How to Evaluate and Minimize the Risk of COVID-19 Transmission within Public Transportation Systems

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16. Abstract

During the COVID-19 outbreak, serious concerns were raised over the risk of spreading the infection on public transportation systems. As the pandemic recedes it will be important to determine optimal timetable design to minimize the risk of new infections as systems resume full service. In this study, we developed an integrated optimization model for service line reopening plans and timetable design. Our model combines a space-time passenger network flow problem and compartmental epidemiological models for each vehicle and platform in the transit system. The algorithm can help policy makers to design schedules under COVID-19 more efficiently. The report develops an optimized timetable for the Bay Area Rapid Transit system. We found that if passengers choose other mode of transportation when closing part of the system or decreasing the frequency of service can prevent the spread of infections, otherwise, if passengers choose to use the closest open station, closings will lead to longer waiting times, higher passenger density and greater infection risk. We found that the goal of stopping the spread of infection could be achieved by minimizing the total delay when infections were similar in different districts across the service area. Where infection rates are different in different districts, minimizing the risk of exposure can be achieved by minimizing weighted travel time where higher weights are applied to areas where the infection rate is highest.

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Executive Summary

In response to the COVID-19 pandemic, local governments and public transportation agencies implemented various policies during the recovery phase to control the spread of infection, such as reducing service frequency and reducing network coverage. However, it is not entirely clear which policies are needed, and the extent to which they should be implemented. Policies should prevent COVID-19 transmission, comply with social distancing rules, promote safe travel, and be financially feasible. The infection risk of COVID-19 is not considered in pre-pandemic network planning and scheduling methods, which focus on travel time or budget reduction. Furthermore, safety measures such as social distancing and vehicle disinfection need to be considered when drafting network reopening plans and new schedules for public transportation under COVID-19.

Taking social distancing orders and the risk of new infections into consideration, we address the following problems simultaneously in this research project: (1) Network reopening design: how to determine which lines and stations to open in a given region; (2) Timetable design: how to design a timetable to satisfy travel demand and minimize the risk of new COVID-19 cases; (3). Policy evaluation: What is the risk of COVID-19 transmission given an existing timetable and reopening plan? This study summarizes previous models and proposes a general model framework for evaluating COVID-19 risks in public transportation systems which consists of combining two models: (1) a contact tracing model, using a space-time passenger flow network to estimate the contact and travel history of each passenger, and (2) a pandemic mechanism using a spatial compartmental model used in epidemiology, where the results from the contact tracing model are used to predict the estimated number of new infections. Some of our key assumptions are that (1) travel demand is deterministic and known to the decision makers, (2) passengers are boundedly rational, that we can solve a system optimal problem with constraints that each passenger can deviate from the shortest path by only a small amount of time. For example, we can force one passenger to wait for the next train, but the extra waiting time must be smaller than a bound, like 30 minutes. (3) walking time inside stations is ignored, (4) bus/train travel time between two stations are fixed (4) the train dwell time is ignored, and passenger boarding takes no time. For more details, please refer to our unpublished work (*19*). Throughout our discussion, "train capacity" means the limit of the number of riders per car.

We verify our algorithm with a small size example involving varying budget constraints and on-board passenger limits. We also apply our algorithm to the Bay Area Rapid Transit (BART) system to develop an optimal timetable and a reopening plan. Finally, we also compared our results to other studies to provide further suggestions to policy makers.

We find that reducing service would result in running too few trains to meet travel demand if we assume that demand is fixed. Therefore, riders would have to use a limited number of trains, leading to an increase in waiting time and therefore COVID-19 cases. We also observed that the projected number of new cases is less sensitive to changes in service when train capacity is high. Therefore,

reducing capacity on lightly used lines, even during peak hours, will not result in a surge in COVID-19 cases, but doing so when the buses/trains are crowded will significantly increase the number of COVID-19 infections.

One strategy used is to restrict the number of riders per train. If the budget and fleet constraints are tight, reducing the capacities of each car is equivalent to reducing the system capacity. If we reduce capacities of each vehicle when the capacity constraints are tight, passengers are forced to choose alternative lines or wait at the same platform for the next vehicle available. We found that the new COVID cases is sensitive to the capacity of each vehicle when many of the capacity constraints are tight. In other words, if most of the vehicles are already running at their capacities, further reducing the capacity of each vehicle will result in a great increase in the number of COVID cases. If only some of the vehicles are at their capacity, further reducing the capacity of each car will force passengers to choose alternative routes, but this change will not be significant since passengers can easily find other available trains.

Based on our model's formulation, we also conclude that:

1. If the COVID-19 infection rates of different districts in the city are almost the same, the goal of minimizing possible exposure in public transportation networks can best be addressed by maintaining existing service thus reducing the total travel time and crowd levels on the system. If passengers spend more time in a more crowded system, the COVID-19 infection rate will be higher.

2. If the infection rates are different among different districts in the city, the goal of minimizing exposure in public transportation networks can be addressed by reducing the weighted sum of the travel time and crowd level, to provide more service in higher-risk regions.

Our results contradict some studies but agree with other studies. This is due to different assumptions made about travel demand. If we assume no passengers will stop using the public transportation system (they will all find the nearest open station and take a train from there) as in our model, closing some lines/stations or restraining capacity beyond social distancing rules will lead to more crowded stations/vehicles in the remainder of the system and increase the risk of transmission. This can be shown from our numerical examples and Islam et al.'s data-driven study (16) or Kumar's simulation (13).

However, if we assume some customers will choose other modes of transportation if station/lines they used before the pandemic are closed, then it would be beneficial to close lines/stations, as is shown by Mo et al.(14). For policy makers, it would be better to test different policies to see how the demand actually changes or to use more advanced demand models to predict the possible impact of policies on travel demand.

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Introduction

In response to the COVID-19 outbreak, the Center of Disease Control and Prevention (CDC) has promulgated guidance, including the practice of social distancing. The state of California also made intensive efforts to implement these protocols and became the first state to issue a mandatory shelter-in-place order, which has largely mitigated the spread of the disease. Consequently, both the demand and supply for public transport decreased drastically from March to September 2020.

During the COVID-19 pandemic, public transportation systems worldwide have faced significant challenges while gradually recovering from total shutdown, including drops in passenger ticket income and the increased risk of spreading COVID-19. According to a report published in September 2020 by the American Public Transportation Association ([1](#)) and other reports ([2](#)), public transit systems across the U.S. are safe for travel owing to the applied response measures, and as of mid-August 2021 there had been no direct evidence of COVID-19 outbreaks linked to intracity public transit. However, there could be new cases related to public transit that have not been identified by contact tracing after open-up in 2021. In China, public-transportation-related outbreaks have been traced to intercity buses in Zhejiang ([3](#)) and Hunan ([4](#)).

To deal with the infection risk, local governments and public transportation agencies have implemented various COVID-19-related policies during the recovery phase to control the spread of the pandemic, such as reducing service frequency and reducing network coverage. For example, in the Bay Area, Bay Area Rapid Transit (BART) reduced service frequency from 15 minutes to 30 minutes on certain lines in April 2020 ([5](#)). New York City Metropolitan Transit Authority cancelled overnight subway services in May 2020 ([6](#)) ([7](#)). In Washington D.C., the Washington Metropolitan Areas Transit Authority closed 19 stations in April 2020 ([8](#)).

However, it is not entirely clear which policies are needed, and the extent to which they should be implemented. Policies should be able to prevent COVID-19 transmission, comply with social distancing rules, and promote safe travel, but policies also need to be budget-friendly. It is worth noting that although many efforts have been made to model the spread of the disease, and the public transportation network design problem has been thoroughly studied in the transportation literature, no clear guidance is available to public transit agencies to produce safe and efficient transit schedules and devise network reopening plans for public transportation systems. The risk of COVID-19 infection is not considered in existing network planning and scheduling methods. Furthermore, safety measures such as social distancing and vehicle disinfection procedures reduce system capacity and increase operating costs. These factors need to be considered when drafting network reopening plans and new schedules.

Taking the active social distancing orders and the risk of new infections into consideration, we address the following problems simultaneously in this project: (1) Network reopening design: How to restore service curtailed during the pandemic and determine which lines and stations to open in a

given region?; (2) Timetable design: How to design a timetable to satisfy travel demand and still minimize the risk of new cases?; (3). Policy evaluation: How to estimate the risk of COVID-19 for a given timetable and reopening plan? In this report we present an optimization framework to minimize the risk given budget, demand and fleet constraints, and evaluate different policies under this framework. We also compare the results from our model and other studies to provide additional suggestions to policy makers.

Closing transit systems is the best strategy to eliminate the spread of COVID-19 but that may not always be practical or desirable. Closing or limiting service on high-demand lines would help to control the spread of COVID-19, though it would reduce the number of passengers served which could actually increase the spread depending on how passengers react.

COVID-19 Policy Evaluation Methods for Public Transportation Systems

This section summarizes different COVID-19 policy evaluation methodologies for public transportation systems based on epidemiological research published worldwide in 2020 and early 2021. These studies can be classified into two categories:

(1) Research with transportation models; and
(2) Research without transportation models.

Transportation models here includes travel demand models, route choice models, and/or delay estimation models. In studies considering COVID-19 and transportation models, some researchers (9) (10) (11) (12) (13) (14) have considered the risks of COVID-19 while building public transportation models. Using this approach, it is possible to quickly evaluate the impacts of different policies and provide additional insights into the COVID-19 risks in public transportation systems. The disadvantage of this type of work is that it usually requires many assumptions regarding the pandemic and passenger route choice properties. If these assumptions are incorrect, the results may be incorrect.

Studies without transportation models (e.g., (3) (4) (15) (16) (17)) are usually based on background knowledge of statistics/biostatistics and epidemiology. Researchers have evaluated policies implemented in 2020 and provided policy suggestions using data-driven approaches such as regression analysis and causal inference. This approach, however, is not able explain the mechanisms behind the correlations identified in their data analyses or make predictions since their analysis is based on existing pre-pandemic data.

We know that COVID-19 can be contracted on public transportation systems when a susceptible passenger encounters an infectious passenger. Studies of COVID-19 epidemiology that utilize transportation models typically address the following issues:

1. Contact tracking: How many infectious passengers did each susceptible passenger encounter? Where did they meet? How long were they in contact?

2. Pandemic mechanism: Given the contact history, how many susceptible passengers contracted COVID-19 after staying in the same location with an infectious passenger for a given period of time?

In other words, contact tracking helps us identify passengers' contact histories in the transportation system, and the pandemic mechanism helps us predict the risk of COVID-19 in the system given such

contact histories. A public transportation model can usually solve the first issue, and epidemiological models play critical roles in addressing the second issue.

Contact Tracing

As COVID-19 is transmitted through contact between individuals, the first task is to track the passenger contacts inside the system. Contact occurs when two passengers stay in the exact same vehicle or on the same platform for a certain period of time. The intensity of the contact depends on the duration and the character of the contact. The longer the passengers spend time together, the higher the risk. If this contact occurs in an open outdoor space or with both persons wearing masks, the risk will be lower. To track possible contacts, we can consider (1) a data-based approach, (2) a multiagent simulation-based approach, and (3) a network-flow-based approach.

A data-based approach does not make assumptions regarding passenger route choices. This type of model uses real-world data such as smart card data (14) (18), and automatic passenger count (APC) data (13) to directly track contacts. Smart card data provide complete information for bus systems if the dataset includes tap-on and tap-off records. It becomes much more challenging to track encounters in a metro/train system, as the smart card data usually only provides entry/exit information at stations. We can match individual trips to trains with mild assumptions regarding passenger behavior. For example, Liu et al. (18) assumed that passengers swiping-out from the same vehicle will form a cluster, then recovered the contact history using clustering and matching algorithms.

For metro/train systems, some researchers have made assumptions on passenger route choice and run multiagent simulations or solved network flow problems on public transportation networks to trace possible contacts. Qian et al. (10) ran a simulation model for the New York metro system. They constructed a network based on the observed metro network layout, demand profile, mobility patterns, and smart card data. Talekar et al. (12) generated travel demand data using a synthetic city based on Mumbai, and ran a cohorting strategy for transit users. Luo et al. (9) used a metapopulation model to estimate travel reproduction numbers under different policies. They assumed that the route choices for passengers remained the same. Qian and Ukkusuri (11) used a deterministic queuing network to model an intracity mobility pattern. They assumed that the departure rate (probability that people leave their living area), split ratio (probability that people move from one region to a different region), and arrival rates were known.

Pandemic Mechanism

With passengers' encounter histories, we can apply a modeling framework for infectious diseases to the transportation system. The population can be split into groups according to health conditions, such as under the susceptible-infectious-recovered (SIR) and susceptible-exposed-infectious-

recovered (SEIR) approaches. The number of new infections is defined as the difference between the susceptible (S) population from time t to time $t + 1$.[1]

Once the contact intensity between susceptible passengers and infected passengers is known, different pandemic mechanisms can be applied to estimate the number of new infections. A pandemic mechanism is a set of equations describing how each group's size evolves, given its contact history. There are two mechanisms that have been used in policy evaluation models for public transportation during the pandemic: (1) individual-level mechanisms based on contact networks, and (2) group-level mechanisms based on compartmental models in epidemiology.

For agent-based contact tracking models (14) (10) (12) (data-based and multiagent simulation-based contact tracking), we can build an individual-level contact network. A contact network is an undirected weighted graph where each node is a passenger, and each link is a contact between two passengers. The weight of the link represents the intensity of contact. For example, if passengers A, B, and C were in the first bus together for 30 minutes, passenger B, D, and E were in the second bus for 20 minutes, and passengers D, E, and F were in the third bus for 15 min, their contact network is shown in Figure 1.

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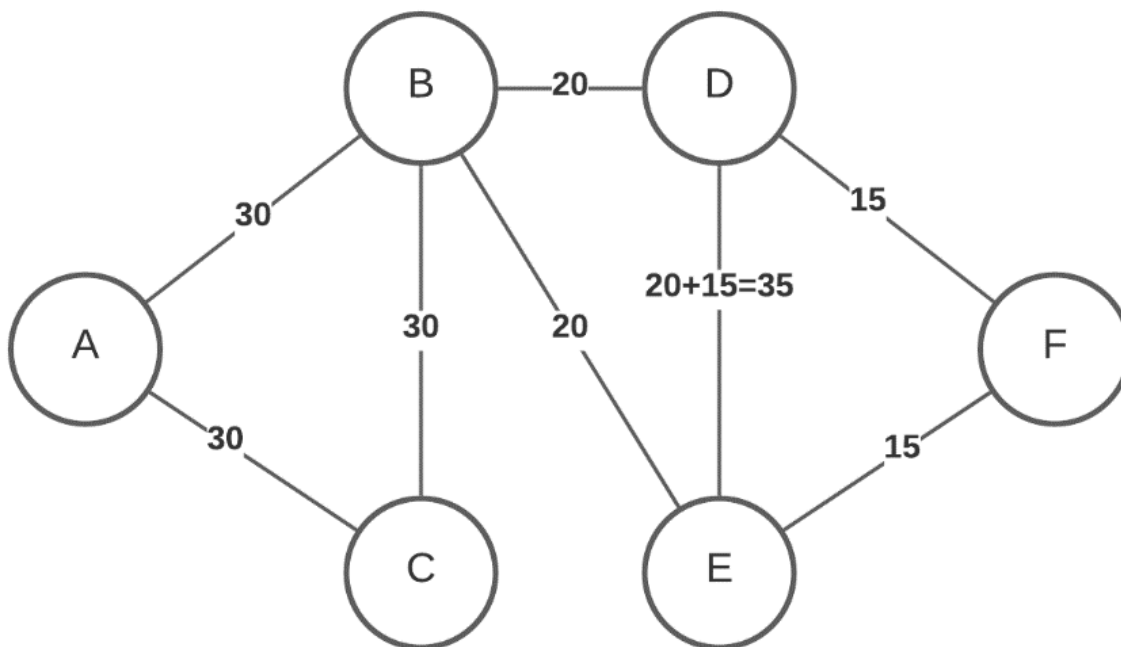
[1] Unlike the case with a long-range epidemiological model, since the time interval here is too small for population growth or immigration to take effect, the change in the S population is determined solely by the pandemic.

Figure 1. A contact network example

Once we have constructed the contact network, the probability of contracting COVID-19 can be modeled as a non-decreasing function of the contact intensity. This approach is useful but cannot be applied to crowded or large-scale systems because the model is agent-based, and the number of nodes will increase rapidly when analyzing a large system.

Another approach is the aggregated-level epidemic model used in network flow-based contact tracking (9) (11). In this type of approach, the dynamics of COVID-19 are modeled as a set of differential equations, where the derivative of the susceptible (S) population indicates the number of new infections. Like the individual-level model, the number of expected new infections is a non-decreasing function of the contact intensity. Our model is based on the aggregated level approach to simplify the calculation of potential infections and to facilitate the optimization procedures.

Report Methodology

In this section we briefly introduce our model and the optimization model. Mathematical details are omitted and interested readers can refer to our paper in the repository arXiv ([19]) for the complete model.

Contact Tracing Model: Space-Time Transit Network with the Pandemic

We use a time-space passenger flow network to estimate the travel and contact history of passengers with given travel demand.

Physical Network

In general, people travelling from one district to another by transit will use the stations nearest to their origin and destination. Each station may have one or multiple platforms. We can model the transit network as in Figure 2.

In Figure 2, we assume the city has a residential district and a commercial district. There are several stations in the public transportation system including station A, B, C, and D. Line 1 (red) connects B and A, line 2 (blue) connects A and C, while line 3 (green) connects A to C. Station B is the nearest station to the residential district while station D is nearest to the commercial district.

Consider a passenger travelling from the residential district to the commercial district. We use a solid red arc in Figure 2 to represent the trajectory of this passenger. He/she can take line 1 from station B to station A, transfer to line 2 at station A, transfer to line 3 at station C, take line 3 from station A to station C, and walk to the commercial area. Note that in station A, there are two platforms so that transfer passengers need to change platforms, while in station C, all lines use the same platform.

Space-Time Network

During a pandemic like COVID-19, the capacity of each vehicle may be strictly limited according to social distancing rules. Therefore, for purposes of constructing this model we need to know the exact number of passengers permitted on each vehicle given a strict social distancing capacity constraint, rather than merely calculating the average number of passengers and comparing it with the vehicle capacity as would have been done in a pre-pandemic situation. In addition, we need to track all passengers who boarded the same vehicle or waited at the same platform with infected individuals, since transmission can occur on platforms or vehicles. We also need to know the exact waiting time to optimize the timetable so that passengers can transfer between different lines smoothly. We cannot answer these questions using the physical network in Figure 2, which can only give network flows averaged over time. We need to add a time dimension to the network and model the city as a space-time network similar to the models in Liu and Zhou ([20]) and Fan et al. ([21]). The idea is to make many

copies of the nodes of the physical network and connect them using different types of arcs representing in-vehicle travelling, waiting time on the platforms, walking, and early/late departure/arrival.

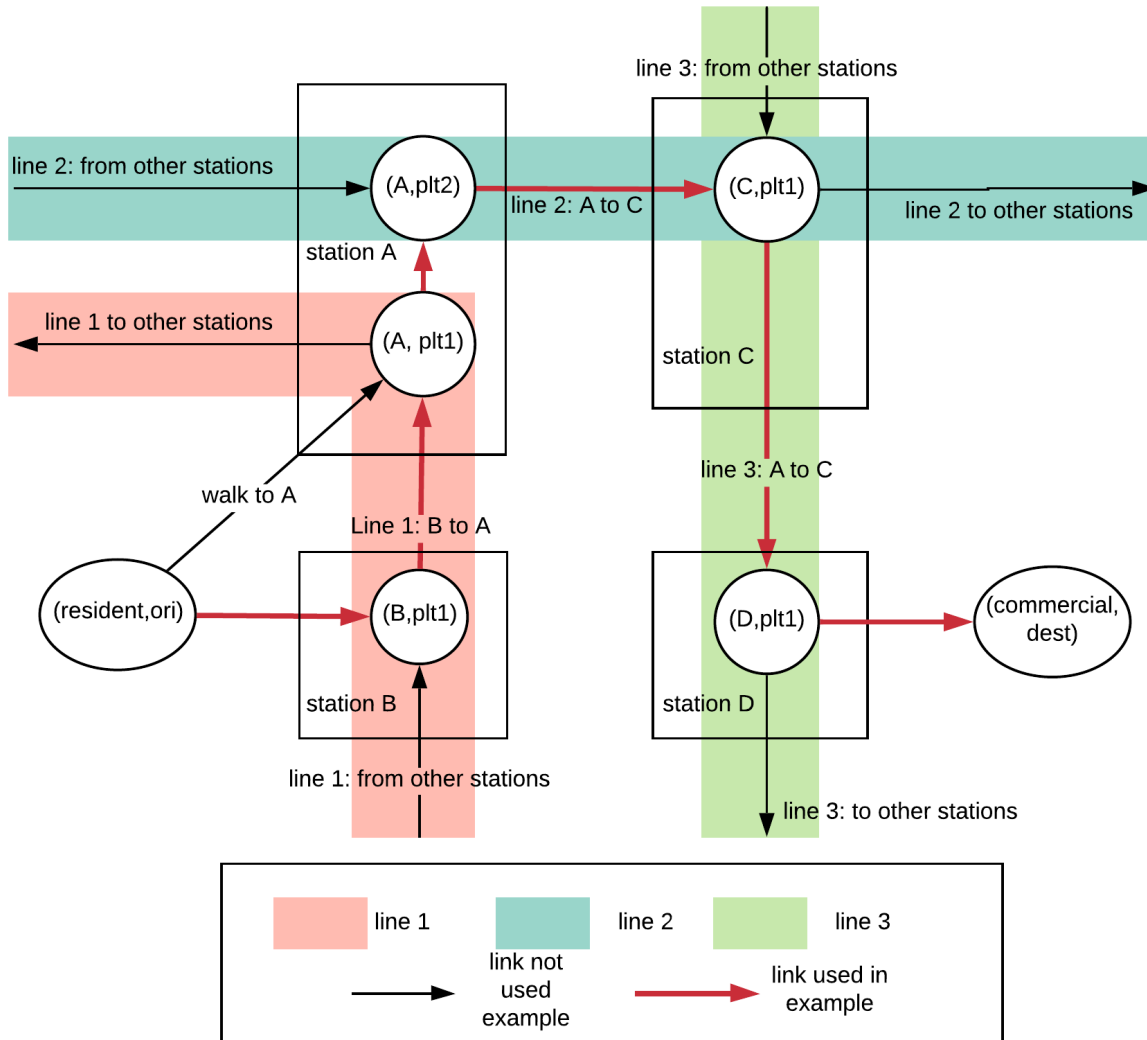

Figure 2. Physical network

If we make T copies of the physical network, we can make connections so that each arc represents a trip from one location at time t_1 to another location at time t_2, where $t_2 - t_1$ is the travel time, or

waiting at same location from t_1 to t_2. In
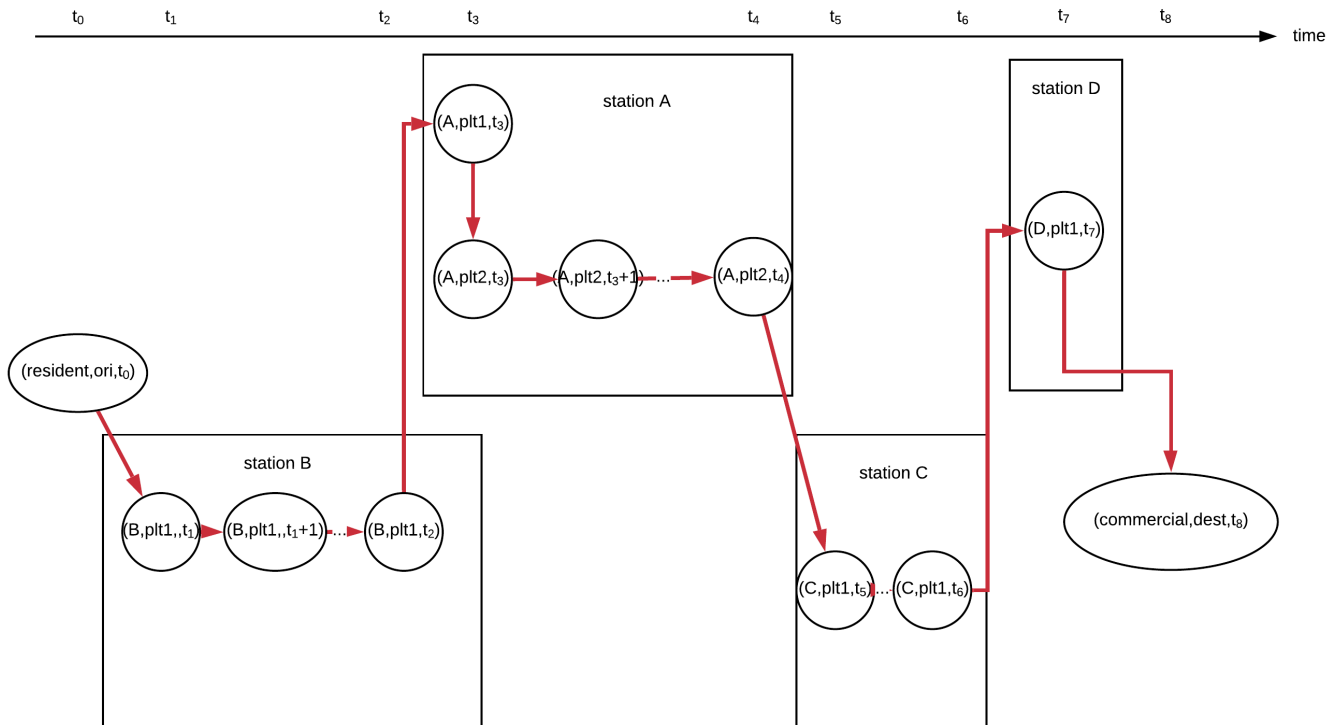
Figure 3, we show how the passenger in Figure 2 will travel in the time-space network. He/she will

- walk from the residential district to station B's platform 1 with travel time $t_1 - t_0$.
- wait at the platform until the next train from line 1 arrives at station B for $t_2 - t_1$, since the next available train will arrive at station B at t_2
- ride a train/bus from B to A using line 1 with travel time $t_3 - t_2$
- transfer from line 1 to line 2 at station A, assuming the transfer takes $t_4 - t_3$.
- wait at station A for the next available train from line 2
- ride line 2 to station C, assuming the travel time from station A to C on line 2 is $t_5 - t_4$
- wait for line 3 at the same platform, and travel to station D
- walk to the commercial area from station D.

Figure 3. Trajectory on the space-time network

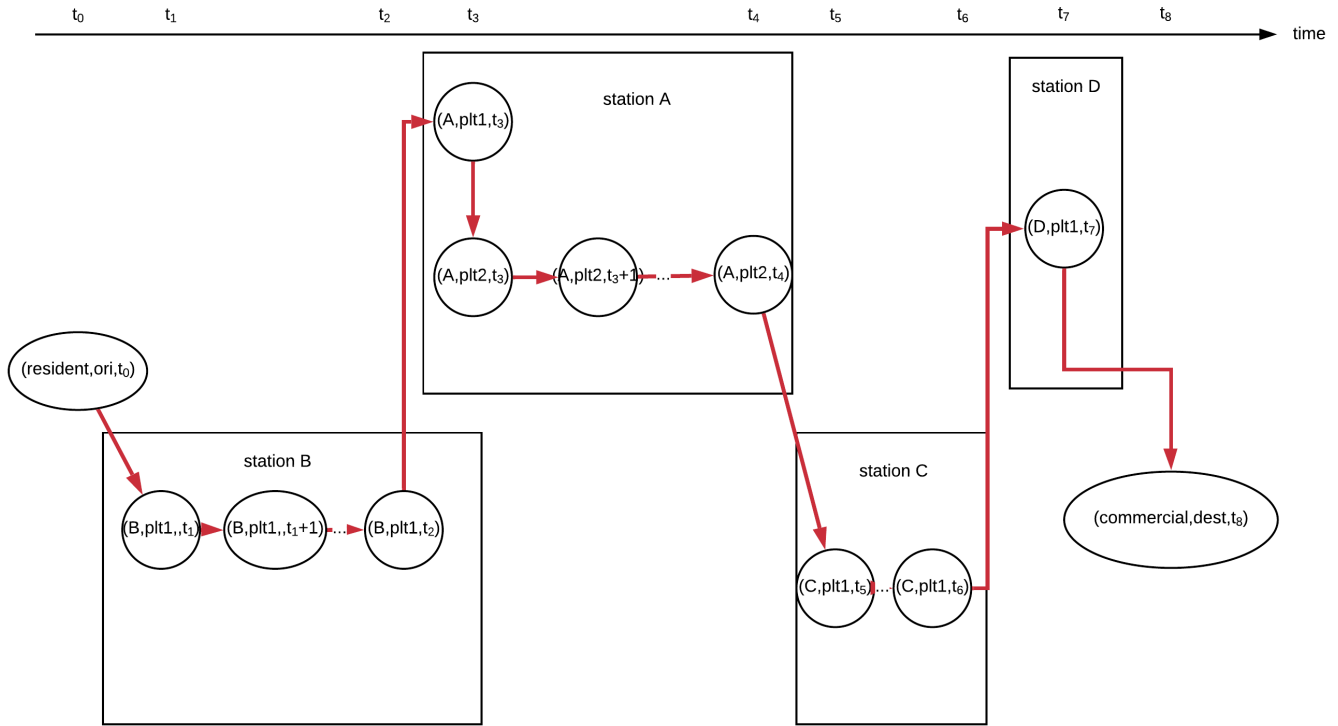# Pandemic Mechanism: Space-Time Transit Network with the Pandemic

Contact Intensities on the Space-Time Network

Assume arc (i,j) in the space-time network represents an in-vehicle trip, passengers using this arc can be divided into different types according to their origin and destination (OD). For example, if we assume there are two types of passengers on this vehicle, i.e. passengers are can be divided into two groups where each group has the same OD. Let the passenger flow of OD pair 1 on arc (i,j) be u_{ij}^1, and the flow of OD pair 2 be u_{ij}^2. Let the travel time on the arc be c_{ij}. Let the infection rate of passengers from OD pair 1 and 2 be q_1 and q_2 respectively. The infection rate on that vehicle/platform represented by arc (i,j) will be $\frac{q_1 u_{ij}^1 + q_2 u_{ij}^2}{u_{ij}^1 + u_{ij}^2}$, and the time that one passenger will spend on this vehicle is c_{ij}. The contact intensity for one passenger on the vehicle will be $c_{ij} \frac{q_1 u_{ij}^1 + q_2 u_{ij}^2}{u_{ij}^1 + u_{ij}^2}$.

Estimate the Total Number of New Infections

If we assume there are three types of passengers: susceptible (S), infectious (I), and recovered or vaccinated (R). If we assume the proportion of susceptible is q_s, the expected number of new infections on that vehicle would be $c_{ij}q_s(u_{ij}^1 + u_{ij}^2)\frac{q_1u_{ij}^1+q_2u_{ij}^2}{u_{ij}^1+u_{ij}^2} = \beta c_{ij}q_s(q_1u_{ij}^1 + q_2u_{ij}^2)$.

If there are multiple types of passengers, the expectation of total new infections would be $\sum_{(i,j)}\sum_w c_{ij}\beta q_s q_w u_{ij}^w$, where w represents different OD pairs.

Optimizing the Timetable and Network Designs

Assumptions

We make these key assumptions on passenger route choice:

1. Users are boundedly rational.
2. Travel demand (from each origin to each destination) at each time step is fixed, and the system serves all the demand.
3. Walking time inside stations is ignored.
4. Bus/train travel time between two stations are fixed.
5. Buses/trains need to be cleaned and disinfected at the last station after one run.
6. The train dwell time is ignored, and passenger boarding takes no time.

In assumption 1, we assume users are boundedly rational, like the model in Liu and Zhou ([20]), in that users are willing to deviate from taking the shortest path if it involves only a limited amount of additional time. The transit agency can control the route by changing the ticket price, enforcing capacity rules so that passengers are forced to change routes or wait for the next train when a car reaches its capacity, or releasing real-time on-board crowd data so that passengers know the risk and can avoid crowded areas out of concern for their own safety.

In assumption 2, we assume that all time-dependent OD demand must be satisfied. Since people who need public transportation during a pandemic like COVID-19 usually don't have access to other means of transportation such as private cars, carpool service may be suspended, and taxis are expensive, we need to satisfy this part of demand using the transit system. In addition, we assume that during the pandemic, remain-at-home orders allow people to 'go out only when necessary' so whatever travel demand exists is deemed 'necessary.'

In assumption 3, we ignore the walking time inside the station so that we only need to consider the possibility of disease transmission on platforms and vehicles. Noting that passengers spend most of their time waiting on platforms or riding in vehicles the time that passengers spend walking inside the stations (usually less than 1 minute) can be ignored compared to waiting time and traveling time.

Assumption 4, 5, and 6 are common assumptions in public transportation models to simplify the calculation of possible new infections.

Input of the model

To determine the optimal timetable design the following input is required:

- Delay tolerance bound in rational bound model. We set this value to 45 minutes in all our numerical examples.
- Time dependent user demand, as is stated in assumption 2. Each passenger has a origin, a destination and a desired departure time.
- Total budget for operational cost.
- Total available trains/buses.
- The network: line and station.

Constraints

There are several constraints we considered in our network flow model, including:

1. Budget constraint: the total cost of bus runs and line/station opening must be less or equal to the budget.
2. Fleet constraints: we can dispatch one train run from the first station on the line only when there are idle trains in that station at that time.
3. Network flow constraints: For station platforms, the incoming flow of passengers must be the same as the outgoing flow. For city districts, the incoming/outgoing flow must be equal to the passenger demand originating from or going to that district.
4. Boundedly rational customers: Passengers are only willing to deviate from the shortest path for a limited amount of time. For example, if the shortest path travel time is 30 minutes, and the bounded rational constraint time is 10 minutes, then the passenger can accept a new path for 30+10=40 minutes. Note that we will not abandon any customers in our optimization model, if this demand cannot be satisfied, the optimization problem will be not feasible.

Minimizing the Total Number of New Infections

We used state-of-the-art optimization methods to obtain a solution for the optimal timetable design and the network reopening plan with the minimal number of potential new infections in each system. To solve the problem, we performed Lagrangian Relaxations (LR) where the integer constraints are relaxed. This is an iterative heuristic algorithm which can help us find lower and upper bound of the problem. At each iteration, we find a lower bound according to the relaxed problem with multipliers, while we can also get an upper bound by solving an integer programming as our heuristic. We then update the multipliers using subgradient method. Please refer to Huang and Shen (19) for the complete algorithms.

Note that the total infections is positively related to the total travel time, in the optimal solution of our mathematical programming problem, two lines will be automatically synchronized at each transfer stations.

Numerical Experiments and Policy Insights

In this section, we verify the accuracy of our design algorithm on a hypothetical toy network. Then we will present our morning peak-hour timetable design for BART.

Numerical Experiment on a Toy Model

Verifying the algorithm
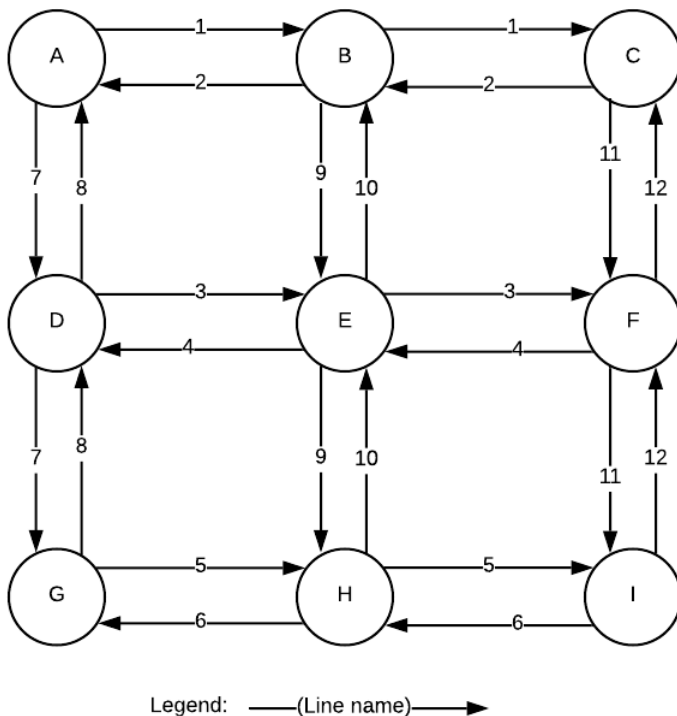
Figure 4. Toy system map

Consider designing a timetable for the small-size transit system given in Figure 4 to minimize the total infection with budget, fleet and demand constraints. The total infection is estimated using the expected number given in the previous section. As this is an small-scale toy problem, we can find the exact result. using GUROBI solver ([22]) and compared with the lower bound determined using our algorithm. The lower/upper bounds after 1000 iterations are (1.4497, 1.4947), while the exact solution of the solver is 1.4947 infections per day in the toy problem. The proposed algorithm has a relatively tight bound (< 4%) for the toy problem, and the optimal solution is found when solving the upper bound. Therefore, our algorithm can correctly identify the optimal solution in this example.

Policy Evaluation: Reducing Service

Figure 5. Analysis on budget

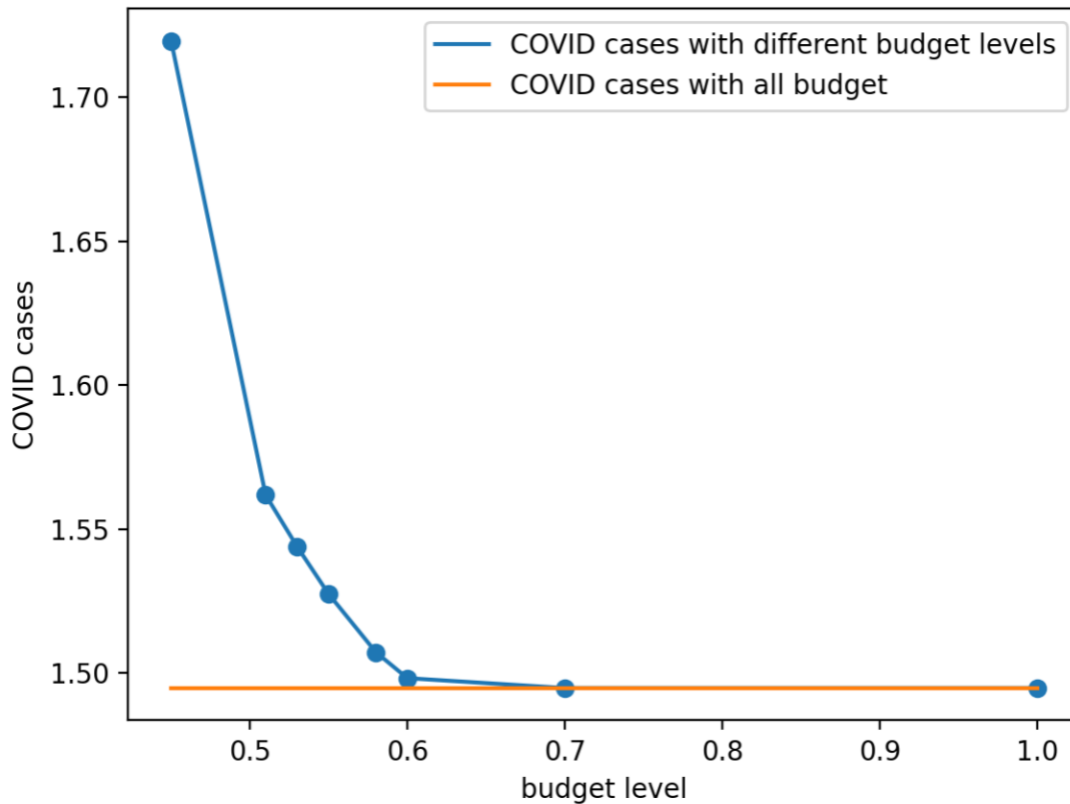The transit agency may decide to reduce service due to the reduced ridership or economic difficulties during the pandemic. For now, the system has enough budget as the pre-pandemic scenario, but the demand is assumed to be in-pandemic. Therefore, the demand constraint is not tight, and we can further reduce the budget while assuming the demand is fixed and is small. We solve the problem under different levels of operational budget as is shown in Figure 5. The budget level set a limit on the number of dispatches, resulting less frequent trains, while the network size, the total number of vehicle available remains the same. Under each operational budget level, we run our optimization algorithm again to find out the optimal timetable design to minimize the risk of COVID-19, and the corresponding optimal number of infections per day is given in Figure 5.

In this example, we use the stochastically generated demand representing the transit demand during the pandemic. The demand is fixed throughout the experiment under different budget levels. The number of available cars is also fixed. The time tolerance of each boundedly rational passenger is assumed to be 45 minutes.

In this example, when the budget is greater than 70 percent of the total budget provided in the last toy model example, we can see the number of COVID-19 cases will not change since the budget constraint is loose, that is, there is still budget in the system to accommodate the travel demand. When budget is smaller than 70 percent, the pandemic infection rate will start to rapidly increase when we reduce service further as passengers begin to crowd onto fewer and fewer trains. When the budget is smaller than 45 percent of the original total budget, it is not possible to find a feasible solution satisfying all travel demand. Therefore, the budget transition point is < 45 percent.

Policy Evaluation: Cutting Train Capacity
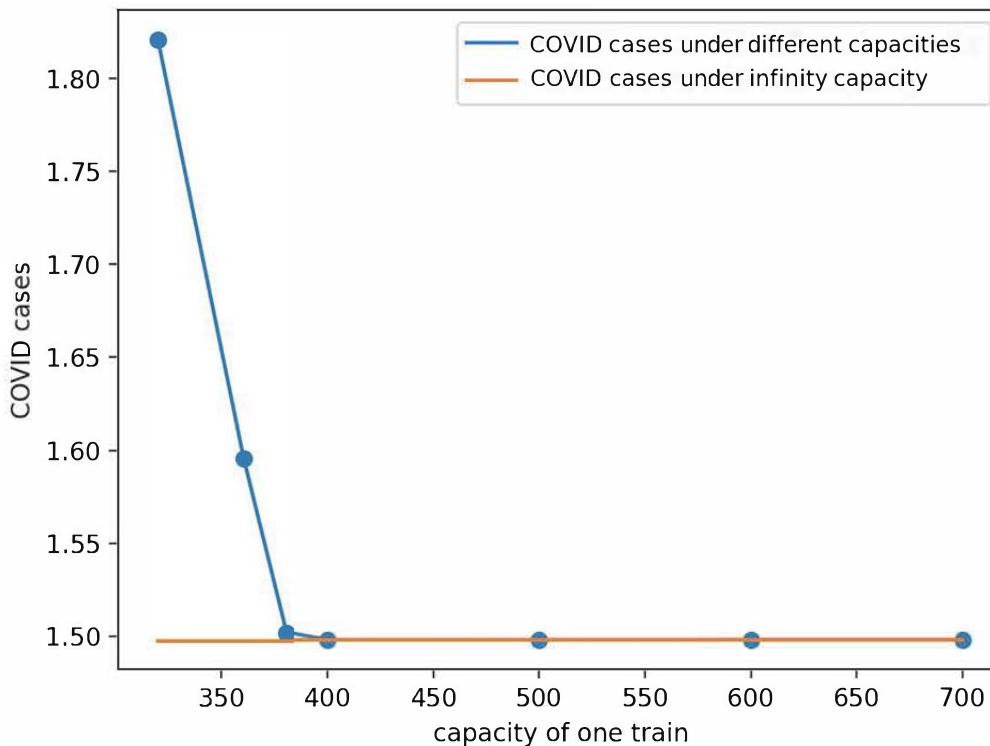
Figure 6. Analysis on capacity

Under the reduced demand during the pandemic, the carrying capacity of one train is also limited according to the social distancing protocols. More train runs would have to be put in service satisfy all the demand even if the demand is small compared to the pre-pandemic. Assume we are running the 10-car trains where the reduced capacity for one car is 60 according to the social-distancing protocol, while the budget and the demand is fixed. If we further reduce the capacity of one car due to a stricter social-distancing rule, with the budget constraint, the number of open lines and train runs would be limited, and the overall system capacity will be reduced.

We changed the capacity of one train in this model to see how the number of new cases changes. For each experiment, we set the budget level to be 60 percent, performed 1000 iterations of our

algorithm and used the best upper bound as the result. Our experiments show that when the train car capacity is 400 or more passengers, the number of potential new cases remain the same, because in the uncapacitated problem, there are at most ~ 400 riders on one train.

The number of new cases will increase rapidly when the capacity for each train is restricted to less than 400 passengers (40 riders per car for 10-car trains). This is due to the increasing waiting time and crowdedness on platforms. As with the budget constraint in the previous example, there is a minimum capacity where this problem become infeasible, and both new cases and total passenger travel are more sensitive when the capacity gets closer to the minimum capacity. When car capacity is limited to less than 320, there is no feasible solution found after 1000 Lagrangian Relaxations (LR) iterations, so the transition point is < 320. In other words, if the capacity is set to 32 riders per car for 10-car trains, with the given budget, we cannot satisfy all the demand since we assume that passengers are boundedly rational, some of the passengers must have delays longer than our limit of 45 minutes, no matter how we adjust the timetable.

Optimizing BART Timetable and Reopening Plans

BART is a rapid transit system that serves the San Francisco Bay Area. It spans 121 miles of double track, and has 48 stations (5). The network is shown in Figure 7. The time horizon consists of the peak travel hours of operation (8:00-11:00 AM) and is divided into one-minute intervals. According to the current timetable, in-vehicle travel time is much longer than the dwell time at each station, so the assumptions listed above are satisfied. There are many parallel links in BART, and transfers between lines occur frequently.

During the pandemic, the demand has been reduced by more than 70%. We use the historical time-dependent workday demand pattern in the pandemic (in March 2020). According to BART, the social distancing protocol requires the number of riders on each car to be less than 60 people. The total system operating cost budget was $767.8 million in 2020 (23). Assume that operating costs are proportional to the length of the operating time of each train dispatched for each line, the cost of one dispatch can be estimated, as is shown in

Table 1. BART line operation cost. The fleet
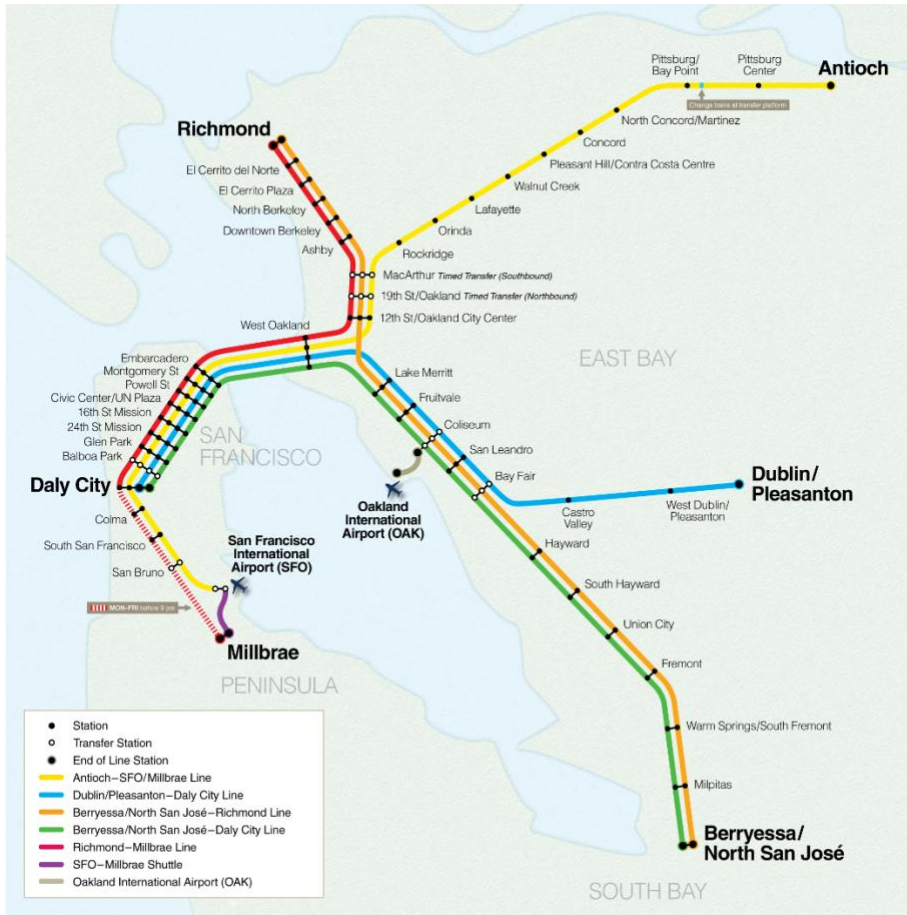
Figure 7. BART map ([24])

Table 1. BART line operation cost

Line number	Color	Line name	Cost ($ millions)
0	gray	Oakland Airport to Coliseum	848
1	gray	Coliseum to Oakland Airport	848
2	blue	Daly City to Dublin/Pleasanton	4179
3	blue	Dublin/Pleasanton to Daly City	4118
4	green	Daly City to Berryessa/North San Jose	5390
5	green	Berryessa/North San Jose to Daly City	5330
6	orange	Berryessa/North San Jose to Richmond	4966
7	orange	Richmond to Berryessa/North San Jose	5330
8	purple	Millbrae to SFIA	545

Line number	Color	Line name	Cost ($ millions)
9	purple	SFIA to Millbrae	545
10	red	Millbrae/Daly City to Richmond	4361
11	red	Richmond to Daly City/Millbrae	4542
12	yellow	Millbrae/SFIA to Antioch	6299
13	yellow	Antioch to SFIA/Millbrae	6481

According to our algorithm, the optimal timetable is shown in Table 2. BART optimal timetable design. The objective function is the expected number of COVID-19 infections during morning peak hour per day. After 1000 LR iterations, the gap is approximately 5.4 percent (LB: 0.470605, UB: 0.496054). The upper bound solution is better than the cost-calculated solution using the current running timetable (COVID-19 cases per day during morning peak hour: 0.500649), which is calculated based on the timetable of BART weekday operation (24) using our column generation algorithm to find the optimal network flow and cost.

In the proposed optimal solution (with the minimum COVID cases while satisfying demand, capacity, budget and fleet constraints), all train lines and stations will be open. The reason is that we need to cover all stations (that there's at least one line accessing to each station) to have a feasible solution and we must open most of lines in BART system to make sure all stations are covered. Although passengers can walk directly from their origin to an open station, the average walking distance is great since the distance between any two stations in the BART system is quite large. Therefore, passengers using the system must have access to a station nearby, and most of the lines need to be open to cover all stations. Another reason for opening all lines is that parallel line makes it possible to set different frequencies for different line sections, thus reducing waiting time and new infections.

In our optimal timetable design, most of the trains will have two to four runs per hour, while the current timetable has only two runs. Lines 11 and 13 (Richmond to Millbrae, and Antioch to SFIA) have many more runs than lines 10 and 12 (Millbrae to Richmond, and SFIA to Antioch). During the morning peak hour, people need to leave their homes in the East Bay for travel to the San Francisco CBD area, and our design captured this demand pattern.

Compared to the original timetable, our timetable has higher run frequency in the first two hours (8:00-10:00 AM). This is because the demand in the first two hours is much higher than the demand from 10:00 AM to 11:00 AM. According to BART data, during the pandemic (March 2020), the total weekday daily demand is 2200 riders from 8:00-9:00 AM, 1649 from 9:00-10:00AM and 1337 from 10:00-11:00AM. We assume the demand is half of the March 2020 demand during the pandemic.

Another major difference between our timetable and the existing timetable is that we have far fewer operations for line 8 and 9 (Millbrae to SFIA and SFIA to Millbrae). There are two reasons for this: (1) These lines serve only passengers between Millbrae and SFIA, and the number of passengers is small

compared to the whole system (approximately 0.1%, with 14.75 passengers from MLBR to SFIA and 15.57 passengers from SFIA to MLBR while the whole system has an average of 27677.38 daily users in weekday March 2020); (2) Our algorithm favors long distance lines since we assume all trains need to be cleaned after one operation, and the cleaning cost and time is fixed. Therefore, the per-mile cost is higher for short lines compared to other lines, and the algorithm tends to favor long distance lines.

Table 2. BART optimal timetable design

Line number	Line name	Optimal timetable (minutes from 8:00AM)	Current timetable (minutes from 8:00AM)
2	Daly City to Dublin/Pleasanton	0,40,60,70,80,110,120,150	21,51,81,111,141,171
3	Dublin/Pleasanton to Daly City	10,30,40,70,80,100,120,140	9,39,69,99,129,159
4	Daly City to Berryessa/North San Jose	10,30,40,60,90,110,130	12,42,72,102,132,162
5	Berryessa/North San Jose to Daly City	0,20,40,70,100	0,30,60,90,120,150
6	Berryessa/North San Jose to Richmond	20,40,60,70,90,100,170	12,42,72,102,132,162
7	Richmond to Berryessa/North San Jose	20,30,50,90,110,120,140,150	3,33,63,93,123,153
8	Millbrae to SFIA	40,110	27,57,87,117,147,177
9	SFIA to Millbrae	120,170	9,39,69,99,109,129,159
10	Millbrae/Daly City to Richmond	20,50,70,100	18,48,78,108,138,168
11	Richmond to Daly City/Millbrae	10,30,40,50,80,90,100,130,140,170	9,39,69,99,129,159
12	Millbrae/SFIA to Antioch	20,50,60,70,80,110,120,170	24,54,84,114,144,174
13	Antioch to SFIA/Millbrae	10,20,60,70,80,90,100,120,140,150,170	0,30,60,90,120,150

Policy Evaluation: Reopening Closed Lines

In this experiment we looked at optimizing reopening the entire system from a hypothetical closed condition using BART as an example. BART never actually closed any lines, but some other systems closed their lines and some researchers like Mo et al. (14) proposed that we should close lines to prevent COVID-19 transmission. In this experiment, we will have a look at the effect of line closure during the pandemic. We assume that passengers will go to the closest open station from their origin and leave the system from the station closest to their destination using the shortest path in the physical network, so the total OD demand is the fixed after closing lines, but some passengers may choose other modes of transportation to get to the closest open stations. We open the lines one by one following Table 3. The proposed algorithm is then executed for 1000 LR iterations to find the upper/lower bound of the COVID-19 risk, optimal timetable design, and network flow assignment for each experiment (plans 1-5). The resulting number of new cases, and average travel time spent travelling within the transit system and the time spent outside the transit system (traveling to/from the nearest open station) are shown in Table 4.

Table 3. Experiment setting for reopening closed lines

Plan	Yellow	Blue	Green	Orange	Red	Purple
1	Open	Open				
2	Open	Open	Open			
3	Open	Open	Open	Open		
4	Open	Open	Open	Open	Open	
5	Open	Open	Open	Open	Open	Open

Table 4. What happens if we reopen closed lines?

PLAN	1	2	3	4	5
NEW CASES (PER 1000 RIDES PER MORNING PEAK HOURS)	0.158	0.172	0.226	0.209	0.183
TRAVEL TIME IN THE SYSTEM (MIN PER PERSON)	21.925	23.975	31.079	28.892	25.297

From the table above, we can see when only two lines are open, the demand is huge, and the system cannot satisfy all the demand in the initial example above (experiment plan 1). We notice that the number of new cases or travel time inside the system is not monotone increasing or decreasing with respect to number of lines. It will increase then decrease as we add lines to the system.

When there are only a few open lines, not all stations are covered, and many people would have to spend most of their time walking or using other modes of transportation to access the system. Therefore, the number of new infections occurring inside the system will be small. Opening a new line result in an increase in passenger distance traveled on the system, and the number of new cases will also increase. Note that although we assume the total demand is fixed, passengers can choose different modes of transportation if they don't have access to the transit network.

With most of the network opened, all the stations are covered, and there are many parallel lines. This enables us to generate flexible timetables such that the frequencies for different route sections can be different using overlapping parallel lines, and the optimal new infection number can be minimized. Closing some existing lines and stations at that point would force some passengers to go to nearby open stations and the number of queuing passengers would increase, leading to more infections.

Policy Recommendations

In this section we summarize our policy recommendations and compare them with other studies.

Closing Lines/Stations

Model Results

Transit agencies should close high-demand lines if the primary concern is to control the spread of COVID-19, but this could make it harder to serve all travel demand, and it could also lead to an increase in COVID-19 cases as passengers crowd onto the open parts of the system. Our study shows that even if service is curtailed by closing some lines or reducing train frequency, if the timetable is optimized it is possible to accommodate passenger demand (i.e., passengers can wait for the next available train) and the number of new cases will not be significant as long as the system capacity remains large. Therefore, if we observe that buses/trains are lightly used even during peak hours, cutting service will not result in a surge in COVID-19 cases, but cutting service when the buses/trains are crowded could significantly increase the health risk of COVID-19.

Our model also shows that when demand remains high closing stations could lead to an increase in COVID-19 cases as most passengers will simply choose the closest alternative stations, which will lead to higher levels of crowding on open platforms which may increase the risk of infection. In short, if the agency decides to maintain service rather than closing down, it is better to keep all lines and stations open while adjusting the timetable and train frequencies instead of closing some lines or stations.

Comparing Our Result with Other Studies

Some systems were entirely shut down during the pandemic, like the bus systems in Wuhan in 2020. However, Islam et al.'s data-driven study of 149 countries (16) showed no evidence that public transport closures had any additional effect on the number of cases when four other physical distancing measures were implemented concurrently.

Mo et al. (14) showed that the viral reproduction rate could be reduced by 15.3 percent by closing the top 40 percent of high-demand routes, and that this approach is more effective where the busiest lines are closed first. This study differs from ours, as the authors did not attempt to optimize the timetables of those lines and accordingly allowed for some unsatisfied demand.

On-Board Capacity Restrictions

Model Results

COVID-19 risks can also be minimized by limiting the number of passengers on individual train cars and buses. Here, though, unless additional vehicles are put in service, system capacity will decrease and travel demand may not be adequately served as passengers may be dissuaded by having to wait longer to board a vehicle. Our model shows that as long as the timetable is optimized, if the car capacity is not too restricted the number of new cases will not increase significantly. However, new cases will rise quickly if the capacity is set too low while the budget constraints are tight, since large numbers of rush hour passengers would not be able to board and must wait on the platforms for the next train, leading to crowd gathering and higher risk of infection.

Comparing Our Result with Other Studies

Kumar's simulation (13) showed a similar result to our model, i.e., the risk was higher when the vehicle capacity was restricted while the demand, one-car capacity and service frequencies were the same, and the risk was less sensitive to capacity limits when the vehicle capacity was higher. However, Mo et al. (14) showed that if we assume that passenger demand drops as a result of reducing overall system capacity, COVID-19 infections will decrease because some passengers will simply give up their trips. If we simply cut service to reduce exposure, it may not work if the same number of passengers are just crowding onto a smaller system. Note that Kumar's simulation (13) used a nonlinear contact-network based pandemic infection model, but the results agreed with our linear infection model.

Different models showed different results due to their assumptions about the impact of COVID on travel demand. The elasticity in demand is a complicated problem itself and is city specific. It is possible that the total demand remains the same if only one or two lines are closed, as passengers will simply go to the nearest unclosed line/stations.

Changes in Travel Demand Leading to Different Results

If we assume no passengers will quit using public transportation (they will find the nearest open station and take a train from there), and we can re-optimize the timetable with the same budget and fleet after closing lines. Closing lines/stations or restraining capacity beyond social distancing rules is not recommended, because this will simply lead to more crowded stations/vehicles on the remainder of the system. This can be shown from our numerical example in "Policy Evaluation: Reopening Closed Lines" and Islam et al.'s data-driven study (16) or Kumar's simulation (13).

However, if we assume customers will abandon the system if station/lines they used before the pandemic are closed, then it would be beneficial to close lines/stations, as is shown by Mo et al. (14). Policy makers need to study historical data to see how travel demand actually changes or use more

advanced demand models to predict the possible impact of different health policies on system demand.

Further Insights Obtained from Our Model Formulation

COVID-19 Risk Minimization vs. Travel Time Minimization

Although the assumptions and the recommended strategies are different, there are some valuable common insights obtained from our research and other studies, as discussed below.

> If the COVID-19 infection rate in a city is nearly the same across different districts, the goal of minimizing COVID-19 infection in public transportation networks coincides with reducing the total travel time and crowd level. If passengers spend more time in a more crowded system, the COVID-19 infection will be higher.
> If the COVID-19 infection rate in a city is highly different in different districts terms of, the goal of minimizing COVID-19 infection in public transportation networks coincides with reducing the weighted sum of the travel time and crowd level, to provide more service in high-risk regions.

To prove the first argument, note our objective function $\sum_{(i,j)} \sum_w c_{ij} \beta q_s q_w u_{ij}^w$ based on spatial-compartmental model, where u_{ij}^w is the link flow on (i,j) from OD pair w. If we assume a fixed time travel cost in the system, the total travel time will be $\sum_{(i,j)} \sum_w c_{ij} u_{ij}^w$. If q_w is the same for all OD pairs, i.e. the city is homogeneous, the total number of COVID infection will be $\beta q_s q_w \sum_{(i,j)} \sum_w c_{ij} u_{ij}^w \propto \sum_{(i,j)} \sum_w c_{ij} u_{ij}^w$. Then the expected number of COVID-19 will be proportional to the total travel time in the system.

If the model is based on contact network (like Mo et al. ([14](#)) or Kumar's simulation ([13](#))), the objective function becomes $\sum_{(i,j)} c_{ij} \sum_{w_2} q_s u_{ij}^{w_2} \sum_{w_1} q_w u_{ij}^{w_1}$ in expectation, if the city is homogeneous as of COVID-19 infection, then $\sum_{(i,j)} c_{ij} \sum_{w_2} q_s u_{ij}^{w_2} \sum_{w_1} q_w u_{ij}^{w_1} = q_s q_w \sum_{(i,j)} c_{ij} u_{ij}^2$. If we model the travel cost using Bureau of Public Roads (BPR) link performance function with $\beta = 2$, the total travel time is $\sum_{(i,j)} \frac{c_{ij} u_{ij}^2}{c_{ij}^2} \propto q_s q_w \sum_{(i,j)} c_{ij} u_{ij}^2$ when all trains have the same capacity. Therefore, no matter what epidemic model we used, the goal of minimizing COVID-19 infection in public transportation networks coincides with reducing the total travel time and crowd level if the city is homogeneous.

If the city is not homogeneous, considering minimizing weighted sum of travel time with weight of q_w for each OD pair. The weighted sum of travel time is $\sum_{(i,j)} \sum_w c_{ij} q_w u_{ij}^w$, which is proportional to our derived COVID-19 infections. However, this is not proportional to $\sum_{(i,j)} \sum_w c_{ij} u_{ij}^w$ if the weights are different from one OD to another.

Therefore, if the city is almost homogeneous in terms of the COVID-19 infection rate, some techniques used in public transportation network design can also be applied to the COVID-19 risk minimization problem, such as parallel lines, express-local lines.

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