

Predicting Acceptable Wait Times for Patrons at Transit Bus Stops by Time of Day

Stephen Arhin, PhD

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REPORT 19-30

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October 2019

DOI: 10.31979/mti.2019.1801

A publication of **Mineta Transportation Institute**
_{Created by Congress in 1991}

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TECHNICAL REPORT DOCUMENTATION PAGE

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Digital Object Identifier Number: 10.31979/mti.2019.1801

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ACKNOWLEDGMENTS

The authors thank Editing Press, for editorial services, as well as MTI staff, including Executive Director Karen Philbrick, PhD; Deputy Executive Director Hilary Nixon, PhD; Graphic Designer Alverina Eka Weinardy; and Executive Administrative Assistant Jill Carter.

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EXECUTIVE SUMMARY

The time spent waiting by bus patrons at bus stops is a primary measure for assessing the reliability of transit services. Uncertainty associated with waiting affects bus patrons' perception of quality of the service provided. Studies on bus wait times have therefore been of interest to transit service agencies and officials in a bid to gain more insight into improving quality of service.

This report presents the findings of a study conducted to determine patrons' maximum and minimum acceptable wait times at bus stops in Washington, DC and to develop prediction models for providing decision-makers with additional tools for improving patronage. The research relied primarily on a combination of field surveys and videobased data collection efforts. Field surveys were conducted to obtain patrons' suggested acceptable wait times at bus stops, while video-based data collection was used to obtain bus operational characteristics.

A total of 3,388 bus patrons were surveyed, at 71 selected bus stops. Additionally, operational data was extracted via video playback for 2,070 bus arrival events on 226 routes. Data was collected for AM peak (7:00 AM -9:30 AM), PM peak (4:00 PM- 6:30 PM) and mid-day periods (10:00 AM – 2:30 PM) over a nine-month duration, from May 2018 through January 2019.

The following are the summary results of the survey conducted at the bus stops:

- The least reported acceptable wait time beyond the scheduled bus arrival time was 1 minute.
- The highest reported acceptable wait time beyond the scheduled bus arrival time was 20 minutes.
- The mean of the reported maximum acceptable wait time for female patrons was 8.5 minutes. The mean of the reported maximum acceptable wait time for male patrons was 8 minutes.
- The modal acceptable wait time, reported by approximately 33% of the patrons, was 5 minutes.
- In decreasing order, the mean of the maximum acceptable wait times of patrons categorized by ethnicity were as follows: African American (8.5 minutes), Asian (8.4 minutes), Hispanic (8.3 minutes), and White (7.0 minutes).
- Patrons are willing to wait longer in warmer temperatures.
- Patrons are willing to wait longer at bus stops with longer headways.

Tables 1 and 2 present some data about the acceptable wait times and alternate mode choices of bus patrons.

Table 1. Averages of Reported Acceptable Wait Times

Table 2. Preferred Alternate Mode Choice of Bus Patrons

Regression analyses were conducted to develop models to predict the maximum acceptable wait time of patrons based on factors including temperature, presence of shelter at the bus stops, average headway of buses, and patrons' knowledge of bus arrival times. The models were developed for A.M., P.M., and mid-day periods. The F Statistics for all three models were determined to be statistically significant with *p*-values <0.001 at a 5% significance level. The models had values (percentage of variance explained) of 64%, 79% and 82%.

Although female patrons generally had lower maximum acceptable wait times than male patrons, the difference was not statistically significant. However, the mean differences between the maximum acceptable wait times of patrons grouped by ethnicity were determined to be statistically significant at a 5% significance level. The study revealed that White patrons had significantly lower maximum acceptable wait times than did patrons of other ethnic groups.

I. INTRODUCTION

Urban areas typically have access to several transportation modes, including bus transit. Transit buses offer short-distance transportation between bus stops on different routes, mostly in dense urban areas. Transit agencies strive to keep patrons satisfied by improving the punctuality of bus arrivals at bus stops and by reducing wait times. Bus transit travel time and wait times are two of the critical factors that influence patrons' decision whether to use buses or to use another mode of transportation. If transit buses arrive at scheduled times, patrons are less likely to have the need to find alternative modes of transportation. However, if buses are chronically late at bus stops, patrons may feel that the bus system is unreliable and may seek alternative modes of transportation.

Henderson (1972) conducted a survey which showed that patrons waiting at a bus stop perceive wait time to be three times more bothersome than the time spent riding on the bus. Consequently, wait times are more likely than transit times to lead travelers to change from using buses to using another transportation mode. It is therefore necessary to be able to predict the maximum acceptable wait time of patrons in order to optimize bus headways, bus dwell times and the adequate spread of bus stops along a route.

This study aimed at modeling patrons' maximum acceptable wait times at bus stops in Washington, DC as functions of various predictor variables, in order to provide decisionmakers with new tools for increasing ridership.

II. LITERATURE REVIEW

This section presents the finding of the review of literature on wait time of bus patrons. The review focuses on the patrons' perception of waiting as well as the on factors that may influence bus patrons' wait times. Previous models developed to predict wait time are also examined.

PERCEIVED AND ACTUAL WAITING TIMES

Several studies have determined that waiting at bus stops is one of the most onerous components for riders of using bus transit services. A survey on bus user preferences was conducted in Australia, and found that among factors such as vehicle quality, trip quality and information quality, waiting time was the top most concern of patrons. A study reported that the time patrons spend waiting at a bus stop is perceived to be more burdensome than the same amount of time spent in-vehicle. The study further attributed the discomfort experienced by patrons to the uncertainty associated with waiting. The inconsistencies in waiting times can lead to large variabilities in travel time. Waiting time was found to be a statistically significant factor in explaining travel time variability. Also, several studies have sought to develop models to quantify waiting time of patrons at bus stops.

The literature has also revealed that perceived wait time of passengers differ from their actual wait times. Bus patrons subjectively overestimate their actual wait times and place more value on this perceived wait time than on any other investigated component of their trip. Perceived wait times tend to have a greater influence on rider discomfort and preference towards bus services than do actual wait times. A study was conducted to estimate the relationship between perceived and actual waiting times of patrons at bus stops on the campus of Ohio State University. The mean difference between the perceived and actual waiting time of passengers was estimated to be 0.84 minutes. A valuation of wait time and service headway among some public transport users in the United Kingdom found that patrons perceive one minute of waiting at a bus stop to be equivalent to 4.4 minutes of in-vehicle time. An earlier study conducted in the United States, found one minute of wait time to be perceived as equivalent to 8.4 minutes of in-vehicle time for a 30-minute journey and equivalent to 13 minutes for a 45 minutes journey.

Passenger perception of bus service performance and the effect of the perception on ridership were measured in a study conducted in Nanjing, China. An exploratory factor analysis found that, of the factors investigated, wait time was the most negatively perceived. A similar study in Nanjing, China determined that when wait times exceed passengers' tolerance; they tend to associate "long wait" as the primary service attribute. Thus, perceived wait times negatively influenced the overall transit service satisfaction. Another study investigated passenger wait time perceptions in Athens, Greece and concluded that on average, perceived wait times are 1.5 times higher than actual wait times. Previously, it had also been found that the disparity depended on whether passengers made a conscious decision to wait, or whether the wait was imposed on them by transit agencies; when imposed by the transit system, wait times were overestimated on average by a factor of two.

FACTORS AFFECTING WAIT TIME PERCEPTION

The perception of passage of time is highly subjective and depends on the innate characteristics of the individual. Thus, different individuals under the similar conditions and with similar demographic traits may nevertheless still perceive the passage of time differently. Nevertheless, several studies have identified certain key external factors that predictably influence the perceived wait times of bus patrons. These include bus stop features and surroundings, trip purpose, period of day, transit service attributes, weather and patrons' demographics. The following sections discuss these factors.

Bus Stop features and surroundings

Bus stop features such as shelter, the presence of a bench, lighting, and the presence of security affect the perceived wait times of patrons. A study showed that transit users in Grenoble, France overestimated their actual wait times at bus stops where light and heat/ventilation where absent. Lesser estimates were recorded where adequate lighting and ventilation were present. Lighting, music, and aesthetics have also been found to influence the perceptions of waiting time, with transit users of a Dutch railway service generally preferring bright lighting, calming music and warm colors. A study was conducted to compare transit riders' actual and self-reported waiting times at 36 bus stations in the cities of Minneapolis and St. Paul in Minnesota, and concluded that wait times at bus stops without amenities such as shelter and benches are perceived to be about 1.3 times longer than they actually are; they also found that females who waited at insecure bus stops for more than 10 minutes overestimated their waiting times by a larger amount than did their male counterparts. Moreover, passengers were found to be more reluctant to wait at transfer stations without available amenities such as shelter and bench than stations with amenities. Bus stop or station amenities not only reduce perceived wait time but also have the potential of making additional actual wait times of transit users more bearable; a study revealed that commuters in Naples, Italy were willing to accept an additional 7 minutes of waiting time to use a new rail line with improved stations.

Studies conducted in California have found that passengers prefer short and predictable wait times in a safe environment, while the attractiveness of bus infrastructure did not really matter.

The availability of real-time arrival and departure information at bus stops also affects the perceived wait times of patrons. The addition of real-time departure information signs to bus and streetcar stops in London reduced perceived waiting times by more than 20%, and the reduction in perceived wait time was estimated to be equivalent to reducing headways from 10 to 8 minutes. A study also found that the availability of real-time arrival information reduced the ratio of perceived wait time to actual wait time.¹⁴ It is recommended that the provision of real-time arrival information by transit services would reduce uncertainty and perceived waiting time.

Period of day

The time of day has been determined to affect bus patron's perception of wait times at bus stops. A study revealed that although perceived and actual wait times are longer during the afternoon than the evening and morning, the ratio of perceived to actual wait times is highest during the morning—perhaps due to riders' anxiety about getting to work on time with patrons tending to overestimate their waits by a factor of 1.74 during the morning, by a factor of 1.63 during the evening and by a factor of 1.41 during the afternoon.⁹ Another study conducted in Harbin, China also revealed that the period of the day to be a statistically significant predictor of waiting time.

Trip purpose

Possible trip purposes include education, entertainment, shopping, returning home, and "personal". Among these, patrons travelling to work are most likely to overestimate their waiting times by a factor of about 1.53.⁹ A study showed that trip purpose is a significant predictor of perceived wait time, also finding that passengers traveling for work purposes overestimated their waiting time the most.19

Transit Service frequency and reliability

A study conducted to identify the effects on waiting times of transit users of service frequency and reliability determined that, buses' strict adherence to their schedule allows transit users to coordinate their arrivals with those of the bus, resulting in average wait times that are less than half the scheduled headway. Uncertainty regarding the arrival times of buses due to unreliability therefore increases both the actual and the perceived wait time of transit users.

Patrons' demographics

Psarros et al. (2011) found that male patrons tend to overestimate their wait times on average by a factor of 1.61, while female patrons who overestimate their wait times on average by a factor of 1.52.⁹ However, Gurmu and Fan and Feng et al.¹⁹ did not find a statistically significant effect of gender on wait time.

WAIT TIME AS A MEASURE OF TRANSIT SERVICE RELIABILITY

In assessing the reliability of transit services, transit agencies and officials have, among other indicators, used passenger wait times as a performance measure. A study concluded that passengers' perception of transit service quality are affected by uncertainty in wait times. Wait time is considered an appropriate measure of service reliability for high frequency routes where the arrival of passengers is random. For low frequency services, it was shown that passengers usually synchronize their arrival time at bus stops with the arrival of buses, thus minimizing wait times.

In a study, waiting cost functions, which took into account headway and service reliability, were developed. The study further found that, by analyzing the behavior of passengers, the cost of waiting could be broken down into two components: the actual mean time spent waiting and the "potential waiting time", a measure of the additional time passengers have to budget for waiting, defined as the 95th percentile waiting time. Potential waiting time was found to be very sensitive to service reliability; therefore, by minimizing the waiting cost function, service reliability can be improved. A similar conclusion was reached in a study, which analyzed the service reliability of a high frequency bus line in Helsinki, using AVL and APC data. It was found that passengers assessed the reliability of bus services mainly in terms of additional waiting and travel time, and suggested that reduction in wait and travel times would increase passenger satisfaction and thereby lead to increases in patronage.

RELATIONSHIP BETWEEN WAITING TIME AND HEADWAY

Several studies have sought to establish the relationship between headway and waiting times of passengers. One of the earliest studies which developed a model for estimating the average wait time of passengers was conducted by Welding.²³ That study focused (among other issues) on passengers wait times for bus services with short headways. It concluded that the average waiting time of passengers who randomly arrive at a boarding point is lowest when the service is perfectly regular. The following model to estimate average wait time was suggested:

$$
Average\ wait\ time = \frac{\sum {h_i}^2}{2\sum h_i}
$$

where $h_{_{\scriptscriptstyle{I}}}$ = headway (in seconds).

A study of the behavior of passengers of a bus network in Stuttgart, Germany found that passengers' arrival at a bus stop is schedule-dependent when headways exceed 8 minutes; most passengers synchronize their arrivals with those of the buses, reducing the time spent waiting.

Further, a study conducted in Leeds, England, developed a model to estimate waiting times from headways, which they called "service intervals", improving an earlier model developed by Holroyd and Scraggs. Flaherty and Mangan assumed that the arrival of passengers is random during mid-day periods, but not during the morning and evening peak periods. Mathematically, and modifying their notation for consistency in this paper, their model is expressed as:

Average waiting time = $1.79 + 0.14h$

Osuna and Newell developed a model which took into consideration the random arrival of passengers during the peak periods. The random waiting time w_r was related to the headway h by the relation:

$$
w_r = \frac{h}{2} \left[1 + \left(\frac{\sigma}{h} \right)^2 \right]
$$

where

 h = the bus headway

σ = the standard deviation of bus headway.

A study analyzed passenger wait times and headways of buses in Manchester, England, and found a linear relationship between wait time and headway. The finding of the study also corroborated that of previous study, concluding that the arrival behavior of passengers is schedule-dependent when headways exceed 8 minutes. Another study used data from London, England, and found that passengers' arrival behavior was schedule-dependent for longer headways, but found that that the threshold headway duration was 12 minutes.

When passenger arrival at bus stops is random and headways are perfectly regular, the average wait times of passengers is one-half of the headway. When headways are not regular, however, the expected wait time increases with the headway variance: a more general model for expected wait times is

$$
\overline{w} = \frac{\overline{h}}{2} [1 + c_v(h)]
$$

where \bar{w} is the average wait time, \bar{h} is the average headway, and $c_v(h)$ is the coefficient of variation of headway. Note that even this more sophisticated model still assumes that passengers arrive at stops randomly, and won't be valid when this assumption fails.

Table 3 summarizes the above-discussed studies related to headway and passenger wait times.

Table 3. Relationship between Waiting Time and Dwell Time

COMPONENTS OF WAITING TIME

Most studies have measured wait time as the time between the arrival of a passenger at the bus stop and the time they board the bus. However, it was argued in another study that this measure is inadequate as a measure of the "real cost" to passengers of waiting, and that a better measure of this cost should take into account four main components: platform waiting, potential waiting, schedule inconvenience, and synchronization cost, with the latter two applying specifically to bus services with long headways.²⁴

Platform waiting time

Platform waiting time is defined as the time between the passenger's arrival at the bus stop and the time they board the bus; this component is the one which most other studies have simply referred to as "wait time." When bus service is reliable, passengers can reduce their platform wait times by synchronizing their arrivals with bus arrivals.

Potential waiting time

Potential waiting time is the additional time that passengers must be prepared to wait beyond the bus's expected departure time, in order to reduce the risk of missing the bus to 5%. The potential waiting time is defined as the difference between the 95th percentile wait time and the mean wait time. Potential waiting time constitutes a genuine form of "waiting" because it is the mean amount of time by which the passenger will arrive to their destination earlier than expected—for example, by waiting to clock in at their workplace.

Schedule inconvenience

Schedule inconvenience is defined as the difference between a passenger's most-desired departure time and the best departure time available on the bus schedule. Like potential waiting time, this component can lead to waiting in the form of early arrival. Unlike potential waiting time, this component is predictable, and so less costly; its predictability means that passengers can more easily plan to use their earliness productively—for example, by planning to buy a coffee and a breakfast sandwich before work.

Synchronization cost

This component of waiting cost measures the burden on passengers of having to adjust their schedule so as to conform to the service time table, quantified in terms of the "equivalent" number of minutes of wait time that they would find equally burdensome. This measure combines burdens including: the psychological stress of conforming to a timetable; the difference between actual arrival time at a bus stop and intended arrival time, necessary because people must always plan to arrive early if they want to guarantee arriving on time; and stress about missing the bus; and the full-headway wait time which occurs when a bus is missed. Furth and Muller suggest that these costs are all either constant or proportional to headway, and that overall synchronization cost, converted into psychologically equivalent minutes of in-vehicle time, be given by

WAIT TIME AND ALTERNATE MODE CHOICE

When passengers wait longer than their threshold of willingness, they tend to begin considering other options of transportation mode. Long waits thus lead to decreases in user confidence and ultimately to reductions in ridership. A study revealed that high variation in waiting time for transit users was leading travelers to select different transportation modes. Travel time has always been known to be the lead factor in mode choice of commuters; however, reliability, which is influenced by the consistency of wait time, was also found to be a major factor in mode choice. In a study on passengers' travel mode choice behavior when waiting at bus stations in Jinan, China, it was argued that passengers choose to end the waiting process and find a different mode choice when they reach a state of strong negative feelings due delay of bus arrivals. Other transportation mode options considered by passengers include transferring to another bus route, taking a taxi, and carpooling. Travel mode choice behavior was also determined to be a consumer choice attribute, which can vary across individuals. Thus, the individual characteristics of passengers determine their waiting time threshold and mode choice behaviors. Individual characteristics such as gender, level of education, occupation, and cultural differences influence the alternate modes of transport chosen by individuals once their acceptable waiting time has passed.

In a study aimed at modeling the choice behavior of passengers waiting on a subway platform where services have temporarily been suspended, it was found that some passengers preferred to wait for services to resume over choosing another mode choice with a shorter travel time. Another study conducted in Ghana examined the factors that influenced commuters travel mode choice, and concluded that safety, travel distance, transport fare, comfort and waiting time influenced commuters' mode choice.

PASSENGER WAIT TIME DISTRIBUTION AND MODELING

A number of studies have examined the distribution of passenger wait times and developed models to estimate wait time. Holroyd and Scraggs developed a model to predict wait time as a function of the distance between the bus stop and the destination, and found that the function was increasing. The wait time in their model was given by

Wait time = $0.48d + 1.8$

with wait time measured in minutes and distance *d* measured in kilometers. Distance was the only predictor variable that was considered. The correlation between wait time and distance, however, was not found to be statistically significant, at a 5% significance level.

Arrival distribution curves were developed based on data collected at 28 bus, tram and commuter rail stations in a study conducted in Zurich, Switzerland. The stations were served by scheduled public transits with headways ranging from 2.33 to 30 minutes. The observations were made on weekdays during the morning, evening and mid-day periods. The analysis of the results showed that both passenger arrivals and wait times have a logarithmic relationship with headway. It further concluded that passengers begin to arrive at stations near the scheduled departure times, even for very short headways. Another study fitted the arrival rate of passengers transferring from rail to buses to normal, exponential, lognormal and gamma distributions. The study concluded that the lognormal and gamma distributions had the best. A similar conclusions was reached in a study conducted in Beijing, China. In this study, passenger arrival times were fitted to extreme value, exponential, lognormal, gamma and normal distributions; it was found that the arrival time of passengers at bus stops connected to rail stations were best fitted with the lognormal distribution, while arrival time of passengers at bus stops not connected to rail stations were best fitted with the gamma distribution.

The distributions of actual passenger wait times and perceived wait times were developed based on data collected at bus stops in London, England. It was found that, the actual wait times of passengers followed the gamma distribution while the perceived wait time of passengers followed the lognormal distribution. In addition, multiple linear regression model were developed to predict perceived wait time of passengers based on data for 234 passengers surveyed at three bus stops in Harbin, China. Factors considered in the development of the model included gender, level of education, possession of a time device, presence of a companion, travel purpose, riding frequency, walking time, reserved waiting, waiting mood, waiting behavior, and time of day (morning or evening peak). The ANOVA results did not find statistically significant effects on perceived waiting time of gender, level of education, or walking time at a 5% significance level.¹⁹ Their best-fitting multiple linear regression model for perceived wait time was

$$
Y = 7.604 - 1.526X_{WT} - 1.474X_{WC} - 1.545X_{TP} + 1.054X_{RF} + 0.894X_{RW} - 0.027X_{RW} - 1.104X_{BC} + 1.104X_{MA} + 2.866X_{MW}
$$

where

Y = perceived wait time

 X_{WT} = having a time device

 $X_{\mu\nu}$ = presence of companion *X_{TP}* = Trip purpose X_{RF} = Riding frequency $X_{\mu\nu}$ = Reserved waiting time X_{BC} = Waiting behavior $X_{\overline{M}A}$ = Waiting mood.

Beyond the regression models, other studies have used machine learning techniques to develop passenger wait time models. A study used artificial neural networks to develop passenger wait time models based on data collected on passengers using a high-speed train service in Beijing. The predictors used in the model were trip distance, transportation mode, travel time, familiarity with the service facility, and level of education. The Artificial Neural Network (ANN) model developed consisted of one input layer with 5 neurons, two hidden layers with 8 and 3 neurons respectively, and an output layer with a single neuron. The sigmoid function and purelin transfer function were used as the respective activation functions in the hidden and output layers. The model was trained with data set of 720 samples, and validated with a data set of 336 samples. The model developed predicted passenger wait time with an average error of 9.2%.

SUMMARY OF LITERATURE REVIEW

The above review of the literature shows that the wait times of bus patrons generally follow lognormal and gamma distributions, and are impacted by several environmental and bus operational factors. It is also pointed out that when wait times exceed acceptable limits, patrons tend to consider other mode choices which provide comparatively efficient service. However, although several studies have sought to model both wait times and perception of wait times as functions of various predictor variables, none has sought to model the maximum acceptable wait time of patrons beyond the schedule arrival time of buses. This study therefore focuses on determining the average maximum wait time of patrons and on developing models for its prediction. Three different models were developed, for three different periods of the day: AM peak (7:00 AM -9:30 AM), PM Peak (4:00 PM- 6:30 PM) and mid-day (10:00 AM – 2:30 PM).

III. RESEARCH METHODOLOGY

 DESCRIPTION OF THE STUDY JURISDICTION

This research is based on data obtained in Washington, D.C. The city is divided into four quadrants of unequal area: Northwest (NW), Northeast (NE), Southeast (SE), and Southwest (SW). The city is also divided into eight Wards, which overlap the boundaries of the quadrants. As of 2017, the population of Washington, D.C. was approximately 694,000 with an annual growth rate of approximately 1.41%. The City is highly urbanized and is ranked as the sixth most congested city in the United States with each driver spending an average of 63 hours per year in traffic. The Washington Metropolitan Area Transit Authority (WMATA) is the agency that oversees the operations of Metrobus service in the jurisdiction. WMATA has a bus fleet of 1,595 buses that make more than 400,000 trips each week day. These buses serve about 11,500 bus stops and operate on 325 routes in Washington D.C., in portions of Maryland, and Northern Virginia, covering a total land area of about 1,500 square miles. Of the total number of bus stops, 2,556 (22.2%) have shelters, while the remainder do not. A map of Washington, D.C. showing the city divided into Wards is shown in Figure 1.

Figure 1. Map of the Washington, D.C.

DATA COLLECTION

Selection of Bus Stops

Bus operational data and patron survey data were collected at seventy-one selected bus stops in Washington, D.C. Both bus stops with shelter and bus stops without shelter were considered. The bus stops were selected based on the following criteria:

- Those on bus routes with longer headways: bus stops on routes with longer head ways were selected to ensure that data collection technicians were able to complete the survey of bus patrons before the arrival of a bus. Such bus stops were identified using the published time tables available on WMATA's website.
- Bus stops with high patronage: selection of bus stops with high patronage ensured that the required minimum number of patrons were surveyed at each bus stop.
- Proximity to metro rail station: bus stops in proximity to railways are viable locations to have access to bus patrons with variable characteristics. In addition, such locations usually have a high number of bus patrons waiting to board a bus.
- Roadway functional classification: bus stops on arterial and collector roads were selected for this study since they usually serve more than two bus lines.

Bus Stop Characteristics

The bus stops selected for this study were categorized in relation to their location on the roadway and to the type of design.

Bus Stop Placement and Location

The WMATA Guidelines for the Design and Placement of Transit Stops defines bus stop locations relative to an intersection. Three types of location are specified: near-side (upstream) of the intersection; far-side (downstream) of the intersection; and mid-block (midway between intersections). Near-side bus stops are usually located at least 5 feet from the intersection, far-side bus stops are at least 50 feet from the intersection and midblock bus stops are positioned midway between two intersections. The selected bus stops for this study are comprised of only near-side and midblock bus stops, which are the predominant types of bus stops in Washington, D.C. Figure 2 depicts typical locations of near-side and mid-block bus stops used in this study.

Figure 2. Near-side and Mid-block Bus Stop Locations

Design Type

Bus stops can be further categorized as being either curb-side bus stops or bus bays, depending on their position in relation to the travel lanes. Curb-side bus stops service patrons from a travel or a parking lane. Bus bays are constructed as insert into the curbs with tapered ends for acceleration and deceleration, and with reinforced concrete pavement. Figure 3 illustrates these two bus stop design types.

Figure 3. Curb-side Bus Stop and Bus Bay

Amenities

Bus stops in Washington, D.C. vary in their possession of amenities such as sign posts, shelters, and information cases. The amenities are described as follows:

• Bus stop sign post: WMATA bus sign posts are usually red plaques mounted on the

top of white poles. The plaque provides the patron with information regarding route numbers, stop ID numbers, WMATA's web site address and a telephone number for patrons to call for assistance.

- *• Information case:* this is a rectangular or cylindrical glass casing containing information such as system maps, neighborhood maps, and/or bus schedules.
- *• Shelters:* these are covering structures that provide protection against weather for passengers waiting at a bus, together with a bench located in the shelter.

The primary bus stop amenity considered in this research was the presence or absence of a shelter.

A summary of the description of the 76 selected bus stops is presented in Appendix A. The information obtained include regarding bus stop ID, design type, placement, amenities, and service route numbers. Figures 4 to 6 present pictures of some bus stops at which data was collected.

Figure 4. Curb-side Bus Stop with Shelter and Bench

Figure 5. Curb-side Bus Stop without Shelter

Figure 6. Bus Bay with Shelter and Bench

DATA COLLECTION

Data collection at the selected bus stops was conducted over a nine-month duration from May 2018 through January 2019. Data was collected during the AM peak (7:00 AM–9:30 AM), PM peak (4:00 PM–6:30 PM) and mid-day periods (10:00 AM–2:30 PM). Two forms of data collection were performed: bus patrons were surveyed and bus operational data was collected. The data collection schedule was organized so as to maximize the number of survey participants and achieve a robust sample size.

Survey Data Collection

Passengers waiting for the arrival of the next bus at the selected bus stops were randomly selected and interviewed during the morning, evening and mid-day periods on weekdays. The field technicians conducted the survey by use of electronic forms on computer tablets. However, where computer tablets where unavailable, paper questionnaires were used. The survey procedure was conducted as follows:

i. Upon arrival at the bus stop, the interviewer first obtained the temperature at the bus stop location from the National Oceanic and Atmospheric Administration (NOAA) weather service website and recorded it in the designated field on the questionnaires accordingly. The date of survey and the name of the interviewer were also recorded.

- ii. The availability or unavailability of a bus shelter and the direction of travel (northbound, southbound, eastbound, westbound, southwest-bound or northeast-bound) the bus stop serves were then observed and recorded in their designated fields.
- iii. When a patron arrived at the bus stop, his/her arrival time and gender were recorded.
- iv. The field technician then approached the patron(s) and, if they were willing to participate, asked the following questions to complete the survey:
	- Whether the passenger was aware of the scheduled bus' arrival time.
	- At bus stops that serve more than one line, the patron was asked which route he/ she intended to take. Where the bus stop served only one line, such information was simply obtained from the bus stop sign post.
	- The patron's minimum acceptable wait time beyond the bus scheduled arrival time for which the patron is willing to wait.
	- The maximum acceptable wait time beyond which the patron would consider an alternative transportation mode.
	- What alternate mode(s) of transportation the patron would consider if the bus was delayed beyond their maximum acceptable wait time.
- v. The bus line(s) that serve the bus stop were recorded.

Table 4 summarizes the variables collected from each bus stop and patron, as well as operational characteristics obtained during the survey, and their associated abbreviations used in the analysis.

A total of 3,388 patrons were surveyed over the period of the study. In the event that the minimum number of responses was not obtained during a particular peak period due to inclement weather or low patron turnout, additional patrons were surveyed on the same day and period the following week. The survey questionnaire is presented in Figure 7. Figure 8 presents a photograph of a patron being surveyed at one of the selected study locations.

Figure 7. Survey Form

Figure 8. Patron Surveyed at a Selected Bus Stop

Bus Operational Data

Bus operational data was collected at each of the 78 selected bus stops. The data was collected by installing video recording cameras at the bus stops. The video recordings took place on weekdays, over a 12-hour duration from 6:30 AM to 6:30 PM. The following data was obtained of each bus arrival event via video playback:

- i. Bus arrival time: a bus was determined to have arrived at a bus stop when it came to a complete stop for boarding and alighting of passengers.
- ii. Bus departure time: a bus was determined to have departed a bus stop when the last passenger had either boarded or alighted the bus and the doors were shut.

A total of 2,070 bus arrival events on 226 routes were extracted, computed and compiled in an EXCEL spreadsheet for further analysis. Figure 9 presents a photograph of one of the video cameras mounted at a selected bus stop.

Figure 9. Video Camera Installed at a Selected Bus Stop

From the collected data, bus arrival and departure times were used to compute headway by finding the difference between the arrival time of a bus and that of the preceding bus on the same route. Therefore, headway was computed as:

$$
H_{A} = AT_{B} - AT_{A}
$$

where H_A is the actual bus headway, AT_A is the arrival time of bus A, and AT_B is the arrival time of bus B.

DATA ANALYSIS

Descriptive Statistics

The frequencies, mean, median, and standard deviation were computed for the bus stop characteristics, passenger characteristics and bus operational characteristics, for each bus stop.

Model Development

Regression Analysis

To investigate the relationship between the maximum acceptable waiting time and variables such as average headway, knowledge of bus arrival time, presence of shelter, and temperature at the bus stops, multiple linear regression analyses were conducted. The regression models were developed for A.M. peak, mid-day off-peak, and P.M. peak periods, indexed as 1, 2, and 3 respectively. The multiple regression models for maximum acceptable wait time take the following form:

$$
MAWT_i = \beta_{oi} + T\beta_{1i} + AH\beta_{2i} + KBAT\beta_{3i} + PS\beta_{4i} + \varepsilon
$$

where

MAWT = Maximum Acceptable wait time

AH = Average Headway

T = Temperature

KBAT = Knowledge of Bus Arrival Time

PS = Presence of Shelter

MAWT is the dependent variable while *T, AH*, *KBAT* and *PS* are the independent variables. The constants *βki* are the regression coefficients, with an associated approximately normally distributed error of ε with mean of zero and variance of σ^2 , denoted as [ε~N(0, σ^2)] and indexed as $k = 0, 1, 2, 3, 4$ for the first, second, third, fourth and fifth regression coefficients respectively.

The variables were tested to ensure that they satisfied the assumptions of normality of errors, no multicollinearity, and homoscedasticity required for the soundness of multiple regression.

Normality of errors

A sound multiple regression model requires that the errors or residual terms are randomly distributed. Therefore, the residuals should approximate the random errors that establish the relationship between the explanatory variables and the response variables. The normality of errors assumption is tested using a normal probability plot. The observed cumulative probabilities of the standardized residuals are plotted against the expected cumulative probabilities of the standardized residuals. If the errors are normally distributed, the plotted points will approximate a straight diagonal line as shown in Figure 10.

Figure 10. Example Normal Probability Plot

Multicollinearity

Multicollinearity is a state of high intercorrelations among the independent variables. If present in a data set, statistical inferences made from the data may not be valid. In addition, multicollinearity may cause the regression coefficients to not be estimated precisely, and the standard errors to be increased. Overall, it reduces the degree of confidence of the resulting model. Multicollinearity is tested using a correlation matrix and Variance Inflation Factor (VIF) (the ratio of the variance of the model with multiple variables to the variance of the model with one variable). Usually, correlation between variables which are higher than 0.5 are considered highly correlated. Also, multicollinearity is considered to be present when the VIF of a variable is greater than $10^{[54]}$
Homoscedacity

Homoscedacity is the condition where the residual term is the same across all values of the independent variables. Violations of this condition, termed heteroscedacity, cause significance tests of the regression coefficients and estimations of confidence intervals to be inaccurate. The test for homoscedacity is conducted by observing the scatter plot of the regression standardized residuals against the predicted values. An even distribution about the zero line, as shown in Figure 11, indicates that the assumption of homoscedacity is met.

Figure 11. Example Scatter Plot showing Homoscedacity

Model Evaluation

The multiple regression models were evaluated using the *p*-values of the F statistics, R², and adjusted $R²$. These evaluating tool are used to assess the performance of the models.

F-test

The F-test evaluates the null hypothesis that, for the population from which a sample was drawn, all regression coefficients are equal to zero, against the alternative hypothesis that at least one regression coefficient is not. Thus, the F-test determines whether the proposed relationship between the response variable and the set of predictors is statistically significant. The F-test is conducted by taking the ratio

$$
F \; statistic \; = \; \frac{MSM}{MSE}
$$

where MSM is the mean of squares for model and MSE is the mean of squares for the error. The statistical significance of the F statistic is then determined using the *p*-value. The significance level for this study was set at 5%.

R² (Coefficient of Determination)

The coefficient of determination, R^2 , is a measure of the goodness-of-fit of a model. It is defined as the percentage of the variance of the dependent variable that can be explained by the model. R-squared is expressed mathematically as:

$$
R^2 = \frac{SST - SSE}{SST}
$$

where *SST* = Sum of Squares Total (sum of the squares of the difference of the dependent variable and its mean)

 SSE = Sum of Squares of Error (sum of the squares of the difference of the predicted dependent variable from actual values of the data)

Generally, increases as predictors are added to the model. However, this increase does not always result in the actual improvement of the model, as this could also be an indication of overfitting of the model. To remedy this effect, an adjusted is also used to assess the model.

Adjusted R²

Like, \bm{R}^2 , \bm{R}^2 $_{\sf adjusted}$ is a measure of the percentage of total variance in the dependent variable that is explained by the model. Unlike \bm{R}^2 , \bm{R}^2 $_{\it adjusted}$ takes into account the model's degrees of freedom, paying a penalty when too many predictor variables are added; *R² adjusted* will decrease as independent variables are added, if the increase in model fit is not enough to make up for the loss of degrees of freedom. It is given by

$$
R_{adjusted}^2 = 1 - \frac{MSE}{MST}
$$

where *MSE* = Mean of Squares Total

MST = Mean of Squares for Error

Model Testing

Model testing is the process of determining if the regression model provides acceptable explanation of variations in the data. The proposed prediction model was tested using the Kolmogorov-Smirnov (K-S), used to determine whether data follows a specified distribution; in this case it was used to check the normality assumption required by ANOVA. A random sample (the variable whose normality is being tested) *x1, x2, ..., xm* is taken from a population and is compared to the hypothesized distribution function to determine if the random sample's distribution is equal to the hypothesized distribution,

$$
H_0: F_{1,n}(x_i) = F_{2,n}(x_2);
$$

for all $\bm{\chi},$ where $F_{_{1,n}}$ and $F_{_{2,n}}$ are empirical cumulative distribution functions of the observed and predicted datasets. The *K-S* test calculates a test statistic *D*, defined as

$$
D_{n,n'} = \sup_{x} |F_{1,n}(x) - F_{2,n}(x)|
$$

where, *sup* is the supremum function. The null hypothesis is rejected at significance level *α* if

$$
D_{n,n'} > c(\alpha) \sqrt{\frac{n+n'}{nn'}}
$$

where, n and n' are the sample sizes of the respective samples. For each α level, Table 5 presents the critical, c value (*α)*.

Table 5. Critical Values of (α)

Hypothesis Testing

The test statistic used in this study for the comparison is the mean. The hypotheses that there is a significant difference in the average MAWT of passengers based on their gender and ethnicity was also tested, at a 5% significance level.

Difference in AWT Based on Gender

The null hypothesis for this test was that there is no effect of a patron's gender on their MAWT, and the alternative hypothesis was that there is an effect:

$$
H_o: X_i = X_2
$$

$$
H_A: X_i \neq X_2
$$

where

 $X_{_I}$ = mean MAWT of female patrons

 X_{\circ} = mean MAWT of male patrons

Difference in AWT Based on Ethnicity

The null hypothesis was that there is no effect of a patron's ethnicity on their MAWT, and the alternative hypothesis was that there is an effect:

$$
H_o: Y_1 = Y_2 = Y_3 = Y_4 = Y_5
$$

$$
H_A: \sim (Y_1 = Y_2 = Y_3 = Y_4 = Y_5)
$$

where, Y_1 = mean MAWT of Black/African American patrons

 Y_{ρ} = mean MAWT of White patrons

Y ³ = mean MAWT of Hispanic patrons

Y ⁴ = mean MAWT of Asian patrons

Y ⁵ = mean MAWT of Other ethnicity patrons

A preliminary analysis of the data to test for the parametric assumptions of normality and equality of variance using the Shapiro-Wilk and Levine Test respectively indicated a statistically significant violation of these assumptions. Because the assumption of normality did not hold, non-parametric tests were needed in order to test for statistically significant differences in MAWT of passengers. A non-parametric Wilcoxon rank-sum test was used to test for an effect of gender, and a non-parametric Kruskal-Wallis test was used to test for an effect of ethnicity.

Wilcoxon rank-sum Test

The Wilcoxon rank-sum test is a statistical analysis used to determine whether two independent samples were drawn from the same distribution; it is commonly thought of as a non-parametric alternative to the two-sample t-test, which tests for whether two samples' means significantly differ. This method tests the null hypothesis by comparing the ranks of the observations of the two groups of variables to decide whether or not the difference between the mean ranks are statistically significant. The statistical significance of the Wilcoxon rank-sum test statistic $W_{_{\!S}}$ is determined as follows;

$$
\bar{W}_s = \frac{n_1 (n_1 + n_2 + 1)}{2}
$$

The Standard Error of the test statistic is then computed as,

$$
SE_{\overline{W_S}} = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}
$$

Further, the z score of the test statistic, $W_{\rm g}$ is computed as,

$$
z = \frac{W_s - \overline{W_S}}{SE_{\overline{W_S}}}
$$

where,

 W_s = Wilcoxon rank-sum test statistics

- $\overline{W_S}$ = the mean of the test statistics
- $SE_{\overline{W_S}}$ = standard error of the test statistic
- $n_{_I}^{}$ = sample size of the male patrons

 $n_{_2}^{}$ = sample size of female patrons

 $Z = z$ score of the test statistic

A z score values greater than 1.96 corresponds to a p-value below 0.05; thus, at a 5% significance level, z scores greater than 1.96 are deemed statistically significant.

Kruskal-Wallis Test

The Kruskal-Wallis Test is used to determine whether two or more independently drawn samples were drawn from the same distribution; a statistically significant result on this test suggests that at least one sample may have been drawn from a different distribution. This method compares the ranks of the observations of three or more groups of a variable to decide whether or not the difference between the mean ranks are statistically significant. The statistical significance of the Kruskal-Wallis test statistic, is determined as follows;

$$
H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1)
$$

where,

H = Kruskal-Wallis test statistics

 N = total sample size

 $R_{_{l}}^{}$ = the sum of ranks for group i

 N_i = the sample size of group i

The H statistic is then compared to a critical value, $\mathsf{H}_{_\mathrm{C}}$, which approximates the chi-square distribution, for the same number of degrees of freedom. If H is higher than $\mathsf{H}_{\scriptscriptstyle\rm c}$, then the result is statistically significant and we reject the null hypothesis.

IV. RESULTS

SURVEY RESULTS

This section presents an overview of the results of the surveys conducted at the selected bus stops.

Summary Survey Statistics

Table 6 presents a summary of the characteristics of patrons and locations surveyed in this study. Figure 12 represents the percentage of surveys conducted by quadrant.

Total # of patrons surveyed	3,388
Total # of females	1,753
Total # of males	1,635
Total # of White patrons	778
Total # of Black patrons	771
Total # of Hispanic patrons	747
Total # of Asian patrons	545
Total # of "Other" patrons	546
Total # of Locations surveyed	76
Total # of Locations with Shelter	40
Total # of Locations without Shelter	31

Table 6. Survey Statistics Quick Facts

Table 6 shows that majority of the patrons that were surveyed were females. Most of the patrons that participated in the survey were White, followed by Black and Hispanic patrons. Most of the surveyed bus stops were located in the North-West (NW) quadrant of the Washington, D.C; as a result, 46.6% of the surveys were conducted in the NW quadrant.

Figure 12. Percentage of Surveys per Quadrant

Survey Statistics and Trends

This section presents the descriptive statistics for the surveys conducted at 76 selected bus stops in Washington, D.C. The patrons' characteristics analyzed included gender, ethnicity, arrival time to the bus stop, knowledge of the bus arrival time to the bus stop, minimum and maximum acceptable wait time and choice of alternative mode of transportation. The location characteristics included: presence of bench at bus stop, quadrant, ward and weather temperature.

The analysis focused on following:

- *• Time:* patron's time of arrival at the bus stop and time of the day;
- *• Location:* survey location identified by pre-defined areas such as Ward, Quadrant, and bus stop type;
- *• Patron's Characteristics:* gender, ethnicity and patron's knowledge of bus arrival time;
- *• Patron's alternate mode:* a patron's next-most-preferred mode of transportation;
- *• Environmental Factors:* temperature

Time

The tables and figures in this section present the frequencies and distributions of patron's minimum and maximum acceptable wait times, categorized by day of the week and by time of day. The highlighted cells represent the modal choices.

Acceptable Wait Times by Time of the Day

Tables 7 and 8 present the minimum and maximum acceptable wait times for patrons by time of the day. From the Table 7, it can be seen that the modal minimum acceptable wait time for the morning period was 1 minute, while the modal minimum acceptable wait time was 2 minutes during the mid-day and evening periods. Also, the modal maximum acceptable wait time during morning, mid-day and afternoon periods were 7 minutes, 10 minutes and 5 minutes respectively.

Figures 13 and 14 show patrons' choices for minimum and maximum acceptable wait times categorized by time of day. Figure 15 presents the summary of minimum and maximum acceptable wait times by time of the day.

Figure 13 shows that patrons chose shorter acceptable wait times during the morning and afternoon periods. Similarly, Figure 14 shows that maximum acceptable wait times were also lower during the morning and afternoon periods. From Figure 15, it can be observed that most patrons were most likely to wait a longer period of time during the mid-day period, followed by the morning and afternoon periods, respectively.

Table 7. Patrons' Acceptable Wait Times by Time of the Day

Figure 13. Patrons' Minimum Acceptable Wait Times by Time of the Day

Figure 14. Patrons' Maximum Acceptable Wait Times by Time of the Day

Figure 15. Patrons' Acceptable Wait Times by Time of the Day

From Table 8, it can be observed that patrons tended to indicate shorter acceptable wait times during the afternoon and morning periods than during the mid-day period (between 4 and 7 minutes).

Acceptable Wait Times by Hour of the Day

Table 9 along with Figures 16 through 18 present the minimum and maximum acceptable wait times for patrons by hour of the day. From the table, the highest number of patrons chose 1 minute as the minimum acceptable wait time at 7:00 AM, 8:00 AM and 4:00 PM, while 5 minutes was the maximum acceptable wait time for patrons at 4:00 PM.

Table 9. Patrons' Acceptable Wait Times by Hour of the Day

Figure 16. Minimum Acceptable Wait Times by Hour of the Day

Figure 17. Maximum Acceptable Wait Times by Hour of the Day

Figure 18. Patrons' Acceptable Wait Times by Hour of the Day

Location

The tables and figures in this section present the frequencies of patron's acceptable wait times by Quadrant, Ward and bus stop type. The highlighted cells are the modal choices.

Acceptable Wait Times by Quadrant

This section presents the patrons' minimum and maximum acceptable wait times, organized by Quadrant. The summary of the acceptable wait times by quadrant is presented in Table 10 and shown in Figures 19 through 21. From the table and figures, it can be seen that the modal acceptable wait time in the NW and the SE quadrants was 1 minute, while the modal acceptable wait times for the NE and SW quadrants were 2 and 3 minutes respectively. The modal maximum acceptable wait time was 5 minutes for the NW and NE quadrants, and 7 minutes for the SE and SW quadrants.

Note: NW=Northwest, NE=Northeast, SE=Southeast, SW=Southwest, BR=Border

Figure 19. Patrons' Minimum Acceptable Wait Times by Quadrant

Figure 20. Patrons' Maximum Acceptable Wait Times by Quadrant

Figure 21. Patrons' Acceptable Wait Times by Quadrant

Acceptable Wait Times by Ward

This section presents the patrons' minimum and maximum acceptable wait times reported in each Ward. Table 11 and Figures 22 through 24 present the summaries of the acceptable wait times by ward. From the table and figures, it can be seen that the modal acceptable wait time was 2 minutes for Wards 1, 3, 5 and 7, and 1 minute for Wards 2, 6 and 8. The modal maximum acceptable wait time was 7 minutes for patrons in Wards 1, 5, 6, 7 and 8.

			Ward								
# Minutes		1	$\overline{2}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{4}}$	5	66	$\overline{7}$	$\boldsymbol{8}$		
	$\overline{1}$	80	133	118	111	169	56	39	49		
	$\overline{2}$	95	120	133	121	197	40	45	38		
	3	88	113	99	122	153	50	43	47		
	4	53	67	58	62	109	28	37	30		
	5	20	26	21	30	46	17	16	13		
	6	20	29	20	25	48	14	17	12		
Min AWT	$\overline{7}$	30	23	21	19	58	15	16	12		
	8	11	8	17	11	11	$\boldsymbol{2}$	4	$\,6\,$		
	9	\overline{c}	3	4	$\mathbf{1}$	4	$\mathbf 0$	0	$\boldsymbol{2}$		
	10	0	6	4	1	5	$\boldsymbol{2}$	\overline{c}	4		
	12	0	$\pmb{0}$	$\mathsf 0$	0	1	$\pmb{0}$	0	$\pmb{0}$		
	13	0	1	$\mathsf 0$	1	0	$\pmb{0}$	0	$\pmb{0}$		
	14	1	1	1	0	0	$\mathbf 0$	0	$\pmb{0}$		
	$\overline{\mathbf{2}}$	0	1	0	$\pmb{0}$	1	0	0	$\pmb{0}$		
	$\overline{\mathbf{3}}$	12	5	5	4	6	$\pmb{0}$	0	$\pmb{0}$		
	5	86	180	204	125	266	41	20	45		
Max AWT	$\overline{7}$	140	142	101	151	190	79	75	61		
	10	95	108	75	139	237	71	70	56		
	12	55	49	78	62	57	28	42	30		
	15	11	41	27	18	41	5	12	19		
	20	1	4	6	5	3	$\pmb{0}$	$\mathsf 0$	$\boldsymbol{2}$		

Table 11. Patrons' Acceptable Wait Times by Ward

Figure 22. Patrons' Minimum Acceptable Wait Times by Ward

Figure 23. Patrons' Maximum Acceptable Wait Times by Ward

Figure 24. Patrons' Acceptable Wait Times by Ward

Acceptable Wait Times by Bus Stop Type

The minimum and maximum acceptable wait times were reported by bus stop type in this section. Table 12 and Figures 25 through 27 present the summary of the acceptable wait times by bus stop type. From the table and figures, it can be seen that the modal maximum acceptable wait time was lower for patrons at bus stops without a bench (5 minutes) than for those at bus stops with a bench (7 minutes).

		Presence of Bench				
	# Minutes	Yes	No			
	$\mathbf{1}$	348	407			
	$\overline{\mathbf{2}}$	349	440			
	3	337	378			
	4	239	205			
	5	101	88			
	6	102	83			
Min AWT	$\overline{7}$	103	91			
	8	52	18			
	9	12	$\overline{\mathbf{4}}$			
	10	18	$\,6\,$			
	12	1	$\pmb{0}$			
	13	\overline{c}	$\pmb{0}$			
	14	$\overline{\mathbf{c}}$	$\mathbf{1}$			
	$\overline{\mathbf{2}}$	$\pmb{0}$	$\overline{2}$			
	3	$\mathbf{3}$	29			
	5	322	645			
Max AWT	$\overline{7}$	508	431			
	10	431	420			
	12	260	141			
	15	122	52			
	20	$20\,$	1			

Table 12. Patrons' Acceptable Wait Times by Bus Stop Type

 \blacksquare 1 \blacksquare 2 \blacksquare 3 \blacksquare 4 \blacksquare 5 \blacksquare 6 \blacksquare 7 \blacksquare 8 \blacksquare 9 \blacksquare 10 \blacksquare 12 \blacksquare 13 \blacksquare 14

Figure 25. Patrons' Minimum Acceptable Wait Times by Bus Stop Type

Figure 26. Patrons' Maximum Acceptable Wait Times by Ward

Figure 27. Patrons' Acceptable Wait Times by Presence of Bench

Patrons' Characteristics

The tables and figures in this section present the frequencies of acceptable wait times categorized by gender, ethnicity and knowledge of bus arrival time.

Acceptable Wait Times by Gender

This section presents the minimum and maximum acceptable wait times by gender. The summary of the acceptable wait times by gender is presented in Table 13 and shown in Figures 28 through 30. From the table and figures, it can be seen that the modal maximum wait time was 5 minutes for female patrons, and 7 minutes for male patrons. From Figure 30, it can be observed that male patrons were more willing to wait for longer periods of time than female patrons.

Table 13. Patrons' Acceptable Wait Times by Gender

Figure 28. Patrons' Minimum Acceptable Wait Times by Gender

Figure 29. Patrons' Maximum Acceptable Wait Times by Gender

Figure 30. Patrons' Acceptable Wait Times by Gender

Acceptable Wait Times by Ethnicity

The minimum and maximum acceptable wait times by ethnicity are presented in this section. The summary of the acceptable wait times by ethnicity is presented in Table 14 and shown in Figures 31 through 33. From the table and figures, it can be seen that the modal acceptable wait times ranged from 1 to 2, for all ethnicities. The modal maximum acceptable wait time was 5 minutes for white patrons, 7 minutes for Black and Asian patrons, and 10 minutes for Hispanic patrons and patrons whose ethnicities were listed as "Other."

		Ethnicity						
	# Minutes		Black	Hispanic	Asian	Other		
	$\overline{1}$	210	152	162	120	111		
	$\overline{\mathbf{2}}$	200	193	153	111	132		
	$\mathbf{3}$	179	157	156	118	105		
	4	80	116	83	81	84		
	5	26	35	58	41	29		
	66	33	38	52	21	41		
Min AWT	$\overline{7}$	28	50	56	30	30		
	8	18	17	14	14	7		
	$\boldsymbol{9}$	$\pmb{0}$	3	$\,6\,$	$\,6\,$	1		
	10	$\ensuremath{\mathsf{3}}$	8	$\,6\,$	$\overline{2}$	5		
	12	$\pmb{0}$	1	$\pmb{0}$	$\pmb{0}$	$\pmb{0}$		
	13	$\pmb{0}$	1	1	$\pmb{0}$	0		
	14	1	$\pmb{0}$	$\pmb{0}$	1	1		
	$\overline{2}$	$\mathbf 0$	0	1	$\mathbf 0$	1		
	$\mathbf{3}$	12	$\sqrt{5}$	$\boldsymbol{9}$	$\overline{2}$	$\overline{\mathbf{4}}$		
	5	379	185	187	118	98		
Max AWT	$\overline{7}$	152	266	174	195	152		
	10	114	176	233	136	192		
	12	86	93	98	59	65		
	15	30	40	42	31	31		
	20	$\mathbf 5$	6	$\mathsf 3$	$\overline{\mathbf{4}}$	$\ensuremath{\mathsf{3}}$		

Table 14. Patrons' Acceptable Wait Times by Ethnicity

Figure 31. Patrons' Minimum Acceptable Wait Times by Ethnicity

Figure 32. Patrons' Maximum Acceptable Wait Times by Ethnicity

Figure 33. Patrons' Acceptable Wait Times by Ethnicity

Acceptable Wait Times by Knowledge of Bus Arrival Time

The minimum and maximum acceptable wait times by knowledge of bus arrival time are presented in this section. The summary of the acceptable wait times by knowledge of bus arrival time is presented in Table 15 (the heavily shaded cells are the modal choices and the lightly shaded ones are the runners-up) and shown in Figures 34 through 36. From the table and figures, it can be seen that the modal acceptable wait time for patrons that did not know the bus arrival time was 2 minutes, with 4 minutes being the second most common answer; in contrast, for patrons who did know the time when the bus was supposed to arrive, while the modal time was again 2 minutes, the second most common answer was only 1 minute. Similarly, the modal maximum acceptable wait time was 12 minutes for patrons who knew the bus arrival time was, but only 5 minutes for patrons that did not know the planned bus arrival time.

Table 15. Patrons' Acceptable Wait Times by Knowledge of Bus Arrival Time

Figure 34. Patrons' Minimum Acceptable Wait Times by Knowledge of Bus Arrival Time

Figure 35. Patrons' Maximum Acceptable Wait Times by Knowledge of Bus Arrival Time

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Figure 36. Patrons' Acceptable Wait Time by Knowledge of Bus Arrival Time

Acceptable Wait Times by Choice of Alternative Transportation Mode

This section presents the minimum and maximum acceptable wait times by choice of alternative mode of transportation. The summary of the acceptable wait times by choice of alternative mode of transportation is presented in Table 16 and shown in Figures 37 through 39. From the table and figures, it can be seen that most patrons chose train and rideshare as their alternative mode of transportation after waiting between 2 and 7 minutes.

# Minutes		Alternative Mode of Transportation						
		Ride Share	Bike	Train	Walk	Other		
	1	192	75	207	134	147		
	$\overline{2}$	197	83	219	124	166		
	3	185	66	213	114	137		
	4	101	58	120	71	94		
	5	35	33	45	26	50		
	6	46	27	53	22	37		
Min AWT	$\overline{7}$	29	29	60	23	53		
	8	14	11	14	19	12		
	9	$\sqrt{5}$	$\overline{2}$	$\mathbf{1}$	$\overline{2}$	$\,6\,$		
	10	6	$\pmb{0}$	$\overline{2}$	4	12		
	12	0	$\pmb{0}$	1	0	$\pmb{0}$		
	13	0	$\pmb{0}$	1	$\pmb{0}$	$\mathbf{1}$		
	14	$\mathbf 0$	$\boldsymbol{2}$	1	$\pmb{0}$	$\pmb{0}$		
	$\overline{\mathbf{2}}$	$\mathbf 0$	$\pmb{0}$	1	1	$\pmb{0}$		
	3	12	$\overline{7}$	$\overline{7}$	3	3		
	5	301	76	279	142	169		
Max AWT	$\overline{7}$	220	88	281	169	181		
	10	160	143	238	119	191		
	12	61	57	102	77	104		
	15	52	11	21	26	64		
	20	4	4	8	$\overline{2}$	$\mathsf 3$		

Table 16. Patrons' Acceptable Wait Times by Knowledge of Bus Arrival Time

Figure 37. Patrons' Minimum Acceptable Wait Times by Alternative Mode of Transportation Choice

Figure 38. Patrons' Maximum Acceptable Wait Times by Alternative Mode of Transportation Choice

Environmental Factors

This section presents the frequencies of acceptable wait times by temperature at the time of the survey.

Acceptable Wait Times by Temperature

Table 17 and Figures 39 through 41 present the summary of responses of preferred minimum and maximum acceptable wait times by the temperature at the time of the survey. From the table, it can be observed that patrons were tended to choose longer maximum acceptable wait times as the temperature increased.

Figure 39. Patrons' Minimum Acceptable Wait Times by Temperature

Figure 40. Patrons' Maximum Acceptable Wait Times by Temperature

Figure 41. Patrons' Acceptable Wait Times by Alternative Mode of Transportation

Mineta Transportation Institute

REGRESSION ANALYSIS

This section presents results of the regression analyses to develop to predict the MAWT's of bus patrons. Three models were developed: one for the A.M. peak period (7:00 AM–9:30 AM), one for the mid-day off-peak period (10:00 AM–2:30 PM), and one for the P.M. peak period (4:00 PM–6:30 PM). The models were multiple linear regressions taking the form:

$MAWT_i = \beta_{oi} + T\beta_{1i} + AH\beta_{2i} + KBAT\beta_{3i} + PS\beta_{4i} + \varepsilon$

A 5% significance level was chosen for testing the models. The models were evaluated for statistical significance, using the p-values of the models' F statistics and a 5% significance level, and for goodness-of-fit, using both the $R_{_2}$ and the adjusted $R_{_2}$ values. Also, the statistical significance of the models' predictors where evaluated using the p-values of the predictors' t-statistics. The t-test for each regression coefficient tests the model against the null hypothesis that the true regression coefficient is zero. The F-test of overall significance for a regression model compares the fit of the regression with the fit of a null model with an intercept (β_{α}) but no predictor variables, that is, to a null model where all regression coefficients are zero.

In order to achieve the best-fitting linear relationship between the dependent variable, Maximum Acceptable Wait Time (MAWT), and transformations of the independent variables, Temperature (T), Average Headway (AH), Period of Day (PS), and Knowledge of Bus Arrival Time (KBAT), several curve estimations between the dependent variable and each independent variable were performed. The transformations were necessary to obtain the best relationship between the dependent and independent variable. The formulae used to transform each independent variable are shown in Table 18. Logistic and cubic transformations of AH and T respectively, resulted in the most favorable relationships with the MAWT while PS and KBAT remained untransformed. The summaries of the results of the regression analyses are presented in the following sections. The detailed results are attached as appendix A.

Table 18. Data Transformation
Wait Time Prediction Models for AM Peak Period

The results of the regression analyses for A.M. peak period are presented in Table 19. After transformation of the variables, the model took the form

$$
MAWT_{A.M.} = \beta_{o1} + (T)^3 \beta_{11} + ln(\frac{1}{AH})\beta_{21} + KBAT\beta_{31} + PS\beta_{41}
$$

The results showed that the fit of this regression model for the A.M. peak period was statistically significant, at a pre-chosen 5% significance level but with its F statistic actually corresponding to a very low p-value of <0.001. The effects of KBAT, PS and the transformed variables AH_{Tr} and T_{Tr} were determined to be statistically significant, again with a pre-chosen significance level of 5% but with very low p-values of <0.001. The bestfitting model, with an adjusted R^2 value of 0.64, was determined to be

$$
MAWT_{AM} = -0.40 + (1.07 \times 10^{-5})T^3 - 0.642ln\left(\frac{1}{AH}\right) - 2.22KBAT + 3.265PS
$$

Table 19. Results of the Regression Analyses for AM Peak Period

Wait Time Prediction Models for Mid-Day Period

The results of the regression analyses for Mid-day period are presented in Table 20. The model took the form

$$
MAWT_{mid-day} = \beta_{o1} + (T)^3 \beta_{11} + Ln(\frac{1}{AH})\beta_{21} + KBAT\beta_{31} + PS\beta_{41}
$$

The results show that the fit of this model is statistically significant, at a pre-chosen 5% significance level but with its F statistic actually corresponding to a very low p-value of

<0.001. The effects of KBAT, PS and the transformed variables AH_{Tr} and T_{Tr} were determined to be statistically significant, also with a pre-chosen 5% significance level but actually with very low p-values of <0.001. The best-fitting model, with an adjusted R^2 value of 0.821, was determined to be

$$
MAWT_{offpeak} = -2.678 + (1.37 \times 10^{-5})T^3 - 0.904Ln\left(\frac{1}{AH}\right) - 1.048KBAT + 2.705PS
$$

Table 20. Results of the Regression Analyses for Mid-day Period

Wait Time Prediction Models for P.M. Peak Period

The results of the regression analyses for P.M. peak period are presented in Table 21. After transformation of the variables the model took the form

$$
MAWT_1 = \beta_{o1} + (T)^3 \beta_{11} + Ln(\frac{1}{AH})\beta_{21} + KBAT\beta_{31} + PS\beta_{41}
$$

The results show that the fit of this model is statistically significant, at a pre-chosen 5% significance level but with its F statistic actually corresponding to a very low p-value of $<$ 0.001. The effects of KBAT, PS and the transformed variables AH_{$_{T}$} and T_r were determined to be statistically significant, also with a pre-chosen 5% significance level but actually with very low p-values of <0.001. The best-fitting model, with an adjusted R^2 value of 0.793, was determined to be:

$$
MAWT_{PM} = 1.372 + (1.37 \times 10^{-5})T - 0.326Ln\left(\frac{1}{AH}\right) - 1.171KBAT + 2.074PS
$$

MODEL SUMMARY

Table 21. Results of the Regression Analyses for P.M. Peak Period

MODEL TESTING

Kolmogorov-Smirnov (K-S) Test

The results of the K-S tests for MAWT show that the maximum difference D between the cumulative distributions of the predicted and observed MAWTs, for all the models, was less than the critical value of 1.36 at a 5% level of significance. Thus, the distributions of predicted values and observed values do not significantly differ. The tables presenting the observed MAWT and the predicted MAWT are presented in Appendix B.

Normality of Errors

The assumption of normality of errors was tested using the normal probability plot. The observed cumulative probabilities of the standardized residuals are plotted against the expected cumulative probabilities of the standardized residuals. Visual inspection of the plots, presented in Appendix B, for all models, shows the observed curve closely follows the diagonal of the plot, indicating that the errors are normally distributed.

Multicollinearity

The test for multicollinearity showed that the VIF for all the variables in models were less than the maximum value of 10. Thus, multicollinearity was not present between the independent variables.

Homoscedacity

Visual inspection of the residual plots of the three models (presented in appendix B) show an even distribution about the zero line, confirming the variance of the residuals of the dependent variable is the same for all values of the independent variable.

Table 22 presents a summary of the models developed.

Table 22. Summary of Results

HYPOTHESIS TESTS

This section presents the results of the Wilcoxon rank-sum and Kruskal-Wallis tests for significant differences in the mean MAWT of bus patrons based on their gender and ethnicity respectively. The results were obtained by comparing the mean of the ranks of the MAWT of the patrons of each group of the independent variables. The z scores of the Wilcoxon W statistic and the Kruskal-Wallis H statistic were tested at a 5% significance level.

Results of the Test for Significant Difference in Mean Rank of MAWT Based on Gender

Tables 23 and 24 present the results of the test for a significance difference in MAWT based on gender. The mean rank of female patrons was calculated to be 1,683.63 while the mean rank of male patrons was calculated to be 1,705.11, as shown in Table 23. The Wilcoxon W statistic was computed to be 2,949,722.00 as observed in Table 24. This statistic had a z score of -0.660, which was determined to be statistically non-significant at a 5% significance level (p= 0.509). Thus, the analysis found no statistically significant effect of passengers' gender on their MAWT.

Table 24. Wilcoxon Rank-Sum Test Statistics - Gender

Results of the Test for Significant Difference in Mean Rank of MAWT Based on Ethnicity

The results of the test for a significant difference in MAWT based on ethnicity are presented in Tables 25 and 26. As Table 25 shows, White patrons had the lowest mean rank of 1,365.61, while patrons of ethnicity "Other" had the highest mean rank of 1,895.57. The Kruskal-Wallis H statistic was calculated to be 131.91 as shown in Table 26. This statistic was determined to be statistically significant, at a pre-chosen 5% significance level but actually with a very low p-value <0.001. Thus, the analysis showed there are significant differences in the MAWT of patrons based on ethnicity.

Table 25. Rankings by Gender

Table 26. Kruskal-Wallis Test Statistics - Ethnicity

A post-hoc analysis was used to investigate the between-ethnicity differences in MAWT of patrons. Since the data did not meet the parametric assumptions of normality and homogeneity of variance, the Games-Howell post-hoc test was used for the analysis. The results of the analysis are shown in Table 27. The table shows the differences between the mean ranks MAWT for each ethnicity with that for each other ethnicity. The statistical significances of these differences are also presented in the table.

Table 27. Results of Post-Hoc Test

**The mean difference is significant at the 0.05 level.*

The results show that there were statistically significant differences in MAWT rank between White patrons and patrons of all other ethnic groups, at a pre-chosen 5% significance level but with a very low p-value of <0.001, with White patrons having the lowest mean MAWT rank.

V. DISCUSSION

The research aimed at developing regression models for predicting the maximum acceptable wait times of bus patrons based on the weather condition (temperature), average headway of buses, patrons' knowledge of bus arrival times and the presence of shelter at the bus stops. Previous literature has shown that the actual wait times of patrons at bus stops differed from their perceived wait times, and that perceived wait time tend to matter more for dictating rider discomfort and preference towards bus services than do actual wait times. The factors that were found to affect the actual wait times of patrons included bus stop features and surroundings, period of day, the purpose of trip, and the patrons' demographics. The wait time of bus patrons has also been used by transit agencies and officials as a measure of transit service reliability; this was especially the case in jurisdictions with high frequency services, where the arrival of passengers is random, and the average wait time approximates half the headway. Although most studies define "wait time" simply as the time spent waiting at the bus stop, a study argued that a better measure of the real cost to patrons of waiting for buses would take into account four components: platform waiting time; potential waiting time; scheduled inconvenience; and synchronization cost. The review of literature also revealed that when wait times exceed acceptable limits, patrons tend to consider other mode choices. The alternate mode choice of passengers ranges from transferring to another bus route to other travel modes such as taxis and carpooling. Actual wait times have been reported in previous literature to follow the lognormal and gamma distributions.

The data used in this study was obtained by surveying 3,388 bus patrons at 71 selected bus stops in Washington, D.C. over an eight-month period. Data obtained from patrons included their ethnicity, gender, minimum and maximum acceptable wait times, alternate transportation mode choice, and knowledge of bus arrival times. Additionally, data on the operational characteristic of the buses were obtained via video playback of video recordings of cameras installed at the selected bus stop. The data extracted from the video recordings included the bus arrival and departure times; the headway of each bus route was then computed from these metrics. In addition, information and conditions at each bus stop at the time of each survey was recorded, including bus stop ID number, bus direction of travel, the availability of shelter and the temperature at the bus stop at the time of survey.

The results of the analysis of the survey data obtained confirmed some of the findings in previous literature and also provided new insight about the acceptable wait time of bus patrons. Analysis of the maximum acceptable wait times of patrons showed lognormal distribution, as was reported for actual wait times in previous literature (Guo et al, 2011). Most patrons' acceptable wait times ranged from 5 to 15 minutes. The mean maximum acceptable wait time of patrons during the mid-day period was found to be higher than during the A.M. and P.M peak periods, was recorded occurred during the A.M. peak period, which may be explained by the fact that most patrons during the A.M. are probably commuting to work and need to arrive on time. The mean acceptable wait times of patrons waiting at bus stops with shelter was approximately 28% higher than mean acceptable wait times of patrons at bus stops without shelter, a value close to the 30% previously reported by Fan et al. (2016). However, with regards to gender, the mean maximum acceptable wait times of male and female patrons was approximately equal, suggesting that the gender of a patron may not influence their waiting tolerance level. In terms of race, White patrons reported the lowest mean maximum acceptable wait time; Black and "Other" ethnicity patrons recorded the highest mean maximum acceptable wait time; and Hispanic and Asian patrons had approximately the same, intermediate, mean maximum acceptable wait times. Bus patrons who had knowledge of the arrival time of the bus were observed to have lower acceptable wait times than those who had no knowledge of the arrival time, thus tending to be less tolerant when buses do not arrival as scheduled. About 40% of the patrons surveyed considered using a train as an alternate transportation mode while about 35% considered using a ridesharing service. Patrons at bus stops close to train stations mostly preferred using a train. Also, 10% preferred to walk to their destination if it is within a walking distance. In addition, 11% of the patrons preferred to use a bicycle.

Statistical analyses of the data were conducted in order to develop predictive models for acceptable wait times based on weather condition (temperature) and average headway of buses. Also, statistically significant differences in the average maximum acceptable wait times of patrons based on gender and ethnicity were investigated. The analyses were conducted at a 5% significance level, although many of the significant p-values turned out to be considerably lower than this. Regression models were developed for the A.M. peak, mid-day off-peak, and P.M. peak periods. All the models were found to fit the data with statistically significant F statistics (with very low p-values <0.001). In addition, the effects of temperature, average headways, presence of shelter, and knowledge of bus arrival time of all the models were determined to be statistically significant (with very low p-values <0.001). In general, the maximum acceptable wait of passengers also tend to increase as temperature increased. The maximum acceptable wait times of patrons increase with increasing bus headway. Moreover, patrons who are aware of the arrival times of buses had shorter maximum acceptable wait times. In addition, at bus stops with shelter, patrons are more likely to wait longer. The Kolmogorov-Smirnov tests showed that the effects of all the models' regression coefficients were statistically significant at a 5% significance level. The models had values (percentage of variance explained) of 64%, 79%, and 82%.

The results of the Wilcoxon rank-sum test showed that the difference in the acceptable wait times of patrons based on the gender was not statistically significant (*p*-values = 0.509). In contrast, the Kruskal-Wallis test did show at least one statistically significant difference in the mean acceptable wait times of patrons based on ethnicity, reporting a statistically significant H statistic of 15.544 (*p*-value=0.000). A post-hoc analysis then revealed that there is a statistically significant difference of the acceptable wait times between White patrons and all other ethnicities.

VI. CONCLUSIONS AND RECOMMENDATIONS

The models developed in this research are potentially useful tools that transit agencies could use to improve bus scheduling and operations in order to ultimately retain and improve ridership. It is recommended that future research explore the use of machine learning and artificial intelligence techniques to predict the wait time of patrons and also incorporate other factors such as average dwell time of buses.

APPENDIX A

*SWB – Southwest-bound, SB - Southbound, NWB – Northwest-bound, NB - Northbound, EB – Eastbound, WB – Westbound

AM PEAK MODEL TESTING

K-S Test for Model Accuracy

Normal Probability Plot

MID PEAK MODEL TESTING

Normal P-P Plot of Regression Standardized Residual

Scatterplot

Normal P-P Plot of Regression Standardized Residual

Normal Probability Plot

Scatter Plot

ENDNOTE

- 1. Hensher, D., Golob, T. (2008). Bus rapid transit systems: a comparative assessment. Transportation.Vol.35, No.4, pp. 501–518.
- 2. Ben-Akiva, M., S.R. Lerman. (1985). Discrete Choice Analysis: Theory and Application to Travel Demand, Cambridge, MA: MIT Press.
- 3. Durán-Hormazábal E., Tirachini E. (2016). Estimation of travel time variability for cars, buses, metro and door-to-door public transport trips in Santiago, Chile. Transportation Economics, ISSN: 0739-8859, Vol. 59, Page: 26–39
- 4. Mishalani, McCord. (2006). Passenger Wait Time Perceptions at Bus Stops: Empirical Results and Impact on Evaluating Real-Time Bus Arrival Information. Journal of Public Transportation, Vol. 9 (2): 89–106.
- 5. Wardman, M. (2004). Public Transport Values Of Time. Transport Policy, Vol. 11, Issue 4, P. 36 3–377.
- 6. Horowitz, A. (1981). Subjective Value of Time in Bus Transit Travel. Transportation, Vol. 10, 149– 164.
- 7. Hu, X., Zhao, L., & Wang, W. (2015). Impact of perceptions of bus service performance on mode choice preference. Advances in Mechanical Engineering. https://doi. org/10.1177/1687814015573826
- 8. Yang, Min & Zhao, Jingyao & Wang, Wei & Liu, Zhiyuan & Li, Zhibin. (2014). Metro commuters' satisfaction in multi-type access and egress transferring groups. Transportation Research Part D: Transport and Environment. Vol. 34. 10.1016/j. trd.2014.11.004.
- 9. Psarros et al (2011), An Empirical Investigation of Passenger Wait Time Perceptions Using Hazard-Based Duration Models. Journal of Public Transportation. Vol. 14. 109– 122. 10.5038/2375-0901.14.3.6.
- 10. Hess et al. 2004. Waiting for the Bus. Journal of Public Transportation, Vol. 7 (4): 67–84.
- 11. Moreau, A., (1992). Public transport waiting times as experienced by customers. Public Transp. Int. 41 (3), 52–71
- 12. Van Hagen, M., (2011). Waiting Experience at Train Stations [Doctoral dissertation]. Eburon Uitgeverij BV.
- 13. Fan et al. (2016). Factors of Perceived Waiting Time and Implications on Passengers' Satisfaction with Waiting Time. PROMET – Traffic &Transportation. 28. 10.7307/ptt. v28i2.1726.
- 14. Daskalakis, N.G., Stathopoulos, A., (2008). Users' perceptive evaluation of bus arrival time deviations in stochastic networks. Journal of Public Transport. Vol. 11 (4), 25–38.
- 15. Cascetta, E., Cartenì, A., (2014). The hedonic value of railways terminals. A quantitative analysis of the impact of stations quality on travellers behaviour. Transport. Res. Part A: Policy Pract. 61, 41–52.
- 16. Iseki, H., Taylor, B., (2010). Style versus service? An analysis of user perceptions of transit stops and stations. J. Public Transport. Vol. 13 (3), 23–48
- 17. Dziekan, K., Kottenhoff, K., (2007). Dynamic At-Stop Real-Time Information Displays for Public Transport: Effects on Customers. Transportation Research Part A: Policy and Practice, Vol. 41, Issue 6, pp. 489–501.
- 18. Padmanaban, R.P.S. & Divakar, K. & Vanajakshi, L. & Subramanian, Shankar. (2010). Development of a real-time bus arrival prediction system for Indian traffic conditions. Intelligent Transport Systems, IET. 4. 189 - 200. 10.1049/iet-its.2009.0079.
- 19. Feng, Shumin & Wu, Haiyue & Sun, Xianglong & Li, Zhenning. (2016). Factors of Perceived Waiting Time and Implications on Passengers' Satisfaction with Waiting Time. PROMET – Traffic &Transportation. 28. 10.7307/ptt.v28i2.1726.
- 20. Turnquist M., Bowman L., (1979). Service Frequency, Schedule Reliability, and Passenger Wait Times at Transit Stops. Transportation Research Part A General 15(6):465–471
- 21. Gurmu, K., Fan, W. (2014). Artificial Neural Network Travel Time Prediction Model for Buses Using Only GPS Data. Journal of Public Transportation, 17 (2): 45–65.
- 22. Reed, B. (1995). "Reduction in the Burden of Waiting for Public Transit due to Real-Time Schedule Information: A Conjoint Analysis Study." Vehicle Navigation and Information Systems Conference, 1995. Proceedings. In conjunction with the Pacific Rim TransTech Conference, 6th International VNIS, "A Ride into the Future": 83–89.
- 23. Welding P. I. (1957). The Instability of a Close-Interval Service. Operational Research Quarterly, Vol.8, No.3, 133–148.
- 24. Furth, G., Muller, T. (2006). Service reliability and hidden waiting time Insights from automatic vehicle location data. Journal of the Transportation Research Board, No. 1955, Transportation Research Board of the National Academies, Washington, D.C, (1955), pp. 79–87.
- 25. Tsegaye Firew (2016), Analysis of Service Reliability of Public Transportation in the Helsinki Capital Region: The Case of Bus Line 550. http://urn.fi/ URN:NBN:fi:aalto-201612226305.
- 26. Weber W. (1966) The travel time of passengers of public transport depending on

railway type and location, Stuttgart University of Technology.

- 27. Holroyd, M., Scraggs, A. (1964) "Journey Times by Car and Bus in Central london", Traffic Engineering and Control, 1964, Vol.6, No.3, 169–173
- 28. O'Flaherty, A., Mangan O., (1970) Bus Passenger Waiting Time in Central Areas, Traffic Engineering and Control, Vol. 11, No.9, 419–421.
- 29. Osuna,.E., Newell, F., (1972). Control strategies for an idealized bus system. Transportation Science 6(1), 5 2–71.
- 30. Seddon P , Day M., (1974). Bus Passenger Waiting Times in Greater Manchester. Traffic Engineering and Control, Vol. 15, No. 9, 1974, pp. 442–445.
- 31. Weber W. (1966) The travel time of passengers of public transport depending on railway type and location, Stuttgart University of Technology.
- 32. Jolliffe, K., Hutchinson, K. (1974). A behavioral explanation of the association between bus and passenger arrivals at a bus stop. Transportation Science Vol 9, 1975, pp 248–282.
- 33. Cham C. (2006). Understanding Bus Service Reliability: a practical framework using AVL/APC data. DSpace@MIT: Massachusetts Institute of Technology. [Online]. Available at: https://dspace.mit.edu/handle/1721.1/34381
- 34. Chen et al. (2017) Influence of travel time variability on train station choice for parkand-rider users. World Conference on Transport Research - WCTR 2016 Shanghai.
- 35. Handy et al. (1998). The effectiveness of land use policies as a strategy for reducing automobile dependence: A study of Austin neighborhoods. Research Report SWUTC/98/46750 1–1. Austin: University of Texas Press.
- 36. Nam, D., Park, D., Khamkongkhun, A., (2005). Estimation of value of travel time reliability. Journal of Advanced Transportation 39(1), 39–61
- 37. Han et al (2018). Research on Passenger's Travel Mode Choice Behavior Waiting at Bus Station Based on SEM-Logit Integration Model. Sustainability. 10. 1996.
- 38. Xue et al. (2013). A Traffic Mode Choice Model for the Bus User Groups Based on SP and RP Data. 13th COTA International Conference of Transportation Professionals.
- 39. Train K. (2003). Discrete Choice Methods with Simulation. Cambridge: Cambridge University Press.
- 40. Takada, K.; Kobayashi, M (2008). Analysis on Choice behavior of railway passengers when railway service stops. Infrastructure. Plan. Rev. 2010, 25, 763–768.
- 41. Baidoo, I. K., & Nyarko, E. (2015). A Discrete Choice Modeling of Service Quality Attributes in Public Transport. Research Journal of Mathematics and Statistics 7(1), 6–10.
- 42. Luethi, M., Weidmann U., Nash A,. Passenger arrival rates at public transport stations. 86th Annual Meeting of the Transportation Research Board, 2007.
- 43. Guo et al. (2011). Modeling Waiting Time for Passengers Transferring from Rail to Buses. Transportation Planning and Technology, Vol. 34, No. 8, 29 2011, pp. 795–809.
- 44. Chen et al. (2014) An Application-Oriented Model of Passenger Waiting Time 5 Based On Bus Departure Time Intervals. TRB 2014 Annual Meeting.
- 45. Kaparias, I., Rossetti, C. and Trozzi, V. (2015). Analyzing passenger arrivals rates and waiting time at bus stops. Paper presented at the 94th Annual Meeting of the Transportation Research Board, 11-01-2015 - 15-01-2015, Washington, DC, USA.
- 46. Chien, Steven I-Jy & Ding, Yuqing & Wei, Chienhung. (2002). Dynamic Bus Arrival Time Prediction with Artificial Neural Networks. Journal of Transportation Engineeringasce - J TRANSP ENG-ASCE. 128. 10.1061/(ASCE)0733-947X(2002)128:5(429).
- 47. <https://ddot.dc.gov/ddot/cwp/view,a,1250,q,642575.asp>

BIBLIOGRAPHY

- Baidoo, I. K., & Nyarko, E. (2015). A Discrete Choice Modeling of Service Quality Attributes in Public Transport. Research Journal of Mathematics and Statistics $7(1), 6 - 10.$
- Ben-Akiva, M., S.R. Lerman. (1985). Discrete Choice Analysis: Theory and Application to Travel Demand, Cambridge, MA: MIT Press.
- Cascetta, E., Cartenì, A., (2014). The hedonic value of railways terminals. A quantitative analysis of the impact of stations quality on travellers behaviour. Transport. Res. Part A: Policy Pract. 61, 41–52.
- Cham C. (2006). Understanding Bus Service Reliability: a practical framework using AVL/APC data. DSpace@MIT: Massachusetts Institute of Technology. [Online]. Available at: https://dspace.mit.edu/handle/1721.1/34381
- Chen et al. (2014) An Application-Oriented Model of Passenger Waiting Time 5 Based On Bus Departure Time Intervals. TRB 2014 Annual Meeting
- Chen et al. (2017) Influence of travel time variability on train station choice for park- andrider users. World Conference on Transport Research - WCTR 2016 Shanghai
- Chien, Steven I-Jy & Ding, Yuqing & Wei, Chienhung. (2002). Dynamic Bus Arrival Time Prediction with Artificial Neural Networks. Journal of Transportation Engineeringasce - J TRANSP ENG-ASCE. 128. 10.1061/(ASCE)0733-947X(2002)128:5(429).
- Daskalakis, N.G., Stathopoulos, A., (2008). Users' perceptive evaluation of bus arrival time deviations in stochastic networks. J. Public Transport. 11 (4), 25–38.
- Durán-Hormazábal E., Tirachini E. (2016). Estimation of travel time variability for cars, buses, metro and door-to-door public transport trips in Santiago, Chile. Transportation Economics, ISSN: 0739-8859, Vol: 59, Page: 26–39
- Dziekan, K., Kottenhoff, K., (2007). Dynamic At-Stop Real-Time Information Displays for Public Transport: Effects on Customers. Transportation Research Part A: Policy and Practice, Volume 41, Issue 6, pp. 489–501.
- Fan et al. (2016). Factors of Perceived Waiting Time and Implications on Passengers' Satisfaction with Waiting Time. PROMET – Traffic &Transportation. 28. 10.7307/ ptt.v28i2.1726.
- Feng, Shumin & Wu, Haiyue & Sun, Xianglong & Li, Zhenning. (2016). Factors of Perceived Waiting Time and Implications on Passengers' Satisfaction with Waiting Time. PROMET – Traffic &Transportation. 28. 10.7307/ptt.v28i2.1726.
- Furth, G., Muller, T. (2006). Service reliability and hidden waiting time Insights

from automatic vehicle location data. Journal of the Transportation Research Board, No. 1955, Transportation Research Board of the National Academies, Washington, D.C, (1955), pp. 79–87.

- Guo et al. (2011). Modeling Waiting Time for Passengers Transferring from Rail to Buses. Transportation Planning and Technology, Vol. 34, No. 8, 29 2011, pp. 795–809.
- Gurmu, K., Fan, W. (2014). Artificial Neural Network Travel Time Prediction Model for Buses Using Only GPS Data. Journal of Public Transportation, 17 (2): 45–65.
- Han et al (2018). Research on Passenger's Travel Mode Choice Behavior Waiting at Bus Station Based on SEM-Logit Integration Model. Sustainability. 10. 1996.
- Handy et al. (1998). The effectiveness of land use policies as a strategy for reducing automobile dependence: A study of Austin neighborhoods. Research Report SWUTC/98/46750 1–1. Austin: University of Texas Press.
- Hensher, D., Golob, T. (2008). Bus rapid transit systems: a comparative assessment. Transportation.Vol.35, No.4, pp. 501–518.

Hess et al. 2004. Waiting for the Bus. Journal of Public Transportation, 7 (4): 67–84.

- Holroyd, M., Scraggs, A. (1964) "Journey Times by Car and Bus in Central london", Traffic Engineering and Control, 1964, Vol.6, No.3, 169–173
- Horowitz, A. (1981). Subjective Value of Time in Bus Transit Travel. Transportation, 10, 149– 164.
- Hu, X., Zhao, L., & Wang, W. (2015). Impact of perceptions of bus service performance on mode choice preference. Advances in Mechanical Engineering. https://doi. org/10.1177/1687814015573826
- Iseki, H., Taylor, B., (2010). Style versus service? An analysis of user perceptions of transit stops and stations. J. Public Transport. 13 (3), 23–48
- Jolliffe, K., Hutchinson, K. (1974). A behavioral explanation of the association between bus and passenger arrivals at a bus stop. Transportation Science Vol 9, 1975, pp 248–282.
- Kaparias, I., Rossetti, C. and Trozzi, V. (2015). Analyzing passenger arrivals rates and waiting time at bus stops. Paper presented at the 94th Annual Meeting of the Transportation Research Board, 11-01-2015 - 15-01-2015, Washington, DC, USA.
- Luethi, M., Weidmann U., Nash A,. Passenger arrival rates at public transport stations. 86th Annual Meeting of the Transportation Research Board, 2007.

Mishalani, McCord. (2006). Passenger Wait Time Perceptions at Bus Stops: Empirical

Results and Impact on Evaluating Real-Time Bus Arrival Information. Journal of Public Transportation, 9 (2): 89–106.

- Moreau, A., (1992). Public transport waiting times as experienced by customers. Public Transp. Int. 41 (3), 52–71
- Nam, D., Park, D., Khamkongkhun, A., (2005). Estimation of value of travel time reliability. Journal of Advanced Transportation 39(1), 39–61
- O'Flaherty, A., Mangan O., (1970) Bus Passenger Waiting Time in Central Areas, Traffic Engineering and Control, Vol. 11, No.9, 419–421.
- Osuna,.E., Newell, F., (1972). Control strategies for an idealized bus system. Transportation Science 6(1), 5 2–71.
- Padmanabhan et al. (2009) Development of a Real-Time Bus Arrival Time Prediction System under Indian Traffic Conditions
- Psarros et al (2011), An Empirical Investigation of Passenger Wait Time Perceptions Using Hazard-Based Duration Models. Journal of Public Transportation. 14. 109– 122. 10.5038/2375-0901.14.3.6.
- Reed, B. (1995). "Reduction in the Burden of Waiting for Public Transit due to Real-Time Schedule Information: A Conjoint Analysis Study." Vehicle Navigation and Information Systems Conference, 1995. Proceedings. In conjunction with the Pacific Rim TransTech Conference, 6th International VNIS, "A Ride into the Future": 83–89.
- Seddon P , Day M., (1974). Bus Passenger Waiting Times in Greater Manchester. Traffic Engineering and Control, Vol. 15, No. 9, 1974, pp. 442–445.
- Takada, K.; Kobayashi, M (2008). Analysis on Choice behavior of railway passengers when railway service stops. Infrastructure. Plan. Rev. 2010, 25, 763–768.
- Train K. (2003). Discrete Choice Methods with Simulation. Cambridge: Cambridge University Press.
- Tsegaye Firew (2016), Analysis of Service Reliability of Public Transportation in the Helsinki Capital Region: The Case of Bus Line 550
- Turnquist M., Bowman L., (1979). Service Frequency, Schedule Reliability, and Passenger Wait Times at Transit Stops. Transportation Research Part A General 15(6):465–471
- Van Hagen, M., (2011). Waiting Experience at Train Stations [Doctoral dissertation]. Eburon Uitgeverij BV
- Wardman, M. (2004). Public Transport Values Of Time. Transport Policy, Volume 11, Issue 4, P. 36 3–377.
- Weber W. (1966) The travel time of passengers of public transport depending on railway type and location, Stuttgart University of Technology.
- Welding P. I. (1957). The Instability of a Close-Interval Service. Operational Research Quarterly, Vol.8, No.3, 133–148.
- Xue et al. (2013). A Traffic Mode Choice Model for the Bus User Groups Based on SP and RP Data. 13th COTA International Conference of Transportation Professionals
- Yang, Min & Zhao, Jingyao & Wang, Wei & Liu, Zhiyuan & Li, Zhibin. (2014). Metro commuters' satisfaction in multi-type access and egress transferring groups. Transportation Research Part D: Transport and Environment. 34. 10.1016/j. trd.2014.11.004.

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