routing guidance offered by the CP, every driver would get better off (at least not worse off) compared to their initial preference based on current traffic information. Furthermore, this study proposes the D-AL, an effective distributed algorithm to quickly solve the routing problem for an online real-time navigation service. The conducted numerical experiments demonstrate the merits of the D-AL in its computation speed. The experimental results also indicate that the CeRM drives the traffic equilibrium to a state better than that under IR and uoER regarding the objective of the system cost. To the best of our knowledge, this is one of the first studies to design an online routing mechanism based on correlated game and distributed optimization to mitigate network traffic congestion. The methodology and findings of this study will significantly contribute to traffic congestion mitigation areas in both literature and practice.

2.8 Appendix

Appendix A

Lemma 13: The optimization problem has a convex objective function and a nonconvex feasible region.

Proof: To check the convexity of the objective function, we exam its Hessian and noticed the Hessian of Z is positive definite.

$$
H(Z)=\begin{bmatrix}
\frac{\partial^2 Z}{\partial p_s^{v_1,i_1}\partial p_s^{v_1,i_1}} & \frac{\partial^2 Z}{\partial p_s^{v_1,i_1}\partial p_s^{v_1,i_2}} & \cdots & \frac{\partial^2 Z}{\partial p_s^{v_1,i_1}\partial p_s^{v_m,i_{k_m}}} \\
\frac{\partial^2 Z}{\partial p_s^{v_1,i_2}\partial p_s^{v_1,i_1}} & \frac{\partial^2 Z}{\partial p_s^{v_1,i_2}\partial p_s^{v_1,i_2}} & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\frac{\partial^2 Z}{\partial p_s^{v_m,i_{k_m}}\partial p_s^{v_1,i_1}} & \frac{\partial^2 Z}{\partial p_s^{v_m,i_{k_m}}\partial p_s^{v_1,i_2}} & \cdots & \frac{\partial^2 Z}{\partial p_s^{v_m,i_{k_m}}\partial p_s^{v_m,i_{k_m}}}
\end{bmatrix}
$$

$$
=\Omega\begin{bmatrix}
2c_1'(f_s^L)+f_s^L c_1''(f_s^L) & & & \\
 & 2c_2'(f_s^L)+f_s^L c_2''(f_s^L) & & \\
 & & \cdots & \\
 & & & 2c_n'(f_s^L)+f_s^L c_n''(f_s^L)
\end{bmatrix}\Omega^T
$$

$$
=\Omega\Lambda\Omega^T
$$

where Ω is the link-path relation matrix, i.e., $\Omega=\begin{pmatrix} \delta_{v_1,i_1}^1 & \delta_{v_1,i_1}^2 & \cdots & \delta_{v_1,i_1}^n \\ \delta_{v_1,i_2}^1 & \delta_{v_1,i_2}^2 & \cdots & \delta_{v_1,i_2}^n \\ \cdots & \cdots & \cdots & \cdots \\ \delta_{v_m,i_{k_m}}^1 & \delta_{v_1,i_2}^2 & \cdots & \delta_{v_m,i_{k_m}}^n \end{pmatrix}$.

Since the link cost function is strictly increasing and convex according to assumption 6, $c_l{}'$ and $c_l{}''$ would be positive. Thus, every element in matrix Λ is positive. Given any diagonal matrix Λ with positive elements, for any vector x and matrix M, $x^T M\Lambda M^T x = (M^T x)^T \Lambda (M^T x) > 0$ must hold. Hence, the hessian matrix $H(Z)$ is positive definite.

Next, we check the rationality constraint's convexity and notice that the Hessian of rationality constraints is indefinite. More exactly, we first calculate the elements in the Hessian and then prove it is neither positive (semi) definite nor negative definite.

The first derivatives of the rationality constraint of vehicle v^* are:

$$
\frac{\partial r_{v^*}}{\partial p_s^{v^*,i^*}}=\sum_{L\in r^{v^*,i^*}} c_L(f_s^L)+\frac{1}{\beta_{v^*}}\left(1+\ln\left(p_s^{v^*,i^*}\right)\right)+\sum_{i=1}^{k_{v^*}} p_s^{v^*,i}\sum_{L\in r^{v^*,i}} c_L{}'(f_s^L)\,\delta_{v^*,i^*}^L \quad ,i^*=1,\dots,k^{v^*}
$$

$$
\frac{\partial r_{v^*}}{\partial p_s^{v',i'}}=\sum_{i=1}^{k_{v^*}} p_s^{v^*,i}\sum_{L\in r^{v^*,i}} c_L{}'(f_s^L)\delta_{v',i'}^L-\sum_{i=1}^{k_{v^*}} p_o^{v^*,i}\sum_{L\in r^{v^*,i}} c_L'(f_o^L)\delta_{v',i'}^L \quad ,v'\neq v^*,i'=1,\dots,k^{v'}
$$

the elements in the Hessian matrix of the rationality constraint of vehicle v^* are:

$$
\frac{\partial r_{v^*}}{\partial p_s^{v^*,i^*}\partial p_s^{v^*,i^*}}=2\sum_{L\in r^{v^*,i^*}} c_L'(f_s^L)\delta_{v^*,i^*}^L+\sum_{i=1}^{k_{v^*}} p_s^{v^*,i}\sum_{L\in r^{v^*,i}} c_L''(f_s^L)\delta_{v^*,i^*}^L+\frac{1}{\beta_{v^*} p_s^{v^*,i^*}} \quad ,i^*=1,\dots,k^{v^*}
$$

$$\frac{\partial r_{v^*}}{\partial p_s^{v^*,i^*} \partial p_s^{v^*,j}} = 2 \sum_{L \in r^{v^*,i^*}} c_L{}'(f_s^L)\delta_{v^*,j}^L + \sum_{i=1}^{k_{v^*}} p_s^{v^*,i} \sum_{L \in r^{v^*,i}} c_L''(f_s^L)\delta_{v^*,i^*}^L \delta_{v^*,j}^L \quad ,j \neq i^*$$

$$\frac{\partial r_{v^*}}{\partial p_s^{v^*,i^*} \partial p_s^{v',i'}} = \sum_{L \in r^{v^*,i^*}} c_L{}'(f_s^L)\delta_{v',i'}^L + \sum_{i=1}^{k_{v^*}} p_s^{v^*,i} \sum_{L \in r^{v^*,i}} c_L{}''(f_s^L)\delta_{v^*,i^*}^L \delta_{v',i'}^L \quad ,v' \neq v^*, i' = 1, \dots, k^{v'}$$

$$\frac{\partial r_{v^*}}{\partial p_s^{v',i'} \partial p_s^{v^*,j}} = \sum_{L \in r^{v^*,j}} c_L{}'(f_s^L)\delta_{v',i'}^L + \sum_{i=1}^{k_{v^*}} p_s^{v^*,i} \sum_{L \in r^{v^*,i}} c_L{}''(f_s^L)\delta_{v^*,j}^L \delta_{v',i'}^L \quad ,v' \neq v^*, i' = 1, \dots, k^{v'}$$

$$\frac{\partial r_{v^*}}{\partial p_s^{v',i'} \partial p_s^{v',j}} = \sum_{i=1}^{k_{v^*}} p_s^{v^*,i} \sum_{L \in r^{v^*,i}} c_L{}''(f_s^L)\delta_{v',i'}^L \delta_{v',j}^L - \sum_{i=1}^{k_{v^*}} p_o^{v^*,i} \sum_{L \in r^{v^*,i}} c_L''(f_{v^*}^L)\delta_{v',i'}^L \delta_{v',j}^L \quad ,v' \neq v^*, i' = 1, \dots, k^{v'}$$

Take a simplified scenario for the demonstration. Assume there are two simple routes (routes that contain only one link) 1 and 2 connecting O and D, and there are only two vehicles a, b traveling in between. Vehicle a has two possible routes 1 and 2, while vehicle b has only route 1. Denote the Hessian matrix as Φ, then for any vector x:

$$\begin{aligned} x\Phi x^T = {} & {x_1}^2 \left(\underbrace{2c_1'(f_s^1) + p_s^{v_a,i_1} c_1''(f_s^1) + \frac{1}{\beta_{v_1} p_s^{v_1,i_1}}}_{A} \right) \\ & + 2x_1x_3 \left(\underbrace{c_1'(f_s^1) + p_s^{v_a,i_1} c_1''(f_s^1)}_{B} \right) \\ & + {x_2}^2 \left(\underbrace{2c_2'(f_s^1) + p_s^{v_a,i_2} c_2''(f_s^1) + \frac{1}{\beta_{v_1} p_s^{v_1,i_2}}}_{C} \right) \\ & + x_2x_4 \left(\underbrace{c_2'(f_s^1) + p_s^{v_a,i_2} c_2''(f_s^1)}_{D} \right) + {x_3}^2 (\underbrace{p_s^{v_a,i_1} c_1''(f_s^1) - p_o^{v_a,i_1} c_1''(f_o^1)}_{E}) \end{aligned}$$

where $f_s^1 = p_s^{v_a,i_1} + p_s^{v_b,i_1}$ is the flow on link 1 if both vehicle a and b follow the guidance and $f_a^1 = p_o^{v_a,i_1} + p_s^{v_b,i_1}$ is the flow on link 1 if only vehicle b follows the guidance while a sticks to its original choice. Suppose $p_s^{v_1,i_1} > p_o^{v_1,i_1}$ (vice versa), and $x_1, x_2, x_3 > 0$. Follow assumption 4, c'' is an increasing function of f. Since $f_s^1 > f_o^1$, we have $c_1''(f_s^1) > c_1''(f_o^1)$. Thus, $p_s^{v_a,i_1} c_1''(f_s^1) - p_o^{v_a,i_1} c_1''(f_o^1) > 0$, and all the 5 items $A, B, C, D, E > 0$.

Thus, we have $x\Phi x^T \begin{cases} < 0, & if\ x_4 < \frac{x_1{}^2A+2x_1x_3B+x_2{}^2C+x_3{}^2E}{-2x_2D} \\ \geq 0, & if x_4 \geq \frac{x_1{}^2A+2x_1x_3B+x_2{}^2C+x_3{}^2E}{-2x_2D} \end{cases}$. So, the hessian matrix Φ is indefinite, and the feasible region defined by the constraint is nonconvex.
Q.E.D

Appendix B

Proof of Theorem 1:

We denote a solution point $\boldsymbol{P}_s = \{p_s^{v,i}\} = \boldsymbol{X} = (x_1, \dots, x_{km})$ and $\boldsymbol{X}^i = (x_1, \dots, x_i + dx_i, \dots, x_{km})$, then the numerical calculation of a single partial derivative for a vehicle in n-DS takes the form:

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{\mathcal{L}(\boldsymbol{X}^i)-\mathcal{L}(\boldsymbol{X})}{dx_i} = \frac{\sum_{v=1}^{m} \nabla f_v(\boldsymbol{X}^i)-\sum_{v=1}^{m} \nabla f_v(\boldsymbol{X})}{dx_i}, i = k(v-1)+1, \dots, kv.$$

In the contrast, the numerical calculation of a single partial derivative for a vehicle in c-DS takes the form:

$$\frac{\partial f_v}{\partial x_i} = \frac{f_v(\boldsymbol{X}^i) - f_v(\boldsymbol{X})}{dx_i}, i = 1, \dots, km$$

An individual vehicle under c-DS has to compute km partial derivatives, while under n-DS has to compute k partial derivatives. Consider the time complexity it takes to numerically calculate $f_v(\boldsymbol{X})$ as l. Then the computation costs measured by the time complexity of vehicle j under c-DS and n-DS are given below:

c-DS time complexity: $(km+1)l$

- the vehicle needs to calculate km different $\frac{\partial f_v}{\partial x_i}$, and thus needs to calculate $\underbrace{f_{v_j}(\boldsymbol{X}^1), \dots, f_{v_j}(\boldsymbol{X}^{km})}_{km}, \underbrace{f_{v_j}(\boldsymbol{X})}_{1}$.

n-DS time complexity: $(k+1)ml$

- the vehicle needs to calculate k different $\frac{\partial \mathcal{L}}{\partial x_i}$, and thus needs to calculate $\underbrace{\underbrace{\sum_{v=1}^{m} f_v\left(X^{k(j-1)+1}\right)}_{m}, \dots, \underbrace{\sum_{v=1}^{m} f_v\left(X^{kj}\right)}_{m}}_{k}, \underbrace{\sum_{v=1}^{m} f_v(X)}_{m}$.

Then, we claim a vehicle in c-DS undertakes $\frac{(k+1)ml-(km+1)l}{(k+1)ml} = \frac{1-\frac{1}{m}}{k+1}$ less workload than in n-DS.

Q.E.D

Appendix C

Independent Routing (IR)

In an independent routing mechanism, each individual driver independently does the best response to the real-time traffic conditions. In this study, we adopt the commonly used multinomial logit-based (MNL) behavior choice model. According to the real-time traffic information $C_{v,o}^{i}$, individual vehicle's independent routing choice preference could then be calculated by the MNL model:

$$p^{v,i} = \frac{e^{-V_{v,i}}}{\sum_{i=1}^{k_v} e^{-V_{v,i}}},$$

where

$$V_{v,i} = \alpha^v + \beta^v C_v^i ,$$

is the measured utility of route r_v^i for vehicle v and α^v, β^v are vehicle-specific constant scalars representing the characteristics of each individual. Readers can refer to Sec 3.1 for a detailed introduction.

User-oriented Equilibrium Routing (uoER)

There are several approaches to derive an uoER. Under the assumption of logit-choice model, this study adopts the coordinated routing mechanism proposed in [10]. Specifically, the route preference is calculated by:

$$\min_{p} \sum_{l \in L} \int_0^{f_l} c_l(w) dw + \sum_{v=1}^{m} \sum_{i=1}^{k^v} \frac{1}{\beta_v} p^{v,i} \ln\left(p^{v,i}\right)$$

$s.t$

$$\sum_{i=1}^{k_v} p^{v,i} = 1, \ \ \forall v = 1, \dots, m$$

$$\sum_{i=1}^{k_v} p^{v,i} = 1, \ \ \forall v = 1, \dots, m$$

$$f_l = \sum_{v=1}^{m} \sum_{i=1}^{k^v} p^{v,i}\, \delta_{v,i}^{L}, \ \ , \ \ \forall l \in L$$

And the corresponding system cost is calculated by $C_{sys} = \sum_{l=1}^{n} f_l c_l(f_s^l)$.

System Optimum Routing (SOR)

We consider there is a centralized agent to systemically generate the route preference for each driver, aiming to minimize the expected system travel cost C_{sys}. Then the SOR routing preference could be calculated by solving the following:

$$\min_{p} C_{sys} = \sum_{l=1}^{n} f_l c_l(f_l)$$

$s.t$

$$\sum_{i=1}^{k_v} p^{v,i} = 1, \ \ \forall v = 1, \dots, m$$

$$p^{v,i} \geq 0, \ \ \forall v = 1, \dots, m, \forall \ i = 1, \dots, k^v$$

$$f_l = \sum_{v=1}^{m} \sum_{i=1}^{k^v} p^{v,i} \, \delta_{v,i}^{L}, \ \ , \ \ \forall l \in L$$

To be noted, the agent generates route choice probability not a route choice for individual vehicles to make it consistent to the setup of our CeRM in this study.

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3.0 RECOMMENDATIONS

There are several possible extensions stemmed from this work. For Task 1, In the future, numerical studies could be conducted on larger-scale traffic networks such as Sioux Falls or real-world networks such as the city of Atlanta. Another direction to explore is to build and test the smartphone-based framework with human subjects to understand the effect of the behavioral change solutions. For task 2, the convergence speed of the D-AL now heavily depends on the computation load of searching the step size. The future study can explore the distributed step size calculation scheme to improve the convergence efficiency dramatically. In addition, this study assumes drivers make decisions purely based on information provided by the CP without using their prior knowledge. However drivers' ex-ante knowledge may affect their compliance to the routing guidance. Therefore, a possible future work is to incorporate individual drivers' ex-ante beliefs into the correlated routing game.