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Multi-Criteria Hazmat Routing on Rail with Railroad-Highway Crossing Considerations

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16. Abstract

This project analyzes different approaches for multi-criteria route selection for hazardous materials (hazmat) transportation on rail. Considering that hazmat transportation planning should regard many safety and risk factors, selecting a route for a shipment can be a challenging decision making problem. Furthermore, the safety and risk factors should be considered in an unbiased objective manner. Three unsupervised approaches are described route selection. A solution methodology is presented with these approaches. A set of numerical analyses is used to compare different approaches. Based on the results of the numerical analyses, it is discussed that, while each approach will output the best route based on a specific value, one of the approaches is more robust compared to the other two approaches. These approaches can be used by decision makers to select routes without a need for ranking and/or comparing various criteria.

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Abstract

This project analyzes different approaches for multi-criteria route selection for hazardous materials (hazmat) transportation on rail. Considering that hazmat transportation planning should regard many safety and risk factors, selecting a route for a shipment can be a challenging decision making problem. Furthermore, the safety and risk factors should be considered in an unbiased objective manner. Three unsupervised approaches are described route selection. A solution methodology is presented with these approaches. A set of numerical analyses is used to compare different approaches. Based on the results of the numerical analyses, it is discussed that, while each approach will output the best route based on a specific value, one of the approaches is more robust compared to the other two approaches. These approaches can be used by decision makers to select routes without a need for ranking and/or comparing various criteria.

Chapter 1 Introduction and Literature Review

1.1 Hazmat Transportation on Rail and Safety Factors

Majority of hazardous materials (hazmat) are transported with rail, especially long distance shipments. Indeed, based on the most recent statistics, rail is the second in ton-miles of hazmat shipments (3). An important problem of rail carriers of hazmat is planning and routing of the hazmat. As noted by Gordon and Young (14), hazmat routing is a challenging problem due to cost, safety, and complexity issues. Especially, hazmat incidents can be very dangerous to the population as well as the environment. While hazmat shipment by rail has the third most incidents, the cost of the damages of hazmat shipment incidents by rail is second compared to other methods of transportation (30). Therefore, in addition to cost and time considerations in routing hazmat on rail, the carriers need to consider the safety aspects of transporting hazmat.

In regarding safety for hazmat transportation, accident risks and the potential impact of accidents are commonly accounted for routing planning (8, 9, 10). Furthermore, multi-modal hazmat routing studies, mostly truck-rail interactions, has focused on transferring the shipment from one mode to the other (36, 37, 38). As reviewed in the following subsection, there are many risk factors to take into account when planning routes for hazmat transportation on rail. Indeed, Code of Federal Regulations (CFR) 49 - Part 172 (Appendix D) lists the following 27 factors to include in the safety and risk analyses of hazmat routing planning (1):

1. Volume of hazardous material transported;

2. Rail traffic density;

3. Trip length for route;

4. Presence and characteristics of railroad facilities;

5. Track type, class, and maintenance schedule;

6. Track grade and curvature;

7. Presence or absence of signals and train control systems along the route ("dark" versus signaled territory);

8. Presence or absence of wayside hazard detectors;

9. Number and types of grade crossings;

10. Single versus double track territory;

11. Frequency and location of track turnouts;

12. Proximity to iconic targets;

13. Environmentally sensitive or significant areas;

14. Population density along the route;

15. Venues along the route (stations, events, places of congregation);

16. Emergency response capability along the route;

17. Areas of high consequence along the route, including high consequence targets

18. Presence of passenger traffic along route (shared track);

19. Speed of train operations;

20. Proximity to en-route storage or repair facilities;

21. Known threats, including any non-public threat scenarios provided by the Department of Homeland Security or the Department of Transportation for carrier use in the development of the route assessment;

22. Measures in place to address apparent safety and security risks;

23. Availability of practicable alternative routes;

24. Past incidents;

25. Overall times in transit;

26. Training and skill level of crews; and

27. Impact on rail network traffic and congestion.

For instance, factor 9 is the number and type of crossings on the route. Indeed, railroad-highway crossings are subject to high safety risks and significant number of all rail incidents are highway-rail incidents (28, 29). The initial focus of this project was to exclusively focus on railroad-highway crossings in routing planning. However, based on the number of safety and risk factors to consider and the literature review presented next, a more generic approach is taken. In particular, the models and the analyses in this project focus on making a route selection when many factors are to be simultaneously considered. That is, given alternative routes, selecting one route considering many different safety risk and safety factors, including number of crossings, is the focus in this project.

In particular, selecting one route, from a set of many alternative routes, is a challenging task when many factors should be accounted for. A route might be preferred over another based on a specific factor while the other route can be more attractive based on another factor. Therefore, considering many safety and risk factors in addition to cost and time criteria, selecting one route is difficult. One challenge is the large number of factors. When the number of factors (evaluation criteria) is large, most of the alternative routes cannot be easily ruled out. Another challenge is how to objectively account for the various factors while assuring an effective selection.

The project analyzes three different approaches to make an effective and objective route selection and provides tools to use each approach. Furthermore, these approaches are computationally compared. The tools and the findings can be used by rail carriers to make

routing plans for hazmat transportation by accounting for many safety and risk factors as well as other operational factors such as cost and time considerations.

1.2 Multi-criteria Hazmat Transportation on Rail and Risk Considerations

Here, multi-criteria decision making models for hazmat routing on rail and risk reduction models are reviewed. This review of the existing literature fostered the recognition of two approach avenues to the risk-considerate selection of hazmat route alternatives. Varying in the procedures used to define risk and additional metrics, the methodologies collectively sample the industry's dispersive efforts to model this complex and evolving transportation problem.

The first approach category is comprised of works seeking to minimize or control the level of risk assumed by the network, while also giving deference to the financial ramifications of selected strategies. This pairing of risk and economic considerations has emerged as a natural point of investigation, stemming from the economically sensitive nature of the entities responsible for hazmat shipment.

Verma formulated a bi-objective optimization model, working to minimize both the cost and transportation risk associated with hazmat shipment (40). Hazmat specific expected risk was used in assessing transport risk; stemming from a consideration of each route's estimated accident rate, shipment characteristics, and expected population exposure. Fixed-radius exposure bands were used in determining population exposure values. Glickman, Erkut, and Zschoke conducted a study to demonstrate the impact of risk-averse rerouting on incurred transportation costs (13). The work used travel distance as a surrogate for cost and recognized risk as the population within a specified radius from the hazardous material during its transport. The paper did not explicitly perform an optimization utilizing the two objectives; instead, it postulated that risk-informed rerouting may award incremental risk reduction benefits compared to associated

increases in cost. The authors did note a bi-objective optimization model (risk, cost) as an obvious extension of their efforts and presented their methods in a manner that facilitates the development of the proposed dual model. Creation of the model would successfully integrate their strategies into a methodology for risk and cost informed route selection.

Verma, Verter, and Gendreau developed a bi-objective optimization model, considering risk and cost objectives, to determine which routes, yards, and train types minimize the objective functions while meeting the demands of the analyzed network (39). Cumulative exposure along routes and within yards was used as the risk metric, with risk values generated by an application of the Gaussian plume model (GPM). Zhao and Verter proposed a similar model, using the GPM to develop an environmental risk factor relating to the transport of used oil products (43). This metric was employed in a bi-objective model looking to minimize cost and environmental risk in determining a route and location schedule for this specific hazmat material.

Liu, Saat, and Barkan proposed an optimization model, considering two risk reduction strategies, in developing an integrated framework to optimize risk reduction (23). Namely, broken rail reduction and tank car safety design enhancement efforts were examined via a Pareto-optimality technique to maximize risk reduction for a given investment level. Lai et al. developed an integrated mathematical model to consider a combination of risk reduction approaches (22). Routing, design, and maintenance considerations were included in the model, in addition to cost considerations associated with the risk reduction and routing measures. These objectives were combined into a single objective function, used to determine the optimal combination of risk reduction strategies.

Verma, Verter, and Zufferey proposed a bi-objective optimization model, explicitly considering rail-truck intermodal hazmat transportation (38). The method assessed routing

alternatives based on cost and exposure risk expectation, incorporating factors inherent of multimodal transportation, including inbound and outbound drayage. Verma and Verter formulated a bi-objective model, looking to minimize the risk and cost associated with a developed routing schedule, while also meeting the demand due-dates necessitated of the shipments (36). The model estimated risk through employment of the GPM to determine the population exposure expected in the event of an accidental release along each route alternative.

Xie, Lu, Wang, and Quadrifoglio looked to optimize the hazmat location and routing plans carried out over multiple modes of transport in terms of overall risk and cost (41). In addition to optimally siting disposal and treatment facilities, this work also sought to optimally place transfer yards. The risk metric used in the optimization was calculated as a combination of the probability of accident and the expected population exposure along each of the transfer routes and at joining yards. Sun, Lang, and Wang developed a bi-objective model to minimize total generalized costs and the social risks associated with the transportation of hazmat on a multimodal network (35). The method calculated social risk as the product of population exposure and the volume of hazmat being transported; deriving population exposure values from the GPM. A normalized weighted sum approach was then used to determine the best performing routes.

Fang, Ke, and Verma developed a mathematical model to minimize the weighted sum of earliness and tardiness costs for each demand shipment and the holding cost at each yard (10). Dealing with hazmat, the minimization was subject to risk threshold constraints developed from historical accident and speed data. Romero, Nozick, and Xu formulated a model to aid in facility location and routing decisions of hazmat (31). In addition to more traditional risk and cost considerations, the equity of the assumed risk among household income cohorts was integrated

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into the model. The model was demonstrated on a realistic example, minimizing total canister-miles (a means of incorporating transportation costs) and transportation accident risk, in addition to determining alternatives meeting equity thresholds.

The second approach category is made up of research focused more explicitly on risk reduction, without a direct consideration of incurred costs. These works may be regarded as standalone contributions to the body of hazmat transportation risk reduction literature; in addition to being candidates for integration with cost considerations in future, commensal works. Hosseini and Verma applied a Value-at-Risk (VaR) methodology, originally developed for portfolio management, to the routing of hazmat rail (17). Route characteristics and decision-maker risk preference were incorporated into a bi-level optimization yielding distinct route and train configuration recommendations based on determined VaR values. Hosseini and Verma expanded upon their work in (17) by addressing VaR's tendency to ignore the tail-ends of used probability distributions (16). This disregard is detrimental to the method's effectiveness as the discounted region is where catastrophic event outcomes reside. A Conditional Value-at-Risk (CVaR) approach was developed to address this issue, yielding a problem instance validated methodology with demonstrated advantages over its predecessor.

Kawprasert and Barkan developed a flexible route rationalization model aimed at reducing the total risk of transporting hazmat over a rail network (19). The model was applied to a case study, finding the most favorable hazmat flow produced by the minimization of car miles, release probability, and annual risk. The performance of each objective generated flow was compared to a baseline flow in terms of distance, probability, and risk metrics. The proposed model is flexible in that it allows for different objective functions to be considered, serving as a framework to determine strategies and routes that promote reduced risk.

Bersani, Papa, Sacile, Sallak, and Terribile formulated a mixed integer linear program to minimize population exposure risk through the rescheduling of hazmat train shipments (2). The approach does not modify the paths upon which the material flows, only when the flow occurs. The dynamic nature of population dispersion was used to accordingly route hazmat shipments in a manner posing the least risk of population exposure. Faghih-Rhoohi, Ong, Asian, and Zhang created a dynamic CVaR model for hazmat transportation networks (9). The developed model aids in the selection of minimal risk routes while considering time schedules for transported material based on the CVaR of accidents in the network. The later consideration broadened the scope of the approach, incorporating a temporal factor through the identification of beneficial departure and transportation times for shipped material.

Relating most directly to the contents of this research, Gordon and Young discussed industry's use of the Rail Corridor Risk Assessment System (RCRMS) as a near-standard tool for risk assessment in relation to the transport of hazmat shipments (14). Describing RCRMS as a useful but resource intensive and informationally sensitive assessment model, the authors posed a simplified methodology, grouping the 27 required risk factors into seven groups. Through this grouping, the authors sought to provide a less onerous evaluation scheme, enabling an easier comparison of candidate routes.

1.3 Multi-criteria Decision Making

Multi-criteria decision making problems (multi-objective optimization problems) are generally characterized by the existence of Pareto efficient solutions. Koopmans effectively defined the Pareto efficient solution set as that composed of non-dominated solutions emerging from an objective-wise examination of the solution alternatives (21). To navigate the complexities of this characteristic, two approach avenues have been developed.

The first set of approach methodologies, including non-preference and a priori methods, conclude with the selection of a single solution to the multi-objective problem. Non-preference methods do not utilize expert or decision maker input in selecting the recommended solution (18). The method of global criterion, in which the distance to some reference solution is minimized, is an example of a non-preference method (42). A priori methods incorporate predefined preference information toward the selection of the recommended solution (18). A fundamental example is the Weighted Sum Method, which transforms a multi-objective problem into a single objective optimization by applying weighted coefficients to each objective function (12). Decision maker input is used to assign the weights for each objective function.

The ε-Constrained Method, another a priori scheme, requires the development of bounds for each objective function (15). In addition, a single objective function from the multiple-objective set is selected as the objective to be optimized. The remaining objectives are applied as constraints, obliging the solution to exist between their defined bounds. The weighted min-max approach seeks to minimize the maximum weighted potential loss between any of a solution's objective function values and their associated reference values (24). The ideal, or *utopia*, point is a commonly selected candidate for the reference values. Objective/Goal Programming Methods require the predetermination of desired values for each objective function (11). These values are then incorporated into a single objective function looking to minimize the total difference between the desired and actual values. This model is well-suited for solving linear problems but experiences difficulty when faced with nonlinear behavior (4). While appropriately developed a priori methodologies are beneficial in that they produce a single, Pareto efficient solution, their necessity of preassigned decision maker preference can serve problematic. The determination of meaningful and feasible expert input may be both cumbersome and difficult to develop,

particularly in large or novel problem instances. Furthermore, the nature of the utilized input is near inherently subjective.

The second approach avenue, including a posteriori and interactive methods, seek to solve the multi-objective optimization by a development of a set of Pareto efficient solutions to the problem. Depending on the method employed, the entire Pareto efficient set or a representative subset thereof is expected to be produced (26). Posteriori methods generally take on the form of mathematical programming-based methods or evolutionary algorithms. The former categorization repeatedly runs an algorithm with each run producing a single Pareto optimal solution (24). Its counterpart categorization is able to determine a set of Pareto efficient solutions with one algorithmic run (45).

The Normal Boundary Intersection method, its successor the Modified Normal Boundary Intersection method, and the Normalized Normal Constraint method are commonly applied examples of mathematical programming-based a posteriori methods (5, 25, 32). Through successive runs, the algorithms aim to give an evenly distributed approximation of the problem's Pareto front. Evolutionary algorithms are generally inspired by behaviors in nature and may incorporate a variety of comparative mechanisms mimicking natural selection to determine sets of Pareto efficient points (4). Prominent examples of genetic algorithms applied to solve multi-objective problems include the Vector-Evaluated Genetic Algorithm, the Non-Dominated Sorting Genetic Algorithm, the Strength Pareto Evolutionary Algorithm, and their inspired successors (33, 34, 44). Other biologically inspired algorithms including Particle Swarm Optimization and Artificial Immune System methods have also been utilized (6, 7, 20). Interactive approaches have also been developed to utilize expert knowledge during the optimization procedure. These processes integrate elicited decision maker input to determine

objective function value preference and provide updated solutions in an iterative process (27). Models of this broader avenue are beneficial in that they provide decision makers with a more complete view of well-performing alternatives and a sense of the model's feasible solution-space. However, these models are often computationally intensive and can provide decision makers with a large set of Pareto efficient solutions from which a single alternative may still need to be selected.

Chapter 2 Route Selection Model and Solution Approach

This chapter models the multi-criteria route selection problem and explains the solution approach.

2.4 Multi-criteria Route Selection Model

Given a required hazmat shipment, there may be many alternative routes that can be used to complete the shipment. Suppose that there are n alternative routes that can be used and let these routes be indexed by $i = 1, 2, \ldots, n$. That is, it is assumed that the decision maker knows all possible routes, which satisfy the operational requirements on the network as well as the shipment requirements. In making a selection, the decision maker needs to consider cost and time objectives as well as many safety and risk factors as listed in Chapter 1. In particular, suppose that there are m criteria that should be taken into account while selecting a route and let these criteria be indexed by $j = 1, 2, \ldots, m$. Here, it is assumed that the decision maker can evaluate any route with respect to each criterion. In particular let r_{ij} be route i's value considering criterion j and decision maker knows r_{ij} for all routes $i = 1, 2, \ldots, n$ and for all criteria $j = 1, 2, \ldots, m$. Without loss of generality, it is assumed that the lower the route i's value for a specific criterion, the better the route is based on that criterion.

Given one criterion, selecting a route for the shipment is an easy task. In particular, suppose that the decision maker wants to find the route that is best with respect to criterion j. Then, the best route for criterion j, denoted by $i(j)$, is as defined in equation 2.1:

$$i(j) = argmin\{r_{ij}: i = 1, 2, \ldots, n\}. \qquad (2.1)$$

However, there are m criteria and different routes can be the best for different criteria. Therefore, which route to select is challenging. Particularly, the multi-criteria route selection problem is as defined in equation 2.2:

$$argmin\{r_{ij}: i = 1,2, \dots, n\} \, \forall j = 1,2, \dots, m. \qquad (2.2)$$

Next, the overview of the solution approach is described.

2.5 Multi-criteria Route Selection Solution Approach

As reviewed in Chapter 1, there are two common approaches for solving multi-criteria decision making problems: Pareto front generation and reduction to single-criterion problem.

The Pareto front generation approach determines the Pareto efficient solution alternatives. A solution is Pareto efficient if there is no other solution alternative which is better in terms of all of the criteria of interest. Formally put, route i' is Pareto efficient if $\nexists i'' \neq i', i'' \in \{1,2, \dots, n\}$ such that $r_{i'j} \geq r_{i''j} \, \forall j \in \{1,2, \dots, n\}$ with $r_{i'j} > r_{i''j}$ for at least one $j \in \{1,2, \dots, n\}$. Given r_{ij} values, Pareto efficient solutions can be generated using an iterative procedure. In Chapter 3, a method for finding Pareto efficient solutions is presented. The problem with this approach is that, when the number of criteria is large, there will be too many Pareto efficient solutions. Therefore, the decision maker will still need to make a selection from a large set of solution alternatives.

Reduction to single-criterion approach reduces the multi-criteria model into a single-criterion model by using specific approaches. In particular, depending on the approach used, for each route, say route i, a single value, say P_i, is defined using the route's criterion values, i.e., r_{ij} values, to calculate route i's P_i value. Then, the route with the best P_i value is selected as the

solution alternative to implement. The problem with this approach is defining P_i value using r_{ij} values. Especially, given that r_{ij} values mostly consist of safety and risk factors, a decision maker should objectively and equally consider these while assuring that an efficient selection is made.

In this project, these two approaches are combined. Specifically, the method, discussed in detail in the next chapter, first generates Pareto efficient routes and then selects one route from the set of Pareto efficient routes by reducing the problem into a single-criterion selection model. To reduce the problem into a single-criterion selection model, three approaches are considered: equal weighting approach, maximum deviation approach, and distance to ideal solution approach. These approaches are selected because they are unsupervised approaches, i.e., they do not require input from decision maker for comparing different criteria. The reason for focusing on unsupervised approaches is to assure that all safety and risk factors are equally and objectively considered in making an efficient selection. This avoids a situation where the decision maker is liable. For instance, if one safety factor is weighted more than another one, and an incident happens because of a less-weighted factor, the decision maker's input would be a bad judgement. Therefore, the three unsupervised approaches, which are explained next, are accepted.

2.5.1 Equal Weighting Approach

Weighting approaches are commonly used for reducing multi-criteria decision making problems into a single-criterion one. Typically, these approaches seek input from the decision maker for ranking/comparing different criteria. However, as noted, the approach should be unsupervised for treating each criterion equally and objectively. Therefore, under equal-weighting approach, it is assumed that each criterion is equally weighted. Given there are *m*

criteria, a route's single value will be the average of its values over all criteria. Let P_i^1 be the

route i's single value based on equal-weighting approach. Then, P_i^1 is as defined in equation 2.3.

$$P_i^1 = \frac{\left(\sum_{j=1}^m r_{ij}\right)}{m}.$$

(2.3)

2.5.2 Maximum Deviation Approach

     Under maximum deviation approach, each route is assigned a single value based on the

deviations of its criterion values from the best criterion values. Particularly, each route's value

under a specific criterion will deviate from the best value for that criterion by at least 0. Recall

from equation 2.1 that route $i(j)$ is the route alternative which has the best value for criterion j.

Therefore $r_{i(j)j}$ defines best value available for criterion j. Note that $r_{i(j)j} = \min\{r_{ij}: i = $

$1,2, ..., n\}$ and, for notational simplicity, let $R_j = r_{i(j)j} = \min\{r_{ij}: i = 1,2, ..., n\}$. Then, route i's

deviation from the best criterion j value, denoted by d_{ij}, reads as

$$d_{ij} = r_{ij} - R_j.$$

(2.4)

Now, let P_i^2 be the route i's single value based on maximum deviation approach. Then, P_i^2 is as

defined in equation 2.5:

$$P_i^2 = \max\{d_{ij}: j = 1,2, ..., m\}.$$

(2.5)

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2.5.3 Distance to Ideal Solution Approach

In this approach, ideal point is taken as the reference point and a route's distance to the ideal point on the criteria space is considered in making a selection. Specifically, ideal point defines a point, not a solution alternative, on the criteria space. This point has the best values for each specific criteria. That is, ideal point's criterion j value is R_j (recall that $R_j = r_{i(j)j} = \min\{r_{ij}: i = 1,2,\ldots,n\}$). In most cases, there will not be a route alternative corresponding to the ideal point. To find the distance to ideal point in m dimensions, this project uses the Euclidean distance measure. In particular, let P_i^3 be the route i's single value based on distance to ideal solution approach. Then, P_i^3 is as defined in equation 2.6:

$$P_i^3 = \sqrt{\sum_{j=1}^{m}(r_{ij} - R_j)^2}. \qquad (2.6)$$

In the next chapter, algorithmic descriptions are discussed for solving the multi-criteria route selection problem using the methodology defined above with each approach.

Chapter 3 Algorithmic Descriptions

This chapter gives the detailed algorithmic descriptions of the solution approach. As noted above, the solution approach consists of two stages: generating Pareto efficient solutions and reduction to single-criteria model.

3.6 Stage 1 - Pareto Efficient Solutions

Suppose that a set of n routes is given with their respective criterion values, i.e., r_{ij} values are given for all $i = 1,2, \dots, n$ and $j = 1,2, \dots, m$. As is defined above, route i' is Pareto efficient if $\nexists i'' \neq i', i'' \in \{1,2, \dots, n\}$ such that $r_{i'j} \geq r_{i''j} \ \forall j \in \{1,2, \dots, n\}$ with $r_{i'j} > r_{i''j}$ for at least one $j \in \{1,2, \dots, n\}$. Let S be the set of routes given and $PE(S)$ be the set of Pareto efficient routes within set S. Given r_{ij} values, Pareto efficient routes in set S can be generated using the iterative procedure defined in table 3.1.

Table 3.1 Iterative procedure to generate Pareto efficient routes

Step:	Process:
0:	Let S be a given set of n routes and r_{ij} be the ith route's criterion j value.
1:	Set $k = 1$.
2:	While $k \leq n - 1$
3:	Set $w = k + 1$
4:	While $w \leq n$
5:	If $r_{kj} = r_{wj} \ \forall j = 1,2, \dots, m$, set $w =: w + 1$
6:	Else, if $r_{kj} \leq r_{wj} \ \forall j = 1,2, \dots, m$, set $S =: S\backslash\{w\}$
7:	Else, if $r_{kj} \geq r_{wj} \ \forall j = 1,2, \dots, m$, set $S =: S\backslash\{k\}, w = k + 1$
8:	Else $w = w + 1$
9:	End
10:	Set $k = k + 1$
11:	End
12:	Return $PE(S) = S$.

The iterative procedure generates all of the Pareto efficient routes given alternative routes. This procedure is illustrated with figure 3.1. Figure 3.1 (a) illustrates 8 route alternatives to be evaluated with 2 criteria and route i's values for the two criteria are given in vector $[r_{i1}, r_{i2}]$ next to the point representing the route for each route. In figure 3.1 (b), Pareto efficient solutions are circled with red.
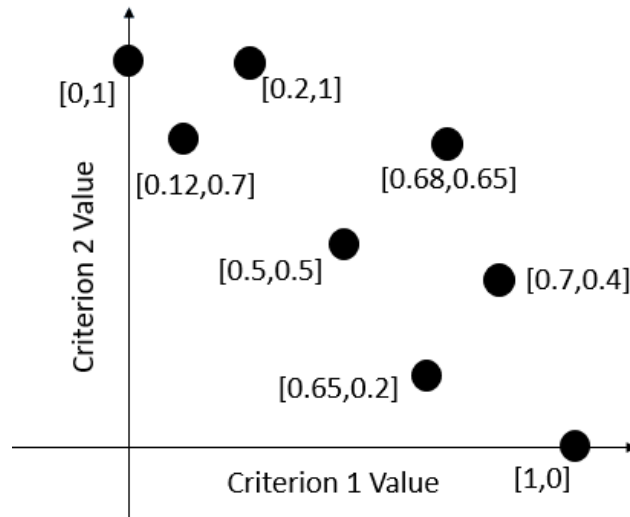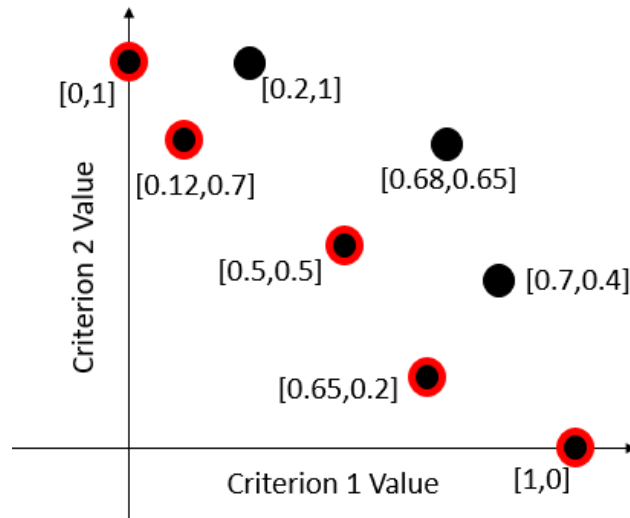
Figure 3.1 Illustration of Pareto efficient routes: routes (a), Pareto efficient routes (b)

3.7 Stage 2 - Single Value Assignments

Suppose that a set of n Pareto efficient routes is given with their respective criterion values, i.e., r_{ij} values are given for all $i = 1,2, \dots, n$ and $j = 1,2, \dots, m$. Then, using equations 2.3, 2.5, and 2.6, one can determine the P_i^1, P_i^2, and P_i^3 values for each Pareto efficient route i. Then, using each approach, the best route can be selected.

In particular, let route i^1 be the best route with respect to equal weighting approach. Then, i^1 is as defined in equation 3.1:

$$i^1 = argmin\{P_i^1 : i = 1,2, \dots, n\} = argmin\left\{\frac{\left(\sum_{j=1}^m r_{ij}\right)}{m} : i = 1,2, \dots, \right\}. \tag{3.1}$$

Now let route i^2 be the best route with respect to equal weighting approach. Then, i^2 is as defined in equation 3.2:

$$i^2 = argmin\{P_i^2 : i = 1,2, \dots, n\} = argmin\{\max\{d_{ij} : j = 1,2, \dots, m\} : i = 1,2, \dots, \}. \tag{3.2}$$

Finally, let route i^3 be the best route with respect to equal weighting approach. Then, i^3 is as defined in equation 3.1:

$$i^3 = argmin\{P_i^3 : i = 1,2, \dots, n\} = argmin\left\{\sqrt{\sum_{j=1}^m (r_{ij} - R_j)^2} : i = 1,2, \dots, n\right\}. \tag{3.3}$$

Equations 3.1, 3.2, and 3.3 will not necessarily return the same route. That is, it is possible that $i^1 \neq i^2 \neq i^3$. This is illustrated with the set of Pareto efficient routes given in figure

3.1 (a). Specifically, figure 3.2 (a) show the route numbers for the Pareto efficient routes, where, without loss of generality, it is assumed that routes 1 to 5 are Pareto efficient. Furthermore, the ideal point is represented with a red point in the figure. Recall that an ideal point is defined such that its criterion j value is $R_j = r_{i(j)j} = \min\{r_{ij}: i = 1,2, \dots, n\}$. In this example, the ideal point is therefore $[0,0]$. Figure 3.2 (b) shows $P_i^1/P_i^2/P_i^3$ values next to each Pareto efficient route.
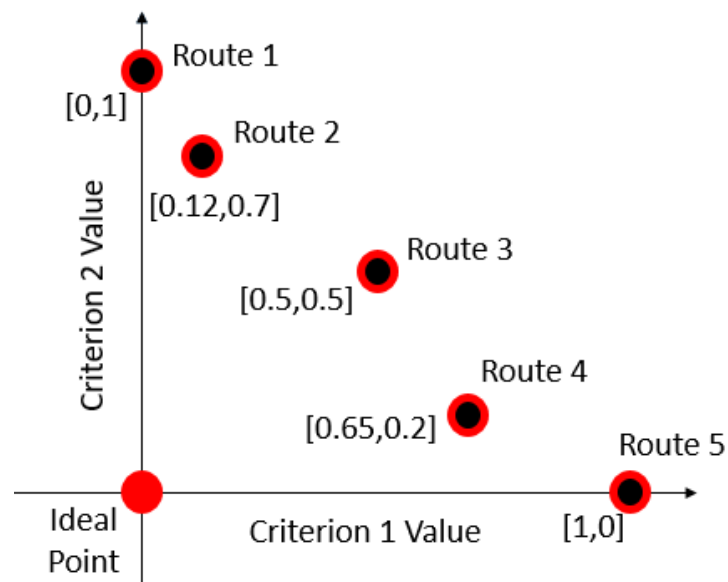
(a)
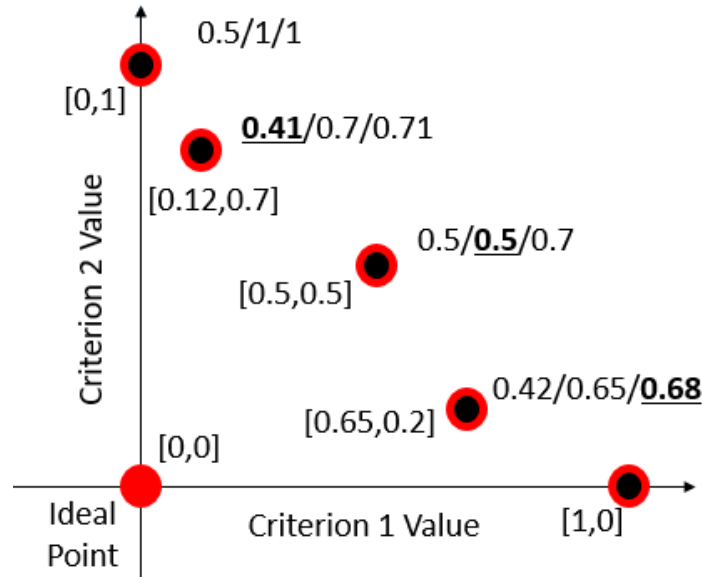
(b)

Figure 3.2 Illustration of single value assignments: routes (a), assignments (b)

Table 3.2 gives the data for the routes in figure 3.2 (b). Based on table 3.2 and equations 3.1, 3.2, and 3.3, we have $i^1 = 2$, $i^2 = 3$, and $i^3 = 4$.

Table 3.2 Single value assignments of Pareto efficient routes

Route i	r_{i1}	r_{i2}	P_i^1	P_i^2	P_i^3
1	0	1	0.50	1	1
2	0.12	0.70	0.41	0.70	0.71
3	0.5	0.50	0.50	0.50	0.71
4	0.65	0.20	0.43	0.65	0.68
5	1	0	0.50	1	1

The overall procedure to solve the multi-criteria route selection problem is summarized in table 3.3. Next chapter discusses the coding efforts for the solution approach and conducts numerical analyses.

Table 3.3 Overall solution approach for multi-criteria route selection

Step:	Process:
0:	Determine r_{ij} values for the given set of routes S
1:	Generate $PE(S)$ using the iterative procedure in Table 3.1
2:	Calculate P_i^1, P_i^2, and P_i^3 using equations 2.3, 2.5, and 2.6, respectively, for each route in $PE(S)$
3:	Determine and return i^1, i^2, and i^3 using equations 3.1, 3.2, and 3.3, respectively.

Chapter 4 Codes and Numerical Results

This chapter explains the coding efforts for the solution approach discussed above and documents the results of extensive numerical investigation.

<u>4.8 Solution Codes</u>

The solution approaches are coded in Matlab. The following Matlab function files are provided in the data supplements as well as in the Appendix.

1. ParetoFinder.m – This Matlab function is the code of the iterative procedure that determines the Pareto efficient routes given a set of routes and each route's criterion values. That is, it is the code of description given in table 3.1, which is used in step 1 of the overall solution approach depicted in table 3.3. Details are explained in Appendix A.

2. IdealPointFinder.m – This Matlab function finds the ideal point given a set of solutions. Its output is used in calculating distance to the ideal point. Details are explained in Appendix B.

3. RouteMetricFinder.m – This Matlab function is the code to calculate P_i^1, P_i^2, and P_i^3 values given a set of solutions. This code is used in step 2 of the overall solution approach depicted in table 3.3. Details are explained in Appendix C.

4. BestRouteFinder.m – This Matlab function is the code to determine i^1, i^2, and i^3 values given a set of routes and each route's P_i^1, P_i^2, and P_i^3 values. This code is used in step 3 of the overall solution approach depicted in table 3.3. Details are explained in Appendix D.

The output of ParetoFinder.m is input to RouteMetricFinder.m, and the output of RouteMetricFinder.m is input to BestRouteFinder.m. The output of the BestRouteFinder.m is a

set of three routes, each of which is the best for the three single value assignment approach discussed.

4.9 Test Problems

In total, 25 problem classes are defined for testing the solution approach. Each problem class corresponds to a combination of $n = \{50,100,150,200,250\}$ (number of alternative routes) and $m = \{5,10,15,20,25\}$ (number of criteria). 10 problem instances are randomly generated within each problem class. That is, in total, 250 problem instances are evaluated. DataGenerator.m (see Appendix E) is the Matlab function used to generate problem instances. For each route's each criterion value is randomly generated from a continuous uniform distribution with range 0 to 10, that is, r_{ij} values are randomly generated between 0 and 10. Finally, the distance to the ideal point is divided by the number of criteria to normalize the distance among different number of criteria.

The problem data (input data) and solution data (output data) for each problem instance are included in the excel file, titled Data_and_Solution.xlsx, supplemented. Particularly, tabs 1 to 250 are the problem data generated such that tabs 1-10 are the problem instances in problem class $n = 50$ and $m = 5$, tabs 11-20 are the problem instances in problem class $n = 100$ and $m = 5$, and so on. Tabs 251-260 are the tabs including the solutions for problem instances in 1-10; tabs 261-270 include the solutions for problem instances 11-20 and so on. Table 4.1 gives the ranges of the tabs for problem data (input data) and problem solution (output data) for each problem class.

Appendix F explains a problem data tab and Appendix G explains a problem solution tab. Tab 501 of Data_and_Solution.xlsx is the tab summarizing the average statistics for each class

over the 10 problem instances within the problem class as well as the average statistics over all

250 problem instances. These results are discussed in detail next.

Table 4.1 Input and output data file details

Problem Class		Input Tabs		Output Tabs	
n	m	start	end	start	end
50	5	1	10	251	260
50	10	11	20	261	270
50	15	21	30	271	280
50	20	31	40	281	290
50	25	41	50	291	300
100	5	51	60	301	310
100	10	61	70	311	320
100	15	71	80	321	330
100	20	81	90	331	340
100	25	91	100	341	350
150	5	101	110	351	360
150	10	111	120	361	370
150	15	121	130	371	380
150	20	131	140	381	390
150	25	141	150	391	400
200	5	151	160	401	410
200	10	161	170	411	420
200	15	171	180	421	430
200	20	181	190	431	440
200	25	191	200	441	450
250	5	201	210	451	460
250	10	211	220	461	470
250	15	221	230	471	480
250	20	231	240	481	490
250	25	241	250	491	500

4.10 Numerical Results

Here, the results of the numerical analyses are summarized. Specifically, the average

values over all 10 problem instances within each problem class, and the average values overall

250 problem instances are given for the equally weighted, maximum deviation, and distance to

ideal solution values for the best routes based on equally weighted, maximum deviation, and

distance to ideal solution values. That is, the averages of $P_{i^1}^1$, $P_{i^1}^2$, and $P_{i^1}^3$ values for the best

route based on the equal weighting value; the averages of $P_{i^2}^1$, $P_{i^2}^2$, and $P_{i^2}^3$ values for the best

route based on the maximum deviation value; and the averages of $P_{i^3}^1$, $P_{i^3}^2$, and $P_{i^3}^3$ values for the

best route based on the ideal solution value are summarized.

Table 4.2 gives the summary statistics over all 250 problem instances solved. Next, these

results are explained in detail.

Table 4.2 Overall average statistics

		Single Value		
		P_i^1	P_i^2	P_i^3
Route	i^1	3.60	6.63	3.22
	i^2	4.10	5.67	3.47
	i^3	3.66	6.22	3.15

Based on the definition of single value assignments, one can note that $P_{i^1}^1 = min\{P_{i^1}^1, P_{i^2}^1, P_{i^3}^1\}$,

$P_{i^2}^2 = min\{P_{i^1}^2, P_{i^2}^2, P_{i^3}^2\}$, and $P_{i^3}^3 = min\{P_{i^1}^3, P_{i^2}^3, P_{i^3}^3\}$. These can be observed in table 4.2.

Therefore, if a specific single value approach is chosen, the best route under that single value

approach is readily available. However, comparing different approaches is important in terms of

which approach to use. The following observations are from table 4.2 based on average over all

250 problem instances solved.

- Route i^1 vs. Route i^2: By definition, route i^1 is the best for P_i^1 value with a P_i^1 value of

  3.60 on average, and thus; based on P_i^1 value, route i^1 is better than route i^2, which has

  P_i^1 value of 4.10 on average. By definition, route i^2 is the best for P_i^2 value with a P_i^2

value of 5.67 on average, and thus; based on P_i^2 value, route i^2 is better than route i^1, which has a P_i^2 value of 6.63 on average. On the other hand, if these two routes are compared based on P_i^3 value (for which neither is the best), it can be seen that route i^1 is better, with a P_i^3 value of 3.22 on average, than route i^2, which has a P_i^3 value of 3.47.

- Route i^1 vs. Route i^3: By definition, route i^1 is the best for P_i^1 value with a P_i^1 value of 3.60 on average, and thus; based on P_i^1 value, route i^1 is better than route i^3, which has P_i^1 value of 3.66 on average. By definition, route i^3 is the best for P_i^3 value with a P_i^3 value of 3.15 on average, and thus; based on P_i^3 value, route i^3 is better than route i^1, which has a P_i^3 value of 3.22 on average. On the other hand, if these two routes are compared based on P_i^2 value (for which neither is the best), it can be seen that route i^3 is better, with a P_i^3 value of 6.22 on average, than route i^1, which has a P_i^3 value of 6.63.

- Route i^2 vs. Route i^3: By definition, route i^2 is the best for P_i^2 value with a P_i^2 value of 5.67 on average, and thus; based on P_i^2 value, route i^2 is better than route i^3, which has a P_i^2 value of 6.22 on average. By definition, route i^3 is the best for P_i^3 value with a P_i^3 value of 3.15 on average, and thus; based on P_i^3 value, route i^3 is better than route i^2, which has a P_i^3 value of 3.47 on average. On the other hand, if these two routes are compared based on P_i^1 value (for which neither is the best), it can be seen that route i^3 is better, with a P_i^1 value of 3.66 on average, than route i^1, which has a P_i^1 value of 4.10.

The above observations, which can be seen in table 4.2, imply the following:

$$Average\ P_{i^1}^1 < Average\ P_{i^3}^1 < Average\ P_{i^2}^1,$$

$$Average\ P_{i^2}^2 < Average\ P_{i^3}^2 < Average\ P_{i^1}^2,$$

$$Average\ P_{i^3}^3 < Average\ P_{i^1}^3 < Average\ P_{i^2}^3.$$

In particular, each route is the best based on one single value assignment approach. This follows from the definition of routes i^1, i^2, and i^3. Furthermore, route i^3 is the second best twice based on the two other single value assignment approaches, for which it is not the best. On the other hand, Route i^1 is the second best once based on the two other single value assignment approaches, for which it is not the best. Finally, route i^2 is not the second best at all based on the two other single value assignment approaches, for which it is not the best. These are illustrated in table 4.3.

Table 4.3 Average robustness

	i^1	i^2	i^3
Number of times it is the best	1	1	1
Number of times it is the 2nd best	1	2	0
Number of times it is the 3rd best	1	0	2

Based on these discussions, it can be suggested that using distance to ideal point approach as the single value assignment approach is robust because it performs better based on other approaches compared to the performances of the other routes, which are not the best for the corresponding approach. In addition, table 4.4 summarizes the percentage of problem instances, where different approaches resulted in the same route selection.

Table 4.4 Average equality comparison

% of problems such that $i^1 = i^2$	25.60%
% of problems such that $i^1 = i^3$	66%
% of problems such that $i^2 = i^3$	38.80%

Table 4.3 indicates that route i^3 overlaps with route i^1 more than route i^2 does (66 vs. 25.6) and route i^3 overlaps with route i^2 more than route i^1 does (38.80 vs. 25.6). These support the robustness of distance to ideal point approach illustrated in table 4.2.

For each problem class, Appendix H lists the average statistics over 10 problem instances solved within that problem class, similar to tables 4.2 and 4.4.

Chapter 5 Conclusion

This project focused on multi-criteria route selection problem for transporting a hazmat shipment on rail. Hazmat transportation on rail is a challenging problem considering the many safety and risk factors that should be accounted for. Coupling the multi-criteria nature of the problem with the existence of many alternative routes, making a route selection for shipping the hazmat becomes a cumbersome process. While the literature has focused on developing optimization models and their solutions under a few factors, there is a lack on providing a simple decision support tool that will recommend only a few routes by objectively considering many of the safety and risk factors at the same time.

This project analyzes three different approaches to reduce the multi-criteria route selection problem into a single-criterion route selection problem. Each of these approaches are unsupervised, meaning that they do not require input from the decision maker. This assures that the decision makers' biases towards some specific criteria are eliminated. The first approach equally weighs all of the criteria and assigns a single value to each route based on this equal weighting. The second approach takes a route's maximum deviation from the best values of each criteria as the route's single value. The third approach measures the distance of a route from the ideal point, which represents the best values for each criterion, and assigns the distance as the route's single value.

The solution approach proposed considers Pareto efficient route alternatives. Then, one route is selected based on each single value assignment approach. As a result, the overall solution approach returns three alternative routes: best one based on equal weighting, best one based on maximum deviation, and best one based on distance to the ideal point. A set of numerical studies suggest that distance to ideal point is a robust approach as it outperforms the

other routes on the approaches that they are not the best. Furthermore, the best route under this approach overlaps more frequently with the best routes under different approaches.

The project has limitations as well. The comparison of the approaches are based on a set of numerical experiments, for which the data is randomly generated. The project team was not able to acquire data from a railroad carrier about hazmat shipments, particularly, due to the sensitivity of such data. However, the detailed description of the solution approach with the given algorithmic descriptions and the codes, can be used easily for any setting.

Several future research directions include to compare these approaches within a multi-criteria route optimization setting. It is estimated that similar results will be observed. Another research direction would be to combine supervised and unsupervised approaches for multi-criteria route selection.

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Appendix A Matlab code for ParetoFinder.m

```matlab
%% FINDS THE SET OF PARETO EFFICIENT ALTERNATIVES GIVEN A SET OF ROUTES
% This Matlab function is the code for the procedure given in Table 3.1.

% INPUT to the function:
% --> RouteSet: This is the (n+1)x(m+1) matrix where entry (i,j) is route i's
%               criterion j value, the (m+1)th column is the route numbers,
%               and the (n+1)th row from 1 to m describes the ideal point
%               of the problem instance
% --> x       : This is the dimensions of RouteSet not including the
%               (m+1)th column or the (n+1)th row; [n x m]

% OUTPUT of the function:
% ---> RouteSet: This is the set of Pareto efficient solutions given in a
%                matrix, of which entry (i,j) is Pareto efficient route i's
%                criterion value j. The (m+1)th column keeps the original
%                route numbers of the corresponding Pareto efficient route
%                and the final row from 1 to m describes the ideal point
%                of the problem instance

function [RouteSet] = ParetoFinder(RouteSet, x)
try
    L = 1;
    M = L + 1;
    while L <= x(1)-1
        while M <= x(1)
            W=RouteSet(L,1:x(2))==RouteSet(M,1:x(2));
            WW=RouteSet(L,1:x(2))<=RouteSet(M,1:x(2));
            WWW=RouteSet(L,1:x(2))>=RouteSet(M,1:x(2));
            if all(W==1)
                M=M+1;
            elseif all(WW==1)
                RouteSet(M,:)=[];
            elseif all(WWW==1)
                RouteSet(L, :)=[];
                M=L+1;
            else
                M=M+1;
            end
        end
        L=L+1;
        M=L+1;
    end
end
```

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Appendix B Matlab code for IdealPointFinder.m

```matlab
%% FINDS THE IDEAL POINT GIVEN A SET OF ROUTES
% This Matlab function is the code for finding ideal point

% INPUT to the function:
% --> RouteSet: This is the (n)x(m) matrix where entry (i,j) is route i's
%               criterion j value

% OUTPUT of the function:
% --> RouteSet: This is the updated (n+1)x(m) matrix where entry (i,j) is
%               route i's criterion j value and the (n+1)th row describes
%               the ideal point of the problem instance

function [RouteSet] = IdealPointFinder(RouteSet)
RouteSet(length(RouteSet)+1,:)=min(RouteSet);
end
```

Appendix C Matlab code for RouteMetricFinder.m

```matlab
%% FINDS THE SINGLE VALUE ASSIGNMENTS FOR EACH GIVEN ROUTE
% This Matlab function is the code for the equations 2.3, 2.5, and 2.6

% INPUT to the function:
% --> RouteSet: This is the (n+1)x(m+1) matrix where entry (i,j) is Pareto
%               efficient route i's criterion j value, the (m+1)th column
%               is the route numbers, and the (n+1)th row from 1 to m
%               describes the ideal point of the problem instance
% --> x       : This is the dimensions of RouteSet not including the
%               (m+1)th column or (n+1)th row; [n x m]
% Note: given RouteSet will be output of ParetoFinder.m, that is, the route
%       alternatives considered are Pareto efficient alternatives

% OUTPUT of the function:
% ---> RouteSet: This is the updated (n+1)x(m+4) matrix where entry (i,j) is
%                Pareto efficient route i's criterion value j.
%                The (m+1)th column keeps the original route numbers of the
%                corresponding Pareto efficient route. The (m+2)th column
%                describes the Pi1 value of the route, the (m+3)th column
%                describes the Pi2 value of the route, the (m+4)th column
%                describes the Pi3 value of the route. The (n+1)th row
%                from 1 to m describes the ideal point of the problem
%                instance.

% Finds the performance metrics of each route
function [RouteSet] = RouteMetricFinder(RouteSet, x)
for n=1:x(1)
    % Equal weighting approach: equation 2.3, i.e., Pi1 value
    RouteSet(n, x(2)+2)=mean(RouteSet(n,1:x(2)));

    % Max deviation approach: equation 2.5, Pi2 value
    RouteSet(n, x(2)+3)=max(RouteSet(n,1:x(2))-RouteSet(x(1)+1,1:x(2)));

    % Distance to ideal point approach: equation 2.6, Pi3 value
    RouteSet(n, x(2)+4)=sqrt(sum((RouteSet(n,1:x(2))-
RouteSet(x(1)+1,1:x(2))).^2)/x(2));

end
end
```

Appendix D Matlab code for BestRouteFinder.m

```matlab
%% FINDS THE BEST ROUTE FOR EACH SINGLE VALUE ASSIGNMENT
% This Matlab function is the code for the equations 3.1, 3.2, and 3.3

% INPUT to the function:
% --> RouteSet: This is the (n+1)x(m+4) matrix where entry (i,j) is
%               Pareto efficient route i's criterion value j.
%               The (m+1)th column keeps the original route numbers of the
%               corresponding Pareto efficient route. the (m+2)th column
%               describes the Pi1 value of the route, the (m+3)th column
%               describes the Pi2 value of the route, the (m+4)th column
%               describes the Pi3 value of the route. The (n+1)th row from
%               1 to m describes the ideal point of the problem instance
% --> x       : This is the dimensions of RouteSet not including the
%               (m+1)th through (m+4)th column or the (n+1)th row; [n x m]
% Note: given RouteSet will be output of ParetoFinder.m, that is, the route
%       alternatives considered are Pareto efficient alternatives

% OUTPUT of the function:
% ---> R: The route number corresponding to the route performing the best
%         by the equally weighting value minimum (equation 3.1)
% ---> S: The route number corresponding to the route performing the best
%         by the maximum deviation value minimum (equation 3.2)
% ---> T: The route number corresponding to the route performing the best
%         by the distance to ideal point value minimum (equation 3.3)
% ---> RouteSet: This is updated the (n+1)x(m+4) matrix where entry (i,j)
%               is Pareto efficient route i's criterion value j.
%               The (m+1)th column keeps the original route numbers of the
%               corresponding Pareto efficient route. The (m+2)th column
%               describes the Pi1 value of the route, The (m+3)th column
%               describes the Pi2 value of the route, The (m+4)th column
%               describes the Pi3 value of the route. The (n+1)th row from
%               1 to m describes the ideal point of the ideal point of the
%               problem instance. The (n+1)th row from (m+2) to (m+4) now
%               describes the single value assignment value of the best
%               performing route by equation 3.1, equation 3.2, and
%               equation 3.3, respectively. These values are,
%               respectively, N, O, and P.

function [R, S, T, RouteSet] = BestRouteFinder(RouteSet, x)

% Finds best performing route of each single value assignment
% Equally weighting value minimum: equation 3.1
[N, R]=min(RouteSet(1:x(1),x(2)+2));
RouteSet(x(1)+1,x(2)+2)=N;

% Maximum deviation value minimum: equation 3.2
[O, S]=min(RouteSet(1:x(1),x(2)+3));
RouteSet(x(1)+1,x(2)+3)=O;

% Distance to ideal point value minimum: equation 3.3
[P, T]=min(RouteSet(1:x(1),x(2)+4));
RouteSet(x(1)+1,x(2)+4)=P;
end
```

Appendix E Matlab code for DataGenerator.m

```matlab
%% THIS CODE GENERATES RANDOM PROBLEM INSTANCES FOR THE GIVEN PROBLEM CLASS
%   AND THE DESIRED NUMBER OF INSTANCES FROM EACH CLASS

% INPUT to the function:
% --> ObjFun: A comma-separated list of the criteria (objective function)
%             number variations to include; an ObjFun input of "5, 10, 15"
%             dictates that problem classes of 5, 10, and 15 criteria
%             values will be created
% --> Rts   : A comma-separated list of the route number variations to
%             include; an Rts input of "50, 100, 150" dictates that problem
%             classes of 50, 100, and 150 routes will be created
% --> Iter  : The number of instances of each problem class to generate

% OUTPUT of the function:
% ---> MasterMatrix: A three-dimensional matrix containing the randomly
%                    generated criterion values for each dictated problem
%                    instance
% ---> i           : The number of criteria variations included in ObjFun;
%                    an ObjFun input of "5, 10, 15" yields an i value of 3
% ---> j           : The number of route variation included in Rts; an Rts
%                    input of "50, 100, 150" yields a j value of 3
% ---> k           : The number of instances of each problem class
%                    generated; k = Iter upon generation of all problem
%                    class instances

% Note: Dimensions in accordance with the input parameters
function [MasterMatrix, i, j, k] = DataGenerator(ObjFun, Rts, Iter)
m=1;
for i=1:length(ObjFun)
    for j=1:length(Rts)
        k=1;
        for k=1:(Iter)
            RandMatrix=1+(10-1)*rand(Rts(j), ObjFun(i));
            MasterMatrix{m}=RandMatrix;
            m=m+1;
        end
    end
end
end
```

Appendix F Problem data tab explanation

Figure F.1 shows a problem data from problem class $n = 50, m = 5$. The first row gives the column titles. Columns 1 to m are the criteria values for the corresponding route defined in the selected row. Column $m + 1$ gives the P_i^1 value, column $m + 2$ gives the P_i^2 value, and column $m + 3$ gives the P_i^3 value. The last row is the ideal point solution with route number 0. Note that a problem data tab will have $n + 2$ rows and $m + 4$ columns.

A	B	C	D	E	F	G	H	I
Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Equally Weighted Average	Minimax	Distance to Ideal	Route Number
8.450223745	1.872623996	2.894005374	8.32113809	3.780686612	5.063735563	7.200597416	4.650228625	1
2.744971711	4.178389142	2.57615323	6.846660594	3.690062638	4.007247463	5.72611992	3.243186923	2
2.673576429	9.279610343	4.191297607	4.533501957	8.197507663	5.7750988	8.174814869	5.30254051	3
⋮	⋮	⋮	⋮	⋮		⋮	⋮	⋮
4.98282373	2.600159967	6.925792002	4.237822371	7.713434482	5.29200651	6.706141147	4.500866916	49
9.354362934	8.434523107	3.156054355	4.578284347	8.795108596	6.863666668	7.787815262	6.135969651	50
1.759613298	1.104795474	1.097066564	1.120540674	1.007293334	2.993265419	3.011605157	2.160518982	0

Figure F.1 Problem data tab representation

Appendix G Problem solution tab explanation

Figure G.1 shows the first part of the solution data from problem class $n = 50, m = 5$.

The first row gives the column titles. Columns 1 to m are the criteria values for the

corresponding Pareto efficient route defined in the selected row. Column $m + 1$ gives the P_i^1

value, column $m + 2$ gives the P_i^2 value, and column $m + 3$ gives the P_i^3 value. Note that these

are the data for only Pareto efficient routes, not every route.

	A	B	C	D	E	F	G	H	I
	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Equally Weighted Average	Minimax	Distance to Ideal	Route Number
	8.450223745	1.872623996	2.894005374	8.32113809	3.780686612	5.063735563	7.200597416	4.650228625	1
	2.744971711	4.178389142	2.57615323	6.846660594	3.690062638	4.007247463	5.72611992	3.243186923	2
	5.92012148	6.959471918	6.584217583	4.890884273	2.001557256	5.271250502	5.854676444	4.402270835	4
	2.838373103	6.657795267	7.476129168	2.305460719	2.452904247	4.346132501	6.379062604	3.903480384	5
	2.218651885	1.134401559	8.793965994	1.702857851	1.499864218	3.069948301	7.69689943	3.465128371	6
	5.087683988	3.972575755	2.770907905	6.767413703	8.687827987	5.457281868	7.680534653	4.753519762	9
	8.598549245	6.009221307	5.551687613	1.906150475	4.06321404	5.225764536	6.838935947	4.486065547	11
	9.371942907	8.974384177	5.547698845	8.899385651	1.007293334	6.760140983	7.869588703	6.327662211	12

Figure G.1 Problem data tab representation – part 1

Figure G.2 shows the second part of the solution data from problem class $n = 50, m = 5$.

Column L gives the description of the given data in the rows. In Column M, the route numbers

for the best routes are given. That is, these are i^1, i^2, and i^3 values. Column N gives the

identifier 1 if the corresponding route is Pareto efficient and 0 otherwise. Column O gives $P_{i^1}^1$,

$P_{i^2}^1$, and $P_{i^3}^1$ values; Column P gives $P_{i^1}^2$, $P_{i^2}^2$, and $P_{i^3}^2$ values; and Column Q gives $P_{i^1}^3$, $P_{i^2}^3$, and

$P_{i^3}^3$ values. In the first 4 rows. Bottom rows gives an identifier in Column P whether $i^1 = i^2$,

$i^1 = i^3$, and $i^2 = i^3$. Finally, the last part gives the number of Pareto efficient solutions

considered. Note that these column letters will change depending on the length of the first part.

PerformanceMatrix	Route Number	Pareto Efficient	Equally Weighted Average	Minimax	Distance
Equally Weighted Minimum	38	1	2.993265419	3.011605157	2.160518982
Minimax	38	1	2.993265419	3.011605157	2.160518982
Distance Minimum	38	1	2.993265419	3.011605157	2.160518982
Equally Weighted = Minimax				1	
Equally Weighted = Minimum Distance				1	
Minimax = Minimum Distance				1	
Pareto Efficient Solutions				29	

Figure G.2 Problem data tab representation – part 2

Appendix H Average statistics for each problem class

Figures H.1 to H.25 show the summary statistics for each problem class with the specified n and m values. The values are the averages over 10 problem instances solved within each problem class. Objective functions number is the number of criteria m, and routes number is the number of routes n.

Objective Functions: 5, Routes: 50				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	2.98	4.43	2.38
Minimax	100%	3.22	3.65	2.40
Distance to Ideal	100%	3.10	3.93	2.32
Equally Weighted = Minimax				50%
Equally Weighted = Minimum Distance				70%
Minimax = Minimum Distance				70%

Figure H.1 Average statistics for problem class n=50, m=5

Objective Functions: 5, Routes: 100				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	2.966204868	4.549193633	2.474924118
Minimax	100%	3.432625586	3.593185491	2.617091734
Distance to Ideal	100%	3.05122932	3.87223184	2.31202612
Equally Weighted = Minimax				20%
Equally Weighted = Minimum Distance				50%
Minimax = Minimum Distance				40%

Figure H.2 Average statistics for problem class n=100, m=5

Objective Functions: 5, Routes: 150				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	2.657330057	4.326727132	2.207655095
Minimax	100%	2.97542169	3.198706221	2.156307918
Distance to Ideal	100%	2.74555418	3.24547208	2.01406511
Equally Weighted = Minimax				40%
Equally Weighted = Minimum Distance				50%
Minimax = Minimum Distance				80%

Figure H.3 Average statistics for problem class n=150, m=5

Objective Functions: 5, Routes: 200				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	2.373267009	2.829687528	1.611601474
Minimax	100%	2.488032102	2.450006364	1.655986031
Distance to Ideal	100%	2.396193461	2.631590657	1.580411104
Equally Weighted = Minimax				50%
Equally Weighted = Minimum Distance				80%
Minimax = Minimum Distance				60%

Figure H.4 Average statistics for problem class n=200, m=5

Objective Functions: 5, Routes: 250				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	2.372323223	2.72914327	1.616027354
Minimax	100%	2.480556535	2.398462752	1.631133772
Distance to Ideal	100%	2.387962156	2.50249071	1.574482049
Equally Weighted = Minimax				60%
Equally Weighted = Minimum Distance				80%
Minimax = Minimum Distance				60%

Figure H.5 Average statistics for problem class n=250, m=5

Objective Functions: 10, Routes: 50				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	3.681403539	6.391969841	3.260138428
Minimax	100%	4.163737953	5.755801765	3.467075721
Distance to Ideal	100%	3.75428916	6.106274094	3.19667533
Equally Weighted = Minimax				50%
Equally Weighted = Minimum Distance				70%
Minimax = Minimum Distance				60%

Figure H.6 Average statistics for problem class n=50, m=10

Objective Functions: 10, Routes: 100				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	3.41651606	6.836629486	3.134068315
Minimax	100%	4.189700092	5.571424807	3.49598656
Distance to Ideal	100%	3.471740118	6.415640634	3.094466401
Equally Weighted = Minimax				20%
Equally Weighted = Minimum Distance				80%
Minimax = Minimum Distance				20%

Figure H.7 Average statistics for problem class n=100, m=10

Objective Functions: 10, Routes: 150				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	3.369273566	5.702515265	2.972849601
Minimax	100%	3.536195655	4.988075423	2.924575319
Distance to Ideal	100%	3.495765173	5.032980872	2.893977561
Equally Weighted = Minimax				60%
Equally Weighted = Minimum Distance				60%
Minimax = Minimum Distance				90%

Figure H.8 Average statistics for problem class n=150, m=10

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Objective Functions: 10, Routes: 200				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	3.140189342	5.658605964	2.705494286
Minimax	100%	3.501894769	4.357194321	2.750432588
Distance to Ideal	100%	3.269199118	4.553444166	2.625274391
Equally Weighted = Minimax				50%
Equally Weighted = Minimum Distance				60%
Minimax = Minimum Distance				70%

Figure H.9 Average statistics for problem class n=200, m=10

Objective Functions: 10, Routes: 250				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	3.254631527	5.718906676	2.865547797
Minimax	100%	3.572425149	4.54582879	2.86624283
Distance to Ideal	100%	3.367586255	4.849141699	2.73527036
Equally Weighted = Minimax				30%
Equally Weighted = Minimum Distance				50%
Minimax = Minimum Distance				70%

Figure H.10 Average statistics for problem class n=250, m=10

Objective Functions: 15, Routes: 50				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	3.741355539	7.219381117	3.356255741
Minimax	100%	4.341165543	6.354645766	3.793152055
Distance to Ideal	100%	3.762520146	7.278058804	3.34109645
Equally Weighted = Minimax				30%
Equally Weighted = Minimum Distance				80%
Minimax = Minimum Distance				20%

Figure H.11 Average statistics for problem class n=50, m=15

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Objective Functions: 15, Routes: 100				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	3.839208481	7.541612714	3.650815381
Minimax	100%	4.551964962	6.269943767	3.964385474
Distance to Ideal	100%	3.93421375	7.070606063	3.526564855
Equally Weighted = Minimax				0%
Equally Weighted = Minimum Distance				50%
Minimax = Minimum Distance				30%

Figure H.12 Average statistics for problem class n=100, m=15

Objective Functions: 15, Routes: 150				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	3.82684722	7.472007403	3.541700903
Minimax	100%	4.577536219	6.202393787	4.005396603
Distance to Ideal	100%	3.927529079	7.039009694	3.456607656
Equally Weighted = Minimax				20%
Equally Weighted = Minimum Distance				40%
Minimax = Minimum Distance				20%

Figure H.13 Average statistics for problem class n=150, m=15

Objective Functions: 15, Routes: 200				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	3.795066125	7.443009073	3.527219307
Minimax	100%	4.215406779	6.184030966	3.713656912
Distance to Ideal	100%	3.840502935	6.689070404	3.416151281
Equally Weighted = Minimax				30%
Equally Weighted = Minimum Distance				70%
Minimax = Minimum Distance				60%

Figure H.14 Average statistics for problem class n=200, m=15

Objective Functions: 15, Routes: 250				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	3.656746801	6.897943801	3.322439726
Minimax	100%	4.058634801	6.033879852	3.512580604
Distance to Ideal	100%	3.681384536	6.625993896	3.289347915
Equally Weighted = Minimax				20%
Equally Weighted = Minimum Distance				80%
Minimax = Minimum Distance				40%

Figure H.15 Average statistics for problem class n=250, m=15

Objective Functions: 20, Routes: 50				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	4.111253323	7.548340858	3.696649338
Minimax	100%	4.524001087	6.9294155	4.006884676
Distance to Ideal	100%	4.111889982	7.454253806	3.687946216
Equally Weighted = Minimax				30%
Equally Weighted = Minimum Distance				90%
Minimax = Minimum Distance				30%

Figure H.16 Average statistics for problem class n=50, m=20

Objective Functions: 20, Routes: 100				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	4.166515871	8.253309824	3.916069913
Minimax	100%	4.868429169	7.036196448	4.342584221
Distance to Ideal	100%	4.223514244	7.856015519	3.802693744
Equally Weighted = Minimax				10%
Equally Weighted = Minimum Distance				40%
Minimax = Minimum Distance				10%

Figure H.17 Average statistics for problem class n=100, m=20

Objective Functions: 20, Routes: 150				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	3.900855122	7.502127259	3.574180021
Minimax	100%	4.614878214	6.877325218	4.025204154
Distance to Ideal	100%	3.900855122	7.502127259	3.574180021
Equally Weighted = Minimax				20%
Equally Weighted = Minimum Distance				100%
Minimax = Minimum Distance				20%

Figure H.18 Average statistics for problem class n=150, m=20

Objective Functions: 20, Routes: 200				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	4.030778587	7.951561465	3.763448494
Minimax	100%	4.471435273	6.604639962	3.964398519
Distance to Ideal	100%	4.058272813	7.713279551	3.720596466
Equally Weighted = Minimax				10%
Equally Weighted = Minimum Distance				80%
Minimax = Minimum Distance				20%

Figure H.19 Average statistics for problem class n=200, m=20

Objective Functions: 20, Routes: 250				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	3.832204625	8.090744115	3.581550919
Minimax	100%	4.705808757	6.600305631	4.154496029
Distance to Ideal	100%	3.85200314	7.648073056	3.50923381
Equally Weighted = Minimax				0%
Equally Weighted = Minimum Distance				70%
Minimax = Minimum Distance				10%

Figure H.20 Average statistics for problem class n=250, m=20

Objective Functions: 25, Routes: 50				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	4.344949716	8.169346811	3.951095744
Minimax	100%	4.826659655	7.406175633	4.243262979
Distance to Ideal	100%	4.409367663	7.955089763	3.887035764
Equally Weighted = Minimax				10%
Equally Weighted = Minimum Distance				60%
Minimax = Minimum Distance				20%

Figure H.21 Average statistics for problem class n=50, m=25

Objective Functions: 25, Routes: 100				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	4.234836473	8.254467147	3.893044223
Minimax	100%	4.941636744	7.343034687	4.399054216
Distance to Ideal	100%	4.25986127	8.029127771	3.871825211
Equally Weighted = Minimax				10%
Equally Weighted = Minimum Distance				80%
Minimax = Minimum Distance				30%

Figure H.22 Average statistics for problem class n=100, m=25

Objective Functions: 25, Routes: 150				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	4.126872452	7.771269218	3.825948822
Minimax	100%	4.792228053	7.102485403	4.276679367
Distance to Ideal	100%	4.175742188	7.680318797	3.808972469
Equally Weighted = Minimax				20%
Equally Weighted = Minimum Distance				70%
Minimax = Minimum Distance				20%

Figure H.23 Average statistics for problem class n=150, m=25

Objective Functions: 25, Routes: 200				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	4.032629741	8.163277894	3.892853766
Minimax	100%	4.651541492	7.208203057	4.22877131
Distance to Ideal	100%	4.101566193	8.013200156	3.809576892
Equally Weighted = Minimax				0%
Equally Weighted = Minimum Distance				50%
Minimax = Minimum Distance				10%

Figure H.24 Average statistics for problem class n=200, m=25

Objective Functions: 25, Routes: 250				
Route Metric	Pareto Efficiency	Weighted Average	Minimax	Distance to Ideal
Equally Weighted Average	100%	4.104233887	8.368516773	3.875785517
Minimax	100%	4.795460239	7.082344259	4.249543428
Distance to Ideal	100%	4.194153829	7.886407988	3.775432899
Equally Weighted = Minimax				0%
Equally Weighted = Minimum Distance				40%
Minimax = Minimum Distance				10%

Figure H.25 Average statistics for problem class n=250, m=25