



Center for Advanced Multimodal Mobility Solutions and Education

Project ID: 2022 Project 08

Prioritizing People - Mixed Equilibrium Assignment for AV Based on
Occupancy (Phase II)

Final Report

by

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September 2023

ACKNOWLEDGEMENTS

This project was funded by the Center for Advanced Multimodal Mobility Solutions and Education (CAMMSE @ UNC Charlotte), one of the Tier I University Transportation Centers that were selected in this nationwide competition, by the Office of the Assistant Secretary for Research and Technology (OST-R), U.S. Department of Transportation (US DOT), under the FAST Act. The authors are also very grateful for all of the time and effort spent by DOT and industry professionals to provide project information that was critical for the successful completion of this study.

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Table of Contents

EXECUTIVE SUMMARY	vii
Chapter 1. Introduction	1
1.1 Problem Statement	1
1.2 Objectives	3
1.3 Contributions.....	3
1.4 Report Overview	3
Chapter 2. Literature Review	4
2.1 Introduction.....	4
2.2 AV Survey	4
2.3 Mixed Traffic Equilibrium Modeling.....	5
2.4 AV Policy.....	6
2.5 Summary	6
Chapter 3. Methodology	7
3.1 Introduction.....	7
3.2 Mixed Equilibrium Assignment.....	7
3.2.1 Strategy Algorithm.....	11
3.2.2 Sioux Falls Network.....	12
3.2.3 Eastern Massachusetts Network.....	12
3.2.4 Chicago Sketch Network:.....	13
3.3 Summary	13
Chapter 4. Results and Analysis	14
4.1 Introduction.....	14
4.2 Results and Analysis	14
4.2.1 Sioux Falls – Strategy 1	14
4.2.2 Sioux Falls – Strategy 2	14
4.2.3 Sioux Falls – Strategy 3	15
4.2.4 Eastern Massachusetts – Strategy 1.....	16
4.2.5 Eastern Massachusetts – Strategy 2.....	17
4.2.6 Eastern Massachusetts – Strategy 3.....	17
4.2.7 Chicago Sketch – Strategy 1	19
4.2.8 Chicago Sketch – Strategy 2	20
4.3 Summary	21
Chapter 5. Summary and Conclusions	22
5.1 Introduction.....	22
5.2 Discussion and Conclusion	22
5.3 Directions for Future Research	22

References23

List of Figures

Figure 1: Example Network.....	2
Figure 2: Flow of the Strategy	10
Figure 3: Sioux Falls Network.....	12
Figure 4: Eastern Massachusetts Network.....	12
Figure 5: Average and Maximum Delay ($\mu r, s$) of the OV's and UV's – Strategy 1.....	14
Figure 6: Average and Maximum Delay ($\mu r, s$) of the OV's and UV's – Strategy 2.....	15
Figure 7: Average and Maximum Delay ($\mu r, s$) of the OV's and UV's – Strategy 3.....	15
Figure 8: Total System Travel Time Plots – Sioux Falls Network.....	16
Figure 9: Average and Maximum Delay ($\mu r, s$) of the OV's and UV's – Strategy 1.....	16
Figure 10: Average and Maximum Delay ($\mu r, s$) of the OV's and UV's – Strategy 2.....	17
Figure 11: Average and Maximum Delay ($\mu r, s$) of the OV's and UV's – Strategy 3.....	17
Figure 12: Total System Travel Time Plots – Eastern Massachusetts.....	18
Figure 13: Number of OD pairs with delays and no-delays	19
Figure 14: Average and Maximum Delay ($\Delta r, s$) of the OV's and UV's.....	19
Figure 15: Representation of delays in OV's and UV's.....	20
Figure 16: Total System Travel Time – Chicago Network.....	20
Figure 17: Deadheading vehicle delay by trip purpose	21

List of Tables

Table 1: Comparison of results for Sioux Falls and Eastern Massachusetts	18
Table 2: Effect on occupied vehicles and unoccupied vehicles – Strategy 1	26
Table 3: Effect on occupied vehicles and unoccupied vehicles – Strategy 2	26
Table 4: Effect on occupied vehicles and unoccupied vehicles – Strategy 3	27
Table 5: Effect on occupied vehicles and unoccupied vehicles – Strategy 1	27
Table 6: Effect on occupied vehicles and unoccupied vehicles – Strategy 2	28
Table 7: Effect on occupied vehicles and unoccupied vehicles – Strategy 3	28

EXECUTIVE SUMMARY

The transition of the vehicle fleet to incorporate AV will be a long and complex process. AVs will gradually form a larger and larger share of the fleet mix, offering opportunities and challenges for improved efficiency and safety. At any given point during this transition a portion of the AV fleet will be consuming roadway capacity while unoccupied (operating without passengers). Should these unoccupied vehicles simply utilize shortest paths to their next destination, they will contribute to congestion for the rest of the roadway users without providing any benefit to human passengers. There is an opportunity to develop routing strategies for unoccupied AVs that mitigate or eliminate their contribution to congestion while still serving the mobility needs of AV owners or passengers. Some of the AV fleet will be privately owned, some will be part of a shared AV fleet. In the former, some AVs will be owned by households that are lower-income and benefit from the ability to have fewer vehicles to serve the mobility needs of the household. In these cases, it is especially important that unoccupied AVs can meet household mobility needs while also limiting the contribution to roadway congestion. The Policy Analysis and Development Team of FHWA is actively conducting research on Transportation Scenario Planning for Connected and Automated Vehicles. This study informs the policy associated with the routing strategies of unoccupied AVs. This study proposes a bi-objective program to evaluate and balance tradeoffs between congestion reduction and the mobility needs of households. Three strategies are proposed to deploy a AV unoccupied methodology to route unoccupied vehicles on longer paths, reducing congestion for occupied vehicles, while still meeting the trip making needs of households. Case studies on transportation networks are presented alongside their practical implications and computational requirements. The methods devised and results obtained through the modeling process indicated that a) for most of the occupied vehicles there will be a reduction in travel times when compared to user equilibrium traffic assignment, b) there were no significant delays for any of the AV owners studied after applying the threshold, and c) The overall performance of the system is improved, resulting in a reduction in total travel time.

Chapter 1. Introduction

1.1 Problem Statement

Autonomous vehicles (AVs) have the potential to bring substantial safety and operational change to the current transportation system. There is widespread optimism, and criticism, that introduction of AVs will have dramatic impacts on traffic congestion and may vastly improve safety. The exact impacts on traffic congestion are still debatable and highly reliant on how vehicles are utilized and prioritized in the future. While literature suggests that AVs will help mitigate traffic congestion and vastly improve safety [1] the transition to a fully autonomous fleet will take time. This era of a mixed fleet of self-driving and human driven vehicles will present many complex problems that will require new methods and models to solve. This progressively changing blend human-driven vehicles, occupied AVs, and unoccupied AVs will present new opportunities to achieve real-time route choice and traffic equilibrium decisions. This is due to connected and automated vehicles collecting, transmitting, and receiving real-time knowledge of current conditions at a scale and density that currently is not possible.

Existing literature suggests that as the concept of shared AVs becomes more prevalent, accessible, and possible the perceived benefits of a Shared Autonomous Vehicle (SAVs) will create a more dynamic ridesharing system [2]. The benefits of shared riding system were evaluated by Ford using fixed pickup and drop off stations. [3] Although there is an extensive research and literature related to SAVs, the transition will take significant time, effort, and a change in culture here in the United States. The United States has a strong desire to remain independent and the use of private AVs tend to still be the primary preference according to recent AV surveys. An internet-based survey was conducted in Austin concluded that only 41% of the respondents were willing to use shared AVs at a cost of 1\$ per mile at least once a week [4].

One of the perceived benefits of an AV is that a single vehicle can meet all the travel needs of an entire family, thus allowing households to own fewer vehicles at a considerable cost savings. In theory, this SAV would drive one family member to an initial destination, then leave unoccupied and travel to the pickup location of another family member. This model, however, only works efficiently if family members are willing to wait and plan their trips accordingly. Under this scenario a single AV could be used to meet the mobility needs of multiple household members. With vehicles that need a human driver, this shared mobility would be impossible. However, while the number of vehicles in a household may decrease, the number of trips made by an individual vehicle may increase dramatically. This shared mobility means that trips from drop off to pickup will be unoccupied and may increase the vehicle miles traveled for a household, depending on the activity level and commuting distance of the household. It is predicted that unoccupied AVs may impact congestion on heavily traveled routes and the travel times of occupied AV and human-driven vehicles will increase. This research follows a vehicle scheduling algorithm by Zhang et al. to calculate the vehicle reduction potential of the household i.e., the potential number of cars that can be reduced for effectively managing the household demand [5].

Levin et al. discusses the rerouting of unoccupied AV trips to less traveled routes or longer paths resulted in the reduction of the congestion on the downtown Austin network [6]. Levin et al. provided a genetic algorithm for shifting unoccupied AVs routes by encouraging AVs to park at cheaper locations further away from the travelers' destination. The findings of that study suggest adjusted parking fees results in reducing the congestion caused by the unoccupied AVs. [7]

This reduction in car ownership is attributed to a surge in the phenomenon of 'deadheading' where the unoccupied AVs consume capacity and contribute to the congestion of the roads with a different travel objective than an occupied vehicle. To effectively manage this challenge, it is important to develop appropriate routing strategies for unoccupied vehicles.

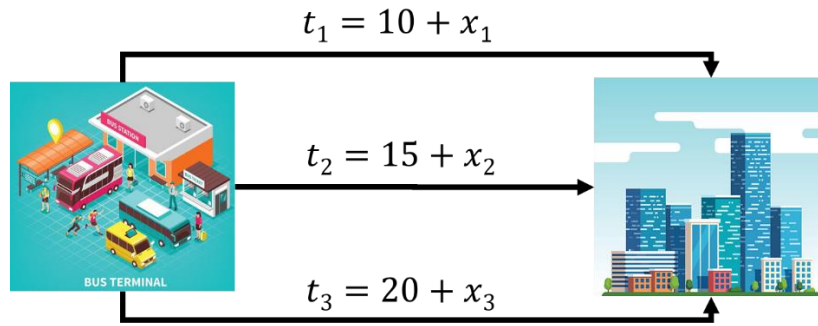


Figure 1: Example Network

Imagine the following situation.

- 15 occupied vehicles wish to travel from a bus terminal (A) to the same office building (B) in the city.
- If they follow the user equilibrium UE assignment, $x_1 = 10, x_2 = 5, x_3 = 0$
- Then $t_1 = t_2 = t_3 = 20$.

Under that initial condition the travel time on each link is equal to 20.

- If 10 of them are occupied vehicles and 5 vehicles unoccupied vehicles traveling from A to B,
- The unoccupied vehicles can follow the path 3 and decrease the travel times of the occupied vehicles. The volumes would be $x_1 = 7.5, x_2 = 2.5, x_3 = 5$ and travel times $t_1 = t_2 = 17.5$ and $t_3 = 25$.

This would reduce the total system travel time from 300 to 237.5 units.

However, if one or all of the unoccupied AVs is owned by a household that is using their AV as a SAV, those vehicles would need to return home to pick up another household member for another trip. This would result in delays imposed on the second household member waiting for an unoccupied vehicle returning to pick up the second household member. This type of assignment scheme would result in these household(s) absorbing the cost of a missed trip or other transportation services. Therefore, if unoccupied AVs are assigned routes that become too lengthy, or require significant delays, it defeats all the benefits of unmanned vehicles. Planners are taking the obvious approach of prioritizing the travel time and needs of occupied vehicles, while minimizing the impacts of empty AVs on occupied vehicles. However, there needs to be

parameters established to ensure that the penalty imposed on unoccupied vehicles is not so severe that it outweighs their benefits.

This research implements a differential route assignment methodology based on AV vehicle occupancy, with the goal of reducing the impacts of unoccupied AV route choice on all drivers and while not discouraging SAV operations in the future. This was accounted for by rerouting unoccupied AVs to minimize the potential impacts on occupied vehicles without disproportionately influencing households that will utilize SAVs in the future.

1.2 Objectives

The proposed research focuses on the following topics:

- This study proposes a bi-objective program to evaluate and balance tradeoffs between congestion reduction and the mobility needs of households.
- Conduct an analysis of this new methodology application based on sample networks as case studies.

1.3 Contributions

The proposed project makes the following contributions:

- Proposing a differential route assignment for occupied versus unoccupied vehicles while considering the impacts of unoccupied AV route choice on AV owners.
- The travel time restrictions of unoccupied vehicles are incorporated and accounted for in the proposed solution so that SAV are not discouraged through inflated travel penalties yet the trip making needs of the household can be achieved without adversely impacting occupied vehicles.

1.4 Report Overview

The remainder of this report is organized as follows. Chapter 2, outlines literature related to the work conducted. Chapter 3, describes that methodologies used and a solution procedure is presented. Chapter 4, details the numerical experiments that were conducted using two different networks to evaluate the performance of the system. Chapter 5 concludes the report, summarizing the major results and propositions for future research.

Chapter 2. Literature Review

2.1 Introduction

Autonomous Vehicles (AV) have the potential to revolutionize transportation operations mode choice. The Federal Highway Administration (FHWA) has led communication and outreach efforts with highway stakeholders, including state departments of transportation (DOTs), public agencies and industry groups to build understanding of the potential impacts of AVs on transportation, society, and the economy [8]. The Policy Analysis and Development Team is actively conducting research on Transportation Scenario Planning for Connected and Automated Vehicles [9]. Although the transition to a complete autonomous fleet is in the distant future, the integration of human driven and AVs is a critical scenario and requires careful planning and coordination. This chapter reviews and synthesizes the current practice and exploration of the potential impacts of autonomous vehicles on transportation planning methods and models. AV implementation has accelerated rapidly over the last 10 years. With car companies like Tesla pushing the boundaries of what can be done and what is allowed to be tested on public roads. The impacts of this new mobility are predicted to have a wide variety of impacts on not only how we travel but how we have to plan the development of our cities as well as transportation policies and infrastructure. Most of the existing literature focuses on perception and adoption surveys focused on understanding demand and adoption rates or the price per mile tipping point at which vehicle ownership becomes unlikely due to cheap unmanned ride sharing services. Many forward-looking mobility experts expect there to evolve a shared autonomous fleet which replicates the current human driven rideshare systems.

2.2 AV Survey

Spieser et. al proposed a shared vehicle mobility on demand system that allows users to rent vehicles on demand. This type of service has the potential to reduce traffic congestion and provide a more convenient option for users [10]. Narayanan et al. provided a comprehensive review of the SAV literature on demand modeling, fleet management, economic impacts and regulatory challenges. Although there is limited research on private autonomous vehicles, autonomous vehicle surveys indicate that younger people in urban areas who are more educated and tech-savvy are more likely to be the first to adopt AV technology, which favors a shared-ride model over individual ownership. [11]. Batur et al. found that people who are interested in using AVs to run errands are more likely to be interested in owning AVs, even after accounting their socio-economic and demographic background, as well as their attitudes towards AVs [12].

Menon et al. describes the likelihood a person would be willing to reduce their current household vehicle ownership by one vehicle in the presence of SAVs.[13] The results indicate that there are key parameters which differ by single vehicle and multivehicle homes with which indicate whether a person will adopt SAVs. However, Menon et al. caution that AV technology becomes more common, personal experiences, media reports, crashes, publicity, and information gathering will change an individuals perception of SAVs. Therefore, future studies are needed to track and understand shifts as they take place. Nazari et al.'s (2018) stated preference survey and socio-economic characteristics that effect choosing a shared versus private AV[14]. Finding that individuals with larger inter-trip travel times are more inclined toward SAVs. Hoboucha et.al developed a vehicle choice model that provided owners the ability to chose between regular, shared

and private AVs. This stated preference survey across Israel and North America found that even if the SAVs were to be completely free, only 75% of the individuals would be willing to use SAV [15].

It is predicted that when AVs become available, private households will not own a significant portion of the fleet. Schoettle et.al used the 2009 U.S. National Household Travel Survey to observe a 43% reduction of vehicle fleet, while personal vehicle usage dropped from 2.1 to 1.2 per household. This was attributed to simply eliminating existing trip overlap [16]. Zhang et al. conducted a study which predicts a 9.5% reduction of private vehicles due to households switching to private AVs. Their study concluded household efficiency gains will be realized though a SAVs being able to serve multiple household trips. However, they also noticed that this shift to SAVs will generate nearly 30 unoccupied Vehicle Miles Traveled (VMT) per day per vehicle eliminated [5]. Nair et. al proposes a model to predict the number of deadheading trips and the pick-up locations for ride-hailing service autonomous vehicles. The findings of this study suggest that the proposed model can be a valuable tool for AV operators. [17]

2.3 Mixed Traffic Equilibrium Modeling

For this study the occupancy of the vehicle is studied and its impact on the network and other travelers is evaluated. Zhou et al. proposes a system of SAVs combined with park-and rides in residential areas to which the deadheading AV was assumed to return to its initial point until the next request was made [18]. Existing research on competition and cooperative traffic assignment was pioneered by Haurie in 1985 [19]. However, in 2017 Chen et al. proposed the use of a mixed equilibrium model, where segments of the road network were dedicated as AVs only [20]. Yang et al. formulated a mixed behavior network equilibrium model as variational inequalities (VI) that simultaneously describe the routing behaviors of user equilibrium (UE), system optimum (SO) and Counter Nash (CN) players [21]. Bagloee et al. proposed a UE-SO mixed equilibrium strategy in which the network assignment was based Connected Vehicles being treated as SO users, and conventional vehicles were modeled as UE users. That research developed a mathematical formulation for the UE-SO mixed traffic assignment methods [22]. Sharon et al. conducted a mixed equilibrium assignment and concluded that optimal flow can be achieved with as low as 13% and as high as 54% of agents in compliance [23]. Wang et al. proposed a bilevel programming model to compute the worst-case equilibrium flow and network performance in a mixed traffic network of human driven vehicles and AVs.[24] Zhang and Nie investigated the issue of a mixed fleet model where human-driven and AV are modeled and since AVs are controllable, they could be dynamically assigned suboptimal routes to prioritize and improve the travel time of human-driven vehicles. [25] They made the assumption that human-driven vehicles will choose the shortest path and behave in a UE manner, but the AVs would be assigned a system optimal (SO) routing that minimizes the total travel time for all users. [26][27]

The proposed framework generates a bi-level model where one group of AVs are assigned using the UE method and the lower-level unoccupied AVs are assigned a different class based on SO. Therefore, instead of treating all AVs as equal, classes are assigned based on occupancy. The priority in the assignment is set as the ability for household trip making and travel time restrictions of unoccupied vehicles are incorporated in the solution method to optimize their usage and benefit to the system.

2.4 AV Policy

The findings of this research aim to inform and guide the development of policies and regulations concerning the routing strategies of unoccupied AVs. The literature in terms of autonomous vehicle policy recommendations is explored in different perspectives. Fraedrich et al. explored the effects of autonomous vehicles (both private and shared) and their compatibility to municipalities' existing objectives. This study concludes that autonomous vehicle development should align with the current public transport [28]. Walker et al. suggested a Dynamic Adaptive Policymaking (DAP) framework for governing the expansion of AV policies given the unpredictability involved. [29] Foldes et al. suggested coordinating among the operation center of AVs, traffic control center and infrastructure operators. The study describes two step planning namely preliminary service planning and operative planning [30].

2.5 Summary

The review of recent literature has indicated that the methods used for traffic assignment in a mixed fleet of occupied and unoccupied AVs need to be updated. Our traditional models will not generate an optimal solution based on the number and frequency of unoccupied AVs trips generated and the potential use and benefits of SAVs. The next section of this report outlines the methods used to conduct the analysis and build this new form of traffic assignment model.

Chapter 3. Methodology

3.1 Introduction

The user equilibrium (UE) assignment procedures are based on Wardrop's principle which makes the assumption that all drivers are uniform in their perception of costs. Therefore, no driver can reduce their cost, or travel time, by unilaterally changing their route. The introduction of AVs allows planners to introduce non-uniform decisions as one of the major strategies of AVs is to effectively replace private car trips and can potentially reduce car ownership. This reduction in car ownership is attributed to a surge in the phenomenon of 'deadheading' where the unoccupied AVs consume capacity and contribute to the congestion of the roads with a different travel objective than an occupied vehicle. To effectively manage this challenge, it is important to develop new routing assignment for unoccupied vehicles. Therefore, new methods need to be developed to account for this shift in assignment. The increasing market share of AVs provides the opportunity to create different classes and assignment assumptions to vehicles based on their occupancy, or lack thereof. The methods described below work to generate a solution to this proposed mixed traffic assignment theory.

3.2 Mixed Equilibrium Assignment

Consider a transportation network $G(N, A)$ with N nodes and A arcs. During the transition to a fully autonomous fleet, two classes of vehicles are using the network: occupied vehicles and unoccupied vehicles. The former is assigned to follow a User Equilibrium (UE) traffic assignment whereas unoccupied vehicles follow a System Optimum (SO) traffic assignment. UE assignment is where each user chooses a route that minimizes their own travel time and SO assignment is a model in which the total system travel time is minimized. The objective of the unoccupied vehicles is restricted by a return window ensuring AV owner's household travel needs are met. The time window is defined as the vehicle delay threshold (θ). The model in the research was adopted from Zhang and Nie and was modified to accommodate delay in the unoccupied vehicles [25]. Should delay in unoccupied vehicles exceed a certain threshold, these vehicles are then treated as occupied vehicles. This accommodation was performed using a heuristic approach. Three different strategies were proposed in this study.

Strategy 1: Mixed Equilibrium Assignment with no vehicle delay threshold for the unoccupied vehicles.

Strategy 2: Mixed Equilibrium Assignment with no vehicle delay more than 95th percentile of all vehicles.

Strategy 3: Mixed Equilibrium Assignment with no vehicle delay of more than 5 minutes.

Table of Notation

Symbol	Description
N	Nodes
A	Arcs
x_a	Flow on arc a
x_o	flow on arc of occupied vehicles
x_d	flow on arc of unoccupied vehicles

t_o	Arc travel time of occupied vehicles
t_d	Arc travel time of unoccupied vehicles
$k^{r,s} \in K^{r,s}$	Path set k from r to s
$r \in R$	Origins
$s \in S$	Destinations
(r', s')	Origin-destination pair that have delay
$f_k^{r,s}$	Flow on path k between origin r and destination s
$k^{*r,s}$	Shortest travel time path from r to s [31]
$q^{r,s}$	Demand between origin r and destination s
e	Percentage demand of unoccupied vehicles
$q_u^{r,s}$	Unoccupied vehicle demand between origin r and destination s $= e * q^{r,s}$
$q_o^{r,s}$	Occupied vehicle demand between origin r and destination s $= (1 - e) * q^{r,s}$
$p^{r,s}$	Optimal number of paths in the system optimum assignment in the increasing order of travel time = $\{k_1, k_2, k_3, k_4 \dots k_p\}$
δ	binary variable indicating if arc a is on path k between origin r and destination
$\delta_{a,k}^{r,s}$	$\begin{cases} 1 & \text{if arc } a \text{ is on path } k \text{ between } r \text{ and } s \\ 0 & \text{if arc } a \text{ is not on path } k \text{ between } r \text{ and } s \end{cases}$
t_0	Free flow travel time on the arc a
C_a	Capacity of arc a
t_a	Travel time function on arc a $t_a = t_0 + \left[1 + 0.15 \left(\frac{x_a}{C_a} \right)^4 \right]$
x_{ak}^*	Flow on arc a on the shortest path k^*
t_{ak}^*	Travel time on arc a on the shortest path k^*
$TSTT$	Total System Travel Time = $\sum_a x_a t_a$
$SPTT$	Shortest Path Travel Time = $\sum_a x_a^* t_a^*$ on the shortest path k^*
γ	Relative Gap = $\gamma = \frac{TSTT}{SPTT} - 1$
λ	Accuracy
z_1	Lower-level objective function of occupied vehicles
z_2	Upper-level objective function of unoccupied vehicles
$T_k^{UE(r,s)}$	Travel time on path k between origin r and destination s in user equilibrium traffic assignment
$T_k^d(r,s)$	Travel time on path k between origin r and destination s of unoccupied vehicles
$\Delta^{r,s}$	Delay of the unoccupied vehicles compared to user equilibrium traffic assignment. $= T_k^d(r,s) - T_k^{UE(r,s)}$
$\mu^{r,s}$	Delay of the vehicle compared to user equilibrium traffic assignment for each OD pair
θ	Delay threshold for the unoccupied vehicle
$\Delta_e^{r,s}$	95 th percentile of the $\Delta^{r,s}$, delay of the unoccupied vehicles with e as % of the demand of unoccupied vehicles

h unoccupied vehicle delay threshold in strategy 3

The model adopted follows the traditional UE and SO formulations that can be found in Sheffi (1985), with modifications similar to those found in Zhang and Nie (2018) [25, 32]. The upper-level objective function is:

$$(1) \quad \min_{x_0} z_1(x) = \sum_a \int_0^{x_a} [t_a(x_0 + x_d)] dx$$

The lower-level objective function is:

$$\min_{x_d} z_2(x) = \sum_a [t_a(x_0 + x_d)(x_0 + x_d)] \quad (2)$$

These are constrained by the standard path flow constraints requiring the flows across all paths $k \in K$ between origin $r \in R$ and destination $s \in S$ satisfy demand between origin and destination. It is assumed that the demand between an OD pair remains constant and the demand is uniformly distributed between occupied and unoccupied vehicles according to the value of e in the ten scenarios.

$$\sum_k f_k^{r,s} = q_k^{r,s} \quad \forall k, r, s \quad (3)$$

$$\sum_k f_k^{r,s} = q_o^{r,s} + q_d^{r,s} \quad \forall k, r, s \quad (4)$$

$$x_a = \sum_r \sum_s \sum_k f_k^{r,s} \delta_{a,k}^{r,s} \quad (5)$$

The mapping (5) produces arc flows f_k using the path flows and the binary indicator δ , which takes the value 1 if link is a on path k between origin r and destination s . This indicator can also be used to compute path travel time on an arc.

$$\sum_r f_k^{(r',s')} = q^{r',s'} \quad \forall (r',s'): T_k^d(r',s') - T_k^{UE}(r',s') > \theta \quad \text{where } r' \subseteq r, s' \quad (6)$$

Constraint (6) will be used to ensure unoccupied vehicle travel times are falling within the necessary time windows for the household owners of the AVs. Should delay in unoccupied vehicles exceed a certain threshold (θ), these vehicles are then treated as occupied vehicles. This accommodation is performed using a heuristic approach to stratify vehicles assignment for the lower level problem. The methodology is implemented using three different strategies and the flow of the strategy is given in **Figure 2**.

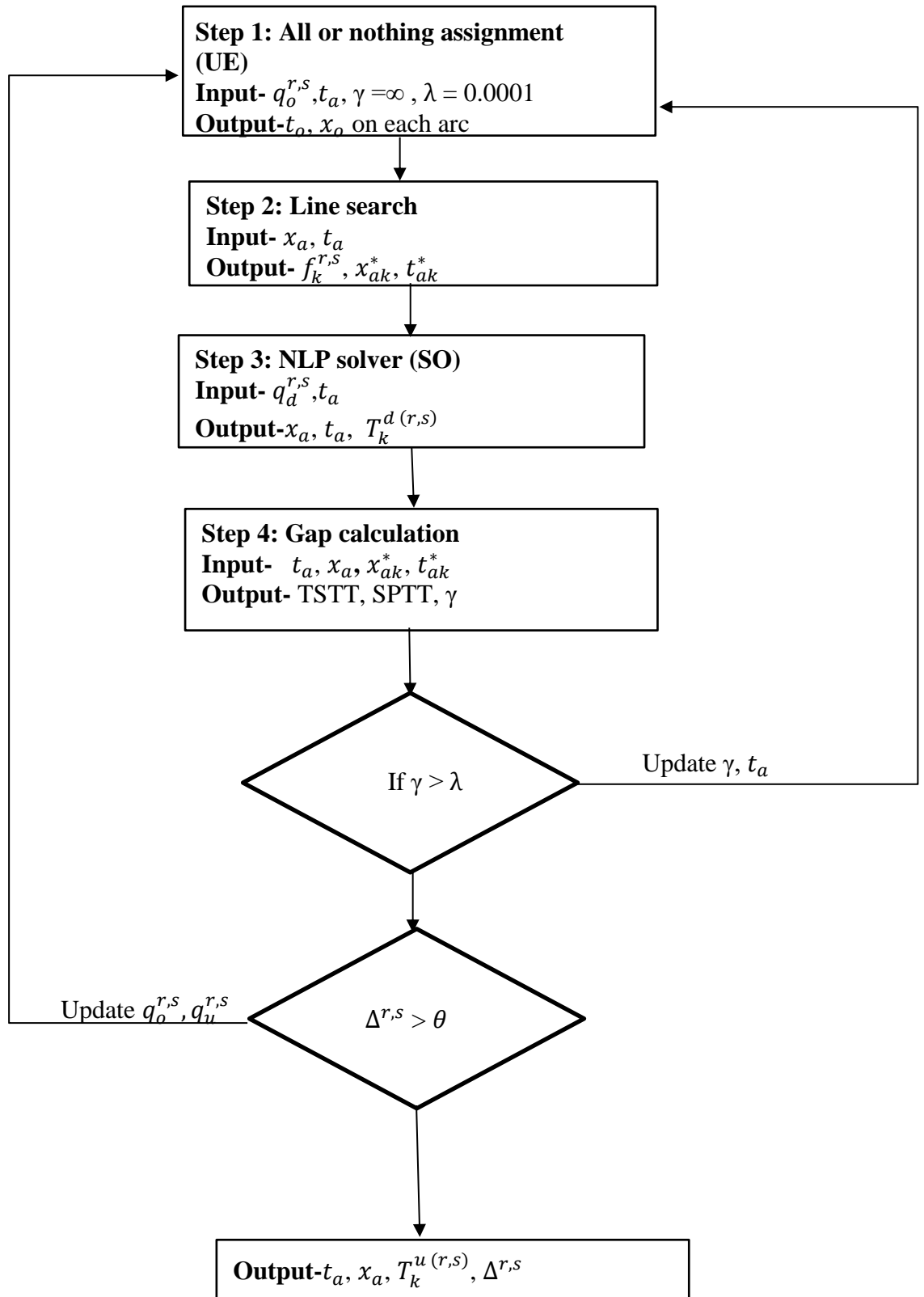


Figure 2: Flow of the Strategy

3.2.1 Strategy Algorithm

Initialization: $q_o^{r,s} = e * q^{r,s}$, $q_u^{r,s} = (1 - e) * q^{r,s}$, $\gamma = \infty$ and $\lambda = 0.0001$, $it = 1$, $\bar{x} = 0$, $x_a = 0$;
 $\Delta^{r,s} = \infty$, $T_k^{UE(r,s)} = \sum_a t_a \delta_{a,k}^{r,s}$ from UE assignment, $\Delta_e^{r,s} = \infty$,

$\theta = \infty$,

$\theta = \Delta_e^{r,s}$ for Strategy 2

$\theta = 5$ for Strategy 3

Begin:

for each $(r, s): r \in R$ and $s \in S$

While: $\gamma > \lambda$ do

for each $(r, s): r \in R$ and $s \in S$

$f_{k^*}^{r,s} = q_o^{r,s}$

for each $a \in A$

$\bar{x} = \sum_r \sum_s f_{k^*}^{r,s} \delta_{a,k^*}^{r,s}$

$x_o = \alpha * \bar{x} + (1 - \alpha) * x_o$

if $i = 1$

$\alpha = 1$

else:

$z_1(x(\alpha)) = z_1((1 - \alpha)x + \alpha\bar{x})$, where $\alpha \in (0,1)$

end

$t_a = t_o + \left[1 + 0.15 \left(\frac{x_o}{c_a}\right)^4\right]$

end

end

$\min_{x_u} z_2(x) = \sum_a [t_a (x_o + x_u)(x_o + x_u)]$

for each $(r, s): r \in R$ and $s \in S$

$\sum_{k \in p^{r,s}} f_k^{r,s} = q_d^{r,s}$

for each $a \in A$

$x_d = \sum_r \sum_s \sum_k f_k^{r,s} \delta_{a,k}^{r,s}$

$t_a = t_o + \left[1 + 0.15 \left(\frac{x_u + x_o}{c_a}\right)^4\right]$

$x_a = x_d + x_o$

end

end

TSTT = $\sum_a x_a t_a$

SPTT = $\sum_a x_{ak}^* t_{ak}^*$

$i = i + 1$

end

$T_k^d(r,s) = \sum_a t_a \delta_{a,k}^{r,s}$

$\Delta^{r,s} = T_k^d(r,s) - T_k^{UE(r,s)}$

If $\Delta^{r,s} < \theta$,

end

else, $q_o^{r,s} = q^{r,s}$ and $q_d^{r,s} = 0$

Continue

3.2.2 Sioux Falls Network

Sioux Falls network has 24 nodes, 76 links and 528 Origin Destination pairs.[33]

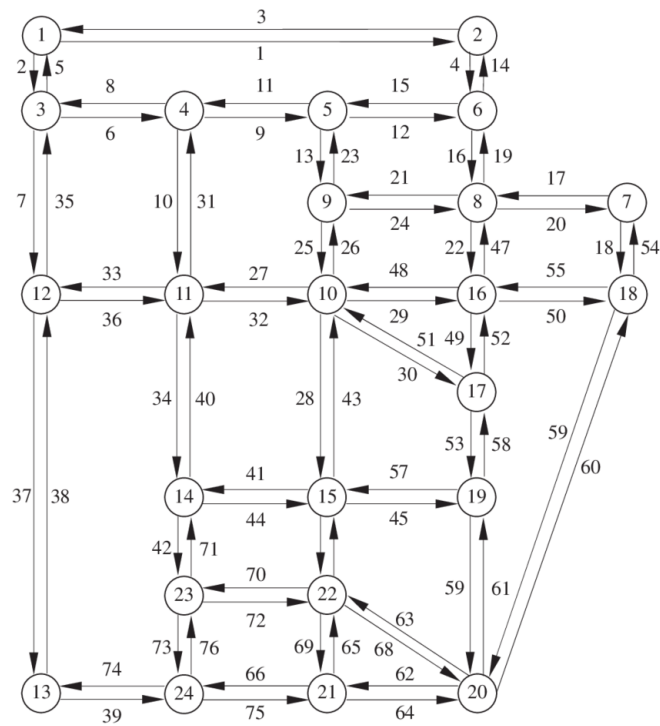


Figure 3: Sioux Falls Network

3.2.3 Eastern Massachusetts Network

Eastern Massachusetts has 74 nodes, 258 links and 5402 OD pairs [33]

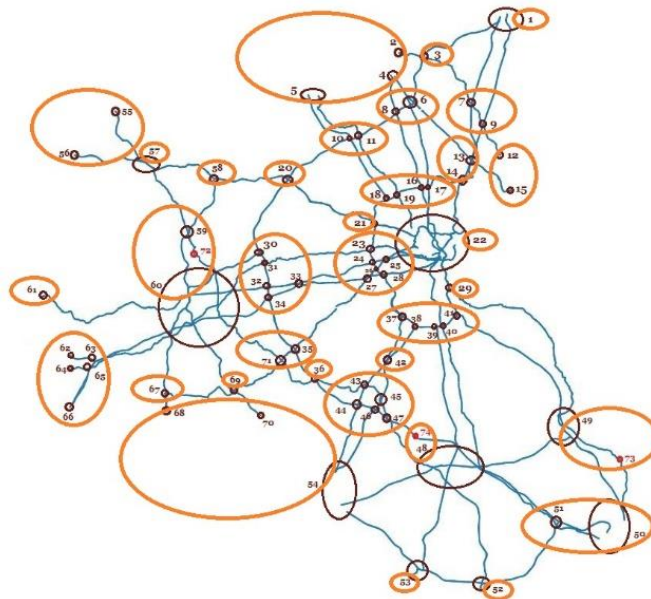


Figure 4: Eastern Massachusetts Network

3.2.4. Chicago Sketch Network:

Data: Two datasets are used for Chicago network, including

1) Northeast Illinois Region travel survey data [34] provided by Chicago Metropolitan Agency for Planning (CMAP). This travel survey data is a comprehensive travel and activity survey occurred between 2018 and April 2019. A total of 12,391 households participated in the survey. The survey was conducted over 5 days. According to survey data, each household owns 1.34 cars on average. The origin and destination of the trips have already been geocoded with longitudes and latitudes. This data is used to determine the purpose of the trip of the methodology.

2) Transportation test networks [33] – Chicago sketch network is used to apply mixed equilibrium assignment. It has 933 nodes, 2950 links and 142,512 OD pairs, the OD pairs that have at least one unit of demand are considered for this current study.

3.3 Summary

The developed algorithms are scripted in python and then applying the corresponding code to three sample transportation networks was successful. The models created were able to generate travel times for links and calculate the predicted travel time and then corresponding delay experienced by each of the groups of vehicles. The next chapter outlines the results of each scenario, network, and system optimization.

Chapter 4. Results and Analysis

4.1 Introduction

Using the developed python code for the mixed equilibrium methodology, the three scenarios were run on each of the two different transportation networks. The results generated are described below. For the Chicago network, household vehicle reduction potential results and the mixed equilibrium model are presented below. The next chapter will discuss the conclusions that were drawn from this analysis conducted in Chapter 4.

4.2 Results and Analysis

4.2.1 Sioux Falls – Strategy 1

Occupied vehicles are assigned according to the principles of UE assignment and deadheading vehicles are assigned using SO where $\theta = \infty$ which means there is no precise threshold. Occupied vehicles saw a reduction in travel time due to the rerouting of the unoccupied vehicles in most cases. Even for vehicles that experienced a delay, the maximum delay is 3 minutes. The 95th percentile of the delay is 2 minutes. Some deadheading vehicles did experience a delay; however, as long as the deadheading vehicle reached the destination in the time window, the value of time of the unoccupied vehicle was zero. The maximum delay in the case of 10% of unoccupied vehicles and 90% of occupied vehicles was 10.23 minutes. **(Figure 5 and Appendix- TABLE 2)**

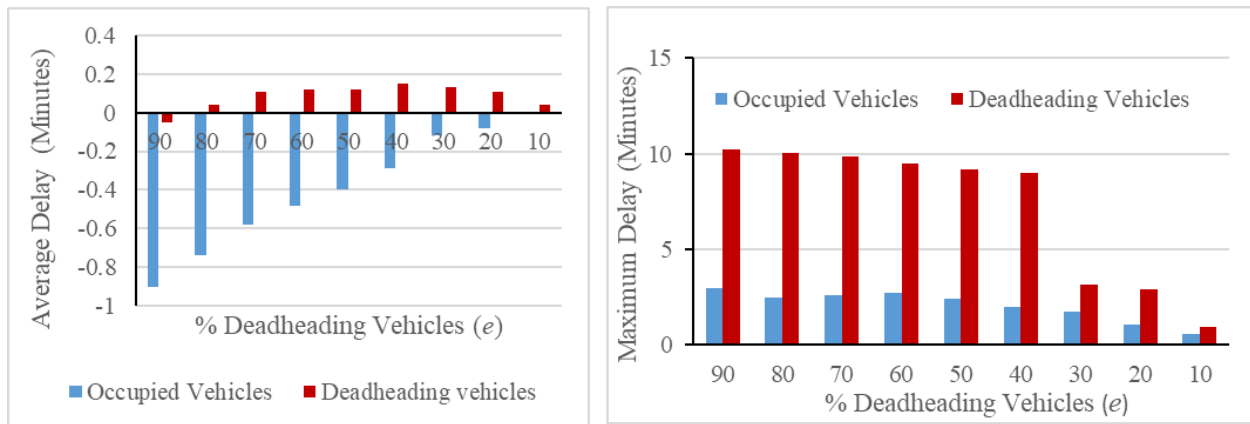


Figure 5: Average and Maximum Delay ($\mu^{r,s}$) of the OVs and UVs – Strategy 1

4.2.2 Sioux Falls – Strategy 2

Deadheading vehicles that have a delay of more than $\Delta_e^{r,s}$ are removed from the deadheading assignment and rerouted according to UE with occupied vehicles. The average delay of occupied vehicles is negative which indicates that most of the occupied vehicles have a faster travel time. The maximum delay of occupied vehicles is 2.8 minutes. The maximum delay for unoccupied vehicles is 2.1 minutes in this case. **(Figure 6 and Appendix- TABLE 3)**

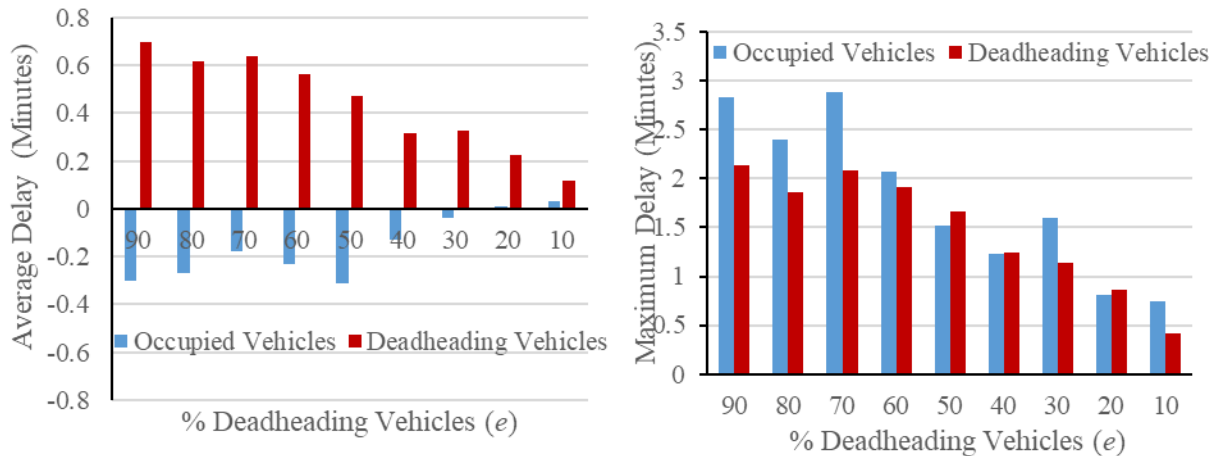


Figure 6: Average and Maximum Delay ($\mu^{r,s}$) of the OV and UVs – Strategy 2

4.2.3 Sioux Falls – Strategy 3

Restricting the deadheading vehicles’ delay threshold to 5 minutes, the average delay of occupied vehicles is negative which indicates that most of the occupied vehicles have a faster travel time. the maximum delay of occupied vehicles is 3.3 minutes. The maximum delay for unoccupied vehicles is also 3.2 minutes in this case. (Figure 7 and Appendix- TABLE 4)

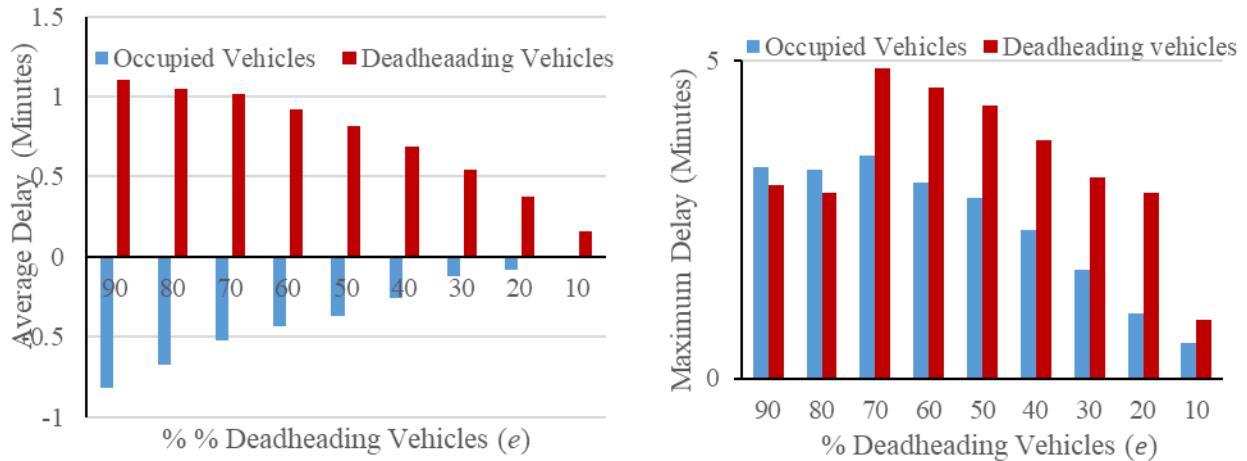


Figure 7: Average and Maximum Delay ($\mu^{r,s}$) of the OV and UVs – Strategy 3

Sioux Falls - Total System Travel Time Results:

In all three cases, the TSTT improved compared to the UE traffic assignment. Note that $e = 0.0$ represents a UE Assignment for all vehicles, as they all have occupants or drivers. As expected, Figure 8 displays that by routing deadheading vehicles as SO, there is a reduction in TSTT until the process converges on the SO solution of $e = 1.0$ (Figure 8)

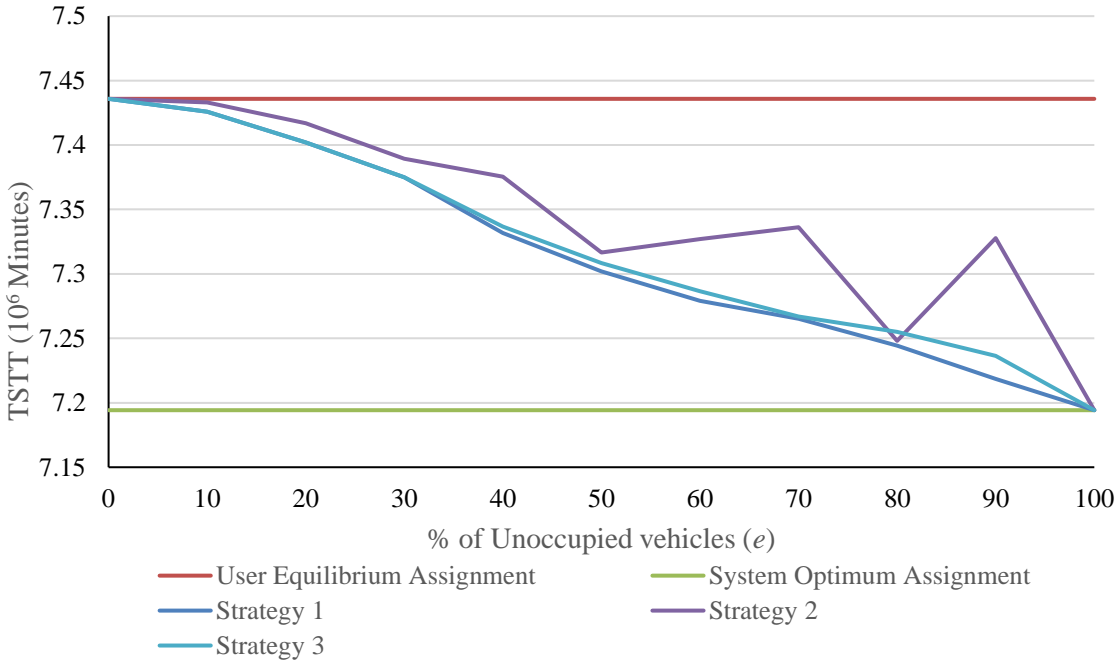


Figure 8: Total System Travel Time Plots – Sioux Falls Network

4.2.4 Eastern Massachusetts – Strategy 1

Occupied vehicles are assigned according to the principles of UE and deadheading vehicles are assigned using SO. Most of the occupied vehicles saw a reduction in travel time due to the rerouting of deadheading vehicles. Some unoccupied vehicles experienced delayed however, as long as the vehicle reached the destination in the time window, the value of time of the empty vehicle is considered to be zero. The maximum delay, in the case of 80:20 ratio of occupied to deadheading vehicles, was 12 minutes. (Figure 9 and Appendix- TABLE 5)

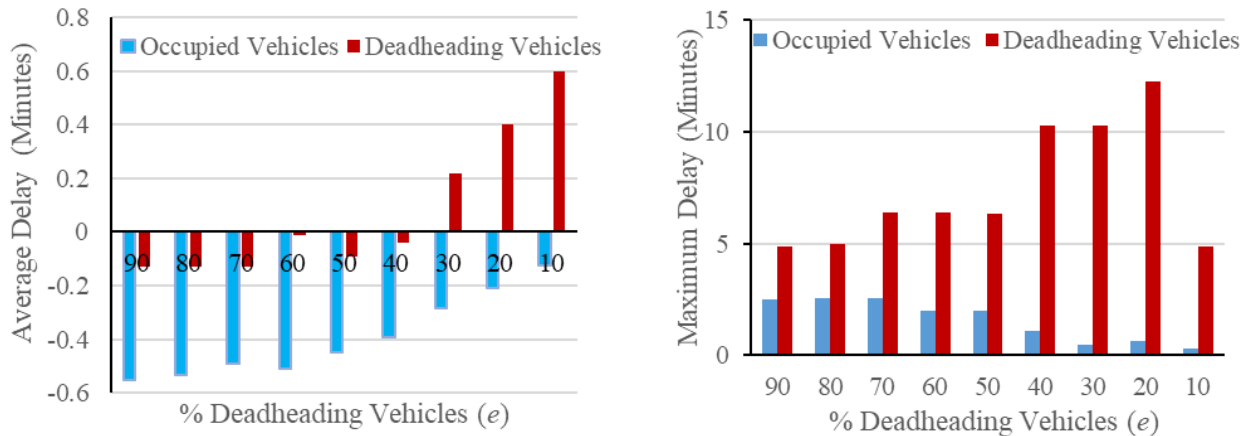


Figure 9: Average and Maximum Delay ($\mu^{r,s}$) of the OVs and UVs – Strategy 1

4.2.5 Eastern Massachusetts – Strategy 2

Deadheading vehicles that have a delay of more than $\Delta_e^{r,s}$ are removed from the deadheading assignment and rerouted according to UE with occupied vehicles. The average delay of occupied vehicles is negative which indicates that most of the occupied vehicles have a reduction in travel time. The maximum delay of occupied vehicles is 2.3 minutes. The maximum delay for unoccupied vehicles is 2.4 minutes in this case. (Figure 10 and Appendix- TABLE 6)

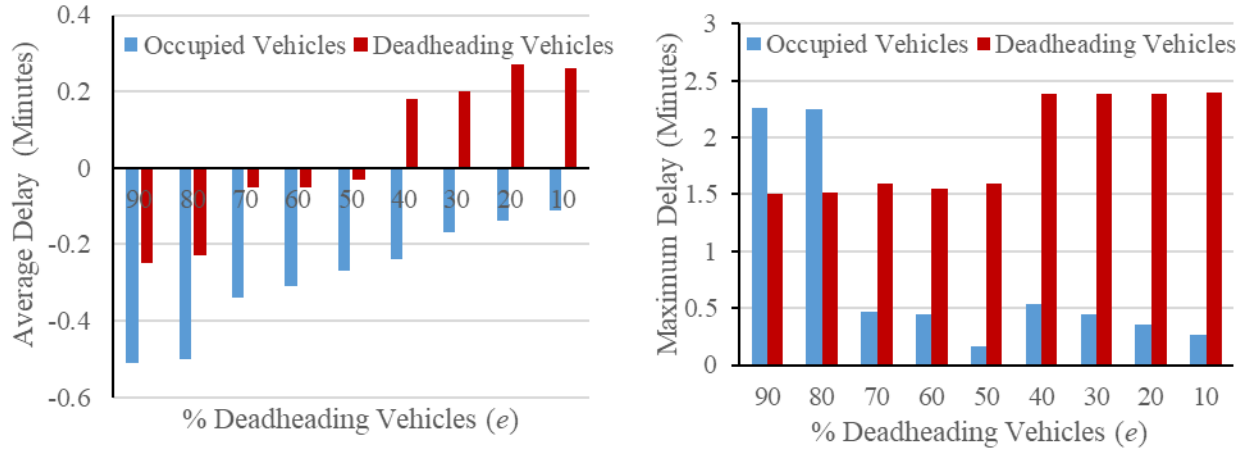


Figure 10: Average and Maximum Delay ($\mu^{r,s}$) of the OV and – Strategy 2

4.2.6 Eastern Massachusetts – Strategy 3

Restricting the deadheading vehicles’ delay threshold to 5 minutes, minimal impact was observed on occupied vehicles, as their maximum delay was still 2.6 minutes. The average delay of occupied vehicles is negative which indicates that most of the occupied vehicles have a reduction in travel time. (Figure 11 and Appendix- TABLE 7)

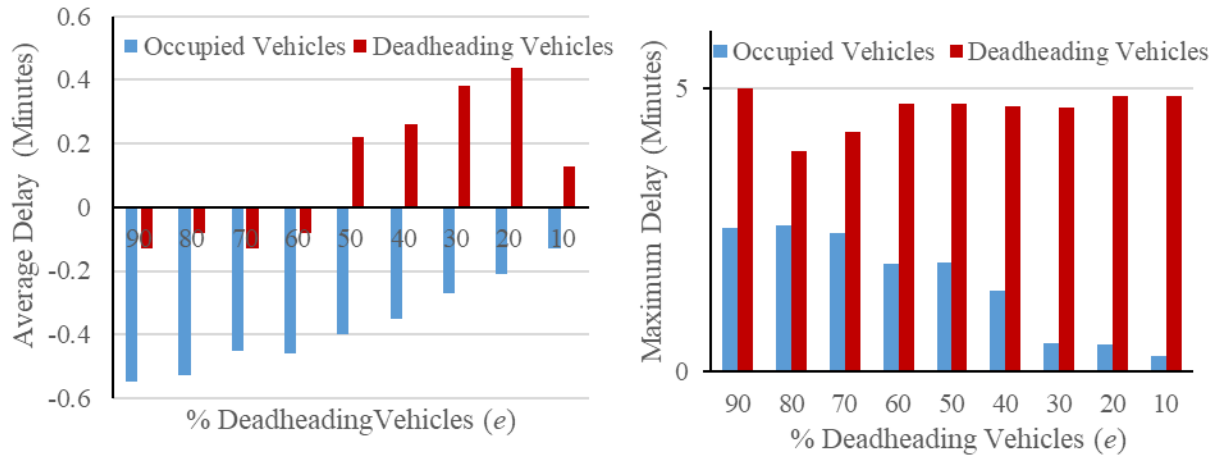


Figure 11: Average and Maximum Delay ($\mu^{r,s}$) of the OV and – Strategy 3

Eastern Massachusetts Network- TSTT Results:

In all three cases, the TSTT improved compared to the UE traffic assignment. Note that $e = 0.0$ represents a UE Assignment for all vehicles, as they all have occupants or drivers. As expected, Figure 8 displays that by routing deadheading vehicles as SO, there is a reduction in TSTT until the process converges on the SO solution of $e = 1.0$ (Figure 12)

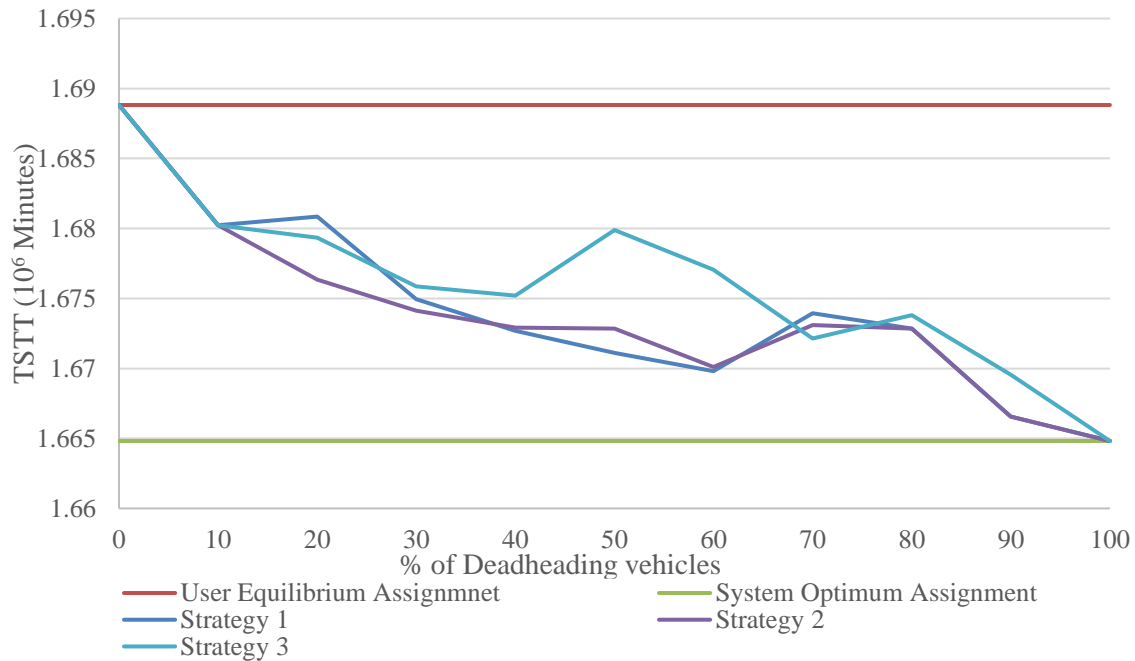


Figure 12: Total System Travel Time Plots – Eastern Massachusetts

Most occupied vehicles faced shorter travel times compared to the regular user equilibrium traffic assignment. None of the AV owners faced any significant delays when θ is enforced. Unoccupied vehicles can be restricted to no more than 5 minutes' delay. There is a gain in the total travel time of the system. Another important aspect of this study is confirmation that AV owners would be able to accommodate multiple trips and thus potentially own fewer vehicles. The computational time of the algorithm is higher for Eastern Massachusetts than Sioux Falls network as the complexity of the network is larger.

Table 1: Comparison of results for Sioux Falls and Eastern Massachusetts

		Sioux Falls	Eastern Massachusetts
Strategy 1	TSTT (10^6 Minutes)	7.19 - 7.43	1.66 - 1.68
	Average $\Delta^{r,s}$ (Minutes)	-0.1 - 0.15	0.01 - 0.6
	Computational Time (hours)	0.75 - 0.95	6.5 - 8.25
Strategy 2	Total System Travel time (10^6 Minutes)	7.19 - 7.43	1.66 - 1.68
	Average $\Delta^{r,s}$ (Minutes)	0.11 - 0.63	-0.25 - 0.27
	Computational Time (hours)	0.75 - 1.25	12.5 - 19.25
Strategy 3	Total System Travel time (10^6 Minutes)	7.19 - 7.43	1.66 - 1.68
	Average $\Delta^{r,s}$ (Minutes)	0.16 - 1.11	0.08 - 0.44
	Computational Time (hours)	5.0 - 7.5	6.5 - 12.5

4.2.7 Chicago Sketch – Strategy 1

Occupied vehicles are assigned according to the principles of UE assignment and deadheading vehicles are assigned using SO where $\theta = \infty$ which means there is no precise threshold. Most of the occupied vehicles did not experience delays due to the rerouting of the unoccupied vehicles (**Figure 13**). Even for vehicles that experienced a delay, the maximum delay is 1.5 minutes. Some deadheading vehicles did experience a delay; however, as long as the deadheading vehicle reached the destination in the time window, the value of time of the unoccupied vehicle was zero. The maximum delay in the case of 10% of unoccupied vehicles and 90% of occupied vehicles was 7.38 minutes (**Figure 14**). The heat map representing the delays in occupied and deadheading vehicles is shown below. (**Figure 15**). Most of the delays are negative, which indicates that a majority of OD pairs experienced a shorter travel time. The Total System Travel Time (TSTT) of the network is less than the user equilibrium assignment $e = 100$ as expected. (**Figure 16**).

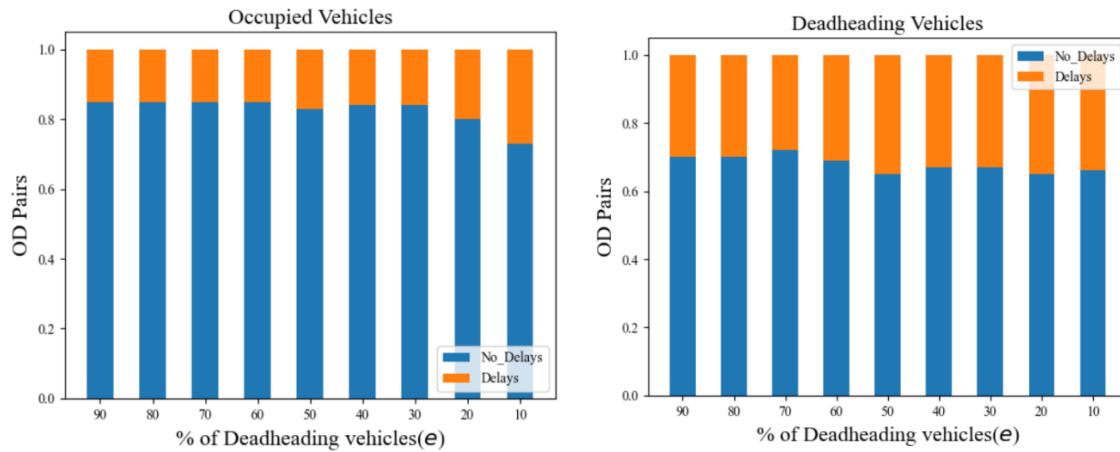


Figure 13: Number of OD pairs with delays and no-delays

Figure 3 Number of OD pairs with delays and no-delays

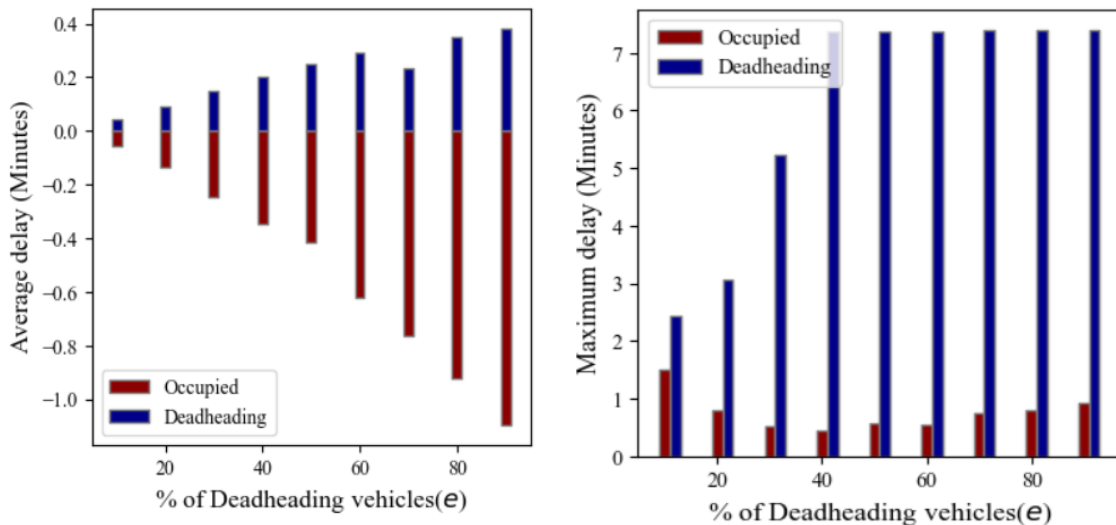


Figure 14: Average and Maximum Delay ($\Delta^{r,s}$) of the OVs and UVs

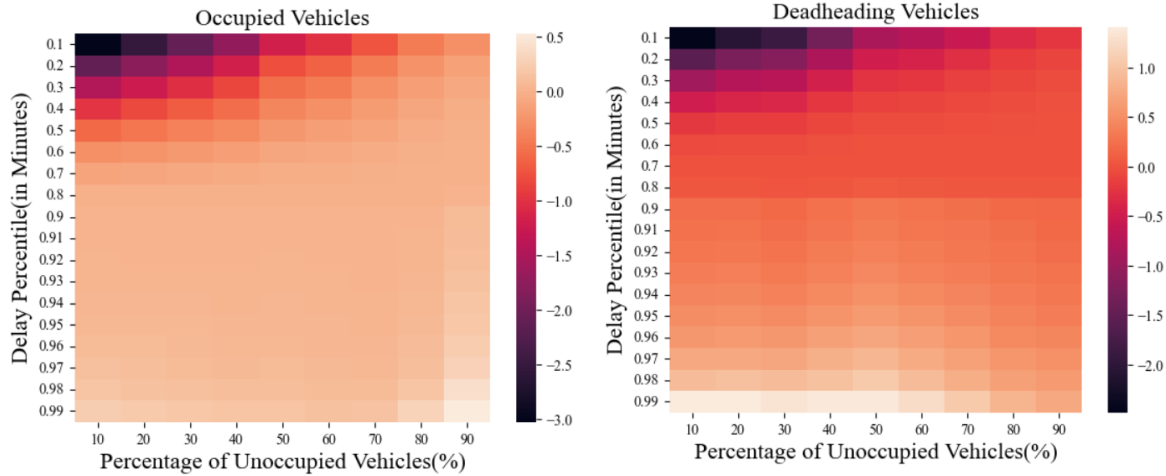


Figure 15: Representation of delays in OVs and UVs

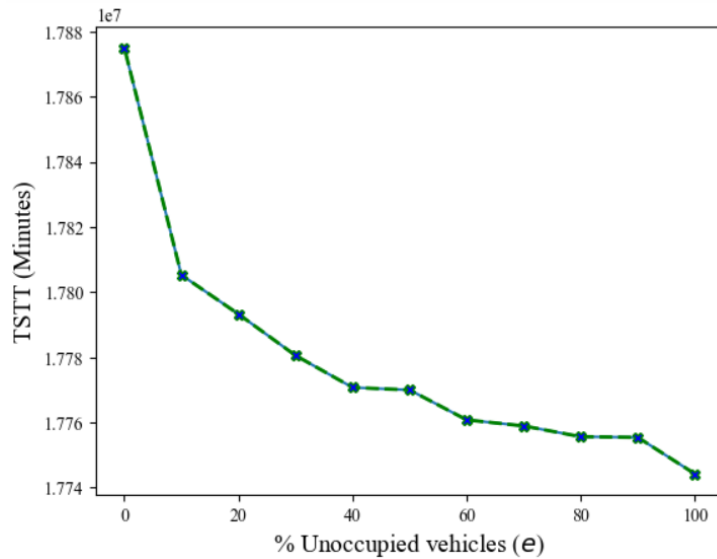


Figure 16: Total System Travel Time – Chicago Network

4.2.8 Chicago Sketch – Strategy 2

Unoccupied vehicles that face considerable delays due to the SO routing will be rerouted to avoid unnecessary delays. After initial mixed equilibrium assignment, the delay of the unoccupied vehicles compared to the user equilibrium assignment is examined. Travel delays are restricted by (θ) which varies based on the purpose of the trip. This indicates that the unoccupied vehicle can take a longer route as long as they are not delayed by more than the threshold (θ) compared to the route that would be chosen in a user equilibrium traffic assignment. The threshold for the four different trip purposes is derived from the distribution of the delay of the deadheading vehicles and is application specific. **Figure 17** shows the distribution of delays and θ for $e = 10$. Other networks may require a different threshold value. Deadheading vehicles that have a delay of more than θ are removed from the deadheading assignment and rerouted according to UE with occupied vehicles. The results show that after rerouting deadheading vehicles and adding a delay threshold, the TSTT for $e = 90$ is 1.779×10^7 and TSTT for $e = 10$ is 1.774×10^7 minutes. The above results

showed the TSTT for $e = 20$ to 80 are greater than of $e = 10$ and less than $e = 90$ for Sioux Falls and Eastern Massachusetts network. Future research is to apply the rerouting strategy for $e = 20$ to 80 .

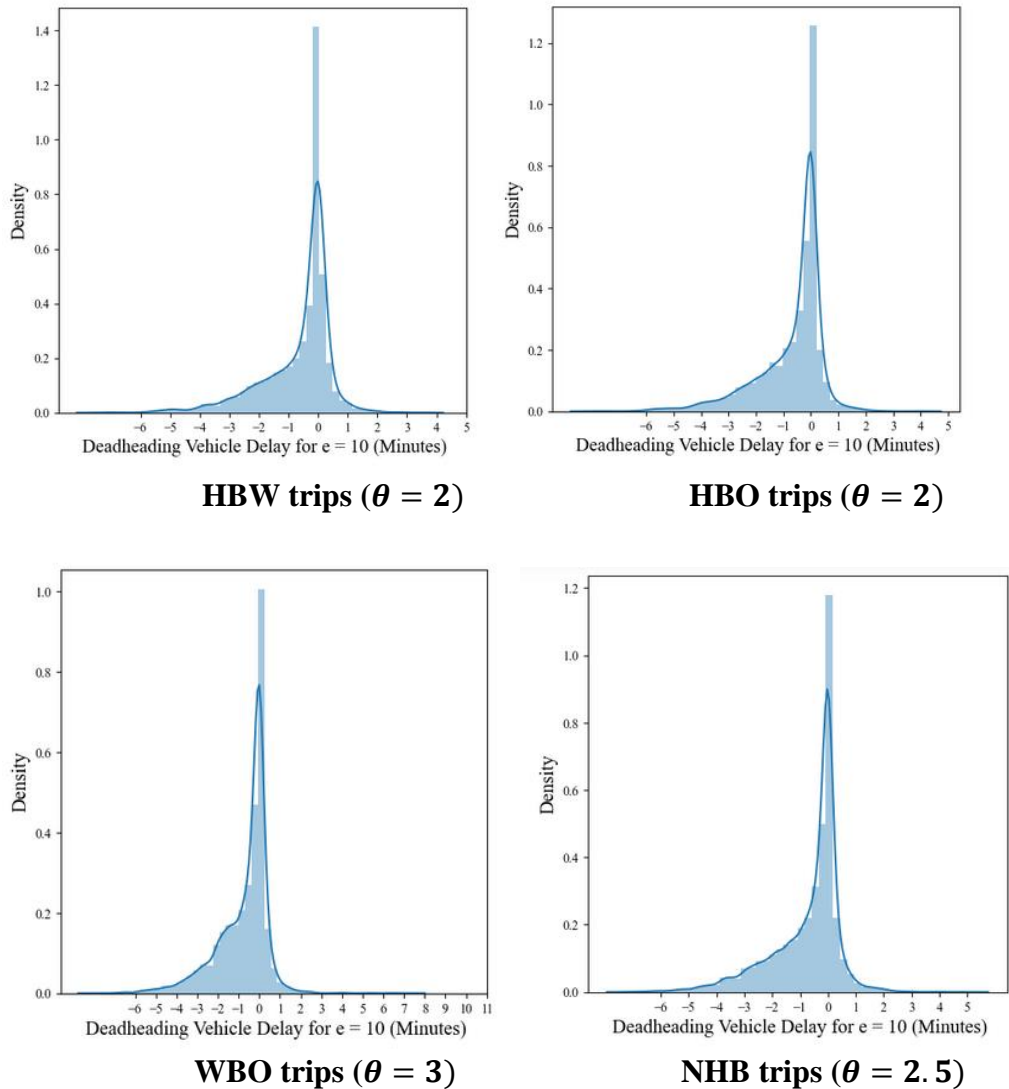


Figure 17: Deadheading vehicle delay by trip purpose

4.3 Summary

The results from the algorithms developed and deployed on the two sample networks indicated that there is an overall improvement in travel times when compared to the user equilibrium model. This method of using a mixed model approach appears to be successful, and promising when developing new models to account for AVs and SAVs scenarios. The final chapter discusses conclusions and outcomes from the research.

Chapter 5. Summary and Conclusions

5.1 Introduction

The methods and analysis conducted as part of this research effort are critical to understanding how the future for planning and modeling the impacts of AVs on our system. Furthermore, this research is important when quantifying and achieving the full benefits of AV adoption. The findings of this research aim to inform and guide the development of policies and regulations concerning the routing strategies of unoccupied AVs.

5.2 Discussion and Conclusion

The objectives of this research effort were to minimize the travel delays experienced by occupied vehicles by minimizing the impact of unoccupied AVs route choice. This was conducted through the implementation of a mixed equilibrium methodology, and then validated by conducting an analysis of this new methodology using sample networks (Sioux Falls, Eastern Massachusetts and Chicago Sketch) as case studies. The improvements of a mixed equilibrium model changed the regular selfish routing of occupied vehicles to allow for unoccupied AVs follow the system optimum traffic assignment. Thereby rerouting unoccupied vehicles to optimize based on a variety of parameters. Another important aspect of this study was to confirm that future AV owner will not face inconveniences due to this dynamic rerouting of unoccupied vehicles.

By adapting the assignment methodology to account for the behavior and ability to assign unoccupied vehicles to preferred routes, the major findings are:

- a) Occupied vehicles will most often see a reduction in travel times when compared to regular user equilibrium traffic assignment.
- b) AV owners who want to use their vehicles as SAVs will not face any significant delays and the unoccupied vehicles delays can be restricted to a threshold without significant negative impacts on occupied vehicles.
- c) The benefits of AVs can be realized since the total system travel time is predicted to be less when compared to regular traffic assignment output from in any of the scenarios tested.

5.3 Directions for Future Research

For future research, the algorithms developed can be applied to larger more complex networks. Furthermore, the parameters for each of the scenarios can be updated and refined to generate a more real time pickup tolerance time windows based on travel survey studies. The expansion of this research can be used to enhance the algorithms developed and allow for more accurate benefits. Future enhancements would be beneficial to understand the real-time benefits that dynamic assignment of vehicles might have on travel times and network efficiency. Future research should include a sensitivity analysis on the parameter of delay threshold. The vehicle delay threshold can be improved using exact methods such as Bender's decomposition.

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APPENDIX

Table 2: Effect on occupied vehicles and unoccupied vehicles – Strategy 1

e (%)	% of OD pairs with faster travel times		Vehicle delay $\mu^{r,s}$ (minutes)			
	Occupied Vehicles (OV)	Unoccupied Vehicles (UV)	Average		Maximum	
			OV	UV	OV	UV
90	58	42	-0.90	-0.05	2.94	10.23
80	59	42	-0.74	0.04	2.49	10.06
70	58	40	-0.58	0.11	2.57	9.83
60	58	41	-0.48	0.12	2.71	9.48
50	57	41	-0.40	0.12	2.39	9.17
40	56	40	-0.29	0.15	1.96	8.98
30	56	40	-0.12	0.13	1.71	3.12
20	49	35	-0.08	0.11	1.03	2.91
10	49	40	-0.01	0.04	0.56	0.90

Table 3: Effect on occupied vehicles and unoccupied vehicles – Strategy 2

e (%)	% of OD pairs with faster travel times		Vehicle delay $\mu^{r,s}$ (minutes)			
	OV	UV	Average		Maximum	
			OV	UV	OV	UV
90	45	71	-0.30	0.70	2.84	2.14
80	59	44	-0.27	0.62	2.40	1.86
70	46	61	-0.18	0.64	2.89	2.09
60	57	44	-0.23	0.57	2.07	1.92
50	50	58	-0.31	0.47	1.52	1.66
40	47	59	-0.13	0.32	1.23	1.24
30	47	59	-0.04	0.33	1.60	1.14
20	40	42	0.01	0.23	0.81	0.86
10	37	47	0.03	0.12	0.75	0.42

Table 4: Effect on occupied vehicles and unoccupied vehicles – Strategy 3

e (%)	% of OD pairs with faster travel times		Vehicle delay $\mu^{r,s}$ (minutes)			
	Occupied Vehicles (OV)	Unoccupied Vehicles (UV)	Average		Maximum	
			OV	UV	OV	UV
90	59	47	-0.82	1.108	3.32	3.05
80	59	47	-0.67	1.049	3.28	2.92
70	57	44	-0.52	1.013	3.5	4.87
60	56	42	-0.43	0.923	3.09	4.58
50	56	43	-0.37	0.813	2.83	4.29
40	55	43	-0.26	0.687	2.34	3.74
30	56	40	-0.12	0.543	1.71	3.16
20	49	35	-0.08	0.376	1.03	2.91
10	49	40	-0.01	0.162	0.56	0.92

Table 5: Effect on occupied vehicles and unoccupied vehicles – Strategy 1

e (%)	% of OD pairs with faster travel		Vehicle delay $\mu^{r,s}$ (minutes)			
	OV	UV	Average delay		Maximum	
			OV	UV	OV	UV
90	73	60	-0.55	-0.13	2.52	4.86
80	74	60	-0.53	-0.13	2.58	4.99
70	73	56	-0.49	-0.13	2.58	6.38
60	68	56	-0.51	-0.01	1.98	6.37
50	64	50	-0.45	-0.09	1.98	6.31
40	61	46	-0.39	-0.04	1.08	10.28
30	60	40	-0.28	0.22	0.48	10.30
20	57	38	-0.21	0.40	0.66	12.26
10	61	36	-0.12	0.60	0.30	4.85

Table 6: Effect on occupied vehicles and unoccupied vehicles – Strategy 2

e (%)	% of OD pairs with faster travel times		Vehicle delay $\mu^{r,s}$ (minutes)			
	OV	UV	Average		Maximum	
			OV	UV	OV	UV
90	67	59	-0.51	-0.25	2.26	1.51
80	65	61	-0.50	-0.22	2.25	1.52
70	67	54	-0.34	-0.05	0.47	1.50
60	67	57	-0.31	-0.05	0.44	1.57
50	64	52	-0.27	-0.02	0.16	1.60
40	58	43	-0.24	0.18	0.53	2.30
30	58	39	-0.17	0.20	0.45	2.30
20	58	37	-0.14	0.27	0.36	2.30
10	60	37	-0.11	0.26	0.26	2.30

Table 7: Effect on occupied vehicles and unoccupied vehicles – Strategy 3

e (%)	% of OD pairs with faster travel times		Vehicle delay $\mu^{r,s}$ (minutes)			
	OV	UV	Average		Maximum	
			OV	UV	OV	UV
90	73	60	-0.55	-0.13	2.52	4.86
80	74	60	-0.53	-0.13	2.58	4.99
70	71	56	-0.45	-0.13	2.45	4.23
60	66	53	-0.46	-0.08	1.91	4.73
50	62	49	-0.40	0.22	1.92	4.71
40	58	46	-0.36	0.26	1.44	4.68
30	57	41	-0.27	0.38	0.50	4.66
20	57	40	-0.21	0.44	0.49	4.85
10	61	36	-0.12	0.60	0.30	4.85