Multiday User Equilibrium with Intelligent Travelers

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Background

- User Equilibrium (UE) typically is the equilibrium of one-shot games / each traveler makes one-shot decisions

- In this research, we propose a new framework -- Multiday User Equilibrium (MUE), where farsighted travelers make sequential decisions on their travel choices for multiple days

- Sequential decision-making: plan out a sequence of actions for a horizon considering long-term effects

Why need to be farsighted

In some scenarios, one can benefit from sequential decision-making

- Travel within a fixed budget[1]
- Parking search[2]
- Ride-hailing vehicles routing[3,4]

[1] Lin, X., Yin, Y., & He, F. (2021). Credit-based mobility management considering travelers' budgeting behaviors under uncertainty. Transportation Science, 55(2), 297–314.
[2] Boyles, S. D., Tang, S., & Unnikrishnan, A. (2015). Parking search equilibrium on a network. Transportation Research Part B: Methodological, 81, 390-409.
[3] Urata, J., Xu, Z., Ke, J., Yin, Y., Wu, G., Yang, H., & Ye, J. (2021). Learning ride-sourcing drivers' customer-searching behavior: A dynamic discrete choice approach. Transportation research part C: emerging technologies, 130, 103293.
[4] Zhang, K., Mittal, A., Djavadian, S., Twumasi-Boakye, R., & Nie, M. (2021). RIde-hail VEhicle Routing (RIVER) as a congestion game. Available at SSRN 3974957.

Why need to be farsighted

Scenario: trip planning

- Minimizing total travel cost considering user inertia over a planning horizon
- Empirical studies support the existence of user inertia in route choices[1] and departure time choices
- Need to plan ahead to strike a balance between { Avoiding congestion / Avoiding adjustments }

[1] Srinivasan, K. K., & Mahmassani, H. S. (2000). Modeling inertia and compliance mechanisms in route choice behavior under real-time information. Transportation Research Record, 1725, 45–53.

Why need to be farsighted

Scenario: trip planning

- E.g. psychological burden when changing departure time choices

I departed at
8:00 today

Easy to depart at
7:50 tomorrow

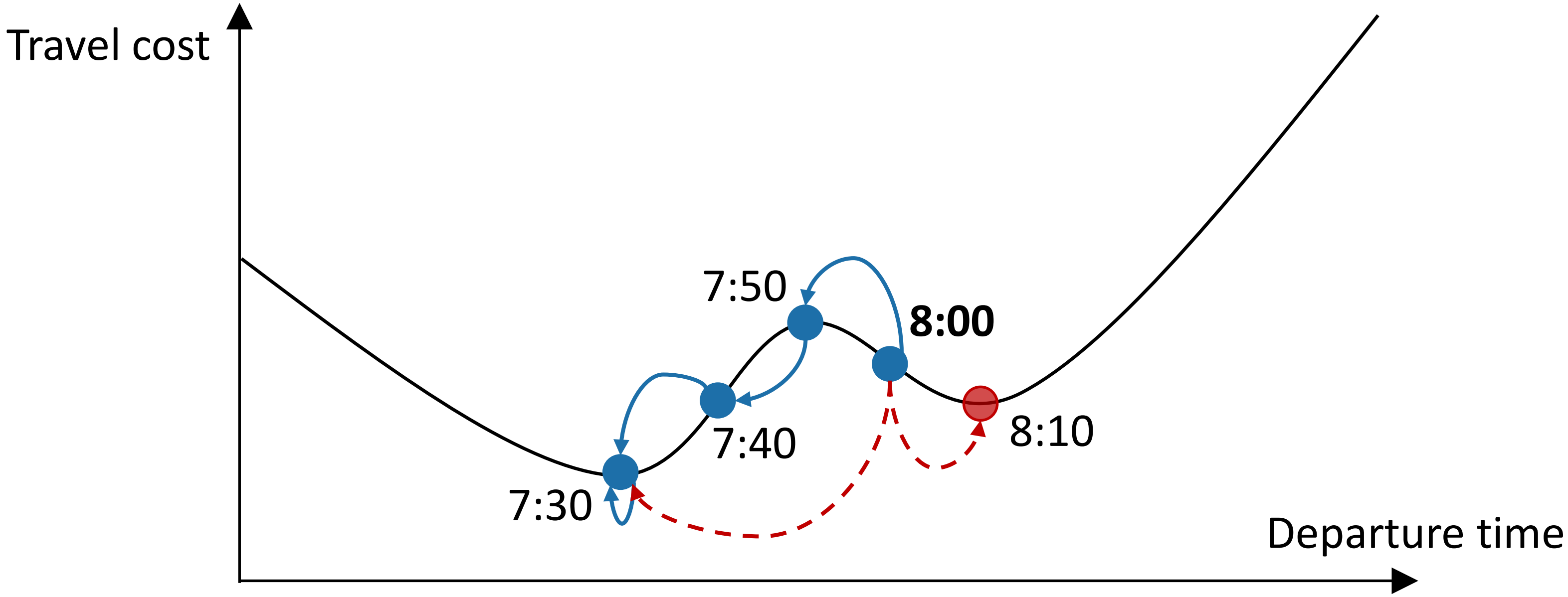
Hard to depart at
7:00 tomorrow

Why need to be farsighted

Scenario: trip planning

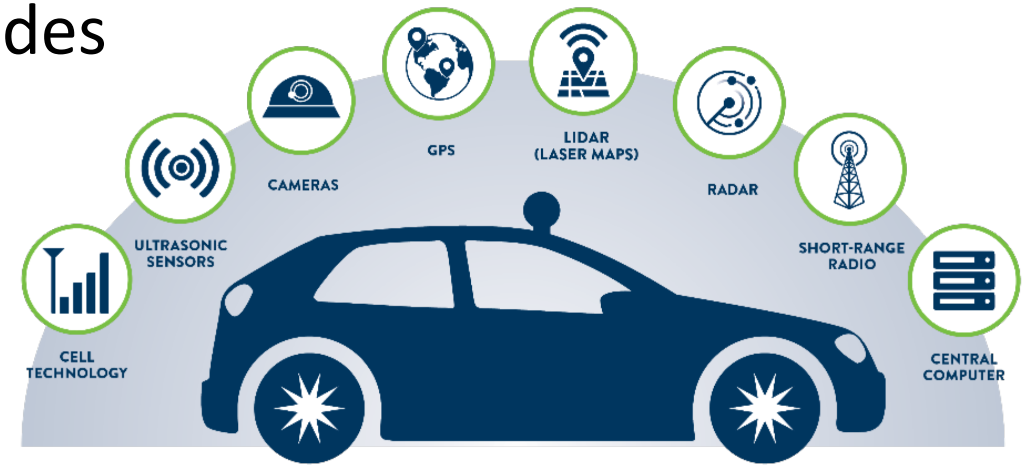
- E.g. a farsighted traveler should plan a trajectory in the future, resulting a lower total cost

- Suppose one can only change 10min per day
- Suppose others' departure profile is fixed



How to be farsighted

- Connected and automated mobility provides
  - Broader information source
  - Stronger computation power



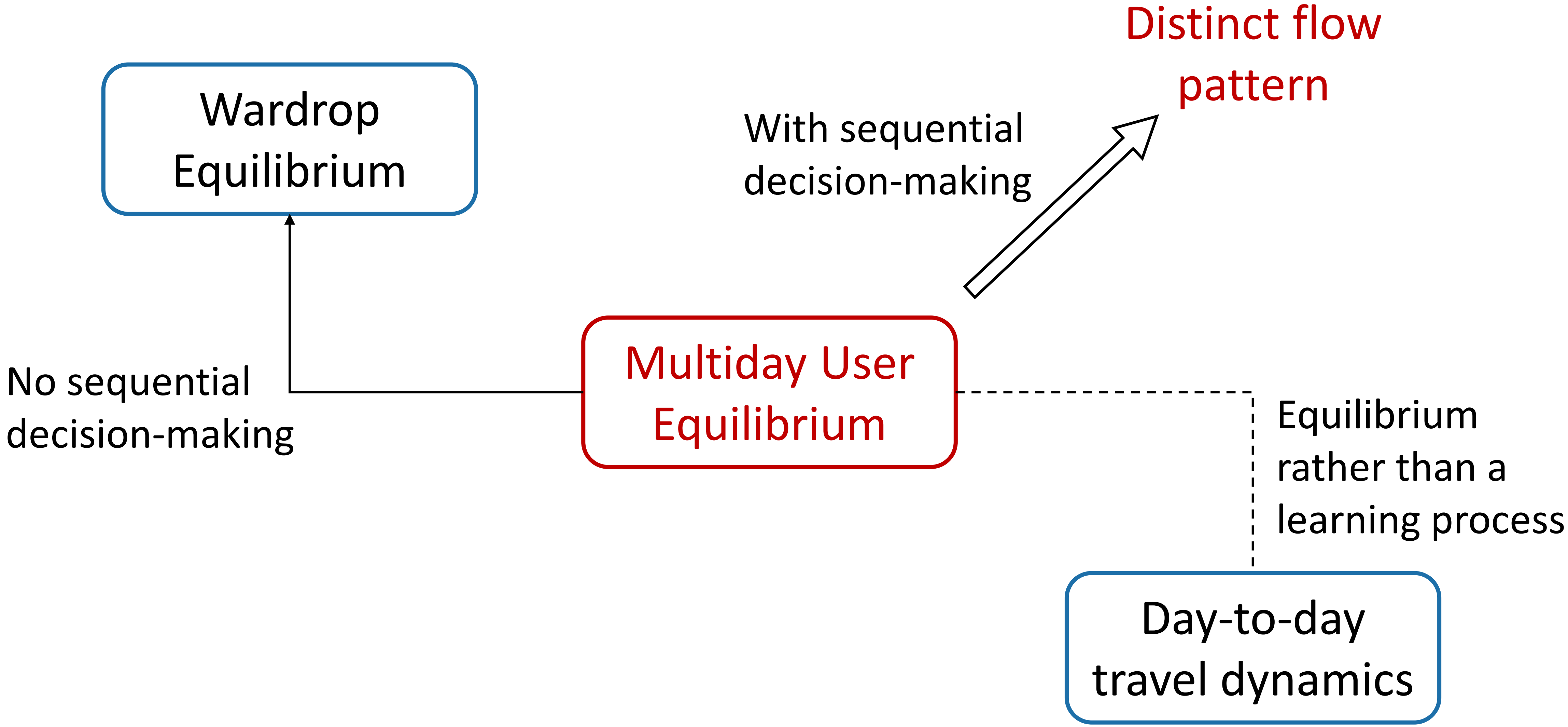
Picture source: MnDOT

- However, it still represents human interests
  - E.g. departure time still largely influences ones' daily routine
  - Intelligent commuters should still consider user inertia

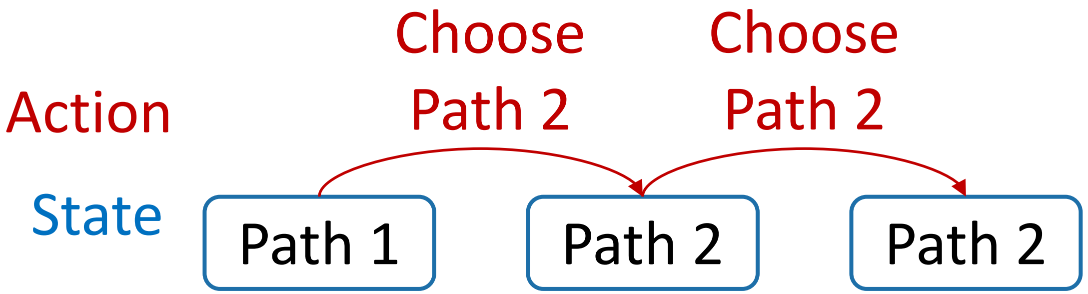
Research overview

- When all travelers are doing sequential decision-making, it will dictate a new traffic flow pattern
- In this research, we propose a mathematical framework for modeling interactions of intelligent travelers who make sequential decisions on their travel choices over a planning horizon

Research overview



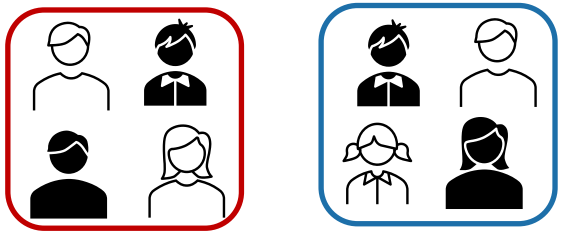
Model setting



- Planning horizon $\mathcal{N} = \{0,1,\dots,N-1\}$, with day $n \in \mathcal{N}$
- State space $\mathcal{S} = \{s_1,\dots,s_M\}$, with state $s \in \mathcal{S}$

  Travel choices (routes, departure time, or both)
- Action space $\mathcal{A} = \mathcal{S}$, with action $a \in \mathcal{S}$

  Choosing the option for the next day
- Mean field (MF) distribution $\mu_n \in \mathcal{P}(\mathcal{S})$
- MF sequence $\boldsymbol{\mu} = \{\mu_n\}_{n\in\mathcal{N}}$

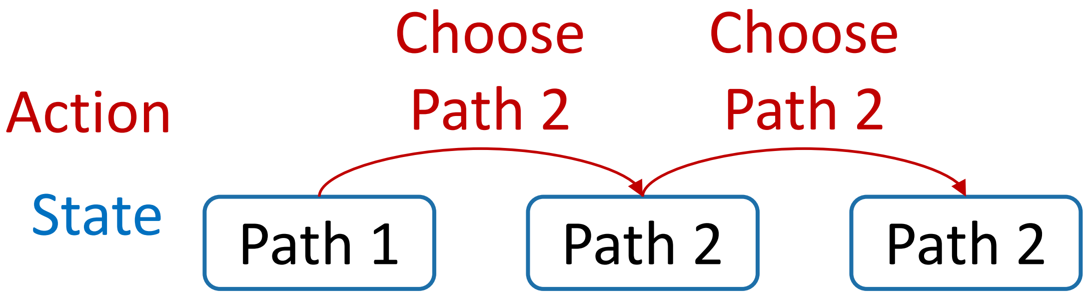


10 on path 1 10 on path 2

$\Rightarrow$ MF = $\left[\frac{1}{2},\frac{1}{2}\right]$

Lasry, J. M., & Lions, P. L. (2007). Mean field games. Japanese Journal of Mathematics, 2(1), 229–260.

Model setting

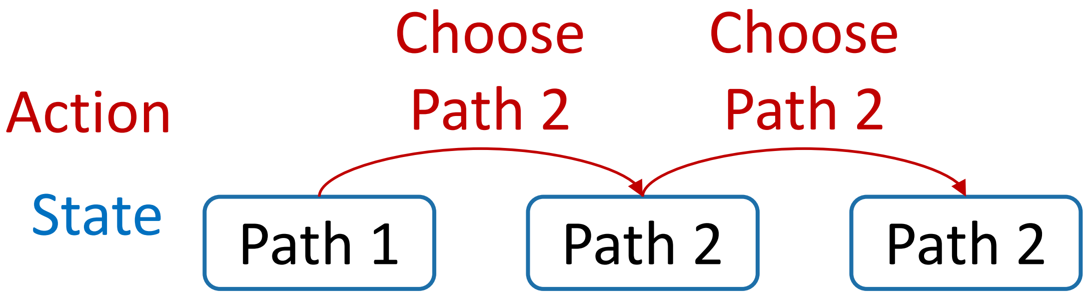


- Policy $\pi_n(a|s) = P(a_n = a|s_n = s)$
- Policy sequence $\boldsymbol{\pi} = \{\pi_n\}_{n\in\mathcal{N}}$

- Transition $p(s'|s,a) = P(s_{n+1} = s'|s_n = s, a_n = a)$
- Here we consider $p(s'|s,a) = \begin{cases} 1, & s' = a \\ 0, & o.w. \end{cases}$

If I decide to choose path 1 tomorrow, I will choose it accordingly then

Model setting



- Cost function

$$c(s,a,\mu,\pi) = f(s,\mu) + d(s,a) + \frac{1}{\theta}\ln \pi(a|s)$$

Travel cost

e.g. travel time in routing

Switching cost[1] for inertia

e.g. $d(s,a) = \epsilon \cdot 1_{s\neq a}$

$d(s,a) = \epsilon \|s-a\|$

Entropy term for random residue

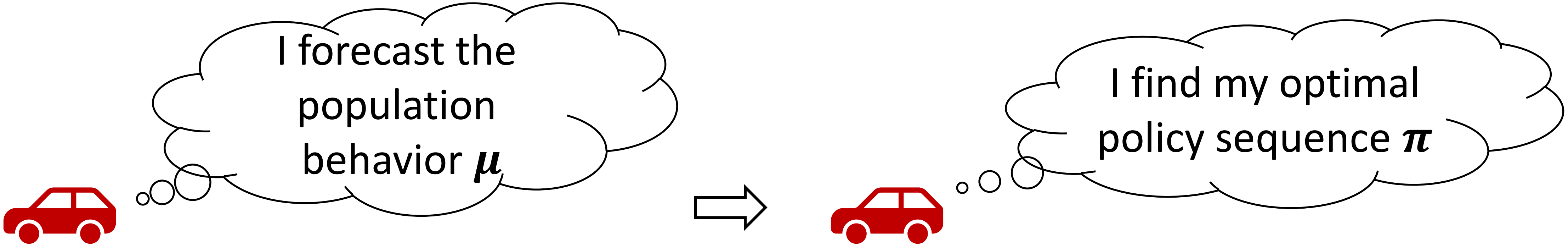
[1] Delle Site, P. (2018). A mixed-behaviour equilibrium model under predictive and static Advanced Traveller Information Systems (ATIS) and state-dependent route choice. Transportation Research Part C: Emerging Technologies, 86, 549–562.

Model setting

- **Individual response**

Given the population behavior $\boldsymbol{\mu}$

$$\boldsymbol{\pi}^* = \Phi(\boldsymbol{\mu}) = \arg\min E\left[\sum_n c(s_n, a_n, \mu_n, \pi_n)\right]$$

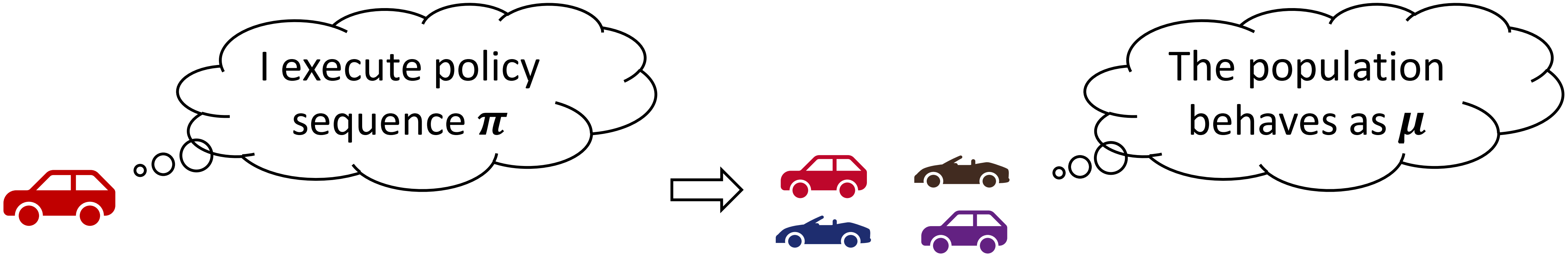


Model setting

- **Population behavior**

Given a policy sequence $\boldsymbol{\pi}$ and an initial distribution μ_0

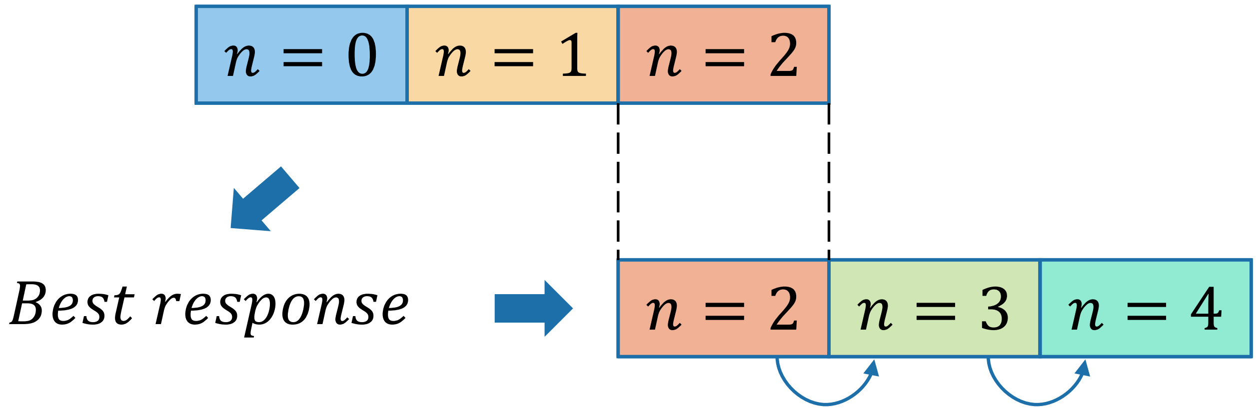
$\boldsymbol{\mu} = \Psi_{\mu_0}(\boldsymbol{\pi})$ can be recursively calculated as $\mu_{n+1}(s) = \sum_{x \in \mathcal{S}} \mu_n(x)\pi_n(s|x)$



Multiday User Equilibrium

- A motivating example of interaction protocol

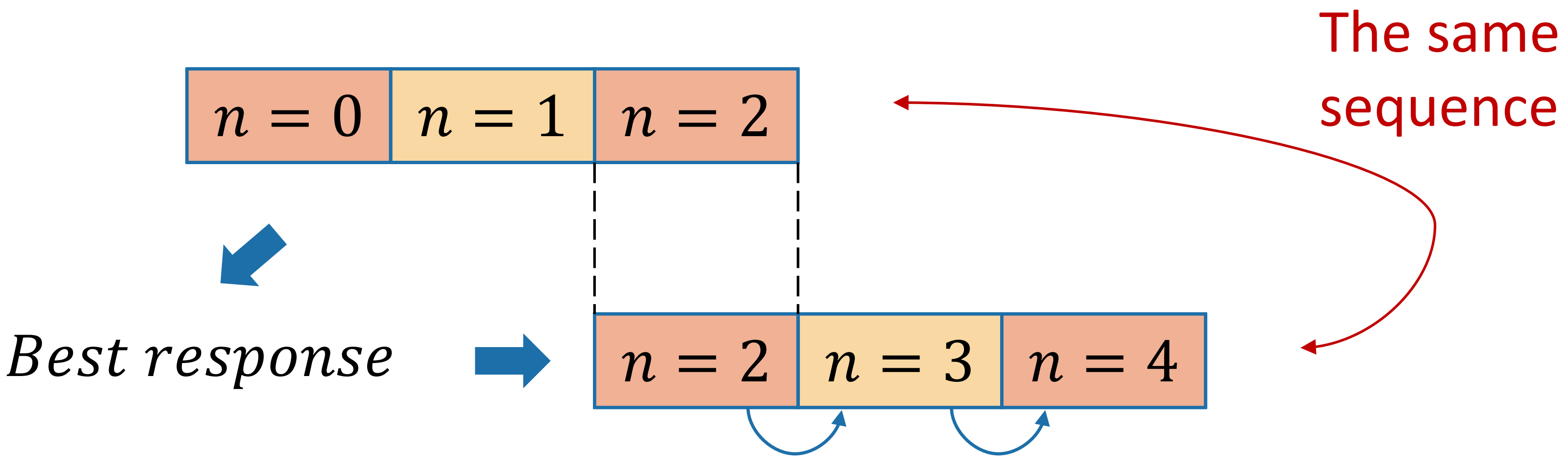
Suppose $N = 3$



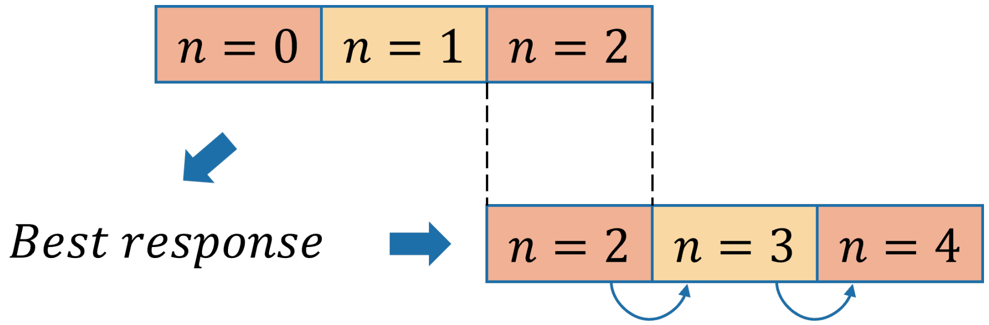
Multiday User Equilibrium

- A motivating example

Consider the steady-state



Multiday User Equilibrium



Definition

A pair $(\boldsymbol{\pi}, \boldsymbol{\mu})$ is called a <u>multiday user equilibrium (MUE)</u> if

$$\boldsymbol{\pi} = \Phi(\boldsymbol{\mu}), \boldsymbol{\mu} = \Psi_{\mu_{N-1}}(\boldsymbol{\pi})$$

- By definition, the MF distribution sequence $\boldsymbol{\mu}$ should have the same initial and final distribution

Multiday User Equilibrium

- The existence is generally ensured

Proposition

Under continuous cost $f(s, \mu)$, there always exists an MUE

Connection with Wardrop Equilibrium

No sequential decision-making: no user inertia

- MUE and logit-SUE are equivalent

Proposition

When $d(s, s') = 0$, there exists a unique MUE, and it repeats the logit-SUE every day

Intuition: the framework reduces to Wardrop Equilibrium when there is no need for forward-looking

Connection with Wardrop Equilibrium

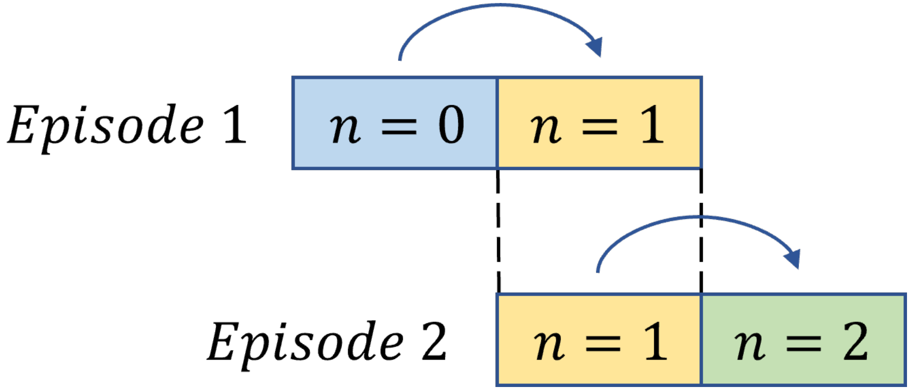
No sequential decision-making: short planning horizon

Proposition

When $d(s,s') \neq 0$ and $N = 2$, the resulting MUE repeats SDSUE

State-dependent SUE: Similar to logit-SUE but consider switching costs[1,2]

Intuition: When $N = 2$, travelers essentially only consider the next day, letting the framework reduces back to Wardrop Equilibrium



[1] Castaldi, C., Delle Site, P., & Filippi, F. (2019). Stochastic user equilibrium in the presence of state dependence. EURO Journal on Transportation and Logistics, 8(5), 535–559.

[2] Castaldi, C., Site, P. D., & Filippi, F. (2017). Stochastic user equilibrium in the presence of inertia. Transportation Research Procedia, 22, 13–24.

Distinct flow pattern

- **With sequential decision-making**: generally, MUE cannot be stationary

Proposition

When $d(s, s') = \epsilon \cdot 1_{s \neq s'}$ and $N > 2$, generally the MUE cannot have time-invariant MF distribution

Intuition: the resulting MUE always have within-horizon fluctuations

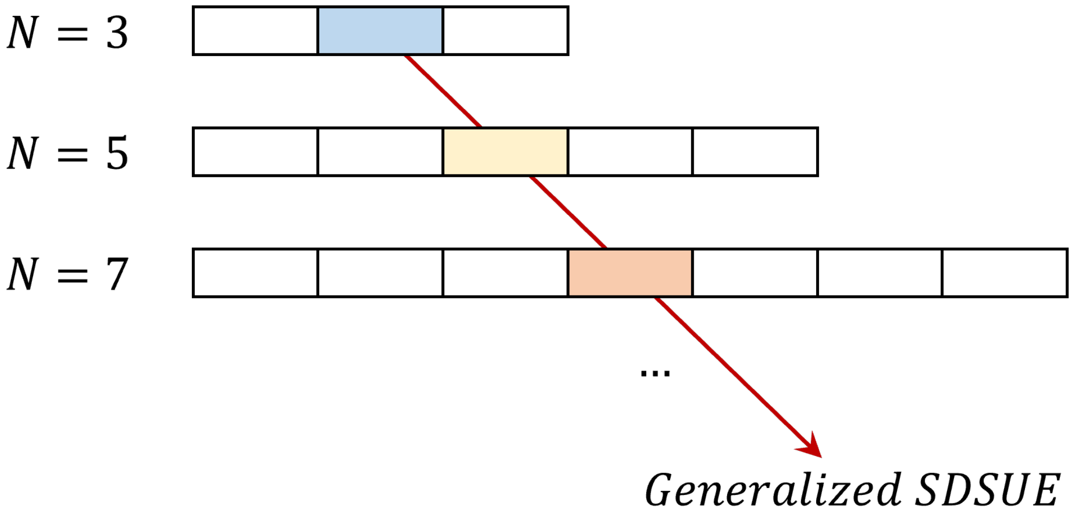
Distinct flow pattern

- **With sequential decision-making**: MUE converges to a generalized SDSUE with longer horizon length (formal definition skipped here)

Proposition

Assume the episode length is odd, and denote it as $\mathcal{N} = \{0, \ldots, N-1\}$. Denote the corresponding MUE as $(\boldsymbol{\pi}^N, \boldsymbol{\mu}^N)$.

$$\mu_K^{(N-1)/2} \to \bar{\mu} \text{ as } K \to \infty$$



Intuition: $\bar{\mu}$ maintains a time-invariant MF distribution and policy

Numerical experiments: departure time

Total number of commuters: 6,000

Bottleneck capacity: 3,000 per hour

Planning horizon: 7 days

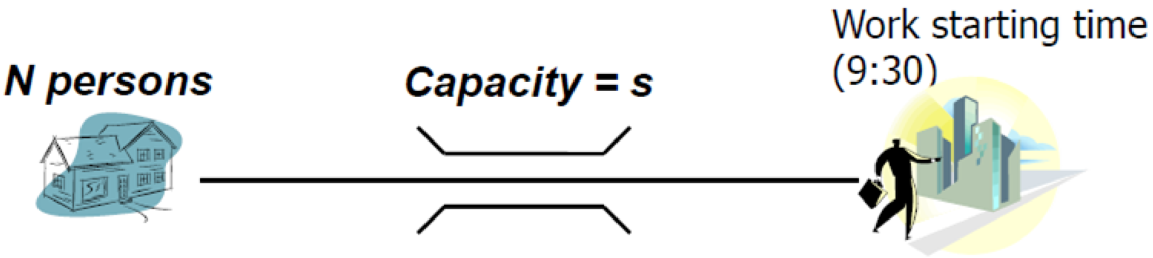
Departure time range: [0,3] hour

Discretized to 40 slices

Desired arrival time: 2 hour

Scheduling cost: $\alpha = 10, \beta = 5, \gamma = 15$

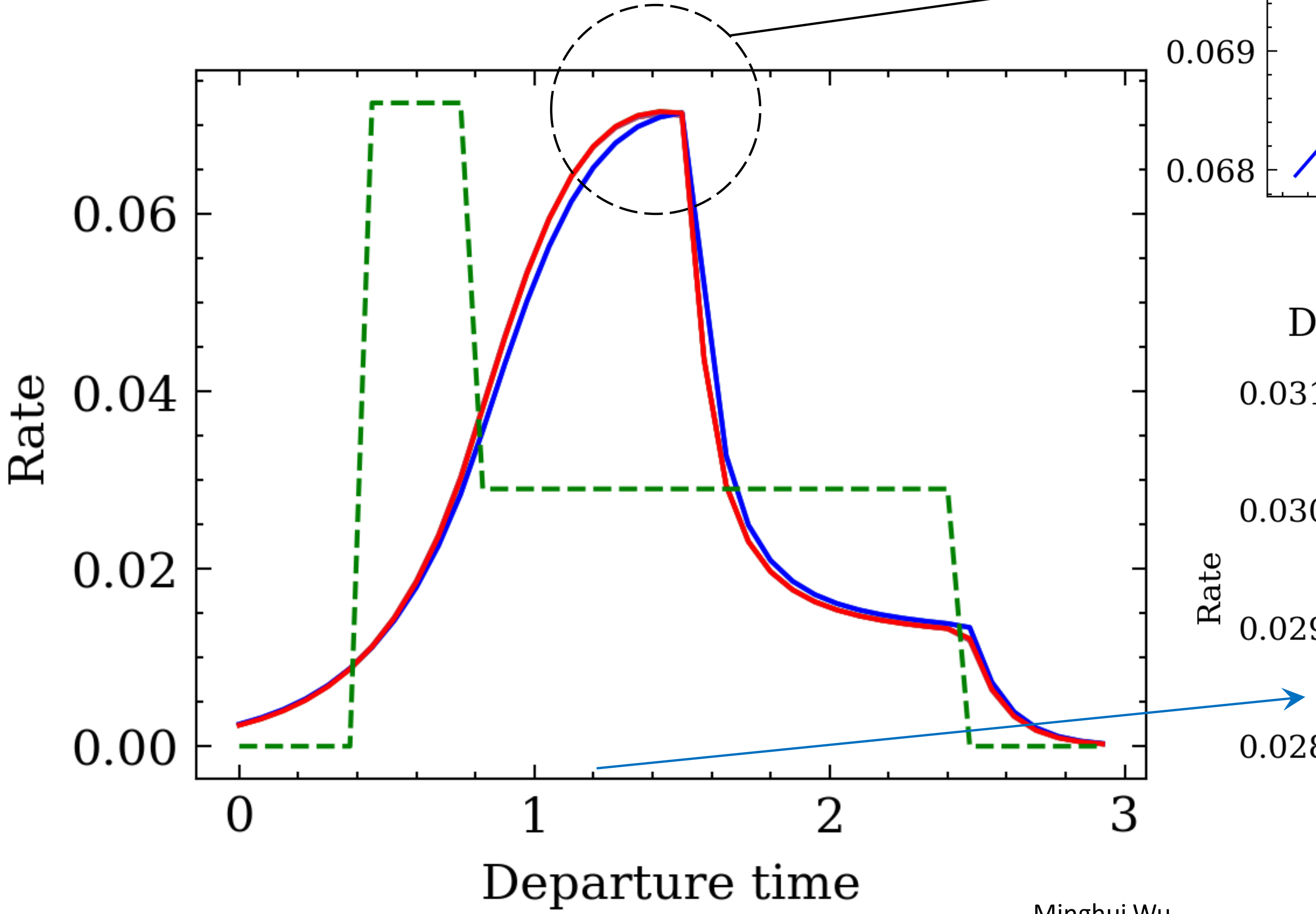
Switching cost: $\epsilon|s - s'|$

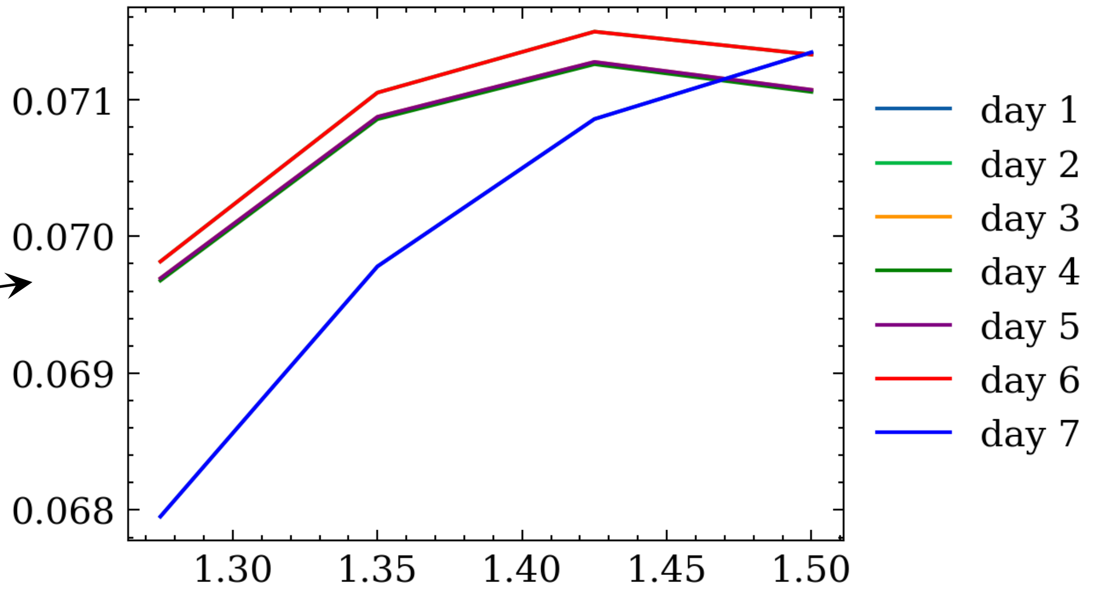


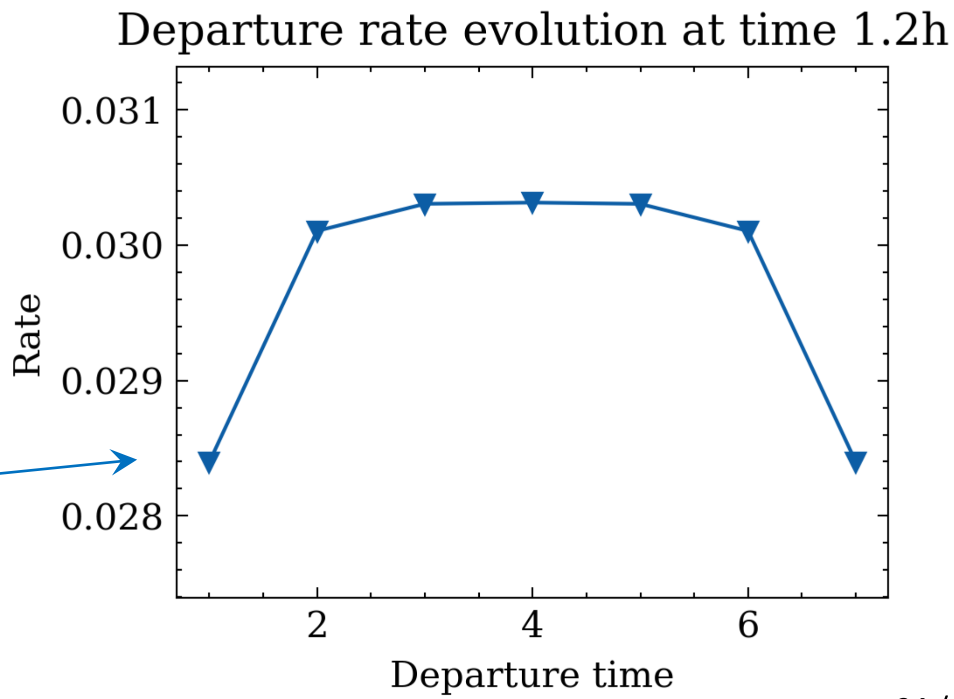
Guo, R. Y., Yang, H., Huang, H. J., & Li, X. (2018). Day-to-day departure time choice under bounded rationality in the bottleneck model. Transportation Research Part B: Methodological, 117, 832–849.

Inertia $\epsilon = 1$, Entropy $\theta = 0.5$

Resulting departure rate in MUE







Numerical experiments: routing

A grid network: 12 links, 6 paths

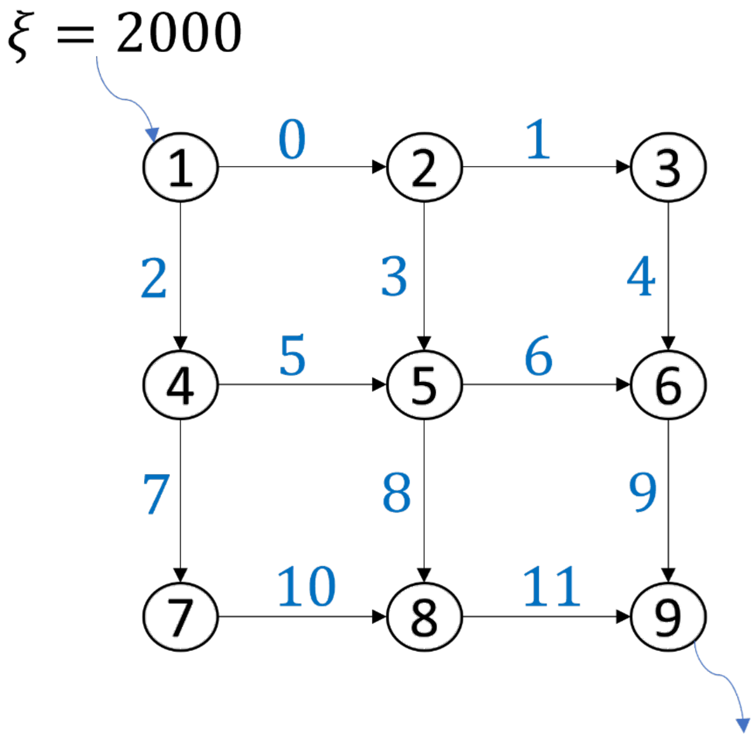
Link travel time: BPR type

Switching cost: $\epsilon \cdot \mathbf{1}_{s \neq s'}$

Entropy term: $\frac{1}{\theta} \ln \pi(s'|s)$

Planning horizon: 7 days

Total inflow: 2000 from Node 1 to Node 9

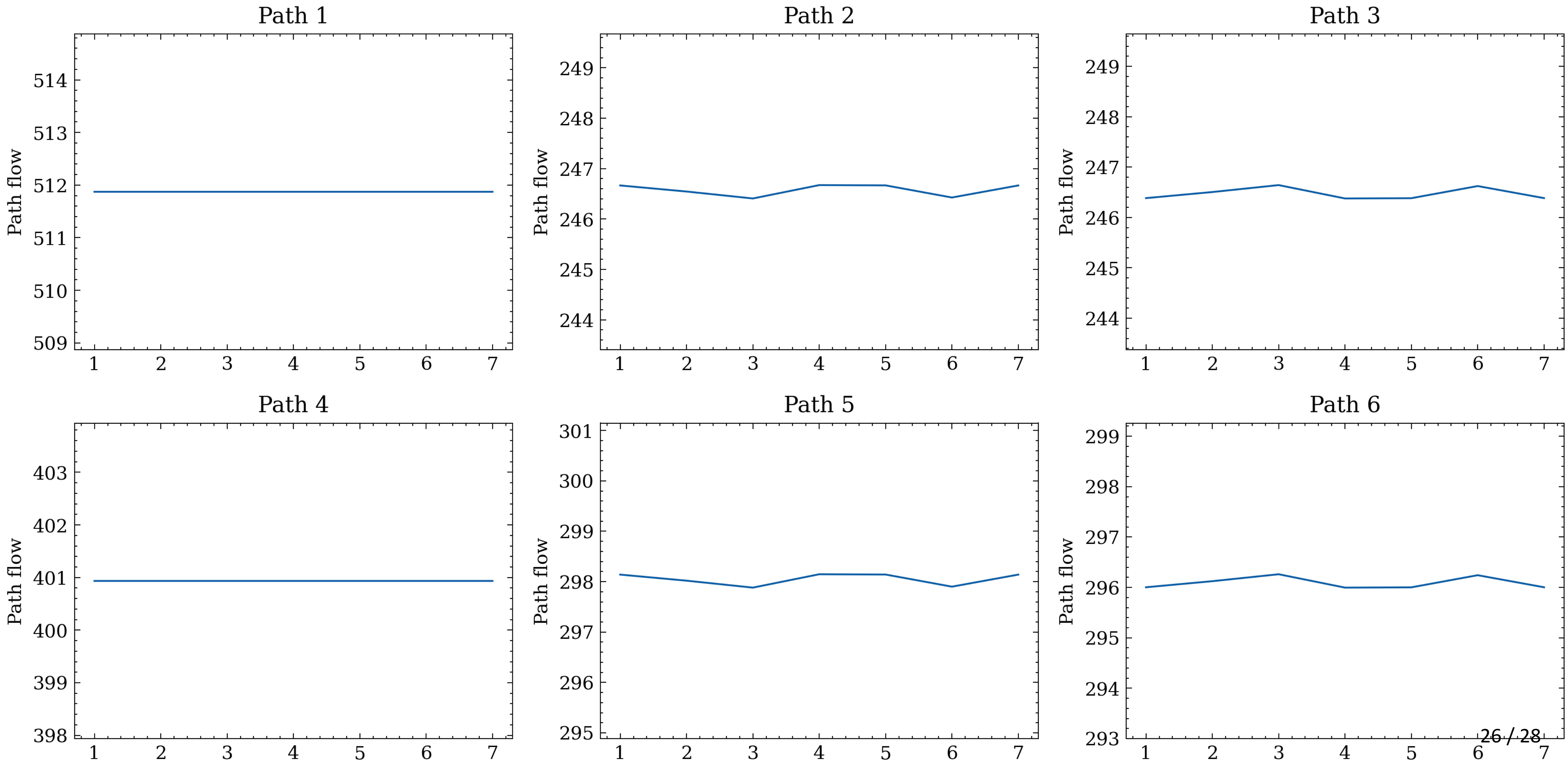


He, X., & Peeta, S. (2016). A marginal utility day-to-day traffic evolution model based on one-step strategic thinking. Transportation Research Part B: Methodological, 84, 237–255.

- No inertia $\epsilon = 0$
- Entropy parameter $\theta = 1$

Resulting path flow evolution in MUE

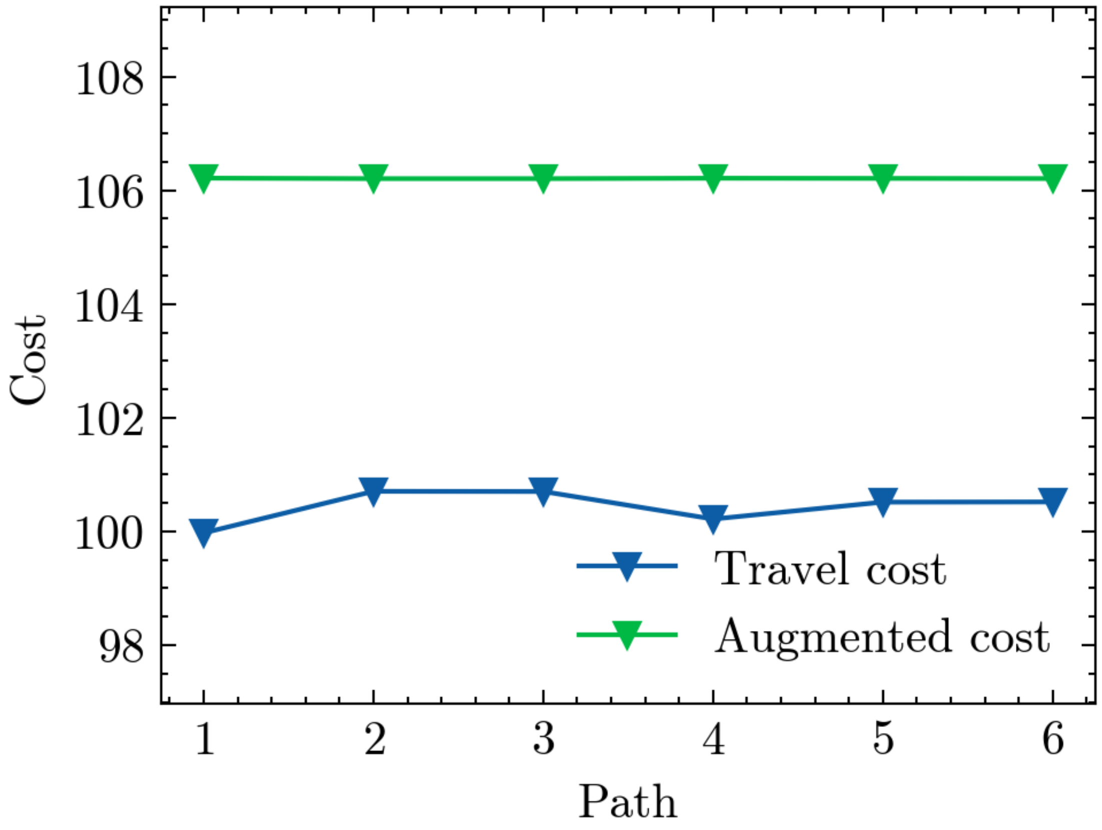
Stationary for the entire episode



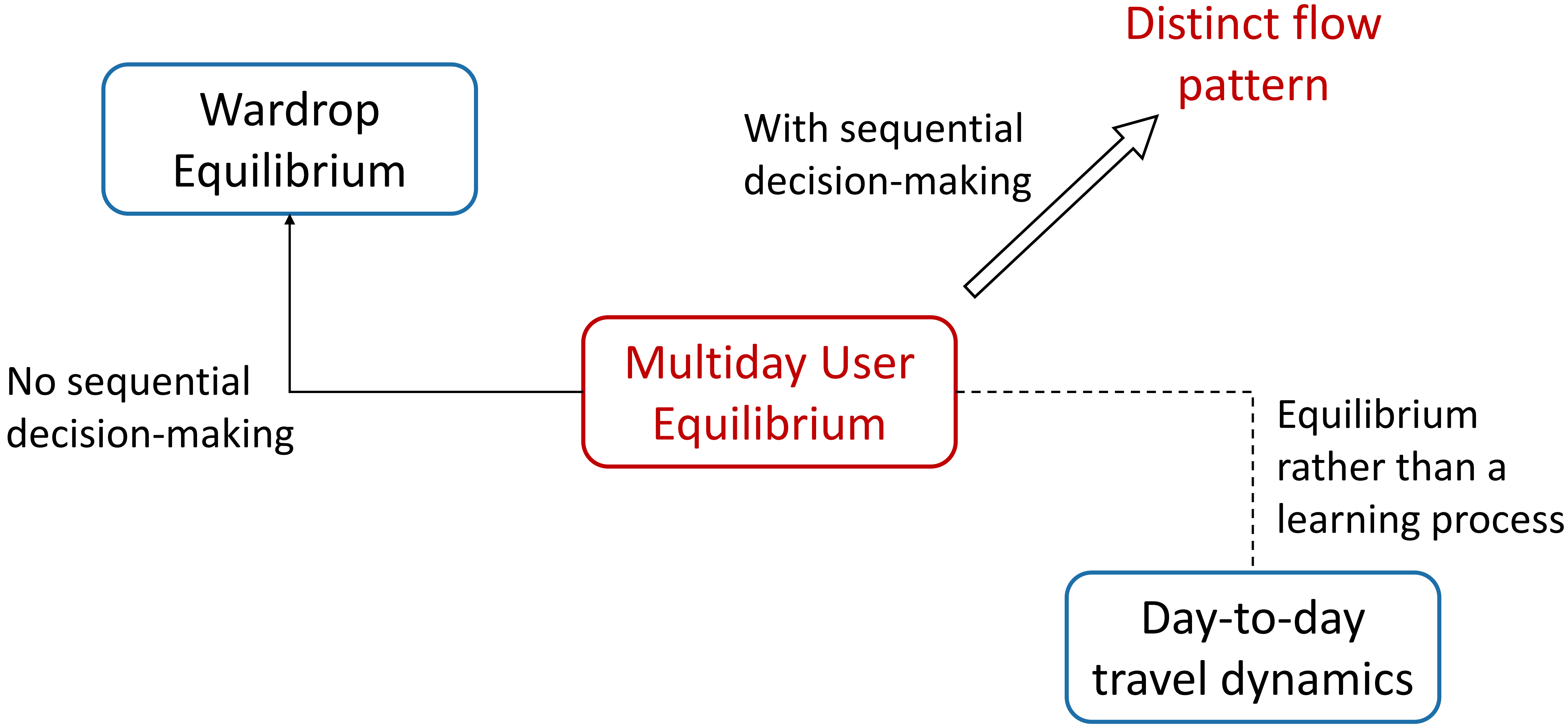
Numerical experiments: routing

- No inertia ($\epsilon = 0$), $\theta = 1$: resulting MUE

Equivalent to logit-SUE



Summary


Wardrop Equilibrium
Distinct flow pattern
With sequential decision-making
No sequential decision-making
Multiday User Equilibrium
Equilibrium rather than a learning process
Day-to-day travel dynamics

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Thank You!

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