U.S. Department
of Transportation
**United States
Coast Guard**

Commandant
United States Coast Guard

2100 Second Street, S.W.
Washington, DC 20593-0001
Staff Symbol: G-MOC
Phone: 202-267-1464

COMDTPUB P16700.4
NVIC

NAVIGATION AND VESSEL INSPECTION CIRCULAR NO.

Subj: LOADING CONSIDERATIONS FOR EXISTING INLAND TANK BARGES

Ref: (a) ABS Rules for Building and Classing Steel Vessels for Service on Rivers and Intracoastal Waterways
(b) Hughes, O. F., *Ship Structural Design*, SNAME, Jersey City, NJ, Second Edition, 1988

1. PURPOSE. This Circular advises of the potential for inland tank barges to buckle under certain non-uniform loading conditions and introduces a methodology for calculating the collapse strength of a barge. With the assistance of a naval architect or professional engineer, a tank barge owner or operator can use the enclosed guidance to calculate the collapse strength of a barge. By knowing the collapse strength, loading guidance can be developed to maintain safe deck stress levels and suit operational needs. This guidance applies to tank barges loading/discharging in still water conditions as well as barges operating on lakes, bays and sounds (LBS) and limited coastwise service (such as offshore routes within the boundary line along the Gulf coast). Development and use of written loading guidance based on the information presented in this NVIC will substantially reduce the risk of catastrophic structural failure due to buckling of the deck structure.

2. DIRECTIVES AFFECTED. None.

3. APPLICABILITY.

DISTRIBUTION – SDL No. 135

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
A																										
B		2	10		1			1							1		5									30
C					*							1	*													
D	1	1		1*							1	*														
E														2	2											
F	1		1								1															
G																										
H																										

*NON-STANDARD DISTRIBUTION: (See page 6.)

a. This guidance is for tank barges, regulated under Title 46, Code of Federal Regulations (46 CFR) Subchapter D, having the following characteristics:

1. Registered length between 175 and 300 feet;
2. Integral cargo tanks; single or double hull; flush or raised-trunk decks;
3. Designed and constructed in accordance with reference (a) or similar standards; and
4. Subject to "abnormal" midship bending stresses.

b. *"Abnormal" bending stresses* are those caused by extreme distribution of cargo, such as loaded midship tanks with empty end tanks. This includes intermediate conditions of loading or discharging, even at terminals in calm water conditions. However, it does not include barges which are slightly trimmed to facilitate cargo operations.

c. *"Abnormal" bending stresses* can also be wave-induced. The Coast Guard has traditionally allowed tank barges designed and constructed to ABS River Rules to operate on LBS and near-coastal routes. However, the wave-induced stresses encountered on some of these exposed or partially-exposed waters may be beyond that intended by the ABS River Rules. Therefore, operation in these waters, regardless of loading practices, may warrant a structural evaluation.

d. When determining whether this guidance applies to a particular vessel, its structural condition must also be considered. A vessel's ability to withstand bending stresses, even under normal loading or operating conditions, will be degraded if there is substantial corrosion or significantly deformed stiffeners within the 40-percent midship length. It is recognized that some degree of corrosion and stiffener deformation exists on the majority of inland tank barges. A single deformed stiffener or localized corrosion is not necessarily a concern, but such conditions over broad areas of the midship deck structure will reduce the integrity of the barge structure. Therefore, these conditions must be considered when calculating collapse strength and developing loading guidance.

4. BACKGROUND.

a. In March and May, 1996, two barges, with the characteristics described above, experienced catastrophic buckling of their decks, resulting in rupture of several cargo tanks and causing significant pollution incidents. In support of the Formal Boards of Marine Investigation for these casualties, the U.S. Coast Guard Marine Safety Center (MSC) launched an intensive engineering analysis of these failures. This analysis revealed that sole compliance with current regulations pertaining to structural design and cargo loading may not adequately guard against catastrophic buckling failures.

b. Current regulations do not address cargo load distribution for tank barges less than 300 feet in length. According to 46 CFR 32.60-1, certificated tank barges on inland routes must meet the design and construction standards required by ABS. Designs meeting these standards have a minimum deck plating thickness requirement to deter buckling. However, these rules do not impose specific minimum requirements for hull section modulus, which relates to deck stress and ability to resist buckling. ABS developed the "River Rules" for barges intended to operate in comparatively smooth waters and loaded in a condition that would limit abnormally severe hull stresses.

Certain tank barges however, require more operational flexibility than permitted by this “still water/uniform loading” limitation. For such barges, cargo loading procedures and load distribution require further consideration.

c. As part of their analysis, the MSC conducted a comprehensive study to determine the collapse strength of barges built to the inland rules, using the “Hughes methodology” of reference (b). They analyzed 25 tank barge designs representative of the inland fleet, 22 longitudinally framed and 3 transversely framed, ranging in length from 150 feet to 300 feet, both single and double hull, flush and raised-trunk decks. The average collapse strength calculated for the longitudinally framed barges was 17,000 psi, with a standard deviation of approximately 700 psi. The standard deviation was very low because the design parameters relative to buckling strength varied very little. The estimated collapse strength calculated for the transversely framed barges was substantially lower, ranging from 4,000 to 6,000 psi.

d. Because initial distortion of a stiffener reduces its collapse strength, the MSC also surveyed a typical in-service tank barge. Eighty-four points on 12 stiffeners within one midship tank were measured. Statistically, the findings indicate that there is an 80-percent chance that the initial deflection will be $1/8^{th}$ inch or less, and only a 5-percent chance that the deflection will approach ¼ inch. In their structural analyses, the MSC assumed an initial stiffener deflection of $1/8^{th}$ inch. This reduced the average collapse strength for a longitudinally-framed barge from 20,000 psi to 17,000 psi.

e. Although corrosion can also play a significant role in the strength of a barge, the MSC study assumed that deck scantlings remained at their original “as built” thickness. The average collapse strength of 17,000 psi was calculated using this assumption. When corrosion of scantlings is factored into the equation, the reduction of strength can be significant. With the deck plating and deck longitudinal stiffeners corroded 25%, the ultimate strength is reduced to approximately 14,000 psi.

f. The Hughes methodology assumes fully-effective attachment of structural components (i.e. continuously-welded stiffeners). However, it was common practice in the past to use serrated stiffeners in barge construction, and intermittent welding is still the normal construction practice. To determine the effect of this construction, the MSC contracted with the U.S. Naval Academy to conduct collapse tests on scaled model panels of typical deck construction. Six panels were tested, four of which were constructed with serrated stiffeners or intermittent welding. The results suggest that actual collapse strength of a barge constructed with serrated stiffeners and/or intermittent welds is reduced by approximately ten percent below predictions by the Hughes methodology.

g. A review of similar structural failures over the past ten years has not identified any singular cause or deficient design factor. However, it is apparent that questionable cargo loading practices were significant contributing causes. It is important to note that two of these failures occurred on barges as small as 168 feet in length. Failures seem to be independent of vessel size or age. The only general conclusions from this analysis thus far are that deck buckling failure is a risk to be taken seriously throughout the entire inland tank barge fleet, and the risk can be reduced through careful cargo loading practices.

h. The results of this comprehensive study found that inland tank barges are susceptible to buckling under moderate to extreme non-uniform loading conditions, even in still water. This vulnerability is further exacerbated when wave-induced stresses are introduced.

i. For those interested, a copy of the formal report of the MSC findings entitled "Ultimate Strength Analysis of Inland Tank Barges" will soon be available for a nominal fee through National Technical Information Services at (703) 487-4650 (phone) or (703) 321-8547 (fax).

5. DISCUSSION.

a. The Marine Safety Center findings have concluded that it is possible to exceed the buckling strength of an inland tank barge while operating within existing regulatory limits. Although casualty history suggests that the probability of a catastrophic tank barge buckling failure is minimal, the environmental consequences are quite considerable. Although small, this probability presents an undesirable risk. However, this risk is easily managed using cargo loading practices to keep deck stress within safe limits.

b. To assist owners/operators in evaluating the collapse strength limits of their barges, enclosure (1) presents the Hughes methodology of reference (b) and enclosure (2) provides example calculations for a typical longitudinally-framed tank barge. It should be noted that this methodology only determines the collapse limits of the hull structure. To ensure that the barge is loaded within acceptable limits, the bending moment stresses from specific loading conditions must be calculated. It is recommended that barge owners and operators retain the services of a naval architect or professional engineer to calculate bending moment stresses and develop appropriate loading guidance. This will enable the barge to be loaded within acceptable stress limits and meet the operational needs of the owner or operator.

c. It is emphasized that the material condition of a barge is critical to its structural integrity, including corrosion and distortion of stiffeners (i.e. latent distortion in still water while unloaded). The Hughes methodology allows latent distortion of stiffeners to be factored into the calculations. Also, corrosion can be factored in by using average "existing" scantling dimensions rather than "as built" dimensions.

d. The Hughes methodology also assumes fully effective attachment between structural members (i.e. continuously-welded connections). Serrated stiffeners or intermittent welding, or cracked welds will reduce the actual collapse strength below levels predicted by the Hughes methodology.

6. IMPLEMENTATION.

a. All owners and operators of tank barges falling within the applicability of this NVIC are advised to conduct a structural evaluation to determine the longitudinal strength limits of their barges. The evaluation should consider the loading practices common to each individual barge. If the compressive deck stress encountered during common loading practices approaches or exceeds the calculated collapse strength, loading guidance should be developed to limit hull stress. Again, the assistance of a naval architect or professional engineer is strongly recommended for this process.

b. Enclosure (1) presents the Hughes methodology for evaluating the collapse strength of a barge, and includes a sample calculation. Other accepted methods for calculating collapse strength may be used.

c. These guidelines are minimum recommendations. Consideration should be given to more stringent safeguards if the condition or operation of the vessel warrants. This would include the case of excessive corrosion over broad areas of the deck structure, structural deformation of deck framing (greater than 1/8th inch over several transverse frames), and/or routine operation in wave conditions exceeding four feet. Also, the impact of certain construction techniques, such as serrated stiffeners and intermittent welding, should be considered. It is the responsibility of the owner or operator to determine an appropriate factor of safety based on these considerations.

7. ACTION.

a. Owners and operators of barges with the characteristics listed in paragraph 3 above should evaluate their current loading operations.

b. Owners and operators may continue non-uniform loading practices provided that a material condition survey of the barge verifies its structural integrity (particularly with respect to corrosion and deformation) and loading calculations indicate that bending stresses will remain within the barge's collapse strength limits.

c. Where operational conditions require careful attention to cargo loading and distribution, it is recommended that the personnel responsible for cargo loading and unloading be provided with clear, carefully-written (considering the end-user) loading guidance. Tank barge owners and operators should ensure that their tankermen completely understand the cargo loading/unloading and distribution procedures contained in this written guidance. This information should be kept readily available aboard the barge (or towing vessel) for use during cargo operations.

d. Officers in Charge, Marine Inspection (OCMI) shall bring this guidance to the attention of appropriate individuals of the tank barge industry within their zones.

e. Adoption and proper application of a loading procedure is the responsibility of the vessel owner. Coast Guard review and approval of strength calculations and cargo load configurations is not required.

Encl: (1) Methodology for Computing Deck Collapse Strength
(2) Sample calculation for typical barge

Non-Standard Distribution:

B:a Commandant (G-MOC), Commandant (G-MSE), CG Marine Safety Center (5)

C:e New Orleans (90); Hampton Roads (50); Baltimore (45); San Francisco, Puget Sound (40); Philadelphia, Port Arthur, Honolulu (35); Miami, Houston, Mobile, Long Beach, Morgan City, Portland OR (25); Jacksonville (20); Boston, Portland ME, Charleston, Galveston, Anchorage (15); Cleveland (12); Louisville, Memphis, Paducah, Pittsburgh, St. Louis, Savannah, San Juan, Tampa, Buffalo, Chicago, Detroit, Duluth, Milwaukee, San Diego, Juneau, Valdez (10); Providence, Huntington, Wilmington, Corpus Christi, Toledo, Guam, Sault Ste. Marie (5).

C:m New York (70); Sturgeon Bay (4).

D:d Except Baltimore, Moriches and Grand Haven.

D:1 CG Liaison Officer MILSEALIFTCOMD (Code N-7CG), CG Liaison Officer RSPA (DHM-22), CG Liaison Officer MARAD (MAR-742), CG Liaison Officer JUSMAGPHIL, CG Liaison Officer ABS, Maritime Liaison Office Commander U.S. Naval Forces Central Command (1).

NOAA Fleet Inspection Officer (1).

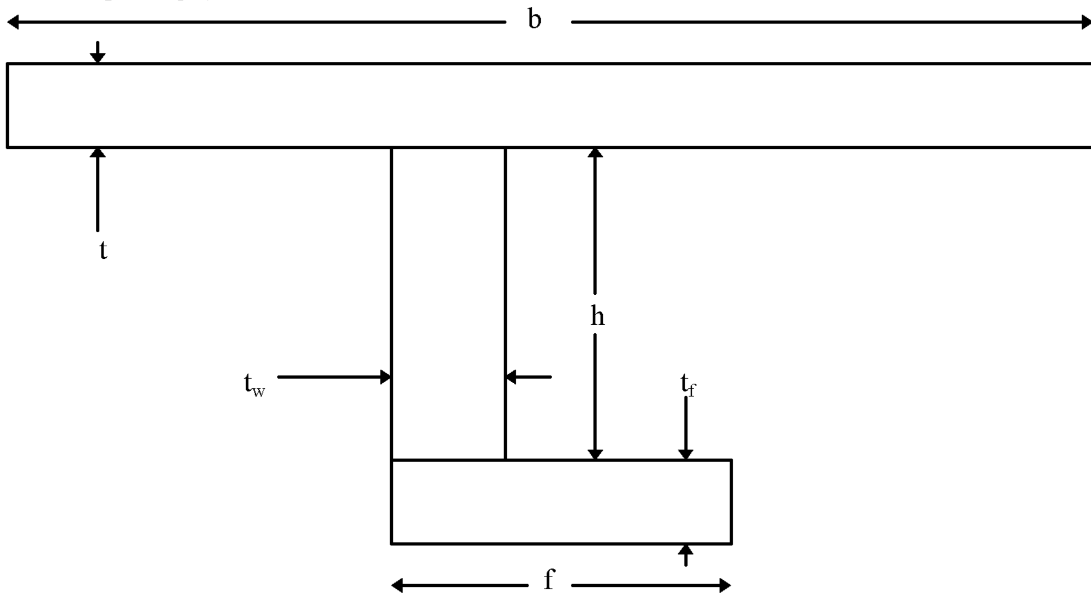
U.S. Merchant Marine Academy (1).

Methodology for Computing Deck Collapse Strength

A. Calculations for Longitudinally Framed Barges

Required Input Information:

Variable	Definition
t	Plate thickness (in)
h	Web height (in)
t_w	Web thickness (in)
f	Flange breadth (in)
t_f	Flange thickness (in)
a	Transverse frame spacing [distance between transverses] (in)
b	Longitudinal stiffener spacing (in



Parameters for Strength Analysis:

(1) Plate flexural rigidity $D = 2{,}747{,}000 * t^3$ (5)

(2) Slenderness parameter $\beta = 0.03367 * \frac{b}{t}$ (6)

(3) $$C_r = \frac{1}{1 + 0.4 * \left(\frac{t}{t_w}\right)^3 * \frac{h}{b}}$$ (7)

(4) $$I_{sp} \approx h^2 * \left[\left(t_f * f\right) + \frac{t_w * h}{3}\right]$$

$$J = 0.33 * \left(t_w{}^3 * h + t_f{}^3 * f \right)$$

$$\varepsilon = 1 + \frac{2.75}{\beta^2}$$

$$T = 0.25 * \left(2 + \varepsilon - \sqrt{\varepsilon^2 - \frac{10.4}{\beta^2}} \right)$$

Combined Section (Plate & Stiffener):

Member	Thickness of member (t_i)	Length of member	Area of member (A_i)	Distance from centroid to deck plate (d_i)	A_i*d_i	$A_i*d_i^2$	Moment of Inertia (I_i)
Flange	t_f	f	$A_f= t_f* f$	$d_f = t + h + \frac{t_f}{2}$	$=A_f*d_f$	$=A_f*d_f*d_f$	$= \frac{f * t_f^3}{12}$
Web	t_w	h	$A_w= t_w* h$	$d_w = t + \frac{h}{2}$	$=A_w*d_w$	$=A_w*d_w*d_w$	$= \frac{t_w * h^3}{12}$
Plate	t	b	$A= t* b$	$d = \frac{t}{2}$	$=A*d$	$=A*d*d$	$= \frac{b * t^3}{12}$
Variables	-	-	$\mathbf{A_c} = \sum A_i$		$\mathbf{B_c} = \sum A_i*d_i$	$\mathbf{C_c} = \sum A_i*d_i^2$	$\mathbf{I_c} = \sum I_i$

Calculations required after filling out above chart:

(1) $$N_c = \frac{B_c}{A_c}$$

(2) $$I_{bl} = I_c + C_c$$

(3) $$I_N = I_{bl} - \left(A_c * N_c^{\ 2}\right)$$

(4) $$y_p = N_c - \frac{t}{2}$$

(5) $$y_f = N_c - t - h - \frac{t_f}{2}$$

(6) $$\rho = \sqrt{\frac{I_N}{A_c}}$$

Stiffener Only:

Member	Thickness of member (t_i)	Length of member	Area of member (A_i)	Distance from centroid to deck plate (d_i)	A_i*d_i	$A_i*d_i^2$	Moment of Inertia (I_i)
Flange	t_f	f	$A_f = t_f * f$	$d_f = h + \frac{t_f}{2}$	$=A_f*d_f$	$=A_f*d_f*d_f$	$= \frac{f*t_f^3}{12}$
Web	t_w	h	$A_w = t_w * h$	$d_w = t + \frac{h}{2}$	$=A_w*d_w$	$=A_w*d_w*d_w$	$= \frac{t_w*h^3}{12}$
Variables	-	-	$\mathbf{A_{st}} = \sum A_i$		$\mathbf{B_{st}} = \sum A_i*d_i$	$\mathbf{C_{st}} = \sum A_i*d_i^2$	$\mathbf{I_{st}} = \sum I_i$

Calculations required after filling out above chart:

(1) $$N_{st} = \frac{B_{st}}{A_{st}}$$

(2) $$I_{bl\,st} = I_{st} + C_{st}$$

(3) $$I_{N_{st}} = I_{bl_{st}} - \left(A_{st} * N_{st}^{\ 2}\right)$$

(4) $$\Delta p = A_{st} * \left(N_{st} + \frac{t}{2}\right) * \left(\frac{1}{A_{tr}} - \frac{1}{A_c}\right)$$

(5) $$\bar{y} = \frac{A_w * \frac{t_w}{2} + A_f * \frac{f}{2}}{A_{st}}$$

(6) $$I_{sz} = \frac{t_f * f^3}{12} + A_f * \left(\frac{f}{2} - \bar{y}\right)^2 + \frac{h * t_w^{\,3}}{12} + A_w * \left(\bar{y} - \frac{t_w}{2}\right)^2$$

Transformed Section:

Member	Thickness of member (t_i)	Length of member	Area of member (A_i)	Distance from centroid to deck plate (d_i)	A_i*d_i	$A_i*d_i^2$	Moment of Inertia (I_i)
Flange	t_f	f	$A_f = t_f * f$	$d_f = t + h + \frac{t_f}{2}$	$=A_f*d_f$	$=A_f*d_f*d_f$	$= \frac{f * t_f^3}{12}$
Web	t_w	h	$A_w = t_w * h$	$d_w = t + \frac{h}{2}$	$=A_w*d_w$	$=A_w*d_w*d_w$	$= \frac{t_w * h^3}{12}$
Plate	t	b	$A = t * b * T$	$d = \frac{t}{2}$	$=A*d$	$=A*d*d$	$= \frac{(T*b)*t^3}{12}$
Variables	-	-	$\mathbf{A_{tr}} = \sum A_i$		$\mathbf{B_{tr}} = \sum A_i*d_i$	$\mathbf{C_{tr}} = \sum A_i*d_i^2$	$\mathbf{I_{tr}} = \sum I_i$

Calculations required after filling out above chart:

(1) $$N_{tr} = \frac{B_{tr}}{A_{tr}}$$

(2) $$I_{bl_{tr}} = I_{tr} + C_{tr}$$

(3) $$I_{N\,tr} = I_{bl\,tr} - \left(A_{tr} * N_{tr}^{\ 2}\right)$$

(4) $$y_{p\,tr} = N_{tr} - \frac{t}{2}$$

(5) $$y_{f\,tr} = N_{tr} - t - h - \frac{t_f}{2}$$

(6) $$\rho_{tr} = \sqrt{\frac{I_{N_{tr}}}{A_{tr}}}$$

Mode I: Compression Failure of the Stiffener:

Collapse Strength:

$$\text{ULT}_I = R_I * \sigma_{FI}$$

Where: For "m" = 1,2,3,4,5: Elastic Tripping Stress

(1) $$\sigma_{a,T} = \frac{1}{I_{sp} + \frac{2 * C_r * b^3 * t}{97.41}} \left(11{,}538{,}462 * J + \frac{m^2 * 9.87}{a^2} * 30{,}000{,}000 * I_{sz} * h^2 + \frac{4 * D * C_r}{9.87 * b} * \left(\frac{a^2}{m^2} + b^2 \right) \right)$$

(2) $$\sigma_{FI} = \min\left\{ (34{,}000), \left(\sigma_{a,T_{\min}} \right) \right\}$$

(3) $$\lambda = \frac{a}{3.14 * \rho} * \sqrt{\frac{\sigma_{FI}}{30{,}000{,}000}}$$

(4) $$\eta = \frac{-0.125 * y_f}{\rho^2}$$

(5) $$\zeta = 1 + \left(\frac{1 + \eta}{\lambda^2} \right)$$

(6) $$R_I = \frac{\zeta}{2} - \sqrt{\frac{\zeta^2}{4} - \frac{1}{\lambda^2}}$$

Mode II: Compression Failure of the Plating:

Collapse Strength:

$$\mathrm{ULT_{II}} = \frac{A_{tr}}{A_c} * R_{II} * \sigma_{FII}$$

Where:

(1) $$\sigma_{au,wc} = \frac{21{,}420}{\beta^2}$$

(2) $$\sigma_{au} = 34{,}000 * (T - 0.1)$$

(3) $$\sigma_{ay_u} = \frac{b}{a} * \sigma_{au} + \left(1 - \frac{b}{a}\right) * \sigma_{au,wc}$$

(4) $$\sigma_{FII} = \frac{T - 0.1}{T} * 34{,}000 * \left(1 - \frac{1{,}000}{\sigma_{ay_u}}\right)$$

(5) $$\lambda_{tr} = \frac{a}{3.14 * \rho_{tr}} * \sqrt{\frac{\sigma_{FII}}{30{,}000{,}000}}$$

(6) $$\eta_{tr} = \frac{0.125 * y_{ptr}}{\rho_{tr}^{\ 2}}$$

(7) $$\eta_{ptr} = \frac{\Delta p * y_{ptr}}{\rho_{tr}^{\ 2}}$$

(8) $$\zeta_{II} = \frac{1}{1 + \eta_{ptr}} + \frac{1 + \eta_{ptr} + \eta_{tr}}{(1 + \eta_{ptr}) * \lambda_{tr}^{\ 2}}$$

(9) $$R_{II} = \frac{\zeta_{II}}{2} - \sqrt{\frac{\zeta_{II}^{\ 2}}{4} - \frac{1}{(1 + \eta_{ptr}) * \lambda_{tr}^{\ 2}}}$$

B. Calculations for Transversely Framed Barges

Required Input Information:

Variable	Definition
t	Plate thickness (in)
a	Transverse frame spacing [distance between transverses] (in)
b	Panel width [distance between longitudinal support, typically bulkheads or sideshells] (in)
δ	Initial deflection (in)

Collapse Strength Calculations:

$$ULT_T = \frac{a}{b} * \left(\sigma_{a,u}\right)_L + \left(1 - \frac{a}{b}\right) * \left(\sigma_{a,u}\right)_{wc}$$

$$ULT_T = \frac{a}{b} * \left(\sigma_{a,u}\right)_L + \left(1 - \frac{a}{b}\right) * \left(\sigma_{a,u}\right)_{wc}$$

$$\beta = \frac{a}{t} * 0.0337$$

$$\left(\sigma_{a,u}\right)_L = 0.25 * \left(1.6 + \zeta - \sqrt{\zeta^2 - \frac{10.4}{\beta^2}}\right) * \sigma_y$$

$$\zeta = 1 + \frac{2.75}{\beta^2}$$

Example Calculations for a Typical Longitudinally Framed Barge

The following calculations are taken from the second edition of *Ship Structural Design* (SNAME, 1988) by Owen F. Hughes. Exact references specifying the precise location of each equation are made when possible.

The example calculations are for a typical longitudinally framed inland tank barge built to the standards of the ABS Rules for Building and Classing Steel Vessesl for Service on Rivers and Intracoastal Waterways (ABS Inland Rules).

Assumptions:

1. The transverse stiffeners are stiff enough to provide support to the longitudinals without deflecting (Safe assumption for longitudinally framed tank barges built to the standards of the ABS Inland Rules).
2. No lateral loads are placed on the deck (Safe assumption for tank barges without a vapor recovery system).
3. All tanks have "open" venting. No P/V valves are installed.

Sign Convention:

Stiffener Deflection: Negative (-) if deflected toward plating (stiffener-induced failure) & positive (+) if deflected toward stiffener (plate-induced failure).

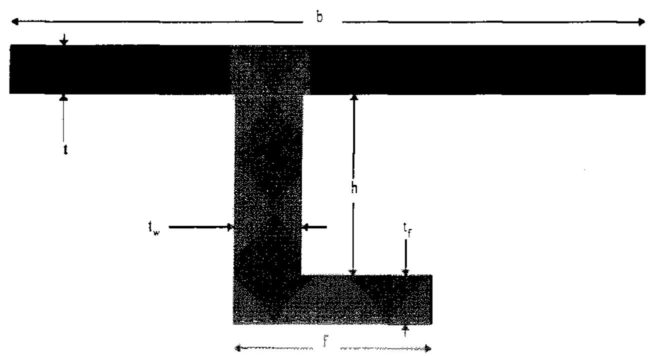
Stresses: Tensile stresses are negative (-) & compressive stresses are positive (+).

Example Calculations for a Longitudinally Framed Barge

(Based on the method outlined in Ship Structural Design: A Rationally-Based, Computer-Aided, Optimization Approach, (SNAME, 1988))

Required Input Information:

Variable	Definition		
t	Plate thickness (in) = 0.313"	E	Modulus of Elasticity = 30,000,000 lb/in²
h	Web height (in) = 3.687"	υ	Poisson's Ratio = 0.3
t_w	Web thickness (in) = 0.313"	σ_{yield}	Material Yield Strength = 34,000 psi
f	Flange breadth (in) = 3"	$\sigma_{transverse}$	Transverse Stress in Deck = 1,000 psi
t_f	Flange thickness (in) = 0.313"		(positive (+) if compressive)
a	Transverse frame spacing (in) = 81"	Δ	Initial Deflection of Stiffeners = +/- 0.125"
b	Longitudinal stiffener spacing (in) = 24"		(positive (+) if deflected toward stiffeners)



Parameters for Strength Analysis:

(1) $$D = \frac{E * t^3}{12 * (1 - \upsilon^2)} = \frac{(30E6)(0.313)^3}{12(1 - 0.3^2)}$$
(eq. 9.1.5)
$$= 83{,}840$$

(2) $$\beta = \frac{b}{t} * \sqrt{\frac{\sigma_{yield}}{E}} = \frac{24}{0.313}\sqrt{\frac{34{,}000}{30E6}}$$
(eq. 9.3.1)
$$= 2.585$$

(3) $$C_r = \frac{1}{1 + 0.4 * \left(\frac{t}{t_w}\right)^3 * \frac{h}{b}} = \frac{1}{1 + 0.4\left(\frac{0.313}{0.313}\right)^3\left(\frac{3.687}{24}\right)}$$
(eq. 13.1.17)
$$= 0.942$$

(4) $$I_{so} \approx h^2 * \left[(t_f * f) + \frac{t_w * h}{3}\right] = 3.687^2\left[(0.313)(3) + \frac{(0.313)(3.687)}{3}\right]$$
(pg 447)
$$= 17.971$$

(5) $$J = 0.33 * (t_w^3 * h + t_f^3 * f) = 0.33\left[(0.313)^3(3.687) + (0.313)^3(3)\right]$$
(eq. 6.1.5)
$$= 0.067$$

(6) $$\varepsilon = 1 + \frac{2.75}{\beta^2} = 1 + \frac{2.75}{(2.585)^2} = 1.411$$
(eq. 14.2.8)

(7) $$T = 0.25 * \left(2 + \varepsilon - \sqrt{\varepsilon^2 - \frac{10.4}{\beta^2}}\right) = 0.25\left[2 + 1.411 - \sqrt{1.411^2 - \frac{10.4}{2.585^2}}\right]$$
(eq. 14.2.8)
$$= 0.688$$

Combined Section (Plate & Stiffener):

Member	Thickness of member (t_i)	Length of member	Area of member (A_i)	Distance from centroid to deck plate (d_i)	A_i*d_i	$A_i*d_i^2$	Moment of Inertia (I_i)
Flange	t_f 0.313	f 3	$A_f = t_f * f$ (0.313)(3) =0.939	$d_f = t + h + \frac{t_f}{2}$ 0.313+3.687 +0.313/2 =4.156	$=A_f*d_f$ (0.939)(4.156) =3.902	$=A_f*d_f*d_f$ $(0.939)(4.156)^2$ =16.219	$=\frac{f*t_f^3}{12}$ $\frac{(3)(0.313)^3}{12}$ =0.008
Web	t_w 0.313	h 3.687	$A_w = t_w * h$ (0.313)(3.687) =1.154	$d_w = t + \frac{h}{2}$ $0.313+\frac{3.687}{2}$ =2.156	$=A_w*d_w$ (1.154)(2.156) =2.488	$=A_w*d_w*d_w$ $(1.154)(2.156)^2$ =5.364	$=\frac{t_w*h^3}{12}$ $\frac{(0.313)(3.687)^3}{12}$ =1.307
Plate	t 0.313	b 24	$A = t*b$ (0.313)(24) =7.512	$d = \frac{t}{2}$ $\frac{0.313}{2}$ =0.156	$=A*d$ (7.512)(0.156) =1.172	$=A*d*d$ $(7.512)(0.156)^2$ =0.183	$=\frac{b*t^3}{12}$ $\frac{(24)(0.313)^3}{12}$ =0.061
Variables			$A_c = \sum A_i$ 0.939+1.154 +7.512 =9.605		$B_c = \sum A_i*d_i$ 3.902+2.488 +1.172 =7.562	$C_c = \sum A_i*d_i^2$ 16.219+5.364 +0.183 =21.766	$I_c = \sum I_i$ 0.008+1.307 +0.061 =1.376

Calculations required after filling out "Combined Section" chart:

(1) $N_c = \frac{B_c}{A_c} = \frac{7.562}{9.605} = 0.787$

(2) $I_{bl} = I_c + C_c = 1.376 + 21.766 = 23.142$

(3) $I_N = I_{bl} - (A_c * N_c^2) = 23.142 - [(9.605)(0.787)^2] = 17.193$

(4) $y_p = N_c - \frac{t}{2} = 0.787 - \frac{0.313}{2} = 0.631$

(5) $y_f = N_c - t - h - \frac{t_f}{2} = 0.787 - 0.313 - 3.687 - \frac{0.313}{2} = -3.370$

(6) $\rho = \sqrt{\frac{I_N}{A_c}} = \sqrt{\frac{17.193}{9.605}} = 1.338$

Stiffener Only:

Member	Thickness of member (t_i)	Length of member	Area of member (A_i)	Distance from centroid to deck plate (d_i)	A_i*d_i	$A_i*d_i^2$	Moment of Inertia (I_i)
Flange	t_f 0.313	f 3	$A_f = t_f * f$ (0.313)(3) = 0.939	$d_f = h + \frac{t_f}{2}$ $3.687 + \frac{0.313}{2}$ = 3.844	$=A_f*d_f$ (0.939)(3.844) = 3.610	$=A_f*d_f*d_f$ (0.939)(3.844)^2 = 13.875	$= \frac{f*t_f^3}{12}$ $\frac{(3)(0.313)^3}{12}$ = 0.008
Web	t_w 0.313	h 3.687	$A_w = t_w * h$ (0.313)(3.687) = 1.154	$d_w = \frac{h}{2}$ 3.687/2 = 1.844	$=A_w*d_w$ (1.154)(1.844) = 2.128	$=A_w*d_w*d_w$ (1.154)(1.844)^2 = 3.924	$= \frac{t_w*h^3}{12}$ $\frac{(0.313)(3.687)^3}{12}$ = 1.307
Variables	-	-	$A_{st} = \sum A_i$ 0.939+1.154 = 2.093		$B_{st} = \sum A_i*d_i$ 3.610+2.128 = 5.738	$C_{st} = \sum A_i*d_i^2$ 13.875+3.924 = 17.799	$I_{st} = \sum I_i$ 0.008+1.307 = 1.315

Calculations required after filling out "Stiffener Only" chart:

(1) $$N_{st} = \frac{B_{st}}{A_{st}} = \frac{5.738}{2.093} = 2.742$$

(2) $$I_{bl\,st} = I_{st} + C_{st} = 1.315 + 17.799 = 19.114$$

(3) $$I_{N_{st}} = I_{bl_{st}} - \left(A_{st} * N_{st}^2\right) = 19.114 - \left[(2.093)(2.742)^2\right] = 3.378$$

(4) $$\bar{y} = \frac{A_w * \frac{t_w}{2} + A_f * \frac{f}{2}}{A_{st}} = \frac{(1.154)\left(\frac{0.313}{2}\right) + (0.939)\left(\frac{3}{2}\right)}{2.093} = 0.759$$

(5) $$I_{s2} = \frac{t_f * f^3}{12} + A_f * \left(\frac{f}{2} - \bar{y}\right)^2 + \frac{h * t_w^3}{12} + A_w * \left(\bar{y} - \frac{t_w}{2}\right)^2 :$$

$$= \frac{(0.313)(3)^3}{12} + (0.939)\left(\frac{3}{2} - 0.759\right)^2 + \frac{(3.687)(0.313)^3}{12} + (1.154)\left(0.759 - \frac{0.313}{2}\right)^2$$

$$= 1.648$$

Transformed Section:

Member	Thickness of member (t_i)	Length of member	Area of member (A_i)	Distance from centroid to deck plate (d_i)	A_i*d_i	$A_i*d_i^2$	Moment of Inertia (I_i)
Flange	t_f 0.313	f 3	$A_f = t_f*f$ (0.313)(3) = 0.939	$d_f = t + h + \frac{t_f}{2}$ 0.313 + 3.687 + 0.313/2 = 4.157	$=A_f*d_f$ (0.939)(4.157) = 3.903	$=A_f*d_f*d_f$ $(0.939)(4.157)^2$ = 16.227	$= \frac{f*t_f^3}{12}$ $\frac{(3)(0.313)^3}{12}$ = 0.008
Web	t_w 0.313	h 3.687	$A_w = t_w*h$ (0.313)(3.687) = 1.154	$d_w = t + \frac{h}{2}$ $0.313 + \frac{3.687}{2}$ = 2.157	$=A_w*d_w$ (1.154)(2.157) = 2.489	$=A_w*d_w*d_w$ $(1.154)(2.157)^2$ = 5.369	$= \frac{t_w*h^3}{12}$ $\frac{(0.313)(3.687)^3}{12}$ = 1.307
Plate	t 0.313	b 24	$A = t*b*T$ [(0.313)(24)] × (0.688) = 5.168	$d = \frac{t}{2}$ 0.313/2 = 0.157	$=A*d$ (5.168)(0.157) = 0.811	$=A*d*d$ $(5.168)(0.157)^2$ 0.127	$= \frac{(T*b)*t^3}{12}$ $\frac{(0.688)(24)(0.313)^3}{12}$ = 0.042
Variables			$A_T = \sum A_i$ 0.939 + 1.154 + 5.168 = 7.261		$B_T = \sum A_i*d_i$ 3.903 + 2.489 + 0.811 = 7.203	$C_T = \sum A_i*d_i^2$ 16.227 + 5.369 + 0.127 = 21.723	$I_T = \sum I_i$ 0.008 + 1.307 + 0.042 = 1.357

Calculations required after filling out "Transformed Section" chart:

(1) $N_u = \frac{B_{tr}}{A_{tr}} = \frac{7.203}{7.261} = 0.992$

(2) $I_{N_t} = I_t + C_{tr} = 1.357 + 21.723 = 23.080$

(3) $I_{Nw} = I_{Ntr} - \left(A_{tr} * N_u^2\right) = 23.080 - (7.261)(0.992)^2 = 15.935$

(4) $y_{pr} = N_u - \frac{t}{2} = 0.992 - \frac{0.313}{2} = 0.836$

(5) $y_{fr} = N_u - t - h - \frac{t_f}{2} = 0.992 - 0.313 - 3.687 - \frac{0.313}{2} = -3.165$

(6) $\rho_w = \sqrt{\frac{I_{Nw}}{A_{tr}}} = \sqrt{\frac{15.935}{7.261}} = 1.481$

(7) $\Delta p = A_p * \left(N_u + \frac{t}{2}\right) * \left(\frac{1}{A_{tr}} - \frac{1}{A_s}\right) = 2.093\left(2.742 + \frac{0.313}{2}\right)\left(\frac{1}{7.261} - \frac{1}{9.605}\right) = 0.204$
(eq 14.2.18)

Mode I: Compression Failure of the Stiffener:

(1) For "m" = 1,2,3,4,5:

$$\sigma_{a,T} = \frac{1}{I_{sp} + \frac{2 * C_r * b^3 * t}{\pi^4}} * \left(\frac{E * J}{2 * (1+\upsilon)} + \frac{m^2 * \pi^2}{a^2} * E * I_{sz} * h^2 + \frac{4 * D * C_r}{\pi^2 * b} * \left(\frac{a^2}{m^2} + b^2 \right) \right)$$

(eq. 13.1.20)

for m=1: $\frac{1}{17.971 + \frac{(2)(0.942)(24)^3(0.313)}{(3.14)^4}} \left[\frac{(30E6)(0.067)}{2(1+0.3)} + \frac{(1)^2(3.14)^2}{81^2}(30E6)(1.648)(3.687)^2 + \left(\frac{4(83840)(0.942)}{(3.14)^2(24)} \right) \left(\frac{(81)^2}{(1)^2} + 24^2 \right) \right]$

$= 111,081$

repeat for m = 2,3,4,5 $\sigma_{a,T_{min}} = \min\{(111,081),(76,321),(114,057),(179,375),(226,796)\} = 76,321$

(2) $\sigma_{FI} = \min\{(\sigma_{yield}),(\sigma_{a,T_{min}})\}$

(eq. 14.2.1) $= \min\{(34,000),(76,321)\} = 34,000$

(3) $\lambda = \frac{a}{\pi * \rho} * \sqrt{\frac{\sigma_{FI}}{E}} = \frac{81}{(3.14)(1.338)} \sqrt{\frac{34,000}{30E6}}$

(eq. 14.2.3)

$= 0.649$

(4) $\eta = \frac{\Delta * y_f}{\rho^2} = \frac{(-0.125)(-3.370)}{(1.338)^2}$

(eq. 14.2.3)

$= 0.235$

(5) $\zeta = 1 + \left(\frac{1+\eta}{\lambda^2} \right) = 1 + \left(\frac{1 + 0.235}{(0.649)^2} \right)$

(eq. 14.2.5)

$= 3.932$

(6) $R_I = \frac{\zeta}{2} - \sqrt{\frac{\zeta^2}{4} - \frac{1}{\lambda^2}} = \frac{3.932}{2} - \sqrt{\frac{(3.932)^2}{4} - \frac{1}{(0.649)^2}}$

(eq. 14.2.5)

$= 0.745$

(7) $ULT_I = R_I * \sigma_{FI} = (0.745)(34,000)$

(eq. 14.2.6)

$= 25,330$ psi

Mode II: Compression Failure of the Plating:

(1) $\sigma_{au,wc} = \frac{0.63 * \sigma_{yield}}{\beta^2} = \frac{(0.63)(34{,}000)}{(2.585)^2} = 3205$

(eq. 12.6.8)

(2) $\sigma_{au} = \sigma_{yield} * (T - 0.1) = (34{,}000)(0.688 - 0.1) = 19{,}992$

(eq. 12.6.5)

(3) $\sigma_{ay_e} = \frac{b}{a} * \sigma_{au} + \left(1 - \frac{b}{a}\right) * \sigma_{au,wc} = \left(\frac{24}{81}\right)(19{,}992) + \left(1 - \frac{24}{81}\right)(3205) = 8179$

(eq. 12.6.12)

(4) $\sigma_{FII} = \frac{T - 0.1}{T} * \sigma_{yield} * \left(1 - \frac{\sigma_{transverse}}{\sigma_{ay_e}}\right) = \left(\frac{0.688 - 0.1}{0.688}\right)(34{,}000)\left(1 - \frac{1000}{8179}\right) = 25{,}505$

(eq. 14.2.14)

(5) $\lambda_{tr} = \frac{a}{\pi * \rho_{tr}} * \sqrt{\frac{\sigma_{FII}}{E}} = \frac{81}{(3.14)(1.481)}\left(\sqrt{\frac{25{,}505}{30E6}}\right) = 0.508$

(eq. 14.2.16)

(6) $\eta_{tr} = \frac{\Delta * y_{ptr}}{\rho_{tr}^2} = \frac{(0.125)(0.836)}{(1.481)^2} = 0.048$

(eq. 14.2.16)

(7) $\eta_{ptr} = \frac{\Delta p * y_{ptr}}{\rho_{tr}^2} = \frac{(0.204)(0.836)}{(1.481)^2} = 0.078$

(eq. 14.2.16)

(8) $\zeta_{II} = \frac{1}{1 + \eta_{ptr}} + \frac{1 + \eta_{ptr} + \eta_{tr}}{(1 + \eta_{ptr}) * \lambda_{tr}^2} = \frac{1}{(1 + 0.078)} + \frac{(1 + 0.078 + 0.048)}{(1 + 0.078)(0.508)^2} = 4.975$

(eq. 14.2.20)

(9) $R_{II} = \frac{\zeta_{II}}{2} - \sqrt{\frac{\zeta_{II}^2}{4} - \frac{1}{(1 + \eta_{ptr}) * \lambda_{tr}^2}} = \frac{4.975}{2} - \sqrt{\frac{(4.975)^2}{4} - \frac{1}{(1 + 0.078)(0.508)^2}} = 0.877$

(eq. 14.2.20)

(10) $ULT_{II} = \frac{A_{tr}}{A_c} * R_{II} * \sigma_{FII} = \left(\frac{7.261}{9.605}\right)(0.877)(25{,}505) = 16{,}909 \text{ psi}$

(eq. 14.2.22)

ULTIMATE STRENGTH = min {ULT$_I$, ULT$_{II}$}

$= \min\{(25{,}330), (16{,}909)\}$

$= 16{,}909$ psi