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# Blowup of a Concrete Pavement Adjoining a Rigid Structure

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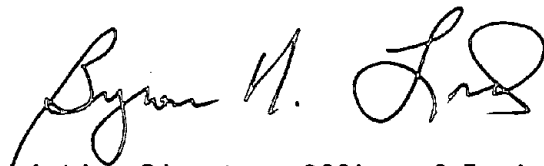
## FOREWORD

There is general agreement that pavement blowups are caused by axial compression forces induced in the pavement by a rise in temperature and moisture, and that they usually occur at traverse joints or crack; although blowups of a long continuously reinforced concrete pavement (CRCP) with a traverse "hinge" is equivalent to lift-off buckling of the pavement.<sup>(1)</sup> A related analysis was presented in 1985.<sup>(2)</sup>

This paper contains an analysis of the case when a long continuous reinforced concrete pavement is subjected to temperature and moisture increases and adjoins a rigid structure. The analysis is similar to the one used in reference 1. The closed form solutions are evaluated numerically. They are then compared with those of the jointless pavement analyzed previously, to show the effect of the rigid structure on pavement response.

This report will be of interest to researchers and engineers concerned with the assessment of blowups of concrete pavements.

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16. Abstract <p>The main cause of concrete pavement blowups are axial compression forces induced into the pavement by a rise in temperature and moisture. Recent analyses by this writer and his students were based on the notion that blowups are caused by lift-off buckling of the pavement. The cases analyzed were: (1) continuously reinforced concrete pavement and (2) concrete pavement weakened by a traverse joint or crack.</p> <p>The present paper contains an analysis of another case, when a long continuously reinforced concrete pavement adjoins a rigid structure, like a bridge abutment. The analysis is similar to the ones described above. The resulting formulation is non-linear and is solved exactly, in closed form. The obtained results are evaluated numerically and are compared with those of a long continuously reinforced pavement, in order to show the effect of the rigid structure on the pavement response.</p>					
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# SI\* (MODERN METRIC) CONVERSION FACTORS

## APPROXIMATE CONVERSIONS TO SI UNITS

## APPROXIMATE CONVERSIONS FROM SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
<b>LENGTH</b>				
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
<b>AREA</b>				
in <sup>2</sup>	square inches	645.2	square millimeters	mm <sup>2</sup>
ft <sup>2</sup>	square feet	0.093	square meters	m <sup>2</sup>
yd <sup>2</sup>	square yards	0.836	square meters	m <sup>2</sup>
ac	acres	0.405	hectares	ha
mi <sup>2</sup>	square miles	2.59	square kilometers	km <sup>2</sup>
<b>VOLUME</b>				
fl oz	fluid ounces	29.57	milliliters	ml
gal	gallons	3.785	liters	l
ft <sup>3</sup>	cubic feet	0.028	cubic meters	m <sup>3</sup>
yd <sup>3</sup>	cubic yards	0.765	cubic meters	m <sup>3</sup>
<b>MASS</b>				
oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams	Mg
<b>TEMPERATURE (exact)</b>				
°F	Fahrenheit temperature	5(F-32)/9 or (F-32)/1.8	Celcius temperature	°C
<b>ILLUMINATION</b>				
fc	foot-candles	10.76	lux	l
fl	foot-Lamberts	3.426	candela/m <sup>2</sup>	cd/m <sup>2</sup>
<b>FORCE and PRESSURE or STRESS</b>				
lbf	poundforce	4.45	newtons	N
psi	poundforce per square inch	6.89	kilopascals	kPa

Symbol	When You Know	Multiply By	To Find	Symbol
<b>LENGTH</b>				
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
<b>AREA</b>				
mm <sup>2</sup>	square millimeters	0.0016	square inches	in <sup>2</sup>
m <sup>2</sup>	square meters	10.764	square feet	ft <sup>2</sup>
m <sup>2</sup>	square meters	1.195	square yards	ac
ha	hectares	2.47	acres	mi <sup>2</sup>
km <sup>2</sup>	square kilometers	0.386	square miles	
<b>VOLUME</b>				
ml	milliliters	0.034	fluid ounces	fl oz
l	liters	0.264	gallons	gal
m <sup>3</sup>	cubic meters	35.71	cubic feet	ft <sup>3</sup>
m <sup>3</sup>	cubic meters	1.307	cubic yards	yd <sup>3</sup>
<b>MASS</b>				
g	grams	0.035	ounces	oz
kg	kilograms	2.202	pounds	lb
Mg	megagrams	1.103	short tons (2000 lb)	T
<b>TEMPERATURE (exact)</b>				
°C	Celcius temperature	1.8C + 32	Fahrenheit temperature	°F
<b>ILLUMINATION</b>				
lx	lux	0.0929	foot-candles	fc
cd/m <sup>2</sup>	candela/m <sup>2</sup>	0.2919	foot-Lamberts	fl
<b>FORCE and PRESSURE or STRESS</b>				
N	newtons	0.225	poundforce	lbf
kPa	kilopascals	0.145	poundforce per square inch	psi

\* SI is the symbol for the International System of Units. Appropriate rounding should be made to comply with Section 4 of ASTM E380.

BLOWUP OF A CONCRETE PAVEMENT  
ADJOINING A RIGID STRUCTURE

TABLE OF CONTENTS

<u>Chapter</u>	<u>Page</u>
I. INTRODUCTION . . . . .	1
II. CONCEPTUAL AND ANALYTICAL PRELIMINARIES . . . . .	3
III. ANALYSIS OF PAVEMENT . . . . .	9
IV. NUMERICAL EVALUATION AND DISCUSSION OF RESULTS . . . . .	15
REFERENCES . . . . .	20

## LIST OF FIGURES

<u>Figure</u>	<u>Page</u>
1. Problem under consideration. . . . .	3
2. Spalled joint or crack at abutment. . . . .	4
3. Axial resistance-displacement response. . . . .	5
4. Sliding frictional resistance as a function of pavement thickness (based on test data presented in references 3 and 4) . . . . .	6
5. Typical equilibrium branches for pavement. . . . .	7
6. Equilibrium branches and axial forces in pavement. (For the determination of $\tilde{N}_t$ , $b = 24$ ft (725 cm) was used.) . . . . .	16
7. Dependence of $T_t$ on pavement thickness $h$ . . . . .	17
8. Dependence of $T_t$ on $r_0^*$ . . . . .	18

## LIST OF TABLES

<u>Table</u>	<u>Page</u>
1. Dependence of $r_0^*$ on $h$ . . . . .	15

## CHAPTER I. INTRODUCTION

There is general agreement that pavement blowups are caused by axial compression forces induced in the pavement by a rise in temperature and moisture. In 1984, analyses were presented for a long continuously reinforced concrete pavement (CRCP) and for a pavement with a transverse "hinge," based on the assumption that blowup is equivalent to lift-off buckling of the pavement.<sup>(1)</sup> A related analysis was presented in 1985.<sup>(2)</sup>

These two papers established the mechanism that leads to blowups. The resulting formulations are nonlinear, but it was possible to solve them exactly and in closed form. These analyses revealed a number of important parameters, like the pavement thickness  $h$ , the sliding frictional resistance at the interface of pavement and soil  $r_0^*$ , the effective flexural stiffness of the pavement  $EI = EI/(1-\nu^2)$ , the coefficient of linear thermal expansion of pavement  $\alpha$ , and the rotational and axial stiffness of the transverse joint or crack.

The present paper contains the analysis for another case; when a long continuously reinforced concrete pavement, that is subjected to temperature and moisture increases, adjoins a rigid structure, like a bridge abutment. The analysis is similar to the one used in reference 1. The derived closed form solutions are evaluated numerically. They are then compared with those of the jointless pavement analyzed previously, in order to show the effect of the rigid structure on pavement response.





CHAPTER II. CONCEPTUAL AND ANALYTICAL PRELIMINARIES

Consider a continuous concrete pavement that is adjoining an abutment as shown in figure 1(a). It is assumed at first that the pavement may rotate freely at the abutment but is constrained there from moving axially. This is the most unfavorable condition for blowup occurrence. The anticipated blowup mode is shown in figure 1(b). It is located near the abutment, because the "hinge," that may form by a badly spalled transverse joint or crack, shown in figure 2, weakens the pavement there and therefore reduces the temperature increase that may cause the shown blowup.

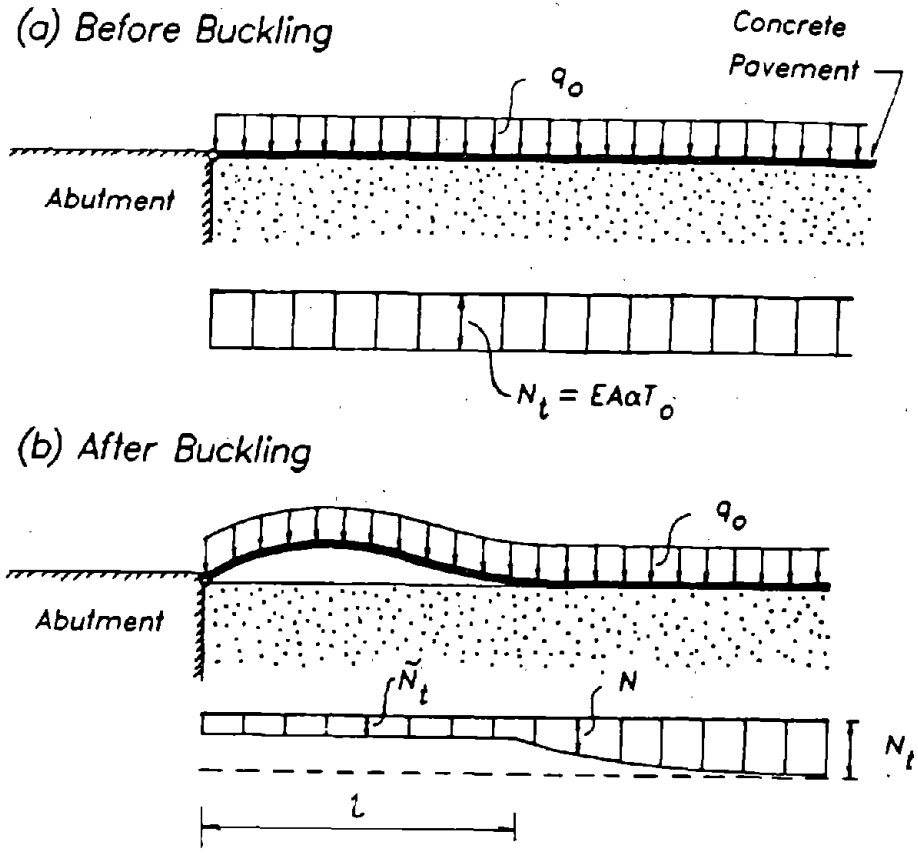


Figure 1. Problem under consideration.

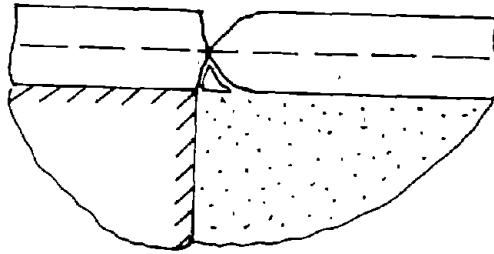


Figure 2. Spalled joint or crack at abutment

A moisture increase (or drop) in the concrete slab may be expressed by an equivalent temperature rise (or drop). Therefore, in the following only temperature changes will be discussed.

A uniform temperature increase above neutral induces, due to constrained expansions, a uniform axial compression force  $N_t$ , as shown in figure 1(a). For sufficiently large values of  $N_t$  the pavement may buckle upward. Then in the *lift-off region of length l*, part of the constrained expansions are released. This results in a drop of the axial force in the lift-off region to  $\tilde{N}_t$ . In the *adjoining region*, due to resistance to axial displacements at the interface of pavement and base, the constrained axial expansions vary; so does the axial force  $\tilde{N}_t < N < N_t$ , as shown in figure 1(b).

In the following analysis, the concrete pavement is replaced by a beam of rectangular cross section. Because the ratio of pavement width to thickness is generally  $b/h > 20$ , for numerical evaluations the bending stiffness is assumed to be  $EI = EI/(1-\nu^2)$ , to account for plate action. The x-axis is placed through the centroid as reference axis. Note that x is a Lagrange coordinate.

Graphs of the axial shearing resistance at the interface of pavement and base, caused by axial displacements, are shown in figure 3. The test results

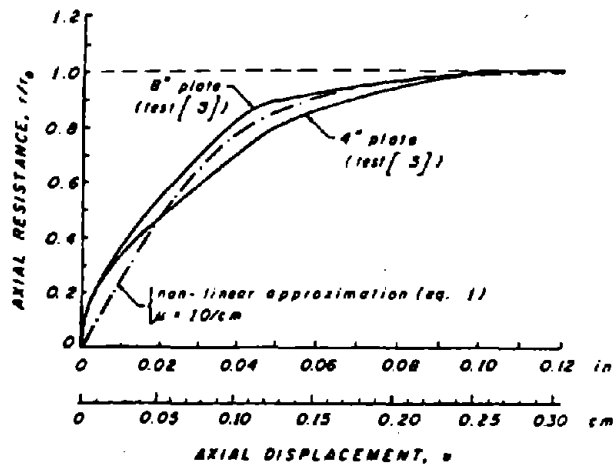


Figure 3. Axial resistance-displacement response.

are shown as solid curves.<sup>(3)</sup> In the following this response is represented by the non-linear relation:

$$r(x) = r_0 \operatorname{tgh}[\mu u(x)] \quad (1)$$

shown as the dash-dot-dash curve. The parameter  $r_0$  is the sliding frictional resistance and  $\mu$  is a second parameter for fitting the analytical expression with the test data. The shown curve is for  $\mu = 10/\text{cm}$ . Note that  $r_0 = br_0^*$  where  $b$  is the width of pavement under consideration and  $r_0^*$  is the sliding frictional resistance per unit area. Values for  $r_0^*$ , based on test data from references 3 and 4 are shown in figure 4. Although the resistance response shown in figure 3 is non-elastic, the use of relation (1) is justified because during blowup the axial displacements are monotonically increasing.

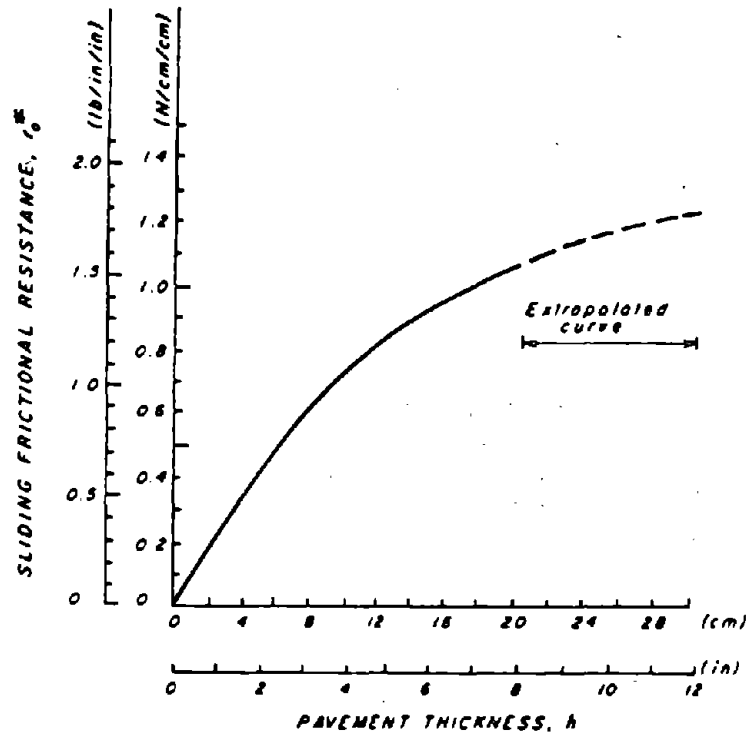


Figure 4. Sliding frictional resistance as a function of pavement thickness (based on test data presented in references 3 and 4).

In the following analysis it is assumed that the pavement is subjected to a uniform temperature increase,  $T_0$ , above neutral and a uniformly distributed own weight,  $q_0$ , per unit length of pavement axis. Because the vertical deflections in the adjoining region are very small, it is assumed that the base is rigid. As shown in reference 5 for a related problem, this appears to be justified. Furthermore it is assumed that prior and during buckling the response of the concrete pavement is elastic.

An important feature of the formulation to be used is that, although the resulting differential equations are non-linear, they can be solved in closed form. The solution yields the post-buckling displacements and axial forces in the pavements. The anticipated results are shown schematically in figure 5.

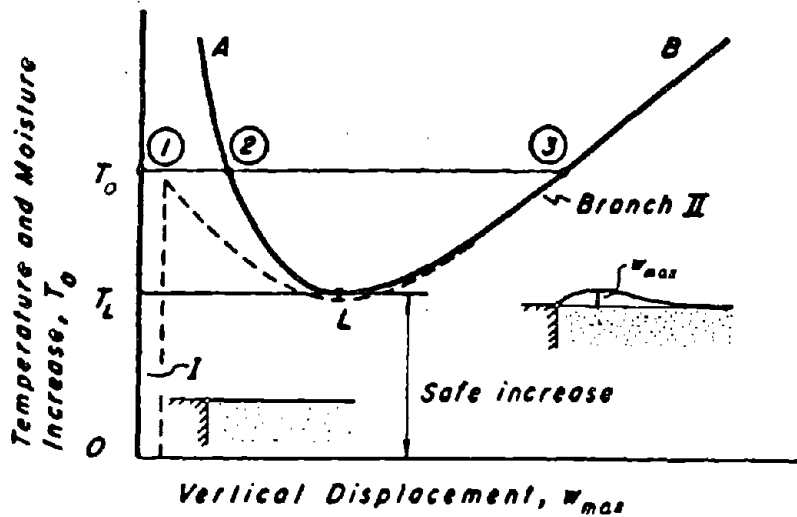


Figure 5. Typical equilibrium branches for pavement.

Each point on the shown equilibrium branches corresponds to an equilibrium configuration of the pavement. Branch I corresponds to the straight unbuckled equilibrium states and Branch II to the vertically deformed states. When the pavement is subjected to a temperature increase  $T_0 < T_L$ , there exists only the straight (stable) equilibrium configuration. However, to a temperature increase  $T_0 > T_L$ , there correspond three states of equilibrium: The (stable) straight state  $\textcircled{1}$ , the (unstable) vertically deformed configuration  $\textcircled{2}$  on branch AL, and the (stable) vertically deformed configuration  $\textcircled{3}$  on branch LB. Thus, when the pavement buckles at a temperature increase  $T_0 > T_L$ , it will move to the deformed equilibrium configuration  $\textcircled{3}$  on branch LB.

A static stability analysis of a concrete pavement subjected to axial compression forces consists of two parts: (1) The determination of all equilibrium states and (2) the investigation of which of the determined equilibrium states are stable and which are not. From the nature of the post-buckling equilibrium branches, and their stability, it follows that the "safe range" of temperature increases to prevent pavement buckling may be determined solely from the post-buckling equilibrium branch, as shown in figure 5. It is:

$$0 < T_0 < T_L \quad (2)$$

This concept was used in the two earlier analyses, references 1 and 2, and will be used in the following study.

Because pavements are usually not "perfectly" straight, it is of interest to know the effect of vertical geometric imperfections on the safe temperature range. An equilibrium branch for relatively small imperfections is shown schematically in figure 5, as a dashed line. Noting that the  $T_c$ -value for this branch is very close to the  $T_c$ -value for the perfectly straight pavement it is concluded that criterion (2) is also valid for a pavement with small vertical imperfections.

CHAPTER III. ANALYSIS OF PAVEMENT

Following the methodology developed in references 1 and 2, the equilibrium states of the vertically deformed pavement, shown in figure 1(b), are described by the differential equations:

$$\left. \begin{aligned} (\overline{EI}w_1'')'' - [EA(\epsilon_1 - \alpha T_0)w_1']' &= q_0 & (a) \\ [EA(\epsilon_1 - \alpha T_0)]' &= 0 & (b) \end{aligned} \right\} 0 < x < l \quad (3)$$

and

$$\left. \begin{aligned} w_a(x) &= 0 & (a) \\ -(EAu_a')' + r_0 \operatorname{tgh}(\mu u_a) &= 0 & (b) \end{aligned} \right\} l < x < \infty \quad (4)$$

The corresponding boundary and matching conditions are:

$$w_1(0) = 0 \quad ; \quad w_1''(0) = 0 \quad ; \quad u_1(0) = 0 \quad (5)$$

$$\left. \begin{aligned} w_1(l) = 0 \quad ; \quad w_1'(l) = 0 \quad ; \quad w_1''(l) = 0 \\ u_1(l) = u_a(l) \quad ; \quad u_1'(l) = u_a'(l) \end{aligned} \right\} \quad (6)$$

and the regularity condition:

$$\lim_{x \rightarrow \infty} [u_a(x), u_a'(x)] \rightarrow 0 \quad (7)$$

The above formulation consists of three non-linear differential equations and nine conditions for the determination of the eight integration constants (4 for  $w_1$ , 2 for  $u_1$ , and 2 for  $u_a$ ) and the lift-off length  $l$ .

In equations (3) to (7),  $u_n(x)$  and  $w_n(x)$  are the axial and vertical displacements at point  $x$  of the pavement reference axis  $x$ , the subscript  $n$  denotes the pavement region, "l" refers to the buckled region and "a" refers to the adjoining region,

$$\epsilon_n = u_n' + \frac{1}{2} w_n'^2$$

( )' = d( )/dx,  $q_0$  is the constant pavement weight per unit length,  $\alpha$  and  $\mu$  are the axial resistance parameters, as defined by relation (1).  $E$  is Young's modulus of pavement,  $A$  is its cross-sectional area, and  $EI$  is its bending

stiffness.  $\alpha$  is the coefficient of linear thermal expansion of pavement and  $T_0$  is the uniform temperature increase in the pavement above the "neutral" temperature.

The corresponding axial forces, bending moments, and shearing forces in the pavement are:

$$\left. \begin{aligned} N_N(x) &= -EA(\epsilon_n - \alpha T_0) \quad ; \quad N > 0 \text{ compression} \\ M_N(x) &= -\overline{EI}w_N'' \\ V_N(x) &= -(\overline{EI}w_N'')' + EA(\epsilon_n - \alpha T_0)w_N' \end{aligned} \right\} \quad (8)$$

Next, the formulation in equations (3) to (7) is solved. The differential equations in (3) and (4) are *non-linear*. However, since equation (3a), when integrated, yields:

$$EA(\epsilon_1 - \alpha T_0) = \text{const.} = -\tilde{N}_t \quad 0 < x < l \quad (9)$$

the equation (3b) reduces, for  $\overline{EI} = \text{constant}$ , to

$$\overline{EI}w_1'''' + \tilde{N}_t w_1'' = q_0 \quad ; \quad 0 < x < l \quad (10)$$

a linear ordinary differential equation with *constant* coefficients. The differential equation (4b) is also *non-linear*. However, since it is of the form  $u'' - F[u]$  it may be easily solved. These analytical features make it possible to solve the above non-linear formulation in closed form.

According to the equations in (8), the left-hand side of equation (9) is the axial force in the buckled region. It is denoted by  $-\tilde{N}_t$ . Thus, the axial compression force in the buckled region  $0 < x < l$  is  $\tilde{N}_t$  and is constant in this domain.

The general solution of equation (10) is

$$w_1(x) = A_1 \cos \lambda x + A_2 \sin \lambda x + A_3 x + A_4 + \frac{q^*}{2\lambda^2} x^2 \quad (11)$$

where

$$\lambda = \sqrt{\tilde{N}_t / \overline{EI}} \quad ; \quad q^* = q_0 / \overline{EI} \quad (12)$$

Since for the problem under consideration  $\tilde{N}_t > 0$ , it follows that  $\lambda$  is a real number.

The integration constants  $A_1$  to  $A_4$  are determined from the first two conditions in equation (5) and in equation (6). They are:

$$\left. \begin{aligned} A_1 &= \frac{q^*}{\lambda^4} \quad ; \quad A_2 = \frac{q^*}{\lambda^4} \frac{\lambda l \sin \lambda l + \cos \lambda l - (\lambda l)^2 / 2 - 1}{\lambda l \cos \lambda l - \sin \lambda l} \\ A_3 &= \frac{q^*}{\lambda^3} \frac{\lambda l (\sin \lambda l - \lambda l \cos \lambda l / 2) + \cos \lambda l - 1}{\lambda l \cos \lambda l - \sin \lambda l} \quad ; \quad A_4 = -\frac{q^*}{\lambda^4} \end{aligned} \right\} \quad (13)$$



when  $\operatorname{tg} \lambda l \neq \lambda l$ .

Substituting the obtained  $w_1(x)$  into the condition  $w_1''(l) = 0$  given in equation (6), it follows that it is satisfied when:

$$2(1 - \cos \lambda l) = \lambda l \sin \lambda l \quad (14)$$

The non-zero roots of this equation are:

$$\lambda l = 2\pi, 8.987, \dots \quad (15)$$

It may be shown that the first root  $2\pi$  corresponds to the deformation shape shown in figure 1(b). This is the condition for the determination of  $l$ . Namely:

$$l = \frac{2\pi}{\lambda} = 2\pi \sqrt{\frac{EI}{\tilde{N}_t}} \quad (16)$$

With  $\lambda l = 2\pi$ , the constants  $A_1$  to  $A_4$  simplify and  $w_1(x)$  becomes:<sup>(6)</sup>

$$w_1(x) = \frac{q^*}{\lambda^4} \left[ \cos \frac{2\pi x}{l} - \pi \sin \frac{2\pi x}{l} - 2\pi^2 \frac{x}{l} + 2\pi^2 \left( \frac{x}{l} \right)^2 - 1 \right] \quad (17)$$

The above expression for  $w_1(x)$ , in conjunction with condition (16),  $\lambda = 2\pi/l$ , still contain the unknown  $\lambda = \sqrt{\tilde{N}_t/EI}$ . It is determined in the following as part of the solution of the remaining equations in the foregoing formulation; namely those for  $u_1(x)$  and  $u_a(x)$ .

The first integral of equation (3b), equation (9), is the *non-linear* differential equation of the first order:

$$EA(u_1' + \frac{1}{2} w_1'^2 - \alpha T_0) = -\tilde{N}_t \quad 0 < x < l \quad (9')$$

Since at this point of the analysis  $w_1(x)$  is a known function, given in equation (17), the above equation reduces to the linear ordinary differential equation for  $u_1$

$$u_1'(x) = \left( \alpha T_0 - \frac{\tilde{N}_t}{EA} \right) - \frac{1}{2} w_1'^2(x) \quad (18)$$

Integrating it from 0 to  $x$ , and noting that according to equation (5)  $u_1(0)=0$ , the following is obtained:

$$u_1(x) = \left( \alpha T_0 - \frac{\tilde{N}_t}{EA} \right) x - \frac{1}{2} \int_0^x w_1'^2(\xi) d\xi \quad (19)$$

Because  $EA = \text{constant}$  and  $w_a(x) \equiv 0$ , equation (4b) reduces to the *non-linear* differential equation:

$$EAu_a'' = r_0 \operatorname{tgh}[\mu u_a(x)] \quad l < x < \infty \quad (20)$$

This equation is a special case of  $u''(x) = F[u(x)]$ ; a well known equation from non-linear vibration theory. It is solved, in closed form, by noting the identity  $2u'' = d(u'^2)/du$ . With it, equation (20) may be written as:

$$d(u_a'^2) = \frac{2r_0}{EA} \operatorname{tgh}(\mu u_a) du_a \quad (21)$$

Integrating it yields:

$$\begin{aligned} u_a'^2(x) &= \frac{2r_0}{EA} \int \operatorname{tgh}(\mu u_a) du_a + B_1 \\ &= \frac{2r_0}{EA} \ln\{\cosh[\mu u_a(x)]\} + B_1 \end{aligned} \quad (22)$$

Because the regularity condition, equation (7), it follows that  $B_1 = 0$ . Thus:

$$u_a'(x) = (\pm) \sqrt{\frac{2r_0}{\mu EA} \ln(\cosh[\mu u_a(l)])} \quad (23)$$

The positive sign is retained, because the anticipated axial displacements  $u_a(x)$  for  $x > 0$  will be negative, decreasing in magnitude with increasing  $x$ .

Substituting  $u_1'(x)$  given in (18) and  $u_a'(x)$  given in (23) into the fifth condition of (6),  $u_1'(l) = u_a'(l)$ , and noting that according to the second condition of (6)  $w_1'(l) = 0$ , we obtain:

$$\alpha T_0 - \frac{\tilde{N}_T}{EA} - \sqrt{\frac{2r_0}{\mu EA} \ln(\cosh[\mu u_a(l)])} = 0 \quad (24)$$

According to equation (6) the condition  $u_1(l) = u_a(l)$  still has to be satisfied. The expression  $u_1(l)$  is obtained by integrating equation (18) from 0 to  $l$ . Since  $u_1(0) = 0$  it follows that:

$$u_1(l) = \left[ \alpha T_0 - \frac{\tilde{N}_T}{EA} \right] l - \frac{1}{2} \int_0^l w_1'^2(x) dx \quad (25)$$

Eliminating  $u_a(l)$  in equation (24) using the remaining matching condition  $u_a(l) = u_1(l)$  and then utilizing equation (25), we obtain:

$$\alpha T_0 - \frac{\tilde{N}_t}{EA} = \sqrt{\frac{2r_0}{\mu EA} \ln \left[ \cosh \left\{ \mu \left[ \left( \alpha T_0 - \frac{\tilde{N}_t}{EA} \right) l - J \right] \right\} \right]} \quad (26)$$

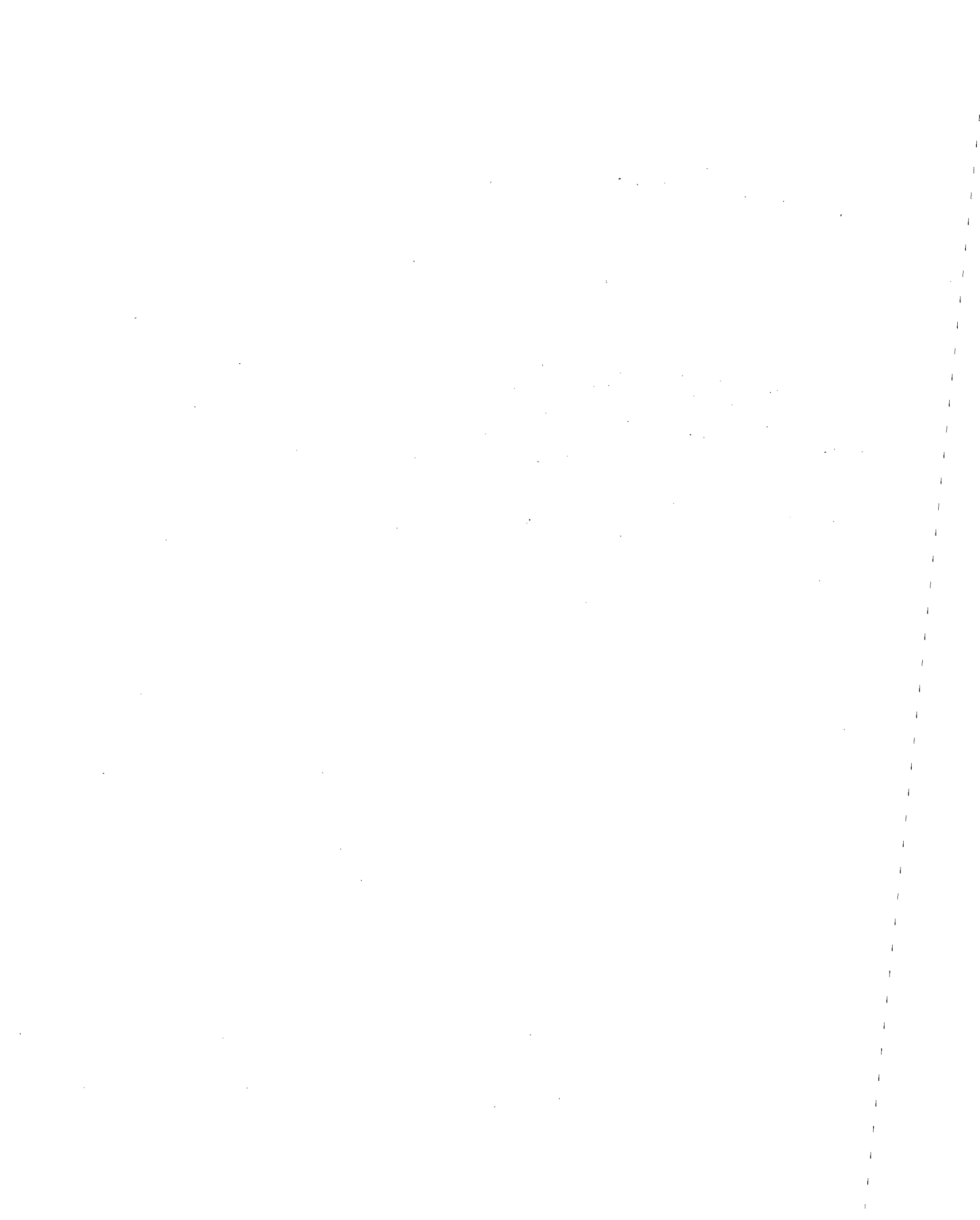
where: (6)

$$J = \frac{1}{2} \int_0^l w_1'^2(\xi) d\xi = 0.00008715 q_*^2 l^7. \quad (27)$$

The solution for the problem shown in figure 1 is just completed. For a given concrete pavement (i.e. for known values of  $E$ ,  $A$ ,  $I$ ,  $\nu$ ,  $\alpha$ ,  $q_0$ ,  $r_0$  and  $\mu$ ) the numerical evaluation of the obtained solution consists of the following steps: Choose a positive value of  $\tilde{N}_t$  and determine the corresponding  $\lambda = \sqrt{\tilde{N}_t / EI}$  and then  $\gamma = 2r_0 / \lambda$ . For this  $(\lambda, \gamma)$  pair calculate the corresponding  $T_0$  value from equation (26) in conjunction with equation (27). The corresponding value of  $w_{max}$  is calculated from equation (17), by first forming  $dw_1/dx = 0$  which yields  $x/l = 0.3464$  and then by substituting this value into  $w_1(x)$ . The result is:

$$w_{max} = - 0.005532 q_0 l^4 / EI \quad (28)$$

These steps are repeated for a range of  $N_t$  values of interest.



## CHAPTER IV. NUMERICAL EVALUATION AND DISCUSSION OF RESULTS

The numerical evaluation of the derived solution was performed for a pavement of constant rectangular cross section, for pavement thicknesses  $h = 6$  in (15 cm), 8 in (20 cm), 10 in (25 cm), 12 in (30 cm) and 14 in (35 cm). It may be easily verified that the obtained results are independent of the pavement width  $b$  because  $r_o$ ,  $A$ ,  $I$ , and  $\bar{N}_l$  are linearly varying with  $b$ . The chosen pavement parameters are:

$$\left. \begin{aligned} E &= 4.35 \times 10^6 \text{ lb/in}^2 \text{ (3,000 kN/cm}^2\text{)} \quad ; \quad \nu = 0.3 \\ \alpha &= 0.5 \times 10^{-5}/^\circ\text{F (} 0.9 \times 10^{-5}/^\circ\text{C)} \quad ; \quad \gamma = 150 \text{ lb/ft}^3 \text{ (23.6 kN/m}^3\text{)} \end{aligned} \right\} \quad (29)$$

$\gamma$  being the unit weight of the pavement material.

Since generally the reinforcement ratio in the pavement is very low (0.5 to 0.75 percent), and it is usually placed near the mid-plane of the cross section, the effect of the reinforcing bars was neglected when calculating the cross-sectional area  $A$  and the effective flexural stiffness  $EI - EI/(1 - \nu^2)$ .

The sliding frictional resistance values  $r_o^*$  at the interface of pavement and base per unit length and unit width of pavement, as defined in equation (1), was determined from the test data in reference 3 and are shown in table 1.

Table 1. Dependence of  $r_o^*$  on  $h$ .<sup>(3)</sup>

$h$	in	6	8	10	12	14
	(cm)	(15)	(20)	(25)	(30)	(35)
$r_o^*$	lb/in <sup>2</sup>	1.34	1.55	1.65	1.74	1.81
	(N/cm <sup>2</sup> )	(0.92)	(107)	(1.14)	(1.20)	(1.25)

To closely approximate the test data in references 3 and 4, it was assumed that  $\mu = 25.4/\text{in}$  (10/cm) for all five pavement thicknesses, as shown in figure 3.

The obtained solution was evaluated for the above parameters. The results are shown in figure 6. Note the drop of the axial force in the buckled region. Also note that the magnitude of the lift-off displacement,  $w_{\max}$  depends on the temperature increase  $T_o$  at which buckling takes place; the higher the temperature increase the larger is  $w_{\max}$ . For example for a pavement with  $h = 25$  cm, to an increase  $T_o = T_L = 56^\circ\text{C}$  there corresponds a  $w_{\max} = 36$  cm where as to  $T_o = 65^\circ\text{C}$  there corresponds  $w_{\max} = 75$  cm; thus more then twice as large.

To show the effect of pavement thickness  $h$  on the safe tempture increases above neutral, these results are plotted in figure 7 as a solid line. Also shown in this figure, as a dashed line, is the corresponding curve.

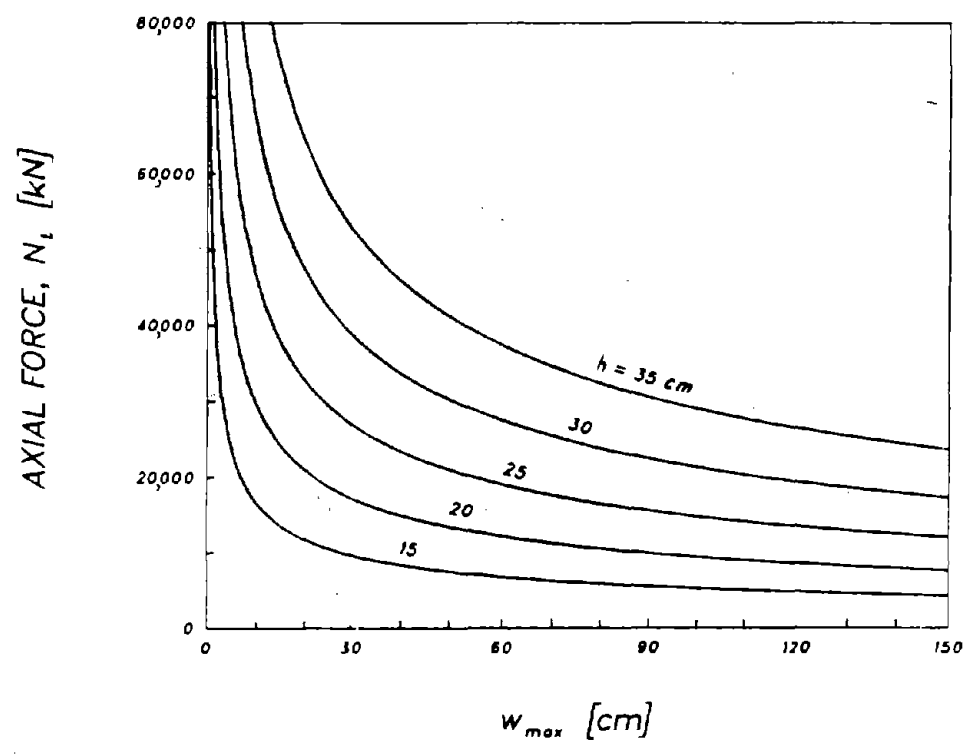
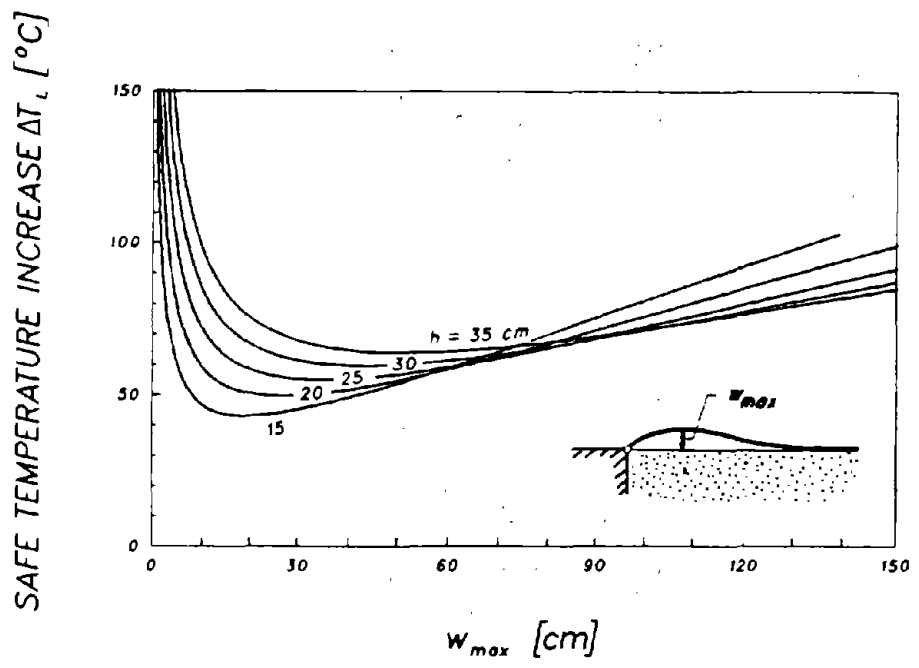


Figure 6. Equilibrium branches and axial forces in pavement. (For the determination of  $\hat{N}_1$ ,  $b = 24$  ft (725 cm) was used.)

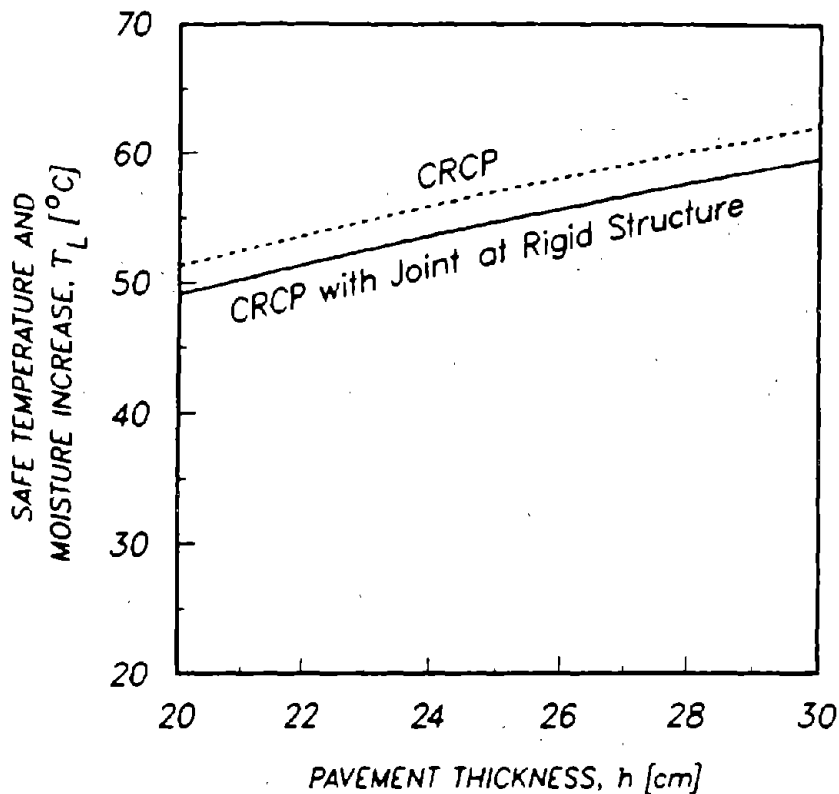


Figure 7. Dependence of  $T_L$  on pavement thickness  $h$ .

for the continuously reinforced concrete pavement (CRCP) far away from the bridge abutment as determined in references 1 and 2. Note the effect of the hinge and "rigid" abutment on the safe temperature increase.

Next, consider the case when the pavement that was rigidly attached to the abutment exhibits, after a time, a transverse crack at this location. This will correspond to the case shown in figure 1, but with rotational resistance in the hinge that will depend on the degree of spalling, as indicated in figure 2. This resistance will increase the safe temperature increase above the values of the solid curve shown in figure 7.

The axial compression force that a CRCP will generate is  $N_T = EA \alpha T_0$ . For the pavement parameters used above,  $h = 10$  in (25 cm) and  $b = 24$  ft (725 cm), this corresponds to an axial compression force, per °C, of:

$$\frac{N_T}{T_0} = EA\alpha = 3,000 \times 25 \times 725 \times 0.9 \times 10^{-6} = 490 \text{ kN/}^\circ\text{C}$$

This rather large compression force (per °C) demonstrates the importance of the *neutral* temperature on pavement blows. The higher the *neutral* temperature the less likely is the possibility that blowup will occur. Note, however, that although a higher *neutral* temperature may prevent blowups, it will lead (in CRCP's) to higher axial tensile forces in the pavement during the winter, that

may cause pavement ruptures. This indicates that the construction season has an effect on the pavement response.

Some pavement engineers are of the opinion that the shearing resistance at the interface contributes to the formation of transverse cracks and thus should be minimized; for example, by placing plastic sheets at the interface. To establish this effect on blowups, the obtained solution was evaluated numerically for different values of the sliding frictional resistance parameter,  $r_0^*$ , without changing the other parameters. The results are shown in figure 8. Note that, according to the above analysis and those of

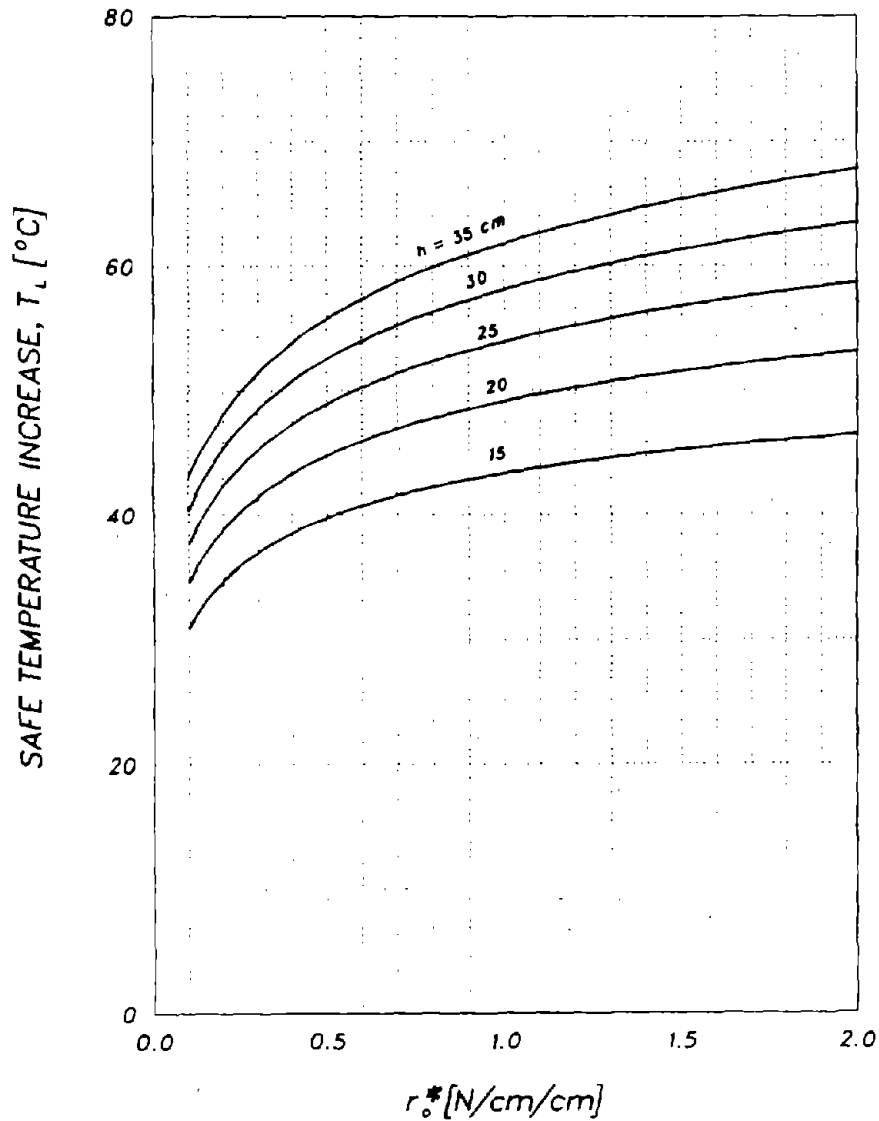


Figure 8. Dependence of  $T_s$  on  $r_0^*$



references 1 and 2, a reduction of this resistance,  $r_0^*$ , also reduces the safe temperature range,  $0 < T < T_c$ , and thus has an adverse effect on pavement blowups. For preventing blowups, the sliding frictional resistances should be as high as possible.

To reduce the axial forces that a concrete highway pavement will exert on a bridge pier, short section of a black top slab is often inserted between the pavement and the abutment. This allows the end zone of the pavement, during periods of high temperatures, to expand toward the bridge, preventing the build up of high compression forces in the vicinity of the bridge abutment. However, the continuous expansion and contraction of the pavement in this region creates maintenance problems, especially when the shearing resistance between pavement and subgrade is very small.

## REFERENCES

1. A. D. Kerr, and P. J. Shade, "Analysis of Concrete Pavement Blowups," *Acta Mechanica*, Springer Verlag, Vienna, Austria, Vol. 52, 1984.
2. A. D. Kerr, and W. A. Dallis, Jr., "Blowup of Concrete Pavements," *Journal of Transportation Engineering*, ASCE, Vol. 111, No. 1, 1985.
3. L. W. Teller, and E. C. Sutherland, "Observed Effects of Variations in Temperature and Moisture on the Size, Shape and Stress Resistance of Concrete Pavement Slabs," *Public Roads*, Public Roads Administration, Washington, D.C., Vol. 16, Nr. 9, 1935.
4. E. F. Kelley, "Application of the Results of Research to the Structural Design of Concrete Pavements," *Public Roads*, Public Roads Administration, Washington, D.C., Vol. 20, Nr. 6, 1939.
5. Y. M. El-Aini, "Effect of Foundation Stiffness on Track Buckling," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 102, EM 3, 1976.
6. A. D. Kerr, "An Improved Analysis for Thermal Track Buckling," *International Journal for Non-Linear Mechanics*, Pergamon Press, Vol. 15, 1980, page 107.