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16. Abstract

This is Volume 2 of a two-volume report of the results of a detailed sensitivity analysis for the FHWA flexible pavement structure model VESYS IIM.

Volume 1 reports the results of studies preparatory to the sensitivity analysis, comparisons of predicted and measured performance and recommendations for improvements to VESYS IIM. The studies preparatory to the sensitivity analysis were primarily aimed at developing realistic values for the many input variables, but also included certain improvements to the computer program. This volume reports the results of the sensitivity analysis itself.

Procedures were developed and applied for conducting a sensitivity analysis for the rather complex VESYS IIM that had 30 independent variables. As a full factorial consideration at only two levels would have involved some 2^{30} solutions, it was necessary to screen out relatively insignificant independent variables in stages and by various methods, separate the analysis into two factorials and to use fractional factorial techniques and other carefully constructed innovations to reduce the task to a manageable level.

The sensitivity analysis was completed with a very minimum of lost information despite the staggering sizes of the full factorials represented. Separate analyses were conducted for cracking damage, rut depth, slope variance, Present Serviceability Index (PSI) and Service Life. The procedures employed, details of the specific analyses and the results of the sensitivity analysis are described in this volume (Volume 2).

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PREFACE

This is Volume 2 of a two-volume report of the results of a detailed sensitivity analysis for the FHWA structural model VESYS II (M). Volume 1 reports the results of studies preparatory to the sensitivity analysis, comparisons of predicted and measured performance and recommendations for improvements to VESYS II (M). The studies preparatory to the sensitivity analysis were primarily aimed at developing realistic values for the many input variables, but also included certain improvements to the computer program. This volume reports the results of the sensitivity analysis itself.

This work was accomplished by a team of engineers and other professionals including Harvey J. Treybig, Thomas W. Kennedy, R. C. G. Haas, R. Franklin Carmichael III, Harold L. Von Quintus, Robert P. Smith, Jack P. Randall and the authors.

Special appreciation is extended to Ms. Shirley Selz for her efforts in development of procedures and a working support computer program for the sensitivity analysis and to Dr. Virgil Anderson for his critical review of the proposed techniques and valuable suggestions for their application. Support for the contract was provided by the Federal Highway Administration, Offices of Research and Development, Contract No. DOT-FH-11-8258. We are grateful for the valuable technical coordination provided by Mr. William J. Kenis, FHWA Contract Manager.

J. Brent Rauhut
John C. O'Quin
W. Ronald Hudson

The work under this contract was conducted to delineate those major areas where additional work would be necessary to improve the operating and predictive capabilities of the VESYS computer programs. Since the conclusion of this contract, some major changes have been made to the program by FHWA. The resulting version is called VESYS IIM(1-4). This version, along with the recently developed FHWA VESYS design users manual, has been distributed to five agencies to be used in a trial implementation design and analysis capacity and as a working tool to evaluate the potential field performance of new materials. The implementation of VESYS IIM is being conducted through Office of Development contracts and HPR studies to determine its suitability as a standard design tool for highway departments.

The findings from this work and the results obtained from new studies currently underway will be used to aid FHWA in making continuing updates and improvements to the VESYS computer program and design users manual.

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CHAPTER I

INTRODUCTION

Background

The FHWA Research Project "New Methodology for Flexible Pavements" has as its objective the development of a rationally based pavement design procedure, which has the capability to predict performance of the pavement over its useful life. As an initial step, a pavement design system was developed under an FHWA contract with the Massachusetts Institute of Technology (MIT) (Ref 1[1], 2[2], and 3[3]). This included concepts for sophisticated design optimization procedures and a computer program called VESYS that performed a structural analysis for a three-layer pavement system on a probabilistic basis and predicted fatigue cracking, rut depth, slope variance, Present Serviceability Index (PSI), and service life with time. Subsequent improvements by MIT and the FHWA resulted in an improved version (August 1974) of the computer program called VESYS IIM. A current FHWA contract with the University of Utah will expand VESYS IIM to a five-layer capability.

VESYS IIM is a long and extremely complex computer program currently requiring some 67 input values, about 27 of which are program control variables and the rest actual independent variables. It is not possible to understand clearly the characteristics of such a large simulative model except through a well defined and laborious sensitivity analysis. Accordingly, the FHWA contracted with Austin Research Engineers Inc to conduct such an analysis. The stated FHWA objective was:

"To determine the sensitivity of the VESYS IIM computer program input variables on predicted pavement serviceability and to evaluate predicted serviceability in terms of realistic pavement performance."

In addition to the sensitivity analysis, FHWA wished to learn if the pavement performance predicted by VESYS IIM realistically reflects

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[1] Moavenzadeh, F., Soussou, J. E., Findakly, H. K., "Synthesis for Rational Design of Flexible Pavements", Part I, School of Engineering, Massachusetts Institute of Technology, Cambridge, Mass., 02139, January 1973.

[2] Soussou, J. E., F. Moavenzadeh and H. K. Findakly, "Synthesis for Rational Design of Flexible Pavements, Part II", FHWA Contract No. FH-11-776, January 1973.

[3] Moavenzadeh, F., Soussou, J. E., Findakly, H.K., Brademeyer, B., "Synthesis for Rational Design of Flexible Pavements", Part III, "Operating Instructions and Program Documentation", February 1974.

performance measurements on real pavements and to obtain recommendations for improvements to VESYS IIM.

Work Preparatory to the Sensitivity Analysis

The development of the sophisticated mathematical idealization called VESYS IIM for complex flexible pavement structures has led to definition of independent variables not previously used in engineering practice and for which limited practical data exists. Also, limited stochastic data exists for independent variables in more common use. If the sensitivity analysis is to be meaningful, all input variables must be varied over realistic and consistent ranges. Also, the various mathematical models comprising VESYS IIM must function properly. The major portion of the research effort was aimed at defining realistic input values for the independent variables and improving or correcting the mathematical models to predict performance responses more accurately. This effort preparatory to conducting the sensitivity analysis is reported in Volume 1 of this report. The results from comparisons of calculated performance responses to field measurements from the AASHO Road Test and the Brampton Test Road, identification of deficiencies in the VESYS IIM system, recommendations for research needed to improve it and cost estimates for the research needs identified also appear in Volume 1.

The Sensitivity Analysis

There is no established procedure for conducting a sensitivity analysis for a system having 30 independent variables. A full factorial consideration would entail in the order of 3^{30} VESYS IIM solutions for three-levels of each independent variable or 2^{30} for two levels, either number being impossibly large. It was necessary to screen out relatively insignificant independent variables in stages and by various methods, separate the analysis into two factorials and to use fractional factorial techniques and other carefully constructed innovations to reduce the task to a manageable level.

The sensitivity analysis was completed with a very minimum of lost information despite the staggering sizes of the full factorials represented. Separate analyses were conducted for cracking damage, rut depth, slope variance, Present Serviceability Index (PSI) and Service life. The procedures employed, details of the specific analyses and the results of the sensitivity analysis are described in this volume (Volume 2).

The sensitivity analysis was conducted on the June 1975 version of VESYS IIM that incorporated certain improvements described in Volume I of this report. Subsequent reference to VESYS IIM will generally refer to this version.

CHAPTER II

STATISTICAL PROCEDURES EMPLOYED

The difficulty in performing a sensitivity analysis on such a complex model as VESYS II lies in the large number of complex and interrelated input variables. The object of the analysis is to determine the effects individually and in combinations of these many variables on the model's response. As there are now some 67 different input parameters (many of these are program control parameters) which are recognized by the program, it is evident that an enormous amount of screening must be performed in order to reduce the task to manageable proportions. The methods by which this reduction was carried out are described in this chapter. They include: the screening of variables by the use of a small preliminary factorial, fractional factorial techniques to reduce the number of solutions required in the main factorial, division of the model into its two main components, factorial analysis of variance to weed out insignificant factors and interactions, and finally, regression analysis to approximate the model with a simple function involving the most significant variables and their interactions.

Preliminary Screening

The task of selecting which variables were to be considered for the main sensitivity analysis was performed on the basis of a set of preliminary sensitivity runs. Estimates were made on the average value and the high and low extremes of each input to the program. Each variable was then run at its high and low extrema, while holding all of the other inputs at the average value which had been determined. The effect of this variation was observed for each response of interest. A crude measure of the sensitivity of each response to the different variables could thus be obtained by ranking the variables in order of the magnitude of the effect produced on the level of each dependent variable. On the basis of these rankings, a number of variables were dropped from further consideration in the sensitivity analysis. See Chapter III for a more detailed discussion of the preliminary sensitivity analyses.

Division of Task Into Two Factorials

Despite the reduction which was achieved by means of the preliminary runs, some nineteen variables still remained for consideration in the analysis. To run all the combinations of these factors at three levels would have required more than a billion solutions of the program. To run all of the combinations at two levels would still have required more than half a million runs. The reason for the staggering size of such experiments is that the number of solutions required increases exponentially with the number of factors involved. This is due to the proliferation of interactions which is caused by the addition of each new factor to the experiment. However, the cracking and rutting models are independent in VESYS IIM and the slope variance model uses data calculated by the rutting model. On the basis of this separation, it was possible to divide the task into two experiments, one for the cracking model called the cracking factorial involving eleven factors, and the other for the rutting and slope variance models called the roughness factorial involving fifteen variables. This allowed an enormous savings in computational effort without any loss of information.

Fractional Factorial Techniques

Even with this significant reduction in size, the experiment is far too large to be run using a full factorial. Fifteen variables at two levels each still require 32,768 solutions in order to obtain all possible interactions of all fifteen variables. Since all of this multitude of interactions are not of interest anyway, an extremely valuable technique exists for reducing the number of solutions required without destroying the information which is desired. Through careful experiment design it is possible to run far fewer solutions than those required for a full factorial and still retain all of the main effects and interactions which are of interest. This is accomplished through the use of fractional factorials. Appendix A gives a detailed explanation of the techniques which were used to design and analyze fractional factorials on this project. Interested persons are referred to that section for examples and references on this rather complex process.

These techniques permitted enormous reduction in the number of solutions required for the sensitivity analysis. The fifteen variable factorial was run at a 1/128 fractional, requiring only 256 solutions of the program to produce the needed data. Similarly the eleven variable cracking factorial was run as a 1/16 fractional, involving 128 solutions. The treatment combinations which were run and the reponses which were obtained from each solution are listed in Appendix C. The values of the dependent and independent variables for each of these runs were punched on cards to be used on statistical analyses for determining the relative sensitivities of the factors in the experiment.

Factorial Analysis of Variance

The 15 factors in the roughness factorial have 105 two-way interactions and 17 three-way interactions which are unconfounded[1] with other lower-order effects in the fractional blocking scheme used. This is too many terms to be analysed by any standard statistical regression package. The same difficulty holds to a lesser extent with the smaller cracking factorial. Some method is required for reducing the number of terms in the model to a manageable size. We wish to delete only those terms which are likely to be the result of "measurement error"

[1] Confounded terms are those whose effects are indistinguishable due to the fractional experiment design.

and retain all those which produce a significant effect. Analysis of variance provides a systematic procedure for selecting only those terms whose effect on a dependent variable are measureable with some high degree of confidence.

In order to perform this analysis some estimate of the measurement error is required. In a computer sensitivity analysis there is no measurement error in the normal sense (except for round-off error which is negligable in this context). However there is some uncertainty in the results due to the confounding scheme required for a fractional factorial. The main effects and two-way interactions are confounded with many higher-order terms, and the responses measured for each term are indistinguishable from those due to its aliases (confounded terms). Since we are assuming that these many aliases make a neglibible contribution to the response caused by a significant main effect or interaction, we need to test the reliability of this assumption.

For this purpose we construct an error pool by averaging the sums of squares due to terms which contain confounded three-ways and higher-order interactions; no main effects, two-way interactions or single three-ways are included. This mean square for error represents the "background noise" in the experiment, the contribution which we expect from the higher-order aliases of a term which we wish to measure. Since we cannot determine the magnitude of this contribution in each specific case, we must treat it as a random variable whose mean is equal to the mean square for error described above. Since this random variable is a combination of a great many independent random factors we may apply the Central Limit theorem (Ref. 4)[1], claiming that it approximates a normal distribution. The ratio of the mean square due to some factor or interaction with this mean square for error may be approximated by a random variable having the F distribution, yielding an estimate of the probability that the factor or interaction in question may be nothing more than a fortuitous combination of confounded higher-order effects. This probability that a term is not significant in itself, but may instead be attributed to random experimental variation is known as the α-level. For each of the analyses of variance on the results of the 15-factor roughness factorial, only terms whose F-ratio (mean square divided by mean square for error) was greater than that required for an α-level of .001 were accepted for further analysis. This means that there is less than one chance in a thousand that the response being measured is due to the 127 aliases which are present because of the 1/128 fractional factorial which was used. The analysis of damage index response from the cracking factorial did not require such a conservative α-level because the 1/16 fractional which was run causes only 15 aliases for each term of interest.

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[1] Page 17; Draper, N.R., and Smith, H, Applied Regression Analysis, John Wiley & Sons, Inc., New York, 1967.

In practice it often happens that two or three terms which contain confounded three-way interactions have considerably higher sums of squares than the rest of the terms in the error pool. Usually, investigation reveals that one of the three-way interactions present involves three of the more significant factors in the experiment, or involves three variables which are obviously physically interrelated (such as the various materials properties for a single layer). This situation does not invalidate the fractional factorial assumptions because such terms are very unlikely to be included in the final regression model. Once the relevant main effects have been entered into the equation, their three-way interaction will not significantly improve the fit. When a term of this type appears, however, it is inconsistant to accept it as significant while retaining it in the error pool. To avoid this situation all terms in the error pool with F-ratios which were significant at an α-level of .01 were deleted from the error pool. The higher α-level avoids overlap between terms in the error pool and terms considered significant.

Orthogonal Regression Techniques

The final step in the statistical reduction of sensitivity data is the development of a set of regression equations which describe the model's responses in terms of the levels of the input variables. These equations provide a first-order approximation to the model which is sufficiently accurate to provide meaningful information, yet simple enough to be readily understandable. Most of the methods of ranking the input variables according to the responses they produce are based on the regression equations.

Comparisons between variables are confused by differences in units and in absolute magnitude. The effects due to an increase of one inch in the top layer are obviously not comparable to those produced by increasing traffic intensity by one axle per day. For this reason, it is convenient to recode all variables to common units and magnitudes. This is easily done by making the following transformation on each independent variable:

$$Z_i = \frac{X_i - \overline{X}}{\sigma_X}$$

where:

X_i is the i^{th} observation of variable x
Z_i is its transformed value,

$\overline{X}$ is the mean of X, and
σ_X is its standard deviation.

The recoded variable, Z, which results from this transformation has a mean of zero and is unitless. A change of one in its value represents a variation of one standard deviation in the value of X.

The primary advantage of this recoding is that when the factorial is run at two levels, representing the mean plus and minus the standard deviation, the Z's will have values of -1 or 1. This results in an orthogonal input matrix to the multiple regression routine. That is, each variable (column in the matrix) is linearly independent of all of the other variables. Such a matrix avoids the computer round-off error which can be so troublesome when regression is performed on highly correlated variables. It also causes the calculated regression coefficients to be uncorrelated.

This is important because it means that the coefficient on each term in the resulting equation represents only the effect due to that factor or interaction. In a non-orthogonal system, a variable in the equation will frequently be correlated with some term not in the equation, and its coefficient will have been adjusted to "explain" the response due to that other variable as well. If the second variable is then entered into the equation in a later step, the coefficient on the first term will often be changed drastically. This situation would be intolerable in a sensitivity analysis, because the coefficients form the basis for sensitivity rankings.

The techniques described above could not be followed strictly in this project. A few input variables had grossly skewed distributions. One standard deviation below the mean of COEFK1, for example, would have been an unrealistic value. In these cases the input values selected were such that a change from the high to low levels represented a variation over the central 67% of the variable's probability distribution. This is the same range which would be traversed by a normally distributed variable being varied from its mean plus one standard deviation to its mean minus one standard deviation. Hence such values could be coded to 1 and -1, producing valid comparisons with the other factors in the analysis. This meant that some factors, such as COEFK1, which appeared in only one factorial would not have a recoded value of zero for their average levels used in the other factorial. No difficulties arose from the slight correlations which were produced in such cases (the problem only arose when the results from the two factorials were combined for the analyses on serviceability index and service life).

A potentially more serious problem was caused by the constrained factor spaces employed for ALPHA(1) and GNU(1). Since these variables are highly correlated in practice, it would have been unrealistic to vary them independently in the sensitivity runs. Very high levels of ALPHA(1) do not occur in combination with very low levels of GNU(1), and vice versa; yet these factors are not fully correlated and relative varation does occur. The solution to this difficulty was to use different values for the high and low levels of ALPHA(1) depending on the level of GNU(1). Table 1 shows the values which were used. This constraint on the values of ALPHA(1) introduced a correlation of .707 between these two factors, which could have presented considerable difficulties had one of these variables been included in a regression

Table 1. Values of ALPHA(1) as a Function of the level of GNU(1)

		GNU(1)	
		low = .2	high = .6
ALPHA(1)	low	.68	.75
	high	.75	.82

equation but not the other, because the effects due to these factors could not have been completely separated. As was expected, however, both of these factors were highly significant for each response except damage index (which did not depend on these permanent deformation parameters at all). Consequently there were no problems caused by this deviation from strict orthogonal coding.

CHAPTER III

PRELIMINARY SENSITIVITY ANALYSES

The consideration of a possible 26 independent variables at two levels in a full factorial would require over 67 million VESYS IIM solutions and was clearly not practical. To reduce the sensitivity analysis to a manageable level, it was necessary to eliminate as many independent variables of minor significance as possible before designing the main sensitivity analysis. It was also useful for gaining insight concerning the relative sensitivity of the VESYS IIM model to the independent variables.

First Preliminary Sensitivity Analysis

A preliminary sensitivity analysis was designed to consist of a solution for a set of "average" input values and a set of solutions varying individual input variables separately to low and high values within their range. This provides a set of three values for each output response, one from the solution with all values at their selected averages and two from the runs in which that variable was set at its low and high values. The ranges from high to low generally represent the range believed to occur in practice, except that the traffic and layer thicknesses were held to the range expected for a busy rural interstate highway. The variation in traffic and thickness design from a typical rural highway to an urban interstate highway or freeway is so great that it was believed more meaningful responses would result if these variables were handled in three separate traffic/thickness design cases. This busy rural section of interstate highway selected represented the "mid-range".

The fatigue coefficient STRNCOEF and the exponent STRNEXP were inadvertently held constant for this preliminary analysis so the response for Damage Index was not considered to be reliable. As these variables have no effect on the rutting model and cracking has only a minor impact on the present serviceability index, the other responses were considered to be reliable.

K1K2CORL, the correlation coefficient for STRNCOEF and STRNEXP was known to be heavily negatively correlated and not too variable based on plots for the two values from fatigue tests. A small factorial of solutions was run with KLK2CORL as a variable without any significant effect on calculated responses. Therefore, a value of -.867 was derived by analysis of the results of many fatigue tests and this value was used as a constant rather than varied in the sensitivity analysis.

The responses of primary interest from the array of computer solutions were plotted at one and twenty years for the low, average and high values of each input variable whose variation appeared important.

These responses were rut depth, slope variance, present serviceability index (PSI) and service life. Each page of plots included responses to the average solution and the two solutions varying the value input for a single variable from its minimum to its maximum.

The horizontal plots for the responses rut depth, slope variance, PSI and service life indicated that the input values across the practical range for the variables listed below have little or no effect:

1. DURATION - duration of the wheel load in seconds.
2. VCDUR - variance of DURATION.
3. RADIUS - radius of the circle representing the tire "footprint", defined as the circle having the area obtained by dividing the wheel load by the "tire pressure", called AMPLITUD.
4. Permanent deformation coefficients GNU(2) and GNU(3) for the base course and subgrade, respectively.
5. BETA - a variable describing time-temperature shift for the asphaltic concrete surface layer.

The elimination of these five variables reduced the potential full factorial to 2^{21} or about 2 million solutions, still an unmanageable number. Other variables appeared to be insensitive, but not at a sufficient confidence level to elimate them from the analysis at that point.

It was clear that additional effort was required to reduce the number of variables and that both fractional factorial analysis and separate analyses for the cracking model and for the rutting model (including slope variance) would be required. Separate analyses would be beneficial as the models did not share all the variables. This is illustrated in Table 2.

The slope variance model is heavily dependent on the variance of rutting from the rutting model, but it also depends on CORLEXP (a value used in obtaining an estimate of slope variance in terms of the variance in rut depth). For this reason, CORLEXP was added to the rutting variables to form a single large factorial for analysing both rutting and slope variance response.

Second Preliminary Sensitivity Analysis

As the development of rational values for the independent variables progressed, the values for the permanent deformation coefficients and other variables of major importance varied sufficiently that it appeared necessary to run a second preliminary factorial to check the results from the first and see if other variables might be

Table 2. Identification of significant independent variables for the cracking and roughness models.

Variables Significant only to Cracking Model	Variables Significant to both Models	Variables Significant only to Roughness Model
STRNCOEF and STRNEXP*	LAYER1	ALPHA(1)
COEFK1	LAYER2	GNU(1)
COEFK2	LAYER3	ALPHA(3)
VCAMP	THICK1	THICK2
	AMPLITUD	CORLEXP**
	LAMBDA	VARCOEF1
	TEMPS	VARCOEF2
		VARCOEF3

* These two variables considered together as one independent variable called NFAIL or Fatigue Life Potential

** Required for slope variance analysis

Cracking Variables + Shared Variables = 11 Variables

Roughness Variables + Shared Variables = 15 Variables eliminated. The independent variables DURATION, VCDUR, BETA and RADIUS were not varied as they were clearly insensitivite based on the first sensitivity analysis. STRNCOEF and STRNEXP, inadvertently held constant during the first preliminary sensitivity analysis, were included and varied for the second analysis.

The only additional variable eliminated as a result of the second preliminary sensitivity analysis was ALPHA(2). GNU(2) and GNU(3) were varied again as a further check and again proved to be insensitive. Thus the permanent deformation coefficients for the base material were not found to be significant to calculated responses from use of VESYS IIM over their rational range. While both ALPHA(3) and GNU(3) are required to define the permanent deformation characteristics of the subgrade, GNU(3) varies over such a small range for these materials that it had little effect.

The results of the second preliminary analysis appear in Table 3. The values shown represent the change in the various responses (dependent variables) for Year 1 and Year 10 as each independent variable is varied across its practical range with all other variables at their mean values. The sensitivity rankings for each response are shown in Table 4.

Selection of Variables for the Main Analysis

It had been established through study and design for the fractional factorials and review of Tables 3 and 4 that fifteen independent variables could be used for the rutting model and eleven for the cracking model. Those variables selected for the main sensitivity analysis included four separate and seven shared variables for the cracking factorial and eight separate and the seven shared for the roughness factorial as shown in Table 2. This cut off is also shown in Table 4, all variables above the dotted lines being selected.

The statistical analyses described in Chapter 2 and Appendix A indicated that 21 unconfounded three-way interactions would be available for the rutting model and 17 for the cracking model. The independent variables selected for consideration with these three-way interactions are shown in Table 5. The assignment of letters to variables was accomplished in such a way that as many of these interactions as possible involved variables which seemed likely to interact significantly. This provides a check on the validity of the assumption underlying the fractional factorial, i.e. that three-way and higher-order interactions are negligable in the final analysis. Since none of the measureable three-way interactions which might be expected to be important (such as ALPHA(1) · GNU(1) · LAYER1 or THICK1 · TEMPS · NFAIL) appeared in any of the regression equations, it is reasonable to conclude that the fractional factorials used in the main sensitivity analysis were relatively free from errors introduced by significant higher-order aliases (i.e., confounded interactions).

Table 3. Results of Second Preliminary Sensitivity Analysis

Variable	Calculated Responses									
	Rut Depth		Slope Variance		Damage Index		Serviceability Index		Service Life	
	Year 1	Year 10	Year 1	Year 10	Year 1	Year 10	Year 1	Year 10	Year 1	Year 10
ALPHA(1)	-232	-145	-672	-26×10^6	0	0	3.0	1.5	3.0	3.0
GNU(1)	4.3	12.4	318	2543	0	0	-42	-317	.09	.09
ALPHA(2)	-.04	0	-2.8	-14	0	0	.4	1.7	0	0
GNU(2)	.02	.04	2.2	10.0	0	0	-.3	-1.2	0	0
ALPHA(3)	-.01	-.23	-7.6	-51	0	0	1.0	6.3	0	0
GNU(3)	.02	.04	1.6	7.8	0	0	-.3	-1.0	0	0
AMPLITUD	1.23	3.49	37	702	6.3	63	-11.4	-87	-.72	-.72
BETA	Known to be insensitive from 1st preliminary analysis									
COEFK1	0	0	0	0	-302	-302	.3	.4	.01	.01
COEFK2	0	0	0	0	-1.1	-10.1	0	0	0	0
CORLEXP	0	0	-141	428	.01	-.07	.9	.9	.03	.03
DURATION	Known to be insensitive from 1st preliminary analysis.									
LAMBDA	1.10	3.12	83	667	4.77	47.7	-10.8	-83	.07	.07
LAYER1	2.43	6.86	171	1369	.99	9.94	-24.05	-182.9	.14	.14
LAYER2	.40	1.15	38	303	3.6	36.4	-4.1	-31.0	.03	.03
LAYER3	.68	1.93	57	465	1.4	13.7	-7	-52	.05	.05
RADIUS	Known to be insensitive from 1st preliminary analysis.									
STRNCOEF and STRNEXP	0	0	0	0	-19.1	-191.1	.3	0	.02	.02
TEMPS	.33	.92	25	199	-66	-655	3.2	-24.7	-.03	.03
THICK1	1.55	4.32	113	904	-57	-566	-12.4	-90	.20	.20
THICK2	-.04	-.12	-37	-297	-.77	-7.7	.68	3.9	-.05	-.05
VARCOEF1	-.04	-.11	75	600	.55	5.5	-.2	-1.1	.11	.11
VARCOEF2	-.03	-.07	86	690	-.3	-3	-.5	0	.12	.12
VARCOEF3	-.08	-.23	339	2734	-.11	-1.1	-1.5	-1.5	.47	.47
VCAMP	0	0	19	153	1.83	18.3	-2.3	-18.8	1.6	1.6
VCDUR	Known to be insensitive from frist preliminary analysis									
STDEVO	0	0	0	0	0	0	0	0	0	0

Table 4. Preliminary Response Rankings

Cracking Factorial	Roughness Factorial	
Damage Index	Rut Depth	Slope Variance
TEMPS	ALPHA(1)	ALPHA(1)
THICK1	GNU(1)	VARCOEF3
COEFK1	LAYER1	GNU(1)
STRNCOEF	THICK1	LAYER1
STRNEXP	AMPLITUD	THICK1
AMPLITUD	LAMBDA	AMPLITUD
LAMBDA	LAYER3	VARCOEF2
LAYER2	LAYER2	LAMBDA
VCAMP	TEMPS	VARCOEF1
LAYER3	ALPHA(3)	LAYER3
COEFK2	VARCOEF3	CORLEXP
LAYER1	THICK2	LAYER2
- - - - -	VARCOEF1	THICK2
THICK2	VARCOEF2	TEMPS
VARCOEF1	- - - - -	- - - - -
VARCOEF2	GNU(2)	VCAMP
VARCOEF3	GNU(3)	ALPHA(3)
CORLEXP	ALPHA(2)	ALPHA(2)
ALPHA(1)	COEFK1	GNU(2)
GNU(1)	COEFK2	GNU(3)
ALPHA(2)	CORLEXP	COEFK1
GNU(2)	{STRNCOEF	{COEFK2
ALPHA(3)	STRNECP}	STRNCOEF}
GNU(3)	VCAMP	STRNEXP
STDEVO	STDEVO	STDEVO

All variables above the dotted lines were included in the main sensitivity analysis.

Table 5. Measureable Three-Way Interations

Cracking Factorial	Roughness Factorial
AMPLITUD · LAMBDA · LAYER2	ALPHA(1) · GNU(1) · THICK1
THICK1 · TEMPS · COEFK1	ALPHA(1) · GNU(1) · LAYER1
THICK1 · TEMPS · NFAIL*	LAYER1 · THICK1 · ALPHA(1)
THICK1 · COEFK1 · NFAIL*	LAYER2 · VARCOEF3 · AMPLITUD
TEMPS · COEFK1 · NFAIL*	CORLEXP · AMPLITUD · VARCOEF2
THICK1 · LAYER1 · VCAMP	LAMBDA · ALPHA(3) · CORLEXP
AMPLITUD · NFAIL* · VCAMP	AMPLITUD · ALPHA(1) · LAMBDA
THICK1 · LAMBDA · LAYER3	AMPLITUD · GNU(1) · TEMPS
LAMBDA · NFAIL* · COEFK2	LAMBDA · THICK1 · VARCOEF1
LAYER2 · NFAIL* · LAYER3	TEMPS · LAYER1 · LAYER2
TEMPS · LAYER1 · COEFK2	VARCOEF1 · LAYER1 · CORLEXP
COEFK1 · LAYER1 · LAYER3	VARCOEF2 · ALPHA(1) · VARCOEF3
VCAMP · COEFK2 · LAYER3	VARCOEF3 · GNU(1) · ALPHA(3)
LAMBDA · COEFK2 · LAYER3	CORLEXP · GNU(1) · LAYER3
AMPLITUD · LAYER2 · LAYER1	LAYER3 · ALPHA(1) · THICK2
AMPLITUD · COEFK1 · COEFK2	
LAMBDA · LAYER2 · LAYER1	
AMPLITUD · LAYER2 · LAYER1	
AMPLITUD · LAMBDA · LAYER1	

*NFAIL is the name which has been given to the fatigue life potential variables, STRNCOEF and STRNEXP, which are coupled to form a single factor.

CHAPTER IV

SENSITIVITY ANALYSIS FOR CRACKING DAMAGE

The independent variables determined from the preliminary sensitivity analysis to importantly affect the prediction of cracking in the surface layer were:

1. Creep compliance vectors for the asphaltic concrete surface layer, base (and subbase if included) and subgrade, called LAYER1, LAYER2, and LAYER3, respectively.
2. Thickness of the first layer, THICK1.
3. The fatigue coefficient $K_1(T)$ and exponent $K_2(T)$, combined as one independent variable and termed "Fatigue Life Potential" or NFAIL. These variables are input as STRNCOEF and STRNEXP, respectively.
4. The term AMPLITUD, which represents tire pressure but includes wheel load data as well because of the fixed radius of load used by VESYS II (M) for a specific solution.
5. The variance of AMPLITUD (called VCAMP) that takes into account variability of both tire pressure and wheel loads.
6. The coefficients of variation COEFK1, and COEFK2 for the Fatigue Life Potential variables $K_1(T)$ and $K_2(T)$, respectively.
7. The array of average monthly temperatures called TEMPS.
8. Average axles per day called LAMBDA.

The dependent variable selected to represent cracking damage was the expected value of Damage Index $E[D_j]$ described in Volume I of this report. For simplicity, Damage Index or [DI] will be used to represent the expected value of Damage Index $E[D_j]$ in subsequent discussion.

Using the techniques for fractional factorials discussed in Chapter 2 and Appendix A, a 1/16 replicate was selected for the 2^{11} factorial resulting from assignment of two levels for each of the eleven independent variables considered (STRNCOEF and STRNEXP are input separately although treated in the sensitivity analysis as coupled to represent a single independent variable). The 128 combinations of independent variables, selected to minimize loss of information, are identified by the appearance of numbers in the blocks for the full 2^{11} factorial shown in Figure 1. The confounding scheme allowed sensitivity evaluation of all main effects, two-way interactions and selected three-way interactions.

The 128 separate solutions identified in Figure 1 were obtained using VESYS IIM. The numbers appearing in the blocks are the calculated values of Damage Index. Actual low and high values used for the independent variables appear in Tables 6 , 7 and 8 and generally

Legend:

1. Subscript "o" means low level and "1" means high
2. Identification of Independent Variables:

A=AMPLITUD	C=TEMPS	E=COEFK1	G=NFAIL	J=VCAMP	K=COEFK2
B=THICK1	D=LAMBDA	F=LAYER2	H=LAYER1	L=LAYER3	

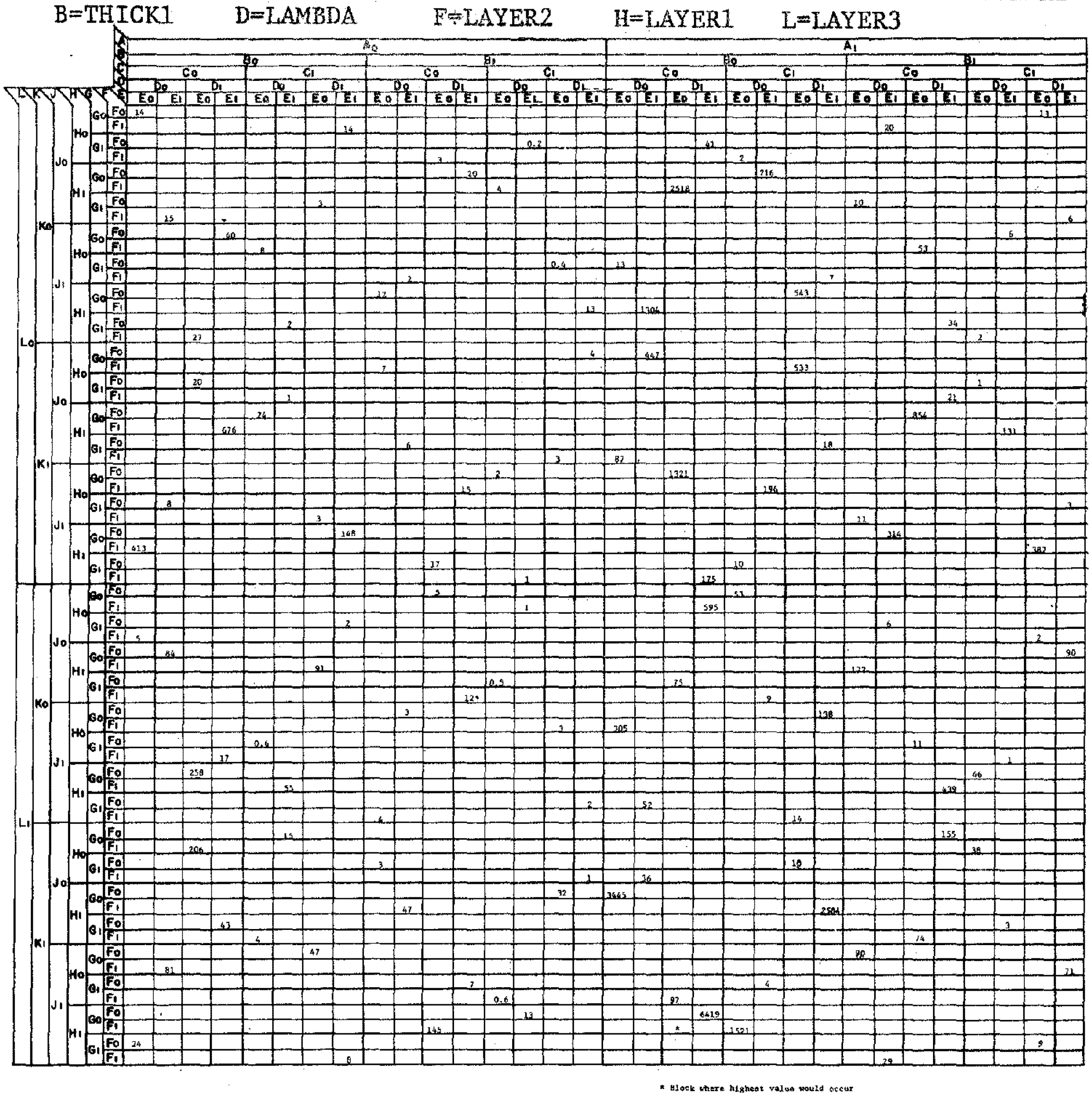

Figure 1. Fractional factorial used for cracking damage analysis (numbers in the blocks identify combinations considered and are themselves the calculated Damage Index after five years).

Table 6. Values for temperatures and fatigue life potential used for the sensitivity analysis.

					Fatigue Life Potential					
					Low		Medium		High	
Month		Temp	$\frac{K_1(T)}{K_1(70)}$	$\frac{K_2(T)}{K_2(70)}$	K_1	K_2	K_1	K_2	K_1	K_2
TEMPS LOW	J	21.5	.0033	1.049	2.04×10^{-16}	5.25	6.11×10^{-8}	3.19	2.93×10^{-6}	2.77
	F	23.5	.0036	1.046	2.22×10^{-16}	5.23	6.66×10^{-8}	3.18	3.2×10^{-6}	2.76
	M	25.5	.0043	1.044	2.65×10^{-16}	5.22	7.96×10^{-8}	3.17	3.82×10^{-6}	2.76
	A	45.0	.030	1.025	1.85×10^{-15}	5.13	5.55×10^{-7}	3.12	2.65×10^{-5}	2.71
	M	60.5	.22	1.009	1.36×10^{-14}	5.05	4.07×10^{-6}	3.07	1.95×10^{-4}	2.66
	J	69.0	.90	1.001	5.56×10^{-14}	5.01	1.67×10^{-5}	3.04	7.99×10^{-4}	2.64
	J	73.0	1.60	.997	9.89×10^{-14}	4.99	2.96×10^{-5}	3.03	1.42×10^{-3}	2.63
	A	75.0	2.70	.995	1.67×10^{-13}	4.98	5.0×10^{-5}	3.02	2.4×10^{-3}	2.63
	S	71.0	1.20	.999	7.42×10^{-14}	5.0	2.22×10^{-5}	3.04	1.07×10^{-3}	2.64
	O	67.5	.14	1.003	8.65×10^{-15}	5.02	2.59×10^{-6}	3.05	1.24×10^{-4}	2.65
	N	44.5	.03	1.025	1.85×10^{-15}	5.13	5.55×10^{-7}	3.12	2.66×10^{-5}	2.71
	D	39	.015	1.031	9.27×10^{-16}	5.16	2.78×10^{-7}	3.13	1.33×10^{-5}	2.72
TEMPS HIGH	J	49.7	.052	1.020	3.21×10^{-15}	5.10	9.62×10^{-7}	3.10	4.62×10^{-5}	2.69
	F	53.3	.082	1.017	5.07×10^{-15}	5.09	1.52×10^{-6}	3.09	7.28×10^{-5}	2.68
	M	59.5	.175	1.011	1.08×10^{-14}	5.06	3.24×10^{-6}	3.07	1.55×10^{-4}	2.67

Table 6. Values for temperatures and fatigue life potential used for the sensitivity analysis (cont.)

Month			Temp	$\frac{K_1(T)}{K_1(70)}$	$\frac{K_2(T)}{K_2(70)}$	Fatigue Life Potential					
						Low		Medium		High	
						K_1	K_2	K_1	K_2	K_1	K_2
TEMPS	HIGH	A	68.6	.90	1.001	5.56×10^{-14}	5.01	1.67×10^{-5}	3.04	7.99×10^{-4}	2.64
		M	75.2	2.72	.9948	1.68×10^{-13}	4.97	5.03×10^{-5}	3.02	2.42×10^{-3}	2.63
		J	81.6	14.0	.9884	8.65×10^{-13}	4.94	2.59×10^{-4}	3.0	1.24×10^{-2}	2.61
		J	84.6	32.0	.9854	1.98×10^{-12}	4.93	5.92×10^{-4}	3.0	2.84×10^{-2}	2.60
		A	84.7	33.0	.9853	2.04×10^{-12}	4.93	6.11×10^{-4}	3.0	2.93×10^{-2}	2.60
		S	78.9	6.4	.9911	3.96×10^{-13}	4.96	1.18×10^{-4}	3.01	5.68×10^{-3}	2.62
		O	70.1	1.05	1.000	6.49×10^{-14}	5.00	1.94×10^{-5}	3.04	9.32×10^{-4}	2.64
		N	59.1	.17	1.011	1.05×10^{-14}	5.06	3.15×10^{-6}	3.07	1.51×10^{-4}	2.67
		D	52.3	.074	1.018	4.57×10^{-15}	5.09	1.37×10^{-6}	3.09	6.57×10^{-5}	2.69

Fatigue Life Potential Input Variables at 70°F:

	Level of NFAIL		
	Low	Medium	High
$K_1(70^oF)$	6.18×10^{-14}	1.85×10^{-5}	8.88×10^{-4}
$K_2(70^oF)$	5.00	3.04	2.64

Table 7. Creep compliance arrays used for the sensitivity analysis

LOAD DURATION (sec.)		.001	.003	.01	.03	.1	.3	1	3	10	30	100
Asphaltic Concrete (LAYER1) ($PSI^{-1} \times 10^{-5}$)	low	.037	.052	.086	.19	.25	.40	.62	.86	1.23	1.58	1.9
	high	.061	.088	.145	.24	.40	.64	1.06	1.56	2.4	3.5	4.0
Base Material (LAYER2) ($PSI^{-1} \times 10^{-5}$)	low	4.55	4.55	4.55	4.55	4.55	4.55	4.55	4.55	4.55	4.55	4.55
	high	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25
Subgrade Material (LAYER3) ($PSI^{-1} \times 10^{-5}$)	low	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.1	11.1
	high	20	20	20	20	20	20	20	20	20	20	20

Table 8. Summary of average effects* (sensitivity of damage index) cracking factorial.

Independent Variables	Low-Level Value	High-Level Value	Average Effects on Distress Index (Including Interactions)	Average Effects as Percent of Overall Mean of D.I. (224.11)	Ranking Based on Average Effects
AMPLITUD, Wheel Load Pressure in PSI	58	95	361	160.6	2
THICK1, Thickness of layer 1 in inches	3.5	5.0	-339	-151.3	3
TEMPS, Temperature Array in ^{o}F	(See Table 6)		-222	-99.1	5
LAMBDA, Truck traffic in axles/day	2000	4500	152	67.8	7
COEFK1-Coefficient of Variation for fatigue coefficient K_1	.30	1.24	18	8.0	11
LAYER2, Creep Compliance array for base material, PSI^{-1}	(See Table 7)		-32	-14.3	9
Fatigue life Potential (K_1 & K_2)	(See Table 6)		-413	-184.3	1

Table 8. Summary of average effects* (sensitivity of damage index) cracking factorial (cont.)

Independent Variables	Low-Level Value	High-Level Value	Average Effects on Distress Index (Including Interactions)	Average Effects as Percent of Overall Mean of D.I. (224.11)	Ranking Based on Average Effects
LAYER1, Creep Compliance array for asphaltic concrete, PSI^{-1}	(See Table 7)		312	139.2	4
VCAMP, Variance of wheel load pressure (AMPLITUD) distribution in $(PSI)^2$	196	529	21	9.4	10
COEFK2, Coefficient of variation for fatigue exponent K_2	0.04	0.1	213	95.0	6
LAYER3, Creep compliance array for subgrade	(See Table 7)		108	48.2	8

*The average effect is the difference between the average Damage Index calculated for all solutions carried out at the low level of that factor and that of all solutions carried out at the high level.

represent one standard deviation either side of the mean value, or the equivalent where a normal distribution failed significantly to represent the actual distribution.

Statistical Analysis

Analysis of Variance

Predictions of damage index after five years of pavement service were analyzed using the factorial analysis of variance program FAØV-01. An initial error pool was constructed by adding the sums of squares of all terms (groups of confounded factors or interactions) in the factorial except those involving single factors, two-way interactions or three-way interactions which are only confounded with higher-order interactions. Using this estimate it was found that all of the unconfounded three-way interactions except two had F-ratios which were not significant at an α-level of .10 (the α-level is the probability of an effect being produced by random variation). Consequently, an augmented error pool was constructed using all of the insignificant three-way interactions as well as the confounded three-ways and higher.

Five of the confounded three-way interactions which had originally been put in the error pool turned out to be significant at an α-level of .10. These terms were examined and in each case at least one of the three-ways which were present consisted solely of factors whose main effects (responses with other factors held constant) were very important. All four of the three-way interactions between the variables AMPLITUD, THICK1, NFAIL and LAYER1 appeared among these significant pairs of three-way interactions. The fifth pair, whose sum of squares was considerably less than the other four, consisted of the three-way interaction of AMPLITUD, LAYER1, and LAYER3 confounded with that of AMPLITUD, COEFK2, and LAYER3. Since all of these variables and their interactions make good physical sense, it was decided that the effects being measured were real. Hence, these five terms were removed from the error pool and included among the variables which were to receive further analysis. On the basis of this corrected error pool, some 31 main effects and interactions, including these five confounded three-ways, were found to be significant at an α-level of .05. These terms and their F-ratios are listed in Table 9.

Regression Analysis

The 31 main effects and interactions which had been selected on the basis of the analysis of variance were further analyzed using the regression sub-program of the Statistical Package of the Social Sciences (SPSS). For purposes of the regression analysis, the high and low values of each variable were recoded to 1 and -1, respectively. Such

Table 9. Damage Index Analysis of Variance

Variable name(s)	Factor(s)	F ratio
AMPLITUD	A	29.93
THICK1	B	26.44
TEMPS	C	11.38
LAMBDA	D	5.28
NFAIL	G	39.26
LAYER1	H	20.27
$COEFK_2$	K	10.46
AMPLITUD, THICK1	AB	17.52
AMPLITUD, TEMPS	AC	6.63
AMPLITUD, NFAIL	AG	26.76
AMPLITUD, LAYER1	AH	13.6
AMPLITUD, $COEFK_2$	AK	6.72
THICK1, TEMPS	BC	6.57
THICK1, NFAIL	BG	23.83
THICK1, LAYER1	BH	11.42
THICK1, $COEFK_2$	BK	6.37
TEMPS, LAYER2	CF	6.12
TEMPS, NFAIL	CG	8.82
TEMPS, LAYER1	CH	4.84
LAMBDA, NFAIL	DG	4.43
LAYER2, $COEFK_2$	FK	5.49
NFAIL, LAYER1	GH	18.46
NFAIL, COEFK2	GK	9.39
LAYER1, COEFK2	HK	5.07
LAYER1, LAYER2	HL	4.14
$COEFK_2$, LAYER3	KL	7.78
(confounded three-ways)	ABG & HKL	16.32
(confounded three-ways)	ABH & GKL	7.98
(confounded three-ways)	AGH & BKL	12.77
(confounded three-ways)	AKL & BGH	10.91
(confounded three-ways)	BGK & AHL	5.96

error pool: degrees of freedom = 55
sum of squares = 7657183
mean square = 139222
$F_{.05}(1,55)$ = 4.02

a transformation results in an orthogonally coded system, which has the important property that addition of new variables to a regression model does not affect the coefficients on the terms which are already present.

It was impossible to achieve a good fit performing regression directly on the damage index itself. This was due to the skewed distribution of the dependent variable. Since damage index has been observed to approximate a log-normal distribution, it was decided to attempt regression on the log of this variable rather than its arithmetic value. A very good fit was obtained in this fashion. The model which has been selected uses only twelve terms plus a constant. It had an R^2 of .989 and a coefficient of variation of 7.5%. Every term in the model is significant at an α-level of .001 on the basis of its F-ratio. Furthermore, the standard deviations of the coefficients are sufficiently small to permit meaningful comparisons between the terms on the basis of magnitudes of their coefficients.

The resulting equation for Damage Index is:

$$\begin{aligned}\text{Log (DI)} \simeq\ & 1.35 + .38\ \text{AMPLITUD} - .34\ \text{THICK1} + .28\ \text{LAYER1} \\ & -.54\ \text{NFAIL} + .19\ \text{COEFK2} + .18\ \text{LAMBDA} \\ & -.34\ \text{TEMPS} - .12\ \text{AMPLITUD} \cdot \text{NFAIL} \\ & +.10\ \text{THICK1} \cdot \text{NFAIL} - .089\ \text{TEMPS} \cdot \text{NFAIL} \\ & -.056\ \text{NFAIL} \cdot \text{COEFK2} - .086\ \text{NFAIL} \cdot \text{LAYER1} \qquad (1)\end{aligned}$$

and:

$$\text{DI} = 10^{\text{Log (DI)}} \qquad (2)$$

where all independent variables are on a scale from -1 to 1.

The regression equation for DI above may be manipulated directly to study the response of DI to the various levels (low, mean, high) of the independent variables and the significant two-way interactions included.

It should be noted that only seven of the eleven original independent variables were found to be significant and appear in the equation, and that each of the five significant two-way interactions included NFAIL (Fatigue Life Potential).

It is especially convenient that independent variables at their mean values, which implies a value of zero in the orthogonally-coded equation, fall out of the equation, leaving the effects of independent variables at their low or high values as the basis for calculating DI. As the low value of any variable is -1 and the high value is 1, the variables themselves only serve in Equation (2) to control the signs for the regression coefficients. This allows ready consideration of a single main effect or any combination of main effects and interactions.

For example, consideration of the effect of varying AMPLITUD from its low to high values while holding all other terms at their mean values yields from Equation (2):

$$\Delta DI(AMPLITUD) \doteq 10^{(1.35 + .38 + 0 + \ldots + 0)} - 10^{(1.35 - .38 + 0 + \ldots + 0)}$$

$$\simeq 53.70 - 9.33 = 44.37$$

As a further illustration and dropping the zero terms, the very important variable NFAIL may be added at its high or low levels. Calculations for NFAIL at a high level are:

$$\Delta DI(AMPLITUD) \simeq 10^{(1.35 + .38 - .54 - .12)} - 10^{(1.35 - .38 - .54 + .12)}$$

$$\simeq 11.7 - 3.5 \simeq 8.2$$

Similarly, the effects of any combination of low, mean and high levels of the variables may be studied.

Such studies have been conducted to assess the nature and magnitude of sensitivity for each of the significant main effects and two-way interactions.

Sensitivity Rankings

There are a number of possibilities for arriving at the levels of sensitivity of cracking damage to the seven significant main effects and five significant two-way interactions. It is necessary to consider several of these to obtain the desired insight to sufficiently explain the calculated response of cracking damage to variations in the significant independent variables.

Ranking by Magnitudes of Regression Coefficients

The simplest and most obvious means of ranking is by considering the coefficients of the terms in the orthogonally-coded Equation (1). The magnitudes of these coefficients indicate the effects of the independent variables or separate two-way interactions on the logarithm of Damage Index. This provides a valid basis for comparison between terms because the values of each independent variable have been recoded to represent the difference from the mean in units of the standard deviation (or some equivalent transformation in the case of variables which are not distributed normally). Consequently the effects due to changing a term from -1 to 1 are the result of varying the input over the central 67% of its probability distribution. Each coefficient in the regression model is independent of the units and the absolute magnitude of the variable or pair of variables with which it is associated.

If the distribution of Damage Index were not approximately log-normal, the regression equation could have been in terms of DI instead of Log (DI). The regression coefficients would then have been more directly indicative. As DI = $10^{\text{Log (DI)}}$, the coefficients contribute as an exponent of 10 and their relative contribution is still of interest, and are plotted in Figure 2.

The coefficients fall in about four ranges of importance as listed below in order of high to lower importance:

1. NFAIL (or STRNCOEF, STRNEXP)
2. AMPLITUD
   THICK1
   TEMPS
   LAYER1
3. COEFK2
   LAMBDA
4. AMPLITUD · NFAIL
   THICK1 · NFAIL
   TEMPS · NFAIL
   NFAIL · LAYER1
   NFAIL · COEFK2

In summary, the relationship of initial strain to cycles-to-failure, called fatigue life potential, and input as (STRNCOEF, STRNEXP) is the most significant individual independent variable and it is included in all significant two-way interactions. The two-way interactions were all less significant than the separate independent variables or main effects.

Ranking by Average Effects

Another means of ranking is by averaging all the values of DI calculated from the 64 VESYS IIM solutions for a particular factor at its high level and the same for the other 64 solutions at its low level. The differences of these two averages are then called "average effects" and their magnitudes used to rank the factors. This means of ranking is independent of the analysis of variance and the multiple regression results.

The rankings by average effects appear in Table 8. It is of interest to note that these rankings are the same as those derived from the coefficients of the multiple regression equation, except that LAYER1 for this analysis ranks just higher than TEMPS instead of the reverse as for the multiple regression coefficient ranking.

Rankings by Main Effects (No Interactions)

The magnitudes of changes in DI as each independent variable in the

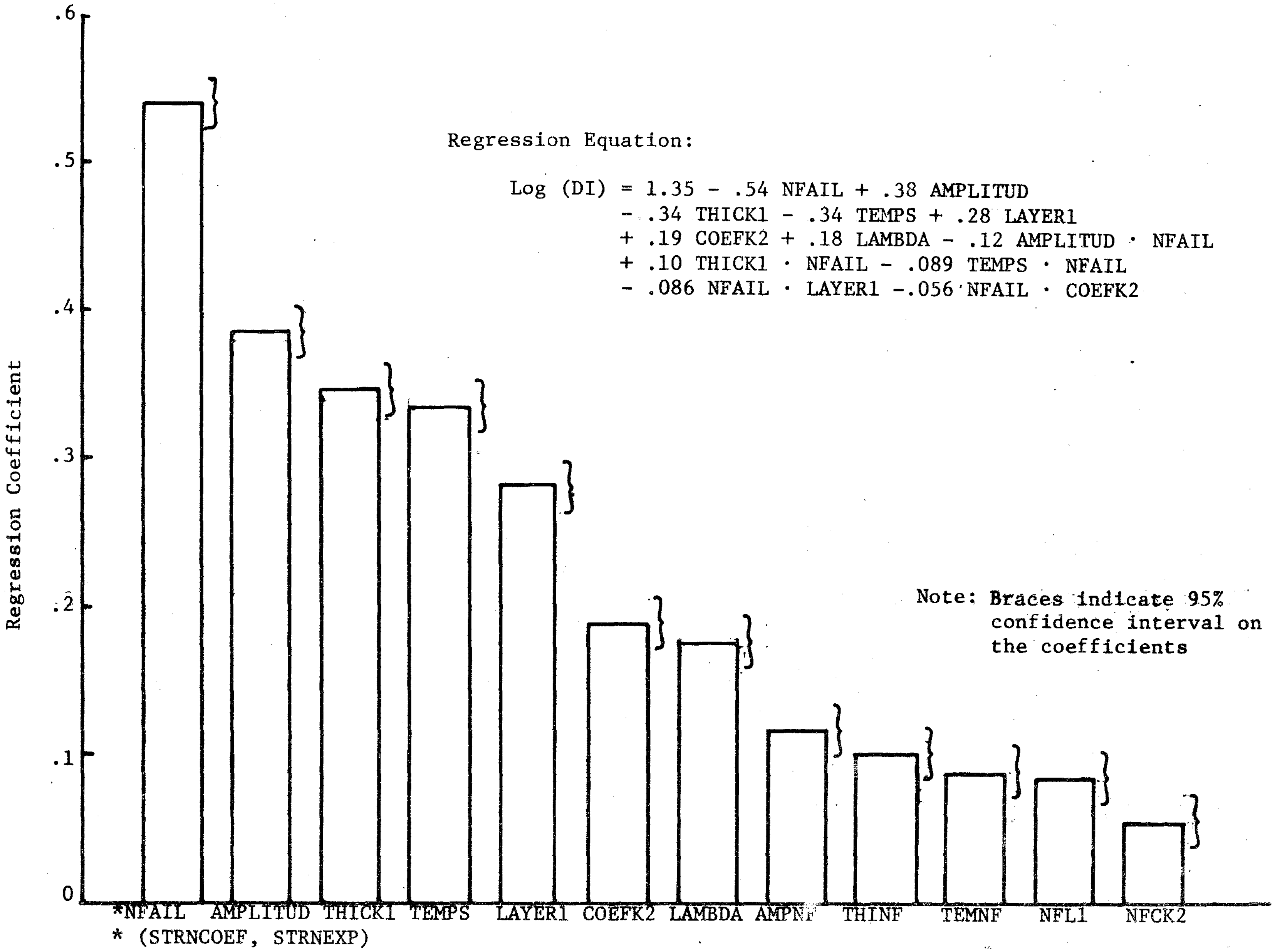

Figure 2. Plot of relative magnitudes of regression coefficients to indicate contribution of significant main effects and interactions.

regression model is varied from its low to high values with all other values at their means may be obtained from Equation (2) as previously discussed. These values, called main effects, are plotted in Figures 3, 4 and 5 in different manners to provide as much insight as possible.

In Figure 3, the main effects appear as the central of three bars for each factor. An arrow appears in each bar to indicate whether increasing the magnitude of the factor increased or decreased the Damage Index (an arrow pointing right indicates increasing DI as in the scale at the bottom of the plot).

Ignoring rankings for the moment, although the needed information is available in Figure 3, it can be seen from the arrows that increasing AMPLITUD, LAMBDA, LAYER1 or COEFK2 results in an increased DI or more cracking. Increasing THICK1, NFAIL or TEMPS decreases DI and consequently the predicted cracking. All of the phenomena described above are physically logical.

As fatigue life potential is so powerful a variable, it and its interaction with each factor has been included in Figure 3. The upper bar represents the main effect of the factor modified by NFAIL at its high level and its interaction with the factor. The lower bar represents the same except with NFAIL at its low level. Note the overpowering effect of NFAIL. When at its high level, DI is dramatically decreased and vice-versa.

Because of the relative importance of NFAIL, its main effect has been plotted in Figure 4 as the central bar. The upper and lower bars then include the main effect for a specific factor at its high or low values, respectively, and its interaction with NFAIL as it is varied from low to high. The modifications to the main effect of NFAIL were very significant, indicating that changes in magnitude of both NFAIL and a second factor may be much more significant than a change in NFAIL alone.

As would be expected, interactions with high values of AMPLITUD, TEMPS, LAYER1 and COEFK2 increased DI, as did a low value of THICK1. The reverse was also true. Although the interaction with LAMBDA was not found to be significant and does not appear in the equation, the bars vary in length because the constant effect of LAMBDA in Equation(1) produces different effects on the antilog values from Equation (2) for high and low levels of NFAIL.

Having gleaned a considerable amount of insight from Figures 3 and 4 as to the nature of DI response to the seven factors and their significant interactions, the matter of rankings according to main effects may best be determined from Figure 5, which shows plots of change in Damage Index for each factor without interactions (bars denoted "NI").

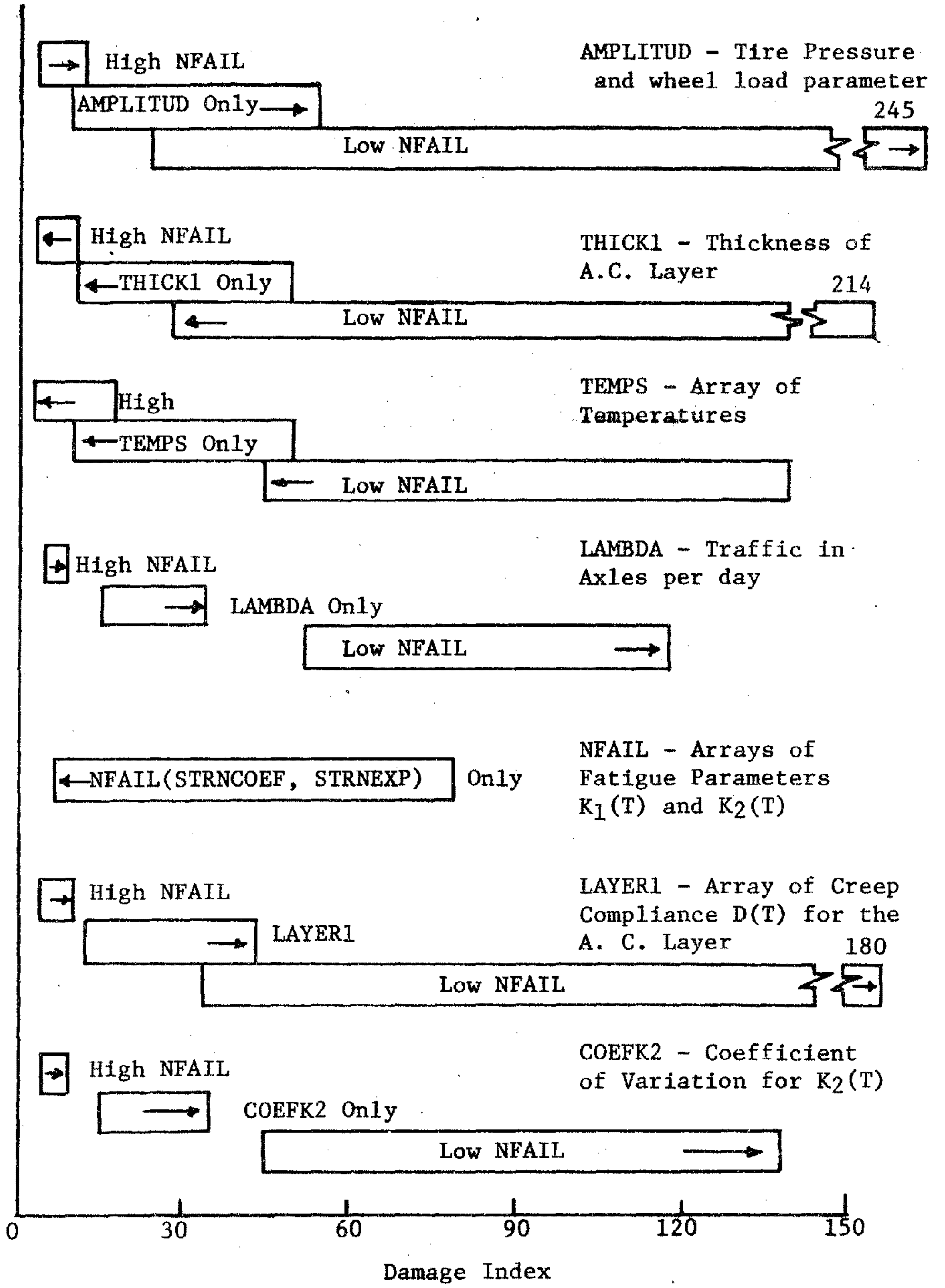

Figure 3. Change in Damage Index while each factor is varied from low to high levels without and with interaction of the factor with fatigue life potential NFAIL at its high and low levels.

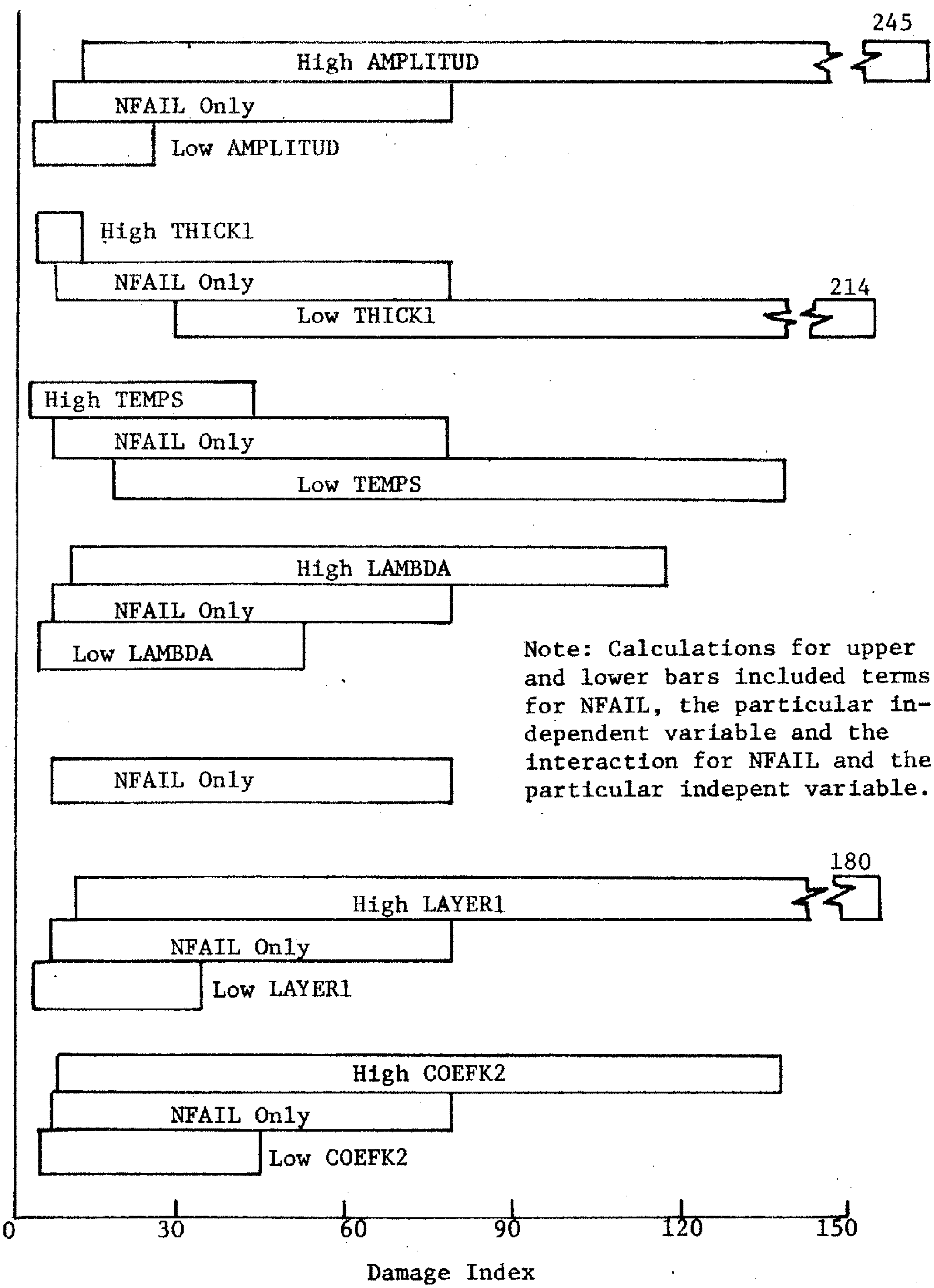

Figure 4. Change in Damage Index while "Fatigue Life Potential" NFAIL is varied from low to high without and with the interaction for each of the other independent variables individually at their low and high values.

The rankings appear in parentheses adjacent to the bars denoted "NI". As would be expected, these rankings are the same as those obtained from the regression coefficients.

Ranking by Span of Effects

Ranking by main effects omits the important effect of a factor's contribution through significant interactions. Consideration of the interactions as well as the main effects results in changes in the Damage Index that may be designated as "Span of effects". As the significant interactions are all with NFAIL, it is sufficient to plot the span of effects in Figure 5 for low and high levels of NFAIL for the factors included in the significant two-way interactions.

As NFAIL (STRNCOEF, STRNEXP) would have no interaction plots with itself, this space in Figure 5 was used to show the maximum span of effect for NFAIL with all variables at whichever combination of levels that would make the largest change in D.I. (ΔDI). Maximization of ΔDI requires a low level of TEMPS and THICK1 and high levels of the other factors. Substituting these values into Equation (1) we get:

$$\begin{aligned}\log(DI) \approx\ & 1.35 + .38(1) - .34(-1) + .28(1) - .54(NFAIL) \\ & + .19(1) + .18(1) - .34(-1) - .12(1)(NFAIL) \\ & + .10(-1)(NFAIL) - .089(-1)(NFAIL) - .056\ (NFAIL)(1) \\ & - .086(NFAIL)(1)\end{aligned}$$

As a function of NFAIL, this yields the following span:

$$\begin{aligned}DI(\text{high NFAIL}) &\approx 10^{(1.35 + .38 + .34 + .28 - .54 + .19 + .18 + .34)} \\ &\quad \cdot 10^{(-.12 - .10 + .089 - .056 - .086)} \\ &\approx 177\end{aligned}$$

$$\begin{aligned}DI(\text{low NFAIL}) &\approx 10^{(1.35 + .38 + .34 + .28 + .54 + .19 + .18 + .34)} \\ &\quad \cdot 10^{(.12 + .10 - .089 + .056 + .086)} \\ &\approx 7464\end{aligned}$$

$$\begin{aligned}\Delta DI(NFAIL) &= DI(\text{high NFAIL}) - DI(\text{low NFAIL}) \\ &\approx 177 - 7464 = -7287\end{aligned}$$

By selecting a combination of factor levels to minimize Equation (2), a Damage Index of 0.297 is calculated to represent the best conditions. This indicates that about 30% of the traffic required to cause cracking failure had occurred. Under the worst conditions, the calculated damage index was 7464, indicating that the truck traffic experienced had been 7464 times that required for cracking failure.

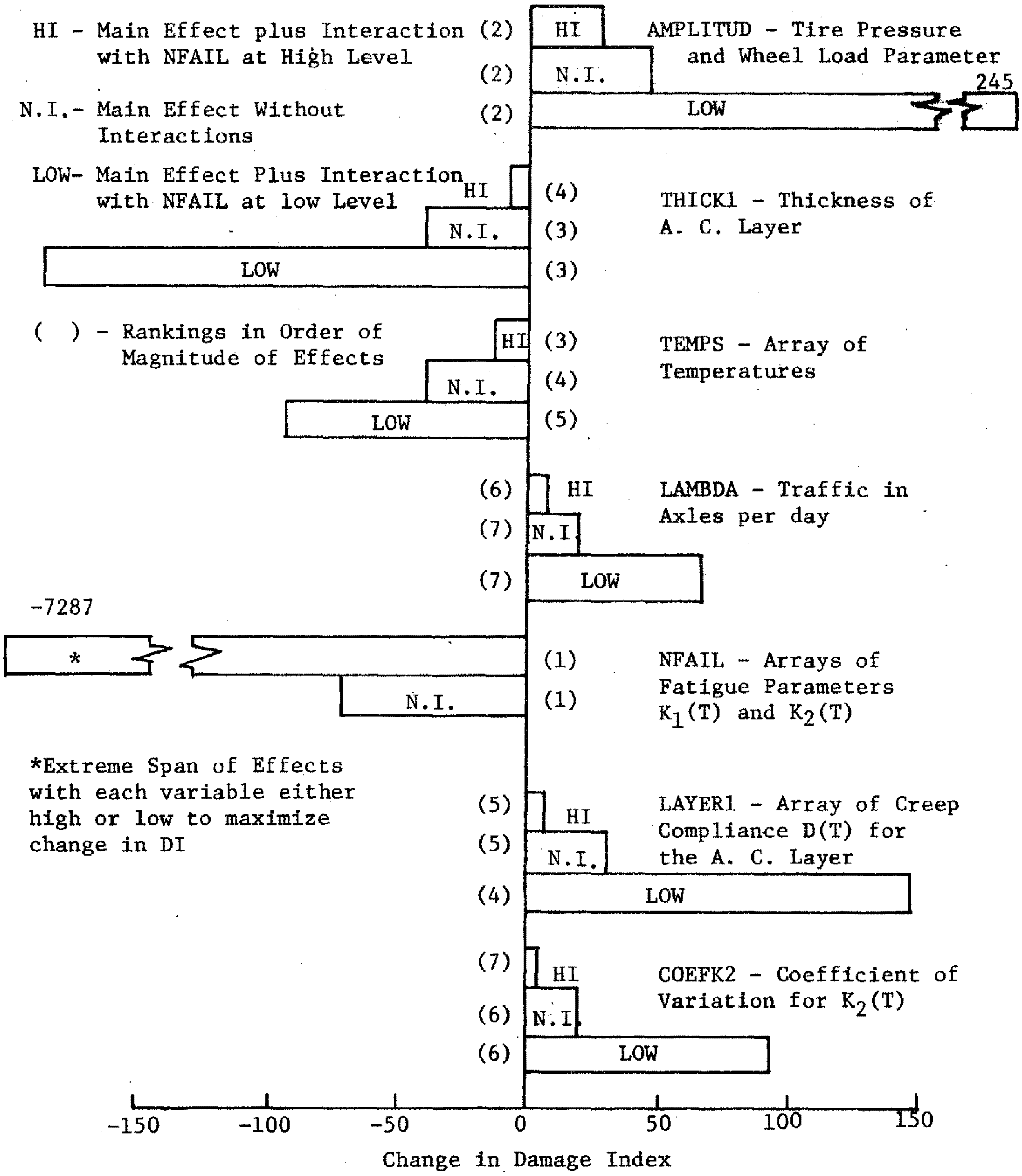

Figure 5. Span of effects for each factor with the fatigue life potential NFAIL low, with no interactions with NFAIL and with NFAIL high.

The "Hi" and "Low" bars above and below the main effect bars labeled "NI" in Figure 5 show the effects of interactions with NFAIL at high and low levels, respectively. Interactions with the high level of NFAIL always served to decrease DI, except for the interaction THICK1 · NFAIL. As for the main effects (bar labeled "NI"), the rankings of span of effects at low and high levels of NFAIL appear in parentheses adjacent to the appropriate bars.

Comparing the three rankings for each factor separately, it is seen that there are differences between rankings for main effects and span of effects and between span of effects at low and high values of NFAIL. This is to be expected as the two-way interactions can considerably affect results due to the exponential nature of the Equation (2).

For convenience, the rankings by all methods of analysis are included in Table 10 and the calculated variations in DI as each factor is varied from low to high (basis for rankings) appear in Table 11. Comparing the rankings using span of effects, it can be seen that for low NFAIL the rankings were the same as for "average main effects" and varied only from the "main effects" ranking in the importance of LAYER1 and TEMPS. Review of the values in Table 11 indicates the effect of LAYER1 to be much the stronger.

For a high level of NFAIL (implies high fatigue life potential), TEMPS became sufficiently important to rank just below AMPLITUD, and LAMBDA and COEFK2 reversed their relative positions from those applying for the other rankings.

Summary Analysis for Cracking Damage

There can be little question as to the primary importance of fatigue life potential, NFAIL, which includes "coupled" values of $K_1(T)$ and $K_2(T)$, as it is physically realistic and supported by all methods of ranking.

AMPLITUD, which includes tire pressure and wheel load, appears to be next in importance. The thickness of the first layer, THICK1, and its creep compliance characterization, LAYER1, follow in that order. TEMPS will either follow or lead these two factors in importance according to level of NFAIL, ranging apparently from fifth to third as NFAIL increases.

The coefficient of variation of the exponent $K_2(T)$, called COEFK2, follows in ranking due to its importance in the stochastic formulation for expected damage index. Truck traffic has the least importance of any of the significant main effects (except in the case of high NFAIL) and has no significant interaction.

Table 10. Comparisons of sensitivity rankings for expected damage index derived from different methods of sensitivity analysis.

Multiple Regression Coefficients	Average Main Effects	Main Effects	Span of Effects: Low NFAIL	Span of Effects: High NFAIL
NFAIL	NFAIL	NFAIL	NFAIL	NFAIL
AMPLITUD	AMPLITUD	AMPLITUD	AMPLITUD	AMPLITUD
THICK1	THICK1	THICK1	THICK1	TEMPS
TEMPS	LAYER1	TEMPS	LAYER1	THICK1
LAYER1	TEMPS	LAYER1	TEMPS	LAYER1
COEFK2	COEFK2	COEFK2	COEFK2	LAMBDA
LAMBDA	LAMBDA	LAMBDA	LAMBDA	COEFK2
AMPLITUD · NFAIL				
THICK1 · NFAIL				
TEMPS · NFAIL				
NFAIL · LAYER1				
NFAIL · COEFK2				

Table 11. Calculated variations in damage index from different methods of sensitivity analysis, variations of each factor from low to high.

Variable or Interation	Regression* Coefficients	Average Main Effects	Main Effects	Span of Effects Low NFAIL	Span of Effects High NFAIL
NFAIL	-.54	-413	-71	-7287**	
AMPLITUD	.38	361	44	245	25
THICK1	.34	-339	-39	-186	-8
TEMPS	.34	-222	-39	- 95	-15
LAYER1	.28	312	31	147	6
COEFK2	.19	213	20	93	4
LAMBDA	.18	152	19	66	6
AMPLITUD · NFAIL	.12				
THICK1 · NFAIL	.10				
TEMPS · NFAIL	.09				
NFAIL · LAYER1	.09				
NFAIL · COEFK2	.06				

*The regression coefficients are from the model for Log DI. They are meaningful for ranking because the antilog function is strictly increasing (i.e., $x > y$ inplies $10^x > 10^y$).

**Span of effects for NFAIL calculated with each other variable at its high or low values to maximize change in D.I.

Described differently for additional clarity, the ranking is in terms of function as follows:

1. Relative susceptibility of the A.C. surface layer to fatigue cracking as indicated by fatigue life potential, NFAIL.

2. Those factors controlling magnitude of horizontal strain: AMPLITUD, THICK1, LAYER1 and TEMPS. TEMPS affects load cycles to cracking failure predicted by NFAIL as well as the magnitude of creep compliance.

3. COEFK2 - Stochastic variation of the important fatigue exponent $K_2(T)$ used in fatigue characterization of the A.C. surface layer material.

4. LAMBDA - Truck traffic in axles per day defining the load cycles experienced at any point in time.

CHAPTER V

SENSITIVITY ANALYSIS FOR RUT DEPTH

The independent variables determined from the preliminary sensitivity analysis to significantly affect the prediction of rut depth in the surface layer were:

1. Permanent deformation parameters ALPHA(1), GNU(1) and ALPHA(3) for the A.C. Surface layer and subgrade, respectively.
2. AMPLITUD, representing tire pressure and wheel load data.
3. LAMBDA, truck traffic in axles per day.
4. Creep compliance vectors for the A.C. surface layer, base layer and subgrade, called LAYER1, LAYER2 and LAYER3, respectively.
5. Thicknesses of the A.C. and base layers called THICK1 and THICK2, respectively.
6. The array of average monthly temperatures TEMPS.
7. CORLEXP, the value C in the exponent for the system's spatial auto correlation function, the second partial derivative of which is used to get an estimate for slope variance in terms of the variance for rut depth.
8. The variances of LAYER1, LAYER2, and LAYER3, called VARCOEF1, VARCOEF2 and VARCOEF3, respectively.

Using the techniques for fractional factorials discussed in Chapter 2 and Appendix A, a 1/128 replicate was selected for the 2^{15} factorial resulting from assignment of two levels for each of the fifteen independent variables considered. The 256 combinations of independent variables selected to minimize loss of information are identified in Appendix C. The confounding scheme allowed sensitivity evaluation of all main effects, two-way interactions and selected three-way interactions.

The 256 separate solutions were obtained using VESYS II (M) and the calculated results appear in Appendix C. Actual low and high values used for the independent variables appear in Tables 6, 7 and 12 and generally represent one standard deviation either side of the mean value, or the equivalent where a normal distribution failed significantly to represent the actual distribution.

Statistical Analysis

Analysis of Variance

Rut depth response from the 15-variable roughness factorial was analyzed using the factorial analysis of variance program FAØV-01. An initial error pool was constructed using all terms containing only confounded three-ways and higher-order interactions. A few of the 119 terms in this group had higher mean squares than the rest. The two

Table 12. Summary of average effects* (sensitivity of rut depth) roughness factorial

Independent Variables	Low-Level Value	High-Level Value	Average Effects* on Rut Depth (Including Interactions)	Average Effects* as Percent of Overall Means of Rut Depth (0.557)	Ranking Based on Average Effects
ALPHA(1), Permanent Deformation Parameter for Surface Layer	0.68 (Also .75)	0.82	-.402	-72	1
AMPLITUD, Wheel Load Pressure in PSI	58	95	.135	24	2
GNU(1), Permanent Deformation Parameter for Surface Layer	0.20	0.60	.124	22.3	3
LAMBDA, Truck Traffic in Axles/Day	2000	4500	.100	18.0	4
ALPHA(3), Permanent Deformation Parameter for Subgrade	0.69	0.94	-.0677	-12.2	5
LAYER3, Creep Compliance Array for Subgrade, PSI^{-1}	(See Table 7)		.0657	11.8	6
LAYER1, Creep Compliance Array for Surface Layer, PSI^{-1}	(See Table 7)		.0407	7.3	7

Table 12. Summary of average effects* (sensitivity of rut depth) roughness factorial (cont.)

Independent Variables	Low-Level Value	High-Level Value	Average Effects* on Rut Depth (Including Interactions)	Average Effects* as Percent of Overall Means of Rut Depth (0.557)	Ranking Based on Average Effects
LAYER2, Creep Compliance Array for Base Material, PSI^{-1}	(See Table 7)		.0265	4.8	8
THICK2, Thickness of Base Layer in Inches	15.0	21.0	-.0177	-3.2	9
TEMPS	(See Table 6)		.0173	3.1	10
THICK1, Thickness of Surface Layer in Inches	3.5	5.0	-.0123	-2.2	11
VARCOEF1, Coefficient of Variation of creep compliance for the A.C. Surface Layer.	0.1	0.3	-.003	-0.5	12
VARCOEF2, Coefficient of Variation of creep compliance for the Base Material	0.1	0.3	-.001	-0.2	13
VARCOEF3, Coefficient of Variation of Creep Compliance for the subgrade	.25	.40	.001	0.2	14

Table 12. Summary of average effects* (sensitivity of rut depth) roughness factorial (cont.)

Independent Variables	Low-Level Value	High-Level Value	Average Effects* on Rut Depth (Including Interactions)	Average Effects* as Percent of Overall Means of Rut Depth (0.557)	Ranking Based on Average Effects
CORLEXP, the Value C in the exponent for the system's spatial auto correlation function	.044	.072	0	0	15

*The average effect is the difference between the average Rut Depth calculated for all solutions carried out at the low level of a factor and that of all solutions carried out at the high level.

terms which were significant at an α-level of .01 were deleted from the pool. This rather restricted cut-off criterion was adequate because only those terms which were significant at the .001 level were to be retained for the regression step. Table 13 lists the terms which were significant according to this analysis.

The two terms involving confounded three-way interactions which had been deleted from the initial error pool had F-ratios which were significant.

three-way interactions confounded	F-ratio
VARCOEF3, TEMPS, CORLEXP ALPHA(1), THICK1, AMPLITUD	42.71
LAYER3, ALPHA(3), THICK2 VARCOEF3, CORLEXP, VARCOEF2 LAYER2, LAYER1, GNU(1)	14.86

In each of these groups, the last three-way interaction seems reasonable from a physical standpoint. Therefore, these confounded terms were included in the regression analysis.

Regression Analysis

The data were recoded on a scale from -1 to 1. This did not result in a strictly orthogonal system because of the constrianed factor space involving ALPHA(1) and GNU(1). However, except for the correlation of .707 between these two terms, the orthogonality is maintained. This means that once these two main effects have been entered into the model, any further additions will not change the coefficients on the terms which are already present.

Stepwise regression was run using the Statistical Package for the Social Sciences (SPSS). Since both ALPHA(1) and GNU(1) entered the model at an early stage, there was no difficulty caused by their correlation. The first 25 terms to be introduced into the model were accepted. The next five were left out because their coefficients, though meaningful, were too small to be significant in the sensitivity rankings. These were:

term	coefficient
THICK1 · LAYER2	.00709
LAYER2 · AMPLITUD	.00672
LAMBDA · LAYER1	.00665
LAYER3 · THICK2	-.00663
ALPHA(3) · THICK1	.00627

Table 13. Significant terms in rut depth analysis of variance.

Variance Name	Factor	F-Ratio
LAYER 3	A	15778.1
LAMBDA	B	36878.3
ALPHA(3)	C	16783.9
LAYER2	D	2558.1
THICK2	E	1149.6
TEMPS	G	1099.4
ALPHA(1)	H	*
THICK1	J	555.1
LAYER1	K	6045.4
GNU(1)	L	*
AMPLITUD	O	66241.7
LAYER3 & LAMBDA	AB	581.0
LAYER3 & ALPHA3	AC	713.0
LAYER3 & LAYER2	AD	84.9
LAYER3 & THICK2	AE	160.9
LAYER3 & ALPHA(1)	AH	221.4
LAYER3 & LAYER1	AK	71.3
LAYER3 & GNU(1)	AL	11.4
LAYER3 & VARCOEF1	AM	15.6
LAYER3 & AMPLITUD	AO	779.0
LAMBDA & ALPHA(3)	BC	313.9
LAMBDA & LAYER2	BD	75.1
LAMBDA & THICK2	BE	50.7
LAMBDA & ALPHA(1)	BH	716
LAMBDA & LAYER1	BK	161.7
LAMBDA & VARCOEF1	BM	35.6
LAMBDA & AMPLITUD	BO	2091.6
ALPHA(3) & THICK2	CE	598.3
ALPHA(3) & TEMPS	CG	17.4
ALPHA(3) & THICK1	CJ	143.7
ALPHA(3) & LAYER1	CK	13.4
ALPHA(3) & VARCOEF1	CM	26.6
ALPHA(3) & AMPLITUD	CO	993.7
LAYER2 & THICK2	DE	13.4
LAYER2 & ALPHA(1)	DH	147.0
LAYER2 & THICK1	DJ	87.0
LAYER2 & AMPLITUD	DO	165.6
THICK2 & ALPHA(1)	EH	12.0
THICK2 & THICK1	EJ	38.6
THICK2 & AMPLITUD	EO	69.7
TEMPS & ALPHA(1)	GH	36.6
TEMPS & AMPLITUD	GO	41.9
ALPHA(1) THICK1	HJ	716.9
ALPHA(1) LAYER1	HK	366
ALPHA(1) GNU(1)	HL	*

Table 13. Significant terms in rut depth analysis of variance (cont.)

Variance Name	Factor	F-Ratio
ALPHA(1) & AMPLITUD	HO	2219.6
THICK1 & LAYER1	JK	183.1
THICK1 & GNU(1)	JL	44.6
THICK1 & AMPLITUD	JO	33.1
LAYER1 & GNU(1)	KL	20.7
LAYER1 & AMPLITUD	KO	355.7
GNU(1) & AMPLITUD	LO	119.6
LAMBDA & ALPHA & AMPLITUD	BHO	41.0
ALPHA1 & THICK1 & LAYER1	HJK	63.7
(confounded three-ways)	FGN & HJO	42.6
(confounded three-ways)	ACE & FNP & DKL	14.9

*Note: ALPHA(1) and GNU(1) are not independent factors because of their constrained factor spaces. Hence these main effects and their interactions must be pooled for analysis of variance.

Pool of ALPHA(1), GNU(1) and ALPHA(1) & GNU(1)	H, L and HL	13166.4

error pool:

degrees of freedom = 117
sum of squares = .00822
mean square = .000070
$F_{.001}(117,1)$ = 11.41

The next two terms, ALPHA(1) · LAYER1 and LAYER1 · GNU(1), had larger coefficients and were included in the model. All of the remaining terms were left out.

The fit obtained using these 27 terms achieved an R^2 of .986 with a coefficient of variation of 5.3 percent. The standard deviations of the coefficients were small enough to permit meaningful comparisons between terms on the basis of the magnitudes of their coefficients. The model which has been selected is:

$$\begin{aligned} RD \simeq\ & .569 + .135\ AMPLITUD + .100\ LAMBDA \\ & - .0677\ ALPHA(3) + .0657\ LAYER3 - .201\ ALPHA(1) \\ & + .124\ GNU(1) + .0407\ LAYER1 + .0265\ LAYER2 \\ & + .0239\ LAMBDA \cdot AMPLITUD - .0177\ THICK2 - .0173\ TEMPS \\ & - .0165\ ALPHA(3) \cdot AMPLITUD + .0146\ LAYER3 \cdot AMPLITUD \\ & - .0140\ LAYER3 \cdot ALPHA(3) - .0493\ ALPHA(1) \cdot AMPLITUD \\ & + .0304\ GNU(1) \cdot AMPLITUD + .0128\ ALPHA(3) \cdot THICK2 \\ & + .0127\ LAYER3 \cdot LAMBDA - .0123\ THICK1 \\ & - .0245\ ALPHA(1) \cdot GNU(1) + .00986\ LAYER1 \cdot AMPLITUD \\ & - .0134\ LAMBDA \cdot ALPHA(1) - .00926\ LAMBDA \cdot ALPHA(3) \\ & - .0280\ ALPHA(1) \cdot THICK1 + .0175\ THICK1 \cdot GNU(1) \\ & - .0200\ ALPHA(1) \cdot LAYER1 + .0124\ LAYER1 \cdot GNU(1) \end{aligned} \quad (3)$$

Sensitivity Rankings

The same methods of ranking used for the cracking damage sensitivity analysis and described in Chapter IV have also been used to arrive at the sensitivity of rut depth to the eleven significant main effects and sixteen significant two-way interactions.

Rankings by Magnitudes of Regression Coefficients

The magnitudes of the regression coefficients for the eleven main effects included in the multiple regression model are exactly one-half of the values appearing as "Main Effects" in Table 14 and the rankings are the same as for "Main Effects." These rankings appear in Table 15. The coefficients of the sixteen interaction terms are also included in Table I for comparison with the main effects.

Rankings by Average Effects

The rankings by average effects, which are independent of the analysis of variance and multiple regression results, are also listed in Table 15. Notice that the rankings are generally the same as for "regression coefficients" and "main effects", except that the constrained factor space for ALPHA(1) and GNU(1) makes the ranking of these variables inaccurate by this method.

Table 14. Calculated variations in rut depth from different methods of sensitivity analysis, variations of each factor from low to high

Variable	Regression Coefficients	Average Effects	Main # Effects	Span of Effects
ALPHA (1)	-.201	-.402*	-.402	.672
AMPLITUD	.135	.269	.270	.560
GNU(1)	.124	.047**	.248	.418
LAMBDA	.100	.201	.200	.319
ALPHA(3)	-.066	-.135	-.135	.241
LAYER3	.166	.131	.133	.214
LAYER1	.041	.081	.081	.166
LAYER2	.027	.053	.052	.053
THICK2	-.018	-.035	-.035	.061
TEMPS	.017	.035	.034	.035
THICK1	.012	-.025	-.024	.115
LAMBDA · AMPLITUD	.024			
ALPHA(3) · AMPLITUD	-.017			
LAYER3 · AMPLITUD	.015			
LAYER3 · ALPHA(3)	-.014			
ALPHA(1) · AMPLITUD	-.049			
GNU(1)· AMPLITUD	.030			
ALPHA(3) · THICK2	.013			
LAYER3 · LAMBDA	.013			
ALPHA(1) ·GNU(1)	-.025			
LAYER1 · AMPLITUD	.010			
LAMBDA · ALPHA (1)	-.013			
LAMBDA · ALPHA (3)	-.009			
ALPHA(1) · THICK1	-.028			
THICK1 ·GNU(1)	.018			
ALPHA (1) · LAYER1	-.020			
LAYER1 ·GNU(1)	.012			

*Calculated average main effect was .201, but three levels of ALPHA(1), were considered rather than two, thus averaging was over one standard deviation instead of two. For oomparison, ALPHA(1) must be multiplied by two.

**ALPHA and GNU were applied as four coupled pairs in a constrained factor space. Two levels of GNU were used with three levels of ALPHA (1). Consequently, the meaning of the average main effect of this constrained factor is not clear and cannot be used for ranking.

#Note that magnitudes of change are equivalent to twice the applicable multiple regression coefficients.

Table 15. Comparisons of sensitivity rankings for expected rut depth derived from different methods of sensitivity analysis.

Multiple Regression Coefficients	Average Effects	Main Effects	Span of Effects
ALPHA(1)	ALPHA(1)	ALPHA(1)	ALPHA(1)
LAYER3	AMPLITUD	AMPLITUD	AMPLITUD
AMPLITUD	LAMBDA	GNU(1)	GNU(1)
GNU(1)	ALPHA(3)	LAMBDA	LAMBDA
LAMBDA	LAYER3	ALPHA(3)	ALPHA(3)
ALPHA(3)	LAYER1	LAYER3	LAYER3
ALPHA(1) · AMPLITUD	LAYER2**	LAYER1	LAYER1
LAYER1	GNU(1)	LAYER2	THICK1
GNU(1)· AMPLITUD	THICK2	THICK2	THICK2
ALPHA(1) · THICK1	TEMPS	TEMPS	LAYER2
LAYER2	THICK1	THICK1	TEMPS
ALPHA(1) · GNU(1)			
LAMBDA · AMPLITUD			
ALPHA(1) · LAYER1			
THICK2			
THICK1 · GNU(1)			
TEMPS			
ALPHA(3) · AMPLITUD			
LAYER3 · AMPLITUD			
LAYER3 · ALPHA(3)			
ALPHA(3) · THICK2			
LAYER3 · LAMBDA			
LAMBDA · ALPHA(1)			
THICK1			
LAYER1 · GNU(1)			
LAYER1 · AMPLITUD			
LAMBDA · ALPHA(3)			

** Magnitude of change for this average effect may not be used for ranking (See Table 14.)

In explanation, three levels of ALPHA(1) were considered with two levels of GNU(1) to produce four sets of "coupled" values instead of two levels for each variable independently. The coupled values were as follows:

1.	Low ALPHA(1) Low GNU(1)	(.68,.20)
2.	High ALPHA(1), Low GNU(1)	(.75,.20)
3.	Low ALPHA(1), High GHU(1)	(.75,.60)
4.	High ALPHA(1), High GNU(1)	(.82,.60)

This was done to more fully explore the significance of the very important "permanent deformation potential" of the asphaltic concrete surface layer. Also, values of ALPHA(1) and GNU(1) are not independent as they are developed in pairs to represent the growth of permanent deformations with load cycles. Consequently, averaging for ALPHA(1) was over one standard deviation (-1 to 0) instead of two standard deviations (-1 to +1) as for other variables, so multiplication by two clarified its ranking. However, GNU(1) had been used at two levels to combine with three levels of ALPHA(1), so the meaning of its average effects are unclear.

Rankings by Main Effects (No Interactions)

The changes in rut depth due to a single main effect may be obtained by multiplying the multiple regression coefficient for the main effect only from Equation (3) by two. This is illustrated for ALPHA(1) below:

$$RD \approx .569 - .201\ \text{ALPHA}(1)$$

$$\Delta RD(\text{ALPHA}(1)) \approx [\cancel{.569} - .201(1)] - [\cancel{.569} - .201(-1)]$$
$$\approx -.201 - .201$$
$$\approx 2(\text{Multiple Regression Coefficient})$$

Consequently, the rankings for multiple regression coefficients and main effects are identical.

The calculated variations in rut depth for the main effects, average effects, and span of effects are shown in Table 14. Note that the values (except for GNU(1) for reasons previously discussed) are almost identical for the separate independent variables whether arrived at by use of the multiple regression model or by averaging effects, which is independent of the multiple regression model. This adds to the confidence in the multiple regression model.

The calculated rut depths for a factor as its value increases from the low to high levels appear in Figure 6. The "main effect" for a factor entered in Table 14 is equal to the "length " of its bar in Figure 6. The arrows indicate whether rut depth decreases or increases as the factor increases in magnitude.

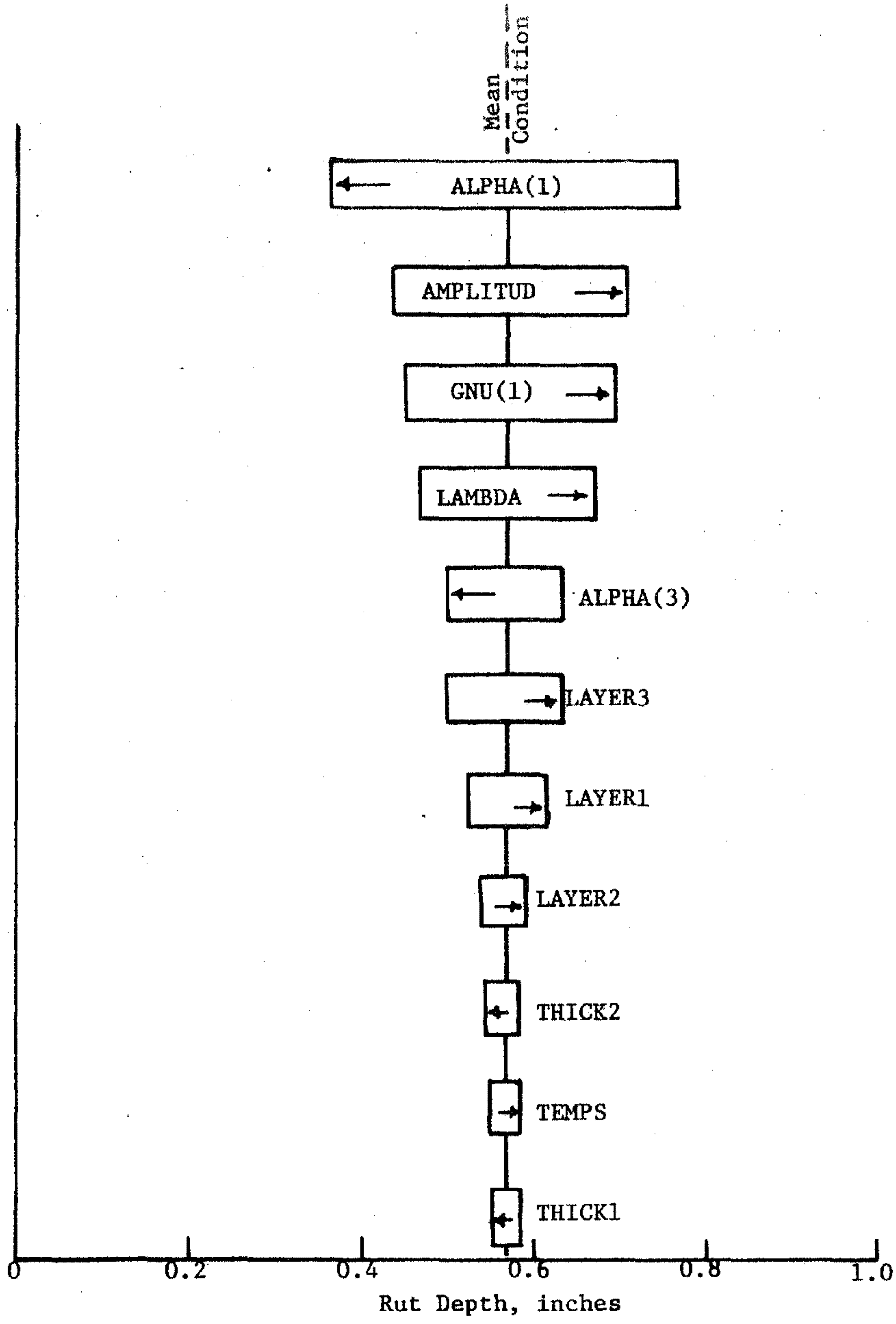

Figure 6. Change in rut depth while each factor is varied from low to high with all other factors at their means.

These plots show very clearly how much variation from the mean condition of .569 inches or rut depth may be introduced by varying each factor separately by one standard deviation either side of the mean.

The same information is plotted differently in Figure 7 to show only the change in rut depth caused by each factor. This plot also shows the direction of the changes as the magnitude of each factor increases and the ranking in parentheses for each factor next to the corresponding bar. Sensitivity rankings by "main effects" also appear in Table 15.

An interesting point to note from review of Table 14 is that a number of the two-way interactions in the multiple regression model represented by Equation (3) have more affect on rut depth than the less sensitive main effects. For instance, the interaction of the strong variables ALPHA(1) and AMPLITUD has a coefficient of -.049, which is larger than the coefficients of LAYER1, LAYER2, THICK2, TEMPS and THICK1. THICK1 is so weak by itself that it could almost be left out of the model, but it has fairly significant interactions with ALPHA(1) and GNU(1).

Ranking by Span of Effects

The addition of the interactions to the main effects analysis allows assessment of how each factor and all its interactions may affect the calculation or prediction of rut depth. The calculations are made such that all its interactions have the same sense as the main effect of interest so that the full range of possible effect is obtained. As illustration, the full span of effect is developed below for ALPHA(1):

$$\begin{aligned} RD \approx\ & .569 - .201\ ALPHA(1) - .0493\ ALPHA(1) \cdot AMPLITUD \\ & - .0245\ ALPHA(1) \cdot GNU(1) - .0134\ LAMBDA \cdot ALPHA(1) \\ & - .0280\ ALPHA(1) \cdot THICK1 - .0200\ ALPHA(1) \cdot LAYER1 \end{aligned}$$

$$\begin{aligned} \Delta RD(ALPHA(1)) \approx\ & [.569 - .201(1) - .0493(1)(1) - .0245(1)(1) \\ & - .0134(1)(1) - .028(1)(1) - .02(1)(1)] \\ & - [.569 - .201(-1) - .0493(-1)(1) - .0245(-1)(1) \\ & - .0134(-1)(1) - .028(-1)(1) - .02(-1)(1)] \\ \approx\ & -.672 \end{aligned}$$

$$\Delta RD(ALPHA(1) \approx 2\ [\text{Sum of absolute values of coefficients for ALPHA(1) and Interactions with ALPHA(1)}]$$

In this case, all factors having interactions with ALPHA(1) are at their high levels while ALPHA(1) changes from low to high. Such would not have been the case had the coefficients on the interaction terms not all been negative.

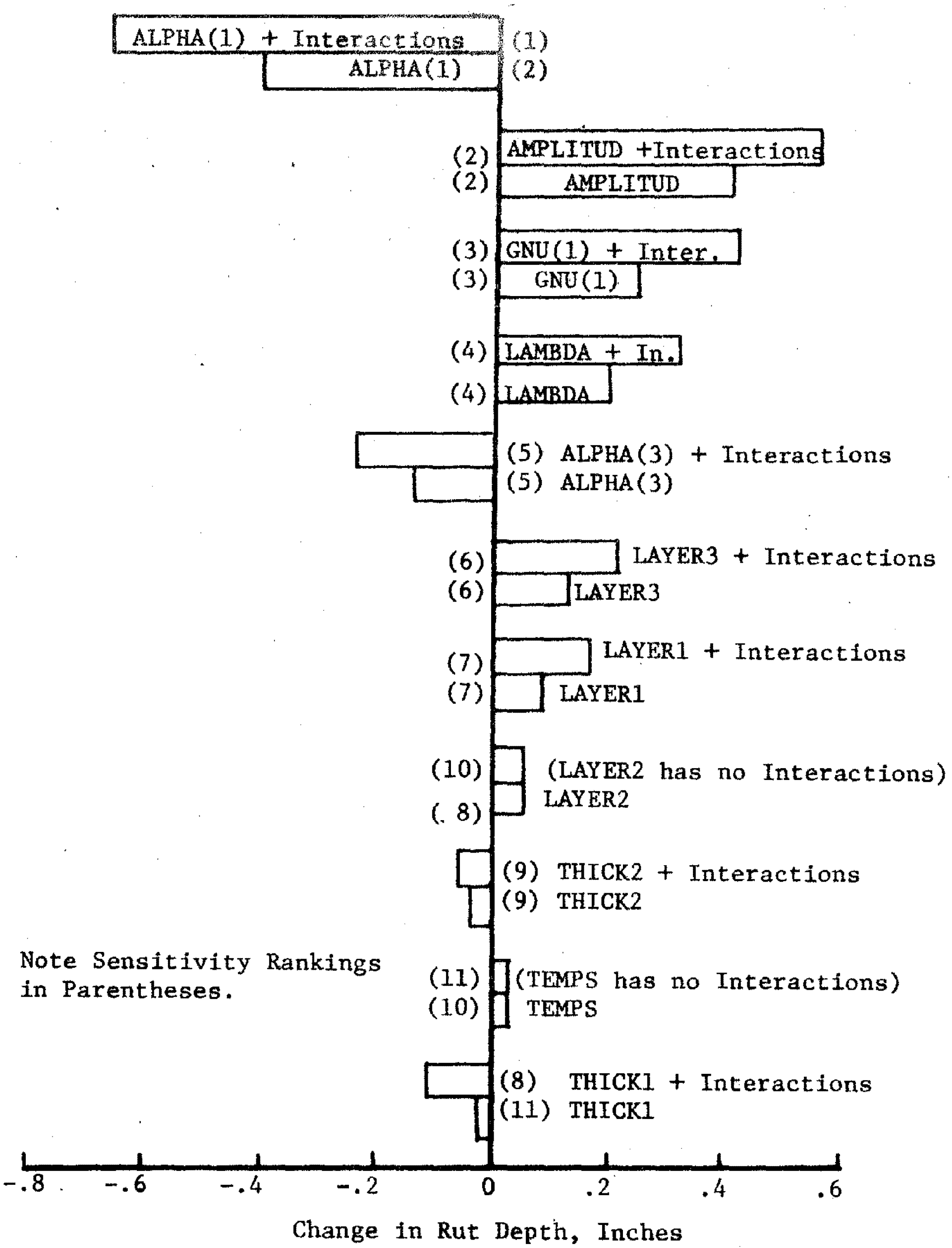

Figure 7. Span of effects for each factor compared to the main effects without interactions.

As for the main effects, the calculated span of effects appears in Table 14 and are plotted in Figure 7. It can be seen readily that the interactions may considerably expand the effects of the independent variables, but they may also reduce them as illustrated for ALPHA(1) below using the equation for rut depth above:

$$
\begin{aligned}
RD &\simeq .569 - .201(1) - .0493(1)(-1) \\
&\quad - .0245(1)(-1) - .0134(-1)(1) - .028(1)(-1) \\
&\quad - .0200(1)(-1) \\
&\simeq .503 \text{ inches}
\end{aligned}
$$

Note that increasing ALPHA(1) alone from its mean to high level would have decreased rut depth by 0.201 inches to .370 inches, but the occurrance of the interaction terms at their low levels limited the decrease to .066 inches to result in a rut depth of .503 inches.

A perhaps more interesting case would be THICK1 as illustrated below:

$$
\begin{aligned}
RD &\simeq .569 - .0123 \text{ THICK1} - .0280 \text{ ALPHA(1)} \cdot \text{THICK1} \\
&\quad + .01749 \text{ THICK1} \cdot \text{GNU(1)}
\end{aligned}
$$

As can be seen, increasing THICK1 from its mean to high level alone could only decrease rutting by .0123 inches while ALPHA(1) at its low level and GNU(1) at its high level through interactions with THICK1 may increase rutting by .0455 inches, thus more than cancelling out the effects from THICK1.

As the number of possible combinations of factor magnitude levels is large, to attempt to analyze all of them in relation to each other would be confusing and tend to obscure the insight possible from inclusion of the interaction terms. For this reason, total span of effects is considered more meaningful for sensitivity rankings.

The sensitivity rankings in terms of span of effects appear in Figure 7 and Table 15.

Summary Analysis for Rut Depth

Comparing the rankings by the several methods listed in Table 15, it can be seen that agreement is complete for the more significant factors ALPHA(1), AMPLITUD, GNU(1), LAMBDA, ALPHA(3), LAYER3 and LAYER1. Due to its significant interactions with ALPHA(1) and GNU(1), THICK1 has risen in significance above LAYER2, THICK2 and TEMPS. Similarly, THICK2 has risen above LAYER2, which has no significant interactions.

The rankings translated into physical terms appear to suggest rather reasonably that permanent deformation potential of the asphaltic concrete surface layer, wheel loading, truck traffic and permanent deformation potential of the subgrade are very important, the stiffnesses of the materials have appreciable effect and layer thicknesses and temperature have only minor importance. While layer thicknesses may appear intuitively to be important for spreading the load and reducing vertical strain, it must be remembered that a thicker layer also has more "gauge length" for the permanent unit strains to accumulate.

The insignificance of temperature is more difficult to account for physically because it is an established fact that rutting increases at least in the asphaltic concrete as temperature increases. The probable reason why temperature ranked low is that its effects are not considered by VESYS II for varying ALPHA and GNU internal to the program as is done for LAYER1, creep compliance for the asphaltic concrete. Such consideration must be taken in the laboratory by using some "average" temperature value for testing that will simulate average surface layer temperatures in the field.

CHAPTER VI

SENSITIVITY ANALYSIS FOR SLOPE VARIANCE

The sensitivity analysis for slope variance was obtained directly from the calculated responses from the roughness factorial. The independent variables considered and the 1/128 replicate fractional factorial have been described previously in Chapter V. The values used for the independent variables appear in Tables 6, 7 and 12.

Statistical Analysis

Analysis of Variance

A factorial analysis of variance was performed on the slope variance responses from the 15-variable roughness factorial in exactly the same manner described in Chapter V for rut depth. The initial error pool of confounded three-ways and higher-order interactions contained three terms which were significant at an α-level of .01, and these were removed from the pool. All terms whose F-ratios were significant at an α-level of .001 are listed in Table 16. They were retained for the regression step of the analysis.

Two of the confounded three-ways which had been deleted from the original error pool turned out to have significant F-ratios. They were:

three-way interactions confounded	F-ratio
LAYER3, LAMBDA, LAYER2 VARCOEF3, CORLEXP, AMPLITUD TEMPS, ALPHA(1), THICK1	14.5
VARCOEF3, TEMPS, THICK1 ALPHA(1), CORLEXP, AMPLITUD	14.1

In each group the last three-way interaction could be expected to represent a genuine effect because the factors involved make good physical sense. The remaining interactions are not so easy to visualize, but the large F-ratios for the main effects of many of the factors involved suggest that they may contribute some to the magnitude of the response. Because these terms were logical, it was decided to retain them for the regression step, even though it seemed unlikely that they would be included in the final model.

Regression Analysis

The data were recoded on a scale from -1 to 1. As was explained

Table 16. Slope variance analysis of variance

Variable Name	Factor	F-Ratio
LAYER3	A	195.6
LAMBDA	B	409.9
ALPHA(3)	C	206.1
LAYER2	D	38.0
THICK2	E	70.9
VARCOEF3	F	471.1
TEMPS	G	12.5
ALPHA(1)	H	*
LAYER1	K	50.5
GNU(1)	L	*
VARCOEF1	M	30.1
CORLEXP	N	734.1
AMPLITUD	O	688.1
VARCOEF2	P	30.0
LAYER3 & LAMBDA	AB	22.3
LAYER3 & ALPHA(3)	AC	27.6
LAYER3 & VARCOEF3	AF	22.2
LAYER3 & ALPHA(1)	AH	16.9
LAYER3 & CORLEXP	AN	36.1
LAYER3 & AMPLITUD	AO	37.5
LAMBDA & ALPHA(3)	BC	21.2
LAMBDA & VARCOEF3	BF	35.5
LAMBDA & ALPHA(1)	BH	25.6
LAMBDA & VARCOEF1	BM	14.2
LAMBDA & CORLEXP	BN	64.9
LAMBDA & AMPLITUD	BO	77.0
ALPHA(3) & THICK2	CE	22.6
ALPHA(3) & VARCOEF3	CF	26.6
ALPHA(3) & CORLEXP	CN	41.1
ALPHA(3) & AMPLITUD	CO	44.7
THICK2 & VARCOEF3	EF	15.2
THICK2 & CORLEXP	EN	13.9
THICK2 & AMPLITUD	EO	17.4
VARCOEF3 & ALPHA(1)	FH	42.1
VARCOEF3 & CORLEXP	FN	79.1
VARCOEF3 & AMPLITUD	FO	89.0
ALPHA(1) & THICK1	HJ	15.5
ALPHA(1) & GNU(1)	HL	*
ALPHA(1) & CORLEXP	HN	70.0
ALPHA(1) & AMPLITUD	HO	81.8
CORLEXP & AMPLITUD	NO	139.6
(confounded three-ways)	FGJ & HNO	14.1
(confounded three-ways)	ABD & FNO & GHJ	14.5

Table 16. Slope variance analysis of variance (cont.)

Varible Name	Factor	F-Ratio

*Note: ALPHA(1) and GNU(1) are not independent factors because of their constrained factor spaces. Hence these main effects and their interaction must be pooled for analysis of variance.

Pool of ALPHA(1), GNU(1) and ALPHA(1) and GNU(1)	H, L and HL	153.54

error pool:

degrees of freedom = 116
sum of squares = 828.538
mean square = 7.14
$F_{.001}$ (116,1) = 11.42

in Chapter V, this did not result in a strictly orthogonal system because of the correlation between ALPHA(1) and GNU(1), but it was close enough for practical purposes. Stepwise regression was run using the Statistical Package for the Social Sciences.

Three different models were tried in an attempt to get the best fit. First the straightforward linear model involving the arithmetic value of slope variance was attempted, but the fit was too unstable, having a coefficient of variation around 45 percent (depending on the number of terms included). Regression on the log of slope variance yielded a more stable model, but only two of the interaction terms were significant enough to be included in the model. This was not considered believeable, furthermore it provided little information of use in the sensitivity rankings. Consequently a third model, involving the log of one plus slope variance was selected.

The 26 terms whose partial F-ratios were significant at an α-level of .01 were incorporated into the equation. Included in these were all of the main effects except THICK1 (which was eliminated in the analysis of variance). Five of the interactions in this equation also appeared in the equation for rut depth (see Chapter V). The seven which did not all include either CORLEXP or VARCOEF3, two factors which were very important for slope variance but did not affect rut depth at all. Table 17 shows the coefficients of the main effects as shown by each of the three models which were investigated. Note that the rankings produced are the same in all three cases except for THICK2 and LAYER2 in the linear model. This deviation is not considered significant because the confidence intervals on the coefficients in the linear model were quite wide and the terms which are interchanged differ only slightly.

The model which is used for the comparisons has an R squared of .983 and a coefficient of variation of 5.9%. The predictive equation is:

$$\begin{aligned}\log(1 + SV) \approx\ & .881 - .175\ \text{CORLEXP} + .165\ \text{AMPLITUD} \\ & + .138\ \text{VARCOEF3} + .131\ \text{LAMBDA} - .0887\ \text{ALPHA(3)} \\ & + .0844\ \text{LAYER3} - .264\ \text{ALPHA(1)} + .162\ \text{GNU(1)} \\ & + .0479\ \text{LAYER1} - .0458\ \text{THICK2} + .0368\ \text{VARCOEF2} \\ & + .0363\ \text{VARCOEF1} + .0362\ \text{LAYER2} + .0227\ \text{TEMPS} \\ & + .0131\ \text{ALPHA(3)} \cdot \text{THICK2} - .0129\ \text{CORLEXP} \cdot \text{AMPLITUD} \\ & - .0114\ \text{VARCOEF3} \cdot \text{CORLEXP} - .0109\ \text{LAMBDA} \cdot \text{CORLEXP} \\ & - .0107\ \text{LAYER3} \cdot \text{ALPHA(3)} + .0102\ \text{VARCOEF3} \cdot \text{AMPLITUD} \\ & - .0143\ \text{ALPHA(1)} \cdot \text{THICK1} + .0096\ \text{LAMBDA} \cdot \text{AMPLITUD} \\ & - .0092\ \text{THICK2} \cdot \text{VARCOEF3} - .0182\ \text{ALPHA(1)} \cdot \text{GNU(1)} \\ & + .0087\ \text{LAMBDA} \cdot \text{VARCOEF3} + .0086\ \text{ALPHA(1)} \cdot \text{CORLEXP} \qquad (4)\end{aligned}$$

Table 17. Comparison of coefficients in the three regression models considered.

Variable Name	Coefficient log(1 + SV)	log SV	SV
ALPHA(1)	-.264	-.326	-6.78
CORLEXP	-.175	-.214	-4.52
AMPLITUD	.165	.202	4.38
GNU(1)	.162	.200	4.30
VARCOEF3	.138	.170	3.63
LAMBDA	.131	.161	3.38
ALPHA(3)	-.0887	-.110	-2.40
LAYER3	.0844	.104	2.34
LAYER1	.0479	.0592	1.19
THICK2	-.0458	-.0556	-1.41
VARCOEF2	.0368	.0464	.915
VARCOEF1	.0363	.0457	.914
LAYER2	.0362	.0450	1.03
TEMPS	.0227	.0276	.589

Sensitivity Rankings

The same methods of ranking used for cracking damage and rut depth as described in Chapter IV have also been used to arrive at the sensitivity of slope variance to the fourteen significant main effects and twelve significant two-way interactions.

Rankings by Magnitudes of Regression Coefficients

The magnitudes of the regression coefficients for the fourteen main effects included in the multiple regression model appear in Table 18. Note that in this case all main effects were more significant than any of the interactions. The rankings are shown in Table 19. The coefficients of the twelve interaction terms are also included in Table 18 for comparison with the main effects.

Rankings by Average Effects

The rankings by average effects, which are independent of the analysis of variance and multiple regression results, are also listed in Table 19 and the changes in slope variance for each independent variable (basis for rankings) in Table 18. Notice that the rankings are generally the same as for "regression coefficients" and "main effects", except that the constrained factor space for ALPHA(1) and GNU(1) makes the ranking of GNU(1) inaccurate by this method. (This is explained in Chapter V). Also, LAYER2 was slightly more significant than VARCOEF2 and VARCOEF1 instead of the reverse in other methods of ranking.

Rankings by Main Effects (No Interactions)

The change in rut depth due to a single main effect is obtained by solving Equation (4) with all terms considered zero except for the constant and that one containing the independent variable of interest alone (no interaction terms). This is illustrated below for ALPHA(1):

$$\text{Log}(1 + SV) \approx .881 - .264\ \text{ALPHA}(1)$$

$$(1 + SV) \approx 10^{(.881 - .264\ \text{ALPHA}(1))}$$

$$\Delta SV(\text{ALPHA}(1)) \approx \left[10^{(.881 - .264(1))}-1\right] - \left[10^{(.881 - .264(-1))}-1\right]$$

$$\approx 4.14 - 1 - 13.96 + 1$$

$$\approx -9.82$$

The change in slope variance appearing in Table 18 under "Main Effects" is calculated similarly for each independent variable. As might

Table 18. Calculated variations in slope variance from different methods of sensitivity analysis, variations of each factor from low to high

Variable	Regression Coefficients	Average Effects	Main Effects	Span of Effects
ALPHA(1)	-.264	-13.56*	-9.82	-15.4
CORLEXP	-.175	- 9.05	-6.30	-13.8
AMPLITUD	.165	8.76	5.92	20.3
GNU(1)	.162	1.82**	5.80	11.9
VARCOEF3	.138	7.25	4.92	9.1
LAMBDA	.131	6.76	4.66	13.2
ALPHA(3)	-.089	-4.79	-3.13	-8.7
LAYER3	.084	4.67	2.97	4.1
LAYER1	.048	2.37	1.68	1.68
THICK2	-.046	-2.81	-1.61	-2.5
VARCOEF2	.037	1.83	1.29	1.29
VARCOEF1	.036	1.83	1.28	1.28
LAYER2	.036	2.06	1.26	1.26
TEMPS	.023	1.18	0.79	0.79
THICK1	0***	0.43	0***	.88
ALPHA(3) · THICK2	.013			
CORLEXP · AMPLITUD	-.013			
VARCOEF3 · CORLEXP	.011			
LAMBDA · CORLEXP	-.011			
LAYER3 · ALPHA(3)	-.011			
VARCOEF3 · AMPLITUD	.010			
ALPHA(1) · THICK1	-.014			
LAMBDA · AMPLITUD	.010			
THICK2 · VARCOEF3	-.009			
ALPHA(1) · GNU(1)	-.018			
LAMBDA · VARCOEF3	.009			
ALPHA(3) · CORLEXP	.009			

*Calculated Average Effect was -6.78, but three levels of ALPHA(1) were considered rather than two, thus averaging was over one standard deviation instead of two. For comparison, ALPHA(1) must be multiplied by two.

**ALPHA(1) and GNU(1) were applied as four coupled pairs in a constrained factor space. Two levels of GNU(1) were used with three levels of ALPHA(1). Consequently, the meaning of the average effect of this constrained factor is not clear and cannot be used for ranking.

***THICK1 does not appear as a main effect in the regression equation, but does have a significant interaction that produces changes in slope variance for "Average Effects" and "Span of Effects".

Table 19. Comparisons of sensitivity rankings for slope variance derived from different methods of sensitivity analysis

Multiple Regression Coefficients	Average Effects	Main Effects	Span of Effects
ALPHA(1)	ALPHA(1)	ALPHA(1)	AMPLITUD
CORLEXP	CORLEXP	CORLEXP	ALPHA(1)
AMPLITUD	AMPLITUD	AMPLITUD	CORLEXP
GNU(1)	VARCOEF3	GNU(1)	LAMBDA
VARCOEF3	LAMBDA	VARCOEF3	GNU(1)
LAMBDA	ALPHA(3)	LAMBDA	VARCOEF3
ALPHA(3)	LAYER3	ALPHA(3)	ALPHA(3)
LAYER3	LAYER1	LAYER3	LAYER3
LAYER1	THICK2	LAYER1	THICK2
THICK2	LAYER2	THICK2	LAYER1
VARCOEF2	VARCOEF2	VARCOEF2	VARCOEF2
VARCOEF1	VARCOEF1	VARCOEF1	VARCOEF1
LAYER2	GNU(1)*	LAYER2	LAYER2
TEMPS	TEMPS	TEMPS	THICK1
ALPHA(1) · GNU(1)	THICK1		TEMPS
ALPHA(1) · THICK1			
ALPHA(3) · THICK2			
CORLEXP · AMPLITUD			
VARCOEF3 · CORLEXP			
LAMBDA · CORLEXP			
LAYER3 · ALPHA(3)			
VARCOEF3 · AMPLITUD			
THICK2 · VARCOEF3			
LAMBDA · VARCOEF3			
ALPHA(3) · CORLEXP			

* Magnitude of change for this "average effect" may not be used for ranking (See Table 18 for explanation).

be expected, these rankings are identical with those for the regression coefficients.

The calculated values of slope variance for a factor as its value increases from the low to high levels appear in Figure 8. The arrow indicates whether slope variance decreases or increases as the factor increases in magnitude. These plots show very clearly how much variation from the mean condition of 6.60 radians x 10^6 of slope variance may be introduced by varying each factor separately by one standard deviation either side of the mean.

The same information is plotted differently in Figure 9 to show only the change in slope variance caused by each factor. This plot also shows the direction of the changes as the magnitude of each factor increases and the ranking in parentheses for each factor next to the corresponding bar. Sensitivity rankings by "main effects" also appear in Table 19.

It is interesting to note that the main effect THICK1 was not significant and does not appear in Equation (4), but its interaction with ALPHA(1) was the second most significant interaction and was included in the regression model.

Ranking by Span of Effects

The addition of the interactions to the main effects analysis allows assessment of how each factor and all its interactions may affect the calculation or prediction of slope variance. The calculations are made such that all interactions have the same sense as the main effect of interest so that the full range of possible effect is obtained. As an illustration, the full span of effect is developed below for ALPHA(1):

$$\text{Log}(1 + \text{SV}) \simeq .881 - .264\ \text{ALPHA}(1) - .0143\ \text{ALPHA}(1) \cdot \text{THICK1} - .0182\ \text{ALPHA}(1) \cdot \text{GNU}(1) + .162\ \text{GNU}(1)$$

To maximize the span we set GNU(1) and THICK1 to their high levels and compute the difference caused by varying ALPHA(1) from high to low.

$$\Delta\text{SV}(\text{ALPHA}(1)) \simeq \left(10^{[.881 - .264(1) - .0143(1)(1) - .0182(1)(1) + .162(1)]} - 1\right) - \left(10^{[.881 - .264(-1) - .0143(-1)(1) - .0182(-1)(1) + .162(1)]} - 1\right)$$

$$\simeq 4.77 - 20.13 = -15.4$$

The calculated span of effects appear in Table 18 and Figure 9. The sensitivity rankings in terms of span of effects appear in Figure 9 and Table 19.

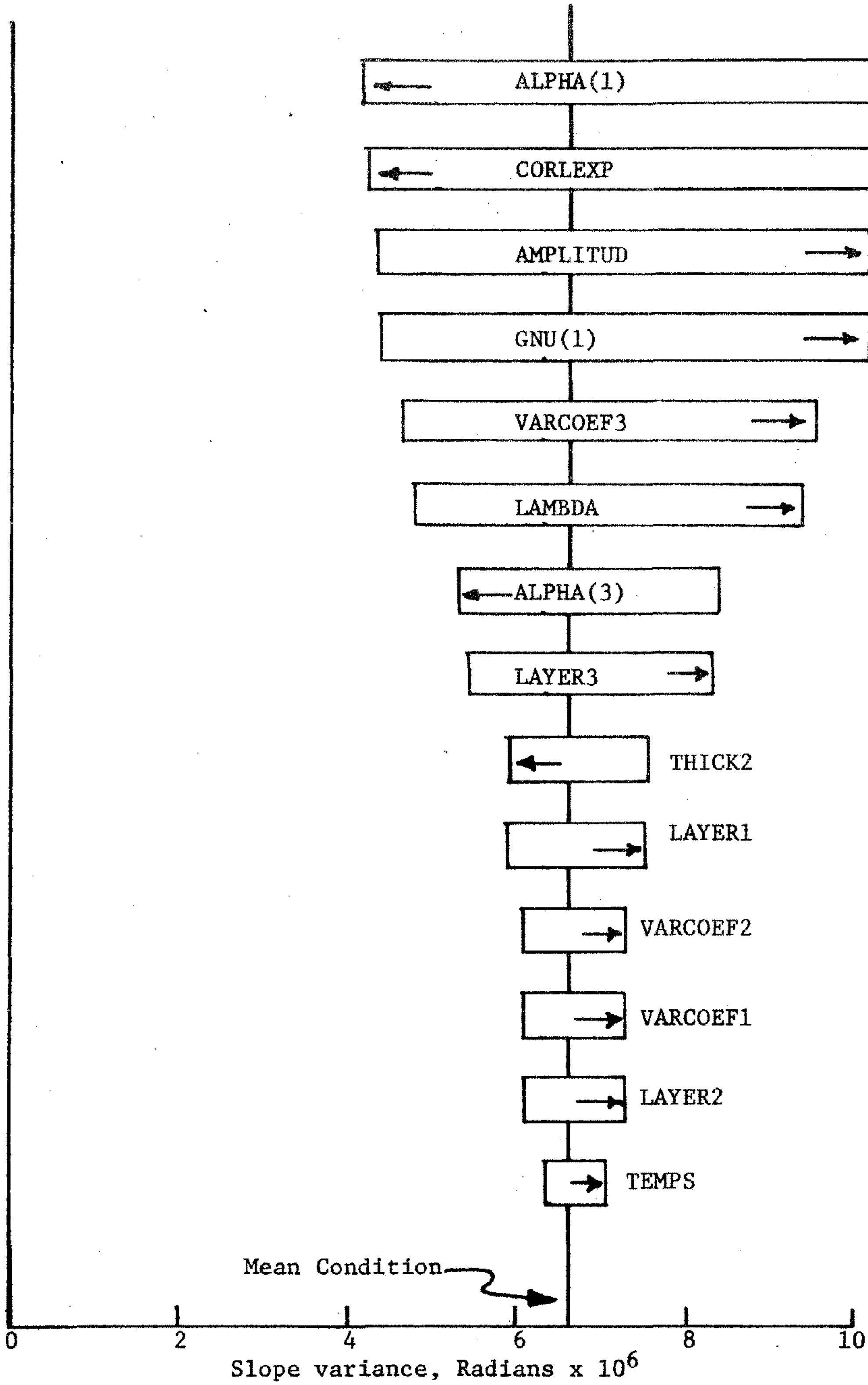

Figure 8. Change in slope variance while each factor is varied from low to high levels with all other factors at their means.

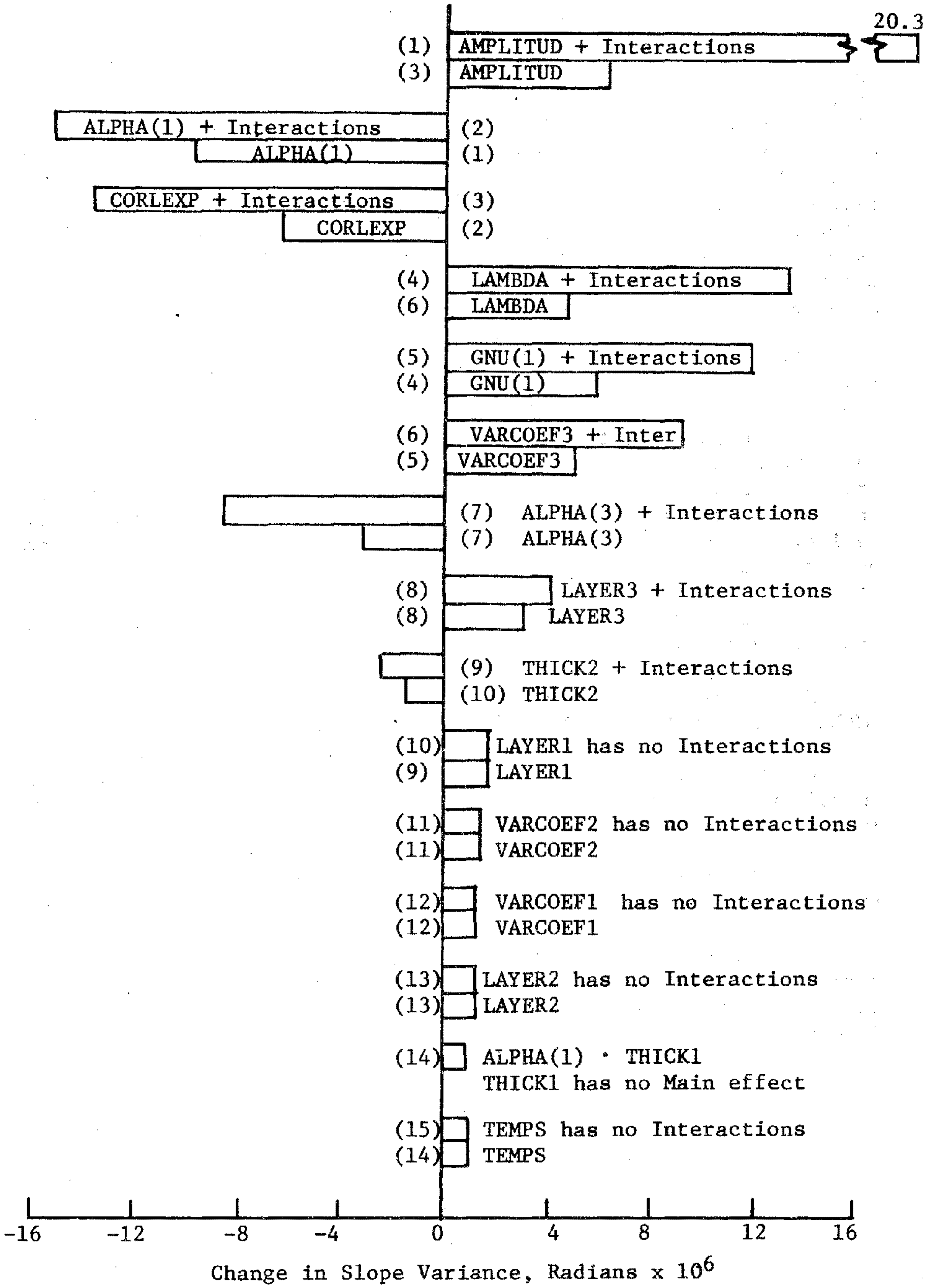

Figure 9. Span of effects for each main effect compared to the main effects with interactions.

Summary Analysis for Slope Variance

The rankings in Table 19 for "main effects" and "average effects" agree for all factors except GNU(1) and LAYER2. As discussed previously, GNU(1) is not measureable using average effects because of its constrained factor space. LAYER2 had a somewhat larger average effect than main effect, and this was sufficient to place it slightly above VARCOEF1 and VARCOEF2 in that ranking; but the differences between these terms are probably not measurable. Looking at "span of effects", we see that in extreme circumstances AMPLITUD and LAMBDA can have considerably greater significance due to their interactions with other strong factors. THICK2 exhibits some increase in importance when ALPHA(3) is at a low level.

The rankings translated into physical terms appear to suggest that:

1. The permanent deformation characteristics of the A.C. surface layer is quite important, while those for the subgrade are fairly important and those for the base materials are not very important. The lack of importance of the base material in this case is because a well-compacted base generally has little relative permanent deformation potential unless badly overstressed.

2. CORLEXP is important in estimation of the variation in rut depth with distance along the wheel path, so it is very significant.

3. Truck traffic LAMBDA and the wheel load representation AMPLITUD are quite important for obvious reasons.

4. The stiffness of the subgrade is significant and its variation in stiffness is even more important as it contributes to the variance in rut depth and hence slope variance.

5. The thicknesses, stiffnesses and variability in stiffnesses of the A. C. surface layer and the base layer were not very significant.

6. Temperature appears insignificant because the permanent deformation characterization for the A.C. surface layer as currently used in VESYS II (M) is not temperature dependent (as previously discussed in Chapter V).

CHAPTER VII

SENSITIVITY ANALYSIS FOR PRESENT SERVICEABILITY INDEX

The sensitivity analysis for Present Serviceability Index (PSI), sometimes called simply serviceability, requires input from both cracking and roughness calculations. The independent variables considered in the cracking and roughness factorials have been previously identified and described.

Statistical Analysis

Selection of Variables

Since PSI depends on the results of both the cracking and roughness modules, it was necessary to combine the results from both factorials in order to study this response. It is not possible to perform factorial analysis of variance on both factorials together, so variable selection was somewhat more complicated.

An easy way to pick terms which are to be used in the regression analysis is suggested by the fact that only 12 terms appear in the damage index model from the cracking factorial. Since the only contribution of the cracking module to the serviceability calculations is the area cracked, we shall assume that any terms which appear only in the cracking factorial and do not significantly affect the damage index need not be considered for determining serviceability response. This means that we can safely ignore all cracking variables except the 12 terms which appear in the equation for damage index. There is little danger in making this simplification since the AASHO equation, which is used for serviceability, is rather insensitive to cracking anyway.

The task of selecting terms for the serviceability regression could be completed by performing factorial analysis of variance on the data from the roughness factorial to select the terms from that factorial which would be added to the 12 terms already selected from the cracking factorial.

Analysis of Variance on Roughness Factorial

The initial error pool was constructed out of confounded three-way and higher-order interactions as explained in Chapter II. Two terms, which were significant at an α-level of .01, were deleted from this pool

Only terms which were significant at a α-level of .001 were retained for the regression analysis. They are listed in Table 20. The two

Table 20. Analysis of variance for serviceability index

Variable Name	Factor(s)	F-Ratio
LAYER3	A	6239.68
LAMBDA	B	15708.01
ALPHA(3)	C	6598.50
LAYER2	D	1113.58
THICK2	E	1157.07
VARCOEF3	F	4863.02
TEMPS	G	234.25
ALPHA(1)	H	*
THICK1	J	103.89
LAYER1	K	2762.08
GNU(1)	L	*
VARCOEF1	M	353.89
CORLEXP	N	8013.43
AMPLITUD	O	26761.95
VARCOEF2	P	362.75
LAYER3, LAMBDA	AB	261.29
LAYER3, ALPHA(3)	AC	290.40
LAYER3, LAYER2	AD	46.16
LAYER3, THICK2	AE	37.46
LAYER3, VARCOEF3	AF	14.22
LAYER3, ALPHA(1)	AH	114.70
LAYER3, THICK1	AJ	16.11
LAYER3, LAYER1	AK	36.68
LAYER3, VARCOEF1	AM	14.12
LAYER3, CORLEXP	AN	12.99
LAYER3, AMPLITUD	AO	328.96
LAMBDA, ALPHA(3)	BC	137.87
LAMBDA, LAYER2	BD	39.30
LAMBDA, THICK2	BE	36.40
LAMBDA, VARCOEF3	BF	28.28
LAMBDA, TEMPS	BG	50.11
LAMBDA, ALPHA(1)	BH	336.53
LAMBDA, LAYER1	BK	17.65
LAMBDA, VARCOEF1	BM	37.29
LAMBDA, CORLEXP	BN	33.10
LAMBDA, AMPLITUD	BO	551.03
ALPHA(3), THICK2	CE	280.66
ALPHA(3), VARCOEF3	CF	19.92
ALPHA(3), THICK1	CJ	48.75
ALPHA(3), VARCOEF1	CM	35.27
ALPHA(3), CORLEXP	CN	23.21
ALPHA(3), AMPLITUD	CO	396.06
LAYER2, ALPHA(1)	DH	72.83
LAYER2, THICK1	DJ	23.25
LAYER2, CORLEXP	DO	81.01

Table 20. Analysis of variance for serviceability index (cont.)

Variable Name	Factor(s)	F-Ratio
THICK2, VARCOEF3	EF	27.85
THICK2, ALPHA(1)	EH	13.47
THICK2, THICK1	EJ	-18.86
THICK2, AMPLITUD	EO	50.07
THICK2, VARCOEF2	EP	17.08
VARCOEF3, ALPHA(1)	FH	33.58
VARCOEF3, VARCOEF1	FM	32.81
VARCOEF3, CORLEXP	FN	11.74
VARCOEF3, AMPLITUD	FO	46.36
VARCOEF3, VARCOEF2	FP	12.70
TEMPS, ALPHA(1)	GH	16.02
TEMPS, THICK1	GJ	52.36
TEMPS, LAYER1	GK	61.09
TEMPS, AMPLITUD	GO	90.56
ALPHA(1), THICK1	HJ	290.61
ALPHA(1), LAYER1	HK	188.33
ALPHA(1), GNU(1)	HL	*
ALPHA(1), CORLEXP	HN	35.93
ALPHA(1), AMPLITUD	HO	892.45
THICK1, LAYER1	JK	208.71
THICK1, GNU(1)	JL	28.41
THICK1, VARCOEF2	JP	33.25
LAYER1, AMPLITUD	KO	43.24
GNU(1), AMPLITUD	LO	62.19
VARCOEF1, AMPLITUD	MO	20.09
CORLEXP, AMPLITUD	NO	770.41
LAMBDA, ALPHA(1), AMPLITUD	BHO	30.38
ALPHA(1), THICK1, LAYER1	HJK	22.09
(confounded three-ways)	ACE & FNP & PKL	13.98
(confounded three-ways)	FGN & HJO	13.12

* note: Because of the constrained factorial, ALPHA(1), GNU(1) and their interaction are pooled for the analysis of variance.

Pool of ALPHA(1), GNU(1) and ALPHA(1) & GNU(1) — H, L and HL — 5155.79

error pool:

- degrees of freedom = 117
- sum of squares = .382454
- mean square = .003267
- $F_{.001}(1,117)$ = 11.41

confounded three-way terms which had been removed from the error pool were included in this group:

Three-Way Interactions Confounded	F-ratio
LAYER3, ALPHA(3), THICK2 VARCOEF3, CORLEXP, VARCOEF2 LAYER2, LAYER1, GNU(1)	13.98
VARCOEF3, TEMPS, CORLEXP ALPHA(1), THICK1, AMPLITUD	13.12

These are the same two confounded terms which were significant in the rut depth analysis of variance, and they are included here for the same reasons. The last three-way interaction in each group seems reasonable from a physical standpoint.

Regression Analysis

Stepwise regression was run on the group of variables listed in Table 7A plus the terms from the damage index regression equation (see Chapter IV). The first 31 terms to be entered into the model were accepted for the serviceability equation. All of these terms had partial F-ratios which were significant at an α-level of .02. The remaining terms considered for the analysis did not contribute much to the fit and had small coefficients whose confidence intervals were too large to permit meaningful comparisons.

The equation which has been selected has an R^2 of .967 and a coefficient of variation of 10.3%. It contains 31 terms plus the constant. Of these terms, 15 represent main effects and the rest are two-way interactions.

$$\begin{aligned}
\text{PSI} \approx\ & 1.90 - .557\ \text{AMPLITUD} - .433\ \text{LAMBDA} \\
& - .267\ \text{LAYER3} + .320\ \text{CORLEXP} + .290\ \text{ALPHA(3)} \\
& - .249\ \text{VARCOEF3} + .858\ \text{ALPHA(1)} - .532\ \text{GNU(1)} \\
& - .177\ \text{LAYER1} - .120\ \text{LAYER2} + .122\ \text{THICK2} \\
& - .0844\ \text{LAMBDA} \cdot \text{AMPLITUD} - .0635\ \text{LAYER3} \cdot \text{AMPLITUD} \\
& + .0711\ \text{ALPHA(3)} \cdot \text{AMPLITUD} - .0573\ \text{LAYER3} \cdot \text{LAMBDA} \\
& - .0681\ \text{VARCOEF2} - .0672\ \text{VARCOEF1} - .0585\ \text{TEMPS} \\
& + .0609\ \text{LAYER3} \cdot \text{ALPHA(3)} - .0599\ \text{ALPHA(3)} \cdot \text{THICK2} \\
& - .0489\ \text{THICK1} \cdot \text{LAYER1} - .0671\ \text{NFAIL} + .214\ \text{ALPHA(1)} \cdot \text{AMPLITUD} \\
& - .135\ \text{GNU(1)} \cdot \text{AMPLITUD} + .0607\ \text{LAMBDA} \cdot \text{ALPHA(1)} \\
& + .0420\ \text{LAMBDA} \cdot \text{ALPHA(3)} - .0329\ \text{TEMPS} \cdot \text{AMPLITUD} \\
& - .0261\ \text{LAYER1} \cdot \text{AMPLITUD} + .0300\ \text{CORLEXP} \cdot \text{AMPLITUD} \\
& + .122\ \text{ALPHA(1)} \cdot \text{THICK1} - .0800\ \text{THICK1} \cdot \text{GNU(1)} \qquad (5)
\end{aligned}$$

The only independent variables which do not produce significant main effects are VCAMP, COEFK1, COEFK2 and THICK1. VCAMP and COEFK1 were not significant in the damage index analysis (see chapter IV), hence they were not even considered here. COEFK2 and THICK1 were quite significant in the cracking model, but as mentioned above the AASHO serviceability equation does not respond much to cracking damage. Even if cracking reaches its maximum level of 1000 square yards cracked per 1000 square yards of pavement, it only decreases the serviceability index by about .31. Hence it is not surprising to see these terms left out of the serviceability model. NFAIL, by far the most important factor in the cracking model, appears late in the equation and has a rather small coefficient (indicating that its effect was not great).

THICK1 makes a small contribution to the rutting response (see Chapter V), but it appears only as an interaction in the slope variance equation, and serviceability is primarily responsive to slope variance. It is interesting to note that this variable does appear in the model, however, through its three significant interactions with LAYER1, ALPHA(1), and GNU(1). Apparently this variable does not act directly on the response, but it has considerable influence on serviceability by modifying the effects due to these three important variables.

Sensitivity Rankings

The same methods of ranking used for the cracking damage sensitivity analysis and described in Chapter IV have also been used to arrive at the sensitivity of PSI to the fifteen significant main effects and sixteen significant two-way interactions.

Rankings by Magnitudes of Regression Coefficients

The magnitudes of the regression coefficients for the fifteen main effects included in the multiple regression model are exactly one-half of the values appearing as "Main Effects" in Table 21 and the rankings are the same as for "Main Effects." These rankings appear in Table 22. The coefficients on the sixteen interaction terms are also included for comparison to the main effects. It is interesting to note that a number of the two-way interactions were more significant than several of the less significant independent variables.

Rankings by Average Effects

The rankings by average effects, which are independent of the analysis of variance and multiple regression results, are also listed in Table 22. Notice that the rankings are generally the same as for "regression coefficients" and "main effects", except that the constrained

Table 21. Calculated variations in serviceability from different methods of sensitivity analysis, variations of each factor from low to high

Variable	Regression Coefficients	Average Effects	Main Effects	Span of Effects
ALPHA(1)	.858	1.72*	1.72	2.51
AMPLITUD	-.557	-1.17	-1.11	-2.43
GNU(1)	-.532	- .21**	-1.06	-1.49
LAMBDA	-.433	- .90	-0.87	-1.35
CORLEXP	.320	.64	.64	.70
ALPHA(3)	.290	.58	.58	1.05
LAYER3	-.267	- .56	- .53	- .90
VARCOEF3	-.249	- .50	- .50	- .50
LAYER1	-.177	- .38	- .35	- .50
THICK2	.122	.24	.24	.36
LAYER2	-.120	- .24	- .24	- .24
VARCOEF2	-.068	- .14	- .14	- .14
VARCOEF1	-.067	- .13	- .13	- .13
NFAIL	-.067	***	- .13	- .13
TEMPS	-.058	- .11	- .12	- .18
THICK1	****	.07	****	.50
ALPHA(1) · AMPLITUD	.214			
GNU(1) · AMPLITUD	-.135			
ALPHA(1) · THICK1	.122			
LAMBDA · AMPLITUD	-.084			
THICK1 · GNU(1)	-.080			
ALPHA(3) · AMPLITUD	.071			
LAYER3 · AMPLITUD	-.064			
LAYER3 · ALPHA(3)	.061			
LAMBDA · ALPHA(1)	.061			
ALPHA(3) · THICK2	-.060			
LAYER3 · LAMBDA	-.057			
THICK1 · LAYER1	-.049			
LAMBDA · ALPHA(3)	.042			
TEMPS · AMPLITUD	-.033			
CORLEXP · AMPLITUD	.030			
LAYER1 · AMPLITUD	-.026			

* Calculated average main effect was 0.86, but three levels of ALPHA(1) were considered rather than two, thus averaging was over one standard deviation instead of two. For comparison, ALPHA(1) must be multiplied by two.

**ALPHA(1) and GNU(1) were applied as four coupled pairs in a constrained factor space. Two levels of GNU(1) were used with three levels of ALPHA(1). Consequently, the meaning of the average main effect of this constrained factor is not clear and cannot be used for ranking.

Table 21. Calculated variations in serviceability from different methods of sensitivity analysis, variations of each factor from low to high (cont.)

***NFAIL was not included in the regression equation only as an interaction, so it has no main effect calculated.

****THICK1 appeared in the regression equation only as an interaction, so it has no main effect calculated.

Table 22. Comparisons of sensitivity rankings for serviceability derived from different methods of sensitivity analysis

Multiple Regression Coefficients	Average Effects	Main Effects	Span of Effects
ALPHA(1)	ALPHA(1)	ALPHA(1)	ALPHA(1)
AMPLITUD	AMPLITUD	AMPLITUD	AMPLITUD
GNU(1)	LAMBDA	GNU(1)	GNU(1)
LAMBDA	CORLEXP	LAMBDA	LAMBDA
CORLEXP	ALPHA(3)	CORLEXP	ALPHA(3)
ALPHA(3)	LAYER3	ALPHA(3)	LAYER3
LAYER3	VARCOEF3	LAYER3	CORLEXP
VARCOEF3	LAYER1	VARCOEF3	VARCOEF3
ALPHA(1) · AMPLITUD	THICK2	LAYER1	LAYER1
LAYER1	LAYER2	THICK2	THICK1**
GNU(1) · AMPLITUD	GNU(1)*	LAYER2	THICK2
THICK2	VARCOEF2	VARCOEF2	LAYER2
ALPHA(1) · THICK1	VARCOEF1	VARCOEF1	TEMPS
LAYER2	TEMPS	NFAIL	VARCOEF2
LAMBDA · AMPLITUD	THICK1	TEMPS	VARCOEF1
THICK1 · GNU(1)			NFAIL
ALPHA(3) · AMPLITUD			
VARCOEF2			
VARCOEF1			
NFAIL			
LAYER3 · AMPLITUD			
LAYER3 · ALPHA(3)			
LAMBDA · ALPHA(1)			
ALPHA(3) · THICK2			
TEMPS			
LAYER3 · LAMBDA			
THICK1 · LAYER1			
LAMBDA · ALPHA(3)			
TEMPS · AMPLITUD			
CORLEXP · AMPLITUD			
LAYER1 · AMPLITUD			

*Magnitude of change for this "average effect" may not be used for ranking. See Table 21 for explanation.

** THICK1 has no main effect, but interacts strongly with other factors to become significant.

factor space for ALPHA(1) and GNU(1) explained in Chapter V makes the ranking of GNU(1) inaccurate by this method.

Rankings by Main Effects (No Interactions)

The changes in PSI due to varying a single factor from its lows to its high levels may be obtained by multiplying the multiple regression coefficient for the factor only from Equation (5) by two. This is illustrated for ALPHA(1) below:

$$\text{PSI} \approx 1.90 + .858\ \text{ALPHA}(1)$$

$$\begin{aligned}\Delta\text{PSI}(\text{ALPHA}(1)) &\approx \left[\cancel{1.90} + .858(1)\right] - \left[\cancel{1.90} + .858(-1)\right] \\ &\approx .858 + .858 \\ &\approx 2(\text{Multiple Regression Coefficient})\end{aligned}$$

Consequently, the rankings for multiple regression coefficients and main effects are identical.

The calculated variations in PSI for the main effects, average effects, and span of effects are shown in Table 21. Note that the values (except for GNU(1) for reasons previously discussed) are almost identical for the separate independent variables whether arrived at by use of the multiple regression model or by averaging effects, which is independent of the multiple regression model. This adds to the confidence in the multiple regression model.

The calculated PSI for a factor as its value increases from the low to high levels appear in Figure 10 . The "main effect" of a factor entered in Table 21 is equal to its "length" in Figure 10. The arrows indicate whether rut depth decreases or increases as the factor increases in magnitude. These plots show very clearly how much variation from the mean condition of PSI = 1.90 may be introduced by varying each factor separately by one standard deviation either side of the mean.

The same information is plotted differently in Figure 11 to show only the change in PSI caused by each factor. This plot also shows the direction of the changes as the magnitude of each factor increases and the ranking in parentheses for each factor next to the corresponding bar. Sensitivity rankings by "main effects" also appear in Table 22.

Ranking by Span of Effects

The addition of the interactions to the main effects analysis allows assessment of how each factor and all its interactions may affect the calculation or prediction of PSI. The calculations are made such that all its interactions have the same sense as the main effect of

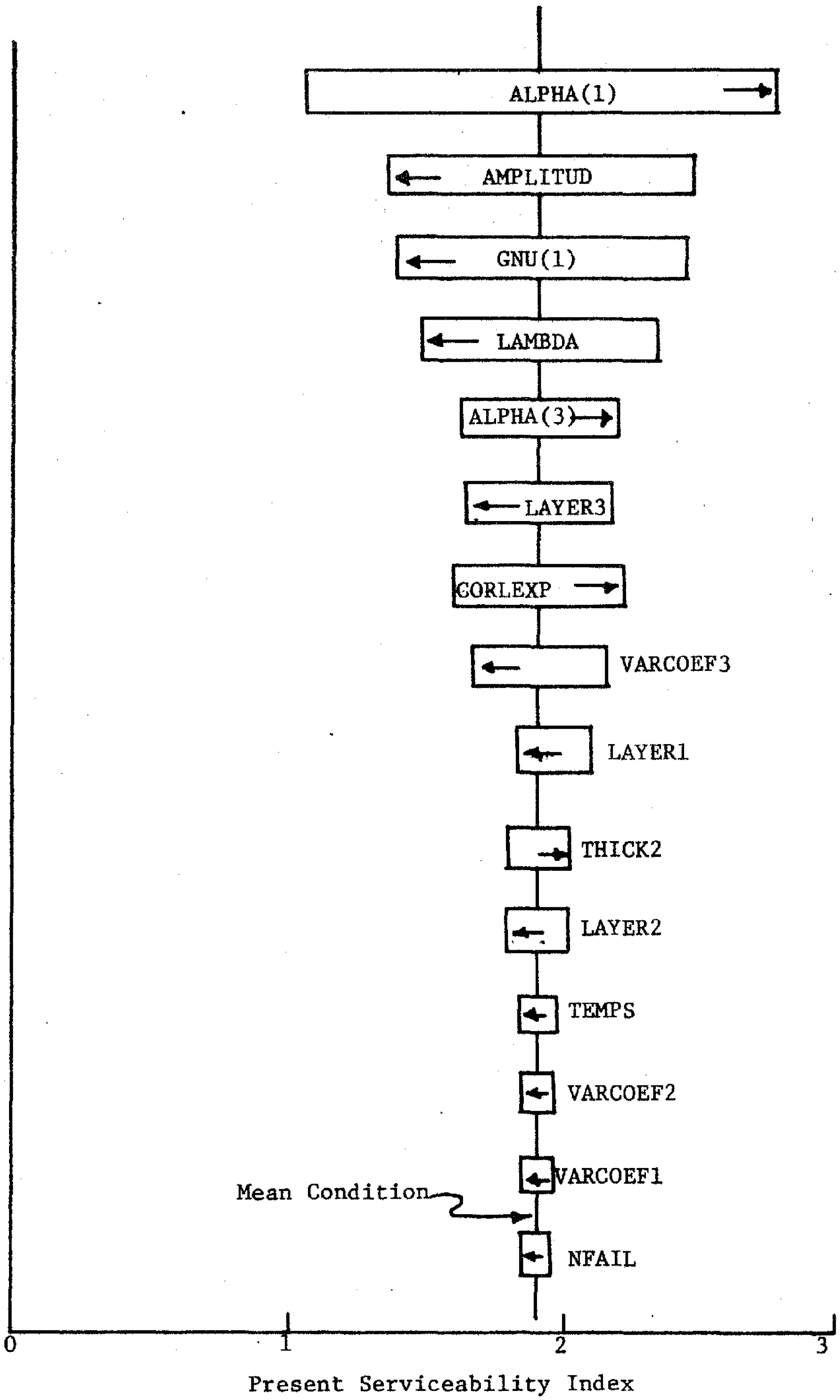

Figure 10. Change in PSI while each factor is varied from low to high levels with all other factors at their means.

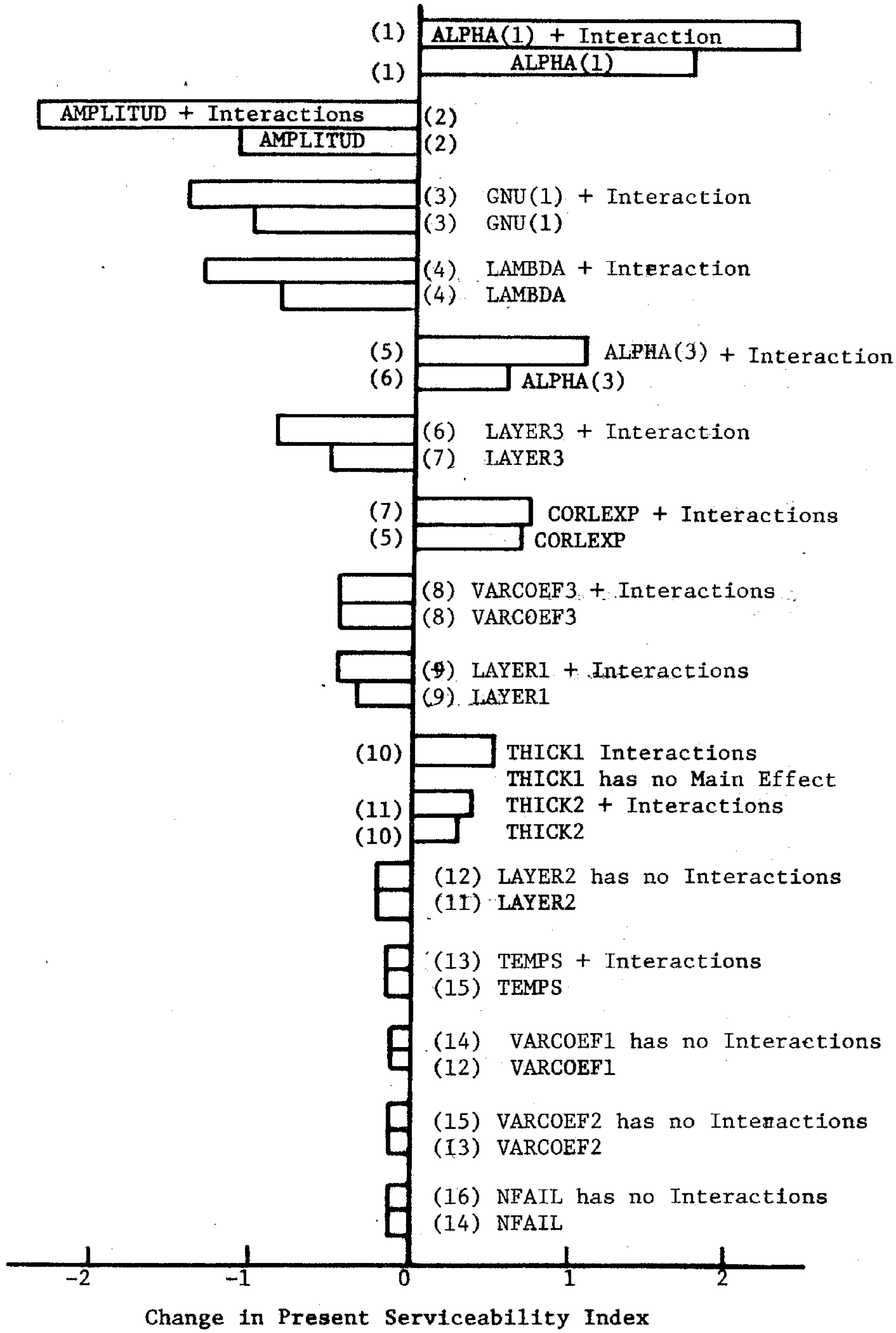

Figure 11. Span of effects for each factor compared to the factor without interactions.

interest so that the full range of possible effect is obtained. As illustration, the full span of effect is developed below for ALPHA(1):

$$\text{PSI} \simeq 1.90 + .858\ \text{ALPHA(1)} + 2.14\ \text{ALPHA(1)} \cdot \text{AMPLITUD} + .0607\ \text{LAMBDA} \cdot \text{ALPHA(1)} + .122\ \text{ALPHA(1)} \cdot \text{THICK1}$$

$$\Delta\text{PSI(ALPHA(1))} \simeq [1.90 + .858(1) + .214(1)(1) + .0607(1)(1) + .122(1)(1)] - [1.90 + .858(-1) + .214(-1)(1) + .0607(1)(-1) + .122(-1)(1$$

$$\simeq 3.155 - .645$$

$$\simeq 2.51$$

In this case, all factors having interactions with ALPHA(1) are at their high levels while ALPHA(1) changes from low to high. Such would not have been the case had the coefficients on the interaction terms not all been of the same sign (positive in this case).

As for the main effects, the calculated spans of effect appear in Table 21 and are plotted in Figure 11. As can be seen from Figure 11 and as discussed previously, the interactions for PSI add significantly to the main effects, especially for AMPLITUD as it appears in eight of the sixteen significant interactions.

The sensitivity rankings in terms of span of effects appear in Figure 11 and Table 22.

Summary Analysis for Present Serviceability Index

Comparing the rankings by the several methods listed in Table 22. it can be seen that agreement is complete for ALPHA(1), AMPLITUD, GNU(1), and LAMBDA, except that GNU(1) was out of order for "average effects" for the reason previously discussed. The relatively heavy effects from interactions caused the following changes for span of effects in relation to average and main effects:

1. ALPHA(3) and LAYER3 were ranked above CORLEXP.

2. THICK1, which has no significant main effect, was ranked 10th due to its significant interaction with ALPHA(1), GNU(1) and LAYER1.

3. TEMPS was ranked above VARCOEF2, VARCOEF1, and NFAIL.

The rankings translated into physical terms appear to suggest that:

1. The permanent deformation characteristics of the A.C. surface layer is quite important to predicting rut depth and hence slope variance

and is therefore very important in predicting PSI. The permanent deformation parameter ALPHA(3) is fairly important while permanent deformation characteristics for the base materials are not very important. The lack of importance of the base material in this case is because a well-compacted base generally has little relative permanent deformation potential unless badly overstressed.

2. CORLEXP is important in estimating the variation in rut depth with distance along the wheel path and thus slope variance so it is very significant.

3. Truck traffic LAMBDA and the wheel load representation AMPLITUD are quite important for obvious reasons.

4. The stiffness of the subgrade is almost as important as its permanent deformation characteristics and its variation in stiffness is also important as it contributes to the variance in rut depth and hence slope variance.

5. The thicknesses, stiffnesses and variability in stiffnesses of the A.C. surface layer and the base layer were not very significant.

6. Temperature appears insignificant because the permanent deformation characterization for the A.C. surface layer as currently used in VESYS IIM is not temperature dependent (as previously) discussed in Chapter V).

CHAPTER VIII

SENSITIVITY ANALYSIS FOR SERVICE LIFE

The sensitivity analysis for service life requires input from both cracking and roughness calculations. The independent variables considered in the cracking and roughness factorials have been previously identified and described.

Statistical Analysis

Selection of Variables

Variables were selected in the manner described in Chapter VII for the serviceability analysis. Each of the terms in the damage index model (see Chapter IV) were used for regression analysis. The remaining factors and interactions on which regression was run were selected from the roughness factorial by means of factorial analysis of variance.

An initial error pool was constructed using confounded three-way and higher-order interaction terms, as described in Chapter II. One term in this group was significant at an α-level of .01, and it was deleted from the pool. Only terms which were significant at an α-level of .001 were used for regression. They are listed in Table 23. The confounded three-way interaction term which had been removed from the error pool was among this group. The second three-way interaction which is present in this term seems to represent a genuine interaction, hence it was retained for further analysis.

three-way interactions confounded	F-ratio
LAYER3, LAYER2, VARCOEF3 LAMBDA, CORLEXP, AMPLITUD	12.79

Regression Analysis

Stepwise regression was run on the variables selected from the deformation factorial analysis of variance and the cracking factorial damage index model. All terms having partial F-ratios which were significant for $\alpha = .01$ (the first 35 terms entered) were included in the equation except for the two-way interaction of THICK1 and NFAIL. The coefficient on this term was unstable, its 95% confidence interval ranging from -.037 to + .42. Because of this it would have been useless for drawing conclusions concerning sensitivity rankings.

Table 23. Service life analysis of variance

Variable Name	Factor	F-Ratio
LAYER3	A	395.63
LAMBDA	B	972.29
ALPHA(3)	C	409.56
LAYER2	D	74.10
THICK2	E	72.11
VARCOEF3	F	388.04
ALPHA(1)	H	*
THICK1	J	31.33
GNU(1)	L	*
VARCOEF1	M	28.33
CORLEXP	N	682.41
AMPLITUD	O	1513.94
VARCOEF2	P	27.70
LAYER3, LAMBDA	AB	54.30
LAYER3, VARCOEF3	AF	21.76
LAYER3, ALPHA(1)	AH	49.44
LAYER3, CORLEXP	AN	40.52
LAYER3, AMPLITUD	AO	87.08
LAMBDA, ALPHA(3)	BC	47.78
LAMBDA, LAYER2	BD	12.07
LAMBDA, THICK2	BE	13.34
LAMBDA, VARCOEF3	BF	51.87
LAMBDA, ALPHA(1)	BH	126.17
LAMBDA, CORLEXP	BN	103.00
LAMBDA, AMPLITUD	BO	223.45
ALPHA(3), VARCOEF3	CF	16.12
ALPHA(3), CORLEXP	CN	30.31
ALPHA(3), AMPLITUD	CO	65.41
THICK2, AMPLITUD	EO	14.68
VARCOEF3, ALPHA(1)	FH	59.21
VARCOEF3, LAYER1	FK	29.97
VARCOEF3, CORLEXP	FN	24.75
VARCOEF3, AMPLITUD	FO	61.47
TEMPS, THICK1	GJ	11.72
ALPHA(1), THICK1	HJ	30.61
ALPHA(1), LAYER1	HK	11.95
ALPHA(1), CORLEXP	HN	92.20
ALPHA(1), AMPLITUD	HO	134.13
THICK1, LAYER1	JK	16.10
THICK1, AMPLITUD	JO	13.42
LAYER1, CORLEXP	KN	29.93
LAYER1, AMPLITUD	KO	35.00
CORLEXP, AMPLITUD	NO	126.10
AMPLITUD, ALPHA(1)	OH	75.31
LAMBDA, ALPHA(1), AMPLITUD	BHO	13.93
	AOF & BHO	12.79

*Note: Because of the constrained factorial, ALPHA(1), GNU(1), and their interaction are pooled for the analysis of variance.

Pool of ALPHA(1), GNU(1) and ALPHA(1), GNU(1)	H, L and HL	299.91

Error pool:

degrees of freedom = 118
sum of squares = 95.7463
mean square = .81141
$F_{.001}(1,118)$ = 11.40

The model which has been selected has an R^2 of .884 and a coefficient of variation of 35.9%. This rather large variation probably indicates a lack of normality in the service life response, but the coefficients in the model had small enough standard deviations to permit valid comparisons between factors. It was decided that the ease of comparison using this straightforward model justified its use here even though theoretical considerations suggest that service life is more likely to approximate an inverse exponential or Weibull distribution. The equation selected contains 34 terms plus the constant.

$$\begin{aligned} SL \simeq\ & 4.40 - 2.16\ \text{AMPLITUD} - 1.73\ \text{LAMBDA} + 1.47\ \text{CORLEXP} \\ & - 1.09\ \text{LAYER3} + 1.14\ \text{ALPHA(3)} - 1.11\ \text{VARCOEF3} \\ & + .84\ \text{LAMBDA} \cdot \text{AMPLITUD} - .74\ \text{LAYER1} + 3.30\ \text{ALPHA(1)} \\ & - 2.01\ \text{GNU(1)} + .53\ \text{LAYER3} \cdot \text{AMPLITUD} - .63\ \text{CORLEXP} \cdot \text{AMPLITUD} \\ & - .51\ \text{LAYER2} - .57\ \text{LAMBDA} \cdot \text{CORLEXP} + .40\ \text{LAYER3} \cdot \text{LAMBDA} \\ & + .48\ \text{THICK2} - .46\ \text{ALPHA(3)} \cdot \text{AMPLITUD} + .44\ \text{VARCOEF3} \cdot \text{AMPLITUD} \\ & + .36\ \text{LAYER1} \cdot \text{AMPLITUD} + \text{LAMBDA} \cdot \text{VARCOEF} - .39\ \text{LAMBDA} \cdot \text{ALPHA(3)} \\ & - .53\ \text{ALPHA(1)} \cdot \text{AMPLITUD} - .36\ \text{LAYER3} \cdot \text{CORLEXP} - .47\ \text{LAMBDA} \cdot \text{ALPHA(3)} \\ & + .31\ \text{ALPHA(3)} \cdot \text{CORLEXP} + .31\ \text{VARCOEF3} \cdot \text{LAYER1} - .31\ \text{LAYER1} \cdot \text{CORLEXP} \\ & + .43\ \text{ALPHA(1)} \cdot \text{CORLEXP} - .30\ \text{VARCOEF1} - .30\ \text{VARCOEF2} \\ & - .40\ \text{VARCOEF3} \cdot \text{ALPHA(1)} - .23\ \text{THICK1} \cdot \text{LAYER1} - .28\ \text{VARCOEF3} \\ & \cdot \text{CORLEXP} + .26\ \text{LAYER3} \cdot \text{VARCOEF3} \end{aligned} \tag{6}$$

Sensitivity Rankings

The same methods of ranking used for the cracking damage sensitivity analysis and described in Chapter IV have also been used to arrive at the sensitivity of service life to the thirteen significant main effects and twenty-one significant two-way interactions.

Rankings by Magnitudes of Regression Coefficients

The magnitudes of the regression coefficients for the thirteen main effects included in the multiple regression model are exactly one-half of the values appearing as "Main Effects" in Table 24 and the rankings are the same as for "Main Effects." These rankings appear in Table 25. The coefficients on the interaction terms are also included for comparison to the main effects. It is interesting to note that a number of the two-way interactions were more significant than several of the less significant main effects.

Rankings by Average Effects

The rankings by average effects, which are independent of the analysis of variance and multiple regression results, are also listed in Table 25. Notice that the rankings are generally the same as for "regression coefficients" and "main effects", except that the

Table 24. Calculated variations in service life from different methods of sensitivity analyses, variations of each factor from low to high

Variable	Regression Coefficients	Average Effects	Main Effects	Span of Effects
ALPHA(1)	3.30	6.60*	6.60	10.26
AMPLITUD	-2.16	-4.38	-4.32	-11.90
GNU(1)	-2.01	- .73**	-4.02	- 4.02
LAMBDA	-1.73	-3.51	-3.46	- 9.62
CORLEXP	1.47	2.94	2.94	8.72
ALPHA(3)	1.14	2.28	2.28	4.60
VARCOEF3	-1.11	-2.22	-2.22	- 6.42
LAYER3	-1.09	-2.24	-2.18	- 5.28
LAYER1	- .74	-1.51	-1.48	- 3.90
LAYER2	- .51	- .97	-1.02	- 1.02
THICK2	.48	.96	.96	.96
VARCOEF2	- .30	- .59	- .60	- .60
VARCOEF1	- .30	- .60	- .60	- .60
THICK1	***	.63	***	.46
TEMPS	****	- .24	****	****
LAMBA · AMPLITUD	.84			
CORLEXP · AMPLITUD	- .63			
LAMBDA · CORLEXP	- .57			
LAYER3 · AMPLITUD	.53			
ALPHA(1) · AMPLITUD	- .53			
LAMBDA · ALPHA(1)	- .47			
ALPHA(3) · AMPLITUD	- .46			
VARCOEF3 · AMPLITUD	.44			
ALPHA(1) · CORLEXP	.43			
LAMBDA · VARCOEF3	.41			
LAYER3 · LAMBDA	.40			
VARCOEF3 · ALPHA(1)	.40			
LAMBDA · ALPHA(3)	- .39			
LAYER1 · AMPLITUD	.36			
LAYER3 · CORLEXP	- .36			
ALPHA(3) · CORLEXP	.31			
VARCOEF3 · LAYER1	.31			
LAYER1 · CORLEXP	- .31			
VARCOEF3 · CORLEXP	- .28			
LAYER3 · VARCOEF3	.26			
THICK1 · LAYER1	- .23			

*Calculated average main effect was 3.30, but three levels of ALPHA(1) were considered rather than two, thus averaging was over one standard deviation instead of two. For comparison, ALPHA(1) must be multiplied by two.

**ALPHA(1) and GNU(1) were applied as four coupled pairs in a constrained factor space. Two levels of GNU(1) were used with three levels of

Table 24. Calculated variations in service life from different methods of sensitivity analyses, variations of each factor from low to high (cont.)

ALPHA(1). Consequently, the meaning of the average main effect of this constrained factor is not clear and cannot be used for ranking.

***THICK1 appeared in the regression equation only as an interaction, so it has no main effect calculated.

****TEMPS was not included in the regression model so it has no main or span of effects calculated.

Table 25. Comparisons of sensitivity rankings for service life derived from different methods of sensitivity analysis

Multiple Regression Coefficients	Average Effects	Main Effects	Span of Effects
ALPHA(1)	ALPHA(1)	ALPHA(1)	AMPLITUD
AMPLITUD	AMPLITUD	AMPLITUD	ALPHA(1)
GNU(1)	LAMBDA	GNU(1)	LAMBDA
LAMBDA	CORLEXP	LAMBDA	CORLEXP
CORLEXP	ALPHA(3)	CORLEXP	VARCOEF3
ALPHA(3)	LAYER3	ALPHA(3)	LAYER3
VARCOEF3	VARCOEF3	VARCOEF3	ALPHA(3)
LAYER3	LAYER1	LAYER3	GNU(1)
LAMBDA · AMPLITUD	LAYER2	LAYER1	LAYER1
LAYER1	THICK2	LAYER2	LAYER2
CORLEXP · AMPLITUD	GNU(1)*	THICK2	THICK2
LAMBDA · CORLEXP	VARCOEF1	VARCOEF1	VARCOEF1
LAYER3 · AMPLITUD	VARCOEF2	VARCOEF2	VARCOEF2
ALPHA(1) · AMPLITUD	TEMPS		THICK1
LAYER2			
THICK2			
LAMBDA · ALPHA(1)			
ALPHA(3) · AMPLITUD			
VARCOEF3 · AMPLITUD			
ALPHA(1) · CORLEXP			
LAMBDA · VARCOEF3			
LAYER3 · LAMBDA			
VARCOEF3 · ALPHA(1)			
LAMBDA · ALPHA(3)			
LAYER1 · AMPLITUD			
LAYER3 · CORLEXP			
ALPHA(3) · CORLEXP			
VARCOEF3 · LAYER1			
LAYER1 · CORLEXP			
VARCOEF1			
VARCOEF2			
VARCOEF3 · CORLEXP			
LAYER3 · VARCOEF3			
THICK1 · LAYER1			

*Magnitude of change for this "average effect" may not be used for ranking. See Table 24 for explanation.

constrained factor space for ALPHA(1) and GNU(1) explained in Chapter V makes the ranking of GNU(1) inaccurate by this method.

Rankings by Main Effects (No Interactions)

The changes in service life due to a single main effect may be obtained by multiplying the multiple regression coefficient for that main effect in Equation (6) by two as previously illustrated for rut depth and PSI. Consequently, the rankings for multiple regression coefficients and main effects are identical.

The calculated variations in service life for the main effects, average effects, and span of effects are shown in Table 24. Note that the values (except for GNU(1) for reasons previously discussed) are almost identical for the separate independent variables whether arrived at by use of the multiple regression model or by averaging effects, which is independent of the multiple regression model. This adds to the confidence in the multiple regression model.

The calculated service life for a factor as its value increases from the low to high levels appear in Figure 12 . The arrows indicate whether rut depth decreases or increases as the factor increases in magnitude. These plots show very clearly how much variation from the mean condition of 4.40 years of service life may be introduced by varying each factor separately by one standard deviation either side of the mean.

The same information is plotted differently in Figure 13 to show only the change in service life caused by each factor. This plot also shows the direction of the changes as the magnitude of each factor increases and the ranking in parentheses for each factor next to the corresponding bar. Sensitivity rankings by "main effects" also appear in Table 25.

Ranking by Span of Effects

The addition of the interactions to the main effects analysis allows assessment of how each factor and all its interactions may affect the calculation or prediction of service life. The calculations are made such that all its interactions have the same sense as the main effect of interest so that the full range of possible effect is obtained. As illustration, the full span of effect is developed below for ALPHA(1):

$$SL \simeq 4.40 + 3.30\ ALPHA(1) - .53\ ALPHA(1) \cdot AMPLITUD - .47\ LAMBDA \cdot ALPHA(1) + .43\ ALPHA(1) \cdot CORLEXP - .40\ VARCOEF3 \cdot ALPHA(1)$$

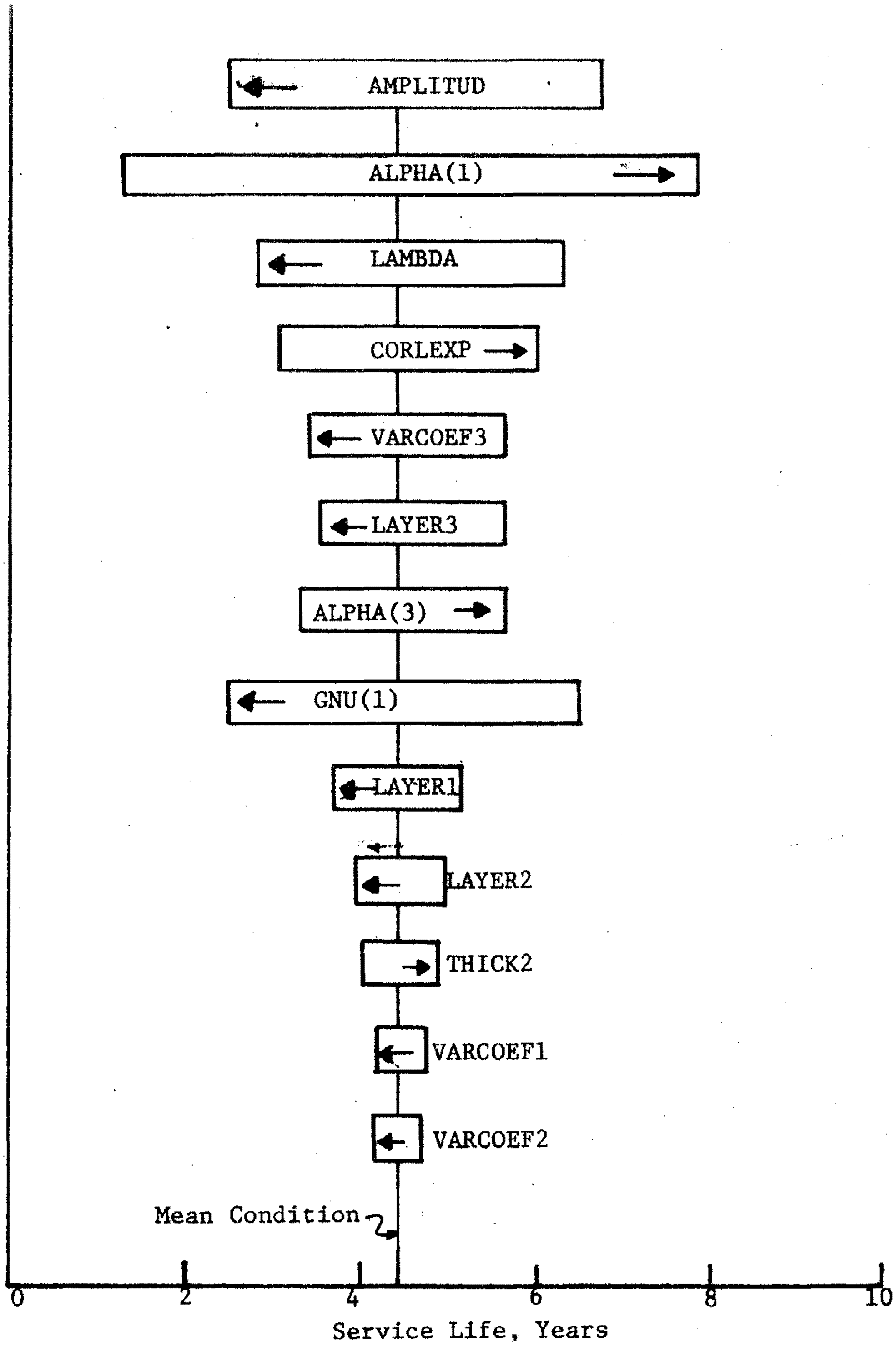

Figure 12. Change in service life while each factor is varied from low to high levels with all other factors at their means.

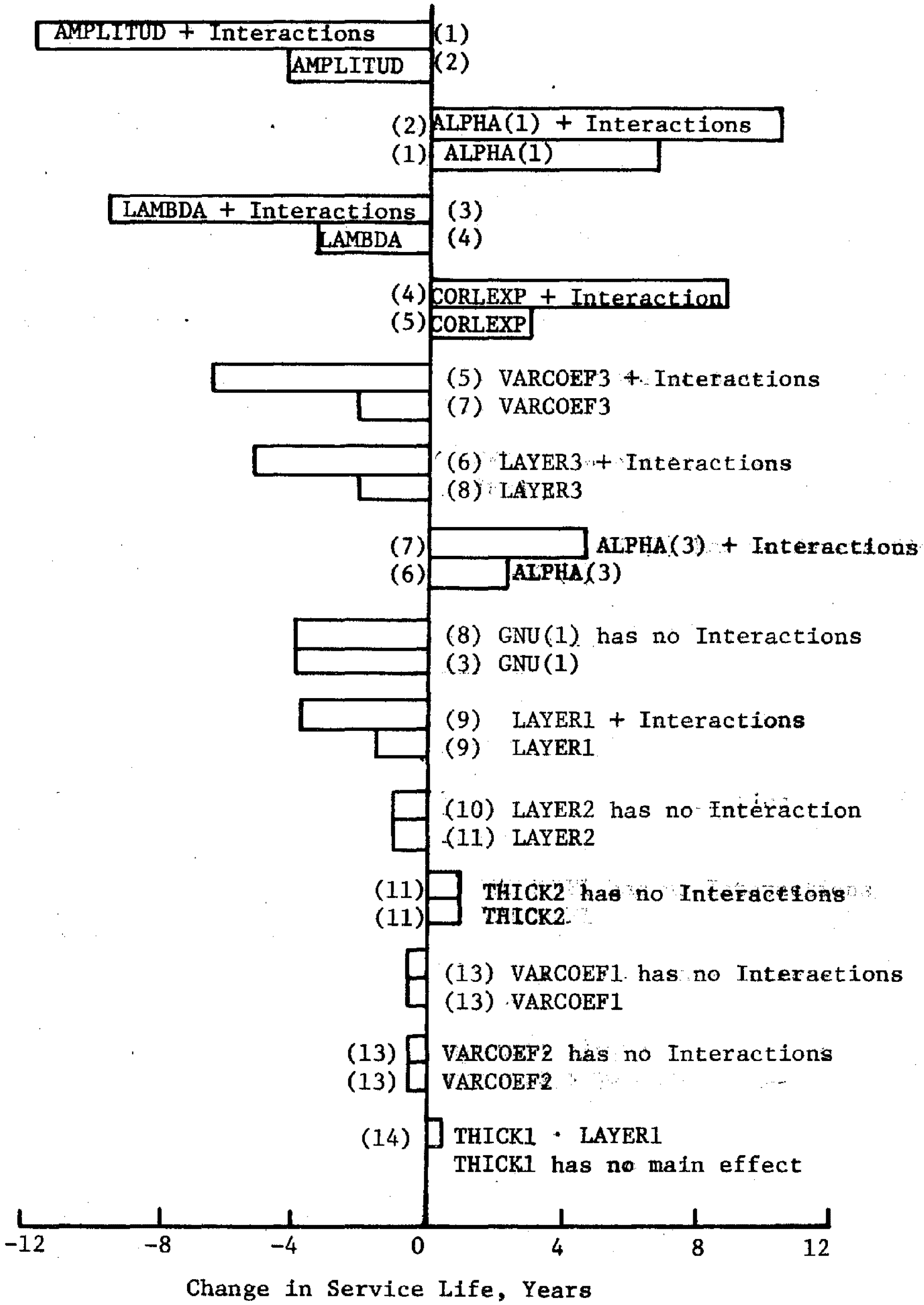

Figure 13. Span of effects for each main effect compared to the main effects without interactions.

$$
\begin{aligned}
\Delta SL(ALPHA(1)) &\approx [4.40 + 3.30(1) - .53(1)(-1) - .47\ (-1)(1) \\
&\quad + .43(1)(1) - .40(-1)(1)] - [4.40 + 3.30\ (-1) \\
&\quad - .53(-1)(-1) - .47(-1)(-1) + .43(-1)(1) \\
&\quad - .40(-1)(-1) \\
&\approx 9.53 - (-.73) \\
&\approx 10.26
\end{aligned}
$$

As for the main effects, the calculated spans of effect appears in Table 24 and are plotted in Figure 13. As can be seen from Figure 13 and as discussed previously, the interactions for service life at their extremes add very significantly to the main effects. The seven interactions including AMPLITUD are sufficiently important for AMPLITUD to displace ALPHA(1) on this basis as the most significant factor although ALPHA(1) was the most significant main effect and had the most significant span of effects for rut depth, slope variance and PSI. LAMBDA also greatly increased its relative significant through its six interactions. VARCOEF3 also increased in apparent significane with its six interactions. GNU(1) had no interactions so it dropped in apparent significance on the basis of span of effects.

The sensitivity rankings in terms of span of effects appear in Figure 12 and Table 25.

Summary Analysis for Service Life

Reviewing changes in calculated service life appearing in Table 24 and Figures 12 and 13 and the rankings by the several methods listed in Table 25, the significant potential for modifying main effects invested in the interactions for this regression model is striking compared to that for rut depth, slope variance and PSI. The span of effects for interactions with NFAIL in the cracking damage model was relatively much larger, but NFAIL does not appear in this model due to relative insignificance of cracking in the AASHO serviceability equation.

No major difference in rankings exists between "average effects" and "main effects" and the changes in service life shown in Table 24 are almost identical. This is to be expected when a fairly good regression model is obtained. However, rather serious differences exist between the rankings by these two methods and "span of effects".

ALPHA(1) has consistently dominated in sensitivity throughout previous analyses for the roughness factorial and AMPLITUD has consistently followed as the second or third most sensitive factor.

However, in terms of the extreme of effects for the interactions (i.e., combination of independent variables at high and low levels such that the response is the highest or lowest possible) called span of effects, AMPLITUD became slightly more important then ALPHA(1) and GNU(1) dropped to a much lesser position of relative significance.

It finally becomes necessary to apply judgement if a single recommended ranking is desired. In this case, ALPHA(1) is clearly more sensitive as a main effect and only becomes less sensitive than AMPLITUD under relatively extreme conditions. Also, ALPHA(1) has clearly been the most sensitive for rut depth, slope variance and PSI, upon which service life is based. Consequently, it is reasonable to infer that ALPHA(1) for the most probable set of conditions or combinations of values for independent variables will produce more change in service life than AMPLITUD. It is also reasonable to expect for similar reasons that GNU(1) would rank around 4th after ALPHA(1), AMPLITUD and LAMBDA.

The rankings approximate those for slope variance and the apparent physical meanings of these rankings are generally as described for slope variance in Chapter IV.

CHAPTER IX

SUMMARY RESULTS FROM SENSITIVITY ANALYSIS

The important results of the sensitivity analysis to be reported are:

1. The relative sensitivity of the five responses to changes in the magnitudes of the 30 independent variables.

2. The sensitivity rankings of the 17 variables found to significantly affect at least one of the five responses in terms of the magnitude of their effect on a calculated resonse.

These results will be discussed in detail subsequently.

Limitations on the Sensitivity Analysis

While the authors feel that the accuracy of this sensitivity analysis is very good considering the number of independent variables involved, it appears worthwhile to briefly summarize the limitations on such an analysis.

There are three primary sources or error in this or any similar sensitivity analysis: fractional factorial confounding, lack of fit in regression, and selection of levels for independent variables. The confounding error caused by running a fractional factorial appears to be small here, because none of the measurable three-way or four-way interactions were significant in any of the regression models. Since these measurable terms were selected to involve likely combinations of factors (see Chapter III), it is concluded that the three-way and higher-order interactions with which the main effects and interactions are confounded are unlikely to cause significant error. Lack of fit in regression can be detected from the R^2 (fraction of the variance of the independent variable which is explained by the equation) and the coefficient of variation. All of the equations developed in this analysis fit very well except for the service life equation, which has a rather large coefficient of variation. As was explained in Chapter VIII, this was probably caused by a lack of normality in the dependent variable. This variation is reflected in the larger confidence intervals on the regression coefficients. These intervals were small enough to provide meaningful (but not "hair-splitting") comparisons between factors on the basis of their contribution to the equation.

By far the most important source of experimental error in this analysis was the selection of values for the dependent variables. Much time and effort was spent on collecting and analyzing data (see detailed discussion in Volume I) from which estimates of the mean, standard deviation and probability distribution of each factor could be estimated. However, the sparcity of data for many variables and their stochastic variations was such that no more then a good first approximation to the time population distribution can be claimed for the limited samples available for those variables. Even so, it is believed that they are sufficiently accurate to allow the assessment of relative sensitivity and to correctly rank the variables.

Due to an error in definition of the truck traffic variable LAMBDA as trucks per day in VESYS documentation in lieu of axles per day as actually used in the computer program, it is felt that the range selected (2000 to 4500) for this variable is more representative of a rural state highway or less travelled interstate highway with say 500 to 1200 trucks per day than the typical rural interstate highway intended. The effect of an increase in truck traffic over a range of higher magnitude (say 5,000 to 10,000 axles per day) might be expected to increase the sensitivity of the responses to LAMBDA somewhat.

Results of the Sensitivity Analysis

The separate analyses for the five responses have been discussed in detail in Chapters IV through VIII with sensitivity rankings by several methods and documentation of relative change in responses due to variation of a factor from one standard deviation below to one standard deviation above its mean value.

Table 26 provides a summary of the sensitivity analysis for all thirty independent variables and for each of the five responses. This table shows both the sense of the effect (i.e., increase or decrease) due to an increase in magnitude of a factor and also shows degree of the effect. A factor is designated as "Insensitive" if its variation caused no significant change in the response. A change less than one-third of the maximum change calculated for any variable is designated as a slight increase or decrease, that between one third and two thirds an increase or decrease and that above two thirds a great increase or decrease. Where there were differences between methods for analysis (such as "main effects" or "span of effects"), degree of the effect was based on relative magnitudes of the calculated effects, knowledge of the variable and its ranking for other related responses (service life is dependent on PSI, PSI on rut depth and slope variance).

Table 27 provides a set of rankings for the seventeen factors found to significantly affect at least one of the five calculated responses. The selection of rankings between factors having different rankings by different methods of analysis was again by judgement as discussed above.

Table 26. Summary of sensitivity analysis for VESYS IIM based on increasing magnitudes of independent variables

Independent Variables	Effects on Output Responses				
	Damage Index	Rut Depth	Slope Variance	Present Serviceability Index	Service Life
Permanent Deformation Parameters:					
ALPHA(1), Surface Layer	Insensitive	Great Decrease	Great Decrease	Great Increase	Great Increase
GNU(1), Surface Layer	Insensitive	Increase	Increase	Decrease	Decrease
ALPHA(2), Base Material	Insensitive	Insensitive	Insensitive	Insensitive	Insensitive
GNU(2), Base Material	Insensitive	Insensitive	Insensitive	Insensitive	Insensitive
ALPHA(3), Subgrade	Insensitive	Decrease	Decrease	Increase	Increase
GNU(3), Subgrade	Insensitive	Insensitive	Insensitive	Insensitive	Insensitive
AMPLITUD, Wheel Load Pressure in PSI	Increase	Great Increase	Great Increase	Great Decrease	Great Decrease
BETA, Time-Temperature Shift Parameter for A.C. Surface Layer	Insensitive	Insensitive	Insensitive	Insensitive	Insensitive

Table 26. Summary of sensitivity analysis for VESYS IIM based on increasing magnitudes of independent variables (cont.)

Independent Variables	Effects on Output Responses: Damage Index	Rut Depth	Slope Variance	Present Serviceability Index	Service Life
COEFK1 - Coefficient of Variation for fatigue coefficient K_1	Insensitive	Insensitive	Insensitive	Insensitive	Insensitive
COEFK2, Coefficient of variation for fatigue exponent K_2	Slight Increase	Insensitive	Insensitive	Insensitive	Insensitive
CORLCOEF, Value B Representing Materials in the System's Spatial Auto Correlation Coefficient	Insensitive	Insensitive	Insensitive	Insensitive	Insensitive
CORLEXP, the Value C in the exponent for the system's spatial auto correlation function	Insensitive	Insensitive	Great Decrease	Increase	Increase
DURATION, Duration of Wheel Load at a point	Insensitive	Insensitive	Insensitive	Insensitive	Insensitive

Table 26. Summary of sensitivity analysis for VESYS IIM based on increasing magnitudes of independent variables (cont.)

	Effects on Output Responses				
Independent Variables	Damage Index	Rut Depth	Slope Variance	Present Serviceability Index	Service Life
K1K2CORL	Insensitive	Insensitive	Insensitive	Insensitive	Insensitive
LAMBDA, Truck Traffic in axles/day	Slight Increase	Increase	Increase	Decrease	Great Decrease
Creep-Compliance Arrays:					
LAYER1, Surface Layer	Increase	Slight Increase	Slight Increase	Slight Decrease	Slight Decrease
LAYER2, Base Material	Insensitive	Slight Increase	Slight Increase	Slight Decrease	Slight Decrease
LAYER3, Subgrade	Insensitive	Increase	Slight Increase	Decrease	Decrease
PSIFAIL	Insensitive	Insensitive	Insensitive	Insensitive	Great Increase
QUALITYO, Initial Present Serviceability Index	Insensitive	Insensitive	Insensitive	Great Increase	Great Increase
RADIUS, Radius of Assumed Circular Tire "Footprint"	Insensitive	Insensitive	Insensitive	Insensitive	Insensitive

Table 26. Summary of sensitivity analysis for VESYS IIM based on increasing magnitudes of independent variables (cont.)

Effects on Output Responses

Independent Variables	Damage Index	Rut Depth	Slope Variance	Present Serviceability Index	Service Life
ST DEVO, standard Deviation of QUALITYO	Insensitive	Insensitive	Insensitive	Insensitive	Insensitive*
Fatigue Life Potential Array (NFAIL):					
STRNCOEF, Fatigue Coefficient K_1; STRNEXP, Fatigue Exponent K_2	Great Decrease	Insensitive	Insensitive	Slight Increase	Insensitive
TEMPS, Temperature Array	Decrease	Slight Increase	Slight Increase	Slight Decrease	Slight Decrease
THICK1, Thickness of layer 1	Decrease	Slight Decrease	Slight Decrease	Slight Increase	Slight Increase
THICK2, Thickness of Base Layer	Insensitive	Slight Decrease	Slight Decrease	Slight Increase	Slight Increase

Table 26. Summary of sensitivity analysis for VESYS IIM based on increasing magnitudes of independent variables (cont.)

Effects on Output Responses

Independent Variables	Damage Index	Rut Depth	Slope Variance	Present Serviceability Index	Service Life
TOLERANCE, Minimum Reliability for Service Life Predictions	Insensitive	Insensitive	Insensitive	Insensitive	Slight Decrease
VARCOEF1, Variance of Creep compliance for the A.C. Surface Layer	Insensitive	Insensitive	Slight Increase	Slight Decrease	Slight Decrease
VARCOEF2, Variance of Creep compliance for the Base Material	Insensitive	Insensitive	Slight Increase	Slight Decrease	Slight Decrease
VARCOEF3, Variance of Creep compliance for the subgrade	Insensitive	Insensitive	Increase	Slight Decrease	Decrease
VCAMP, Variance of AMPLITUD	Insensitive	Insensitive	Insensitive	Insensitive	Insensitive
VCDUR, Variance of DURATION	Insensitive	Insensitive	Insensitive	Insensitive	Insensitive

*ST DEVO is insensitive at Tolerance = 50, but for higher values would cause a slight decrease.

Table 27. List of sensitive independent variables by output response and in order of sensitivity ranking (insensitive variables omitted).

Cracking Damage Index	Rut Depth	Slope Variance	Present Serviceability Index	Service Life
NFAIL	ALPHA(1)	AMPLITUD	ALPHA(1)	ALPHA(1)
AMPLITUD	AMPLITUD	ALPHA(1)	AMPLITUD	AMPLITUD
THICK1	GNU(1)	CORLEXP	GNU(1)	LAMBDA
TEMPS	LAMBDA	GNU(1)	LAMBDA	GNU(1)
LAYER1	ALPHA(3)	LAMBDA	ALPHA(3)	CORLEXP
COEFK2	LAYER3	VARCOEF3	CORLEXP	VARCOEF3
LAMBDA	LAYER1	ALPHA(3)	LAYER3	LAYER3
	LAYER2	LAYER3	VARCOEF3	ALPHA(3)
	THICK1	THICK2	LAYER1	LAYER1
	THICK2	LAYER1	THICK2	LAYER2
	TEMPS	VARCOEF2	THICK1	THICK2
		VARCOEF1	LAYER2	VARCOEF1
		LAYER2	TEMPS	VARCOEF2
		THICK1	VARCOEF2	THICK1
		TEMPS	VARCOEF1	
			NFAIL	

Reviewing Tables 26 and 27, it can be seen that:

1. Fatigue life potential is of paramount importance for cracking, while those factors controlling horizontal strain are next in importance. Truck traffic and the stochastic variation of the fatigue exponent K_2 are also significant.

2. The permanent deformation characteristics of the surface layer and the wheel loading are very important for rut depth, slope variance, serviceability and service life. The truck traffic; CORLEXP; the permanent deformation characteristics, stiffness and variation in stiffness of the subgrade are also important, while the thicknesses and stiffnesses of the surface and base layers are only of slight importance.

3. Temperature is fairly important to prediction of cracking, but only of slight importance to the calculation of the other responses. Its effect as used in the roughness model of VESYS IIM is only to alter the layer stiffnesses. If ALPHA(1) and GNU(1) were correctly characterized as temperature dependent, temperature might be expected to rise in importance.

Further review of Table 26 indicates that primary emphasis must be placed on reliable values of ALPHA(1) and GNU(1), which are "coupled" values obtained from the same dynamic test on the A.C. surface material and on reliable estimates of the wheel load and tire pressure distribution. If prediction of cracking damage (a minor term in the AASHO serviceability equation) is of special interest, a reliable relation for fatigue life potential must also be obtained.

A number of variables appear to have so little effect that much less effort is warranted to obtain relatively high accuracy for their magnitudes. This fortunately includes a number of variables that are difficult to evaluate such as the time-temperature shift function for the A.C., wheel load duration and its distribution, permanent deformation characterization for the base material, the correlation between the fatigue coefficients K_1 and the exponent K_2, creep-compliance for the various layers and stochastic distributions for several of the variables. Estimates will suffice for a number of these variables and could be added to the computer program as constants for use when values were not available and not furnished as input.

For the most part, the importance of the independent variables to the computed responses is consistent with known physical realities. The low degrees of importance of some of the factors are surprising at first glance and a few are discussed below:

1. The stiffnesses of the various layers (as defined in this case by LAYER1, LAYER2 and LAYER3) are usually considered to be quite important to an elastic layer analysis as they affect considerably the horizontally stresses and strains at the bottom of the surface layer.

However, horizontal stresses and strains primarily affect cracking, which has little effect on serviceability as defined by the AASHO equation. In reality, cracking does have a long-term effect in expediting deterioration of the pavement through decreased stiffness of the surface layer due to cracks and of the base and subgrade due to moisture infiltration through surface cracks. As PSI is calculated at a particular time point and these sources of deterioration are not modeled in VESYS IIM, the calculated service life also is less sensitive to cracking and hence layer stiffnesses than it probably is in reality. Creep itself was not significant because the moving wheel loads are at a point such a short time that the band of their variation is fairly narrow when converted to time on a creep-compliance curve.

2. Thicknesses of the surface and base layers are obviously important to the performance of a pavement, but their range is highly correlated to truck traffic and the variation between projects at a particular traffic level is not very large in practice or in this sensitivity analysis. Also, both in reality and VESYS IIM, the increase in a layer thickness increases the "gauge length" for vertical unit strain to apply while at the same time reducing the vertical strain. Therefore, the increase in layer thickness is partially compensatory in effect and may not be very effective in decreasing rut depth, assuming that the materials are not overstressed and failure underway in either case.

The results of the sensitivity analysis reported appear to be valid and may be used by designers and researchers utilizing VESYS IIM to establish priorities in their efforts to define the input values for the many independent variables. They also offer valuable insight into the nature of the VESYS IIM analytical and predictive model itself.

REFERENCES

1. Moavenzadeh, F., Soussou, J. E., Findakly, H. K., "Synthesis for Rational Design of Flexible Pavements", Part I, School of Engineering, Massachusetts Institute of Technology, Cambridge, Mass., 02139, January 1973.

2. Soussou, J. E., F. Moavenzadeh and H. K. Findakly, "Synthesis for Rational Design of Flexible Pavements, Part II", FHWA Contract No. FH-11-776, January 1973.

3. Moavenzadeh, F., Soussou, J. E., Findakly, H. K., Brademeyer, B., "Synthesis for Rational Design of Flexible Pavements", Part III, "Operating Instructions and Program Documentation", February 1974.

4. Draper, N. R., and Smith, H., Applied Regression Analysis, John Wiley & Sons, Inc., 1967.

5. Anderson, Virgil L., and McLean, Robert, Design of Experiments, A Realistic Approach, Marcel Dekker, Inc., New York, 1974.

6. "Fraction Factorial Experiment Designs for Factors at Two Levels," U. S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series - 48, April 15, 1957.

7. Berger, P. D., Technometrics 14:971 (1972).

APPENDIX A

ANALYSIS OF 2^n FRACTIONAL FACTORIALS

Fractional Factorial Concepts

Introduction

It is not practical to attempt to run an experiment containing all the multitudes of combinations of n variables when n is large. In order to accomplish the required goals with a reasonable level of effort, a systematic reduction in the numbers of combinations of variables (factors) to be tested must be applied.

The first reduction may be applied by establishing two levels of values (high and low) for each variable so that the treatment combinations may be limited to 2^n.

When 2^n separate experiments are impractical, further reductions may be made by systematic use of "fractional factorials", which implies omitting selected experiments in such a manner that the major factors or effects of interest are retained in the experiment. The basis for fractional factorials is essentially the concept that the higher-order terms contribute negligibly to the calculated dependent variables in a polynomial series. Fortunately, classical statistical methods exist for rationally designing fractional factorial experiments such that loss of information is minimized.

Statistical Background

A discussion of fractional factorials is given by Anderson and McLean (Ref. 5)[1] and a confounding scheme may be determined by the researcher using the procedure described. However, for experiments using 5-16 factors, designs which minimize the loss of information have already been determined and published by the National Bureau of Standards (NBS). Their publication (Ref. 6)[2] provides the defining effects (or contrasts), the treatment combinations in the principal block, and a list of two-factor interactions, if any, which are confounded.

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[1] Chapter 10; Anderson, Virgil L., and McLean, Robert A., Design of Experiments, Marcel Dekker, Inc., New York, 1974.

[2] "Fractional Factorial Experiment Designs for Factors at Two Levels," U.S. Department of Commerce, National Bureau of Standards, Applied Mathematics Series - 48, April 15, 1957.

Notation

The notation in the NBS publication for treatment combinations uses small letters to denote factors measured at their high levels. Absence of letters means those factors are measured at their low levels. The symbol (1) means all factors are at their low levels.

For example:

Treatment Combination	A	B	C	D	E	F	G
(1)	low	low	low	low	low	low	low
a	high	low	low	low	low	low	low
bg	low	high	low	low	low	low	high
abdg	high	high	low	high	low	low	high
cefg	low	low	high	low	high	high	high

A method is described below to use a standard factorial analysis of variance program to perform the calculations and a separate program to make the correspondence between the analysis of variance output and the confounded effects. The maximum size analysis using the programs described in this paper is a 1/128 replicate of a 2^{15} fractional factorial An example problem for a 1/4 replicate of a 2^7 fractional factorial is included to demonstrate the method. The following definitions will be used in this appendix:

n = total number of factors

2^n = number of treatment combinations if the entire factorial were run

$\frac{1}{r}$ = fractional replicate (r must be a multiple of 2)

m = number of defining effects (see page 260 of Ref 1), power of 2 such that $2^m = r$ (i.e. $\log_2 r = m$)

k = n-m, or number of factors submitted to the analysis of variance program. For program FAØV01*, k must be 8 or less.

2^k = number of treatment combinations actually run in the experiment.

Therefore,

$$\frac{1}{r} \text{ replicate of } 2^n \text{ factorial} = \frac{2^n}{r} \text{ treatment combinations}$$

For example,

If n = 7 and r = 4, m = 2 and k = 5

$$\frac{1}{4} \text{ replicate of } 2^7 = \frac{2^7}{4} = 2^5 = 32 \text{ treatment combinations}$$

*Based on BMD2V from the 1968 edition of Biomedical Computer Programs, UCLA, this program was developed by the Center for Highway Research, University of Texas at Austin.

The observations on these 32 treatment combinations would be submitted to the factorial analysis of variance program as if they represented a complete 2^5 factorial.

Procedure Used

Overview

The general procedure described in this section is explained in greater detail following this overall view. An example problem follows which furnishes specific directions.

The researcher first calculates a defining contrast or selects a confounding scheme from the National Bureau of Standards publication (Ref 2). The 2^k treatment combinations which will actually be measured in the experiment can then be identified by the procedure in Reference 1 or from the NBS publication.

The observations on the 2^k combinations are submitted to a standard analysis of variance program. This appendix assumes that FAØVO1, or BMD2V will be used. The data will appear to FAØVO1 as a 2^k factorial. Each effect in the analysis of variance will be confounded with as many other effects as appear in the defining effects and generalized interactions. To interpret the FAØVO1 output, it is necessary to know with which effects each term in the analysis of variance is confounded.

A separate program identifies all the confounded effects for the user. Program FRACT (See Appendix B for program listing, input guide and example input) requires the user to provide the defining effects and a list of terms corresponding to the terms in the 2^k analysis of variance. FRACT then computes all the effects with which each term is confounded, and prints all those through the 4-factor interactions. Printing is suppressed for 5-factor and higher interactions.

The general detailed procedure is as follows:

Identify the Treatment Combinations to be Run

When the defining effects have been selected, the 2^n treatment combinations can be assigned to the blocks. The researcher may choose at random which block to run. For each design in the NBS publication, however, a list of treatment combinations in the principal block is provided. The principal block is that which contains the treatment combination in which every factor appears at the low level. While it is very convenient to use the list provided in the publication, the selection of the block is immaterial for the analysis of variance technique described below. The example problem assumes the treatment combinations listed in the NBS publication are to be run.

Define the 2^k Analysis of Variance

The observations of the 2^k treatment combinations will be submitted to FAØV01 as a complete factorial with k factors. It is necessary to select the k factors and make a list of all the main effects and interactions.

Standard analysis of variance programs require a full factorial. For analysis of a fractional factorial, it is necessary to present the input data so that it appears to be a full 2^k factorial, where k is equal to n-m. This implies that m factors must be eliminated before the fractional factorial may be run as a full factorial on FAØV01. Care must be taken when selecting the particular m letters which are to be ignored in the reduced factorial. One method for selecting the factors to be removed (frequently called "dead letters") is given by Berger (Ref 7)[1]. The selection process may best be explained by example.

Consider a ¼ replicate of a 2^7 factorial with the 7 factors labelled A, B, C, D, E, F, and G. The selected defining effects are ABCE and ABDFG. These defining effects determine the dead letters. First select a letter from the first defining effect which preferably is not in the other defining effect. Suppose E is selected because E appears in ABCE, but not in ABDFG (C could just as well have been used). Next choose D because it occurs in ABDFG and not in ABCE. (for the same reason F or G could have been selected). Since two dead letters are required to reduce the 2^7 fractional factorial to a full 2^5 factorial, D and E will be the designated dead letters.

Dead letter E is associated with the three-way interaction ABC in the defining effect ABCE. Similarly, D is associated with ABFG. Based on this concept we must identify the 32 treatment combinations in the reduced factorial. Treatment combinations are denoted by lower-case letters, which indicate the factors present at a high level (all other factors having low values). The admissable five letters (a, b, c, f and g) are associated with the two dead letters (d and e) as follows:

1. a is with dead letters d and e
2. b is with dead letters d and e
3. c is with dead letter e
4. f is with dead letter d
5. g is with dead letter d

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[1]Berger, P.D., <u>Technometrics</u> 14:971 (1972).

The treatment combinations in the principle block of the fractional factorial are listed on page 19 of Reference 2. Using the above relationships, they can be associated with the terms in the full 2^5 factorial by ignoring all occurrances of the letters d and e as indicated in Table 28.

Print Analysis of Variance Terms With Their Aliases

Program FRACT prints each term in the 2^k analysis of variance and the terms in the 2^n design with which it is confounded. Five-factors and higher interactions are not printed.

If one of the 2^k analysis of variance terms corresponds to a term in the defining contrast, an incorrect set of factors has been chosen. The program prints a message and stops. The user must then redefine the factors in the 2^k analysis of variance and rerun FRACT.

The program may be run from a remote terminal or on cards. The input guide may be interpreted as a reference to a physical card or to a line typed on a remote terminal.

The output of FRACT lists each analysis of variance term and its aliases. It suppresses the printing for any aliases which are 5-factor or higher. Each term or one of its aliases must be identified as the effect of interest. Many terms and aliases will consist entirely of interactions which are not of interest, however. These should be identified for pooling into the error term.

Run Analysis of Variance

Run a 2^k analysis of variance program. If FAØV01 is used, a precise order of cards is required. It is critical to arrange the data exactly in order, or wrong results will be produced. See the example problem for a detailed sample of the input order.

Perform Analysis of Variance Tests

Label each term in the analysis of variance table with the main effect or interaction which this term represents. Refer to the output of FRACT to do the labeling. Pool the higher order interactions to form an error term. Pooling is accomplished by adding the sums of squares for each term to be pooled, and then calculating the pooled mean square by dividing the new sum of squares by the sum of the degrees of freedom associated with these terms (see the example problem).

Table 28. Order of data cards for FAØV01

Card	2^5	2^{7-2}	Col.	1 A	2 B	3 C	4 D	5 E	6 F	7 G	9 + 80 Y's
				Treatment Combinations							
1	(1)	(1)		0	0	0	0	0	0	0	
2	g	dg		0	0	0	1	0	0	1	
3	f	df		0	0	0	1	0	1	0	
4	fg	fg		0	0	0	0	0	1	1	
5	c	ce		0	0	1	0	1	0	0	
6	cg	cdeg		0	0	1	1	1	0	1	
7	cf	cdef		0	0	1	1	1	1	0	
8	cfg	cefg		0	0	1	0	1	1	1	
9	b	bde		0	1	0	1	1	0	0	
10	bg	beg		0	1	0	0	1	0	1	
11	bf	bef		0	1	0	0	1	1	0	
12	bfg	bdefg		0	1	0	1	1	1	1	
13	bc	bcd		0	1	1	1	0	0	0	
14	bcg	bcg		0	1	1	0	0	0	1	
15	bcf	bcf		0	1	1	0	0	1	0	
16	bcfg	bcdfg		0	1	1	1	0	1	1	
17	a	ade		1	0	0	1	1	0	0	
18	ag	aeg		1	0	0	0	1	0	1	
19	af	aef		1	0	0	0	1	1	0	
20	afg	adefg		1	0	0	1	1	1	1	
21	ac	acd		1	0	1	1	0	0	0	
22	acg	acg		1	0	1	0	0	0	1	
23	acf	acf		1	0	1	0	0	1	0	
24	acfg	acdfg		1	0	1	1	0	1	1	
25	ab	ab		1	1	0	0	0	0	0	
26	abg	abdg		1	1	0	1	0	0	1	
27	abf	abdf		1	1	0	1	0	1	0	
28	abfg	abfg		1	1	0	0	0	1	1	
29	abc	abce		1	1	1	0	1	0	0	
30	abcg	abcdeg		1	1	1	1	1	0	1	
31	abcf	abcdef		1	1	1	1	1	1	0	
32	abcfg	abcefg		1	1	1	0	1	1	1	

Note that the cards are ordered according to factors A, B, C, F, and G without regard to factors D and E.

Example Problem

Suppose a ¼ replicate of a 2^7 factorial is to be analyzed. The experimenter will run $\frac{2^7}{4} = 2^5 = 32$ treatment combinations instead of $2^7 = 128$ combinations required if a full factorial were run. The NBS publication, page 19, will be used to determine the defining contrast and the list of treatment combinations in the principal block.

In this problem:

$n = 7$

$2^n = 128$

$\frac{1}{r} = \frac{1}{4}$

$m = 2$

$k = 5$

$2^k = 32$

Factors: A,B,C,D,E,F,G

Defining contrast: I = ABCE = ABDFG = CDEFG

2-factor interactions not measureble: AB, AC, AE, BC, BE, CE

Select the Treatment Combinations to be Run

Assume the principal block will be selected. The treatment combinations listed on page 19 of the NBS publication (Ref 2) will be run. These are:

(1)	abcdef	abcdeg	fg
abce	df	dg	abcefg
adefg	bcg	bcf	ade
bcdfg	aeg	aef	bcd
bdefg	acg	acf	bde
acdfg	beg	bef	acd
ab	cdef	cdeg	abfg
ce	abdf	abdg	cefg

Define the 2^5 Factorial for FAØV01

There are 21 possible ways that 5 factors can be chosen from the 7 total factors. Those which will produce an interaction term which matches a term in the defining contrast cannot be used. In this example problem, the factors A, B, C, F, and G are chosen to identify the 2^5 factorial for FAØV01.

List the Terms in the 2^5 Analysis of Variance With Their Aliases

The list of treatment combinations to be run will be entered as part of the input for program FRACT. For FRACT, the terms need not be in the same order as they will appear in the output of FAØV01. However, it is much more convenient if the order is consistent with FAØV01. List the main effects first, then the 2-factor interactions, and then the 3, 4, and 5-factor interactions. The letters in each term must be arranged alphabetically.

Note that FAØV01 labels the terms with numeric rather than letter codes. Factor A = 1, factor B = 2, etc. For this procedure, it is less confusing to use letters. FRACT requires the input to be in letter form.

The analysis of variance table for this problem will contain the following terms. Beside each alphabetic code is the numeric label which FAØV01 will print.

Letter codes	FAØV01 Labels	Letter Codes	FAØV01 Labels
A	1	ABF	124
B	2	ABG	125
C	3	ACF	134
F	4	ACG	135
G	5	AFG	145
AB	12	BCF	234
AC	13	BCG	235
AF	14	BFG	245
AG	15	CFG	345
BC	23	ABCF	1234
BF	24	ABCG	1235
BG	25	ABFG	1245
CF	34	ACFG	1345
CG	35	BCFG	2345
FG	45	ABCFG	12345 (Residual)
ABC	123		

For this problem, n = 7, r = 4, and the defining contrast contains the terms ABCE, ABDFG, and CDEFG. These values, along with the list of terms in the 2^5 analysis of variance table are input to FRACT. See Appendix B for the input guide to FRACT, and Table 29 for a copy of the input for this example problem.

Table 29 Input to FRACT for example problem

User Identification Card
Password Card
Job Card
EXECPF 1466 FRACT
7-8-9 card (end-of-record)
7 4 1/4 replicate of 7 factors

ABCE ABDFG CDEFG

1	A	B	C	F	G	AB	AF	AF	AG
2	BC	BF	BG	CF	CG	FG	ABC	ABF	ABG
3	ACF	ACG	AFG	BCF	BCG	BFG	CFG	ABCF	ABCG
4	ABFG	ACFG	BCFG	ABCFG					

6-7-8-9 card (end-of-record)

The output from FRACT lists each analysis of variance term and its aliases. For each term, identify either the term or one of its aliases as the term of interest. Identify those which will be pooled as an error term. Table A3 shows how this was done in our example problem.

In this example, some of the 2-factor interactions are confounded with each other. Unless the experimenter has reason to believe one of the pair is negligible, the effect represented by that term will be attributed to both 2-factor interactions since it is impossible to separate them. The sample problem contains these terms because a ¼ replicate of a 2^7 was chosen for illustration. Ordinarily a researcher would select a design in which all 2-factor interactions are measurable.

Run the Analysis of Variance

If FAØV01 is used, great care must be exercised to make sure the 32 data cards are submitted to FAØV01 in the correct order. Otherwise, the results will be wrong.

Each data card must be carefully identified so it can be easily arranged in its proper order. In the example problem, the data cards are punched as follows:

Col.	1	0 =	low	level	of	factor	A; 1 =	high	level	of	factor	A
	2	0 =	"	"	"	"	B; 1 =	"	"	"	"	B
	3	0 =	"	"	"	"	C; 1 =	"	"	"	"	C
	4	0 =	"	"	"	"	D; 1 =	"	"	"	"	D
	5	0 =	"	"	"	"	E; 1 =	"	"	"	"	E
	6	0 =	"	"	"	"	F; 1 =	"	"	"	"	F
	7	0 =	"	"	"	"	G; 1 =	"	"	"	"	G

9 to 80: dependent variable(s). Punch in as many columns as needed.

As explained above we are concerned only with factors A, B, C, F and G for this 2^5 factorial. Factors D and E will be ignored in ordering the data cards.

The order of cards for FAØV01 is listed in Table 28. In general, the levels of the highest numbered factor (or factor having the letter nearest the end of the alphabet) vary most rapidly. The levels of the next lower numbered factor (or factor with the letter next closest to the first of the alphabet) are varied next most rapidly, etc.

Table 28 shows the order of cards for the example problem. If the cards are to be arranged by sorting them on a sorting machine, sort first on column 7, then on column 6, 3, 2, and 1, respectively. The order can be easily checked by viewing columns 1, 2, 3, 6, and 7 as a 5-digit number, and then making sure this 5-digit number is arranged in increasing order of magnitude.

Analysis of Variance Tests

The analysis of variance terms and aliases as identified by FRACT are listed at the left margin of Table 30. The corresponding terms printed by FAØV01 are printed to the right, and relabeled with an alias where appropriate. Interactions of three or more factors are to be pooled. Table 31 shows the form of the final analysis of variance.

Table 30. Terms in 2^5 analysis of variance

Analysis of Variance Terms and Aliases	FAØV01 Labels	Relabeled
A = BCE = BDFG	1	A
B = ACE = ADFG	2	B
C = ABE = DEFG	3	C
F = ABDG = CDEG	4	F
G = ABDF = CDEF	5	G
AB = CE = DFG	12	AB or CE
AC = BE	13	AC or BE
AF = BCEF = BDG	14	AF
AG = BCEG = BDF	15	AG
BC = AE	23	BC or AE
BF = ACEF = ADG	24	BF
BG = ACEG = ADF	25	BG
CF = ABEF = DEG	34	CF
CG = ABEG = DEF	35	CG
FG = ABD = CDE	45	FG
ABC = E = CDFG	123	E
ABF = CEF = DG	124	DG
ABG = CEG = DF	125	DF
ACF = BEF = BCDG = ADEG	134	Pool
ACG = BCDF = ADEF	135	Pool
AFG = BD = ACDE	145	BD
BCF = AEF = ACDG = BDEG	234	Pool
BCG = AEG = ACDF = BDEF	235	Pool
BFG = AD = BCDE	245	AD
CFG = ABCD = DE	345	DE
ABCF = EF = CDG	1234	EF
ABCG = EG = CDF	1235	EG
ABFG = CEFG = D	1245	D
ACFG = BEFG = BCD = ADE	1345	Pool
BCFG = AEFG = ACD = BDE	2345	Pool
ABCFG = EFG = CD = ABDE	12345	CD

Note that aliases for 5-factor and higher-order interactions are not listed because they are not printed by program FRACT.

Table 31. Analysis of variance table

FAØV01 Labels	Sources	Degrees of Freedom	Sum of Squares	Mean Squares	F Ratio
1	A	1			
2	B	1			
3	C	1			
1245	D	1			
123	E	1			
4	F	1			
5	G	1			
12	AB or CE	1			
13	AC or BE	1			
245	AD	1			
23	AE or BC	1			
14	AF	1			
15	AG	1			
145	BD	1			
24	BF	1			
25	BG	1			
12345	CD	1			
34	CF	1			
35	CG	1			
345	DE	1			
125	DF	1			
124	DG	1			
1234	EF	1			
1235	EG	1			
45	FG	1			
134 + 135 + 234 + 235 + 1345 + 2345	Error	6			
		31			

The sums of squares and mean squares are printed by FAØV01. The mean square error is calculated by dividing the total of the sums of squares for the pooled terms by 6. F-tests can then be made by dividing each mean square by the mean square for error.

APPENDIX B

DESCRIPTION OF PROGRAM FRACT

Function of the Program

Program FRACT was written by Shirley Selz as an aid in designing the fractional factorials to be used in the main sensitivity analysis. It was specifically written for this project and is dependent on Control Data computer hardware. A description of the program is published here as an aid to other researchers, who are welcome to use it either as a whole or in part. Austin Research Engineers makes no formal guarantee as to the correctness of this program, those who use it do so at their own risk. We do affirm, however, that it has been carefully checked out and there are no known bugs. A listing of the program is provided in Table 32.

The purpose of the program is to identify the main effects and interactions which are confounded in each term of a 2^n fractional factorial. This is necessary for performing factorial analysis of variance using packaged statistical routines. It can also be of great assistance in assigning letter-names to the independent variables in an experiment. If particular interactions are of interest, and not all interactions of that type are measurable in the fractional, then knowing the confounded interactions makes it possible to assign letters to variables in such a way that the desired interactions are clear of low-order aliases.

FRACT accepts as input the number of factors, the fractional replicate to be run, the defining contrast and generalized interactions, and the letters by which the terms will be known to the factorial analysis of variance program. It prints out each such term followed by all main effects and interactions in the actual fractional factorial with which it is confounded. To avoid excessively lengthy printout, five-factor and higher interactions are not listed. An example input deck is shown in Table 29.

Input Guide For Program Fract

I. Order of Cards

1. System Cards
2. Problem Card
3. Defining Contrast Card(s)
4. Analysis of Variance terms
5. End of file

II. Preparation of Cards

1. System Cards

User Identification Card
Password Card
Job Card
EXECPF 1466 FRACT or RUN(S) / LGO. if deck is used
7-8-9 card

2. Problem Card

Col. 1, 2 n = total number of factors

3-5 r = fractional replicate

6-80 any information to be printed on the output as a title

3. Defining Contrast Card(s)

There will be r-1 terms in the defining contrast.

Col. 1-15 first term in defining contrast (alphabetic letters, left justified)

16	blank
17-31	second term in defining contrast
32	blank
33-47	third term in defining contrast
48	blank
49-63	fourth term in defining contrast
64	blank
65-79	fifth term in defining contrast
80	blank

If there are more than 5 terms in the defining contrast, continue punching in the same format on subsequent cards.

4. Analysis of Variance Terms for 2^k Analysis

There will be $2^k - 1$ terms in the 2^k analysis of variance. FAØV01 handles a maximum of 8 factors.

Col. 1-5 any identification, such as card sequence number.
6-13 first term in 2^k analysis of variance (left justified)
14-21 second " " " " " " " "
22-29 third " " " " " " " "
30-37 fourth " " " " " " " "
38-45 fifth " " " " " " " "
46-53 sixth " " " " " " " "
54-61 seventh" " " " " " " "
62-69 eighth " " " " " " " "
70-77 ninth " " " " " " " "

Continue punching analysis of variance terms in the same format on subsequent cards.

5. <u>End of File</u>

Col. 1 = 6-7-8-9

Table 32. Listing of Program FRACT

```
      PROGRAM FRACT(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION ID(255,15),L(15),NAME(255,15)
      INTEGER REP,TERM(255,4)
      INTEGER TITLE(8),DC(255,15),SUM
      COMMON/LL/LETTER(15)
      DATA LETTER/1HA,1HB,1HC,1HD,1HE,1HF,1HG,1HH,1HJ,1HK,
     11HL,1HM,1HN,1HO,1HP/
      READ(5,10) NFACT,REP,TITLE
   10 FORMAT (I2,I3,7A10,A5)
C     N = NUMBER OF TERMS IN DEFINING CONTRAST
      N = REP - 1
C     READ DEFINING CONTRAST (5 TERMS PER CARD)
      READ (5,20) ((NAME(I,J),J=1,15),I=1,N)
   20 FORMAT (5(15A1,1X))
C     COMPUTE NUMBER OF TERMS IN FAOV01
C     M = NUMBER OF ANOVA TERMS
      M = ((2**NFACT)/REP) - 1
      WRITE (6,100) TITLE, NFACT,REP,M
  100 FORMAT (1H1,2X,8A10/3X,*NUMBER OF FACTORS =*I3/
     13X,*FRACTIONAL REPLICATE =*I4/3X,*NUMBER OF TERMS*
     21X,*IN ANALYSIS OF VARIANCE OUTPUT =*I4)
      WRITE (6,101)
  101 FORMAT (*0  TERMS IN DEFINING CONTRAST*)
      WRITE (6,102) ((NAME(I,J),J=1,15),I=1,N)
  102 FORMAT (6X,8(15A1,1X))
      WRITE (6,103)
  103 FORMAT (//)
C
C     CALL SUBROUTINE INTERP TO CONVERT LETTERS IN DEFINING
C     CONTRAST TO NUMBER CODES
      CALL INTERP(NAME,DC,N)
C
      DO 22 I = 1,255
      DO 22 J = 1,15
   22 NAME(I,J) = 1H
C     READ FAOV01 TERMS
      READ (5,25) ((NAME(I,J),J=1,8),I=1,M)
   25 FORMAT (5X,9(8A1))
      CALL INTERP(NAME,ID,M)
   30 DO 60 I = 1,M
      WRITE (6,130)  (NAME(I,J),J=1,8)
  130 FORMAT (1X,8A1)
      NN = 0
```

```
C       J = COUNTER FOR N TERMS IN DEFINING CONTRAST
C       K = COUNTER FOR FACTORS
        DO 34 J = 1,N
        SUM = 0
        DO 40 K = 1,15
        L(K) = DC(J,K) + ID(I,K)
        IF (L(K).EQ.2)  L(K) = 0
        SUM = SUM + L(K)
   40   CONTINUE
        IF (SUM.GT.4) GO TO 34
C       TERM WITH WHICH THIS FAOV TERM IS CONFOUNDED IS A 4-WAY
C       INTERACTION OR LESS.
C       CHECK TO SEE IF SUM.GT.0.  IF NOT, AN ANOVA TERM IS
C       THE SAME AS A TERM IN THE DEFINING CONTRAST.  WRITE
C       MESSAGE AND STOP.
        IF (SUM.GT.0)  GO TO 41
        WRITE (6,113) I,J
  113   FORMAT (//*  ANOVA TERM NUMBER*I4,* IS THE SAME AS TERM NUMBER*
       1I4,* IN THE DEFINING CONTRAST.*/*  PROGRAM STOP.  CHOOSE A*
       21X,*DIFFERENT SET OF ANOVA FACTORS AND RERUN PROGRAM FRACT.*)
        STOP
   41   CONTINUE
        NN = NN + 1
        TERM(NN,1)=TERM(NN,2)=TERM(NN,3)=TERM(NN,4) = 1H
        KK = 0
        DO 42 K = 1,15
        IF (L(K).EQ.0)  GO TO 42
        KK = KK + 1
        TERM(NN,KK) = LETTER (K)
   42   CONTINUE
   34   CONTINUE
        IF (NN.EQ.0)  GO TO 60
        WRITE (6,115) ((TERM(J,K),K=1,4),J=1,NN)
  115   FORMAT (2X,22(4A1,2X))
   60   CONTINUE
        STOP
        END
```

Table 32. Listing of Program FACT (cont.)

```
      SUBROUTINE INTERP(ALPHA,NUM,M)
      INTEGER ALPHA(255,15),NUM(255,15)
      COMMON/LL/LETTER(15)
      DO 1 I = 1,255
      DO 1 J = 1,15
    1 NUM(I,J) = 0
      DO 40 IJK = 1,M
      J1 = 1
      DO 30 I = 1,15
      IF (ALPHA(IJK,I).EQ.1H )  GO TO 42
      DO 10 J = J1,15
      IF (ALPHA(IJK,I).EQ.LETTER(J))  GO TO 20
   10 CONTINUE
      WRITE (6,200) LETTER
  200 FORMAT (1X,15A1)
      WRITE (6,120) (ALPHA(IJK,II),II=1,15)
  120 FORMAT (///*  CHECK THIS TERM FOR ILLEGAL CHARACTERS*,3X,15A1)
      STOP
   20 NUM(IJK,J) = 1
      J1 = J + 1
   30 CONTINUE
   42 CONTINUE
   40 CONTINUE
      RETURN
      END
```

APPENDIX C

TREATMENT COMBINATIONS AND CALCULATED RESPONSES

A listing of the treatment combinations run and the responses which were obtained from each combination is given here for reference. This information constitutes the "raw data" of the sensitivity analysis. The VESYS IIM computer program was temporarily modified to punch out the treatment combinations, input data values and the levels of the five responses which were of interest. Being in machine-readable form, this information was immediately available for statistical analysis. The files which are printed here were condensed from the data which was punched by the program while the solutions were being calculated.

The units of rut depth are inches, slope variance is in radians times 10^6, and service life is measured in years. Damage index and serviceability index are dimensionless. The treatment combinations which were run and the responses which were observed are listed in Tables 34 and 36.

The notation used there for the treatment combinations is the same as that used in reference 2 except that upper-case letters are used because the computer employed does not recognize lower-case characters. Each combination is described by a list of the letters for the factors which are present at a high level in that particular run. All factors not named are therefore at their low levels. The special notation (1) is used to indicate the treatment combination in which every factor is at its low level (otherwise it would be represented by no letters at all). By convention the letter "I" is not used in naming the factors in an experiment.

Tables 33 and 35 list the independent variables and the letters by which they are labeled in the two factorials. Note that a single variable may appear in each factorial under different letter designations. This is because letters were assigned to the variables with an eye to placing likely three-ways into positions in the factorial where they would be measurable. Since this is a function of the defining contrast and generalized interactions of the fractional design, it was not practical to keep the letter designations constant between the two factorials. Also note that only the factors which are active in the factorial are described by their presence or absence in the treatment combination notation. A variable which is active in one factorial but not the other is held constant at its average value through the set of solutions in which it is not a factor.

Table 33. Variable Assignments for the Cracking Factorial

Factor	Variable Name
A	AMPLITUD
B	THICK1
C	TEMPS
D	LAMBDA
E	COEFK1
F	LAYER2
G	NFAIL (STRNCOEF and STRNEXP)
H	LAYER1
J	VCAMP
K	COEFK2
L	LAYER3

Table 34. Cracking Factorial Runs

Treatment Combination	Damage Index	Treatment Combination	Damage Index
(1)	13.6271	BDFG	2.7275
GHJKL	24.2640	BDEH	19.8614
FHJK	412.6657	BDEGJKL	6.5690
FGL	4.8243	BDEFJK	14.6220
EHL	84.3967	BDEFGHL	11.7758
EGJK	7.9517	BCJK	2.2879
EFJKL	81.3475	BCGHL	.4677
EFGH	14.7654	BCFH	4.4852
DHJL	258.2512	BCFGJKL	.5810
DGK	19.9553	BCEHJKL	12.6544
DFKL	206.2555	BCEG	.2273
DFGHJ	26.8972	BCEFL	.9715
DEJ	39.5149	BCEFGHJK	1.0656
DEGHKL	43.4693	BCDHKL	32.0095
DEFHK	676.3494	BCDGJ	.4205
DEFGJL	16.9514	BCDFJL	2.9659
CHK	73.4700	BCDFGHK	2.6445
CGJL	.6422	BCDEK	3.7869
CFJ	8.4683	BCDEGHJL	1.6541
CFGHKL	3.8232	BCDEFHJ	13.2954
CEKL	15.1065	BCDEFGKL	1.0338
CEGHJ	1.8767	AHKL	3445.3830
CEFHJL	54.7941	AGJ	13.2404
CEFGK	1.2398	AFJL	304.7649
CDJKL	46.5867	AFGHK	86.9922
CDGH	2.6331	AEK	446.7571
CDFHL	91.2515	AEGHJL	52.0059
CDFGJK	3.4886	AEFHJ	1304.1740
CDEHJK	147.6692	AEFGKL	35.8595
CDEGL	1.7987	ADJK	1320.7090
CDEF	14.1451	ADGHL	74.9404
CDEFGHJKL	7.9835	ADFH	2518.3950
BHJ	12.0828	ADFGJKL	96.9681
BGKL	3.3087	ADEHJKL	6419.4440
BFK	7.3698	ADEG	41.0560
BFGHJL	4.3181	ADEFL	595.1730
BEJL	2.5685	ADEFGHJK	174.6929
BEGHK	5.8992	ACL	52.7833
BEFHKL	47.3986	ACGHJK	9.5756
BEFGJ	1.8366	ACFHJKL	1521.0190
BDL	4.5906	ACFG	2.0700
BDGHJK	16.8807	ACEH	215.6753
BDFHJKL	145.2592		

Table 34. Cracking Factorial Runs (cont.)

Treatment Combination	Damage Index	Treatment Combination	Damage Index
ACEGJKL	4.0908	ABEHJK	52.9662
ACEFJK	195.9966	ABEGL	73.9618
ACEFGHL	8.6668	ABEF	155.0836
ACDHJ	543.0631	ABEFGHJKL	33.7219
ACDGKL	10.3634	ABDHK	438.9135
ACDFK	532.6311	ABDGJL	20.9609
ACDFGHJL	13.4887	ABCHJL	45.5438
ACDEJL	137.8559	ABCGK	1.4131
ACDEGHK	18.3813	ABCFKL	38.1323
ACDEFHKL	2583.9940	ABCFGHJ	1.7541
ACDEFGJ	7.2677	ABCEJ	6.4652
ABJKL	90.0857	ABCEGHKL	3.3333
ABGH	9.8761	ABCEFHK	130.6495
ABFHL	171.7088	ABCEFGJL	1.1410
ABFGJK	11.4130	ABCD	12.8751
ABDFJ	313.5884	ABCDGHJKL	8.9331
ABDFGHKL	6.4005	ABCDFHJK	387.3965
ABDEKL	20.0530	ABCDFGL	1.6846
ABDEGHJ	28.7766	ABCDEHL	90.2544
ABDEFHJL	853.5584	ABCDEGJK	2.7617
ABDEFGK	10.6857	ABCDEFJKL	70.9534
		ABCDEFGH	5.5527

Table 35. Variable Assignments for the roughness factorial.

Factor	Variable Name
A	LAYER3
B	LAMBDA
C	ALPHA(3)
D	LAYER2
E	THICK2
F	VARCOEF3
G	TEMPS
H	ALPHA(1)
J	THICK1
K	LAYER1
L	GNU(1)
M	VARCOEF1
N	CORLEXP
O	AMPLITUD
P	VARCOEF2

Table 36. Roughness factorial runs

Treatment Combination	Rut Depth	Slope Variance	Serv. Index	Service Life
(1)	.3408	2.2723	2.7970	7.6233
HJKL	.2892	1.9106	2.9372	10.0637
GHMNOP	.4576	2.4532	2.5965	5.6196
GJKLMNOP	.7883	7.6985	1.2511	1.4841
FKLMOP	.6998	30.1321	.4139	.5103
FHJMOP	.3680	9.6637	1.8280	2.0860
FGHKLN	.3334	1.9964	2.8785	8.6270
FGJN	.3480	2.7330	3.[illegible]169	9.9253
EKMNO	.5831	2.4151	2.4110	4.5705
EHJLMNO	.3671	1.2829	3.0302	11.0358
EGHKP	.2831	2.2046	2.8710	8.5996
EGJLP	.3719	3.4018	2.8530	8.3176
EFLNP	.3456	2.2265	2.8104	8.0179
EFHJKNP	.2435	1.2041	3.1898	15.7928
EFGHLMO	.4354	8.1839	1.8499	2.2628
EFGJKMO	.6460	21.2295	.7947	1.0287
DHKMO	.5252	7.0578	1.8182	2.4006
DJLMO	.6482	13.6185	1.1251	.9734
DGKNP	.4526	1.9777	2.7327	6.7574
DGHJLNP	.2828	.7766	3.6304	20.0345
DFHLP	.3057	5.3854	2.3315	3.9362
DFJKP	.4171	11.5030	1.6754	1.7034
DFGLMNO	.7331	11.7589	1.0700	1.2321
DFGHJKMNO	.4875	6.1354	1.9937	2.7766
DEHN	.2661	.4154	3.4999	25.0603
DEJKLN	.4587	1.4844	2.8603	8.5049
DEGMOP	.6250	11.9079	1.2692	1.4024
DEGHJKLMOP	.5024	7.9852	1.7677	2.0106
DEFHKLMNOP	.5122	5.1983	2.0621	3.0080
DEFJMNOP	.5456	7.0092	1.8081	2.2327
DEFGKL	.4671	7.8488	1.8690	2.2393
DEFGHJ	.2438	2.9803	3.0628	10.8432
CLMP	.2901	2.7976	2.7374	7.2138
CHJKMP	.1949	1.3361	3.1764	14.8100
CGHLNO	.3396	.7947	3.2468	13.7405
CGJKNO	.5694	2.6094	2.3906	4.4649
CFKO	.4926	11.0141	1.5546	1.7631
CFHJLO	.2865	4.6875	2.4028	4.3921
CFGHKMNP	.2201	1.1779	3.2175	14.6598
CFGJLMNP	.3185	2.8606	3.0269	10.7229
CEKLNOP	.5466	2.8941	2.3685	4.3125
CEHJNOP	.2602	.5868	3.4137	20.8313

Table 36. Roughness factorial runs (cont.)

Treatment Combination	Rut Depth	Slope Variance	Serv. Index	Service Life
CEGHKLM	.2332	1.0965	3.2289	15.9526
CEGJM	.2701	1.9712	3.2556	13.7523
CEFMN	.2499	1.0661	3.2313	15.9043
CEFHJKLMN	.2128	.9112	3.3170	21.6639
CEFGHOP	.3108	4.5517	2.3946	4.4059
CEFGJKLOP	.6544	21.9952	.7507	.8873
CDHKLOP	.4113	4.1242	2.3385	4.1419
CDJOP	.4663	5.3325	2.0977	3.0815
CDGKLMN	.3957	1.5868	2.9139	8.8876
CDGHJMN	.1849	.4387	3.8607	28.3503
CDFHM	.1999	2.4794	2.8831	8.6975
CDFJKLM	.3995	11.6721	1.6905	1.5802
CDFGNOP	.5218	5.5346	2.0056	2.9231
CDFGHJKLNOP	.4099	3.9213	2.3905	4.3658
CDEHLMNP	.2199	.5833	3.4501	24.8504
CDEJKMNP	.3440	1.4822	3.0054	10.6542
CDEGLO	.5833	5.0382	1.9523	2.7099
CDEGHJKO	.3688	2.4231	2.7057	6.5541
CDEFHKNO	.3794	1.8241	2.8520	7.6828
CDEFJLNO	.5370	5.1001	2.0337	2.7821
CDEFGKMP	.3513	6.9340	2.1130	3.0210
CDEFGHJLMP	.2069	2.8556	3.1192	11.7395
BHKLMNP	.4521	2.5544	2.6004	5.6762
BJMNP	.4593	2.7978	2.5401	5.2816
BGKLO	1.0473	19.5274	-.1454	.5959
BGHJO	.5801	7.3457	1.6990	2.2306
BFHNO	.6383	7.2045	1.6186	2.1214
BFJKLNO	1.0127	21.5541	-.1280	.4970
BFGMP	.5228	18.1170	1.1747	1.4494
BFGHJKLMP	.4323	13.9338	1.5156	1.9102
BEHLOP	.5968	8.6682	1.5520	1.8253
BEJKOP	.8636	17.9727	.4311	.7153
BEGLMN	.5104	2.1372	2.6065	5.7247
BEGHJKMN	.3762	1.3341	3.0143	10.1598
BEFHKM	.3932	6.4305	2.1093	3.0878
BEFJLM	.4719	12.8064	1.5148	1.4187
BEFGKNOP	.8922	13.8971	.5557	.9587
BEFGHJLNOP	.5626	6.1411	1.8675	2.4872
BDLNOP	.9715	8.2372	.7295	1.1894
BDHJKNOP	.6639	4.1452	1.9379	2.8955
BDGHLM	.4687	6.3385	2.0124	2.8045
BDGJKM	.6328	13.2335	1.2024	1.5888

Table 36. Roughness factorial runs (cont.)

Treatment Combination	Rut Depth	Slope Variance	Serv. Index	Service Life
BDFKMN	.6066	8.1211	1.6156	1.9746
BDFHJLMN	.3658	3.7403	2.5031	5.02[illegible]9
BDFGHKOP	.8246	37.1452	-.0409	.5144
BDFGJLOP	.9746	58.7970	-.8174	.3689
BDEKLMP	.6131	12.3943	1.2898	1.2688
BDEHJMP	.3260	3.5867	2.5581	5.4438
BDEGHKLNO	.7960	3.2973	1.8005	2.7635
BDEGJNO	.8434	4.9113	1.4309	2.0426
BDEFO	.8433	26.0332	.1929	.6145
BDEFHJKLO	.6628	19.9402	.8084	.7392
BDEFGHMNP	.4152	3.6986	2.4433	4.6854
BDEFGJKLMNP	.6675	10.8337	1.2856	1.6706
BCHKN	.3227	.7340	3.2946	14.4233
BCJLN	.4157	1.4939	2.9158	9.1461
BCGKMOP	.7710	18.7697	.6211	.9121
BCGHJLMOP	.4533	6.8857	1.9436	2.6224
BCFHLMNOP	.4757	5.2937	2.1081	3.2863
BCFJKMNOP	.7596	15.1850	.8193	.9979
BCFGL	.4411	9.7573	1.7563	2.1760
BCFGHJK	.3105	5.7444	2.2849	3.8312
BCEHMO	.4434	4.0731	2.3045	4.0440
BCEJKLMO	.8407	16.8319	.5389	.6688
BCEGNP	.3975	1.5224	2.9305	8.7060
BCEGHJKLNP	.3300	1.0391	3.1661	13.9238
BCEFHKLP	.3266	5.2815	2.3199	3.9520
BCEFJP	.3657	7.3585	2.0550	2.7836
BCEFGKLMNO	.8097	9.6149	1.0397	1.3236
BCEFGHJMNO	.4096	3.4045	2.4749	4.8643
BCDMNO	.7087	5.1070	1.7106	2.5026
BCDHJKLMNO	.5482	3.4995	2.2525	3.8184
BCDGHP	.3339	2.7618	2.6992	6.4461
BCDGJKLP	.6047	9.7611	1.4799	1.8462
BCDFKLNP	.5365	6.2166	1.9273	2.6046
BCDFHJNP	.2585	1.6354	3.0480	11.2505
BCDFGHKLMO	.6422	23.0233	.7406	.8643
BCDFGJMO	.7158	36.2522	.2351	.7209
BCDEK	.4872	3.5075	2.3623	4.3034
BCDEHJL	.2794	1.5310	3.0505	11.9473
BCDEGHKMNOP	.6094	4.3159	2.0151	3.2014
BCDEGJLMNOP	.8047	7.6993	1.2124	1.5788
BCDEFLMOP	.7696	32.1370	.2061	.4889
BCDEFHJKMOP	.5011	15.4435	1.2960	1.3311

Table 36. Roughness factorial runs (cont.)

Treatment Combination	Rut Depth	Slope Variance	Serv. Index	Service Life
BCDEFGHLN	.3462	1.7360	2.9311	8.9499
BCDEFGJKN	.5290	5.0237	2.0790	3.1319
AHLMN	.3689	1.3333	3.0231	10.3120
AJKMN	.5071	2.7884	2.4688	4.8318
AGLOP	.8511	17.9823	.4622	.8736
AGHJKOP	.6198	9.5870	1.4376	1.7115
AFHKNOP	.6544	8.7676	1.4492	1.8172
AFJLNOP	.7831	13.1357	.8726	.9462
AFGKM	.5338	15.3549	1.2781	1.4203
AFGHJLM	.3494	8.1712	2.3216	3.9688
AEHKLO	.5974	5.5771	1.8593	2.5742
AEJO	.6361	7.2879	1.6036	1.9297
AEGKLMNP	.5127	3.3082	2.3621	4.2437
AEGHJMNP	.2900	1.0357	3.5229	16.3277
AEFHMP	.3112	6.0044	2.2557	3.6325
AEFJKLMP	.5272	17.5003	1.1926	.9805
AEFGNO	.6886	7.5609	1.4835	1.8852
AEFGHJKLNO	.5984	5.7390	1.8486	2.4458
ADKLNO	.9858	6.5788	.8507	1.4041
ADHJNO	.5336	2.2414	2.5335	5.2048
ADGHKLMP	.4764	7.6027	1.8768	2.4153
ADGJMP	.4914	8.6448	2.0801	3.1146
ADFMNP	.4974	6.1905	1.9968	2.7965
ADFHJKLMNP	.4038	4.5037	2.3413	4.0437
ADFGHO	.6771	22.0841	.7012	.9088
ADFGJKLO	1.0686	62.8269	-1.1582	.3244
ADELM	.4908	5.7820	2.0399	2.8189
ADEHJKM	.3409	3.0433	2.6369	6.0361
ADEGHLNOP	.6333	3.6015	2.0852	3.3200
ADEGJKNOP	.8761	6.8369	1.1206	1.5899
ADEFKOP	.8166	32.6549	.0807	.5279
ADEFHJLOP	.5279	14.5156	1.2966	1.2068
ADEFGHKMN	.4033	2.7990	2.6180	5.8003
ADEFGJLMN	.5131	5.8597	2.3227	4.0420
ACHNP	.2367	.5508	3.4535	21.0966
ACJKLNP	.4668	2.1535	2.6691	6.3180
ACGMO	.5670	7.9587	1.6648	2.1686
ACGHJKLMO	.4742	6.1598	1.9894	2.7894
ACFHKLMNO	.4621	4.0578	2.2958	4.0051
ACFJMNO	.5213	6.4165	1.9090	2.6386
ACFGKLP	.4425	11.3619	1.6464	1.8391
ACFGHJP	.2195	2.9259	3.0948	10.6082

Table 36 .Roughness factorial runs (cont.)

Treatment Combination	Rut Depth	Slope Variance	Serv. Index	Service Life
ACEHKMOP	.4109	5.3703	2.1684	3.4813
ACEJLMOP	.5876	10.7450	1.4131	1.3979
ACEGKN	.3712	.8526	3.1929	12.8731
ACEGHJLN	.2329	.3869	3.8572	25.6283
ACEFHL	.2366	2.5313	2.8410	8.1540
ACEFJK	.3856	6.7569	2.0863	2.9169
ACEFGLMNOP	.5909	7.6375	1.6682	2.0303
ACEFGHJKMNOP	.4085	3.7070	2.4261	4.6041
ACDKMNOP	.6742	5.3657	1.7484	2.5543
ACDHJLMNOP	.3985	2.0028	2.7819	7.1867
ACDGHK	.3104	1.8524	2.9318	8.6420
ACDGJL	.4378	4.2846	2.6275	5.8658
ACDFLN	.4137	3.2613	2.5183	5.1304
ACDFHJKN	.2630	1.5049	3.0804	11.6120
ACDFGHLMOP	.4845	14.8601	1.3512	1.4578
ACDFGJKMOP	.7406	38.3434	.1349	.6462
ACDEP	.3603	3.3094	2.5665	5.4693
ACDEHJKLP	.2882	2.1003	2.8909	9.0126
ACDEGHMNO	.4366	1.6114	2.8421	7.3576
ACDEGJKLMNO	.8413	6.5283	1.2406	1.5725
ACDEFKLMO	.7187	22.4765	.5991	.7033
ACDEFHJMO	.3521	6.9779	2.0694	2.9662
ACDEFGHKLNP	.3193	1.9740	2.8996	8.6154
ACDEFGJNP	.3706	2.8296	2.9703	8.9427
ABKP	.7360	15.0724	.8834	1.081
ABHJLP	.4696	5.5582	2.0984	3.0473
ABGHKMNO	1.0047	8.5738	.6049	1.3490
ABGJLMNO	1.1495	14.3771	-.2417	.8507
ABFLMO	1.1314	70.1623	-1.4554	.2047
ABFHJKMO	.8449	43.4911	-.2179	.4413
ABFGHLNP	.5736	7.1531	1.7660	2.3506
ABFGJKNP	.7715	13.7269	.8715	1.4475
ABEMNOP	.9403	10.2338	.6565	1.2053
ABEHJKLMNOP	.7938	7.3754	1.2687	1.6249
ABEGH	.4832	4.2156	2.2556	3.8209
ABEGJKL	.7735	10.7840	1.0456	1.5641
ABEFKLN	.6964	7.3861	1.4952	1.8125
ABEFHJN	.4019	2.9314	2.5943	5.6578
ABEFGHKLMOP	.9030	46.1061	-.4180	.4081
ABEFGJMOP	.9611	56.5359	-.7462	.4462
ABDHMOP	.9229	26.9026	-.0407	.6338
ABDJKLMOP	1.3699	62.5882	-2.2309	.1731

Table 36. Roughness factorial runs (cont).

Treatment Combination	Rut Depth	Slope Variance	Serv. Index	Service Life
ABDGN	.7658	4.3639	1.6891	2.4781
ABDGHJKLN	.6216	3.2320	2.1824	3.6177
ABDFHKL	.6610	21.4212	.7807	.8504
ABDFJ	.6531	24.7208	.6871	.7941
ABDFGKLMNOP	1.4699	50.1552	-2.5148	.2745
ABDFGHJMNOP	.8344	18.2039	.4987	1.0722
ABDEHKNP	.5632	3.2071	2.2947	4.0518
ABDEJLNP	.6656	3.9935	1.9667	2.7932
ABDEGKMO	1.2358	31.3194	-1.1471	.4543
ABDEGHJLMO	.7988	17.0304	.6310	1.0835
ABDEFHLMNO	.8609	13.3406	.6687	1.0577
ABDEFJKMNO	1.1599	26.5012	-.7684	.4907
ABDEFGLP	.7437	29.1578	.3559	.7683
ABDEFGHJKP	.5301	15.4574	1.2797	1.6102
ABCKLM	.6055	8.8409	1.5494	1.8498
ABCHJM	.3108	2.9820	2.6809	6.3154
ABCGHKLNOP	.7419	5.3962	1.6026	2.4009
ABCGJNOP	.8333	6.1667	1.2999	1.9294
ABCFOP	.8078	35.8862	.0277	.5849
ABCFHJKLOP	.6556	25.0736	.6484	.7308
ABCFGHMN	.3795	3.1213	2.5886	5.5460
ABCFGJKLMN	.6835	11.2501	1.2235	1.6319
ABCELNO	.8149	4.2223	1.5996	2.3387
ABCEHJKNO	.5859	2.1785	2.4631	4.8347
ABCEGHLMP	.3742	4.5992	2.3504	4.1890
ABCEGJKMP	.5873	11.4594	1.3964	1.9474
ABCEFKMNP	.5190	6.0214	1.9786	2.8714
ABCEFHJLMNP	.3171	2.4323	2.7976	7.6239
ABCEFGHKO	.6570	16.6179	.9582	1.2536
ABCEFGJLO	.8749	35.1182	-.1247	.5953
ABCDHLO	.6891	8.9215	1.3553	1.8282
ABCDJKO	1.0158	21.7950	-.1376	.6660
ABCDGLMNP	.6389	5.1030	1.8650	2.6402
ABCDGHJKMNP	.4261	2.3964	2.6710	6.0879
ABCDFHKMP	.4465	13.1176	1.5372	1.8244
ABCDFJLMP	.5663	23.7489	.8917	.7995
ABCDFGKNO	1.0608	19.4553	-.1937	.7632
ABCDFGHJLNO	.6304	8.1238	1.5505	1.9864
ABCDEHKLMN	.4508	1.6505	2.8227	7.3001
ABCDEJMN	.4977	2.6173	2.5186	5.1195
ABCDEGKLOP	1.1027	31.0484	-.6884	.4546
ABCDEGHJOP	.5782	7.6119	1.6771	2.2563
ABCDEFHNOP	.6292	7.3599	1.6215	2.2685
ABCDEFJKLNOP	1.0989	23.4265	-.4646	.4748
ABCDEFGM	.5547	15.7071	1.2239	1.4648
ABCDEFGHJKLM	.4393	10.7787	1.6886	2.0449

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