

76-14.4  
**Implementation  
Package**

**WAVE  
EQUATION  
ANALYSIS  
OF PILE  
DRIVING**

**WEAP  
PROGRAM**

**Volume IV — Narrative Presentation**

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16. Abstract <p>A computer program was written and tested that performs a realistic Wave Equation Analysis of Piles driven by any type of impact hammer. Conventional pile and soil models were used in addition to both a thermodynamic model for diesels and refined mechanical hammer models.</p> <p>The program development was aimed at providing a simple input and both a flexible and extensive output that includes automatic plotting capabilities. Pile Driving Hammer data were prepared and stored in a file for most of the commonly encountered models. The computer language is FORTRAN IV.</p> <p>The program was extensively tested against measured pile top force and velocity data and against measured diesel combustion pressure and stroke.</p> <p>This volume is the fourth in a series. The others in the series are:</p> <table border="1"> <thead> <tr> <th><u>Vol No.</u></th> <th><u>FHWA No.</u></th> <th><u>Short Title</u></th> <th><u>NTIS (PB) No.</u></th> </tr> </thead> <tbody> <tr> <td>1</td> <td>IP-76-14.1</td> <td>Background</td> <td></td> </tr> <tr> <td>2</td> <td>IP-76-14.2</td> <td>User's Manual</td> <td></td> </tr> <tr> <td>3</td> <td>IP-76-14.3</td> <td>Program Documentation</td> <td></td> </tr> </tbody> </table>			<u>Vol No.</u>	<u>FHWA No.</u>	<u>Short Title</u>	<u>NTIS (PB) No.</u>	1	IP-76-14.1	Background		2	IP-76-14.2	User's Manual		3	IP-76-14.3	Program Documentation	
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**WAVE EQUATION ANALYSIS OF  
PILE DRIVING**

**Narrative Presentation**

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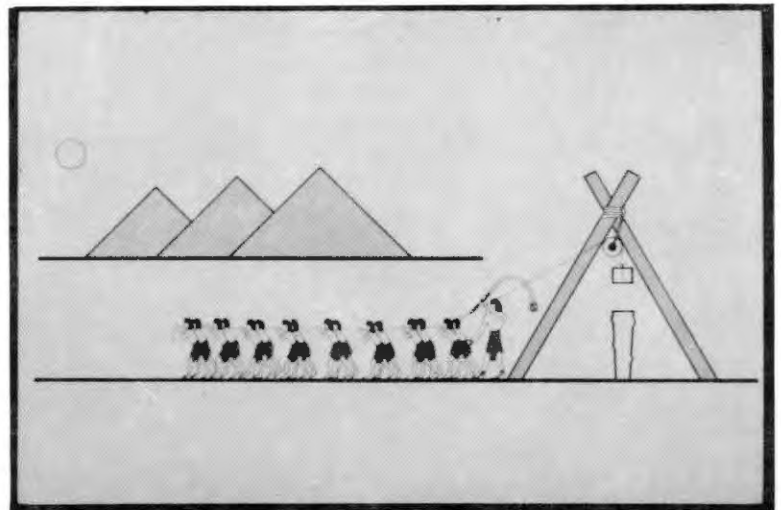
## **Part I**

### **Background**

- A1. When foundation engineers view pile driving and reflect on the fact that the driving operation induces failure in the pile-soil system, it is natural that they should attempt to use measurements made during driving to determine bearing capacity.

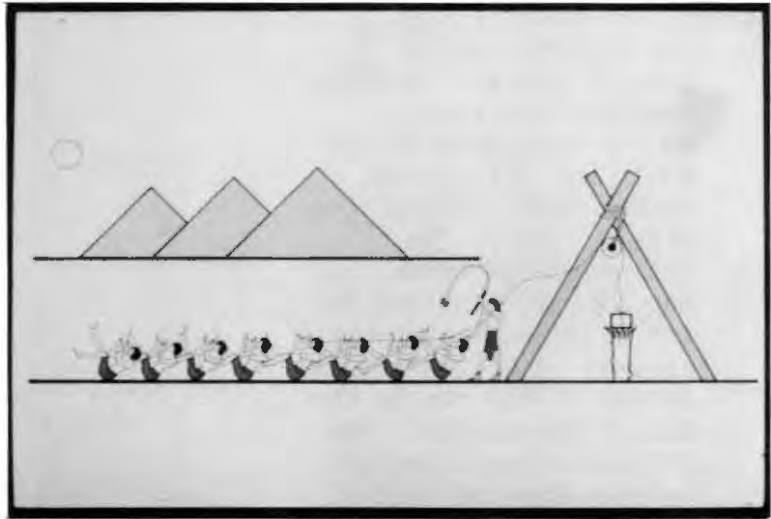


- A2. Piles have been used as a foundation element since antiquity. They were driven with primitive equipment and were designed by a completely intuitive approach.





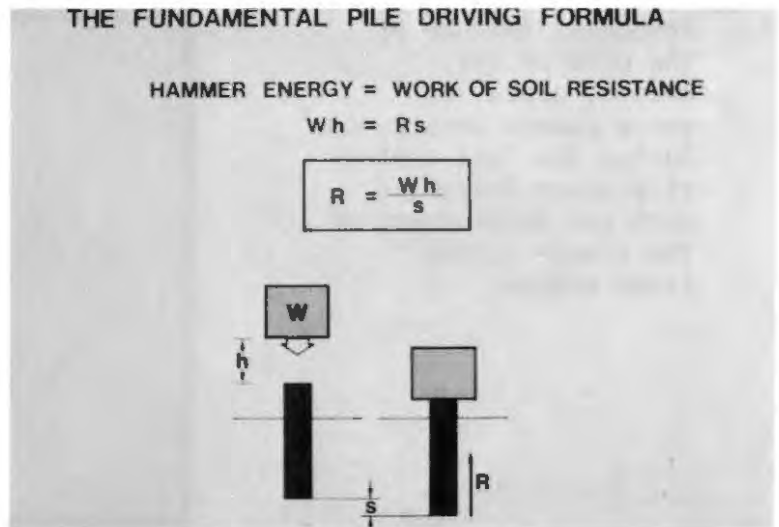
- A3. Probably, failure of the pile or the driving operation was a common occurrence. During the last century pile usage increased with the development of the single acting steam hammer.



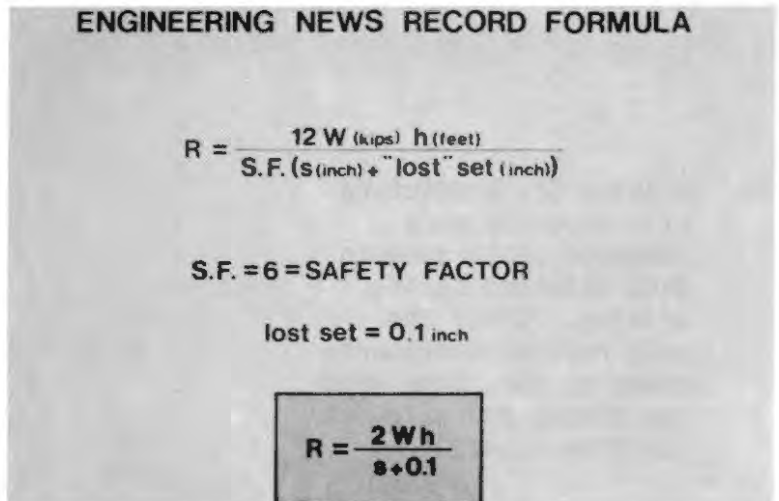
- A4. Methods for predicting pile strength were proposed using dynamic data obtained during driving. Since the only realistic measurements at that time were ram stroke and pile set (or blow count)



- A5. it was natural that the engineer would turn to energy concepts, equating ram energy,  $Wh$ , to work done on the soil,  $Rs$ . Of course, rather gross assumptions were implied. The resistance of force,  $R$ , was assumed constant and the delivered energy was assumed to be the potential energy of the ram at the top of the stroke. The effects of cushions and helmets were neglected.



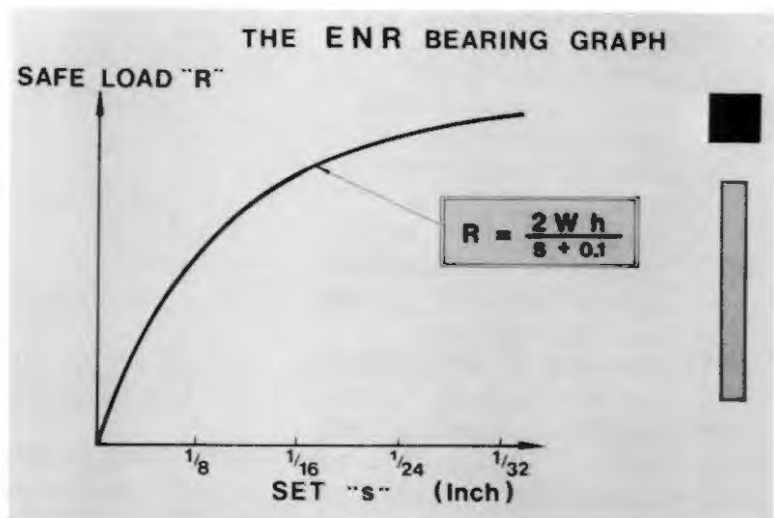
- A6. Since poor results were achieved, a large factor of safety was introduced in order to insure against failure and the "losses" were estimated in terms of "lost" set.



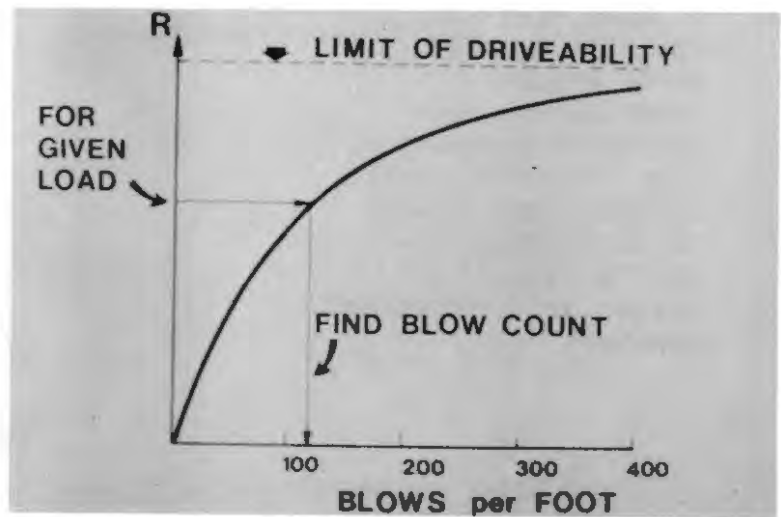
- A7. A large number of formulas appeared. While dynamic formulas have been generally discredited, they are still widely applied today because of their simplicity and the lack of a better, well-recognized approach.



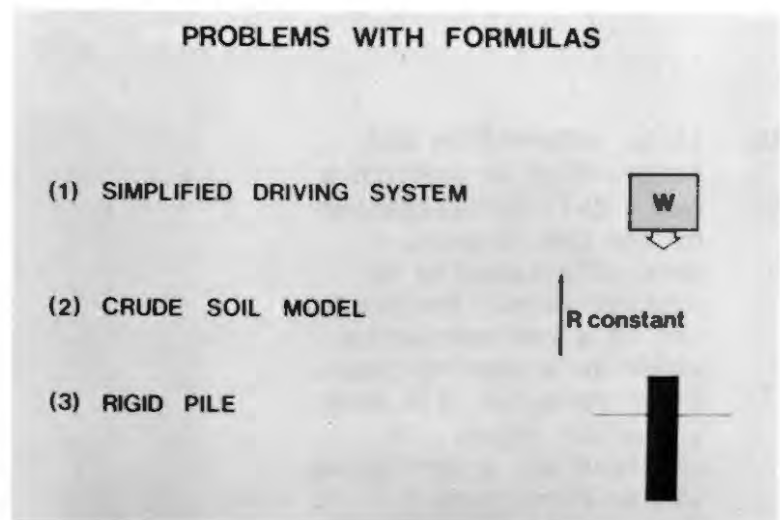
- A8. It is interesting and instructive to examine a graphical representation of the ENR formula. Here pile capacity is plotted versus permanent set in a representation known as a bearing graph. Other formulas will have a similar shape. In application, a particular set or blow count (hammer blows per foot) is related to the pile design load.



- A9. Alternatively, for a specified capacity the required blow count can be determined. Also, an equipment selection can be made. For example, a particular system can only overcome a certain maximum resistance. At the upper range of the curve, a large change in blow count does not indicate a comparable change in capacity.



- A10. Now consider the causes for problems with the dynamic formulas. First, the derivation is not based on a realistic treatment of the driving system nor does it recognize the variability of equipment performance. Typical driving systems can include many elements in addition to the ram such as helmet, capblock, cushion, and anvil. Second, the soil resistance is very crudely treated. The assumption is made that the soil resistance is a constant force and this assumption neglect even the most obvious characteristics of real soil performance. Third, the pile is assumed to be rigid. This neglects completely the real flexibility and the wide variations it can have.



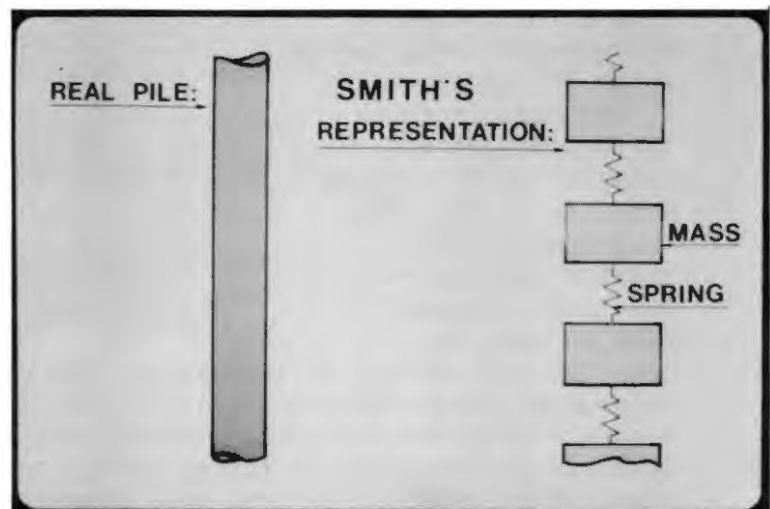
- A11. As an alternative, the Wave Equation Analysis was proposed in the last century. The Wave Equation is actually a linear differential equation which was derived from Newton's Second Law. Since real pile driving problems are not easily solved by the Wave Equation.

### THE WAVE EQUATION

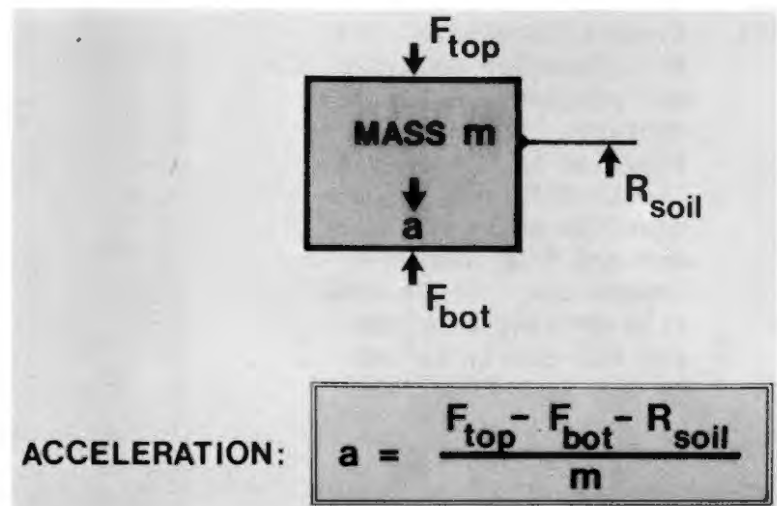
$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

**u** Displacement  
**c** Wave Speed  
**t, x** Time, Length Coordinates

- A12. E.A.L. Smith proposed, in the 1950's, to divide the pile into relatively large segments (say a few feet each) and represent each of them by a discrete mass and a spring. The total pile and driving system is so represented and then the motion of the pile is analyzed by applying Newton's Second Law successively over very short time increments.



- A13. The forces acting on a typical pile element are those due to the strain at its top and bottom and to the soil resistance. Thus, from Newton's Second Law, the elements' acceleration at a particular time is equal to the sum of  $F_{top}$ ,  $F_{bottom}$ , and  $R_{soil}$ , divided by the mass of the element.



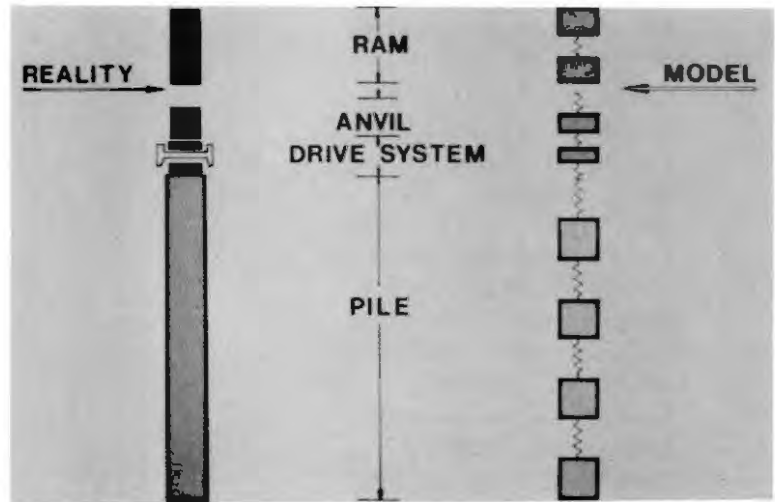
- A14. Having determined the acceleration of the element, its velocity and displacement can be determined by integration. For this integration a small time increment is used during which the acceleration is assumed constant. The results of the integration become more accurate as the time increment is made shorter. Of course, as the time increment is made shorter the computational effort is increased. The cycle is completed by the determination of the compression in the spring between neighboring elements from the calculated element displacements. The compression then can be converted to forces acting on the elements and the computation repeated for the next time increment.

$$\text{VELOCITY} = \text{ACCELERATION} \times \text{TIME INCREMENT}$$

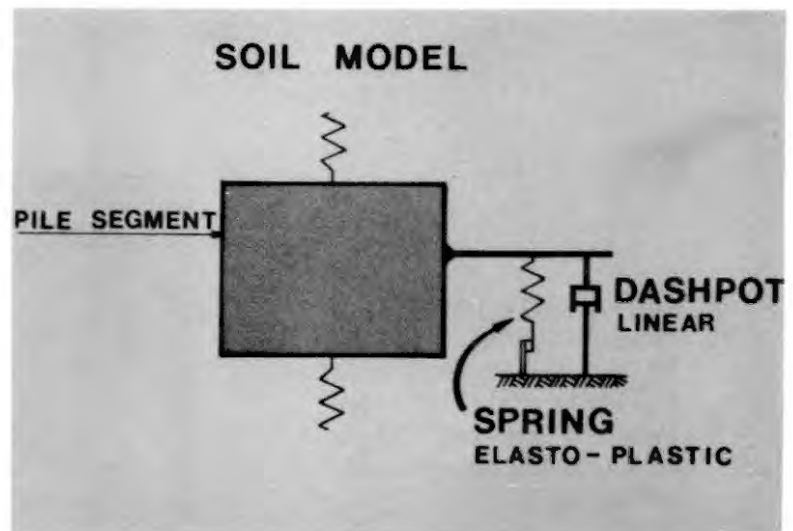
$$\text{DISPLACEMENT} = \text{VELOCITY} \times \text{TIME INCREMENT}$$

- A15. In summary, the total pile driving system is represented by a series of masses and springs. It is analyzed by giving the ram an initial velocity determined by the manufacturer's stated characteristics and then analyzing the system using the force-inertia balance over the required time period consisting of a large number of time increments. The

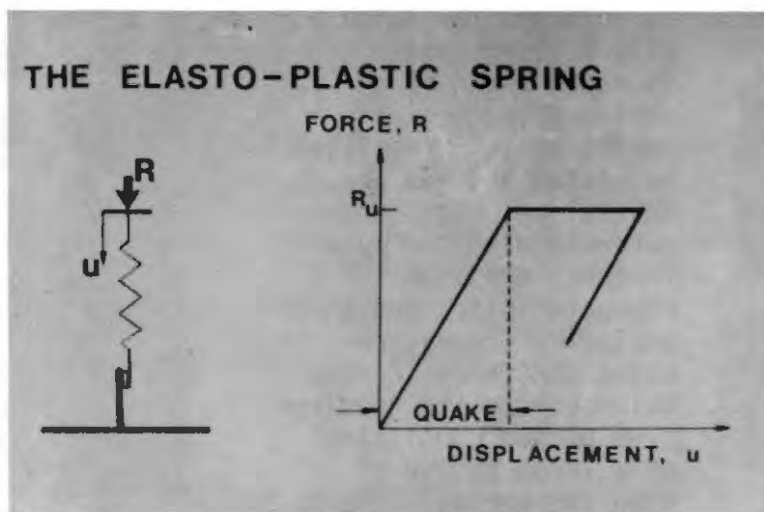
mass and spring characteristics can be calculated from the physical characteristics of the pile and the driving system.



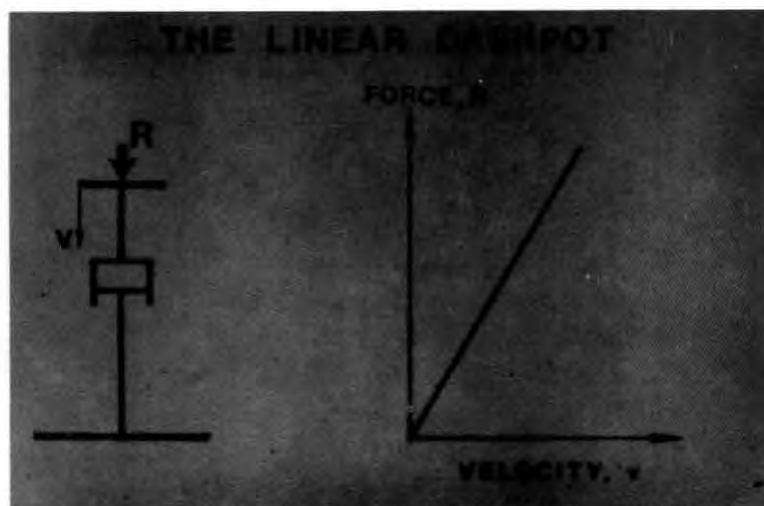
- A16. But pile driving analysis must include more than just the pile and the driving system. A soil resistance force has been mentioned. Now consider some of the details of the description of this force. It is assumed to be composed of a static and a dynamic portion and it can be represented by a spring and a dashpot.



- A17. The spring or static portion of the resistance relates the soil resistance to the pile element displacement. The soil resistance force is assumed to increase linearly with the displacement until a displacement called the quake is reached. With further displacement the force remains constant. Unloading occurs along a line parallel to the loading line.

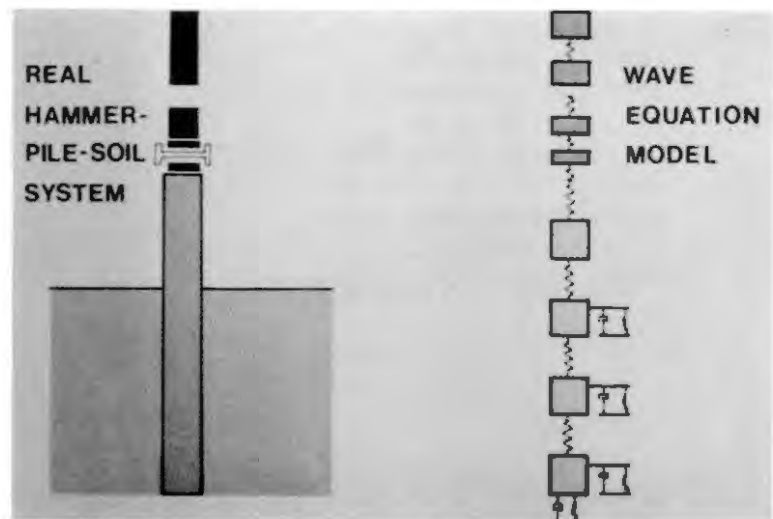


- A18. The dynamic portion of the resistance is represented by a dashpot that assumes a linear relationship between the resistance force and the element velocity.



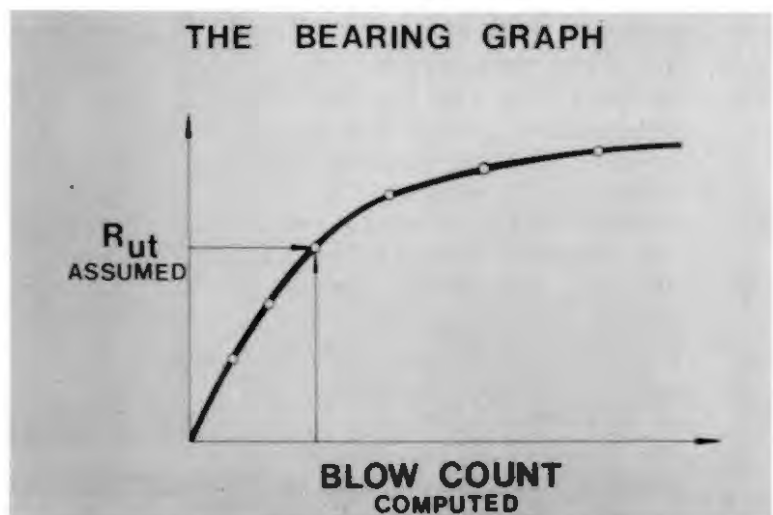


- A19. So the total pile driving analysis problem is represented by the series of masses and springs representing the pile and the driving system with the soil resistance on the embedded portion modeled by elastic plastic springs and linear dashpots attached to the appropriate elements.

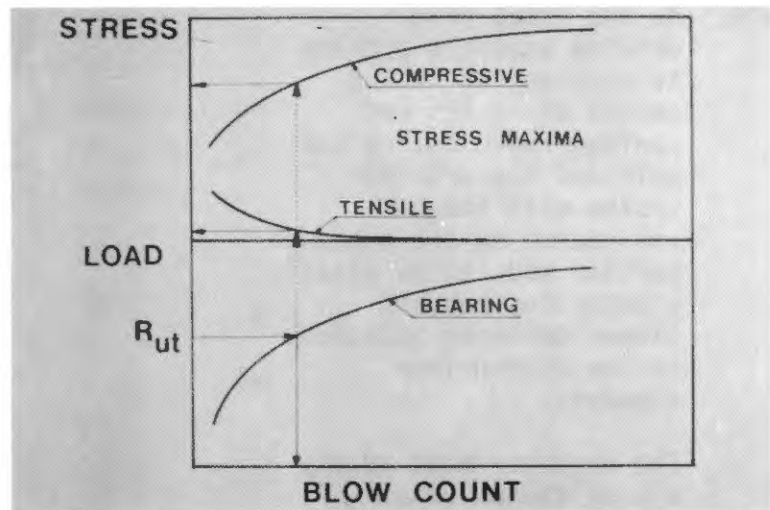


The engineer must supply all of the physical characteristics of the system including the soil. Computer programs have been prepared to perform the analysis and these programs have come to be known as Wave Equation programs. In this presentation the name wave equation will refer to computer programs of this general type.

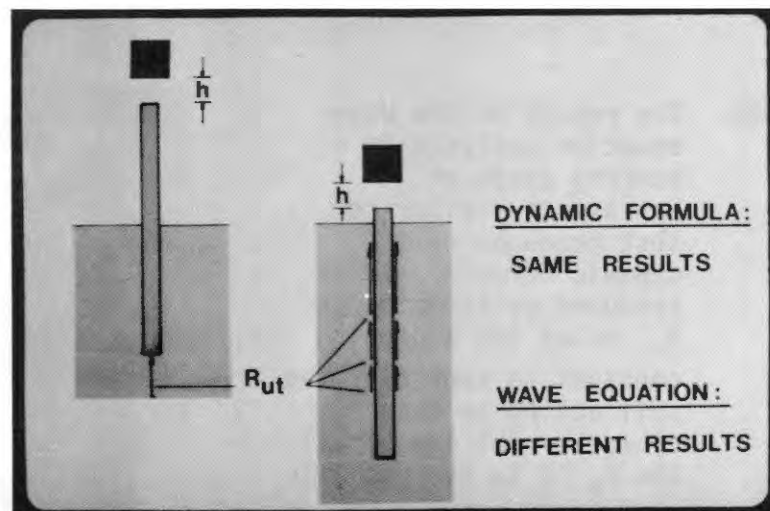
- A20. The result of the wave equation analysis is a bearing graph of appearance similar to that produced by a dynamic formula. It is produced by assuming an  $R_u$  value and a damping constant to represent the soil acting on each element. The sum of all the  $R_u$ 's is the ultimate static capacity of the pile,  $R_{UT}$ . For a particular  $R_{UT}$  the wave equation analysis is performed and the permanent pile displacement is calculated. The bearing graph is constructed from a set (say 5 or 6) of these points by interpolation.



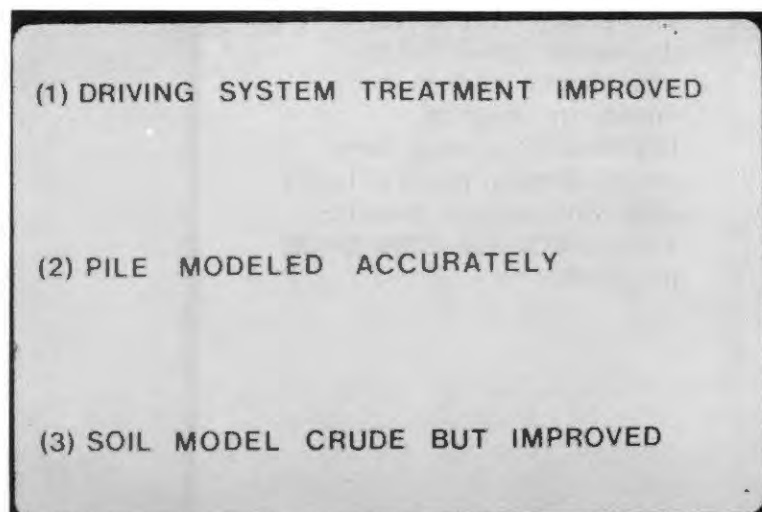
A20a. In addition to the bearing graph the critical stresses in the pile can also be determined since the forces in the pile at each element for every time increment were found during the analysis.



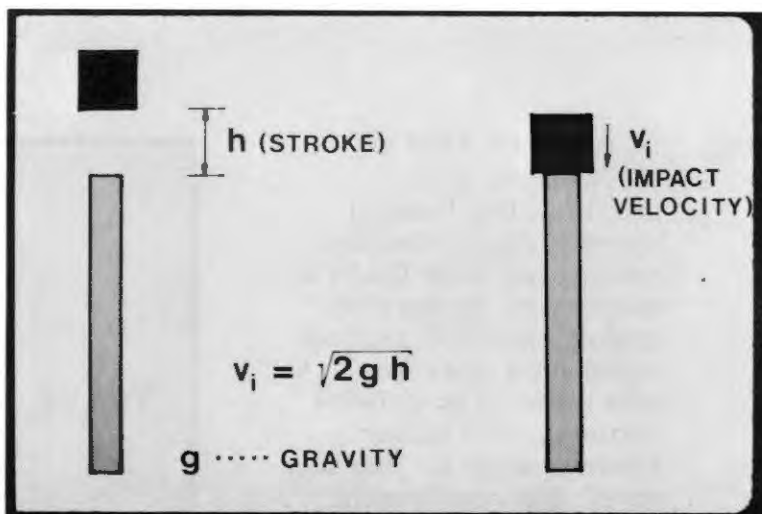
A21. It should be noted that a bearing graph is associated with a particular pile penetration. While it is similar in appearance to the graph generated by a dynamic formula, it represents something quite different. The dynamic formula has as its only variable, hammer energy with, perhaps, an estimate of losses. The wave equation includes pile flexibility, an estimate of soil performance, and driving system characteristics. In the case shown here the dynamic formula would give the same bearing graph in both cases while the wave equation bearing graph would be different.



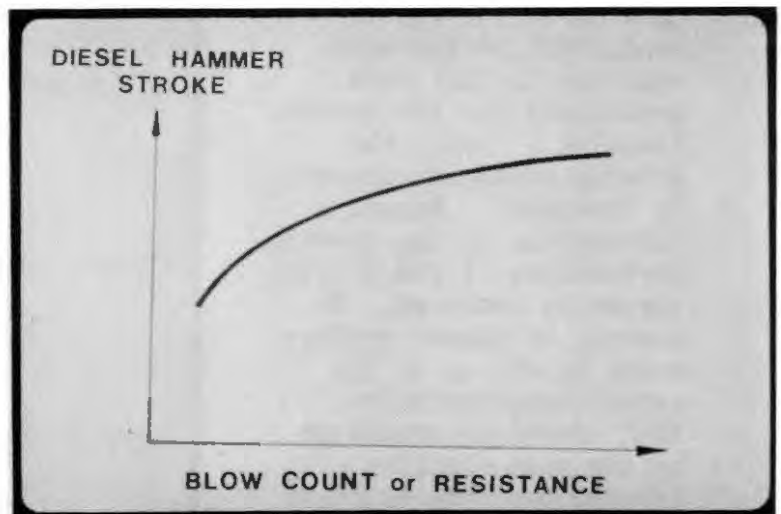
- A22. Consider now a similar evaluation of the wave equation to that made previously for the dynamic formulas. First, the driving system treatment is improved. Accurate information on the dynamic performance of the driving system is required. Of course, if hammer performance is not up to its rated characteristics, that cannot be predicted by the wave equation. Second, the pile is accurately modeled. Third, the soil model, while crude, is clearly improved over that used in the dynamic formula. More accurate representations of the soil could be created but they are not justified due to the difficulty in determining the required parameters.



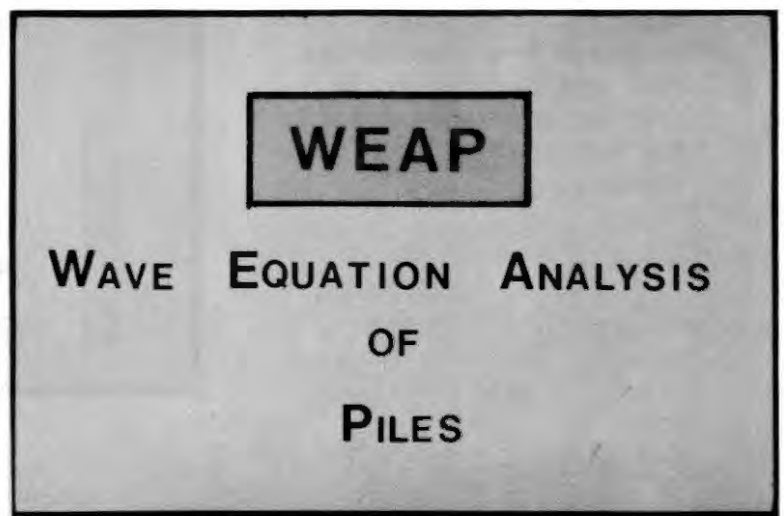
- A23. The computer programs which have previously been available have used as basic input the ram impact velocity as computed from the stroke. This characteristic is particularly undesirable for application to diesel hammers



- A24. where the stroke is dependent on driving resistance and is not known in advance. Difficulties have been encountered, particularly with the stress predictions obtained from these programs.

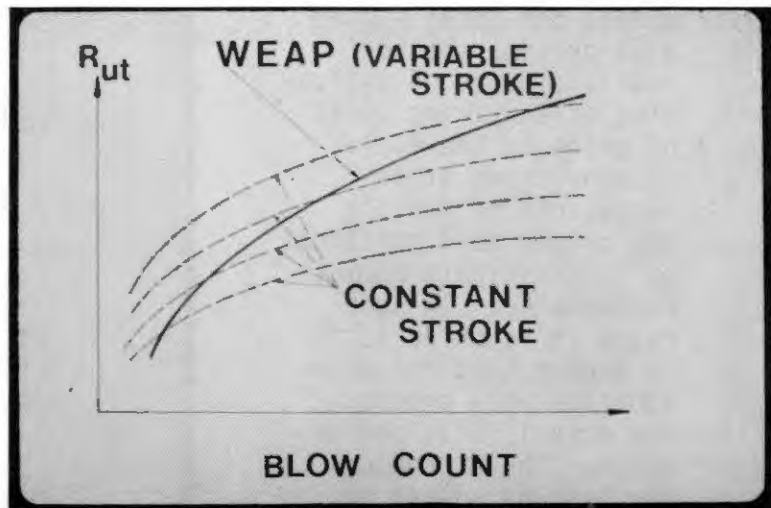


- A24a. In order to find a solution to this problem, the Federal Highway Administration contracted with Goble & Associates to develop a wave equation program based on a more realistic model for pile driving hammers, particularly diesel hammers. Furthermore, the requirement was included to test the program against the large volume of force and velocity measurements made during the Case



Western Reserve University Pile Driving Research Program. The result of this contract is a computer program called WEAP.

A25. The basic program is illustrated by the bearing graph shown here. If a separate bearing graph were obtained for each possible stroke, the family of curves shown dashed would be obtained. Now if the stroke were measured in the field together with the blow count, the appropriate curve could be used to determine the capacity. A series of strokes and related blow counts would produce the solid curve which is the actual bearing graph. Unfortunately, a driving system could not be evaluated prior to going to the field.



A26. The WEAP Program provides an improved treatment of the combustion forces and a more accurate representation of air/steam hammers in addition to analyzing the total thermo-mechanical cycle of the diesel hammer and determining stroke (or bounce chamber pressure for closed end diesels) as an output quantity.

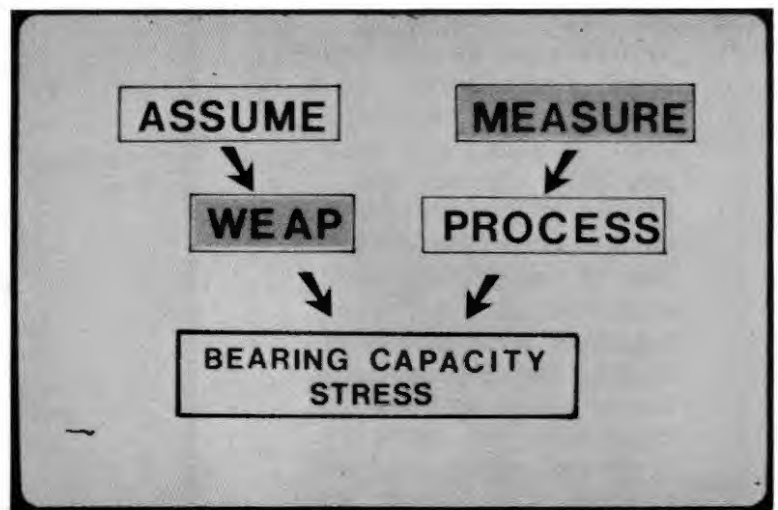
A good wave equation program which realistically

WEAP

### IMPROVEMENTS

1. METAL TO METAL IMPACT SIMULATION
2. AIR/STEAM MODEL MORE DETAILED
3. THERMODYNAMIC CYCLE INCLUDED
4. STROKE AN OUTPUT FOR DIESELS

- A27. models the total system will provide the best possible means of evaluating pile driving short of going to the field. In many cases it is impossible to provide the proper soil constants or to anticipate hammer performance. In such cases it is unrealistic to expect that the wave equation will predict the actual field performance. These problems can best be solved by field measurements using modern electronic devices which are now available.



## **Part II**

### **Models**

- B1. In the previous section of this presentation the use of the wave equation program in the selection of equipment and driving criteria for pile foundations was discussed. Only very general considerations were covered. In this section a greater level of detail will be presented.

# **WEAP**

## **PART 2**

### **MODELS**

- B2. The operation of pile driving hammers and equipment will be discussed and the model used to represent the equipment, the pile and the soil will be described in detail.

When creating a mathematical model of a pile driving hammer, it is necessary that the hammer operation be clearly understood. First, single acting air/steam hammers will be treated.

## **MATHEMATICAL MODELS**

- 1. HAMMER**
- 2. PILE**
- 3. SOIL**

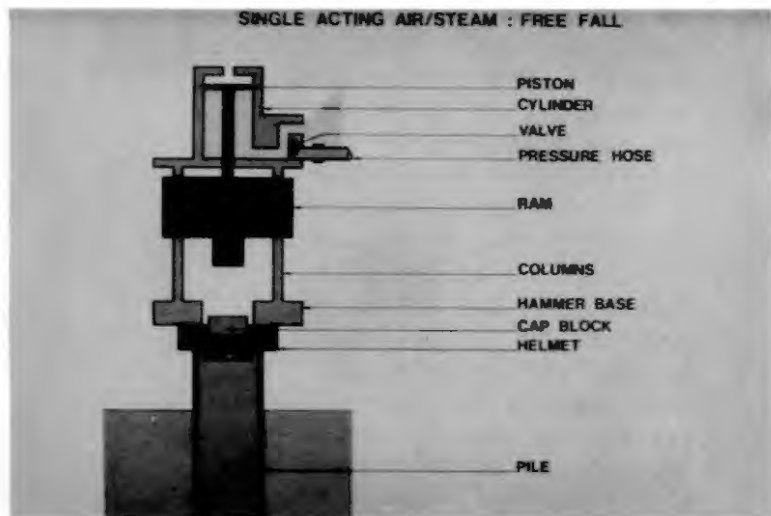


- B3. The driving system consists of the hammer and a helmet holding the top of the pile in position and containing a capblock on which the ram impacts. For concrete piles a cushion is inserted between the pile top and the helmet.

Assume that the cycle begins with the ram in the free fall down stroke sliding on the columns.

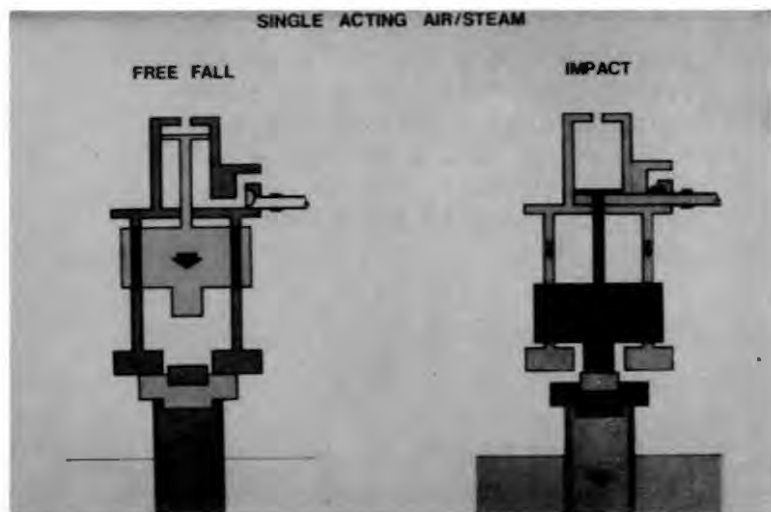
The exhaust valve is open allowing the cylinder under the piston to exhaust directly to the atmosphere. At this time the hammer assembly

consisting of the hammer base, columns, and cylinder are all resting on the helmet (and, hence, on the pile top).

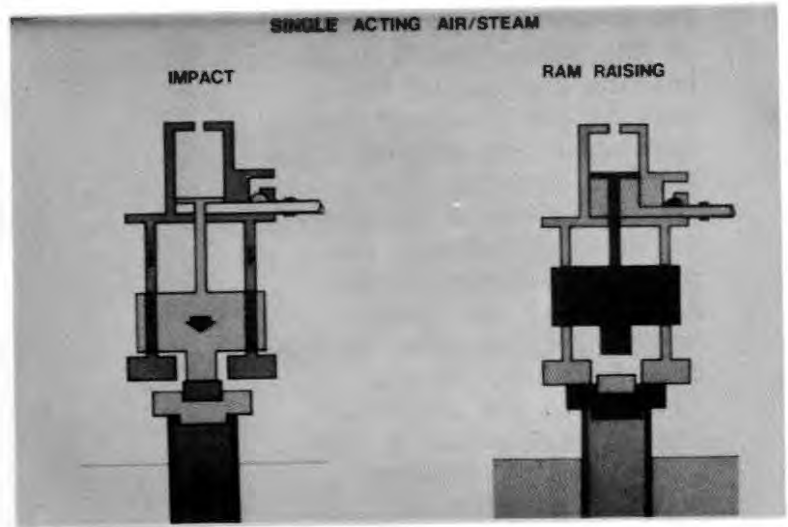


- B4. At the bottom of the down stroke the ram impacts the capblock and drives it rapidly down together with the helmet and the pile top. The assembly is left up in the air falling under the action of gravity. Slightly before reaching the bottom of the stroke the intake valve is opened allowing the active gas (either compressed air or steam) to enter the cylinder. Therefore,

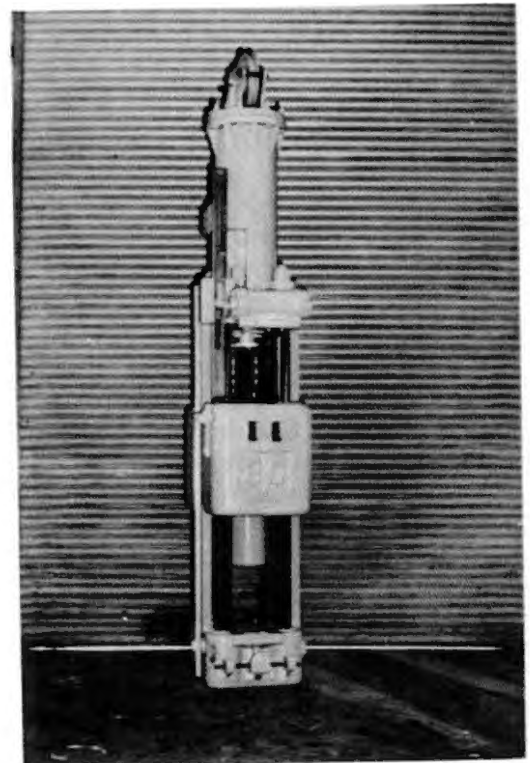
since the gas pressure acts on the bottom of the cylinder it provides a force in addition to gravity which drives the assembly down.



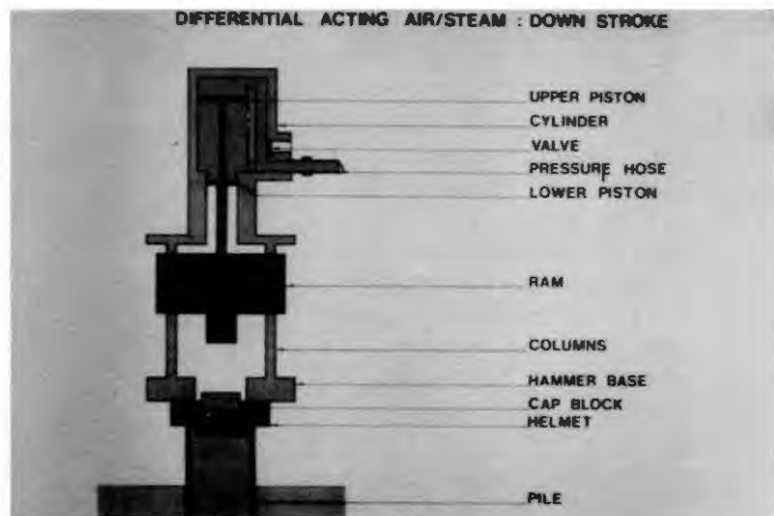
- B5. The assembly impact with the helmet can be substantial since its weight is as great as the ram and the pile rebound velocity is often comparable to the ram impact velocity. The ram gains its upward velocity due to the pile rebound and the active gas pressure in the cylinder. At some point during the upstroke the exhaust valve opens and the ram "coasts" on up to the top of its stroke. Therefore, the stroke of an air/steam hammer is not necessarily constant.



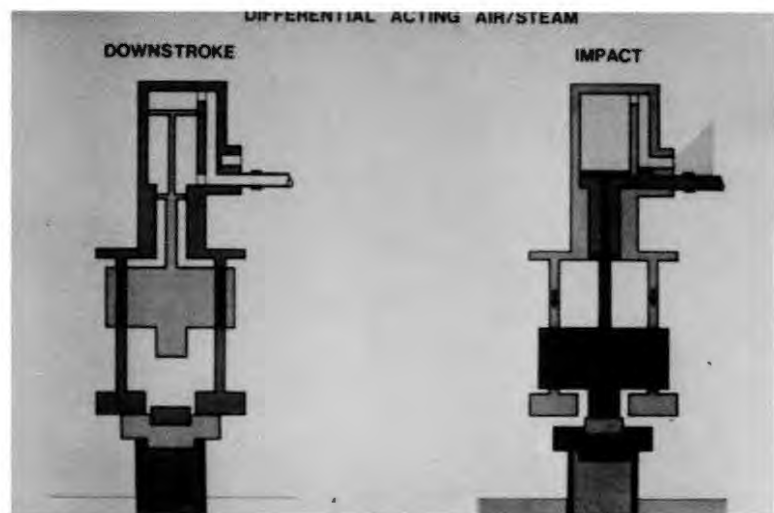
- B6. In this slide a typical single acting air/steam hammer is shown. This particular machine has a 6,000 pound ram, a stroke of about three feet, and an operating speed of about 60 blows per minute.



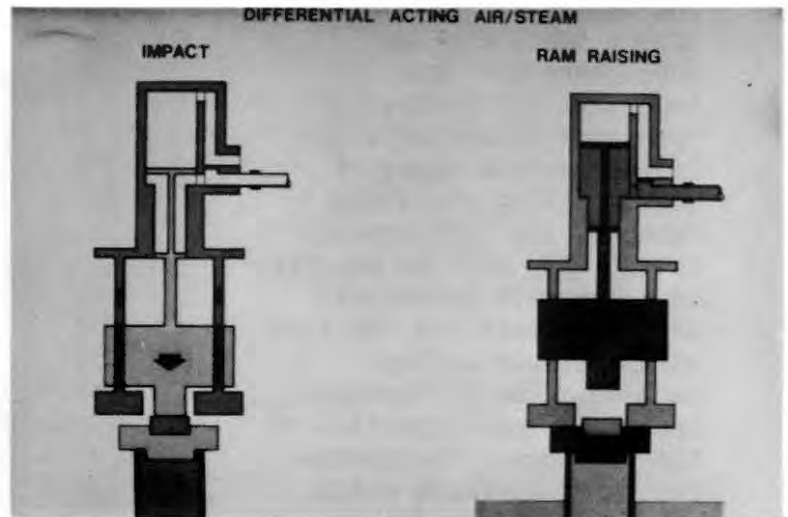
- B7. The double acting air/steam hammer has a similar appearance but some important differences in operating characteristics. Of the various types of double acting air/steam hammers, the differential acting type will be described. The main components of this hammer are the same as the single acting machine. The differences are in the configuration of the cylinder. The piston has three surfaces which are subjected to gas pressure, the upper and lower surfaces of the upper piston and the upper surface of the much smaller, lower piston. Thus, since the same pressure acts on all surfaces there is a net down force on the piston which is reacted by the weight of the hammer assembly. The ram is acted on by both gravity and the applied force and is accelerated more quickly than for the single acting hammer. The upward acting force on the assembly must be limited so as not to lift the assembly off the helmet. This provides a limitation on the force which can be applied to the ram.



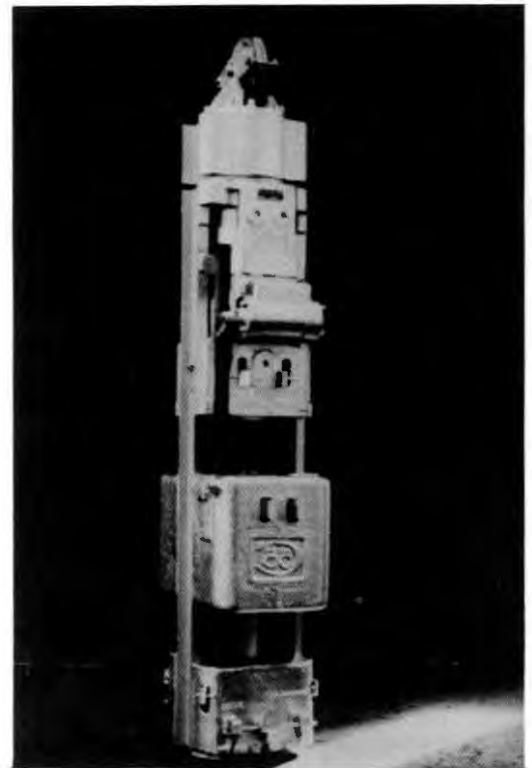
- B8. When the ram impacts on the capblock the helmet is driven out from under the hammer assembly. The assembly then falls to the helmet for a second impact similar to the single acting hammer. Shortly before reaching the bottom of the stroke the valve changes so as to release all gas pressure from the upper chamber. Now with the upper chamber at ambient pressure the net force on the ram is up.



- B9. The cycle is completed at the end of the up stroke when gas pressure is again applied in both the upper and lower chambers. The purpose of the double acting concept is to shorten the stroke and speed up the operation while maintaining about the same ram impact velocity. Typically, double acting hammers operate at about twice the speed of single acting hammers. As mentioned above, the differential acting hammer has been described. There are, of course, other types of double acting hammers.



- B10. The machine shown here is double acting and runs at about 110 blows per minute.



- B11. This slide shows another example of the double acting hammer. This particular machine operates on hydraulic fluid supplied by a pump. The system is a closed one since the spent fluid is returned to the pump. The pump is equipped to allow the control of the fluid pressure.



- B12. The rating of both the single and double acting hammer assumes a predictable and constant energy. To include any energy losses the impact velocity is usually calculated using a reduced energy obtained by multiplication with an efficiency factor which is less than one. From the reduced energy an impact velocity can be calculated for input to the Wave Equation. Both hammer types can be handled in the same Computer model.

ENERGY  $E = W h e$

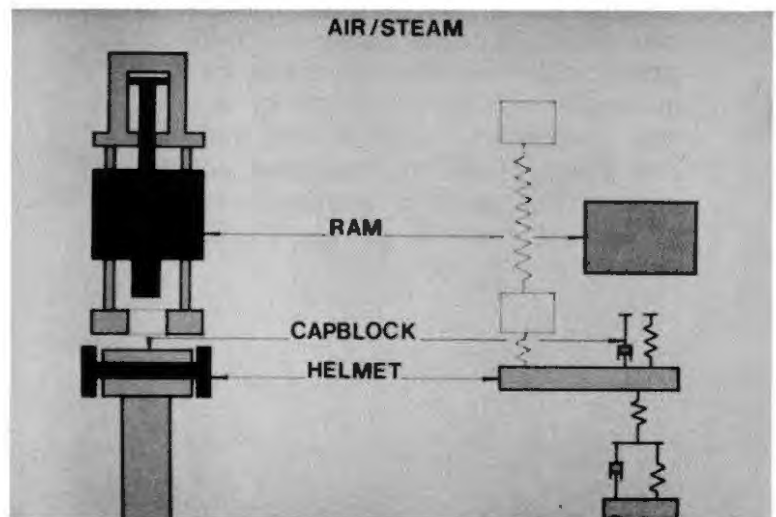
W...WEIGHT OF RAM

h...HEIGHT OF FALL

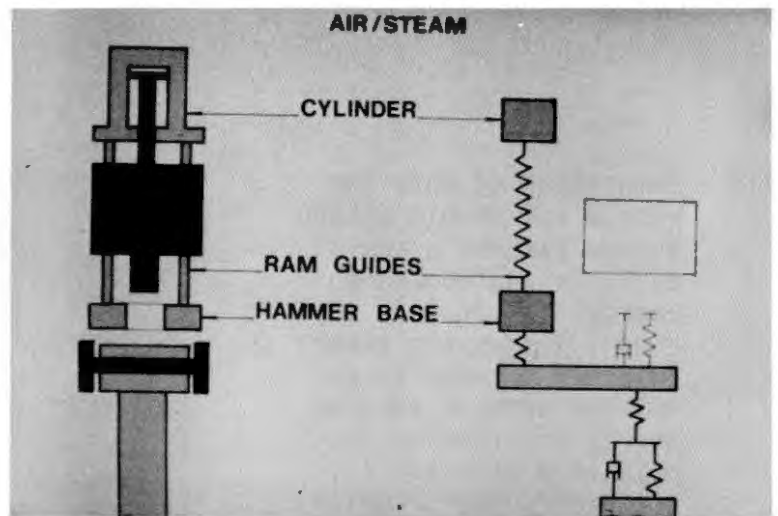
e...EFFICIENCY

IMPACT VELOCITY =  $\sqrt{2 g h e}$

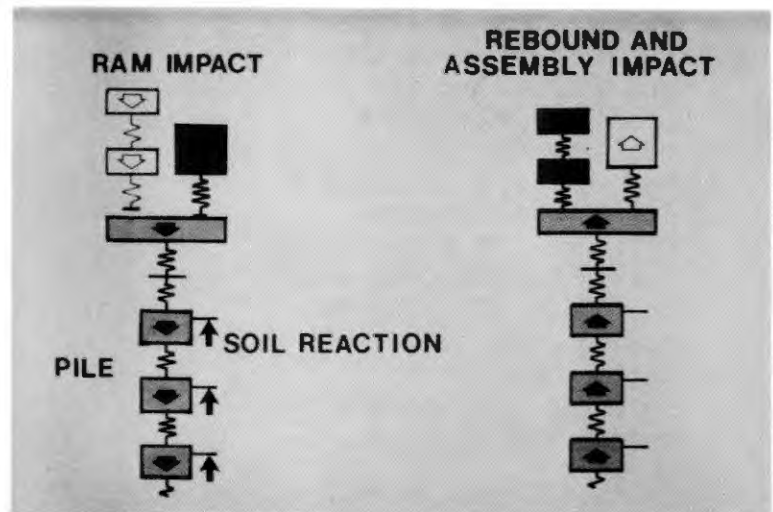
- 13a. The mathematical model used in the WEAP program is more detailed than that which has been used previously. The ram can be represented by a series of masses and springs. The spring shown connecting the bottom ram element and the helmet is capable of carrying compression only. For currently available air/steam hammers the rams are so stocky that they can be represented by a single mass element. The details of the modeling of the helmet-capblock assembly will be discussed later.



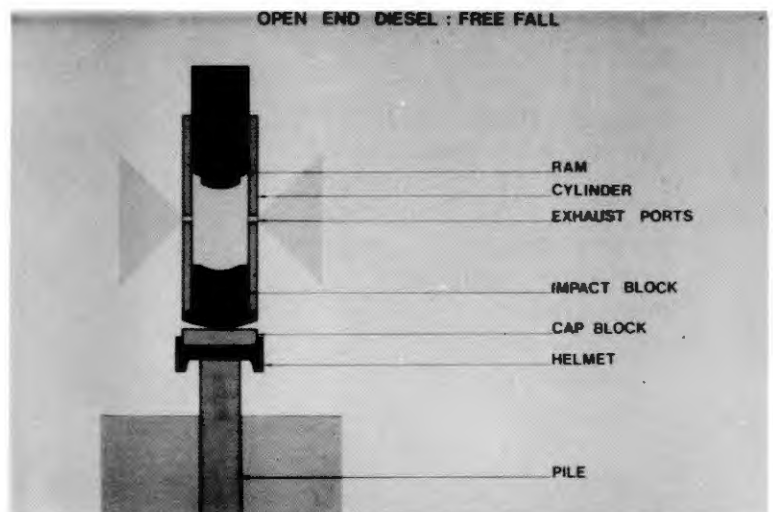
- 13b. The hammer assembly is also modeled by a series of masses and springs with the bottom spring at the interface of the assembly and the helmet also a compression-only spring. For the majority of currently available hammers, two masses are required; one representing the cylinder and the other, the hammer base. The connecting spring gets its flexibility primarily from the hammer columns.



- B14. The wave equation program begins by giving the ram element (or elements) the rated impact velocity. As impact occurs all segments of the wave equation model obtain a velocity and, therefore, move downward. The portions of the assembly segments are also continuously computed in their free fall. The motion of the pile segments causes the soil resistance forces to be activated. Once the pile has rebounded and all soil springs are elastic, computations are terminated.

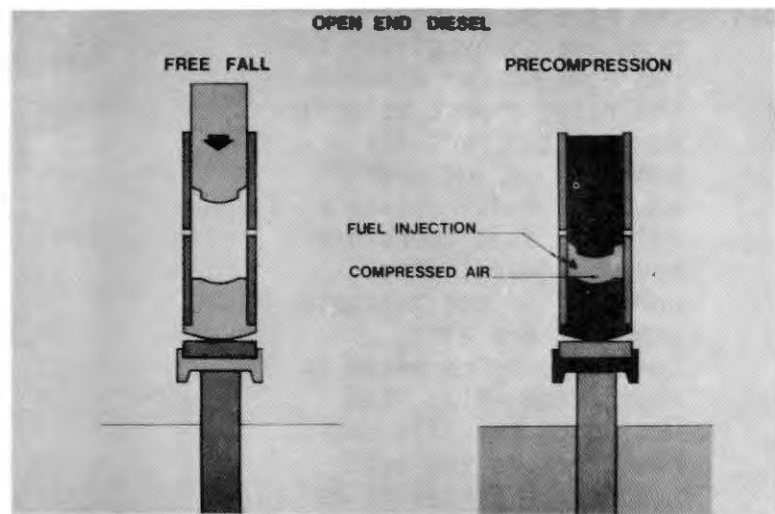


- B15. Consider now the single acting or open end diesel hammer. It consists of three important elements: a ram, an impact block or anvil, and a cylinder all resting on a capblock and helmet similar to that used for air/steam hammers. To start the hammer the ram is lifted by the crane and dropped. Excess air is blown out of the exhaust ports or if this is a downstroke during operation combustion products and air are blown out.

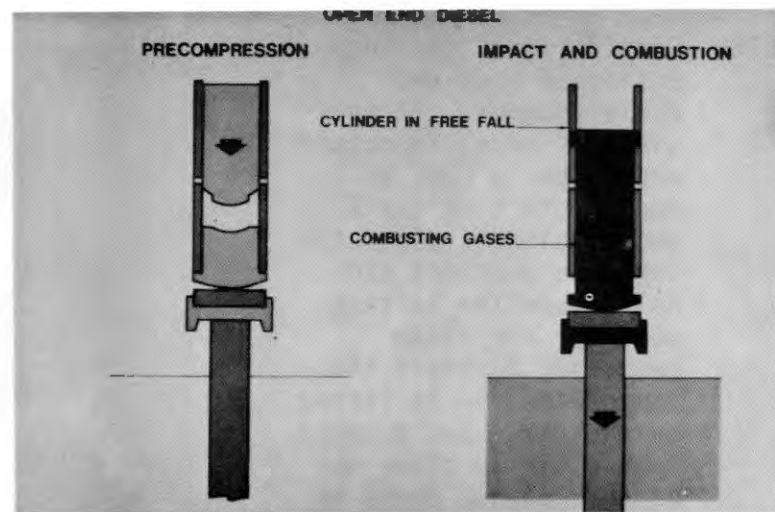




- B16. When the ram falls past the exhaust ports they are blocked and the precompression phase begins. The falling ram activates a cam to inject fuel into the cylinder. This may be in liquid form falling onto the impact block or it may be atomized. In the latter case it is injected only shortly before impact.

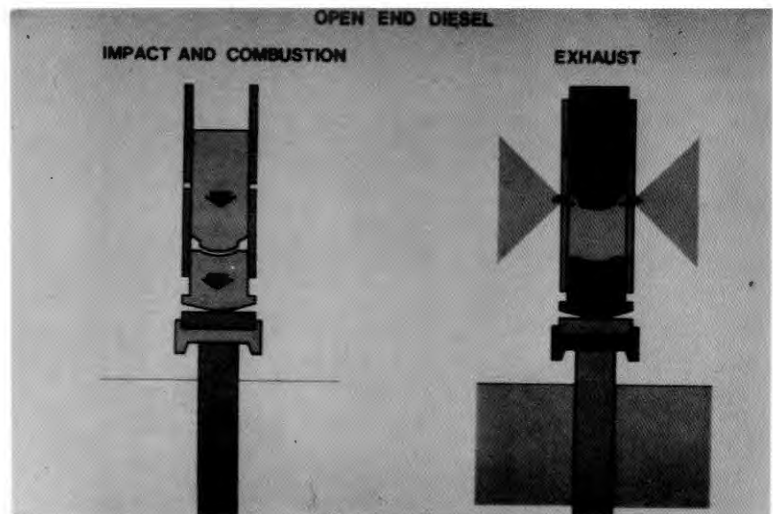


- B17. The ram is shaped so that at impact a small combustion chamber remains with a high compression ratio. If the fuel is injected as a liquid it is atomized on impact. Combustion occurs causing a very rapid increase in pressure.

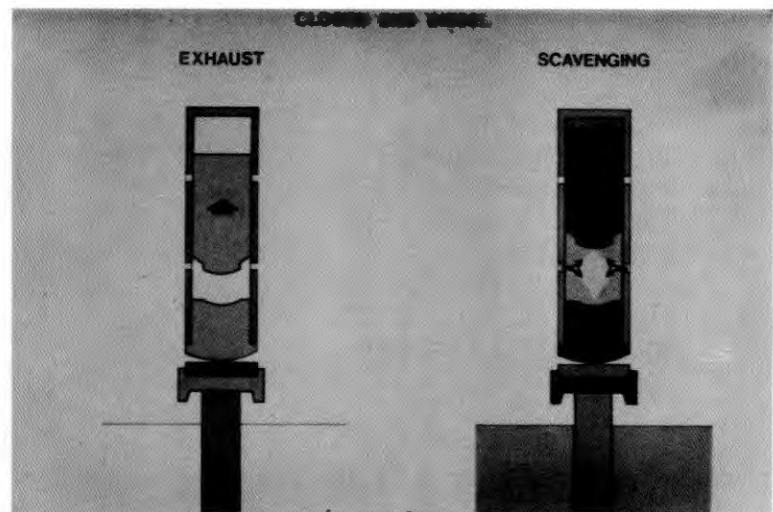




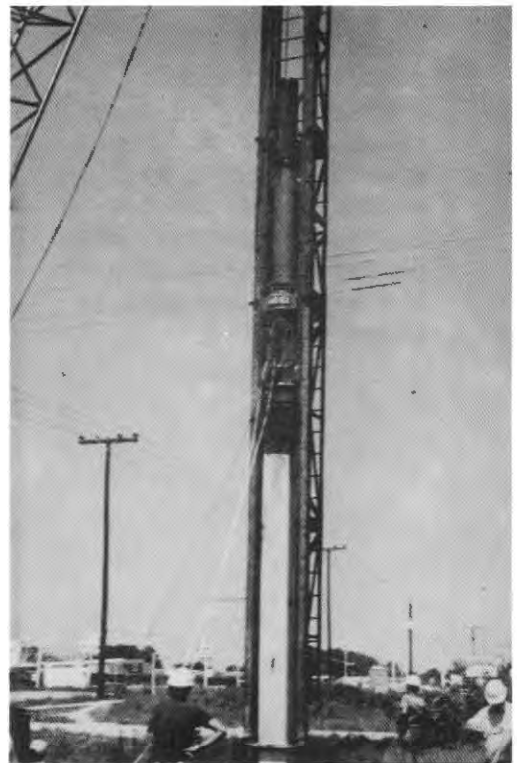
- B18. As a result of the combined effect of the combustion chamber pressure and the pile rebound, the ram moves upward allowing an expansion of the combustion products. When it passes the exhaust ports the excess pressure is blown off.



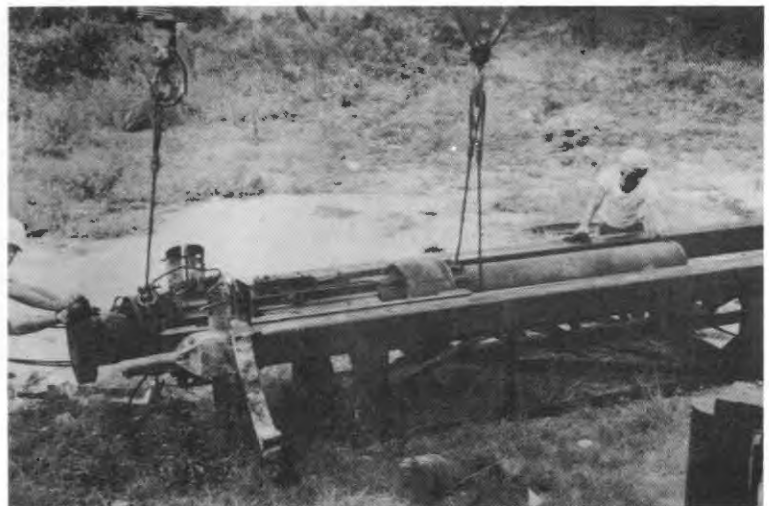
- B19. However, the ram has an upward velocity so it continues upward in a free fall condition, in the process drawing in fresh air in a scavenging phase. The maximum height that the ram achieves depends on the pile rebound as well as the combustion effect. In normal operation the stroke can vary over a wide range depending on the pile rebound and stiffness. Strokes ranging between four feet and nine feet are not abnormal for various hammer types.



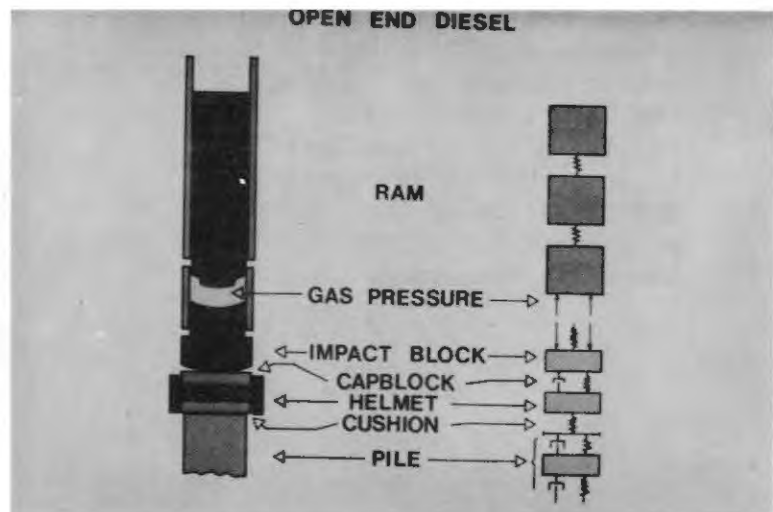
- B20. An open end diesel hammer is shown here. The photograph was taken when the ram was near the top of the stroke and a large part of the ram extends beyond the top of the cylinder.



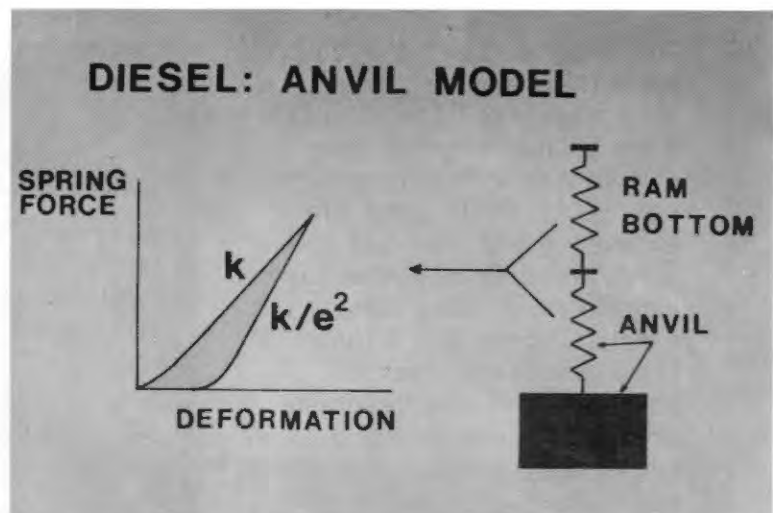
- B21. In this photograph, another open end diesel hammer is shown. The impact block is partially extended and clearly visible at the bottom of the hammer.



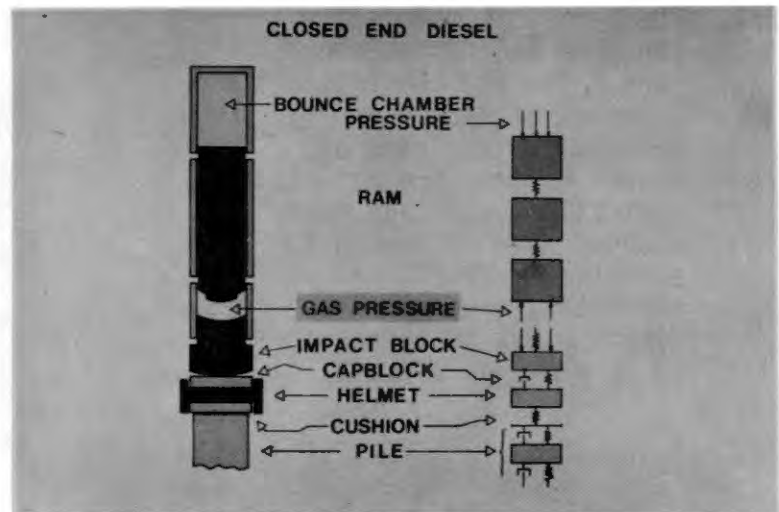
- B22. The hammer is modeled in the WEAP program as shown here. The ram consists of a series of masses and springs. Since rams of diesel hammers are typically longer and more slender than is common for air/steam hammers, several elements are used. The impact block is represented by a single element.



- B23. The steel to steel impact of the ram on the impact block represents a particular problem and one which must be handled carefully. The research conducted during the development of the WEAP program leads to the use of an impact block spring which represents simply the combined stiffness of the last ram segment and the anvil. The spring is modeled with non-linear properties as are the springs of all cushion materials. The change of slope is identified by a coefficient of restitution,  $e$ . The model below the impact block is identical with that used for the air/steam hammer.



B24. The combustion chamber pressure must also be active between the ram and the impact block. The resulting forces can best be described by discussing the operation of the program as far as the hammer is concerned.



B25. The program begins with either an assumed or a user specified ram stroke. With a knowledge of the stroke the velocity of the ram as it falls past the exhaust ports can be calculated from elementary kinematics. When the exhaust ports are closed the combustion chamber pressure and ram velocity are determined using a rigid body assumption and step-wise calculating ram velocity resulting from the action of gravitational and combustion forces.

1. ASSUME STROKE
2. COMPUTE RAM VELOCITY AT PORTS
3. FIND RAM VELOCITY AT IMPACT

B26. As the ram moves down the combustion chamber pressure is calculated from the gas law. When the ram reaches the impact block the velocity is known and the wave analysis can be performed in a fashion similar to that described earlier.

**GAS LAW:**

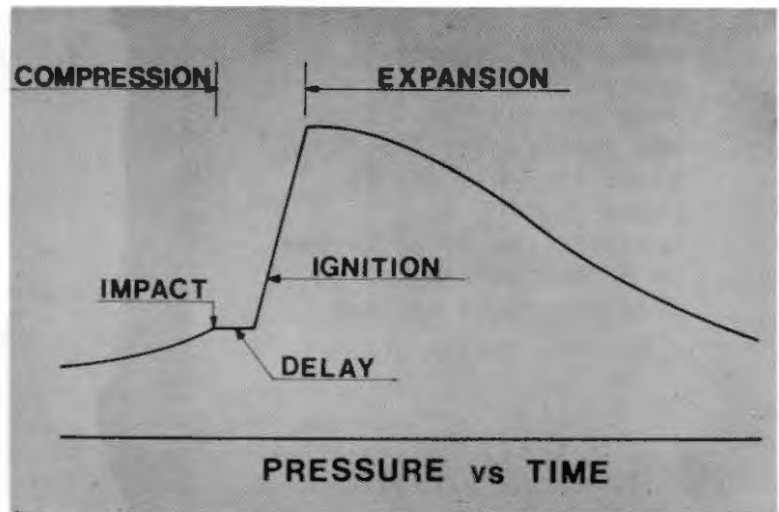
**PRESSURE:** 
$$p = p_{atm} \left( \frac{V_i}{V} \right)^{exp}$$

$P_{atm}$     Atmospheric Pressure  
 $V_i, V$     Initial, Current Volume  
 $exp$       Gas Property

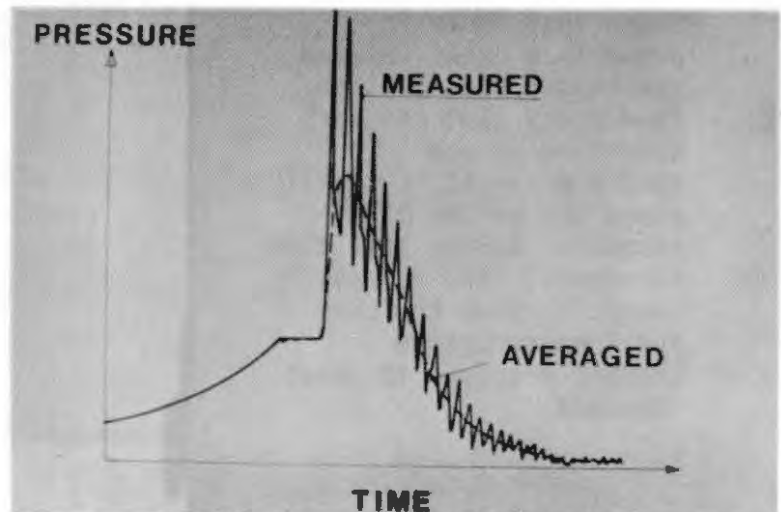
B27. Measurements of combustion chamber pressure have shown that there is normally a delay between ram impact and ignition. This delay time can vary depending on job conditions. It is usually about one or two milliseconds. During this time the impact induced wave is traveling down the pile and the combustion chamber pressure is about constant.

1. ASSUME STRIKE
2. COMPUTE RAM VELOCITY AT PORTS
3. FIND RAM VELOCITY AT IMPACT
4. IMPACT AND COMBUSTION DELAY
5. IGNITION; GAS FORCE TO MAXIMUM
6. EXPANSION

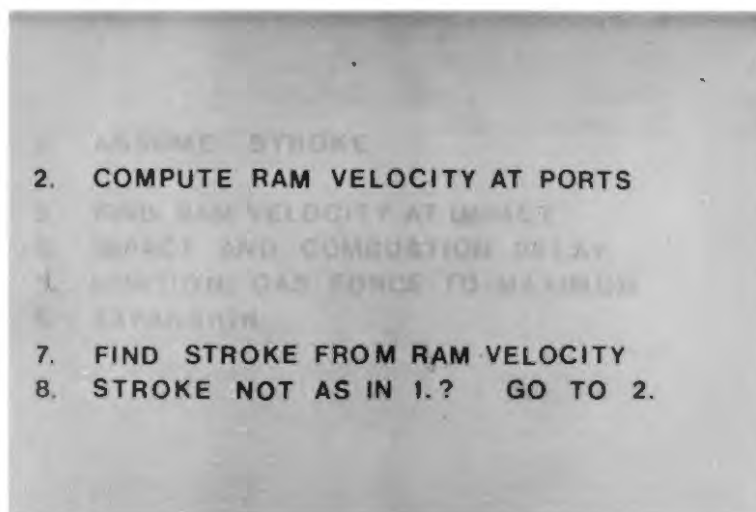
B28. When ignition occurs the pressure increases very rapidly to some maximum value. This pressure can be calculated from combustion laws. However, due to the imperfect nature of diesel hammer combustion the computed pressure is not reliable. Therefore, measured maximum pressures are used in the WEAP program. From this time on the combustion chamber pressure used in WEAP is determined by the volume of the chamber using the gas law.



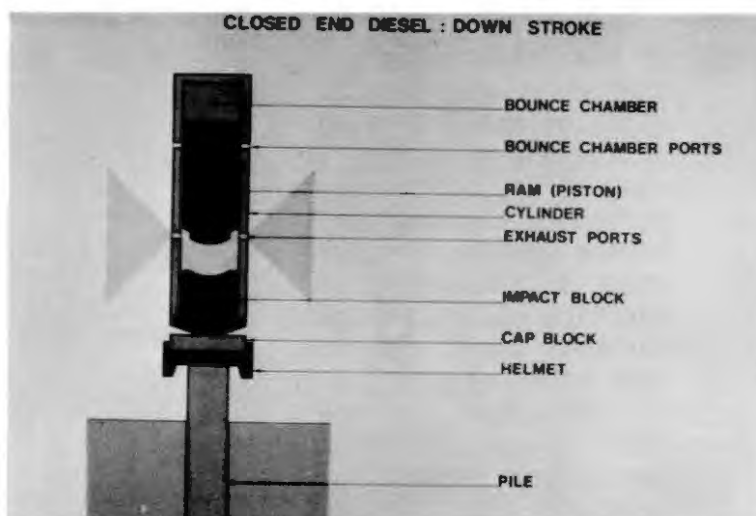
B29. Actually, the pressure will have a high frequency fluctuation which is induced by ignition. However, this phenomenon cannot be realistically modeled and it has no significant effect on the pile dynamic performance.



B30. When the ram clears the exhaust ports the pressure is dropped to zero and the rigid body velocity calculated. The stroke can then be determined. If it agrees with the starting stroke, computation is stopped. Otherwise, a new calculation cycle is started using the rebound stroke as the beginning stroke. Convergence is usually rapid.

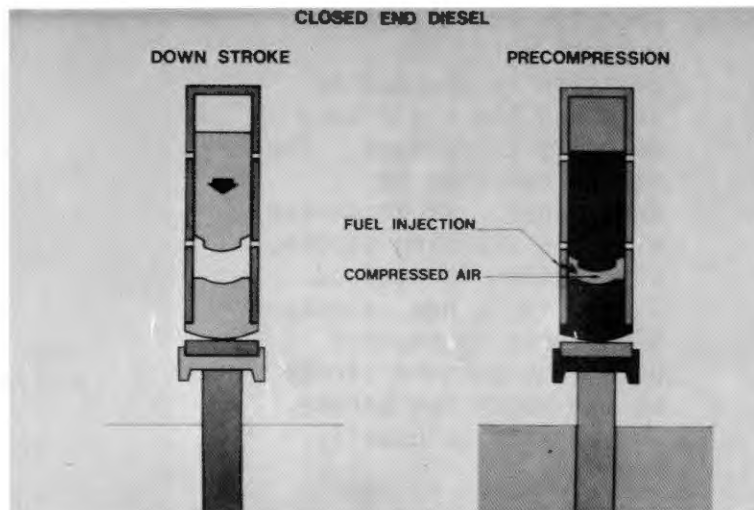


B31. The closed end diesel hammer is in some important ways different from the open end hammer. The top of the cylinder in this case is closed so that on the up stroke air is trapped in the resulting chamber, known as the bounce chamber. The hammer is again started by lifting the ram with the crane. During the down stroke the ram accelerates under the action of gravity and the air pressure in the bounce chamber.

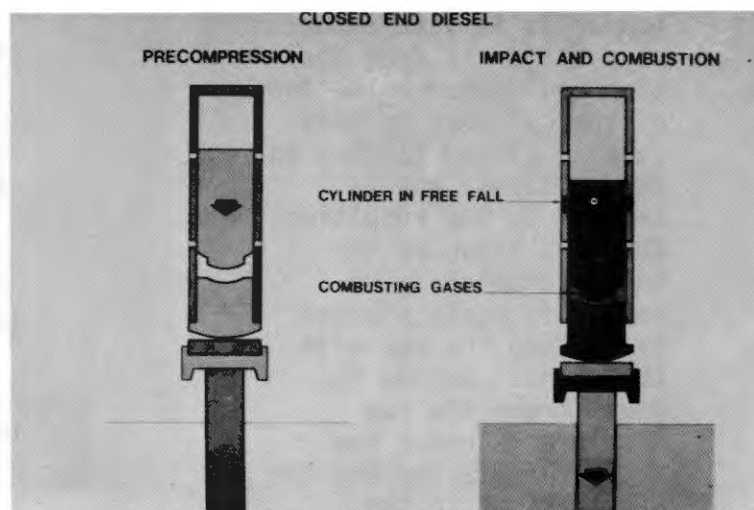




- B32. Again when the ram passes the exhaust ports the gas in the combustion chamber is compressed.

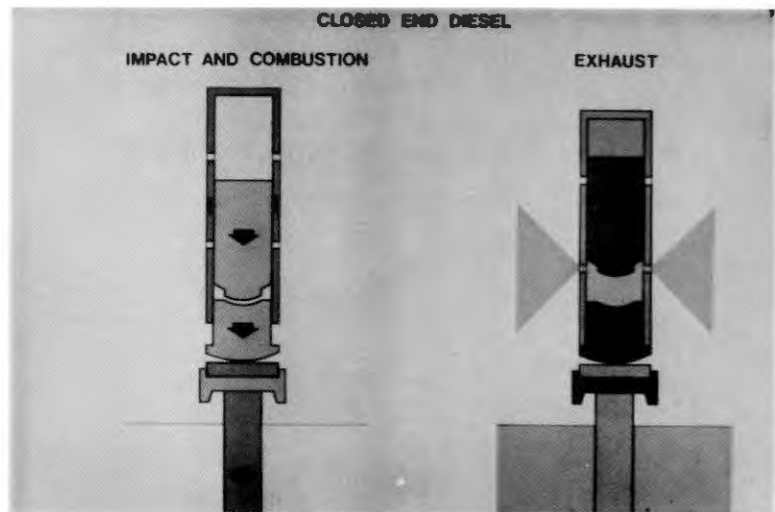


- B33. At the bottom of the stroke the ram impacts with the anvil driving is and the pile top down. The bounce chamber ports are cleared and the bounce chamber pressure comes to atmospheric. Ignition occurs at about the same time. The ram then moves upward due to the action of both the pile rebound and the combustion chamber pressure.

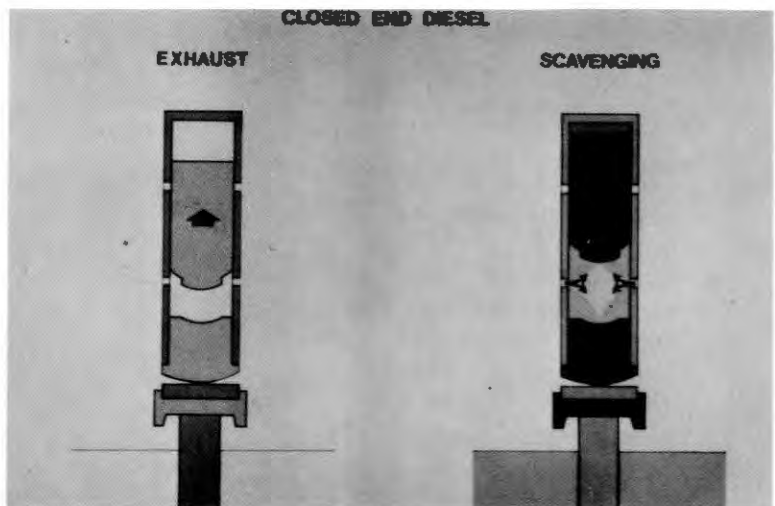




- B34. When the exhaust ports are cleared the combustion chamber goes down to the ambient atmospheric pressure. Also, a pressure is built up in the bounce chamber slowing the ram down more quickly than would gravity alone.



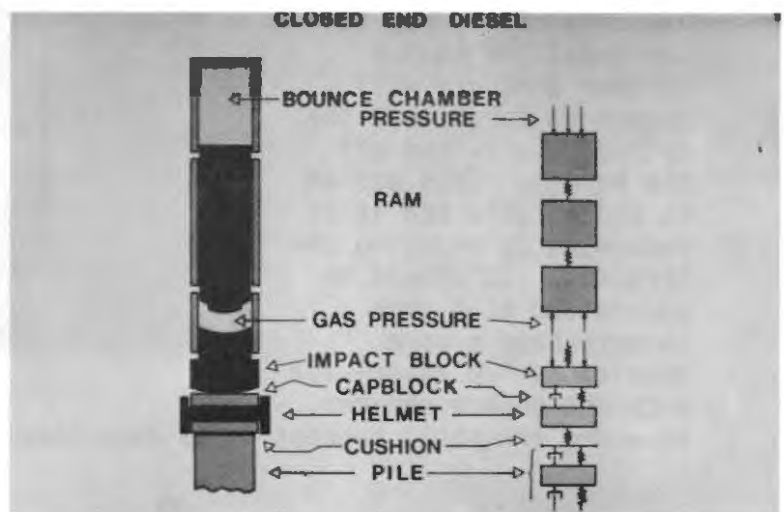
- B35. As the ram continues on up fresh air is drawn into the combustion chamber and scavenging occurs. If the upward velocity of the ram is too great the bounce chamber pressure will become so large that the cylinder is lifted off the helmet. This action is undesirable and it is prevented by reducing the throttle. It should be understood that some hammers have a more complex system associated with the bounce chamber. However, the basic concept is as described.



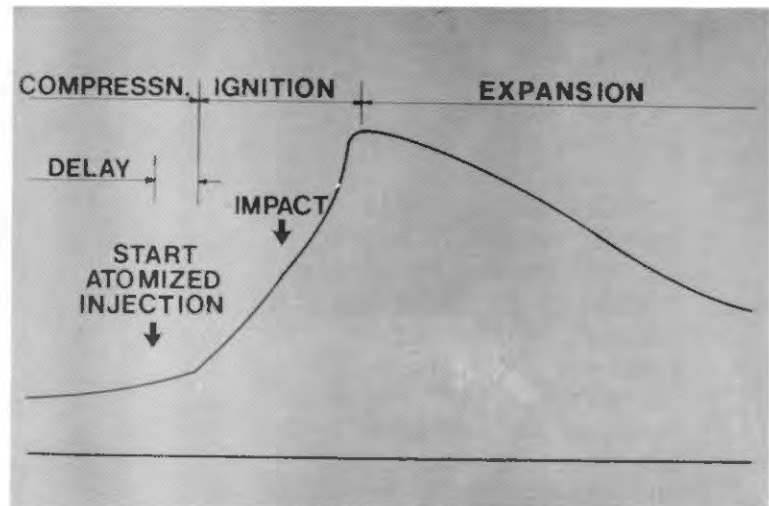
- B36. In this slide a double acting diesel hammer is shown. Characteristic for this particular hammer is a pressure tank at the top of the unit which provides for a larger bounce chamber.



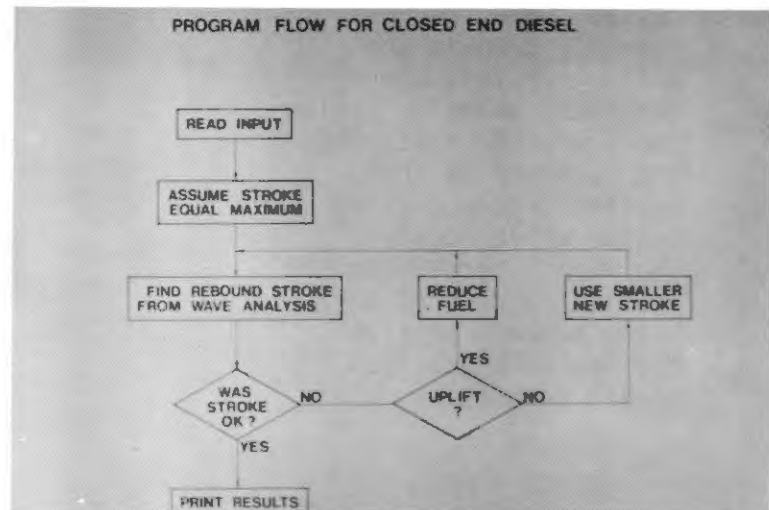
- B37. The model used to represent the hammer is shown here. It is similar to that used for the open end diesel except for the bounce chamber pressure. The resulting force acting on top of the hammer must be calculated from the gas law and included in the computation of ram motion.



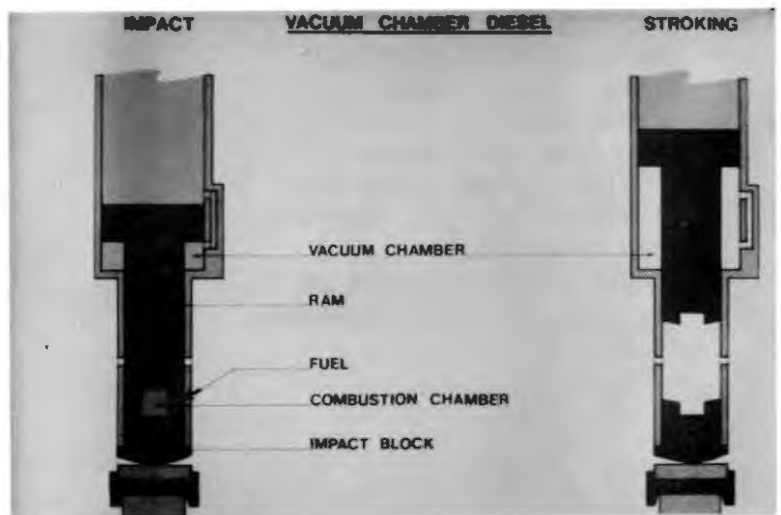
- B38. Some hammers introduce the fuel in an atomized state. Thus, ignition occurs shortly after injection in a manner similar to the conventional diesel engine. Burning then takes place over a longer time depending on the rate of fuel injection.



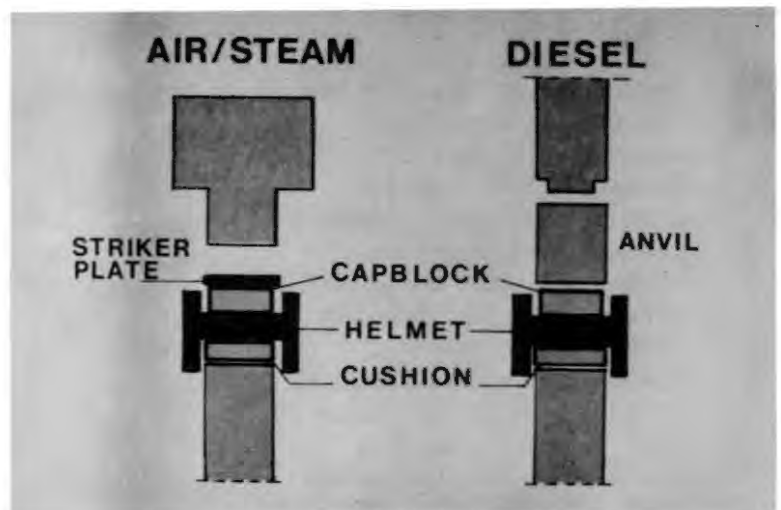
- B39. The computer program for the closed end diesel hammer must operate somewhat differently than for open end hammers. If the open throttle stroke is such that lift off is induced then the combustion pressure, associated with a reduced throttle, must be found so that lift off is only incipient. On the other hand, if lift off does not occur then the computation is the same as for open end hammers where the stroke must converge.



- B40. An additional diesel hammer type known as the vacuum chamber diesel is also modeled in the program. The hammer is illustrated here schematically. This hammer is double acting in that on the up stroke a reduced pressure is induced in the vacuum chamber. This force increases rapidly to a near vacuum and then continues to increase only gradually. The stroke of the hammer is not limited by a hammer top so the computation convergence is only on stroke.



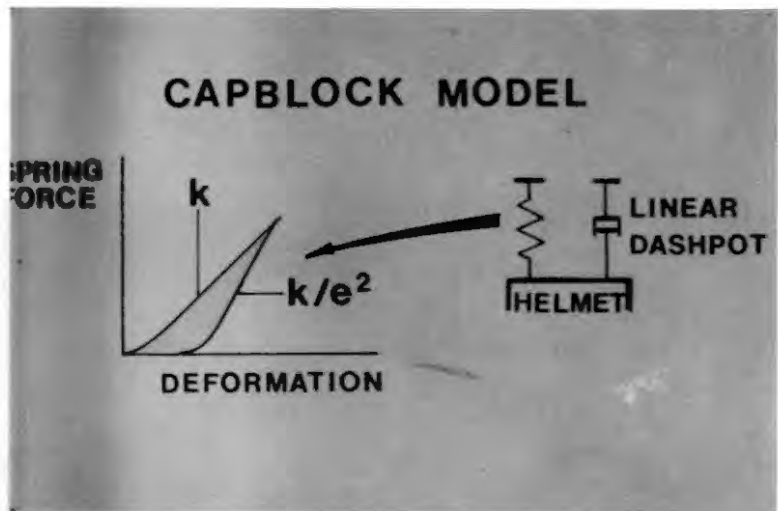
- B41. So far only hammer models have been discussed. Now the next elements down in the driving system, the capblock, helmet, cushion model will be described. The capblock is that portion of the driving system which receives the impact force from the ram in the case of air/steam hammers or from the impact block from diesel hammers.



- B42. Capblocks formerly consisted of hardwood or cable coils. Increasingly artificial composite materials are used. These newer materials have the advantage of offering more uniform and reproducible mechanical properties.



- B43. The capblock is modeled as a spring with a steeper unloading portion to represent the coefficient of restitution. Also, a dashpot is used in its model. Comparison of measured with computed force and velocity curves proved this model to be valid.



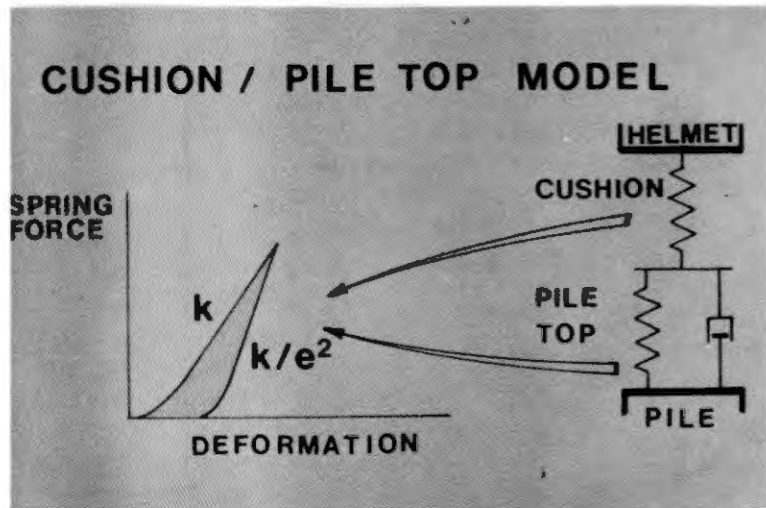
- B44. The helmet is that steel element which contains the capblock on top and holds the pile in alignment with the appropriate recess in the bottom. It is modeled as a mass together with the mass of the capblock, strikerplates and pile adaptors, if present. If steel or timber piles are being driven the helmet fits directly on top of the pile.



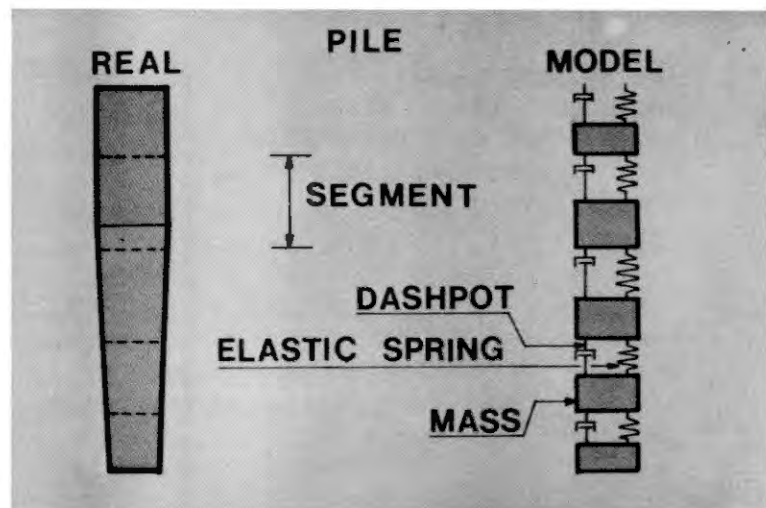
- B45. For concrete piles a softwood cushion is placed between the helmet and the pile. The thickness of this cushion may vary widely.



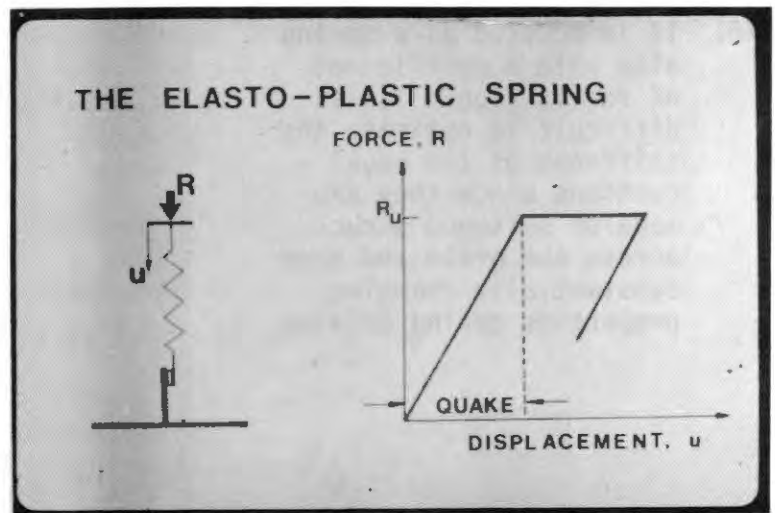
- B46. It is modeled as a spring also with a coefficient of restitution. It is difficult to estimate the stiffness of the usual cushions since they are made of softwood struck across the grain and have substantially changing properties during driving.



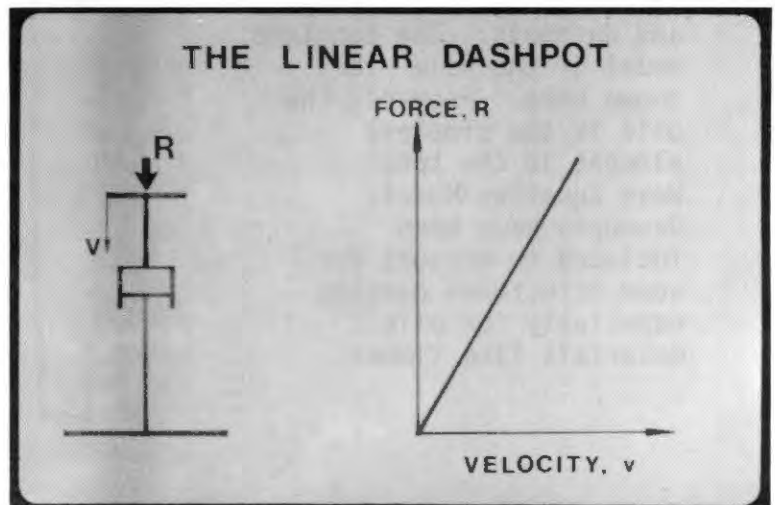
- B47. The pile is modeled as a series of masses, springs, and dashpots. The complete model of the pile is shown here. Probably the pile is the simplest element in the total Wave Equation Model. Dashpots have been included to account for some structural damping especially for pile materials like timber.



- B48. The soil model was already introduced in the first section of this presentation. It consists of an elasto plastic spring which reaches an ultimate resistance force at a compression value called the quake.

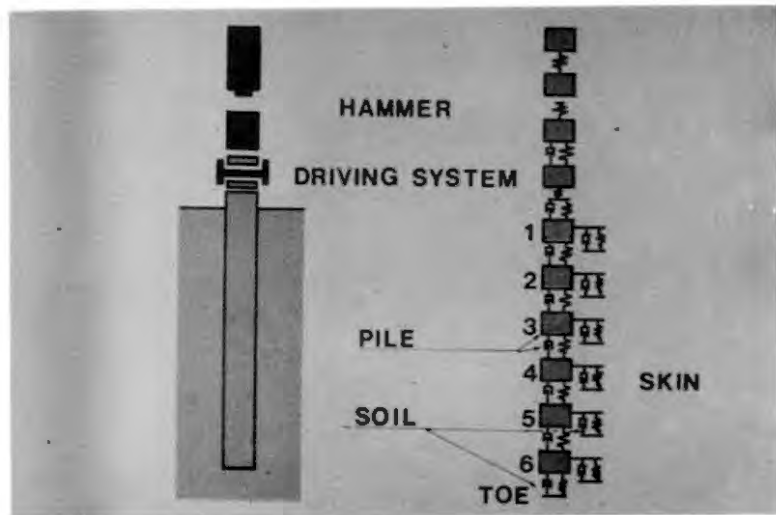


- B49. The soil model also includes a linear dashpot which produces a resistance force proportional to the pile velocity.





B50. The total wave equation model is shown here for the case of a diesel hammer driving a pile which is represented by six elements. A complete spring-dashpot soil model acts at each pile segment. An additional soil model unit acts at the toe to represent end bearing. The sum of the individual ultimate resistance forces acting at each pile segment, here six, is equal to the total ultimate skin resistance. Skin resistance plus end bearing equals the total pile resistance.

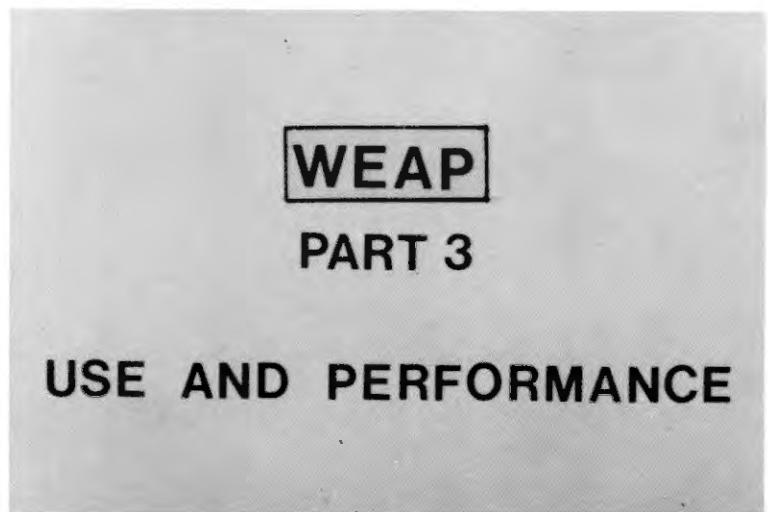


Part III  
Program Use and Performance

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- C1. In this, the third section of this presentation, further details of the WEAP program will be presented. An example of data input will be described, three sample problems will be presented and some results of the correlation between measured and calculated force and velocity curves will be given. The total data input structure with all options is too lengthy to describe in detail. Rather, an example which can be run with minimal input data will be described.

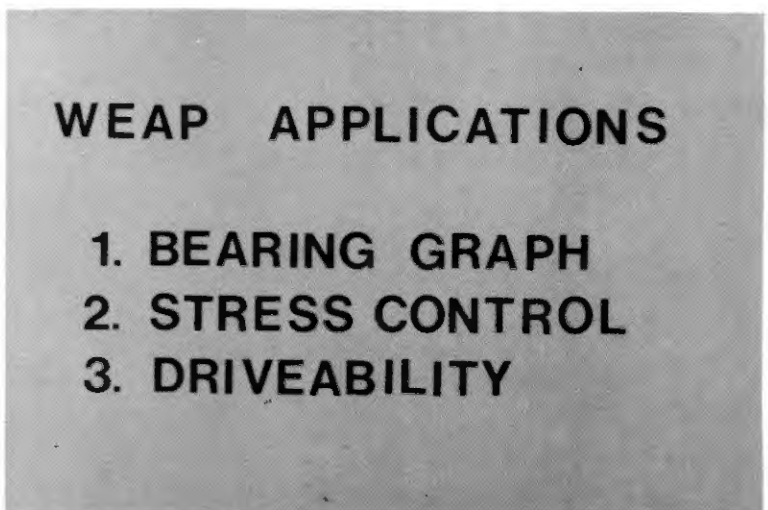
More detail is given in the User's Manual.



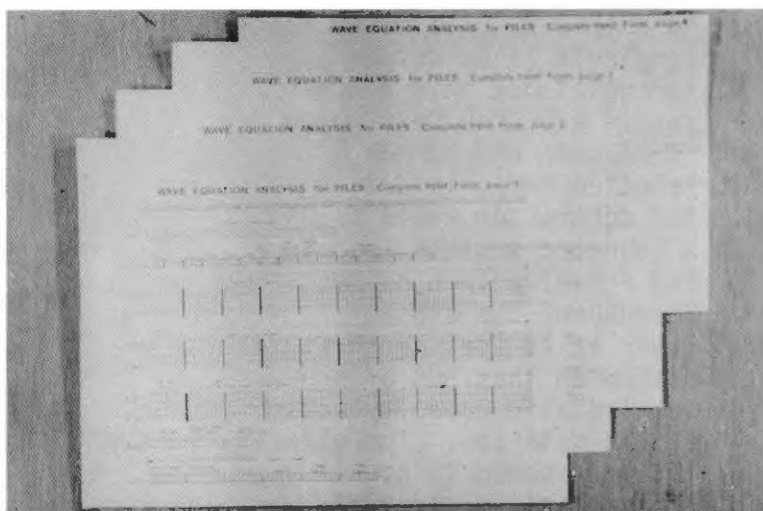
- C2. There are three typical applications for a Wave Equation Program:

(a) The Construction of a Bearing Graph; (b) The Control of Stresses, in particular tension stresses in concrete and compression stresses in steel piles; and (c) The Analysis of driveability. By driveability we are referring to a check on a driving system, given a required pile bearing capacity.

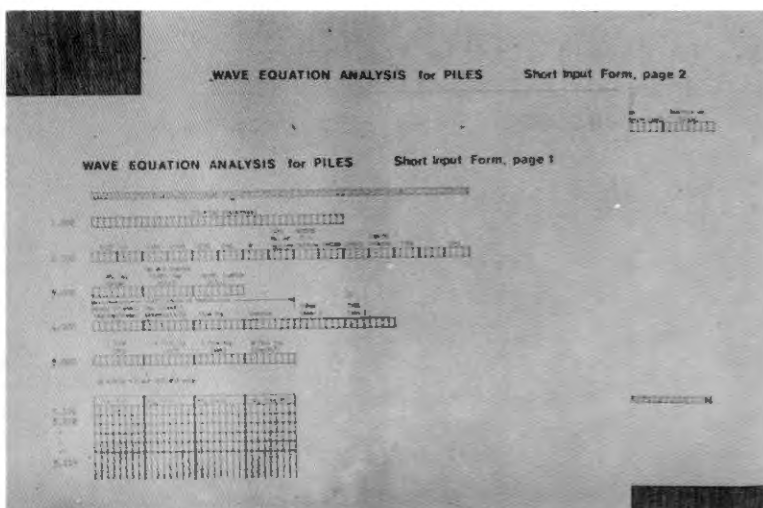
For each of these three groups an example will be given.



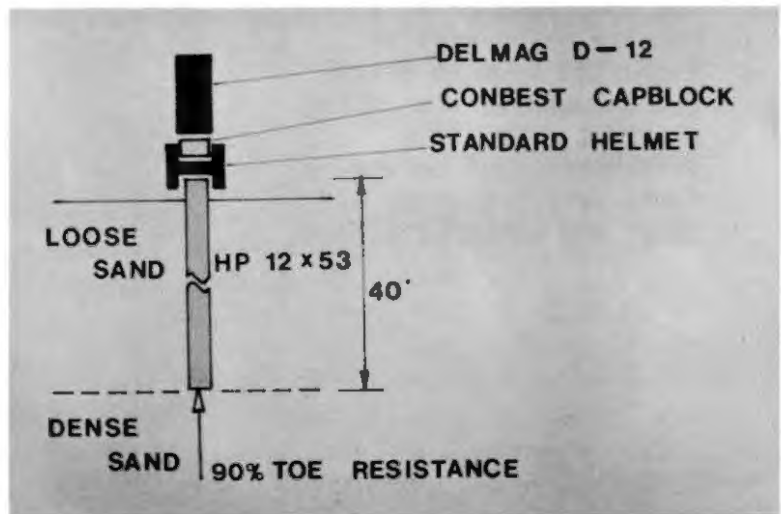
- C3. The construction of a bearing graph will be discussed in more detail to demonstrate the coding of input data. Two different input coding forms were prepared, a SHORT and a COMPLETE form;



- C4. only the short one is demonstrated here. The first problem can be stated as follows:



- C5. An HP12 x 53 pile of 40 feet length is to be driven through loose into dense sand. A DELMAG D-12 which is an open end hammer is selected for pile driving. The contractor wants to use a standard conbest capblock and a twelve inch helmet. Because of the dense layer at 40 feet depth it is estimated that 90% of the total static capacity acts at the pile toe. The other 10% is assumed to be triangularly distributed along the side of the pile.



Input preparation is as follows:

- C6. On card 1 a title is entered that identifies the problem.

CARD NO. 1.000

TITLE

EXAMPLE OPEN END DIESEL HAMMER

- C7. Card 2 allows the input of various options. The first two of these options deal with the type and the quantity of output. Leaving both data fields blank causes a small but sufficiently detailed output to be printed.

CARD NO. 2,000

OUTPUT OPTIONS

IOUT IJJ

--	--	--	--	--	--	--	--

- C8. The next three fields of the option card are devoted to the hammer. IHAMR is the hammer identifier. A 2 is inserted here as given in the User's Manual for a D-12 hammer. Hammer information is stored on about 80 different hammers. Of course, all of the hammer information could be input, using data cards.

Leaving IOSTR blank causes the computer to find the proper hammer stroke. IOSTR set to -1 or 1 would produce an iteration with constant stroke or no iteration at all.

IFUEL determines where or not full combustion pressures are to be used. A blank field specifies normal operation.

CARD NO. 2,000

HAMMER OPTIONS

IHAMR IOSTR IFUEL

	2						
--	---	--	--	--	--	--	--

DELMAG D-12; IDENTIFIER = 2

- C9. The next five options deal with the pile input. Normally, segment stiffnesses and masses are determined by the program, and therefore, IPEL and N, the number of pile segments, can be left blank. There are no splices and the pile is uniform, thus ISPL and NCROSS are left blank. Finally, IBEDAM, the pile damping parameter, is left blank because a steel pile is being analyzed and its small damping is usually neglected.

CARD NO. 2.000

PILE OPTIONS

IPEL	N	ISPL	NCROSS	IBEDAM


- C10. Three more parameters dealing with the soil are given on card no. 2. IPERCS is the percentage of skin friction, here 10. ISMITH is left blank since viscous rather than Smith's soil damping approach is chosen. During the course of the piling research project at Case Western Reserve University a different approach to the treatment of soil damping was developed. This model, referred to here as viscous, uses the pile impedance to non-dimensionalize the damping constant. It has produced good results. Both methods are contained in the program. ITYS is set to 6 to distribute the resistance in a triangular manner. The resistance distributions which are available are given in the User's Manual.

CARD NO. 2.000

SOIL OPTIONS

IPERCS	ISMITH	ITYS
10		6

SOIL RESISTANCE DISTRIBUTION NO. 6



- C11. The driving system is identified on card no. 3. For this standard system one finds in the manual values of 0.95 kips for the helmet-capblock weight and 21,000 kips/inch for the conbest capblock stiffness. There is no cushion on top of the pile.

CARD NO. 3.000

**CAP AND CUSHION**

WEIGHT OF CAP kips	STIFFNESS OF CAP k/in.	STIFFNESS OF CUSHION k/in.
0.95	21,000	

- C12. Coefficients of restitution are left blank on card no. 4 except for the capblock where a 0.8 is inserted as appropriate for conbest. The computer will make a reasonable choice for all quantities that are left blank on this card.

CARD NO. 4.000

**COEFFICIENTS OF RESTITUTION**

ANVIL	CAP	PILE TOP	CUSHION
	0.8		



- C13. The pile has to be described on card no. 5. For the uniform case the following input is sufficient: the length equal to 40 feet, the cross sectional area equal to 15.5 square inches, the steel's elastic modulus of 30,000 ksi and the steel's specific weight of 492 lbs/cubic foot.

CARD NO. 5.000

PILE DESCRIPTION

LENGTH ft.	AREA in. <sup>2</sup>	MODULUS ksi	SPEC. WEIGHT lbs/ft <sup>3</sup>
40.	15.5	30000.	492.

- C14. Card no. 6 is used for soil parameter input. Quakes for both skin and toe bearing are set to 0.1 inches, soil damping was selected as 0.3 and 0.15 for the skin and toe, respectively. The ultimate soil bearing capacity was set to -1.0 since more than one value has to be analyzed. The soil's coefficient of restitution is left blank for normal operation.

CARD NO. 6.000

SOIL PARAMETERS

QUAKE SKIN in.	QUAKE TOE in.	DAMPING SKIN	DAMPING TOE	RULT	COEFFICIENT OF RESTITUTION
0.1	0.1	0.3	0.15	-1.0	

- C15. No other input except for card no. 9 is required. On this card four ultimate capacities; namely, 30, 60, 90, and 120 tons are specified. These four values are sufficient for the construction of a bearing graph.

CARD NO. 9.000

ULTIMATE BEARING CAPACITIES

tons

30.	60.	90.	120.
-----	-----	-----	------

- C16. The output consists in this case of seven pages. There are two pages giving hammer, pile, soil, and option details. Four pages list the extrema of forces, stresses, velocities, and displacements for each element.

W E A P - WAVE EQUATION ANALYSIS FOR PILES

THIS PROGRAM WAS PREPARED FOR THE FEDERAL HIGHWAY ADMINISTRATION  
BY GOBLE & ASSOCIATES, CLEVELAND, OHIO

EXAMPLE OPEN END DIESEL HAMMER  
PILE DESCRIPTION

A DEL. TON (FT)	.0	40.0
A (SQ. IN.)	15.5	15.5
E (KSI)	30000.	30000.
WMMR (LB/FT)	492.0	492.0

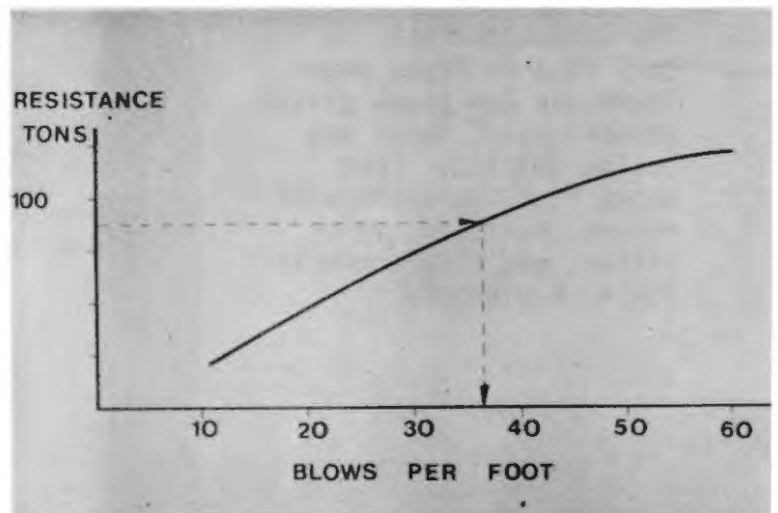
  

HAMMER MODEL DEL.0-12			
ELEMENT	WEIGHT (KIPS)	STIFFNESS (K/IN)	COEFF. RESTITUTION
1	.000		
2	.000	121250.0	
3	.000	121250.0	
4	.000	121250.0	
ANVIL	.010	86833.3	.800
CAK	.050	21000.0	.800
CUSHION		.0	1.000
PILE TOP			.800

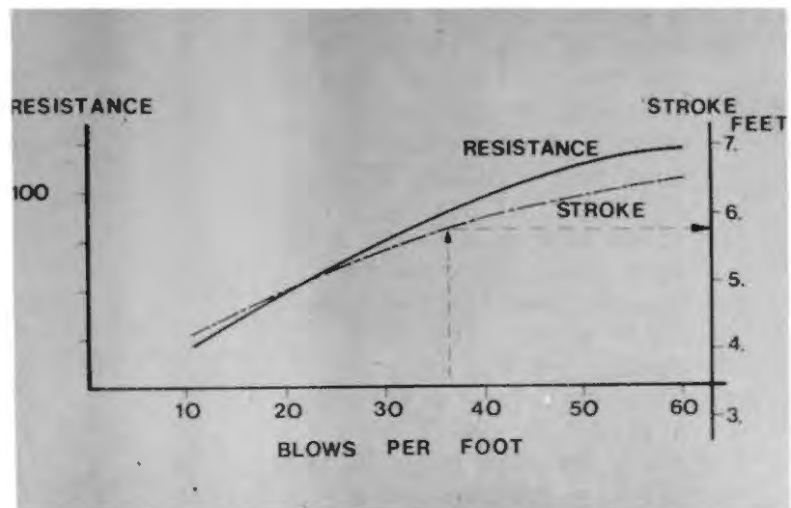
- C17. On the last page a summary is printed containing the ultimate resistance, RULT, analyzed, and the corresponding blow count, stroke, minimum and maximum stresses and the speed of the hammer in blows per minute.

EXAMPLE      OPEN END DIESEL HAMMER							
SUMMARY							
NO	R ULT TONS	BLOW CT 1/FT	STROKE FT	MIN STR KSI	MAX STR KSI	BLOWS/ MINUTE	
1	30.0	14.	4.35	.00	18.11	56.3	
2	60.0	24.	5.22	.00	21.72	51.4	
3	90.0	40.	5.66	.00	23.43	49.5	
4	120.0	54.	6.23	.00	25.29	47.2	

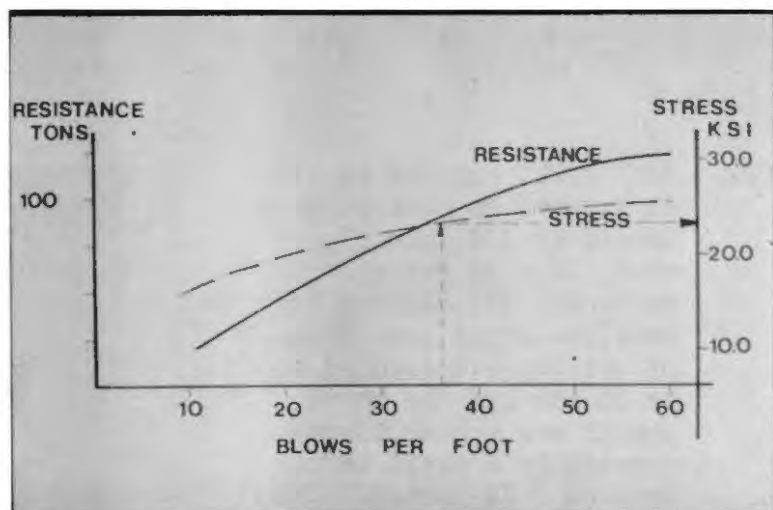
- C18. The blow count results were used to construct a bearing graph as shown here. If an ultimate resistance of 90 tons was desired, then the blow count should be 36 blows per foot as indicated by the arrows.



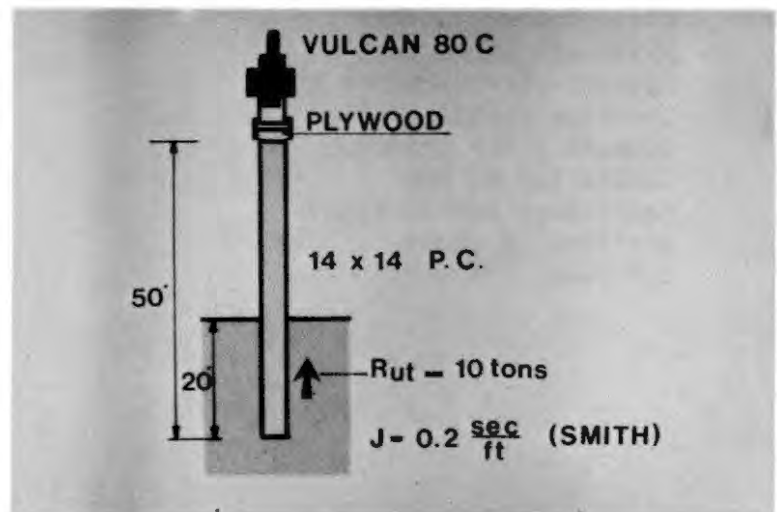
- C19. Also plotted in this slide are the stroke results corresponding to the blow counts of the summary. The expected stroke for 90 ton resistance and 36 blows per foot is about 5.8 feet.



- C20. In a similar manner the maximum compressive stresses are plotted and one finds a maximum stress of 24 ksi at 36 blows per foot driving resistance. It should be noted that the WEAP program produces the bearing graph automatically, if so desired.

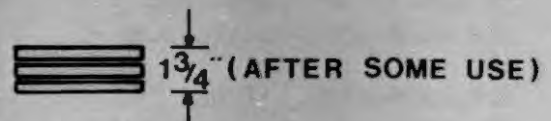


- C21. A second type of wave equation application is for the control of tension stresses in concrete piles. As an example consider a Vulcan 80C differential acting air/steam hammer driving a 50 foot long, 14 x 14 inch square, prestressed concrete pile. The soil overlaying the bearing strata is very soft and a situation is to be analyzed with only 10 tons skin resistance and a Smith skin damping value of 0.2 at a penetration of 20 feet. The cushion thickness is to be determined such that the tension stresses are less than 1.0 ksi.



- C22. In a first run the cushion is assumed to consist of 3 sheets of 3/4 inch plywood, 14 x 14 cross section. Its elastic modulus--after some time of driving--is assumed to be 30 ksi and the three sheets are probably compressed to a total thickness of 1.75 inches. The resulting cushion stiffness is, therefore, 3360 kips/inch. The coefficient of restitution is 0.5.

**CUSHION: 3 SHEETS  $\frac{3}{4}$  PLYWOOD**

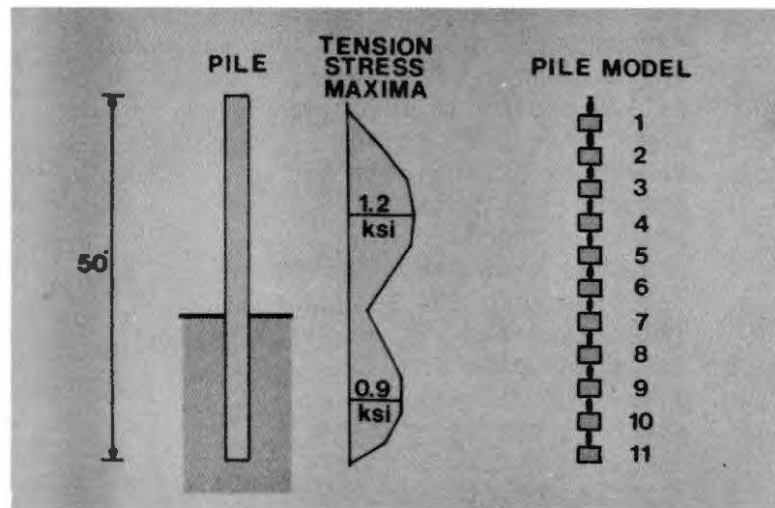


$$A = 14 \times 14 = 196 \text{ in}^2$$

$$E = 30 \text{ ksi}$$

$$k = \frac{196(30)}{1.75} = 3,360 \text{ kips/inch}$$

- C23. In this slide the maximum tension stresses are plotted as determined by the computer for each element. The highest tension stress occurs at a depth of 18 feet and is 1.2 ksi.



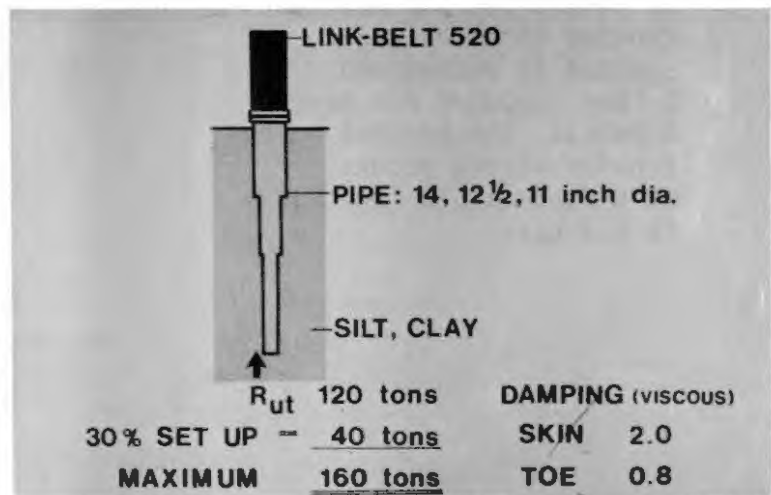
- C24. Thus the tension stresses are too high. A rerun of this problem with 6 cushion sheets and, therefore, a cushion stiffness which is only 1680 kips/inch, produces a tension stress of 0.6 ksi.

## TENSION STRESS RESULTS

3 CUSHION SHEETS: **1.2 ksi**  
 $k = 3,360$  kips/inch

6 CUSHION SHEETS: **0.6 ksi**  
 $k = 1,680$  kips/inch

- C25. The third problem to be demonstrated deals with driveability. A thin-walled pipe pile which is step tapered is to be driven by a Link-Belt 520 hammer. Estimates of bearing capacity in a cohesive silt and clay are 120 tons with 33% set up. This means that if driving is interrupted and the pile left at rest for an extended period of time, it would obtain a static capacity of 160 tons.



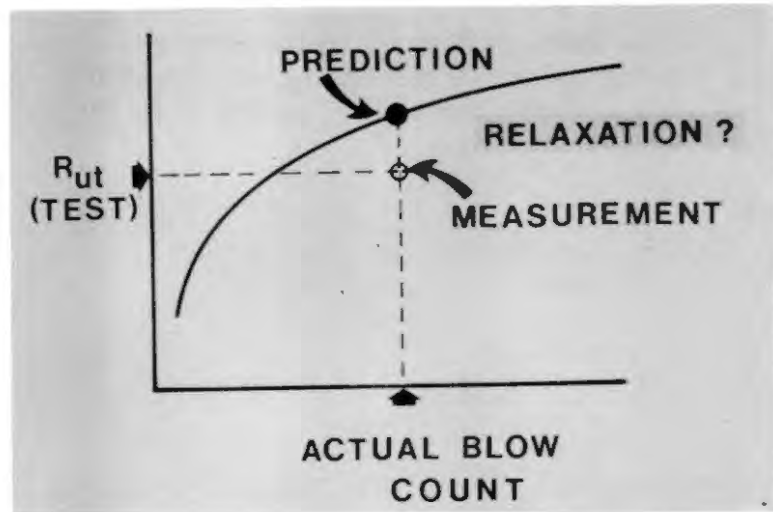
- C26. A WEAP analysis with relatively high viscous damping factors of 2.0 and 0.8 for skin and toe, respectively, shows that the driving resistance would be 221 blows per foot at 120 tons resistance. It would be infinite, that is there would be no set, at the set up capacity of 160 tons. The conclusion is that the hammer/pile system would be insufficient if the pile had to be driven after a waiting period.

EXAMPLE DRIVEABILITY CHECK LH 520

SUMMARY

NO	R. ULT TONS	BLOW/CT 1/FT	STROKE FT	MIN STM KSI	MAX STM KSI	BLOWS/ MINUTE	H.C. PER PSI
1	120.0	221.	3.03111	0.00	30.54	73.9	24.7
2	160.0	999999.	3.03121	0.00	30.80	73.9	24.7

C27. Previously developed wave equation programs have been tested by comparing the predicted pile capacity with the value obtained from static load tests. This comparison could be meaningless for a number of reasons. The pile may gain or lose strength between driving and load testing.



C28. The dynamic soil constants may be incorrectly estimated. The driving system constants may not be correctly known and the hammer performance may not be as expected. Thus, a very large number of constants can be adjusted to achieve a satisfactory correlation with capacity. The value of these constants are certainly not unique.

#### WAVE EQUATION ERROR SOURCES

##### SOIL :

1.  $R_{ut}$  ... DISTRIBUTION
2.  $J$  ... DAMPING
3.  $q$  ... QUAKE

##### DRIVING SYSTEM :

1.  $k$  ... STIFFNESS
2.  $e$  ... COEFF. of RESTITUTION

##### HAMMER :

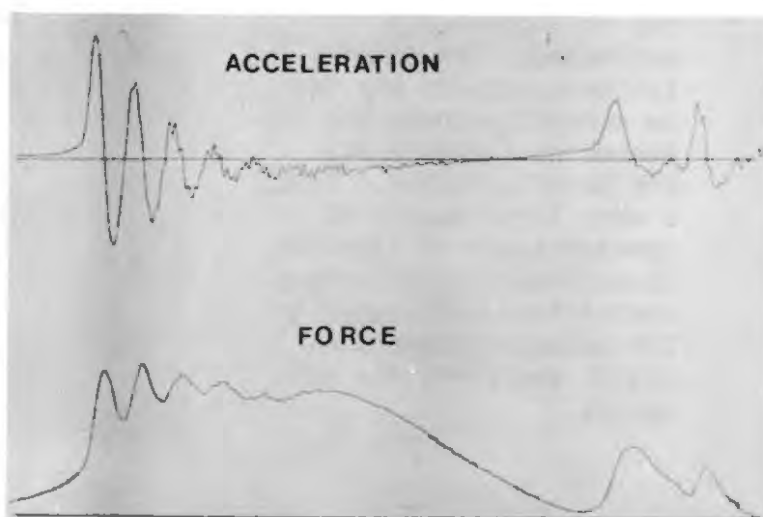
1. PERFORMANCE



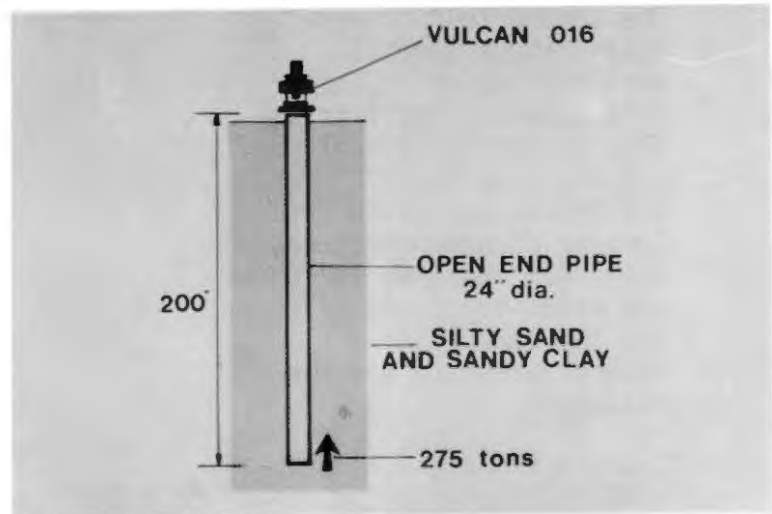
- C29. During the Piling Research Program at Case Western Reserve University a large volume of force and acceleration measurements were made at the pile top during driving.



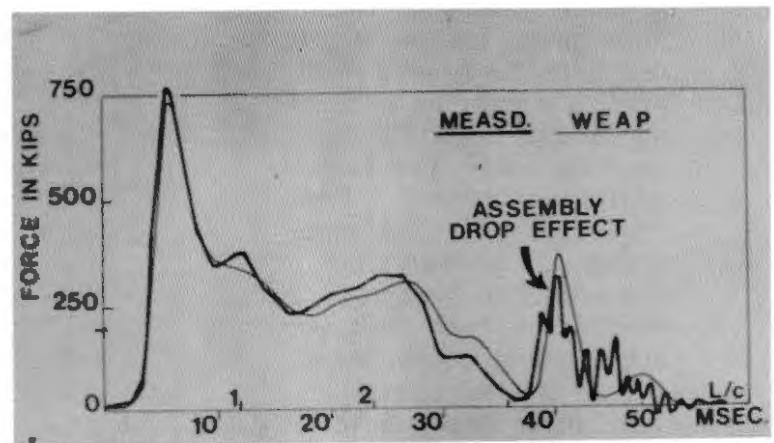
- C30. This huge volume of data (several thousand hammer blows) was available to test the program. The available measurements consist of force and acceleration records at the pile top. These measurements were compared with the values obtained by WEAP. One of the major activities of the project was to provide recommendations on the values to be used for the hammer dynamic constants. Sixteen different jobs were so tested and the results are presented in the final report. Three of these examples will be shown here.



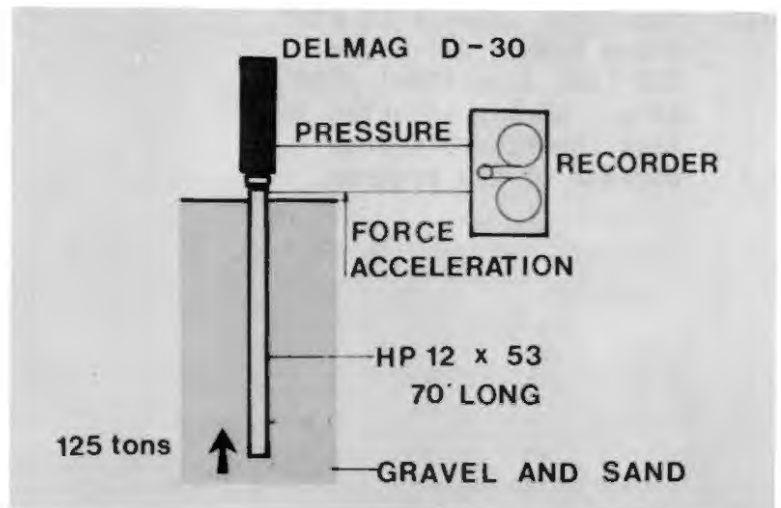
C31. The first case is an air/steam hammer driving a 200 foot long steel pipe pile. It was selected to test the assembly drop portion of the program.



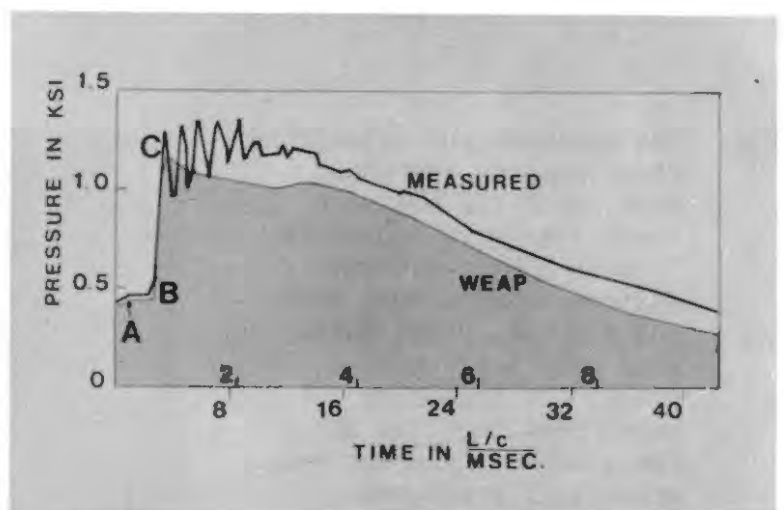
C32. The measured and calculated force records are shown here, with the measured force the heavy line. It is seen that the first maximum matches very well and the two curves agree well over their entire range. It should be noted that the time of the assembly drop was very accurately predicted.



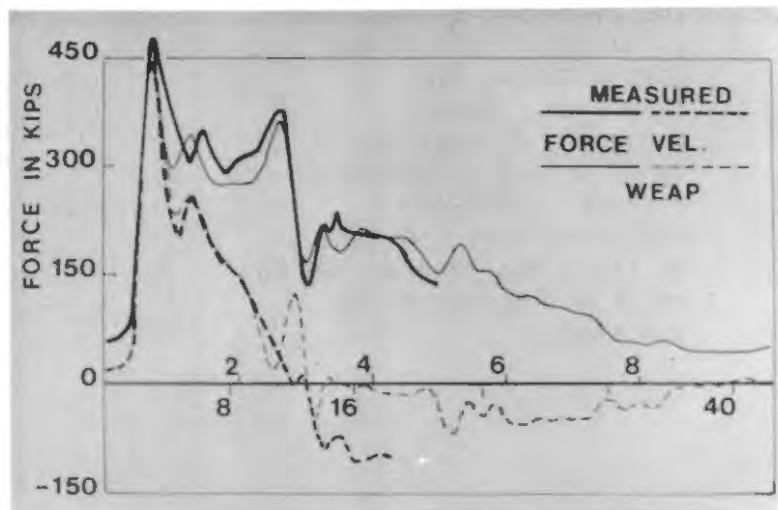
- C33. The second test case was taken from a special test program on a DELMAG D-30 hammer. These tests were performed by Goble & Associates in 1971, for the Foundation Equipment Company of Newcomerstown, Ohio. In addition to force and velocity at the pile top the combustion chamber pressure was also measured.



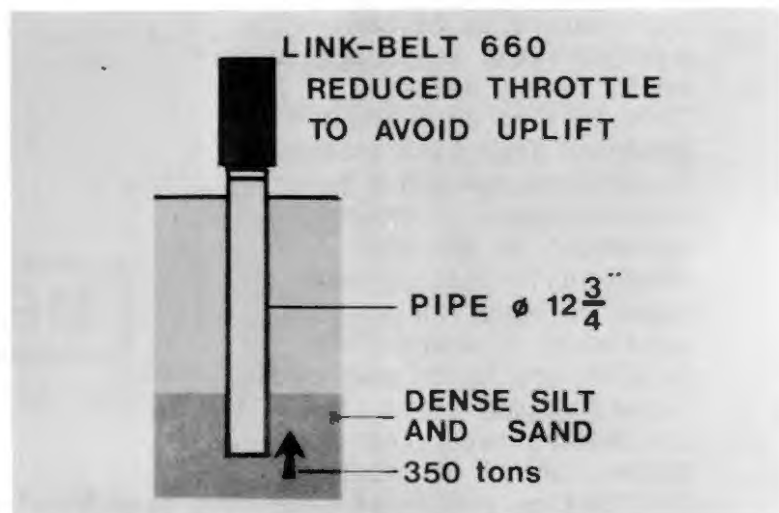
- C34. The comparison of the pressure is shown here. Ram impact occurs at point A. Most of the precompression phase has not been included. The pressure remains constant for about 1 1/2 milliseconds until ignition begins at point B. Then it increases rapidly to point C. At ignition the measured record exhibits a high frequency oscillation characteristic of diesel engine ignition. Since this oscillation is unimportant for hammer performance no attempt is made to represent it. It can be seen that the pressure calculated by WEAP is similar but somewhat smaller than the measured value.



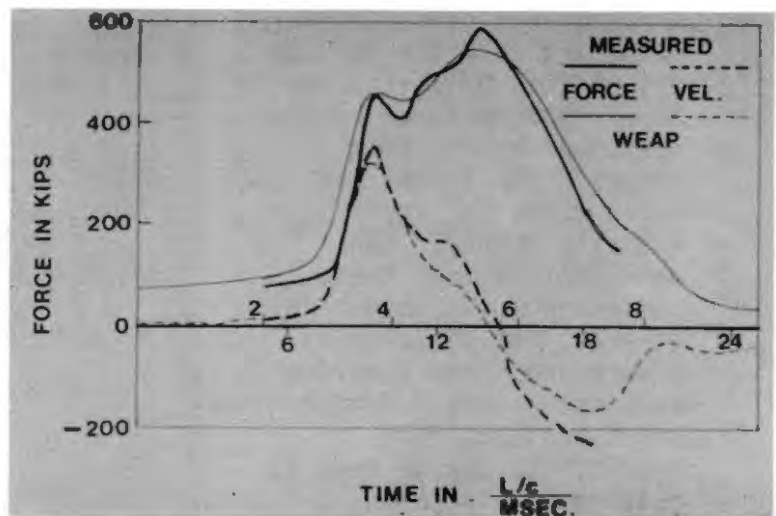
- C35. A comparison of measured force and velocity for the top of this pile is shown here. The measured values are shown by the bold curves. The forces are solid lines and the velocity dashed. The agreement between these curves is quite good. In general, the velocity is a more sensitive quantity and, therefore, a better one to match. This sensitivity can be seen in this example at about 14 milliseconds where the measured and calculated velocities differ substantially.



- C36. As a third example consider a pile driven by a closed end diesel hammer, the Link-Belt 660. The pile was a 43 foot long pipe of 12-3/4 inch outside diameter. It was driven into very dense silt and sand.



- C37. The results are shown here as before. This hammer uses atomized fuel injection, thereby producing an earlier ignition and a smoother transition between precompression and impact on the force record. Again, excellent agreement is achieved.



- C38. The WEAP program is the first wave equation program to be extensively and systematically tested against field measurements of force and velocity. This testing activity has produced important changes in the program and a more correct model of driving systems. It has also shown again that correct input information must be available if meaningful results are to be obtained. In many cases the state-of-the-art makes the determination of input information problematical. The user should not be surprised if such cases produce results which differ from the field experience. The wave equation concept is another engineering tool, the results of which should be evaluated by a knowledgeable engineer. Happy Pile Driving!

