Optimizing Combined Truck Routing and Parking based on Parking Availability Prediction

Final Report

METRANS Project 17-01

October 2018

Petros Ioannou
Filipe de Almeida Araujo Vital

Viterbi School of Engineering
Ming Hsieh Department of Electrical Engineering
University of Southern California
Los Angeles, CA 90089-2560

METRANS
Transportation Center
USC | CSULB

About the METRANS Tier 1 UTC

METRANS UTC is a Tier 1 University Transportation Center funded under the US Department of Transportation's University Transportation Centers Program. Established in 2013, METRANS UTC is a partnership of the University of Southern California and California State University, Long Beach.

METRANS UTC conducts an integrated, multidisciplinary program of research, education and technology transfer aimed at increasing the economic competitiveness of large metropolitan areas through improved transportation system performance. Our research approach is uniquely integrative: we address passengers and freight across all surface transportation modes.   Passengers and freight often share the same infrastructure and compete for the same capacity. The research challenge is how all urban functions can best be managed together. By developing strategies that promote productivity and better integrate modes and users, METRANS UTC will contribute to more efficient, sustainable metropolitan transportation.

U.S. Department of Transportation (USDOT) Disclaimer

The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the information presented herein. This doc-ument is disseminated under the sponsorship of the United States Department of Transportation's University Transportation Centers program, in the interest of information exchange. The U.S. Government assumes no liability for the contents or use thereof.

Acknowledgments

Abstract

According to the U.S. Department of Transportation, 36 states are experiencing shortages in rest areas, affecting the truck drivers' ability to comply with working hour regulations. This issue points to the need of better utilizing the existing truck parking capacity, as expanding the infrastructure would require significant capital investment. In this study we presented a mixed integer programming model to the truck driver scheduling problem under the USA Hours-of-Service regulations which include the parking availability of the rest areas along a route as time-windows conditioned to the scheduling of a rest stop. We also include in our MIP model the USA weekly working hours constraint. This constraint is required for long trips, but is usually not treated as most papers limit the trip's duration and required on-duty time. In order to alleviate scalability issues due to the model's complexity, we also studied when can the optimality of a solution found by a simplified model be guaranteed. In addition, we proposed a formulation for the vehicle shortest path and truck driver scheduling problem, which was modeled as a shortest path problem with resource constraints.

i

Contents

List of Figures

List of Tables

1 Introduction

1.1 Background

It was estimated that, in the year of 2013, trucks were responsible for carrying around 70% (in weight) of USA's total freight shipments, without considering multimodal shipments that use trucks at some point [1]. It is expected that this value will still be as high as 66% by the year of 2040, despite substantial increases in multimodal and rail shipments [1]. This shows just how important trucks are to the USA economy. However, the increasing demand for trucks comes with a need for supporting infrastructure and legislation.

A survey by the American Transport Research Institute (**ATRI**) determined the top issues in the trucking industry, among which are the Hours-of-Service (**HOS**) rules, Compliance, Safety and Accountability scores and Truck Parking [2]. These issues are strongly linked. The HOS rules caused an increase in the demand for parking as the drivers cannot exceed a certain number of hours driving. The increase in demand made the already existing truck parking shortage even more pronounced, making some drivers opt to park illegally, leading to a decrease in safety conditions. The lack of appropriate and convenient parking locations has been the cause of several safety issues over the past years as drivers might be forced to either drive while tired and increase the risk of accidents or park illegally in unsafe locations, which might also pose a safety hazard to them and other drivers. This issue is specially grave for long-haul truck drivers, who can stay weeks or months at a time on the road. For them it is crucial to have proper rest locations.

Over the past years some states evaluated their truck parking availability and the impact of shortages in parking locations. The state of California is one of the states with the largest number of parking spaces. However, due to the large highway network and heavy truck traffic, these parking spaces are too sparse compared to the real necessities of the state. As of 2000, California had estimated the state's total number of parking spaces as 8600, which is 38% of the estimated demand of 22700 parking spaces [3]. Similarly, a 2015 report by the Virginia Department of Transportation calculated a statewide deficit of nearly 5000 parking spaces, which means that the state only satisfies around 60% of the calculated demand (12500 spaces) [4]. According to the U.S. Department of Transportation (**USDOT**), 36 states are experiencing shortages in rest areas, either public or private, which negatively affect truck parking [5]. During a survey, a large percentage of drivers reported difficulty in finding safe parking from 4PM to 5AM, while less than 10% reported difficulty from 5AM to 4PM [5]. However, another survey showed that less than 50% of truck stops operate overcapacity from 7PM to 5AM [5]. These results suggest that the existing parking capacity is not being fully utilized, possibly due to drivers not having enough information on where parking is available, and redistributing the parking demand in time and space can mitigate the truck parking shortage.

Due to the size and importance of the trucking industry, the truck parking shortage has several negative consequences to society, such as the increase of

the following factors:

Illegal Parking Surveys carried out by some states have identified several hundred illegal or unofficial parking locations such as freeway shoulders, freeway entrance and exit ramps, roadways accessing freeway ramps, local streets and commercial areas [5]. The use of these locations poses serious safety hazards to other motorists and truck drivers themselves and expose drivers to become targets of ill-intentioned people.

Unsafe Driving With the driving time limits imposed by the HOS rules, a driver unable to find a suitable parking location may choose to either park illegally or drive illegally and tired. A study by the AAA Foundation for Traffic Safety found that 21% of all accidents in which a person was killed involved a drowsy driver [6]. Although the data used was not specific to trucks, it shows how dangerous drowsy driving can be.

Environmental Impact The shortage of parking spaces forces drivers to drive around looking for parking and/or park at inappropriate locations. Both actions result in an increase in fuel consumption and emissions. While in some truck stops the drivers are able to plug in their vehicles to the grid and avoid idling, no illegal parking location will have this kind of service available, forcing the truck to idle for several hours. Idling is a large source of emissions, fuel expenditure and engine wear, so many states already have laws and incentives for idling reduction [7]. If the drivers often need to find parking in the local streets, they might impact the air quality and health of the nearby communities [8].

Cost As mentioned before, the shortage of parking can have a substantial impact on fuel consumption, be it because of the time spent looking for parking or the time spent idling for lack of proper infrastructure. A study by the University of California, Davis has estimated idling time to be responsible for 8.7% of the total fuel consumption of long-haul trucks [9]. Fuel is responsible for a large share of the operational costs in the trucking industry, making the overall cost highly dependent on fuel costs [10]. Other than the fuel consumption there is still the cost related to vehicle maintenance (10% of total cost in 2015) [10], which can be increased by almost $2,000 a year due to idling [11]. Insurance premiums are another possibility of impacted costs as they can be affected by the number of accidents and robberies involving this kind of vehicle.

These problems affect society as a whole and any improvement could be reflected as economic gains to multiple stakeholders. Reference [12] presents an assessment of the possible benefits of the implementation of Intelligent Truck Parking systems to reduce the parking shortage. This study focus on including parking availability as part of the shipment's planning from the very beginning. We aim to help drivers avoid the parking shortage by planning the routes such

that their required stops are scheduled at locations and times where parking is likely to be available.

1.2 Motivation

The existent work on the truck driver scheduling problem (TDSP) and on the vehicle routing truck driver scheduling problem (VRTDSP) usually assumes that the drivers can park and rest at client locations, without any restriction on the time, and if rest areas are included they are considered to be always available. Some papers don't even consider suitable parking locations at all, they assume that the vehicle can stop anywhere along the route. If the current context were of abundant truck parking this wouldn't be an issue. Nevertheless, many drivers often face difficulty finding parking at certain regions and times of the day. Therefore, the parking availability of the parking locations should be considered in order for the generated schedules to be feasible in practice.

In addition, current work usually assumes that trips are short enough, generally less than a week, so that they are not affected by certain restrictions. However, long-haul truck drivers can stay for several weeks on the road, so they do need to account for those regulations in their planning. Even if we consider that drivers might not have several weeks worth of work scheduled in advance, they should still be able to factor in the impact of their previous and current jobs when planning for the future ones.

In this study we propose optimization models that integrate parking availability in the process of planning truck shipments. In particular, we focus on long-haul truck drivers that might need to stay several weeks on the road and need that their schedules comply with the regulations controlling longer trips.

1.3 Structure of the Report

The rest of this report is organized as follows. Section 2 presents a literature review of the research done on scheduling and routing methods considering HOS rules, and the HOS regulations that were used in most related papers. Section 4 presents the description and model of the truck driver scheduling problem with parking availability. Section 5 presents the description and model of the truck driver routing and scheduling problem with parking availability. Section 6 presents the conclusion.

2 Literature Review

2.1 HOS Regulations

Most of the surveyed work was developed based on the regulations of the USA or European Union, with papers mentioning Canadian and Australian regulations. The different regulations have many similarities, so models developed for one may be very similar to the models for the others depending on which parts of

the regulation are taken into account. In this section we will introduce the USA and European regulations.

2.1.1 USA

The current USA HOS regulation [13, 14] differentiates between driving time, on-duty time and off-duty time. Driving time is all time spent operating a commercial motor vehicle, on-duty time is all the time from when the driver begin to work or is required to be ready for it until the time when the driver is relieved from work, and off-duty time is all the time when the driver is not on-duty. The regulation restricts when the driver can and cannot drive according to the accumulated driving time or the elapsed time during a certain period. Each restriction can be reset by an off-duty period with a minimum duration specified in the regulation. The regulation uses 3 different types of off-duty periods, one with a minimum duration of 30 consecutive minutes, which we will refer to as a *break*, one with a minimum duration of 10 consecutive hours, which we will refer to as a *rest* or *daily rest*, and one with a minimum duration of 34 consecutive hours, which will be referred to as a *weekly rest*. Note that the longer off-duty periods can be used to reset the restrictions related to the shorter ones. The USA HOS regulation can be summarized as follows [13] :

- Daily Driving Time Limit: A driver may drive at most 11 hours in between 2 consecutive *daily rests*.

- 14-Hour Elapsed Time Limit: A driver is not allowed to drive after 14 consecutive hours have elapsed since the last *daily rest* ended.

- Sleeper Berth Provision: Drivers using the sleeper berth provision must take at least 8 consecutive hours in the sleeper berth, plus a separate 2 consecutive hours either in the sleeper berth, off duty, or any combination of the two.

- Rest Breaks: A driver is not allowed to drive after 8 consecutive hours have elapsed since the last *break* ended.

- 60/70-Hour Limit: A driver is not allowed to drive after having been on duty for 60/70 hour in any period of 7/8 consecutive days. The 7/8 consecutive days period can be restarted by taking a *weekly rest*.

The Sleeper Berth Provision, which allows the driver to split a *daily rest* in 2 parts will not be considered in this study.

2.1.2 European Regulations

The European regulation has more restrictive time limits, but it allows for more flexibility a limited number of times each week. Similar to the American regulation, 3 types of off-duty periods are used to reset the different constraints. Most of the restrictions are applied to the driving time, but there are also some

regarding the working/on-duty time. The regulation defines a week as the time between Monday 00:00 and Sunday 24:00. Night time is defined as a period of at least 4 hours between 00:00 and 7:00. The regulation can be summarized as follows:

Driving time [15, 16]

- Breaks: *Break* periods are at least 45 minutes long, and can be split into an off-duty period of at least 15 minutes and another one of at least 30 minutes.

- Daily rest: The *daily rest* period shall be at least 11 hours long, with an exception of going down to 9 hours maximum three times a week. *Daily rests* can be split into 3 hours rest followed by 9 hour rest to make a total of 12 hours daily rest.

- Weekly rest: The *weekly rest* consists of at least 45 continuous hours, which can be reduced every second week to 24 hours. Compensation arrangements apply for reduced *weekly rest* periods. *Weekly rest* is to be taken after six days of working.

- Continuous Driving Limit: After driving for 4.5 hours a driver must stop for a *break* or *daily rest*.

- Daily Driving Time Limit: A driver may drive at most 9 hours in between 2 consecutive *daily rests*. It may be extended to 10 hours twice a week.

- Weekly Driving Time Limit: The driving time may not exceed 56 hours in a single week, and it may not exceed 90 hours in 2 consecutive weeks.

- Interval between Daily rests: The elapsed time between the end of a *daily rest* and the end of the following *daily rest* must not exceed 24 hours.

- Interval between Weekly rests: The elapsed time between the end of a *weekly rest* and the start of the following *weekly rest* must not exceed 144 hours.

Working time [17, 18]

- Maximum weekly working time: The maximum weekly working time is 60 hours, but the average weekly working time over 4 months cannot exceed 48 hours.

- Continuous Work Limit: The driver cannot work for more than 6 hours without a *break*.

- Night work: When working during night time, the maximum daily working time is 10 hours.

2.2 Scheduling Methods with HOS Rules

The TDSP under HOS regulations has been studied both as a part of a vehicle routing problem and by itself. Different versions of the TDSP were considered, the main differences being: where the driver is allowed to rest, the HOS regulation considered, the planning horizon, whether service time at client locations is considered, whether the travel times are time-dependent, and if the objective is to obtain a feasible, sub-optimal or optimal schedule. Reference [19] presented a column generation approach for the Practical Pickup and Delivery Problem, which include both routing and scheduling under specific constraints. The scheduling constraints included the old USDOT HOS rules on driving and working time and a maximum trip time of 6 days. It was assumed that the driver could stop at any point during the trip and a near-optimal schedule was calculated heuristically. [20] considered a TDSP where the vehicle is allowed to park anywhere, with a single time-window per location, and a simplified version of USA's HOS constraints (only the driving time between rests is limited and no service times are considered). A backward search method was used to find a feasible schedule. In [21], Goel also aimed to find a feasible schedule for a fixed route with a single time-window for each location, and presented a breadth-first search algorithm that considers the European HOS regulation for trips of up to 6 days. In [22], Goel and Kok presented a similar method, but now considering USA's HOS regulation and multiple time-windows. A limit of 70 hours of on-duty time per trip was assumed, eliminating the need to consider the rules that regulate longer trips. Similarly to the methods mentioned before, these 2 breadth-first search algorithms do not restrict the allowed parking locations. In [23], a mixed integer programming (MIP) formulation to the scheduling problem with multiple time-windows was presented, together with a dynamic programming approach to solve it more efficiently. This formulation considered the most common types of restrictions present in HOS regulations and can be configured to model or approximate different regulations. Different from the previous ones, this model considers that the driver may rest only at customer locations; rest areas can be modeled as customer locations with zero service time and an unbounded time-window. Similar MIP formulations were used in [24, 25], where the author restricts parking only to rest areas and includes an environmental impact factor dependent on the types of idling used in each stop. The infrastructure available at each rest area and in the truck define which types of idling can be used during the stops. The idling costs are estimated based on CO_2 emissions, equipment costs, and fuel and electricity prices. The model considers a maximum of 60 hours of on-duty time needed for the trip, a planning horizon of 7 days, and that each rest area has a single very-wide time-window. In [26], Kok et al. study the problem of optimizing the departure time and schedule for a fixed route with time-dependent travel times. An integer linear programming model based on the European Union regulation is proposed to optimize the schedule of a single day, with breaks restricted to customer locations. The time-dependent travel times are assumed to be piece-wise linear and are modeled as sets of affine functions controlled by binary variables.

In addition, an integrated approach to solve the routing and scheduling problem using an insertion heuristic and the proposed scheduling algorithm is presented. The model can be extended to include multi-day trips and model rest areas as customer locations with zero service time and unbounded time-windows.

2.3 Routing & Scheduling Methods with HOS Rules

The TDSP often appears as part of the VRTDSP, such as in [27, 28, 29, 30, 31]. Due to the complexity of the combined problem, most authors use methods similar to the ones mentioned in the previous section to calculate the schedules and costs of routes generated heuristically. In [28], the author combines the scheduling model from [25] with an adaptive large neighborhood search to solve the routing and scheduling problem with idling options. In [29] the routing and scheduling of one-day trips with time-dependent travel times under the European regulation was treated. A dynamic programming heuristic was used to generate vehicle routes and an heuristic is used to optimize the vehicle departure time and schedule. Reference [27] presented the only exact method for the routing and scheduling problem. The problem was modeled as a shortest path problem with resource constraints over an extended auxiliary network and was solved using a branch and price algorithm. The scheduling part allowed the vehicle to stop at any point of the route, without considering specific parking locations. They considered the USA and European regulations and a planning horizon of 6 days. In [30], Gaddy applied a modified Clarke-Wright Savings Heuristic to a VRP in order to generate HOS compliant routes with certain restrictions to parking locations. The client locations are used as parking, but only part of the client locations are made available for this purpose.

The TDSP and VRTDSP have seen a lot of progress in the past decade. However there are still areas that can be improved. The surveyed papers usually limit the planning horizon and/or the total trip length in order to avoid the rules that regulate longer trips. Many methods already restrict parking to suitable areas as customer sites or rest areas. However, the rest areas are considered to be always available and already on the route. The possibilities of needing to take a detour to find a suitable rest area or that the rest areas might be full/unavailable at certain times are not considered.

3 Project Objective

The objective of this study is to develop methods to improve the planning of long-haul truck shipments by integrating routing and scheduling algorithms with parking availability information, and by extending the considered planning horizon. When a company needs to plan their shipments they must allocate each order to a certain truck driver, decide the order in which the orders are gonna be fulfilled, the route/path to be taken and the driver's schedule. This complete problem is the VRTDSP usually treated in the literature, the models for this problem generally consider that the shortest path in between any 2 customer

locations is known. The routing part refers to choosing the clients and the order in which to visit them, not the actual path to be taken between client locations. In this study we do not consider the client allocation step. We refer to routing as deciding the actual path that the driver needs to take. Therefore, we will call it vehicle shortest path and truck driver scheduling problem (VSPTDSP) to avoid misunderstandings.

We consider the problem of planning a trip for a single long-haul truck with a single client to be served, so a single origin and destination. The truck moves along a road network with known time-independent travel times. The driver's schedule must comply with the USA HOS regulation. On the network, there is a set of suitable truck parking locations (TPL) which the driver can use to rest. These locations are not always available, but we assume that the time-windows in which they are available are known by the planner.

This problem can be divided into 2 interdependent parts: the routing, which decides the path that the vehicle will use to get to the destination, and the scheduling (TDSP), that decides when to depart from the origin, in which TPLs to stop and for how long.

The TDSP will be treated on section 4, assuming that we already have a pre-defined route and know which TPLs are located along that route. The integration with routing (VSPTDSP) will be treated on section 5.

4 The Truck Driver Scheduling Problem with Parking Availability

In this section we consider the problem of scheduling the rest stops for a long-haul truck trip with a known route and a single client while taking into account the USA HOS regulations and estimated parking availability windows for all rest areas along the route. It is assumed that the rest areas are located on the route and require no detours to be accessed. The route has $n+1$ nodes, 2 of which are the origin, node 0, and destination of the truck, node n. The other $n-1$ are rest areas located along the truck route. For each node $i \in \{0, 1, \ldots, n\}$ the variable $x_i = (x_{i,a}, x_{i,d})$ represents the arrival and departure times of the truck at that node. Each rest area i has T_i parking availability time-windows $[t_{i,\tau}^{min}, t_{i,\tau}^{max}]$, where $\tau \in \{1, 2, \ldots, T_i\}$ indicates the time-window's index. The time-windows restrict the arrival time at that node and are only in effect when the truck has to stop at that specific node, driving by it is not constrained by the time windows. For each location and time-window, a binary variable $y_{i,\tau}$ is used to define if that specific time window is being used (yes:1, no:0). Driving by without stopping is represented by the variable $y_{i,0}$ (drive by:1, stop:0). The travel time $d_{i,i+1}$ in between nodes is considered known and independent of time. The planning horizon is denoted by t_{hor}. The driver must reach its destination before the specified planning horizon. Figure 1 shows an example of a route with origin v_0, 3 rest areas v_1, v_2 and v_3 with 3 time-windows each, and a destination v_4 also with 3 time-windows.
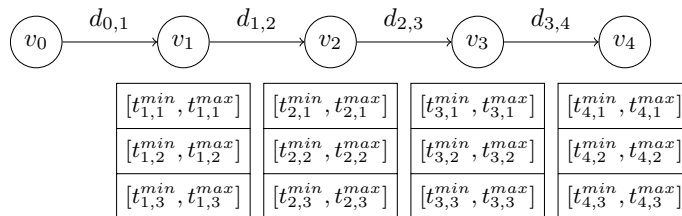
8

Figure 1: Simple route with 5 locations (origin, 3 rest areas and destination) with 3 time-windows each.

The schedule has to comply with the hours-of-service regulations. Here we will consider the USA regulations [14] without the sleeper berth provision. Any regulation that follows a similar structure can be implemented. R is defined as the set of different types of rest period described in the regulation. For each $r \in R$, t_r defines the minimum duration of that type of rest period. C is the set of constraints imposed by the regulation. $C_1 \subseteq C$ is the set of constraints controlling the maximum elapsed time between off-duty periods. $C_2 \subseteq C$ is the set of constraints controlling the maximum accumulated driving time between off-duty periods. $C_3 \subseteq C$ is the set of constraints controlling the maximum accumulated on-duty time during a rolling time-window; the width of the time-window for a constraint $c \in C_3$ is represented by δ_c. In the USA regulation δ_c is 7 or 8 days, so these rolling window constraints will be referred to as weekly constraints. For each constraint $c \in C$, t_c is the time limit imposed by the regulation and $R_c \subseteq R$ is the set of rest types that can reset this counter. The binary variable $z_{i,r}$ indicates whether a rest of type r is taken at location i (yes:1, no:0). A truck cannot take more than 1 different type of rest at the same location and drivers cannot stop and wait at locations where they are not going to take any type of rest. The departure time from the origin must be within the interval $[t_0, t_{dep}]$, t_0 was set to 0. It is assumed that the driver has been off-duty for long enough before the departure time, so that all constraints' counters were restarted before departure. Table 1 lists all the variables and parameters used in the model.

4.1 Model for Short Trips

This problem can be modeled as a mixed integer linear programming problem when the weekly constraints are not considered and as a quadratically constrained mixed integer programming problem when the weekly constraints are needed. This section presents the model used for short trips and focus in showing how the parking availability is modeled by time-windows, before the weekly constraints are included in the next section. MIP models for the TDSP under HOS regulations were proposed previously in [23, 25], where they model rest areas as customer locations with unbounded time-windows and no service time.

9

Table 1: Variables

Variables		
Symbol	Description	Unit
$x_{i,a}$	Arrival time at location i	hours (h)
$x_{i,d}$	Departure time from location i	hours (h)
$y_{i,\tau}$	Used time-window τ at location i?	-
$y_{i,0}$	Drove by location i?	-
$z_{i,r}$	Rest of type r was taken at location i?	-
$\lambda_{i,j,c}$	Accumulated driving time generated by trip departing location i at time $x_{j,a}$, relative to rolling time-window constraint c	hours (h)
$\psi_{i,j,c}$	Accumulated driving time generated by trips departing locations 0 to i at time $x_{j,a}$, relative to rolling time-window constraint c	hours (h)
$\alpha_{i,j,p,c}$	Auxiliary variable for ramp constraint	-
$\beta_{i,j,q,c}$	Auxiliary variable for ramp constraint	-
Parameters		
Symbol	Description	Unit
T_i	Number of time-windows at location i	time-window
$t_{i,\tau}^{min}$	Lower limit of τ-th time-window at location i	hours (h)
$t_{i,\tau}^{max}$	Upper limit of τ-th time-window at location i	hours (h)
R	Set of rest types defined in the regulation	-
C	Set of constraints defined in the regulation	-
t_c	Time limit related to constraint $c \in C$	hours (h)
δ_c	Rolling time-window's width for constraint $c \in C_3$	hours (h)
R_c	Set of rest types that can reset the counter t_c from constraint $c \in C$	-
t_r	Minimum duration for rest of type $r \in R$	hours (h)
$d_{i,i+1}$	Travel time from location i to location $i+1$	hours (h)
t_{hor}	Planning time horizon	hours (h)
t_{dep}	Maximum departure time from the origin	hours (h)

In this model we aim to include the parking availability of the rest areas in the model, so we cannot use unbounded time-windows anymore. However, unlike customer locations, the rest areas are not required stops, and if the driver is not stopping at a certain rest area there is no need to restrict the schedule with that rest area's parking availability. Therefore, the time-windows must be conditioned to the scheduling of off-duty periods at the rest areas. The formulation is as follows:

$$\text{Minimize} \quad \text{Total travel time} = x_{n,a} - x_{1,d} \tag{1}$$

$$x_{i,d} + d_{i,i+1} = x_{i+1,a} \quad \forall 0 \leq i \leq n-1 \tag{2}$$

$$x_{i,a} + \sum_{r \in R} r z_{i,r} \leq x_{i,d} \quad \forall 1 \leq i \leq n \tag{3}$$

$$x_{i,d} \leq x_{i,a} + (1 - y_{0,\tau}) t_{hor} \quad \forall 1 \leq i \leq n \tag{4}$$

$$y_{i,0} + \sum_{\tau=1}^{T_i} y_{i,\tau} = 1 \quad \forall 1 \leq i \leq n \tag{5}$$

$$\sum_{\tau=1}^{T_i} y_{i,\tau} = \sum_{r \in R} z_{i,r} \quad \forall 1 \leq i \leq n-1 \tag{6}$$

$$\sum_{\tau=1}^{T_i} y_{i,\tau} t_{i,\tau}^{min} \leq x_{i,a} \quad \forall 1 \leq i \leq n \tag{7}$$

$$x_{i,a} \leq t_{hor} - \sum_{\tau=1}^{T_i} [y_{i,\tau}(t_{hor} - t_{i,\tau}^{max})] \quad \forall 1 \leq i \leq n \tag{8}$$

$$x_{k,a} - x_{i,d} \leq t_c + t_{hor} \sum_{j=i+1}^{k-1} \sum_{r \in R_c} z_{j,r} \quad \forall 0 \leq i < k \leq n, c \in C_1 \tag{9}$$

$$\sum_{j=i}^{k-1} d_{j,j+1} \leq t_c + t_{hor} \sum_{j=i+1}^{k-1} \sum_{r \in R_c} z_{j,r} \quad \forall 0 \leq i \leq k \leq n, c \in C_2 \tag{10}$$

$$\sum_{r \in R} z_{i,r} \leq 1 \quad \forall 1 \leq i \leq n \tag{11}$$

$$x_i \in [0, t_{hor}]^2, y_i \in \{0,1\}^{T_i+1}, z_i \in \{0,1\}^{|R|} \quad \forall 1 \leq i \leq n \tag{12}$$

$$x_{0,d} \in [0, t_{dep}], y_{n,0} = 0 \tag{13}$$

The objective function (1) is set to minimize the total trip duration. Constraint (2) guarantees that the arrival time equals the departure time of the previous location plus the required driving time. Constraint (3) states that the vehicle must not depart before the arrival time plus the minimum rest time decided for that location. Constraint (4) controls what happens when the driver does not stop at a certain location. If the vehicle does not stop at location

i, the arrival time equals the departure time for that location. This constraint works with constraints (3,5,6) to assure this. Equality will hold when $y_{i,0}$ equals 1. When this happens, constraint (3) turns into $x_{i,a} \leq x_{i,d}$ and constraint (4) turns into $x_{i,d} \leq x_{i,a}$, so we get $x_{i,a} = x_{i,d}$. If $y_{i,0}$ equals 0, then constraint (4) is always true as t_{hor} is large. Constraint (5) states that at any location, either exactly 1 of the time windows is used or the vehicle does not stop. Constraint (6) states that, for any location, if a rest period is not taken at that location, the vehicle will not stop. Constraints (7) and (8) check the time-windows. Arrival must happen after the beginning and before the end of the chosen time window, and before the maximum time horizon considered in the problem. Constraint (9) checks that time elapsed since the last rest in $R_c, c \in C_1$ is less than t_c. (Multiplying by t_{hor} makes it true whenever a rest of type c is taken in between the nodes i and k). Constraint (10) checks if the accumulated driving time in between rest periods in $R_c, c \in C_2$ is less than t_c. Constraint (11) guarantees that only 1 rest period can be scheduled per location. Constraint (12) sets the variables' domains, and (13) guarantees that the departure time from the origin is within the required period and that the vehicle will stop at the destination. This formulation considers a single customer, i.e. single required stop, but more required stops can be included by setting the variable $y_{i,0}$ of the desired location to zero. This way the algorithm will force the driver to stop at that location.

4.2 Model for Long Trips

For short trips it is enough to consider the constraints of elapsed and accumulated driving time between rest/break periods. However, when dealing with trips longer than 1 week, it is necessary to consider the regulations that limit the working hours over longer periods of time. Usually, complying with the short trip constraints does not guarantee compliance with the long trip ones. In the USA this regulation is defined as a rolling time-window of 7 or 8 days in which the driver can drive for at most 60 or 70 hours, respectively. This restriction can be described as follows:

$$\lambda_{i,c}(t) = R(t-x_{i,d}) - R(t-x_{i+1,a}) - R(t-x_{i,d}-\delta_c) + R(t-x_{i+1,a}-\delta_c), \quad \forall 0 \leq i \leq n-1, c \in C_3 \tag{14}$$

$$\lambda_c(t) = \sum_{i=0}^{n-1} \lambda_i(t), \quad \forall c \in C_3 \tag{15}$$

$$\lambda(t)_c \leq t_c, \quad \forall t \in \{x_{1,a}, x_{2,a}, \ldots, x_{n,a}\}, c \in C_3 \tag{16}$$

where $R(t)$ is the unit ramp function. $\lambda_{i,c}(t)$ represents the accumulated driving time generated by the displacement between location i and $i+1$ at time t and $\lambda_c(t)$ represents the accumulated driving time at time t, both relative to constraint $c \in C_3$. It is sufficient to check these constraints at the arrival times x_i, a. If the constraints are broken anywhere they will also be broken at the arrival time that follows.

4.2.1 MIP formulation for the ramp constraints

The function $\lambda_c(t)$, which represents the accumulated driving time over the last week at time t, needs to be evaluated at all arrival times $x_{j,a}$, so each of its component functions $\lambda_{i,c}(t)$ must also be evaluated at these times. $\lambda_{i,c}(t)$ represents the accumulated driving time generated only by the displacement from location i to location $i+1$. Constraints will be defined separately for each evaluated time according to the method for writing piecewise linear functions in MIP models described in [32]. The domain of the functions $\lambda_{i,c}(t)$ will be divided in sections according to when the slope of the function changes and auxiliary variables are used to write t according to where it is located relative to these sections boundaries. For each function and required evaluation time $\lambda_i(x_{j,a})$, the sets of variables $\{\alpha_{i,j,p}\}$, $\{\beta_{i,j,q}\}$, and $\{\lambda_{i,j}\}$ are defined as follows:

$$\alpha_{i,j,p,c} \in \{0,1\}, \beta_{i,j,q,c} \in [0,1]$$
$$\forall 0 \leq i < j \leq n, 0 \leq p \leq 4, 1 \leq q \leq 5, c \in C_3 \tag{17}$$

$$1 \geq \alpha_{i,j,0,c} \geq \beta_{i,j,1,c} \geq \alpha_{i,j,1,c} \geq \cdots \geq \alpha_{i,j,4,c} \geq \beta_{i,j,5,c}$$
$$\forall 0 \leq i < j \leq n, c \in C_3 \tag{18}$$

$$\alpha_{i,j,p,c} < \beta_{i,j,p+1,c} + 1 \quad \forall 0 \leq i < j \leq n, c \in C_3 \tag{19}$$

$$x_{j,a} = x_{i,d}\beta_{i,j,1,c} + d_{i,i+1}\beta_{i,j,2,c} + (\delta_c - d_{i,i+1})\beta_{i,j,3,c}$$
$$+ d_{i,i+1}\beta_{i,j,4,c} + t_{hor}\beta_{i,j,5,c} \quad \forall 0 \leq i < j \leq n, c \in C_3 \tag{20}$$

$$\lambda_{i,j} = d_{i,i+1}\beta_{i,j,2} - d_{i,i+1}\beta_{i,j,4} \quad \forall 0 \leq i < j \leq n \tag{21}$$

$$\sum_{i=0}^{j-1} \lambda_{i,j} \leq t_c \quad \forall 1 \leq j \leq n, c \in C_3 \tag{22}$$

where the α's and β's are auxiliary variables used to model the piecewise definition of $\lambda_{i,c}(t)$. The α's determine in which section of the function domain t is and the β's define its exact position. $\lambda_{i,j} = \lambda_i(x_{j,a})$, δ_c is the width of the time-window associated with constraint c in hours, t_c is the time limit, in hours, associated to constraint c and $d_{i,i+1}$ is the travel time, in hours, between locations i and $i+1$. Constraint (17) defines the domains of the α's and β's, (18) forces the β of any section to only be able to take non-zero values after the α of the previous section is set to 1, and the α of any section to only be able to be set to 1 after the β of that same section reaches 1. Constraint (19) says that the α of any section cannot take a non-zero value while the β of that section is still zero. Constraint (20) writes the time instant to be evaluated $x_{j,a}$ as a function of the α's and β's. Constraint (21) uses the α's and β's to calculate the $\lambda_i(x_{j,a})$, and constraint (22) calculates and limits the accumulated driving time over the moving time-window relative to regulation $c \in C_3$. This set of constraints substitutes constraints (14), (15) and (16), and guarantees that the accumulated driving time in any period of δ_c consecutive hours is kept below t_c.

13

Due to (20) this problem would be a quadratically constrained problem. However, as that constraint is only defined for $j > i$, $\beta_{i,j,1,c}$ will always be 1 and can be defined as a constant, making the constraint linear. It is also possible to reduce the number of new variables by analyzing the possible values of the α's and β's at the required evaluation points. As (20) only considers $j > i$, the variables $\alpha_{i,j,p,c}$ for $p < 2$ and $\beta_{i,j,q,c}$ for $q < 3$ will be always 1 and can be defined as constants.

4.2.2 Reset for weekly constraint

According to USA's regulation, a driver may restart the 168 consecutive hours (7 days) period, by taking an off-duty time of 34 or more consecutive hours. When this type of rest is taken the system should be able to set the weekly accumulated driving time at the end of that rest to zero and start counting again from there. Two possible ways of modeling this behavior are by using quadratic constraints or indicator constraints controlled by the variables $z_{i,r}$. For both formulations a set of variables $\{\psi_{i,j}\}$ will be created to represent the accumulated driving time generated by all trips starting at locations 0 to i measured at time x_j, a. The quadratic constraint formulation is the following:

$$\psi_{i,j} = \left(1 - \sum_{r \in R_c} z_{i,r}\right) \psi_{i-1,j} + \lambda_{i,j} \quad \forall 1 \leq i < j \leq n, c \in C_3 \tag{23}$$

$$\psi_{0,j} = \lambda_{0,j}, \quad \forall 1 \leq i < j \leq n \tag{24}$$

$$\psi_{j-1,j} \leq t_c, \quad \forall 1 \leq j \leq n, c \in C_3 \tag{25}$$

where constraint (23) defines $\psi_{i,j}$ and sets to zero all contributions from nodes before location i when a long rest is taken at location i. As mentioned before, this model assumes that the driver was off-duty for long enough before departure, so constraint (24) considers the initial accumulated driving time as being zero. These 3 constraints replace constraint (22). This formulation is non-linear and non-convex, which makes the problem a lot harder to solve. Furthermore, it cannot be used on CPLEX, so the experiments ran on CPLEX used indicator constraints instead. The formulation using indicator constraints can be obtained by substituting constraint (23) by the following constraint:

$$\psi_{i,j} = \begin{cases} \psi_{i-1,j} + \lambda_{i,j} & \text{if } \sum_{r \in R_c} z_{i,r} = 0 \\ \lambda_{i,j} & \text{if } \sum_{r \in R_c} z_{i,r} = 1 \end{cases} \quad \forall 1 \leq i < j \leq n, c \in C_3 \tag{26}$$

It is reasonable to assume that $|R_c| = 1$ for $c \in C_3$, so in this case the conditions would turn into $z_{i,r} = 1$ and $z_{i,r} = 0$. Otherwise, auxiliary variables, equal to $\sum_{r \in R_c} z_{i,r}$, can be created and used as conditions.

4.2.3 Remarks

Including the rolling time-window constraints greatly increases the complexity of the problem, due to the large number of extra variables and constraints required. Adding the possibility of resetting the counter increases the complexity even further, making the problem non-linear and non-convex. Therefore, it is important to analyze the impact of each part of the model on the complexity, cost and feasibility in different scenarios.

Weekly driving time constraint (without reset)

Feasibility　This part of the model only affects the feasibility of the solutions when the total driving time of the trip exceeds the weekly limits imposed by the regulations, i.e. in the USA, any trip with a total driving time below 60 hours is not affected by those constraints and feasible (legal) schedules can be found even if they are not included. However, it is important to note that, when these constraints are not present, simply limiting the total trip duration or planning horizon to one week is not enough to guarantee a feasible schedule. A constraint on the accumulated driving time over that week would still be necessary, but in this case it could be implemented through constraint (10).

Cost　When used for short trips, this part of the model will not have any impact on the solution cost as the weekly constraints are not active. In the case of long trips, the simple model does not generate HOS compliant solutions so its costs are not realistic and do not need to be considered for comparison. The cost of the solutions generated by the model with weekly constraint but without reset will be equal to or greater than the cost obtained when the constraint reset is included.

Complexity　The number of extra variables and constraints needed to implement this part of the model increases with n^2. Therefore, this model is not very scalable. For very long trips it might be necessary to find ways to reduce the number of variables and constraints, like grouping parking lots by region instead of considering them individually and removing unrealistic time-windows.

Weekly constraint reset

Feasibility　The constraint reset does not affect the feasibility of the problem, it only improves the solution cost. Therefore, it is possible to omit this option to reduce problem complexity. The total trip duration might increase, but the schedules generated will still comply with the HOS regulations.

Cost　How much this reset can improve the cost depends on the regulation used. Usually, the sum of the minimum time a driver takes to reach the weekly driving limit and the minimum rest time needed to reset the weekly constraint

is less than a week (or less than the considered moving time-window width), so the difference between these values is the time saved by resetting the constraint. The more time is saved, the more advantageous it is to use the constraint reset. In the case of the USA, a driver takes at least 112.5h to drive 60h. When resting for 34h they can reset all constraints, which means that, on average, they can drive for 60h every 146.5h, instead of every 168h. This represents an increase of more than 14% on the average weekly driving time.

Complexity   The number of extra variables and constraints needed to implement this part of the model increases with n^2. In addition to that, some of the new constraints are non-linear and non-convex, aggravating the scalability issue.

4.2.4   Trip Duration Bounds

The schedule is affected by the truck stops' locations, by their availability windows and by the HOS regulation. In order to calculate lower bounds for this problem we can consider an ideal scenario where time-windows are not an issue, and only consider the HOS regulation. As they are dependent only on the regulation being used, it is possible to calculate these bounds offline, and use them as a way to evaluate the solutions obtained during the schedule optimization. The lower bounds depend on the used regulation. Here we will consider the same structure of the USA regulation. We calculated the lower bounds by optimizing the total trip duration for trip lengths that do not use *daily rests*, then used this as a building block to optimize trips that do not need *weekly rests*, then for trips of any length. The parameters used are defined as follows:

t_b minimum *break* duration;

t_r minimum *rest* duration;

t_w minimum *weekly rest* duration;

t_{eb} limit for elapsed time between *breaks*;

t_{ar} limit for accumulated driving time between *rests*;

t_{aw} limit for accumulated driving time between *weekly rests*;

δ_w moving time-window width;

ϵ arbitrarily small positive constant.

First we calculate the minimum trip duration for a trip with less than a day's worth of driving time, $f_d(\cdot)$.

$$f_d(x) = x + t_b \cdot \left\lfloor \frac{x - \epsilon}{t_{eb}} \right\rfloor \Bigg(\quad 0 \le x \le t_{ar} \tag{27}$$

16

where x represents the trip length in hours, i.e. the total driving time of the trip. As x was limited to less than the daily driving limit t_{ar}, we just need to calculate the number of *breaks* that will be necessary during the day. The ϵ is used to avoid including a *break* when the driving time is exactly on the limit.

Then we can use $f_d(\cdot)$ to describe the trip duration for trips with less than a week's worth of driving time, $f_w(\cdot)$, as follows:

$$u(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \end{cases} \tag{28}$$

$$g_w(\overline{y}) = f_d(y_0) + \sum_1^n \left[t_r u(y_i) + f_d(y_i) \right], \quad \overline{y} \in \mathbb{R}^{n+1}, 0 \leq \|\overline{y}\|_\infty \leq t_{ar} \tag{29}$$

$$A(x) = \left\{ \overline{y} \in \mathbb{R}^{n+1} \; 0 \leq \|\overline{y}\|_\infty \leq t_{ar} \;,\; \|\overline{y}\|_1 = x, n = \left\lfloor \frac{x - \epsilon}{t_{eb}} \right\rfloor \right\} \tag{30}$$

$$f_w(x) = \min_{\overline{y} \in A(x)} g_w(\overline{y}), \quad 0 \leq x \leq t_{aw} \tag{31}$$

The 'shorter than a week' trips described here can be divided in multiple 'less than a day' trips separated by *daily rests*. Function $g_w(\cdot)$ optimizes these smaller sections using $f_d(\cdot)$ and adds a *daily rest* for each non-zero section, calculating the minimum trip duration given a vector $\overline{y} = (y_0, y_1, \ldots, y_n)$ composed of the $n+1$ section lengths y_i. Equation (31) optimizes the trip duration over all valid combinations of section lengths, with up to n *daily rests*. Equation (30) defines the valid section length vectors, choosing n such that the optimization considers enough *daily rests* to account for the case of taking a *daily rest* every time a *break* is needed.

The same approach is used to calculate the trip duration for longer trips, but now we want to consider 2 different scenarios: being able to reset the weekly constraint with a *weekly rest*, and using only the rolling time-window constraint. The trip duration for a trip with more than a week's worth of driving time when *weekly rests* are allowed, $f_{l1}(\cdot)$, is calculated as follows:

$$g_{l1}(\overline{y}) = f_w(y_0) + \sum_1^n \left[t_w u(y_i) + f_w(y_i) \right], \quad \overline{y} \in \mathbb{R}^{n+1}, 0 \leq \|\overline{y}\|_\infty \leq t_{aw} \tag{32}$$

$$B(x) = \left\{ \overline{y} \in \mathbb{R}^{n+1} \; 0 \leq \|\overline{y}\|_\infty \leq t_{aw} \;,\; \|\overline{y}\|_1 = x, n = \left\lfloor \frac{x - \epsilon}{L} \right\rfloor \right\} \tag{33}$$

$$f_{l1}(x) = \min_{\overline{y} \in B(x)} g_{l1}(\overline{y}), \quad 0 \leq x \tag{34}$$

where $t_{eb} \leq L \leq t_{aw}$ should be chosen in a way that creates a vector long enough to test the different possibilities of moving driving hours between weeks to avoid *breaks* and *daily rests* when possible. This value affects the maximum number of *weekly rests* that the problem will consider in the optimization. Taking $L = t_{eb}$ certainly works as it is the maximum driving time allowed without any kind of rest, so , in the case of USA regulations, the optimization would consider

even the case of taking a *weekly rest* every 8h of driving. For long trips this resolution might be excessive and will unnecessarily slow down the calculations due to the large number of possible vectors. An optimal schedule will have a high driving time to trip duration ratio, so we suggest taking L as the length of the trip when the intermediate weekly cycles (the ones that are followed by a *weekly rest*) would be the most efficient, i.e. $L = \arg\max\limits_{x \le t_{aw}} \dfrac{x}{f_w(x) + t_{aw}}$.

The trip duration for a trip with more than a week's worth of driving time when *weekly rests* are not allowed, $f_{l2}(\cdot)$, is calculated as follows:

$$f_{l2}(x) = \delta_w \cdot \left\lfloor \frac{x - \epsilon}{t_{aw}} \right\rfloor + f_w(mod(x - \epsilon, t_{aw})) \tag{35}$$

where $mod(\cdot, \cdot)$ is the modulo operator, which returns the remainder of the division between its arguments. In this case, as we know during any period of δ_w at most t_{aw} driving hours can be schedule, every chunk of t_{aw} driving hours will generate a trip duration of δ_w. Therefore, we just need to optimize the the driving time that remains after taking these chunks out.

We can then calculate the minimum trip duration for when the driver is allowed both behaviors by combining $f_{l1}(\cdot)$ and $f_{l2}(\cdot)$:

$$f_{l1,l2}(x_1, x_2) = \begin{cases} f_{l1}(x_1), & \text{if } x_2 = 0 \\ f_{l2}(x_2), & \text{if } x_1 = 0, x_2 > 0 \\ f_{l1}(x_1) + t_w + f_{l2}(x_2), & \text{if } x_1, x_2 > 0 \end{cases} \tag{36}$$

$$f_l(x) = \min\limits_{x_1 + x_2 = x} f_{l1,l2}(x_1, x_2), \quad x_1, x_2 \ge 0 \tag{37}$$

In order to force the usage of the rolling time-window we can restrict equation (37) to using x_2 greater than t_{aw}.

4.2.5 Simplified Model for Long Trips

Due to the scalability issues caused by the rolling time-window constraint, we will also study a simplified way to implement the weekly constraint with the reset. Some authors model the weekly constraint as an accumulated driving time constraint between 2 consecutive weekly rests. This formulation does not consider all possible solutions generated by the rolling time-window constraint, but it does guarantee a valid schedule. This model would be almost the same as the one presented in section 4.1, only adding the accumulated driving time between weekly rests constraint to the set of constraints C_2. Therefore, all the extra variables and constraints presented in section 4.2 would not be needed anymore.

In the USA regulation, under ideal conditions, resetting the weekly constraint is more efficient than decreasing the average daily driving time and using the rolling time-window. The rolling time-window constraint is only used when it is convenient for the driver to be on-duty for more than 60 hours over
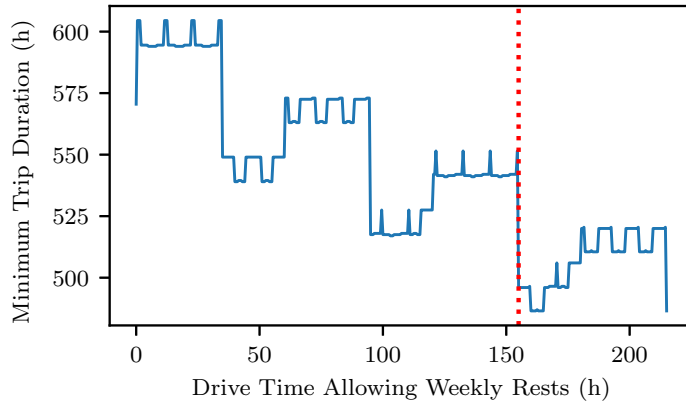
Figure 2: Minimum trip duration for a trip with a total of 215h of driving time according to how much of the trip allows weekly rests to be taken.

a time interval longer than 168 hours without taking a weekly rest. This might be caused by inconvenient time-windows or long work times needed during this interval. Therefore, it is possible that a solution found by the simplified weekly constraint will have a lower cost than the lower bound for solutions that require the usage of the rolling time-window at some point during the trip.

We can use the model in section 4.2.4 to estimate the lower bound for the duration of trips varying how much of the driving time allows the use of weekly rest and how much does not. For example, a trip with 215h of total driving time could have 115h of driving using weekly rests and 100h not using them, and we would need to consider a weekly rest in between those 2 parts of the trip to reset all constraints. Each part of this trip would behave like the respective case described by equations (34) and (35). An example of how the minimum trip duration varies with the length of these parts is shown on Figure 2. It can be seen that the shortest trips happen when most of the time is allocated to using weekly rests. The rolling time window option is only used on the points of the plot to the left of the red line. The red line marks when more than 60h of driving time do not allow the use of rests, therefore requiring the use of the rolling time window. In order to find the lower bound of schedules that actually use the rolling time window we can calculate the minimum of everything to the left of the red line. We can guarantee optimality for any schedule calculated using the simplified weekly constraint for which the trip duration is lower than this bound. Section 4.3.2 presents experiments simulating long-haul trips, and we can see how this lower bound compares with the experimental results.

4.3 Experiments

4.3.1 Parking Availability Impact

This section describes the setup of the experiment used to test the impact of considering availability windows for every parking lot along a truck route. A route, approximately 1960Km long, going from San Diego to Seattle was chosen. Data from the FHWA was used to find rest areas and truck stops located close to the route and position them along the route. Figure 3 shows the parking lots along the route (gray circles), as well as the chosen parking locations for the base case (triangles and squares) and for one of the tested scenarios (crosses). 94 rest stops and rest areas were considered. This trip takes less than a week, so the rolling time-window constraints are not considered. In order to simulate parking availability, time windows with start and end times normally distributed were considered for each rest area/truck stop. The distribution used for the start times had mean 5 hours (5am) and standard deviation of 0.5 hours, and the one for the end time had mean 20 hours (8pm) and standard deviation of 1 hour. It was considered that the truck must depart from the origin during the first 24 hours and that the final destination has daily time-windows from 8am to 6pm. For this experiment multiple scenarios with different parking availability time-windows were generated. The problem was solved using the solver CPLEX, both for the base case without the time-windows and for each scenario using the parking availability constraints. The number of scenarios used (N), average trip duration (avg_trip), average arrival (avg_arr) and departure times (avg_dep) and the feasibility rate ($feas$) of each model are shown on Table 2. In this experiment the base case found a solution that scheduled a stop at a time when the chosen parking lot is very likely to be full, which caused the feasibility rate to drop to zero. The feasibility rate for this experiment is highly dependent on the probability distribution used to define the availability windows. In this experiment, the distribution parameters were chosen so that the times when the time-windows are closed resemble the times when truck drivers report difficulty in finding parking more often. The time-windows do not reflect the real availability of the truck stops in the used route, so these results can only be used as an example of how easy it is for a schedule to be almost always infeasible if the parking availability is not considered during planning. On the other hand, considering parking availability did not affect the total trip duration in this experiment, possibly due to the large number of parking lots to choose from.

4.3.2 Long-Haul Trips

Due to the scalability issues mentioned before, this experiment did not consider a real route and the surrounding truck stops. A route was generated with equally spaced truck stops, the travel time between two adjacent truck stops was set to 1 hour. Like in the previous experiment, normal distributions were used to define the time-windows for each truck stop. The distribution used for the start times had mean 4 hours (4am) and standard deviation of 1 hour, and the one
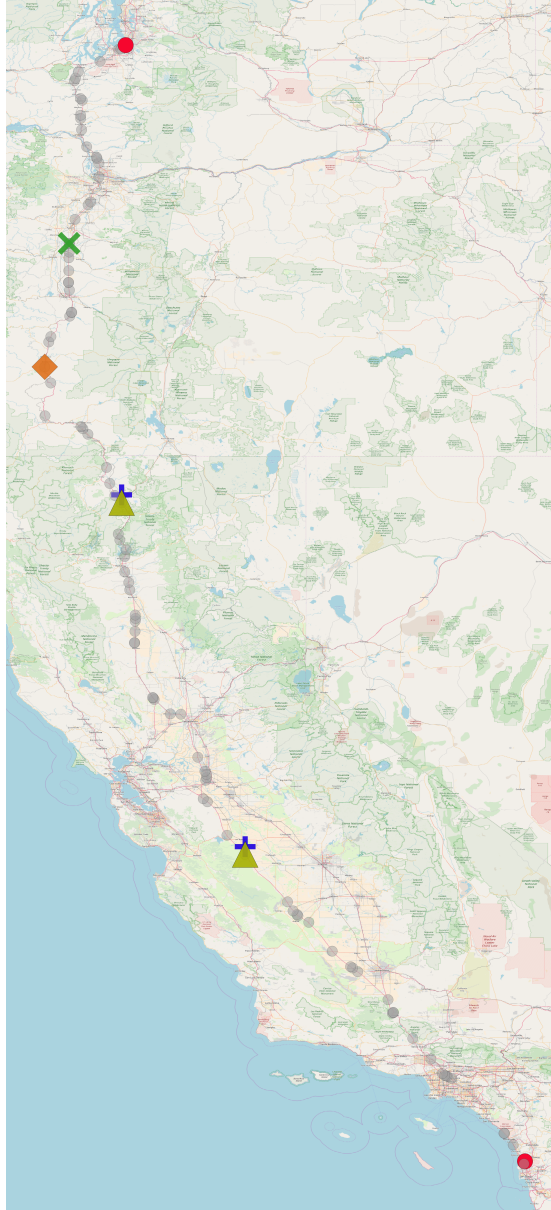
Figure 3: Route used on short trip experiment. San Diego to Seattle through the I-5 freeway. The triangles (base model) and +s (new model) represent truck stops chosen for daily rests, and the square (base model) and × (new model) represent the ones chosen for short breaks. The gray circles represent the truck stops near the chosen route.

Table 2: Comparison between schedules obtained with and without parking availability windows.

N	Parking	avg_trip (h)	$feas$	avg_arr (h)	avg_dep (h)
30	No	46.67	0%	56	9.23
30	Yes	46.67	100%	57.93	11.26
1000	No	46.67	0%	56	9.23
1000	Yes	46.67	100%	57.98	11.30

for the end time had mean 21 hours (9pm) and standard deviation of 2 hours. It was considered that the truck must depart from the origin during the first 24 hours and that the final destination has daily time-windows from 8am to 6pm.

This experiment tested the performance of 3 different methods to generate long-haul schedules. The USA HOS regulation defines a rolling time-window constraint for long-haul drivers; they cannot drive after being on-duty for more than 60/70 hours during 7/8 consecutive days, and can take a rest of at least 34 consecutive hours in order to reset this counter. We used the regulation for 60h on-duty during the past 7 days. We considered 3 different ways of generating HOS compliant schedules. First, we can implement the rolling time windows as shown on section 4.2, without including the possibility of resetting the counter; this is referred to as the 'No Reset' method. Second, for the 'Reset' method, the reset for the rolling time-window was included as shown on section 4.2.2. Third, we used the simplified weekly constraint described in section 4.2.5, referred to as 'Simplified Const'.

4.3.3 Performance

It can be seen on Figure 4 that when the reset option is not implemented the total trip duration gets automatically increased to more than 1 week when the needed driving time is larger than the weekly limit. When the reset is implemented the trip duration only gets increased by the duration of the weekly rest needed to reset the counter. This is the reason why the 'Simplified Const' is likely to find an optimal schedule for the USA regulations. In general, it is more efficient for the drivers to take the 34h rest and reset the counter than to reduce their average daily driving hours to match the rolling time-window. However, this is not necessarily true for every regulation. Figure 4 also shows, as 'TW Bound', the theoretical lower bound for solutions that need the rolling time-window to be found. We can see that, on average, the results for 'Simplified Const' are significantly lower than this bound. Therefore we can prove optimality for most of the results found by this method without needing to use the complete model. For schedules that exceed this bound we can only show what is the maximum possible improvement to the solution if the complete model were used, and use this information to decide whether to accept the current solution or try to improve it by using the complete model. The presented model only treated the case with a single client and the parking availability windows are
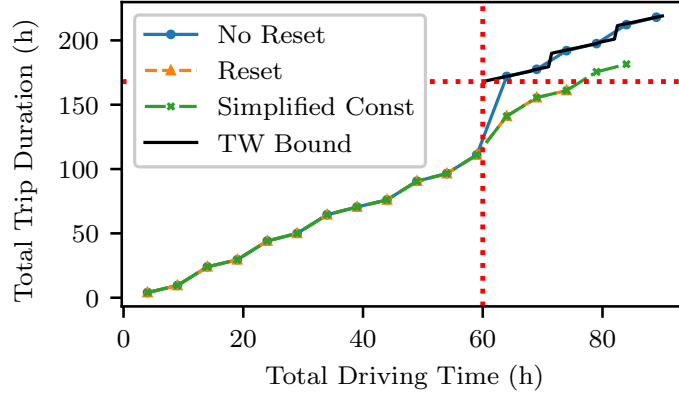
Figure 4: Average trip duration of schedules calculated by the 3 methods with varying total driving time, and the theoretical lower bound of solutions that need the rolling time to be found. The vertical dotted line represents when the total driving time reaches the weekly limit (60h) and the horizontal dotted line represents when the trip duration reaches 1 week (168h).

fairly wide, so they do not affect the scheduling very much. However, if more clients are added to the route, with more restrictive time-windows, the solution costs obtained by using weekly rests should lose some of their advantages.
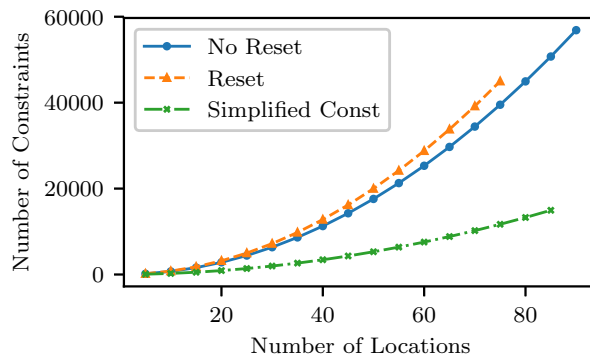
4.3.4 Complexity

Figure 5 shows how the number of locations used and the total driving time affect the problem's complexity. It can be seen on Figure 5a that the solve time for all 3 methods are almost the same when the number of locations used is smaller than 60. At 60 locations the total driving time reaches the weekly driving limit (60h) and the weekly constraints start being needed. The solve time for the 'Reset' method rises sharply after that threshold. The solve time for the 'No Reset' model also increases exponentially, but at a slower rate. Figures 5b and 5c show that those 2 methods have a similar number of constraints and variables, however some of the constraints used to reset the weekly driving time counter are indicator constraints; this is likely the cause of the significantly longer solve time for the 'Reset' model. Unexpectedly, even though the 'Simplified Const' method has a notably smaller number of variables and constraints, the variation of its solve time was very inconsistent and did not show a significant improvement compared to the 'No Reset' method. Nevertheless, its solve time is still shorter than the 'Reset' method while finding solutions of same or similar costs, which are significantly better than the 'No Reset' solutions.

23
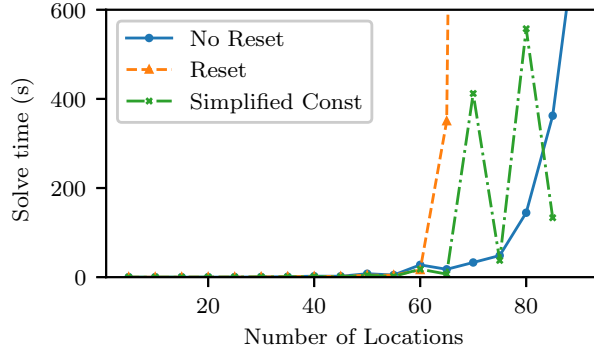
(a)
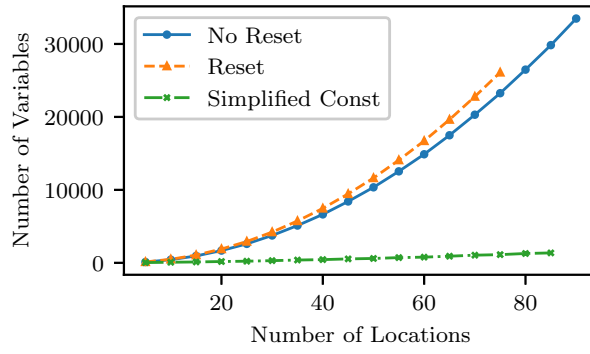
(b)

(c)

Figure 5: Results showing how the complexity of the 3 methods vary with the number of locations and driving time.

4.4 Extensions

4.4.1 Multiple Clients

If the trip has more than 1 customer location to visit, it is necessary to add a required stop at the customer location and some work time might also be needed for loading/unloading. The original model doesn't consider stops for work purposes so some modifications are needed to implement this. All required stops will be assigned a parameter w_i representing the work time required for that location. Let N be the set of required stop locations. The following constraints need to be included in the problem:

$$x_{i,a} + w_i = x_{i,d} \quad \forall i \in N \tag{38}$$

$$\sum_{r \in R} z_{i,r} = 0 \quad \forall i \in N \tag{39}$$

$$y_{i,0} = 0, \forall i \in N \tag{40}$$

$$\sum_{j=i}^{k-1} d_{j,j+1} + \sum_{j=i}^{k-1} v_j \leq t_c + t_{hor} \sum_{j=i+1}^{k-1} \sum_{r \in R_c} z_{j,r} \quad \forall 0 \leq i \leq k \leq n, c \in C_3 \tag{41}$$

$$w_i = 0, \forall i \notin N \tag{42}$$

Constraints (38) to (40) substitute constraints (3) and (6) for the locations with required stops ($i \in N$), so a $i \notin N$ should be included in the original constraints to avoid conflicts. Note that all $z_{i,r}$'s are set to zero due to the assumption that drivers cannot rest at customer locations. As the set of constraints C_3 measures the on-duty time, not the driving time, constraint (42) needs to be added to the problem when work time is considered.

5 The Vehicle Shortest Path and Truck Driver Scheduling Problem with Parking Availability

5.1 Problem Description

The problem consists of planning the trip of a single truck from an origin to a single destination, while complying with the USA HOS regulations and only scheduling off-duty time at TPLs which are expected to have available parking at the time of arrival. The context is the same as for the TDSP presented previously, but now the route taken is not given and the TPLs cannot be reached without deviating from the main road. The travel times are given and still considered time-independent, the parking availability of the TPLs are also assumed known. We modeled this problem as a shortest path problem with resource constraints (SPPRC)[33], where the time, trip duration or cost, and the counters for the different HOS regulations will be treated as resources. The simplified road network is defined as an acyclic directed graph G=(V,A), where V is the set of nodes of the graph and A is the set of arcs. The nodes represent locations

of interest in the road network, as TPLs, client locations, intersections, and road branching spots. The arcs represent road segments. Let $d_{i,j}$ be constants associated with each arc (i, j) representing the travel time between locations i and j. Let v_0 denote the origin node and v_n the destination node, so every path must start at v_0 and end at v_n. To avoid an overly complex network, the graph is built considering only the main routes the driver can take for that specific trip and the TPLs around them. TPLs are included in the network as nodes branching out from one of the main paths, that merge back into the path at a downstream node. The shortest paths from the main road to the TPLs are assumed known. In case the shortest path requires the truck to re-enter the road at a point upstream of the branching out point, the difference between the real merge point and the wanted merge point can be added to the cost of the edge connecting the truck stop to the main path. Figure 6 shows an example network, the nodes with a number index are road nodes and the ones with a letter index are TPLs. The edges connected to TPLs were represented as dashed arrows and the main paths as continuous arrows.
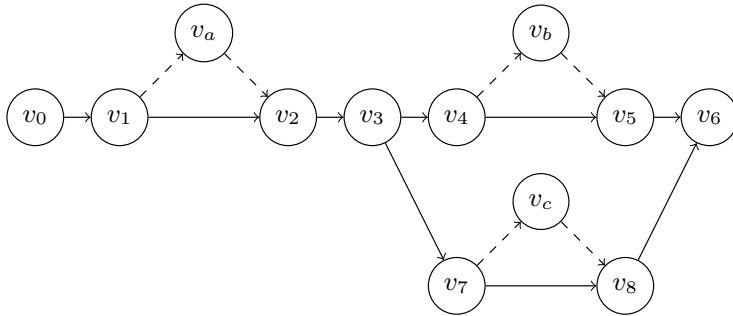

Figure 6: Example of simplified road network.

In order to treat the different possible actions a driver can take at a TPL, an extended network G'=(V',A') is defined, where each node v_i representing a TPL is substituted by the sub-network in Figure 7.

Node v_i^{in} represents the moment of arrival at the parking lot, v_i^b, v_i^r and v_i^w represent the type of off-duty period the driver chose to take (*break*, *daily rest* and *weekly rest*, respectively). The arcs going from v_i^{in} to each one of these nodes have as cost the minimum time required by each type of off-duty period, i.e. t_b, t_r and t_w for *break*, *daily rest* and *weekly rest*, respectively. v_i^{out} represents the moment when the driver is ready to leave the parking lot and the arcs leading to it have a variable cost $\delta_{i,*,k}$, instead of the constant d_{ij}, representing the extra time the driver spent in the parking lot beyond the minimum time required.

The variables $x_{i,k}$ and $a_{i,k}$ are associated with each node $v_i \in V'$ and partial solution k. $x_{i,k}$ represents the time at which node v_i is visited in k. $a_{i,k}$ indicates if node v_i is visited in k.

For each truck stop entrance node v_i^{in} and for the destination v_n a set of time-windows is defined. For the rest areas they represent the times when the
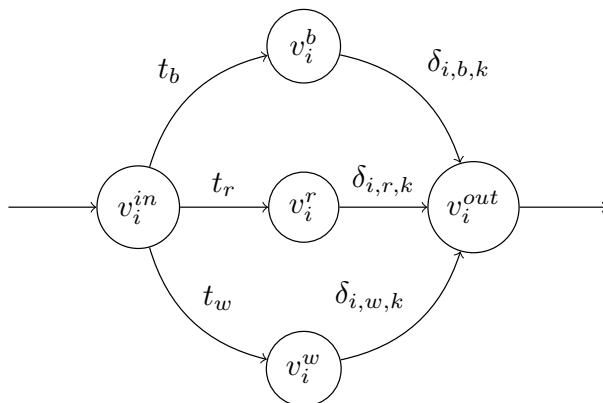
Figure 7: Sub-network used to expand each truck stop node.

parking lot is not full, and for the destination they represent the times when the customer can receive the delivery.

As in the TDSP model, T_i time-windows are defined separately for each node i, restricting the allowed arrival time for vehicles that intend to stop/rest at that location. Each time-window is defined by a tuple $(t_{i,\tau}^{min}, t_{i,\tau}^{max})$ representing the minimum and maximum arrival times allowed by that time-window, where $\tau \leq T_i$ is the index of that window.

As the driver schedule must comply with the HOS regulations, each regulation constraint is modeled by a different resource that must be kept below the limits described in the regulation throughout the whole path. For each node i and path k, the resources considered are:

- time when node was visited $(x_{i,k})$

- elapsed time since trip start $(\eta_{i,k}^s)$

- elapsed time since last break $(\eta_{i,k}^b)$

- elapsed time since last rest $(\eta_{i,k}^r)$

- elapsed time since last weekly rest $(\eta_{i,k}^w)$

- accumulated driving time since last rest $(\psi_{i,k}^r)$

- accumulated driving time since last weekly rest $(\psi_{i,k}^w)$

- accumulated driving time over the last 7 days $(\psi_{i,k}^m(t))$

For each arc (v_i, v_j) and partial solution k, a variable $b_{i,j,k}$ indicates if arc (v_i, v_j) is visited in k. A resource extension function (REF) $f_{i,j}(\cdot)$ defines how each resource is updated when arc (v_i, v_j) is visited. $f_{i,j}(\cdot)$ can be defined as 4 different functions depending on what kind of activity the

27

Table 3: Resource Extension Functions

Resource	f^d	f^b	f^r	f^w
$x_{jk} =$	$x_{ik} + d_{ij}$	$x_{ik} + d_{ij}$	$x_{ik} + d_{ij}$	$x_{ik} + d_{ij}$
$\eta^s_{jk} =$	$\eta^s_{ik} + d_{ij}$	$\eta^s_{ik} + d_{ij}$	$\eta^s_{ik} + d_{ij}$	$\eta^s_{ik} + d_{ij}$
$\eta^b_{jk} =$	$\eta^b_{ik} + d_{ij}$	0	0	0
$\eta^r_{jk} =$	$\eta^r_{ik} + d_{ij}$	$\eta^r_{ik} + d_{ij}$	0	0
$\eta^w_{jk} =$	$\eta^w_{ik} + d_{ij}$	$\eta^w_{ik} + d_{ij}$	$\eta^w_{ik} + d_{ij}$	0
$\psi^r_{jk} =$	$\psi^r_{ik} + d_{ij}$	ψ^r_{ik}	0	0
$\psi^w_{jk} =$	$\psi^w_{ik} + d_{ij}$	ψ^w_{ik}	ψ^w_{ik}	0
$\psi^m_{jk}(t) =$	$\psi^m_{ik}(t)$ $+$ $g(x_{ik}, d_{ij}, \delta_c, t)$	$\psi^m_{ik}(t)$	$\psi^m_{ik}(t)$	0

arc represents (driving, *break*, *daily rest*, *weekly rest*). For arcs connected to *break*, *daily rest* and *weekly rest* nodes, the REF will be f^b, f^r and f^w, respectively. The other arcs represent when the driver is driving and use f^d to update the resources. Table 3 shows the expressions for each REF. The function $g(.,t)$, used to calculate $\psi^m_{i,k}(t)$, represents the accumulated driving time generated by a single arc in the network, it can be defined as $g(x_{ik}, d_{ij}, \delta_c, t) = R(t - x_{ik}) - R(t - x_{ik} - d_{ij}) - R(t - x_{i,d} - \delta_c) + R(t - x_{ik} - d_{ij} - \delta_c)$, where $R(t)$ is the unit ramp function. Figure 8 shows which REF is used in each arc of an example network with origin v_a, destination v_d and an already expanded truck stop node v_i.
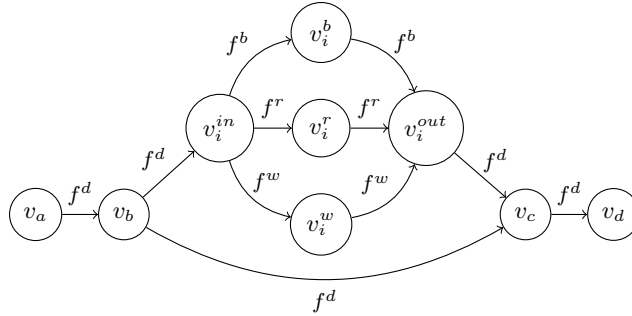
Figure 8: Example network with REFs used in each node.

The evolution of resource $\psi^m_{i,k}(t)$ is dependent on the previous decisions on that path, so a function is used in order to consider all information necessary. This function's value must be less or equal than 60 hours everywhere for the solution to be feasible. This function is non-decreasing on driving periods and non-increasing on resting periods, so it suffices to verify that $\psi^m_{i,k}(x_{i,k}) \leq 60$ for every visited node representing a truck stop entrance. Feasibility windows are

Table 4: Resource Feasibility Windows

Resource	Window
x_{jk}	$\bigcup_{\tau}[w_{j\tau}^{min}, w_{j\tau}^{max}]$
η_{jk}^{s}	-
η_{jk}^{b}	$[0, t_{eb}]$
η_{jk}^{r}	$[0, t_{er}]$
ψ_{jk}^{r}	$[0, t_{ac}]$
ψ_{jk}^{w}	$[0, t_{aw}]$

defined for each resource according to the HOS regulations. We will consider the simplified long trip model from section 4.2.5, which will give us the resource windows shown on Table 4, where the parameters are the same ones defined in section 4.2.4. t_b, t_r and t_w are the minimum durations of *breaks*, *daily rests* and *weekly rests*, respectively. t_{eb} is the limit for elapsed time between *breaks* , t_{ar} is the limit for accumulated driving time between *daily rests* , and t_{aw} is the limit for accumulated driving time between *weekly rests*. Note that resources $\eta_{i,k}^{w}$ and $\psi_{i,k}^{m}(t)$ and their REFs were defined as they can be needed for certain regulations, but were not included in the table because they are not used in the simplified long trip model.

A feasible solution is composed by a path (nodes and arcs) which is feasible according to the network structure and structural constraints, and a schedule (resource vectors) which is resource feasible according to the time-windows and HOS constraints. Let Θ_i be the vector of resources for a node i, a path $\pi = (v_0, v_1, \ldots, v_p)$ is resource feasible if there exists a feasible resource vector Θ_i for all nodes $i = 0, \ldots, p-1$, such that $f_{v_i,v_{i+1}}(\Theta_i) = \Theta_{i+1}$ for all $i = 0, \ldots, p-1$. The path must obey the network structure, so for all $v_i \in \pi \backslash \{v_p\}$ it is required that $(v_i, v_{i+1}) \in A'$. The optimal solution will be the one with minimum cost among all feasible solutions.

5.2 Proposed Approach

A dynamic programming algorithm using a label correcting method will be used to solve this problem. Labels are defined for partial solutions ending at each node and dominance rules are used to decide which partial solutions are kept and used to continue the search. The resources used to define the labels of the model are $x_{i,k}$, $\eta_{i,k}^{s}$, $\eta_{i,k}^{b}$, $\eta_{i,k}^{r}$, $\psi_{i,k}^{r}$, and $\psi_{i,k}^{w}$. The full version of the regulation would need to use the resource $\psi_{i,k}^{m}(t)$ instead of $\psi_{i,k}^{w}$. The next section defines the dominance rules used. The rules include all resources, but only the ones needed by the labels of the chosen HOS regulation model should be considered.

Resource	general case	v_*^b nodes	v_*^r nodes	v_*^w nodes
x_{jk}	$= x'_{jk} - \gamma$	$= x'_{jk} - \gamma$	$= x'_{jk} - \gamma$	$= x'_{jk} - \gamma$
η_{jk}^s	$< \eta_{jk}^{s\prime} - \gamma$	$< \eta_{jk}^{s\prime} - \gamma$	$< \eta_{jk}^{s\prime} - \gamma$	$< \eta_{jk}^{s\prime} - \gamma$
η_{jk}^b	$\leq \eta_{jk}^{b\prime} - \gamma$	$\leq \eta_{jk}^{b\prime}$	$\leq \eta_{jk}^{b\prime}$	$\leq \eta_{jk}^{b\prime}$
η_{jk}^r	$\leq \eta_{jk}^{r\prime} - \gamma$	$\leq \eta_{jk}^{r\prime} - \gamma$	$\leq \eta_{jk}^{r\prime}$	$\leq \eta_{jk}^{r\prime}$
η_{jk}^w	$\leq \eta_{jk}^{w\prime} - \gamma$	$\leq \eta_{jk}^{w\prime} - \gamma$	$\leq \eta_{jk}^{w\prime} - \gamma$	$\leq \eta_{jk}^{w\prime}$
ψ_{jk}^r	$\leq \psi_{jk}^{r\prime} - \gamma$	$\leq \psi_{jk}^{r\prime} - \gamma$	$\leq \psi_{jk}^{r\prime}$	$\leq \psi_{jk}^{r\prime}$
ψ_{jk}^w	$\leq \psi_{jk}^{w\prime} - \gamma$	$\leq \psi_{jk}^{w\prime} - \gamma$	$\leq \psi_{jk}^{w\prime} - \gamma$	$\leq \psi_{jk}^{w\prime}$
$\psi_{jk}^m(x_{jk})$	$\leq \psi_{jk}^{m\prime}(x'_{jk}) - \gamma$	$\leq \psi_{jk}^{m\prime}(x'_{jk}) - \gamma$	$\leq \psi_{jk}^{m\prime}(x'_{jk}) - \gamma$	$\leq \psi_{jk}^{m\prime}(x'_{jk})$

5.2.1 Dominance Rules

In order to reduce the search space, dominance rules were defined to remove from the search partial solutions that cannot generate better results than other partial solutions already included in the search space. These rules are defined based on the resource vector, also used as the label, Θ_i, where i is the last location visited by the partial solution being analyzed.

Let Θ_i and Θ'_i denote the labels for two different partial solutions ending at the same node i and let $\gamma = x'_i - x_i \geq 0$. The dominance rules are described on Table 5. We can say that Θ_i dominates Θ'_i if they satisfy all the inequalities of a column of Table 5.

For the general case, when these conditions are true, solutions generated from Θ_i will not break any HOS constraint that Θ'_i would not break. However, it is possible that due to the difference between x_i and x'_i, some path extensions generated from Θ'_i might not be feasible with Θ_i due to time-window constraints. Therefore we intend to check for dominance only on nodes representing off-duty periods (v^b, v^r and v^w), as they can extend their off-duty period in order to satisfy a time-window if necessary. In addition, as some resources can be reset to zero at these nodes and the extension of the off-duty time does not affect the reset resources, we can generate stricter rules.

These dominance rules do not consider the impact of past decisions on the future values of $\psi_i^m(t)$. If the rolling time window constraint is being considered, then it is possible that labels that could generate optimal solutions will be removed and the final solution will be suboptimal.

5.2.2 Choosing $\delta_{i,*}$

For each partial solution ending at an off-duty node v_i^*, and at the origin, the variable cost $\delta_{i,*}$ of the outgoing arc has to be chosen. These variables affect the arrival time and elapsed time resources, so they also affect the feasibility of the downstream paths. These variables are continuous and their discretiza-

tion might cause a great increase in the search space. In order to reduce the search space, we propose to generate a list of all time-window constrained nodes reachable within a t_b hours drive and consider only the minimum $\delta_{i,*}$ needed to reach each one of these nodes within a valid time-window. The only reason to take a longer than necessary rest is to satisfy time-window constraints, therefore any increase of $\delta_{i,*}$ beyond these values would only increase the final solution cost. The list of nearby nodes can be generated offline during pre-processing and can be pruned online according to that partial solution's resource vector, which affects the maximum allowed driving time.

6   Conclusion and Recommendations

This study extended the MIP models for the truck driver scheduling problem (TDSP), integrating information on the expected parking availability at each rest area and truck stop along the route, as well as extending the allowed planning horizon allowed by the model. In addition to proposing a way to model the weekly constraints from the USA HOS regulation, usually avoided in the literature, this study also presented a sufficient condition to check the optimality of a solution achieved using a simplified weekly constraint.

A truck route going from San Diego to Seattle was used to the test the impact of the parking availability on the trip duration. Random availability windows were set to each parking location near the chosen route. The experiment results suggest that the integration of parking availability in the TDSP does not impact negatively the total trip duration. Historical truck parking availability data is needed in order to simulate more realistic scenarios before implementation is possible. Experiments with fictitious routes with varying lengths and number of parking locations were used to study the complexity of the model. The experiments revealed a scalability issue that can hinder the usage of the model in some cases. This can limit the number of parking locations that can be considered in long-trips, and also affect its usage to update the schedule in the middle of the trip when necessary.

We also proposed a model for the vehicle shortest path and truck driver scheduling problem (VSPTDSP), which differs from the vehicle routing truck driver scheduling problem usually treated in the literature by focusing not on choosing the order in which to visit the customers, but on which path to take to reach the next customers, including the detours needed to reach available parking locations. The problem was formulated as a shortest path problem with resources constraints. Labels and dominance rules were proposed to be used in a label correcting approach to solve this problem.

With the possibility of future implementation in mind, we consider as future research directions to include time-dependent travel times in the TDSP model, to work on improving its scalability, and to further develop the proposed approach for the VSPTDSP.

7 Implementation

The proposed model can be solved by MIP solvers like CPLEX, so interested companies could test it in practice. However, there are still some issues that need to be addressed before it can be implemented properly. The biggest obstacles for practical applications would be the acquisition of truck parking availability data and the lack of time-dependent travel times.

The parking availability data could be obtained through partnerships with parking locations that already have this kind of data available. For this model, data on the times when the parking lots achieve capacity and when they become available again would be enough. The exact availability at each time of the day is not required. It would be advantageous for private truck stops to provide this information to trucking companies. When included in the route planning these parking locations could be assigned as the scheduled rest stops for certain trips, and receive preferential treatment from the drivers/companies using the system.

The significance of time-dependent travel times may be lower for routes having travel times with smaller variances, but we are not aware of how often this is the case, and if this kind of route would also face parking shortages. This issue requires further research to extend the models and include time-dependent travel times. Another issue which requires further work is the scalability of the model. As the number of parking locations and trip length increase, the solve time increases exponentially. This planning is done off-line, so it is a less pressing issue, but it still significantly limits the number of parking locations that can be considered in long trips. In addition, it hinders the possibility of using the system to update the schedule during the trip when necessary.

References

[1] U.S. Department Of Transportation, "Freight Facts and Figures 2015," Tech. Rep., 2015.

[2] American Transportation Research Institute, "Critical Issues in the Trucking Industry – 2017," ATRI, Arlington, VA, Tech. Rep. October, 2017.

[3] S. Shaheen and C. J. Rodier, "Commercial Vehicle Parking in California: Exploratory Evaluation of the Problem and Possible Technology-Based Solutions," Tech. Rep. August, 2007.

[4] Kimley Horn, "Virginia Truck Parking Study," Virginia Department of Transportation, Tech. Rep., 2015.

[5] U.S. Department of Transportation, "Jason's Law Truck Parking Survey Results and Comparative Analysis," Tech. Rep. August, 2015.

[6] B. C. Tefft, "Prevalence of Motor Vehicle Crashes Involving Drowsy Drivers, United States, 2009-2013," AAA Foundation for Traffic Safety, Washington, DC, Tech. Rep. November, 2014.

[7] American Transportation Research Institute, "Compendium of idling regulations," ATRI, Arlington, VA, Tech. Rep. July, 2016.

[8] M. Palaniappan, D. Wu, and J. Kohleriter, "Clearing the Air: Reducing Diesel Pollution in West Oakland," Pacific Institute, Oakland, CA, Tech. Rep. November, 2003.

[9] N. Lutsey, C.-J. Brodrick, D. Sperling, and C. Oglesby, "Heavy-Duty Truck Idling Characteristics: Results From a Nationwide Truck Survey," *Transportation Research Record*, no. 1880, pp. p. 29–38, 2004.

[10] I. W. Ford Torrey and D. Murray, "An Analysis of the Operational Costs of Trucking: 2016 Update," ATRI, Arlington, VA, Tech. Rep. September, 2016.

[11] United States Environmental Protection Agency, "What You Should Know About Truck Engine Idling, April 2002," 2002.

[12] J. Sochor and G. Mbiydzenyuy, "Assessing the Benefits of Intelligent Truck Parking," *International Journal of Intelligent Transportation Systems Research*, vol. 11, no. 2, pp. 43–53, 2013.

[13] Federal Motor Carrier Safety Admnistration, "Summary of Hours of Service Regulations." [Online]. Available: https://www.fmcsa.dot.gov/regulations/hours-service/summary-hours-service-regulations [2018-03-12]

[14] Code of Federal Regulations, "Title 49 Part 395 (49CFR3.395) (2013), Hours of Service of Drivers."

[15] European Parliament and Council, "Regulation (EC) No 561/2006," *Official Journal of the European Union*, vol. L 102, pp. 1–13, 2006.

[16] European Comission, "Driving time and rest periods." [Online]. Available: https://ec.europa.eu/transport/modes/road/social_provisions/driving_time_en [2018-03-12]

[17] The European Parliament, "Working Time Directive - Directive 2002/15/EC," no. 11, pp. 35–39, 2002.

[18] European Comission, "Working time." [Online]. Available: https://ec.europa.eu/transport/modes/road/social_provisions/working_time_en [2018-03-12]

[19] H. Xu, Z.-L. Chen, S. Rajagopal, and S. Arunapuram, "Solving a Practical Pickup and Delivery Problem," *Transportation Science*, vol. 37, no. 3, pp. 347–364, 2003.

[20] C. Archetti and M. Savelsbergh, "The Trip Scheduling Problem," *Transportation Science*, vol. 43, no. 4, pp. 417–431, 2009.

[21] A. Goel, "Truck Driver Scheduling in the European Union," *Transportation Science*, vol. 44, no. 4, pp. 429–441, 2010.

[22] A. Goel and L. Kok, "Truck Driver Scheduling in the United States," *Transportation Science*, vol. 46, no. 3, pp. 317–326, aug 2012.

[23] A. Goel, "The minimum duration truck driver scheduling problem," *EURO Journal on Transportation and Logistics*, vol. 1, no. 4, pp. 285–306, 2012.

[24] Ç. Koç, T. Bektas, O. Jabali, and G. Laporte, "The Truck Driver Scheduling Problem with Idling Options," CIRRELT, Montreal, Canada, Tech. Rep., 2016.

[25] Ç. Koç, T. Bektaş, O. Jabali, and G. Laporte, "A comparison of three idling options in long-haul truck scheduling," *Transportation Research Part B: Methodological*, vol. 93, pp. 631–647, nov 2016.

[26] A. L. Kok, E. W. Hans, and J. M. J. Schutten, "Optimizing departure times in vehicle routes," *European Journal of Operational Research*, vol. 210, no. 3, pp. 579–587, 2011.

[27] A. Goel and S. Irnich, "An Exact Method for Vehicle Routing and Truck Driver Scheduling Problems," *Transportation Science*, vol. 51, no. 2, pp. 737–754, may 2017.

[28] Ç. Koç, O. Jabali, and G. Laporte, "Long-haul vehicle routing and scheduling with idling options," *Journal of the Operational Research Society*, vol. 69, no. 2, pp. 235–246, feb 2018.

[29] A. L. Kok, E. W. Hans, J. M. J. Schutten, and W. H. M. Zijm, "A dynamic programming heuristic for vehicle routing with time-dependent travel times and required breaks," *Flexible Services and Manufacturing Journal*, vol. 22, no. 1-2, pp. 83–108, jun 2010.

[30] A. C. Gaddy, S. Hernandez, and S. Nurre, "Incorporating Truck Parking and Hours of Service (HOS) into a Truck Routing Heuristic," in *Transportation Research Board 97th Annual Meeting*, Washington DC, United States, 2018.

[31] M.-E. Rancourt, J.-F. Cordeau, and G. Laporte, "Long-Haul Vehicle Routing and Scheduling with Workig Hour Rules," *Transportation Science*, vol. 47, no. 1, pp. 81–107, 2013.

[32] B. A. Schoomer, "The Incorporation of Step Functions and Ramp Functions into a Linear Programming Model," *Operations Research*, vol. 12, no. 5, pp. 773–777, 1964.

[33] S. Irnich and G. Desaulniers, "Shortest Path Problems with Resource Constraints," in *Column Generation*, G. Desaulniers, J. Desrosiers, and M. M. Solomon, Eds. Springer US, 2005, ch. 2, pp. 33–65.