FINAL REPORT

Urban System Modeling and Performance Measurement Using Multiple Data Sources

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16. Abstract

As technologies advance, emerging urban data are increasingly available for wide urban areas.), which are referred to as *Urban Hybrid Traffic Data* (*U-HTD*) in this report. U-HTD provides great opportunities for urban transportation/traffic system performance evaluation, modeling, and management. It also poses great challenges in data collection, processing, storage, and use, including data privacy and security issues. This research investigates the issues of applying U-HTD for urban traffic modeling, including (i) fusion methods of mobile and fixed data, (ii) freight transportation modeling using multiple sources of data, and (iii) new econometric method for analyzing interaction behavior data.

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1. Introduction

As technologies advance, emerging urban data are increasingly available for wide urban areas. They include data from dedicated loop detectors, license plate readers, vehicle re-identification, electric toll collection (ETC), Bluetooth, cellular phones, GPS-enabled devices (such as navigation devices and smart phones), potentially Connected Vehicles, and so on. Such data are inherently heterogeneous, including both fixed-location data (e.g.., those from loops) and mobile data (e.g., those from GPS), which are referred to as *Urban Hybrid Traffic Data* (*U-HTD*) in this report. U-HTD provides great opportunities for urban transportation/traffic system performance evaluation, modeling, and management. They can directly measure how the urban system performs and help pinpoint problem areas (such as bottlenecks). Integrated with other data from surveys / interviews / experiments, they can help better understand how the system components behave and interact with each other. They can also help make more informed decisions about how to allocate resources and investment to manage urban systems more effectively and efficiently. On the other hand, however, U-HTD poses great challenges in data collection, processing, storage, and use, including data privacy and security issues.

Two research themes were conducted in this project: (i) fusion methods of mobile and fixed data, (ii) freight transportation modeling using multiple sources of data, and (iii) new econometric method for analyzing interaction behavior data. The first research theme aims to tackle some of the fundamental challenges when developing methods on how to best mine the different data elements in U-HTD, how to protect privacy when processing and using U-HTD, and how to developing novel methods that can best utilize U-HTD for critical urban transportation applications. For this purpose, the project team focuses on fusing the two crucial data elements in U-HTD, namely the fixed-location data (loop data were used in the project) and mobile data. A data analytic methods was developed for the data fusion and the results are expected to help develop methods to integrate all data elements in U-HTD in the near future.

The second research theme focuses on estimating freight trip generation models that could accurate describe and predict the flows of cargo and amounts of freight trips generated from the interactions between shippers and receivers, their applicability and transferability across different time periods, the analysis of the effects of time on freight trip generation model parameters and its implications for planning and policy making purposes within the transportation system. Accordingly, as part of the study presented here, a set of econometric models are developed in order to estimate the number of freight trips attracted by receivers as well as the number of freight trips produced by shippers. A formal definition of freight trip attraction and production is presented.

The third research theme aims to develop an innovative econometric model that is able to behavioral-consistently capture these new phenomena in the information era which include intricate matching networks, mutual selection, and intensive joint decision making between transportation agents.The proposed model consists of two parts: The first part explains the matching process in a many-to-one matching structure; the second part characterizes the joint decision making process of mutually-selected decision makers. The two parts are integrated by recognizing their dependency by a sample selection process: a joint response is only observed for matched decision makers.

2. A Data Fusion and Information Integration Framework

It is a longstanding goal for transportation researchers to model and understand how traffic distribute and propagate along road networks, based on collected traffic data. With respect to the way data are collected, traffic data can be broadly categorized into fixed-location data (e.g., inductive loop, magnetic, radar, infrared, acoustic, cameras, etc.) and mobile sensing data (e.g., Global Positioning System (GPS) data). Fixed-location data can provide traffic information (e.g., volume and occupancy) at pre-defined locations, but cannot fully capture the traffic progression over space. Mobile sensing data offer more detailed spatial coverage (e.g. travel time and speed) but are in general sparsely sampled over time (Sun & Ban, 2013).

While both data sources have their own advantages and provide some insights of the traffic, each source itself is usually incomplete and biased. To resolve these issues, recent research has suggested the use of data fusion techniques that can take the advantage of each data source and use both sources to validate each other. Indeed, data fusion techniques have been widely applied in many transportation fields, such as vehicle tracking, positioning and motion prediction (Shan et al., 2013; Salameh et al., 2013), safety control and risk assessment (Logi & Ritchie, 2001; Bifulco et al., 2011; Ahmed & Abdel-Aty, 2013), vehicle classification (Junghans & Jentshel, 2007), urban freight performance measurement (Yang et al., 2014), eco-driving/eco-routing (Boriboonsomsin et al., 2012), in addition to traffic state estimation and prediction (Van Lint & Hoogendoorn, 2009; Treiber et al., 2011; Deng et al., 2013; Antoniou et al., 2013; Zhou & Mirchandani, 2015).

2.1 Review of Existing Data Fusion Methods

Among the existing traffic data fusion models, many of them focus on aggregated characteristics such as average volume, travel time and speed (Sun et al., 2009; Kong et al., 2009; Ou et al., 2010; Du et al., 2012; Bachmann et al., 2013; Box et al., 2013; Antoniou et al., 2013). These models mainly concern macroscopic/large-scale traffic patterns and do not look into the details of traffic problems. As novel technologies continue to emerge, high resolution traffic data (e.g., event-based fixed-location data, second-by-second GPS traces) have become increasingly available. This has enabled fine-grained traffic modeling (Ban et al., 2009; Ban et al., 2011; Ban & Gruteser, 2012; Hao et al., 2012; Sun & Ban, 2013; Sun et al., 2013; Sun et al., 2015), which targets detailed urban traffic states and performance measures, such as individual travel times, vehicle trajectories/emissions, and real-time performance of urban traffic signals. To date, most research on this topic is conducted using single-source data. And only a handful of studies (Berkow et al., 2009; Sun & Ban, 2011; Wolfermann et al., 2011; Mehran et al., 2012) attempted to combine fixed-location and mobile sensing data for fine-grained traffic modeling.

There are some challenges that have not been fully addressed in the existing literature. Firstly, the estimated travel times or trajectories of individual vehicles (if not captured by the mobile sensing data) are usually governed by macroscopic traffic flow model (e.g. the Lighthill-Whitham-Richards model, Newell's simplified kinematic wave theory, etc.), which cannot capture microscopic/individual behavior (e.g., the detailed acceleration and deceleration processes of a vehicle). Secondly, existing models are usually restricted to lane-by-lane estimation. Overtaking is prohibited in most cases by assuming first-in-first-out (FIFO). Lane changing/splitting and traffic merging from side streets are rarely considered. These challenges may lead to erroneous estimations along a traffic corridor/network. Last but not least, data error, mismatching (between fixed-location data and mobile sensing data), and detector failure problems are not fully considered. These challenges will be studied in this research.

In this research, a data fusion and information integration approach is proposed to model and interpret traffic along urban arterial corridors based on heterogeneous data sources. The method aims to integrate fixed-location data with mobile sensing data and match the vehicle records at upstream with those collected at downstream detector locations. To make the model more realistic, traffic knowledge such as lane choice decision, traffic merging and travel time information are calibrated using the historical dataset and then integrated into the modeling framework. By doing so, the model can obtain individual travel times of the "most probable" matching results, which can be directly used to estimate corridor travel times of individual vehicles. The proposed method is not restricted to specific sensor configuration and lane-by-lane estimation. Instead, overtaking and lane choice/traffic merging can be properly captured.

The results indicate that at 20% mobile penetration, the proposed method can correctly match on average about 50% vehicle records at incoming sub-systems that approach the intersections, and about 95% vehicle records at outgoing sub-systems that depart from the intersections. At the corridor-level, 42% of vehicle records are correctly matched, which can enable more accurate estimation of individual corridor travel times. The root mean square error (RMSE) of the estimated individual corridor time is about 7.4 seconds, which is about 6% of the average corridor travel time. Similar trends are observed for the "inaccurate data" and the "detector failure" scenarios. Per the specific application, 0.5 seconds were identified as the cut-off GPS (time) accuracy. Incorporating mobile sensing records with poorer accuracy may not enhance the performance of the proposed model. Moreover, it was found that the "detector failure" issue greatly affects the modeling performance. The proposed approach may also be extended to other fine-grained urban traffic modeling, such as estimation of vehicle trajectories, vehicle-based fuel consumption/emissions, and help infer real-time queuing processes at signalized intersections.

The rest of the Chapter is organized as follows. Section 1.2 introduces a method to decompose the problem and provides a description of the heterogeneous traffic data that are considered in the research. A probability-based method is then introduced in Section 1.3 to extract useful traffic patterns from historical traffic data. Section 1.4 proposes a data fusion and information integration approach that formulates the problem as a combinatorial optimization problem. And a tabu-search-based heuristic approach is then applied to solve this problem. Section 1.5 presents the numerical experiment results using the NGSIM data. Discussions, conclusions, and future research directions are provided in Section 1.6.

2.2. Problem decomposition and data description

An arterial corridor can be considered as a multi-input, multi-output system which interacts with its neighboring corridors/networks. To model traffic in such an open system, the relationships between its inputs and outputs need to be carefully considered. In this section, a decomposition method is introduced to simplify the problem. Then the heterogeneous traffic data are described, which are considered in this research.

2.2.1 Problem decomposition

To model traffic flow along an arterial corridor/network, a problem decomposition approach is proposed to divide a large open traffic system into a few sub-systems so that the traffic problem in each sub-system becomes easier to solve. In particular, a corridor is divided into a group of incoming sub-systems that approach the intersections (see Figure 1(A)), and a group of outgoing sub-systems that depart from the intersections (see Figure 1(B)). The locations of fixed-location traffic sensors are indicated by solid boxes.

As shown later, the traffic problem in each sub-system can be solved separately. Since the downstream detectors (denoted by the boxes) at incoming sub-systems can be (partially) considered as the upstream detectors at outgoing sub-systems. The solutions (i.e., vehicle indices) at an incoming sub-system (e.g., intersection upstream or Figure 1(A)) can be (partially[1]) treated as the input to an outgoing sub-system (e.g., intersection downstream or Figure 1(B)), and vice versa. In this manner, traffic flow can be linked along the entire corridor/network.

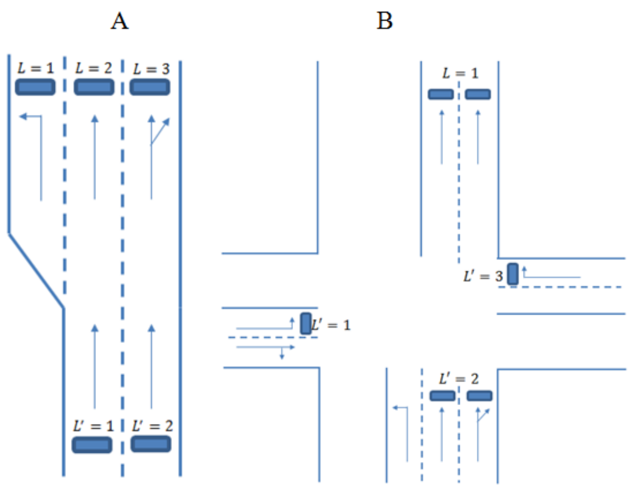

Figure 1: Problem decomposition; (A) Incoming sub-system; (B) Outgoing sub-system

The two decomposed sub-systems describe two different problems that are essential to arterial corridor traffic. At the incoming sub-systems, the primary interest is on vehicles' lane-choice decisions. That is, some vehicles will take the left-turn lane (i.e., $L = 1$) or make a right-turn (i.e., at $L = 3$) and leave the corridor, in addition to the ones that are making the through movement (i.e., at $L = 2$ or $L = 3$). At the outgoing sub-systems, the interest is on vehicles merging from the incoming sub-systems (i.e., $L' = 2$) and from the side streets (i.e., $L' = 1$ and $L' = 3$). These two traffic problems can be captured using the same framework as introduced in the subsequent sections.

2.2.2 Data description

The traffic data considered in this research come from two different sources: (i) fixed-location data that are collected using fixed-location traffic sensors as indicated by the boxes in Figure 1, which could include data collected using inductive loop detectors, magnetic sensors, or virtual loops from camera detectors that record the time whenever a vehicle passes a predefined location; such data

[1] "Partially" here means a part of the solutions from an incoming sub-system (i.e., $L = 2$ and $L = 3$ as in Figure 1(A)) is treated as the input to the adjacent outgoing subsystem (i.e., $L' = 2$ as in Figure 1(B)).

are also known as event-based traffic data; and (ii) mobile sensing data that are collected using devices with tracking capabilities, which could include the GPS trace or departure time records for the same vehicle (e.g., via Bluetooth records or Vehicle-to-Infrastructure communications) collected at discrete locations.

The fixed-location data considered in this research are lane-specific. That is, whenever a vehicle passes a fixed-location sensor on one lane, the timestamp and corresponding lane information are recorded. The fixed-location data from all the lanes at one location (e.g., from $L' = 1$ and $L' = 2$ in Figure 1(A)) are then grouped and sorted by time. Therefore a sequence of fixed-location records can be obtained at each sensor location. Noteworthy that the records from multiple traffic movement in Figure 1(B), i.e., $L' = 1$, $L' = 2$, and $L' = 3$ are also grouped and sorted. They are considered as a single input to the outgoing sub-system. Within each sub-system, the input and output data are sequences of records collected at upstream and downstream locations respectively. Each record embodies the index of the vehicle in the traffic stream, the lane/movement indicator, and the time that the vehicle leaves a fixed-location sensor.

In addition to fixed-location data, mobile sensing data (vehicle traces or travel time pairs) can also be collected within each sub-system. Using mobile sensing data, the corresponding time records can be readily found at each fixed-location sensor location (where fixed-location data are collected). The mobile sensing dataset are sorted by the timestamp at the each sensor location.[2] Mobile sensing data are used to help validate the fixed-location records and make connections between the input and output data. This research assumes that lane information or movement direction is not available from the mobile sensing data. In the cases that lane information or movement directions are available from mobile sensing data (e.g., via map matching), the problem will be greatly simplified. In this regard, the proposed method offers solutions to a harder problem.

Noteworthy that for the same vehicle, its fixed-location record and mobile sensing record (if this vehicle is equipped with mobile sensing devices) may not perfectly coincide. [3] This problem, called "inaccurate data" problem, is captured by the proposed model in the subsequent sections. In practice, it is also possible that some lanes/lane groups are not covered by fixed-location sensors,

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[2] Since only time records are considered in this research, the collection frequency of mobile sensing data is not deemed a big issue, as long as they can be used to infer the times when vehicle passes the fixed-location locations.

[3] This is not rare in real applications due to the measurement error (e.g., GPS inaccuracy, low data collection frequency) and poor synchronization between heterogeneous sources.

e.g., the downstream detector of the left-turning lane ($L = 1$) is missing in Figure 1(A); and/or the upstream detector of the right-turning approach ($L' = 3$) is missing in Figure 1(B). Similarly, the detectors may fail to detect some passing-by vehicles. This problem, called "detector failure" problem, is also investigated using the proposed modeling framework.

The collected data can be visualized in a time-space diagram, as shown in Figure 2. Figure 2(A) illustrates the traffic data under an ideal scenario, in which the mobile sensing data and fixed-location data are free of error and perfectly synchronized. Figure 2(B) represents the "inaccurate data" scenario in which the probe record and the fixed-location record of the same vehicle do not coincide. Figure 2(C) shows the "detector failure" scenario in which the downstream detectors are not functioning well and can only detect a proportion of the passing-by vehicles. Numerical experiment and results of these special scenarios are provided in Section 5.

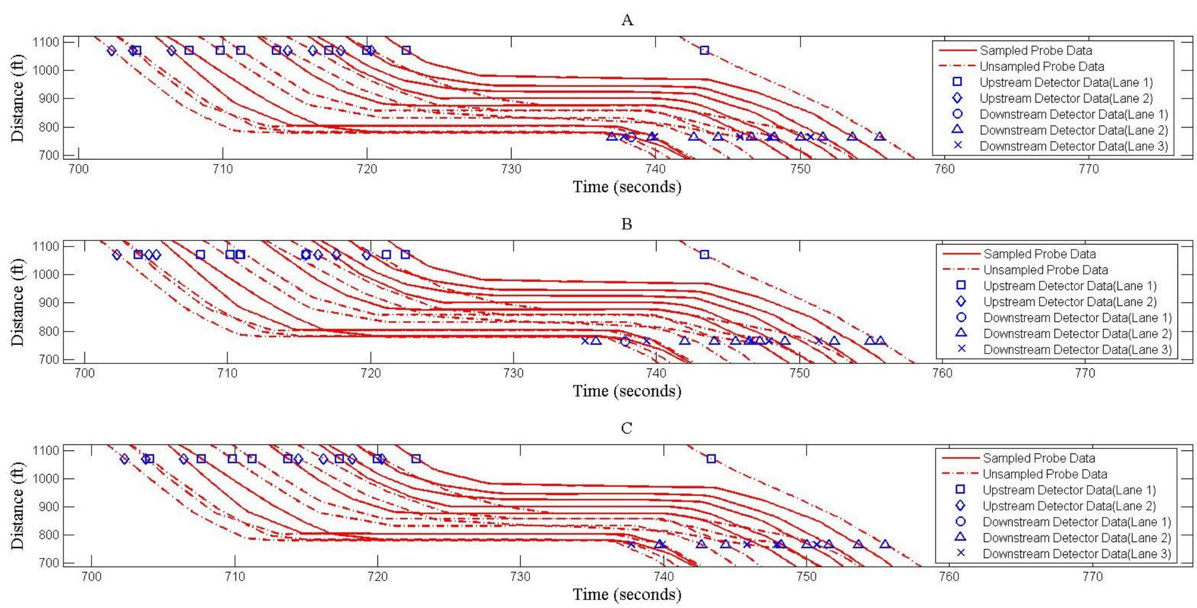


Figure 2: Visualization of heterogeneous traffic data (incoming sub-system); (A) Ideal scenario; (B) "Inaccurate data" scenario; (C) "Detector failure" scenario

Given the two types of data, the fundamental goal is to match the upstream records with the downstream ones, so that a complete picture of the traffic problem can be revealed. For two fixed-location records that are marked by the mobile sensing records at the upstream and downstream locations (i.e., they correspond to the same probe vehicle), these two fixed-location records are directly matched. Therefore the original problem is reduced to a matching problem between the upstream and downstream fixed-location records that are not marked by the mobile sensing records. The model formulation and its solution method will be discussed in Section 4.

2.3. Information acquisition from historical data

The proposed modeling approach is to find the "most probable" matching results between the upstream and downstream fixed-location records. To make the model more realistic, traffic knowledge such as lane choice decision, traffic merging and travel time information are calibrated using the historical dataset and then integrated into the model.

2.3.1 Lane choice and traffic merging

For an incoming sub-system (e.g., Fig. 1(A)), consider a vehicle on lane l at downstream, the lane choice probability indicates how likely this vehicle comes from lane l' at upstream ($L' = l'$), which is represented as equation (1). Note that hereafter in the chapter, capitalized symbols are used to denote random variables, and lower case symbols to indicate values of the random variables. In equation (1), $q_l^{l'}$ is the number of historical data samples that go through both downstream lane l and upstream lane l'; and the denominator is the total number of historical data samples that pass through lane l at downstream. These values are considered as historical data for model calibration purposes. In practice, they could be acquired from field data collections (e.g., camera data). In this research, they are known from the NGSIM data in the numerical experiment.

$$P(L = l, L' = l') = p_{L'}(l'; l) = \frac{q_l^{l'}}{\sum_{L'} q_l^{l'}} \tag{1}$$

With respect to the outgoing sub-systems, equation (1) can also be directly applied to represent the traffic merging from the incoming sub-system (i.e., the through movement) and the turning movements. For example, consider a vehicle at downstream (i.e., $L = 1$; see Figure 1(b)), the traffic merging likelihood denotes how likely this vehicle comes from the side streets or through movement at upstream locations.

2.3.2 Travel time information

Given certain lane choice or traffic merging, the probability density of travel time is denoted as $f_{T'}(t'; l, l', t)$. This term concerns with the probability of taking travel time τ ($\tau = t - t'$) from an upstream lane or movement direction l' to a downstream lane or lane group l. To illustrate the concept, Figure 3 shows the travel time histograms of all possible lane choices for an incoming sub-system. The lane configuration in this case is identical to that in Figure 1(A).

In Figure 3, each subplot represents the travel time histogram for a possible lane choice. A bi-modal travel time pattern can be observed in most cases. This is because some vehicles traverse the sub-system without experiencing much delay (free-flowing), which correspond to the samples that have lower travel times. Some vehicles get delayed before they pass the downstream detector location (e.g., due to red signals), which correspond to the samples that have higher travel times. It is found that the frequencies for certain lane choices are quite low, which is due to the small sample sizes in the historical dataset. An extreme case is found in Figure 3(D), in which no travel time samples are available. This is because it is unlikely to have a vehicle that travels from the rightmost lane at upstream ($L' = 2$) to the left-turn lane ($L = 1$) at downstream.

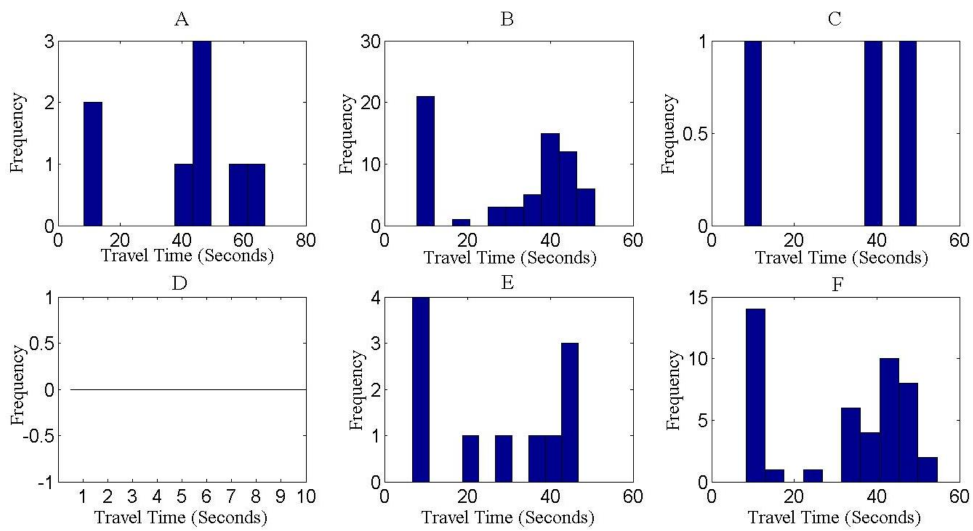


Figure 3: Travel time plots at an incoming sub-system; (A) $L = 1, L' = 1$; (B) $L = 2, L' = 1$; (C) $L = 3, L' = 1$; (D) $L = 1, L' = 2$; (E) $L = 2, L' = 2$; (F) $L = 3, L' = 2$

The travel time distributions are estimated using a non-parametric method called kernel smoothing density estimation, as in equation (2). Here $h_\tau^{l' \to l}$ represents the set of historical travel times (assume n historical records, each record is denoted as τ_n) for the vehicles that pass lane l' at upstream and lane l at downstream, in sequence; h is the bandwidth, which is selected to minimize the mean integrated squared errors (MISE) based on the historical dataset. Given a pair of fixed-location records, the corresponding probability density can be found at travel time $\tau = t - t'$. More details can be found in Bowman & Azzalini (1997).

$$f_{T'}(t';l,l',t)=\frac{1}{nh}\sum_{h_\tau^{l'\to l}}\frac{1}{\sqrt{2\pi}}e^{\frac{1}{2}(\frac{t-t'-\tau_n}{h})^2} \tag{2}$$

2.4. Traffic data fusion and information integration

A combinatorial optimization problem is formulated in this section to model the matching problem between the fixed-location data and mobile sensing data. The model is able to capture the traffic problems at both incoming and outgoing sub-systems. Information extracted historical datasets are integrated into the modeling framework.

2.4.1 Matching between fixed-location data and mobile sensing data

First denote a set of random variables as $\{M_j \in M \big| j=1,2,\dots N_m\}$, which indicates the downstream fixed-location index of the jth vehicle in the downstream mobile dataset. That is, for the jth vehicle in the downstream mobile dataset (out of the total number of m mobile sensing records), its corresponding index in the fixed-location dataset at downstream. Similarly, another set of random variable $\{M'_k \in M' | k=1,2,\dots N_m\}$ can be defined, which represents the upstream fixed-location index of the kth vehicle in the upstream mobile dataset. Note that the number of mobile sensing records at upstream equals to the number at downstream (N_m), however, different subscripts (i.e., j and k) are used to denote the indices of vehicles in the downstream and upstream mobile dataset. This is because for the same probe vehicle, e.g., $ID(j)=ID(k)$, its indices in the upstream and downstream mobile dataset may not be the same ($j \neq k$), where $ID(\cdot)$ denotes the unique ID of a probe vehicle.

Further denote $p_M(m_j)$ and $p_{M'}(m'_k)$, which stand for the discretized matching probabilities between a fixed-location record and a mobile sensing record, at downstream and upstream locations, respectively. In practice, these two types of probability can be calculated based on historical data or provided by the domain experts, which are related to the accuracy of both data sources and how well these two datasets are synchronized with each other. Assuming independent matching between mobile sensing records and fixed-location records, the joint probability of matching between the fixed-location dataset and the mobile sensing dataset can be represented as equation (3) for downstream and equation (4) for upstream.

In equation (3), the discretization scheme converts the probability density of the arrival time difference between a mobile sensing record and a fixed-location record (e.g., $f_T(t_{m_j};\hat{t}_j)$) to a normalized matching probability $p_M(m_j)$. Where α_M is the normalization term. In this project, the probability density function of travel time difference is assumed to follow a normal distribution, as in equation (5). Here $\hat{t}_j$ is the time for the jth mobile sensing record to pass a downstream location; and t_{m_j} represent the time for the m_jth record in downstream fixed-location dataset. Same analysis can also be applied to $p_{M'}(m'_k)$. Note that $m_j \neq m_{j'}$ for any $j \neq j'$, and $m'_k \neq m'_{k'}$ for any $k \neq k'$ are needed to ensure that each mobile sensing record is matched only once to a fixed-location record.

$$P(M = m) = \prod_{j=1}^{N_m} p_M(m_j) = \prod_{j=1}^{N_m} \alpha_M f_T(t_{m_j}; \hat{t}_j) \text{, } \forall j \neq j', m_j \neq m_{j'} \tag{3}$$

$$P(M' = m') = \prod_{j=1}^{N_m} p_{M'}(m'_k) = \prod_{k=1}^{N_m} \alpha_{M'} f_T(t_{m'_k}; \hat{t}_{k'}) \text{, } \forall k \neq k', m'_k \neq m'_{k'} \tag{4}$$

$$f_T(t_{m_j}; \hat{t}_j) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\hat{t}_j - t_{m_j})^2}{2\sigma^2}} \tag{5}$$

2.4.2 Matching between upstream and downstream fixed-location data

Within each sub-system, to describe the matching probability between the fixed-location data at downstream and upstream, another set of random variables $\{S_i \in S | i = 1,2, \dots N_s\}$ is used to represent the upstream fixed-location index of the ith vehicle in the downstream fixed-location dataset. Here the total number of downstream fixed-location records is denoted by N_s. Consider a downstream fixed-location record (l_i, t_i), and an upstream fixed-location record $\left(l'_{s_i}, t'_{s_i}\right)$, the matching probability between these two records can be expressed as $p_S(s_i)$. In equation (6), this is formulated as the product of lane choice/traffic merging likelihood and the travel time probability density as in equation (2), multiple the normalization term α_S.

$$p_S\left(s_i; l_i, l'_{s_i}, t_i, t'_{s_i}\right) = \alpha_S p_{L'}\left(l'_{s_i}; l_i\right) f_{T'}\left(t'_{s_i}; l_i, l'_{s_i}, t_i\right) \tag{6}$$

Assuming independent matching between fixed-location records, the joint probability of matching between the upstream fixed-location dataset and the downstream fixed-location dataset can be expressed in equation (7). Similarly, need $s_i \neq s_{i'}$ for any $i \neq i'$ to ensure that each downstream fixed-location record is matched only once to an upstream fixed-location record.

$$\boldsymbol{P(S = s) = \prod_{i=1}^{N_s} p_S\left(s_i; l_i, l'_{s_i}, t_i, t'_{s_i}\right) \text{, } \forall\, i \neq i', s_i \neq s_{i'}} \tag{7}$$

2.4.3 Model formulation

Assuming independency for matching between fixed-location records and matching between fixed-location records and the mobile sensing records, the overall matching problem can be formulated as a combinatorial optimization model as in (8.1) – (8.5).

$$\max_{s,m,m'} P(M = m)P(M' = m')P(S = s) \tag{8.1}$$

$$s.t.\ (j - j')(m_j - m_{j'}) > 0\ \ for\ \ j \neq\ j' \tag{8.2}$$

$$(k - k')(m'_k - m'_{k'}) > 0\ \ for\ \ k \neq\ k' \tag{8.3}$$

$$s_{m_j} = m'_k \qquad for\ \ ID(j) = ID(k) \tag{8.4}$$

$$(i - i')(s_i - s_{i'}) > 0\ \ for\ i \neq\ i'\ \ if\ l_i = l_{i'}\ and\ l'_{s_i} = l'_{s_{i'}} \tag{8.5a}$$

$$s_i - s_{i'} \neq 0\ \ for\ i \neq\ i' \tag{8.5b}$$

In this optimization model, the objective (8.1) is to maximize the overall matching probability. Constraint (8.2) and (8.3) are used to make sure that for two matched mobile sensing records, their relative sequences in the mobile dataset and in the fixed-location dataset are consistent. Constraint (8.4) is used to make sure the matching between the upstream fixed-location records and downstream fixed-location records are only conducted for these records that are not marked by the mobile sensing records. For a pair of fixed-location records that are marked by the same mobile sensing record (i.e., $ID(j) = ID(k)$), they are directly matched (i.e., $s_{m_j} = m'_k$, where m'_k is matched with the kth vehicle in the mobile sensing dataset at upstream; m_j is matched with the jth vehicle in the mobile sensing dataset at downstream). Constraint (8.5a) is the first-in-first-out (FIFO) condition, which is used to prohibit overtaking for vehicles that have the same lane choice/movement direction. In other words, if two vehicles are on the same upstream lane and the same downstream lane (or traveling in the same movement direction; see Figure 1(B)), their relative sequences in the upstream fixed-location dataset and the downstream fixed-location dataset need to be consistent. Among different lanes/lane groups (either upstream or downstream), overtaking is allowed under this requirement. It is worthy to mention that the FIFO condition is not a “must-have” in the model. In theory, overtaking happens when one vehicle passes another vehicle and gets back to the original lane/lane group. In the cases that overtaking is not prohibited (FIFO does not hold), Constraint (8.5b) applies, which ensures one-to-one matching between fixed-location records. The one-to-one matching constraints – as in equation (3), (4), and (7) – are imbedded in the constraints (8.2), (8.3), and (8.5).

2.4.4 Solution method

The proposed optimization model can be solved using a two-step approach. In the first step, all possible outcomes of M and M' can be enumerated based on constraints (8.2) and (8.3). In practical terms, consider a real time application of 15 or 30 minutes interval, the penetration of mobile sensing data is usually low and the size of M or M' is not large within a sub-system. Also, M and M' are also tightly bounded by the constraints. Therefore, enumerating all possible outcomes of M and M' is not computational demanding. The second step starts from constraint (8.4), which attempts to find the matching between downstream and upstream fixed-location data (i.e., S), based on a possible outcome of M and M'. Due to constraint (8.4), some downstream and upstream fixed-location records are already labeled by the mobile sensing matching result (M and M'). Therefore this task only needs to be done for the unlabeled fixed-location records. The best solution of the problem is to maximize the overall matching probability (i.e., the objective function (8.1)), which is the most probable matching result deduced based on the historical traffic patterns and the observed heterogeneous data.

Noteworthy that the second step of the matching problem is difficult to be solved exactly. Since the total number of mobile sensing records is usually low but the total number of fixed-location records could be quite large (e.g., hundreds of records in a 15-30 minutes time interval), enumerating all possible outcomes of this matching problem can be extremely computational expensive. [4] To avoid enumerating all possible outcomes, the problem is solved using a heuristic procedure called tabu search (TS). The TS procedure is applied to use flexible structure memories to guide the searching process. It can therefore avoid from being trapped at local optimal solutions. The tabu-search-based solution procedure is summarized in Figure 4.

[4] For example, consider 15 (unmarked) upstream and 15 downstream fixed-location records, enumerating all possible matching outcomes ($15! = 1.31 \times 10^{12}$) is too computational expensive. The complexity of the problem grows explosively as the number of vehicles increases.

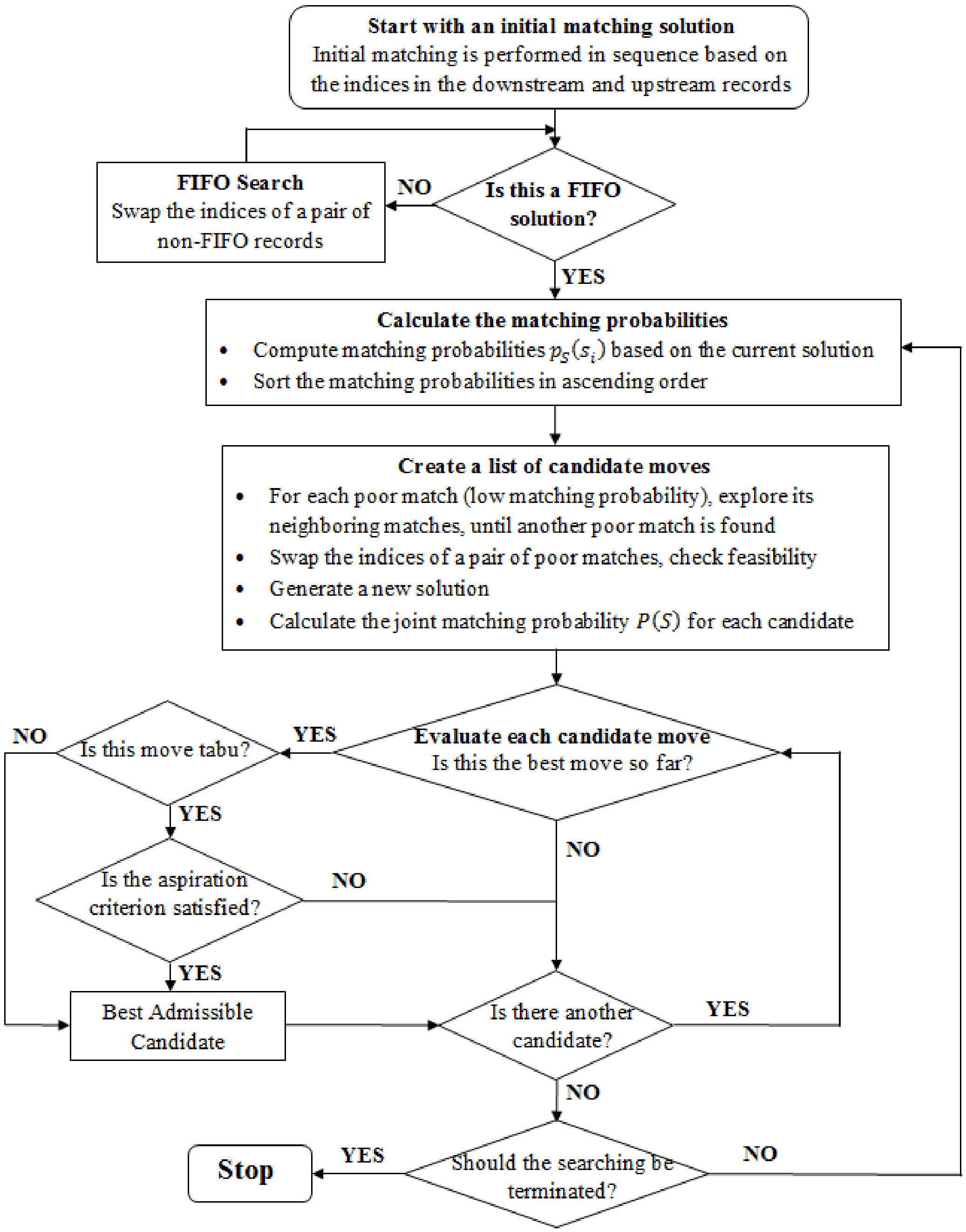


Figure 4: Searching procedures

The searching algorithm starts with an initial solution, which is based on the indices of the fixed-location records (that are not marked by the mobile sensing records) at the upstream and downstream locations. For example, the first vehicle in the downstream fixed-location dataset will be matched with the first vehicle in the upstream fixed-location dataset, and so forth. As shown later, this initial matching solution (i.e., "naïve matching" solution) is usually not a bad starting point, however, it certainly can be improved by integrating other traffic information such as lane choice, traffic merging, and travel time. Based on the initial solution, the next step is to find a FIFO solution. This is done by swapping the indices (matching results) of a pair of non-FIFO records, until a FIFO solution is reached; this step is skipped for the problems without a FIFO constraint. Given this solution, a core TS procedure is performed to find the solution that maximizes the overall matching probability. This is done by identifying a list of candidate moves (i.e., each move represents a swap of indices of a pair of matching results) and finding out the best move that corresponds to the best matching probability. The candidate moves need to satisfy the feasibility requirements, i.e., Constraint (8.4) and (8.5). In order to prevent from falling into the local optimal solutions, a tabu restriction is adopted to disable the reversal/repetitive moves, i.e., if a fixed-location record has been swapped in any of the previous N iterations (N is set to be 3 in the research), it cannot be swapped again this soon. The tabu restriction is subject to an important exception. When a tabu move yields a sufficiently attractive improvement, e.g., it improves the joint matching probability by more than 1%, the tabu restriction may be overridden. This procedure is repeated until the stop criterion is satisfied, i.e., the number of iterations is large enough or the solution cannot be further improved. For theoretical discussions of the tabu search method, one can refer to Glover (1990) and Glover et al. (1995).

2.5. Experiment and numerical results

Numerical experiments were conducted using the NGSIM dataset collected at Peachtree St. in Atlanta, Georgia (Cambridge Systematics, 2007). This arterial corridor was decomposed into 6 sub-systems, including 3 incoming sub-systems, and 3 outgoing sub-systems. The sensor locations and traffic merging situations are similar to the examples in Figure 1. One can refer to Sun & Ban (2013) for more information of this dataset.

During the 15 minutes data collection period, the traffic volume that passes each sub-system is in the range of 100 to 200 vehicles. Trajectories of the entire traffic population are available in the NGSIM dataset. The fixed-location sensors were (virtually) deployed close to the stop bars, and the upstream locations of the road segments. They were designated to represent the locations of the stop bar detectors and advance detectors that are widely adopted in practice. Fixed-location records (i.e., time, lane number/movement direction) were collected at these sensor locations. To understand the impacts of mobile sensing data, the experiments were conducted using mobile sensing data with different penetration rates.[5] The ground truth NGSIM data were applied as the historical dataset, based on which the travel time distribution, lane choice and traffic merging likelihood were empirically calculated. The problem was formulated based on (8-1) to (8-5) for each sub-system, which was solved using the proposed solution method. The matching accuracy was used to indicate the performance of the proposed models, which is defined as the percentage of downstream fixed-location records that have been correctly matched with the upstream fixed-location records. The matching results of all sub-systems were then integrated to obtain the individual corridor travel time of the vehicles that pass the entire corridor. The travel times of the vehicles that enter or leave the corridor from the intermediate intersections are not considered in the analysis.

2.5.1 Ideal scenario

In this scenario fixed-location records and mobile sensing records are assumed to perfectly coincide, which means M and M' can be readily determined. The overall matching problem is thus reduced to an optimization problem represented by the objective (8.1), constraints (8.4) and (8.5).

The results of the models with FIFO constraint (8.5a) were first compared to the ones without FIFO constraint (8.5b). Figure 5 shows the matching results for two incoming sub-systems, one with short segment length (Figure 5(A)) that restricts overtaking, and the other with relative long segment length (Figure 5(B)). In Figure 5(A), it is found that the solutions with FIFO constraints outperform the ones without FIFO constraints, about 5% on average. This is because the sub-system is not long enough to permit frequent overtaking activities. In Figure 5(B), the plots indicate that for this longer segment, the solutions without FIFO constraints are better than the ones with FIFO constraints. Since the FIFO constraint prohibits overtaking, it has led to incorrect matching results in this case. Hereafter in the experiment, the FIFO constraint was applied for the sub-systems with infrequent overtaking (less than 2%), and the non-FIFO constraint was used for the sub-systems with frequent overtaking (larger than or equal to 2%). And a generic term “Tabu search (TS)” is used to represent the results of the proposed models, no matter whether FIFO constraint or non-FIFO constraint was applied.

[5] This was done by randomly sampling vehicle trajectories based on the penetration rate. Each data point shown in this section was the mean of results based on 20-time random sampling, except 0 and 100% penetration.

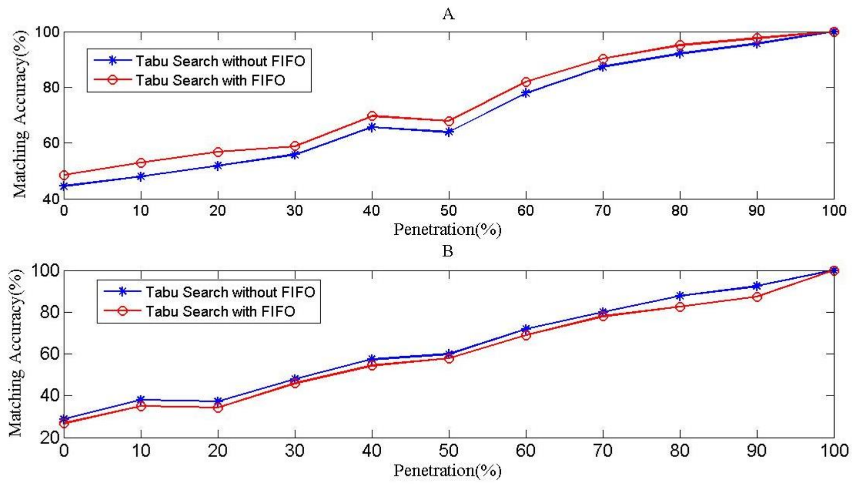


Figure 5: Matching accuracy of FIFO solutions vs. Non-FIFO solutions; (A) Intersection 1 incoming; (B) Intersection 3 incoming

To justify the effectiveness of the proposed searching approach, the results of the TS approach are compared with the naïve matching approach. The latter one refers to the initial matching solution generated at the first step in Figure 4. [6] Figure 6 indicates the matching results for all the 6 sub-systems, with respect to the change of mobile penetration.

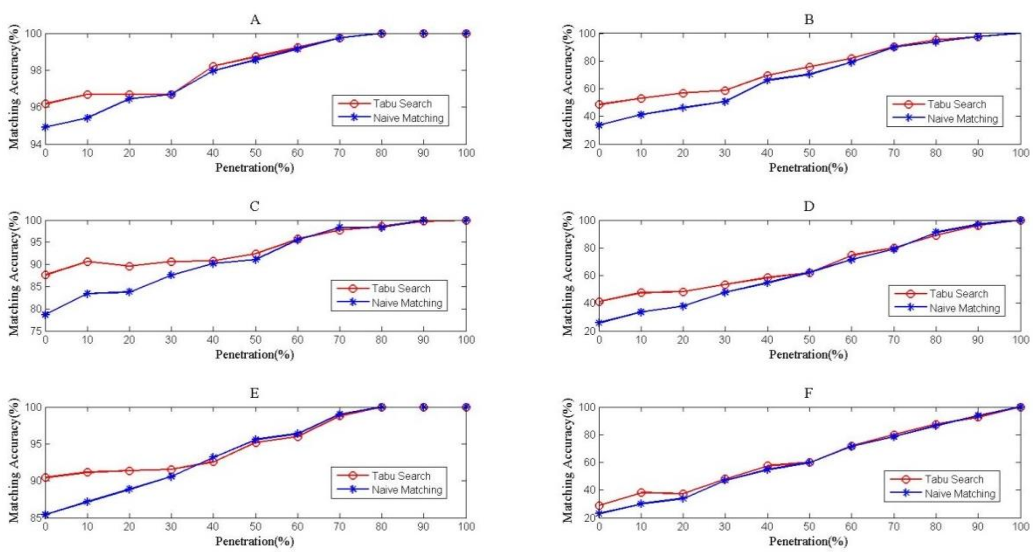


Figure 6: Matching results at sub-systems; (A) Intersection 1 outgoing; (B) Intersection 2 incoming; (C) Intersection 2 outgoing; (D) Intersection 3 incoming; (E) Intersection 3 outgoing; (F) Intersection 4 incoming

[6] The naïve matching is based on the indices of the fixed-location records (that are not marked by the mobile sensing records) at the upstream and downstream locations. For example, the first vehicle in the downstream fixed-location dataset will be matched with the first vehicle in the upstream fixed-location dataset, so on and so forth.

It is shown that the matching results at outgoing sub-systems (see Figure 6(A), (C), and (E)) are much better compared to the ones at incoming sub-systems (i.e., Figure 6(B), (D), and (F)). In specific, the naïve matching results of outgoing sub-systems are usually as high as 80%-90%, even at low penetration. This is mainly because the traffic discharging processes are regulated by traffic signals at intersections. In other words, for the traffic that are in the same movement direction (e.g., left-turn traffic from side street), a dedicated signal phase is assigned to these vehicles and their indices in the traffic stream will usually remain the same. Despite the satisfying matching results associated with the naïve matching solutions at outgoing sub-systems, it is found that the TS solutions are more accurate than or at least similar to the naïve matching solutions.

At incoming sub-systems, it can be observed that the solutions obtained using the TS method are significantly better (15% in the best case) than the naïve matching method, especially at low mobile penetration. For example in Figure 6(D), at zero penetration (no mobile data), the accuracy is 25.7% for naïve matching, but about 41.1% for TS; at 10% penetration, the matching accuracies increase to 33.7% and 47.5%, respectively. As mobile penetration goes up, the numbers also increase accordingly. The results clearly indicate that the data fusion approach yields better matching results at each sub-system, compared to using fixed-location data only.

By linking the matching results at each sub-system with the input data at the downstream sub-system, the fixed-location data can be connected along the corridor. The corridor-level matching results (i.e., for the vehicles that traverse the entire corridor) were then compared with the ground truth, and the matching accuracies are summarized in Table 1. In contrast to using mobile sensing data only,[7] it is observed that the naïve matching method can improve the corridor matching accuracy by 13.3% to 19.0%, depending on the mobile penetration; while the proposed TS method improves the results by 13.6% to 22.4%, which outperforms the naïve matching method. The corridor-level matching accuracy is much lower compared to the numbers at individual sub-systems, as the matching errors accumulated over the entire corridor.

A straightforward application of the proposed approach is to estimate individual corridor travel times. For the vehicle records that are correctly matched, the travel time error is zero; for the vehicles that are mismatched, the travel time errors are the absolute differences between the estimations and the actual travel times; for the vehicle records that are linked outside of the corridor,

[7] For example, statistically speaking, 20% mobile penetration would result in a 20% corridor-level matching accuracy.

their travel time errors are not considered in the comparison. The Root Mean Square Error (RMSE) of the estimated individual travel times are also provided in Table 1. Compared with the naïve matching method, the TS method is able to estimate individual corridor travel times more accurately. At 20% mobile penetration, the proposed method correctly matches 41.9% of vehicle records; the RMSE of the estimated individual corridor travel time is 7.4 seconds, which is about 6% of the average corridor travel time.

Table 1: Corridor-level matching results and individual travel time

Mobile penetration	Naïve matching		Tabu search	
	Matching accuracy	RMSE of individual corridor travel time (seconds)	Matching accuracy	RMSE of individual corridor travel time (seconds)
0	19.0%	9.5	22.4%	9.3
20%	37.4%	9.4	41.9%	7.4
50%	66.2%	6.7	69.7%	6.3
80%	93.3%	1.7	93.6%	1.7

2.5.2 "Inaccurate data" scenario

The above results indicate the performance of the model for an ideal scenario, i.e., the fixed-location data and mobile sensing data are perfect and well synchronized. In practice, however, due to GPS inaccuracy, these two datasets may not coincide. For example, consider a probe vehicle that passes a point detector, the timestamp recorded by the point detector might be different from that in the probe data. The so called "inaccurate data" problem can be captured by equation (3) and equation (4), at downstream and upstream locations, respectively. And thus it is included in the objective function (8.1). To simulate the "inaccurate data" problem, the mobile sensing records are perturbed by a Gaussian noise, using equation (9). Here t_j is an original mobile sensing record at a downstream location, for record j, $\hat{t}_j$ is a perturbed one; Similar perturbations are conducted for the mobile sensing records at upstream locations. Different values of σ are tested in the experiment to see how GPS accuracy affects the modeling performance. [8]

$$\hat{t}_j - t_j \sim N(0, \sigma^2) \qquad (9)$$

The "inaccurate data" problem was then solved using the proposed model (8). The results reveal similar trend compared to the ideal scenario (i.e., σ equals to zero). In Figure 7, the sub-system

[8] Here the time accuracy of GPS records is considered. It is however directly comparable to conventionally used spatial accuracy. For example, assuming an operation speed of 10 meter per second, $\sigma = 0.5$ seconds is equivalent to a spatial accuracy (standard error) of 5 meters.

matching results are shown for the cases with $\sigma = 0.2$ seconds. It is found that the TS approach outperforms the naïve matching approach, especially at low penetration. However, the matching results are typically lower in this case since the mobile sensing data and fixed-location data are not perfectly matched. In Figure 8, corridor-level results are analyzed with respect to different GPS accuracy (i.e., σ = 0, 0.2, 0.5, 0.8, and 1.0 seconds). In particular, Figure 8(A) shows that when σ exceeds 0.5, the matching results do not necessarily increase while more mobile sensing data become available. This implies that if GPS accuracy is bad (i.e., standard deviation larger than 0.5 seconds), the data fusion approach may fail to yield better matching results, compared to using fixed-location data only. Figure 8(B) presents the results of individual corridor travel time estimation. Again, $\sigma = 0.5$ seconds can be identified as a cut-off point. For comparison purpose, the RMSE of corridor travel time was also plotted, estimated using the average travel time aggregated based on mobile sensing data only (the dash-dot line). The results clearly evidenced that fined-grained (individual-based) travel time estimation yields smaller estimation error and therefore can better describe individual travel pattern.

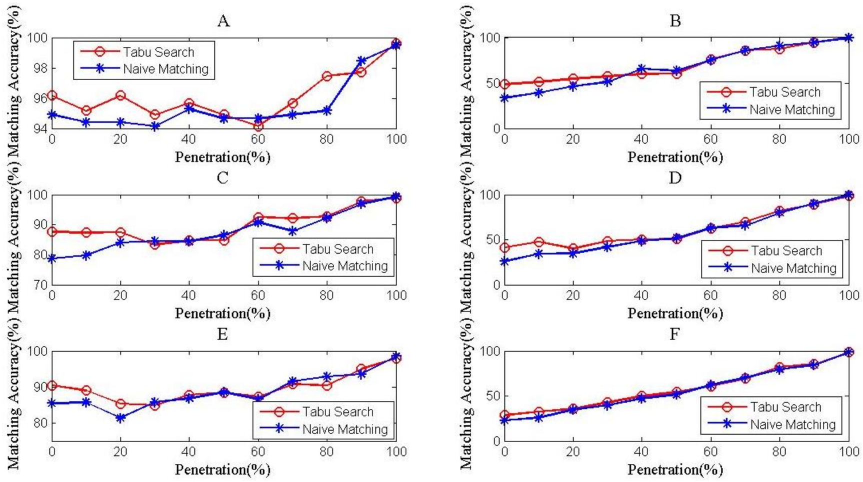


Figure 7: Matching results at sub-systems (inaccurate data, sigma = 0.2); (A) Intersection 1 outgoing; (B) Intersection 2 incoming; (C) Intersection 2 outgoing; (D) Intersection 3 incoming; (E) Intersection 3 outgoing; (F) Intersection 4 incoming

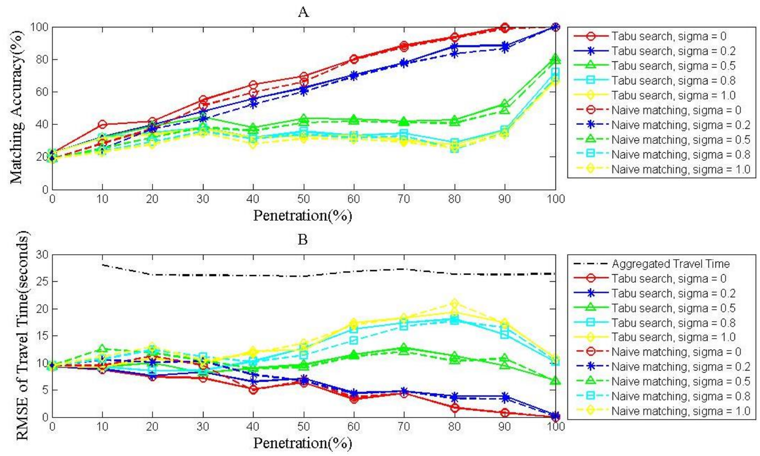


Figure 8: Corridor-level results (inaccurate data); (A) Matching accuracy; (B) RMSE of individual corridor travel time

2.5.3 "Detector failure" scenario

The "detector failure" scenario represents that the fixed-location sensors fail to detect some vehicles, or completely break down. It also applies to the situation where detector is not deployed for a specific lane. To model this scenario, an incoming sub-system was selected and some fixed-location records were randomly filtered out at downstream detector locations, while the detector data at other sub-systems remain the same. Results of the "detector failure" scenario are provided in Figure 9. In Figure 9(A), it is found that the matching accuracy at the sub-system reduces significantly if the number of undetected vehicles becomes larger. For example, at 20% penetration, if 5% of vehicles cannot be detected, the matching accuracy reduces to 36.4% from 37.8% in the ideal scenario; if 20% of vehicles cannot be detected, this number further reduces to 18.4%, which is lower compared to the penetration rate. This is mainly because some fixed-location records have been intentionally removed, and the mobile sensing records that match with the removed fixed-location records cannot be used in the matching process. Similar trend has been observed for the corridor-level results, as in Figure 9(B). In most cases, the proposed TS method performs better than the naïve matching method. In Figure 9(C), it is found if 20% or more vehicles are undetected, the estimated individual travel times become less reliable and are close to the estimations using average travel time. Nonetheless, it was found the data fusion approach always led to better results compared to using single data source only.

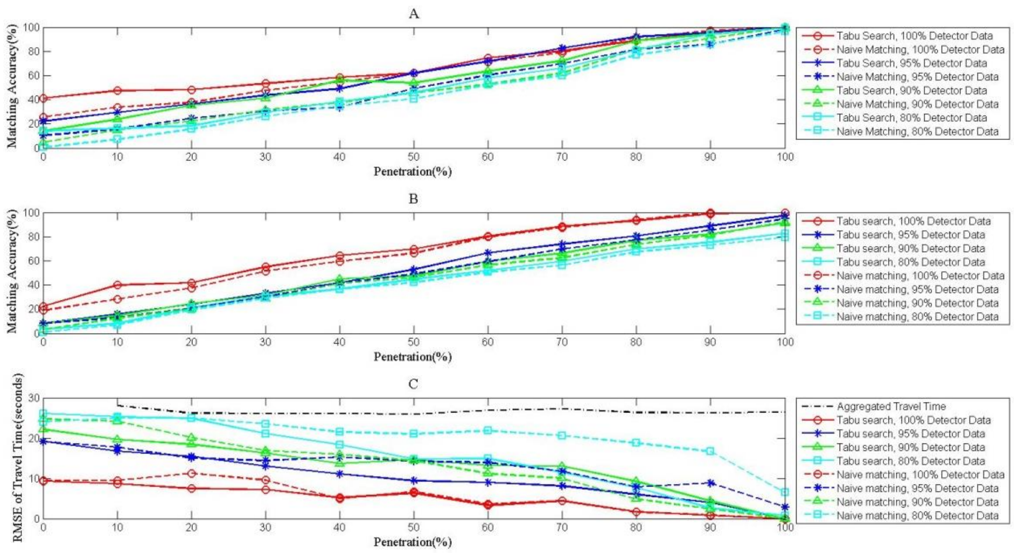


Figure 9: Results of Detector Failure Scenario; (A) Matching accuracy at Intersection 2 incoming sub-system; (B) Corridor-level matching accuracy; (C) RMSE of individual corridor travel time

2.6. Discussions

In this research, a data fusion and information integration approach was proposed to interpret traffic flow and matching of data between different locations or sources along arterial corridor. This was done by making connections between the upstream and downstream traffic data at various sensor locations. Historical traffic information such as travel time, lane choice, and traffic merging were integrated into the matching algorithms. By validating the fixed-location data against mobile sensing data, some upstream and downstream fixed-location records can be directly matched, and the matching results can therefore be significantly improved. In addition to the ideal scenario which assumed the fixed-location data and mobile sensing data are accurate and perfectly matched, some practical issues were also studied related to the use of heterogeneous traffic data, such as data errors, and detection failures.

Compared with the naïve matching method (i.e., matching only based on vehicle indices), the results indicated that the proposed method yields better matching results and can estimate individual corridor travel time more accurately, especially at lower penetration. This feature of the proposed model is practically favorable since the mobile penetration is very low in current practices. In the ideal scenario, at 20% mobile penetration, the proposed method correctly matches 41.9% of vehicle records (corridor-level); the RMSE of the estimated individual corridor travel time is 7.4 seconds, which is about 6% of the average corridor travel time. Similar trends were observed for the “inaccurate data” and the “detector failure” scenarios. Per the specific application, 0.5 seconds (standard deviation) were identified as the cut-off GPS accuracy. Incorporating mobile sensing records with poorer accuracy may not enhance the performance of the proposed model. Moreover, it was found that the “detector failure” issue drastically affects the modeling performance. A tentative solution to this problem is to consider some missing-data imputation method that helps to fill in the undetected records.

The results provided in this research are only based on the experiments conducted using the 15-minute NGSIM data, with small sample size. It will be interesting to see how the proposed data fusion framework performs given larger network and larger datasets. The proposed method may also be extended to other fine-grained urban traffic applications. For example, the matching results can be used to further infer queue length information, estimate vehicle trajectories and energy/emissions. These traffic applications will be pursued in the future study.

3. Time-Dependent Effects in Freight Trip Attraction Models

3.1 Review of Freight Trip Generation Models

Some of the first applications of temporal stability analysis in transportation related models were presented for the case of vehicle trip generation models. Parameters in most of such cases were found to be very stable in time [6]. Depending on the type of models and nature of the predictors (independent variables) considered the models may be more or less stable over a time period. Also, conditional to whether or not the model parameters are stable over the time, the models could be used to forecast the travel demand in future instances in the design of new investment projects. The amount of research devoted to the analysis of transferability of trip generation models and the study of time-dependent effects in trip generation model parameters is very scarce and even less research has been done in the case of time-dependent effects in freight trip generation model parameters and its implications in transportation planning and policy making processes.

Since trip generation models are commonly estimated as a function of different socioeconomic variables that could be easily collected via surveys or using census data, the use of such type of models is quite common among transportation practitioners for passenger trip demand modeling. It is also commonly assumed that those models can be used to predict future conditions. In those cases only changes in variables such as population and population income for aggregate models, and family size or car ownership in the case of disaggregate models are expected. The underlying assumption is that the marginal effects of these variables in the production of vehicle trips do not change over time. However, as some early studies show, this is not always the case. In the study presented by Doubleday [7], the parameters for different socioeconomic variables used as predictors in a passenger trip generation model are analyzed and their temporal stability is tested. The study also presents an analysis of the nature of the common assumptions of temporal stability of trip generation models. One of the main results of this study is that variables such as employment status could present significant time-dependent changes for certain categories of models and specific groups in the population such as housewives, employed females, and retired persons, especially when disaggregate models are implemented with the individual as the behavioral unit.

The hypothesis of temporal stability of trip generation model parameters have been extensively tested in the literature related to vehicle trip modeling. Most studies presented in the related literature, demonstrate the stability of vehicle trip generation model parameters [6, 8, 9]. Different approaches are used to test the temporal stability of passenger trip generation model parameters.

One of these approaches is the collection of data from a particular population through surveys in two different time periods. Then passenger trip generation models are estimated from either the older or most recent data set and using the estimated models, predictions are drawn for its counterpart and compared with the real observations. Hypothesis tests could be used to statistically prove the similarity between the predicted trips and the real observations. This approach was implemented by Kannel and Heathington [6]; they implemented along with a parameter values comparison, a chi-square contingency analysis and proved that there were no statistical difference between the estimated and observed number of trips. The study was made for a time period of 7 years predicting both future and past observations for disaggregate travel demand models.

McCarthy [10] used before and after data sets to estimate multinomial logit models of work trip modal choice as well. His study shows that the estimates of the coefficients of the model are transferable in time, especially for short term forecasting applications, since all variable coefficients in his model were stable through short periods of time. Another approach commonly used to test the temporal stability of parameters in trip generation models is either to include time dependent variables in the trip generation models and test their statistical significance or to perform hypothesis tests on parameters from a base period model against a design or end period model parameters. By the means of a hypothesis test McCarthy [10] also proves at a 95% level of confidence, the temporal stability of the model coefficients for estimated for 1973-4 and 1975. The study corresponds to a before and after implementation of the Bay Area Rapid Transit (BART) in the city of San Francisco.

The research conducted by Gunn [8] used the same approach to test the transferability of model coefficients of disaggregate travel demand models at the beginning and end of a ten years period. The study was applied for different European regions, in order to test special transferability as well. In this study, estimates were drawn from both data sets which used revealed and stated preference data respectively. Contrasting with previous cases presented, they compared the model parameters of the different variables in the beginning and ending period data sets by looking for scaling factors, where a scaling factor of 1.0 would indicate that the model parameters are transferable. The results of the study confirmed the transferability of travel demand models, suggesting that these set of models were suitable for forecasting purposes.

The study presented by Mwakalonge, Waller and Perkins [11], also confirmed the short term temporal stability for linear-regression trip generation models for total travel time. The approach they used to test the temporal stability of the model parameters was to run a t-test on the coefficients of the independent variables (predictors) for the base year models against the design year models. The results show that the parameters for variables such as certain family structures, household size, and constant term were found to be temporally stable. They also present a comparative analysis of observed and predicted values of trips. One of the key findings of this work is that although regression trip generation models for total travel time present short-term temporal stability, their ability to correctly predict future scenarios decrease with time, which reveals that long term predictions should not be obtained with these models. The study was mainly centered in analyzing the non-motorized vehicles trip generation, for which temporal stability could not be proved. Huntsinger [9] also presents evidence of temporal stability for mode choice and logistic regression models for trips generation. His study also shows the implication of changes in trips rates for project level planning processes. Similar to the previous works presented this research also implements statistical tests on model parameters to assess their stability over time using data from the period between 1995 and 2006. Parameter equality between the two models tested is statistically proved in almost all cases studied via t-tests. This study also measures the precision of the prediction of future observed behavior in trip making processes, where a percentage error is calculated for the predicted versus the observed cases.

The analysis of different studies presented in the literature of passenger trip generation suggests that in general, we could say that for the specific case of passenger vehicle trip generation the parameters of predictor variables tend to be stable over short periods of time. This however, is not the case for long period studies, which is commonly the case in both design and investment projects and policy making processes. In such cases, the implementation of old models from previous studies whose parameters are no longer applicable to current conditions in new project studies could lead to serious under or overestimation of travel demand, which in turn would have a negative impact on the transportation system as a whole.

Parameter stability studies in transportation demand modeling aim primarily at demonstrating the extent to which the parameters of models are either geographically or temporally transferable. Geographic transferability refers to the ability of the model parameters to produce accurate estimations in different geographic regions than those for which the model was initially estimated.

On the other hand, temporal stability refers to the ability of a model to be used to produce estimations in a different time period. In other words, these types of studies could be applied in order to evaluate the validity of models developed in transportation studies from past years, when used for estimations in current conditions and determine whether or not time-dependent effects are present affecting the marginal effect of the variables used as estimators in actual conditions. Neglect of these time-dependent effects when using models to forecast traffic demand could lead to serious under/overestimation errors that could cause regulations and policies based on these estimations to be completely ineffective.

The implementation of models for different time periods to make comparisons by direct observation of the values of the parameters to determine whether or not significant changes are present from period to period is one of the first approaches employed to analyze temporal stability. The downside of these models is obviously that they neither provide any means of measuring the time-dependent effects nor present a way to test the statistical significance of time-dependent effects. One of the few studies analyzing time-dependent effects specifically in freight trip generation models is presented by [12], Holguín-Veras, Sarmiento, González, Thorson and Sánchez [13], [14]. In this study conducted using freight origin-destination samples collected by the Colombian Government, the stability of parameters of freight generation, distribution and empty trip models is tested. Statistically significant time-dependent effects were found, suggesting that for the case of Colombia, freight trip generation and distribution models are not stable throughout time, and a time-dependent effect has to be taken into account in the modeling process for freight demand forecasting. For this study, a set of fixed time-dependent effects for the different years considered were considered which were found to be very significant at a 95% confidence level, for most of the cases presented. A direct comparison between the estimated parameter values for each year is also presented where we can clearly observe a difference of around 1.4 times the value for the first year compared with the last year. This implies a yearly increase in the rates of freight trips produced in the country. In the case of freight demand and freight distribution models, rates of change with respect to the previous year are calculated as well. Freight demand models were found to have higher percentages of change with respect to freight trip distribution models [14]. One of the most important conclusions of this study suggests that regardless of the functional form of the model used to estimate freight trips demand, the parameters of the model increase over time. This holds true for the case of trip production and attraction models as well as the case of trip distribution models.

One limitation that can be observed in the case of these studies in the assessment of the stability of freight trip generation model parameters is that the analysis presented on parameter stability is only for models using aggregate data and not for establishment based data. This is sufficient to draw general conclusions regarding the identification of time-dependent effects in the model parameters, however, not sufficient to measure these effects at a more disaggregate level, e.g., industry type. Another limitation could be the lack of inclusion of more explanatory variables such as establishment sales, as well as interaction terms between the different socioeconomic variables included, which could better explain the changes in the values of model parameters over the time. Hence, a set of models produced using more disaggregate data as well as assessing the effect of more explanatory variables could yield more accurate ways to estimate the freight trip demand models.

The work presented by Oliveira-Neto, Chin and Hwang [15] concludes that aggregated models for freight demand are not suitable to produce "reasonable predictions of freight for a future year horizon" (Oliveira-Neto, Chin and Hwang [15], p. 2). They also suggest that more time-dependent factors such as some productivity related variables should be included in the modeling process for freight demand in order to increase the quality of the predictions drawn, and better analyze and explain the time-dependent factors affecting freight demand model parameters. The study utilized aggregate data from the Commodity Flow Survey (CFS) and the County Business Patterns of the U. S. Census. The results showed that although the models produced for all of the U.S. states which includes 27 industry sectors and common socioeconomic variables (such as the state payroll) account for a significant portion of the freight generated, these models lack enough predictive power to be implemented in future year studies.

3.2 Data used in this research

This section explains in detail the process followed in this study to obtain the data, and the preparation process conducted in order to use the data gathered in the modeling process of freight trip generation with time-dependent effects. The origin and characteristics of each one of the data sets used is explained along with the variables selected to be included in the set of models developed. A brief discussion about the distribution and descriptive statistics of these variables is also presented.

The data used for this study corresponds to different datasets from a series of surveys collected in 2005, 2006, 2011 and 2014, by the Center for Infrastructure, Transportation, and the Environment (CITE) at Rensselaer Polytechnic Institute (RPI), as part of different research projects regarding subjects such as freight trip generation and land use, service trips, off hour deliveries initiatives among others. The data collected contains disaggregated data at the establishment level from New York City. The type of establishments surveyed corresponds to shippers receivers and carriers located in the New York City metropolitan area. Since the purpose of each survey was different for each project, the entire set of variables contained in each data set varies, however, a consolidated database was produced out of the four survey datasets available, with the trip attraction, production, establishment and economic related data necessary for this study. Table 2 presents a summary of the datasets used in this study, the different projects for which they were gathered and the number of records for each case. The economic and business characteristic related information of the surveyed companies was obtained mainly from Dun & Bradstreet.

Table 2: Relevant variables by dataset

Year	Related Project	Relevant Variables	Carriers	Receivers	Total records
2005	OHD Pilot	Amount of shipments, deliveries, type of business, type of transporter, fleet size, average time of the operation, employment, location, contact information, revenue	192	180	372
2006	OHD Pilot	Amount of shipments, deliveries, type of business, type of transporter, fleet size, average time and cost of the operation, employment, location, contact information, revenue	139	200	339
2011	OHD Implementation	Amount of shipments, deliveries, type of business, type of transporter, shipment size, average time of the operation, employment, location, contact information, revenue	-	-	263
2014	NCFRP25	Amount of shipments, deliveries, service trips,type of business, type of transporter, shipment size, average payload, fleet size and type, employment, location, contact information, revenue	-	-	450

The present study processed and analyzed shipment and delivery data from the surveyed businesses which contains an important number of variables with relevance for freight trip generation studies. However, due to the heterogeneity of the four datasets some variables could not be included since there were not observations for all four periods. The set of variables used in this study include time-related variables, the number of daily shipments and deliveries received by each business, the number of part and full time employees in the actual location for each surveyed establishment, and the industry type associated with the business obtained from the North American Industry Classification System (NAICS) code. As part of the information obtained from Dun & Bradstreet, data regarding business reported revenue (in million dollars), location, including state, county, and ZIP code. In Table 3 a summary of the descriptive statistics of the four principal variables used in the modeling process of this study is presented.

From the mean and standard deviation values, we can observe how heterogeneous the industry types are, especially for the case of employment and revenue, which varies between the industry sectors. From the standard deviation, we can also observe heterogeneity in terms of variability among industry sectors. Even within subgroups of industry sectors such as NAICS 31-33 corresponding to manufacturing businesses and NAICS 44-45 corresponding to retailers, we can observe considerable differences in terms of central tendency and variability. This suggests that different set of models to describe and predict freight trip generation patterns should be estimated by industry type. Although the statistical difference between industry types could be easily proven by means of hypothesis tests, the evidence from the descriptive statistics and current practices reported in the literature of freight trip generation modeling go in favor of the above-mentioned approach.

Table 3: Relevant variables descriptive statistics

Revenue (Mi. UDS)											
NAICS	**23**	**31-33**	**31**	**32**	**33**	**42**	**44-45**	**44**	**45**	**48-49**	**72**
Obs	44	123	37	33	53	146	179	121	58	12	87
Mean	18.86	60.94	22.94	17.54	142.35	93.58	7.41	11.37	3.46	37.73	2.30
Std. Dev.	41.69	764.15	49.65	18.54	762.31	817.37	66.87	64.92	16.04	42.64	7.41
Min	0.2800	0.0003	0.0010	0.0029	0.0003	0.0002	0.0002	0.0002	0.0002	5.8500	0.0006
Max	244	5543	210	61	5543	9700	650	650	120	153	58

Employment											
NAICS	**23**	**31-33**	**31**	**32**	**33**	**42**	**44-45**	**44**	**45**	**48-49**	**72**
Obs	45	157	43	44	70	156	228	154	74	14	100
Mean	35.67	37.71	35.44	32.37	45.31	25.09	18.44	17.38	19.51	45.76	27.88
Std. Dev.	46.23	90.27	48.49	43.29	62.64	40.25	41.44	24.86	33.15	60.22	31.86
Min	5.00	1.00	2.45	2.45	1.00	1.00	1.00	1.00	1.00	3.45	4.00
Max	250	350	200	200	350	305	202	202	173	195	180

Deliveries											
NAICS	**23**	**31-33**	**31**	**32**	**33**	**42**	**44-45**	**44**	**45**	**48-49**	**72**
Obs	45	156	42	44	70	155	228	154	74	14	99
Mean	4.33	4.93	2.99	4.50	7.29	6.06	4.64	4.78	4.49	11.26	3.87
Std. Dev.	5.90	14.66	2.47	6.06	13.11	11.17	10.96	6.73	8.65	16.31	3.99
Min	0.20	0.18	0.40	0.18	0.20	0.20	0.20	0.20	0.20	0.20	0.36
Max	30	80	15	25	80	85	60	50	60	50	25

Shipments											
NAICS	**23**	**31-33**	**31**	**32**	**33**	**42**	**44-45**	**44**	**45**	**48-49**	**72**
Obs	19	70	14	22	34	48	66	45	21	7	13
Mean	3.79	7.98	5.93	11.64	6.37	6.41	5.33	4.59	6.06	10.86	4.63
Std. Dev.	4.52	16.92	8.25	12.89	7.22	7.49	12.23	9.24	8.02	11.68	5.81
Min	1.00	0.18	1.00	0.18	0.18	0.36	0.18	0.18	0.18	1.00	0.18
Max	20	40	25	40	25	35	40	40	25	30	20

Some of the limitations encountered with the data used for this study include the fact that, since the datasets for each year were collected with a different purpose, the set of variables available in each case is not the same. Thus, while for the case of 2014 data set, contains variables a lot of variables of interest to include in the modeling process, it could not be included as most of these variables were not available in the 2005, 2006, and 2011 datasets. A set of interaction variables were also generated between the dummy and integer variables representing the time in years with both employment and revenue. Another limitation encountered is the considerable amount of missing values in both dependent (deliveries and shipments) and independent variables (employment and revenue), which reduced the number of observations available by about half. A high number of outliers due to typing errors resulted in high values for shipment which significantly reduced number of employees; this, also contributed to the reduction of the number of observations available, since these observations had to be removed. Nonetheless, conveniently, the remaining number of observations was still sufficient to produce statistically valid models.

As remarked in the previous section, the raw data received from the surveys contained a lot of unit mismatches, missing observations, and outliers and a large set of variables that are not relevant to the present study. As such, a data preparation process was necessary in order to produce a clean and consolidated database combining all datasets from the four years considered and keeping only the set of variables relevant for this study. The first step in this process was to identify the set of variables relevant for this study included in all datasets for the different years. The set of variables included is: the number of deliveries, the number of shipments, the number of employees in the actual location, the industry type, year, and revenue. The observations with missing values were identified for the selected variables and removed. The next step was to identify outliers within the database, ensuring consistency between employment and revenue of the establishment with the reported amount of shipments and deliveries. This process, however, was not an easy task since for the case of employment and revenue a considerable amount of observations with high values lying above the 1.5 interquartile range were found, as Figure 10 and Figure 11 shows. This could suggest the presence of sub categories of revenue for wholesale and retail sectors (NAICS 44-45) especially. In the case of employment, the same pattern for these two sectors can also be observed. However, for simplicity purposes and in order to avoid an excessively high number of models to estimate, only models categorized by industry sectors were estimated. Conversely, some observations in industry sectors with high revenue and number of employees which could look like outliers, are just some of the handful of cases of large companies involved in the transportation business either as receivers or shippers that own a fleet of vehicles and conduct their own logistics operations. Therefore, outliers had to be carefully checked and contrasted with the data gathered from Dun & Bradstreet.

Figure 10: Distribution of revenues by industry sectors

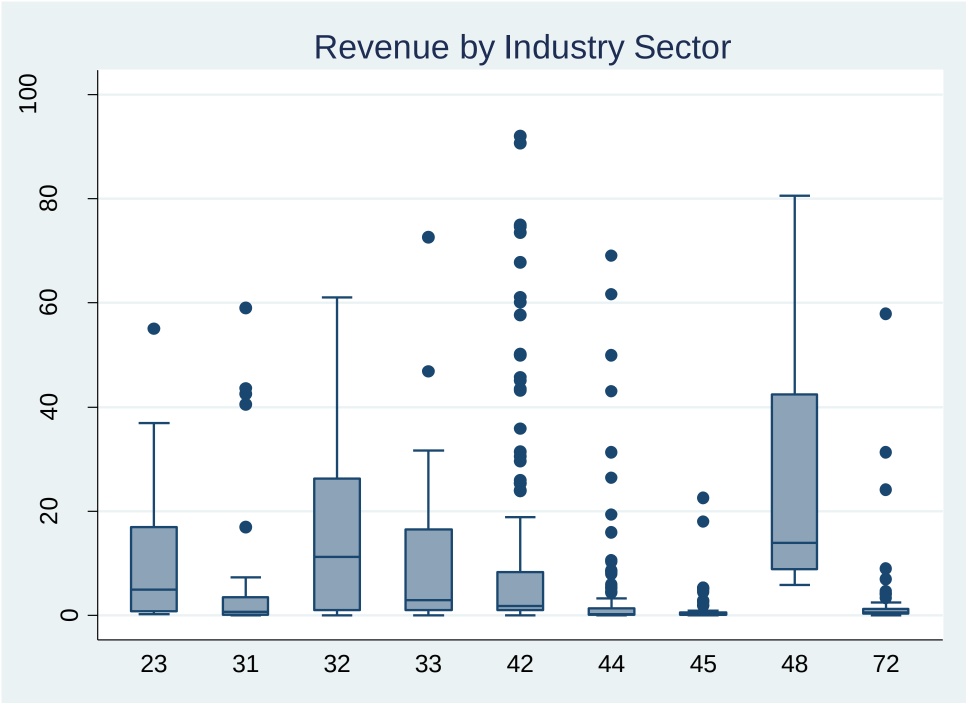
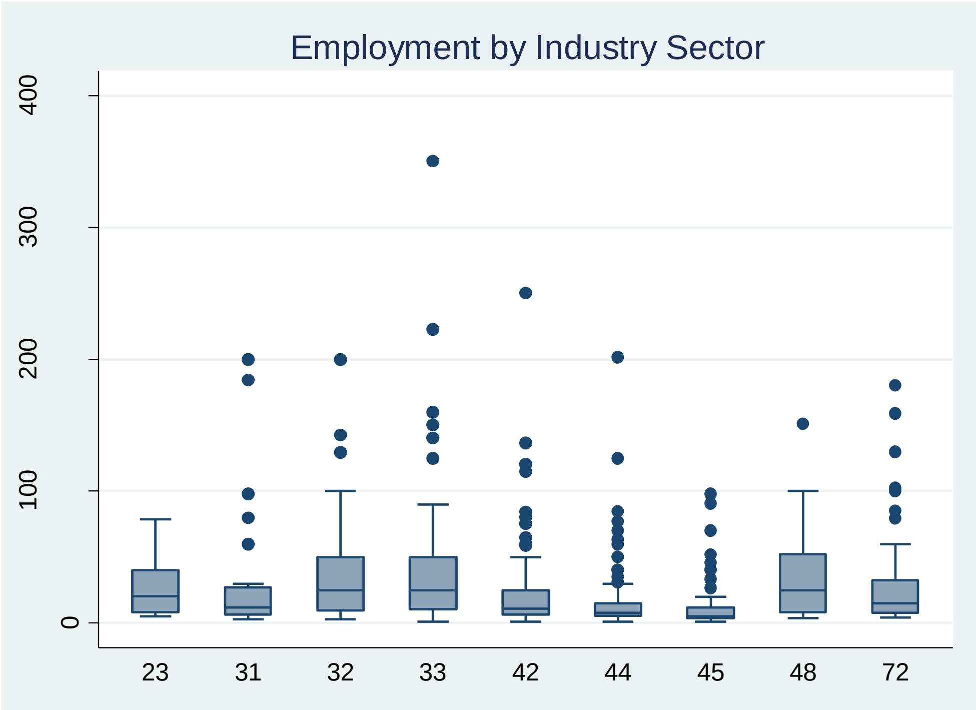


Figure 11: Distribution of employment by industry sector



After removing missing values and outliers from the database, the next step consisted of transforming all variables into a single unit of measurement. For instance, while most observations reported the number of shipments or deliveries per day, some only reported this value per week. Therefore, a conversion factor of 5.5 days per week was implemented, to transform the shipments per week into shipments per day. This factor, as reported in [5], considers the five weekdays and a working time of half a business day for Saturdays as commonly implemented in many transportation related companies. The same procedure was implemented in the case of deliveries. For employment, since most of the cases reported both full time and part time employees, a conversion factor of 0.45 was applied, in order to produce a full-time equivalent for the variable employment.

The next set of preprocessing steps performed on the database were to keep the variables needed for the modeling process and generate the set of dummy and interaction variables needed to estimate models in different functional forms as will be explained in the methodology. Firstly, a set of dummy binary variables was generated for the time to represent the different years available as $\delta_y = 1$ if year = y, 0 otherwise. An integer version of the time variable was generated in order to not only consider a fixed effect of the time but to also analyze the effect of the course of time on freight trip generation model parameters. For this case, the year 2005 was set as the origin, 2006 as 1, 2011 as 6, and 2014 as 9, corresponding to 1, 6 and 9 years elapsed from the initial year 2005. However, this would consider only an incremental effect of time on freight trip generation models. Therefore, a case for piecewise regression was considered; this would break the time variable into two periods comprised from 2005-2011 and from 2011-2014, as historical evidence suggest the presence of the great recession which occurred in 2008, affecting the first period and a reactivation of the economy affecting the second period. Finally, a set of interaction variables between the dummy variables and the independent variables (employment, revenue), of the form $\delta_y X = \delta_y * X$ was also created, were X represents the independent variable. A second set of interaction variables was similarly generated, between the integer time variables, employment, and revenue, as well as an interaction term between revenue and employment as $T_1 X = T_1 * X$.

After generating this set of variables the final preparation step consisted of generating variables corresponding to the natural logarithm of the employment and revenue, along with its different interaction terms in order to be able to estimate linear and nonlinear functional forms of the freight trip generation models. The final database was then kept for the modeling process which will be discussed in the following section.

3.3 Methodology and Modeling Process

This sections explains the approach followed to model freight trip generation including time dependent effects and revenue in order to assess and analyze temporal effects in such types of models along with its interaction with employment and revenue. The procedures implemented to model freight trip generation with fixed time-dependent effects are first discussed. Then, a second approach implementing continuous time effects is discussed, along with its limitations and an alternative approach using piecewise functions. Finally a brief discussion on the different functional forms used is presented.

The estimation and analysis of time-dependent effects in freight trip generation models performed in this study comprise a series of modeling processes including different approaches. Models considering fixed time effects are estimated in order to test the presence of significant time-dependent effects in freight trip generation models. A set of models considering continuous time effects is also estimated with the objective of testing significant effects in trip rates with the passing of time. Finally, another approach including piecewise linear functions is tested. The procedure followed to implement these approaches as well as the advantages and limitations of each method are presented in the following sections. The modeling process developed considered in the first place, a model including all variables deemed relevant to the generation of freight trips. Then, variables found to be statistically non-significant were systematically removed. After this process the conceptual validity of remaining variables was tested; as part of this, independent variables with negative effect on the estimation of freight trips generated were systematically removed as well. The reason behind this is that the employment, revenue, and time along with their interactions, which were considered as independent variables in this study, tend to have a positive effect in the generation of freight trips. Reductions in the number of trips generated as a result of an increase in the number of employees or the revenue of an establishment are neither probable nor consistent with the observed behavior of the type of freight intensive businesses considered in this study. Finally, the bests models with statistically and conceptually valid variables were selected and tabulated for freight trip attraction and production in each one of the approaches considered.

To test the statistical significance of fixed time-dependent effects on freight trip generation models, a set of econometric models were estimated including fixed time variables. From the reported year of each observation, a binary variable was created for each observation in the dataset obtained from the surveys conducted on carriers and receivers from New York City. Variables were created for 2005, 2006, 2011 and 2014, where the statistical significance of these variables reveals the presence of a time-dependent effect for the particular year represented for each variable. This method quantifies the fixed time-dependent effects in freight trip generation models, that is to say, the number of additional trips included in the model when a time-dependent effect is found to be statistically significant for a given time period. To include these variables in the estimated freight trip generation models, the year 2005 was selected as the base year, hence removed from the variables included in the models to avoid collinearity problems. These type of models were estimated using OLS as presented in Equation (15):

$$FTG = \beta_0 + \beta_i x_i + \beta_j \delta_j + \beta_k \delta_j x_i$$

(1)

Where β represent the parameters, δ stands for the Dummy variable representing time period and x the independent variables. Significant effects on the time effect variables created were tested at a 95% of confidence for the majority of the models. Some models included variables significant at a 90% of confidence following the process explained at the beginning of this section. A clear disadvantage of this method is that it only involves fixed time effects for the different periods considered in the modeling process. Therefore, although it is able to quantify time effects, this method lacks predictive power which prevents the models that incorporate this method from being efficient for forecasting purposes.

Since fixed time effects cannot be used for forecasting purposes, which is the case of most transportation infrastructure design and transportation system planning studies, an approach where time is treated as an integer variable "T" increasing in one unit of time for each period elapsed was implemented including its interaction with the independent variables denoted here as "I". The objective of this was to test statistically significant effects in variables accounting for the passing of time, in order to take into account time-dependent effects in freight trip generation models used in planning and design applications. These type of models are presented in Equation (16):

$$FTG = \beta_0 + \beta_i x_i + \beta_T T + \beta_I I \qquad \textbf{(2)}$$

These types of models are well-suited not only to quantify the time-dependent effects in freight trip generation models but also to be implemented for forecasting purposes in future scenarios and design projects, since the time is included as an independent variable in the model. With this method, the model is able to quantify the effect of time in the number of trips produced. A clear disadvantage of this approach is a very strong assumption that the time-dependent effects are always increasing, which is not necessarily the case, since different socioeconomic events might arise during specific time periods affecting negatively or positively the traffic demand and then disappear in following periods. Such types of events could not be captured by this type of models. Due to this limitation, the final set of models were estimated as a piecewise function defining two different periods: before and after the financial crisis of 2007-2008, with 2011 as the breaking point. This process is described in the following subsection.

Finally as part of this study a piecewise version of the continuous time effect models was implemented as an approach to avoid the assumption made in the case of considering the time as continuous which only allows increasing time effects. This method, however, can only be implemented in cases in which information is available regarding significant events that may take place in future scenarios that may drastically affect the traffic demand in the transportation system. Equation (17) describes these type of models.

$$FTG = \beta_0 + \beta_i x_i + \beta_{T1} T_1 + \beta_{T2} T_2 + \beta_I I$$

(3)

Where:

T_1= Includes the time periods from the origin to the period of disruption

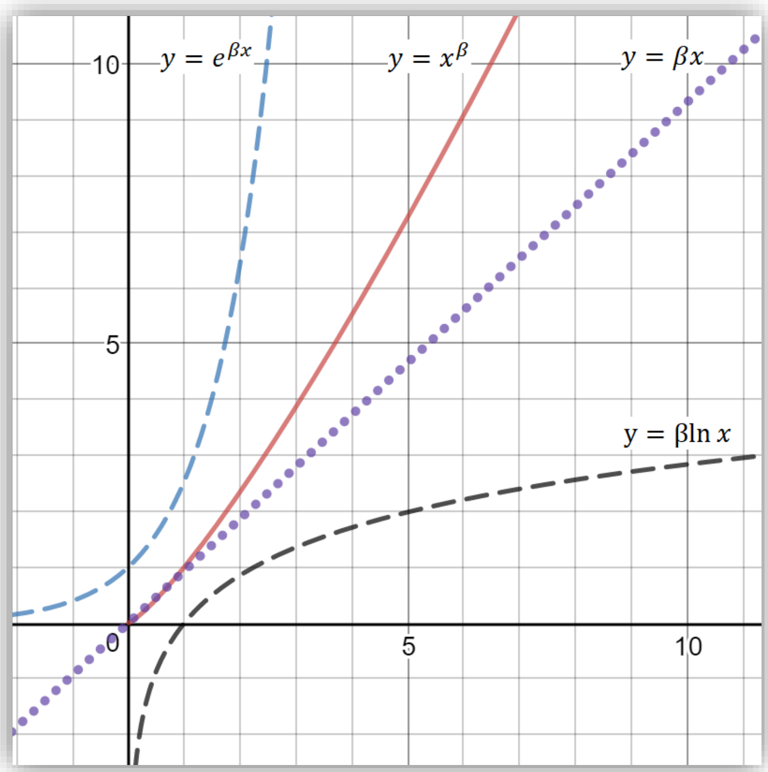
T_2= Includes the time periods starting from the period of disruption onwards

In this study, the period of disruption considered is the financial and economic crisis of 2007-2008 that took place in the United States of America, which triggered the great recession.

Given the inherent heterogeneity of freight trip generation patterns among the different industry sectors and the lack of larger sets of explanatory variables due to data limitations, different functional forms were considered in this study in an effort to develop more accurate freight trips demand models. The rationale behind this is that as previous studies show[5], certain industry sectors could display a constant rate of trips generated while others may increase their rate of trip production proportionally to employment or revenue. Assuming that across all industry sectors only one particular functional form is applicable could lead to serious errors in the estimation of trips. Four functional forms were considered. This allows the model estimation and time-dependent effect analysis to accommodate different types of behaviors within industry sectors which are reflected in different data patterns encountered.

The first type of models is linear in nature since they estimate the number of trips generated as a function of a constant term plus a parameter determining the rate at which changes in the independent variables increase or decrease the number of trips generated. The second set of models are nonlinear and correspond to transformations of nonlinear functions in order to produce linear regression models, which would be able to describe nonlinear patterns in the data. Among these types of models, we have linear-logarithmic models, logarithmic-linear models, and logarithmic-logarithmic models. A graphic representation of these functional forms is presented in Figure 12

Figure 12: Models functional forms



The advantage of considering nonlinear functional forms is that as the equations suggest these type of models are extremely flexible and able to account for different types of patterns that could arise from the data corresponding to the different industry sectors and their interactions with the time-dependent variables analyzed.

3.4 Results and Analysis

A set of linear regression models were estimated using the survey data collected for the New York City area as described in previous sections. Data from 2005, 2006, 2011 and 2014 were included, and a set of econometric models for freight trip attraction and production was estimated using employment as the independent variable. With the intent to capture the time-dependent effect that could take place due to the nature of the behavior of the market, a set of indicator variables were generated and included in the models as binary variables whose statistical significance would imply fixed time-dependent effects taking place in the estimation process of freight trip generation models. Considering the limitations associated with fixed time-dependent effects, a set of integer time-dependent variables were also generated setting the year 2005 as the origin. A second round of econometric models was then estimated and analyzed. Finally, a set of models was estimated for freight trip attraction and production considering both fixed and continuous time-dependent effects, by including time as well as revenue in order to test to what extent the time-dependent effects encountered could be explained by changes in economic factors such as the revenue of the establishments. The results are presented in Table 5.1 to Table 5.8. The values of the t-statistics are presented in parenthesis. The models were estimated at a level of significance of 95% for the majority of the cases. In a few cases, some variables were accepted at a 90% of confidence. For each industry sector, one model for each one of the four functional forms was estimated and the best models (those that best fit to the data) were selected for each industry sector.

A description of the variables and codes used in the tables of results is presented below.

NAICS codes:

- 23 Construction
- 31-33 Manufacturing
- 31 Food, Beverage, Tobacco, Textile, Apparel
- 32 Wood, Paper, Chemical, Plastics Nonmetals
- 33 Metal, Machinery, Electronics, Furniture & Misc.
- 42 Wholesale Trade
- 44-45 Retail Trade
- 44 Motor Vehicle, Furniture, Electronics, Clothing
- 45 Sporting Goods, Hobby, Books, & Music Stores
- 48-49 Transportation and Warehousing
- 72 Accommodation and Food Services

Variables:

- Const Constant term denoting a fixed number of trips

- δ_i Binary variable equal to 1 if the year of the observation is equal to i, 0 otherwise
- Empl Employment
- lnEmpl Natural logarithm of employment
- E Employment
- lnE Natural logarithm of employment
- Rev Revenue
- lnRev Natural logarithm of employment
- ln(Rev*E) Natural logarithm of Revenue*Employment
- T_{11} Number of years elapsed starting from 2005 up to 2011
- T_{14} Number of years elapsed starting from 2011 up to 2014

Type of model:

- Lin Linear-model
- Lin-Lg Linear-logarithmic model
- Lg-Lin Logarithmic-linear model
- Lg-Lg Logarithmic-logarithmic model

The first set of models estimated as part of this study correspond to freight trip attraction models. Aiming at determining the presence of significant fixed and continuous time-dependent effects in freight trip attraction models a set of indicative variables –one for each year– was introduced in the models and their statistical significance was tested. The results reveal that significant time-dependent effects exist throughout the industrial sectors considered, refuting for the case of freight trip attraction models, the common assumption in passenger trip generation models that parameters are stable over the time. In particular, the results show that there is a very significant time-dependent effect, especially for the year 2014, which occurs across all industry sectors, with the exception of accommodation and food services (NAICS 72). As shown in tables, for the year 2014 there is a significant increase in the number of trips produced. In the case of construction, manufacturing and retail businesses, this happens when the employment increases as well since, the time-dependent effect appears in an interaction with the employment. The results are presented in Table 5.1.

Table 4: Freight trip attraction models with fixed time effects

FTA Models with Fixed Time Effects												
Type of model		**Lin**	**Lin**	**Lin**	**Lin**	**Lg-Lg**	**Lg-Lg**	**Lg-Lg**	**Lin**	**Lin**	**Lg-Lg**	**Lin**
NAICS		**23**	**31-33**	**31**	**32**	**33**	**42**	**44-45**	**44**	**45**	**48-49**	**72**
Const	**Coeff.**	2.965	2.620	2.337	1.979	-0.516	-	-	2.975	2.794	-	1.739
	t-stat	(4.17)	(4.92)	(9.79)	(1.84*)	(-1.80*)	-	-	(4.33)	(4.99)	-	(3.51)
δ_6	**Coeff.**	-	-	-	-	-	-	0.685	-	-	-	-
	t-stat	-	-	-	-	-	-	(2.43)	-	-	-	-
δ_{11}	**Coeff.**	-	-	-	-	0.512	-	-	-	-	-	2.694
	t-stat	-	-	-	-	(2.20)	-	-	-	-	-	(2.23)
δ_{14}	**Coeff.**	-	-	-	-	-	0.561	-	-	11.206	1.765	-
	t-stat	-	-	-	-	-	(2.32)	-	-	(4.65)	(4.68)	-
Empl	**Coeff.**	-	-	-	-	-	-	-	0.100	-	-	0.073
	t-stat	-	-	-	-	-	-	-	(4.2)	-	-	(6.11)
lnEmpl	**Coeff.**		-	-	-	0.396	0.387	0.392	-	-	-	-
	t-stat		-	-	-	(4.16)	(13.25)	(14.31)	-	-	-	-
δ_6*E	**Coeff.**	-	-	-	-	-	-	-	0.534	-	-	-
	t-stat	-	-	-	-	-	-	-	(4.07)	-	-	-
δ_{11}*E	**Coeff.**	-	-	-	-	-	-	-	-	-	-	-
	t-stat	-	-	-	-	-	-	-	-	-	-	-
δ_{14}*E	**Coeff.**	0.067	0.101	0.040	0.151	-	-	-	0.184	-	-	-
	t-stat	(3.34)	(11.56)	(7.75)	(8.9)	-	-	-	(4.04)	-	-	-
δ_{14}*lnE	**Coeff.**	-	-	-	-	0.225	-	0.205		-	-	-
	t-stat	-	-	-	-	(3.49)	-	(2.65)		-	-	-
Obs.		57	166	49	50	67	198	232	158	74	12	95
F-stat		11.12	133.60	60.04	79.13	23.99	53.34	34.80	24.96	21.61	0.00	19.80
R2		0.17	0.45	0.56	0.62	0.53	0.21	0.23	0.33	0.23	0.38	0.30
S2		23.93	40.23	2.49	47.20	0.58	1.04	0.89	47.92	21.98	1.42	11.70

All variables shown are significant at a 95% (except for those signaled in the table with the sign "*" whose level of significance is 90%)

Table 5.1 shows that there is a constant number of delivery trips attracted to the businesses regardless of the effect of the employment –the negative value in the case of NAICS 33 corresponds to a very low number of trips since the model is log-log–. In general, most model functional forms are linear. For NAICS 33, 42 and 44-45, the final models are log-log, however, with parameter values between 0.205 and 0.685, the function resembles a linear function with a very small slope. For wholesalers and transportation related businesses, this constant is only present for the year 2014. The significant time-dependent effect present for the year 2014 suggest that construction, manufacture, and retail businesses were recovering from the financial crisis of

2007-2008. In particular, metal, machinery, electronics, furniture & misc. manufacturers and food accommodation services also present significant time-dependent effects for 2011. The significant time-dependent effect for the year 2006 interacting with the employment for the case of motor vehicle, furniture, electronics, and clothing businesses could suggest a growth in this sector just before the occurrence of the financial crisis.

Table 5: Freight trip attraction models with continuous time effects

FTA Models with Continuous Time Effects												
Type of model		**Lin**	**Lin**	**Lin**	**Lin**	**Lg-Lin**	**Lg-Lin**	**Lin**	**Lin**	**Lin**	**Lg-Lg**	**Lin**
NAICS		**23**	**31-33**	**31**	**32**	**33**	**42**	**44-45**	**44**	**45**	**48-49**	**72**
Const	**Coeff.**	2.965	2.620	2.337	1.979	0.510	0.482	3.085	3.220	2.794	-	1.735
	t-stat	(4.17)	(4.92)	(9.79)	(1.84*)	(3.87)	(4.3)	(5.61)	(4.48)	(4.99)	-	(3.49)
T_{11}	**Coeff.**	-	-	-	-	-	-	-	-	-	-	0.445
	t-stat	-	-	-	-	-	-	-	-	-	-	(2.21)
T_{14}	**Coeff.**	-	-	-	-	0.155	0.546	-	-	-	-	-
	t-stat	-	-	-	-	(1.68*)	(4.75)	-	-	-	-	-
Empl	**Coeff.**	-	-	-	-	0.009	0.025	0.082	0.107	-	-	0.073
	t-stat	-	-	-	-	(4.49)	(5.41)	(4.08)	(4.3)	-	-	(6.11)
lnEmpl	**Coeff.**	-	-	-	-	-	-	-	-	-	0.447	-
	t-stat	-	-	-	-	-	-	-	-	-	(3.37)	-
T_{14}*E	**Coeff.**	0.022	0.034	0.013	0.050	-	-0.009	0.053	0.058	3.735	-	-
	t-stat	(3.34)	(11.56)	(7.75)	(8.9)	-	(-4.38)	(4.27)	(3.63)	(4.65)	-	-
Obs.		57	166	49	50	67	198	232	158	74	12	95
F-stat		11.12	133.60	60.04	79.13	21.10	16.57	29.86	26.50	21.61	0.00	19.73
R2		0.17	0.45	0.56	0.62	0.40	0.20	0.21	0.25	0.23	0.09	0.30
S2		23.93	40.23	2.49	47.20	0.74	1.07	45.99	52.73	21.98	2.09	11.71

All variables shown are significant at a 95% (except for those signaled in the table with the sign "*" whose level of significance is 90%)

From the models presented in Table 5.2, produced for freight trip attraction considering continuous time-dependent effects, we can observe that throughout the different industry sectors considered, time-dependent effects are highly significant at a 95% of confidence in the majority of the cases, which corroborates that for the case of freight trip attraction the common assumption of temporal stability of trip generation models parameters does not hold. In this case, the time-dependent variables account for one time period from 2005-2011 (before and during the crisis) as well as another period from 2011-2014 (after the crisis). In general, all sectors except for accommodation and food services, describe a significant time-dependent effect of increased number of trips for the period after the crisis. This is consistent with the results of models with fixed effects. Food and accommodation sector do not show significant time-dependent effects for the period after the crisis which is also consistent with what can be expected since regardless of the effects of the crisis, the food is a basic and necessary good. Time-dependent effects were not significant in the case of the transportation sector according to this models, however, due to the reduced number of observations available for this sector, this results may not be conclusive.

The second set of models estimated corresponds to freight trip production models. The objective as in the case of freight trip attraction models was to determine the presence of significant fixed and continuous time-dependent effects in freight trip production. The results for freight trip production show different patterns for the different industry sectors considered. However, a general conclusion from these models is that for freight trip production models, time-dependent effects are significant and very important to explain the behavior of the production of freight trips in businesses from freight intensive sectors. The results in this case also do not support the common assumption in passenger trip generation models that parameters are stable over the time. All models present interactions between time-dependent variables and employment which suggest that the production of freight trips across all industry sectors is a function of a combined effect between time and employment. Although the models show different behaviors across the industry sectors considered, the year 2014 appears again as a differential factor increasing the production of freight trips. The results for freight trip production models are presented in Table 5.3.

Table 6: Freight trip production models with fixed time effects

FTP Models with Fixed Time Effects												
Type of model		**Lin**	**Lg-Lin**	**Lg-Lg**	**Lg-lin**	**Lg-Lg**	**Lg-Lg**	**Lin**	**Lin**	**Lin-Lg**	**Lg-Lg**	**Lg-Lin**
NAICS		**23**	**31-33**	**31**	**32**	**33**	**42**	**44-45**	**44**	**45**	**48-49**	**72**
Const	**Coeff.**	-	0.562	-	-	0.586	0.923	19.000	17.000	-	-	-
	t-stat	-	(3.57)	-	-	(2.38)	(4.95)	(6.22)	(5.01)	-	-	-
δ_6	**Coeff.**	-	-	-	-	-	-	-17.500	-15.750	-	-	-
	t-stat	-	-	-	-	-	-	(-4.43)	(-3.51)	-	-	-
δ_{11}	**Coeff.**	-	-	-	-	-	-	-17.030	-16.231	-	-	-
	t-stat	-	-	-	-	-	-	(-5.24)	(-4.43)	-	-	-
δ_{14}	**Coeff.**	-	-	1.058	2.070	-	0.957	-	-	11.206	-	0.933
	t-stat	-	-	(2.34)	(5.65)	-	(2.9)	-	-	(4.65)	-	(2.22)
Empl	**Coeff.**	-	-	-	0.156	-	-	-	-	-	-	-
	t-stat	-	-	-	(1.92*)	-	-	-	-	-	-	-
lnEmpl	**Coeff.**	-	-	0.343	-	-	-	-	-	6.552	-	-
	t-stat	-	-	(4.02)	-	-	-	-	-	(4.41)	-	-
δ_6*E	**Coeff.**	0.068	-	-	-0.149	-	-	-	-	-	-	-
	t-stat	(3.18)	-	-	(-1.82*)	-	-	-	-	-	-	-
δ_6*lnE	**Coeff.**	-	-	-0.308	-	-	-	-	-	-5.898	-	-
	t-stat	-	-	(-3.03)	-	-	-	-	-	(-2.88)	-	-
δ_{11}*E	**Coeff.**	-	-	-	-	-	-	0.236	0.297	-	-	-
	t-stat	-	-	-	-	-	-	(5.03)	(4.87)	-	-	-
δ_{11}*lnE	**Coeff.**	-	-	-	-	-	-	-	-	-3.070	-	-
	t-stat	-	-	-	-	-	-	-	-	(-1.88*)	-	-
δ_{14}*E	**Coeff.**	0.095	-	-	-0.150	-	-	-	-	-	-	0.012
	t-stat	(10.06)	-	-	(-1.84*)	-	-	-	-	-	-	(2.22)
δ_{14}*lnE	**Coeff.**	-	0.425	-	-	0.315	-	-	-	-	0.413	-
	t-stat	-	(7.92)	-	-	(3.89)	-	-	-	-	(4.07)	-
Obs.		17	69	15	24	30	44	51	32	19	7	11
F-stat		56.74	62.74	40.03	7.29	15.13	8.39	15.97	12.78	11.14	0.00	13.02
R2		0.79	0.48	0.87	0.52	0.35	0.17	0.50	0.58	0.58	0.34	0.59
S2		4.99	0.93	0.20	1.20	0.99	1.04	37.38	34.60	32.17	0.90	0.62

All variables shown are significant at a 95% (except for those signaled in the table with the sign "*" whose level of significance is 90%)

As the results show, the model for freight trip production in the construction sector is a function of a combined effect between the time and the employment for the case of 2006 and 2014. There are no significant effects for the years 2005 and 2011 in this case. The absence of effects in the year 2011 could suggest a pause in this sector due to the financial crisis. In the case of the manufacturing sector the models are described by a constant term denoting a constant number of trips produced by these establishments –although very small–, and a parameter increasing the number of trips with the interaction between time and employment. Breaking down the industry sectors corresponding to manufacturers we can observe that NAICS 31 and 32 present a similar pattern, with a constant rate of trips produced for the year 2014 and a parameter increasing the number of trips produced with the employment. These models also present a parameter reducing the number of freight trips produced with the number of employees in the establishment in 2006. This means that the effect of the employment in the year 2006 is even smaller than the other years. NAICS 33 is also described by a constant number of trips produced plus a parameter increasing the production of freight trips with the employment in 2014, as is the case in the combined manufacturing sector. Wholesale and retail sectors show a common pattern of decrease in the number of freight trips produced during the crisis and a reactivation after the crisis. In particular, the retail sector which tends to produce more freight trips compared to other sectors, present a significant decrease in 2006 and 2011 in the constant amount of trips produced which then increases again in 2014. For NAICS 44 this reactivation takes place in 2011 in interaction with the employment. Finally, transportation and food sectors only present significant effects in the employment of 2014.

Table 5.4 presents the results for the set of models of freight trip production considering continuous time-dependent effects. The objective with these models is to detect continuous time-dependent effects in these models which in the case of models with fixed effects are only detected as an intercept –base number of freight trips produced–. From the results, we can observe that there are significant time-dependent effects increasing the number of freight trips produced in the establishments either with the number of years elapsed or the interaction between the employment and the number of years elapsed from the initial year. As expected, these effects are very different for the two periods considered, being in most cases very low for the period before and during the crisis and increasing considerably for the period after the crisis.

Table 7: Freight trip production models with continuous time effects

FTP Models with Continuous Time Effects												
Type of model		**Lin**	**Lg-Lin**	**Lg-Lin**	**Lin**	**Lg-Lin**	**Lg-Lin**	**Lin**	**Lin**	**Lin**	**Lin-Lg**	**Lin**
NAICS		**23**	**31-33**	**31**	**32**	**33**	**42**	**44-45**	**44**	**45**	**48-49**	**72**
Const	**Coeff.**	-	0.512	-	-	0.552	0.923	-	-	-	-	-
	t-stat	-	(2.98)	-	-	(2.1)	(4.95)	-	-	-	-	-
T_{11}	**Coeff.**	-	-	-	-	-	-	0.330	-	0.622	-	-
	t-stat	-	-	-	-	-	-	(1.81*)	-	(2.04)	-	-
T_{14}	**Coeff.**	-	0.588	0.878	3.907	0.452	0.319	6.333	5.667	-	-	-
	t-stat	-	(7.31)	(11.34)	(3.08)	(3.65)	(2.9)	(6.28)	(5.15)	-	-	-
Empl	**Coeff.**	-	-	-	-	-	-	-	-	-	2.164	-
	t-stat	-	-	-	-	-	-	-	-	-	(2.72)	-
T_{11}*E	**Coeff.**	0.068	-	0.008	-	-	-	0.039	0.052	0.027	-	-
	t-stat	(3.18)	-	(3.73)	-	-	-	(5.08)	(6.18)	(2.21)	-	-
T_{14}*E	**Coeff.**	0.032	-	-	0.037	-	-	-	-	0.184	-	0.038
	t-stat	(10.06)	-	-	(3.25)	-	-	-	-	(3.94)	-	(5.51)
Obs.		17	69	15	24	30	44	51	32	19	7	11
F-stat		56.74	53.47	73.75	21.09	13.34	8.39	24.44	40.25	7.26	0.00	0.00
R2		0.79	0.44	0.85	0.49	0.32	0.17	0.50	0.57	0.48	0.22	0.54
S2		4.99	1.00	0.22	129.50	1.03	1.04	36.62	32.67	40.34	55.57	16.91

All variables shown are significant at a 95% (except for those signaled in the table with the sign "*" whose level of significance is 90%)

The common pattern in these models is a reactivation increasing the number of freight trips produced yearly by establishment from manufacture, wholesale and retail sectors. For the particular case of retailers, the results show that this sector's freight trip production can be estimated as by a parameter denoting the marginal contribution on freight trips generated by the interaction between the number of years elapsed in the first period –before crisis– and the employment, then, the number of trips start to increase at a much higher rate with the years. Transportation, in this case, does not seem to be affected by time-dependent effects, however, as mentioned before the small number of observations for this sector makes it difficult to consider these results representative of the sector.

As demonstrated by the model's results from previous sections there are significant time-dependent effects affecting both production and attraction of freight trips of the industry sectors considered for this study. Additionally, another aspect inherent to all types of businesses is that during the course of time, several technology changes might take place that could improve the productivity and consequently the revenue of the particular businesses from the sector for which such technology is developed. In such cases where improvements in the revenue of a business take place over the time, the time-dependent effects exposed in the previous sections could be explained by those changes in the revenue of businesses. This section presents a set of models developed aiming at determining whether or not the revenue is significant in freight trip generation models, and if time-dependent effects in could be explained by the presence of such variables. The results reveal that the variable revenue is very significant to explain the dynamics of freight trip generation of the freight intensive sectors considered for this study. The revenue was found significant in 68% of the final models estimated for freight trip generation either by itself or in a combined effect with employment. The results show that for some specific industry sectors, the revenue or the interaction of revenue with employment are the factors explaining the generation of freight trips. However, time-dependent effects are not completely canceled with the inclusion of revenue but rather, is the combination of the interactions of revenue with time-dependent effects and the employment that were found significant to explain the number of freight trips generated by the establishments of freight intensive sector businesses.

Table 8: Freight trip attraction models with fixed time effects and revenue

FTA Models with Fixed Time Effects												
Type of model		**Lg-Lg**	**Lg-Lg**	**Lin**	**Lin**	**Lg-Lg**	**Lg-Lin**	**Lin**	**Lin**	**Lin**	**Lin**	**Lin**
NAICS		**23**	**31-33**	**31**	**32**	**33**	**42**	**44-45**	**44**	**45**	**48-49**	**72**
Const	**Coeff.**	-	-	1.851	3.025	-	0.711	2.150	2.239	2.613	13.083	1.037
	t-stat	-	-	(6.24)	(3.31)	-	(6.16)	(3.71)	(3.77)	(2.52)	(2.67)	(1.84*)
δ_6	**Coeff.**	-	-	-	-	-	1.169	4.869	-	-	-	1.705
	t-stat	-	-	-	-	-	(4.37)	(2.49)	-	-	-	(1.74*)
δ_{14}	**Coeff.**	-	-	-	-	1.877	-	-	-	-	-	-
	t-stat	-	-	-	-	(7.41)	-	-	-	-	-	-
R	**Coeff.**	-	-	-	-	0.140	-	0.860	0.440	1.477	-	-
	t-stat	-	-	-	-	(1.78*)	-	(5.71)	(2.72)	(5.63)	-	-
Empl	**Coeff.**	-		-	-	-	-	0.057	0.087	-	-	0.105
	t-stat	-		-	-	-	-	(2.46)	(4.56)	-	-	(5.60)
lnEmpl	**Coeff.**	-	0.558	-	-	-	-	-	-	-	-	-
	t-stat	-	(5.11)	-	-	-	-	-	-	-	-	-
δ_6*R	**Coeff.**	-	-	0.210	-	-	-	1.476	-	-	-	-
	t-stat	-	-	(2.54)	-	-	-	(2.46)	-	-	-	-
δ_6*E	**Coeff.**	-	-	-	-	-	0.021	-	0.466	-	-	-
	t-stat	-	-	-	-	-	(1.84*)	-	(4.75)	-	-	-
δ_{11}*R	**Coeff.**	-		44.097	-	-	-	-	-	-	-	-
	t-stat	-		(2.00)	-	-	-	-	-	-	-	-
δ_{11}*lnR	**Coeff.**	-	0.114	-	-	-	-	-	-	-	-	-
	t-stat	-	(2.71)	-	-	-	-	-	-	-	-	-
δ_{11}*E	**Coeff.**	-	-	-	-	-	0.023	-	-	-	-	-
	t-stat	-	-	-	-	-	(2.95)	-	-	-	-	-
δ_{14}*R	**Coeff.**	-	-	-	-	-0.139	-	-0.842	-0.416	-1.323	-	-
	t-stat	-	-	-	-	(-1.77*)	-	(-5.58)	(-2.57)	(-4.96)	-	-
δ_{14}*E	**Coeff.**	-	-	0.043	-	-	-	-	-	-	-	-0.047
	t-stat	-	-	(8.51)	-	-	-	-	-	-	-	(-1.95*)
R*E	**Coeff.**	-	-	-	0.002	-	-	-	-	-	-	-
	t-stat	-	-	-	(5.90)	-	-	-	-	-	-	-
ln(R*E)	**Coeff.**	0.186	-	-	-	-	-	-	-	-	-	-
	t-stat	(6.57)	-	-	-	-	-	-	-	-	-	-
Obs.		44	15	36	33	27	145	179	121	58	12	86
F-stat		0.00	19.89	25.11	34.76	13.50	8.60	17.57	19.43	18.30	0.00	17.32
R2		0.16	0.60	0.70	0.53	0.53	0.15	0.34	0.40	0.40	0.00	0.39
S2		0.98	0.25	2.27	21.92	0.94	1.28	40.44	28.02	55.89	288.81	9.44

Note: All variables shown are significant at a 95% (except those with "*" with a level of significance of 90%)

From the result presented in Table 8 we can observe that in the case of models estimated for construction, wood, paper, chemical, plastics and nonmetals sectors (NACIS 23 and 32), the interaction between revenue and employment is the significant factor accounting for the freight delivery trips attracted to these businesses. The interaction between revenue and fixed time-dependent effects and also between employment and fixed time-dependent effect were found significant in the case of retailers, manufacturers and food service related businesses, especially for the year 2014, again presumably, accounting for a reactivation post financial crisis. Significant effects for interactions between the years 2006, 2011 and revenue, as well as the same years with employment were found for wholesale, retail, and manufacture related businesses. In particular, revenue was not significant for industry sectors such as food and accommodation services, transportation, and wholesalers.

Table 9 shows the results corresponding to models considering continuous time-dependent effects. These type of models also show that time-dependent effects have been replaced by the interaction between time and revenue in the case of construction and manufacturing industry sectors. We can also observe that again, very different slopes of the functions corresponding to the time periods before and after the crisis. An interesting patterns found for the attraction of freight trips in retail industry sector is that, the time-dependent effect for the period after the crisis, present in previous versions of these type of models, seems to be replaced by the variable revenue, which means that specifically for this industry sector the attraction of freight trips in the period after the financial crisis can be explained by the changes in the revenue of this establishments.

Table 9: Freight trip attraction models with continuous time effects and revenue

FTA Models with Continuous Time Effects												
Type of model		**Lg-Lg**	**Lg-Lin**	**Lin**	**Lin**	**Lg-Lin**	**Lg-Lg**	**Lin**	**Lin**	**Lg-Lg**	**Lg-Lin**	**Lin**
NAICS		**23**	**31-33**	**31**	**32**	**33**	**42**	**44-45**	**44**	**45**	**48-49**	**72**
Const	**Coeff.**	-	0.591	1.997	3.025	0.627	-	2.680	2.628	1.590	1.845	1.694
	t-stat	-	(5.30)	(6.68)	(3.31)	(4.03)	-	(4.31)	(4.38)	(4.99)	(5.29)	(3.48)
T_{11}	**Coeff.**	-	-	-	-	-	-	-	-	-	-	0.453
	t-stat	-	-	-	-	-	-	-	-	-	-	(2.27)
T_{14}	**Coeff.**	-	-	-	-	0.235	-	-	-	-	-	-
	t-stat	-	-	-	-	(2.28)	-	-	-	-	-	-
R	**Coeff.**	-	-	-	-	-	-	0.027	0.024	-	-	-
	t-stat	-	-	-	-	-	-	(2.77)	(3.01)	-	-	-
Empl	**Coeff.**	-	0.010	-	-	0.007	-	0.095	0.086	-	-	0.072
	t-stat	-	(6.46)	-	-	(3.06)	-	(4.36)	(4.37)	-	-	(6.10)
T_{11}*R	**Coeff.**	-	-	0.200	-	-	-	1.974	2.905	-	-	-
	t-stat	-	-	(2.33)	-	-	-	(3.09)	(4.93)	-	-	-
ln(T_{11}*R)	**Coeff.**	-	-	-	-	-	-	-	-	0.251	-	-
	t-stat	-	-	-	-	-	-	-	-	(3.86)	-	-
ln(T_{11}*E)	**Coeff.**	-	-	-	-	-	0.327	-	-	-	-	-
	t-stat	-	-	-	-	-	(10.52)	-	-	-	-	-
T_{14}*E	**Coeff.**	-	-	0.014	-	-	-	-	-	-	-	-
	t-stat	-	-	(8.02)	-	-	-	-	-	-	-	-
R*E	**Coeff.**	-	6E-06	-	0.002	-	-	-	-	-	-	-
	t-stat	-	(2.22)	-	(5.90)	-	-	-	-	-	-	-
ln(R*E)	**Coeff.**	0.186	-	-	-	-	0.096	-	-	-	-	-
	t-stat	(6.57)	-	-	-	-	(3.63)	-	-	-	-	-
Obs.		44	122	36	33	70	52	179	121	33	12	99
F-stat		0.00	25.30	32.98	34.76	14.77	36.11	15.39	21.25	14.89	0.00	19.76
R2		0.16	0.30	0.67	0.53	0.31	0.42	0.21	0.35	0.32	0.00	0.29
S2		0.98	0.95	2.46	21.92	1.04	0.55	47.69	30.03	0.66	1.46	11.53

Note: All variables shown are significant at a 95% (except those with "*" with a level of significance of 90%)

The production of freight trips was also analyzed. The results of this process are presented in Table 10.

Table 10: Freight trip production models with fixed time effects and revenue

FTP Models with Fixed Time Effects												
Type of model		**Lin**	**Lg-Lin**	**Lg-Lin**	**Lg-Lg**	**Lg-Lg**	**Lg-Lin**	**Lin**	**Lin**	**Lin**	**Lg-Lg**	**Lin**
NAICS		**23**	**31-33**	**31**	**32**	**33**	**42**	**44-45**	**44**	**45**	**48-49**	**72**
Const	**Coeff.**	2.196	-	2.635	-	-	-	-	-	-	-	2.313
	t-stat	(3.73)	-	(11.26)	-	-	-	-	-	-	-	(2.50)
δ_6	**Coeff.**	-	-	-2.462	-	-	-	-	-	-	-	-
	t-stat	-	-	(-7.44)	-	-	-	-	-	-	-	-
δ_{11}	**Coeff.**	-	-	-2.181	-	-	1.273	-	-	-	-	-
	t-stat	-	-	(-6.40)	-	-	(4.05)	-	-	-	-	-
δ_{14}	**Coeff.**	-	2.198	-	-	-	1.850	19.500	18.4	-	-	-
	t-stat	-	(13.81)	-	-	-	(6.43)	(7.01)	(5.79)	-	-	-
R	**Coeff.**	-	-	-	-	-	0.214	-	-	4.718	-	-
	t-stat	-	-	-	-	-	(3.62)	-	-	(6.97)	-	-
δ_6*R	**Coeff.**	-	-	-	-	-	-	-	-	-4.235	-	-
	t-stat	-	-	-	-	-	-	-	-	(-4.56)	-	-
δ_{11}*R	**Coeff.**	-	23.925	16.817	-	-	-	1E+03	9E+02	-	-	-
	t-stat	-	(1.84)	(2.14)	-	-	-	(6.26)	(5.03)	-	-	-
δ_{11}*E	**Coeff.**	-	-	-	-	-	-	-	-	1.034	-	-
	t-stat	-	-	-	-	-	-	-	-	(8.54)	-	-
δ_{14}*R	**Coeff.**	-	-	-	-	-	-0.214	-	-	-	-	-
	t-stat	-	-	-	-	-	(-3.62)	-	-	-	-	-
δ_{14}*lnR	**Coeff.**	-	-	-	-	-	-	-	-	-	0.558	-
	t-stat	-	-	-	-	-	-	-	-	-	(4.98)	-
δ_{14}*E	**Coeff.**	0.040	-	-	-	-	-	-	-	-	-	-
	t-stat	(1.83*)	-	-	-	-	-	-	-	-	-	-
R*E	**Coeff.**	2E-04	-	-	-	-	-	-	-	-	-	0.004
	t-stat	(2.11)	-	-	-	-	-	-	-	-	-	(6.19)
ln(R*E)	**Coeff.**	-	-	-	0.349	0.219	-	-	-	-	-	-
	t-stat	-	-	-	(9.15)	(8.29)	-	-	-	-	-	-
Obs.		19	56	13	16	27	45	52	36	16	7	12
F-stat		33.49	56.81	22.05	0.00	0.00	5.69	52.77	37.69	38.34	0.00	38.32
R2		0.81	0.51	0.88	0.36	0.50	0.29	0.51	0.53	0.86	0.32	0.79
S2		4.42	0.86	0.22	0.93	0.93	1.09	46.43	50.41	12.86	1.04	8.24

Note: All variables shown are significant at a 95% (except those with "*" with a level of significance of 90%)

In the case of freight trip production models, we can observe that indeed the revenue was found to be a very significant variable to explain the production of trips within all industry sectors. Time-dependent effects disappear and are replaced by the effect of revenue only for the manufacturing and food and accommodation industry sectors. The interaction between employment and the year

2014, which was found significant across almost all industry sectors also disappears in all industry sectors except for the construction sector. This pattern shows that the production of freight trips, in general, could rather be explained by the revenue or the interaction between time-dependent effects and the revenue of the establishment for freight intensive sectors.

Table 11: Freight trip production models with continuous time effects and revenue

FTP Models with Continuous Time Effects												
Type of model		Lin	Lg-lin	Lg-lin	Lg-Lg	Lg-Lg	Lg-lin	Lin-Lg	Lin-Lg	Lin-Lg	Lg-Lg	Lin
NAICS		23	31-33	31	32	33	42	44-45	44	45	48-49	72
Const	Coeff.	2.196	-	-	-	-	0.907	-	-	-	-	2.313
	t-stat	(3.73)	-	-	-	-	(4.66)	-	-	-	-	(2.50)
$\ln T_{11}$	Coeff.	-	-	-	-	-	-	13.927	12.911	19.013	-	-
	t-stat	-	-	-	-	-	-	(7.10)	(5.78)	(5.03)	-	-
T_{14}	Coeff.	-	0.733	0.878	-	-	0.323	-	-	-	-	-
	t-stat	-	(12.99)	(10.96)	-	-	(2.78)	-	-	-	-	-
$\ln(T_{11}*R)$	Coeff.	-	-	-	-	-	-	4.193	4.047	5.505	-	-
	t-stat	-	-	-	-	-	-	(6.06)	(4.99)	(4.35)	-	-
$T_{11}*E$	Coeff.	-	0.008	0.008	-	-	-	-	-	-	-	-
	t-stat	-	(2.97)	(3.63)	-	-	-	-	-	-	-	-
$\ln(T_{14}*R)$	Coeff.	-	-	-	-	-	-	-	-	-	0.421	-
	t-stat	-	-	-	-	-	-	-	-	-	(4.75)	-
$T_{14}*E$	Coeff.	0.013	-	-	-	-	-	-	-	-	-	-
	t-stat	(1.83*)	-	-	-	-	-	-	-	-	-	-
R*E	Coeff.	2E-04	-	-	-	-	-	-	-	-	-	0.004
	t-stat	(2.11)	-	-	-	-	-	-	-	-	-	(6.19)
ln(R*E)	Coeff.	-	-	-	0.349	0.219	-	-	-	-	-	-
	t-stat	-	-	-	(9.15)	(8.29)	-	-	-	-	-	-
Obs.		19	70	14	16	27	48	44	29	15	7	12
F-stat		33.49	55.67	64.54	0.00	0.00	7.74	37.39	25.34	19.33	0.00	38.32
R2		0.81	0.45	0.84	0.36	0.50	0.14	0.47	0.48	0.60	0.27	0.79
S2		4.42	0.97	0.23	0.93	0.93	1.25	27.71	27.01	23.84	1.12	8.24

Note: All variables shown are significant at a 95% (except those with "*" with a level of significance of 90%)

Finally, the set of models considering continuous time-dependent effects reveal again the different patterns in the production of trips between the before and after crisis periods. As we can observe from Table 11, the time-dependent effects for the period after crisis are replaced by the effects of revenue in the case of manufacturing, retail, and food services industry sectors. However, some of these results might not be conclusive since due to data limitations, the number of observation available was reduced since not all establishments are shippers, and among the set of shippers included in this study many of them did not report their revenue.

The results, in general, reveal that indeed the time has a significant effect on the parameters of freight trip generation models, thus when not considered, these effects could cause serious estimation errors. We can also observe that the models are consistently remarking an increase in the amount of trips in the year 2014 or the period corresponding to years between 2011 and 2014 which is right after the financial crisis that took place in 2007-2008. This pattern is present especially for the construction and manufacturing industry sectors as well as wholesale and retail. This could suggest a reactivation of the economy for this period which could be explained by the reactivation that took place after the economic crisis from 2007-2008. In the case of freight trip production models, in particular, that trend showing an increase in the number of trips produced in the year 2014 is also present. Models with fixed time-dependent effects also suggest that in the years 2006 and 2011 the number of trips produced was drastically reduced which is also consistent with the effects produced by an economic crisis.

When the revenue is considered, some interesting patterns are also observed. In the first place, we can observe that in general, the variable revenue is very significant to explain the attraction of freight trips, especially in the case of the industry sectors of manufacturing, wholesale and retail. Additionally, combined effects between the revenue and employment. This pattern reveals that both variables contribute to better explain the attraction of trips. We can also observe that the significance of the time variable for the year 2014 in the case of fixed effects, and the period of time between 2011 and 2014 for the case of continuous time effects disappears for most of the industry sectors when the revenue is considered, which suggests that the variable revenue also helps to explain the changes in freight trip generation patterns produced in time.

Finally, an interesting pattern can also be observed with the inclusion of time-dependent effects and the revenue in freight trip generation models. From previous studies, it is known that power models –included in the OLS regression as logarithmic-logarithmic models- tend to better fit to the data in freight trip generation in most of the cases [5]. The results of this study reveal that when the time-dependent effects variable and the revenue are taken into account, linear models tend to produce better results than exponential and power models for most of the cases, which could suggest that the effect of these new variables tend to linearize the shape of the models.

With the objective of testing the overall performance of the estimations of the models presented for this study, a comparative analysis is conducted between models presented in the literature of freight trip generation for New York City by Holguin-Veras, et al. [5] that do not consider time-dependent effects and the models estimated as part of the present work. The comparison is presented by means of two measures of forecast accuracy. First, the root mean squared error (RMSE) of the estimations drawn by both sets of models is used and the results are compared and analyzed. Then the models are compared again using the mean absolute percentage error (MAPE) and some concluding remarks are presented. The results of this comparison are presented in Table 12 and Table 13.

Notation:

Model P25: The type of functional form used for freight trip generation models presented by Holguin-Veras, et al. [5] which do not consider time-dependent effects.

RMSE P25: The root squared mean errors of models presented by Holguin-Veras, et al.[5]

MAPE P25: The mean absolute percentage error of models presented by Holguin-Veras, et al. [5]

Model TE: The type of functional form of models considering time-dependent effects.

RMSE TE: The root mean squared errors of models considering time-dependent effects.

MAPE P25: The mean absolute percentage error of models considering time-dependent effects.

Table 12: Freight trip attraction models RSME comparison

NAICS	FTA			
	Model P25	**RMSE P25**	**Model TE**	**RMSE TE**
23	Log-Log	5.689	Lin	**4.892**
31-33	Lin	8.835	Log-Log	10.001
31	Log-Log	1.970	Lin	**1.506**
32	Lin	5.631	Lin	**4.811**
33	Log-Log	12.379	Log-Log	**11.885**
42	Log-Log	11.012	Log-Log	12.681
44-45	Log-Log	6.794	Lin	**6.359**
44	Log-Log	5.957	Lin	**5.293**
45	Log-Log	8.371	Lin	**7.476**
48	Log-Log	19.831	Log-Log	**15.119**
72	Log-Log	3.560	Lin	**3.073**

Table 13: Freight trip production models RSME comparison

NAICS	FTP			
	Model P25	**RMSE P25**	**Model TE**	**RMSE TE**
23	Lin	2.256	Lin	**2.103**
31-33	Log-Log	9.521	Log-Lin	**8.537**
31	Log-Log	7.032	Log-Lin	**4.629**
32	Log-Log	14.060	Log-Lin	**11.362**
33	Log-Log	6.785	Log-Log	**6.011**
42	Log-Log	7.408	Log-Lin	**6.454**
44-45	Log-Log	11.060	Lin	**6.814**
44	Lin	6.993	Lin	**5.882**
45	Log-Log	6.073	Lin	**3.586**
48	Log-Log	12.302	Log-Log	**12.302**
72	Lin	4.042	Lin	**2.870**

As tables show, except for the case of NAICS 31-33 and 42, all models considering time-dependent effects present lower root squared mean errors, which suggest that these models have a better fit to the data and are able to draw better estimations of freight trip generation in freight intensive sectors than models that do not consider time-dependent effects.

The larger errors obtained for NAICS 31-33 and 42 in the models considering time-dependent effects, could be explained by the fact that the models for those models include the variable revenue as a predictor, and due to a considerable occurrence of missing data points, the number of observations in these models is drastically reduced. When a model without revenue and only accounting for the time-dependent effects in these industry sectors is used to produce the estimations the root squared mean errors reduce and become lower than models not considering time-dependent effects as Table 14 show.

Table 14: RMSE comparison using models without revenue

NAICS	FTA			
	Model 25	**RMSE 25**	**Model N**	**RMSE N**
23	Log-Log	5.6890649	Lin	**4.892247**
31-33	Lin	8.8349	Log-Log	**8.6375**
31	Log-Log	1.9698581	Lin	**1.506466**
32	Lin	5.6311127	Lin	**4.8107**
33	Log-Log	12.379463	Log-Log	**11.88472**
42	Log-Log	11.012207	Log-Log	**10.7939**
44-45	Log-Log	6.7942512	Lin	**6.359065**
44	Log-Log	5.9567097	Lin	**5.293248**
45	Log-Log	8.3712305	Lin	**7.47599**
48	Log-Log	19.831462	Log-Log	**15.11923**
72	Log-Log	3.5598792	Lin	**3.072642**

The Table 15 and Table 16 present the results of the comparison using the mean absolute percentage error (MAPE)

Table 15: Freight trip attraction models MAPE comparison

NAICS	FTA			
	Model	MAPE P25	Model	MAPE TE
23	Log-Log	175%	Lin	194%
31-33	Lin	126%	Log-Log	240%
31	Log-Log	64%	Lin	**48%**
32	Lin	178%	Lin	239%
33	Log-Log	126%	Log-Log	**115%**
42	Log-Log	238%	Log-Log	**100%**
44-45	Log-Log	211%	Lin	**183%**
44	Log-Log	226%	Lin	**164%**
45	Log-Log	157%	Lin	207%
48	Log-Log	674%	Log-Log	**296%**
72	Log-Log	149%	Lin	**127%**

Table 16: Freight trip production models MAPE comparison

NAICS	FTP			
	Model	MAPE P25	Model	MAPE TE
23	Lin	71%	Lin	**63%**
31-33	Log-Log	307%	Log-Lin	**158%**
31	Log-Log	162%	Log-Lin	**39%**
32	Log-Log	362%	Log-Lin	**281%**
33	Log-Log	250%	Log-Log	**147%**
42	Log-Log	278%	Log-Lin	**183%**
44-45	Log-Log	842%	Lin	**183%**
44	Lin	322%	Lin	600%
45	Log-Log	239%	Lin	**222%**
48	Log-Log	183%	Log-Log	**166%**
72	Lin	142%	Lin	195%

The results for MAPE also show that for 72% of the cases the models considering time-dependent effects and revenue present a lower mean absolute percentage error than those not considering time effects. In the case of freight trip attraction, 7 out of 11 of the models considering time effects present a lower value for MAPE, while in the case of freight trip production 9 models were superior to those not considering time-dependent effects. Considering that in general, for all models estimated as part of this study we are adding explanatory variables such as time-dependent effects and revenue which are significant and improve the quality of the models, equal or lower errors should be expected compared to models not considering these effects. However, one disadvantage of the MAPE is that this measure puts a heavier penalty on positive errors than on negative errors which could be the reason why some of the models considering time-dependent effects have larger values for MAPE.

3.5 Discussions

The findings derived from this study could have important implications for planning and policy purposes since freight trip generation models are very often the foundation stone of traffic management studies and operations, traffic demand management, transportation system planning and transportation infrastructure design among other activities of high importance to the economy.

Firsts, freight trip generation models are not temporal stable but rather, the time has a significant effect on the model parameters. This implies that models implemented without taking into account the time effects, serious problems of under or overestimations in the number of freight trips would occur. This will negatively impact the costs incurred in design projects as well as cause an under or over the capacity design of transportation infrastructure. The policies implemented based on such inaccurate models may also result in being ineffective since the base models used in the analysis would not capture the real freight trip generation patterns.

The results also reveal that freight trip generation patterns are highly responsive to market disruptions such as financial and economic crises. The analysis derived from freight trip generation modeling processes including time-dependent effects and revenue could help practitioners to better understand and quantify the effects of major market disruptions in freight trip attraction and production patterns which could be used to increase the effectiveness of transportation policies and traffic management. Additionally, in some industry sectors, the inclusion of revenue in the models seems to help explain or account for the time effects in freight trip generation models. The implication of having this pattern is that in cases when no time-dependent information is available to include in freight trip generation modeling processes, the revenue of the establishments could also be included to ensure the validity of the models to avoid estimation errors that could have a negative impact when models are implemented with planning and policy making processes.

Freight trip generation models also tend to be linear rather than exponential, with the inclusion of time-dependent variables and the revenue. This suggest that these linear models could also be used for forecasting purposes, rather than exponential models, since one disadvantage of such type of models is that although they have a good explanatory power, they are not conceptually valid for forecasting purposes since the generation of trips increases dramatically when high values in explanatory variables are normally present, which is the case in metropolitan areas like New York City, with a great deal of heterogeneity within the industry sectors.

Freight trip generation models are commonly the founding stone of important transportation infrastructure design projects, transportation system planning studies as well as transportation related policy making processes. Therefore, having freight trip generation models able to produce accurate estimations is vital. Stability of model parameters is a common assumption in passenger trip generation, meaning that models are assumed to be transferable over the time and well-suited to produce accurate estimation for different time periods. However, as demonstrated in this study, models estimated for freight trip generation are not stable over the time, but rather present significant time-dependent effects. Not accounting for such time-dependent effect in freight trip generation models could cause serious under or overestimations in the number of trips generated by a certain industry sector; the afore mentioned situation could cause also negatively affect costs in transportation system improvement projects or could lead to the implementation of ineffective transportation policies with the corresponding misuse of public funds.

The results presented in this study also reveal that including continuous time-dependent effects and the revenue of establishments is not only highly significant to improve the accuracy of the estimations produced by freight trip generation models but also provide predictive capabilities to the models allowing them to be implemented in future scenarios studies accounting for time-dependent effects. The comparative analysis conducted reveal that models including time-dependent effects and revenue have a good fit to the data and produce better estimations than those reported in previous studies. Therefore, it is recommended to include time-dependent effects in freight transportation modeling.

Finally, another important implication of this study is that freight trip generation models are very responsive to market disruptions taking place over the time which is captured by including in the models for freight trip generation time-related variables and variables accounting for changes in the revenue of establishments.

4. Modeling Multinomial Outcomes from Partner Selection and Joint Decision Making Processes

The transportation research community is observing a growingly large amount of data in travel decisions that are jointly made by multiple decision makers. These joint behavior data can be widely collected due to burgeoning information technology innovations and adoptions of GPS devices. These new technologies have significantly improved data gathering techniques, which effectively collect data that were not obtainable before. New technologies also enhance the frequency and quality of communication between individuals, enabling extensive individualized collaboration. A typical example is the fast-evolving e-commerce business, which enables frequent small-package deliveries between sellers and buyers (e.g., the merchandise supported by Craigslist). Another example is the mobile phone-based ridesharing, which provides real-time driver-rider matching to improve mobility. These emerging joint behavior examples present a critical feature that has not been investigated: the behavior involves a bidirectional partner selection process preceding the joint decision making, as opposed to the one-side decision making in classic travelers' behavior.

Existing methodologies for analyzing joint responses can be categorized into three groups. The first group is agent-based modeling and cooperative game theory. Both assume certain behavior rules for each decision maker. The second group is optimization-based, seeking to coordinate multiple decision makers to maximize total benefit of the entire system. The third group is econometric modeling, which extracts information from empirical data to explain the relationship between factors of interest. Econometric modeling has been widely used in classic travel demand models, but travel behavior jointly determined by multiple decision makers has not been studied sufficiently. Although studies have pointed out the importance of interaction among multiple decision makers (*1*), sophisticated methodologies are still in need. Most existing regression models pertaining to multiple decision makers focus on intra-household interactions (*2; 3*), assuming that the connections among decision makers are pre-determined and no "partner selection" is involved. To the authors' best knowledge, so far only a few preliminary matching models attempted to explain why decision makers decide to collaborate with each other.

Sorensen (*4*) first proposed a two-equation model framework to consider partner mutual-selection between banks and firms during firms' IPO. The framework uses a matching equation to explain the bank-firm partner selection and a binary outcome model to formulate firms' IPO choice. The matching is a one to many process as one company can be invested by multiple banks but one bank can invest in only one company. Essentially, the framework assumes that a firm starts with assessing the characteristics of all banks that can potentially provide the IPO service and, based on the assessment, selects the banks that fulfill the company's desire. If the company's most favorable banks dislike the company, the company turns to the next best bank until the desired bank also likes the company. As the partner selection process is bidirectional, a similar assessment process occurs on the bank's side at the same time. As for estimating unknown parameters, observed company-bank matching relationship data is used. The observed matching network can be envisioned by that an agent on one side is connected with certain matched and unmatched pairs on the other side. Such matched and unmatched relationships can imply a series of inequality conditions of pairwise utility (e.g., the preference of matching). The relative value of utility can be used to formulate a limited dependent variable model where the dependent variable in a regression is restricted by a range. Estimated coefficients would reveal the attribution of influential factors on the formation of company-bank matching.

The joint response of matched companies and banks is a simultaneous process of matching and only occurs between matched companies and banks. A joint decision is reached based on common interests and compromise of conflicting claims between matched companies and banks. Without considering the counterparty, each agent achieves the response by fulfilling his/her own desire. However, such a decision/response may not be accepted by the counterparty and consequently, efforts are recognized in reaching an agreement by adjustment of different claims. From the perspective of econometric modeling, conflicting claims are captured by taking both sides' attributions into consideration. The joint response is a result of reconcilement of contributions from each side.

This paper extends Sorensen's model (*4*) with binary joint responses to multinomial joint responses, which can potentially solve a wide selection of transportation problems, such as travel mode/route/time-of-day choices. Multinomial logit/probit models are often used in understanding categorical outcomes and have been widely adopted in travel demand modeling, transport project evaluation, and transportation economics. However, coupled with the matching equation, extending from binary outcomes to multinomial outcomes greatly increases the computational complexity, because the number of equations is increasing, leading to a high-dimensional variance-covariance matrix specification in the error term and additional interrelated unknown variables in the estimation process. Therefore, the major contribution of this paper is to mathematically specify the joint response with multinomial outcome with partner selection consideration, propose the innovative estimation approach, and use a set of simulation studies to show the performance of the proposed model. In detail, the proposed model consists of P equations: (P-1) equations to formulate the multinomial outcome (P alternatives), and one equation to formulate partner selection. The P equations are connected by the error terms in each equation, resulting in a (P×P) variance-covariance matrix in the error term.

The specification of the variance-covariance term also roots in the idea of Sorensen (*4*). Sorensen (*4*) posits that the relationship of joint decision making and mutual selection is analogous to a sample selection process: the joint decision outcome is only observed for matched pairs (e.g., a subset of entire sample). The proposed model borrows the idea of sample selection, but relaxes the constraint on the variance of the two equations: Sorensen (*4*) applies a constraint that the outcome equation would always has a larger variance than the matching equation. In contrast, this paper does not apply any constraint on the magnitude of variance: the variance is purely driven by empirical datasets. Therefore, the specification of the flexible variance-covariance matrix is the second contribution to the existing literature.

The paper is organized as follows. The next section reviews important literature regarding the proposed model. Model Specification introduces the proposed model, its estimation approach, and is followed by a Model Simulation section presenting sensitivity analysis. Conclusions are finally reached to conclude the importance of the proposed model.

4.1 Review of Existing Interaction Data Analysis Framework

Methodologies analyzing the formation of collaborative behavior have a relatively short history. In 2010, Dale Mortensen earned the Nobel Prize in Economics for the analysis of markets with search frictions, which explicitly explain the process of partner selection. Matching model frameworks can explain phenomena that are raised by multiple decision makers. When the matching relationship is observed, these models can analyze the effect of influential factors on forming the observed matching. Matching data include which firms do business with which firms, which men are married to which women, and which players are teammates with which players, among other data involving collaboration. The basic economic idea is that one individual would like to match with the most attractive partners, leading to the highest benefits for the individual. Econometricians seek influential factors that determine the observed matching and estimate the parameters of these factors.

An important matching study is Sorensen (*4*), which uses a two-sided matching model to explain firms' IPO with the bank-firm matching. It first uses a limited dependent variable equation to explain the bank-firm partner selection and then uses a binary outcome model to formulate firms'

IPO. The first equation considers matching utility of all possible pairs while the second equation only considers the IPO of matched pairs. A Bayesian MCMC approach is employed to estimate parameters of factors in determining all parameters in the two equations. This modeling framework models each pair's behavior to analyze the selection of partners. Such a method is a fundamental work for this paper, which extends the outcome of joint decisions from binary outcome to multinomial outcome and a restricted variance-covariance matrix to a flexible matrix specification.

Similar econometric matching models have been discussed in other (but a limited number of) empirical studies. Chen (*5*) specifies the utility equations for each side of the partner to analyze the premium of bank loans. This study is an extension of Sorensen (*4*) in which the utility of paired partners is investigated instead of utility of decision makers, respectively. However, this model may suffer from identification problems if extended to discrete outcomes. Except this study, to the author's best knowledge, no other studies have conducted the research in a similar way. Other studies have analyzed the matching process from the market's perspectives, using different estimation methods, or without considering the mutual selection process. For example, Choo and Siow (*6*) investigated the stable matching relationship from the market's perspective rather than from each decision maker's perspective. Similar marriage analyses can be also found in Siow (*7*). Hitsch et al. (*8*) also analyze a marriage dataset, but the methodology does not consider the mutual preference by sorting pairwise utility. Fox (*9*) and Levine (*10*) use maximum score estimators (e.g., a non-parametric model) to identify parameters in the matching relationship.

The proposed model considers the formation of collaboration and the joint response of matched decision makers as two simultaneous processes. The basic idea is to specify two equations with each equation capturing one process. An important issue of the two equations is that the number of samples in each equation is different from each other. The first equation models all potential pairs but the second equation models only the matched pairs. Such a type of cross-equation relationship is analogous to a sample selection process. To analyze such a process, sample selection models can provide important insights. Standard sample selection models use binary outcome models select subsamples, and the objective is to correct the bias resulted from non-random sampling (*11-13*). The selection equation (i.e., the first equation) is a binary outcome model constituting all samples. The outcome equation is a continuous model with only samples that have affirmative answers in the binary outcome model. In the transportation field, Rashidi et al. (*14*) uses a sample selection model to correct the bias resulting from selecting samples from a particular type of neighborhood. Anderson et al. (*15*) employs a sample selection model to identify the occurrence of railway track renewal and analyzes its costs in Sweden. Vance and Iovanna (*16*) investigates the determinants of automobile travel demand by considering gender by a sample selection model. The sample selection bias is tested by the significance of correlation between the selection equation and the outcome equation. If the two equations are found independent, a sample selection model may reduce to a two-part model. The proposed models in this paper can be seen as an extension of the standard sample selection model: the binary outcome model is replaced by a matching equation and the binary model is extended to multinomial outcome models.

Collectively, the proposed model in this paper investigates a multinomial discrete outcome (compared to the binary outcome), and a flexible variance-covariance matrix in the sample selection process (compared to a restricted variance-covariance matrix).

4.2 Model Specification

Matching Equation

The matching equation characterizes the partner selection process between agents in a two-side market (e.g., suppliers and customers in a freight market). This equation is specified based on the following assumptions: (1) Each agent is assumed to have full information of all agents on the other side and intends to look for the best partner from the other side; (2) The matching has a one-to-many structure; (3) Each potential pair has a pairwise utility, which values the preference of mutual selection. And (4) The matching relationship data is stable: no agent prefers to deviate from current pairs and form a new pair with another agent.

Let $i\left(i=1...I\right)$ denote a set of suppliers and $j\left(j=1...J\right)$ denote a set of customers in a two-side freight market. The number of possible pairs in the market is $I\times J$. Let N_0 and N_1 denote the collections of unmatched and matched pairs, respectively. Thus, the collection of supplier i's unmatched pairs is denoted as $N_0\left(i\right)$ and the collection of supplier i's matched pairs is denoted as $N_1\left(i\right)$. Note that a certain matched pair between supplier i and customer j can be stated as either $j\in N_1\left(i\right)$ or $i\in N_1\left(j\right)$. Let u_{ij} denote the matching utility of pair ij. Assume the utility of all possible pairs are distinct.

As the matching relationship is assumed stable, a set of inequality conditions can be inferred to characterize the relative magnitude of pairwise utility u_{ij}. The inequality conditions for each unmatched pair u_{ij} is

$$u_{ij} < \overline{u_{ij}} = max\left[\min_{j' \in N_1(i)} u_{ij'}, u_{N_1(j)j}\right]$$

The term $\overline{u_{ij}}$ is the opportunity cost for supplier i or customer j to deviate from their existing pairs and form a new match together with each other. An unmatched pair remains unmatched in a stable market when at least one side is unwilling in the counterparty. For supplier i's side, the current matched pairs should be better than the proposed pair i-j. Even the worst current matched pair of i is better than the proposed pair. $\min_{j' \in N_1(i)} u_{ij'}$ is the utility of the worst current matched pair of i and serves as a threshold. As long as the utility of the proposed pair is smaller than this threshold, supplier i is unwilling to matched with customer j. As this is a mutual selection process, the same process can be found on the customer j's side: $\boldsymbol{u_{N_1(j)j}}$ is the utility of the current matched pair of j and serves as a threshold. As long as the utility of the proposed pair is smaller than this threshold, customer j is unwilling to matched with supplier i.

If any side is unwilling to match with the counterparty, the pair cannot be matched. Hence, if the utility of the proposed pair is smaller than any of the two thresholds, the pair is unmatched, resulting in a maximum for the outermost parenthesis.

A matched pair u_{ij} is constrained by the following conditions:

$$\boldsymbol{u_{ij} > \underline{u_{ij}} = max\left[\max_{j' \in S(i)} u_{ij'}, \max_{i' \in S(j)} u_{i'j}\right]}$$

where $\boldsymbol{S(i)} = \{\boldsymbol{j \in J}: \boldsymbol{u_{ij} > u_{N_1(j)j}}\}$ and $\boldsymbol{S(j)} = \left\{\boldsymbol{i \in I}: \boldsymbol{u_{ij}} > \min_{\boldsymbol{j' \in N_1(i)}} \boldsymbol{u_{ij'}}\right\}$

The term $\underline{u_{ij}}$ is the opportunity cost for supplier i and customer j to stay with their existing pairs. A matched pair remains matched in a stable market when both sides are unwilling to match with any feasible deviations. The feasible deviations are denoted by $S(i)$ and $S(j)$, respectively. For the supplier i's side, the feasible deviation is a collection of j who would accept i's propose surely if i proposes to them. If the proposed deviation is better than any of the current matched pair of the proposed deviated customer j. $\boldsymbol{u_{N_1(j)j}}$ is the utility of the current pair and serves as a threshold. As long as the utility of the proposed deviation is greater than this threshold, the corresponding j is a feasible deviation of i. The feasible deviations of customer j can be derived using the same method.

Both sides are unwilling to match with any feasible deviation, indicating that the utility of the matched pair should be better than all feasible deviations. Hence, the maximum of the maximum is used to capture such a relationship.

The pairwise utility can be attributed to a series of explanatory variables, which can be expressed as a regression equation (e.g., the matching equation in the proposed models)

$$u_{ij} = \alpha w + \eta_{ij}$$

where

$$\alpha = \begin{bmatrix} \alpha_i & \alpha_j & \alpha_{ij} \end{bmatrix},\ w = \begin{bmatrix} w_i \\ w_j \\ w_{ij} \end{bmatrix}.$$

The explanatory variable w includes factors of supplier i (e.g., denoted as w_i), customer j (e.g., denoted as w_j), and their joint factors (e.g., denoted as w_{ij}). The α consists of the corresponding parameters to be estimated. The error term η_{ij} contains unobserved effects determining the pairwise utility and is assumed to follow a normal distribution. This error term can be also decomposed to include supplier side errors, customer side errors, and joint error. However, such decomposition leads to identification concerns and is left for future works.

Multinomial Outcome Equation

The multinomial outcome y_{ij}, such as mode/route/time choices, can be analyzed using a multinomial probit model. The observed outcome can be modeled as

$$y^*_{ij,p} = \beta_p x_p + \varepsilon_{ij,p}$$

$$y_{ij,p} = p \text{ if } y^*_{ij,p} = \begin{cases} p, & y^*_{ij,p} = \max\left(y^*_{ij,1}, y^*_{ij,2}, \ldots, y^*_{ij,P}\right) \\ 0, & \text{otherwise} \end{cases}$$

where the term x contains influential factors of supplier i, supplier j, and their the joint factors. The β contains the corresponding parameters to be estimated. The error term ε is assumed to follow a normal distribution.

There are $P-1$ equations with P possible choices. As a result, each variable is denoted by the subscript p indicating the alternatives. In a standard multinomial probit model, a base case is first defined and each of the $(P-1)$ equations captures the difference in choice utility between this choice and the base choice. The proposed model implements the same strategy.

The error terms are assumed to follow a multivariate normal distribution.

$$\begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{P-1} \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \omega_{11} & \cdots & \omega_{1,P-1} \\ \vdots & \ddots & \vdots \\ \omega_{P-1,1} & \cdots & \omega_{P-1,P-1} \end{bmatrix} \right)$$

In practice, one of the diagonal elements in the variance-covariance term is often constrained as 1 due to identification concerns. Without such a constraint, the estimated values of the variance-covariance element could be scaled up.

Connection Between the Two Equations

As only matched pairs could make joint decisions, samples in the matching equation are different from samples in the joint decision making equation. Samples in the matching equation are all possible pairs of supplier i and customer j so that the total number of samples is $I \times J$. However, the samples in the joint decision making equation are just a proportion of $I \times J$ where $ij \in N_1$.

Without considering the effect of the matching process on the joint decision making process, the estimation of θ and β in the joint decision making equation would be biased. The bias can be demonstrated by the conditional expectation of y_{ij}

$$E\left(y_{ij} \mid i, j \text{ are matched}\right) = \beta x + E\left(\varepsilon_{ij} \mid \eta_{ij,c} < \overline{u_{ij}} - \alpha w, \eta_{ij,m} > \underline{u_{ij}} - \alpha w\right) \qquad (0.1)$$

where $\eta_{ij,c}$ and $\eta_{ij,m}$ denote the error terms in the matching equation for unmatched pairs and matched pairs respectively. If ε_{ij} is independent to both $\eta_{ij,c}$ and $\eta_{ij,m}$, the estimation of θ and β is unbiased, because the expectation on the right side is zero. However, if ε_{ij} is dependent on any one of $\eta_{ij,c}$ and $\eta_{ij,m}$, the expectation is not zero so that the estimation would be biased. Such a type of estimation bias is usually called a sample selection bias.

Dealing with the sample selection bias can borrow the idea of specifying a sample selection model. Basically, sample selection models assume the error terms of both equations to follow a joint distribution. The proposed model specifies a P-dimensional multivariate normal distribution

$$\begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_{P-1} \\ \eta \end{pmatrix} \sim \Phi(0,\Sigma) = \Phi\left(\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1,P-1} & \sigma_{1,P} \\ \vdots & \ddots & \vdots & \vdots \\ \sigma_{P-1,1} & \cdots & \sigma_{P-1,P-1} & \sigma_{P-1,P} \\ \sigma_{P,1} & \cdots & \sigma_{P,P-1} & 1 \end{bmatrix} \right)$$

The elements in the upper-left $(P-1)\times(P-1)$ sub-matrix capture the correlation among the $(P-1)$ choices. The elements in the last row and the last column capture the correlation between a certain choice and the matching equation. The choice-matching correlations could vary across choices, leading to a high flexibility. Such a specification allows for correlation between the two equations. Using empirical data, the joint distribution can be estimated.

Note that the existing literature specifies the variance-covariance matrix in a binary outcome case as

$$\begin{bmatrix} 1+\delta^2 & \delta \\ \delta & 1 \end{bmatrix}$$

with only one free variable to be estimated. This specification presumes that the variance of the outcome is greater than the variance of the mutual selection process. In contrast, the proposed variance-covariance matrix does not make this assumption, giving the model a higher flexibility.

Model Estimation

This paper employs a Bayesian MCMC approach with data augmentation to estimate parameters in the proposed model. The reasons of using such a method are (1) The matching relationship is too complex to be disentangled using traditional maximum likelihood estimation approaches: the likelihood of one pair is dependent on the likelihood of all other possible pairs. (2) Latent variables are defined as the dependent variables in the matching equation and multinomial outcome equations. Data augmentation can deal with latent variables by treating them as parameters.

The likelihood function of the proposed model is given by

$$f\left(y=p,u\mid\vartheta\right)=\prod_{ij\in N_0}\mathrm{I}\left(u_{ij}<\overline{u_{ij}}\right)\phi\left(\alpha w,1\right)$$

$$\times\prod_{ij\in N_1}\mathrm{I}\left(u_{ij}>\underline{u_{ij}}\right)\phi\left(\alpha w+\frac{\sigma_{12}}{\sigma_{11}}\left(y_{ij}^{*}-\beta x\right),1-\frac{\sigma_{12}^{2}}{\sigma_{11}}\right)$$

$$\times\phi\left(\beta x+\sigma_{12}\left(u_{ij}-\alpha w\right),\sigma_{11}-\sigma_{12}^{2}\right)$$

$$\times\left(I\left(y_{ij}=0\right)I\left(\max\left(y_{ij,p}^{*}\le 0\right)\right)+\sum_{q=1}^{P}I\left(y_{ij}=P\right)I\left(y_{ij}^{*}>\max\left(0,y_{ij,-q}^{*}\right)\right)\right)$$

The following conjugate prior distributions are used for each parameter: $\alpha\sim Normal\left(\alpha_0,\mathrm{A}_0\right)$, $\beta\sim Normal\left(\beta_0,\mathrm{B}_0\right)$, and $\Sigma\sim Inverse\,Wishart\left(\rho R,\rho\right)I\left(\sigma_{PP}=1\right)$. The prior of Σ is an inverse Wishart distribution with one on the last diagonal element.

Given prior distributions and likelihood functions, posterior distributions of parameters and latent variables can be derived. There are five steps to simulate posterior distributions for all parameters

1. Sample y_{ij}^{*}. The latent variable in the joint decision making equation follows a $\left(P-1\right)$-dimensional multivariate normal distribution. The posterior distribution is a truncated multivariate normal distribution

$$y_{ij}^{*}\sim MTN_{R_{ij}\left(y_{ij}\right)}\left(\delta_{y_{ij}^{*}},\Omega_{y_{ij}^{*}}\right)$$

where

$$\delta_{y_{ij}^{*}}=\begin{bmatrix}\beta_1 x_1\\ \vdots\\ \beta_{P-1}x_{P-1}\end{bmatrix}+\begin{bmatrix}\sigma_{1P}\\ \vdots\\ \sigma_{P-1,P}\end{bmatrix}\times\left(u_{ij}-\alpha w\right),$$

$$\Omega_{y_{ij}^{*}}=\begin{bmatrix}\sigma_{11}&\cdots&\sigma_{1,P-1}\\ \vdots&\ddots&\vdots\\ \sigma_{P-1,1}&\cdots&\sigma_{P-1,P-1}\end{bmatrix}-\begin{bmatrix}\sigma_{1P}\\ \vdots\\ \sigma_{P-1,P}\end{bmatrix}\times\begin{bmatrix}\sigma_{P1}&\cdots&\sigma_{P,P-1}\end{bmatrix}.$$

$R_{ij}\left(y_{ij}\right)$ denotes the truncated region. If $y_{ij}=0$, R_{ij} consists of the region where each component of y_{ij}^* is negative. When $y_{ij}=p$, R_{ij} restricts $y_{ij,p}^*$ to be positive and greater than all other $y_{ij,p}^*$.

2. Sample u_{ij}. This latent variable in the matching equation follows a normal distribution. For unmatched pairs, the posterior distribution is exactly the same as that in the ordinal case. For matched pairs, the posterior distribution is

$$u_{ij} \sim N\left(\delta_{u_{ij}}, \Omega_{u_{ij}}\right)$$

where

$$\delta_{u_{ij}} = \alpha w + \left[\sigma_{P1}, \cdots, \sigma_{P,P-1}\right] \times \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1,P-1} \\ \vdots & \ddots & \vdots \\ \sigma_{P-1,1} & \cdots & \sigma_{P-1,P-1} \end{bmatrix}^{-1} \times \begin{bmatrix} y_{ij,1}^* - \beta_1 x_1 \\ \vdots \\ y_{ij,P-1}^* - \beta_{P-1} x_{P-1} \end{bmatrix},$$

$$\Omega_{u_{ij}} = 1 - \left[\sigma_{P,1}, \cdots, \sigma_{P,P-1}\right] \times \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{P,P-1} \\ \vdots & \ddots & \vdots \\ \sigma_{P-1,1} & \cdots & \sigma_{P-1,P-1} \end{bmatrix}^{-1} \times \begin{bmatrix} \sigma_{1,P} \\ \vdots \\ \sigma_{P-1,P} \end{bmatrix}$$

The distribution is truncated below at $\underline{u_{ij}}$.

3. Sample β_p. Parameters in each of joint decision making equation follow a normal distribution of

$$\beta_p \sim N\left(D_{\beta_p} d_{\beta_p}, D_{\beta_p}\right)$$

where

$$D_{\beta_p} = \left(B_0^{-1} + \sum_{ij \in N_1} x_p^{'} \Sigma_p^{-1} x_p\right)^{-1},$$

$$d_{\beta_p} = B_0^{-1}\beta_{p,0} + \sum_{ij \in N_1} x_p \Sigma_p^{-1} \left(y_{ij,p}^* - \sigma_{1,(-p)} \times \sigma_{(-p),(-p)}^{-1} \times \begin{bmatrix} y_{ij,1}^* - \beta_1 x_1 \\ \vdots \\ y_{ij,P-1}^* - \beta_{P-1} x_{P-1} \\ u_{ij} - \alpha w \end{bmatrix}_{(-p)} \right)$$

with $\Sigma_p = \sigma_{pp} - \sigma_{1,(-p)} \times \sigma^{-1}_{(-p),(-p)} \times \sigma_{(-p),1}$.

4. Sample α. The posterior distribution of parameters in the matching equation is a normal distribution of

$$\alpha \sim N\left(D_\alpha d_\alpha, D_\alpha\right)$$

where

$$D_\alpha = \left(A_0^{-1} + \sum_{ij\in N_0} ww' + \sum_{ij\in N_1} w\Sigma_u^{-1}w' \right),$$

$$d_\alpha = \left(A_0^{-1}\alpha_0 + \sum_{ij\in N_0} wu_{ij} + \sum_{ij\in N_1} w\Sigma_u^{-1}\left(u_{ij} - \sigma_{1,(-P)} \times \sigma^{-1}_{(-P),(-P)} \times \begin{bmatrix} y^*_{ij,1} - \beta_1 x_1 \\ \vdots \\ y^*_{ij,P-1} - \beta_{P-1}x_{P-1} \end{bmatrix} \right)\right)$$

with $\Sigma_P = 1 - \sigma_{1,(-P)} \times \sigma^{-1}_{(-P),(-P)} \times \sigma_{(-P),1}$.

5. Sample Σ. The posterior distribution of the variance-covariance matrix is a conditional inverse Wishart distribution

$$\Sigma \sim IW\left(\rho R + \sum_{ij\in N_1} e_{ij}e_{ij}', N + \rho \right) I\left(\sigma^2_{PP} = 1\right)$$

where $e_{ij} = \begin{bmatrix} \varepsilon_{ij,1} \\ \vdots \\ \varepsilon_{ij,P-1} \\ \eta_{ij} \end{bmatrix}$ and N is the number of matched pairs.

4.3 Model Validation

This section presents a series of simulation studies to show the performance of the proposed joint response model. A freight supplier-customer collaboration context is used to help with illustration. A freight supply chain often consists of multiple market players, such as suppliers and customers. A critical business decision of both suppliers and customers is to find the best business partners in the market to best improve profits. Observed cargo movement is a result of player collaboration.

To begin with, one set of simulation data is randomly generated based on pre-defined parameter values. With the simulation data, the proposed estimation approach is implemented and the parameter recovery capability is evaluated to show the model's performance. Then, a more comprehensive sensitivity analysis is conducted to show the sensitivity of model estimation by a set of different parameter values. For each parameter value, the simulation data are randomly generated for 30 times and the parameter recovery capability is evaluated for all simulation data.

The model validation analysis is implemented as the following steps: (1) Define parameter values; (2) Use defined parameter values to generate simulation data; (3) Treat the defined parameters as unknown and use the simulation data to estimate these parameters; (4) Compare the estimated parameters and the pre-defined parameters. If a good parameter recovery capability is found, the model has a good performance.

First, the parameters in the matching equation are defined. The assumed market has 20 suppliers and 100 customers (i.e., $I = 20$ and $J = 100$). Therefore, there are 2,000 (i.e.,$I \times J = 2{,}000$) possible supplier-customer pairs, leading to 2,000 pairwise utility. Each supplier could match with 5 customers and each customer could match with only 1 supplier. Which two agents are matched depends on the relative magnitude of pairwise utility, which is determined by three explanatory variables and disturbance. The first two explanatory variables are attributes of suppliers and customers, respectively (e.g., w_i and w_j which are constant for one certain agent). The third explanatory variable is a joint factor (e.g., w_{ij} , which varies across pairs). These explanatory variables are generated by standard normal distributions. Table 1 summarizes values of parameters in the matching equation.

Table 1. Parameter values in the matching equation

Parameter	True Values	Initial Values
I	20	
J	100	
α	(-0.6, 0.9, -0.3)	(0, 0, 0)

The multinomial outcome equation is assumed to have three categorical outcomes, leading to two equations. Each equation is assumed to have three explanatory variables. Similarly, two of them (i.e., $x_{i,p}$ and $x_{j,p}$) are attributes of each side, respectively (e.g., the value is constant over one certain agent) and the other one (i.e., $x_{ij,p}$) is the joint factor, which varies across pairs. Note that there can be alternative-specific variables, but they make no difference mathematically. All of the explanatory variables are generated by standard normal distributions. Table 2 summarizes values of parameters in the outcome equation. Given the joint decision making has two equations, the error terms follow a tri-variate normal distribution.

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \eta \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1.5 & 0.3 & 0.2 \\ 0.3 & 1.2 & 0.1 \\ 0.2 & 0.1 & 1 \end{bmatrix} \right) \qquad (0.2)$$

Table 2. Parameter values for the multinomial case in the joint decision making equation

Parameter	True Values	Initial Values
β_1	(-0.9, 0.6, 0.3)	(0, 0, 0)
β_2	(0.6, 0.3, -0.9)	(0, 0, 0)
σ_{11}	1.5	1
σ_{22}	1.2	1
σ_{12}	0.3	0
σ_{23}	0.2	0
σ_{13}	0.1	0

Based on the pre-defined values of parameters, the simulation data of the multivariate case are generated. Table 3 summarizes the average, standard deviation, minimum value, and maximum values of the generated data. Given the sizes of agents on both sides, the number of observations in the matching equation is 2,000. The generated data leads to 100 matched pairs.

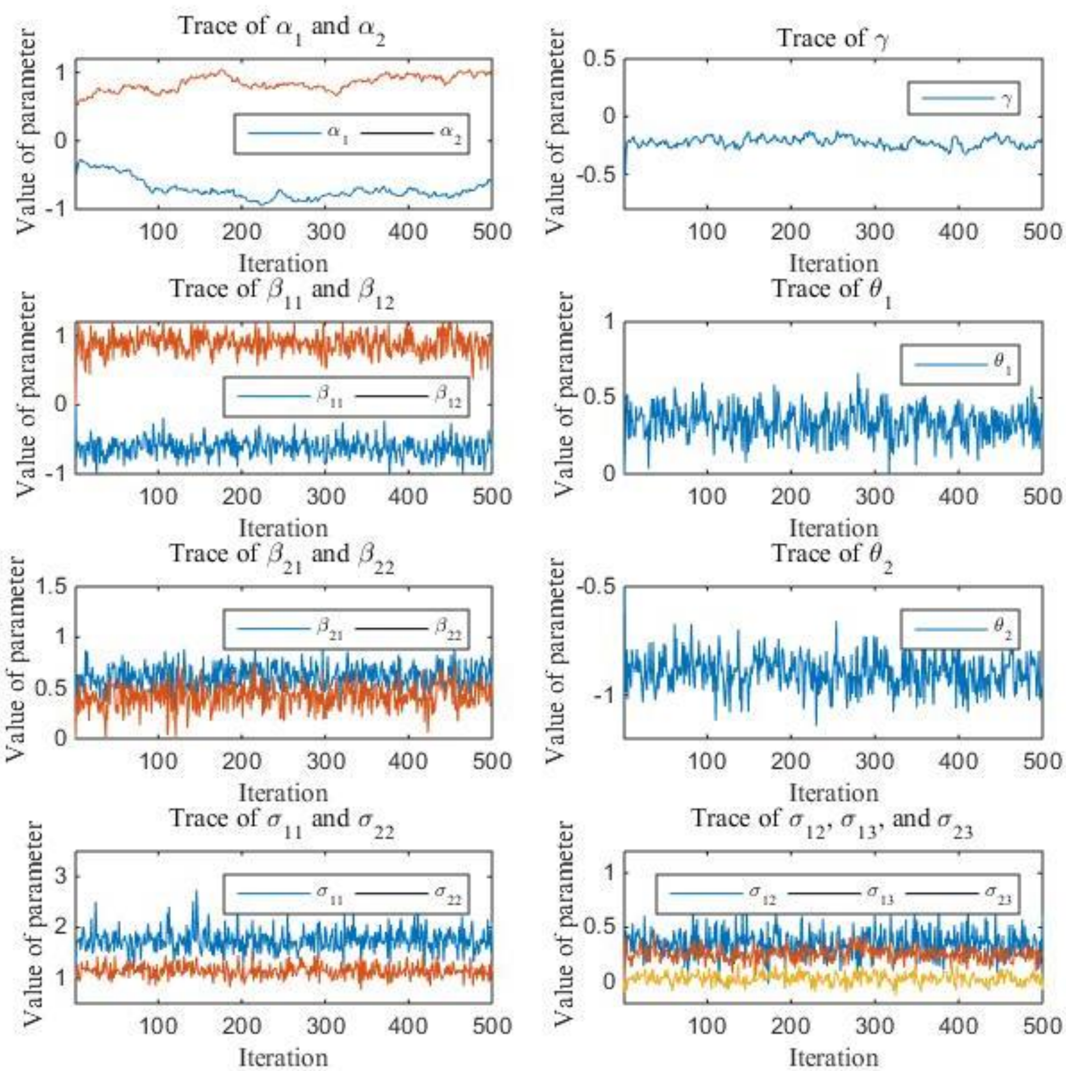
Table 3. Summary statistics of generated data in the multinomial case

Generated Data	Size	Mean	Standard Deviation	Min	Max
w_i	2,000	-0.41	0.94	-1.47	1.68
w_j	2,000	-0.63	0.97	-2.85	1.66
w_{ij}	2,000	0.07	1.01	-3.74	2.98
$x_{i,1}$ $x_{i,2}$	100	-0.41	0.94	-1.47	1.68

$x_{j,1}$ $x_{j,2}$	100	-0.63	0.97	-2.85	1.66
$x_{ij,1}$ $x_{ij,2}$	100	0.07	1.01	-3.74	2.98

The Bayesian MCMC approach is used to estimate the parameters with the input of the generated data. Due to the faster speed of convergence, the estimation of the multinomial case reduces to iterate 500 times with the first 300 as the burn-in period. The traces of estimated parameters are shown in Error! Reference source not found.1.

Figure 1. Estimated parameter traces of the model validation study



Traces show that all parameters converge to their true values fast, which indicates a good parameter recovery capability. The estimation results are summarized in Table 4.

Table 4. Estimation result summary of simulation data in the multinomial outcome case

Parameter	True Value	Mean	Standard Deviation	95% CI Low	95% CI Up
α_i	-0.6	-0.65	0.06	-0.53	-0.76
α_j	0.9	0.91	0.09	0.74	1.08
α_{ij}	-0.3	-0.24	0.04	-0.32	-0.16
$\beta_{i,1}$	-0.9	-0.64	0.14	-0.91	-0.36
$\beta_{j,1}$	0.6	0.87	0.16	0.55	1.19
$\beta_{ij,1}$	0.3	0.33	0.10	0.13	0.53
$\beta_{i,2}$	0.6	0.62	0.11	0.41	0.83
$\beta_{j,2}$	0.3	0.33	0.12	0.20	0.47
$\beta_{ij,2}$	-0.9	-0.90	0.08	-1.05	-0.75
σ_{11}	1.5	1.65	0.21	-	-
σ_{22}	1.2	1.13	0.13	-	-
σ_{12}	0.3	0.35	0.13	-	-
σ_{13}	0.2	0.25	0.06	-	-
σ_{23}	0.1	0.02	0.05	-	-

4.4 Sensitivity Analysis

Validation by only one randomly-generated dataset may not thoroughly evaluate the parameter recovery capability of the proposed estimation approach. Two aspects can be implemented to improve the validation process: measuring the recovery capability at different parameter values and repeating the estimation process by multiple times. Thus, this paper further analyzes a variety of different pre-defined parameter values to discuss the sensitivity of estimation. In order to compare parameter recovery performance, two statistics are introduced, mean absolute percentage error (MAPE) and mean percentage square error (MPSE).

$$MAPE = \sum_{t=1}^{T} \frac{|z_t - z_0|}{|z_0|} / T, \qquad MPSE = \sum_{t=1}^{T} \left(\frac{z_t - z_0}{z_0} \right)^2 / T$$

In the equations, $t\left(t=1\ldots T\right)$ denotes the number of iterations. Term z_t is the value of parameter at iteration t and term t_0 is the true parameter value. APE measures the average absolute deviation from the true values. PSE further uses the squared error to penalize large deviations.

Two matching structures are investigated: (1) 20 vs 100, and (2) 10 vs 200. In specific, in the first matching structure, each supplier i has 5 matched customers and each customer j has 1 matched supplier. In the second matching structure, each i has 20 matched customers and each customer has 1 matched supplier. The variance-covariance matrix has two free variables on the diagonal elements and three free variables on the off-diagonal elements. The diagonal elements are investigated at (1.5, 1.2), (1.5, 0.6), and (0.8, 0.6). The off-diagonal elements are investigated at (0.3, 0.2, 0.1) and (-0.1, -0.2, -0.3). Table 5 summarizes the investigated scenarios.

Table 5. Summary of simulation data in the sensitivity analysis

	Matching Structure	Variance-Covariance term
1	20 vs 100	$\begin{pmatrix} 1.5 & 0.3 & 0.2 \\ & 1.2 & 0.1 \\ & & 1 \end{pmatrix}$
2	20 vs 100	$\begin{pmatrix} 1.5 & -0.1 & -0.2 \\ & 1.2 & -0.3 \\ & & 1 \end{pmatrix}$
3	20 vs 100	$\begin{pmatrix} 1.5 & 0.3 & 0.2 \\ & 0.6 & 0.1 \\ & & 1 \end{pmatrix}$
4	20 vs 100	$\begin{pmatrix} 1.5 & -0.1 & -0.2 \\ & 0.6 & -0.3 \\ & & 1 \end{pmatrix}$
5	20 vs 100	$\begin{pmatrix} 0.8 & 0.3 & 0.2 \\ & 0.6 & 0.1 \\ & & 1 \end{pmatrix}$
6	20 vs 100	$\begin{pmatrix} 0.8 & -0.1 & -0.2 \\ & 0.6 & -0.3 \\ & & 1 \end{pmatrix}$
7	10 vs 200	$\begin{pmatrix} 1.5 & 0.3 & 0.2 \\ & 1.2 & 0.1 \\ & & 1 \end{pmatrix}$
8	10 vs 200	$\begin{pmatrix} 1.5 & -0.1 & -0.2 \\ & 1.2 & -0.3 \\ & & 1 \end{pmatrix}$
9	10 vs 200	$\begin{pmatrix} 1.5 & 0.3 & 0.2 \\ & 0.6 & 0.1 \\ & & 1 \end{pmatrix}$
10	10 vs 200	$\begin{pmatrix} 1.5 & -0.1 & -0.2 \\ & 0.6 & -0.3 \\ & & 1 \end{pmatrix}$
11	10 vs 200	$\begin{pmatrix} 0.8 & 0.3 & 0.2 \\ & 0.6 & 0.1 \\ & & 1 \end{pmatrix}$
12	10 vs 200	$\begin{pmatrix} 0.8 & -0.1 & -0.2 \\ & 0.6 & -0.3 \\ & & 1 \end{pmatrix}$

For each scenario, 30 simulations are randomly generated and estimated and the first 300 iterations for each simulation are treated as the “burn-in” period and the last 200 iterations are used to calculate the estimation results. The sensitivity is then evaluated by taking the average over the 30 random samples. Throughout the twelve scenarios, the parameters can be recovered by the proposed estimation approach successfully. The two matching structures show a slight difference in estimation performance: the second matching structure is better. The reason is that the number of matched pairs in the second structure (200) is greater than the first (100). This rovides a narrower inequality range on the matching utility, leading to a better parameter identification.

4.5 Discussions

This paper develops an innovative econometric model to analyze joint responses with the consideration of partner selection and joint decision making. The proposed model consists of two parts: the first part uses a limited dependent variable equation to explain the matching process in a one-to-many matching structure; the second uses a multinomial probit model to characterize joint responses in the form of categorical values. The two parts are connected with each other by a variance-covariance matrix, which is determined by empirical data.

The necessity of developing such an innovative analytic model is due to the increasing accessibility of joint behavior data and growing interaction behavior of transport users enabled by information technology development and adoption of GPS devices. Existing econometric methodologies cannot effectively analyze such joint behavior. Therefore, this paper contributes to the existing literature by investigating the underlying mechanism of joint behavior. In the transportation research community, to the authors' best knowledge, this paper is the first one attempting to model partner selection and joint decision making in the same manner. Several aspects can be potentially improved in this paper. For example, the matching structure can be extended to a many-to-many matching structure. The model simulation can investigate the multinomial equation with a higher number of alternatives. The cross-equation can be specified in other format, such as structural equation models. Given the importance of understanding joint response, this paper is a valuable start and sheds light on the travel behavior research.

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