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# Curved Steel Bridge Research Project Interim Report I: Synthesis



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Research and Development  
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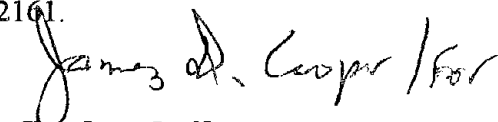
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## FOREWORD

This report is part of the Federal Highway Administration's Curved Steel Bridge Research Project. The report will be of interest to bridge design engineers and structural research engineers alike because of the many aspects of curved bridge loading that were investigated.

This report summarizes the results of a comprehensive literature search. This information is also available in an electronic data base.

Copies may be obtained from the National Technical Information Service, 5285 Port Royal Road, Springfield, Virginia 22161.

A handwritten signature in black ink that reads "James A. Cooper / For". The signature is written in a cursive style.


Charles J. Nemmers  
Director, Office of Engineering and  
Highway Operations Research and Development

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16. Abstract The objectives of the FHWA Curved Steel Bridge Program are (1) to conduct fundamental research into the structural behavior of curved steel flexural members and bridges, and (2) to address construction issues, in order to provide adequate information to develop and clarify design specifications.  The work under this program is a coordinated effort between the Transportation Research Board (TRB), the Federal Highway Administration (FHWA), and participating States under a Highway Planning and Research (HP&R) Pooled Fund Study. This program focuses on four areas: (1) synthesis of work that has been done since the Consortium of University Research Teams (CURT) Project; (2) update of the current specification in a load factor design format; (3) conduct of research recommended by Structural Stability Research Council's (SSRC) Task Group 14 at the April 14-15, 1991 workshop; and (4) development of a load and resistance factor design specification based on research conducted under area 3. Areas 1 and 3 are conducted by FHWA as a pooled fund study with an administrative contract. Area 2 is conducted by TRB under the National Cooperative Highway Research Program (NCHRP) Project 12-38. Area 4 is proposed to be addressed by NCHRP at a future date. This report summarizes the results of a comprehensive literature search under the FHWA research program.				
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## SI\* (MODERN METRIC) CONVERSION FACTORS

### APPROXIMATE CONVERSIONS TO SI UNITS

### APPROXIMATE CONVERSIONS FROM SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol	Symbol	When You Know	Multiply By	To Find	Symbol
<b>LENGTH</b>					<b>LENGTH</b>				
in	inches	25.4	millimeters	mm	mm	millimeters	0.039	inches	in
ft	feet	0.305	meters	m	m	meters	3.28	feet	ft
yd	yards	0.914	meters	m	m	meters	1.09	yards	yd
mi	miles	1.61	kilometers	km	km	kilometers	0.621	miles	mi
<b>AREA</b>					<b>AREA</b>				
in <sup>2</sup>	square inches	645.2	square millimeters	mm <sup>2</sup>	mm <sup>2</sup>	square millimeters	0.0016	square inches	in <sup>2</sup>
ft <sup>2</sup>	square feet	0.093	square meters	m <sup>2</sup>	m <sup>2</sup>	square meters	10.764	square feet	ft <sup>2</sup>
yd <sup>2</sup>	square yards	0.836	square meters	m <sup>2</sup>	m <sup>2</sup>	square meters	1.195	square yards	yd <sup>2</sup>
ac	acres	0.405	hectares	ha	ha	hectares	2.47	acres	ac
mi <sup>2</sup>	square miles	2.59	square kilometers	km <sup>2</sup>	km <sup>2</sup>	square kilometers	0.386	square miles	mi
<b>VOLUME</b>					<b>VOLUME</b>				
fl.oz	fluid ounces	29.57	milliliters	mL	mL	milliliters	0.034	fluid ounces	fl.oz
gal	gallons	3.785	liters	L	L	liters	0.264	gallons	gal
ft <sup>3</sup>	cubic feet	0.028	cubic meters	m <sup>3</sup>	m <sup>3</sup>	cubic meters	35.71	cubic feet	ft <sup>3</sup>
yd <sup>3</sup>	cubic yards	0.765	cubic meters	m <sup>3</sup>	m <sup>3</sup>	cubic meters	1.307	cubic yards	yd <sup>3</sup>
NOTE: Volumes greater than 1000 L shall be shown in m <sup>3</sup>									
<b>MASS</b>					<b>MASS</b>				
oz	ounces	28.35	grams	g	g	grams	0.035	ounces	oz
lb	pounds	0.454	kilograms	kg	kg	kilograms	2.202	pounds	lb
T	short tons (2000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")	Mg (or "t")	megagrams (or "metric ton")	1.103	short tons (2000 lb)	T
<b>TEMPERATURE (exact)</b>					<b>TEMPERATURE (exact)</b>				
'F	Fahrenheit temperature	5(F-32)/9 or (F-32)/1.8	Celsius temperature	°C	°C	Celsius temperature	1.8C + 32	Fahrenheit temperature	°F
<b>ILLUMINATION</b>					<b>ILLUMINATION</b>				
fc	foot-candles	10.76	lux	lx	lx	lux	0.0929	foot-candles	fc
fl	foot-Lamberts	3.426	candela/m <sup>2</sup>	cd/m <sup>2</sup>	cd/m <sup>2</sup>	candela/m <sup>2</sup>	0.2919	foot-Lamberts	fl
<b>FORCE and PRESSURE or STRESS</b>					<b>FORCE and PRESSURE or STRESS</b>				
lbf	poundforce	4.45	newtons	N	N	newtons	0.225	poundforce	lbf
lbf/in <sup>2</sup>	poundforce per square inch	6.89	kilopascals	kPa	kPa	kilopascals	0.145	poundforce per square inch	lbf/in <sup>2</sup>

\*SI is the symbol for the International System of Units. Appropriate rounding should be made to comply with Section 4 of ASTM E380.

(Revised September 1993)

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## LIST OF ABBREVIATIONS AND SYMBOLS

- a = Distance between transverse stiffeners
- b = Width of half of the flange
- $b_f$  = Width of compression flange
- $C_w$  = Warping constant
- d = Depth of the section
- E = Young's modulus of steel
- $F_y$  = Yield stress
- $f_b$  = Allowable bending stress
- G = Shear modulus of steel
- $I_y$  = Moment of inertia about the y-axis
- J = Torsion constant
- L = Length of girder
- $M_u$  = Ultimate moment
- R = Radius of curvature of girder web
- $t_f$  = Flange thickness
- $t_w$  = Web thickness
- $\nu$  = Poisson's ratio for the steel



## CHAPTER I. INTRODUCTION

### GENERAL

During the first half of the 19th century, De Saint Venant published a memoir that marked the birth of all research efforts, which have been published to date, on the analysis and design of horizontally curved girders.<sup>(1)</sup> Since then, thousands of pages of technical papers, reports, and books have been published in the literature concerning various applications in the fields of civil, mechanical, and aerospace engineering.

Despite the considerable progress begun almost 150 years ago, serious studies pertaining to the analysis and design of horizontally curved bridges commenced only 30 years ago when the Federal Highway Administration, in 1969, formed the Consortium of University Research Teams (CURT). This team consisted of Carnegie Mellon University, The University of Pennsylvania, the University of Rhode Island, and Syracuse University, whose research efforts along with those at the University of Maryland resulted in the initial development of working stress design criteria and tentative design specifications. The American Society of Civil Engineers and the American Association of State Highway and Transportation Officials (ASCE-AASHTO) Committee on Flexural Members compiled the results of most of research efforts prior to 1976 and presented a single source set of recommendations pertaining to the design of curved I-girder bridges.<sup>(2)</sup> The CURT research activity was shortly followed by the development of the Load Factor Design criteria adopted by AASHTO (1980) along with the working stress design criteria in the first Guide Specifications for Horizontally Curved Highway Bridges.<sup>(3,4,5)</sup>

A survey of most published works pertaining to horizontally curved bridges was first presented by McManus et al. whose bibliography list contained 202 references, four of which only dealt with box girders.<sup>(6)</sup> McManus' paper was discussed by other authors who added additional references to the original list.<sup>(7,8,9)</sup> Nine years later, the ASCE-AASHTO Committee on Flexural Members (1978a) presented a state-of-the-art report that provides 106 references dealing primarily with horizontally curved box girders.<sup>(10)</sup> The Committee also presented results of a survey pertaining to the geometry, design, detail, construction and performance of box-girder bridges constructed in the United States, Canada, Europe, and Japan (ASCE-AASHTO Committee on Flexural Members 1978b).<sup>(11)</sup> The survey was an update of a more limited survey published by the AASHO-ASCE Committee on Flexural members (1973).<sup>(12)</sup> Very recently, Nakai and Yoo published a book that offers a comprehensive listing of numerous papers, with particular attention to the Japanese literature.<sup>(13)</sup>

The current 1993 AASHTO Guide Specification For Horizontally Curved Highway Bridges is primarily based upon research work conducted prior to 1978.<sup>(14)</sup> Since then, a significant amount of work has been conducted to enhance the specifications and to better understand the behavior of curved girders. Unfortunately, the results of these various research activities are scattered and in some cases unevaluated. Hence, a renewed review of available work on curved bridges would be of significant benefit to the engineering community.

## **OBJECTIVE**

The objective of this task was to perform a comprehensive up-to-date literature search on work related to horizontally curved steel box and I-girders and to provide a synopsis of the information contained in each reference. All references surveyed or assembled in this report were placed in an electronic data base that is easy to access, query , and update as additional research is completed.

## **RESEARCH PROCEDURE**

In order to accomplish the aforementioned objective, a comprehensive literature search using both computerized searching and "old fashioned" manual paging was conducted. This initial search resulted in the discovery of almost 900 references. The title of each, along with their abstracts, were then examined to determine their applicability to curved steel girders. References that were not relevant to curved steel bridges were eliminated. For instance, a paper related to model tests on curved concrete beams was deleted from the initial list. General references on analysis that may apply to steel, concrete, and so forth have been kept in the list. This process resulted in almost 750 references, which were placed in the electronic data base described next.

## **ELECTRONIC DATA BASE**

The information compiled on horizontally curved bridges was stored in the Microsoft ACCESS data base management software, which runs under the WINDOWS environment. This data base program was chosen for its user friendliness, its straightforward, intuitive and graphical approach to design, its ability to generate printed reports and screen forms and its compatibility with almost all the other commercially available data base programs.

It was decided to store the information pertaining to a particular reference in six different fields. The first three fields contain material about the title, author and year of publication. The fourth field contains the source of publication (e.g. journal or conference name, publisher, .etc.), while the fifth field contains some additional information regarding the source such as the order or contract number. The last field gives a summary or abstract of the publication.

In addition to these six major fields, some check-boxes were added to the data base form to act as keywords in this survey. These check-boxes can be used to classify whether a publication is on Box or I-girders, Static, Dynamic, Thermal, or Fatigue analyses, or other aspects of behavior (see figure 1).

The information could then be used to sort and filter required publications. As an example, one could request all (Design) work by (Heins) which was published after (1975) and that information could be sorted according to the (Year) in an ascending order. The result of a filtering and/or sorting process can be on-screen (temporary), or can be saved as a query which can be used at a later time. Additionally, one can also search a certain field for a particular word or name.

A data base file is typically large in size since it contains the information that needs to be stored, as well as the data required to design and tailor the forms, tables, reports and queries used in the data base. This makes it difficult to save the data base file on a regular floppy disk without resorting to compressing and decompressing routines. One of the useful features of ACCESS is that it can save the data base in a file that contains the empty or skeleton form, while saving the stored information as a separate text (ASCII) file. This usually makes it possible to copy both files to a medium or large size floppy disk. This feature also makes it easy to export the information to any other data base management program if it accepts text files.

## **SYNTHESIS**

As stated earlier all collected titles along with their abstracts were placed in an electronic data base. This allows the acquisition of a needed synopsis of any curved-girder article related to any desirable key word(s) (author, title, phenomenon,...etc.) in a few seconds. This synthesis reports only on references that: (1) were felt to have been the key ingredients in the development of the existing guide specifications for horizontally curved highway bridges, and (2) may play important roles in the development of enhanced curved-bridge analysis, design, construction, and fabrication guidelines. For example, 1932 a reference by Moorman entitled "Semi-graphical method of analysis for horizontally curved-beams" can be found in the data base but is not discussed in this report. Therefore, only 540 of the 750 references need be reviewed in this task. A list of works comprising the references that have not been selected for reviews is placed in the bibliography section of this report.

Because of budget and time constraint, about 300 reports were reviewed briefly in this study, about 210 were recommended for further review (140 of these reports were difficult to obtain). All 210 references were placed in the Bibliography and marked with either an astrisk to identify references that were difficult to obtain, or with the # symbol to identify references that were available but were not reviewed.

It should be noted that a synthesis of this nature cannot sufficiently present the details described in each reference, but rather present some of their more interesting highlights for future evaluation.

<b>TITLE</b>	Inelastic Flange Buckling of Curved Plate Girders		
<b>AUTHOR</b>	Culver, C.G. and Nasir, G. (1971)	Year 1971	
<b>INFO</b>	ASCE, Journal of Structural Division, April, vol. 97, no. ST4, pp. 1239-1256	Summary <input type="checkbox"/>	
<b>INFO2</b>	PROC PAPER 8072	Author Abstract <input checked="" type="checkbox"/>	
<b>A B S T R A C T</b>	<p>Local buckling of the flanges of horizontally curved plate girders is studied analytically. Buckling in both the elastic and inelastic range is considered. The mathematical model consists of small deflection plate equations for an isotropic media in the elastic range and an orthotropic media in the inelastic range. Prebuckling stresses due to bending and nonuniform torsion and residual stresses due to the fabrication process are included in the mathematical model. The influence of the stress distribution and yielding of portions of the curved plate on the buckling coefficients is studied. Plate buckling curves relating the internal forces in curved beams to the flange dimensions, width-to-thickness ratio, required to prevent buckling are also presented.</p>	FEM <input type="checkbox"/>	Arbitrary Shape <input type="checkbox"/>
		Box-Girder <input type="checkbox"/>	
		FSM <input type="checkbox"/>	I-Girder <input checked="" type="checkbox"/>
		FDM <input type="checkbox"/>	Static Analysis <input type="checkbox"/>
		S. Defl. <input type="checkbox"/>	Dynamic Analysis <input type="checkbox"/>
		GDE <input type="checkbox"/>	Experimental <input type="checkbox"/>
		P. Grid <input type="checkbox"/>	Design <input type="checkbox"/>
		S. Frame <input type="checkbox"/>	Thermal <input type="checkbox"/>
		V-Load <input type="checkbox"/>	Stability <input checked="" type="checkbox"/>
		Available <input checked="" type="checkbox"/>	Construction <input type="checkbox"/>
Referenced <input checked="" type="checkbox"/>	Fatigue <input type="checkbox"/>		
	Diaphragms <input type="checkbox"/>		
	Bearing <input type="checkbox"/>		

Figure 1. Initial screen output of the electronic data base.

## CHAPTER II. ANALYSIS METHODS

### INTRODUCTION

The analysis methods presented in this report are classified into two major categories: Approximate and Refined methods. The approximate methods require minimal modeling effort on part of the designer, and hence are adequate for preliminary analysis and design purposes. The following approximate methods are surveyed in this study:

- The Plane Grid Method.
- The Space Frame Method.
- The V-Load Method.

The refined methods, on the other hand, are somewhat more elaborate, computationally intensive, and time consuming in terms of modeling. Therefore, the methods that fall in this class should be used for the final or detailed analyses. In this report, the following are classified as refined methods:

- The Finite Element Method.
- The Finite Strip Method.
- The Finite Difference Method.
- Analytical Solution to Differential Equations.
- The Slope Deflection Method.

Some useful details and comparisons on the available analysis techniques were provided by Buckle and Hood on curved box girders and by Heins and Yilmaz et al. on curved box and I-girders.<sup>(15,16,17)</sup>

In succeeding sections of this chapter, references available on the analysis methods pertaining to horizontally curved steel bridges are presented. Reviewed references are grouped into articles that addressed the static and/or the dynamic response of curved bridge decks, curved bridges with arbitrary cross sections, and curved bridges with I- sections, and curved bridges with box sections.

### APPROXIMATE METHODS

**Plane Grid Method:** The structure in this method is modeled as an assemblage of two-dimensional grid members with one translational and two rotational degrees of freedom. This method does not account for nonuniform torsion (warping), and hence, can be used only for initial member sizing.

**Space Frame Method:** This method was first introduced in 1973 by Brennan and Mandel for the analysis of open and closed curved members.<sup>(18)</sup> The curved members are idealized as three-

dimensional straight members, while the diaphragms and lateral bracing are assumed as truss-like members which can carry only axial loads. The effect of warping is not usually included in this analysis, which makes this method practical for initial design purposes.

**V-Load Method:** This method uses equivalent straight girders with span lengths equal to the arc lengths instead of the individual curved girders by adding self-equilibrating vertical shear forces (acting on diaphragm locations) that take the curvature into account. These loads are dependent on the radius of curvature, bridge width, and diaphragm spacing.<sup>(19,20,21)</sup>

## **REFINED METHODS**

**Finite Element Method:** This approach discretizes the structure into small divisions (elements) where each element is defined by a specified number of nodes. The behavior of each element (and ultimately the structure) is assumed to be a function of its nodal quantities (displacements and/or stresses), which serve as the primary unknowns in this formulation. This is one of the most general and accurate methods to use since it does not put any limitation on the geometry, loads or boundary conditions, and can be applied to open/closed girders and static/dynamic analysis. Additionally, the structure's response can always be improved by refining the mesh used and by increasing the number of nodes (or degrees of freedom) for each element. However, the rather involved modeling and analysis efforts required by this method may in some cases make it impractical for preliminary analysis.

**Finite Strip Method:** In this numerical method, the curved bridge is divided into narrow strips in the circumferential direction which are supported in their radial direction. The analysis includes bending and membrane actions as well as warping and distortional effects.<sup>(22)</sup> Although this method provides some simplicity over the finite element method because of the smaller number of unknowns required, it does not offer the flexibility and versatility of the latter method.

**Finite Difference Method:** In this method, a grid is superimposed on the structure and the governing differential equations are replaced by algebraic difference equations that are solved for each grid point.

**Solution to the Governing Differential Equations (GDE):** In this class, an analytical solution to the GDE is obtained. The solution is usually a closed form or a convergent series solution, such as a Fourier series.

**Slope Deflection Method:** In this approach, the partial differential equations are established in terms of slope-deflection equations and the solution is assumed to be a Fourier series. The analysis includes the effects of curvature, nonuniform torsion, and diaphragms. However, the solution is usually applicable to certain number of spans and geometries.

## BRIDGE DECK ANALYSIS

Cheung studied horizontally curved bridge decks where he divided the slab into concentric simply supported strips, and assumed the deflection to be a Fourier series in the longitudinal direction and a beam function in the transverse direction.<sup>(23)</sup> Juhl derived the equations for a statically determinate lateral support system to minimize high internal stresses due to temperature change of the superstructure, and to predict the boundary displacements of a skew and curved structure.<sup>(24)</sup> Abdelraouf and Matlock introduced refined triangular planar and nonplanar elements and compared them with less refined triangular and quadrilateral elements in the analysis of bridge decks.<sup>(25)</sup> Unlike the less refined elements, the proposed elements in Abdelraouf and Matlock proved suitable for coarse meshes. Cheung studied analytically and experimentally the behavior of simply supported curved bridge decks with intermediate column supports.<sup>(26)</sup> His analytical study was based on the finite strip method, the results of which compared favorably with experimental values obtained from testing 30 1:60 scale asbestos cement curved slab decks. Dey conducted a static analysis of orthotropic curved bridge decks with two radial edges simply supported and with the other two curved edges free, using a combination of Fourier series and the finite difference technique. The governing fourth order partial differential equation of orthotropic plates was converted to an ordinary differential equation and solved by the finite difference method. Harik and Pashanasangi presented a solution for the analysis of orthotropic curved decks subjected to uniform, partial uniform and patch loads, line and partial line loads in the radial and tangential directions, and point loads.<sup>(27)</sup> The analysis is based upon an approach similar to that of the finite strip, but does not require the polynomial representation and minimization procedure often associated with the finite strip. The deck was divided into radially supported curved strips, whose deflections and loads were expressed in a Levy Fourier series. Convergence was achieved by increasing the number of modes instead of the number of elements. The spline finite strip method was applied by Cheung et al. to curved slab bridges and the obtained results demonstrated its accuracy.<sup>(28)</sup> A computer-based method of analysis was proposed by Azad et al., and Azad et al. for continuous, curved slab-type bridge decks.<sup>(29,30)</sup> Using the method of finite difference in conjunction with the method of consistent deformation and a Levy-type series formulation, equilibrium difference equations were written along the central radial line of the simply supported deck. Dey and Malhorta introduced a higher-order strip element which utilized a quintic polynomial in the radial direction.<sup>(31)</sup> The method proved easy to program and required little computational effort. Later, Verma and Dey discretized the total potential energy of bridge decks in terms of pivot displacements using finite difference operators with constant order of truncation error.<sup>(32)</sup> The total potential energy was minimized to obtain the force-displacement relationship which was solved for pivot displacements. Lakshmy et al. included in-plane and transverse shear deformations in a nine-noded isoparametric Mindlin-plate bending element.<sup>(33)</sup> Their method was claimed to be effective with both isotropic and orthotropic curved/skewed bridges.

Regarding the dynamic analysis of horizontally-curved bridge decks, Yonezawa modeled the deck as an orthotropic curved plate and calculated its natural frequency.<sup>(34)</sup> Kunukasseil and Ramakrishnan studied the dynamic response of a curved bridge deck and studied the effect of speed on the dynamic deformations of the deck.<sup>(35)</sup> Their analytical model considered the deck as an isotropic annular sector plate with the radial edges simply supported and the circular edges free. Ramakrishnan and Kunukasseil presented an analytical method for calculating the free

vibration frequencies of stiffened curved bridge decks by idealizing the deck as a system comprising a number of isotropic annular plates and circular ring segments rigidly connected to each other.<sup>(36)</sup> Using almost the same mathematical model for their bridge deck, Ramakrishnan and Kunukkasseil developed an analysis method for the dynamic response of a stiffened bridge deck to a moving vehicle idealized as a single degree of freedom system.<sup>(37)</sup> Finally, Dey idealized a curved bridge deck as a number of finite strips with orthotropic properties and the moving vehicle as a sprung mass moving at a constant speed.<sup>(38)</sup> Viscous damping was considered in this study.

## **STATIC ANALYSIS OF CURVED BRIDGES WITH ARBITRARY CROSS-SECTIONS**

**Finite Element Method:** Shore developed a fully compatible three-dimensional flat plate circular element to aid in 70 model tests.<sup>(39,40)</sup> This element was implemented in STACRB and DYNCRB, which are static and dynamic finite-element analysis programs, respectively. During the same time period, a method based on harmonic analysis in the circumferential direction and on the finite-element method in the transverse direction was presented by Meyer and Scordelis.<sup>(41)</sup> The method, however, does not satisfy differential equilibrium and force boundary conditions.

Segmental and quadrilateral elements for plate bending were used by Sawko and Merriman in a FE program for analyzing simply supported continuous isotropic and orthotropic bridges.<sup>(42)</sup> Actual tests and comparisons with previous results proved the accuracy of this method.

Lansberry and Shore and Shore and Lansberry developed an annular conforming and fully-compatible four-noded segment element for thin plates under membrane and bending stresses.<sup>(43,44)</sup> The displacement expansion was the same for all three components and was taken as the product of one-dimensional first order Hermite interpolation polynomials in polar coordinates.

The derivation of a curved beam equilibrium element by Laursen resulted in a stiffness matrix and a consistent modal loading vector.<sup>(45)</sup> This element can be easily implemented in a standard finite element code. Later on, Noor et al. included transverse shear deformation and bending-extensional coupling in mixed curved beam elements used in geometrically nonlinear analysis of deep arches.<sup>(46)</sup> The stiffness matrix was obtained using a modified form of the Hellinger-Reissner mixed variational principle.

In 1979, Yoo presented an exact displacement solution to the governing homogeneous differential equations of curved thin-walled beams.<sup>(47)</sup> The resulting torsional moment was analytically separated into St. Venant and warping torsions. Unlike typical FE procedures, no more accuracy will be gained when using finer meshes in this type of analysis. Stress and displacement (warping) function approaches were employed by Thasanatorn and Pilkey in a FE formulation of composite and anisotropic circularly curved members subject to torsional and constrained warping stresses.<sup>(48)</sup>



The governing differential equations of circularly curved beams were solved by Just and combined with the finite element procedure to formulate an exact and explicit stiffness matrix and displacement transformation matrix.<sup>(49)</sup>

In 1982, two publications addressed the nonlinear behavior of curved girders using finite element analysis. The isoparametric finite element method and the updated Lagrangian approach were used by Nakai et al. to analyze the geometrically and materially nonlinear behavior of horizontally curved bridges.<sup>(50)</sup> Second-order terms and warping effects were fully represented by Hirashima and Yoda in their nonlinear FE analysis applied to curved and twisted thin-walled beams.<sup>(51)</sup>

Bien, J. analyzed curved girders with arbitrary cross section in simple and multispans structures using a combination of Fourier series and a two-dimensional finite element technique.<sup>(52)</sup> Goto et al. presented the theoretical convergence and accuracy of the method of separation of rigid body displacements by applying it to a curved element of variable cross section, and an approximate straight element of constant cross section, and then comparing the solutions to those of the direct Lagrangian equations.<sup>(53)</sup> They proved the effectiveness of this technique for the finite displacement analysis of curved members.

Prathap studied the effect of reduced integration on shallow curved beam elements and the use of coupled displacement fields in finite rings.<sup>(54)</sup> Akoussah et al. used the penalty technique to develop a locking-free curved beam element.<sup>(55)</sup> The updated Lagrangian approach was then used to generalize the formulation to nonlinear analysis. In 1987, Yoo presented consistent stiffness, stability and mass matrices of a discrete curved finite element based on a variational principle.<sup>(56)</sup> Convergence of this model was extremely fast and formed an upper bound to the exact solution. Still dealing with the issue of the nonlinear behavior of curved girders, Elias and Chen employed the assumptions of the shallow beam theory (with shear deformations) to develop a geometrically nonlinear curved beam element for deflection and buckling analysis.<sup>(57)</sup>

A horizontally curved three-noded isoparametric beam element with or without an elastic base throughout its length was proposed by Dasgupta and Sengupta.<sup>(58)</sup> Shear deformations and torsional loading was considered in the formulation and a two point Gaussian quadrature rule was used in numerical integration. Comparisons with other analytical solutions indicated the suitability of the proposed element.

Benedtti and Tralli substituted the axis line by a B-spline in a curved beam model which resulted in the generation of an equilibrated stress resultant field.<sup>(59)</sup> Apart from the error encountered in numerical integration, this method computes the flexibility (or stiffness) matrix exactly. Concurrently, Graves and Mansour-Tehrani experimented with a method combining the finite element and finite strip methods for analyzing partially prismatic structures and applied it effectively to horizontally curved bridges.<sup>(60)</sup>

In 1990, Surana and Nguyen presented a three-dimensional beam element whose axial and transverse displacements can be of arbitrary polynomial order.<sup>(61)</sup> The formulation results in hierarchical approximation functions and nodal variables. The orders of approximation in the

longitudinal direction can be chosen to differ from that used in the transverse direction to optimize the rate of convergence of the FE solution. During the same year, Hsu et al. developed a more exact horizontally curved beam element in which the warping d.o.f. conforms to the bimoment (warping). The method proved applicable for open and closed sections.<sup>(62)</sup> In 1990, Tin-Loi and Pulmano used the static or lower bound theorem of plasticity, discretized the curved grid structure into finite elements, and linearized the nonlinear bending-torsion yield surface, which led to an easy-to-solve linear programming problem.<sup>(63)</sup>

A finite element approach based on combining beam and shell strains was proposed in 1991 by Kanarachos et al.<sup>(64)</sup> New shape functions, which were either exact or polynomial, were introduced and resulted in significantly reducing the number of nodal variables. Yang et al. showed that straight-beam elements can be used to study the buckling of curved beams.<sup>(65)</sup> The moment terms induced by initial bending moments undergoing three-dimensional rotations, and interelement compatibilities were both included in the analysis which resulted in a consistent straight-beam element. The proposed semitangential element proved to be a good substitute for the curved-beam elements.

Pantazopoulou also used new polynomial shape functions along with reduced numerical integration and an enforced geometrical constraint equation in three-dimensional curved beam elements to eliminate the torsional and membrane locking.<sup>(66)</sup>

**Finite Strip Method:** A finite strip method was proposed by Meyer and Scordelis for simply supported folded plate structures.<sup>(41)</sup> Their method used harmonic/Fourier analysis in the circumferential direction and finite element analysis in the other direction.

Arizumi et al. modeled composite curved box/plate girders with complete and incomplete interaction using curved strip elements for the concrete slab and steel girder, and used two-dimensional spring elements for the shear connectors.<sup>(67)</sup> Test and theoretical values compared favorably with the numerical results. Based on the results obtained, the slip behavior of curved composite box girders was also investigated. A two-noded strip element with one integration point and based on the Mindlin shell plate theory was introduced by Onate and Suarez.<sup>(68)</sup> All element matrices were in explicit form, and led to accurate and economical analysis.

**Finite Difference Method:** Heins and Looney presented a finite difference solution for the analysis of curved orthotropic plate and curved girder bridges.<sup>(69)</sup> This method considered interaction of all curved girders of the floor system and the slab in evaluating actual stresses. The analysis of single and continuous span curved girders was presented by Yoo et al., where they solved Vlasov equations using the finite difference technique.<sup>(70)</sup> The solution, which was implemented in a FORTRAN computer program, permitted inclusion of point wise property variations and interaction of the vertical and torsional deformations. Yoo and Heins also gave a listing and user manuals of the CURSGL computer program for the analysis of single and multispan prismatic and nonprismatic curved girders, and for CURSYS which is used for analyzing a system of curved girders with interconnected top deck and diaphragms.<sup>(71)</sup> These programs are based on solving Vlasov equations using the finite difference technique. Later on, Heins and Stroczkowski analyzed continuous curved box, tubular or I bridge girders by also

developing a computer program which solves Vlasov equations using the finite difference technique.<sup>(72)</sup>

Sheinman proposed a procedure which admits large deflections, small strains, moderately small rotations, shear deformation, and geometric imperfections.<sup>(73)</sup> The numerical solution was obtained by the modified Newton's method and finite differences.

**Analytical Solution to Differential Equations:** An analytical technique was developed by Spates and Heins and Heins and Spates to evaluate all internal forces and external deflections on a curved girder with general loading and boundary conditions.<sup>(74,75)</sup> Experimental tests on a single curved girder showed reasonable agreement with theory. Stresses and deformations of curved, open and closed cross-sectional, thin-walled beams or tubes were investigated by Nitzsche and Miller who considered warping of the cross section.<sup>(76)</sup> The concepts of shear center and the self-equilibrating stress system were not necessary since the analysis was based solely upon equilibrium, stress-strain, and strain-displacement relations.

Oran and Lin reformulated the problem of the static analysis of simply supported, fan-shaped, multibeam bridges using a Fourier series.<sup>(77)</sup> A procedure was devised to accelerate the convergence of bending moments in beams located directly under concentrated loads. Furthermore, a method was developed for the determination of influence surfaces for various effects at a specified section of the beam.

Transverse shear deformations were considered by Fettahlioglu and Tabi for curved planar structures.<sup>(78)</sup> Euler equations of motion were transformed to 3 uncoupled differential equations that are functions of normal displacements, rotations and angles of twist. Solutions to the governing differential equations were derived for the general case, with particular solutions given for special loading cases. Examples on full rings and curved beams confirmed the significant effect of shear deformations on displacements.

In 1985, Karamanlidis investigated the strain-displacement equations of a curved beam undergoing arbitrarily large deflections/rotations.<sup>(79)</sup> Two alternative procedures were used: a mathematically consistent approach and an engineering-type approach. It was shown that, upon making certain simplifying assumptions regarding higher-order terms, both approaches lead to identical results.

Kuo and Yang considered a curved beam as a limiting case of an infinite number of straight beams for which the classical straight beam equations was used to derive new curved beam equations that avoided truncation and selection of higher-order terms in conventional curved-beam formulations.<sup>(80)</sup>

**Plane Grid Method:** A computer program was presented by Lavelle and Boick and Lavelle for the analysis of planar grid structures with curved members, loaded normal to their planes.<sup>(81,82)</sup> The equations to develop the required stiffness and load matrices were presented, and a detailed flow chart and program listing were also given.

Culver et al. used the plane grid analysis to show that the behavior of individual curved girders was substantially different from that of straight girders.<sup>(83)</sup> Dawkins replaced the continuous girder by an assemblage of grid-type straight, prismatic elements, representing chords of the original curve.<sup>(84)</sup> A direct recursion-inversion solution procedure was used in a specially written computer program to solve the equilibrium equations. In this program, the section properties, loads and restraints were allowed to vary along the girder. During the same time, Lavelle et al. developed the CUGAR1 computer program for analyzing plane, curved continuous girders by replacing them with an assemblage of straight, prismatic grid-type elements.<sup>(85)</sup>

An experimental and theoretical investigation was made by Douglas et al. to determine the structural behavior of three horizontally curved girder highway bridges located in Birmingham, Alabama.<sup>(86)</sup> The plane grid method for curved prismatic members, which neglects warping, and another rigorous mathematical method, which includes the effect of warping, were both used for the theoretical analysis, and their results were compared to the experimental values. Concurrently, the response of a symmetrical two-span continuous, horizontally curved steel girder bridge was investigated experimentally and theoretically (using planar grid analogy) by Beal and Kissane.<sup>(87)</sup> The results showed that the planar grid analysis method was suitable for predicting deflections and in-plane bending moments. However, the experimental results showed that the magnitude of the lateral flange bending stresses due to dead load was significant although the planar grid cannot determine such stresses.

Fu and Sahin incorporated the AASHTO guide specification provisions (working stress and load factor) into a two-dimensional grid based computer program that was intended for use in the design, rating, and rehabilitation of horizontally curved box and I-girder bridges.<sup>(88)</sup>

**Space Frame Method:** Morris presented the derivation of three-dimensional curved beam stiffness coefficients by calculating nodal reactions resulting from imposing unit displacements on a single degree of freedom while keeping all other nodes fixed.<sup>(89)</sup> This process is applied repeatedly until all coefficients of the 12 x 12 stiffness matrix are obtained.

The applicability of the space frame method for the analysis of thin-walled spatially curved beams was studied by Yoda et al..<sup>(90)</sup> Akhtar introduced the stiffness matrix of the circular beam element to be implemented directly in existing computer programs for analyzing structures composed of straight and circular members.<sup>(91)</sup>

El-Ali developed a simple computer algorithm for the analysis of multi-girder horizontally curved skew multigirder bridges.<sup>(92)</sup> The algorithm was basically a space frame program with a three-dimensional linear beam element formulation.

**Slope Deflection Method:** The equations needed to analyze a curved orthotropic or isotropic deck on curved flexible girders, using a slope-deflection technique were first presented by Heins and were applied to an example bridge problem.<sup>(93)</sup> In the limit (rectangular condition), the proposed equations correlated favorably with the rectangular plate stiffness coefficients. Bell and Heins at the University of Maryland developed a solution to the general equations for analysis of a continuous orthotropic deck on curved flexible girders using the slope-deflection Fourier Series

method.<sup>(94)</sup> A computer program was developed for one-span and two-span continuous structures under static loading. The experimental deflections from a stiffened plate model verified the accuracy of the slope-deflection analysis. Bell and Heins also presented three computer programs based on analytical techniques of the slope deflection Fourier Series for analyzing a statically loaded curved bridge structure.<sup>(95)</sup> The COBRA (Curved Orthotropic Bridge Analysis) programs can be used to study composite or noncomposite girder-slab action. The user can specify any number of concentrated loads and may choose to superimpose a uniformly distributed load over the concentrated loading.

## **DYNAMIC ANALYSIS OF CURVED BRIDGES WITH ARBITRARY CROSS-SECTIONS**

**Finite Element Method:** Petyt and Fleischer studied three finite element models.<sup>(96)</sup> The study showed that rigid body motion should be represented carefully in order to determine the radial vibrations of curved beams. Sabir and Ashwell applied four FE shape functions typically used in static analysis to vibration problems.<sup>(97)</sup> Massoud and Boudreau used a three-dimensional beam element formulation and the state equation approach to obtain the local rotation matrix of the principal torsion-flexure axes.<sup>(98)</sup> The inertia of the beam was expressed as an equivalent mass matrix.

A two-dimensional and three-dimensional beam element was presented by Surana and Nguyen, in which axial and transverse displacements can be of arbitrary polynomial order.<sup>(99,100)</sup> The formulation results in hierarchical approximation functions and nodal variables. It proved effective for very slender and very deep beam dynamic problems since all three stress and strain components were retained. The method permits sharp corners in the geometry and can accurately predict dynamic stress concentrations at such locations.

**Finite Strip Method:** Vibration analysis of prismatic plates and axisymmetric shells was performed by Suarez et al. using a Reissner-Mindlin based finite strip analysis.<sup>(101)</sup> Reduced integration was used for the two-noded elements which led to an explicit form for all element matrices.

**Finite Difference Method:** Tene et al. presented a general analytical and numerical procedure analyzing horizontally curved beams under static and dynamic loadings.<sup>(102)</sup> The formulation included shear deformation and rotary inertia, and the finite difference solution utilized Houbolt's method for numerical time integration. Sheinman proposed a procedure for dynamic analysis which admits large deflections, small rotations, shear deformation, rotary inertia, geometrical initial imperfections and viscous damping.<sup>(103)</sup> The numerical solution was obtained by the modified Newton's method, finite differences, and Houbolt's method in the time domain.

**Analytical Solution to Differential Equations:** The dynamic response of a simple span horizontally curved bridge under a moving mass system (that resembles a vehicle) was investigated by Tan and Tan and Shore.<sup>(104,105,106)</sup> The effects of rolling and rocking were not considered in the analysis. The method of (initial conditions) was used to obtain the solution to

the differential equation. The analysis was based on classical beam theory which neglects shear effects, flexural rotary inertia, and axial forces. The bridge was assumed to be a single prismatic beam whose mass is uniformly distributed, and whose cross-sectional dimensions were small compared to the radius of curvature. Additionally, the load was assumed to act through the shear center (which matches the centroid and center of twist). The support conditions were taken as simply supported but restrained against torsion and free to warp.

An analytical model for the vibration response of a curved beam was presented by Montalvao e Silva and Urgueira.<sup>(107)</sup> The model was based on the closed-form solution of the dynamic differential equations of equilibrium for an infinitesimal element vibrating out of its initial plane of curvature. Rotary inertia and/or shear deformation effects were included in the analysis. During the same time frame, an accurate and general theory for the dynamic analysis of curved beams was presented by Bhimaraddi.<sup>(108)</sup> The derivation accounts for shear deformation and rotary inertia and can be applied to the analysis of thick curved beams and rings. The formulation assumes a parabolic variation for shear strains and involves six displacement components for the center-line: three translations and three rotations. It was shown that for certain composite curved beams, the in-plane and out-of-plane vibrations are coupled. In such cases, complete analysis rather than separate in-plane and out-of-plane analyses is required.

## STATIC ANALYSIS OF CURVED I-GIRDER BRIDGES

**Finite Element Method:** El-Amin and Brotton developed a finite element for horizontally curved beams, which includes warping.<sup>(109)</sup> Formulation of the stiffness matrix was explicit, and hence, avoids numerical integration. Chaudhuri and Shore developed a thin-walled curved beam element based on Vlasov's theory.<sup>(110)</sup> The element satisfied two uncoupled sets of governing differential equations, two for axial bending in the plane of curvature, and two for bending-torsion out of the plane of curvature. Chaudhuri and Shore also obtained the solution of the homogeneous differential equations, which led to the required functions for the seven nodal displacement components of the two-noded curved beam elements.<sup>(111)</sup> The formulation neglects shear deformation and flexural rotary inertia. El-Amin and Kasem proposed a new element for an open thin-walled bisymmetric cross-section.<sup>(112)</sup> This element employs a cubic displacement function for vertical displacement and a seventh order polynomial for the angle of twist along the length of the girder. Later on, a finite element that includes warping was developed by Thornton and Master and implemented in a computer program using the direct stiffness approach.<sup>(113)</sup>

Hirashima et al. investigated the strength of a stiffened web subjected to a concentrated load using the finite element method, and concluded that design criteria for straight girders should be modified for curved I-girders they recommended using stiffeners on the outer surface of the compression side of the web.<sup>(114)</sup> Hiwatashi and Kuranishi studied the behavior of curved plate girders under bending using the finite-element method.<sup>(115)</sup> Their emphasis was on the interactions between the flange plates and the web plate. The results were utilized to propose more reasonable boundary conditions of the web panel. Furthermore, it was shown that vertical buckling of flange plate does not arise from loss of the bending capacity of the plate girder.

Suetake et al. studied the influence of flanges on the strength of I-girders under bending using the mixed finite element method.<sup>(116)</sup> The Newton-Raphson method was used in the geometrical nonlinear analysis. Study showed that the compression flange significantly affected the strength of the girder. At the same time, the geometrically nonlinear behavior of curved web panels with or without stiffeners under pure bending was investigated using an updated Lagrangian isoparametric FEM by Nakai et al..<sup>(117)</sup> A curved beam-column model and a corresponding interaction curve were proposed.

Yang et al. derived two curved beam elements, with different shape functions, for buckling and nonlinear behavior of horizontally curved I-beams. Warping and curvature effects were taken into consideration and an updated Lagrangian approach was adopted to establish the incremental equation of equilibrium. Later on, an analysis algorithm based on finite element analysis was adopted by Bishara and Elmir to determine the forces in cross-frame members of four medium-span, simply supported, four-lane, tangent multigirder bridges.<sup>(118)</sup> The numerical results were utilized with those obtained from the testing of five bridges to propose design procedures for cross frames on multistringers bridges.

**Finite Difference Method:** Murphy and Heins expressed Vaslov's equations in finite difference form to describe the structural response of the individual girders. Diaphragm action was included in the formulation which led to a system of linear algebraic equations that govern the behavior of curved bridge systems.<sup>(119,120)</sup> This method of analysis is incorporated into a computer program, which was then used to conduct a parametric study. The study resulted in a set of design aids for dead load effects. The web distortional response of a single span curved girder bridge system was determined by Heins and Martin and Martin and Heins through the application of a finite difference procedure.<sup>(121,122)</sup> This procedure was then computerized and resulted in design formulas. In 1985, Azad et al. idealized the deck as a curved flat plate supported by a number of flexible I-girders whose flexural and torsional properties are known.<sup>(123)</sup> The girder reactive forces were considered in the analysis. Numerical results of the Levy-type solution showed that torsional stiffness has negligible influence on a deck whose radius is larger than 150 m.

**Slope Deflection Method:** Bell and Heins and Heins and Bell developed and programmed the Fourier series slope-deflection method for analyzing curved orthotropic deck bridge systems.<sup>(124,125)</sup> The method considers the interaction of the curved orthotropic or isotropic plates with curved elastic girders. The effects of warping and pure torsion, bending and radius of curvature were accounted for in the analysis.

In 1974, Douglas and Gambrell introduced a method for predicting stress and deflection of horizontally curved girder highway bridges which gave close agreement between predicted and measured values.<sup>(86)</sup> This technique incorporates equilibrium and compatibility equations, boundary conditions, slope deflection equations, and interior support restraint conditions to generate the solution for internal moments, torques, shear forces, deflection, and rotations.

**Plane Grid Method:** Weissman proposed a method for statically indeterminate analysis of plane grid systems with straight elements which can be used for horizontally curved beams composed of steel I-shaped beams with concrete slab decks.<sup>(126)</sup> Later on, Beal and Kissane showed through an

experimental and numerical study for New York State DOT that the planar grid method with properly specified member properties was adequate for estimating deflections and in-plane bending stresses.<sup>(127)</sup> They also demonstrated that lateral flange bending stresses were significant in spite of the fact that they are unaccounted for by this technique.

Victor studied a horizontally curved, steel- girder bridge of welded I-girder and concrete slab analytically (using the planar grid method for curved members) and through a field test.<sup>(128)</sup> Experimental data demonstrated that the grid method closely predicted the dead load flexural stresses and vertical deflections. Rotations and lateral bending stresses could not be predicted by the grid method for the live load case.

Mondkar and Powell and Powell and Mondkar, at the University of California at Berkeley, presented a computer program CURVBRG based on the plane grid method for analyzing horizontally curved open members.<sup>(129,130)</sup> The effects of warping torsion were considered rationally, and structural components such as diaphragms and braced (truss type) cross frames were idealized by essentially exact procedures. The bridge cross section was allowed to vary along the length of the member, and changes in form of the structure as construction progresses could be considered. The program was developed to analyze the effects of static loads, support settlement and moving live loads.

In 1975, Beal and Ruby presented a design program for single-span horizontally curved plate girder bridges which is based on planar grid analysis.<sup>(131)</sup> During the same time, the method was modified to include warping of open sections and was implemented into the CUGAR2 program (Lavelle and Laska 1975a, b).<sup>(132,133)</sup>

**V-load Method:** The V-load method was initially extended to multigirder bridges by United States Steel.<sup>(134)</sup> Grubb extended the V-load method to composite I-girder bridges with any support configuration.<sup>(19)</sup> Results for noncomposite and composite I-girders were compared with corresponding results from finite-element curved bridge models. The V-load results for dead-loads were very close to the finite element method solutions. The lateral distribution factors used in the analysis had a significant influence on the live-load V-load results. It was also demonstrated that the V-load method should not be used for analyzing closed-framed laterally-braced I-girder systems.

Fiechtl et al. presented the development and an evaluation of the V-load analysis procedure.<sup>(21)</sup> The V-load method was found suitable for approximate analysis of preliminary bridge designs with composite sections, variable radius of curvature and skewed supports. The authors concluded that the following limitations of the method should always be kept in mind: 1) Transverse distribution factors affect the analysis significantly. 2) The method underestimates inner girder stresses. 3) The effects of bracing in the plane of the bottom flange are not considered by this method. 4) The error in the V-load method is greater for skew supports than radial supports. Poellot proposed a straight forward design approach for curved bridges using the V-Load Method, which was implemented into a computer program called V-LOAD.<sup>(20)</sup> The derivation and development of the method was given in detail and illustrated by means of a design example. The example deals with structural analysis, sizing of the sections (according to



AASHTO's Guide Specifications for Horizontally Curved Highway Bridges) and the calculation of lateral flange bending, centrifugal force, and forces in crossframes.

## **DYNAMIC ANALYSIS OF CURVED I-GIRDER BRIDGES**

**Finite Element Method:** A general FE formulation was proposed by Yoo and Fehrenbach to estimate the natural frequencies of thin-walled curved girders.<sup>(135)</sup> The method considers warping, flexural and torsional rotary inertia and antisymmetry of the cross-section .

**Analytical Solution to Differential Equations:** The exact solution of the differential equations for free vibrations of simply supported doubly symmetric, and open prismatic curved beams was given by Culver.<sup>(136)</sup> A method, based on Raleigh-Ritz analysis, was used to calculate the natural frequencies of horizontally curved fixed-fixed or fixed-simply supported beams. Christiano, Christiano and Culver performed a dynamic analysis on single span, horizontally curved bridges subject to moving forcing systems comprised of sprung and unsprung mass components.<sup>(137,138,139)</sup> The bridge was assumed to be continuous, and composed of thin-walled open members. An infinite series was used to obtain the exact solution to the partial differential equations.

## **STATIC ANALYSIS OF CURVED BOX GIRDER BRIDGES**

**Finite Element Method:** Meyer pioneered the development of a new refined finite-element method to analyze general non-prismatic folded plate structures with an incorporated three-dimensional frame.<sup>(140)</sup> The effect of varying major bridge parameters on wheel-load characteristics was studied and design recommendations were proposed. During the same time, box-girder bridges having one or two spans and with curved layout and skewed supports were analyzed using the finite-element method by Sisodiya et al. and then verified by tests on an aluminum bridge model.<sup>(141)</sup>

In 1971, Scordelis reviewed the analysis methods available for box-girder bridges, and described two methods: Direct Stiffness Harmonic Analysis and the Finite-Element Method.<sup>(142)</sup> These methods were found appropriate for straight/curved/skewed multi-cell box girders. An element that has a beam-like n-plane displacement field was developed by Lim et al.<sup>(143)</sup> The element is trapezoidal in shape, and hence can be used to analyze right/skewed/curved box girder bridges with constant depth and width. At the same time, Cheung and Sisodiya derived the in-plane stiffness of a parallelogram element by assuming linear function for the longitudinal displacement and linear/cubic for the displacement normal to the axis of the girder.<sup>(144)</sup> The element was combined with a plate element to create a parallelogram shell element which was used successfully to analyze straight and curved box girders with skewed supports. A stiffness method, which utilizes sector plates and cylindrical shell elements, was developed by Chu and Pinjarkar.<sup>(145)</sup> The stiffness coefficients of sector plates were derived and similar parameters for shell elements were based on Hoff's solution of Donnell's equations. The method can be applied only to simply-supported bridges without intermediate diaphragms, but can be extended to any

other arrangement. Concurrently, Aneja and Roll tested a plastic model of a horizontally curved box-beam highway bridge and compared the results to a finite-element analysis.<sup>(146)</sup> The comparison showed a close agreement between the shapes, but not the magnitudes of the stress plots.

Later on, a finite-element method was introduced by Fam and Turkstra, in which annular, cylindrical and conical elements were developed.<sup>(147)</sup> Analysis proved accurate for static and free-vibration curved box-girder problems. In 1974, a method that uses thin-walled beam elements was proposed by Bazant and El-Nimeiri.<sup>(148)</sup> The method includes warping, and transverse cross-section distortion and can be applied only to single-cell sections (with overhangs). Shear effects due to shear forces and transverse bimoments were eliminated by using low-order polynomials for transverse displacements and distortions.

Evans and Al-Rifaie conducted 18 tests on box girder models with various curvatures and loading conditions and compared the results with theoretical values obtained from the finite-element method.<sup>(149)</sup> This method proved quite accurate in the analysis of curved box girders.

During the same time, Fam and Turkstra introduced a finite-element method which utilized straight and curved elements. In-plane, bending and coupling actions and anisotropic behavior were included in the analysis.<sup>(150)</sup> An experimental study of a single-span horizontally curved plexiglass box-girder beam with diaphragms and flange overhangs, subjected to mid-span loads, was also conducted by Fam and Turkstra.<sup>(151)</sup> Their experimental results compared favorably to the results of a special purpose finite-element program for the analysis of curved boxes.

Ramesh et al. uncoupled in-plane and out-of-plane forces and neglected shear deformation to introduce a curved element with six degrees of freedom (d.o.f) at each node.<sup>(152)</sup> Their method is applicable to single and multi-cell sections. Later on, Turkstra and Fam performed a three-dimensional finite-element analysis on single-cell curved box-girder sections with variable curvature, length, web spacing, number of diaphragms, and loading.<sup>(153)</sup> The numerical procedure showed the importance of warping and distortion in relation to bending. Zhang and Lyons added three extra degrees of freedom to the normal six in the thin-walled beam element formulation to take into account warping, distortion and shear effects.<sup>(154)</sup>

In 1990, Paavola combined the torsion of thin-walled members with the finite-element method to reduce the size of the computational model used in analyzing closed cross sections.<sup>(155)</sup> Shanmugam and Balendra conducted an experimental and theoretical study on the behavior of multicell structures curved in plan.<sup>(156)</sup> The results of eight tests on two perspex models of different span/radius ratios subjected to different loading were compared with theoretical values obtained from the finite-element method; good agreement between the results was observed. Wen and Suhendro averaged the nonlinear part of the axial strain to improve the accuracy in this nonlinear curved three-dimensional beam formulation, which does not include warping.<sup>(157)</sup> The method utilizes the Newton-Raphson scheme and the Fixed Lagrange coordinates to solve this numerically nonlinear curved beam problem.

**Finite Strip Method:** The Finite Strip Method (FSM) was first applied to box girders by Meyer when he analyzed simply supported horizontal folded plates.<sup>(140)</sup> Discussion was given on how to extend the theory to continuous girders. A theoretical and experimental study of the influence of diaphragms on the distortional deformation of box girders was conducted by Sakai and Okumura.<sup>(158)</sup> Vlasov's folded-plate theory was modified and its results were compared with those of the FSM and experimental data for straight and curved box girders.

Buragohain and Agrawa presented a method based on harmonic analysis in the circumferential direction and a modified finite difference method in the transverse direction.<sup>(159)</sup> The total potential energy was divided into extensional/bending and shear/twisting for which two element matrices were obtained.

A computer program based on the FSM and harmonic analysis was presented by Kabir and Scordelis for the analysis of continuous prismatic curved folded plate structures with flexible interior diaphragms or supports.<sup>(160)</sup> Circumferential finite strips were used to represent each plate element, interior diaphragms were modeled as flexible beams, and interior supports were represented as two-dimensional planar frame bents.

Cheung applied the FSM to continuous bridges by introducing a two-noded (straight) and three-noded (curved) strip elements.<sup>(161)</sup> Results showed that the lower order two-noded element tended to underestimate the stresses and moments in the inner edges of the curved bridge.

Lin and Chen proved the validity and efficiency of the FSM and the CCBG (Continuous Curved Box Girders) program by comparing their results against those of plastic model tests of three span continuous curved box girders.<sup>(162)</sup> In fact, the CCBG program is not limited to box sections but rather can be used in analyzing thin-walled girders of open or closed cross sections. Later on, the spline FSM was extended by Li et al., to handle circular and non circular curvatures.<sup>(163)</sup> Thin shells were used to model the webs, while curved flat plates represented the flanges. Piecewise polynomials were used as shape functions in the transverse direction, and products of B-3 splines were adopted in the longitudinal direction. During the same time, Arizumi, et al. studied the distortional and slip behavior of simply supported curved composite box girders using elastic analysis and static tests.<sup>(164)</sup> The results from the FSM (with spring elements for shear connectors), curved beam theory and distortional theory were compared to the static test results and a parametric study was presented to evaluate the effects of cross-sectional deformations on the induced stresses.

In 1991, Shimizu and Yoshida used the FSM to calculate the magnitude of reaction forces to be used in the design of load-bearing diaphragms at the intermediate support of two-span continuous curved box girders.<sup>(165)</sup>

**Finite Difference Method:** Horizontally curved box girders on skewed supports were first analyzed using the finite difference method by Komatsu et al. who used the simple torsion theory and the transfer matrix concept to derive the solution.<sup>(166,22)</sup> Harik and Ekabaram used the classical linear theory of buckling of sector plates in the stability analysis of horizontally and

vertically curved box members.<sup>(167)</sup> Fourier series and finite differences were used to estimate the critical buckling stresses.

**Analytical Solution to Differential Equations:** Kirstek worked on tapered and deformable box girders.<sup>(168)</sup> The cross section was first considered rigid (assuming complementary bracing), and then external forces acting in the direction of the imaginary bracing were applied. The actual solution was basically the combination of the first and second steps. Experimental investigation verified the theory. Kambe extended the modified theory, proposed by R. Heilig, for torsion bending, based on a shear-stress field in equilibrium with warping normal stress, to cover a circularly curved box girder bridge.<sup>(169)</sup> The modified theory was accurately formulated in terms of cylindrical coordinates. In 1985, Waldron used Vlasov's thin-walled beam theory to obtain the flexibility matrix for single-cell curved box girders.<sup>(170)</sup> The formulation accounts for curvature, torsional warping and shear deformation.

**Slope Deflection Theory:** Bonakdarpour et al. and Heins et al. predicted the behavior of a single two-span, three-cell plexiglass model by the slope deflection Fourier series technique.<sup>(171,172,173)</sup> The experimental results showed the effectiveness of the analytical method used and that single-cell properties could be applied in the analysis, while warping effects were neglected. Furthermore, Bonakdarpour et al. and Heins and Bonakdarpour compared the results from the slope-deflection analytical method with those obtained from testing a series of bridge models with different numbers of spans and diaphragm arrangements, and which consisted of either a bare steel frame system, or a steel frame with a noncomposite or composite concrete deck.<sup>(174,175)</sup>

**Plane Grid Method:** The response of a symmetrical, two-span, continuous, horizontally curved steel box-girder bridge was investigated by Kissane and Beal.<sup>(176)</sup> The planar grid method was compared against experimental analysis. Test results show that the in-plane bending moments for dead and static live loads were approximately 86 percent of their respective theoretical values. The total load carried by the individual girder, however, was within 6 percent of that predicted by the grid method. Lavelle and Laska modified the plane grid method to analyze closed cross sections and truss-type diaphragms and implemented the method in the CUGAR3 computer program.<sup>(132,177)</sup> The formulation does not consider the distortion of the cross section and assumes enough bracing is provided at the top of the flange so that the section is truly closed.

In 1989, Hsu developed two curved thin-walled beam elements, based on a Vlasov element and an improved Vlasov element, to analyze horizontally curved steel box girder bridges.<sup>(22)</sup> The box girders were modeled as thin-walled beams which connect the transverse diaphragms as a planar grid system. When compared to other analytical procedures, the analytical results were found to be accurate enough for bridge design. A distortional thin-walled beam element was also developed to facilitate the analysis.

## **DYNAMIC ANALYSIS OF CURVED BOX GIRDER BRIDGES**

**Finite-element Method:** Chang et al. proposed a finite-element which includes a seventh warping degree of freedom to avoid having to use more refined finite-elements.<sup>(178)</sup>

**Finite Strip Method:** Cheung and Li extended the spline finite-strip method for vibration analysis of curved box-girders.<sup>(179)</sup> Computational effort was greatly reduced in comparison with the finite-element method.

**Finite Difference Method:** Heins and Oleinik studied the response of single and multiple span curved single box beam bridges with any number of intermediate diaphragms.<sup>(180)</sup> A computer program was developed to calculate the forces, stresses, and deformations along the curved bridge. The program was later refined by Heins and Sheu to include the composite action with the concrete deck.<sup>(181)</sup>

Heins and Sahin used the finite difference method to obtain the natural frequency of curved box girders. The numerical results of this parametric study aided in developing a simplified natural frequency equation expressed in the form:<sup>(182)</sup>

$$f = \frac{\pi}{2K^2L^2} \sqrt{\frac{EI_x + \frac{EC_w}{R^2} - \frac{GJL^2}{R^2}}{M}} \quad (1)$$

In this equation,  $K = 1$  for simple spans and is defined by a quadratic equation (please see reference 182) for two and three-span bridges, and  $M$  is the mass.

**Space Frame Method:** Ho and Reilly and Ho developed the stiffness matrices for single span simply supported curved box beam bridges using the unit load theorem.<sup>(183,184)</sup> A computer program, CBGB, was written to perform a parametric study that represents ninety bridge configurations with different girder radii, girder spacing, number of lane, number of girders, and span lengths. Two design parameters resulted from this study: 1) Lateral load distribution factors for proportioning wheel loads to a given girder and 2) Modification factors for predicting bending and torsional effects.

Heins and Lin developed a method in which equivalent stiffnesses, that correspond to one rotational and three translational degrees of freedom, were calculated for the entire bridge system.<sup>(185,186)</sup> These parameters were then used to calculate the natural frequencies for a single-degree-of-freedom (sdf) system. Using response spectrum curves, the system accelerations were predicted. These results, were utilized to develop empirical design equations.

Abdel-Salam and Heins and Abdel-Salam presented the results of a comprehensive study of the seismic response of curved steel box-girder bridges.<sup>(187,188)</sup> The El Centro earthquake ground motion acceleration record and its corresponding response spectrum were used as dynamic input and the bridge was modeled using three-dimensional space frame elements in which special elements were introduced to account for the curved geometry and boundary conditions. Both response history and response spectrum techniques were performed and results were compared. The simultaneous application of the three components of the earthquake, the effects of damping, noncomposite sections, rotational inertia, and column height were all studied. The maximum dynamic responses were correlated with a static analysis of the bridge, and equivalent

loads and design curves and criteria were developed. A comparison between computer dynamic analysis and the proposed design criteria showed good correlation.

Yilmaz et al. discussed the analysis techniques (single-mode spectral method and multimode spectral method) along with a proposed design guide for curved steel box girder bridges, which incorporates the interaction between bending and torsional forces.<sup>(189)</sup> The dynamic study was confined to a space frame matrix simulation where a series of typical elements attached rigidly together were used to form a continuous curved box girder bridge. A study of the seismic response of curved steel box girder bridges was conducted by Abdel-Salam and Heins.<sup>(190)</sup> The bridge was modeled using three-dimensional space frame elements, and the El Centro earthquake accelerogram was used as a dynamic input to calculate the maximum responses at the bridge deck and columns. A 6-element curved model having short end elements with appropriate member releases was found to yield satisfactory results.

## CHAPTER III. CURVED I-GIRDER BRIDGES

### LOADS

**Load Distribution:** Heins and Siminou developed equations giving distribution factors for curved-bridge systems consisting of four, six, and eight girders.<sup>(191)</sup> These equations resulted from analytical solutions based upon the slope deflection method.<sup>(192)</sup> Heins and Jin used the computer program SAP to build a three-dimensional space frame model, that included the effect of diaphragms and bottom lateral bracing, to examine the load distribution on horizontally curved I-girders.<sup>(193)</sup> Numerical results were compared with those from two experiments conducted by Brennan and Armstrong, and were then used to develop empirical equations applicable to curved bridges analyzed by the grid method.<sup>(194,195,196)</sup> Brockenbrough used the finite-element code MSC/NASTRAN to analyze three-dimensional models of two-span continuous curved bridges without bottom lateral bracing.<sup>(197)</sup> The model consisted of the entire bridge including the concrete deck and the cross frames. Results of the analysis showed that variations in girder stiffness and cross frame spacing had negligible effects on the distribution factors, while the girder spacing and central angle per span had the largest effects. The authors also concluded that distribution factors in the 1980 AASHTO Guide Specification Commentary were very conservative, the Heins and Siminou factors were unconservative, and recommended that the Heins and Jin factors not be used.<sup>(191,193,198)</sup> An approximate equation based on a V-load modification of the distribution factors for straight girders was found to agree with finite-element results for exterior girders, but was conservative for the interior girders. AISC Marketing extended the work of Brockenbrough and developed a system of individual computer programs for the calculation of load distribution factors for both straight and curved I-girder bridges for use with line-girder programs.<sup>(197,199,200)</sup>

**Impact:** The current specifications regarding the impact factors given in the commentary to the I-Girder portion of the AASHTO guide specification were based on an investigation by Chaudhari and Shore at the University of Pennsylvania.<sup>(201)</sup> In that study, the finite-element method was used with annular plate, thin-walled curved beam, straight prismatic beam, and frame-type diaphragm elements. The inertia properties of the elements were obtained by assuming linear elastic behavior and small displacements. Warping of the cross section, and the centrifugal forces due to the vehicle's movement were included in the analysis and the vehicle was idealized as a sprung mass supported on two unsprung masses. The bridge-vehicle interaction led to a set of differential equations of motion whose coefficients vary with the speed of the vehicle. These equations were solved using the computer program DYNCRB/IG (Dynamic Analysis of Curved Bridges/I-Girders) implemented by Chaudhari and the numerical time integration was performed using the linear-acceleration method.<sup>(202)</sup> This computer program is an extension of another program, STACRB, developed for static analysis of horizontally curved bridges.<sup>(203)</sup> Chaudhari and Shore concluded that the impact factors increase with the speed of the vehicles, but do not significantly change with the span length. Furthermore, this study showed that the impact factors for the support reactions and diaphragm stresses for curved bridges are higher than those for a straight bridge with equal span, and that the factors for a two-span bridge are less than those of a single

span bridge. The study also recommended that the high impact factors for the bimoment in curved I-girders be taken into consideration in the design process.

## STABILITY

**Flange-Slenderness Requirements:** The combination of both bending and warping in an I-shaped curved girder results in a nonuniform distribution of the stress across the compression flange width. Therefore, the outside part of the flange will buckle at a stress value different from that of the inside part. To study the buckling problem of a curved I-shape girder, Culver and Frampton examined analytically the elastic buckling case in which each half of the flange was treated separately as an isotropic sector plate free on one edge and rotationally restrained along the other edge by the web and the other half of the flange.<sup>(204,205)</sup> The fundamental equation of buckling was then written in polar coordinates and solved numerically using the finite difference method. This investigation was later extended by Culver and Nasir to cover not only the elastic but also the inelastic flange local buckling behavior.<sup>(206,207)</sup> In the inelastic range, the mathematical model was based upon the assumption of orthotropic behavior. Their numerical results showed that the influence of curvature is very small for flange curvature ratios  $1 \times 10^{-6} < b/R < 0.01$ , and that as yielding spreads across the flange, both the buckling stress and the buckled wave length decrease. The design recommendations resulting from this study suggested that the width to thickness ratio be limited to that set forth in the 1969 AISC Specification for straight rolled compact beams ( $b/t \leq 1650/\sqrt{F_y}$  where  $F_y$  is in psi).<sup>(208)</sup> Culver presented a summary of his research related to proportioning the compression flange in a horizontally curved I-girder and pointed out that the total stress (warping plus bending) at the flange tip must be limited to  $0.55 F_y$  if the AASHTO  $b/t$  limit for straight girders is used for curved girders.<sup>(209)</sup> In such a case, the factor of safety against local buckling for both straight and curved I-girders becomes essentially the same regardless of the ratio of bending to warping stress. This notion has been adopted in the AASHTO Guide Specification, where the flange local buckling criterion is based only on the  $b/t$  ratio. This notion was challenged recently by Kang and Yoo who used the finite-element code MSC/NASTRAN to demonstrate that in a curved girder, local buckling of the flange is affected not only by the width to thickness ratio, but also by the initial curvature and the warping normal stress.<sup>(210)</sup> Komatsu and Kitada use an elasto-plastic finite displacement analysis to study the ultimate strength of outstanding steel plates with initial imperfections.<sup>(211)</sup> Results of this study were used to conclude that local buckling of the flange will not occur if:<sup>(13, 212,213)</sup>

$$\lambda = \sqrt{\frac{F_y}{F_{cr}}} = \frac{b}{t} \sqrt{\frac{12(1-\nu^2) F_y}{0.43 \pi^2 E}} \leq 0.5 \quad (2)$$

resulting in a limiting flange slenderness ratio:

$$\frac{b}{t} \leq 0.312 \sqrt{\frac{E}{F_y}} \quad (3)$$



**Web-Slenderness Requirements:** The web-slenderness requirements for a curved I-girder differ significantly from those of straight girders. These requirements are often based upon both the bending behavior, shear behavior, combination of the two, and the web out-of-plane deflections. The web in a curved I-girder may be stiffened transversely, longitudinally, or both.

#### Transversely Stiffened Web

Current AASHTO web-slenderness requirements for a curved I-girder are based on the analytical studies conducted by Culver et al.<sup>(214, 215)</sup> In these studies, the web panel was modeled as a series of isolated elastically supported cylindrical strips subjected to appropriate load distributions.<sup>(216)</sup> The stress state in each cylindrical strip was determined from the total potential energy of a nonlinear arch model using the Rayleigh-Ritz method. Numerical results were generated and a curve fitting technique was used to develop the following web-slenderness requirement:

$$\frac{d}{t_w} \leq \frac{36,000}{\sqrt{F_y}} \left[ 1 - 8.6 \left( \frac{a}{R} \right) + 34 \left( \frac{a}{R} \right)^2 \right] \quad (4)$$

where  $d$  and  $t_w$  are the depth and thickness of the web respectively. An extension of this work was published a year later, when Culver et al. used a two-way shell model to check the accuracy of the isolated elastically supported cylindrical strips by treating the panel as a unit rather than as individual strips.<sup>(217)</sup> The work also included the effects of longitudinal stiffeners.

During almost the same time, Abdel-Sayed studied analytically the pre buckling and the elastic buckling behavior of curved webs, and proposed approximate conservative equations for the estimation of the critical load under pure normal loading (stress), pure shear, and combined normal and shear loading.<sup>(218)</sup> He used the linear theory of shells, neglected the effect of torsional rigidity of the flanges, and assumed the transverse stiffeners to be rigid in their directions (thus no strains could be developed along the edges of the panels) to obtain the governing differential equations that were solved using the Galerkin method.

In 1980, Daniels et al. summarized the Lehigh University five-year experimental research program on the fatigue behavior of horizontally curved bridges and concluded that the CURT slenderness limits were too severe, and developed one equation for Load Factor Design in the form:<sup>(219)</sup>

$$\frac{d}{t_w} \leq \frac{36,500}{\sqrt{F_y}} \left[ 1 - 4 \left( \frac{a}{R} \right) \right] \leq 192 \quad (5)$$

and one equation for Allowable Stress Design in the form:

$$\frac{d}{t_w} \leq \frac{23,000}{\sqrt{f_b}} \left[ 1 - 4 \left( \frac{a}{R} \right) \right] \leq 170 \quad (6)$$

The latter equation is now used in the 1993 AASHTO Guide Specifications For Horizontally Curved Highway Bridges.

In 1984, Mikami and Furunishi studied the nonlinear behavior of cylindrical web panels of horizontally curved plate girders subjected to pure bending and combined bending and shear.<sup>(220)</sup> They used the Washisu's nonlinear theory of shells to derive a set of differential equations solved by the finite difference method. Numerical values of the load, deflection, membrane stress, bending stress, and torsional stress were obtained for various values of panel aspect ratio and curvature. The work concluded that as the curvature increases, the membrane stress in the circumferential direction decreases. Furthermore, a curved panel under combined bending and shear will experience higher bending stress and lower level of circumferential membrane stress than those resulting from pure bending.

### Transverse Stiffener Requirements

Culver et al. and Mariani et al. studied analytically curved web panels under pure shear using the Donnell shell equation and the Galerkin method.<sup>(221,222)</sup> They concluded that the required stiffener rigidity for a curved web is less than that for a straight web if the panel aspect ratio  $a/d$  is less than 0.78. Here 'a' is the distance between transverse stiffeners and d is the depth of the web. And, for  $0.78 \leq a/d \leq$  , the required stiffener rigidity increases with curvature in the amount  $+\frac{1}{1,775}\left(\frac{a}{d}-0.78\right)Z^4$  where Z is a curvature parameter defined as  $Z = \left[ \frac{a^2}{Rt} \right] \sqrt{1-\nu^2}$  with  $a/d \leq$  . In these equations, R is the radius to the centerline of the web, t is thickness of the web, and  $\nu$  is Poisson's ratio. The study was limited to curved girders in which  $0 \leq Z \leq 10$ .

In 1984, Nakai, Kitada, Ohminami, and Fukomuto presented analytically a beam-column model to estimate the strength of transverse stiffeners in curved girders.<sup>(223,224)</sup> The analytical results were compared with experiments conducted by Nakai et al., which led to a recommendation that the relative rigidity parameter  $\beta$ , defined as the ratio between required rigidity of a transverse stiffeners in horizontally curved girders to that in straight girders (as specified in the Japanese specification), must be the following:<sup>(225)</sup>

- For stiffeners attached to one side of the web plate:

$$\beta = \begin{cases} 1.0 + (\alpha - 0.69)Z[9.38\alpha - 7.67 - (1.49\alpha - 1.78)Z], & \text{for } 0.69 \leq \alpha \leq 1.0 \\ 1.0 & \text{for } \alpha < 0.69 \end{cases} \quad (7)$$

- For stiffeners attached to both sides of web plate:

$$\beta = \begin{cases} 1.0 + (\alpha - 0.65)Z[12.67\alpha - 10.42 - (1.99\alpha - 2.49)Z], & \text{for } 0.65 \leq \alpha \leq 1.0 \\ 1.0 & \text{for } \alpha < 0.65 \end{cases} \quad (8)$$

where  $\alpha = a/d$ .

### Longitudinally Stiffened Web

In a paper described earlier,<sup>(217)</sup> Culver et al. generated numerical results from their analytical model and then used a curve fitting approach to obtain the following slenderness requirement for a curved web stiffened with one longitudinal stiffener situated in the compressive stress region:

$$\frac{d}{t_w} \leq \frac{46,000}{f_b} \left[ 1 - 2.9 \sqrt{\frac{a}{R}} + 2.2 \frac{a}{R} \right] \quad (9)$$

### Longitudinal Stiffener Requirements

**THE 1986 JAPANESE PAPER BY NAKAI, KITADA, OHMINAMI, AND KAWAI DEALS WITH THE DESIGN OF LONGITUDINAL STIFFENERS FOR CURVED GIRDERS.**<sup>(226)</sup>

**Overall Buckling:** A horizontally curved girder loaded normal to its plane of curvature will bend and twist simultaneously. Early works on the subject were developed by Vlasov and Dabrowski whose formulations were based on the undeformed structure.<sup>(227,228)</sup> Culver and McManus presented a second-order analysis in which the equilibrium equations were formulated on the deformed structure.<sup>(229,230)</sup> Results obtained in this study were compared to those of lateral buckling tests conducted in 1971 by Mozer et al.<sup>(231)</sup> The study recommended a set of formulae that were later adopted, and still are, in the 1993 AASHTO Guide Specifications for Horizontally Curved Bridges.

In 1987, Nishida et al. used the large deflection theory of curved members to derive the following approximate critical elastic moment for a curved beam subjected to two equal end moments:<sup>(232)</sup>

$$M_{cr} \approx \sqrt{\left(1 - \frac{L^2}{\pi^2 R^2}\right) \frac{\pi^2 E I_y}{L^2} \left(GJ + \frac{\pi^2 E C_w}{L^2}\right)} \quad (10)$$

In the above expression, when R approaches infinity,  $M_{cr}$  approaches the elastic critical buckling moment for a straight girder.

Yoo used the minimum potential energy principle to obtain solutions for the determination of the elastic flexural-torsional buckling loads for in-plane and of out-of-plane buckling modes of thin-walled curved beams that do not undergo local buckling.<sup>(233,234)</sup> In 1983, Yoo and Pfeiffer investigated the elastic buckling of thin-walled curved members through a variational-based finite-element formulation.<sup>(235)</sup> Solutions to different cases pertaining to the stability of curved beams were obtained and compared against existing solutions obtained by Timoshenko and Gere, Vlasov, and Culver.<sup>(236,227,209)</sup> Numerical results obtained in this study were significantly different from those of Timoshenko, Vlasov and Culver. The discrepancies were attributed to incorrect formulations in Timoshenko and Vlasov's cases, and to the fact that the governing differential equation was viewed as a deflection-amplification problem rather than a classical eigenvalue problem in Culver.<sup>(209)</sup> Later, Yoo and Pfeiffer presented a solution to the stability of curved beams with in-plane deformation and asserted their earlier conclusion related to the discrepancies

with existing solutions including, this time, the work of Vacharajittuphahn et al. and Vacharajittuphahn and Trahair, which are essentially based on Vlasov's formulation.<sup>(237,238)</sup> In 1985, Yoo and Carbine<sup>(239)</sup> carried out a series of laboratory tests on 12 W10x12 simply supported curved beam specimens subjected to concentrated loads. The length of each specimen was approximately 6.1 m (20 ft) with a subtended angle ranging from 0° to 30°. Specimens were loaded from the top flange in one case and from the bottom flange in another case. All specimens reached their ultimate loads, which were higher than the analytically predicted buckling load values from Yoo and Pfeiffer. Bottom flange loaded specimens had higher ultimate loads and showed lower cross sectional deformations than top flange loaded specimens. Papangelis and Trahair examined experimentally the work of Yoo and Yoo and Pfeiffer by conducting tests on circular aluminum arches. They concluded that the theoretical loads obtained from the work of Yoo differ substantially from various analytical and experimental results of various researchers.<sup>(240,241,242,243,244)</sup> The conflict among these curved beam theories was also discussed in a series of publications by Yang and Kuo, who derived the nonlinear differential equations of equilibrium for horizontally curved I-beams by making use of the principle of virtual displacements to establish the equilibrium of a bar in its deformed or buckled configuration.<sup>(245,246,247)</sup> Numerical results were obtained and compared with those resulting from Yoo's as well as Vlasov's theories. The authors attributed the discrepancy between their results and Yoo's to the fact that Yoo not only neglected both the radial stress effect and the contribution of shear stresses to the potential energy, but also substituted the curvature terms of the curved beam in the potential energy equation of a straight beam. In a recent paper published in 1991, Kuo and Yang further criticized the work of Vlasov and Yoo and supported their argument by solving numerically the buckling problem of a curved beam with solid cross section under uniform bending in one case and uniform compression in another case.<sup>(80)</sup> Based on a finite-element study, Kang and Yoo showed that initial curvature and warping do not affect lateral-torsional buckling strength of curved girders with a subtended angle between two adjacent cross bracings up to 0.1 radian, the maximum value allowed in the current AASHTO Guide Specifications.<sup>(210)</sup>

## ULTIMATE STRENGTH AND DESIGN RECOMMENDATIONS

Culver and McMannus studied analytically and experimentally the elastic and inelastic behavior of horizontally curved girders and made design recommendations that were essentially adopted in the AASHTO Guide Specifications.<sup>(229)</sup>

Fukomoto and Nishida (1981) tested six simply supported single curved I-beams under a concentrated load.<sup>(248)</sup> Curvatures, including out-of-straightness of these sections ( $L/8R$ ), varied from 1/1379 to 1/97, with  $L$  and  $R$  being the curved arc length and the radius of the beam respectively. Both numerical and experimental results of this investigation agreed quite well. For a curved I-girder under two equal end moments, it was shown that as value of the end moments increases, the horizontal displacement of the curved girder increases up to a value approximately equal to 0.8 of the flange width, then it reverses direction, while the vertical deflection as well as the angle of twist increases as the value of the end moment increases. It was also shown that early yielding due to residual stresses decreases the ultimate strength. The authors proposed an approximate ultimate strength formula that requires a solution of the following quartic equation:

$$\lambda^4 \delta^4 - \left\{ \left[ 1 + \frac{P_E (d-t)}{2M_p} \left( \frac{L^2}{2Rb_f} \right) \right] \lambda^4 + 1 \right\} \delta^2 - \left( \frac{L^2}{2Rb_f} \right) \delta + 1 = 0 \quad (11)$$

where  $\lambda = \sqrt{\frac{M_p}{M_E}}$ ,  $\delta = \frac{M_u}{M_p}$ ,  $M_p \cong F_y b_f t (d-t)$ , and  $P_E = \frac{\pi^2 EI_y}{L^2}$

In 1983, Yoshida and Maegawa presented the finite-displacement formulation of the transfer matrix to study the ultimate strength of horizontally curved I-beams.<sup>(249)</sup> The analysis covers elastic and inelastic behavior, and the ensuing numerical results were in good agreement with the experiments conducted by Fukomoto and Nishida.<sup>(248)</sup>

Perhaps the simplest equation predicting the ultimate bending strength of a horizontally curved I-girder is presented by Nakai et al. and also by Kitada et al.<sup>(250,213)</sup> The equation is of the form:

$$M_u = \left[ 1.92 + 0.357 \frac{L^2}{Rb_f} \right] M_a \quad (12)$$

where  $M_a$  is the allowable bending moment, as defined in the Japanese Specification for a straight I-girder subjected to bending and torsion. This equation is empirical and based upon 19 tests in which the elements comprising the cross sections are classified as compact, and the  $a/d$  ratio is less than one. Furthermore, the above equation is valid only when a curvature parameter defined as  $Z_o \approx \frac{a^2}{8Rt_w}$  is less than one.

In addition to the bending strength, the ultimate strength of curved I-girders under shear and under combined shear and bending were addressed by Nakai et al. and also by Kitada et al. who concluded that Rocky's model for straight plate girders can be used to estimate the ultimate shear strength in curved girders as long as the curvature parameter  $Z_o < 1$  (virtually all experiments conducted by the Japanese researchers had  $a/d < 1$ ).<sup>(223,225,212,251)</sup> In addition, interaction curves related to the ultimate strength under combined bending and shear were also provided. The Japanese technical papers by Mikami et al., Kuranishi and Hiwatashi, Nakai et al. form the basis of the ultimate strength studies discussed briefly earlier under this section.<sup>(252,253,50,254)</sup>

## DIAPHRAGMS, CROSS FRAMES, AND LATERAL BRACING

In 1986, Yoo and Littrell used the computer program SAP to study the effects of cross-bracing in curved bridges.<sup>(255)</sup> A single span curved bridge consisting of six 1.52 m (60 in) deep plate girders spaced 2.75 m (9 ft) apart was modeled using eight-noded brick elements and truss members. The bridge model was then analyzed under dead and live loads for different curvatures.

The parametric study results were analyzed by linear and nonlinear regression and resulted in the development of the following design equation for the maximum bracing spacing:

$$S_{\max} = L \left[ -\ln \left( \frac{F_{ws} R}{18.890 L} \right) \right]^{-1.3364} \quad (13)$$

where  $F_{ws}$  is a factor for warping stress due to dead and live loads. It is usually selected arbitrarily, but should be kept less than 0.5.

In 1989, Schelling et al. presented a study concerning the construction effects on bracing on curved I-girders.<sup>(256)</sup> In this study, the authors considered the response of single and continuous-span, horizontally curved, single and multigirder bridges under the girders own self weight and the placement of the concrete deck. The effects of top and bottom lateral bracing system during construction was also studied. The investigation resulted in a set of equations that define the dead load distributions throughout the superstructure system analyzed by the two-dimensional grid method.

## FATIGUE

Akehashi et al. presented the results of an experimental study pertaining to the fatigue strength of the web-to-flange fillet welds in horizontally curved I-girders.<sup>(257)</sup> In this paper, which provides minimum details, three plate-girder models, with web-slenderness ratios of 144, were tested under concentrated loads for 200,000,000 cycles. The authors concluded that there would be no cracks at the web-to-flange fillet weld as long as the web out-of-plane bending stress is limited to that set forth in the Japanese specification.

## HEAT CURVING

Brockenbrough studied the effect of heat curving on the residual stress, strain, and curvature of a bridge girder.<sup>(258,259,260)</sup> In the analytical model, a perfect elastic-plastic, temperature-dependent stress-strain relationship was used for the steel, and heat-transfer theory for a semi-infinite plate was used to obtain the temperature profiles in the girder flanges. Both the experimental and analytical approaches led to close results. It was shown that the residual stresses after heat curving increased, except at the mid-width of the flange where welding reduced the initial stress. The tensile residual stress along the heated flange edges reached the yield point, but was less at the opposite flange tips. A plastic flow of residual strain was noticed in the heated width of the flange, while a linearly varying strain was detected across the flange. The web bowed due to the compressive residual stress. The heat-curving procedure caused an increase in the yield point at the heated edge, while resulted in a decrease at an interior position on the flange, but had an insignificant effect on the yield point elsewhere. Furthermore, the final curvature resulting from the residual strain distribution generally increased with maximum temperature applied during curving.

Criteria for heat curving rolled beams and welded girders were also formulated by Brockenbrough.<sup>(261)</sup> The criteria were adopted by AASHTO in 1969, and were based on two provisions. The first is the minimum radius requirement, which was adopted to prevent the web from excessive bowing. The second is the requirement for increase in vertical camber that was provided to minimize vertical residual deflections, or loss in camber which results from inelastic action under initial loadings. Brockenbrough's studies resulted in the development of fabrication aids for heat curved girders.<sup>(262,263)</sup>

In 1979, Daniels and Batcheler presented the results of a study of the effects of heat curving on the fatigue strength of plate girders.<sup>(264)</sup> Their analytical parametric study of the residual stresses and strains due to heat curving indicated that heat curving had little influence on the fatigue strength of plate girders. A summary of this task along with other tasks of the study was also given by Daniels et al. in which I and box sections were investigated.<sup>(219)</sup>

A simply supported span was monitored during construction of a heat-curved girder bridge by Hilton to investigate the camber loss in the bridge.<sup>(265,266)</sup> Although some construction camber loss occurred shortly after the concrete deck was placed, it was only a quarter of that determined by the AASHTO equation. Furthermore, service camber losses almost ceased after 6 1/2 months. The total loss (construction plus service) was 13 percent of that predicted by AASHTO, which indicated that the equation adopted in the code may only be applicable to certain radii ranges.

## CURVED COMPOSITE I-GIRDERS

The ultimate behavior of horizontally curved composite I-girder bridges was studied in 1972 by Yoo and Yoo and Heins who proposed the following yield equation that accounts for the interaction between the torsional and bending forces:<sup>(267,268)</sup>

$$\left(\frac{M}{M_p}\right)^2 + \left(\frac{T}{T_p}\right)^2 = 1 \quad (14)$$

Where  $M_p$  and  $T_p$  are the plastic moment and plastic torque of the cross-section, respectively. The bending term  $M$  assumes only normal bending moment and neglects the normal warping stresses. Such an assumption is usually justified in curved composite girders. The paper also provides a number of design charts and equations for the plastic design of curved composite girders.

In 1973, Colville conducted experiments on four curved steel-concrete single girders that were simply supported.<sup>(269,270)</sup> Two of the test specimens failed in torsion and the other two failed in combined bending and torsion. Results of tests were used to derive a design procedure for the design of shear connectors in a composite curved girder. This procedure was adopted in the Guide Specifications for Horizontally Curved Highway Bridges.<sup>(14)</sup>





## CHAPTER IV. CURVED BOX GIRDER BRIDGES

### LOADS

**Impact:** The impact factors formerly specified in the 1987 AASHTO Guide Specification for horizontally curved box-girder sections were a result of a study by Rabizadeh, Shore and Rabizadeh, and Rabizadeh and Shore.<sup>(271, 272, 273)</sup> The computer program DYNCRB/BG (Dynamic Analysis of Curved Bridges/Box-Girders) developed by Rabizadeh and Shore was used for the dynamic analysis.<sup>(272,274)</sup> In this study, annular plates and cylindrical shells were used to model the slab, bottom flanges and webs, while rectangular plates and bar elements were used to model the diaphragms. Damping was neglected but centrifugal forces were considered in the analysis. Mass condensation was used to uncouple part of the differential equations, and the linear acceleration method was used to solve the dynamic problem.<sup>(275)</sup> The study covered only limited cases.

Galdos and Schelling et al. investigated the dynamic impact factors for horizontally curved steel box girder bridges under heavy truck loadings.<sup>(276, 277, 278)</sup> The curved bridges were modeled using an improved two-dimensional grid method, which accounts for the dynamic bridge behavior. The truck loads were modeled as a pair of concentrated forces moving along in circumferential paths. Bridge damping in these studies were neglected. Various number of spans and girders were studied and an impact factor criterion which leads to lower impact values, was proposed to extend the former AASHTO criteria. Such criteria were recently adopted in the 1993 AASHTO Guide.

### STABILITY

**Compression Flange Design:** The compression flange of a horizontally curved box girder is subjected to both normal and shearing stresses. The normal stresses are due to bending, non uniform torsion, and distortion of the cross section. The shearing stresses are from both the initial girder curvature, the nonuniform torsion and St. Venant torsion.

In 1970, Culver and Nasir studied analytically the elastic and inelastic buckling behavior of an unstiffened flange of a curved box girder.<sup>(279)</sup> The flange was idealized as a simply supported sector plate subjected to a combination of in-plane normal and shearing stresses. The assumption of a simply supported sector plate is conservative because it neglects the restraint provided by the webs and the restraint resulting from the torsional rigidity when a diaphragm or cross frame is present. The in-plane normal bending stress was assumed to be uniformly distributed across the flange width thereby neglecting the shear lag effects, and the shearing stresses were assumed to have a linear distribution. The energy approach was used to formulate the buckling problem, and the solution was obtained by the Rayleigh-Ritz method. Results of this study showed that yielding of the compression flange significantly reduces the flange buckling stiffness, which appeared to affect the normal stress more than the shear stress. Furthermore, it was shown for practical box-girder proportions that the influence of curvature of the compression flange on the buckling coefficient was not significant.

## ULTIMATE STRENGTH

In 1978, Yonezawa et al. examined experimentally the ultimate behavior of a horizontally curved box girder with a longitudinally stiffened deck plate.<sup>(280)</sup> The experimental model consisted of a welded 600-mm (23.6-in.) wide and 300-mm (11.8-in.) deep box girder. The span of the model was 4200-mm (165-in.) along its centerline with a radius of curvature equal to 4000-mm (15.7 in.). The specimen was tested under two concentrated loads and the top flange failed due to inelastic buckling. A year later, Dogaki et al. tested another specimen of almost the same dimensions to failure, which also occurred in the top flange due to a combination of local and overall buckling.<sup>(281)</sup> The ultimate loads for these two specimens were compared with various existing analytical methods that were primarily developed for straight girders. In 1985, Yonezawa et al. presented an analytical method to predict the elastic buckling of orthogonally stiffened sector plates under uniaxial compression uniformly distributed at the straight edges.<sup>(282)</sup>

Arizumi et al. reported on tests involving three composite box girders with different cross sections and radii, but the same length.<sup>(283)</sup> The boundary conditions were designed so that simple supports for bending and fixed supports for torsion were obtained. A single load was applied at four different locations.

Daniels conducted a full-scale test on a horizontally curved composite box girder that was initially designed for fatigue testing.<sup>(284)</sup> The box section was 965 mm (38 in) wide and 876 mm (34.5 in) deep and the concrete slab was 1524 mm (60 in) wide and 203 mm (8 in) deep. Before casting the concrete slab, the specimen was first tested under a concentrated load located at midspan above the outside web. The load was kept low in the elastic range and a load-deflection behavior of the box was established. In the second stage, a Bridgeform was attached to the box girder using screws. Two different patterns of screws were investigated, and the girder was in each case subjected to low static loads. In the final stage, the box section along with the concrete slab were tested to failure. Test results showed that a steel Bridgeform with a closer screw pattern reduced the vertical deflection and the rotation of the box significantly over those of the open section (without a Bridgeform). For the composite box section, a premature failure in the concrete slab occurred, therefore results pertaining to the ultimate strength were inconclusive.

## DIAPHRAGMS AND CROSS FRAMES

**Intermediate Diaphragms:** The use of intermediate diaphragms in curved box girder bridges subjected to external loads is always necessary in order to provide adequate rigidity, which will in turn minimize the cross-sectional distortions that result in large normal stresses. Oleinik and Heins and Heins and Oleinik used the formulations developed by Vlasov and Dabrowski to examine the load-deformation and cross-sectional deformations of horizontally curved box girders. (See references 285, 286, 180, 227 and 228.) A computer program was written to solve the governing differential equation for cross-sectional deformations using the finite difference numerical technique. The accuracy of the resulting solutions was checked by making a comparison to closed-form results obtained by Dabrowski, and by comparing the numerical results to

experimental data obtained from tests conducted at Carnegie-Mellon University. A parametric study was used to develop a number of design equations. It was pointed out that the addition of diaphragms in a curved box girder will minimize the normal distortional stresses without affecting the flexural stress distribution in the member. In 1980, Nakai et al. studied analytically the distortion of horizontally curved box girders using Dabrowski's equation, whose numerical solution was obtained by the transfer matrix method.<sup>(287)</sup> The theoretical results were compared with experimental data obtained from tests conducted on four plexiglass models of the same span length, radius of curvature, and central angle, but with different numbers of diaphragms. The results of this paper were further utilized by Nakai and Murayama to develop three approximate equations for estimating the distortional warping stresses for horizontally curved box girder bridges subjected to a distributed dead load, a distributed live load, and a line live load.<sup>(288)</sup> In 1989, Yabuki and Arizumi studied analytically and experimentally the normal distribution of the distortional deformations in the cross-section of single span curved box-girder bridges.<sup>(289)</sup> In the analytical part, the distortional warping response of the box girder was modeled as a beam on an elastic foundation. The diaphragms were modeled as additional springs attached to the beam. The solution of the fourth-order differential equation was obtained numerically by the finite difference method. In the experimental part, two steel models were tested. Both models had a radius of 12 m and a central angle of 30°. Rigid diaphragms were provided at the end supports. The first model however had only one intermediate diaphragm at the center, while the second had five equally spaced intermediate diaphragms. The test models were subjected to a single concentrated load at the middle of the box width at mid-span point. Normal stress distributions and load-deformation characteristics were obtained experimentally and showed good agreement with analytical calculations despite the lack of exact modeling of the boundary conditions. Extensive numerical parametric results showed that the distortional warping stress increases as the central angle increases. In addition, it was concluded that the empirical equations proposed by Nakai and Murayama for predicting the distortional stress were in good agreement with the present study for the cases of distributed dead load and line live load only.<sup>(288)</sup> For the case of uniformly distributed live load, the distortional stress equation was modified. Provisions pertaining to the spacing of intermediate diaphragms in steel curved box girder bridges were formulated.

In 1979, Daniels, Abraham, and Yin presented the results of a finite-element study concerning the effect of spacing of the rigid interior diaphragms on the fatigue strength of curved steel box girders.<sup>(290)</sup> The results showed that reducing the interior diaphragm spacing effectively controls the distortional normal and bending stresses, and increases the fatigue strength of curved steel box girders.

## FATIGUE

Inukia et al. investigated the stress history data on the Fort Duquesne Bridge over the Allegheny River in Pittsburgh, Pennsylvania to examine the fatigue strength of the details in the curved composite box girders; in particular, the termination of transverse stiffeners at intermediate diaphragms and the discontinuous backup bars at the flange-to-web junction.<sup>(291)</sup> The bridge had three continuous spans; the centerline arch length of each span was 30.48 m (100 ft) and the radii for each span were 260 m (853 ft), 263 m (863 ft), and 260 m (853 ft), respectively. Transverse

diaphragms were provided every 3.048 m (10 ft). The field testing was conducted on the middle span, whose cross section contained both transverse and longitudinal stiffeners. To simulate AASHTO HS20 loading, the FHWA's truck load was used and run at speeds of 8 km/h (5 mi/h) to simulate the static load and between 69 and 80 km/h (43 to 50 mi/h) to simulate the dynamic load. Experimental data was collected over 5 days. The study concluded that transverse stiffeners used as connection plates for the diaphragm should be rigidly attached to both flanges so that at the transverse stiffener-to-flange gap, local stresses (bending in the vertical direction) are small. In addition, backup bars that are discontinuous are prone to fatigue crack propagation, and thus they must be made continuous and treated as structural components in the bridge, or preferably eliminated.

Daniels and Batcheler evaluated the fatigue performance of eight types of welded details categorized as AASHTO Fatigue Categories B, C, D, and E by testing three full-scale curved box girders subjected to approximately 2,000,000 constant amplitude load cycles.<sup>(292)</sup> Results showed that AASHTO Category B, C, D, and E details for straight girders were appropriate for curved girders as well. It was also recommended that special attention be given to the design of internal diaphragm connections at which secondary fatigue crack growth can occur as a result of out-of-plane forces and displacements if not detailed properly.

## **FIELD TESTS**

Heins and Lee examined the static and dynamic response of a two-span continuous curved box bridge located in Seoul, Korea.<sup>(293)</sup> Measured deflections and mean normal stresses at the bottom flange, top flange, and web correlated reasonably well against analytical values obtained from three computer programs, CURSGL, STRUDL, and SAP.<sup>(71)</sup> The largest values of impact factors calculated from deflection and stress measurements were 0.75 and 0.16, respectively. The experimental natural frequency of the bridge was found to be 2.7 c/s and agreed quite well with the calculated value, 2.6 c/s, using the expression derived by Heins and Sahin.<sup>(182)</sup>

## **DESIGN AIDS**

Heins presented design equations applicable to practical straight and curved steel composite box girder bridges of moderate length from 15.6 to 78 m (50 to 250 ft).<sup>(294)</sup> Included were approximate formulae for the section geometry, load distribution, calculation of forces, and bracing requirements.

## CHAPTER V. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDIES

This task resulted in the identification of approximately 750 references pertaining to a wide range of analytical and experimental investigations on horizontally curved I- and box girder bridges. These references are placed in an electronic data base that is easy to access, query, and update as additional research is completed. Of the 750 surveyed references, 300 references were reviewed briefly in this report, about 210 references were recommended for further review (140 of these references were difficult to obtain), while the remaining 240 references were placed in the bibliography. While specific recommendations can only be made when the references categorized in this report are examined thoroughly in future tasks of this project and in the NCHRP Project 12-38, some recommendations deserving immediate attention, in the writers' views, are summarized in the following sections.

### ANALYSIS METHODS

1- The plane grid and space frame methods treat curved members as straight members, and hence are recommended to be used only for preliminary design purposes. The V-load method, which was applied to I-girders only, underestimates inner girder stresses and does not consider the bracing effect in the plane of the bottom flange. In addition, the reliability of these results is dependent on the selection of the transverse live-load distribution factors.<sup>(20)</sup> Thus the V-load method can only be recommended for preliminary analysis.

Among the refined methods, the finite-element method is probably the most involved and time consuming. However, it is still the most general, accurate and comprehensive technique that has been applied to static/dynamic elastic/inelastic analysis with different mechanical and thermal loading. The other refined methods proved to be as accurate as the finite method, but are limited to certain configurations and boundary conditions and they are generally more cumbersome to use. To facilitate using the finite-element method for the analysis of horizontally curved bridges, it is recommended to incorporate a special-purpose user-friendly graphical interface to the existing computer codes, which would expedite modeling and discretizing of the structure and allow the user to view results and modify the loading, boundary conditions and/or configuration.<sup>(295,296)</sup>

2- Although a number of publications dealt with the geometrically and/or materially nonlinear behavior of horizontally curved bridges, the issue of performing inelastic analysis of the entire bridge system has not been addressed clearly. The basic questions that require answers are summed up in the following: when, why, and how should the bridge engineer pursue a rather involved nonlinear analysis? Also, can simplified approximations be made to take some advantage of reserve strength of those members that do not require detailed inelastic analyses.

3- An up-to-date survey on the available software for analyzing horizontally curved bridges similar to that published in 1978 by Schelling et al. would be extremely useful.<sup>(297)</sup> Such a

survey should clearly state the analytical method adopted in each computer package, the assumptions and limitations, the type of elements available, the design and visual/graphical capabilities of the programs, their abilities to generate elements and/or loads, and some bench mark tests to calibrate the performance of such programs. In addition, a concise document comparing results from various analytical methods, including three-dimensional finite element analysis, to those of field measurements must be prepared.

## **BEHAVIOR OF CURVED GIRDER BRIDGES DURING CONSTRUCTION**

1- The stability issue of curved box and I-girder bridges during construction is limited to the analytical work of Schelling et al.<sup>(256)</sup> This work should be examined further using three-dimensional finite element analysis, and extended to cover box-girder bridges with single and multiple spans. In addition, the analytical study should address the effects of ties, bracing, web stiffeners, etc. on the distortional behavior of horizontally box and I-girder bridges during construction.

2- A field experimental investigation must be carried out to measure the internal forces and deformation in the girders and in the bracing system during construction. Comparison between the analytical and experimental data must be made. Recommendations addressing the analysis method or the level of analysis required for predicting the behavior of curved girder bridges under construction need to be developed.

3- Cost-effective construction methods and erection guidelines that incorporate the experiences of steel fabricators and erectors must be developed.

## **ULTIMATE STRENGTH OF CURVED BOX AND I-GIRDERS**

1- Experiments demonstrating local buckling and/or lateral-torsional buckling limit states for single and multiple curved I-girder system must be conducted. The study should include composite and non-composite I-girders comprised of plate elements with different slenderness ratios. The experimental study must be accompanied by an analytical investigation to address the validity of existing, often controversial, theoretical solutions or it must develop new ones. The same recommendations also apply to curved box girders.

2- Experiments demonstrating the limit states in a transversely and/or longitudinally stiffened web should be conducted. The investigation should include cases in which the transverse stiffeners are placed at distances greater than the web depth ( $a/d > 1$ ).

3- Experiments addressing the effective width of concrete slab for composite curved I- or box girder bridges should be conducted.

## **IMPACT**

The impact factor adopted in the current AASHTO Specifications is based upon the work of Chaudhuri and Shore for the case of I-girders, and the work of Schelling et al. for the case of box girders.<sup>(201,276,277,278)</sup> All of these studies neglect the bridge damping and make various assumptions related to modeling the moving loads. Furthermore, the impact factors developed from these studies are perhaps more suitable when two dimensional grid analysis is performed. The work of Nakai and Kotoguchi must be examined. In addition, it would be extremely useful if all previously developed works were verified analytically using refined analysis methods.<sup>(298)</sup> Attempts must be made to obtain field measurements of the impact factor so that final design recommendations can be made.





## APPENDIX A - USERS MANUAL FOR CURVED BRIDGE DATA BASE

### LOADING CURVED DATA BASE

1. Load Microsoft Access by double-clicking on its icon from the Windows Program Manager.
2. Open the data base that contains the references on curved bridges by selecting "OPEN DATA BASE" from the "FILE" menu and typing in "CURVED.MDB" preceded by the drive and directories under which the data base file is located (e.g. if the file is saved on the B: drive under the root directory, type in "B:\CURVED.MDB").
3. Select the "FORM" button from the CURVED data base window, and choose "OPEN" which will display the first reference on the screen. To move around and display the rest of the references, one can use the forward and backward arrow buttons located at the lower left corner of the screen. (Clicking on the double arrow buttons displays the first or last references).

### SEARCH EXAMPLE

To find a paper by Heins published in 1983 (where Heins might not be the first author) :

1. Click on the "AUTHOR" box.
2. From the "EDIT" menu, select "FIND".
3. Type in "\*Heins\*83\*" (the asterisks are used as wild cards).
4. Click on the "FIND NEXT" button. Access will display the first occurrence of such a paper. Clicking on the "FIND NEXT" button again will display the second occurrence, and so on.

### QUERY EXAMPLE

To find all experimental work by Nakai and Kitada (order is not important) published after 1983 and sort the outcome by the year (in an ascending order):

1. From the "RECORDS" menu, choose "EDIT FILTER / SORT".
2. In the lower window apply the following requirements:

Field:	AUTHOR	YEAR	EXPERIMENTAL
Sort:		Ascending	
Criteria:	*Nakai*Kitada*	> 1983	Yes
or:	*Kitada*Nakai*		

3. Select "APPLY FILTER / SORT" from the "RECORDS" menu.

## **REPORT EXAMPLE**

To print the result of the last query :

1. Instead of executing step 3 from the previous example, choose "SAVE AS QUERY" from the "FILE" menu.
2. Type in a proper name (e.g. "Nakai-Kitada") and click on the "OK" button.
3. Close the query by selecting "CLOSE" form the "FILE" menu.
4. Close the form by selecting "CLOSE" form the "FILE" menu.
5. Click on the "REPORT" button, and decide between "BIBLIOGRAPHY" and "DETAILED" formats (the first is for a bibliography list while the second includes abstracts).
6. Click on the "DESIGN" button.
7. From The "VIEW" menu, select "PROPERTIES".
8. Type in the name of the query in the "RECORD SOURCE" box (e.g. "Nakai-Kitada")
9. Select "PRINT" form the "FILE" menu.

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