



**Center for Advanced Multimodal Mobility  
Solutions and Education**

**Project ID: 2017 Project 06**

**ROBUST ROUTING, ASSIGNMENT, AND SIMULATION  
OF TRANSIT SYSTEMS**

**Final Report**

by

Nicholas Lownes, Ph.D., P.E. (ORCID ID: <https://orcid.org/0000-0002-3885-2917>)  
Associate Professor, Department of Civil and Environmental Engineering  
University of Connecticut  
261 Glenbrook Rd. U-3037  
Storrs, CT 06269  
Phone: 1-860-486-2717; Email: [nlownes@engr.uconn.edu](mailto:nlownes@engr.uconn.edu)

for

Center for Advanced Multimodal Mobility Solutions and Education  
(CAMMSE @ UNC Charlotte)  
The University of North Carolina at Charlotte  
9201 University City Blvd  
Charlotte, NC 28223

**September 2018**



## **ACKNOWLEDGEMENTS**

This project was funded by the Center for Advanced Multimodal Mobility Solutions and Education (CAMMSE @ UNC Charlotte), one of the Tier I University Transportation Centers that were selected in this nationwide competition, by the Office of the Assistant Secretary for Research and Technology (OST-R), U.S. Department of Transportation (US DOT), under the FAST Act. The authors are also very grateful for all of the time and effort spent by DOT and industry professionals to provide project information that was critical for the successful completion of this study.

## **DISCLAIMER**

The contents of this report reflect the views of the authors, who are solely responsible for the facts and the accuracy of the material and information presented herein. This document is disseminated under the sponsorship of the U.S. Department of Transportation University Transportation Centers Program in the interest of information exchange. The U.S. Government assumes no liability for the contents or use thereof. The contents do not necessarily reflect the official views of the U.S. Government. This report does not constitute a standard, specification, or regulation.



# Table of Contents

<b>EXECUTIVE SUMMARY .....</b>	<b>xi</b>
<b>Chapter 1. Introduction.....</b>	<b>1</b>
1.1 Problem Statement.....	1
1.2 Objectives .....	1
1.3 Expected Contributions.....	1
1.4 Report Overview .....	2
<b>Chapter 2. Literature Review .....</b>	<b>3</b>
2.1 Introduction.....	3
2.2 What is Fast-Trips.....	3
2.3 Problems Fast-Trips Meant to Solve.....	3
2.4 What exists already .....	4
<b>Chapter 3. Methodology .....</b>	<b>7</b>
3.1 Introduction.....	7
3.2 Fast-Trips Code Flow Chart.....	7
3.3 Inputs.....	8
3.3.1 Transit Network .....	8
3.3.2 Passenger Demand.....	9
3.3.3 Configuration Parameters .....	9
<b>Chapter 4. Case Study and Result Analysis .....</b>	<b>13</b>
4.1 Introduction.....	13
4.2 Model Inputs .....	13
4.3 Run-time Analysis .....	14
4.3.1 Old Code vs Updated code .....	17
<b>Chapter 5. Summary and Conclusions .....</b>	<b>19</b>
5.1 Summary and Future Work.....	19
<b>References.....</b>	<b>21</b>



## List of Figures

Figure 3.1: Flow chart of the Fast-Trips code run .....	8
Figure 4.1: Run-time difference between different Pathfinding types .....	13
Figure 4.2: Run-time reduction with the increase in Maximum Stop Process Count.....	13
Figure 4.3: Run-time reduction with the increase in Overlap Chunk Size .....	14
Figure 4.4: Run-time difference between different Overlap Variables .....	14
Figure 4.5: Run-time reduction with the change in Minimum Transfer Penalty.....	15
Figure 4.6: Run-time difference between different Simulation types .....	15





## List of Tables

Table 3.1: GTFS-PLUS Transit Network must require files .....	9
Table 3.2: A Dyno-Demand dataset MUST include the following file.....	10
Table 3.3: A Dyno-Demand dataset MAY include these optional files.....	10
Table 3.4: Configuration options for fasttrips.....	10
Table 3.5: Configuration options for pathfinding.....	11
Table 3.6: San Francisco SF-CHAMP Transit network data summary.....	13
Table 4.1: Differences in outputs between old and updated codes in Fast-Trips.....	16



## **EXECUTIVE SUMMARY**

Transit system complexity is a function not only just of the infrastructure but also is strongly tied to user behavior, which is driven by perception of the quality of service (in experiencing waiting, transfers, and travel time variability), which is not always an accurate reflection of the true quality of service. The variability and inconsistency (with reality) of these perceptions can be captured in part by correlating the multiple data sources, observing travel patterns and modeling user behavior under uncertainty.

The effect of service reliability and accessibility will be investigated towards the development of route choice, stop choice, and departure time choice models. Stochastic multimodal network and user behavior models will be leveraged toward the development of routing, assignment, and simulation models for ridership estimation and performance analysis. Robust analysis tools will be developed that take into account system characteristics which are a function of the performance of the auto network, models passenger flow in the network under stochastic rules and predicts system wide travel patterns. Such models will enhance decisions made by transit agencies when allocating resources toward additional capacity, schedule updates, and stop/station location in the long term; and stop skipping, rerouting, and vehicle holding as real-time operational decision.

The objective of this project is to (1) represent the transit users' behavior of Hartford, Connecticut more accurately; (2) model the transit system with high resolution; and (3) produce disaggregate results. To accomplish this project, the research team will work on developing the input network dataset and passenger demand data for Hartford, Connecticut. An open source code-base dynamic transit assignment tool then will be used to simulate transit passenger route-finding and user experiences.



# Chapter 1. Introduction

## 1.1 Problem Statement

Many of the questions confronting urban transportation planners today concern moving people rather than the vehicles. However, most advanced operational planning analysis tools still operate on the vehicle as the primary unit of analysis and use a geographic scale incompatible with the measures of concern to human-scale travel. When our ability to analyze solution A far surpasses our ability to analyze and understand solutions B, C, and D, we may be more likely to pick option A even though it isn't the best.

Great strides in advanced operational planning for vehicles have been widely supported and are starting to be adopted by the mainstream. This includes technology known as Dynamic Traffic Assignment which routes vehicles through an entire area using a second-by-second time resolution. While transit is often represented in Dynamic Traffic Assignment, its performance can only be measured based on how well the vehicle performs in traffic, not the quality of the route to the person taking transit. While this enables engineers to analyze how much faster a bus might get through an intersection with transit signal priority, it doesn't let planners understand the levels of investment necessary to ameliorate over-crowding or engineers and service planners understand the system-level effects of strategies to address transit reliability. To achieve the goal of analyzing human-scale transportation investments, Fast-Trips was developed and further extended by a multi-agency effort to implement a dynamic transit passenger assignment model for travel demand forecasting.

## 1.2 Objectives

The objective of this report is to (1) understand the inputs of this dynamic transit assignment tool Fast-Trips; (2) run for the San-Francisco case study dataset; (3) investigate the computational efficiency of the model.

## 1.3 Expected Contributions

The expected contributions of this report are details regarding the development of a dynamic transit assignment model and the steps taken to integrate into an urban region that

currently uses a traditional four-step planning model. Substantial effort was made in modifying the Fast-trips codebase for implementation in the Hartford, CT region and the development of the required enhanced input data.

## **1.4 Report Overview**

The remainder of this report is organized as follows: Chapter 2 presents a comprehensive review of the state-of-the-art and state-of-the-practice literature on the dynamic transit assignment approaches. Chapter 3 discusses codebase logic and detailed model inputs required to run Fast-Trips. Chapter 4 describes a San-Francisco case study and analyzes the run-times of the outputs that result from various configuration parameters. The difference in results between old and updated codebases is also addressed in this chapter.

## Chapter 2. Literature Review

### 2.1 Introduction

This chapter provides a review on the dynamic transit assignment tool Fast-Trips. The transit assignment tools which already in use worldwide will also be discussed in this section.

### 2.2 What is Fast-Trips?

Fast-Trips (Flexible Assignment and Simulation Tool for Transit and Intermodal Passengers) is a dynamic transit assignment model for regional planning analysis. It assigns each transit traveler a specific transit route considering the published vehicle schedule. It also has the capability of taking into account congestion and simulated vehicle trajectories through integration with a dynamic traffic assignment (DTA) model. The model was developed as a part of an integrated advanced transportation model in SHRP2 C10B project sponsored by the Federal Highway Administration (1, 3, 4) for travel demand forecasting.

The Fast-Trips model is divided into two submodules: assignment and simulation. Passengers are assigned given origin and destination zones and a preferred departure time at the origin or a preferred arrival time at the destination. Assignment can be done using a deterministic trip-based shortest path (TBSP) using travel time or a stochastic trip-based hyperpath (TBHP) using generalized travel costs (2). In either case, vehicle capacity constraints can be included and a user equilibrium (all passengers taking their optimal available path) can be reached with iterations of assignment and simulation. In the passenger simulation module, boarding and alighting of passengers is simulated, along with other aspects of the trip (e.g. access, egress, waiting). Passengers may fail to board a transit vehicle, and the model can reassign passengers to alternate paths. Dwell time is also calculated as a function of passenger boardings and alightings at each stop. Travel statistics are accumulated and experienced skims can be generated.

### 2.3 Problems Fast-Trips Addresses

The original implementation, Fast-Trips, solved several problems that are important to the project stakeholders that earlier transit route choice tools could not. These included the ability to represent a more nuanced schedule of transit vehicle trips and the resulting ridership, including

- Network link travel times that could be informed by dynamic traffic assignment models;
- Passenger queueing at transit stops;
- Stop dwell times that could be a function of boards, alightings and crowding;
- The ability to consider a set of transit paths for each passenger, rather than a single best path, in order to represent the benefit of having more than one option; and
- Transit vehicle capacities, which could result in bumped passengers, affecting transit path quality.

In the Fast-Trips Implementation Project for the SHRP-2 Implementation Assistance Program, stakeholders added a few more requirements to support long-term planning. These included:

- Heterogeneity of riders: different types of riders can value (or devalue) different aspects of a transit trip differently. For example, older travelers may be more bothered by extreme crowding or longer walks. By representing this heterogeneity of riders, Fast-Trips could better assess the demographics of travelers that would benefit most from a planned project.
- Passengers affecting transit: boardings, alightings and crowding often affect transit vehicle dwell times. The relationship between transit dwell times and other variables should be fully configurable based on transit vehicle type.
- Transit affecting passenger experiences: some people will get seats while others will not be able to board crowded vehicles and wait longer. Some people will miss transfers while others may ride a few stops in the wrong direction to get a seat on a crowded line.
- Transit reliability: Missed transfers and reliability issues affect how people perceive the quality of transit.

## **2.4 What exists already**

In practice, transit route choice and assignment models that are in active use today tend to be static models included as part of commercial travel modeling software packages. INRO's Emme has the most documentation in terms of the underlying research behind the model, and several modeling innovations included in this model are referenced in sections below (5,6). PTV Group's



VISUM model includes headway-based and time-table-based assignment, as well as non-additive fares, but it's not clear how these are implemented (7). Caliper's website says that TRANSCAD includes "the broadest set of transit assignment methods including some innovative methods not found in other packages. These include a stochastic user equilibrium method that deals with multiple service alternatives, vehicle capacity, and optionally with dwell time and user's value of time", although the implementation details behind these innovations are not published (8). Citilab's Cube Voyager also comes with two static transit route choice/assignment models, TRNBUILD and Public Transport, the internal details of which are not published either. In order to test the effects of policies affecting transit capacity increases, the San Francisco County Transportation Authority developed a transit assignment model on top of TRNBUILD; this represented capacity constraints and passenger-based transit delays in a limited way by programmatically modifying the inputs to TRNBUILD to deter passengers from crowded vehicles and adjust schedules (9).

Beyond practice, there has been much research into dynamic transit assignment modeling. In broad strokes, most of this is based on lowest-cost path search using network labeling like Fast-Trips. However, Fast-Trips includes a novel approach called Recursive Logit, which corresponds to a sequence of link choices that together form a path choice. This approach is unique in that it avoids path enumeration, and so it can be consistently estimated and used for prediction without limiting the path choice set (10). Although this framework is promising, Fast-Trips is still planning to develop, enhance and test the hyperpath-based model from the original Fast-Trips, because it is closer to practice-ready.



# Chapter 3. Methodology

## 3.1 Introduction

The codebase flow chart of how Fast-Trips works, input and output parameters will be discussed in this chapter.

## 3.2 Fast-Trips Codebase Flow Chart

A flow chart of how the codes work in the Fast-Trips is shown below in the Figure 3.1.

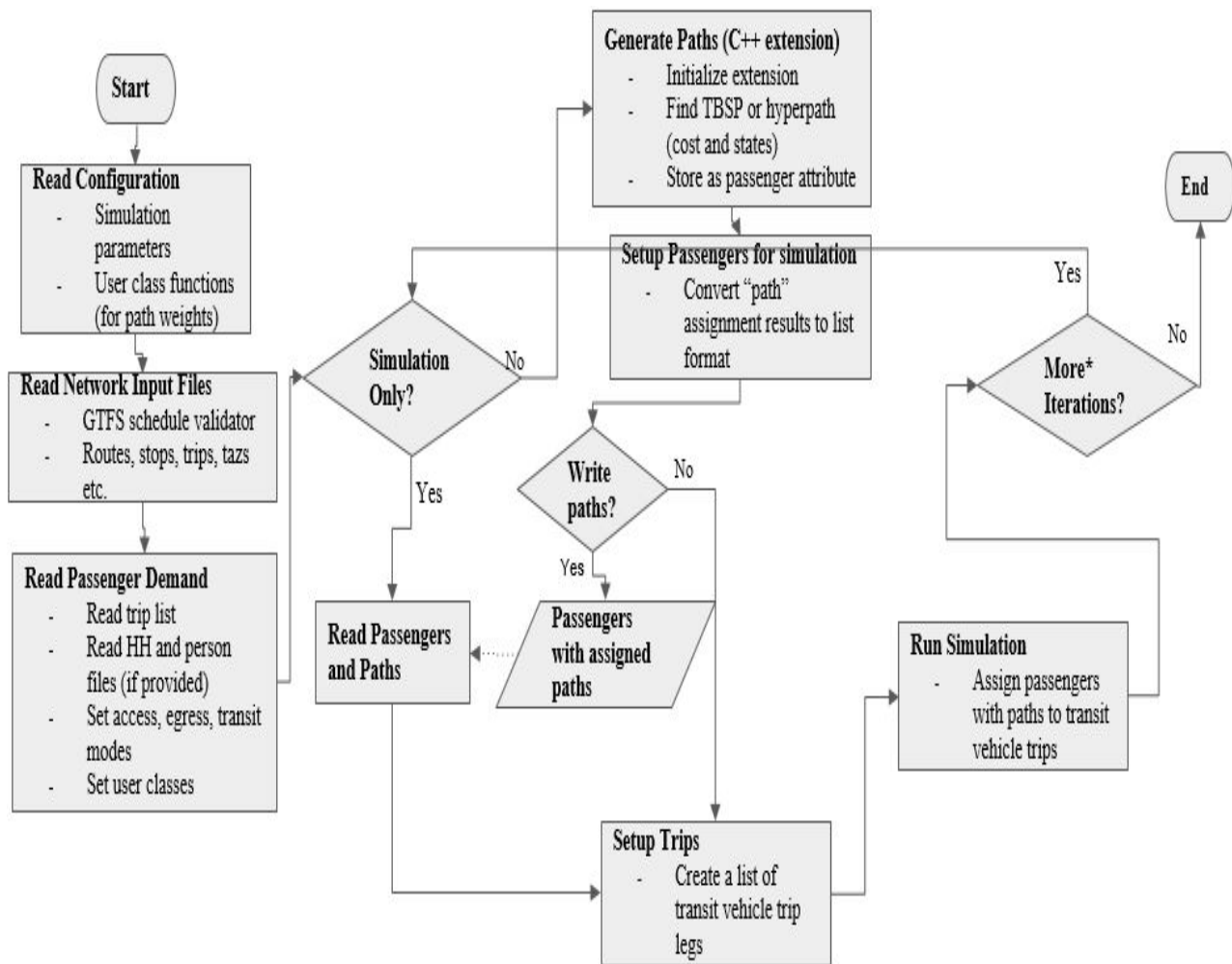


Figure 3.1: Flow Chart of the Fast-Trips codebase

## 3.3 Inputs

### 3.3.1 Transit Network

The input networks are stored in the GTFS-PLUS format. GTFS-PLUS is a GTFS-based transit network data standard suitable for dynamic transit modeling. A GTFS-PLUS transit network consists of required and optional data files that together describe a transit network service.

**Table 3.1: GTFS-PLUS transit network required files**

Filename	Description
<b>agency.txt</b>	Transit agencies (ID, name, URL, agency time-zone etc)
<b>calender.txt</b>	Transit schedule calendar
<b>stops.txt</b>	Transit stops and stations (stop ID, Name, Lat, Lon etc)
<b>stop_times.txt</b>	Transit trip stop times (trip ID, arrival/depart. time etc)
<b>routes.txt</b>	Transit routes (Route ID, route short/long name, type etc)
<b>trips.txt</b>	Transit vehicle trips (Trip ID, Route ID, Service ID, Shape ID etc)
<b>transfers.txt</b>	Transfer links (transfer type, min. transfer time)
<b>routes_ft.txt</b>	Information about route modes (i.e. local bus, express bust), fare class, proof of payment against the route IDs
<b>vehicles_ft.txt</b>	Transit vehicles' details (seated capacity, standing capacity, max. speed, acceleration, deceleration, dwell time etc.)
<b>trips_ft.txt</b>	Information about vehicle types (i.e. Bus, LRT/BRT, Commuter rail) against the trip/route IDs
<b>transfers_ft.txt</b>	Includes transfers' distances from one stop to another in a transit network
<b>walk_access_ft.txt</b>	Includes walk access dist. from origin TAZ to a boarding stop, and walk egress dist. from alighting stop to destination TAZ in a network

The GTFS-PLUS file attributes on the red shaded color above are available and found from the GTFS dataset. Transitfeed is used for parsing and validating the GTFS component. The attributes on the blue shaded color are required to be added to turn GTFS into GTFS-PLUS.

A GTFS-PLUS transit network MAY include the following files:

**drive\_access\_ft.txt** → drive access links

**drive\_access\_points\_ft.txt** → park and ride access links; must be included if provide drive access links

**bike\_access\_ft.txt** → walk access links

**shapes.txt** → transit route shape points

**stops\_ft.txt** → additional transit stop and station information

**stop\_times\_ft.txt** → additional transit trip stop time information

**fare\_attributes\_ft.txt** → fare attributes

**fare\_rules.txt** → fare rules

**fare\_transfer\_rules\_ft.txt** → fare transfer rules

**fare\_periods\_ft.txt** → additional fare rules

### 3.3.2 Passenger Demand

The input demands are stored as Dyno-Demand format which is travel demand data standard suitable for dynamic network modeling.

**Table 3.2: A Dyno-Demand dataset requirements**

Filename	Description
<b>trip_list.txt</b>	Trip attributes of each person (person ID, O/D TAZ, person trip ID, mode, purpose, departure time, arrival time, Time Target, VOT)

**Table 3.3: A Dyno-Demand dataset optional files**

Filename	Description
<b>person.txt</b>	Person attributes (person ID, age, gender, work status, multiple jobs, transit pass, disability etc.) against the HH IDs
<b>household.txt</b>	Household attributes (HH vehicles, HH income, HH size, HH workers, HH preschool/grad school/high school, HH elders etc.) against the HH IDs

### 3.3.3 Configuration Parameters

The configuration files are parsed by python's ConfigParser module and therefore adhere to that format, with two possible sections: *fasttrips* and *pathfinding*.

**Table 3.4: Configuration options for *fasttrips***

Option Name	Type	Default
bump_buffer	float	5
bump_one_at_a_time	bool	False
capacity_constraint	bool	False
create_skims	bool	False
debug_num_trips	int	-1
debug_trace_only	bool	False
fare_zone_symmetry	bool	False
max_iterations	int	1
number_of_processes	int	0

Option Name	Type	Default
output_passenger_trajectories	bool	True
output_pathset_per_sim_iter	bool	False
prepend_route_id_to_trip_id	bool	False
simulation	bool	True
skim_start_time	string	5:00
skim_end_time	string	10:00
debug_output_columns	bool	False
skip_person_ids	string	'None'
trace_ids	string	"None"

**Table 3.5: Configuration options for *pathfinding***

Option Name	Type	Default
bump_buffer	float	5
bump_one_at_a_time	bool	False
capacity_constraint	bool	False
create_skims	bool	False
debug_num_trips	int	-1
debug_trace_only	bool	False
fare_zone_symmetry	bool	False
max_iterations	int	1
number_of_processes	int	0

Option Name	Type	Default
bump_buffer	float	5
bump_one_at_a_time	bool	False
capacity_constraint	bool	False
create_skims	bool	False
debug_num_trips	int	-1
debug_trace_only	bool	False
fare_zone_symmetry	bool	False
max_iterations	int	1
number_of_processes	int	0

### Overlap Path-Size Penalties

The path size overlap penalty is formulated by Ramming (11) and discussed in Hoogendoorn-Lanser et al. (12) When the pathsize overlap is penalized (pathfinding overlap\_variable is not None), then the following equation is used to calculate the path size overlap penalty:

$$PS_i = \sum_{a \in I_i} \frac{l_a}{L_i} \frac{1}{\sum_{j \in C_{in}} \left(\frac{L_i}{L_j}\right)^\gamma \delta_{aj}}$$

Where,

- $I$  is the path alternative for individual  $n$
- $\Gamma_i$  is the set of legs of path variable  $i$
- $l_a$  is the value of the overlap\_variable for leg a. So it is either 1, the distance or the time of leg a depending of if overlap\_scale\_parameter is count, distance or time, respectively.
- $L_i$  is the total sum of the overlap\_variable over all legs  $l_a$  that make up path alternative  $i$
- $C_{in}$  is the choice set of path alternatives for individual  $n$  that overlap with alternative  $i$
- $\gamma$  is the overlap\_scale\_parameter

- $\delta_{ai} = 1$  and  $\delta_{aj} = 0 \forall j \neq i$





## Chapter 4. Case Study and Results

### 4.1 Introduction

The GTFS-PLUS network data and passenger demand data for Hartford, CT aren't fully available yet. Hence, the San Francisco case study for limited demand data has been run to check the outputs and computational efficiency within various configuration parameters. The San Francisco Bay Area GTFS-PLUS Network was taken as network input. The passenger demand data used in this case were derived from the California Household Travel Survey (CHTS).

### 4.2 Model Inputs

Number of Agencies	29
Number of TAZs	2,266
Number of Stops	61,81
Number of Routes	853
Types of Transit Vehicles	19
Number of Vehicle Trips	36,058
Total number of households	8,086
Total number of persons	20,138

Table  
San

3.6:

Francisco SF-CHAMP Transit Network Data Summary

### 4.3 Run-time Analysis

For simplicity and maintaining a low run-time, only 3 person trips have been into consideration. Only one iteration has been specified for all the simulations. The model was run by changing the various configuration parameters to see run-times for various cases.

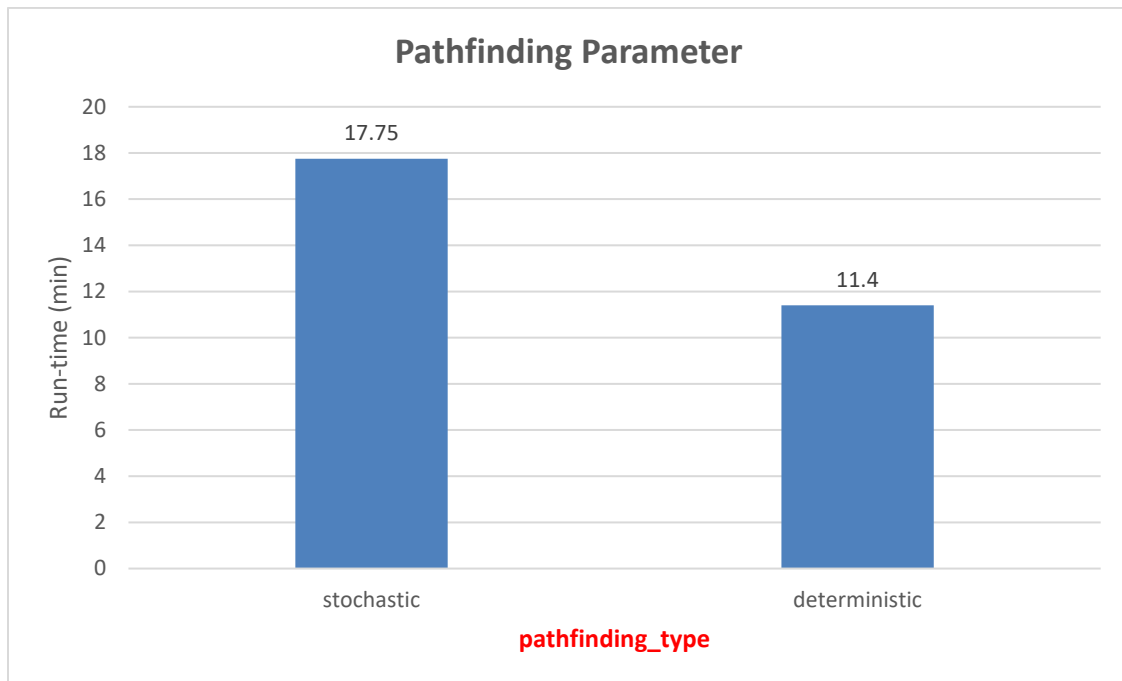


Figure 4.1: Run-time difference between different Pathfinding types

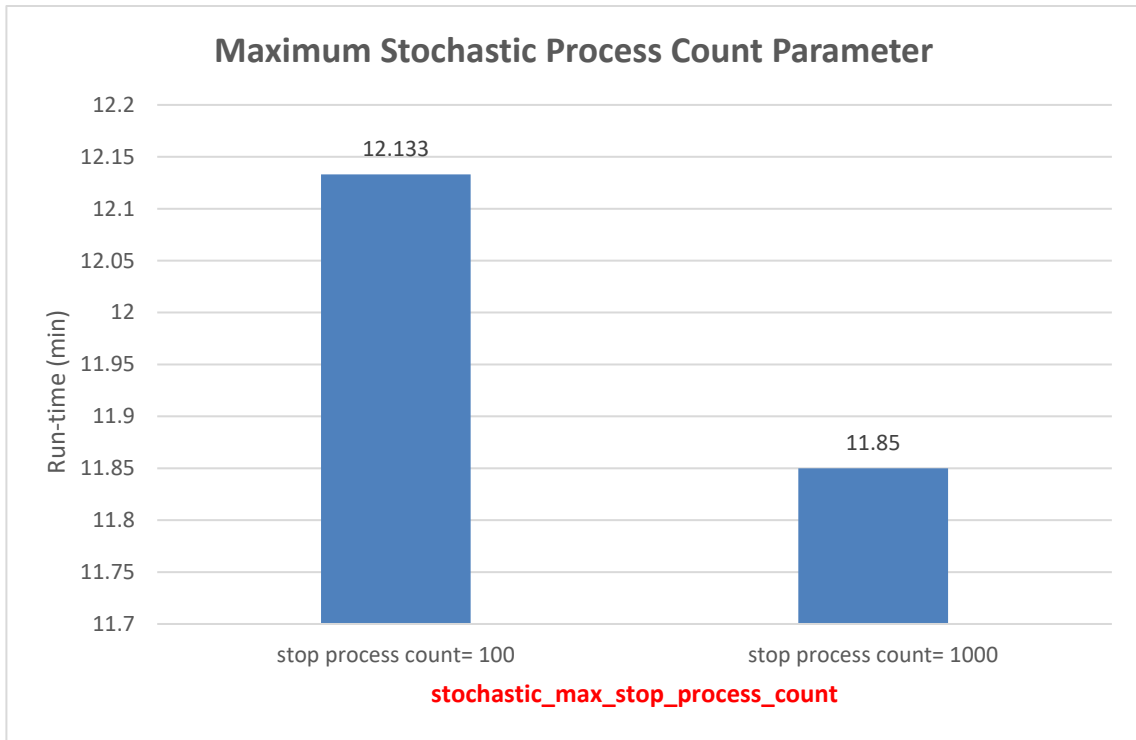


Figure 4.2: Run-time reduction with the increase in Maximum Stop Process Count

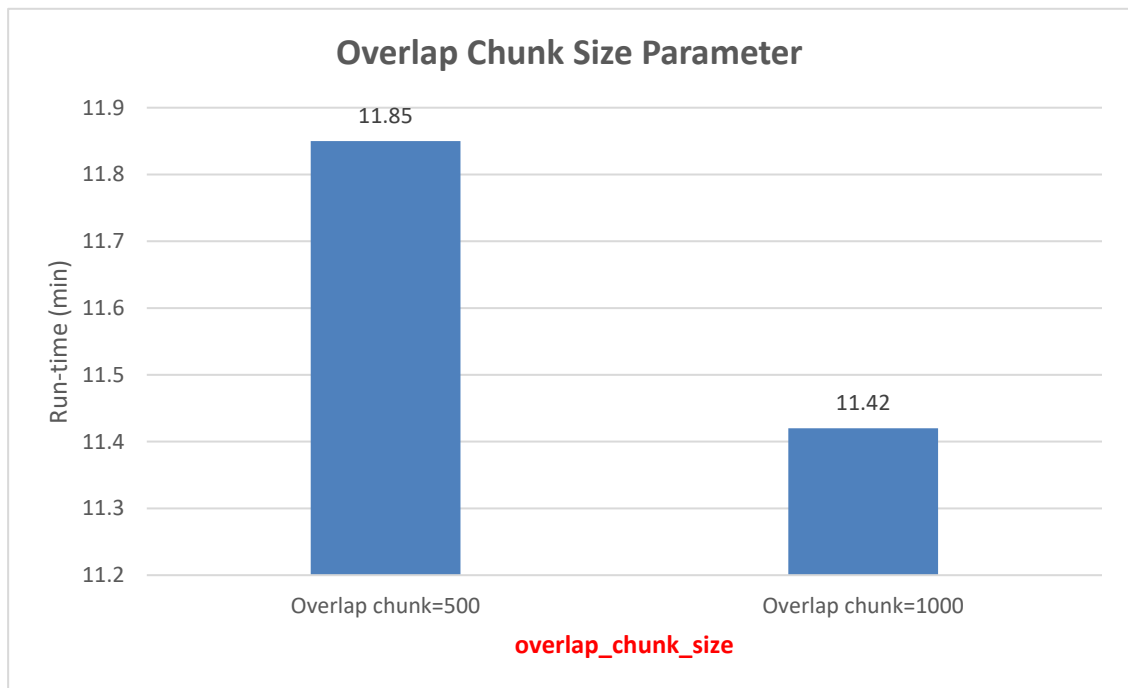


Figure 4.3: Run-time reduction with the increase in Overlap Chunk Size

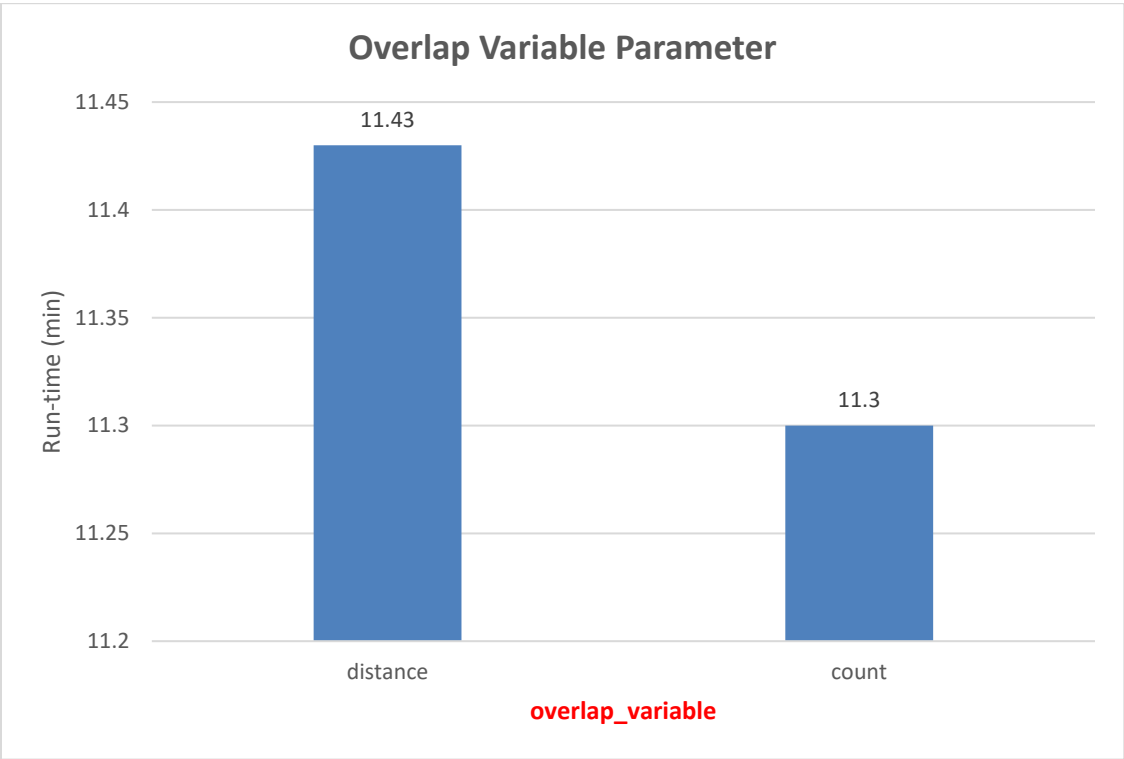


Figure 4.4: Run-time difference between different Overlap Variable types

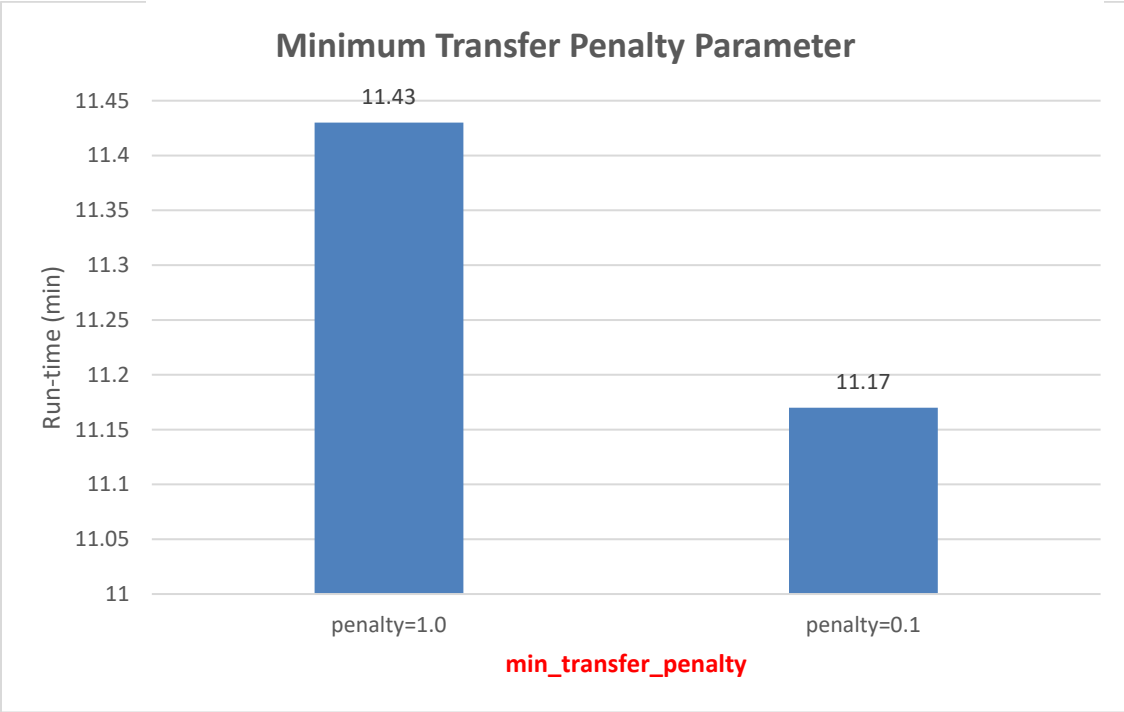


Figure 4.5: Run-time reduction with the change in Minimum Transfer Penalty

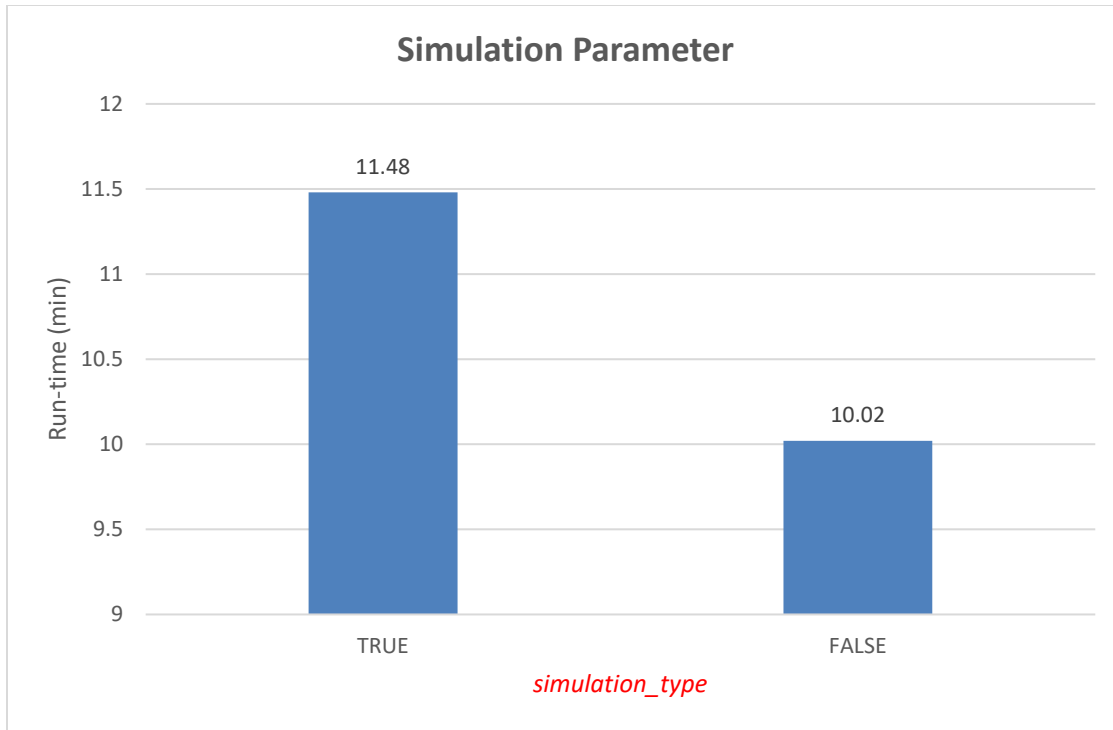


Figure 4.6: Run-time difference between different Simulation types

### 4.3.1 Old Code vs Updated code

The codes in the Fast-Trips have been updated lately. The new codes are producing outputs faster and more efficiently than from the previous codes. The new codes have also resolved some errors that were encountering earlier in the previous code run.

Table 4.1: Differences in outputs between old and updated codes in Fast-Trips

	Old Code	Updated code
<b>Pathsets in default configuration (Stochastic Assignment)</b>	37	4
<b>Run-time (Stochastic)</b>	31 min	17 min
<b>Run-time (Deterministic)</b>	27 min	12 min
<b>Multiple Global Iteration</b>	Error	Error resolved
<b>Pathfinding Iteration Parameter</b>	Takes default value	Can be specified



## Chapter 5. Summary and Conclusions

### 5.1 Summary and Future Work

The goal of this project is to represent the transit users' behavior of Hartford, Connecticut with high resolution by capturing the important phenomena such as crowding, service reliability etc. Data availability issues, which are currently being addressed, prevented a full implementation for the Hartford network. A powerful open-source tool, Fast-Trips is used in this case for simulating transit user route finding. Unlike conventional transit path building models, Fast-Trips doesn't assume a homogenous set of travelers. Different types of riders affect the transit system in different ways. Some are slow to board, some use cash versus a fare-card which makes the bus stop for longer, some may need a spot for their wheelchair. All of these things affect the performance of the transit line. Fast-Trips represents each transit passenger individually, along with their demand attributes and a target origin departure or destination arrival time. Conventional transit passenger route finding methods use a singular set of optimization parameters for all transit riders. The problem with this method is that it results in an all-or-nothing assignment of everybody going between that general origin-destination pair to a singular transit route. Ignoring individual variation makes transit choices unrealistically lumpy. Fast-Trips considers spectrum of perceptions about the route and also reveals that different services are important for different people. Since Fast-Trips models individual riders, one can observe the distribution of experiences rather than the average experience.

Fast-Trips utilizes a trip-based hyperpath model (TBHP) to generate a set of paths with low generalized costs. The TBHP algorithm is a variant of Dijkstra's Algorithm for finding a shortest path with a few variations. TBHP is a stochastic path set generation algorithm because each hyperlink represents a number of actual links which are chosen probabilistically when paths are enumerated. TBHP can be formulated as a frequency-based or schedule-based model, and FAST-Trips applied it to a schedule-based network.

To understand the model outputs and investigate computational efficiency, a San Francisco case study was analyzed within various configuration parameters. The SF-CHAMP GTFS-PLUS network data was taken as network input, and the passenger demand data derived from the California Household Travel Survey (CHTS) as demand input. At first, the model was run for the

default configurations. A sensitivity analysis of the configuration parameters was performed, with the run-time differences observed and documented. The change in pathfinding type from ‘stochastic’ to ‘deterministic’ results in a significant run-time reduction by 36% in producing outputs. The run-times can also be reduced slightly by increasing the overlap chunk size and maximum stop process count. If the simulation type is set as ‘False’ instead of ‘True’, the run-time found to reduce by approximately 13%.

The Hartford GTFS dataset is currently lacking some important attributes (i.e. transfers, fares). These attributes are necessary in simulating transit user route finding using Fast-Trips. Work is ongoing to develop these necessary attribute files in the Hartford GTFS dataset. Further research is necessary to incorporate the additional Fast-Trips network inputs (i.e. routes\_ft.txt, trips\_ft.txt, transfers\_ft.txt, walk\_access\_ft.txt) to modify GTFS into GTFS-PLUS suitable for dynamic transit modeling. The dyno-demand data at disaggregate level of entire Connecticut is currently being synthesized and will be incorporated in the near future.



## References

1. Khani, A., Bustillos, B., Noh, H., Chiu, Y. C., & Hickman, M. (2014a). Modeling Transit and Intermodal Tours in a Dynamic Multimodal Network. In Transportation Research Board 93rd Annual Meeting (No. 14-5567).
2. Khani, A., Hickman, M., & Noh, H. (2014b). Trip-Based Path Algorithms Using the Transit Network Hierarchy. *Networks and Spatial Economics*, 1-19.
3. Khani, A. (2013). Models and Solution Algorithms for Transit and Intermodal Passenger Assignment (Development of FAST-TRIPs Model).
4. SHRP 2 C10(B) (2014). Partnership to Develop an Integrated Advanced Travel Demand Model with Mode Choice Capability and Fine-Grained, Time-Sensitive Networks. Transportation Research Board. Available at <http://apps.trb.org/cmsfeed/TRBNetProjectDisplay.asp?ProjectID=2828>. Accessed August 2014
5. Spiess, H., M. Florian (1989). Optimal strategies: A new assignment model for transit networks. In *Transportation Research Part B*, Vol. 23, pp. 83-102.
6. Constantin, I., D. Florian (2016). Journey Levels in Strategy-Based Transit Assignment: Modeling Integrated Transit Fares and More. Presented at the 95th Annual Meeting of the Transportation Research Board, Washington, D.C.
7. PTV Group, *PTV Visum Functions*, <http://vision-traffic.ptvgroup.com/en-us/products/ptv-visum/functions/> Accessed July 28, 2016.
8. Caliper, *TransCAD Transportation Planning Software: Planning and Travel Demand with TransCAD*. <http://www.caliper.com/TCTravelDemand.htm>. Accessed July 28, 2016.
9. Zorn, L., E. Sall, D. Wu (2012). Incorporating crowding into the San Francisco Activity-Based Travel Model. In *Transportation*, Vol. 39, No. 4, pp. 755-771.
10. Fosgerau, M., E. Frejinger, A. Karlstrom (2013). A Link Based Network Route Choice Model with Unrestricted Choice Set. In *Transportation Research Part B: Methodological*, Vol 56, pp. 70-80.
11. Ramming, M.S (2002). *Network Knowledge and Route Choice*. Ph.D. Thesis. Massachusetts Institute of Technology, Cambridge, Mass.
12. Hoogendoorn-Lanser, S., Nes, R.V., & Bovy, P. (2005). *Path Size Modeling in Multimodal Route Choice Analysis*. 27 In Transportation Research Record: Journal of the transportation Research Board, No 1921, 28 Transportation Research Board of the National Academies, Washington, D.C., pp. 27-34.