

# **UNDERSTANDING INTERDEPENDENCIES BETWEEN SYSTEMS TOWARD RESILIENT CRITICAL LIFELINE INFRASTRUCTURE IN THE PACIFIC NORTHWEST**

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Sponsorship  
PacTrans

for

Pacific Northwest Transportation Consortium (PacTrans)  
USDOT University Transportation Center for Federal Region 10  
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Seattle, WA 98195-2700

In cooperation with US Department of Transportation-Research and Innovative Technology  
Administration (RITA)



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## Technical Report Documentation Page

<b>1. Report No.</b>	<b>2. Government Accession No.</b>	<b>3. Recipient's Catalog No.</b>	
<b>4. Title and Subtitle</b> Understanding Interdependencies between Systems towards Resilient Critical Lifeline Infrastructure in the Pacific Northwest		<b>5. Report Date</b> XX/XX/XXXX	
		<b>6. Performing Organization Code</b>	
<b>7. Author(s)</b> Shangjia Dong, Alireza Mostafizi, Haizhong Wang Oregon State University		<b>8. Performing Organization Report No.</b>	
<b>9. Performing Organization Name and Address</b> PacTrans Pacific Northwest Transportation Consortium University Transportation Center for Region 10 University of Washington More Hall 112 Seattle, WA 98195-2700		<b>10. Work Unit No. (TRAIS)</b>	
		<b>11. Contract or Grant No.</b>	
<b>12. Sponsoring Organization Name and Address</b> United States of America Department of Transportation Research and Innovative Technology Administration		<b>13. Type of Report and Period Covered</b>	
		<b>14. Sponsoring Agency Code</b>	
<b>15. Supplementary Notes</b> Report uploaded at <a href="http://www.pacTrans.org">www.pacTrans.org</a>			
<b>16. Abstract</b> <b>ABSTRACT NEEDED</b>			
<b>17. Key Words</b> <b>KEY WORDS NEEDED</b>		<b>18. Distribution Statement</b> No restrictions.	
<b>19. Security Classification (of this report)</b> Unclassified.	<b>20. Security Classification (of this page)</b> Unclassified.	<b>21. No. of Pages</b>	<b>22. Price</b> NA



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# 1 Introduction

The Critical Infrastructure Protection (CIP) Program sponsored by the U.S. Department of Homeland Security (DHS) has three primary goals (Bush et al. 2005):

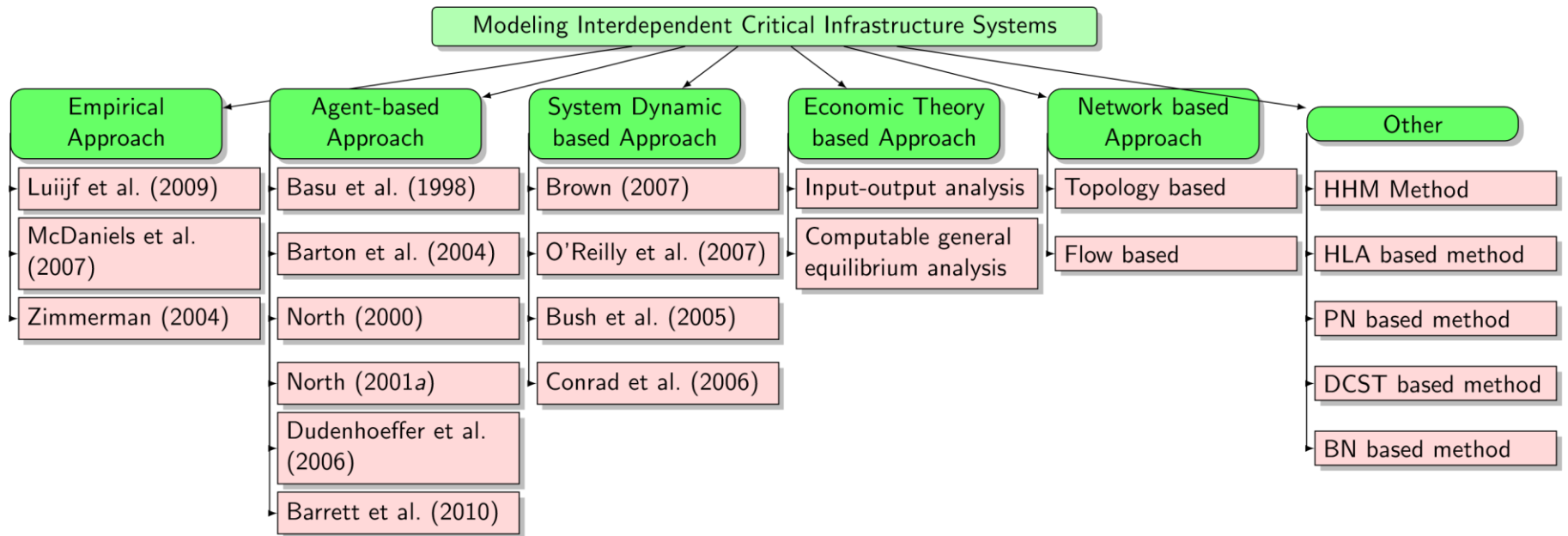
1. Develop, implement, and evolve a rational approach for prioritizing CIP strategies and resource allocations using modeling, simulation, and analyses to assess vulnerabilities, consequences, and risks;
2. Propose and evaluate protection, mitigation, response, and recovery strategies and options;
3. Provide real-time support to decision makers during crises and emergencies.

“Infrastructure interdependencies refer to relationships or influences that an element in one infrastructure imparts upon another infrastructure” (Dudenhoeffer et al., 2006). According to Rinaldi et al. (2001), interdependency is “two infrastructures [that] are interdependent when each is dependent on the other”. Yao et al. (2004) defined “lifeline interaction as the mutual effect between a lifeline system and other lifeline systems in the same district under seismic conditions. The reliability of a lifeline system, in addition to its earthquake resistant performance, still depends on the reliability of other lifeline systems which have functional connections or physical proximity with the lifeline system” (Kakderi et al., 2011). Various definitions of interdependency are summarized in table 1-1.

**Table 1-1.** Summary of interdependency types

<b>Authors</b>	<b>Interdependency Types</b>	<b>Definitions</b>
Rinaldi et al. (2001)	Physical	A physical reliance on material flow from one infrastructure to another
	Cyber	A reliance on information transfer between infrastructure
	Geographic	A local environmental event affects components across multiple infrastructure due to physical proximity
	Logical	A dependency that exists between infrastructures that does not fall into one of the above categories
Buhne et al. (2003)	Requires dependency	The binding of one object implies the need of another object, i.e., a required following
	Exclusive dependency	The binding of one object excludes another object
	Hints dependency	The binding of one object has positive impact on another object
	Hinders dependency	The binding of one object has negative impact on another object
Dudenhoeffer et al. (2006)	Physical	Direct linkage between infrastructures as from a supply/consumption/production relationship
	Geographic	Co-location of infrastructure components within the same footprint
	Policy	A binding of infrastructure components due to policy or high level decisions
	Informational	A binding or reliance on information flow between infrastructures
Pederson et al. (2006)	Policy/Procedural Interdependency	“An interdependency that exist due to policy or procedure that relates a state or event change in one infrastructure sector component to a subsequent effect on another component”
	Societal interdependency	“The interdependencies or influences that an infrastructure component event may have on societal factors such as public opinion, public confidence, fear, and cultural issues”
Zhang et al. (2011)	Functional	Inputoutput and substitution relationships
	Physical	Sharedlocation, capacity or right of way
	Budgetary	Shared public funding
	Market and Economic	Sharedoperating environment
Wallace (2001)	Input	The infrastructure requires as input one or more services fromanother infrastructure in order to provide some other service
	Shared	Some physical components and/or activities of the infrastructureused in providing the service are shared
	Exclusive-or	Only one of two or more services can be provided by aninfrastructure
	Mutually dependent	At least one of the operations of any infrastructurein I is dependent upon each of the other infrastructures in I
	Co-located	Any of their physical components or activities are situated within a prescribed geographical region





**Figure 1-1.** Summary of interdependency modeling approaches

**Table 1-2.** Summary of interdependency studies on real infrastructure systems

<b>Authors</b>	<b>Investigated Infrastructure Systems</b>	<b>Methodology</b>
Zhang et al. (2011)	Producer(electric power, transportation, telecommunication), household	Spatial computable,general equilibrium (SCGE)
Utne et al. (2011)	Electricity, water, transport, and information and communication technology system	Cross-sector risk,and vulnerability analysis
Ouyang et al. (2009)	Electrical network, gas pipeline system	Structural,vulnerability and functional vulnerability
Ouyang et al. (2009)	Energy system,water system	Network flow model
Li et al. (2013)	Nuclear power plants	Integer linear program
Johansson and Hassel (2010)	Electrified railway network	Vulnerability analysis
Dueas-Osorio and Vemuru (2009)	Power transmission system	Network flow numerical simulation method
Barrett et al. (2010)	Cellular and mesh network, transportation network, social phone call network	Interaction based model
Tolone et al. (2004)	Electrical power transmission and distribution, gas distribution, telecommunication, and transportation	Geographical information system (GIS)
Ouyang and Dueas-Osorio (2011 <i>b</i> )	Electrical network, gas pipeline system	Generalized interdependent effects
Halu et al. (2014)	Airport networks, railway networks	Spatial multiplex and spatial interacting networks
Dueas-Osorio et al. (2007)	Electrical network, water distribution network	Network topology based model
North (2001 <i>a</i> )	Electrical power system, natural gas system	Multi-agent social and organizational model
Lee et al. (2007)	Electrical station, telecommunication network, subway network	Network flow model
Dueas-Osorio et al. (2006)	Electrical power network, water distribution	Network topology and network flow model
Wang et al. (2012)	Electrical power network, water network	Interdependent CIS vulnerability analysis
Wang et al. (2012)	Electrical power network, gas network	Interdependent CIS vulnerability analysis
Zio and Ferrario (2013)	Nuclear power plant, power network, water network, transportation network	System of system analysis
Giorgio and Liberati (2012)	Electric distribution network, telecommunication network, SCADA monitoring system	Dynamic Bayesian network

## 2 Literature Review

### 2.1 Empirical Approaches

The empirical approach analyzes interdependencies of critical infrastructure systems (CISs) on the basis of historical accident or disaster data and expert experience. Empirical analysis can help identify frequent and significant failure patterns, generate interdependency strength metrics to facilitate decision making, conduct empirical-based risk analysis, and provide alternatives to minimize the risk (Ouyang, 2014). As some intangible interdependent relationships are undetectable by using standard data collection approaches or only emerge after the occurrence of a disruptive event, the interdependencies among CISs can be hard to discover under normal operation. Thus, historical events can be used to unveil the interdependency structure between CISs. Interdependency incident records are usually collected from newspapers, media reports, official ex-post assessments, and utility operators (Luijff et al., 2009; McDaniels et al., 2007).

McDaniels et al. (2007) defined infrastructure failure interdependency (IFI) as failures in interdependent infrastructure systems that are due to an initial infrastructure failure stemming from an extreme event. McDaniels et al. (2007) examined the IFIs that occurred in three kinds of events: the August 2003 northeastern North American blackout, the 1998 Quebec ice storm, and a set of three 2004 Florida hurricanes. In the paper, a framework was developed to define the context and condition of the initial failure, the nature of the interactions and context that leads to an IFI event, and the severity of the societal consequences. Two indexes were also characterized on the basis of the data set collected from the aforementioned three events: impact index (the product of the failure duration and severity weights) and extent index (the product of the failure spatial extent and number of people affected) (Chang et al., 2009; McDaniels et al., 2008).

Luijff et al. (2009) examined the interdependencies of 1,749 critical infrastructure (CI) failure details in 29 European nations. The events were classified into three categories: cascade initiating events (an event that causes an event in another CI), cascade resulting events (an event that results from an event in another CI), and independent events (an event that is neither a cascade initiating event nor a cascade resulting event). These are not mutually exclusive. The analysis suggested that energy and telecommunication systems were the main cascade initiating sectors; they caused outages in other sectors 60 percent and 24 percent of the time, respectively. The dependencies were found very focused and directional. As a matter of fact, because the data did not exhibit reciprocal relationships occurring frequently, one may want to stop talking about interdependency. In addition, 50 percent of the events in the energy sector triggered a disruption in another CI, and about 40 percent of events in the telecommunications sector triggered a disruption. The analysis also demonstrated that 24 percent of the 1,749 events were a first-level cascade event, 4 percent were the result of a second cascade, and four events were caused by a third cascade. No deeper cascades were found, either in Europe or internationally.

The interdependencies were found to occur far less frequently than analysts have modelled. Only two out of 770 failures were found. However, the data may have been biased for two reasons. First, the European languages that were used to extract the data from media reports were limited. Second, not every serious CI incident is reported by the news media; in other words, the news is more likely to report incidents that are of interest to their audience (Luijff et al., 2009).

In order to explore whether certain combinations of infrastructure failures are more common than others, Zimmerman (2004) created a database of failure sequences and components based on websites, reports from the National Transportation Safety Board, and news

media searches. Several indicators were introduced to inform mitigation and emergency decision making: the types of infrastructure that were more frequently damaged than other infrastructure, the types of infrastructure that are more commonly affected by or damaged by failures in other infrastructure, the ratio between being a cause of a failure and being affected by a failure, the combinations of failures that were most frequent, and the number of people affected and how they are affected. Utne et al. (2011) created a cascade diagram to describe cascading failures across CISs under a specific initiating event. The diagram supports both qualitative and quantitative analysis of consequences and risk, which is helpful when a stakeholder has no strong risk analysis background. Kjolle et al. (2012) also presented a cross-sector risk analysis that included contingency analysis (power flow) and reliability analysis for power systems, as well as an interdependency study with a cascade diagram.

The empirical approaches help to identify potential vital interdependency patterns and increase decision makers' awareness of and capabilities to respond to an event. But they also have some weaknesses (Ouyang, 2014). First, under-reported interdependency failures may have a significant impact. Second, there is no standardized data collection procedure for interdependent CIS performance. A uniform data collection method would assist in collecting the information, thus reducing the time needed to code and sort out the content of incident reports. Third, previous failure records may be able to give accurate prediction of future similar events but not new disasters.

## **2.2 Agent-Based Approaches**

An agent-based approach adopts a bottom-up method and presumes that the complex behavior or phenomenon is generated from relatively simple interactions among the enormous agents. Each agent interacts with others and the environment on the basis of a set of rules, which

approximate the way a real counterpart would react. Most CIS components can be modeled as agents. Agent-based approaches are, therefore, widely employed in CIS interdependencies modeling.

Sandia National Laboratories (SNL) firstly developed Aspen (Basu et al., 1998), an “agent-based” Monte-Carlo method, to simulate the U.S. economy. Individuals in the model represent real-life economic decision makers. It models a large number of individual economic agents with fine detail and a high degree of freedom. In 2000, Aspen-EE (Barton et al., 2000) proposed to simulate the interdependent effects of market decisions and disruptions in the electric power system on other CISs in the U.S. economy. It extended and modified the capabilities of Aspen. Aspen and Aspen-EE both utilized a message-passing mechanism to achieve communication between agents without specially representing the telecommunication system. In 2004, SNL developed CommAspen (Barton et al., 2004), a new agent-based model to simulate the interdependent effects of disruptions in the telecommunications infrastructure on other CIS, such as banking, finance, and electric power. In the meantime, SNL (Schoenwald et al., 2004) also developed a next-generation agent-based economic ‘laboratory’ (N-ABLE) to analyze the economic factors, feedbacks, and downstream effects of infrastructure interdependencies.

Argonne National Laboratory (ANL) developed an agent-based model, Spot Market Agent Research Tool Version 2.0 (SMART II) (North, 2000), in 2000. It used a Swarm agent-based framework. SMART II contained three different kinds of components: (a) generation agents that produce electric power, (b) consumer agents that use electric power, and (c) interconnections that represent the transmission grid. Two kinds of agents were considered, electric power generators and consumers. Unlike the models from SNL, SMART II considered

the topology of the power transmission system. As an extension of SMART II, an integrated model of electric power and natural gas markets, SMART II+, was developed (North, 2001*a*). SMART II+ included new agents and interconnections that represent the electric power marketing and transmission infrastructure, the natural gas marketing and distribution infrastructure, and the interconnections between infrastructures in the form of natural gas fired electric generators. The interdependency analysis suggested that emergency natural gas consumed by electric generators need to be monitored to prevent electric failures from spreading to natural gas infrastructure. Later, Flexible Agent Simulation Toolkit (FAST) was developed. It was an integrated infrastructure model based on SMART II+. FAST included many of the original features of SMART II+ along with improvements in modeling infrastructure, detail, and fidelity (North, 2001*b*).

Idaho National Laboratory (INL) developed the Critical Infrastructure Modeling System (CIMS) (Dudenhoeffer et al., 2006) for critical infrastructure interdependency analysis. CIMS took a command-level approach, aiming to provide decision makers with mission capability information without digging down to the engineering level. It enabled the visualization to update as the simulation ran. This would allow decision makers to quickly evaluate the interrelationships between infrastructure networks and the damaging effects of the events. However, when the size and complexity of the network increased, the visualization might not suffice. Additional search and analysis methods were required to identify event-effect relationships, especially across multiple infrastructures. Therefore, INL and the University of Idaho sought to use artificial intelligence (AI) techniques to facilitate space searching and possible interaction identification. A genetic algorithm (GA) integrated into CIMS was developed to search for the optimal infrastructure assets to protect from attack or to restore in a disaster situation.

Tolone et al. (2004) developed a framework to identify and understand vulnerabilities in the geographic information system (GIS) environment. The simulation contained four critical infrastructures for a fictional town: electrical power transmission and distribution, gas distribution, telecommunications, and transportation. Barrett et al. (2010) developed a conceptual framework for investigating human-initiated interdependencies between critical societal infrastructures. A case study of a chemical plume in the downtown Portland, Oregon, area was conducted to show the evacuation of individuals leading to traffic congestion. This in turn would cause changes in cell phone usage patterns, which would result in an overload of the telecommunications system.

Agent-based approaches model the interactions of participants in the interdependent CISs to provide scenario-based, what-if analysis. They allow us to assess the effectiveness of different control strategies and can also be integrated with other modeling techniques to provide more comprehensive analysis. However, there are several concerns with regard to agent-based approaches. First, the assumptions made by the modeler will heavily affect the quality of simulation, and such assumptions can be hard to justify either theoretically or statistically. Second, there is an obstacle in calibrating the simulation parameters. The relevant data are hard to acquire because stakeholders consider detailed information about each CIS to be very sensitive because of their relevance to their business (Ouyang, 2014).

### **2.3 System Dynamic-Based Approaches**

The system dynamics (SD)-based approach is a top-down method that manages and analyzes complex adaptive systems with interdependencies. SD enables us to model the network abstractly and to evaluate the impacts of that topology on system robustness as a result of different types of failures and attacks. It is useful in finding the tipping point in the dynamic



behavior from stable to unstable operational conditions (Brown, 2007). There are three basic elements in this type of approach: feedback, stock, and flow. Feedback loops represent the interconnections and directions of effects between CIS components. Stocks indicate quantities or states of the system, and flow rates control the level of stocks. Two diagrams are usually used to depict interdependent CISs: a causal-loop diagram captures the causal effects among different variables and a stock-and-flow diagram describes the flow of information and products through the system (Brown, 2007).

Argonne, Los Alamos, and Sandia National Laboratories jointly developed the Critical Infrastructure Protection Decision Support System (CIPDSS). The CIPDSS serves as a decision support system that provides help in critical infrastructure protection decision making (O'Reilly et al., 2007). It allows decision makers to prioritize and invest limited resources and to execute rational strategies to protect various systems and infrastructures on the basis of objective and dynamic modeling, simulation and analysis (Bush et al., 2005). Conrad et al. (2006) applied SD models to the analysis of power distribution and its cascading effect on telecommunications infrastructure, as well as emergency services infrastructure.

By capturing causes and effects under disruptive conditions, SD-based approaches are able to model the dynamic and evolutionary behavior of the interdependent CISs. They also capture the effects of policy and technical factors to reflect system evolution in the long term and to provide investment recommendations. SD does have limitations. First, SD-based approaches are semi-quantitative, since the causal-loop diagram is developed on the basis of the knowledge of experts. Second, SD-based models are hard to calibrate because of a lack of data. Third, SD-based approaches use differential equations to model system-level behavior. They cannot,

however, analyze component-level behaviors, such as changes in infrastructure topologies. Last, S-D based models are hard to validate because of the difficulty in obtaining data (Ouyang, 2014).

## 2.4 Economic Theory-Based Approaches

There are two types of players on the market: households and businesses. Households offer labor and capital to businesses in exchange for income. In turn, businesses use these factors to produce goods and services to sell to households. However, businesses do not just use labor and capital to produce goods and services, but they also need various raw and processed materials and services, referred to as intermediate goods. Infrastructure falls into this category, and CIS interdependencies can be thus analyzed through a model of economic interdependencies (Rose, 2005). Currently the literature mainly contains two types of economic interdependencies models: input-output (I-O) analysis and computable general equilibrium (CGE) analysis.

### 2.4.1 *Input-Output Analysis*

Input-output analysis is the most widely used tool of regional economic impact analysis. The I-O based method is a static and linear model of all purchases and sales between sectors of an economy. The model follows the form:

$$x = Ax + c \Leftrightarrow \{x_i = \sum_j a_{ij}x_j + c_j \forall i\} \tag{2-1}$$

where

- x* refers to the total production output from the industry *i*
- x<sub>i</sub>* *c<sub>i</sub>* represents the industry *i*'s total output for final consumption by end-users
- x<sub>ii</sub>* *a<sub>ij</sub>* is the Leontief technical coefficient that represents the ratio of industry *i*'s inputs to industry *j* in terms of the total production requirements of industry *j*.

Haimes and Jiang (2001) proposed a Leontief-based infrastructure Inoperability Input-output Model (IIM). It would depict a system that consists of  $n$  critical complex intra-connected and interconnected infrastructures, with the output being the risk of inoperability that could be triggered by the input, which could be one or multiple failures due to accidents, natural hazards, or attacks. Here, inoperability of a system was defined as the inability of the system to perform its intended functions. In this model,  $x_i$  stands for the overall degree of inoperability of the  $i$ th infrastructure that can be triggered by various attacks.  $a_{ij}$  is the probability of inoperability that the  $j$ th infrastructure contributes to the  $i$ th infrastructure because of their interconnectedness, and  $c_i$  represents the additional risk of inoperability that is inherent in the intraconnectedness of the  $i$ th critical infrastructure. When a perturbation was inflicted onto one or multiple infrastructures, the proposed model estimated the effects measured by infrastructure or industry inoperability.

However, several weaknesses limited the wide employment of the input-output model (Zhang et al. 2011): spatial characteristics were not considered, the linear risk transmission input-output relationship, an inability to address dynamic issues, and coefficients calibration was difficult. On the basis of the Leontief input-output model, several extended IIMs were proposed to overcome its limitations. They are summarized as follows:

**Demand-Reduction IIM:** This model was derived by combining the insight and intuition gained from the physical IIM with the rigor of proven Bureau of Economic Analysis (BEA) databases. It defines perturbations as a reduction in final demand (i.e., the difference between the as-planned and degraded final demands) to a set of economic sectors and assesses the output reduction or inoperability of each interdependent economic sector (Haimes et al. 2005; Santos and Haimes 2004).

**Dynamic IIM:** Carvajal and Daz (2002) argued that the capital coefficients in traditional Leontief dynamic I-O model must be either zero or negative for an economic system to be stable. Therefore, the capital coefficient can be interpreted as an expression of short-term counter-cyclical policy instead of long-term growth. The model describes the inoperability evolution process and the temporally interdependent recovery of economic sectors after an attack or natural disaster while integrating the industry resilience coefficients to quantify and manage the improvement of various sectors (Haimes et al., 2005; Lian and Haimes, 2006). The classical dynamic IIM was developed to help understand the infrastructure interdependencies of deliberate external attacks or unfortunate natural disasters. However, classical dynamic IIM is a demand-driven model. In the real world, more supply-driven sectors exist than demand-driven sectors in interdependent infrastructure systems (Xu et al., 2011). A supply-driven dynamic IIM was proposed by Xu et al. (2011) to model the behavior of the value-added input perturbation.

**Supply-Side Price IIM:** Ghosh (1958) introduced the supply-driven input-output model, while the interpretation of describing physical output changes that are caused by changes in the physical inputs of primary factors was convincingly argued. Dietzenbacher (1997) showed that Ghosh's model becomes plausible once it is interpreted as a price model. The interpretation of the supply-driven model as a price model also allows for the derivation of the Ghosh quantity model. Supply-side price IIM helps address the cascading impacts among interdependent economic sectors when the initial inoperability is driven by value-added perturbations, and output-side IIM investigates the output levels of industries (Leung et al., 2007).

**International Trade IIM:** Jung et al. (2009) developed an international trade IIM to investigate resulting international trade inoperability for all industry sectors on the basis of losses they could incur.

**Multiregional IIM:** Crowther and Haimes (2010) introduced multiregional IIM by extending the IIM to model multiregional interdependencies among the various regions in the U.S. It measures the cascading regional effects of disruptions to interconnected industries (Pant et al., 2011). Crowther and Haimes (2010) also emphasized the importance of spatial explicitness in interdependency analysis through a case study.

The IIM-based models offer intuitive interpretations of interdependencies and can be utilized to study the inoperability of CISs in relation to different types of perturbations. Perturbation propagation among interconnected infrastructure can be easily captured by the IIM models, which also provide insights into how to implement effective mitigation efforts. In addition, they are all based on large-scale databases and measure the interdependencies among infrastructure systems via economic relationships. Therefore, the IIM-based models are useful for macroeconomic-level and industry-level interdependency analysis after natural hazards or accidental events. Furthermore, the analytical solutions enable parameter sensitivity analysis. However, IIM-based models have several weaknesses (Ouyang, 2014). First, interdependencies at the component level cannot be investigated through input-output based models. Second, because the interdependency matrix is derived from economic databases, the matrix elements only measure interdependent strengths under normal economic operations. Namely, a linear risk transmission input-output relationship is assumed. In fact, the coupling strength among interdependent infrastructures is non-linear and depends on real-time infrastructure or industry outputs. IIM-based models can give a good approximation analysis of cascading failure and recovery processes when the perturbations have small impacts on some economic sectors, but they have limited strength in the face of large or new perturbations on non-recoverable economic sectors.

### 2.4.2 *Computable General Equilibrium Analysis*

Computable general equilibrium (CGE) analysis captures most of the advantageous features of I-O, such as the consideration of interdependencies among economic sectors, while it overcomes many of its weaknesses, such as linear interdependencies among economic sectors, and lack of consumers' and producers' behavioral responses to markets and prices (Ouyang, 2014).

Zhang et al. (2011) proposed a generalized modeling framework to analyze the interdependencies among infrastructure systems under a multilayer infrastructure network (MIN) modeling platform. The various infrastructure systems are modeled as individual networks connected through links representing market interactions. The horizontal links represent the interaction or flows of a CIS across different regions, while the vertical links denote the interdependencies among various CISs in the same region. The interactions can be formulated through supply-demand mechanisms. Combining the MIN platform with the computable general CGE theory and its spatial extension (SCGE), an equilibrium problem is thus formulated. This model considers the physical and operational characteristics of infrastructure systems, the substitutability of infrastructure commodities/services, the decision-making behaviors of producers and system users, and transportation/transmission costs.

CGE-based approaches complement the input-output-based methods, capture the nonlinear interactions among CISs, provide resilience or substitution analysis of a single CIS and the whole economy, and enable the capture of different types of interdependencies in a single framework. However, there are two limitations of CGE-based methods. First, the calibration of production functions and utility functions depends on its function form. Calibration becomes difficult when the relevant data are scant. Second, the resilience analysis for producers relies

heavily on external sources for some of the elasticity values required during the calibration, but studies on elasticity derivation are scarce (Zhang et al., 2011).

## 2.5 Network-Based Approaches

It is intuitive to model the CIS as a network, where nodes represent different CIS components and links denote the interactions among them. Network-based approaches model single CISs by network and describe the interdependencies by links between nodes, providing detailed descriptions of their topologies and flow patterns (Ouyang, 2014). The measurement of the response of CISs to hazards can start by modeling the failures caused by hazards at the component level, then examining the cascading failures within and across CISs at the system level. Current network-based studies can be broadly grouped into topology-based methods and flow-based methods.

### 2.5.1 *Topology-Based Methods*

There are several ways that nodes can fail, such as directly from a hazard, indirectly by disconnection from the source nodes within the CIS, simultaneous failure of the dependent nodes in other CISs, or failure of back-up plans. Topology-based analysis can be approached from two perspectives: an analytical model and a simulation method.

When node heterogeneity is not considered (source nodes, transmission nodes, sink nodes etc.), then each CIS can then be characterized by its node degree of distribution, which is represented by a generating function. Thus the size of a CIS's giant component under random failures can be studied analytically. The degree of distribution  $p_k$ , which denotes the probability that a randomly chosen vertex in a network has degree  $k$ , has the generating function

$$G_0(x) = \sum_{k=0}^{\infty} p_k x^k.$$

For two interdependent networks, one can follow the propagation of the failures back and forth from one network to the other so that the cascading failures can be characterized. Starting from layer A, we consider the random damage to a fraction  $1 - p$  of the nodes that have been inflicted on the network. The size  $S_A^{(0)}$  of the giant component will depend on the average message  $\sigma_A^{(0)}$ , indicating the probability that a node at an end of a link in layer A is in the giant component of the same layer. Then we consider layer B, in which we assume that all the nodes whose replica nodes in layer A are not in the giant component of layer A are damaged. Therefore, we consider all the nodes in layer B that remain in the giant component of layer B after this damage propagating from layer A has been inflicted. The probability that the nodes are not damaged is  $p[1 - G_0(1 - \sigma_A^{(0)})]$ , and using this probability we can calculate the probability  $\sigma_B^{(1)}$  that a random link in layer B reaches a node in the giant component of layer B and the probability  $S_B^{(1)}$  that a random node in layer B is in the remaining giant component of layer B. Iterating this process back and forth from one layer to the other, it is possible to describe the propagation of cascading events in the interconnected network. At the end of this iterative procedure, the remaining nodes are the nodes in the mutually connected component. In the literature, there are several types of networks of networks:

1. Networks of networks with a fixed supernetwork, i.e., networks of networks in which if a layer  $\alpha$  is connected with layer  $\beta$ , then every node  $(i, \alpha)$  is connected to its replica node  $(i, \beta)$  in layer  $\beta$  (Bianconi et al., 2015; Gao et al., 2011);
2. Networks of networks with a given superdegree distribution, in which every node  $(i, \alpha)$  is connected to  $q = q_\alpha$  replica nodes  $(i, \beta)$  of randomly chosen layers  $\beta$  (Bianconi and Dorogovtsev, 2014);



3. Networks of networks with a fixed supernetwork and random permutation of the labels of the nodes, i.e. network of networks in which if a layer  $\alpha$  is connected with layer  $\beta$ , then any node  $(i, \alpha)$  is connected to a single node  $(j, \beta)$ , where  $j$  is taken from a random permutation  $\pi_{\alpha, \beta}$  of the indices  $i$  (Gao et al., 2012, 2013);
4. Networks of networks with multiple interconnections, in which every node  $(i, \alpha)$  has  $k_{i, \alpha, \beta}$  connections with nodes in layer  $\beta$  (Leicht and D'Souza (2009).

The generating function method offers an analytical solution to the interdependent networks under different types of hazards. However, this method can only analyze the random networks with large or infinite size under random attacks with identical failure probabilities for all components by removing the nodes with the largest degrees. In reality, infrastructure networks are spatially embedded (Bashan et al., 2013), and not only are their sizes limited but their component failure probabilities are different. Therefore, other approaches are needed to achieve the goal (Ouyang, 2014).

When node heterogeneity is considered in the CIS modeling, simulation-based methods are normally utilized to examine the performance of interdependent CISs under different network failures. The performance of each network can be measured through different measures, such as the inverse characteristic path length and connectivity loss, clustering coefficient and redundancy ratio (Dueas-Osorio et al., 2007), the number of damaged nodes, the duration of the component unavailability, and the number of customers served. As to system-level performance, factors include lost service hours (Johansson and Hassel, 2010) and the fraction of customers affected (Poljanek et al., 2012). In order to evaluate the direct effects of dependencies, removal of a component is a way of representing that a component is not able to deliver its designed function. For example, Johansson and Hassel (2010) represented the strains to an infrastructure by

removing nodes or edges in the network model of one or several infrastructures, or by removing the dependency edge between two infrastructures, and then investigated the consequences after the failure.

According to Ouyang et al. (2009), the interdependent effect can be defined as “the absolute difference between the independent and interdependent efficiency, and difference is normalized by the maximum independent efficiency attained at any removal fraction.”

$$\text{Interdependent effect} = \frac{|\text{Interdependent Efficiency} - \text{Independent Efficiency}|}{\max(\text{Independent Efficiency})} \quad (2-2)$$

Adjusting and designing the interdependent topologies is an effective strategy to reduce the cascading failure effects across CISs. Winkler et al. (2011) introduced a performance assessment methodology for coupled infrastructures that links physical fragility modeling with the topology of realistic and ideal connecting interfaces. The interface was built on features such as betweenness, clustering, vertex degree, and Euclidean distance. It provides utility owners and operators with new and simple, yet adequate, strategies to enhance routine operation and reduce the probability of widespread interdependent failures following disruptive events. Hernandez-Fajardo and Dueas-Osorio (2011) introduced a new dynamic methodology for the assessment of systemic fragility propagation across interdependent networks. It improves the existing static methodologies. They found that most of the interdependent failure propagation across systems occurs early in the evolution process from transient to steady state performance.

Although the topology-based methodology can capture the topological features of the interdependent CISs and identify the critical CIS components, the topological model alone cannot provide sufficient information about the flow performance of the real CISs. Therefore,

other modeling approaches are needed for overall decision support for real-world CISs (Hines et al., 2010; Ouyang, 2014).

### **2.5.2** *Flow-Based Methods*

Interdependent CISs are viewed as networks, with movement of commodities and services corresponding to flows. Flow-based methods focus on modeling the mechanisms of these flows. Nodes and edges constitute the topology and have the capacities to produce, load, and deliver the services. Lee et al. (2007) incorporated five types of infrastructure interdependencies into a network flows mathematical representation: an interdependent layer network (ILN) model. Each infrastructure system is defined as a collection of nodes and arcs, with commodities flowing from node to node along paths in the network. For each commodity, each node is a supply node that is the source for the commodity, a demand node that requires certain amount of commodity, or a trans-shipment node that neither produces nor consumes but serves as a point through which the commodity passes. They all designed to follow the flow conservation constraints: (1) for supply nodes, total flow out of the node is no greater than the available supply; (2) for demand nodes, the demand should be met; (3) for trans-shipment nodes, the flow into the nodes must equal the flow out of the node. Lee et al. (2007) pointed out that in systems such as transportation and telecommunications, commodities move across the system with specific origins and destinations. The model was implemented by using the lower Manhattan region of New York for illustration. It allowed users to assess post-disruption impacts, provided insights into the effects of restoration plans, and facilitated scheduling and decision making (Cavdaroglu et al., 2013, 2014).

Models are capable of representing the instantaneous failure between infrastructure components such as in an electric power grid, and the telecommunications sector has been

widely investigated. However, buffered characteristics exhibited by networks such as for fuel and food should also be considered. Svendsen and Wolthusen (2007*a,b*) refined the unbuffered model (Svendsen and Wolthusen, 2008) by adding buffered resource models into the model.

While more and more studies try to develop a uniform model to describe the interdependencies among CISs, they neglect that different CISs have different operation mechanisms. By using physical rules, more realistic modeling of CISs can be generated. Ouyang et al. (2009) adopted the direct current (DC) power flow model (Dobson et al., 2001) and gas pipeline model (Kralik et al., 1984) to investigate the interdependencies of a power system and gas system. They found that interdependencies have a small effect on the power network but a larger impact on the gas pipeline network. Results also showed that removing a fraction, 18 percent, can cause the largest interdependent effect. Applying the same flow models to gas and power in Houston, Texas, Ouyang and Dueas-Osorio (2011*a*) proposed an approach to finding a global optimum strategy to designing or retrofitting the interdependent topologies between CISs to minimize cascading failures across urban infrastructure systems under multiple hazards. Two metrics were introduced: the global annual cascading failure effect (GACFE) and the GACFE-based cost improvement (GACI). The GACI metric quantifies the improvement of the strategy's effectiveness per kilometer increment of interdependent link length. When design cost is not considered, the lower the GACFE is, the better the strategy is. When the design cost is considered, the higher the GACI is, the better the interface design strategy is.

In general, flow-based models can capture the flow characteristics of interdependent CISs and realistically describe the operation mechanism. They also could help identify the critical CIS component and provide emergency protection suggestions. However, if more detailed operation mechanisms are pursued, then the computational cost would be very high (Ouyang, 2014).

## 2.6 Other Approaches

In addition to the aforementioned approaches, other methods have been developed to model and analyze interdependent CISs, such as hierarchical holographic modeling (HHM) methods, high level architecture (HLA)-based methods, petri-net (PN)-based methods, dynamic control system theory (DCST)-based methods, Bayesian network (BN)-based methods, and more.

Haimes (1981) introduced HHM to provide a comprehensive theoretical framework for systems modeling. HHM gives a holographic view of a modeled system while also adding both robustness and resilience to the model by capturing different systems aspects and other societal elements. It also adds more realism to the modeling process. The basis of holographic modeling is overlapping among various holographic models with respect to the objective function, constraints, decision variables, and input-output relationships of the CISs. Haimes (2008) employed HHM to study the multiple dimensions of the risks of a System of Systems (SoS). However, because of structural complexity, network evolution, connection diversity, dynamic complexity, node diversity, and the interdependent complexity of interdependent CIS, HHM is hard to implement in interdependent CIS modeling. It is infeasible to provide a mathematical model for some perspective of the system.

“A system-of-systems (SoS) consists of multiple, heterogeneous, distributed, occasionally independently operating systems embedded in networks at multiple levels that evolve over time” (DeLaurentis, 2007). The interdependent feature of CISs fits in the SoS concept (Eusgeld and Nan, 2009). Zio and Ferrario (2013) adopted Muir Wed to represent all the dependencies and interdependencies among the components of the CISs connected to the nuclear power plant, then applied the Monte Carlo simulation to calculate the probability that the nuclear

power plant would enter an unsafe state. An analysis was also conducted to find the critical point at which interdependencies would affect the safety of the nuclear power plant. Eusgeld and Nan (2009) extended the electric power supply (EPS) system to system-of-systems by adding the agent-based model of supervisory control and data acquisition (SCADA). Also, the high level architecture (HLA) based interdependency model was introduced. It was constructed with three layers: the lowest level represents the single CIS models, the middle level describes the interaction among CISs, and the highest level includes the global system-of-systems model.

Petri-net (PN) (Petri, 1966) can be represented by four components:  $PN = (P, T, I, O)$ , where  $P$  denotes a set of places,  $T$  stands for transitions,  $I$  represents input functions, and  $O$  is the output function. “Taking the places of the net together with the tokens to represent the states or conditions of the CISs or their components, and the transitions to represent the impacts across CISs or their components, then the CISs interdependencies are simulated by the flow of the tokens throughout the network” (Beccuti et al., 2012; Ouyang, 2014). This method is similar to network-based approaches. One limitation of this approach is that the computation complexity could be high when detailed information needs to be modeled or the system size is too large (Ouyang, 2014).

The DCST-based method is built on the dynamic control system theory (Agostino et al., 2010; Fioriti et al., 2010; Ouyang, 2014). It describes the investigated CISs by the transfer functions, which express the input/output relationship of two interdependent infrastructures or their components. The Methodology for Interdependencies Assessment EU has adopted DCST to assess interdependencies in information and communications technology (ICT) and power system networks (Casalicchio et al., 2011).

BN-based methods use a directed acyclic graph to depict the interdependencies among CISs. In the graph, nodes denote the random variables, which represent the status of infrastructure components and services and adverse events. Edges stand for the conditional dependencies, which describe the causal relationships among adverse events, CISs components, and infrastructure services (Ouyang, 2014). Giorgio and Liberati (2012) presented a novel approach to analyzing the CIS interdependencies based on the Dynamic Bayesian Network (DBN). The modeling procedure is divided into three stages: the first stage models the adverse events that would impact the analyzed CISs, the second stage captures the interdependencies among CISs, and third stage allows us to monitor the state of provided services. The DBN methodology is considered flexible and able to include experts' opinion. It has also inherited some weakness. For example, computational complexity increases with the number of variables.

### 3 Methodology

#### 3.1 Percolation Process

Given a network, the probability that a randomly chosen node from the network that has degree  $k$  is  $p_k$ . The generating function for this probability distribution  $p_k$  is

$$G_0(x) = p_0 + p_1x + p_2x^2 + p_3x^3 + \dots = \sum_{k=0}^{\infty} p_k x^k \quad (3-1)$$

The average degree  $z$  of a node can be calculated by

$$z = \langle k \rangle = \sum_k k p_k = G'_0(1) \quad (3-2)$$

The  $G_0(x)$  encapsulates all the information contained in the probability distribution  $p_k$ . Say  $G_0(x)$  “generates” the probability distribution  $p_k$  (Newman, 2010). Following a randomly chosen edge, the node at either end of the edge has degree  $k$  with a probability proportional to  $k p_k$ . This is because there are  $k$  times as many edges connected to a node of degree  $k$  than to a node of degree 1. This is called the excess degree of the node. The probability  $q_k$  of having excess degree  $k$  is

$$q_k = \frac{(k+1)p_{k+1}}{\sum_k k p_k} = \frac{(k+1)p_{k+1}}{z} \quad (3-3)$$

Therefore, another generating function,  $G_1(x)$ , can be represented as

$$G_1(x) = \sum_{k=0}^{\infty} q_k x^k = \frac{1}{\langle k \rangle} \sum_{k=0}^{\infty} (k+1) p_{k+1} x^k = \frac{1}{\langle k \rangle} \sum_{k=0}^{\infty} k p_k x^{k-1} = \frac{G'_0(x)}{G'_0(1)} = \frac{G'_0(x)}{z} \quad (3-4)$$

Consider the transportation network as a graph in which intersections are described as nodes and roads are depicted as links. (for a lifeline network in general, stations are the nodes,



and interactions/transmissions are the links.) The percolation process is parametrized by the fraction of the nodes in the network. When more nodes are present, the network tends to be more connected. We define the cluster that has the maximum number of nodes as the giant component. When the percentage of nodes present in the network decreases, there exists a transition point at which the giant component breaks apart. The point at which the percolation transition occurs is called the critical percolation threshold ( $p_c$ ). We identify the threshold at which the giant component size relative to the network size is larger than 0. The logic behind this is that although small clusters can form in the network, they are isolated from the resources and not functioning properly. Only when the cluster size reaches a critical point, namely the percolation threshold, can the network start functioning (partially). Therefore, we use the relative size of the giant component to the size of the whole network. In order to connect to the giant component, node  $A$  must be connected to the giant component via at least one of its neighbors. That is to say,  $A$  does not belong to the giant component if (and only if) it is not connected to the giant component via any of its neighbors. Define  $u$  as the average probability that a node is not connected to the giant component through its neighbors. If node  $A$  has degree  $k$ , then the probability that it is not connected to the giant component via any neighbors is  $u^k$ . Hence, the average probability that a node is not in the giant component is  $\sum_k p_k u^k = G_0(u) = \sum_{k=0}^{\infty} p_k u^k$ .

This probability also equals  $1 - S$ , where  $S$  is the fraction of the nodes that belongs to the giant component (Newman, 2010). Therefore, we have

$$S = 1 - G_0(u) \tag{3-5}$$

Again, the probability that a node is not connected to the giant component via a particular neighboring node is equal to the probability that this node is not connected to the giant

component through any of its neighbors. If there are  $k$  of these neighbors, then the probability is  $u^k$ . Since it connects to a neighboring node through an edge,  $k$  is following the excess degree distribution  $q_k$ . Therefore, it is formulated as

$$u = \sum_{k=0}^{\infty} q_k u^k = G_1(u) \tag{3-6}$$

Equations 3-5 and 3-6 provide a complete solution procedure to identify the size of giant component in a transportation network.

### 3.2 Cascading Failure

There exists a critical coupling of interdependent nodes in interdependent, non-embedded networks above which single failure can invoke cascading failures that may abruptly break down the system. Below this critical coupling, the transition is continuous, and small failures lead only to small system damage but not to a collapse. As a result of this critical coupling, the resilience of these interdependent networks is threatened by the risky abrupt collapse phenomenon.

Therefore, when the coupling of interdependent nodes in a network of networks (NON) is below this critical coupling, the system can be considered in a safe region, free from the risk of abrupt collapse (Gao et al., 2014).

However, many critical infrastructure networks are embedded in space. Interdependent spatially embedded networks, modeled by coupled lattices, have been found significantly more vulnerable than non-embedded networks (Bashan et al., 2013). In contrast to non-embedded networks, there is no critical coupling of interdependence, but any small coupling of interdependent nodes will lead to an abrupt collapse of first-order transition. In such systems there is no safe region (Gao et al., 2014).

In an interdependent system, take two networks  $A$  and  $B$  with the number of nodes  $N_A$  and  $N_B$ , respectively (Parshani et al., 2010). Within network  $A$ , the nodes are connected by  $A$  edges with degree distribution  $P_A(k)$ , while the nodes in network  $B$  are connected by  $B$  edges with degree distribution  $P_B(k)$ . In addition, a fraction  $q_A$  of network  $A$  nodes depends on the nodes in the network  $B$ , and a fraction  $q_B$  of network  $B$  nodes depends on the nodes in network  $A$ .

The process of cascading failures is initiated by randomly removing a fraction  $1 - p$  of network  $A$  nodes and all the  $A$  edges that are connected to them. Because of the interdependence between the networks, the nodes in network  $B$  that depend on removed  $A$  nodes are also removed, together with the  $B$  edges that are connected to them. As nodes and edges are removed, each network breaks up into connected components. We assume that when the network is fragmented, the nodes belonging to the giant component connecting a finite fraction of the network are still functional, while nodes that are parts of the remaining small clusters become nonfunctional. Since each network is connected differently, the nodes that become nonfunctional on each step are different for both networks. This leads to the removal of more dependent nodes from the coupled network, and so on.

The formalism of this process is presented as follows. We define  $p_A$  and  $p_B$  as the fraction of nodes belonging to the giant components of networks  $A$  and  $B$ , respectively. The remaining fraction of network  $A$  nodes after an initial removal of  $1 - p$  is  $\psi'_1 \equiv p$ . The initial removal of nodes will disconnect additional nodes from the giant component. The remaining functional part of network  $A$  therefore contains a fraction  $\psi_1 = \psi'_1 p_A(\psi'_1)$  of the network nodes. Since a fraction  $q_B$  of nodes from network  $B$  depends on nodes from network  $A$ , the number of nodes in network  $B$  that becomes nonfunctional is  $(1 - \psi_1)q_B = q_B[1 - \psi'_1 p_A(\psi'_1)]$ . Accordingly, the remaining

fraction of the network  $B$  is  $\phi'_1 = 1 - q_B[1 - \psi'_1 p_A(\psi'_1)]$ , and the fraction of nodes in the giant component of network  $B$  is  $\phi_1 = \phi'_1 p_B(\phi'_1)$ .

Following this approach we can construct the sequence  $\psi_n$  and  $\phi_n$  of giant components, and the sequence  $\psi'_1$  and  $\phi'_1$  of the remaining fraction of nodes at each stage of the cascade of failures.

$$\begin{aligned}
\psi'_1 &\equiv p & \psi_1 &= \psi'_1 p_A(\psi'_1) \\
\phi'_1 &= 1 - q_B[1 - p_A(\psi'_1)p] & \phi_1 &= \phi'_1 p_B(\phi'_1) \\
\psi'_2 &= p[1 - q_A(1 - p_B(\phi'_1))] & \psi_2 &= \psi'_2 p_A(\psi'_2), \dots \\
\psi'_n &= p[1 - q_A(1 - p_B(\phi'_{n-1}))] & \psi_n &= \psi'_n p_A(\psi'_n) \\
\phi'_n &= 1 - q_B[1 - p_A(\psi'_n)p] & \phi_n &= \phi'_n p_B(\phi'_n)
\end{aligned} \tag{3-7}$$

To determine the state of the system at the end of the cascade process we look at  $\phi'_m$  and  $\psi'_m$  at the limit of  $m \rightarrow \infty$ . This limit must satisfy the equations  $\psi'_m = \psi'_{m+1}$  and  $\phi'_m = \phi'_{m+1}$  since eventually the clusters stop fragmenting, and the fractions of randomly removed nodes at step  $m$  and  $m + 1$  are equal. Denoting  $\psi'_m = x$  and  $\phi'_m = y$ , we arrive at a system of two equations with two unknowns:

$$x = p[1 - q_A[1 - p_B(y)]] \tag{3-8}$$

$$y = 1 - q_B[1 - p_A(x)p] \tag{3-9}$$

The model can be solved through generating functions. The generating functions will be defined for network  $A$  while similar equations describe network  $B$ . The generating function of

the degree distributions  $G_{A0}(\xi) = \sum_k P_A(k) \xi^k$ . Analogously,  $G_{A1}(\xi) = \frac{G'_{A0}(\xi)}{G'_{A0}(1)}$ . Random removal of

fraction  $1 - p$  of nodes will change the degree distribution of the remaining nodes, so the

generating function of the new distribution is equal to the generating function of the original distribution with the argument equal to  $1 - p(1 - \zeta)$  (Newman, 2002). The fraction of nodes that belongs to the giant component after the removal of  $1 - p$  nodes is

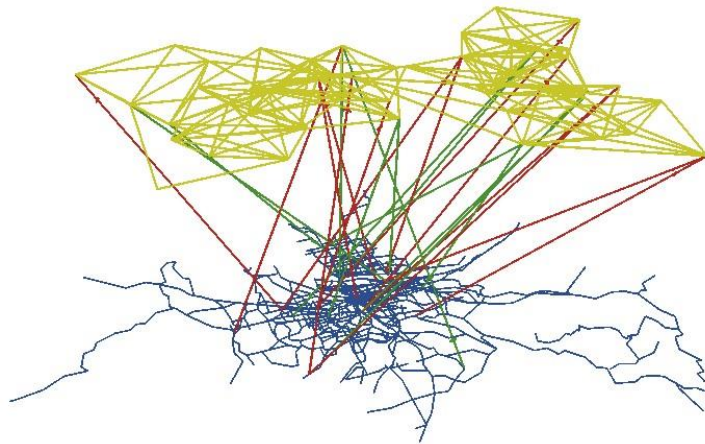
$$p_A(p) = 1 - G_{A0}[1 - p(1 - f_A)] \quad (3-10)$$

where  $f_A = f_A(p)$  satisfies a transcendental equation

$$f_A = G_{A1}[1 - p(1 - f_A)] \quad (3-11)$$

## 4 Agent-Based Simulation Framework

The lifeline network can be abstracted as a graph in which the node represents the infrastructure, and the link denotes the interaction between nodes. In a transportation network, nodes represent the intersection, and links represent the road. Each node has two states: function and fail. Using the concept proposed in Chapter 3, an agent-based modeling framework was developed in NetLogo. The framework is presented in figure 4-1. The yellow layer is network A, and blue layer is network B. The red and green lines represent the interaction from A to B and B to A, respectively. Networks A and B can be any network. Because of the colocation interdependency in infrastructure networks, in this project, network A and B both represented transportation networks as critical lifeline infrastructure. However, this framework is only limited to transportation networks; this interdependency modeling framework can be applied to different lifeline networks.



**Figure 4-1:** Interdependent lifeline infrastructure agent-based modeling framework

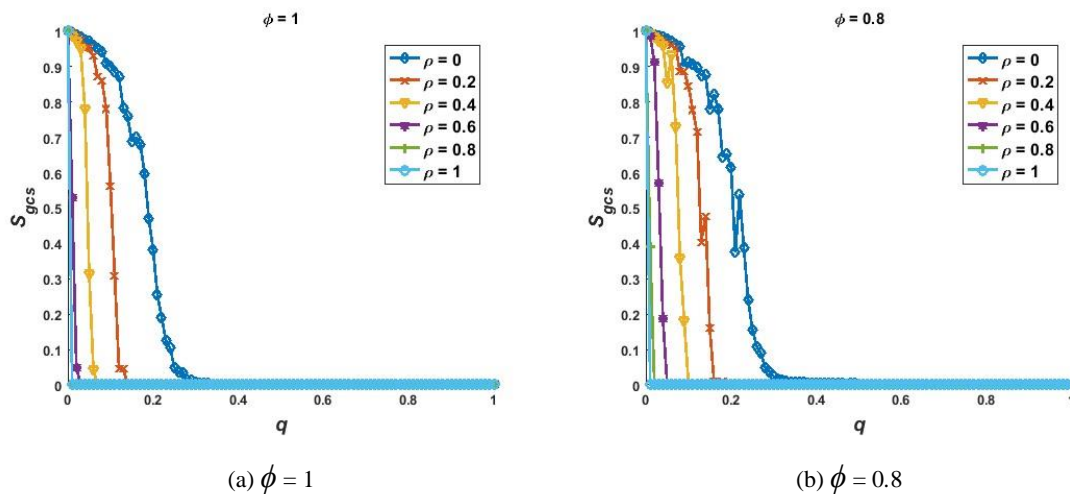


## 5 Results

### 5.1 High Interdependency

Three parameters control the cascading failure within the interdependent lifeline system.  $\phi$  represents the interdependency from network A to network B, i.e.,  $\phi = 0.8$  means that 80 percent of the nodes' normal function in network B depends on network A. Analogously,  $\rho$  represents the interdependency from network A to network B, i.e.,  $\rho = 0.6$  means that 60 percent of the nodes' normal function in network A depends on network B. In this experiment, we examined a failure that initiated in network A and then propagated through the system.  $q$  represented the fraction of nodes in network A that failed during the disaster, i.e.,  $q = 0.4$  means that 40 percent of the nodes in network A failed. In the meantime, we measured the giant component size in network B to examine the interdependency effect on interconnected networks.

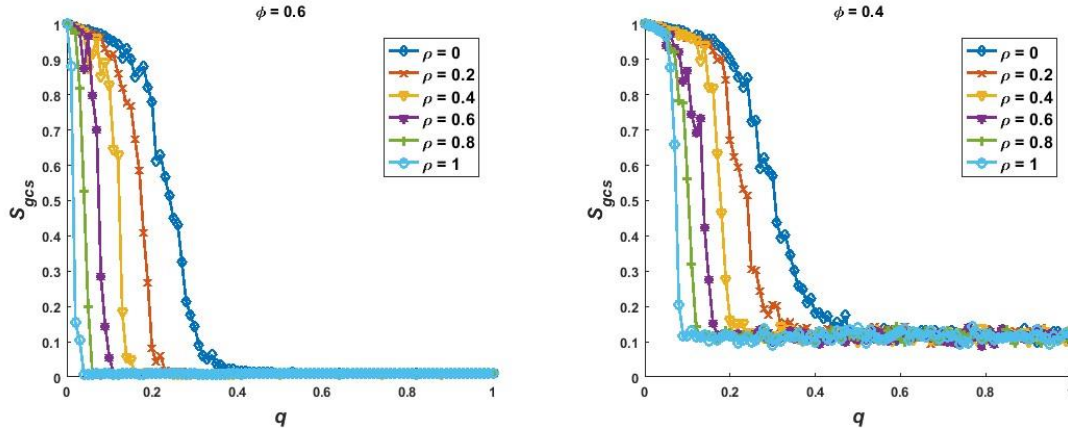
Figure 5-1 presents the scenario in which network A and network B are highly interdependent.



**Figure 5-1:** High interdependency between networks

For example, as shown in figure 5-1(a), when  $\phi = 1$ , then  $\rho = 0.2$ , which means that network B is fully dependent on network A; at  $q = 0.17$ , the  $S_{gcs}$  decreases to 0. Empirically, this





suggests that when the initial failure in network A reaches 17 percent, the giant component size in network B will become 0. We call the point when the giant component diminishes the critical threshold,  $q_c$ . As shown in figure 5-1(b), when  $\phi = 0.8$ , the  $q_c$  at low  $\rho$  does not change much. However, at a high  $\rho$ , such as 0.4 or 0.6, the  $q_c$  increases by 0.04 and 0.02, respectively. This indicates that at a high  $\phi$ , a small increase in  $\rho$  will lead to a critical threshold increase. This suggests that when network B is highly dependent on A, and when network A is increasingly dependent on B, a small disruption in network A will result in catastrophic failure in network B.

## 5.2 Medium Interdependency

Figure 5-2 presents a scenario in which the interdependency is in medium range. When  $\phi = 0.6$ , then  $\rho = 0$ , and  $q_c = 0.4$ . Compare that to the case in which  $\phi = 1$ ,  $\rho = 0$ , and  $q_c = 0.3$ . That is to say, when network A depends on network B at a fixed level, then as the network B increasingly depends on network A, the critical threshold  $q_c$  increases. In the meantime, when  $\phi = 0.4$ , the giant component size stops decreasing at 0.1, which means that when 40 percent of the nodes in network B depend on network A, no matter how much network A depends on network B, the giant component size will always stay at 0.1 percent eventually. Similarly, when  $\rho$  increases, the  $q_c$  increases. For example, when  $\phi = 0.4$ , then  $\rho = 0$ ,  $q_c = 0.5$ .

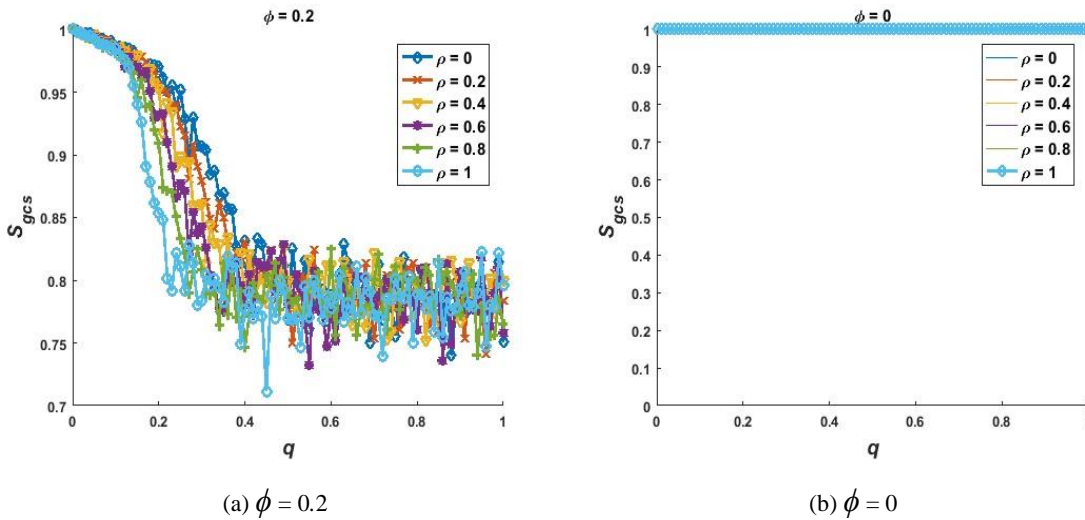
(a)  $\phi = 0.6$

(b)  $\phi = 0.4$

**Figure 5-2:** Medium interdependency between networks

### 5.3 Low Interdependency

When  $\phi$  decreases to a low level, then the different  $\rho$  cases get closer. Similarly, at a low level  $\phi$ , for example,  $\phi = 0.2$  as shown in figure 5-3(a), the giant component size stays steady ( $S_{gcs} = 0.75$ ). When the  $\phi$  decreases to 0, as shown in figure 5-3(b), which means that network B is not dependent on network A, then no matter how many nodes fail in network A, the giant component size will stay at zero. This is because when network B is not dependent on network A, then no matter how big the failure, the failure has no channel to propagate to network B. Essentially, this is a single network percolation.



**Figure 5-3:** Low interdependency between networks

## 6 Conclusion

When two networks are interdependent on each other, then failure occurring in one network will propagate to the other one, and there is a threshold above which a small disturbance will result in a cascading failure that may abruptly break down the system. In this project, we investigated the critical threshold that will prevent a network from breaking down in different scenarios. Multiple networks can be involved in an interdependent system; in this project, only two networks were considered in order to prove the concept. However, the framework can be generalized to multiple networks. An agent-based modeling framework was developed to simulate cascading failure between networks.

The results were as follows:

- At a fixed  $\phi$  (i.e., the fraction of nodes in network B that depends on network A), when  $\rho$  increases (i.e., the fraction of nodes in network A that depends on network B), the critical percolation threshold decreases. That is to say, when network A is more dependent on network B, a smaller disruption in network A will be needed to destroy network B.
- When  $\rho$  is fixed, as  $\phi$  decreases, the critical threshold increases. For example, when  $\phi = 0.8$ ,  $\rho = 0.2$ , and  $q_c = 0.18$ , as  $\phi$  increases to 0.6, then  $q_c = 0.22$  when  $\rho = 0.2$ . This means that when the fraction of nodes in network A that depends on network B is fixed, as the nodes in network B that depend on network A decrease, the fraction of nodes needed to fail in network A to destroy network B increases. This is reasonable because when network B is not strongly dependent on network A, there are fewer channels for failure of network A to propagate to network B. Therefore, the critical threshold for the giant component to diminish increases;

- As  $\phi$  keeps decreasing, the giant component size stops decreasing at a certain level. For instance, when  $\phi = 0.4$ , then  $S_{gcs}$  is around 0.1 for all levels of  $\rho$ , while when  $\phi = 0.2$ , then  $S_{gcs}$  is around 0.75. This phenomenon indicates that when network B is less dependent on network A, then failure in network A can hardly propagate to network B, which leads to higher giant component size. To increase the robustness of one network, the key is to impair its dependency on other networks.

## 7 Future Research

This project explored how different dependencies between networks shape the robustness of network behavior. While experiment demonstrated very insightful results, there are several interesting directions that can be explored in future research. First, more lifeline infrastructure networks can be investigated through the framework. Because of data limitations, we utilized two transportation networks to prove the concept. However, real coupled networks would be more interesting to investigate and would generate insightful ideas. Second, random failure was explored in this project. However, network failure can also result from man-made disasters, so localized failure patterns should be further investigated to validate the results. Third, not only do percolation phenomenon exist in  $\phi$  and  $\rho$ , but when they both change,  $q_c$  also exhibits a phased transition. This could be examined by designing more scenarios.

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