Multiday User Equilibrium with Intelligent Travelers

Minghui Wu
joint work with Dr. Yafeng Yin, Dr. Jerome P. Lynch

University of Michigan
Background

• User Equilibrium (UE) typically is the equilibrium of one-shot games / each traveler makes one-shot decisions

• In this research, we propose a new framework -- Multiday User Equilibrium (MUE), where farsighted travelers make sequential decisions on their travel choices for multiple days

• Sequential decision-making: plan out a sequence of actions for a horizon considering long-term effects
Why need to be farsighted

In some scenarios, one can benefit from sequential decision-making

• Travel within a fixed budget[1]

• Parking search[2]

• Ride-hailing vehicles routing[3,4]


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Why need to be farsighted

Scenario: trip planning

- Minimizing total travel cost considering user inertia over a planning horizon

- Empirical studies support the existence of user inertia in route choices[1] and departure time choices

- Need to plan ahead to strike a balance between [Avoiding congestion, Avoiding adjustments]

Why need to be farsighted

Scenario: trip planning

• E.g. psychological burden when changing departure time choices

I departed at 8:00 today

Easy to depart at 7:50 tomorrow

Hard to depart at 7:00 tomorrow
Why need to be farsighted

Scenario: trip planning

• E.g. a farsighted traveler should plan a trajectory in the future, resulting a lower total cost

- Suppose one can only change 10min per day
- Suppose others’ departure profile is fixed
How to be farsighted

• Connected and automated mobility provides
  • Broader information source
  • Stronger computation power

• However, it still represents human interests
  • E.g. departure time still largely influences ones’ daily routine
  • Intelligent commuters should still consider user inertia
Research overview

• When all travelers are doing sequential decision-making, it will dictate a new traffic flow pattern

• In this research, we propose a mathematical framework for modeling interactions of intelligent travelers who make sequential decisions on their travel choices over a planning horizon
Research overview

- Wardrop Equilibrium
- Multiday User Equilibrium
  - No sequential decision-making
  - With sequential decision-making
    - Distinct flow pattern
  - Equilibrium rather than a learning process
- Day-to-day travel dynamics
Model setting

- Planning horizon $\mathcal{N} = \{0, 1, \ldots, N - 1\}$, with day $n \in \mathcal{N}$
- State space $\mathcal{S} = \{s_1, \ldots, s_M\}$, with state $s \in \mathcal{S}$
  
  Travel choices (routes, departure time, or both)

- Action space $\mathcal{A} = \mathcal{S}$, with action $a \in \mathcal{S}$
  
  Choosing the option for the next day

- Mean field (MF) distribution $\mu_n \in \mathcal{P}(\mathcal{S})$

- MF sequence $\mu = \{\mu_n\}_{n \in \mathcal{N}}$


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Model setting

• Policy $\pi_n(a|s) = P(a_n = a|s_n = s)$

• Policy sequence $\pi = \{\pi_n\}_{n \in \mathcal{N}}$

• Transition $p(s'|s, a) = P(s_{n+1} = s'|s_n = s, a_n = a)$

• Here we consider $p(s'|s, a) = \begin{cases} 1, & s' = a \\ 0, & o.w. \end{cases}$

If I decide to choose path 1 tomorrow, I will choose it accordingly then
Model setting

- Cost function

\[ c(s, a, \mu, \pi) = f(s, \mu) + d(s, a) + \frac{1}{\theta} \ln \pi(a|s) \]

- Travel cost
  - e.g. travel time in routing

- Switching cost\(^1\) for inertia
  - e.g. \( d(s, a) = \varepsilon \cdot 1_{s \neq a} \)

- Entropy term for random residue
  - \( d(s, a) = \varepsilon \| s - a \| \)

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Model setting

• Individual response

Given the population behavior $\mu$

$$\pi^* = \Phi(\mu) = \arg\min E [\sum_n c(s_n, a_n, \mu_n, \pi_n)]$$
Model setting

• Population behavior

Given a policy sequence $\pi$ and an initial distribution $\mu_0$

$$\mu = \Psi_{\mu_0}(\pi)$$
can be recursively calculated as

$$\mu_{n+1}(s) = \sum_{x \in S} \mu_n(x) \pi_n(s|x)$$
Multiday User Equilibrium

• A motivating example of interaction protocol

Suppose $N = 3$
Multiday User Equilibrium

• A motivating example

Consider the steady-state

\[ n = 0 \quad n = 1 \quad n = 2 \]

\[ n = 2 \quad n = 3 \quad n = 4 \]

The same sequence

Best response
Multiday User Equilibrium

A pair \((\pi, \mu)\) is called a multiday user equilibrium (MUE) if
\[
\pi = \Phi(\mu), \mu = \Psi_{\mu_{N-1}}(\pi)
\]

• By definition, the MF distribution sequence \(\mu\) should have the same initial and final distribution
Multiday User Equilibrium

• The existence is generally ensured

Proposition

Under continuous cost $f(s, \mu)$, there always exists an MUE
Connection with Wardrop Equilibrium

No sequential decision-making: no user inertia

- MUE and logit-SUE are equivalent

**Proposition**

When $d(s, s') = 0$, there exists a unique MUE, and it repeats the logit-SUE every day

Intuition: the framework reduces to Wardrop Equilibrium when there is no need for forward-looking
Connection with Wardrop Equilibrium

No sequential decision-making: short planning horizon

Proposition

When $d(s, s') \neq 0$ and $N = 2$, the resulting MUE repeats SDSUE

State-dependent SUE: Similar to logit-SUE but consider switching costs[1,2]

Intuition: When $N = 2$, travelers essentially only consider the next day, letting the framework reduces back to Wardrop Equilibrium


Distinct flow pattern

• With sequential decision-making: generally, MUE cannot be stationary

Proposition

When \( d(s, s') = \epsilon \cdot 1_{s \neq s'} \) and \( N > 2 \), generally the MUE cannot have time-invariant MF distribution

Intuition: the resulting MUE always have within-horizon fluctuations
Distinct flow pattern

• With sequential decision-making: MUE converges to a generalized SDSUE with longer horizon length (formal definition skipped here)

Proposition
Assume the episode length is odd, and denote it as $\mathcal{N} = \{0, \ldots, N - 1\}$. Denote the corresponding MUE as $(\pi^N, \mu^N)$.

$$\mu_k^{(N-1)/2} \rightarrow \bar{\mu} \text{ as } K \rightarrow \infty$$

Intuition: $\bar{\mu}$ maintains a time-invariant MF distribution and policy

$N = 3$

$N = 5$

$N = 7$

Generalized SDSUE
Numerical experiments: departure time

Total number of commuters: 6,000
Bottleneck capacity: 3,000 per hour
Planning horizon: 7 days
Departure time range: [0,3] hour
Discretized to 40 slices
Desired arrival time: 2 hour
Scheduling cost: $\alpha = 10$, $\beta = 5$, $\gamma = 15$
Switching cost: $\epsilon |s - s'|$

Inertia $\epsilon = 1$, Entropy $\theta = 0.5$
Resulting departure rate in MUE
Numerical experiments: routing

A grid network: 12 links, 6 paths
Link travel time: BPR type
Switching cost: $\epsilon \cdot 1_{s \neq s'}$
Entropy term: $\frac{1}{\theta} \ln \pi(s'|s)$
Planning horizon: 7 days
Total inflow: 2000 from Node 1 to Node 9

• No inertia $\epsilon = 0$
• Entropy parameter $\theta = 1$

Resulting path flow evolution in MUE

Stationary for the entire episode
Numerical experiments: routing

• No inertia ($\epsilon = 0$), $\theta = 1$: resulting MUE

Equivalent to logit-SUE

![Graph showing cost versus path]

- Travel cost
- Augmented cost
Summary

Wardrop Equilibrium

No sequential decision-making

Multiday User Equilibrium

With sequential decision-making

Distinct flow pattern

Equilibrium rather than a learning process

Day-to-day travel dynamics
Thank You!

Minghui Wu
minghuiw@umich.edu