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Analysis Guidelines and Examples for Fracture Critical Steel Twin Tub Girder Bridges

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16. Abstract
Twin steel tub girder bridges are an aesthetically pleasing structural option offering long span solutions in tight-radii direct connectors. However, these bridges require a routine two-year inspection frequency, as well as a thorough hands-on inspection because of their fracture critical designation. The heightened inspection requirements for fracture critical bridges come at a significant cost to the Texas Department of Transportation (TxDOT). Recent research has shown that tangent, or nearly tangent, twin steel tub girder sections can redistribute load to the intact girder after fracture of one of the girder bottom flanges. Additional research is required to develop recommendations for practical analysis of typical twin steel tub span configurations with the degree of curvature common to twin steel tub direct connectors. A finite element method (FEM), specifically Abaqus, and SAP2000 can be employed for both rigorous FEM and grillage solutions along with push-down plastic analysis typically available to TxDOT and its consultant bridge designers. These analysis and modeling methods take into account the capacity of the fractured girder, especially at the support locations, and realistically model the load distribution between the intact girder and the fractured girder. The modeling and analysis methods are incorporated in a straightforward manner on a large scale to the inventory of the steel tub bridges. The requirements outlined in the Federal Highway Administration memorandum dated June 12, 2012, and entitled “Clarification of Requirements for Fracture Critical Members” were met in the employed analysis methods.

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DISCLAIMER

This research was performed in cooperation with the Texas Department of Transportation (TxDOT) and the Federal Highway Administration (FHWA). The contents of this report reflect the views of the authors, who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official view or policies of FHWA or TxDOT. This report does not constitute a standard, specification, or regulation.

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1. FOREWORD

The Federal Highway Administration (FHWA 2012) (Lwin 2012) defines a fracture critical member (FCM) as "a steel member in tension, or with a tension element, whose failure would probably cause a portion of or the entire bridge to collapse." FCMs are composed of nonredundant members that make the structure extremely vulnerable to fatigue and/or fracture failures. An engineer may encounter the dilemma of choosing either a steel twin tub girder (STTG) or long-span steel (or concrete) multi-girder superstructure. Despite the advantages that tub girders offer for long-span curved bridges, there may be a reluctance to select this option since it is deemed fracture critical by the above FHWA requirements. The classification of the structure as FCM implies significant ongoing inspection costs due to the hands-on nature of inspection requirements for FCMs. However, it is contended that the current classification based on load path redundancy alone may be markedly conservative if it does not consider the inherent reserve capacity of STTG bridges, especially those continuous structures with multiple spans.

If these bridges are reclassified as nonfracture critical based on the reserve capacity in a damaged state under the design loads, substantial savings in inspection costs will accrue over the life of the structure. At the time of design, certain simplifications are made whereby the degree of redundancy and reserve capacity are not recognized. The quantification of the reserve capacity of each twin tub bridge structure can potentially lead to the reassignment from the default position of a fracture critical bridge to a reclassification as nonfracture critical.

The finite element method (FEM) makes use of the advanced nonlinear elasto-plastic analysis incorporating a detailed mesh refinement and precise element modeling. The results generated are consequently the most accurate and reliable. A second but less rigorous computational method implements a lower-bound analysis method using a nonlinear push-down grillage analysis. This ease in the computational effort comes at the cost of a comparatively less precise simulation of the bridge material properties and loading conditions.

Recent research reported in Texas Department of Transportation (TxDOT) project 0-6937 suggests that many of the bridges may be safely reclassified as nonfracture critical based on the computed overstrength factors. Since this assessment ideally needs to be an economical and rapid process, the applicability of using the simpler computational method such as grillage, aided by the plastic upper and lower-bound solutions serving as a check, is examined in that project.

The feasibility of these simper solutions is established through the comparison of the results obtained from the three independent methods. This volume primarily focuses on the two simpler methods that serve as the preliminary steps in the identification of the reserve capacity of the structure to determine whether the bridge could be reclassified as nonfracture critical.

In spite of the relative simplicity of the nonlinear grillage methods, there remains considerable time and effort to code and run such an analysis. Moreover, once complete, the analyst may remain unsure of the dependability of the result.

To confirm the dependability of results, the simplest approach is to conduct a limit plastic collapse analysis of the bridge deck in a span-by-span fashion. This limit analysis for a fractured twin tub bridge needs to be rooted in the use of yield line theory. As such, analytical solutions using yield line theories may be posed as either lower or upper-bound solutions to the problem (Park and Gamble 2000).

2. ANALYSIS SCHEMA

2.1 SCOPE

This section presents an overview of the analysis procedures that should ideally be followed so that an STTG bridge that is presumed to be fracture critical (meaning, by definition of FCMs as per FHWA, it contains a “steel member in tension, or with a tension element, whose failure would probably cause a portion of or the entire bridge to collapse”) may be reclassified as nonfracture critical. The analysis schema shown in Figure 2.1 depicts the overall procedure and decision points.

In principle, two methods of analysis that are markedly different in their approach are used to arrive at a positive declassification decision. If the first method—the yield line theory that provides a limit analysis solution—does not indicate a positive solution, there is normally little hope the following methods will succeed. Therefore, the analysis may be terminated. The process of the evaluation of the STTG bridges for their declassification is charted using an algorithmic representation, as shown in Figure 2.1. The procedure is as follows:

2.2 COLLECTION AND ORGANIZATION OF DATA:

The necessary details of the bridge are documented by using the design drawings. The reports for the member properties and additional information and other pertinent attributes are systematically recorded as per the requirements of each method. For example, the yield line analysis does not model the 3-D elements, such as diaphragms and stiffeners, that the FEM model does; therefore, the details of those elements need not be documented for the first stage. Thus, a selective documentation of the data is encouraged for efficient collection of useful information.

2.3 STAGE 1: YIELD LINE ANALYSIS

Yield line analysis is an explicit direct method of plastic analysis used to provide an upper-bound solution for the reserve capacity of the bridge in terms of the overstrength factor, Ω. The first stage can be perceived as a litmus test for the reserve capacity of the bridge span under consideration since if this method concludes an overstrength, $\Omega_{Yield\ Line} < 1$, then the chances of

the remaining two methods resulting in a higher reserve capacity are very slim, and the bridge will be deemed fracture critical. If the yield line analysis predicts an overstrength, $\Omega_{Yield\ Line} > 1$, then proceed to the second stage of analysis, which is computational.

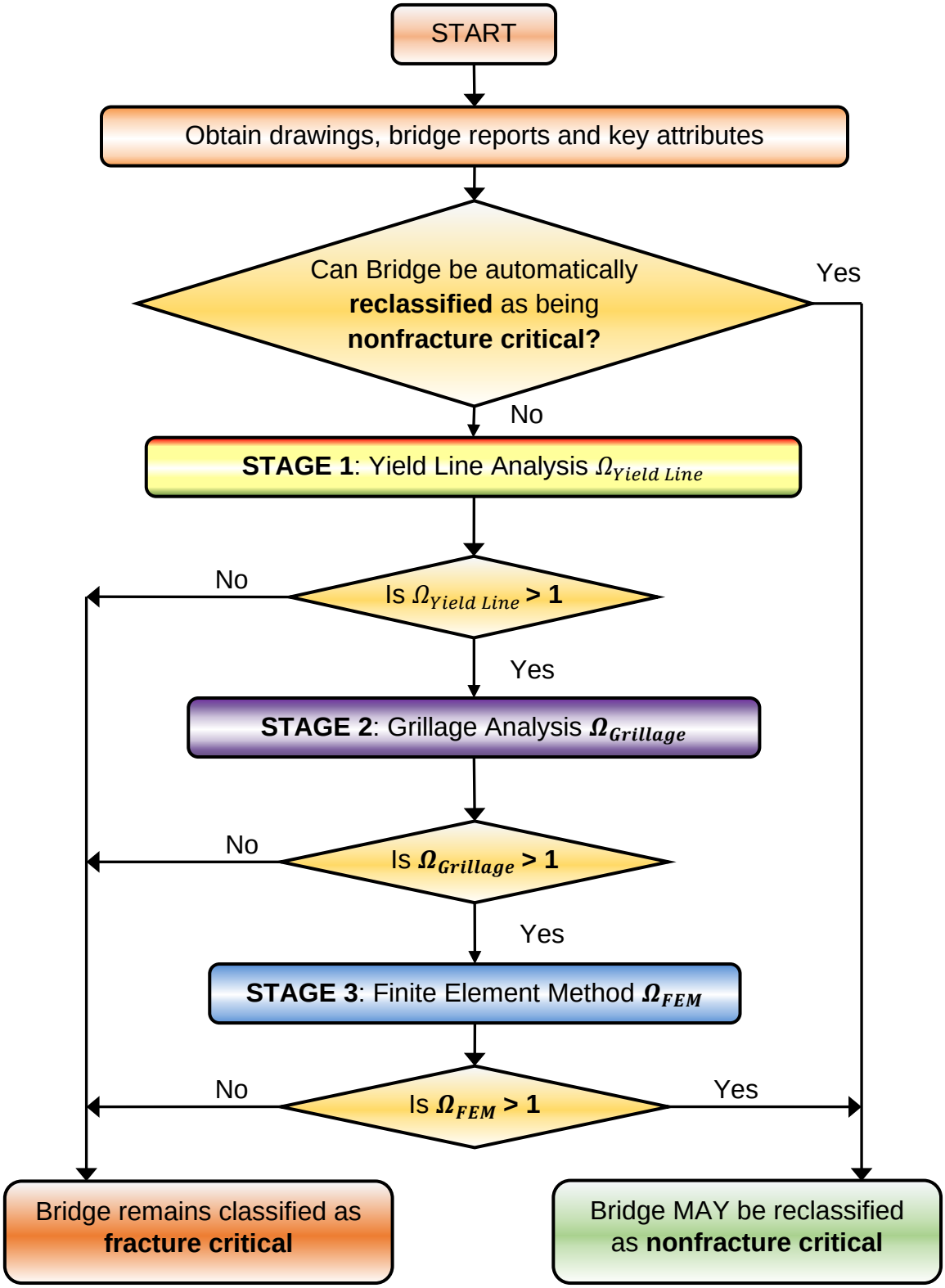


Figure 2.1. Flowchart for Analysis Procedure of STTG Bridges.

2.4 STAGE 2: GRILLAGE ANALYSIS

The nonlinear push-down analysis is a computational method that may be carried out using reputable commercial software such as SAP2000 (Wilson and Habibullah 1997). The concept of the method is the strip method of plastic analysis that incorporates the modeling of structural components as equivalent grillage members whose capacity is assessed through hinge failure in an elasto-plastic analysis. The method generally leads to lower-bound solutions. However, if strain-hardening is used in the constitutive relations, solutions where $\Omega_{Grillage} > \Omega_{Yield\ Line}$ are possible; the lower of the two should be adopted. If $1 > \Omega_{Grillage} > \Omega_{Yield\ Line}$, the evaluation shall be terminated by classifying the bridge as fracture critical. In the event of an overstrength, $\Omega_{Grillage}$>1, the analysis shall be taken to the next stage.

2.5 STAGE 3: FINITE ELEMENT METHOD

It is possible that following the Stage 2 analysis, a result may lead to $\Omega_{Yield\ Line} > 1$ and $\Omega_{Grillage} < 1$. This result does not necessarily mean the bridge cannot be reclassified as nonfracture critical; rather, a more exacting computational analysis is needed as a tie-breaker. Therefore, an advanced modeling and load simulation analysis should then be conducted using general-purpose nonlinear FEM software such as Abaqus (Dassault Systèmes 2017) to capture the complete behavior of the elements and their material and structural properties. This method is the most precise of all the three methods employed but is considerably more costly in terms of time and effort. Because this method is considered the most definitive, then if the overstrength factor $\Omega_{FEM} > 1$, the structure may be reclassified as nonfracture critical. Conservatively, if $\Omega_{FEM} < 1$, the bridge shall continue to be classified as fracture critical.

2.6 BACKGROUND THEORY AND EXAMPLES

Sections 3 and 4 provide background to yield line and grillage approaches, respectively. In Sections 5–7 of these guidelines, overstrength factor results respectively are presented for the following 3 bridges selected from the suite of 15 typical steel twin tub girder (STTG) bridges from the Texas bridge inventory that are described in the TxDOT (0-6937) report of Fracture Critical Steel Twin Tub Girder Bridges:

- Single-span bridges: Bridge 2 is used as an example that has no support fixity.
- Two-span continuous bridges: Both exterior spans of Bridge 5 are used as examples—with one degree of fixity over the interior support.
- Three-span or greater continuous bridges: Both exterior spans of Bridge 10, with one fixity over the interior support, and one interior span of Bridge 10, with fixity over both the interior supports, are provided as examples.

3. PLASTIC ANALYSIS USING YIELD LINE THEORIES

As described in Park and Gamble (2000), the overstrength factor is calculated using virtual work analysis by equating the factored external work to the internal work when a maximum deflection of unity takes place:

$$\Omega EWD_U = IWD_N \tag{3.1}$$

in which IWD_N = internal work done based on nominal material properties; EWD_U = external work done by factored ultimate design load; and Ω=overstrength factor.

The external work done is calculated as follows:

$$EWD_U = W_{ET} L_x^* \frac{\delta}{2} = 0.5 W_{ET} L_x^* \tag{3.2}$$

in which W_{ET} = the total load that effectively participates in the collapse mechanism; δ = 1 = virtual displacement; and L_x^* = the span length under consideration measured on the centerline (CL) of the collapse mechanism, such that:

$$L_x^* = \left(1 + \frac{B}{4R_{℄}}\right) L_x \tag{3.3}$$

where L_x = CL of the bridge (midway between the twin tubs); B = the width of the bridge; and $R_{℄}$ = the radius of curvature measured along the CL of the bridge deck for a straight bridge $L_x^* = L_x$.

For AASHTO HL93 truck and lane loads, this results in:

$$W_{ET} = w_u L_x^* (b + 0.5s) + W_x L_x^* + \left(336 - \frac{523}{\lambda L_x^*} - \frac{2091}{(1-\lambda) L_x^*}\right) K_{lane} \tag{3.4}$$

where s = the width of the area of the slab along which the mechanism under consideration is applied; b = the transverse distance of the interior flange of the fractured girder from the outer edge of the bridge; w_u = the area load consisting of self-weight of the reinforced concrete deck slab and the applied lane load (kip/ft^2 = ksf); W_x = the line load consisting of the self-weight of the fractured tub girder and the guardrail (kip/ft); and λ = the critical location factor for the hinge to occur, normally at the location of maximum moments. For simply supported spans and the interior spans of three- or more-span continuous bridges, λ = 0.5, whereas for two-span bridges or the end span for three- or more-span bridges, λ = 0.4.

Thus, for simply supported spans and for interior spans (λ =0.5):

$$W_T = w_u L_x^*(b + 0.5s) + W_x L_x^* + \left(336 - \frac{5226}{L_x^*}\right) K_{lane} \tag{3.5}$$

For the end spans of continuous bridges (2 spans or greater, where λ = 0.4).

$$W_T = w_u L_x^*(b + 0.5s) + W_x L_x^* + \left(336 - \frac{4793}{L_x^*}\right) K_{lane} \tag{3.6}$$

For a wider bridge, the second lane of trucks may participate (in part) in the collapse mechanism, as depicted in Figure 3.1. The axle loads are required to be increased proportionally to their deflection with respect to the truck position over the fractured girder. Thus, the lane load requires modification through the scalar K_{lane}. For one line of truck wheels participating:

$$K_{lane} = 1 + 0.5\ \frac{y}{s} \tag{3.7}$$

in which y = distance measured from the intact (unfractured) girder to the line of wheels. If both lines of wheels are participating in the mechanism, then:

$$K_{lane} = 1 + \frac{y}{s}\ ; \qquad K_{lane} \leq 2 \tag{3.8}$$

where y = distance to the CL of the truck.

The internal work done is calculated as follows:

$$IWD_N = \left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right) k_{bound} + \left(\frac{m_x b}{\lambda(1-\lambda)L_x^*}\right) + \left(\frac{0.5M_{p1}^-}{(1-\lambda)L_x^*}\right) + \left(\frac{0.5M_{p2}^-}{(1-\lambda)L_x^*}\right) \tag{3.9}$$

where m'_y and m_y are the negative and positive moment capacities per unit width in the *y* direction, respectively, and m'_x, and m_x are the negative and positive moment capacities per unit width in the *x* direction, respectively (units k-in./in. = k-ft/ft, or kN m/m or N-mm/mm); M_{p1}^- and M_{p2}^- are the plastic moment capacities of the composite deck and the intact girders at the ends of the span in consideration (0.5 is used since the outside girder alone takes part in the critical mechanism); and λ = the critical location factor for the hinge to occur, normally at the location of maximum moments, as defined above.

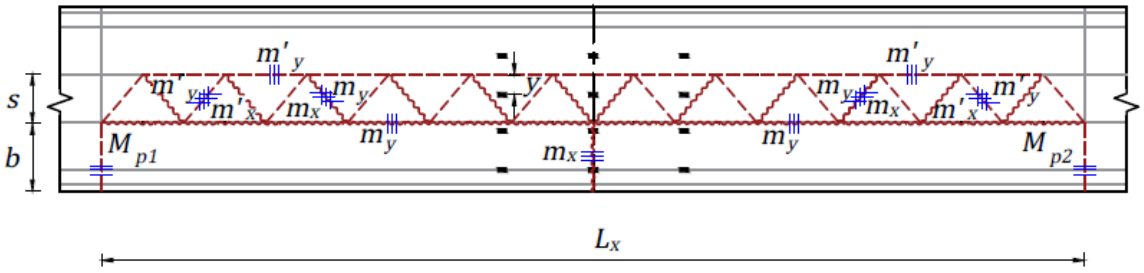


Figure 3.1. Layout of Typical Interior Span with Yield Line Mechanism.

Note that for simply supported spans, there is no end fixity; therefore, M_{p1}^{-} and M_{p2}^{-} are set to zero, whereas exterior spans of the two-span and of the three-span bridges have one fixity at the end support; therefore, one of the M_{p1}^{-} and M_{p2}^{-} are set to zero for that case, and interior spans have fixity at both the end, implying that both M_{p1}^{-} and M_{p2}^{-} are non-zero.

For simply supported spans and the interior spans of three-or-more-span continuous bridges (λ = 0.5):

$$IWD_{Simple\ Spans} = \left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right)k_{bound} + \frac{4}{L_x^*}(m_x b) \tag{3.10}$$

$$IWD_{Int.Spans} = \left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right)k_{bound} + \left(\frac{4m_x b}{L_x^*}\right) + \left(0.5M_{p1}^{-} + 0.5M_{p2}^{-}\right)\left(\frac{2}{L_x^*}\right) \tag{3.11}$$

For two-span bridges or the end-span for three- or more-span bridges (λ = 0.4):

$$IWD_{Ext.Spans} = \left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right)k_{bound} + \left(\frac{25m_x b}{6L_x}\right) + \left(\frac{25M_{p1}^{-}}{12L_x^*}\right) \tag{3.12}$$

The term k_{bound} represents the modifier term representing the upper and lower-bound solutions, as follows:

$$k_{bound}^{upper} = \left[1 + 2\frac{\tan\alpha}{\tan\theta}\right] = 1 + \frac{4s}{L_x}\sqrt{\left(\frac{m'_x + m_x}{m'_y + m_y}\right)} \tag{3.13}$$

and:

$$k_{bound}^{lower} = \left[1 + 2\frac{\tan^2\alpha}{\tan^2\theta}\right] = 1 + \frac{8s^2}{{L_x}^2}\left(\frac{m'_x + m_x}{m'_y + m_y}\right) \tag{3.14}$$

where:

$$\tan\alpha = \frac{s}{L_x/2} \tag{3.15}$$

and:

$$\tan\theta = \frac{sN}{L_x} = \sqrt{\frac{m'_y + m_y}{m'_x + m_x}} \tag{3.16}$$

The overstrength factors are computed using Equation (3.1).

$$\Omega_{Yield\ Line} = \frac{IWD_N}{EWD_U} = \frac{\left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right)k_{bound} + \left(\frac{m_x b}{\lambda(1-\lambda)L_x^*}\right) + \left(\frac{0.5M_{p1}^- + 0.5M_{p2}^-}{2L_x^*(1-\lambda)}\right)}{0.5W_{ET}} \tag{3.17}$$

For simply supported spans:

$$\Omega_{Simply\ Supported} = \frac{\left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right)k_{bound} + \frac{4}{L_x^*}(m_x b)}{0.5W_{ET}} \tag{3.18}$$

For interior spans:

$$\Omega_{Interior} = \frac{\left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right)k_{bound} + \left(\frac{4m_x b}{L_x^*}\right) + \left(0.5M_{p1}^- + 0.5M_{p2}^-\right)\left(\frac{2}{L_x^*}\right)}{0.5W_{ET}} \tag{3.19}$$

For exterior spans:

$$\Omega_{Exterior} = \frac{\left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right)k_{bound} + \left(\frac{25m_x b}{6L_x}\right) + \left(\frac{25M_{p1}^-}{12L_x^*}\right)}{0.5W_{ET}} \tag{3.20}$$

The longitudinal and transverse moment (positive and negative) capacities of the deck slab and the positive capacities of the composite intact section are computed based on the standard U.S. code procedure using the specified compressive strength of concrete and specified or as-built (if known) yield strength of reinforcing steel in the deck and the guardrail and in the structural steel of the twin tub girders. The negative capacities of the composite intact section are computed using plastic analysis of sections via the equal area method, assuming that the concrete has cracked completely and does not contribute to tension. Since the fractured outside girder alone takes part in the postulated critical mechanism, the negative moment capacity of half the section is used for the computation of the overstrength factor of the exterior spans.

The tabulations in the examples in Section 5, 6 and 7 that follow are presented such that the input values to be used depend on bridge geometry, the material properties of the deck and

the guardrail, the reinforcement, and the structural steel. They are indicated by yellow highlighting of the corresponding row number, with the value itself in boldface. Similarly, the values that need to be solved to ensure equilibrium and the corresponding equilibrium checks are indicated by blue highlighting of the corresponding row number, with the value itself in boldface.

The other rows can be automated by feeding the formulae presented in the column named FORMULA/DEFINITION/EQUATION, which also mentions the conditions for which each formula is applicable. Since Bridge 2 does not have support fixity at all, the moment calculations for the positive and negative composite deck and the intact girders are irrelevant for this bridge, and are therefore not included in this section. The results are also presented in boldface.

4. COMPUTATIONAL IMPLEMENTATION OF GRILLAGE ANALYSIS

The computational analysis of the fracture critical twin tub girder bridges (TTGBs) may be implemented using commercial nonlinear structural analysis software. Programs such as SAP2000 may be useful to carry out the following steps.

Step 1: Define Cylindrical Coordinate System for the Grillage Grid

For the TTGB, the longitudinal grids need to be located at the two exterior edges of the bridge, the CL of the two exterior top flanges, and the two interior top flanges. The transverse grillage grids need to be located at the ends on 7 ft spacing increments in the middle of the bridge (for easier assignment of the HS-20 truck load), and at pier locations in the case of a multi-span bridge. An illustration of the grid system for a single-span bridge is shown in Figure 4.1. The transverse spacing increments will need to be converted to a radial spacing in the cylindrical coordinate system using Equation (4.1):

$$Radial\ Spacing\ (\text{deg.}) = \left(\frac{Spacing\ Length}{Radius\ of\ bridge}\right) * \left(\frac{180}{\pi}\right) \tag{4.1}$$

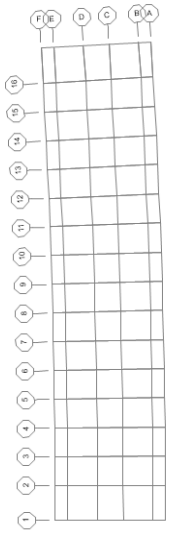
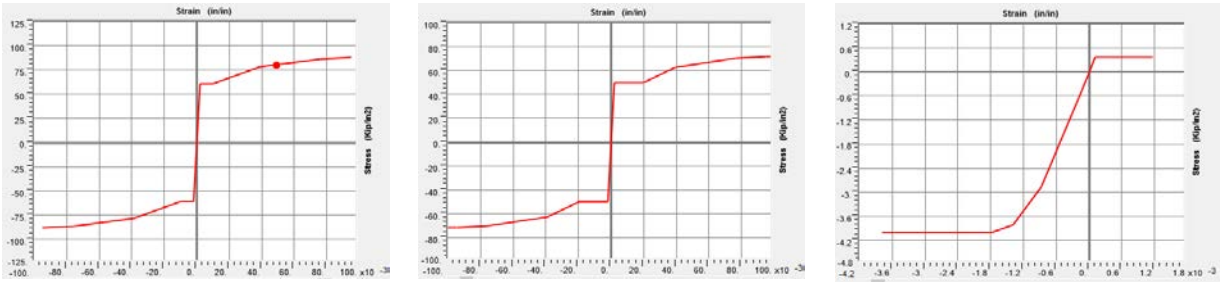


Figure 4.1. Grillage Grid System for a Single-Span Bridge.

Step 2: Nonlinear Material Properties of the Members

The fractured TTGB will be analyzed at ultimate loading conditions; therefore, the steel and concrete components of the bridge will be taken beyond their elastic capacity. The composite

girder and deck system is composed of concrete that will reach cracking and crushing strains and of rebar and steel plates that will reach strains beyond yielding. The material models to be used are represented in Figure 4.2. Nonlinear constitutive material behavior is defined in the advanced properties within the material definition.

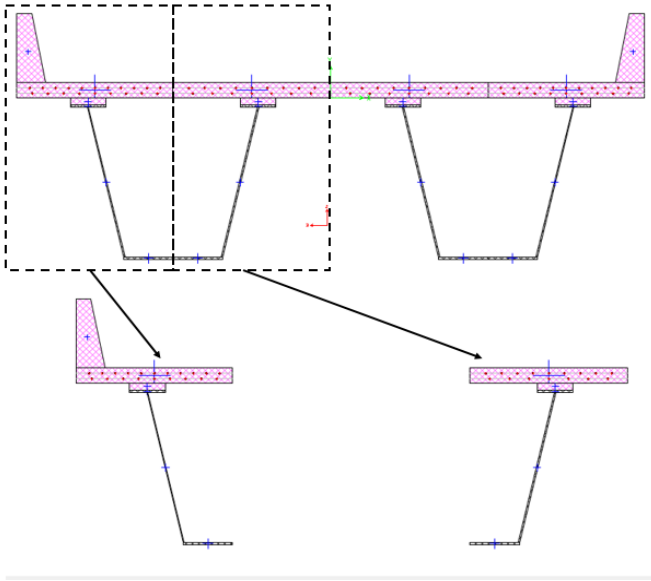


(a) Reinforcing Bar (b) Steel Plate (c) Concrete

Figure 4.2. Constitutive Material Models (SAP2000).

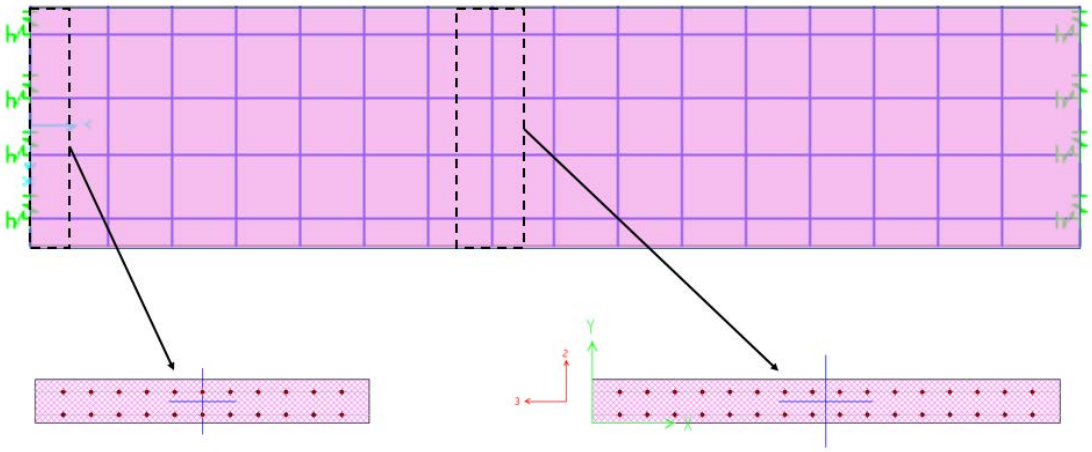
Step 3: Define Section Properties for Longitudinal and Transverse Bridge Members

Using the section designer feature in SAP2000, a composite tub, deck, and railing section can be generated. The exterior longitudinal member in Figure 4.3 consists of the railing, the deck from the CL of the girder to the exterior edge (with corresponding reinforcing bar), one top flange, one web, and half of the bottom flange. The interior longitudinal member consists of the deck from the CL of the bridge to the CL of the girder, with corresponding longitudinal reinforcing bar, one top flange, one web, and one half of the bottom flange. The transverse members in Figure 4.4 consist of concrete deck and transverse reinforcing bar. However, it is critical to set the weight modifier to zero of the transverse section to not double-count the concrete deck weight. It should be noted that as the steel plate members change dimensions and the reinforcing pattern changes throughout the length of the bridge, and new sections will need to be created to represent the new dimensions.



a) Exterior Longitudinal Member b) Interior Longitudinal Member

Figure 4.3. Representative Longitudinal Members.



a) End Transverse Member b) Interior Transverse Member

Figure 4.4. Representative Transverse Members.

The fractured girder can be modeled by simply copying the exterior and interior longitudinal sections and removing the bottom flange, web, and top flange steel plate components.

Step 4: Define Hinge Properties

Following the creation of the necessary longitudinal and transverse members, plastic hinges need to be created for each section. The grillage push-down analysis will generate plastic hinge formation under the ultimate loading condition. Within the section designer of SAP2000, there is a moment curvature response tool that allows the user to generate moment curvature data for

each of the members created in Step 3. The data from SAP2000 is then exported into an Excel spreadsheet. Then the angle on the Moment Curvature window can be changed to 180 to get the negative moment curvature, and once again the data is exported to the same Excel spreadsheet. The moment curvature response is then normalized against the maximum positive and negative moments and their corresponding curvatures and then plotted. The Hinge Definition window in SAP2000 will only allow four normalized positive and negative moment curvature data points per section hinge. Therefore, a best fit plot for each moment curvature response needs to be generated in Excel using only 9 points (4 positive, 4 negative, and 1 zero). The hinge length is assigned as half the member depth. A representative hinge property is depicted in Figure 4.5. For ease of convergence, non-negative slopes are recommended for the hinge properties.

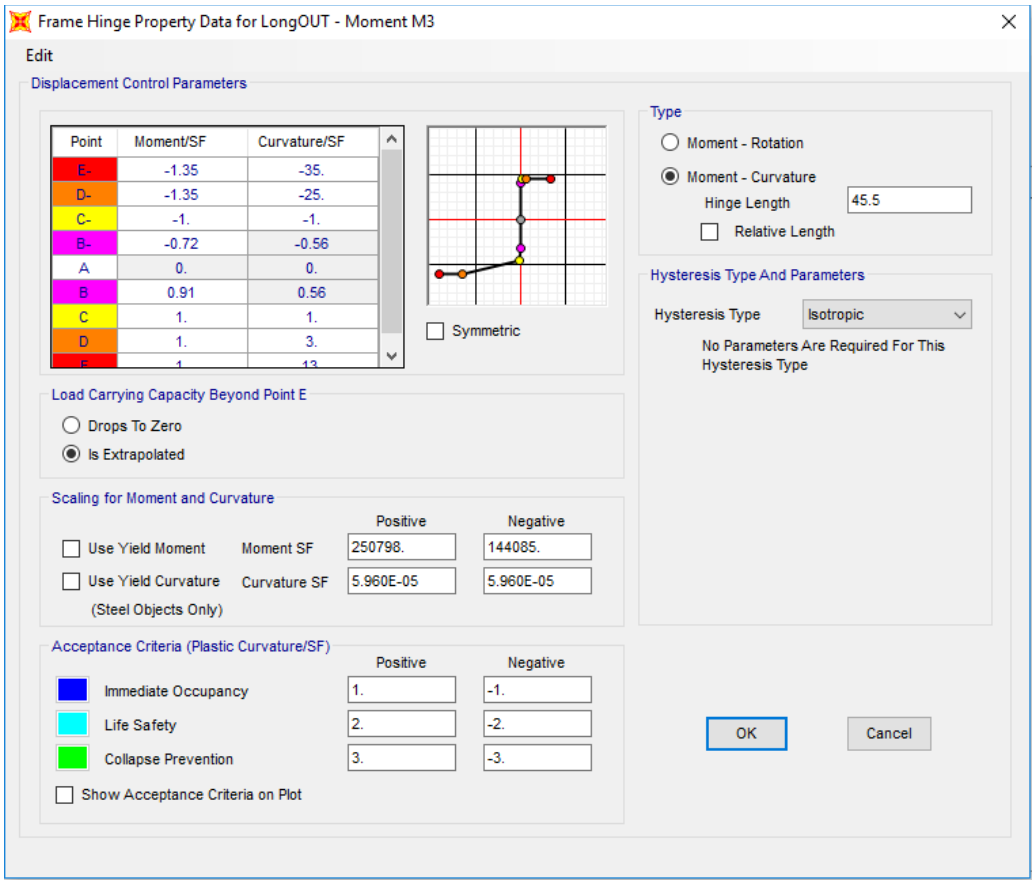


Figure 4.5. Representative Hinge Property.

Step 5: Assign Longitudinal and Transverse Members to the Grid

Using the “quick draw” frame section tool in SAP2000, the various longitudinal and transverse frame sections can be assigned to the grillage grid that was established in Step 1 by merely

selecting the desired section from the drop-down menu and clicking on the appropriate grillage grid. Figure 4.6 shows a screenshot from SAP2000 after all members have been assigned to a simple-span bridge.

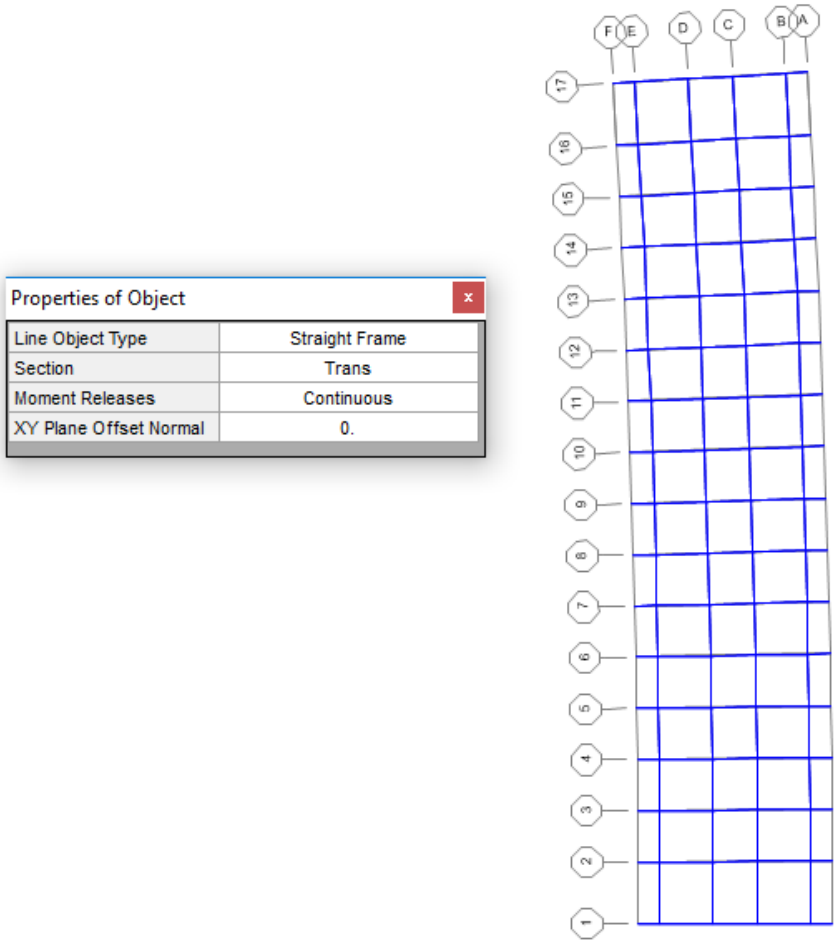


Figure 4.6. Screenshot of SAP2000 Post Frame Section Assignment.

Step 6: Assign Hinges to Frame Members

At this stage, the longitudinal and transverse members are already assigned to the grillage grid. To allow for plastic hinge formation, hinges need to be assigned at the nodal intersection of all members, as represented in Figure 4.7. Longitudinal hinges need be placed at both joints at the end of each member. Transverse hinges need to be assigned at a distance of half a top flange width away from each node.

Figure 4.7. Representative Hinge Assignments.

Step 7: Assign Boundary Conditions

The support conditions of the physical bridge are elastomeric bearing pads. These pads are represented by springs with a lateral stiffness of 6 kip/in. and a vertical stiffness of 3050 kip/in. in the grillage model, as represented in Figure 4.8. For the single-span bridges, springs will be assigned at each end of the longitudinal joints. For the two- and three-span bridges, springs are also assigned to the ends of the longitudinal joints and to the joints at the pier location.

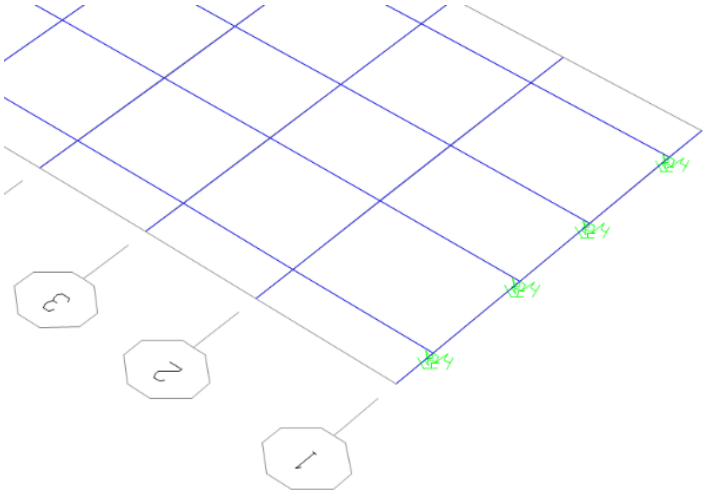


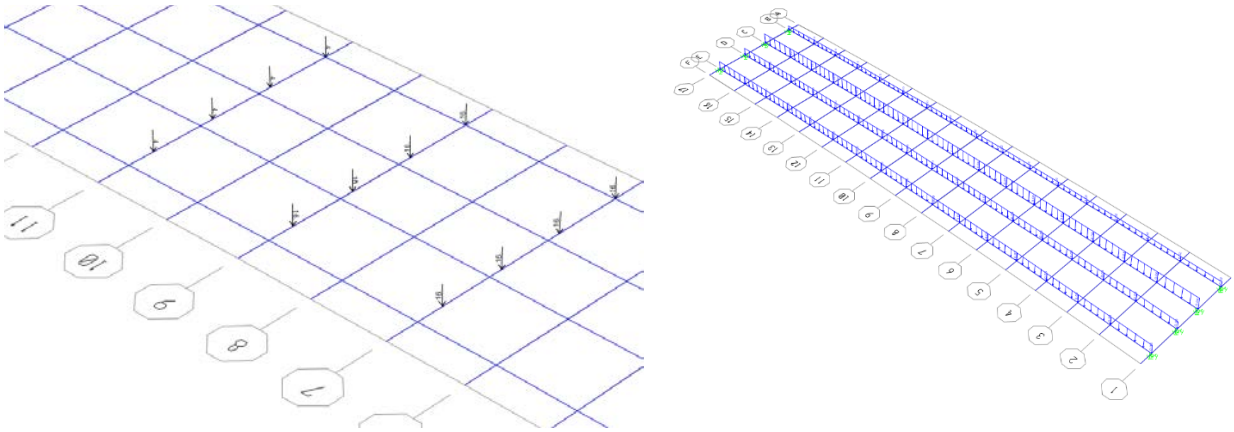
Figure 4.8. Spring Boundary Conditions.

Step 8: Define Load Patterns and Load Cases

For single-span bridges, two additional load patterns need to be created: the truck load and the lane load. For the two- and three-span bridges, a truck load pattern needs to be defined for each span, and the same follows for the lane load. Once load patterns are established, load cases need to be created. Each load case represents a load combination of 1.25*DL + 1.75(LL + IM), where DL = dead load, LL = live load, and IM = impact load, or 1.25*DL + 1.75*LL + 2.33*HS-20, where DL = dead load, LL= lane load, and HS-20 = truck load. Each load case should be set to nonlinear behavior. The first load case should start from a zero initial condition. Each proceeding load case should start from the end of the previous load case. Each span should have its own set of load cases. In addition, each load case should be divided into 20 or more steps.

Step 9: Assign Frame Loads

Two lanes of truck loading should be assigned as a series of point loads, and the two lanes of lane loads should be distributed as lane loads to the longitudinal members (as depicted in Figure 4.9). An HS20 truck load consists of 32 kip middle and rear axles and 8 kip front axle loads spaced 14 ft apart longitudinally and 6 ft transversely. The first line of wheels will be placed 3 ft from the curved edge of the deck (2 ft away from the face of the curb plus 1 ft curb thickness), the second 9 ft from the edge of the deck, the third 15 ft from the edge of the deck, and the fourth 21 ft from the edge of the deck. The middle axle load of each truck should be placed at half the span length on the single-span bridges and interior middle spans of three-span bridges. The middle axle should be placed at approximately 0.4*L from the end of the end spans of two-span and three-span bridges. Two lanes of lane loading (0.64 kip/in. each) should be located—one 8 ft from the edge and the second lane 20 ft from the edge. However, since the longitudinal members are placed according to the girder placement, the lane loads have to be distributed according to the tributary area.



a) Truck Load b) Lane Load

Figure 4.9. Grillage Truck and Lane Load.

Step 10: Assign Data Collection Points at Centerline of Girder

At the location of each of the center axles, the transverse members between the outer longitudinal member and the interior longitudinal member need to be divided into two pieces using the Divide Lines feature in SAP2000. This process allows for the collection of data at the CL of the member.

Step 11: Run Analysis for Dead Load Only

In order to get to dead load value of the data, the intact bridge should be run solely under the dead load case. The reactions should be recorded for all supports.

Step 12: Run Analysis for All Load Cases for Each Intact Span

For the intact bridge, run all HL93 load cases for the span being evaluated. Once the program has run, collect the displacement data for points 1 thru 4 (seen in Figure 4.10) and the CL points of the inside and outside girder at the location of the center axle. Be sure to obtain the results in the step-by-step format so that the load case and step for each displacement point can be collected as well. This process will be completed once for each span.

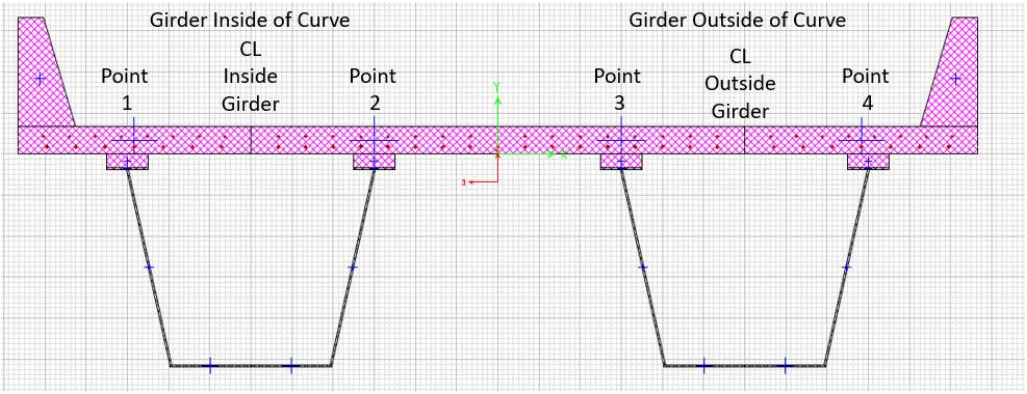


Figure 4.10. Location of Grillage Data Collection Points.

Step 13: Replace Hinges at Fracture

At the location of the center axle for the span being evaluated, replace the longitudinal hinges on the outside girder with their fractured counterparts. The hinge assignment is depicted in Figure 4.11.

Figure 4.11. Fractured Hinge Pattern.

Step 14: Run Analysis for All Load Cases for Each Fractured Span

For the fractured bridge, run all HL93 load cases for the span being evaluated. Once the program has run, collect the displacement data for points 1 thru 4 (seen in Figure 4.10) and the CL points of the inside and outside girder at the location of the center axle. Be sure to obtain the results in the step-by-step format so that the load case and step for each displacement point can be collected as well. This process will be completed once for each span, making certain to replace intact hinges in the preceding span before assigning fractured hinges in the following span.

Step 15: Post-Process the Data

For both the intact and fractured bridge, the following calculations need to be made:

- Omega (Ω):
  - $\Omega_i = \Omega_{i-1} + \left(\frac{1}{\#\ of\ Steps\ in\ Load\ case}\right)$.
- Longitudinal Chord Rotation of Interior and Exterior Girder:
  - $Chord\ Rot._{Single\ Span\ or\ Interior\ Span} = -1 * \left(\frac{\delta_{CL}}{0.5*L}\right)$.
  - $Chord\ Rot._{Exterior\ Span} = -1 * \left(\frac{\delta_{CL}}{0.4*L}\right)$.
  - The above equations are in radians.
- Transverse Deck Rotation:
  - Relative rotation of deck at inside flange of inside girder:
    - $\alpha_{2-3} = \left(\frac{\delta_3-\delta_2}{s}\right) - \left(\frac{\delta_2-\delta_1}{w}\right)$.
    - $\alpha_{3-2} = \left(\frac{\delta_3-\delta_2}{s}\right) - \left(\frac{\delta_4-\delta_3}{w}\right)$.
    - Where s = spacing between the interior top flanges (at the CL of the web) of the inside and outside girders and w = spacing between the top flanges (at the CL of the web) of the same girder.
    - The above rotations are in radians.
- Applied Load:
  - Calculate unit applied load or applied load at 1 Ω:
    - $Unit\ Applied\ Load_{single\ span} = 1.25 * Total\ Reaction\ from\ Dead\ Load\ Case + 2 * (2.33 * HS20\ truck + 1.75 * Lane\ Load)$.
    - $Unit\ Applied\ Load_{multi\ span} = 1.25 * Total\ Reaction\ from\ Dead\ Load\ Case * \left(\frac{L_{span}}{L_{Total}}\right) + 2 * (2.33 * HS20\ truck + 1.75 * Lane\ Load)$.
    - $Applied\ Load = Unit\ Applied\ Load * \Omega$.
- Intact Stiffness of Intact Bridge:
  - $Initial\ Stiffness_{Intact\ at\ \Omega=0.4} = \frac{0.4}{Abs(\delta_{OG-CL})}$.

- Instantaneous Stiffness for Fractured Bridge:
    - $Instantaneous\ Stiffness_{OG-Frac.i} = \frac{\Omega_i - \Omega_{i-1}}{\delta_i - \delta_{i-1}}$.

The criteria above can be organized into an Excel spreadsheet, as noted in Figure 4.12.

	A	B	C	D	E	F	G	H	I	J	K	L	M	V	X	Y	Z	AA	AB	AC	AD
1	Intact			OG-CL		IG-CL		Point 4	Point 3	Point 2	Point 1			Applied		OG		IG			
2	Load Case	Load Step	Ω	Delta (in)	Chord (rad)	Delta (in)	Chord (rad)	Delta (in)	Delta (in)	Delta (in)	Delta (in)	α 23 (rad)	α 32 (rad)	Load (kip)	Ω (cal)	Delta (in)	Chord (deg)	Delta (in)	Chord (deg)	α 23 (deg)	α 32 (deg)
3	LC1	0	0	0	0.00000	0	0.00000	0	0	0	0	0	0	0	0.00	0.00	0.00000	0.00	0.00000	0	0
4	LC1	1	0.05	-0.1037	0.00015	-0.087926	0.00013	-0.10541	-0.10129	-0.0942	-0.07905	7.89E-05	4.94E-05	66	0.05	0.10	0.00861	0.09	0.00730	0.004522	0.002831
5	LC1	2	0.1	-0.2074	0.00030	-0.175851	0.00025	-0.21081	-0.20259	-0.18839	-0.15809	0.000158	9.88E-05	132	0.10	0.21	0.01722	0.18	0.01460	0.009043	0.005662

Figure 4.12. Spreadsheet of Grillage Data.

Failure criteria are as follows:

- The instantaneous stiffness for the fractured outside girder is less than 5 percent of the initial stiffness of intact outside girder.
- The chord angle (deflection at midspan divided by half span length) of the outside girder for simple spans or interior spans is greater than 2 degrees. The chord angle for exterior spans in multi-span bridges is greater than 3 degrees.
- The transverse deck rotation is greater than 5 degrees.

5. BRIDGE 2

5.1 YIELD LINE ANALYSIS EXAMPLE OF BRIDGE 2

This section documents the steps to compute the overstrength factor of Bridge 2 via the yield line analysis (grillage analysis is explained in the next sub-section). Figure 5.1 presents the dimensional details of the simply supported span of Bridge 2. Further details can be found in Appendix A.

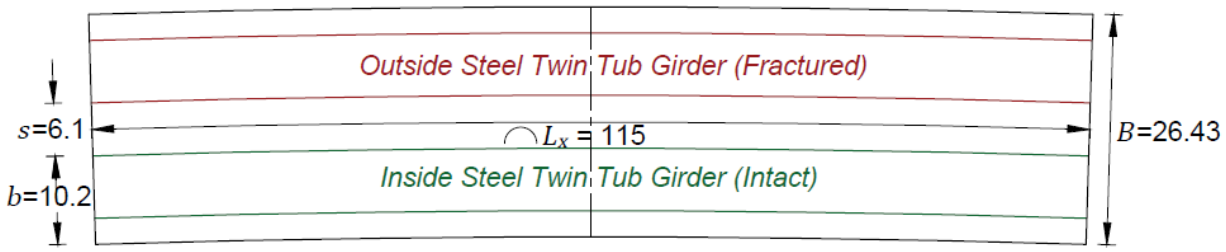


Figure 5.1. Schematic Diagram of Bridge 2 (R_L = 1910 ft, L_x = 115 ft).

This section also presents the stepwise procedure of the yield line analysis conducted to establish the upper-bound and lower-bound solution range for the overstrength factor that is in conjunction with the theory of plastic analysis in Section 3.

The moment capacities (longitudinal and transverse) of the deck slab are calculated using the standard US code-based procedure following the Whitney's stress block approach. The capacities are calculated for one ft wide cross-section of the bridge. The geometric parameters namely B = the total width (breadth) of the deck slab, t = thickness and b = the width of each cross-section are noted from the structural plans. The various material properties of concrete and steel are obtained through the bridge plans and the reports associated with the respective bridges. These properties include f_c' = the specified compressive strength of concrete, ε_{cu} = the maximum strain at the extreme concrete compression fiber (computed as per Section 22.2.2 of ACI-318 (2017) which states the "assumptions for concrete."), f_y = the yield strength of steel of the reinforcement bars, E = the Young's modulus of steel and ε_y = the yield strain that is the ratio of yield strength and Young's modulus of steel in the reinforcement.

β_1 = the factor relating depth of equivalent rectangular compressive stress block to depth of neutral axis is computed using Table 22.2.2.4.3 of ACI-318 (2017). The formula applicable for the strength of concrete used in this study is as follows

$$\beta_1 = 0.85 - 0.05(f_c' - 4) \qquad \text{where } f_c' \text{ is in ksi}$$

The details of the reinforcement such as $\#_{bar}$ = the number of bars per one ft wide section (for the longitudinal capacities) or s = the on-center spacing (for the transverse capacities), d_b = the diameters of the bars, A_s the corresponding areas of steel, cc = the clear cover (from the bridge drawings) and d and d' = the subsequently computed effective depths of the tensile and compressive zones respectively, are recorded. ε_{top} and ε_{bot} = the net tensile strain in the extreme tension reinforcement at the top and bottom of the section respectively, are determined from a linear distribution (ACI-318 2017).

The section of concrete is considered to be divided into compressive and tensile zones by a neutral axis that is located at a depth c from the top fiber. The compressive force due to concrete (with a negative sign) is found using the formula

$$C_c = 0.85 f_c' \mathrm{b} \beta_1 \mathrm{c} = 0.85 f_c' \mathrm{b} \qquad \text{where } a = \beta_1 \mathrm{c}$$

If the reinforcement steel is in compression, the compressive force due to compression steel is denoted by a negative sign, as follows

$$T_{top/bottom} = -(A_s f_s - 0.85 f_c' A_s)$$

The tensile force due to the steel reinforcement is given by

$$T_{top/bottom} = A_s f_s$$

To ensure that the computation of the forces considers whether or not the steel has yielded, the following formula is used for the strength of steel at the extreme fiber of reinforcement

$$f_s = \frac{E \varepsilon_{top/bottom}}{\left\{1 + \left|\frac{E \varepsilon_{top/bottom}}{f_y}\right|^{20}\right\}^{0.05}}$$

The depth of neutral axis, c (indicated by blue highlighting) is obtained by equating the tensile and compressive forces. The moment capacities are found by taking moments of forces about the neutral axis.

The computation of the overstrength factor is based on the procedure explained in the Section 3. The data necessary for the calculations such as the geometric details, the moment capacities, the volume of girder and area of cross-section of rail are listed as input values

indicated by yellow highlighting. The parameters needed to solve the equations in Section 3 are computed using the formulae and allowance factors for dead load, live load and the weight of the stiffeners. The parameters and their corresponding formulae and values are tabulated in a sequential order and can be regenerated using any spreadsheet program.

		MOMENT CAPACITY OF DECK SLAB (BRIDGE 2)					1/4
		Description	Parameter	Data			
DECK SECTION	1	Total Width	B in	**317**			
	2	Thickness	t in	**8**			
	3	Section width	b in	12			
CONCRETE MATERIAL	4	Characteristic Compressive Strength	f_c' ksi	4			
	5		β_1	0.85			
	6	Strain	ε_{cu}	0.003			
REBAR MATERIAL	7	Yield Strength	f_y ksi	60			
	8	Young's Modulus	E ksi	29000			
	9	Strain	ε_y	0.002070			
		Moment Computations		Longitudinal		Transverse	
	10			m_x'	m_x	m_y'	m_y
	11			Top	Bottom	Top	Bottom
REINFORCING BAR DETAIL	12	Bar No.		**5**	**5**	**5**	**5**
	13	Diameter of Bar	d_b in	0.625	0.625	0.625	0.625
	14	Area of Bar	A_Φ in^2	0.31	0.31	0.31	0.31
	15	Spacing	s in	–	–	**5**	**5**
	16	No. of Bars	$\#_{bar}$	**38**	**32**	–	–
	17	Area of Steel	A_s in^2	0.446	0.376	0.744	0.744
	18	Clear Cover	cc in	**2**	**1.25**	**2**	**1.25**
	19	Effective depth (tension)	d in	5.0625	5.8125	5.6875	6.4375
	20	Effective depth (comp.)	d' in	2.1875	2.9375	1.5625	2.3125
	21	Depth of NA	c in	**1.35**	**1.42**	**1.44**	**1.81**

		MOMENT CAPACITY OF DECK SLAB (BRIDGE 2)					2/4
		Moment Computations		Longitudinal		Transverse	
	22			m'_x	m_x	m'_y	m_y
	23			Top	Bottom	Top	Bottom
STRAIN	24		ε_{top}	0.008227	0.003201	0.008829	0.000837
	25		ε_{bot}	0.001851	0.009270	0.000250	0.007682
	26	$\beta_1 c$	a in	1.15	1.21	1.23	1.54
FORCES	27	Compression-Concrete	C_c kip	−46.91	−49.29	−50.03	−62.70
	28	Tension/ Compression-Steel	T_{top} kip	26.76	26.76	44.64	18.06
	29		T_{bot} kip	20.16	22.53	5.39	44.64
	30	Equilibrium Check	$\sum T + C$ kip	**0.00**	**0.00**	**0.00**	**0.00**
RESULTS	31	Moment	M_n k-in/in	**12.71**	**14.98**	**19.30**	**23.41**

Remarks:

Top Yielded	Top Yielded	Top Yielded	Top Not Yielded
Bottom Not Yielded	Bottom Yielded	Bottom Not Yielded	Bottom Yielded
Both steel in Tension	Both steel in Tension	Both steel in Tension	Both steel in Tension

		COMPUTATION OF OVERSTRENGTH FACTOR SINGLE SPAN CASE (BRIDGE 2)		3/4
		Parameter	Formula/Definition/Equation	Data
GEOMETRY	1	L_x	Span length (center line)	**115.00** ft
	2	R_{CL}	Radius of center line	**1910** ft
	3	B	Width	**26.4** ft
	4	L_x^*	Outer region length $L_x^* = \left(1 + \frac{B}{4R_{C_L}}\right) L_x$	115.40 ft
	5	s	Inter-Girder Spacing	**6.1** ft
	6	b	Width of Girder + Edge	**10** ft
	7	t	Deck Thickness	**8** in
INTERNAL WORK DONE, IWD	8	m_x	Longitudinal Positive Moment per ft	**15** k-in/in
	9	m'_x	Longitudinal Negative Moment per ft	**13** k-in/in
	10	m_y	Transverse Positive Moment per ft	**23** k-in/in
	11	m'_y	Transverse Negative Moment per ft	**19** k-in/in
	12	$\tan\theta$	$\tan\theta = \sqrt{\frac{m'_y + m_y}{m'_x + m_x}}$	1.24 (Θ = 50.8˚)
	13	$\tan\alpha$	$\tan\alpha = \frac{2s}{L_x}$	0.11 (Θ = 6.0˚)
	14	k_{bound}^{upper}	$\left[1 + 2\frac{\tan\alpha}{\tan\theta}\right] = 1 + \frac{4s}{L_x}\sqrt{\frac{m'_x + m_x}{m'_y + m_y}}$	1.17
	15	k_{bound}^{lower}	$\left[1 + 2\frac{\tan^2\alpha}{\tan^2\theta}\right] = 1 + \frac{8s^2}{{L_x}^2}\sqrt{\frac{m'_x + m_x}{m'_y + m_y}}$	1.01
	16	IWD_{upper}	$\left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right)k_{bound}^{upper} + \frac{4m_x b}{L_x}$	478 k-ft *
	17	IWD_{lower}	$\left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right)k_{bound}^{lower} + \frac{4m_x b}{L_x}$	415 k-ft *

Note: *: A unit deflection (δ = 1) is considered; therefore, the unit of internal work is in k-ft

		COMPUTATION OF OVERSTRENGTH FACTOR SINGLE SPAN CASE (BRIDGE 2)		4 4
		Parameter	Formula/Definition/Equation	Data
EXTERNAL WORK DONE, EWD	18	DL	Dead Load Factor	1.25
	19	LL	Live Load Factor	1.75
	20	SAF	Stiffener Allowance Factor	1.15
	21	γ_c	Unit weight of reinforced concrete	0.15 kcf
	22	w_u	Area load due to reinforced concrete + lane load $\boldsymbol{DL\gamma_c \frac{t}{12} + LL \cdot \frac{0.64}{12}}$	0.218 ksf
	23	γ_s	Unit weight of steel	0.49 kcf
	24	V_g	Volume of Girder	**148** ft^3
	25	A_r	Area of Rail Cross-Section SSTR	**2.75** ft^2
	26	V_r	Volume of Rail $= L_x A_r$	317 ft^3
	27	W_x	$1.25\left(1.15V_g\gamma_s + V_r\gamma_c\right)/L_x$	1.42 k/ft
	28	y (Lane 2)	$(b + s - 15)$ for $(b+s) < 21$ $(b + s - 18)$ for $(b+s) > 21$	1.25 ft
	29	K_{lane}	$1 + 0.5\frac{y}{s}$ for $(b+s) < 21$; $1 + \frac{y}{s}$ for $(b+s) > 21$	1.10
	30	$EWD_{HS\text{-}20}$	$\left(168 - \frac{2613}{L_x}\right) K_{Lane}$	160 k-ft
	31	W_{ET}	$w_u L_x(b + 0.5s) + W_x L_x + 2EWD_{HS20}$	817 k-ft
	32	EWD	$0.5\ W_{ET}$	409 k-ft *
RESULTS	33	Ω_{upper}	IWD_{upper}/EWD	**1.17**
	34	Ω_{lower}	IWD_{lower}/EWD	**1.02**

Note: *: A unit deflection (δ = 1) is considered; therefore, the unit of external work is in k-ft

5.2 GRILLAGE ANALYSIS EXAMPLE OF BRIDGE 2

The following steps are explained to conduct the computational implementation of grillage analysis of Bridge 2.

1. Gather bridge geometry and material information.

Steel Tub Properties (fy = 50 ksi)						
Location ft	Top Flange		Web		Bottom Flange	
	Width in.	Thickness in.	Width in.	Thickness in.	Width in.	Thickness in.
0–115	18	1.00	79	0.625	50	1.00

Location	Parameter	Description/Value
Bridge	Location	Harris County, I-610
	Year Designed/Year Built	2002/2004
	Design Load	HS25
	Length, ft	115
	Spans, ft	115
	Radius of Curvature, ft	1909.86
Deck	Width, ft	26.417
	Thickness, in.	8
	Haunch, in.	4
	Rail Type	SSTR
Rebar	# of Bar Longitudinal Top Row (#5)	40
	# of Bar Longitudinal Bottom Row (#5)	32
	Transverse Spacing Top Row (#5)	5
	Transverse Spacing Bottom Row (#5)	5
	Rebar Strength (ksi)	60
Girder	CL of Bridge to CL of Girder (in.)	79.5
	CL of Top Flange to CL of Top Flange (in.)	86

2. Determine material constitutive behavior.

Concrete (4 ksi)

Stress (ksi)	Strain (1/in)
−4	−3.79E-03
−4	−3.56E-03
−4	−2.69E-03
−4	−1.78E-03
−3.8205	−1.40E-03
−2.8718	−8.69E-04
−0.6403	−1.78E-04
0	0
0.378	1.06E-04
0.378	1.16E-03

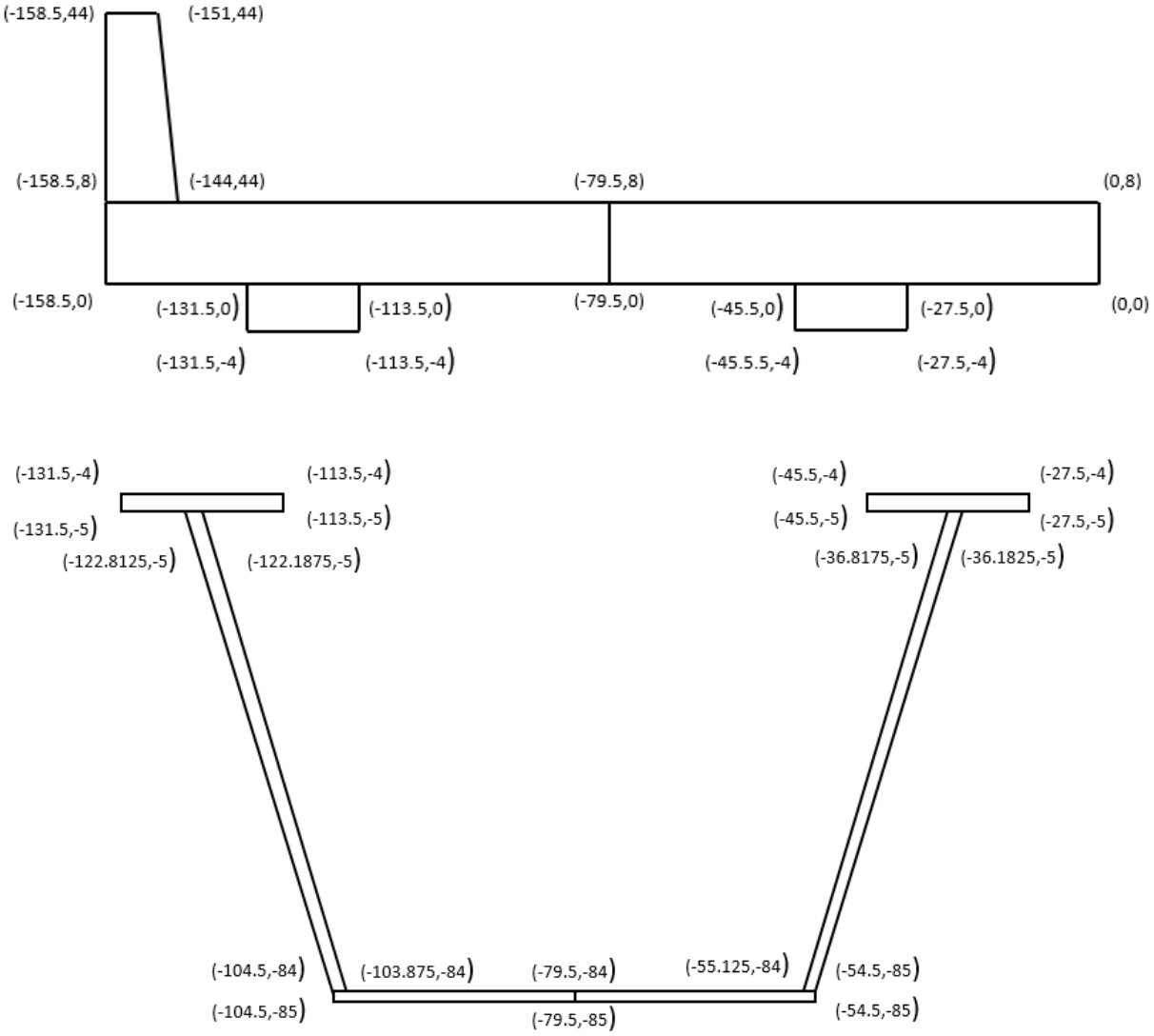
Rebar (60 ksi)

Stress (ksi)	Strain (1/in)
−87.9	−0.095
−87.9	−0.0944
−86.6	−0.0761
−78	−0.0386
−60.7	−9.80E-03
−60.3	−2.08E-03
0	0
60.3	2.08E-03
60.7	9.80E-03
78	0.0386
86.6	0.0761
87.9	0.0944
87.9	0.095

Steel (50 ksi)

Stress (ksi)	Strain (1/in)
−71.6	−0.1
−71.6	−0.097
−71.6	−0.095
−71.6	−0.0946
−70.3	−0.0764
−62.5	−0.039
−50	−0.0196
−50	−1.72E-03
0	0
50	1.72E-03
50	0.0196
62.5	0.039
70.3	0.0764
71.6	0.0946
71.6	0.095
71.6	0.097
71.6	0.1

3. Create a coordinate system for half width of the span.



4. Create a cylindrical coordinate system for the curved bridge and ensure that the middle transverse divisions are 7 ft because it will aid in applying the truck load, whose axles are separated by 14 ft.

   A. $\#\ of\ Segments = \left(\frac{Length\ (ft)*12}{84\ in.(7ft)}\right)\ rounded\ to\ nearest\ even\ number.$

   $$\#\ of\ Segments = \left(\frac{115 * 12}{84}\right) = 16.428, so\ 14\ was\ selected.$$

   B. $End\ Segment\ Length = \frac{\left((Length*12) - (\#\ of\ Segments*84)\right)}{2}.$

   $$End\ Segment\ Length = \frac{\left((115 * 12)\ -\ (14 * 84)\right)}{2} = 102\ in.$$

C. $Theta = \frac{Length}{Radius}$.

$$Theta = \frac{115}{1909.86} = 0.0602\ rad\ or\ 3.450\ degrees$$

D. Determine the radial offsets using the outside edge, the outside flange, the inner flange, and CL of the bridge.

Offsets (in.)	
Edge	158.5
Outside Flange	122.5
Inner Flange	36.5
CL of Bridge	0

Radial Spacing (in.)		
A	23077.3	CL + Edge
B	23041.3	CL + OF
C	22955.3	CL + IF
Center Line	22918.8	or 1909.86 (ft)
D	22882.3	CL − IF
E	22796.3	CL − OF
F	22760.3	CL − Edge

E. Determine the longitudinal spacing along theta by converting the longitudinal segment lengths into degrees.

- The first and last segments are 102 in., and the intermediate segments are 84 in. The total length is 115 ft, or 1380 in.
- $Radial\ Spacing\ (rad) = \frac{Long.Spacing}{Radius}$
- $Radial\ Spacing\ (degree) = Radius\ Spacing\ (rad) * \frac{180}{\pi}$

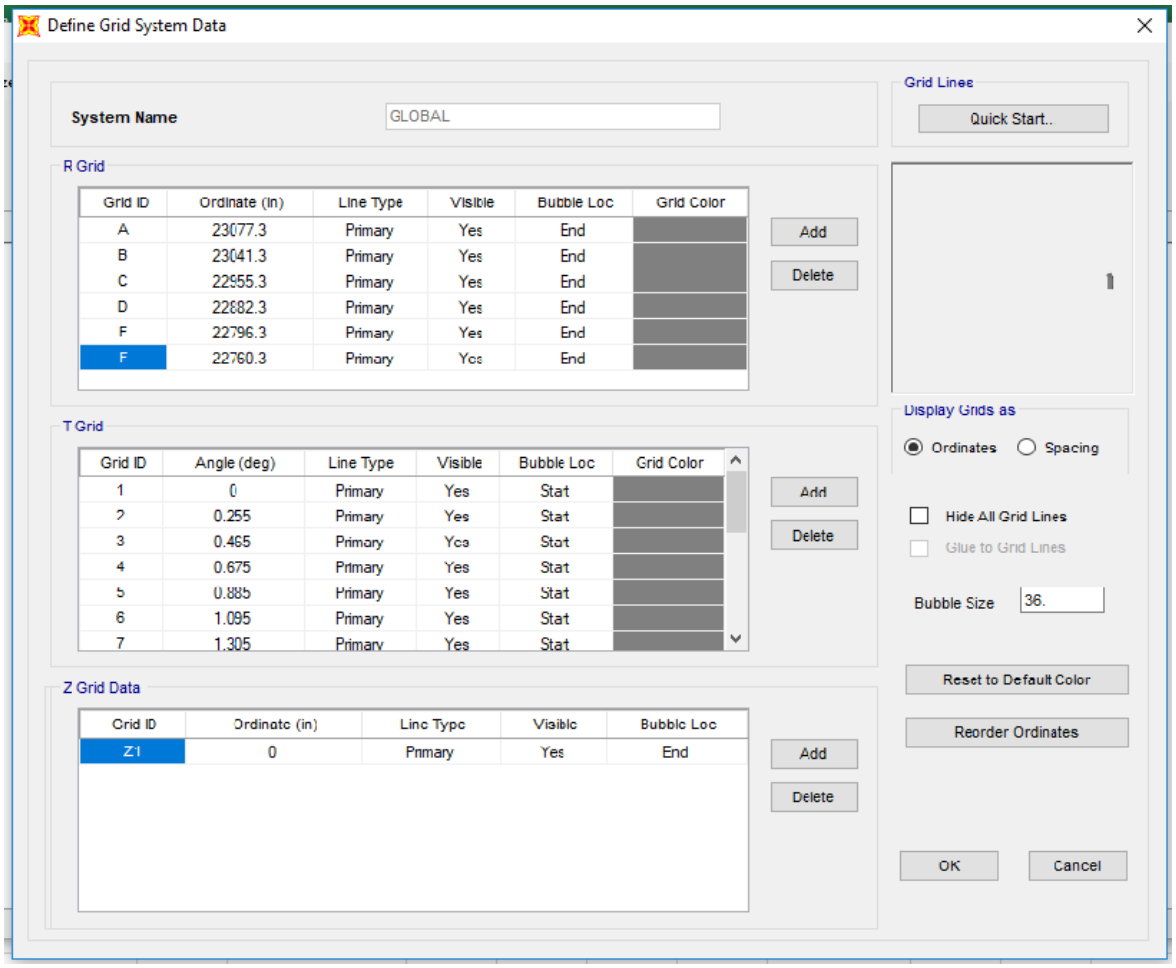
Long. Spacing (in.)	Radial Spacing (rad.)	Radial Spacing (degrees)
0	0.0000	0.000
102	0.0045	0.255
186	0.0081	0.465
270	0.0118	0.675
354	0.0154	0.885
438	0.0191	1.095
522	0.0228	1.305
606	0.0264	1.515
690	0.0301	1.725
774	0.0338	1.935
858	0.0374	2.145
942	0.0411	2.355
1026	0.0448	2.565
1110	0.0484	2.775
1194	0.0521	2.985
1278	0.0558	3.195
1380	0.0602	3.450

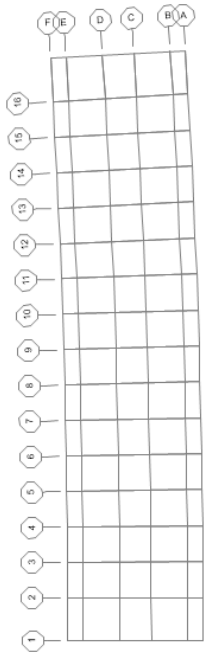
- $Int. Transverse\ Element\ width = 84\ in.$
- $End\ Transverse\ Element = 102 - \left(\frac{84}{2}\right) = 60\ in.$

5. Input the coordinate system into SAP2000.
   A. Select File > New Model > Blank model (making sure units are in kips and in.).
   B. Right click on the blank workspace and select Edit Grid Data > Modify/Show System > Quick Start > Cylindrical.
      - In the Number of Gridlines panel, set "Along Z = 1."
      - In the Grid Spacing panel, set "Along Z = 1."
      - Select OK.
      - Delete all R and T Coordinates that were generated.
   C. Add correct coordinates for R.

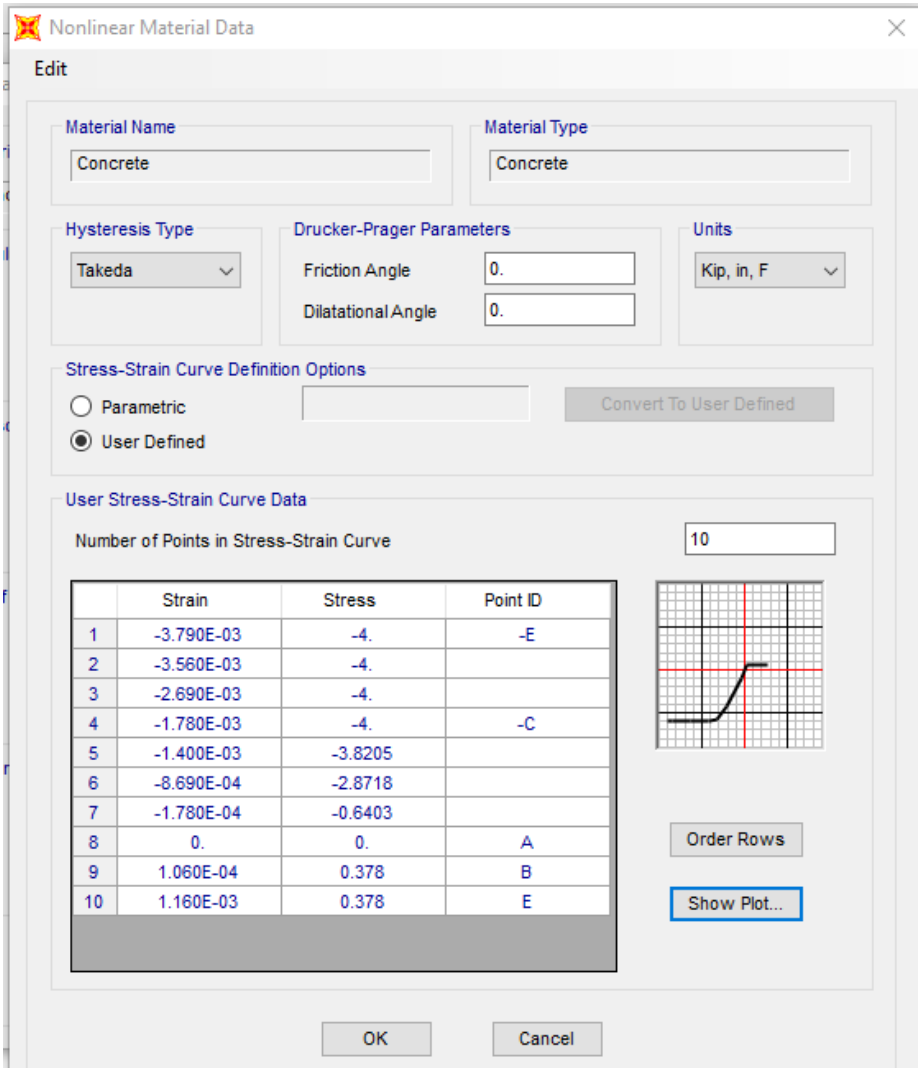
      All radial coordinates (A, B, C, D, E, and F).

   D. Add correct coordinates for T.
      - All theta coordinates for T (0 to 1380 in.).
      - Click OK.
   E. The grid system is now formed.

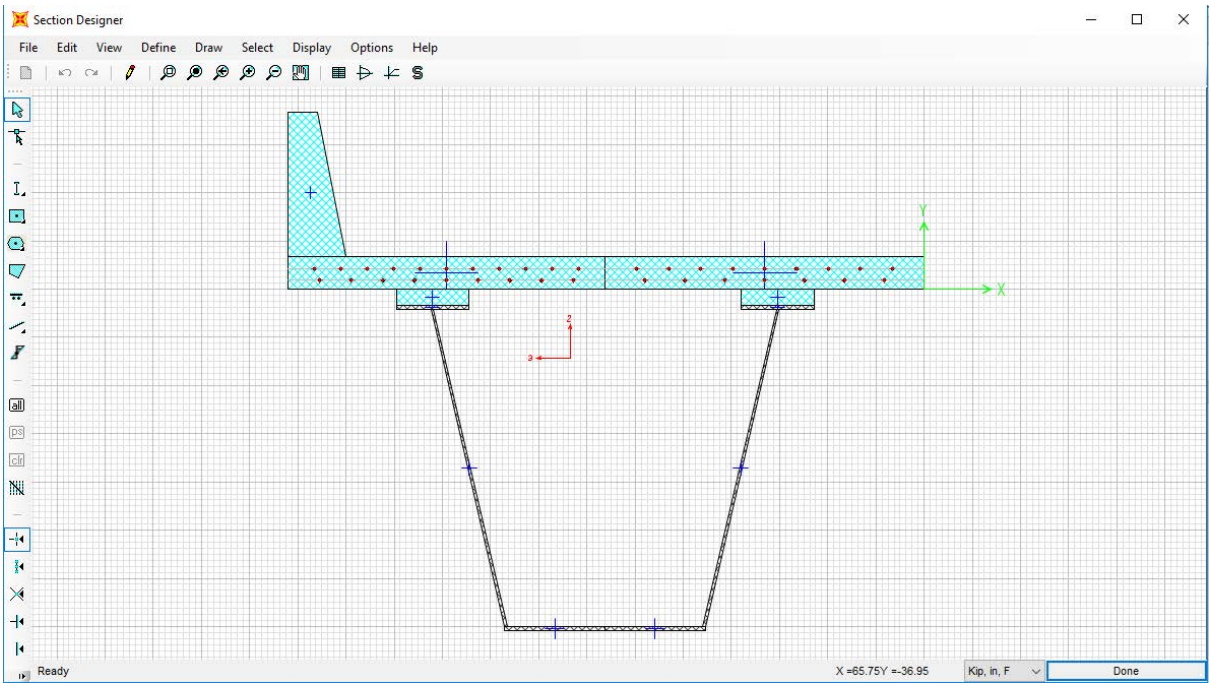

Define Grid System Data
System Name
GLOBAL
Grid Lines
Quick Start..
R Grid
Grid ID | Ordinate (in) | Line Type | Visible | Bubble Loc | Grid Color
A | 23077.3 | Primary | Yes | End
B | 23041.3 | Primary | Yes | End
C | 22955.3 | Primary | Yes | End
D | 22682.3 | Primary | Yes | End
F | 22796.3 | Primary | Yes | End
F | 22760.3 | Primary | Yes | End
Add
Delete
T Grid
Grid ID | Angle (deg) | Line Type | Visible | Bubble Loc | Grid Color
1 | 0 | Primary | Yes | Start
2 | 0.255 | Primary | Yes | Start
3 | 0.465 | Primary | Yes | Start
4 | 0.675 | Primary | Yes | Start
5 | 0.885 | Primary | Yes | Start
6 | 1.095 | Primary | Yes | Start
7 | 1.305 | Primary | Yes | Start
Add
Delete
Z Grid Data
Grid ID | Ordinate (in) | Line Type | Visible | Bubble Loc
Z1 | 0 | Primary | Yes | End
Add
Delete
Display Grids as
Ordinates
Spacing
Hide All Grid Lines
Glue to Grid Lines
Bubble Size
36.
Reset to Default Color
Reorder Ordinates
OK
Cancel


F
E
D
C
B
A

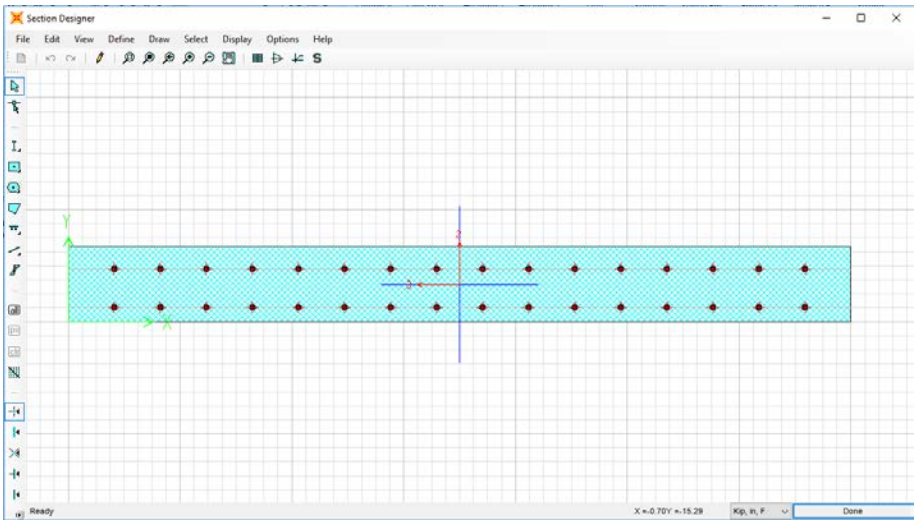
6. Define material in SAP2000.
    A. Click Define > Materials > Add New Material > Material Type (Steel, Concrete, or Rebar) > Standard (User) > OK.
    B. At the bottom of the window, select the box that states, “Switch to Advanced Properties.”
    C. In the open window, name the material “Concrete,” “Steel,” or “Rebar” depending on which material is being defined. Then click “Modify/Show Material Properties.”
    D. In the Material Property Data window, click the “Nonlinear Material Data” icon.
    E. In the Nonlinear Material Data window, select the “Convert to User Defined” icon.
    F. Input the number of data points for the stress strain behavior (10 for concrete, 13 for rebar, 17 for steel).
    G. Input the data points for the stress strain behavior.
    H. Select “OK.”
    I. Repeat this process again for the remaining materials.



7. Define frame cross sections in SAP2000.
    A. Click Define > Section Properties > Frame Sections > Add New Properties.
    B. In the Frame Section Properties drop-down box, select Other and click Section Designer. In the SD Section Designer window, name the section B2Long and click the Section Designer icon.
    C. Using the Polygon feature, draw the features of the half width of the bridge from Step 3. Features include one rail, two concrete deck pieces, two concrete haunches, two top flanges, two webs, and two pieces of the bottom flange.
        - To change material types for the polygons, right-click on the polygon and select the desired material type from the material drop-down menu.
        - To change the coordinates of the polygon's nodes, use the Reshaper tool.
    D. Add in the longitudinal rebar to both concrete deck elements by using the Line Bar from the Draw Reinforcing Shape tool. From the design drawings, it can be determined that there are 11 #5 top bars and nine #5 bottom bars in the outer concrete deck element and nine #5 top bars and seven #5 bottom bars in the inner concrete element, with a 2 in. top cover and transverse reinforcement. The top bars are located at 5.0625 in. with a 1.25-in. bottom cover, and transverse reinforcements are located at 2.1875.
    E. Click Done.

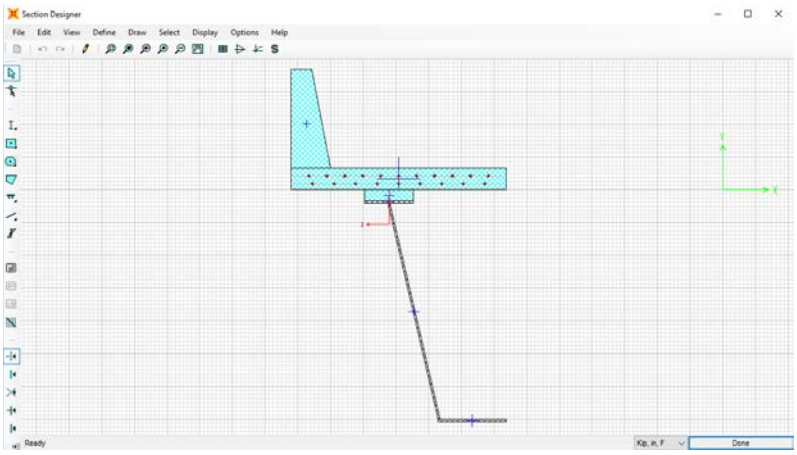


    F. Repeat this process for the transverse elements.
        - At the SD Section Designer window select Modifiers and set Mass and Weight to 0 in order to not double-count the dead weight.
        - The interior transverse members are 84 in. wide (end members are 60 in. wide), with #5 rebar at 5 in. spacing at 5.6875 in. and 1.5625 in.

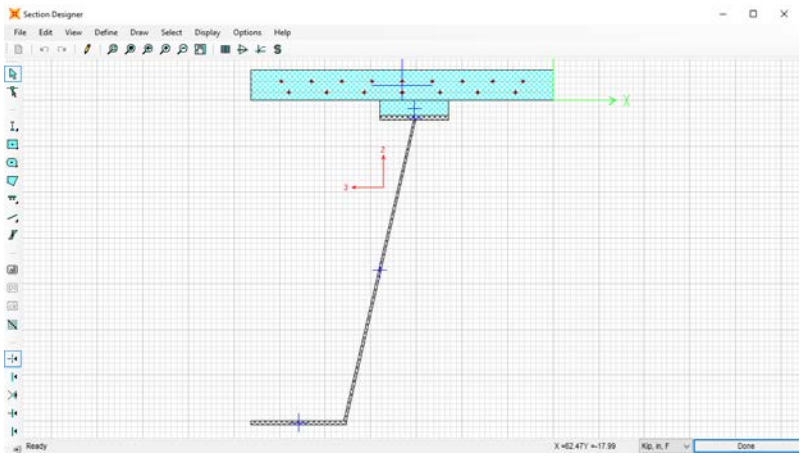


G. To generate an exterior longitudinal member and an interior longitudinal member, make two copies of the B2Long section. Label one LongOut and the other LongInt.

- For the LongOut, delete every element right of the CL.

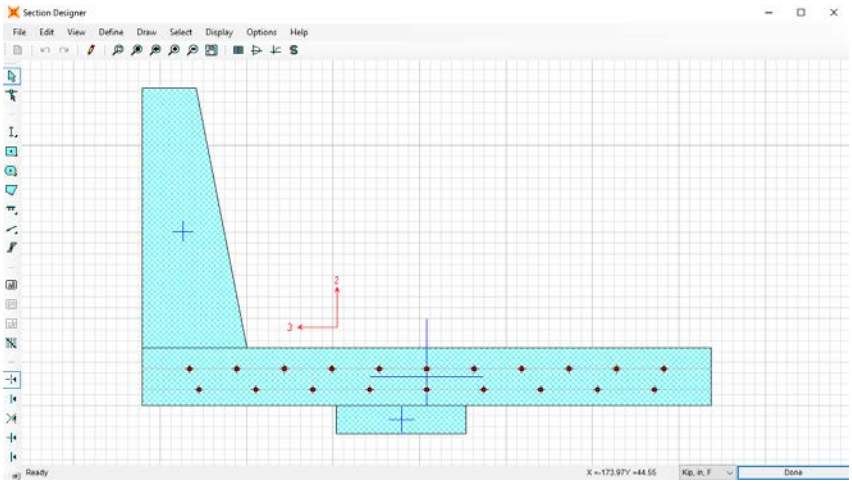


- For the LongInt, delete every element left of the CL.



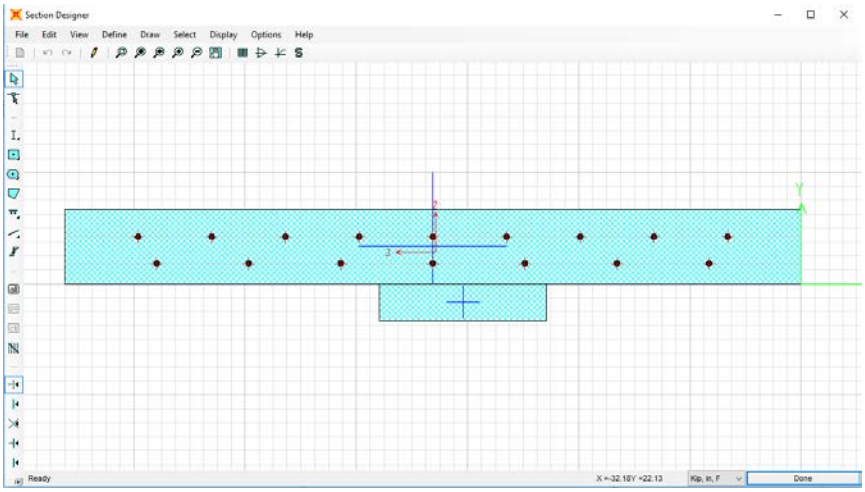
H. To generate a simulated fracture section, make copies of LongOut and LongInt.
   - Name the LongOut copy FracOut.

     Delete the bottom flange, web, and top flange of the steel tub.



   - Name the LongInt copy FracInt.

     Delete the bottom flange, web, and top flange of the steel tub.



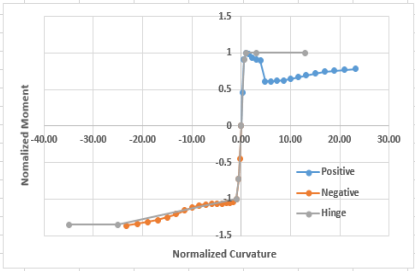
8. Generate plastic hinges for frame elements in SAP2000.
   A. Define > Section Properties > Frame Sections.
   B. Select the desired cross-section. Hinges will need to be made for the LongOut, LongInt, FracOut, FracInt, Trans, and TransEnd.
   C. Once selected, click Modify/Show Property > Section Designer.
   D. Once in the section designer, select the Moment Curvature Curve tool.
   E. In the Moment Curvature Curve window, select Details.
      - Copy the Moment Curvature data to an Excel file.
      - Select OK.
   F. In the Moment Curvature Curve window, change the angle (deg) to 180, then select Details.

- Copy the Moment Curvature data to the same Excel file as was done previously.
- Select OK.

G. Generate a Normalized Moment Curvature diagram.

- Normalize the moments by dividing each of the positive moments by the maximum positive moment and the negative moment by the maximum negative moment. Next, divide the curvatures by the curvatures corresponding to the maximum and negative moments.
- Plot the normalized positive and negative moment curvatures on a chart.
- Create a hinge moment curvature plot on the same chart with four positive moment points and four negative moment points without generating a negative slope.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
1																							
2	Positive												Normalized										
3	Concrete	Neutral A	Steel Stra	Tendon St	Concrete	Steel Com	Steel Ten	Prestress	Net Force	Curvature	Moment		M	C									
4	0	0	0	0	0	0	0	0	0	0	0		0	0.00									
5	-7.54E-04	-0.28	9.53E-04	0	-1185	-98.3936	1283.824	0	0.0125	1.32E-05	113885		0.454091	0.22									
6	-1.84E-03	1.106	2.43E-03	0	-2473	-213.797	2685.704	0	-0.7189	3.31E-05	228216		0.909959	0.56									
7	-3.06E-03	5.3861	4.63E-03	0	-2986	-205.926	3191.444	0	-4.14E-02	5.96E-05	250798		1	1.00									
8	-4.63E-03	6.6997	7.32E-03	0	-3161	-234.918	3396.077	0	-0.1158	9.26E-05	245937		0.980618	1.56									
9	-6.64E-03	6.5511	0.0104	0	-3129	-349.403	3477.927	0	-0.0659	0.000132	234212		0.933867	2.22									
10	-9.02E-03	6.2104	0.014	0	-3004	-514.419	3518.052	0	0.0355	0.000179	228063		0.909349	3.00									
11	-1.19E-02	5.425	0.018	0	-2812	-708.809	3520.728	0	-0.1453	0.000232	223859		0.892587	3.89									
12	-0.0256	-31.1492	0.012	0	0	-1883	1882.917	0	-0.4252	0.000291	153264		0.611105	4.89									
13	-0.0313	-30.9535	0.0148	0	0	-1890	1890.471	0	0.3358	0.000357	153956		0.613865	6.00									
14	-0.0376	-30.7525	0.0179	0	0	-1897	1897.551	0	0.3651	0.00043	154625		0.616532	7.22									
15	-0.0446	-30.8267	0.0211	0	0	-1918	1916.933	0	-0.6171	0.00051	156301		0.623215	8.55									
16	-0.0522	-30.883	0.0247	0	0	-1978	1977.914	0	0.0149	0.000596	161297		0.643135	10.00									
17	-0.0603	-30.9435	0.0285	0	0	-2048	2048.255	0	0.0298	0.000688	167050		0.666074	11.55									
18	-0.0691	-31.0003	0.0325	0	0	-2127	2126.823	0	0.0228	0.000788	173468		0.691664	13.22									
19	-0.0785	-31.1408	0.0368	0	0	-2208	2208.25	0	0.0249	0.000893	180046		0.717892	15.00									
20	-0.0883	-31.1394	0.0414	0	0	-2282	2282.484	0	0.7215	0.001006	186020		0.741712	16.89		M	C						
21	-0.0988	-31.1509	0.0463	0	0	-2327	2326.757	0	0.0915	0.001125	189616		0.756051	18.89		-1.35	-35						
22	-0.1099	-31.156	0.0515	0	0	-2371	2370.534	0	0.0316	0.001251	193172		0.770229	21.00		-1.35	-25						
23	-0.1215	-31.1605	0.0569	0	0	-2414	2414.281	0	0.0211	0.001383	196731		0.78442	23.22		-1	-1						
24																-0.72	-0.56						
25																0	0						
26	Negative															0.91	0.56						
27	Concrete	Neutral A	Steel Stra	Tendon St	Concrete	Steel Com	Steel Ten	Prestress	Net Force	Curvature	Moment					1	1						
28	0	0	0	0	0	0	0	0	0	0	0		0	0.00		1	3						
29	-6.83E-04	20.6702	5.09E-04	0	-413.456	-720.977	307.0412	0	-827.392	1.32E-05	64645		-0.44866	-0.22		1	13						



H. Define Hinge Length.

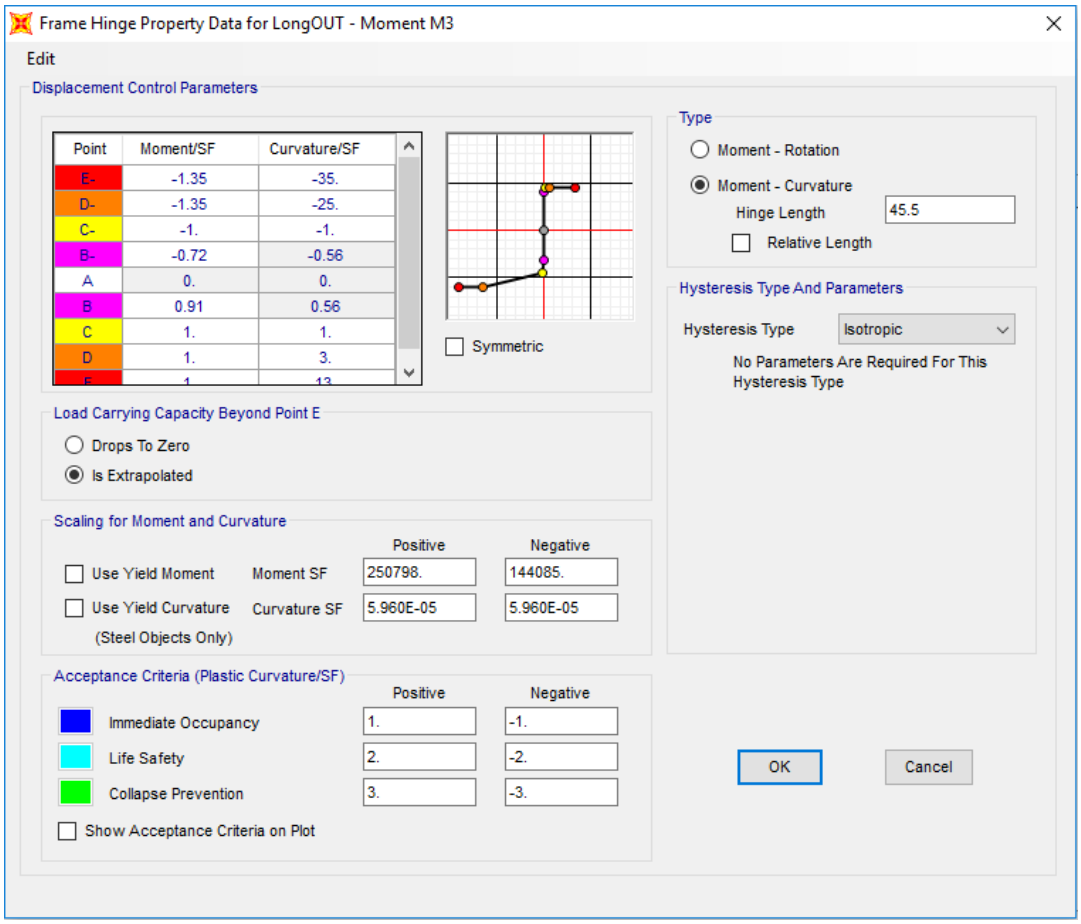
The hinge length is one half of the section depth.

- $Hinge_{long} = 0.5 * (Deck\ thickness\ +\ haunch\ height\ +\ top\ flange\ thickness +\ web\ height +\ bottom\ flange\ thickness)$
  a. $Hinge_{long} = 45.5\ in.$
- $Hinge_{Frac} = 0.5 * (Deck\ thickness + haunch\ height)$
  a. $Hinge_{Frac} = 6\ in.$
- $Hinge_{Trans} = 0.5 * (Deck\ thickness)$
  a. $Hinge_{Trans} = 4\ in.$

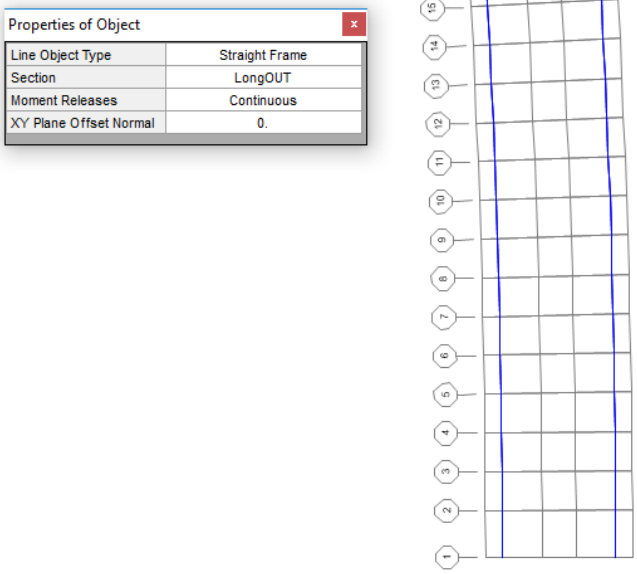
I. Make the plastic hinge in SAP2000.

- Select Define > Section Properties > Hinge Properties > Add New Properties.
- In the Type window, select Moment Curvature and input the corresponding correct hinge length.

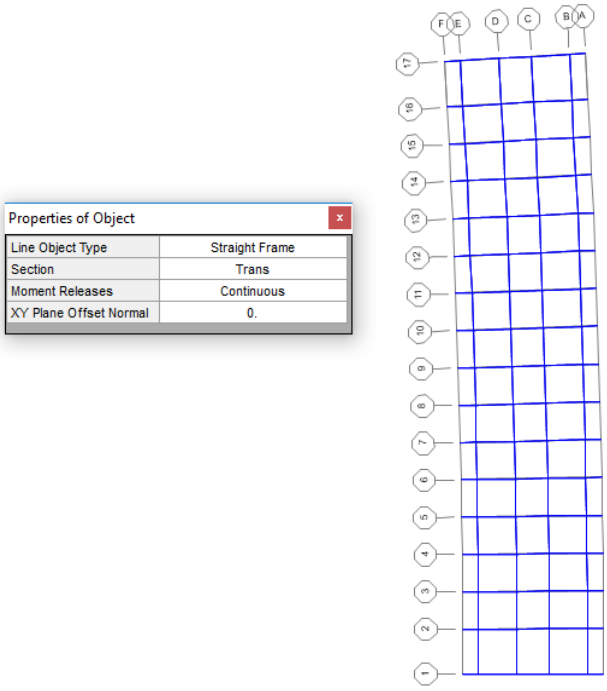
- In the Moment Curvature table, insert the four positive and four negative normalized moment curvatures and the zero point.
- Uncheck the symmetric box and select the Is Extrapolated option in the Load Carrying Capacity beyond Point E window.
- In the Scaling for Moment and Curvature window, insert the maximum positive moment and corresponding curvature as well as the maximum negative curvature and corresponding curvature.
- In the Acceptance Criteria, use the values 1, 2, and 3 for Immediate Occupancy, Life Safety, and Collapse Prevention in the positive column and −1, −2, and −3 for the negative column.
- Repeat for all remaining frame sections (LongOut, LongInt, FracOut, FracInt, Trans, and TransOut).



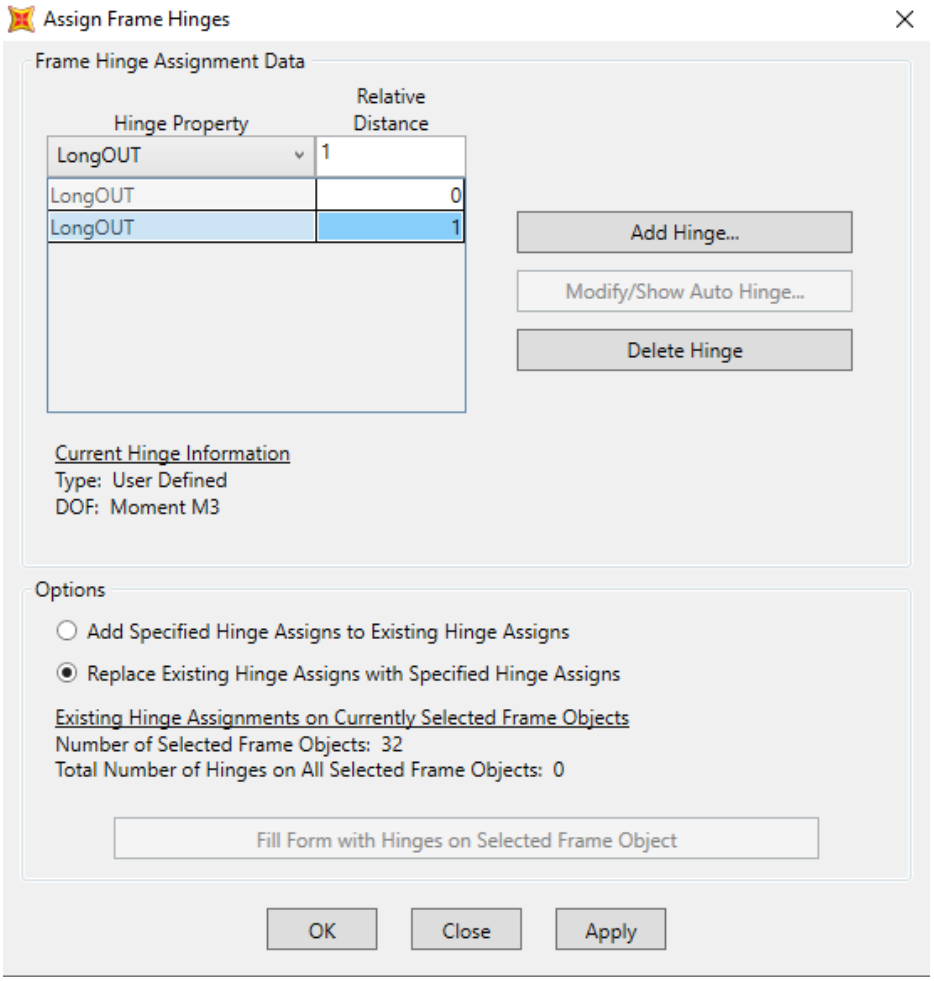
9. Assign frame members to grid.
    A. Select the Draw tab > Quick Draw Frame/Cable/Tendon.
    B. In the Section drop-down menu, select LongOut.
    C. Click on every grid segment on second-to-last longitudinal grids (B and E).



D. Change the section to LongInt and repeat Step C but for the two interior longitudinal grids (C and D).
E. Change the Section to TransEnd and repeat Step C but for the end transverse grids (1 and 17).
F. Change the Section to Trans and repeat Step C but for all other transverse grids (2 to 16).

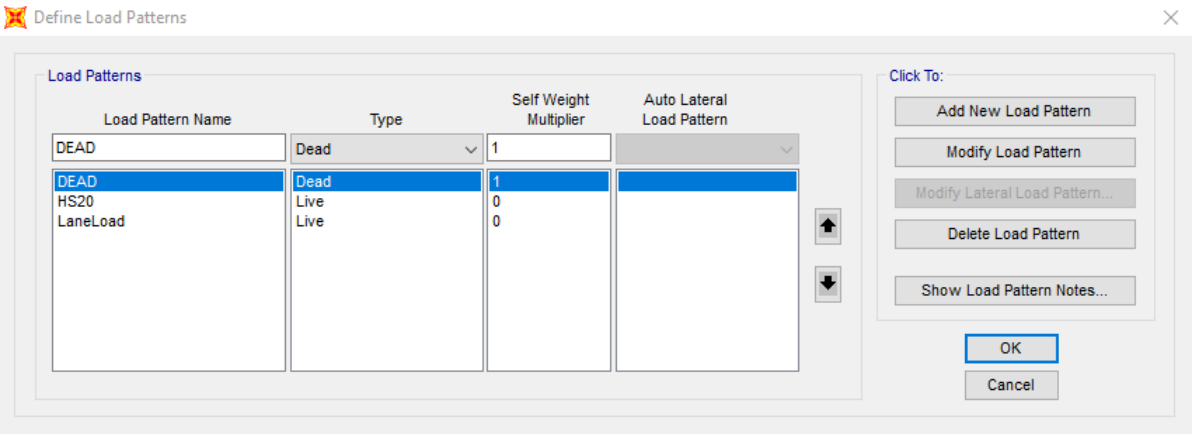


10. Assign hinges to frame elements.
    A. The longitudinal hinges are placed at the ends of the longitudinal frame elements or at a relative distance of 0 and 1.
    B. The transverse hinges are placed at a distance of half a top flange width away from the node.
        - $Hinge\ Loc._{F\ to\ E} = \frac{half\ flange\ width}{Element\ Width} = 1 - \frac{\frac{18}{2}}{36} = 0.75.$
        - $Hinge\ Loc._{E\ to\ D\ and\ C\ to\ B} = \frac{half\ flange\ width}{Element\ Width} = \frac{18/2}{86} = 0.1046\ and\ (1 - 0.1046)\ or\ 0.8954.$
        - $Hinge\ Loc._{D\ to\ C} = \frac{half\ flange\ width}{Element\ Width} = \frac{18/2}{73} = 0.1233\ and\ (1 - 0.1046)\ or\ 0.8767.$
        - $Hinge\ Loc._{B\ to\ A} = \frac{half\ flange\ width}{Element\ Width} = \frac{18/2}{36} = 0.25.$
    C. In SAP2000, assign the hinges to corresponding frame elements.
        - Select the desired frame elements you wish to assign hinges to, such as LongOut. (The elements will turn from blue to yellow).
        - In SAP2000, select the Assign tab > Frame > Hinges.
            - From the drop-down menu, select LongOut and set relative distance to 0 and click Add.
            - From the drop-down menu, select LongOut and set relative distance to 1 and click Add.
            - Click OK.

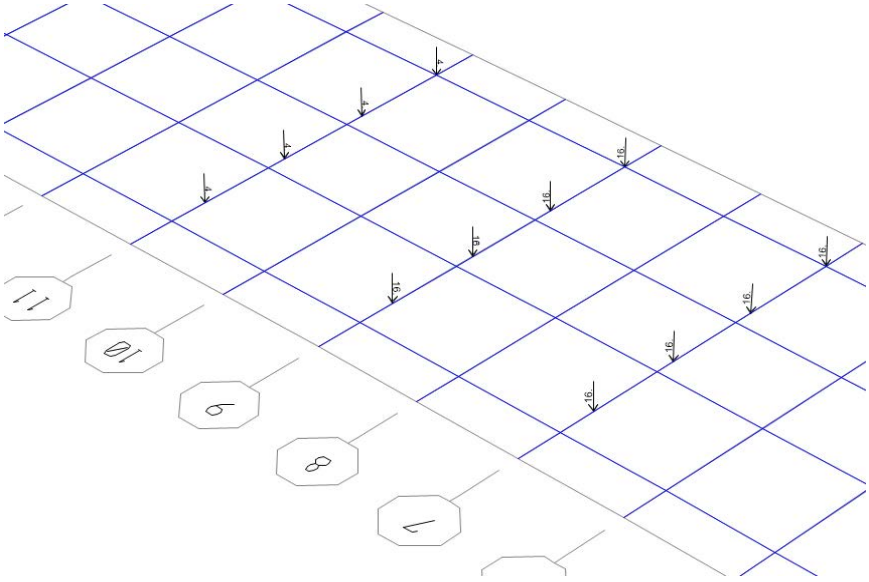


- Repeat the previous step of assigning the hinges for all other frame elements.

11. Assign loads to the frame elements.
    A. Wheel Axle Loads
        - Axle loads will be placed at distances of 36 in., 108 in., 180 in., and 252 in. from the outside of the curved edge.
        - One line of the 16 kip axles will be placed at the halfway point of the bridge, or Transverse Grid 9, with the second line 14 ft away at Transverse Grid 7. One line of 4 kip axles will be placed 14 ft away from the first line of axles, at Gridline 11.
        - In SAP2000, the loads have to be placed at a relative distance, so this value needs to be calculated.
            - $HS20_{Axel\ 1\ Loc\ (B-A)} = \frac{L_1-36}{L_1} = \frac{36-36}{36} = 0.$
            - $HS20_{Axel\ 2\ Loc\ (C-B)} = \frac{L_1+L_2-108}{L_2} = \frac{36+86-108}{86} = 0.1628.$
            - $HS20_{Axel\ 3\ Loc\ (D-C)} = \frac{L_1+L_2+L_3-180}{L_3} = \frac{36+86+73-180}{73} = 0.2055.$
            - $HS20_{Axel\ 4\ Loc\ (E-D)} = \frac{L_1+L_2+L_3+L_4-252}{L_4} = \frac{36+86+73+86-252}{86} = 0.3372.$
    B. Lane Loads
        - Lane Loads are lane loads of 0.640 kip/ft (0.05333 kip/in.) centered at a distance of 96 in. and 240 in. from outside of the curved edge.
        - These lane loads will be placed on the longitudinal frame elements. They will be assigned to elements along the B, C, D, and E longitudinal elements according to the appropriate tributary distance.
            - $LaneLoad_B = \left(\frac{L_1+L_2-96}{L_2}\right) * laneload = \left(\frac{36+86-96}{86}\right) * 0.05333 = 0.016124\frac{kip}{in.}$.
            - $LaneLoad_C = laneload - laneload_B = 0.05333 - 0.016124 = 0.037209\frac{kip}{in.}$.
            - $LaneLoad_D = \left(\frac{L_1+L_2+L_3+L_4-240}{L_4}\right) * laneload = \left(\frac{36+86+73+86-240}{86}\right) * 0.05333 = 0.025426\frac{kip}{in.}$.
            - $LaneLoad_E = laneload - laneload_D = 0.05333 - 0.025426 = 0.027907\frac{kip}{in.}$.
    C. In SAP2000, first define the load patterns.
        - Select the Define tab > Load Patterns.
            - Under the Load Pattern Name, enter HS-20 and change the type in the drop-down menu to Live. The self-weight multiplier should be set to 0. Then click Add New Load Pattern.
            - Under the Load Pattern Name, enter LaneLoad and change the type in the drop-down menu to Live. The self-weight multiplier should be set to 0. Then click Add New Load Pattern.
            - Click OK.

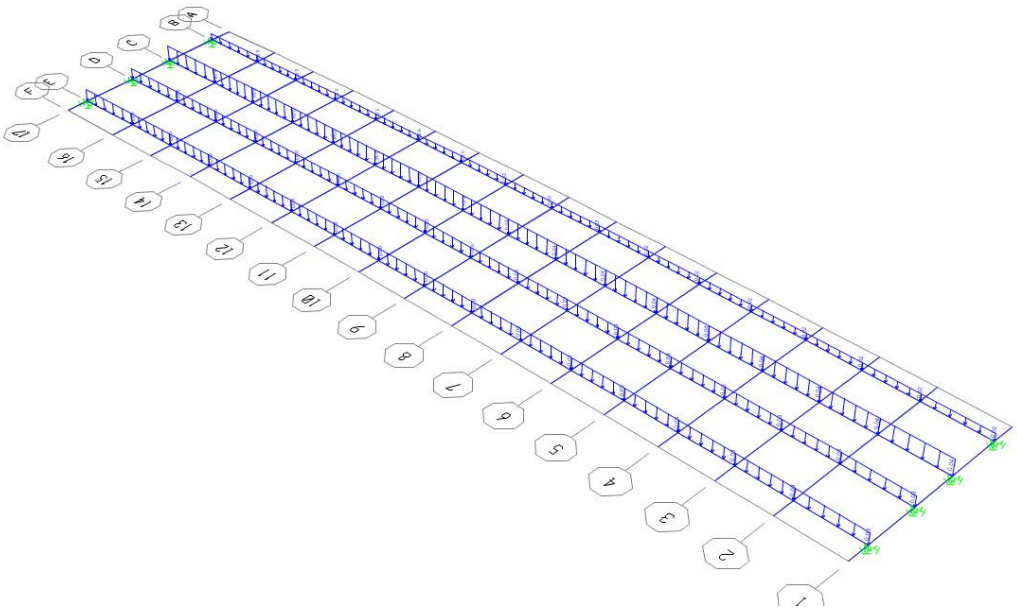


- Assign the wheel loads in SAP2000.
    - Select the exterior transverse element of Gridline 9 and 7.
    - Click Assign > Frame Loads > Point.
    - From the Load Pattern drop-down menu, select HS-20 and verify that the coordinate system is set to Global, the Load Direction is Gravity, and the Load Type is Force.
    - In Column 1, enter a relative distance of 0 (Axle 1, Loc. B-A) and load of 16 kips.
    - Click OK.
    - Repeat for Gridline 11 to assign the 4 kip load.
    - Repeat Steps 1–6 for Axle 2 ,3, 4, Loc. C-B, D-C, and E-D.



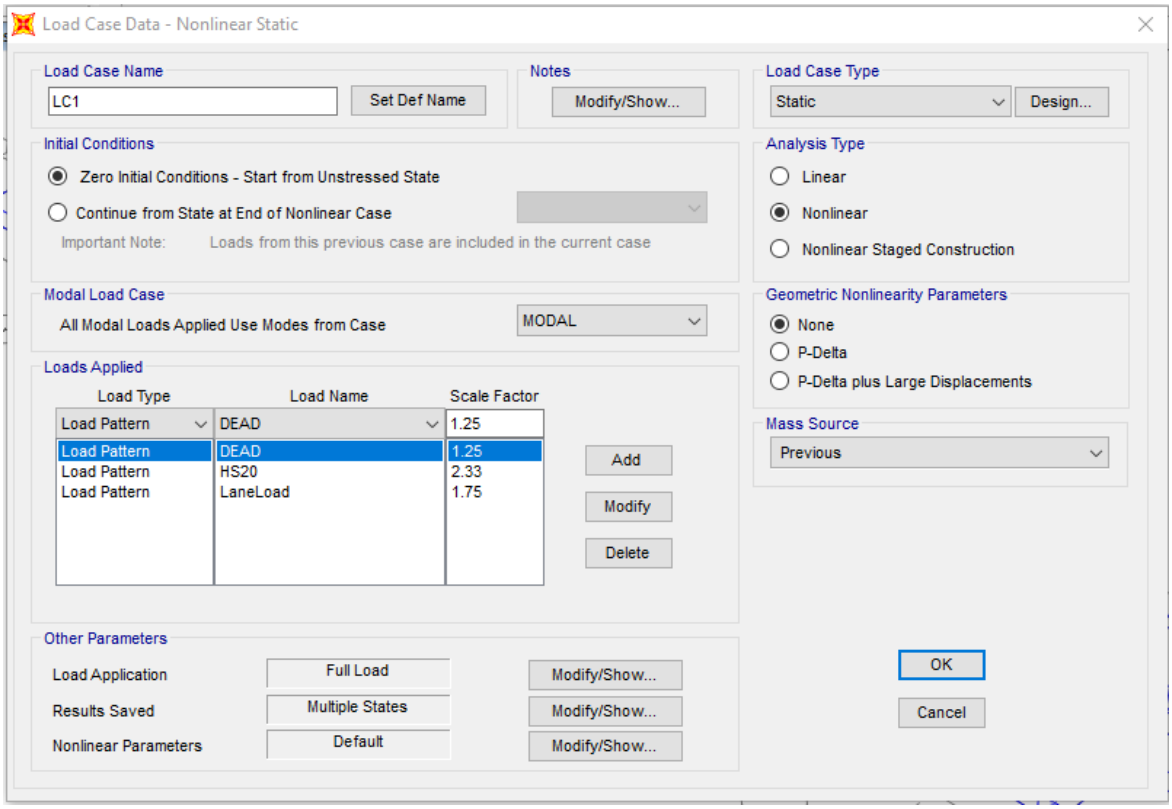
- Assign the Lane Load in SAP2000.
    - Select all exterior longitudinal frame elements along Gridline B.
    - Click Assign > Frame Loads > Distributed.

- From the Load Pattern drop-down menu, select Lane Load and verify that the coordinate system is set to Global, the Load Direction is Gravity, and the Load Type is Force.
- In the Uniform Load box, enter 0.016124 (Lane Load B).
- Click OK.
- Repeat Steps 1–5 for all of the longitudinal elements along gridlines C, D, and E.

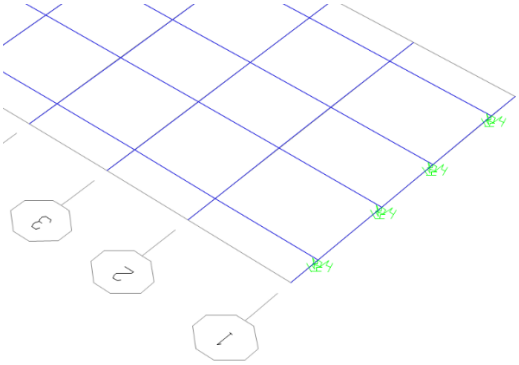


12. Define load cases.
    A. The load case being used to determine redundancy is $1.25DL + 1.75(LL + IM)$. Where DL = Dead Load, LL = Live Load, and IM = Impact Load. When substituting in the truck load and the lane load, the preceding equation reduces to $1.25DL + 1.75LaneLoad + 2.33HS20$.
    B. Generate load cases in SAP2000.
        - Click Define > Load Cases > Add New Load Case.
        - In the Load Case Name Panel, name the load case "LC1."
        - In the Analysis Type, select "Nonlinear."
        - For the LC1 Load Case in the Stiffness to Use panel, select "Zero Initial Conditions."
        - In the Loads Applied panel, leave the Load Type "Load Pattern" in the drop-down Load Name menu, select DEAD and change the Scale Factor to 1.25. Click Add. Change the Load Name menu, select HS-20, and change the Scale Factor to 2.33. Click ADD. Change the Load Name menu, select Lane Load, and change the Scale Factor to 1.75. Click ADD.
        - In the Other Parameters panel, in the Results Saved section, click Modify/Show.

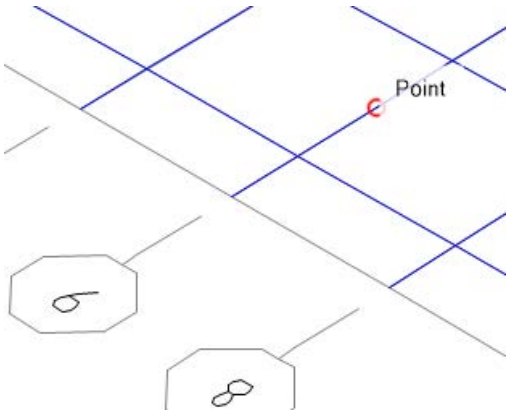
- In the Results Saved for Nonlinear Static Load Cases window, change the Results Saved to Multiple States, and in the For Each Stage panel, change the Minimum Number of Saved Steps and the Maximum Number of Saved Steps to 20. Click OK.
- Click the OK on the Load Case Data window.
- Repeat the previous 8 steps mentioned in B to create an LC2, LC3, and LC4. However, in the Initial Conditions window, select Continue from State at End of Nonlinear Case, and from the drop-down menu, select the preceding load case. (For LC2, the Nonlinear Case LC1 would be selected).



13. Define end supports.
    A. The elastomeric bearing pads for each girder have a lateral stiffness of 12 kip/in. and a vertical stiffness of 6100 kip/in. Since the tub girders are divided in half, the lateral stiffness will be 6 kip/in., and the vertical stiffness will be 3050 kip/in.
    B. Assign spring supports in SAP2000.
        - Select the eight nodes at the very end of the longitudinal members.
        - Click Assign > Joint- >Springs.
        - In the Assign Joint Springs window in the Simple Springs Stiffness panel, enter 6 for Translation 1 and 2 and 3050 for Translation 3.
        - Click OK.



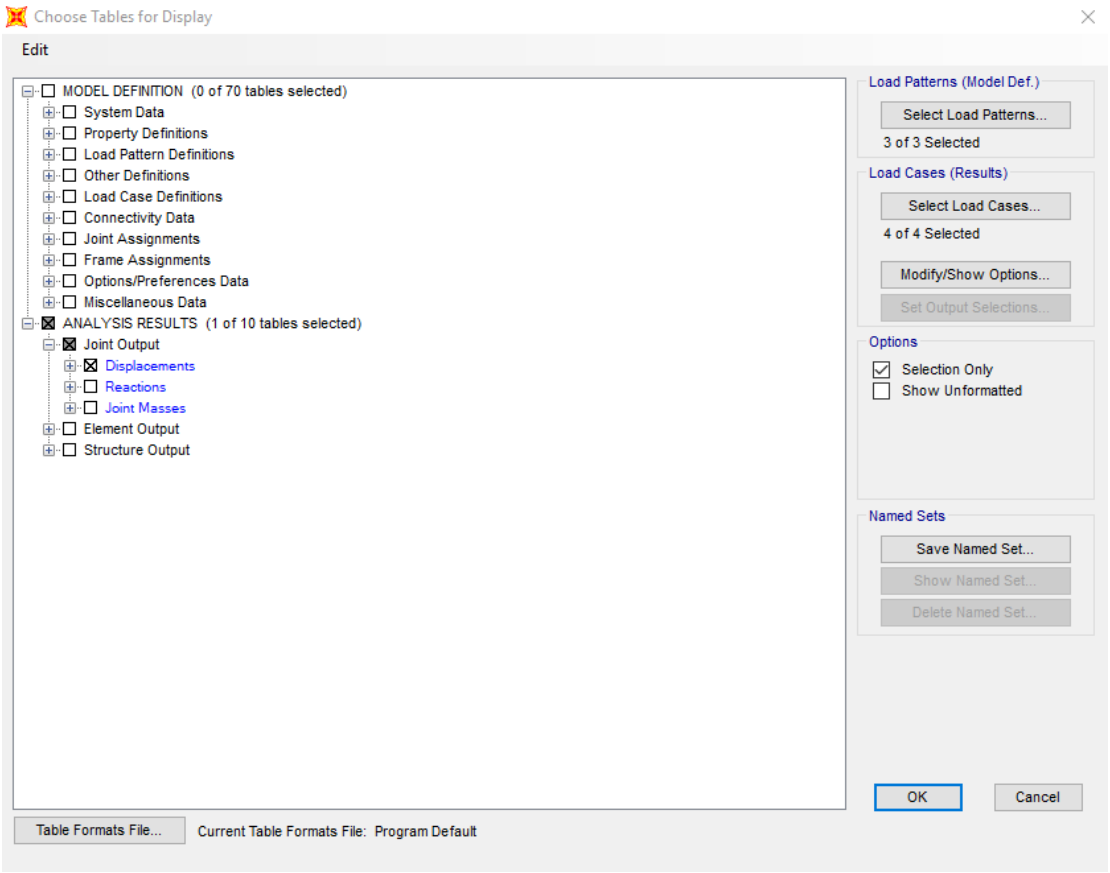
14. Define CL data acquisition points mid-span of girder.
    A. Select the transverse frame elements between B and C and between D and E at mid-span (Gridline 9).
    B. Click Edit > Edit Lines > Divide Frames.
    C. In the Divide into Specified Number of Frames window, enter 2 for Number of Frames.
    D. Click OK.



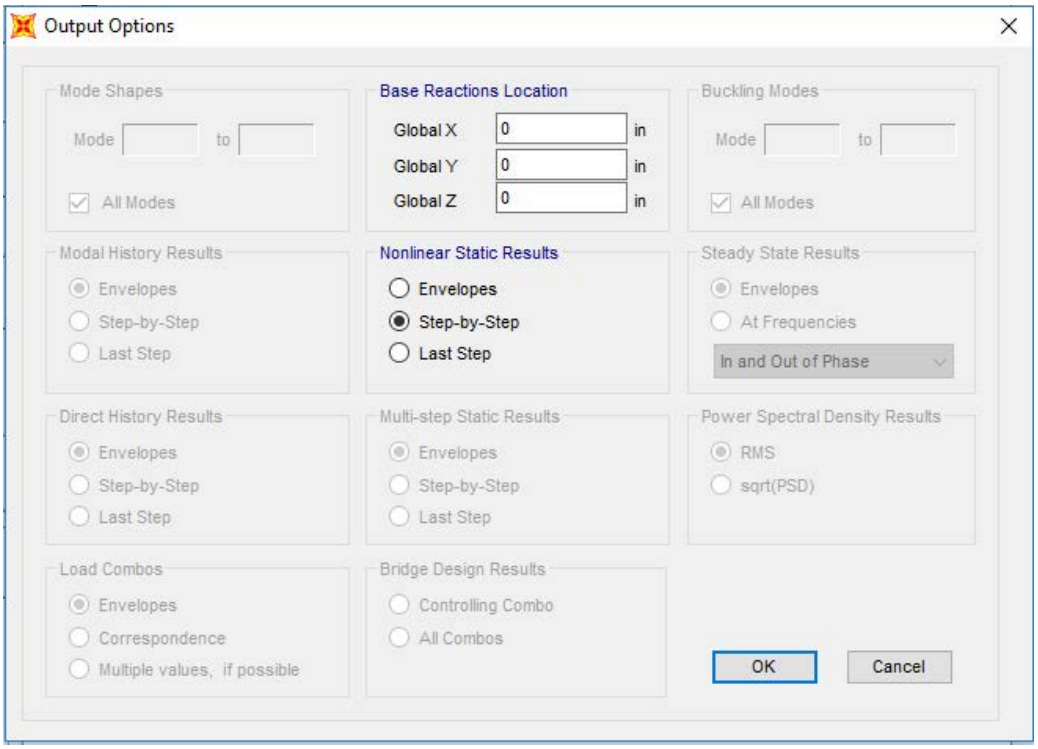
15. Analyze the nonfracture structure for dead load only.
    A. In SAP2000, click Analyze > Run Analysis.
    B. In the Set Load Cases to Run window, click the Run/Do Not Run All button until every Action is Do Not Run.
    C. Select DEAD then click Run/Do Not Run Case until the action is run.
    D. Click Run Now. Let SAP2000 run the load cases until the screen says the analysis is complete.
    E. Once the analysis is complete, select Spring Reactions.
    F. Click Display > Show Tables
    G. In the Choose Table for Display window, click the + symbol beside Joint Output and select the square box beside Reactions.
    H. In the Output Options window in the Nonlinear Static Results panel, select Last Step.
    I. Select and copy the information from the F3 column.

J. The sum of the F3 values is the dead load.
K. Click Done.
L. Unlock the structure.

16. Analyze the nonfractured structure.
    A. In SAP2000, click Analyze > Run Analysis
    B. In the Set Load Cases to Run window, click the Run/Do Not Run All button until every action is Do Not Run.
    C. Select LC1, LC2, LC3, and LC4 and then click Run/Do Not Run Case until the action for all four is run.
    D. Then click Run Now.
    E. Let SAP2000 run the load cases until the screen says the analysis is complete.
    F. Once the analysis is complete, select the data collection point on the transverse member on the outside girder (C-B).
    G. Click Display > Show Tables.
    H. In the Choose Table for Display window, click the + symbol beside Joint Output and select the square box beside Displacements.

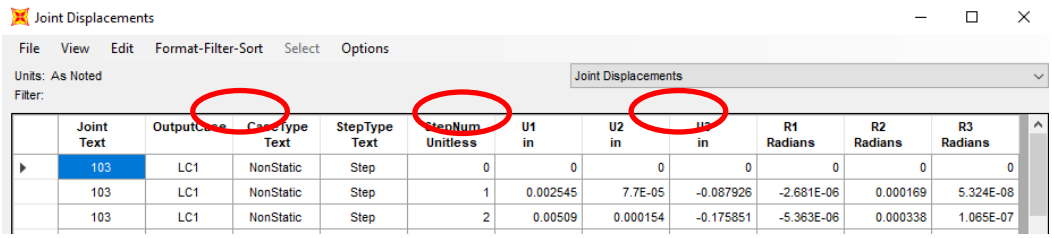


I. Click the Modify/Show Options button.
J. In the Output Options window in the Nonlinear Static Results panel, select Step-by-Step.



K. Click OK.

L. Select and copy the information from the Output Case, StepNum Unitless, and the U3 in. column and paste them into an Excel worksheet. These columns represent Load Case, Step Number, and Deflection for the Outside Girder, respectively.



	Joint Text	OutputCase Text	CaseType Text	StepType Text	StepNum Unitless	U1 in	U2 in	U3 in	R1 Radians	R2 Radians	R3 Radians
▶	103	LC1	NonStatic	Step	0	0	0	0	0	0	0
	103	LC1	NonStatic	Step	1	0.002545	7.7E-05	-0.087926	-2.681E-06	0.000169	5.324E-08
	103	LC1	NonStatic	Step	2	0.00509	0.000154	-0.175851	-5.363E-06	0.000338	1.065E-07

M. Click Done.

N. Select the data collection point on the transverse member on the inside girder (E-D) and repeat Steps G–L. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

O. Select the joint on the transverse member on the outside girder at Transverse Element 9 (at Longitudinal Element B) and repeat Steps G–L. This information goes into the Delta 4 column. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

P. Select the joint on the transverse member on the outside girder at Transverse Element 9 (at Longitudinal Element C) and repeat Steps G–L. This information goes into the Delta 3 column. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

Q. Select the joint on the transverse member on the inside girder at Transverse Element 9 (at Longitudinal Element D) and repeat Steps G–L. This information goes into the Delta 2 column. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

R. Select the joint on the transverse member on the outside girder at Transverse Element 9 (at Longitudinal Element E) and repeat Steps G–L. This information goes into the Delta 1 column. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

	A	B	C	D	E	F	G	H	I	J	K	L	M	V	X	Y	Z	AA	AB	AC	AD
1	Intact			OG-CL		IG-CL		Point 4	Point 3	Point 2	Point 1			Applied		OG		IG			
2	Load Case	Load Step	Ω	Delta (in)	Chord (rad)	Delta (in)	Chord (rad)	Delta (in)	Delta (in)	Delta (in)	Delta (in)	α 23 (rad)	α 32 (rad)	Load (kip)	Ω (cal)	Delta (in)	Chord (deg)	Delta (in)	Chord (deg)	α 23 (deg)	α 32 (deg)
3	LC1	0	0	0	0.00000	0	0.00000	0	0	0	0	0	0	0	0.00	0.00	0.00000	0.00	0.00000	0	0
4	LC1	1	0.05	-0.1037	0.00015	-0.087926	0.00013	-0.10541	-0.10129	-0.0942	-0.07905	7.89E-05	4.94E-05	66	0.05	0.10	0.00861	0.09	0.00730	0.004522	0.002831
5	LC1	2	0.1	-0.2074	0.00030	-0.175851	0.00025	-0.21081	-0.20259	-0.18839	-0.15809	0.000158	9.88E-05	132	0.10	0.21	0.01722	0.18	0.01460	0.009043	0.005662

S. Once all the data are collected, unlock the model by selecting the Lock tool on the left hand side of the SAP2000 screen.

17. Analyze the fractured structure.

A. At mid-span along Gridline 9, replace the hinges of the outside longitudinal element (Gridline B) with FracOUT hinges according to Step 10.

B. At mid-span along Gridline 9, replace the hinges of the first interior longitudinal element (Gridline C) with FracInt hinges according to Step 10.

C. Repeat Step 15 for the Fractured Case and collect the data accordingly.

18. Post-process the data.

A. In the Excel Sheet, the following values need to be calculated for each step.

- Omega (Ω).

$$\Omega_i = \Omega_{i-1} + \left(\frac{1}{\#\ of\ Steps\ in\ Load\ case}\right).$$

- Longitudinal Chord Rotation of Interior and Exterior Girder.

- $Chord\ Rot._{Single\ Span} = -1 * \left(\frac{\delta_{CL}}{0.5*L}\right)$ (rad).

- Transverse Deck Rotation.
  - $\alpha_{2-3} = \left(\frac{\delta_3 - \delta_2}{s}\right) - \left(\frac{\delta_2 - \delta_1}{w}\right)$ (rad).
  - $\alpha_{2-3} = \left(\frac{\delta_3 - \delta_2}{s}\right) - \left(\frac{\delta_2 - \delta_1}{w}\right)$ (rad).
  - Where s = spacing between the interior top flanges of the inside and outside girders and w = spacing between the top flanges of the same girder.
- Applied Load.
  - Calculate unit applied load or applied load at 1 Ω.
  - $Unit\ Applied\ Load_{single\ span} = 1.25 * Total\ Reactions\ from\ Dead\ Load\ Case + 2 * (2.33 * HS20\ truck + 1.75 * Lane\ Load)$.
  - $Applied\ Load = Unit\ Applied\ Load * \Omega$

B. Repeat Step A for the fractured case.

C. Calculate the initial stiffness for the intact bridge and instantaneous stiffness for fractured bridge.
   - For the nonfractured condition (intact bridge), find the absolute displacement for the outside girder at an Ω value of 0.4:

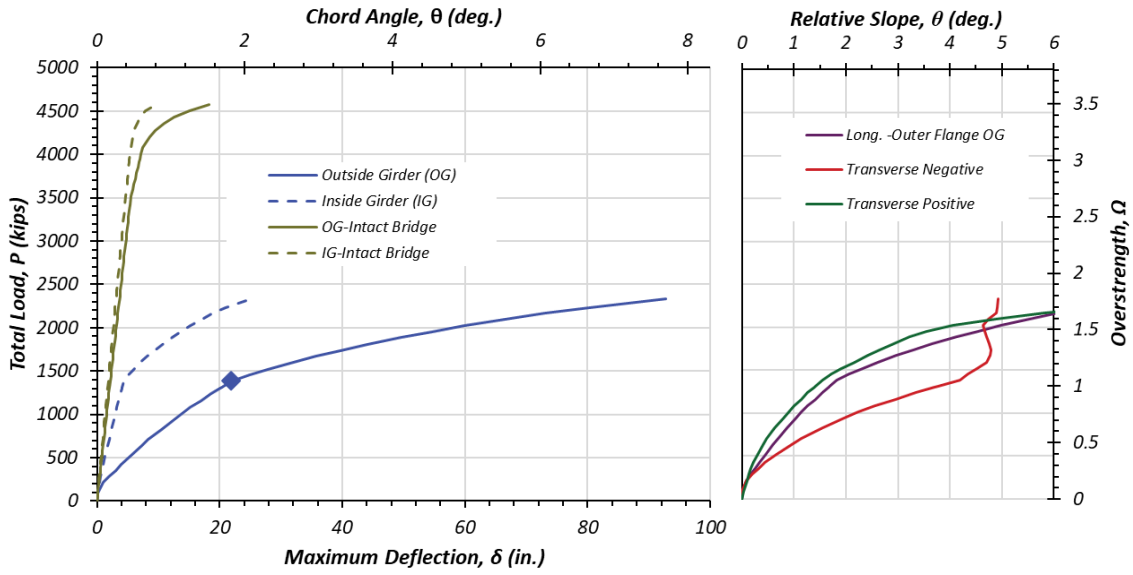
$$Initial\ Stiffness = \frac{0.4}{Absolute\ Displacement\ OG\ (at\ \ \Omega = 0.4)}.$$

   - For the Fractured case, add an additional column labeled stiffness:

$$Instantaneous\ Stiffness_{OG-Frac.i} = \frac{\Omega_i - \Omega_{i-1}}{\delta_i - \delta_{i-1}}.$$

D. Failure of the structure occurs at the Ω of the fractured bridge at the first of the following criteria:
   - The instantaneous stiffness for the fractured outside girder is less than 5 percent of the initial stiffness of the intact outside girder.
   - The chord angle of the outside girder for a simple span or interior spans is greater than 2 degrees. The chord angle for exterior spans of multi-span bridges is greater than 3 degrees.
   - The transverse deck rotation is greater than 5 degrees.

E. On a chart, plot the nonfractured outside and inside girder as well as the fractured outside and inside girder with displacement on the primary *x* axis and the total force on the primary *y* axis and Ω on the secondary *y* axis.

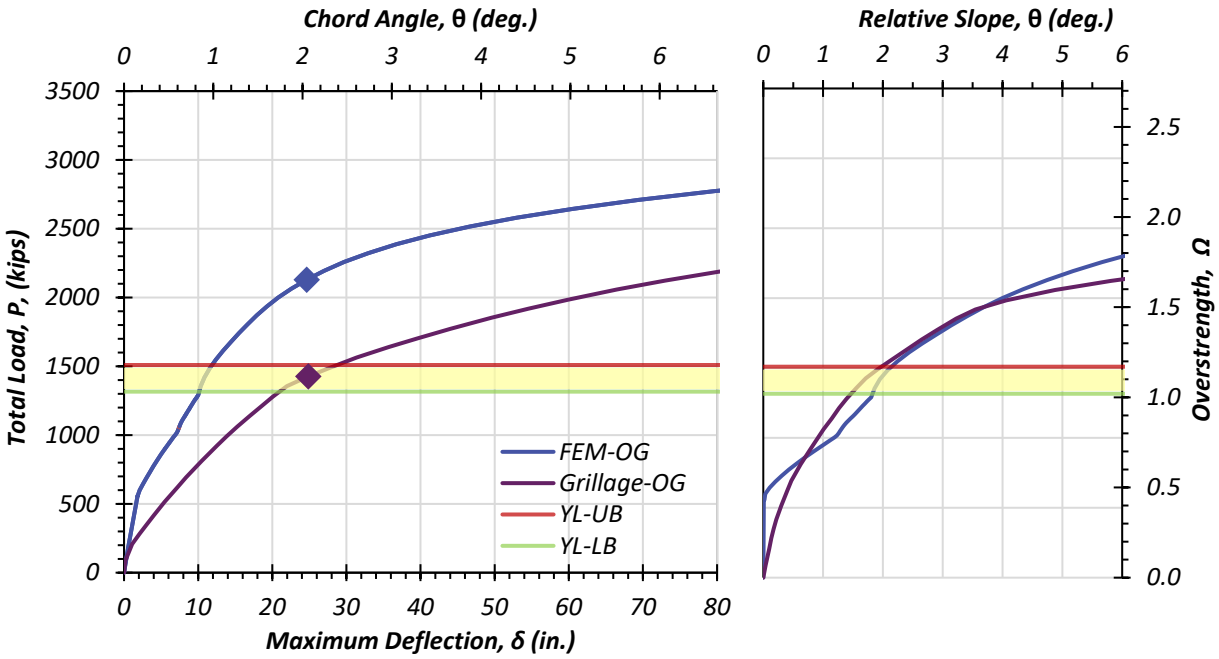


5.3 OVERALL RESULTS FOR BRIDGE 2

Table 5.1 and Figure 5.2 presents the overall results for Bridge 2. In addition to plotting the foregoing upper- and lower-bound yield line solutions and the grillage overstrength (load) versus deflection results, for the sake of completeness, FEM results have also been included in Table 5.1 and in Figure 5.2. Clearly, all methods indicate that Bridge 2 could be reclassified as nonfracture critical. It should be noted that while yield line results provide the collapse load capacity, the analysis is silent on the deformation limitations. Such results, however, are provided by both grillage and FEM solutions.

Table 5.1. Results Summary of Bridge 2.

Method of Analysis	Overstrength Factors	Single Span (115 ft)
Yield Line Theories	$\Omega_{Yield\ Line}^{Upper\ Bound}$	1.17
	$\Omega_{Yield\ Line}^{Lower\ Bound}$	1.02
Grillage Analysis	$\Omega_{Grillage}$	1.11
FEM	Ω_{FEM}	1.65



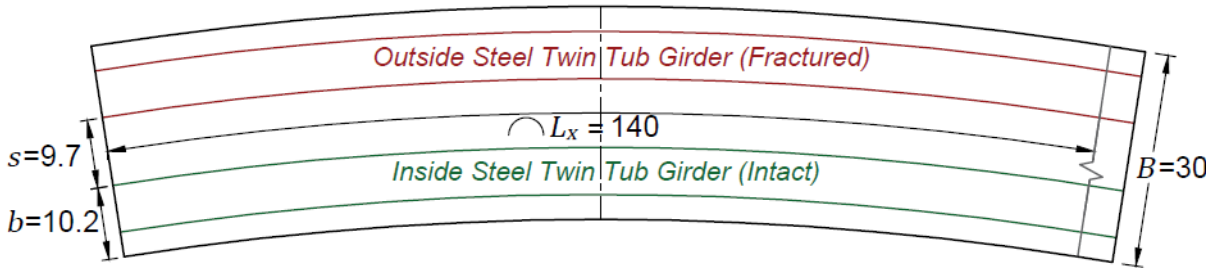
(i) Load displacement *(ii) Deck rotations*

Figure 5.2. Comparison of the Results for Bridge 2, L_x = 115 ft.

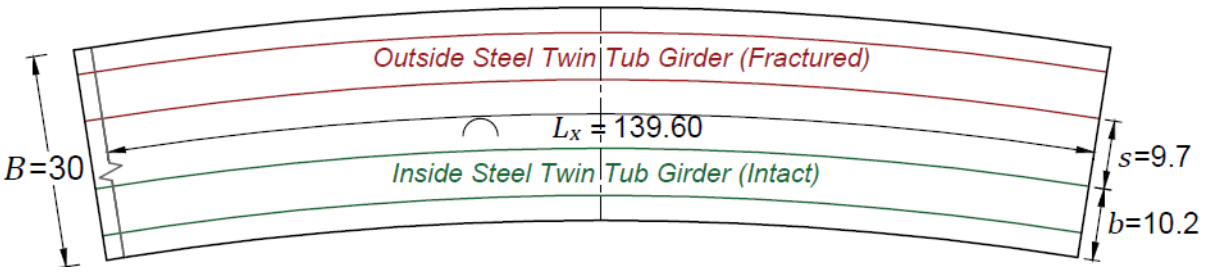
6. BRIDGE 5

6.1 YIELD LINE ANALYSIS EXAMPLE OF BRIDGE 5

This section presents the steps to calculate the reserve capacity (overstrength factor) of Bridge 5 using the yield line analysis (grillage analysis is explained in the next sub-section). Figure 6.1 presents the geometric details of the two exterior spans of Bridge 5, which are further illustrated in detail in Appendix B.



(a) Bridge 5, Span 1 (L_x = 140 ft)



(b) Bridge 5, Span 2 (L_x = 139.60 ft)

Figure 6.1. Schematic Diagrams of Bridge 5 (R_L = 450 ft).

This section presents the stepwise procedure of the yield line analysis conducted to establish the upper-bound and lower-bound solution range for the overstrength factor as per the procedure described in Section 3.

The moment capacities of the deck slab are computed in the same way as described in Section 5. The overstrength factors are also calculated using the same procedure as mentioned in Section 5. Bridge 5 is a two-span bridge which is why the spans have one continuous support. The moment capacities of the intact section at the continuous support are computed as the

positive and negative moment of the intact section using the similar procedure as that followed for calculating the moment capacities of the deck slab alone (explained in Section 5). The difference being the additional elements of the girder cross-section that contribute to the flexural strength. The intact cross-section of the bridge, including the girders, at the continuous supports is considered.

The geometric parameters namely $B =$ the total width (breadth) of the deck slab, $t =$ thickness and h = thickness of haunch are noted from the structural plans. The various material properties of concrete and steel are obtained from the bridge plans and the reports associated with the respective bridges. These properties include f_c' = the specified compressive strength of concrete, ε_{cu} = the maximum strain at the extreme concrete compression fiber (computed as per Section 22.2.2 of ACI-318 (2017) which states the "assumptions for concrete."), f_y = the yield strength of steel of the reinforcement bars, F_y' = the yield strength of the steel of STTG, E = the Young's modulus of steel of and ε_y and ε_s = the yield strain that is the ratio of yield strength and Young's modulus of steel, of the reinforcement and the SSTG, respectively.

There are additional number of bars provided for extra strength at the supports that are denoted by "bent." The details of the reinforcement such as $\#_{top\ bars} =$ the number of top bars, $\#_{top\ bars\ bent} =$ the number of top bars at bent, $\#_{bottom\ bars} =$ the number of bottom bars, $d_{b\ top\ tra}$ = the diameters of the transverse top bars, $d_{b\ bot\ tra}$ = the diameters of the transverse bottom bars, $d_{b\ top\ long}$ = the diameters of the longitudinal top bars, $d_{b\ top\ long}'$ = the diameters of the longitudinal top bars at bent, $d_{b\ bot\ long}$ = the diameters of the longitudinal bottom bars and the clear cover for each type of bar are recorded with suitable subscript to *cc.*

The STTG dimensions such as D_g = depth of girder (overall), b_{tf} = top flange width, t_{tf} = top flange thickness, b_w = web width, t_w = web thickness, b_{bf} = bottom flange width and t_{bf} = bottom flange thickness are also tabulated.

The composite area neutral axis is found by using n which denotes the ratio of yield strength of steel of reinforcement and that of the STTG to express the area computations in terms of an effective area. The positive moment capacity is found by the similar procedure for computing the compressive and tensile forces as explained in Section 5 whereas the negative moment capacity is found by using the equal area method of plastic analysis where the areas of

compression and tension zones are calculated. The neutral axis for the positive moment capacity is obtained by equating the compressive and the tensile forces while the plastic neutral axis for the negative moment capacity is calculated by equating the areas in compression and tension. The depth of compression zones from the top is denoted by c and is indicated by blue highlighting. y = the portion of the width of web in compression and y' = the portion of the width of web in tension. The depths of the compressive and tensile forces from the neutral axis are computed for the positive moment capacity while the depths of the center of gravity of compressive and the tensile areas from the plastic neutral axis are calculated for the negative moment capacity. The neutral axis and the plastic neutral axis may lie either in the web or in the top flanges and those sections are denoted by the subscripts 1 and 2 for the top-half and the bottom-half of the element of girder in which the neutral axes lie, respectively.

For the positive moment capacity, the net tensile strain in the extreme steel components of the section respectively, are determined from a linear distribution (ACI-318 2017). The compressive and tensile forces are computed in the similar method explained in Section 5. The compressive forces and areas are denoted by a negative sign while the tensile forces and areas are denoted by positive sign. The concrete is assumed to be completely cracked for the negative moment capacity and is not considered for the tensile area. The positive moment capacity is obtained by taking the moments about the neutral axis and negative moment capacity is calculated by taking moments about the plastic neutral axis.

The yield line mechanism engages only the outer half of the bridge cross-section. Therefore, only half of the intact moment capacities are used. The positive intact moment capacity is used for obtaining the exact location of fracture in the end spans that is needed to develop the value of the fraction of the span length from the exterior support at which the girder is fractured, λ. The λ is set to 0.4 as a result of a detailed analysis explained in the TxDOT (0-6937) report on Fracture Critical Steel Twin Tub Girder Bridges. The negative intact moment capacity is used for the internal work done at the interior supports.

		MOMENT CAPACITY OF DECK SLAB (BRIDGE 5)						1/9
		Description	Parameter	Data				
DECK SECTION	1	Total Width	B in	**360**				
	2	Thickness	t in	**8**				
	3	Section Width	b in	12				
CONCRETE MATERIAL	4	Characteristic Compressive Strength	f_c' ksi	4				
	5		β_1	0.85				
	6	Strain	ε_{cu}	0.003				
REBAR MATERIAL	7	Yield Strength	f_y ksi	60				
	8	Young's Modulus	E ksi	29000				
	9	Strain	ε_y	0.002070				
		Moment Computations		Longitudinal		Transverse		
	10			m_x'	m_x	m_y'	m_y	
	11			Top	Bottom	Top	Bottom	
REINFORCING BAR DETAIL	12	Bar No.		**4**	**5**	**5**	**5**	
	13	Diameter of Bar	d_b in	0.5	0.625	0.625	0.625	
	14	Area of Bar	A_Φ in^2	0.2	0.31	0.31	0.31	
	15	Spacing	s in	–	–	**5**	**5**	
	16	No. of Bars	$\#_{bar}$	**41**	**33**	–	–	
	17	Area of Steel	A_s in^2	0.273	0.341	0.744	0.744	
	18	Clear Cover	cc in	**2**	**1.25**	**2**	**1.25**	
	19	Effective depth (tension)	d in	5.125	5.8125	5.6875	6.4375	
	20	Effective depth (comp.)	d'in	2.1875	2.875	1.5625	2.3125	
	21	Depth of NA	c in	**1.06**	**1.06**	**1.44**	**1.81**	

		MOMENT CAPACITY OF DECK SLAB (BRIDGE 5)					2/9
		Moment Computations		Longitudinal		Transverse	
	22			m'_x	m_x	m'_y	m_y
	23			Top	Bottom	Top	Bottom
STRAIN	24		ε_{top}	0.011466	0.005115	0.008829	0.000837
	25		ε_{bot}	0.003174	0.013406	0.000250	0.007682
	26	$\beta_1 c$	a in	0.90	0.90	1.23	1.54
FORCES	27	Compression-Concrete	C_c kip	−36.86	−36.86	−50.03	−62.70
	28	Tension/ Compression-Steel	T_{top} kip	16.40	16.40	44.64	18.06
	29		T_{bot} kip	20.46	20.46	5.39	44.64
	30	Equilibrium Check	$\sum T + C$ kip	**0.00**	**0.00**	**0.00**	**0.00**
RESULTS	31	Moment	M_n k-in/in	**9.35**	**12.45**	**19.30**	**23.41**

Remarks:

Top Yielded	Top Yielded	Top Yielded	Top Not Yielded
Bottom Yielded	Bottom Yielded	Bottom Not Yielded	Bottom Yielded
Both steel in Tension	Both steel in Tension	Both steel in Tension	Both steel in Tension

		POSITIVE AND NEGATIVE MOMENT OF INTACT SECTION (BRIDGE 5, SPANS 1 AND 2)				3/9
			Moment			
		Description	Positive		Negative	
DECK SECTION	1	Total width	B in	**180**	B in	180
	2	Thickness	t in	**8**	t in	**8**
	3	Haunch thickness	h in	**4**	h in	**4**
CONCRETE MATERIAL	4	Characteristic Compressive Strength	f_c' ksi	4		
	5		β_1	0.85		
	6	Strain	ε_{cu}	0.003		
REINFORCING BAR DETAILS	7	Yield Strength	f_y ksi	60	f_y ksi	60
	8	Strain	Rebar ε_y	0.002070		
	9	No of Bars at Top	# top bars	**20.5**	# top bars	**20.5**
	10	No of Bars at Top Bent			# top bars bent	**20**
	11	No of Bars at Bottom	# bot bars	**16.5**	# bot bars	**16.5**
	12	Clear Cover Top	cc_{top} in	**2**	cc_{top} in	**2**
	13	Clear Cover Bottom	cc_{bot} in	**1.25**	cc_{bot} in	**1.25**
	14	Transverse Top Diameter	$d_{b\ top\ tra}$ in	**0.625**	$d_{b\ top\ tra}$ in	**0.625**
	15	Transverse Bottom Diameter	$d_{b\ bot\ tra}$ in	**0.625**	$d_{b\ bot\ tra}$ in	**0.625**
	16	Longitudinal Top Diameter	$d_{b\ top\ long}$ in	**0.5**	$d_{b\ top\ long}$ in	**0.5**
	17	Longitudinal Top Diameter at Bents			$d_{b\ top\ long}'$ in	**0.625**
	18	Longitudinal Bottom Diameter	$d_{b\ bot\ long}$ in	**0.625**	$d_{b\ bot\ long}$ in	**0.625**
	19	Effective Depth Top	d_{top} in	2.88	d_{top} in	2.88
	20				$d_{top\ bent}$ in	2.94
	21	Effective Depth Bottom	d_{bot} in	5.81	d_{bot} in	5.81
	22	Area of Steel for #4 Bars	$A_{\#4}$ in^2	0.196	$A_{\#4}$ in^2	0.196
	23	Area of Steel for #5 Bars	$A_{\#5}$ in^2	0.307	$A_{\#5}$ in^2	0.307
STRUCTURAL STEEL	24	Yield Strength	F'_y ksi	50	F'_y ksi	50
	25	Strain	ε_s	0.001720		
	26	Young's Modulus	E ksi	29000		
	27	f_y/F'_y	n	1.2	n	1.2
	28	Effective Area of Steel for #4 Bars	$A_{\#4.\ eff}$ in^2	0.236	$A_{\#4.\ eff}$ in^2	0.236
	29	Effective Area of Steel for #5 Bars	$A_{\#5.\ eff}$ in^2	0.368	$A_{\#5.\ eff}$ in^2	0.368

		POSITIVE AND NEGATIVE MOMENT OF INTACT SECTION (BRIDGE 5, SPANS 1 AND 2)				4 / 9
			Moment			
		Description	Positive		Negative	
GIRDER DETAILS	30	Girder Depth	D_g in	55.75	D_g in	58
	31	Top Flange Width	b_{tf} in	**18**	b_{tf} in	**18**
	32	Top Flange Thickness	t_{tf} in	**1**	t_{tf} in	**2.5**
	33	Web Width	b_w in	**54**	b_w in	**54**
	34	Web Thickness	t_w in	**0.5**	t_w in	**0.6**
	35	Bottom Flange Width	b_{bf} in	**56**	b_{bf} in	**56**
	36	Bottom Thickness	t_{bf} in	**0.75**	t_{bf} in	**1.5**
DEPTHS	37	Compression Zone	c in	**12.89**	c in	**39.29**
	38	b_w in Comp.	y in	0.89	y in	37.79
	39	b_w in Tension	y' in	0.11	y' in	16.21
DISTANCES	40	Distances of Compressive Forces and Tensile Forces from Neutral Axis/ Distances of Areas in Tension and Compression from PNA	$d\ C_{conc}$ in	7.41	$d\ A_{rebar\ top}$ in	27.84
	41		$d\ C_{rebar\ top}$ in	10.02	$d\ A_{rebar\ top\ bent}$ in	27.78
	42		$d\ C_{rebar\ bottom}$ in	7.08	$d\ A_{rebar\ bot}$ in	24.90
	43		$d\ C_{tf1}$ in	0.45	$d\ A_{tf}$ in	17.46
	44		$d\ T_{tf\ 2}$ in	0.05	$d\ A_{w\ 1}$ in	8.11
	45		$d\ T_w$ in	54.05	$d\ A_{w\ 2}$ in	18.89
	46		$d\ T_{bf}$ in	54.48	$d\ A_{bf}$ in	38.54
STRAIN	47		$\varepsilon\ C_{rebar\ top}$	0.002331		
	48		$\varepsilon\ C_{rebar\ bottom}$	0.001648		
	49		$\varepsilon\ C_{tf}$	0.000104		
	50		$\varepsilon\ C_{w\ 1}$	0.000012		
	51		$\varepsilon\ T_{w\ 2}$	0.012578		
	52		$\varepsilon\ T_{bf}$	0.012677		

		POSITIVE AND NEGATIVE MOMENT OF INTACT SECTION (BRIDGE 5, SPANS 1 AND 2)				5/9
			Moment			
		Description	Positive		Negative	
FORCE/AREA	53	Compression F_C/ Tension A_T	$0.85f'_c\,\beta BT$ kip	−4162	$A_{s\,top}$ kip	5
	54				$A_{s\,top\,bent}$ kip	7
	55		$A_{s\,top}f_y$ kip	−273	$A_{s\,bottom}$ kip	6
	56		$A_{s\,bottom}f_y$ kip	−270	A_{tf} kip	90
	57		$2b_{tf}yF'_y$ kip	−97	$A_{w\,1}$ kip	18
	58	Tension F_T/ Compression A_C	$2b_{tf}y'F'_y$ kip	1.39		
	59		$2b_wt_wF'_y$ kip	2700	$A_{w\,2}$ kip	−43
	60		$b_{bf}t_{bf}F'_y$ kip	2100	A_{bf} kip	−84
CHECK	61	Equilibrium of Forces/Areas	F_C kip	−4800	A_T kip	127
	62		F_T kip	4800	A_C kip	−127
	63		$F_C + F_T$ kip	**0**	$A_T + A_C$ kip	**0**
MOMENT	64	(Half Section)	$0.5\,M_p^+$ k-ft	24658	$0.5\,M_p^-$ k-ft	26450
	65	(Full Section)	M_p^+ k-ft	**49317**	M_p^- k-ft	**52901**

		COMPUTATION OF OVERSTRENGTH FACTOR EXTERIOR: 2-SPAN CASE (BRIDGE 5, SPAN 1)		6/9
		Parameter	Formula/Definition/Equation	Data
GEOMETRY	1	L_x	Span Length	**140.00** ft
	2	R_{CL}	Radius of center line	**450** ft
	3	B	Width	**30** ft
	4	L_x^*	Outer region length $L_x^* = \left(1 + \frac{B}{4R_{C_L}}\right) L_x$	142.33 ft
	5	s	Inter-Girder Spacing	**9.7** ft
	6	b	Width of Girder + Edge	**10** ft
	7	t	Deck Thickness	**8** in
INTERNAL WORK DONE, IWD	8	m_x	Longitudinal Positive Moment per ft	**12** k-in/in
	9	m'_x	Longitudinal Negative Moment per ft	**9** k-in/in
	10	m_y	Transverse Positive Moment per ft	**23** k-in/in
	11	m'_y	Transverse Negative Moment per ft	**19** k-in/in
	12	$0.5M_p^-$	Negative Moment at Support	**26450** k-ft
	13	λ	Fraction of Length from the exterior support at which girder is fractured	0.40
	14	tan θ	$\tan\theta = \sqrt{\frac{m'_y + m_y}{m'_x + m_x}}$	1.40 (Θ = 54.4°)
	15	tan α	$\tan\alpha = \frac{2s}{L_x}$	0.14 (Θ = 7.9°)
	16	k_{bound}^{upper}	$\left[1 + 2\frac{\tan\alpha}{\tan\theta}\right] = 1 + \frac{4s}{L_x}\sqrt{\frac{m'_x + m_x}{m'_y + m_y}}$	1.20
	17	k_{bound}^{lower}	$\left[1 + 2\frac{\tan^2\alpha}{\tan^2\theta}\right] = 1 + \frac{8s^2}{{L_x}^2}\sqrt{\frac{m'_x + m_x}{m'_y + m_y}}$	1.02
	18	IWD_{upper}	$(m'_y + m_y)\left(\frac{L_x}{2s}\right) k_{bound}^{upper} + \frac{m_x b}{L_x(\lambda - \lambda^2)} + \frac{0.5M_p^-}{(1-\lambda)L_x}$	689 k-ft *
	19	IWD_{lower}	$\left(m_y^{'} + m_y\right)\left(\frac{L_x}{2s}\right) k_{bound}^{lower} + \frac{m_x b}{L_x(\lambda - \lambda^2)} \quad \frac{0.5M_p^-}{(1-\lambda)L_x}$	634 k-ft *

Note: *: A unit deflection (δ = 1) is considered; therefore, the unit of internal work is in k-ft

		COMPUTATION OF OVERSTRENGTH FACTOR EXTERIOR: 2-SPAN CASE (BRIDGE 5, SPAN 1)		7/9
		Parameter	Formula/Definition/Equation	Data
EXTERNAL WORK DONE, EWD	20	DL	Dead Load Factor	1.25
	21	LL	Live Load Factor	1.75
	22	SAF	Stiffener Allowance Factor	1
	23	γ_c	Unit weight of reinforced concrete	0.15 kcf
	24	w_u	Area load due to reinforced concrete + lane load $DL\gamma_c \frac{t}{12} + LL \cdot \frac{0.64}{12}$	0.218 ksf
	25	γ_s	Unit weight of steel	0.49 kcf
	26	V_g	Volume of Girder	**145** ft^3
	27	A_r	Area of Rail Cross-Section (T4(S))	**1.25** ft^2
	28	V_r	Volume of Rail $= L_x A_r$	178 ft^3
	29	W_x	$1.25\left(1.15V_g\gamma_s + V_r\gamma_c\right)/L_x$	0.95 k/ft
	30	y (Lane 2)	$(b+s-15)$ for $(b+s) < 21$ $(b+s-18)$ for $(b+s) > 21$	4.84 ft
	31	K_{lane}	$1 + 0.5\frac{y}{s}$ for $(b+s) < 21$; $1 + \frac{y}{s}$ for $(b+s) > 21$	1.25
	32	$EWD_{HS\text{-}20}$	$\left(168 - \frac{261.33}{\lambda L_x} - \frac{1045.33}{(1-\lambda)L_x}\right) K_{Lane}$	189 k-ft
	33	W_{ET}	$w_u L_x(b + 0.5s) + W_x L_x + 2EWD_{HS20}$	979 k-ft
	34	EWD	$0.5\ W_{ET}$	490 k-ft *
RESULTS	35	Ω_{upper}	IWD_{upper}/EWD	**1.41**
	36	Ω_{lower}	IWD_{lower}/EWD	**1.29**

Note: *: A unit deflection (δ = 1) is considered; therefore, the unit of external work is in k-ft

		COMPUTATION OF OVERSTRENGTH FACTOR EXTERIOR: 2-SPAN CASE (BRIDGE 5, SPAN 2)		8 9
		Parameter	Formula/Definition/Equation	Data
GEOMETRY	1	L_x	Span Length	**139.60** ft
	2	R_{CL}	Radius of center line	**450** ft
	3	B	Width	**30** ft
	4	L_x^*	Outer region length $L_x^* = \left(1 + \frac{B}{4R_{C_L}}\right) L_x$	141.93 ft
	5	s	Inter Girder Spacing	**10** ft
	6	b	Width of Girder + Edge	**10** ft
	7	t	Deck Thickness	**8** in
INTERNAL WORK DONE, IWD	8	m_x	Longitudinal Positive Moment per ft	**12** k-in/in
	9	m'_x	Longitudinal Negative Moment per ft	**9** k-in/in
	10	m_y	Transverse Positive Moment per ft	**23** k-in/in
	11	m'_y	Transverse Negative Moment per ft	**19** k-in/in
	12	$0.5M_p^-$	Negative Moment at Support	**26450** k-ft
	13	λ	Fraction of Length from the exterior support at which girder is fractured	0.40
	14	tan θ	$\tan\theta = \sqrt{\frac{m'_y + m_y}{m'_x + m_x}}$	1.40 (Θ = 54.4°)
	15	tan α	$\tan\alpha = \frac{2s}{L_x}$	0.14 (Θ = 7.9°)
	16	k_{bound}^{upper}	$\left[1 + 2\frac{\tan\alpha}{\tan\theta}\right] = 1 + \frac{4s}{L_x}\sqrt{\frac{m'_x + m_x}{m'_y + m_y}}$	1.20
	17	k_{bound}^{lower}	$\left[1 + 2\frac{\tan^2\alpha}{\tan^2\theta}\right] = 1 + \frac{8s^2}{{L_x}^2}\sqrt{\frac{m'_x + m_x}{m'_y + m_y}}$	1.02
	18	IWD_{upper}	$\left(m_y^{'} + m_y\right)\left(\frac{L_x}{2s}\right) k_{bound}^{upper} + \frac{m_x b}{L_x(\lambda - \lambda^2)} + \frac{0.5M_p^-}{(1-\lambda)L_x}$	683 k-ft *
	19	IWD_{lower}	$\left(m_y^{'} + m_y\right)\left(\frac{L_x}{2s}\right) k_{bound}^{lower} + \frac{m_x b}{L_x(\lambda - \lambda^2)} \quad \frac{0.5M_p^-}{(1-\lambda)L_x}$	628 k-ft *

Note: *: A unit deflection (δ = 1) is considered; therefore, the unit of internal work is in k-ft

		COMPUTATION OF OVERSTRENGTH FACTOR EXTERIOR: 2-SPAN CASE (BRIDGE 5, SPAN 2)		9/9
		Parameter	Formula/Definition/Equation	Data
EXTERNAL WORK DONE, EWD	20	DL	Dead Load Factor	1.25
	21	LL	Live Load Factor	1.75
	22	SAF	Stiffener Allowance Factor	1
	23	γ_c	Unit weight of reinforced concrete	0.15 kcf
	24	w_u	Area load due to reinforced concrete + lane load $DL\gamma_c \frac{t}{12} + LL \cdot \frac{0.64}{12}$	0.218 ksf
	25	γ_s	Unit weight of steel	0.49 kcf
	26	V_g	Volume of Girder	**152** ft^3
	27	A_r	Area of Rail Cross-Section (T4 (S))	**1.25** ft^2
	28	V_r	Volume of Rail $= L_x A_r$	177 ft^3
	29	W_x	$1.25\left(1.15V_g\gamma_s + V_r\gamma_c\right)/L_x$	0.99 k/ft
	30	y (Lane 2)	$(b+s-15)$ for $(b+s) < 21$ $(b+s-18)$ for $(b+s) > 21$	4.84 ft
	31	K_{lane}	$1 + 0.5\frac{y}{s}$ for $(b+s) < 21$; $1 + \frac{y}{s}$ for $(b+s) > 21$	1.25
	32	$EWD_{HS\text{-}20}$	$\left(168 - \frac{261.33}{\lambda L_x} - \frac{1045.33}{(1-\lambda)L_x}\right) K_{Lane}$	189 k-ft
	33	W_{ET}	$w_u L_x(b + 0.5s) + W_x L_x + 2EWD_{HS20}$	983 k-ft
	34	EWD	$0.5\ W_{ET}$	491 k-ft *
RESULTS	35	Ω_{upper}	IWD_{upper}/EWD	**1.39**
	36	Ω_{lower}	IWD_{lower}/EWD	**1.28**

Note: *: A unit deflection (δ = 1) is considered; therefore, the unit of external work is in k-ft

6.2 GRILLAGE ANALYSIS EXAMPLE OF BRIDGE 5

The following steps are explained to conduct the computational implementation of grillage analysis of Bridge 5.

1. Gather bridge geometry and material information.

Location ft	Top Flange		Web		Bottom Flange	
	Width in.	Thickness in.	Width in.	Thickness in.	Width in.	Thickness in.
0–105	18	1.00	54	0.5	56	0.75
105–122	18	1.00	54	0.5625	56	1.250
122–140	18	1.75	54	0.5625	56	1.250
140–157	18	1.75	54	0.5625	56	1.250
157–174	18	1.57	54	0.5625	56	1.250
174–192	18	1.00	54	0.5625	56	0.75
192–280	18	1.00	54	0.5	56	0.75

Location	Parameter	Description/Value
Bridge	Location	Travis County, I-35
	Year Designed/Year Built	1998/2002
	Design Load	
	Length, ft	279.58
	Spans, ft	140, 139.58
	Radius of Curvature, ft	450
Deck	Width, ft	30
	Thickness, in.	8
	Haunch, in.	4
	Rail Type	T4(S)
Rebar	# of Bar Longitudinal Top Row (#4)	40
	# of Bar Longitudinal Bottom Row (#5)	36
	# of Bar Longitudinal Top Row (#4) @support	41
	# of Bar Longitudinal Top Row (#5) @support	40
	# of Bar Longitudinal Bottom Row (#5) @support	36
	Transverse Spacing Top Row (#5), in.	5
	Transverse Spacing Bottom Row (#5), in.	5
Girder	CL of Bridge to CL of Girder (in.)	56.5
	CL of Top Flange to CL of Top Flange (in.)	83

2. Determine material constitutive behavior.

Concrete (4 ksi)

Stress (ksi)	Strain (1/in)
−4	−3.79E-03
−4	−3.56E-03
−4	−2.69E-03
−4	−1.78E-03
−3.8205	−1.40E-03
−2.8718	−8.69E-04
−0.6403	−1.78E-04
0	0
0.378	1.06E-04
0.378	1.16E-03

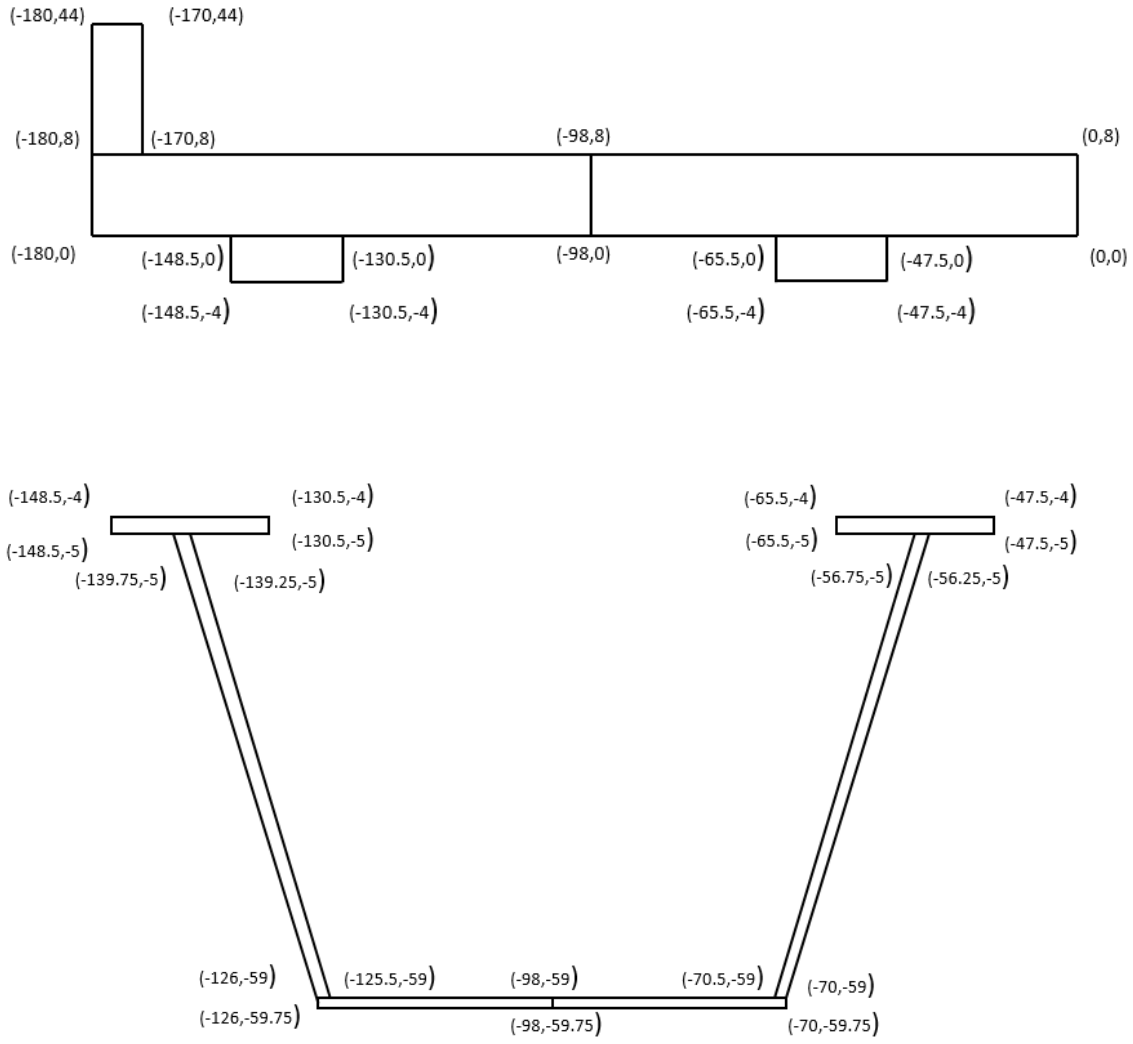
Rebar (60 ksi)

Stress (ksi)	Strain (1/in)
−87.9	−0.095
−87.9	−0.0944
−86.6	−0.0761
−78	−0.0386
−60.7	−9.80E-03
−60.3	−2.08E-03
0	0
60.3	2.08E-03
60.7	9.80E-03
78	0.0386
86.6	0.0761
87.9	0.0944
87.9	0.095

Steel (50 ksi)

Stress (ksi)	Strain (1/in)
−71.6	−0.1
−71.6	−0.097
−71.6	−0.095
−71.6	−0.0946
−70.3	−0.0764
−62.5	−0.039
−50	−0.0196
−50	−1.72E-03
0	0
50	1.72E-03
50	0.0196
62.5	0.039
70.3	0.0764
71.6	0.0946
71.6	0.095
71.6	0.097
71.6	0.1

3. Create a coordinate system for the half width of the span for each cross-section in 1. An example of the first cross-section is shown below.



4. Create a cylindrical coordinate system for the curved bridge and ensure that the middle transverse divisions are 7 ft because this will aid is applying the truck load whose axles are separated by 14 ft.

   A. $\#\ of\ Segments_{per\ span} = \left(\frac{Length\ (ft)*12}{84\ in\ (7ft)}\right)\ rounded\ to\ nearest\ even\ number$:

   $$\#\ of\ Segments_{Span\ 1\&2} = \left(\frac{140 * 12}{84}\right) = 20.$$

   B. $End\ Segment\ Length = \frac{\left((Length*12)-(\#\ of\ Segments*84)\right)}{2}$:

   $$End\ Segment\ Length = \frac{\left((140 * 12) - (14 * 84)\right)}{2} = 0\ in.$$

   Therefore, the end segments are also equal to 84 in.

   C. $Theta = \frac{Total\ Length}{Radius}$.

   $$Theta = \frac{280}{450} = 0.6222\ rad\ or\ 36.65\ degrees.$$

   D. Determine the radial offsets using the outside edge, the outside flange, the inner flange, and CL of the bridge.

Offsets (in.)	
Edge	180
Outside Flange	139.5
Inner Flange	56.5
CL of Bridge	0

Radial Spacing (in.)		
A	5580	CL + Edge
B	5539.5	CL + OF
C	5456.5	CL + IF
Center Line	5400	or 450 (ft)
D	5343.5	CL − IF
E	5260.5	CL − OF
F	5220	CL − Edge

E. The longitudinal spacing along theta is determined by converting the longitudinal segment lengths into degrees.
   - All segments are 84 in. The total length is 280 ft, or 3360 in.
   - $Radial\ Spacing\ (rad) = \frac{Long.Spacing}{Radius}$
   - $Radial\ Spacing\ (degree) = Radius\ Spacing\ (rad) * \frac{180}{\pi}$

Long. Spacing (in.)	Radial Spacing (rad.)	Radial Spacing (deg.)	Cross-Section
0	0.0000	0.000	Long1
84	0.0156	0.891	Long1
168	0.0311	1.783	Long1
252	0.0467	2.674	Long1
336	0.0622	3.565	Long1
420	0.0778	4.456	Long1
504	0.0933	5.348	Long1
588	0.1089	6.239	Long1
672	0.1244	7.130	Long1
756	0.1400	8.021	Long1
840	0.1556	8.913	Long1
924	0.1711	9.804	Long1
1008	0.1867	10.695	Long1
1092	0.2022	11.586	Long1
1176	0.2178	12.478	Long2
1260	0.2333	13.369	Long2
1344	0.2489	14.260	Long2
1428	0.2644	15.152	Long3
1512	0.2800	16.043	Long3
1596	0.2956	16.934	Long4
1680	0.3111	17.825	Long4
1764	0.3267	18.717	Long4
1848	0.3422	19.608	Long3
1932	0.3578	20.499	Long3
2016	0.3733	21.390	Long2
2100	0.3889	22.282	Long2
2184	0.4044	23.173	Long2
2268	0.4200	24.064	Long1
2352	0.4356	24.955	Long1
2436	0.4511	25.847	Long1

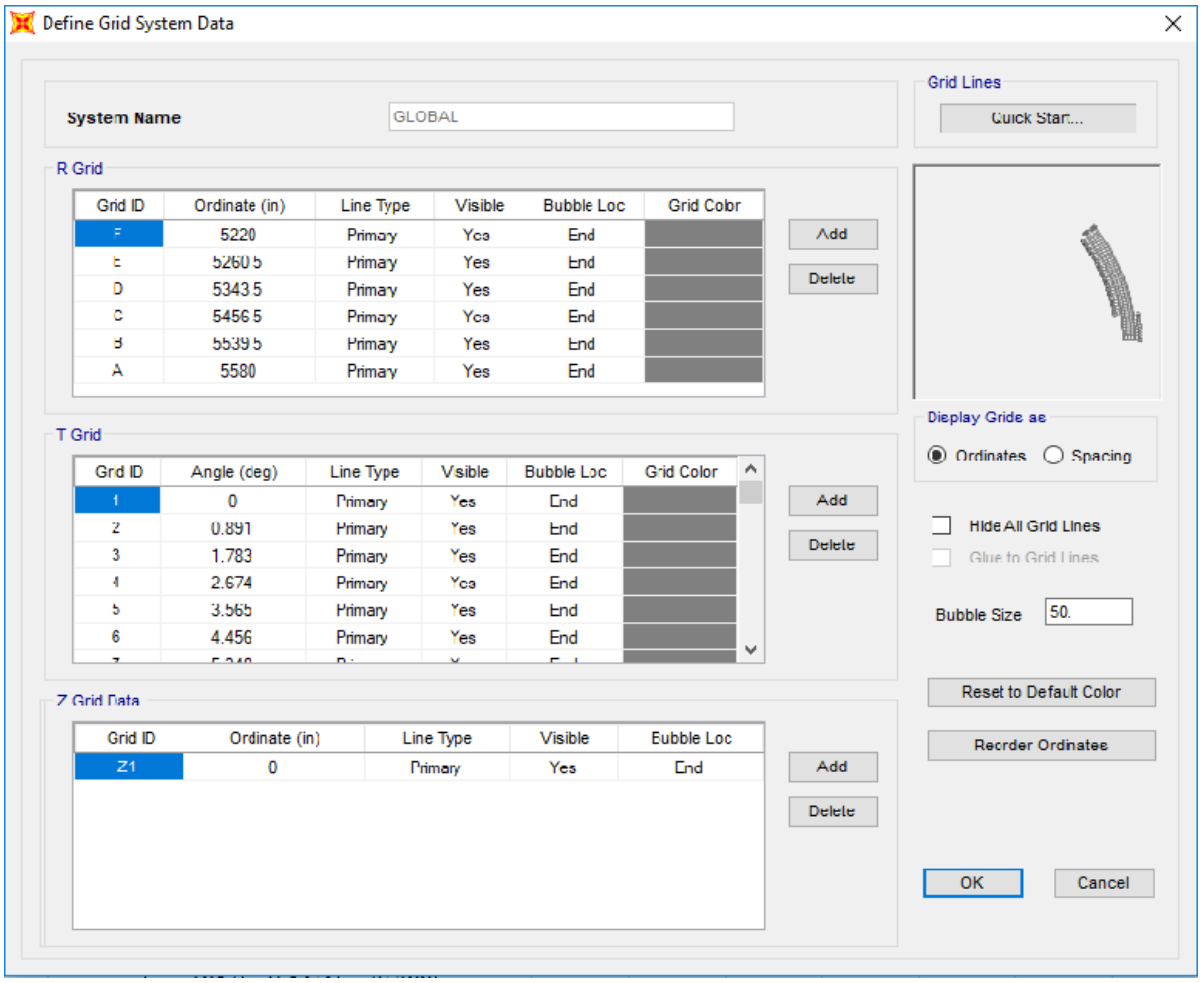
2520	0.4667	26.738	Long1
2604	0.4822	27.629	Long1
2688	0.4978	28.521	Long1
2772	0.5133	29.412	Long1
2856	0.5289	30.303	Long1
2940	0.5444	31.194	Long1
3024	0.5600	32.086	Long1
3108	0.5756	32.977	Long1
3192	0.5911	33.868	Long1
3276	0.6067	34.759	Long1
3360	0.6222	35.651	Long1

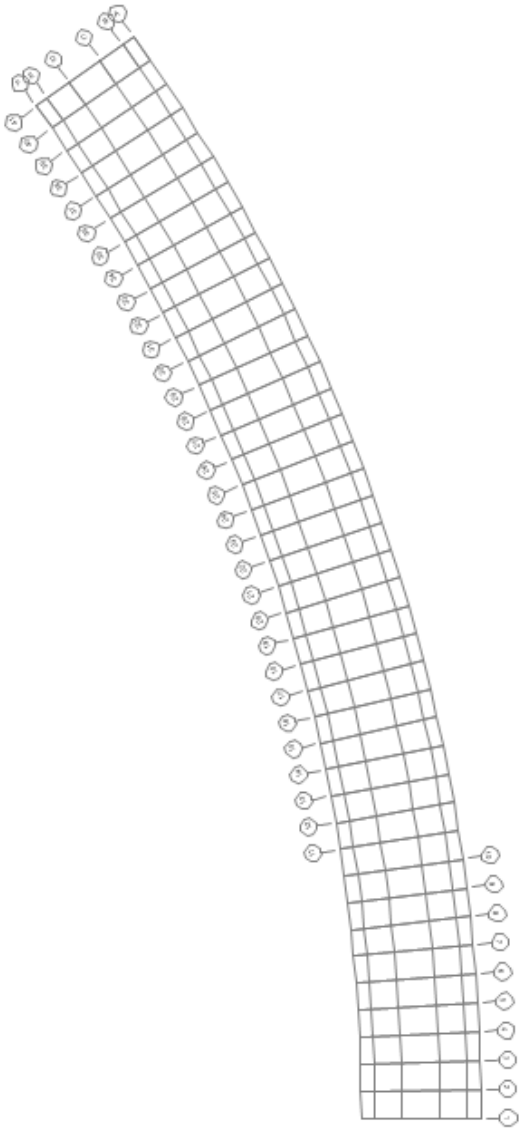
- $Int. Transverse\ Element\ width = 84\ in.$
- $End\ Transverse\ Element = 84 - \left(\frac{84}{2}\right) = 42\ in.$
- $Pier\ Tansverse\ Element = 84 + 84 - 84 = 84\ in.$

5. Input the coordinate system into SAP2000.
   A. Select File > New Model > Blank model (making sure units are in kips and inches).
   B. Right click on the blank workspace and select Edit Grid Data > Modify/Show System > Quick Start >Cylindrical.
      - In the Number of Gridlines panel, set "Along Z = 1."
      - In the Grid Spacing panel, set "Along Z = 1."
      - Select OK.
      - Delete all R and T Coordinates that were generated.
   C. Add correct coordinates for R.

      All radial coordinates are A, B, C, D, E, and F where A = outside edge; B = centerline of outer top flange of outside girder; C = Centerline of inner top flange of outside girder; D = Centerline of inner top flange of inside girder and F = inside edge.

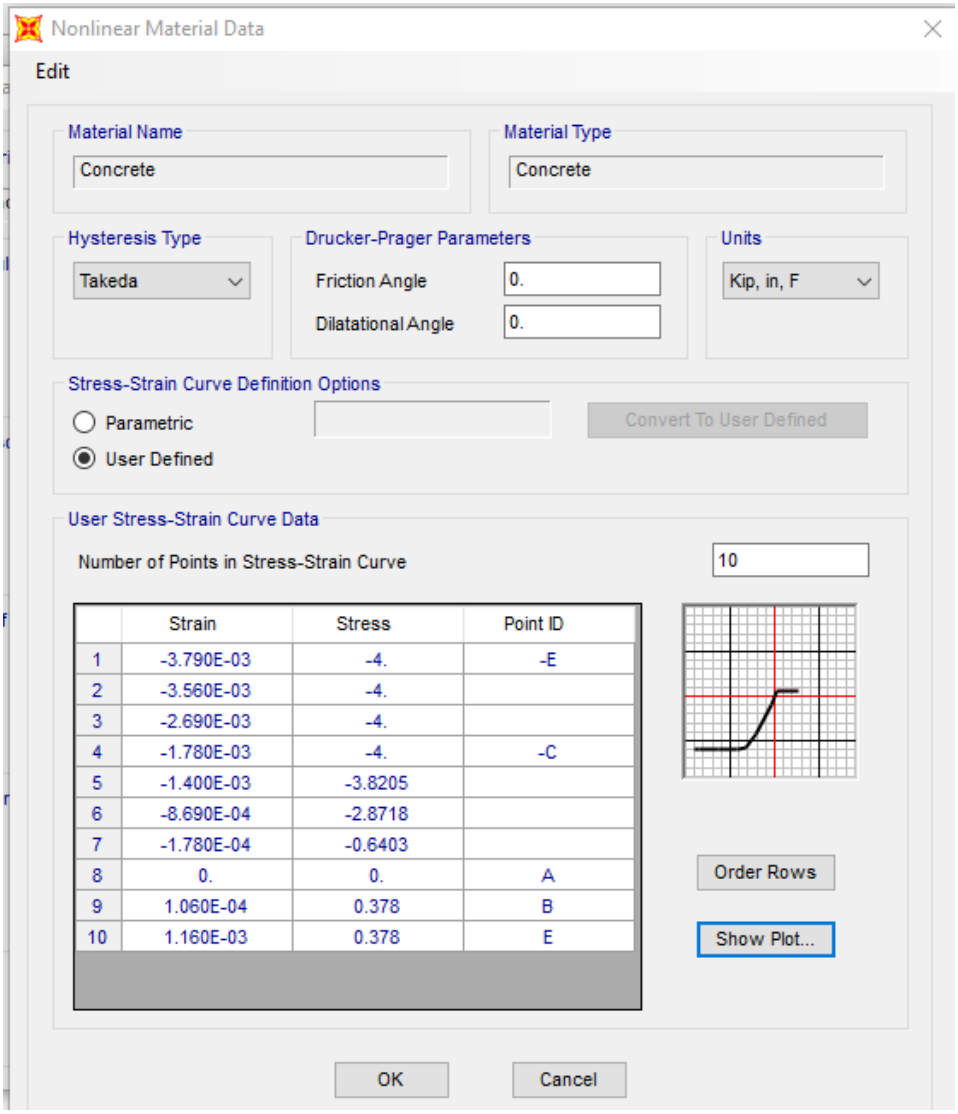
   D. Add correct coordinates for T
      - All theta coordinates for T (0 to 3360 in.).
      - Click OK.
   E. The grid system is now formed.


Define Grid System Data
System Name
GLOBAL
Grid Lines
Quick Start...
R Grid
Grid ID | Ordinate (in) | Line Type | Visible | Bubble Loc | Grid Color
F | 5220 | Primary | Yes | End
E | 52605 | Primary | Yes | End
D | 53435 | Primary | Yes | End
C | 54565 | Primary | Yes | End
B | 55395 | Primary | Yes | End
A | 5580 | Primary | Yes | End
Add
Delete
T Grid
Grid ID | Angle (deg) | Line Type | Visible | Bubble Loc | Grid Color
1 | 0 | Primary | Yes | End
2 | 0.891 | Primary | Yes | End
3 | 1.783 | Primary | Yes | End
4 | 2.674 | Primary | Yes | End
5 | 3.565 | Primary | Yes | End
6 | 4.456 | Primary | Yes | End
Add
Delete
Z Grid Data
Grid ID | Ordinate (in) | Line Type | Visible | Bubble Loc
Z1 | 0 | Primary | Yes | End
Add
Delete
Display Grids as
Ordinates
Spacing
Hide All Grid Lines
Glue to Grid Lines
Bubble Size
50.
Reset to Default Color
Reorder Ordinates
OK
Cancel



6. Define material in SAP200.
    A. Click Define > Materials- > Add New Material > Material Type (Steel, Concrete, or Rebar) >Standard (User) > OK.
    B. At the bottom of the window, select the box that states, “Switch to Advanced Properties.”
    C. In the open window, name the material “Concrete,” “Steel,” or “Rebar” depending on which material is being defined. Then click “Modify/Show Material Properties.”
    D. In the Material Property Data window, click the Nonlinear Material Data icon.
    E. In the Nonlinear Material Data window select the Convert to User Defined icon.
    F. Input the number of number of data points for the stress strain behavior (10 for concrete, 13 for rebar, 17 for steel).
    G. Input the data points for the stress strain behavior.
    H. Select “OK.”

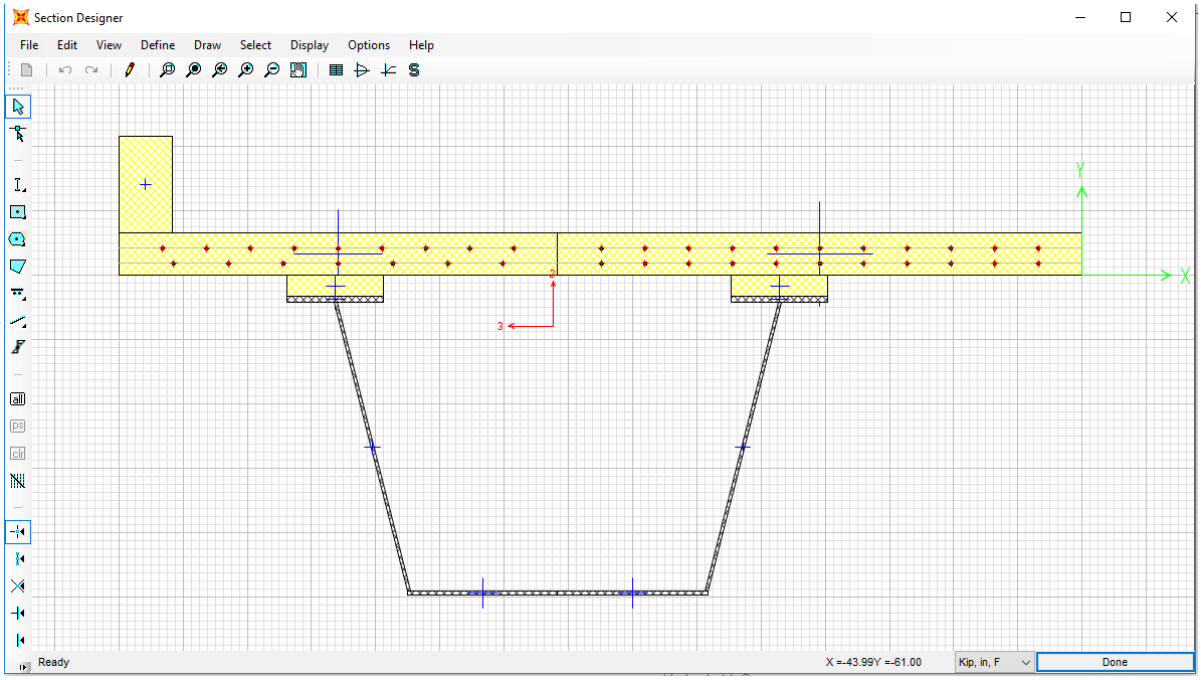
I. Repeat this process again for the remaining materials.



7. Define frame cross-sections in SAP2000.
   A. Click Define > Section Properties > Frame Sections > Add New Properties.
   B. In the Frame Section Properties drop-down box, select “Other” and click Section Designer. In the SD Section Designer window, name the section B5Long1 and click the Section Designer icon.
   C. Using the Polygon feature, draw the features of the half width of the bridge from Step 3. These features include one rail, two concrete deck pieces, two concrete haunches, two top flanges, two webs, and two pieces of the bottom flange.
      - To change material types for the polygons, right-click on the polygon and select the desired material type from the material drop-down menu.
      - To change the coordinates of the polygon’s nodes, use the Reshaper tool.
   D. Add in the longitudinal rebar to both concrete deck elements by using the Line Bar from the Draw Reinforcing Shape tool. From the design drawings, it can be

determined that there are seven #4 top bars and nine #5 bottom bars in the outer concrete deck element and 11 #4 top bars and 11 #5 bottom bars in the inner concrete element. At the pier support, nine additional #5 top bars are added to the top of the outer concrete deck and 10 additional #5 top bars are added to the inner concrete deck with a 2 in. top cover and transverse reinforcement. The top bars are located at 5.0625 in., with a 1.25 in. bottom cover and transverse reinforcement. The top bars are located at 2.1875.

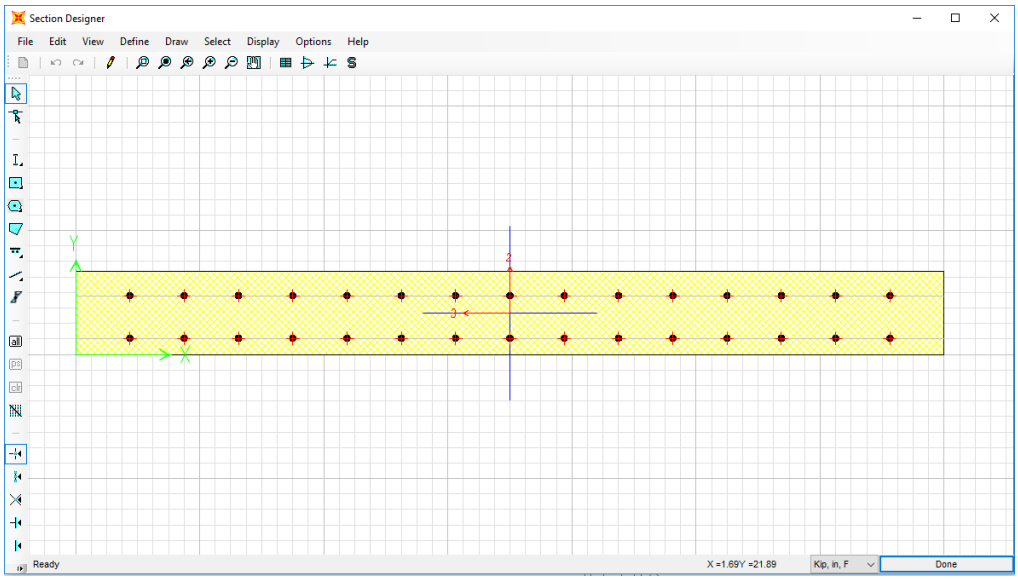
E. Click Done (Below is an example of the 1st cross-section).



F. Repeat this process for the remaining longitudinal elements.

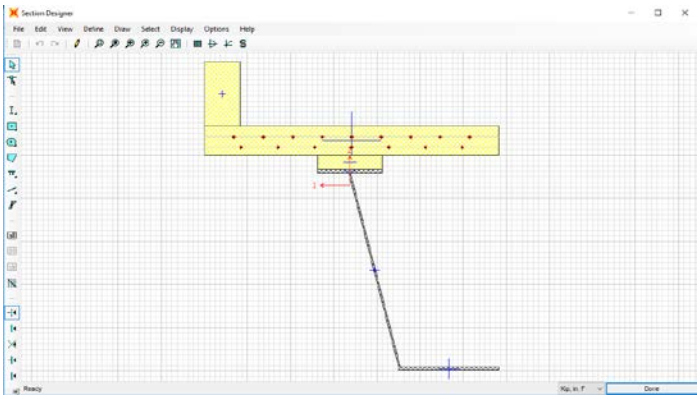
G. Repeat the process for the transverse elements.

- In the SD Section Designer window, select Modifiers and set Mass and Weight to 0 to not double-count the dead weight.
- The interior transverse members are 84 in. wide (end members are 42 in. wide), with #5 rebar at 5 in. spacing at 5.6875 in. and 1.5625 in.

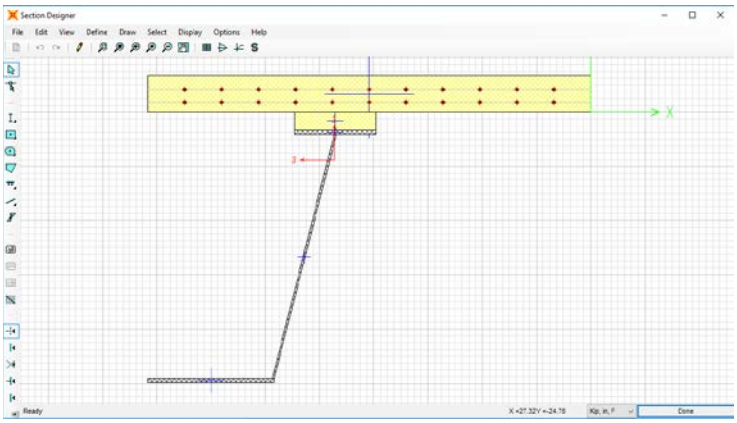


H. To generate an exterior longitudinal member and an interior longitudinal member, make two copies of the B5Long1 section. Label one Long1Out and one Long1Int.

- For the Long1Out, delete every element right of the CL.



- For the Long1Int, delete every element left of the CL.

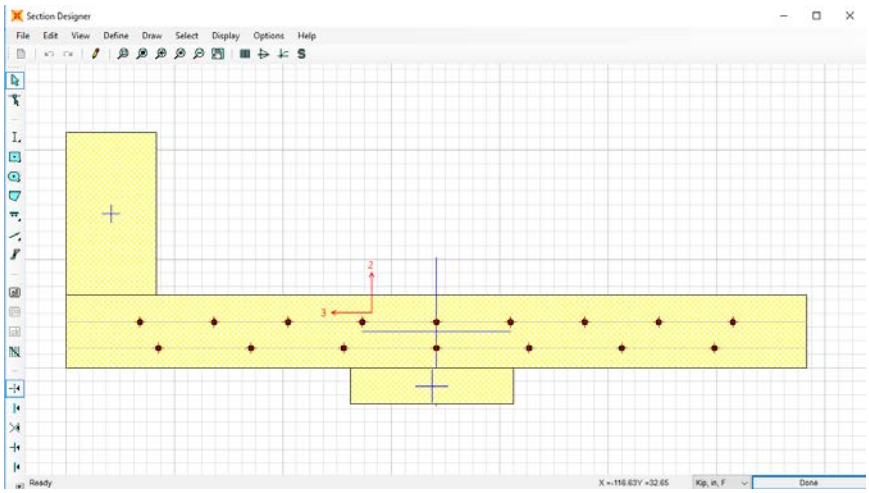


- Repeat this process for all cross-sections.

I. To generate a simulated fracture section, make copies of Long1Out and Long1Int.

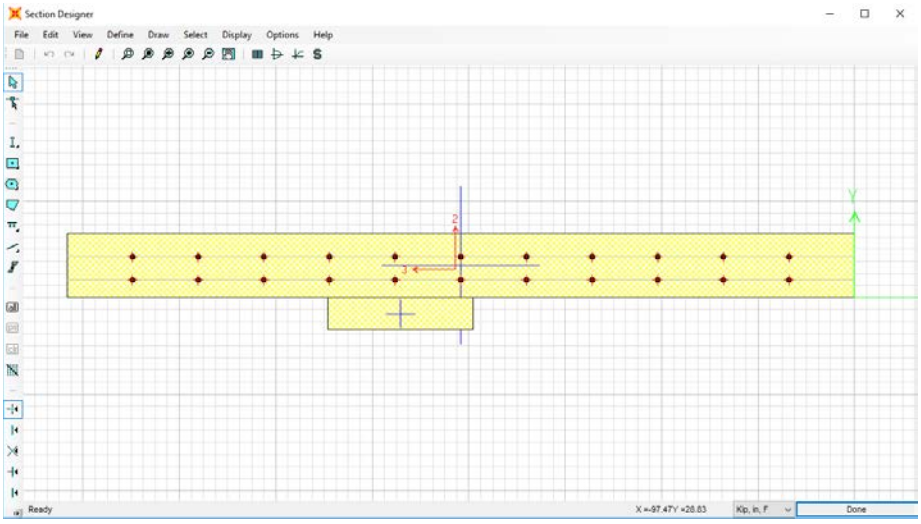
- The reason Long1Out and Long1Int are chosen for the fractured section is because the fracture occurs at 0.4*L, or 672 in. On the radial spacing table, Long1 is the section used at 672 in.
- Name the Long1Out copy Frac1Out.

Delete the bottom flange, web, and top flange of the steel tub.



- Name the LongInt copy FracInt.

Delete the bottom flange, web, and top flange of the steel tub.



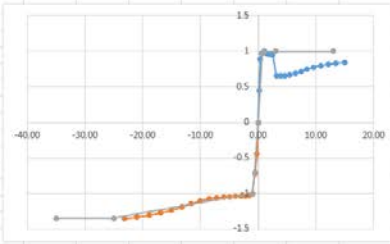
8. Generate plastic hinges for frame elements in SAP2000.
   A. Define > Section Properties > Frame Sections.
   B. Select the desired cross-section. Hinges will need to be made for the Long(1–4)Out, Long(1–4)Int, FracOut, FracInt, Trans, and TransEnd.
   C. Once selected, click Modify/Show Property > Section Designer.
   D. Once in the section designer, select the Moment Curvature Curve tool.
   E. In the Moment Curvature Curve window, select Details.
      - Copy the moment curvature data to an Excel file.

- Select OK.

F. In the Moment Curvature Curve window, change the angle (deg) to 180, then select Details.
   - Copy the moment curvature data to the same Excel file as the one used previously.
   - Select OK.

G. Generate a Normalized Moment Curvature Diagram.
   - Normalize the moments by dividing each of the positive moments by the maximum positive moment and the negative moment by the maximum negative moment, and divide the curvatures by the curvatures corresponding to the maximum and negative moments.
   - Plot the normalized positive and negative moment curvatures on a chart.
   - Create a hinge moment curvature plot on the same chart with four positive moment points and four negative moment points without generating a negative slope.

	A	B	C	D	E	F	G	H	I	J	K	M	N	P	Q
2	Positive														
3	Concrete S	Neutral A	Steel Strai	Tendon St	Concrete S	Steel Com	Steel Ten	Prestress	Net Force	Curvature	Moment	M	C		
4	0	0	0	0	0	0	0	0	0	0	0	0	0.00		
5	-6.42E-04	-0.0653	9.56E-04	0	-849.525	-71.5223	921.2336	0	0.1867	1.86E-05	56261	0.447094	0.14		
6	-1.53E-03	1.4924	2.46E-03	0	-1758	-130.476	1888.462	0	-0.0221	4.66E-05	111681	0.887505	0.36		
7	-2.49E-03	4.6917	4.70E-03	0	-2119	-90.6898	2208.734	0	-5.01E-01	8.39E-05	121852	0.968332	0.64		
8	-3.67E-03	6.2978	7.52E-03	0	-2351	-110.929	2462.295	0	0.0725	0.000131	125837	1	1.00		
9	-5.21E-03	6.4308	0.0108	0	-2439	-154.905	2593.535	0	-0.0456	0.000186	121679	0.966957	1.43		
10	-7.04E-03	6.4221	0.0145	0	-2495	-209.435	2703.974	0	-0.4089	0.000252	119891	0.952748	1.93		
11	-9.17E-03	6.2948	0.0188	0	-2537	-256.469	2793.547	0	-0.0143	0.000326	119168	0.947003	2.50		
12	-0.0225	-20.3633	0.0127	0	0	-1461	1460.759	0	-0.0104	0.00041	82471	0.65538	3.14		
13	-0.0275	-20.1838	0.0157	0	0	-1465	1465.611	0	0.1893	0.000503	82810	0.658074	3.86		
14	-0.0329	-19.9925	0.019	0	0	-1470	1470.333	0	0.2082	0.000606	83150	0.660775	4.64		
15	-0.0393	-20.4119	0.0222	0	0	-1494	1493.947	0	0.0502	0.000718	84541	0.671829	5.50		
16	-0.0464	-20.868	0.0256	0	0	-1533	1533.258	0	0.0173	0.000839	86803	0.689805	6.43		
17	-0.0537	-20.9742	0.0294	0	0	-1590	1590.45	0	0.3643	0.000969	90046	0.715576	7.43		
18	-0.0615	-21.0802	0.0336	0	0	-1653	1653.258	0	0.0119	0.001109	93606	0.743867	8.50		
19	-0.07	-21.2377	0.0379	0	0	-1718	1718.782	0	0.3503	0.001258	97263	0.772928	9.64		
20	-0.0786	-21.1113	0.0428	0	0	-1771	1771.636	0	0.1748	0.001416	100205	0.796308	10.85		
21	-0.0875	-20.8117	0.0484	0	0	-1816	1816.259	0	0.1105	0.001584	102695	0.816095	12.14	-1.35	-35
22	-0.0973	-20.8379	0.0537	0	0	-1849	1849.515	0	0.1005	0.001761	104568	0.83098	13.49	-1.35	-25
23	-0.1077	-20.877	0.0593	0	0	-1883	1882.613	0	0.0149	0.001948	106436	0.845824	14.93	-1	-1
24														-0.7	-0.56
25														0	0
26	Negative													0.97	0.64
27	Concrete S	Neutral A	Steel Strai	Tendon St	Concrete S	Steel Com	Steel Ten	Prestress	Net Force	Curvature	Moment			1	1
28	0	0	0	0	0	0	0	0	0	0	0	0	0.00	1	3
29	-7.09E-04	13.306	4.99E-04	0	-341.349	-577.707	235.8451	0	-683.211	1.86E-05	35333	-0.44896	-0.22	1	13



H. Define Hinge Length.

The hinge length is one half of the section depth.

- $Hinge_{long} = 0.5 * (Deck\ thickness + haunch\ height + top\ flange\ thickness + web\ height + bottom\ flange\ thickness)$.

$$Hinge_{long} = 30\ in.$$

- $Hinge_{Frac} = 0.5 * (Deck\ thickness + haunch\ height)$.

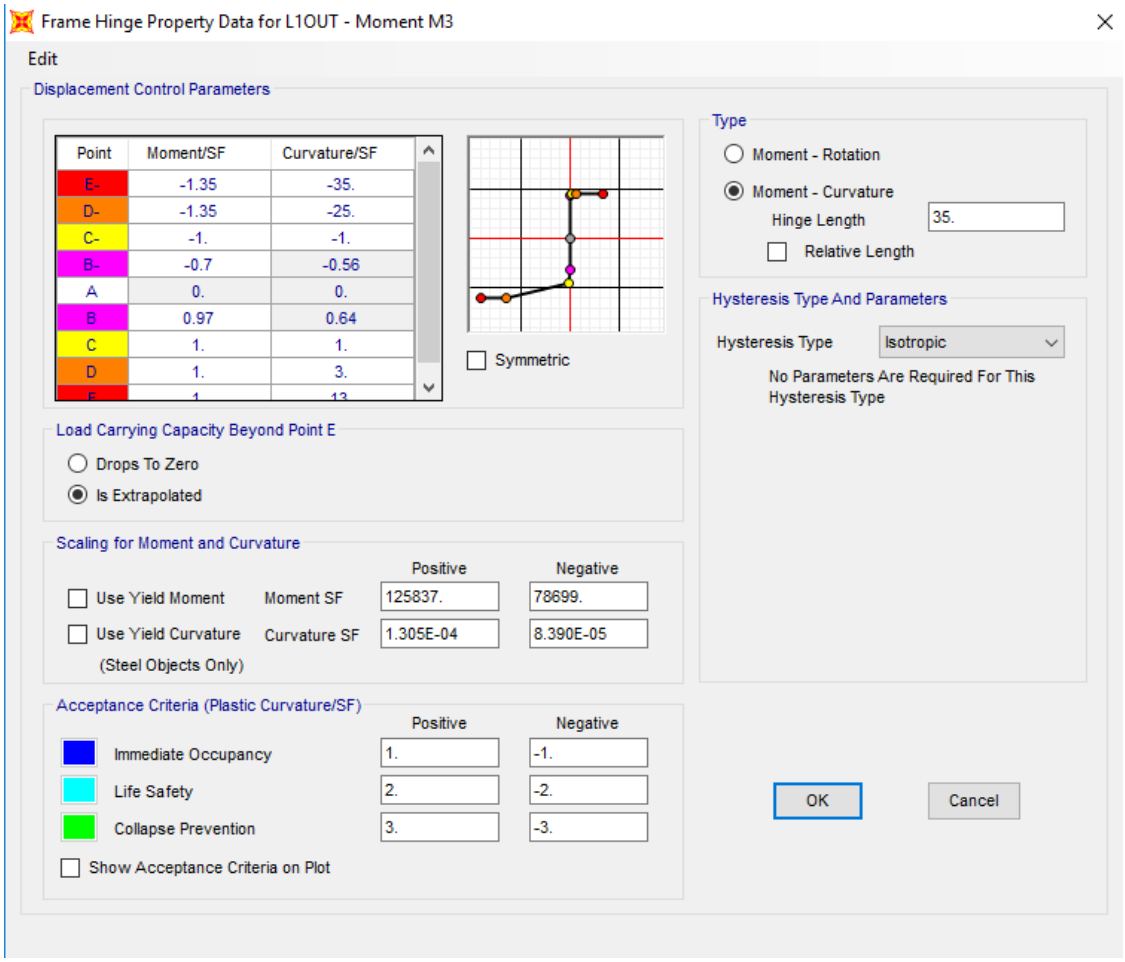
$$Hinge_{Frac} = 6\ in..$$

- $Hinge_{Trans} = 0.5 * (Deck\ thickness)$.

$$Hinge_{Trans} = 4\ in.$$

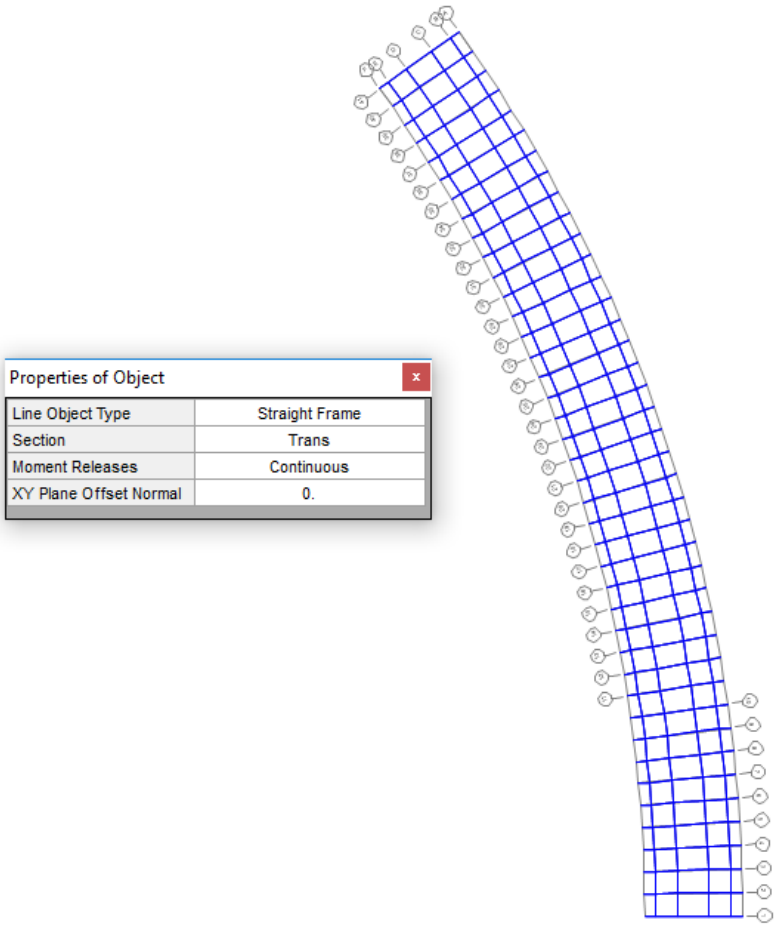
I. Make the plastic hinge in SAP2000.

- Select Define > Section Properties > Hinge Properties > Add New Properties.
- In the Type window, select Moment Curvature and input the corresponding correct hinge length.
- In the Moment Curvature table, insert the four positive and four negative normalized moment curvatures and the zero point.
- Uncheck the symmetric box and select the Is Extrapolated option in the Load Carrying Capacity beyond Point E window.
- In the Scaling for Moment and Curvature window, insert the maximum positive moment and corresponding curvature as well as the maximum negative curvature and corresponding curvature.
- In the Acceptance Criteria, use the values 1, 2, and 3 for Immediate Occupancy, Life Safety, and Collapse Prevention in the positive column and −1, −2, and −3 for the negative column.
- Repeat for all remaining frame sections (Long(1–4)Out, Long(1–4)Int, FracOut, FracInt, Trans, and TransEnd).



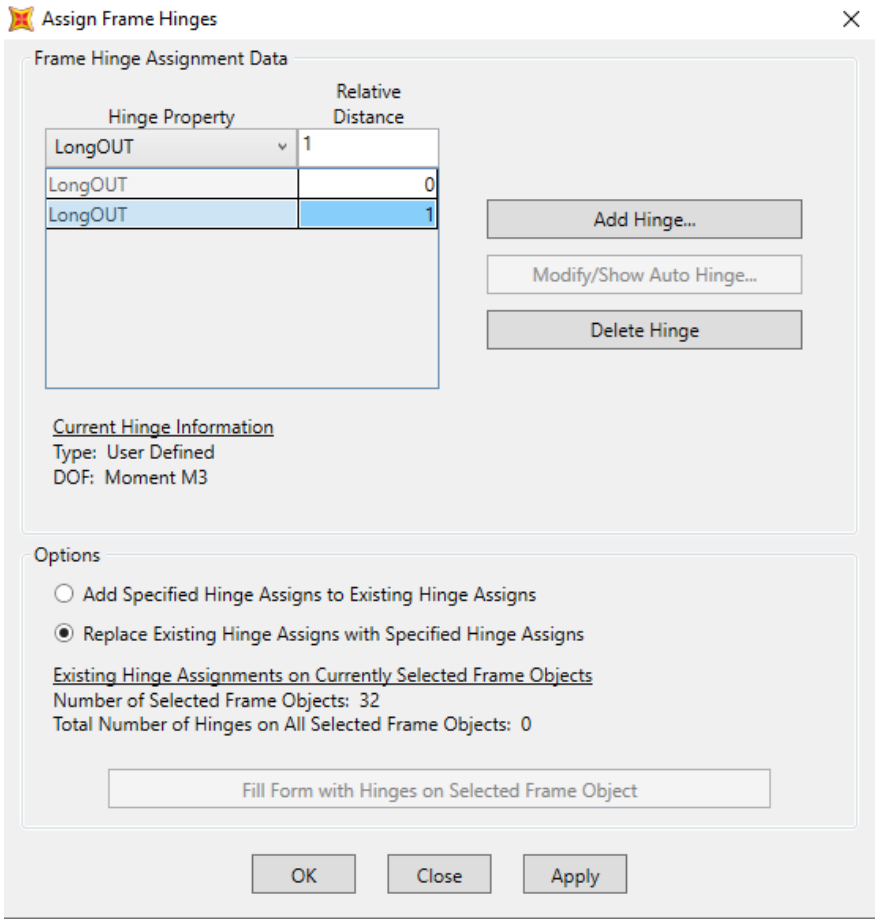
9. Assign frame members to grid.
   A. Select the Draw tab > Quick Draw Frame/Cable/Tendon.

B. In the Section drop-down menu, select Long1Out.
C. Click on every grid segment on the second-to-last longitudinal grids (B and E) from Transverse Grids 1–15 and 28–41.
D. Change the section to Long1Int and repeat Step C but for the two interior longitudinal grids (C and D).
E. In the Section drop-down menu, select Long2Out.
F. Click on every grid segment on the second-to-last longitudinal grids (B and E) from Transverse Grids 15–18 and 25–28.
G. Change the section to Long2Int and repeat Step E but for the two interior longitudinal grids (C and D).
H. In the Section drop-down menu, select Long3Out.
I. Click on every grid segment on the second-to-last longitudinal grids (B and E) from Transverse Grids 18–20 and 23–25.
J. Change the section to Long3Int and repeat Step I but for the two interior longitudinal grids (C and D).
K. In the Section drop-down menu, select Long4Out.
L. Click on every grid segment on the second-to-last longitudinal grids (B and E) from Transverse Grids 20–23.
M. Change the Section to LongInt and repeat Step L but for the two interior longitudinal grids (C and D).
N. Change the Section to TransEnd and repeat Step C but for the end transverse grids (1 and 41).
O. Change the Section to Trans and repeat Step C but for all other transverse grids (2–40).

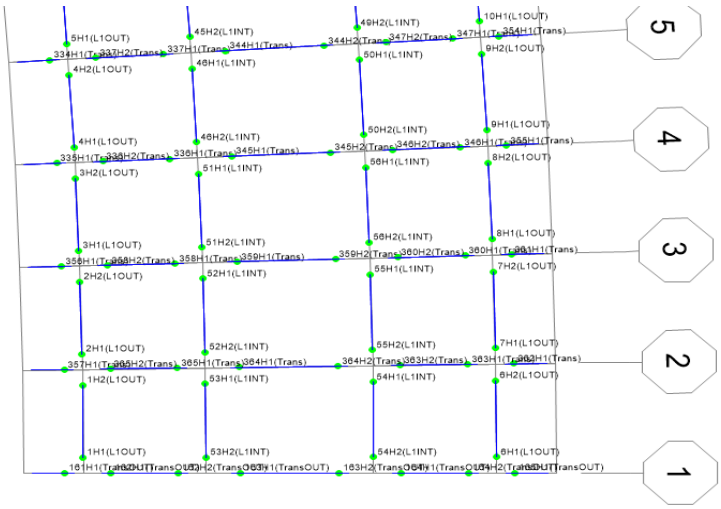


10. Assign hinges to frame elements.
    A. The longitudinal hinges are placed at the ends of the longitudinal frame elements or at a relative distance of 0 and 1.
    B. The transverse hinges are placed at a distance of half a top flange width away from the node.
        - $Hinge\ Loc._{F\ to\ E} = \frac{half\ flange\ width}{Element\ Width} = 1 - \frac{\frac{18}{2}}{40.5} = 0.7778.$
        - $Hinge\ Loc._{E\ to\ D\ and\ C\ to\ B} = \frac{half\ flange\ width}{Element\ Width} = \frac{18/2}{83} = 0.1084\ and\ (1 - 0.1084)\ or\ 0.8916.$
        - $Hinge\ Loc._{D\ to\ C} = \frac{half\ flange\ width}{Element\ Width} = \frac{18/2}{113} = 0.0796\ and\ (1 - 0.0796)\ or\ 0.9204.$
        - $Hinge\ Loc._{B\ to\ A} = \frac{half\ flange\ width}{Element\ Width} = \frac{18/2}{40.5} = 0.2222.$
    C. In SAP2000, assign the hinges to corresponding frame elements.
        - Select the desired frame elements you wish to assign hinges to, such as Long1Out. (The elements will turn from blue to yellow).
        - In SAP2000, select the Assign tab > Frame > Hinges

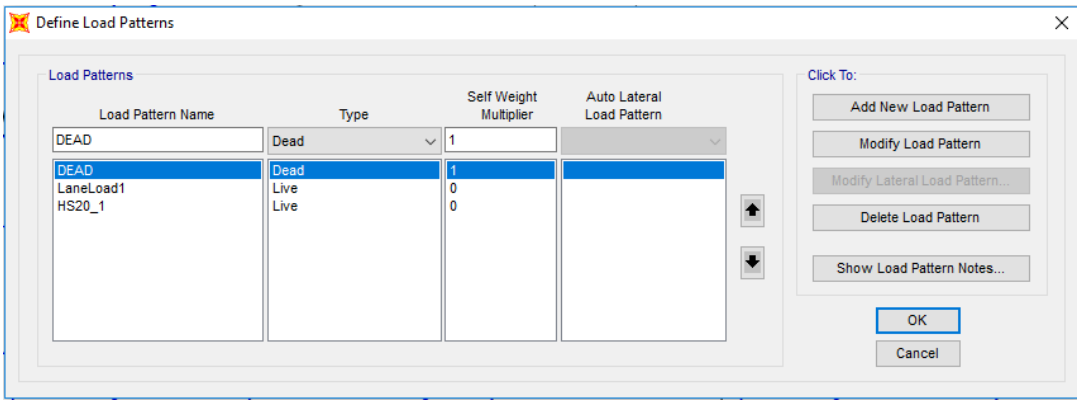
- From the drop-down menu, select Long1Out and set relative distance to 0 and click ADD.
- From the drop-down menu, select Long1Out and set relative distance to 1 and click ADD.
- Click OK.



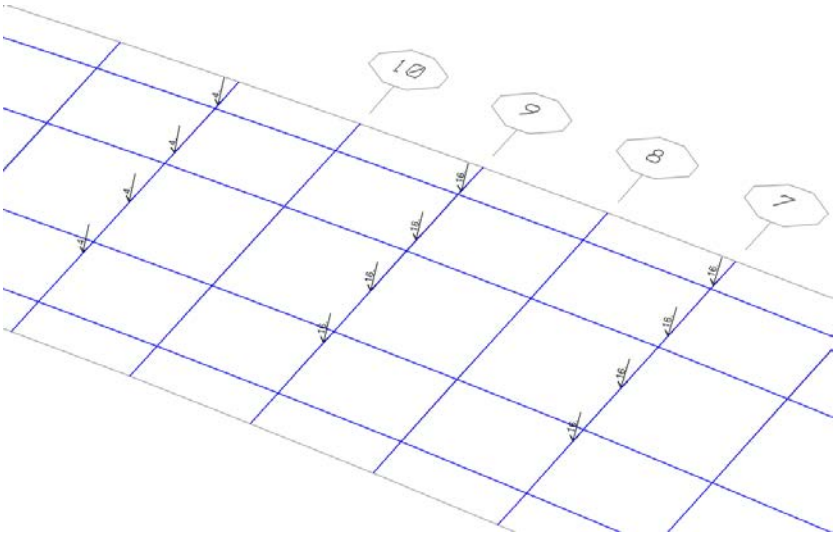
- Repeat previous step of assigning the hinges for all other frame elements.



11. Assign loads to the frame elements.
    A. Wheel Axle Loads.
        - Axle loads will be placed at distances of 36 in., 108 in., 180 in., and 252 in. from the outside of the curved edge.
        - One line of the 16 kip axles will be placed at the 0.4L point of the bridge, or Transverse Grid 9, with the second line 14 ft away at Transverse Grid 7. One line of 4 kip axles will be placed 14 ft away from the first line of axles, at Gridline 11.
        - In SAP2000, the loads have to be placed at a relative distance, so this value needs to be calculated.
            - $HS20_{Axel\ 1\ Loc\ (B-A)} = \frac{L_1-36}{L_1} = \frac{40.5-36}{40.5} = 0.1111.$
            - $HS20_{Axel\ 2\ Loc\ (C-B)} = \frac{L_1+L_2-108}{L_2} = \frac{40.5+83-108}{83} = 0.1867.$
            - $HS20_{Axel\ 3\ Loc\ (D-C)} = \frac{L_1+L_2+L_3-180}{L_3} = \frac{40.5+83+113-180}{113} = 0.5.$
            - $HS20_{Axel\ 4\ Loc\ (E-D)} = \frac{L_1+L_2+L_3+L_4-252}{L_4} = \frac{40.5+83+113+83-252}{83} = 0.8133.$
    B. Lane Loads
        - Lane Loads are lane loads of 0.640 kip/ft (0.05333 kip/in.) centered at a distance of 96 in. and 240 in. from the outside of the curved edge.
        - These lane loads will be placed on the longitudinal frame elements. They will be assigned to elements along the B, C, D, and E longitudinal elements according to the appropriate tributary distance.
            - $LaneLoad_B = \left(\frac{L_1+L_2-96}{L_2}\right) * laneload = \left(\frac{40.5+83-96}{83}\right) * 0.05333 = 0.017671\frac{kip}{in.}$.
            - $LaneLoad_C = laneload - laneload_B = 0.05333 - 0.017671 = 0.035663\frac{kip}{in.}$.
            - $LaneLoad_D = \left(\frac{L_1+L_2+L_3+L_4-240}{L_4}\right) * laneload = \left(\frac{40.5+83+113+83-240}{83}\right) * 0.05333 = 0.051084\frac{kip}{in.}$.
            - $LaneLoad_E = laneload - laneload_D = 0.05333 - 0.051084 = 0.002249\frac{kip}{in.}$.
    C. In SAP2000, the load patterns must first be defined.
        - Select the Define tab > Load Patterns.
            - Under the Load Pattern Name, enter HS20_1 and change the type in the drop-down menu to Live. The self-weight multiplier should be set to 0. Then click Add New Load Pattern.
            - Under the Load Pattern Name, enter LaneLoad1 and change the type in the drop-down menu to Live. The self-weight multiplier should be set to 0. Then click Add New Load Pattern.
            - Click OK.

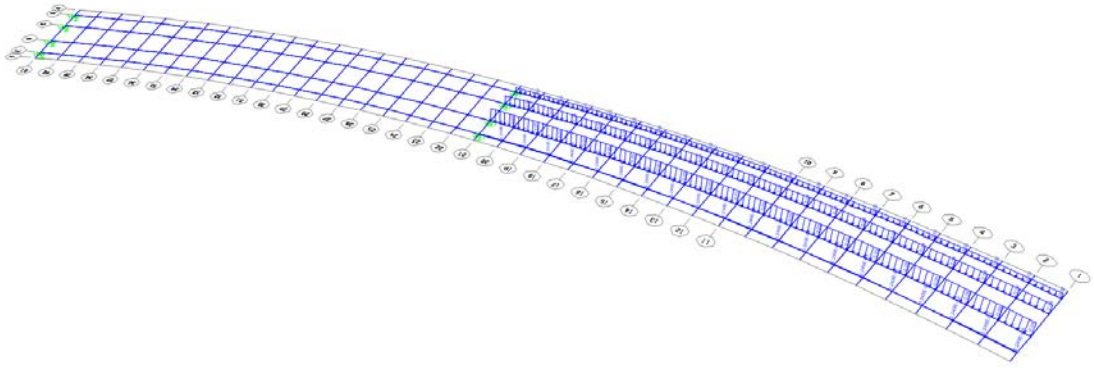


- Assign the HS-20 wheel loads in SAP2000.
    - Select the exterior transverse element of Gridlines 9 and 7.
    - Click Assign > Frame Loads > Point.
    - From the Load Pattern drop-down menu, select HS20 and verify that the coordinate system is set to Global, the Load Direction is Gravity, and the Load Type is Force.
    - In Column 1, enter a relative distance of 0 (Axle 1, Loc. B-A) and load of 16 kips.
    - Click OK.
    - Repeat for Gridline 11 to assign the 4 kip load.
    - Repeat Steps 1–6 for Axle 2, 3, and 4, Loc. C-B, D-C, and E-D.

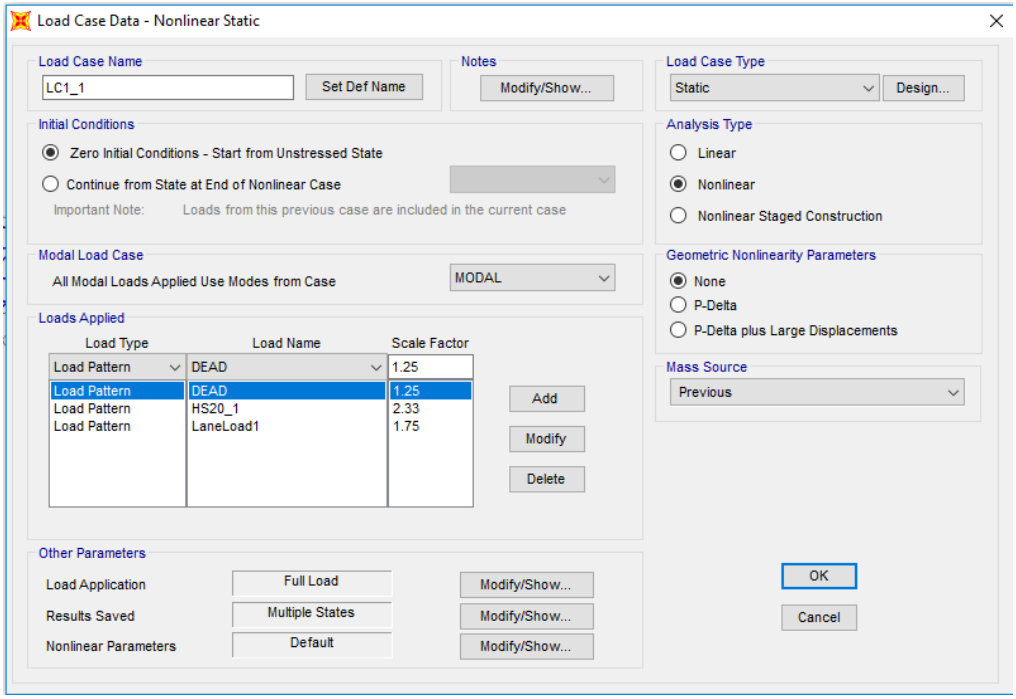


- Assign the Lane Load in SAP2000.
    - Select all exterior longitudinal frame elements along Gridline B.
    - Click Assign > Frame Loads > Distributed.
    - From the Load Pattern drop-down menu, select Lane Load and verify that the coordinate system is set to Global, the Load Direction is Gravity, and the Load Type is Force.
    - In the Uniform Load box, enter 0.016124 (Lane Load B).
    - Click OK.

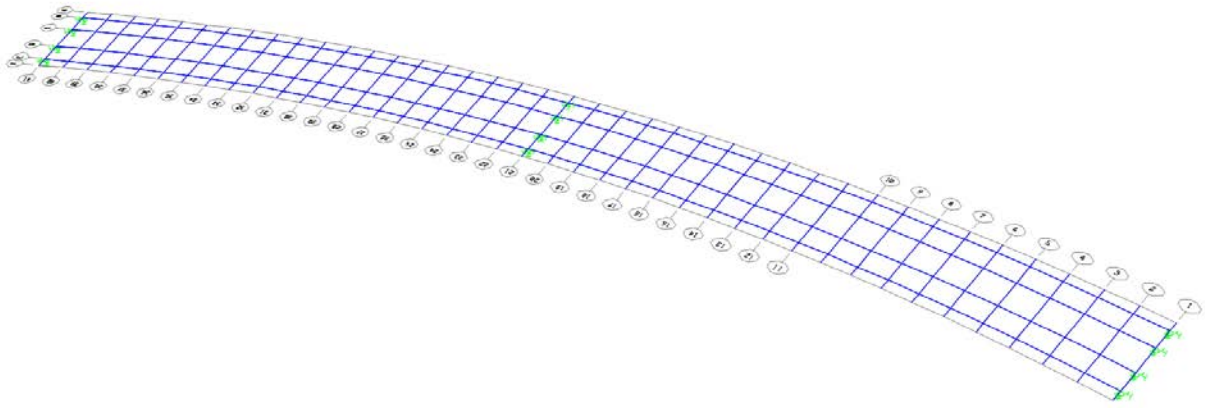
- Repeat Steps 1–5 for all of the longitudinal elements along gridlines C, D, and E.



12. Define load cases.
    A. The load case being used to determine redundancy is $1.25DL + 1.75(LL + IM)$, where DL = Dead Load, LL = Live Load, and IM = Impact Load. When substituting in the truck load and the lane load, the preceding equation reduces to $1.25DL + 1.75LaneLoad + 2.33HS20$.
    B. Generate Load Cases in SAP2000.
        - Click Define > Load Cases > Add New Load Case.
        - In the Load Case Name Panel, name the load case "LC1_1."
        - In the Analysis Type, select "Nonlinear."
        - For the LC1_1 Load Case in the Stiffness to Use panel, select "Zero Initial Conditions."
        - In the Loads Applied panel, leave the Load Type "Load Pattern" in the drop-down Load Name menu, select DEAD and change the Scale Factor to 1.25. Click Add. Change the Load Name menu to select HS20 and change the Scale Factor to 2.33. Click ADD. Change the Load Name menu to select Lane Load and change the Scale Factor to 1.75. Click ADD.
        - In the Other Parameters panel, in the Results Saved section, click Modify/Show.
        - In the Results Saved for Nonlinear Static Load Cases window, change the Results Saved to Multiple States, and in the For Each Stage panel, change the Minimum Number of Saved Steps and the Maximum Number of Saved Steps to 20. Click OK.
        - Click OK on the Load Case Data window.
        - Repeat the previous 8 steps listed under B to create an LC2_1, LC3_1, and LC4_1. However, in the Initial Conditions window, select Continue from State at End of Nonlinear Case, and from the drop-down menu select the preceding load case. (For LC2_1, the Nonlinear Case LC1_1 would be selected).

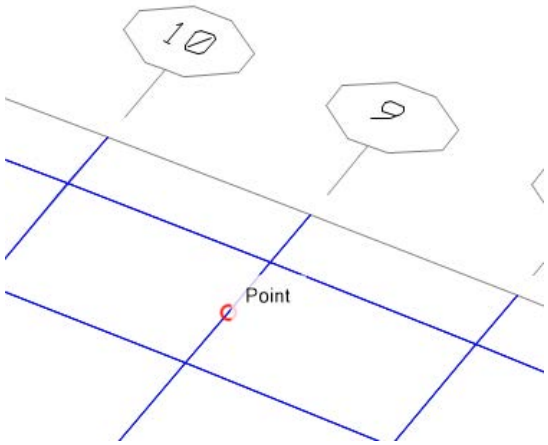


13. Define end supports.
    A. The elastomeric bearing pads for each girder have a lateral stiffness of 12 kip/in. and a vertical stiffness of 6100 kip/in. Since the tub girders are divided in half, the lateral stiffness will be 6 kip/in., and the vertical stiffness will be 3050 kip/in.
    B. Assign spring supports in SAP2000.
        - Select the 12 nodes, eight at the very end of the longitudinal members, four at the location of the pier.
        - Click Assign > Joint > Springs.
        - In the Assign Joint Springs window in the Simple Springs Stiffness panel, enter 6 for Translation 1 and 2 and 3050 for Translation 3.
        - Click OK.



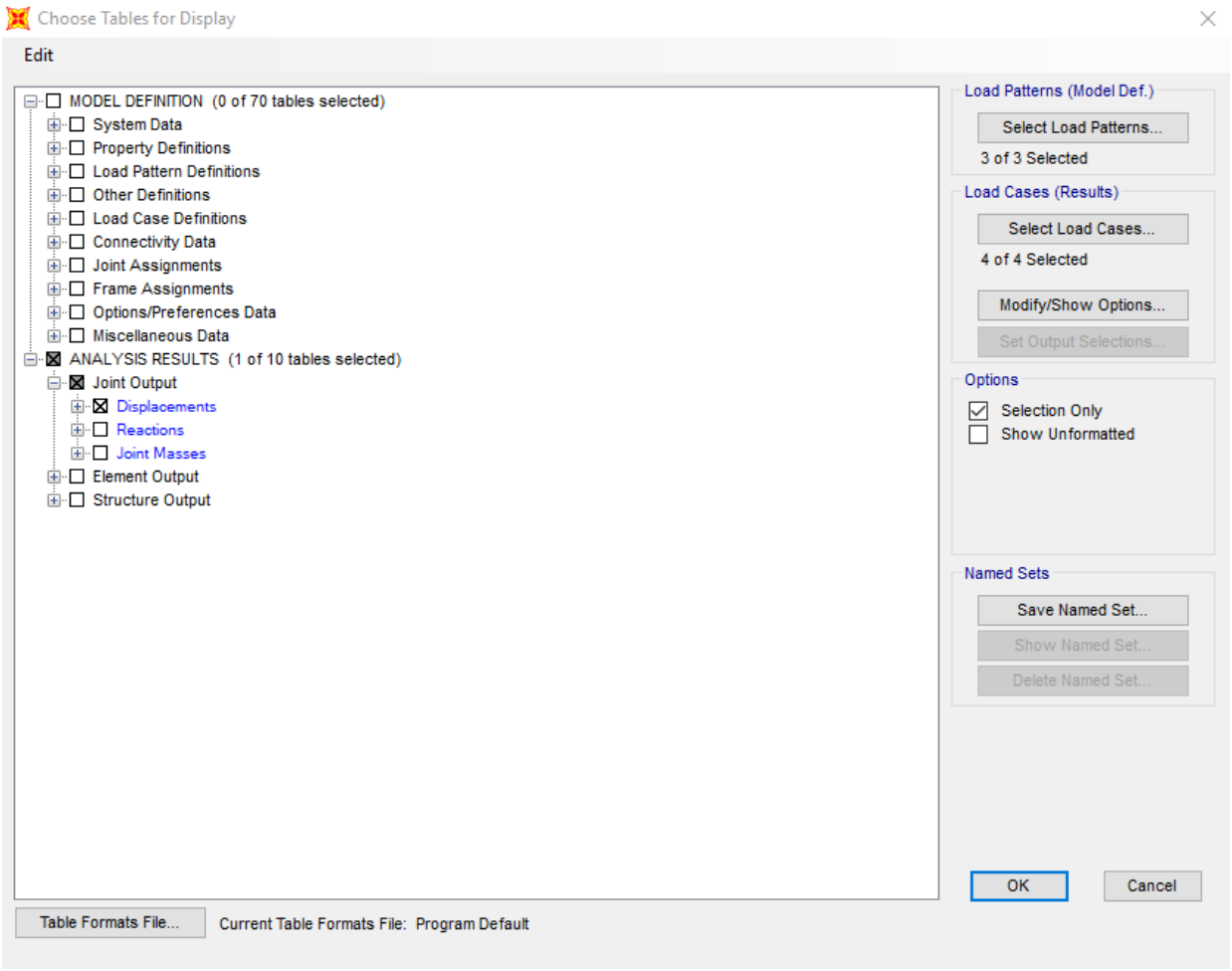
14. Define CL data acquisition points mid-span of girder.

A. Select the transverse frame elements between B and C and between D and E at 0.4L (Gridline 9).
B. Click Edit > Edit Lines > Divide Frames.
C. In the Divide into Specified Number of Frames window, enter 2 for Number of Frames.
D. Click OK.

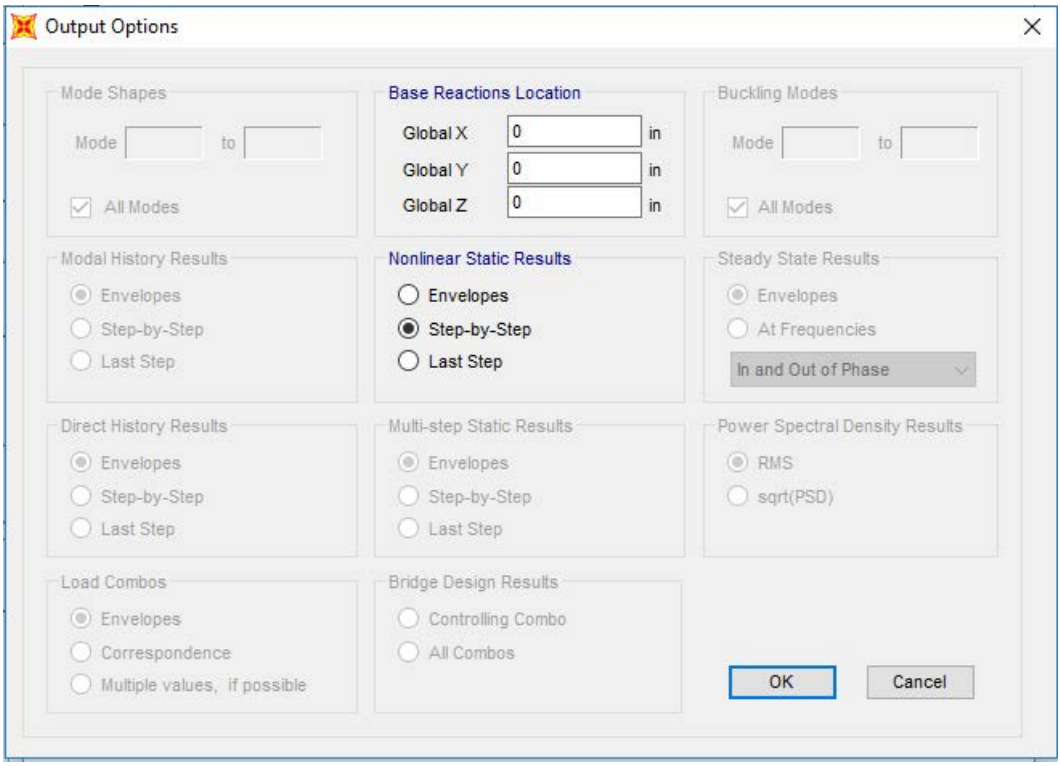


15. Analyze the nonfracture structure for dead load only.
    A. In SAP2000, click Analyze > Run Analysis
    B. In the Set Load Cases to Run window, click the Run/Do Not Run All button until every action is Do Not Run.
    C. Select DEAD then click Run/Do Not Run Case until the action is run.
    D. Click Run Now. Let SAP2000 Run the Load Cases until the screen says the analysis is complete.
    E. Once the analysis is complete, select Spring Reactions.
    F. Click Display > Show Tables.
    G. In the Choose Table for Display window, click the + symbol beside Joint Output and select the square box beside Reactions.
    H. In the Output Options window in the Nonlinear Static Results panel, select Last Step.
    I. Select and copy the information from the F3 column.
    J. The sum of the F3 values is the dead load.
    K. Click Done.
    L. Unlock the structure.
16. Analyze the nonfractured structure.
    A. In SAP2000, click Analyze > Run Analysis.
    B. In the Set Load Cases to Run window, click the run/Do Not Run All button until every action is Do Not Run.
    C. Select LC1_1, LC2_1, LC3_1, and LC4_1 then click Run/Do Not Run Case until the action for all four is run.
    D. Click Run Now.

E. Let SAP2000 Run the Load Cases until the screen says the analysis is complete.
F. Once the analysis is complete, select the data collection point on the transverse member on the outside girder (C-B).
G. Click Display > Show Tables.
H. In the Choose Table for Display window, click the + symbol beside Joint Output and select the square box beside Displacements.

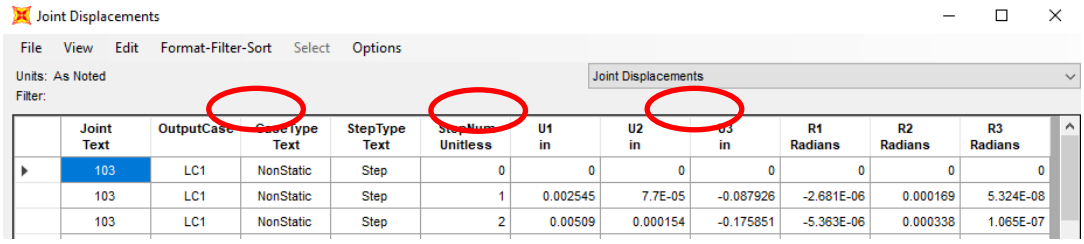


I. Click the Modify/Show Options button.
J. In the Output Options window in the Nonlinear Static Results panel, select Step-by-Step.



K. Click OK.

L. Select and copy the information from the Output Case, StepNum Unitless, and the U3 in. column and paste them into an Excel worksheet. These columns represent Load Case, Step Number, and Deflection for the Outside Girder, respectively.



Joint Text	OutputCase	CaseType Text	StepType Text	StepNum Unitless	U1 in	U2 in	U3 in	R1 Radians	R2 Radians	R3 Radians
103	LC1	NonStatic	Step	0	0	0	0	0	0	0
103	LC1	NonStatic	Step	1	0.002545	7.7E-05	-0.087926	-2.681E-06	0.000169	5.324E-08
103	LC1	NonStatic	Step	2	0.00509	0.000154	-0.175851	-5.363E-06	0.000338	1.065E-07

M. Click Done.

N. Select the data collection point on the transverse member on the inside girder (E-D) and repeat Steps G–L. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

O. Select the joint on the transverse member on the outside girder at Transverse Element 9 (at Longitudinal Element B) and repeat Steps G–L. This information goes into the Delta 4 column. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

P. Select the joint on the transverse member on the outside girder at Transverse Element 9 (at Longitudinal Element C) and repeat Steps G–L. This information goes into the Delta 3 column. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

Q. Select the joint on the transverse member on the inside girder at Transverse Element 9 (at Longitudinal Element D) and repeat Steps G–L. This information goes into the Delta 2 column. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

R. Select the joint on the transverse member on the outside girder at Transverse Element 9 (at Longitudinal Element E) and repeat Steps G–L. This information goes into the Delta 1 column. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

	A	B	C	D	E	F	G	H	I	J	K	L	M	V	X	Y	Z	AA	AB	AC	AD
1	Intact			OG-CL		IG-CL		Point 4	Point 3	Point 2	Point 1			Applied		OG		IG			
2	Load Case	Load Step	Ω	Delta (in)	Chord (rad)	Delta (in)	Chord (rad)	Delta (in)	Delta (in)	Delta (in)	Delta (in)	α 23 (rad)	α 32 (rad)	Load (kip)	Ω (cal)	Delta (in)	Chord (deg)	Delta (in)	Chord (deg)	α 23 (deg)	α 32 (deg)
3	LC1	0	0	0	0.00000	0	0.00000	0	0	0	0	0	0	0	0.00	0.00	0.00000	0.00	0.00000	0	0
4	LC1	1	0.05	-0.1037	0.00015	-0.087926	0.00013	-0.10541	-0.10129	-0.0942	-0.07905	7.89E-05	4.94E-05	66	0.05	0.10	0.00861	0.09	0.00730	0.004522	0.002831
5	LC1	2	0.1	-0.2074	0.00030	-0.175851	0.00025	-0.21081	-0.20259	-0.18839	-0.15809	0.000158	9.88E-05	132	0.10	0.21	0.01722	0.18	0.01460	0.009043	0.005662

S. Once all the data are collected, unlock the model by selecting the Lock tool on the left hand side of the SAP2000 screen.

17. Analyze the fractured structure.

A. At mid-span along Gridline 9, replace the hinges of the outside longitudinal element (Gridline B) with FracOUT hinges according to Step 10.

B. At mid-span along Gridline 9, replace the hinges of the first interior longitudinal element (Gridline C) with FracInt hinges according to Step 10.

C. Repeat Step 15 for the fractured case and collect the data accordingly.

18. Post-process the data.

A. In the Excel Sheet, the following values need to be calculated for each step.

- Omega (Ω):

$$\Omega_i = \Omega_{i-1} + \left(\frac{1}{\#\ of\ Steps\ in\ Load\ case}\right).$$

- Longitudinal Chord Rotation of Interior and Exterior Girder:

$$Chord\ Rot._{Single\ Span} = -1 * \left(\frac{\delta_{CL}}{0.5*L}\right) \text{ (rad).}$$

- Transverse Deck Rotation:
  - $\alpha_{2-3} = \left(\frac{\delta_3 - \delta_2}{s}\right) - \left(\frac{\delta_2 - \delta_1}{w}\right)$ (rad).
  - $\alpha_{2-3} = \left(\frac{\delta_3 - \delta_2}{s}\right) - \left(\frac{\delta_2 - \delta_1}{w}\right)$ (rad).
  - Where s = spacing between the interior top flanges of the inside and outside girders and w = spacing between the top flanges of the same girder.
- Applied Load:
  - Calculate unit applied load or applied load at 1 Ω.
  - $Unit\ Applied\ Load_{single\ span} = 1.25 * Total\ Reactions\ from\ Dead\ Load\ Case + 2 * (2.33 * HS20\ truck + 1.75 * Lane\ Load)$.
  - $Applied\ Load = Unit\ Applied\ Load * \Omega$.

B. Repeat Step A for the fractured case.

C. Calculate the initial stiffness for the intact bridge and instantaneous stiffness for the fractured bridge.
   - For the nonfractured condition (intact bridge), find the absolute displacement for the outside girder at an Ω value of 0.4:

$$Initial\ Stiffness = \frac{0.4}{Absolute\ Displacement\ OG\ (at\ \ \Omega = 0.4)}.$$

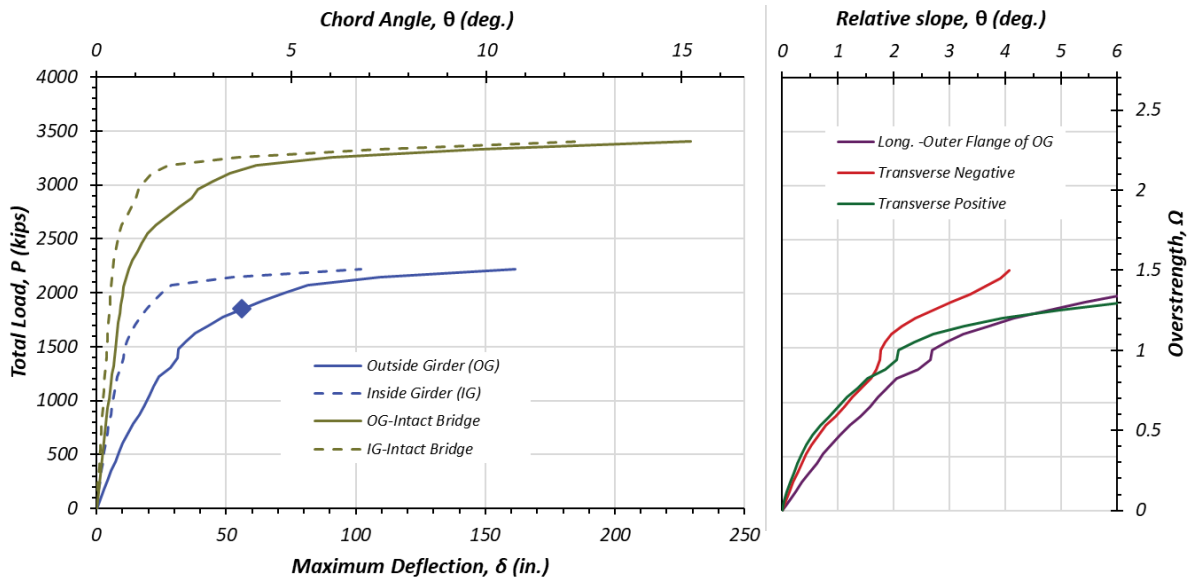
   - For the Fractured case, add an additional column labeled stiffness:

$$Instantaneous\ Stiffness_{OG-Frac.i} = \frac{\Omega_i - \Omega_{i-1}}{\delta_i - \delta_{i-1}}.$$

D. Failure of the structure occurs at the Ω of the fractured bridge at the first of the following criteria:
   - The instantaneous stiffness for the fractured outside girder is less than 5 percent of the initial stiffness of the intact outside girder.
   - The chord angle of the outside girder for a simple spans or interior spans is greater than 2 degrees. The chord angle for exterior spans of multi-span bridges is greater than 3 degrees.
   - The transverse deck rotation is greater than 5 degrees.

E. On a chart, plot the nonfractured outside and inside girder as well as the fractured outside and inside girder with displacement on the primary *x* axis and the total force on the primary *y* axis and Ω on the secondary *y* axis.

F. Repeat Steps 11–18 for Span 2, if a different length. For Bridge 5, both Span 1 and Span 2 are 140 ft long, so no repletion is needed.

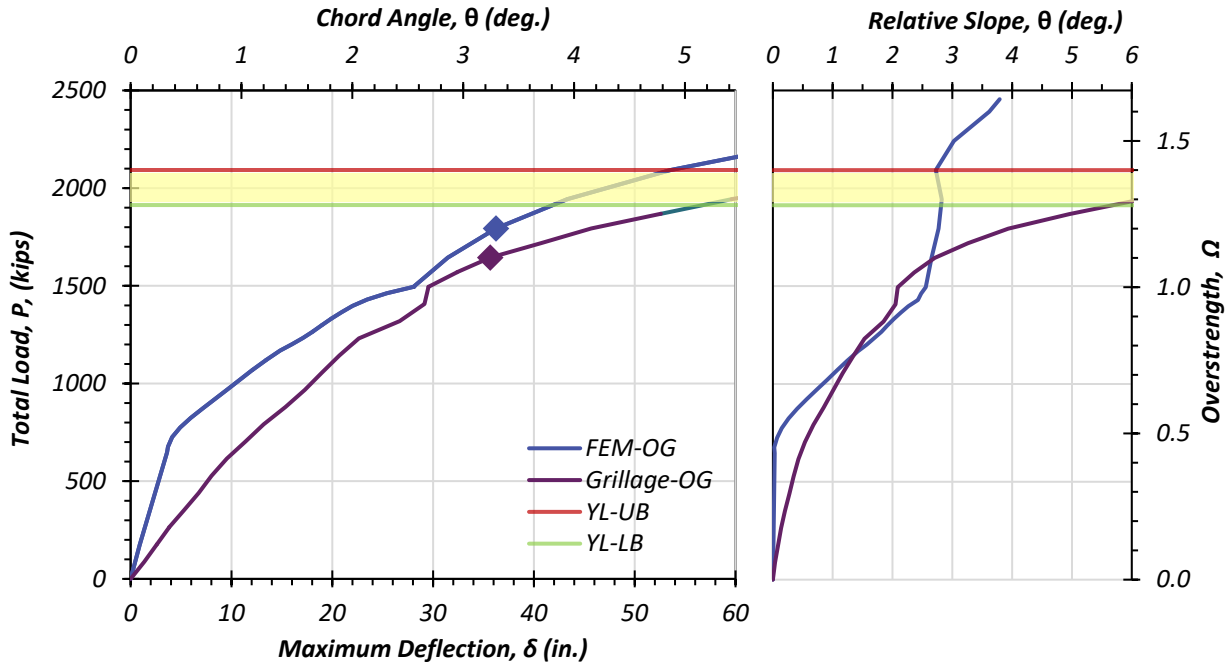


6.3 OVERALL RESULTS FOR BRIDGE 5

Table 6.1 and Figure 6.2 depicts the overall results for Bridge 5, comparing all the three methods using the upper-bound and lower-bound solution range of the yield line theory and the grillage overstrength (load) versus deflection plots. FEM data are also shown in Table 6.1 and in Figure 6.2 to facilitate a thorough comparison. All three methods suggest that Bridge 5 could be reclassified as nonfracture critical.

Table 6.1. Results Summary of Bridge 5.

Method of Analysis	Overstrength Factors	Span 1 (140 ft)	Span 2 (139.60 ft)
Yield Line Theories	$\Omega_{Yield\ Line}^{Upper\ Bound}$	1.40	1.39
	$\Omega_{Yield\ Line}^{Lower\ Bound}$	1.28	1.28
Grillage Analysis	$\Omega_{Grillage}$	1.10	1.10
FEM	Ω_{FEM}	1.20	1.20



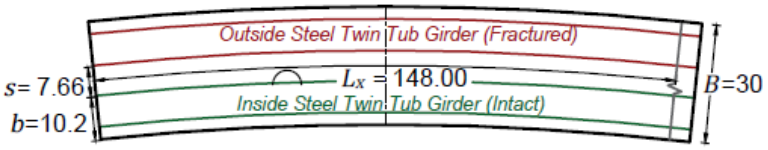
(i) Load displacement *(ii) Deck rotations*

Figure 6.2. Comparison of the Results for Bridge 5, Span 1 and 2, L_x = 140 ft.

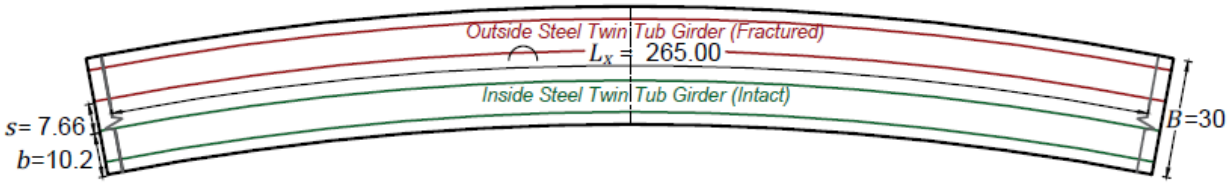
7. BRIDGE 10

7.1 YIELD LINE ANALYSIS EXAMPLE OF BRIDGE 10

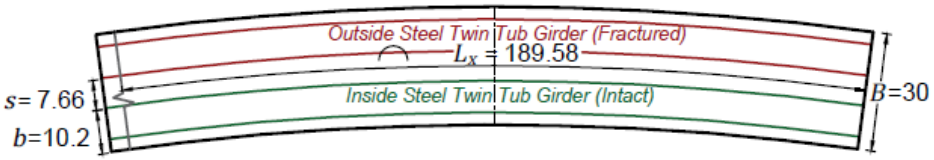
This section documents the steps to compute the overstrength factor of Bridge 10 using the yield line analysis (grillage analysis is explained in the next sub-section). Figure 7.1 presents the dimensional details of the three spans of Bridge 10. Further details may be found in Appendix C.



(a) Bridge 10, Span 1 (L_x = 148 ft)



(b) Bridge 10, Span 2 (L_x = 265 ft)



(c) Bridge 10, Span 3 (L_x = 189.58 ft)

Figure 7.1. Schematic Diagrams of Bridge 10 (R_L = 716 ft).

This section presents the stepwise procedure of the yield line analysis conducted to establish the upper-bound and lower-bound solution range for the overstrength factor using the procedure of Section 3. The moment capacities are computed using the methods explained in Section 5 and 6. It should be noted that this bridge consists of two typical exterior spans with one support fixity and one interior span with fixity at both the supports.

	MOMENT CAPACITY OF DECK SLAB (BRIDGE 10)	1 / 14

		Description	Parameter	Data
DECK SECTION	1	Total Width	B in	**360**
	2	Thickness	t in	**8**
	3	Section width	b in	12
CONCRETE MATERIAL	4	Characteristic Compressive Strength	f_c' ksi	4
	5		β_1	0.85
	6	Strain	ε_{cu}	0.003
REBAR MATERIAL	7	Yield Strength	f_y ksi	60
	8	Young's Modulus	E ksi	29000
	9	Strain	ε_y	0.002070

		Moment Computations		Longitudinal		Transverse	
	10			m_x'	m_x	m_y'	m_y
	11			Top	Bottom	Top	Bottom
REINFORCING BAR DETAIL	12	Bar No.		**4**	**5**	**5**	**5**
	13	Diameter of Bar	d_b in	0.5	0.625	0.625	0.625
	14	Area of Bar	A_Φ in^2	0.2	0.31	0.31	0.31
	15	Spacing	s in	-	-	**6**	**6**
	16	No. of Bars	$\#_{bar}$	**41**	**33**	-	-
	17	Area of Steel	A_s in^2	0.273	0.341	0.620	0.620
	18	Clear Cover	cc in	**2**	**1.25**	**2**	**1.25**
	19	Effective depth (tension)	d in	5.125	5.8125	5.6875	6.4375
	20	Effective depth (comp.)	d' in	2.1875	2.875	1.5625	2.3125
	21	Depth of NA	c in	**1.06**	**1.06**	**1.34**	**1.67**

		MOMENT CAPACITY OF DECK SLAB (BRIDGE 10)					2 / 14
		Moment Computations		Longitudinal		Transverse	
	22			m'_x	m_x	m'_y	m_y
	23			Top	Bottom	Top	Bottom
STRAIN	24		ε_{top}	0.011466	0.005115	0.009770	0.001153
	25		ε_{bot}	0.003174	0.013406	0.000508	0.008561
	26	$\beta_1 c$	a in	0.90	0.90	1.14	1.42
FORCES	27	Compression-Concrete	C_c kip	−36.86	−36.86	−46.34	−57.93
	28	Tension/ Compression-Steel	T_{top} kip	16.40	16.40	37.20	20.73
	29		T_{bot} kip	20.46	20.46	9.14	37.20
	30	Equilibrium Check	$\sum T + C$ kip	**0.00**	**0.00**	**0.00**	**0.00**
RESULTS	31	Moment	M_n k-ft	**9.35**	**12.45**	**16.63**	**20.52**

Remarks:

Top Yielded	Top Yielded	Top Yielded	Top Not Yielded
Bottom Yielded	Bottom Yielded	Bottom Not Yielded	Bottom Yielded
Both steel in Tension	Both steel in Tension	Both steel in Tension	Both steel in Tension

		POSITIVE AND NEGATIVE MOMENT OF INTACT SECTION (BRIDGE 10, SPAN 1)			3 / 14	
			Moment			
		Description	Positive		Negative	
DECK SECTION	1	Total width	B in	**180**	B in	180
	2	Thickness	t in	**8**	t in	**8**
	3	Haunch thickness	h in	**5**	h in	**5**
CONCRETE MATERIAL	4	Characteristic Compressive Strength	f'_c ksi	4		
	5		β_1	0.85		
	6	Strain	ε_{cu}	0.003		
REINFORCING BAR DETAILS	7	Yield Strength	f_y ksi	60	f_y ksi	60
	8	Strain	Rebar ε_y	0.002070		
	9	No of Bars at Top	# top bars	**20.5**	# top bars	**20.5**
	10	No of Bars at Top Bent			# top bars bent	**20**
	11	No of Bars at Bottom	# bot bars	**16.5**	# bot bars	**16.5**
	12	Clear Cover Top	cc_{top} in	**2**	cc_{top} in	**2**
	13	Clear Cover Bottom	cc_{bot} in	**1.25**	cc_{bot} in	**1.25**
	14	Transverse Top Diameter	$d_{b\ top\ tra}$ in	**0.625**	$d_{b\ top\ tra}$ in	**0.625**
	15	Transverse Bottom Diameter	$d_{b\ bot\ tra}$ in	**0.625**	$d_{b\ bot\ tra}$ in	**0.625**
	16	Longitudinal Top Diameter	$d_{b\ top\ long}$ in	**0.5**	$d_{b\ top\ long}$ in	**0.5**
	17	Longitudinal Top Diameter at Bents			$d_{b\ top\ long}$' in	**0.625**
	18	Longitudinal Bottom Diameter	$d_{b\ bot\ long}$ in	**0.625**	$d_{b\ bot\ long}$ in	**0.625**
	19	Effective Depth Top	d_{top} in	2.88	d_{top} in	2.88
	20				$d_{top\ bent}$ in	2.94
	21	Effective Depth Bottom	d_{bot} in	5.81	d_{bot} in	5.81
	22	Area of Steel for #4 Bars	$A_{\#4}$ in^2	0.196	$A_{\#4}$ in^2	0.196
	23	Area of Steel for #5 Bars	$A_{\#5}$ in^2	0.307	$A_{\#5}$ in^2	0.307
STRUCTURAL STEEL	24	Yield Strength	F'_y ksi	50	F'_y ksi	50
	25	Strain	ε_s	0.00172		
	26	Young's Modulus	E ksi	29000		
	27	f_y/F'_y	n	1.2	n	1.2
	28	Effective Area of Steel for #4 Bars	$A_{\#4.\ eff}$ in^2	0.236	$A_{\#4.\ eff}$ in^2	0.236
	29	Effective Area of Steel for #5 Bars	$A_{\#5.\ eff}$ in^2	0.368	$A_{\#5.\ eff}$ in^2	0.368

		POSITIVE AND NEGATIVE MOMENT OF INTACT SECTION (BRIDGE 10, SPAN 1)				4 / 14
			Moment			
		Description	Positive		Negative	
GIRDER DETAILS	30	Girder Depth	D_g in	80	D_g in	83
	31	Top Flange Width	b_{tf} in	**24**	b_{tf} in	**24**
	32	Top Flange Thickness	t_{tf} in	**1**	t_{tf} in	**3**
	33	Web Width	b_w in	**78**	b_w in	**78**
	34	Web Thickness	t_w in	**0.625**	t_w in	**0.9**
	35	Bottom Flange Width	b_{bf} in	**59**	b_{bf} in	**59**
	36	Bottom Thickness	t_{bf} in	**1.25**	t_{bf} in	**2**
DEPTHS	37	Compression Zone	c in	**30.14**	c in	**53.65**
	38	b_w in Comp.	y in	16.14	y in	51.65
	39	b_w in Tension	y' in	61.86	y' in	26.35
DISTANCES	40	Distances of Compressive Forces and Tensile Forces from Neutral Axis/ Distances of Areas in Tension and Compression from PNA	$d\ C_{conc}$ in	17.33	$d\ A_{rebar\ top}$ in	39.48
	41		$d\ C_{rebar\ top}$ in	27.26	$d\ A_{rebar\ top\ bent}$ in	39.41
	42		$d\ C_{rebar\ bottom}$ in	24.32	$d\ A_{rebar\ bot}$ in	36.54
	43		$d\ C_{tf1}$ in	16.64	$d\ A_{tf}$ in	27.85
	44		$d\ T_{w\ 1}$ in	8.07	$d\ A_{w\ 1}$ in	13.18
	45		$d\ T_{w\ 2}$ in	30.93	$d\ A_{w\ 2}$ in	25.82
	46		$d\ T_{bf}$ in	62.49	$d\ A_{bf}$ in	52.65
STRAIN	47		$\varepsilon\ C_{rebar\ top}$	0.00271		
	48		$\varepsilon\ C_{rebar\ bottom}$	0.00242		
	49		$\varepsilon\ C_{tf}$	0.00166		
	50		$\varepsilon\ C_{w\ 1}$	0.00080		
	51		$\varepsilon\ T_{w\ 2}$	0.00308		
	52		$\varepsilon\ T_{bf}$	0.00622		

		POSITIVE AND NEGATIVE MOMENT OF INTACT SECTION (BRIDGE 10, SPAN 1)					5 / 14
			Moment				
		Description	Positive		Negative		
FORCE/AREA	53	Compression F_C/ Tension A_T	$0.85f_c'\beta bt$ kip	−4162	$A_{s\,top}$ kip	5	
	54				$A_{s\,top\,bent}$ kip	7	
	55		$A_{s\,top}f_y$ kip	−273	$A_{s\,bottom}$ kip	6	
	56		$A_{s\,bottom}f_y$ kip	−344	A_{tf} kip	144	
	57		$2b_{tf}t_{tf}F'_y$ kip	−2305	$A_{w\,1}$ kip	46	
	58		$2t_wyF'_y$ kip	−469.82			
	59	Tension F_T/ Compression A_C	$2t_wy'F'_y$ kip	3866	$A_{w\,2}$ kip	−90	
	60		$b_{bf}t_{bf}F'_y$ kip	3688	A_{bf} kip	−118	
CHECK	61	Equilibrium of Forces/Areas	F_C kip	−7554	A_T kip	208	
	62		F_T kip	7554	A_C kip	-208	
	63		$F_C + F_T$ kip	**0**	$A_T + A_C$ kip	**0**	
MOMENT	64	(Half Section)	$0.5\,M_p^+$ k-ft	40008	$0.5\,M_p^-$ k-ft	58368	
	65	(Full Section)	M_p^+ k-ft	**80016**	M_p^- k-ft	**116736**	

		POSITIVE AND NEGATIVE MOMENT OF INTACT SECTION (BRIDGE 10, SPAN 3)				6 / 14
			Moment			
		Description	Positive		Negative	
DECK SECTION	1	Total width	B in	**180**	B in	180
	2	Thickness	t in	**8**	t in	**8**
	3	Haunch thickness	h in	**5**	h in	**5**
CONCRETE MATERIAL	4	Characteristic Compressive Strength	f_c' ksi	4		
	5		β_1	0.85		
	6	Strain	ε_{cu}	0.003		
REINFORCING BAR DETAILS	7	Yield Strength	f_y ksi	60	f_y ksi	60
	8	Strain	Rebar ε_y	0.002070		
	9	No of Bars at Top	# top bars	**20.5**	# top bars	**20.5**
	10	No of Bars at Top Bent			# top bars bent	**20**
	11	No of Bars at Bottom	# bot bars	**16.5**	# bot bars	**16.5**
	12	Clear Cover Top	cc_{top} in	**2**	cc_{top} in	**2**
	13	Clear Cover Bottom	cc_{bot} in	**1.25**	cc_{bot} in	**1.25**
	14	Transverse Top Diameter	$d_{b\ top\ tra}$ in	**0.625**	$d_{b\ top\ tra}$ in	**0.625**
	15	Transverse Bottom Diameter	$d_{b\ bot\ tra}$ in	**0.625**	$d_{b\ bot\ tra}$ in	**0.625**
	16	Longitudinal Top Diameter	$d_{b\ top\ long}$ in	**0.5**	$d_{b\ top\ long}$ in	**0.5**
	17	Longitudinal Top Diameter at Bents			$d_{b\ top\ long}'$ in	**0.625**
	18	Longitudinal Bottom Diameter	$d_{b\ bot\ long}$ in	**0.625**	$d_{b\ bot\ long}$ in	**0.625**
	19	Effective Depth Top	d_{top} in	2.88	d_{top} in	2.88
	20				$d_{top\ bent}$ in	2.94
	21	Effective Depth Bottom	d_{bot} in	5.81	d_{bot} in	5.81
	22	Area of Steel for #4 Bars	$A_{\#4}$ in^2	0.196	$A_{\#4}$ in^2	0.196
	23	Area of Steel for #5 Bars	$A_{\#5}$ in^2	0.307	$A_{\#5}$ in^2	0.307
STRUCTURAL STEEL	24	Yield Strength	F'_y ksi	50	F'_y ksi	50
	25	Strain	ε_s	0.00172		
	26	Young's Modulus	E ksi	29000		
	27	f_y/F'_y	n	1.2	N	1.2
	28	Effective Area of Steel for #4 Bars	$A_{\#4.\ eff}$ in^2	0.236	$A_{\#4.\ eff}$ in^2	0.236
	29	Effective Area of Steel for #5 Bars	$A_{\#5.\ eff}$ in^2	0.368	$A_{\#5.\ eff}$ in^2	0.368

		POSITIVE AND NEGATIVE MOMENT OF INTACT SECTION (BRIDGE 10, SPAN 3)			7 / 14	
			Moment			
		Description	Positive		Negative	
GIRDER DETAILS	30	Girder Depth	D_g in	79.75	D_g in	83.5
	31	Top Flange Width	b_{tf} in	**24**	b_{tf} in	**24**
	32	Top Flange Thickness	t_{tf} in	**1**	t_{tf} in	**3**
	33	Web Width	b_w in	**78**	b_w in	**78**
	34	Web Thickness	t_w in	**0.625**	t_w in	**0.9**
	35	Bottom Flange Width	b_{bf} in	**59**	b_{bf} in	**59**
	36	Bottom Thickness	t_{bf} in	**0.75**	t_{bf} in	**2.5**
DEPTHS	37	Compression Zone	c in	**22.18**	c in	**45.72**
	38	b_w in Comp.	y in	8.18	y in	43.22
	39	b_w in Tension	y' in	69.82	y' in	34.78
DISTANCES	40	Distances of Compressive Forces and Tensile Forces from Neutral Axis/ Distances of Areas in Tension and Compression from PNA	$d\,C_{conc}$ in	12.75	$d\,A_{rebar\ top}$ in	47.91
	41		$d\,C_{rebar\ top}$ in	19.30	$d\,A_{rebar\ top\ bent}$ in	47.84
	42		$d\,C_{rebar\ bottom}$ in	16.36	$d\,A_{rebar\ bot}$ in	44.97
	43		$d\,C_{tf}$ in	8.68	$d\,A_{tf}$ in	36.28
	44		$d\,T_{w\ 1}$ in	4.09	$d\,A_{w\ 1}$ in	17.39
	45		$d\,T_{w\ 2}$ in	34.91	$d\,A_{w\ 2}$ in	21.61
	46		$d\,T_{bf}$ in	70.20	$d\,A_{bf}$ in	44.47
STRAIN	47		$\varepsilon\,C_{rebar\ top}$	0.00261		
	48		$\varepsilon\,C_{rebar\ bottom}$	0.00221		
	49		$\varepsilon\,C_{tf}$	0.00117		
	50		$\varepsilon\,C_{w\ 1}$	0.00055		
	51		$\varepsilon\,T_{w\ 2}$	0.00472		
	52		$\varepsilon\,T_{bf}$	0.00950		

		POSITIVE AND NEGATIVE MOMENT OF INTACT SECTION (BRIDGE 10, SPAN 3)				8 / 14
			Moment			
		Description	Positive		Negative	
FORCE/AREA	53	Compression F_C/ Tension A_T	$0.85f'_c \beta bt$ kip	−4162	$A_{s\ top}$ kip	5
	54				$A_{s\ top\ bent}$ kip	7
	55		$A_{s\ top}f_y$ kip	−273	$A_{s\ bottom}$ kip	6
	56		$A_{s\ bottom}f_y$ kip	−344	A_{tf} kip	144
	57		$2b_{tf}t_{tf}F'_y$ kip	−1634	$A_{w\ 1}$ kip	61
	58		$2t_w y F'_y$ kip	−163.91		
	59	Tension F_T/ Compression A_C	$2t_w y' F'_y$ kip	4364	$A_{w\ 2}$ kip	−76
	60		$b_{bf}t_{bf}F'_y$ kip	2213	A_{bf} kip	−148
CHECK	61	Equilibrium of Forces/Areas	F_C kip	−6577	A_T kip	223
	62		F_T kip	6577	A_C kip	−223
	63		$F_C + F_T$ kip	**0**	$A_T + A_C$ kip	**0**
MOMENT	64	(Half Section)	$0.5\,M_p^+$ k-ft	32207	$0.5\,M_p^-$ k-ft	64603
	65	(Full Section)	M_p^+ k-ft	**64414**	M_p^- k-ft	**129206**

		COMPUTATION OF OVERSTRENGTH FACTOR EXTERIOR: 3-SPAN CASE (BRIDGE 10, SPAN 1)		9 / 14
		Parameter	Formula/Definition/Equation	Data
GEOMETRY	1	L_x	Span Length	**148.00** ft
	2	R_{CL}	Radius of center line	**716** ft
	3	B	Width	**30** ft
	4	L_x^*	Outer region length $L_x^* = \left(1 + \frac{B}{4R_{C_L}}\right) L_x$	149.55 ft
	5	s	Inter Girder Spacing	**7.7** ft
	6	b	Width of Girder + Edge	**11.2** ft
	7	t	Deck Thickness	**8.00** in
INTERNAL WORK DONE, IWD	8	m_x	Longitudinal Positive Moment per ft	**12** k-in/in
	9	m'_x	Longitudinal Negative Moment per ft	**9** k-in/in
	10	m_y	Transverse Positive Moment per ft	**21** k-in/in
	11	m'_y	Transverse Negative Moment per ft	**17** k-in/in
	12	$0.5M_p^-$	Negative Moment at Support	**58368** k-ft
	13	λ	Fraction of Length from the exterior support at which girder is fractured	0.40
	14	tan θ	$\tan\theta = \sqrt{\frac{m'_y + m_y}{m'_x + m_x}}$	1.31 (Θ = 52.5 °)
	15	tan α	$\tan\alpha = \frac{2s}{L_x}$	0.10 (Θ = 5.9°)
	16	k_{bound}^{upper}	$\left[1 + 2\frac{\tan\alpha}{\tan\theta}\right] = 1 + \frac{4s}{L_x}\sqrt{\frac{m'_x + m_x}{m'_y + m_y}}$	1.16
	17	k_{bound}^{lower}	$\left[1 + 2\frac{\tan^2\alpha}{\tan^2\theta}\right] = 1 + \frac{8s^2}{{L_x}^2}\sqrt{\frac{m'_x + m_x}{m'_y + m_y}}$	1.01
	18	IWD_{upper}	$\left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right) k_{bound}^{upper} + \frac{m_x b}{L_x(\lambda - \lambda^2)} + \frac{0.5M_p^-}{(1-\lambda)L_x}$	1070 k-ft *
	19	IWD_{lower}	$\left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right) k_{bound}^{lower} + \frac{m_x b}{L_x(\lambda - \lambda^2)} + \frac{0.5M_p^-}{(1-\lambda)L_x}$	1018 k-ft *

Note: *: A unit deflection (δ = 1) is considered; therefore, the unit of internal work is in k-ft

		COMPUTATION OF OVERSTRENGTH FACTOR EXTERIOR: 3-SPAN CASE (BRIDGE 10, SPAN 1)		10 / 14
		Parameter	Formula/Definition/Equation	Data
EXTERNAL WORK DONE, EWD	20	DL	Dead Load Factor	1.25
	21	LL	Live Load Factor	1.75
	22	SAF	Stiffener Allowance Factor	1
	23	γ_c	Unit weight of reinforced concrete	0.15 kcf
	24	w_u	Area load due to reinforced concrete + lane load $DL\gamma_c \frac{t}{12} + LL \cdot \frac{0.64}{12}$	0.218 ksf
	25	γ_s	Unit weight of steel	0.49 kcf
	26	V_g	Volume of Girder	**252** ft^3
	27	A_r	Area of Rail Cross-Section (T4 (S))	**1.25** ft^2
	28	V_r	Volume of Rail = $L_x A_r$	187 ft^3
	29	W_x	$1.25\left(1.15V_g\gamma_s + V_r\gamma_c\right)/L_x$	1.42 k/ft
	30	y (Lane 2)	$(b+s-15)$ for $(b+s) < 21$ $(b+s-18)$ for $(b+s) > 21$	3.83 ft
	31	K_{lane}	$1 + 0.5\frac{y}{s}$ for $(b+s) < 21$; $1 + \frac{y}{s}$ for $(b+s) > 21$	1.25
	32	$EWD_{HS\text{-}20}$	$\left(168 - \frac{261.33}{\lambda L_x} - \frac{1045.33}{(1-\lambda)L_x}\right) K_{Lane}$	190 k-ft
	33	W_{ET}	$w_u L_x (b + 0.5s) + W_x L_x + 2EWD_{HS20}$	1082 k-ft
	34	EWD	$0.5\ W_{ET}$	541 k-ft *
RESULTS	35	Ω_{upper}	IWD_{upper}/EWD	**1.98**
	36	Ω_{lower}	IWD_{lower}/EWD	**1.88**

Note: *: A unit deflection (δ = 1) is considered; therefore, the unit of external work is in k-ft

		COMPUTATION OF OVERSTRENGTH FACTOR INTERIOR: 3-SPAN CASE (BRIDGE 10, SPAN 2)		11 / 14
		Parameters	Formula/Definition/Equation	Data
GEOMETRY	1	L_x	Span Length	**265.00** ft
	2	R_{CL}	Radius of center line	**716** ft
	3	B	Width	**30** ft
	4	L_x^*	Outer region length $L_x^* = \left(1 + \frac{B}{4R_{C_L}}\right) L_x$	267.78 ft
	5	s	Inter Girder Spacing	**7.7** ft
	6	b	Width of Girder + Edge	**11** ft
	7	t	deck thickness	**8** in
INTERNAL WORK DONE, IWD	8	m_x	Longitudinal Positive Moment per ft	**12** k-in/in
	9	m'_x	Longitudinal Negative Moment per ft	**9** k-in/in
	10	m_y	Transverse Positive Moment per ft	**21** k-in/in
	11	m'_y	Transverse Negative Moment per ft	**17** k-in/in
	12	$0.5M_{p1}^-$	Negative Moment at Support 1	**58368** k-ft
	13	$0.5M_{p2}^-$	Negative Moment at Support 2	**64603**
	14	tan Θ	$\tan\theta = \sqrt{\frac{m'_y + m_y}{m'_x + m_x}}$	1.31 (Θ = 53˚)
	15	tan α	$\tan\alpha = \frac{2s}{L_x}$	0.06 (Θ = 3.3˚)
	16	k_{bound}^{upper}	$\left[1 + 2\frac{\tan\alpha}{\tan\theta}\right] = 1 + \frac{4s}{L_x}\sqrt{\frac{m'_x + m_x}{m'_y + m_y}}$	1.09
	17	k_{bound}^{lower}	$\left[1 + 2\frac{\tan^2\alpha}{\tan^2\theta}\right] = 1 + \frac{8s^2}{{L_x}^2}\sqrt{\frac{m'_x + m_x}{m'_y + m_y}}$	1.00
	18	IWD_{upper}	$\left(m_y^{'} + m_y\right)\left(\frac{L_x}{2s}\right) k_{bound}^{upper} + \frac{4m_x b}{L_x} + \left(0.5M_{p1}^- + 0.5M_{p2}^-\right)\frac{2}{L_x}$	1620 k-ft *
	19	IWD_{lower}	$\left(m_y^{'} + m_y\right)\left(\frac{L_x}{2s}\right) k_{bound}^{lower} + \frac{4m_x b}{L_x} + \left(0.5M_{p1}^- + 0.5M_{p2}^-\right)\frac{2}{L_x}$	1566 k-ft *

Note: *: A unit deflection (δ = 1) is considered; therefore, the unit of internal work is in k-ft

		COMPUTATION OF OVERSTRENGTH FACTOR INTERIOR: 3-SPAN CASE (BRIDGE 10, SPAN 2)		12 / 14	
		Parameter	Formula/Definition/Equation	Data	
EXTERNAL WORK DONE, EWD	20	DL	Dead Load Factor	1.25	
	21	LL	Live Load Factor	1.75	
	22	SAF	Stiffener Allowance Factor	1.15	
	23	γ_c	Unit weight of reinforced concrete	0.15	kcf
	24	w_u	Area load due to reinforced concrete + lane load $DL\gamma_c \frac{t}{12} + LL \cdot \frac{0.64}{12}$	0.22	ksf
	25	γ_s	Unit weight of steel	0.49	kcf
	26	V_g	Volume of Girder	**519**	ft^3
	27	A_r	Area of Rail Cross-Section (T4 (S))	**1.25**	ft^2
	28	V_r	Volume of Rail = $L_x A_r$	335	ft^3
	29	W_x	$1.25\left(1.15V_g\gamma_s + V_r\gamma_c\right)/L_x$	1.60	k/ft
	30	y (Lane 2)	$(b+s-15)$ for $(b+s) < 21$ $(b+s-18)$ for $(b+s) > 21$	3.83	ft
	31	K_{lane}	$1 + 0.5\frac{y}{s}$ for $(b+s) < 21$; $1 + \frac{y}{s}$ for $(b+s) > 21$	1.25	
	32	$EWD_{HS\text{-}20}$	$\left(168 - \frac{2613}{L_x}\right) K_{Lane}$	198	k-ft
	33	W_{ET}	$w_u L_x(b + 0.5s) + W_x L_x + 2EWD_{HS20}$	1701	k-ft
	34	EWD	$0.5\ W_{ET}$	851	k-ft *
RESULTS	35	Ω_{upper}	IWD_{upper}/EWD	**1.90**	
	36	Ω_{lower}	IWD_{lower}/EWD	**1.84**	

Note: *: A unit deflection (δ = 1) is considered; therefore, the unit of external work is in k-ft

		COMPUTATION OF OVERSTRENGTH FACTOR EXTERIOR: 3-SPAN CASE (BRIDGE 10, SPAN 3)		13 14
		Parameters	Formula/Definition/Equation	Data
GEOMETRY	1	L_x	Span Length	**189.58** ft
	2	R_{CL}	Radius of center line	**716** ft
	3	B	Width	**30** ft
	4	L_x^*	Outer region length $L_x^* = \left(1 + \frac{B}{4R_{C_L}}\right) L_x$	191.57 ft
	5	s	Inter Girder Spacing	**7.7** ft
	6	b	Width of Girder + Edge	**11.2** ft
	7	t	Deck Thickness	**8.00** in
INTERNAL WORK DONE, IWD	8	m_x	Longitudinal Positive Moment per ft	**12** k-in/in
	9	m'_x	Longitudinal Negative Moment per ft	**9** k-in/in
	10	m_y	Transverse Positive Moment per ft	**21** k-in/in
	11	m'_y	Transverse Negative Moment per ft	**17** k-in/in
	12	M_p^-	Negative Moment at Support	**64603** k-ft
	13	λ	Fraction of Length from the exterior support at which girder is fractured	0.40
	14	tan θ	$\tan\theta = \sqrt{\frac{m'_y + m_y}{m'_x + m_x}}$	1.31 (Θ=52.5)
	15	tan α	$\tan\alpha = \frac{2s}{L_x}$	0.08 (Θ=4.6°)
	16	k_{bound}^{upper}	$\left[1 + 2\frac{\tan\alpha}{\tan\theta}\right] = 1 + \frac{4s}{L_x}\sqrt{\frac{m'_x + m_x}{m'_y + m_y}}$	1.12
	17	k_{bound}^{lower}	$\left[1 + 2\frac{\tan^2\alpha}{\tan^2\theta}\right] = 1 + \frac{8s^2}{{L_x}^2}\sqrt{\frac{m'_x + m_x}{m'_y + m_y}}$	1.01
	18	IWD_{upper}	$\left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right) k_{bound}^{upper} + \frac{m_x b}{L_x(\lambda - \lambda^2)} + \frac{0.5M_p^-}{(1-\lambda)L_x}$	189.58 k-ft *
	19	IWD_{lower}	$\left(m'_y + m_y\right)\left(\frac{L_x}{2s}\right) k_{bound}^{lower} + \frac{m_x b}{L_x(\lambda - \lambda^2)} + \frac{0.5M_p^-}{(1-\lambda)L_x}$	716 k-ft *

Note: *: A unit deflection (δ = 1) is considered; therefore, the unit of internal work is in k-ft

		COMPUTATION OF OVERSTRENGTH FACTOR EXTERIOR: 3-SPAN CASE (BRIDGE 10, SPAN 3)		14 14
		Parameter	Formula/Definition/Equation	Data
EXTERNAL WORK DONE, EWD	20	DL	Dead Load Factor	1.25
	21	LL	Live Load Factor	1.75
	22	SAF	Stiffener Allowance Factor	1
	23	γ_c	Unit weight of reinforced concrete	0.15 kcf
	24	w_u	Area load due to reinforced concrete + lane load $DL\gamma_c \frac{t}{12} + LL \cdot \frac{0.64}{12}$	0.218 ksf
	25	γ_s	Unit weight of steel	0.49 kcf
	26	V_g	Volume of Girder	**335** ft^3
	27	A_r	Area of Rail Cross-Section (T4(S))	**1.25** ft^2
	28	V_r	Volume of Rail = $L_x A_r$	239 ft^3
	29	W_x	$1.25\left(1.15V_g\gamma_s + V_r\gamma_c\right)/L_x$	1.46 k/ft
	30	y (Lane 2)	$(b+s-15)$ for $(b+s) < 21$ $(b+s-18)$ for $(b+s) > 21$	3.83 ft
	31	K_{lane}	$1 + 0.5\frac{y}{s}$ for $(b+s) < 21$; $1 + \frac{y}{s}$ for $(b+s) > 21$	1.25
	32	$EWD_{HS\text{-}20}$	$\left(168 - \frac{261.33}{\lambda L_x} - \frac{1045.33}{(1-\lambda)L_x}\right) K_{Lane}$	194 k-ft
	33	W_{ET}	$w_u L_x(b + 0.5s) + W_x L_x + 2EWD_{HS20}$	1297 k-ft
	34	EWD	$0.5\ W_{ET}$	648 k-ft *
RESULTS	35	Ω_{upper}	IWD_{upper}/EWD	**1.67**
	36	Ω_{lower}	IWD_{lower}/EWD	**1.59**

Note: *: A unit deflection (δ = 1) is considered; therefore, the unit of external work is in k-ft

7.2 GRILLAGE ANALYSIS EXAMPLE OF BRIDGE 10

The following steps are explained to conduct the computational implementation of grillage analysis of Bridge 10.

1. Gather bridge geometry and material information.

Location ft	Top Flange		Web		Bottom Flange		Sec. Type	
	Width in.	Thickness in.	Width in.	Thickness in.	Width in.	Thickness in.	Section	Rebar
0–50	24	1.00	78	0.625	59	0.750	1	Reg.
50–98	24	1.00	78	0.625	59	1.250	2	Pier
98–131	24	2.00	78	0.75	59	2.000	3	Pier
131–181	24	3.00	78	0.875	59	2.000	4	Pier
181–230	24	1.00	78	0.875	59	1.250	5	Pier
230–247	24	1.00	78	0.75	59	1.000	6	Reg.
247–297	24	1.00	78	0.75	59	1.250	7	Reg.
297–330	24	1.00	78	0.75	59	1.000	8	Reg.
330–380	24	1.00	78	0.875	59	1.250	5	Pier
380–396	24	2.00	78	0.875	59	1.250	5	Pier
396–430	24	3.00	78	0.875	59	2.000	9	Pier
430–447	24	3.00	78	0.875	59	2.000	9	Pier
447–464	24	2.00	78	0.75	59	1.250	10	Pier
464–499	24	1.00	78	0.75	59	1.250	10	Pier
499–602	24	1.00	78	0.625	59	0.750	1	Reg.

Location	Parameter	Description/Value
Bridge	Location	Harris County, IH 10
	Year Designed/Year Built	1998/2002
	Design Load	
	Length, ft	602.58
	Spans, ft	148, 265, 189.58
	Radius of Curvature, ft	716.2
Deck	Width, ft	30
	Thickness, in.	8
	Haunch, in.	5
	Rail Type	T4(s)
Rebar	# of Bar Longitudinal Top Row (#4)	42
	# of Bar Longitudinal Bottom Row (#5)	32
	# of Bar Longitudinal Top Row (#4) @support	42
	# of Bar Longitudinal Top Row (#5) @support	40
	# of Bar Longitudinal Bottom Row (#5) @support	32
	Transverse Spacing Top Row (#5), in.	6
	Transverse Spacing Bottom Row (#5), in.	6
Girder	CL of Bridge to CL of Girder (in.)	45
	CL of Top Flange to CL of Top Flange (in.)	96

2. Determine material constitutive behavior.

Concrete (4 ksi)

Stress (ksi)	Strain (1/in)
−4	−3.79E-03
−4	−3.56E-03
−4	−2.69E-03
−4	−1.78E-03
−3.8205	−1.40E-03
−2.8718	−8.69E-04
−0.6403	−1.78E-04
0	0
0.378	1.06E-04
0.378	1.16E-03

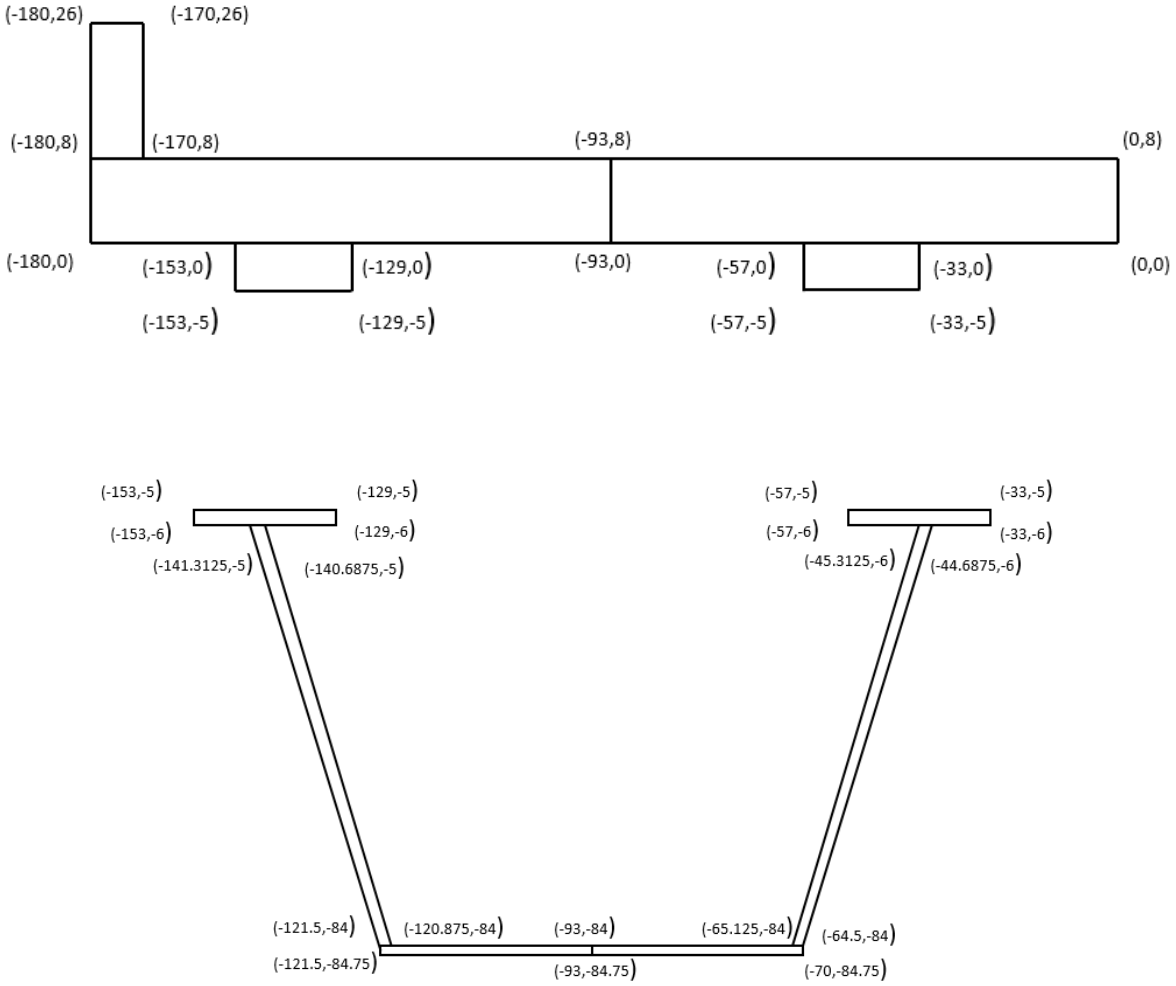
Rebar (60 ksi)

Stress (ksi)	Strain (1/in)
−87.9	−0.095
−87.9	−0.0944
−86.6	−0.0761
−78	−0.0386
−60.7	−9.80E-03
−60.3	−2.08E-03
0	0
60.3	2.08E-03
60.7	9.80E-03
78	0.0386
86.6	0.0761
87.9	0.0944
87.9	0.095

Steel (50 ksi)

Stress (ksi)	Strain (1/in)
−71.6	−0.1
−71.6	−0.097
−71.6	−0.095
−71.6	−0.0946
−70.3	−0.0764
−62.5	−0.039
−50	−0.0196
−50	−1.72E-03
0	0
50	1.72E-03
50	0.0196
62.5	0.039
70.3	0.0764
71.6	0.0946
71.6	0.095
71.6	0.097
71.6	0.1

3. Create a coordinate system for the half width of the span for each cross-section. An example of the first cross-section is shown below.



4. Create a cylindrical coordinate system for the curved bridge and ensure that the middle transverse divisions are 7 ft because this will aid is applying the truck load, whose axles are separated by 14 ft.
   A. $\#\ of\ Segments_{per\ span} = \left(\frac{Length\ (ft)*12}{84\ in\ (7ft)}\right)\ rounded\ to\ nearest\ even\ number$:
      - $\#\ of\ Segments_{Span\ 1} = \left(\frac{148*12}{84}\right) = 21.428, or\ 18.$
      - $\#\ of\ Segments_{Span\ 2} = \left(\frac{265*12}{84}\right) = 37.857, or\ 36.$
      - $\#\ of\ Segments_{Span\ 3} = \left(\frac{190*12}{84}\right) = 27.143, or\ 24.$
   B. $End\ Segment\ Length = \frac{\left((Length*12)-(\#\ of\ Segments*84)\right)}{2}$:
      - $End\ Segment\ Length_{Span\ 1} = \frac{\left((148*12)-(18*84)\right)}{2} = 132\ in.$
      - $End\ Segment\ Length_{Span\ 2} = \frac{\left((265*12)-(36*84)\right)}{2} = 78\ in.$
      - $End\ Segment\ Length_{Span\ 3} = \frac{\left((190*12)-(24*84)\right)}{2} = 132\ in.$
   C. $Theta = \frac{Total\ Length}{Radius}$:
      - $Theta = \frac{603}{716.2} = 0.84194\ rad., or\ 48.24\ degrees.$
   D. Determine the radial offsets using the outside edge, the outside flange, the inner flange, and CL of the bridge.

Offsets (in.)	
Edge	180
Outside Flange	141
Inner Flange	45
CL of Bridge	0

Radial Spacing (in.)		
A	8774.4	CL + Edge
B	8735.4	CL + OF
C	8639.4	CL + IF
Center Line	8594.4	or 450 (ft)
D	8549.4	CL − IF
E	8453.4	CL − OF
F	8414.4	CL − Edge

   E. The longitudinal spacing along theta is determined by converting the longitudinal segment lengths into degrees.
      - The segments vary in length. The total length is 603 ft, or 7236 in.
      - $Radial\ Spacing\ (rad) = \frac{Long.Spacing}{Radius}$.
      - $Radial\ Spacing\ (degree) = Radius\ Spacing\ (rad) * \frac{180}{\pi}$.

Long. Spacing (in.)	Radial Spacing (rad.)	Radial Spacing (deg.)
0	0.0000	0.000
132	0.0154	0.880
216	0.0251	1.440
300	0.0349	2.000
384	0.0447	2.560
468	0.0545	3.120
552	0.0642	3.680
636	0.0740	4.240
720	0.0838	4.800
804	0.0935	5.360
888	0.1033	5.920
972	0.1131	6.480
1056	0.1229	7.040
1140	0.1326	7.600
1224	0.1424	8.160
1308	0.1522	8.720
1392	0.1620	9.280
1476	0.1717	9.840
1560	0.1815	10.400
1644	0.1913	10.960
1776	0.2066	11.840
1854	0.2157	12.360
1938	0.2255	12.920
2022	0.2353	13.480
2106	0.2450	14.040
2190	0.2548	14.600
2274	0.2646	15.160
2358	0.2744	15.720
2442	0.2841	16.280
2526	0.2939	16.840
2610	0.3037	17.400
2694	0.3135	17.960
2778	0.3232	18.520
2862	0.3330	19.080
2946	0.3428	19.640
3030	0.3526	20.200
3114	0.3623	20.760
3198	0.3721	21.320

3282	0.3819	21.880
3366	0.3917	22.440
3450	0.4014	23.000
3534	0.4112	23.560
3618	0.4210	24.120
3702	0.4307	24.680
3786	0.4405	25.240
3870	0.4503	25.800
3954	0.4601	26.360
4038	0.4698	26.920
4122	0.4796	27.480
4206	0.4894	28.040
4290	0.4992	28.600
4374	0.5089	29.160
4458	0.5187	29.720
4542	0.5285	30.280
4626	0.5383	30.840
4710	0.5480	31.400
4794	0.5578	31.960
4878	0.5676	32.520
4956	0.5767	33.040
5088	0.5920	33.920
5172	0.6018	34.480
5256	0.6116	35.040
5340	0.6213	35.600
5424	0.6311	36.160
5508	0.6409	36.720
5592	0.6507	37.280
5676	0.6604	37.840
5760	0.6702	38.400
5844	0.6800	38.960
5928	0.6898	39.520
6012	0.6995	40.080
6096	0.7093	40.640
6180	0.7191	41.200
6264	0.7288	41.760
6348	0.7386	42.320
6432	0.7484	42.880
6516	0.7582	43.440
6600	0.7679	44.000
6684	0.7777	44.560

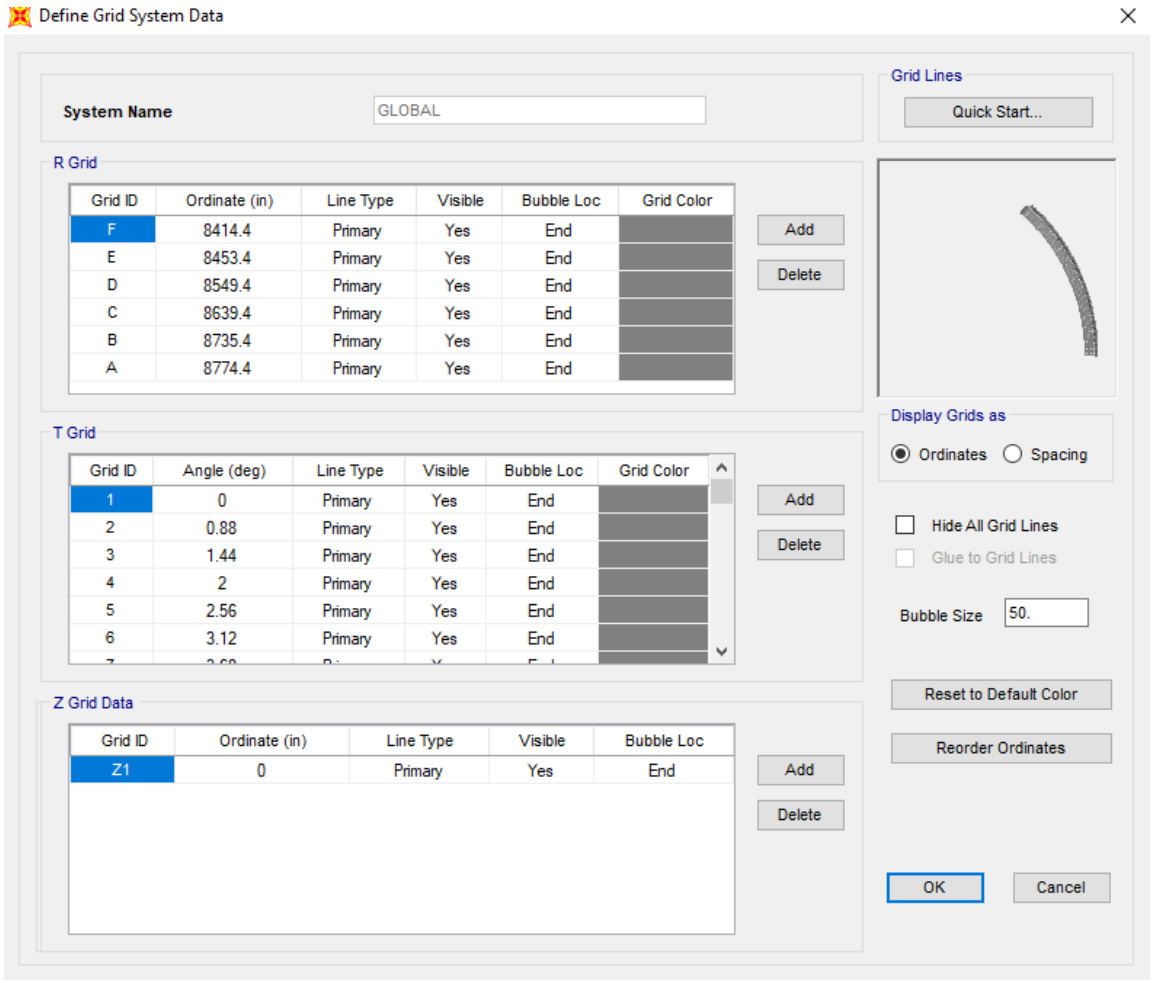
6768	0.7875	45.120
6852	0.7973	45.680
6936	0.8070	46.240
7020	0.8168	46.800
7104	0.8266	47.360
7236	0.8419	48.240

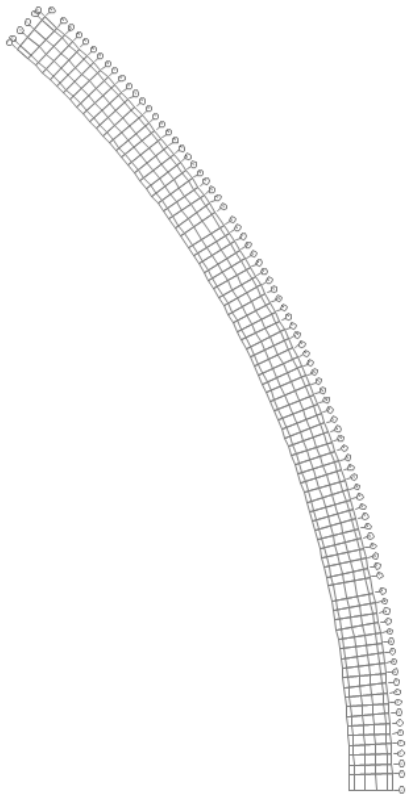
- $Int. Transverse\ Element\ width = 84\ in.$
- $End\ Transverse\ Element = 132 - \left(\frac{84}{2}\right) = 90\ in.$
- $Pier\ Tansverse\ Element = 132 + 78 - 84 = 126\ in.$

5. Input the coordinate system into SAP2000.
   A. Select File > New Model > Blank model (making sure units are in kips and in.).
   B. Right click on the blank workspace and select Edit Grid Data > Modify/Show System > Quick Start > Cylindrical.
      - In the Number of Gridlines panel, set "Along Z = 1."
      - In the Grid Spacing panel, set "Along Z = 1."
      - Select OK.
      - Delete all R and T Coordinates that were generated.
   C. Add correct coordinates for R.

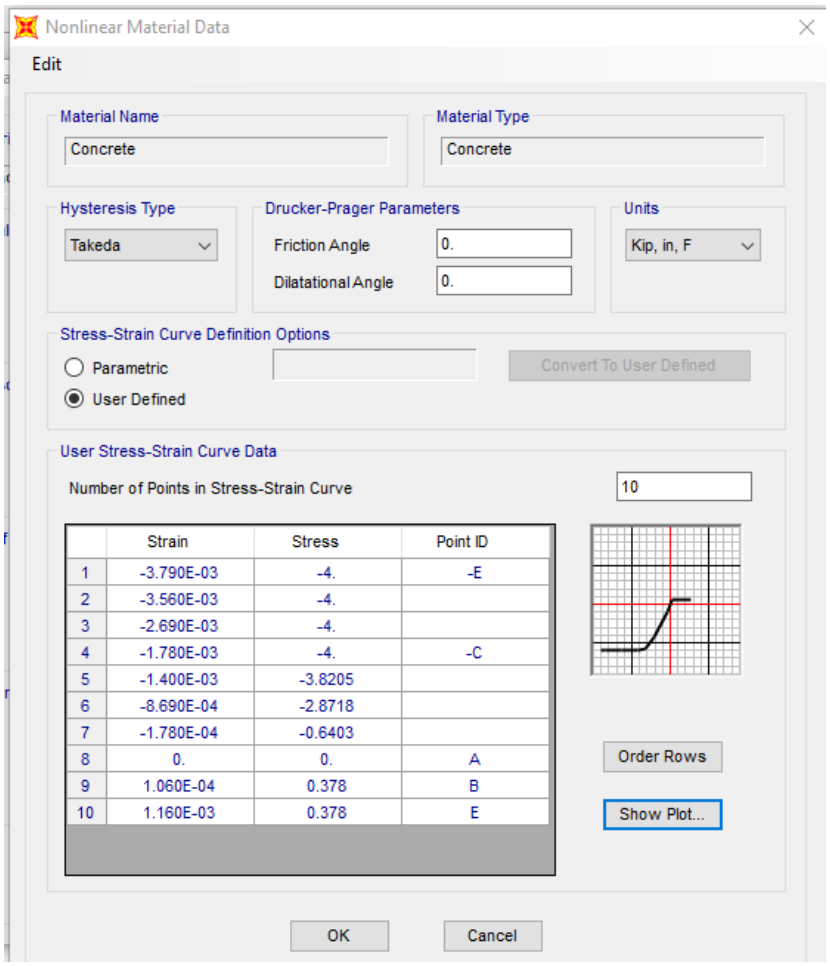
      All radial coordinates (A, B, C, D, E, and F).

   D. Add correct coordinates for T.
      - All theta coordinates for T (0 to 7236 in.)
      - Click OK.
   E. The grid system is now formed.


Define Grid System Data
System Name
GLOBAL
Grid Lines
Quick Start...
R Grid
Grid ID | Ordinate (in) | Line Type | Visible | Bubble Loc | Grid Color
F | 8414.4 | Primary | Yes | End
E | 8453.4 | Primary | Yes | End
D | 8549.4 | Primary | Yes | End
C | 8639.4 | Primary | Yes | End
B | 8735.4 | Primary | Yes | End
A | 8774.4 | Primary | Yes | End
Add
Delete
T Grid
Grid ID | Angle (deg) | Line Type | Visible | Bubble Loc | Grid Color
1 | 0 | Primary | Yes | End
2 | 0.88 | Primary | Yes | End
3 | 1.44 | Primary | Yes | End
4 | 2 | Primary | Yes | End
5 | 2.56 | Primary | Yes | End
6 | 3.12 | Primary | Yes | End
Add
Delete
Z Grid Data
Grid ID | Ordinate (in) | Line Type | Visible | Bubble Loc
Z1 | 0 | Primary | Yes | End
Add
Delete
Display Grids as
Ordinates
Spacing
Hide All Grid Lines
Glue to Grid Lines
Bubble Size
50.
Reset to Default Color
Reorder Ordinates
OK
Cancel



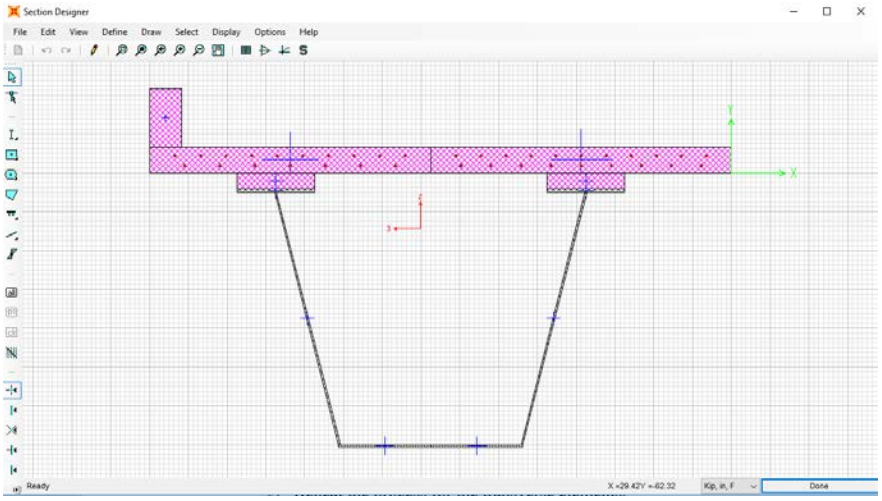
6. Define material in SAP2000.
    A. Click Define > Materials -> Add New Material > Material Type (Steel, Concrete, or Rebar) > Standard (User) > OK.
    B. At the bottom of the window, select the box that states, “Switch to Advanced Properties.”
    C. In the open window, name the material “Concrete,” “Steel,” or “Rebar” depending on which material is being defined. Then click “Modify/Show Material Properties.”
    D. In the Material Property Data window, click the “Nonlinear Material Data” icon.
    E. In the Nonlinear Material Data window, select the “Convert to User Defined” icon.
    F. Input the number of data points for the stress strain behavior (10 for concrete, 13 for rebar, 17 for steel).
    G. Input the data points for the stress strain behavior.
    H. Select “OK.”
    I. Repeat this process again for the remaining materials.



7. Define frame cross-sections in SAP2000.
    A. Click Define > Section Properties > Frame Sections > Add New Properties.
    B. In the Frame Section Properties drop-down box, select Other and click Section Designer. In the SD Section Designer window, name the section B5Long1 and click the Section Designer icon.
    C. Using the Polygon feature, draw the features of the half width of the bridge from Step 3. These features include one rail, two concrete deck pieces, two concrete haunches, two top flanges, two webs, and two pieces of the bottom flange.
        - To change material types for the polygons, right-click on the polygon and select the desired material type from the material drop-down menu.
        - To change the coordinates of the polygon's nodes, use the Reshaper tool.
    D. Add in the longitudinal rebar to both concrete deck elements by using the Line Bar from the Draw Reinforcing Shape tool. From the design drawings, it can be determined that there are 10 #4 top bars and seven #5 bottom bars in the outer concrete deck element and 11 #4 top bars and nine #5 bottom bars in the inner concrete element. At the pier support, 10 additional #5 top bars are added to the top of the outer concrete deck and 10 additional #5 top bars are added to the inner concrete deck, with a 2 in. top cover and transverse reinforcement. The top bars

are located at 5.0625 in., with a 1.25-in. bottom cover and transverse reinforcement. The top bars are located at 2.1875.

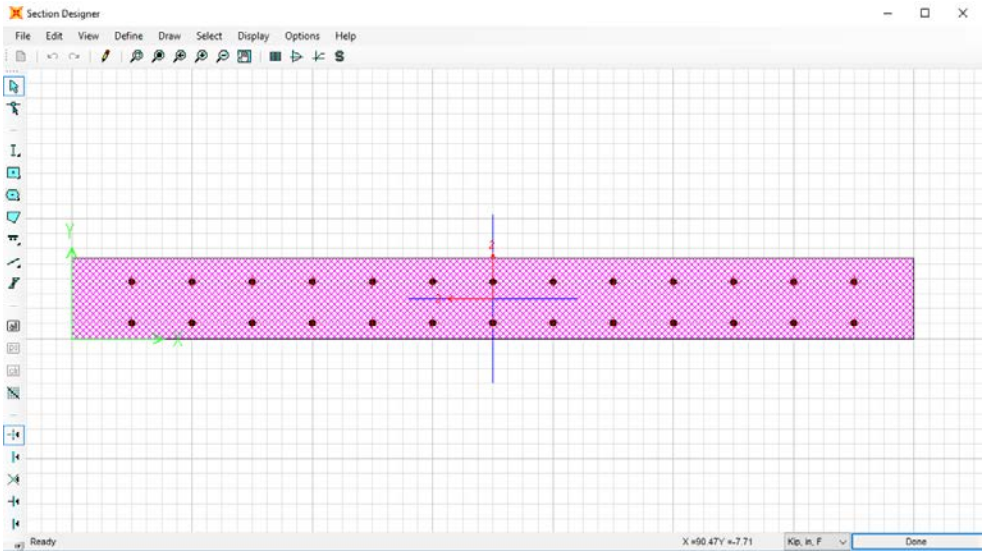
E. Click Done (below is an example of the 1st cross-section).



F. Repeat this process for the remaining longitudinal elements.

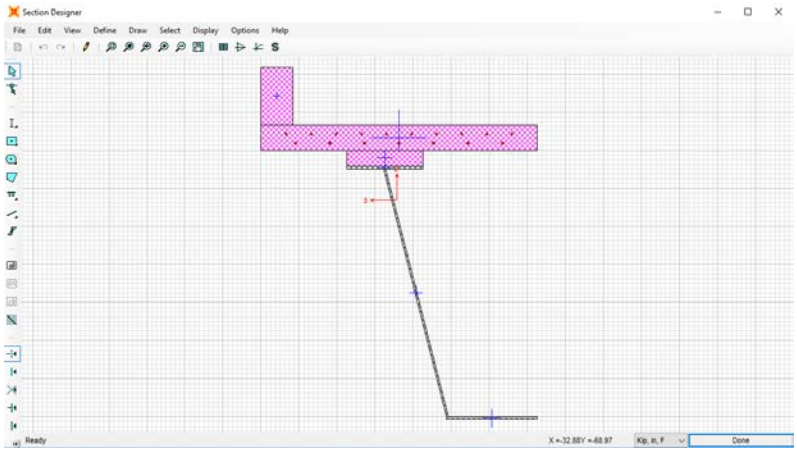
G. Repeat the process for the transverse elements.

- In the SD Section Designer window, select Modifiers and set Mass and Weight to 0 to not double-count the dead weight.
- The interior transverse members are 84 in. wide (end members are 90 in. wide and pier members are 126 in.), with #5 rebar at 5 in. spacing at 5.6875 in. and 1.5625 in.

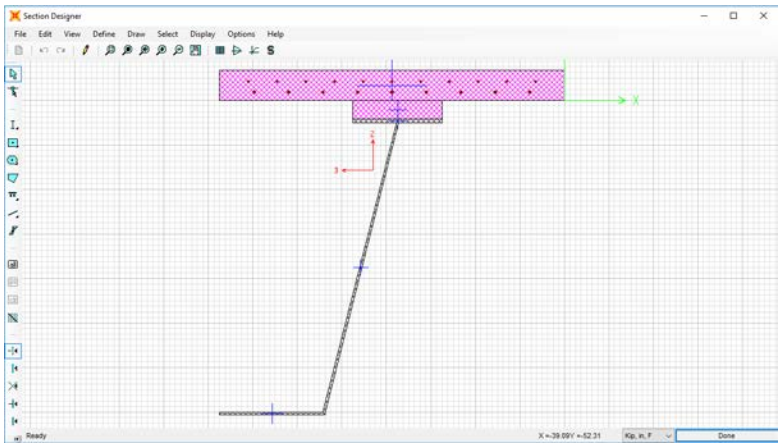


H. To generate an exterior longitudinal member and an interior longitudinal member, make two copies of the B10Long1.1 section. Label one Long1Out and one Long1Int.

- For the Long1Out, delete every element right of the CL.

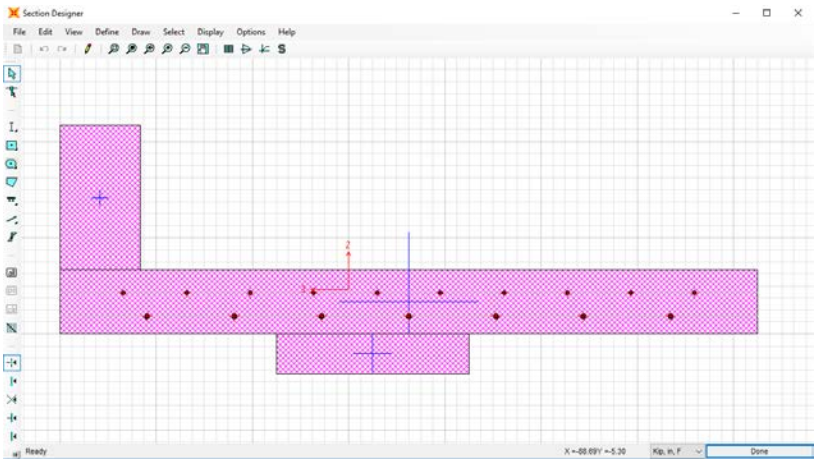


- For the Long1Int, delete every element left of the CL.

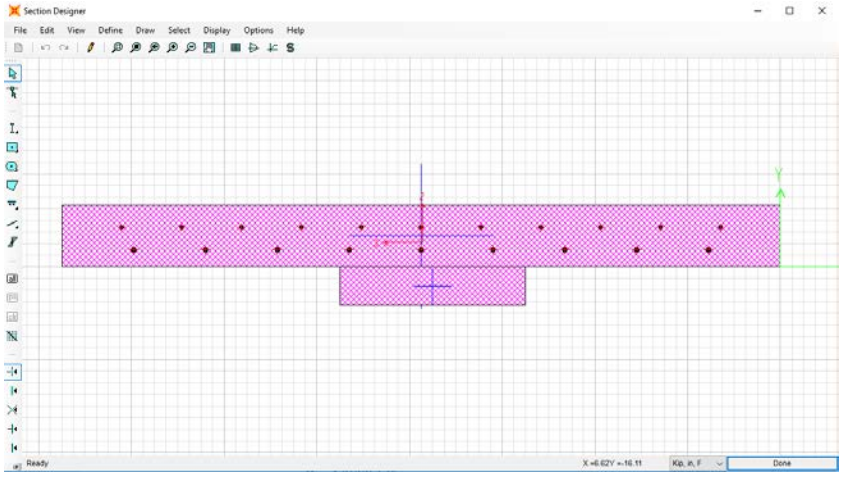


- Repeat this process for all cross-sections.

I. To generate a simulated fracture section, make copies of Long1Out and Long1Int.
   - The reason Long1Out and Long1Int are chosen for the fractured section is because the fracture occurs at 0.4*L, or 710 in. On the radial spacing table, Long1 is the section used at 710 in.
   - Name the Long1Out copy FracInt1.1.
     - Delete the bottom flange, web, and top flange of the steel tub.



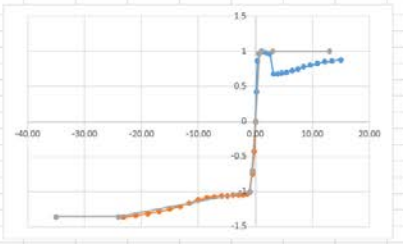
- Name the Long1Int copy FracInt1.1.
    - Delete the bottom flange, web, and top flange of the steel tub.



8. Generate plastic hinges for frame elements in SAP2000.
    A. Define > Section Properties > Frame Sections.
    B. Select the desired cross-section. Hinges will need to be made for all of the necessary cross-sections.
    C. Once selected, click Modify/Show Property > Section Designer.
    D. Once in the section designer, select the Moment Curvature Curve tool.
    E. In the Moment Curvature Curve window, select Details.
        - Copy the moment curvature data to an Excel file.
        - Select OK.
    F. In the Moment Curvature Curve window, change the angle (deg) to 180, then select Details.
        - Copy the moment curvature data to the same Excel file as the one used previously.
        - Select OK.
    G. Generate a Normalized Moment Curvature Diagram.

- Normalize the moments by dividing each of the positive moments by the maximum positive moment and the negative moment by the maximum negative moment, and divide the curvatures by the curvatures corresponding to the maximum and negative moments.
- Plot the normalized positive and negative moment curvatures on a chart.
- Create a hinge moment curvature plot on the same chart with four positive moment points and four negative moment points without generating a negative slope.

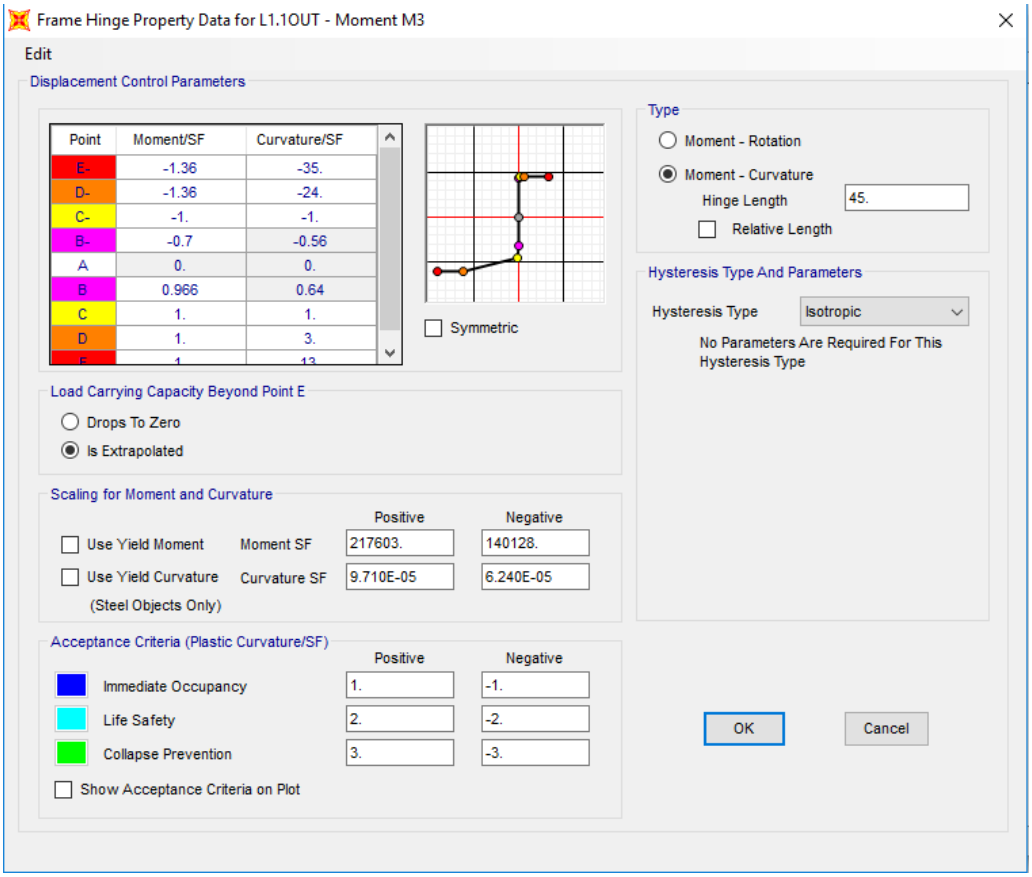
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
2	Positive																						
3	Concrete	Neutral A	Steel Stra	Tendon St	Concrete	Steel Com	Steel Ten	Prestress	Net Force	Curvature	Moment		M	C									
4	0	0	0	0	0	0	0	0	0	0	0		0	0.00									
5	-5.83E-04	-0.202	9.54E-04	0	-1023	-134.148	1157.457	0	0.6999	1.39E-05	92327		0.424291	0.14									
6	-1.39E-03	1.8021	2.46E-03	0	-2182	-278.572	2460.218	0	-0.5396	3.47E-05	188632		0.866863	0.36									
7	-2.20E-03	6.5568	4.72E-03	0	-2709	-258.87	2968.071	0	-4.31E-02	6.24E-05	210140		0.965704	0.64									
8	-3.16E-03	9.2404	7.60E-03	0	-3022	-189.735	3211.643	0	-0.0135	9.71E-05	217603		1	1.00									
9	-4.38E-03	10.2529	0.011	0	-3170	-162.774	3332.986	0	0.022	0.000139	216988		0.997174	1.43									
10	-5.91E-03	10.2344	1.48E-02	0	-3162	-221.373	3383.742	0	0.4763	0.000187	213039		0.979026	1.93									
11	-7.71E-03	10.0496	1.92E-02	0	-3122	-281.487	3403.438	0	-0.0672	0.000243	210855		0.968989	2.50									
12	-0.0198	-22.9408	0.014	0	0	-1854	1854.169	0	-0.0301	0.000305	148132		0.680744	3.14									
13	-0.0242	-22.8196	0.0173	0	0	-1858	1858.773	0	0.3035	0.000375	148558		0.682702	3.86									
14	-0.0292	-22.9173	0.0208	0	0	-1871	1871.122	0	0.052	0.000451	149608		0.687527	4.64									
15	-0.0349	-23.6182	0.0242	0	0	-1906	1906.531	0	0.1244	0.000534	152481		0.70073	5.50									
16	-0.0411	-24.0453	0.028	0	0	-1961	1960.493	0	-9.66E-02	0.000624	156827		0.720702	6.43									
17	-0.0476	-24.1875	0.0323	0	0	-2034	2033.797	0	0.0141	0.000722	162696		0.747674	7.43									
18	-0.0546	-24.3215	0.0369	0	0	-2115	2115.214	0	0.015	0.000826	169211		0.777613	8.50									
19	-0.0618	-24.2485	0.0419	0	0	-2189	2189.226	0	0.1694	0.000937	175107		0.804709	9.64									
20	-0.0693	-23.9243	0.0475	0	0	-2249	2249.521	0	0.0233	0.001055	179894		0.826707	10.86									
21	-0.0772	-23.6615	0.0534	0	0	-2307	2306.81	0	0.2443	0.001179	184442		0.847608	12.14		-1.36	-35						
22	-0.0859	-23.727	0.0593	0	0	-2351	2350.667	0	0.0432	0.001311	187939		0.863678	13.50		-1.36	-24						
23	-0.0951	-23.7901	0.0655	0	0	-2395	2394.726	0	0.0178	0.00145	191454		0.879832	14.93		-1	-1						
24																-0.7	-0.56						
25																0	0						
26	Negative															0.966	0.64						
27	Concrete	Neutral A	Steel Stra	Tendon St	Concrete	Steel Com	Steel Ten	Prestress	Net Force	Curvature	Moment					1	1						
28	0	0	0	0	0	0	0	0	0	0	0		0	0.00		1	3						
29	-7.27E-04	16.5861	5.20E-04	0	-374.912	-698.732	323.5235	0	-750.12	1.39E-05	59719		-0.42617	-0.22		1	13						



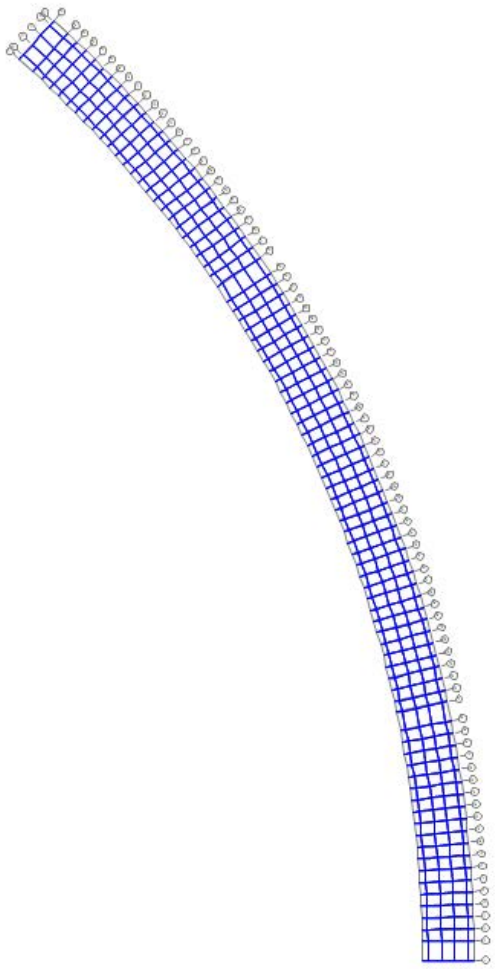
H. Define Hinge Length.
- The hinge length is one half of the section depth.
  - $Hinge_{long} = 0.5 * (Deck\ thickness + haunch\ height + top\ flange\ thickness + web\ height + bottom\ flange\ thickness)$:
    a. $Hinge_{long} = 45\ in.$
  - $Hinge_{Frac} = 0.5 * (Deck\ thickness + haunch\ height)$:
    a. $Hinge_{Frac} = 7\ in.$
  - $Hinge_{Trans} = 0.5 * (Deck\ thickness)$:
    a. $Hinge_{Trans} = 4\ in.$

I. Make the plastic hinge in SAP2000.
- Select Define > Section Properties > Hinge Properties > Add New Properties.
- In the Type window, select Moment Curvature and input the corresponding correct hinge length.
- In the Moment Curvature table, insert the four positive and four negative normalized moment curvatures and the zero point.
- Uncheck the symmetric box and select the Is Extrapolated option in the Load Carrying Capacity beyond Point E window.
- In the Scaling for Moment and Curvature window, insert the maximum positive moment and corresponding curvature as well as the maximum negative curvature and corresponding curvature.

- In the Acceptance Criteria, use the values 1, 2, and 3 for Immediate Occupancy, Life Safety, and Collapse Prevention in the positive column and −1, −2, and −3 for the negative column.
- Repeat for all remaining frame sections.

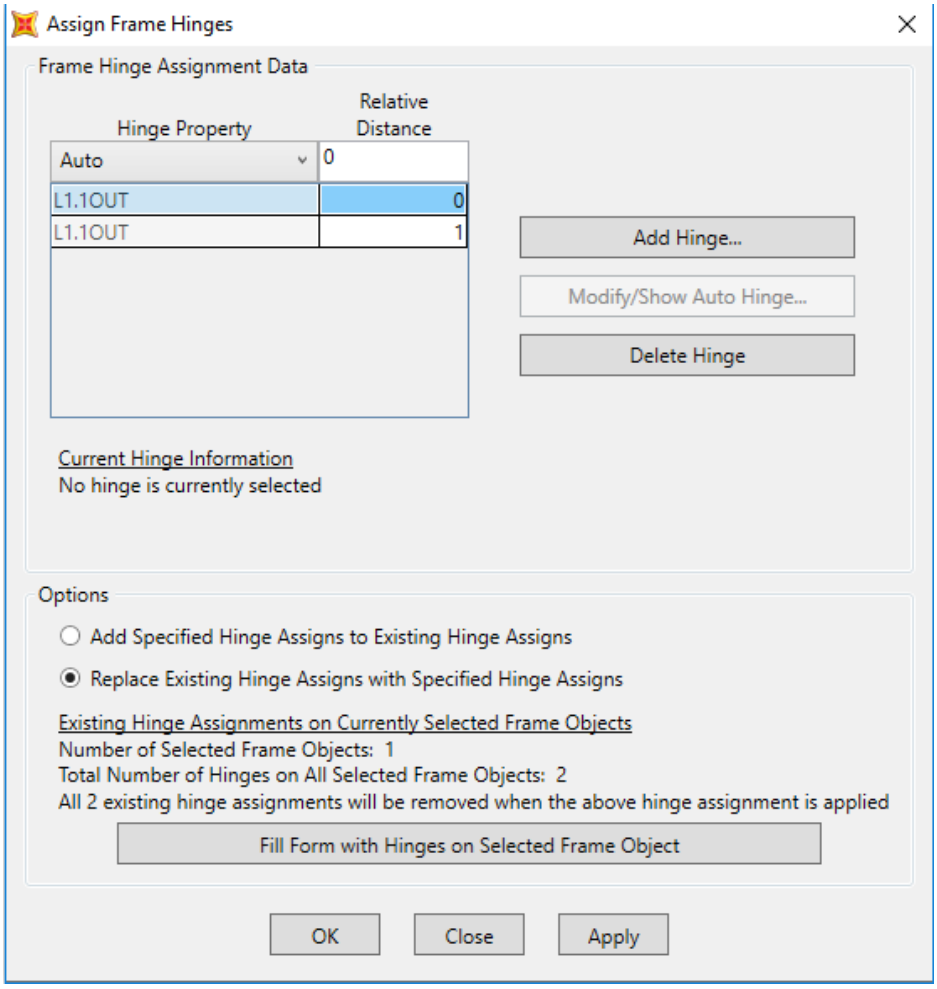


9. Assign frame members to grid.
   A. Select the Draw tab > Quick Draw Frame/Cable/Tendon.
   B. In the Section drop-down menu, select the appropriate cross-section and click on the grillage grid member.

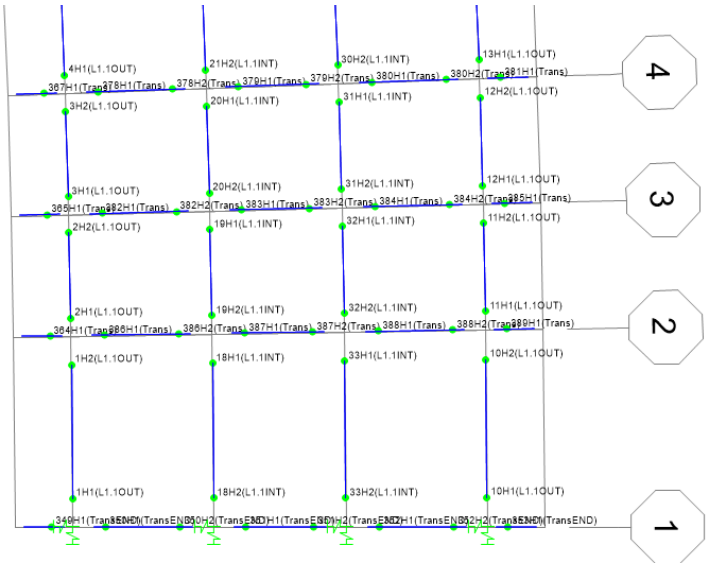


10. Assign hinges to frame elements.
    A. The longitudinal hinges are placed at the ends of the longitudinal frame elements or at a relative distance of 0 and 1.
    B. The transverse hinges are placed at a distance of half a top flange width away from the node.
        - $Hinge\ Loc._{F\ to\ E} = \frac{half\ flange\ width}{Element\ Width} = 1 - \frac{\frac{24}{2}}{39} = 0.6923.$
        - $Hinge\ Loc._{E\ to\ D\ and\ C\ to\ B} = \frac{half\ flange\ width}{Element\ Width} = \frac{24/2}{96} = 0.125\ and\ (1 - 0.125)\ or\ 0.875.$
        - $Hinge\ Loc._{D\ to\ C} = \frac{half\ flange\ width}{Element\ Width} = \frac{24/2}{90} = 0.1333\ and\ (1 - 0.1333)\ or\ 0.8667.$
        - $Hinge\ Loc._{B\ to\ A} = \frac{half\ flange\ width}{Element\ Width} = \frac{24/2}{39} = 0.3077.$
    C. In SAP2000, assign the hinges to corresponding frame elements.
        - Select the desired frame elements you wish to assign hinges to, such as Long1Out. (The elements will turn from blue to yellow).
        - In SAP2000, select the Assign tab > Frame > Hinges.

- From the drop-down menu, select Long1Out and set relative distance to 0 and click ADD.
- From the drop-down menu, select Long1Out and set relative distance to 1 and click ADD.
- Click OK.



- Repeat the previous step of assigning the hinges for all other frame elements.

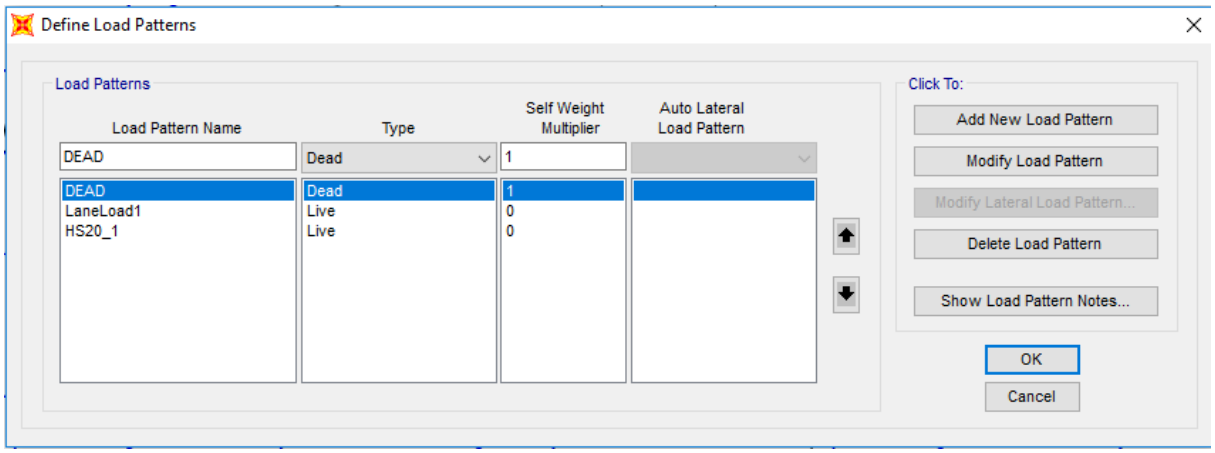


11. Assign loads to the frame elements.
    A. Wheel Axle Loads.
        - Axle loads will be placed at distances of 36 in., 108 in., 180 in., and 252 in. from the outside of the curved edge.
        - One line of the 16 kip axles will be placed at the 0.4L point of the bridge, or Transverse Grid 9, with the second line 14 ft away at Transverse Grid 7. One line of 4 kip axles will be placed 14 ft away from the first line of axles, at Gridline 11.
        - In SAP2000, the loads have to be placed at a relative distance, so this value needs to be calculated.
            - $HS20_{Axel\ 1\ Loc\ (B-A)} = \frac{L_1-36}{L_1} = \frac{39-36}{39} = 0.0792.$
            - $HS20_{Axel\ 2\ Loc\ (C-B)} = \frac{L_1+L_2-108}{L_2} = \frac{39+96-108}{96} = 0.2813.$
            - $HS20_{Axel\ 3\ Loc\ (D-C)} = \frac{L_1+L_2+L_3-180}{L_3} = \frac{39+96+90-180}{90} = 0.5.$
            - $HS20_{Axel\ 4\ Loc\ (E-D)} = \frac{L_1+L_2+L_3+L_4-252}{L_4} = \frac{39+96+90+96-252}{90} = 0.7188.$
    B. Lane Loads
        - Lane Loads are lane loads of 0.640 kip/ft (0.05333 kip/in.) centered at a distance of 96 in. and 240 in. from outside of the curved edge.
        - These lane loads will be placed on the longitudinal frame elements. They will be assigned to elements along the B, C, D, and E longitudinal elements according to the appropriate tributary distance.
            - $LaneLoad_B = \left(\frac{L_1+L_2-96}{L_2}\right) * laneload = \left(\frac{39+96-96}{96}\right) * 0.05333 = 0.021667\frac{kip}{in.}$.
            - $LaneLoad_C = laneload - laneload_B = 0.05333 - 0.021667 = 0.031667\frac{kip}{in.}$.

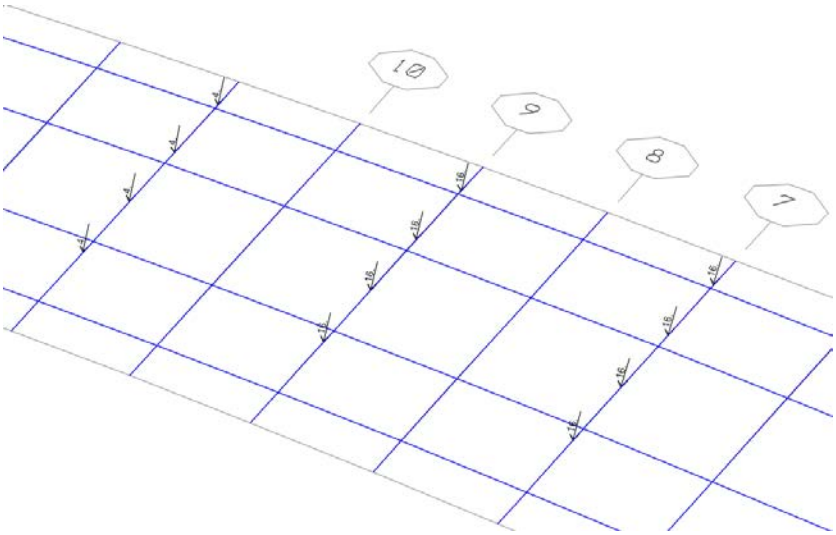
- $LaneLoad_D = \left(\frac{L_1+L_2+L_3+L_4-240}{L_4}\right) * laneload = \left(\frac{39+96+90+96-240}{96}\right) * 0.05333 = 0.045 \frac{kip}{in.}$.
- $LaneLoad_E = laneload - laneload_D = 0.05333 - 0.045 = 0.00833 \frac{kip}{in.}$.

C. In SAP2000, the load patterns must first be defined.

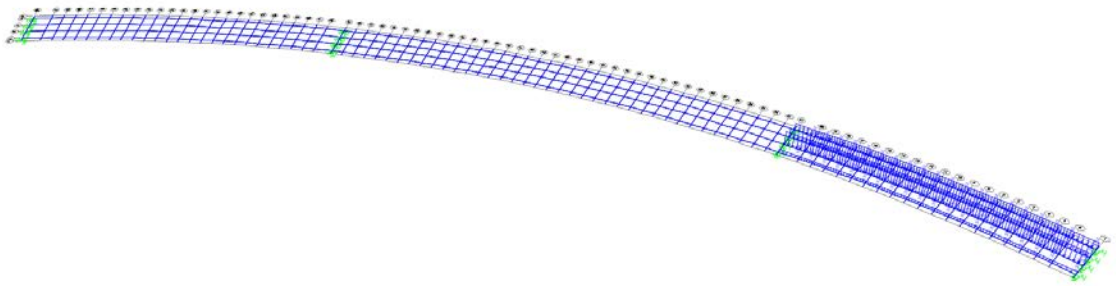
- Select the Define tab > Load Patterns.
  - Under the Load Pattern Name, enter HS20_1 and change the type in the drop-down menu to Live. The self-weight multiplier should be set to 0. Then click Add New Load Pattern.
  - Under the Load Pattern Name, enter LaneLoad1 and change the type in the drop-down menu to Live. The self-weight multiplier should be set to 0. Then click Add New Load Pattern.
  - Click OK.



- Assign the wheel loads in SAP2000.
  - Select the exterior transverse element of Gridlines 9 and 7.
  - Click Assign > Frame Loads > Point.
  - From the Load Pattern drop-down menu, select HS20 and verify that the coordinate system is set to Global, the Load Direction is Gravity, and the Load Type is Force.
  - In Column 1, enter a relative distance of 0 (Axle 1, Loc. B-A) and load of 16 kips.
  - Click OK.
  - Repeat for Gridline 11 to assign the 4 kip load.
  - Repeat Steps 1–6 for Axle 2, 3, and 4, Loc. C-B, D-C, and E-D.

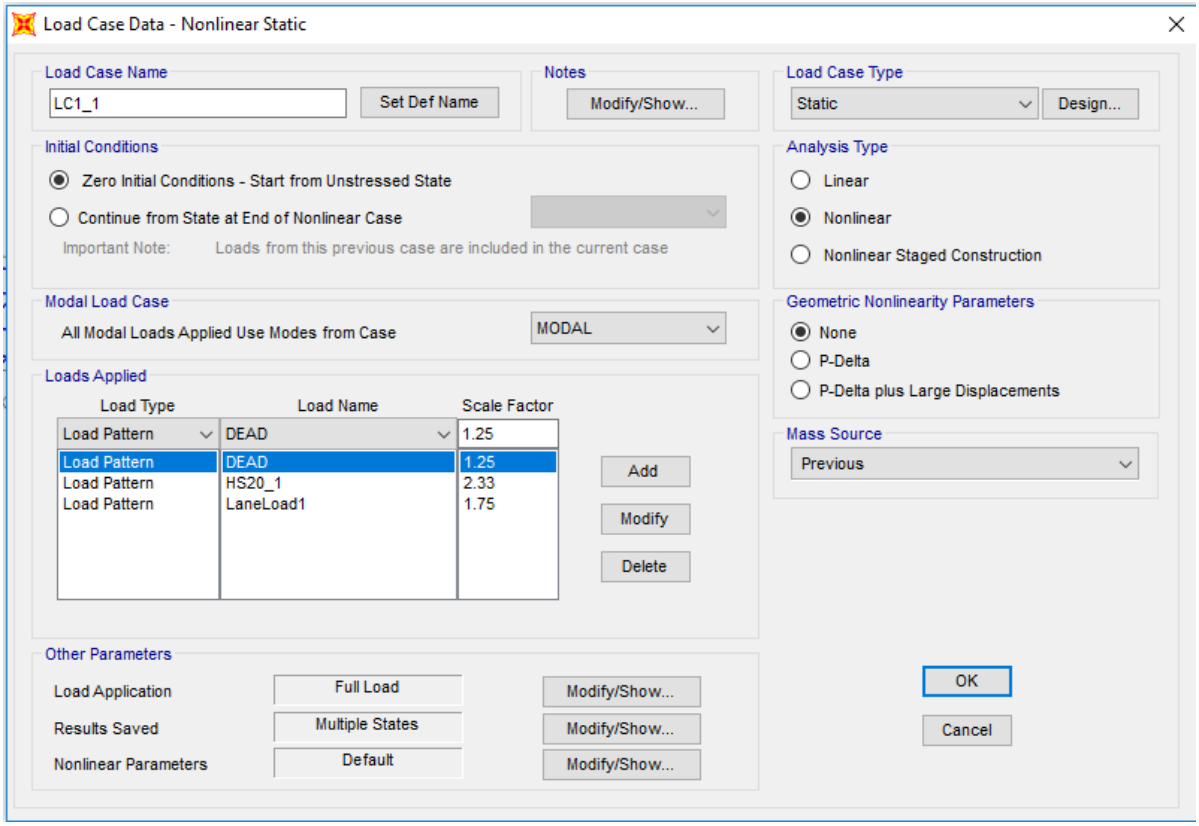


- Assign the Lane Load in SAP2000.
    - Select all exterior longitudinal frame elements along Gridline B.
    - Click Assign > Frame Loads > Distributed
    - From the Load Pattern drop-down menu, select Lane Load and verify that the coordinate system is set to Global, the Load Direction is Gravity, and the Load Type is Force.
    - In the Uniform Load box, enter 0.021667 (Lane Load B).
    - Click OK.
    - Repeat Steps 1–5 for all of the longitudinal elements along gridlines C, D, and E.

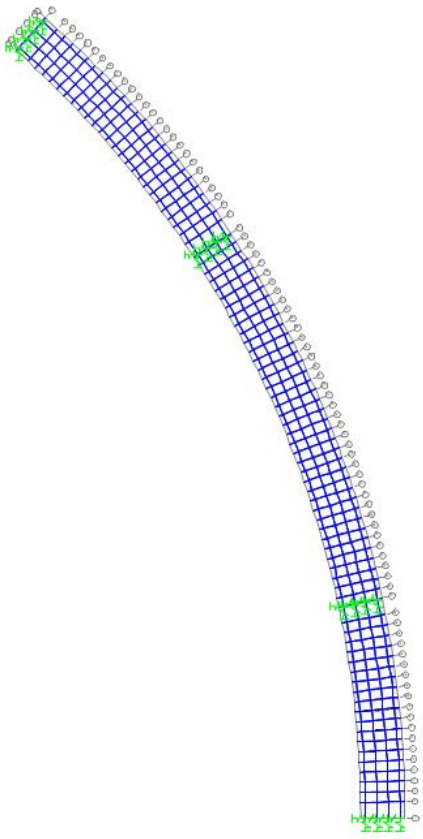


12. Define load cases.
    A. The load case being used to determine redundancy is $1.25DL + 1.75(LL + IM)$. Where DL = Dead Load, LL = Live Load, and IM = Impact Load. When substituting in the truck load and the lane load, the preceding equation reduces to $1.25DL + 1.75LaneLoad + 2.33HS20$.
    B. Generate Load Cases in SAP2000.
        - Click Define > Load Cases > Add New Load Case.
        - In the Load Case Name Panel, name the load case “LC1_1.”
        - In the Analysis Type, select “Nonlinear.”

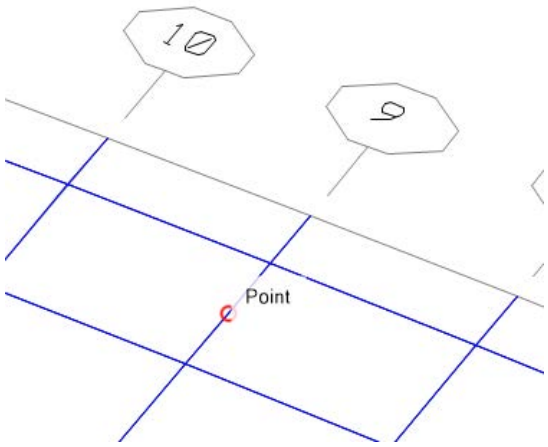
- For the LC1_1 Load Case in the Stiffness to Use panel, select “Zero Initial Conditions.”
- In the Loads Applied panel, leave the Load Type “Load Pattern” in the drop-down Load Name menu, select DEAD and change the Scale Factor to 1.25. Click Add. Change the Load Name menu, select HS20, and change the Scale Factor to 2.33. Click ADD. Change the Load Name menu, select Lane Load, and change the Scale Factor to 1.75. Click ADD.
- In the Other Parameters panel, in the Results Saved section, click Modify/Show.
- In the Results Saved for Nonlinear Static Load Cases window, change the Results Saved to Multiple States, and in the For Each Stage panel, change the Minimum Number of Saved Steps and the Maximum Number of Saved Steps to 20. Click OK.
- Click the OK on the Load Case Data window.
- Repeat the previous 8 steps listed out in B to create an LC2_1, LC3_1, and LC4_1. However, in the Initial Conditions window, select Continue from State at End of Nonlinear Case, and from the drop-down menu select the preceding load case. (For LC2_1, the Nonlinear Case LC1_1 would be selected).
- Repeat the immediately previous step to create LC(1–4)_2 for Span 2 and LC(1–4)_3 for Span 3.



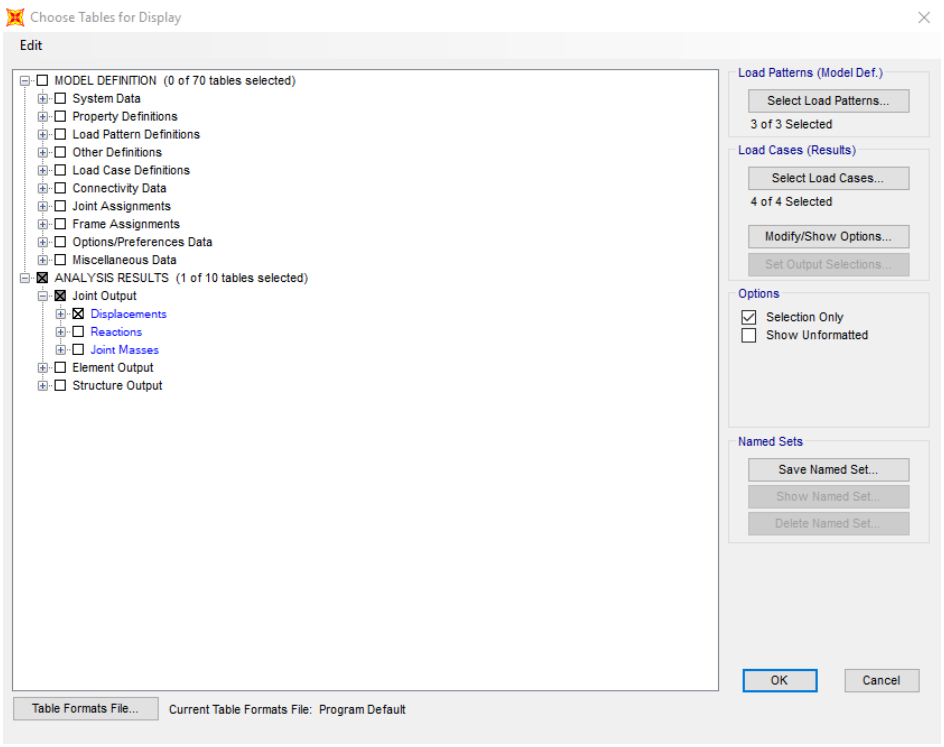
13. Define end supports.
    A. The elastomeric bearing pads for each girder have a lateral stiffness of 12 kip/in. and a vertical stiffness of 6100 kip/in. Since the tub girders are divided in half, the lateral stiffness will be 6 kip/in., and the vertical stiffness will be 3050 kip/in.
    B. Assign spring supports in SAP2000.
        - Select the 16 nodes—8 at the very end of the longitudinal members, 8 at the location of the piers.
        - Click Assign > Joint > Springs.
        - In the Assign Joint Springs window in the Simple Springs Stiffness panel, enter 6 for Translation 1 and 2 and 3050 for Translation 3.
        - Click OK.



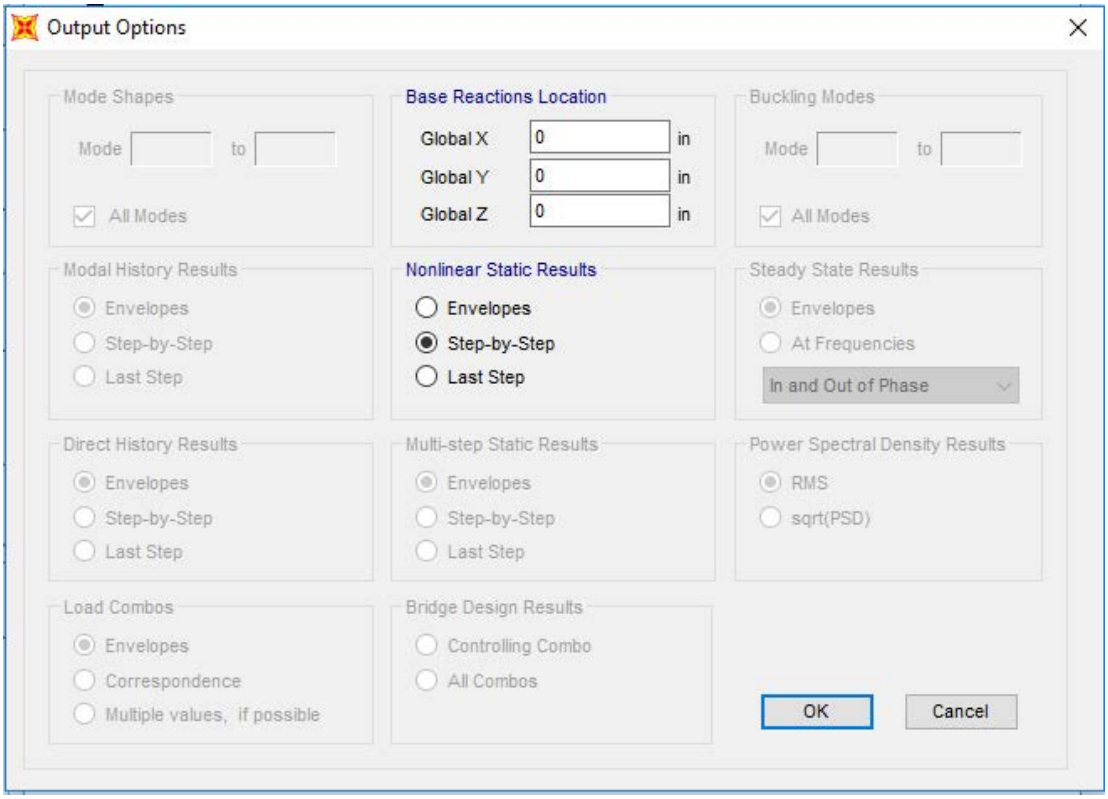
14. Define CL data acquisition points mid-span of girder.
    A. Select the transverse frame elements between B and C and between D and E at 0.4L (Gridline 9).
    B. Click Edit > Edit Lines > Divide Frames.
    C. In the Divide into Specified Number of Frames window, enter 2 for Number of Frames.
    D. Click OK.



15. Analyze the nonfracture structure for dead load only.
    A. In SAP2000, click Analyze > Run Analysis.
    B. In the Set Load Cases to Run window, click the Run/Do Not Run All button until every action is Do Not Run.
    C. Select DEAD then click Run/Do Not Run Case until the action is run.
    D. Then click Run Now. Let SAP2000 Run the Load Cases until the screen says the analysis is complete.
    E. Once the analysis is complete, select Spring Reactions.
    F. Click Display > Show Tables.
    G. In the Choose Table for Display window, click the + symbol beside Joint Output and select the square box beside Reactions.
    H. In the Output Options window in the Nonlinear Static Results panel, select Last Step.
    I. Select and copy the information from the F3 column.
    J. The sum of the F3 values is the dead load.
    K. Click Done.
    L. Unlock the structure.
16. Analyze the nonfractured structure.
    A. In SAP2000, click Analyze > Run Analysis.
    B. In the Set Load Cases to Run window, click the Run/Do Not Run All button until every Action is Do Not Run.
    C. Select LC1_1, LC2_1, LC3_1, and LC4_1, then click Run/Do Not Run Case until the action for all four is run.
    D. Click Run Now.
    E. Let SAP2000 Run the Load Cases until the screen says the analysis is complete.
    F. Once the analysis is complete, select the data collection point on the transverse member on the outside girder (C-B).
    G. Click Display > Show Tables.
    H. In the Choose Table for Display window, click the + symbol beside Joint Output and select the square box beside Displacements.

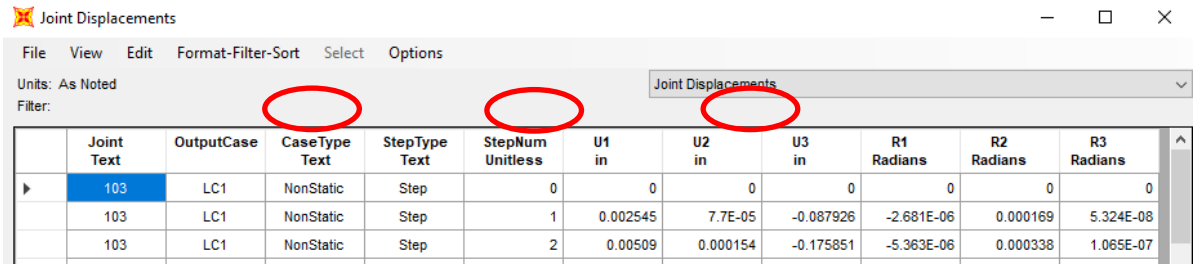


I. Click the Modify/Show Options button.
J. In the Output Options window in the Nonlinear Static Results panel, select Step-by-Step.



K. Click OK.

L. Select and copy the information from the Output Case, StepNum Unitless, and the U3 in. column and paste them into an Excel worksheet. These columns represent Load Case, Step Number, and Deflection for the Outside Girder, respectively.



Joint Displacements

File View Edit Format-Filter-Sort Select Options

Units: As Noted

Filter:

Joint Displacements

	Joint Text	OutputCase	CaseType Text	StepType Text	StepNum Unitless	U1 in	U2 in	U3 in	R1 Radians	R2 Radians	R3 Radians
▶	103	LC1	NonStatic	Step	0	0	0	0	0	0	0
	103	LC1	NonStatic	Step	1	0.002545	7.7E-05	-0.087926	-2.681E-06	0.000169	5.324E-08
	103	LC1	NonStatic	Step	2	0.00509	0.000154	-0.175851	-5.363E-06	0.000338	1.065E-07

M. Click Done.

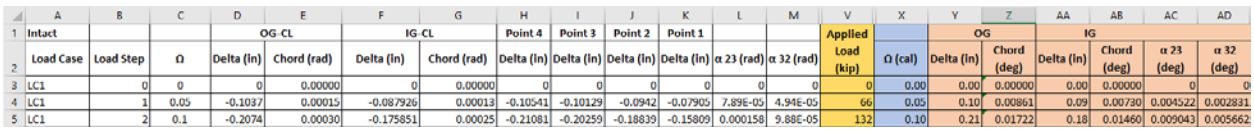
N. Select the data collection point on the transverse member on the inside girder (E-D) and repeat Steps G–L. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

O. Select the joint on the transverse member on the outside girder at Transverse Element 9 (at Longitudinal Element B) and repeat Steps G–L. This information goes into the Delta 4 column. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

P. Select the joint on the transverse member on the outside girder at Transverse Element 9 (at Longitudinal Element C) and repeat Steps G–L. This information goes into the Delta 3 column. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

Q. Select the joint on the transverse member on the inside girder at Transverse Element 9 (at Longitudinal Element D) and repeat Steps G–Ll. This information goes into the Delta 2 column. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.

R. Select the joint on the transverse member on the outside girder at Transverse Element 9 (at Longitudinal Element E) and repeat Steps G–L. This information goes into the Delta 1 column. However, for Step L, there is no need to copy Output Case, Step Num Unitless again.



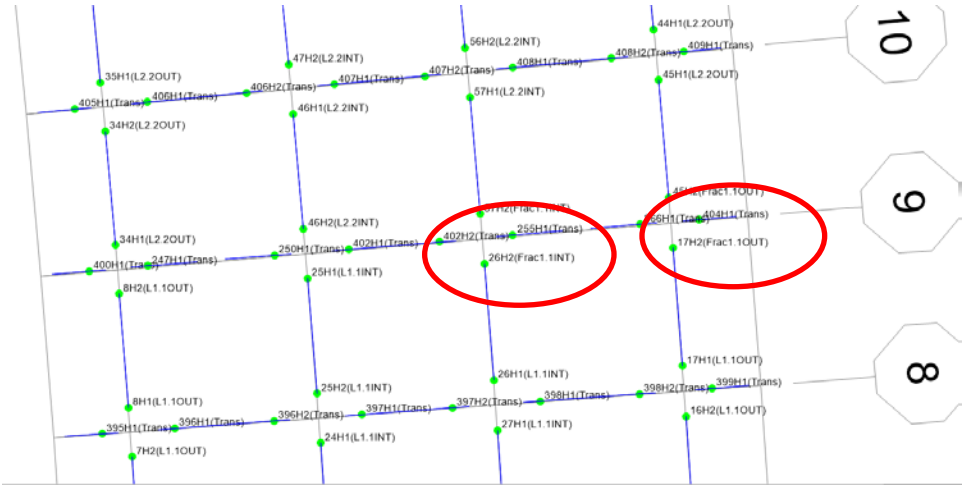
	A	B	C	D	E	F	G	H	I	J	K	L	M	V	X	Y	Z	AA	AB	AC	AD
1	Intact			OG-CL		IG-CL		Point 4	Point 3	Point 2	Point 1			Applied		OG		IG			
2	Load Case	Load Step	Ω	Delta (in)	Chord (rad)	Delta (in)	Chord (rad)	Delta (in)	Delta (in)	Delta (in)	Delta (in)	α 23 (rad)	α 32 (rad)	Load (kip)	Ω (cal)	Delta (in)	Chord (deg)	Delta (in)	Chord (deg)	α 23 (deg)	α 32 (deg)
3	LC1	0	0	0	0.00000	0	0.00000	0	0	0	0	0	0	0	0.00	0.00	0.00000	0.00	0.00000	0	0
4	LC1	1	0.05	-0.1037	0.00015	-0.087926	0.00013	-0.10541	-0.10129	-0.0942	-0.07905	7.89E-05	4.94E-05	66	0.05	0.10	0.00861	0.09	0.00730	0.004522	0.002831
5	LC1	2	0.1	-0.2074	0.00030	-0.175851	0.00025	-0.21081	-0.20259	-0.18839	-0.15809	0.000158	9.88E-05	132	0.10	0.21	0.01722	0.18	0.01460	0.009043	0.005662

S. Once all of the data are collected, unlock the model by selecting the Lock tool on the left hand side of SAP2000 screen.

17. Analyze the fractured structure.

A. At mid-span along Gridline 9, replace the hinges of the outside longitudinal element (Gridline B) with FracOUT hinges according to Step 10.

B. At mid-span along Gridline 9, replace the hinges of the first interior longitudinal element (Gridline C) with FracInt hinges according to Step 10.



C. Repeat Step 15 for the fractured case and collect the data accordingly.

18. Post-process the data.

A. In the Excel sheet, the following values need to be calculated for each step.

- Omega (Ω):
  - $\Omega_i = \Omega_{i-1} + \left(\frac{1}{\#\ of\ Steps\ in\ Load\ case}\right)$.
- Longitudinal Chord Rotation of Interior and Exterior Girder:
  - $Chord\ Rot._{Single\ Span} = -1 * \left(\frac{\delta_{CL}}{0.5*L}\right)$ (rad).
- Transverse Deck Rotation:
  - $\alpha_{2-3} = \left(\frac{\delta_3-\delta_2}{s}\right) - \left(\frac{\delta_2-\delta_1}{w}\right)$ (rad).
  - $\alpha_{2-3} = \left(\frac{\delta_3-\delta_2}{s}\right) - \left(\frac{\delta_2-\delta_1}{w}\right)$ (rad).
  - Where s = spacing between the interior top flanges of the inside and outside girders and w = spacing between the top flanges of the same girder.
- Applied Load:
  - Calculate unit applied load or applied load at 1 Ω.
  - $Unit\ Applied\ Load_{single\ span} = 1.25 * Total\ Reactions\ from\ Dead\ Load\ Case + 2 * (2.33 * HS20\ truck + 1.75 * Lane\ Load)$.
  - $Applied\ Load = Unit\ Applied\ Load * \Omega$.

B. Repeat Step A for the fractured case.

C. Calculate the initial stiffness for the intact bridge and the instantaneous stiffness for the fractured bridge.

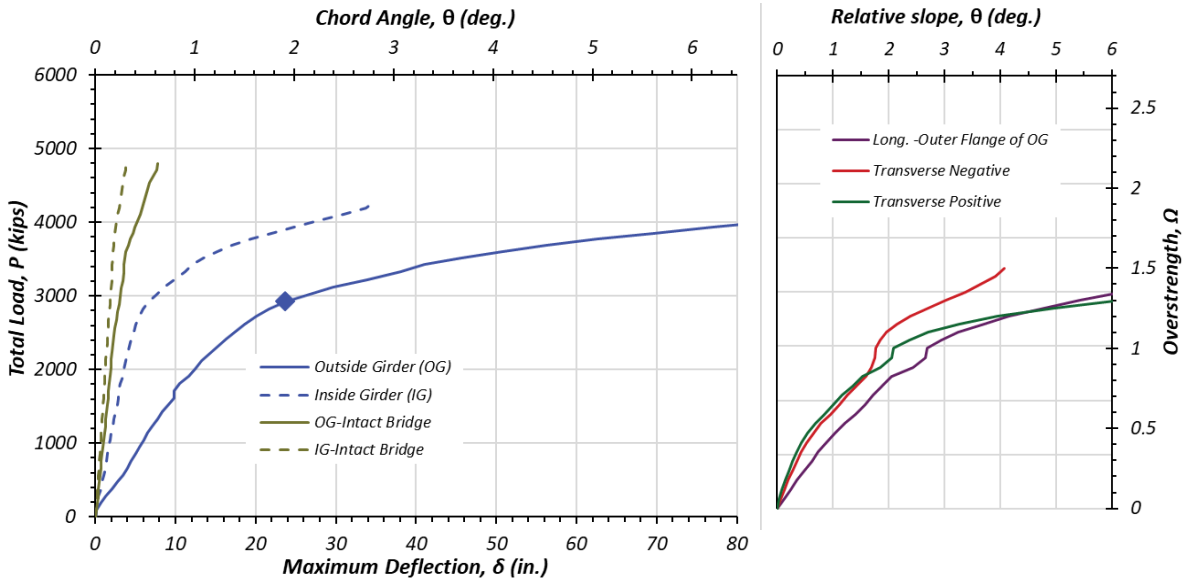
- For the nonfractured condition (intact bridge) find the absolute displacement for the outside girder at an Ω value of 0.4:
  - $Initial\ Stiffness = \frac{0.4}{Absolute\ Displacement\ OG\ (at\ \Omega=0.4)}$.

- For the Fractured case, add an additional column labeled stiffness:
    - $Instantaneous\ Stiffness_{OG-Frac.i} = \frac{\Omega_i - \Omega_{i-1}}{\delta_i - \delta_{i-1}}$.

D. Failure of the structure occurs at the Ω of the fractured bridge at the first of the following criteria:
- The instantaneous stiffness for the fractured outside girder is less than 5 percent of the initial stiffness of the intact outside girder.
- The chord angle of the outside girder for a simple span or interior spans is greater than 2 degrees. The chord angle for exterior spans of multi-span bridges is greater than 3 degrees.
- The transverse deck rotation is greater than 5 degrees.

E. On a chart, plot the nonfractured outside and inside girder as well as the fractured outside and inside girder with displacement on the primary *x* axis and the total force on the primary *y* axis and Ω on the secondary *y* axis.

19. Repeat Steps 11–18 for Span 2 and 3 (Span 1 is pictured below).

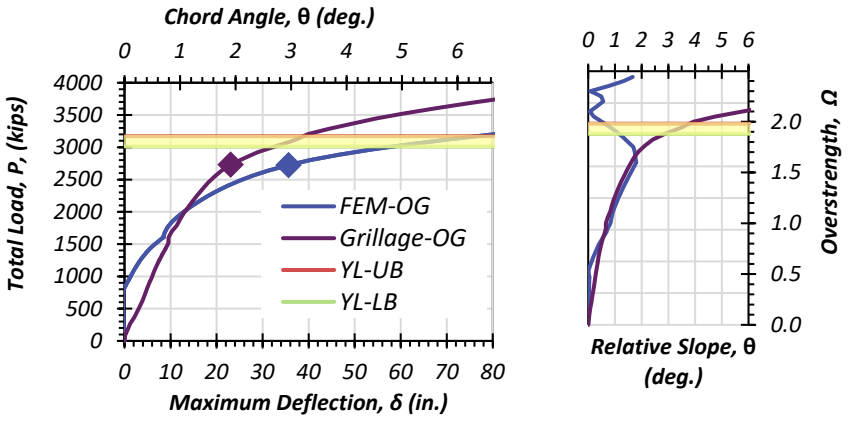


7.3 OVERALL RESULTS FOR BRIDGE 10

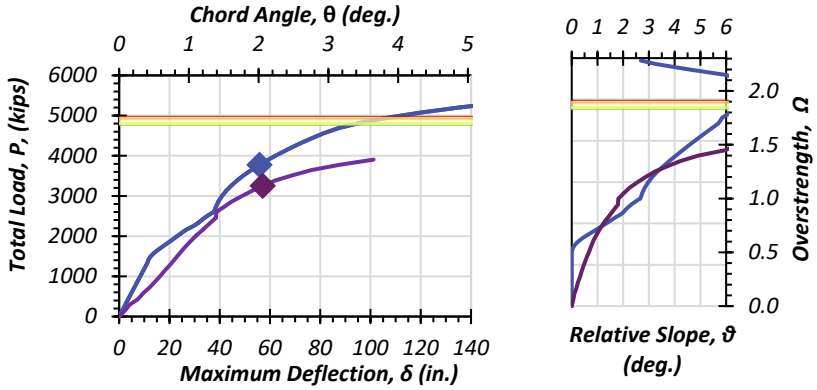
Table 7.1 and Figure 7.2 presents the overall results for Bridge 10. Similar to the previous sections, a comparison between the three methods is drawn using the results of the yield line band, which is flanked by the upper-bound and the lower-bound solutions, and the overstrength (load) versus deflection plots of the results from grillage and FEM analyses. It is evident, as shown in Figure 7.2, that all three methods indicate that Bridge 10 could be reclassified as nonfracture critical.

Table 7.1. Results Summary of Bridge 10.

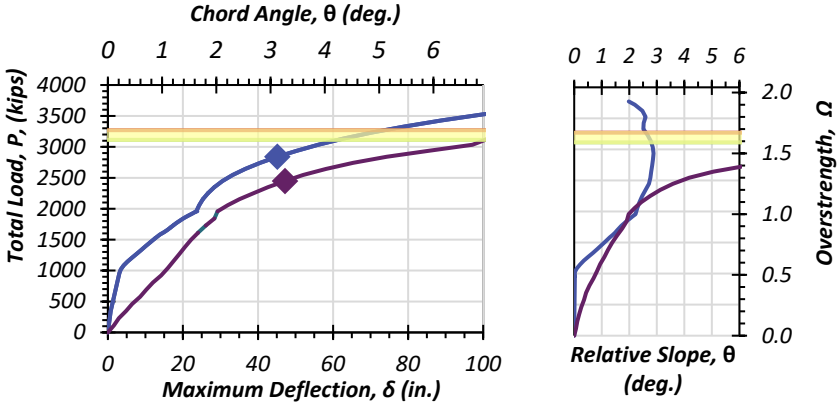
Method of Analysis	Overstrength Factors	Span 1 (140 ft)	Span 2 (139.60 ft)	Span 3 (139.60 ft)
Yield Line Theories	$\Omega_{Yield\ Line}^{Upper\ Bound}$	1.98	1.90	1.67
	$\Omega_{Yield\ Line}^{Lower\ Bound}$	1.88	1.84	1.59
Grillage Analysis	$\Omega_{Grillage}$	1.71	1.25	1.25
FEM	Ω_{FEM}	1.70	1.45	1.45



(a) Span 1, L_x =148 ft



(b) Span 2, L_x = 265 ft



(c) Span 3, L_x = 190 ft

(i) Load displacement *(ii) Deck rotations*

Figure 7.2. Comparison of the Results for Bridge 10.

8. CLOSURE AND CAVEATS

In comparison to the more accurate FEM analysis, the yield line and grillage push-down methods are reputable simplified analysis models for predicting the overstrength capacity of TTGBs with one fractured girder.

When compared to the FEM and yield line methods, the grillage analysis tends to provide a lower-bound solution for overstrength factor and thereby underestimates the reserve capacity. The yield line method is unable to determine displacement limitations. Therefore, either the grillage or FEM approaches are needed to identify the overall deflection limits based on slab hinge (yield line) rotational restrictions.

The key features of the three methods are discussed as follows:

- Yield line method:
    - The least intensive modeling method (by hand) with limited parameters. The yield line method includes the plates of the steel tubs, reinforcing bars, and the deck concrete.
    - Upper-bound and lower-bound yield line solutions are indicative of strength limits.
    - The yield line approaches do not provide information on practical displacement limitations.
- Grillage method:
    - The grillage method is a computational solution that is somewhat similar to the lower-bound strip methods of plastic analysis of slabs.
    - As an incremental analysis, the grillage approach is able to overcome the shortcoming of identifying displacement limits that are absent in the yield line approach.
    - The initial stiffness of the grillage is not captured well and thereby tends to be a lower-bound solution.
- FEM:
    - The nonlinear FEM is the most computationally intensive and accurate modeling approach.

- Because the method is a full 3D approach, other mechanisms that can only be explained by large-deflection theory may be identified.
- At large deflections, it is often observed that the FEM strength may exceed the upper-bound yield line solution. In the FEM, this is attributed to the development of tensile membrane action. Strength from such membrane action cannot be captured by the grillage and yield line methods because they are both predominately based on flexural collapse.
- It should be noted that while membrane action may theoretically be mobilized, the deflection generally exceeds the practical rotation limits that can be achieved in the slab that exists between the twin tubs. Therefore, the reserve strength capacity must be restricted to be not more than the yield line solution.

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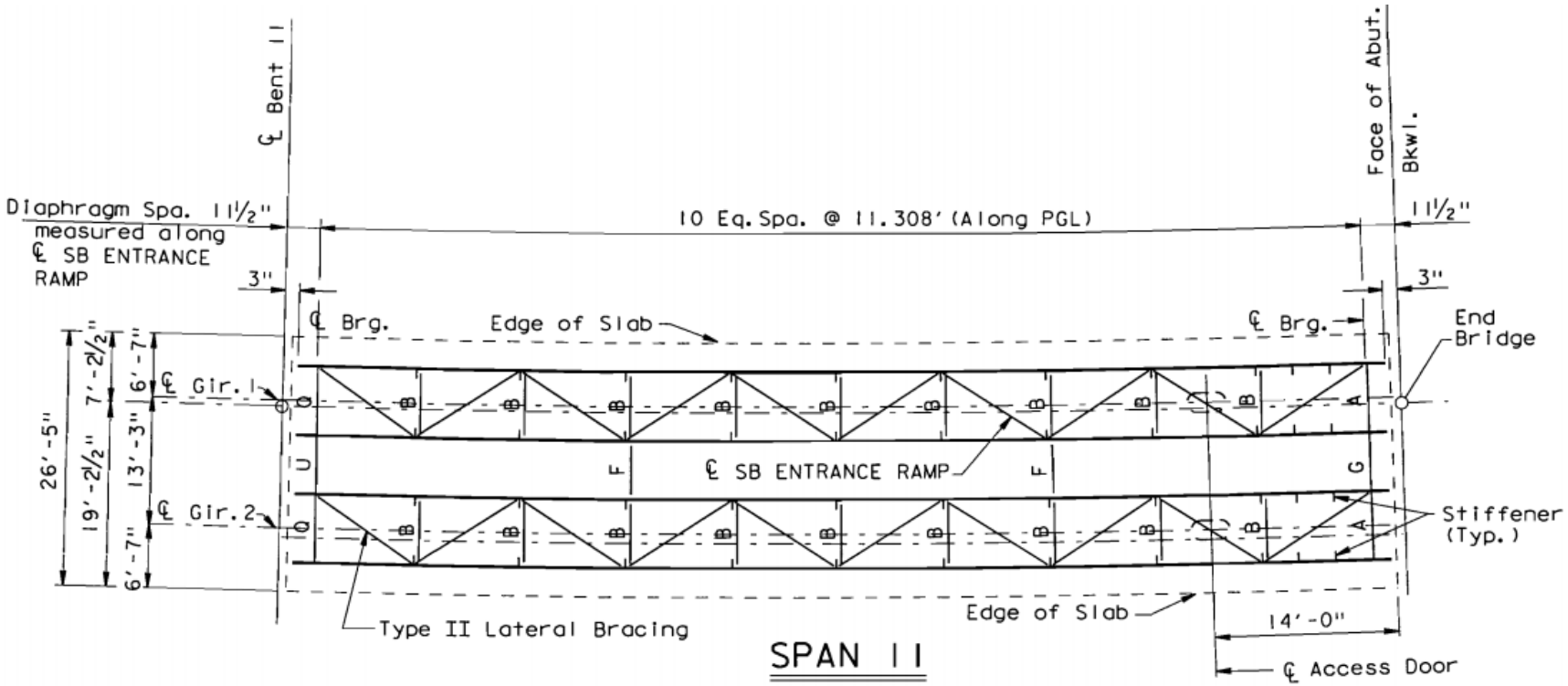
FHWA. (2012). Clarification of requirements for fracture critical members. Memorandum from Office of Bridge Technology, Federal Highway Administration, Washington, DC.

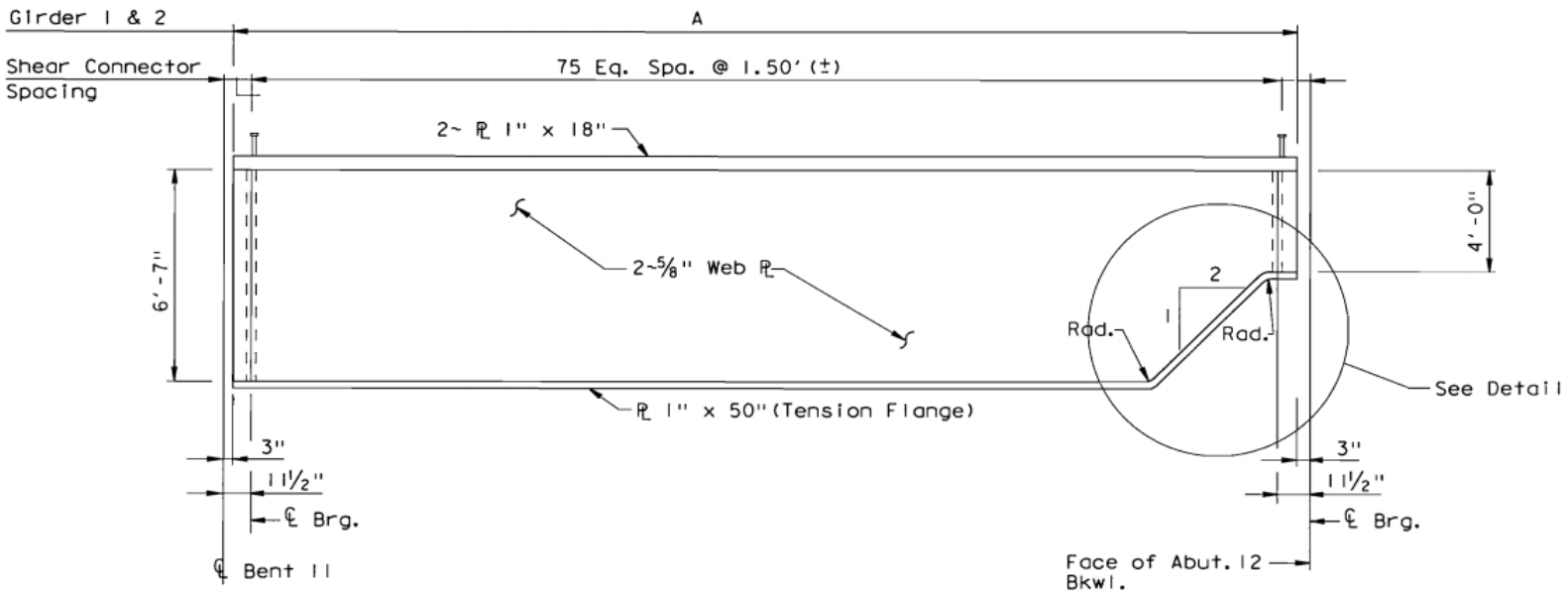
Park, R., and Gamble, W. L. (2000). Reinforced concrete slabs. New York, John Wiley & Sons.

Wilson, E., and Habibullah, A. (1997). SAP2000: integrated finite element analysis and design of structures. Computers and Structures Inc., Berkeley, CA.

APPENDIX A. STRUCTURAL DRAWINGS—BRIDGE 2

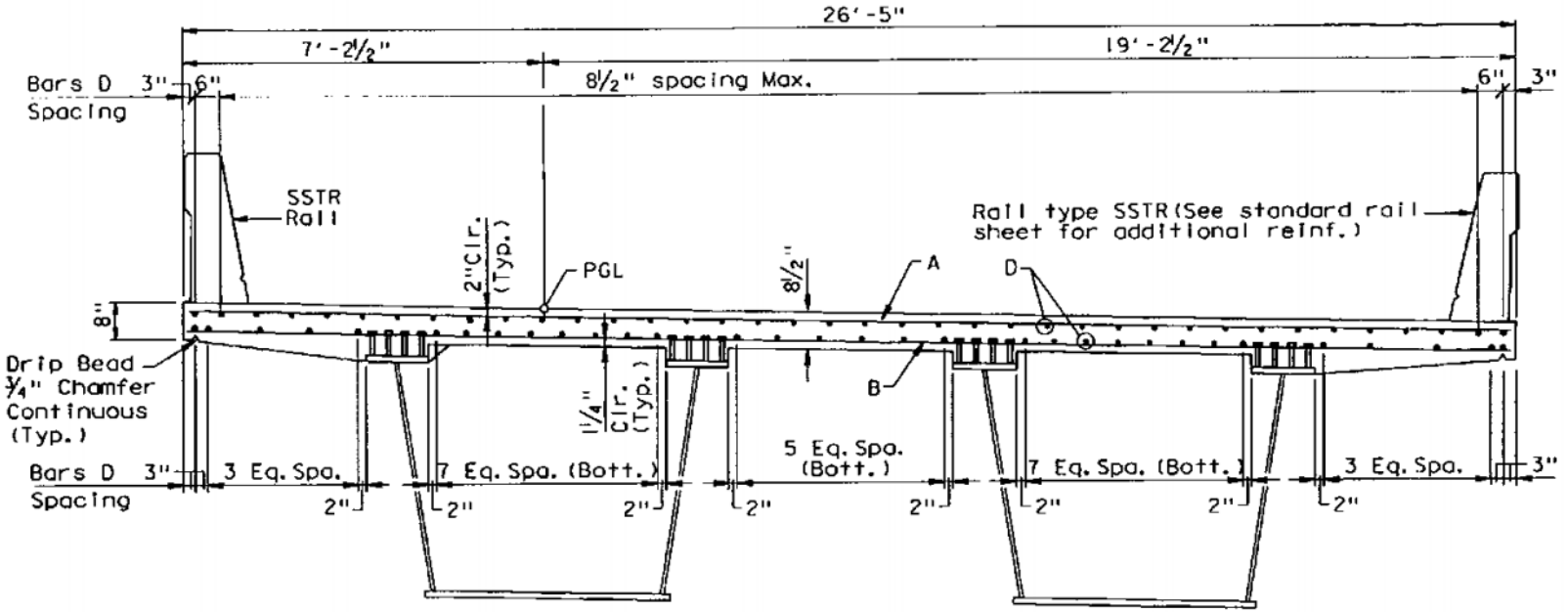
BRIDGE 2: 12-102-0271-17-530





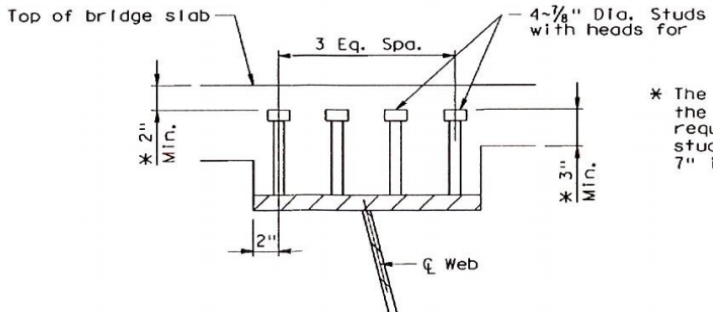
BOX GIRDER ELEVATION (TYP)

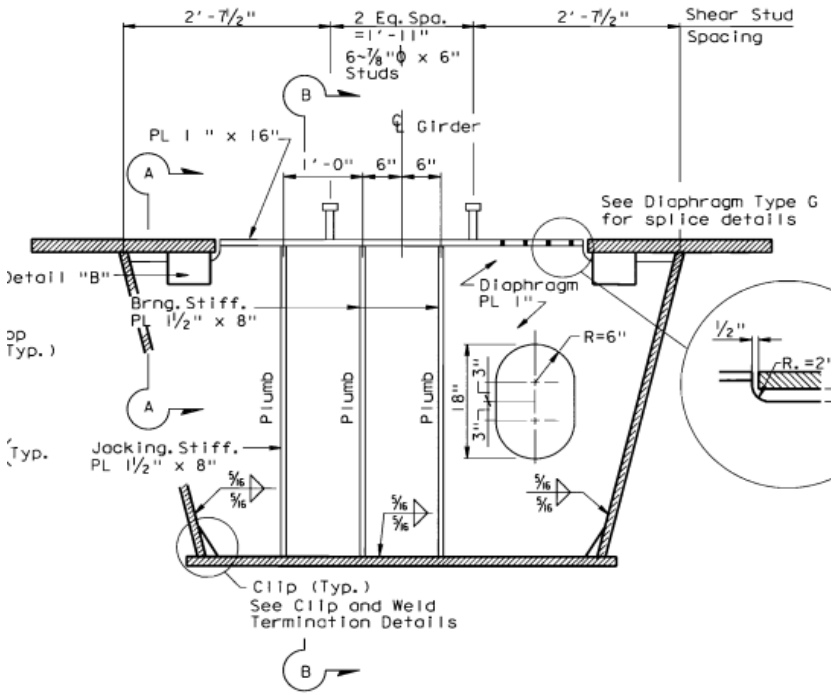
(DIMENSIONS SHOWN ARE ALONG ℄ GIRDER)



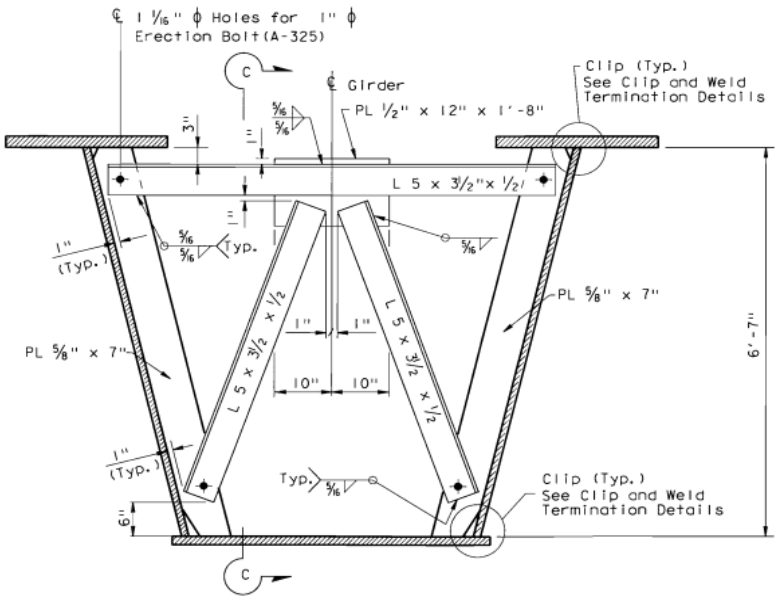
BOX GIRDER ELEVATION (TYP)

(DIMENSIONS SHOWN ARE ALONG ℄ GIRDER)





DIAPHRAGM ~ TYPE A



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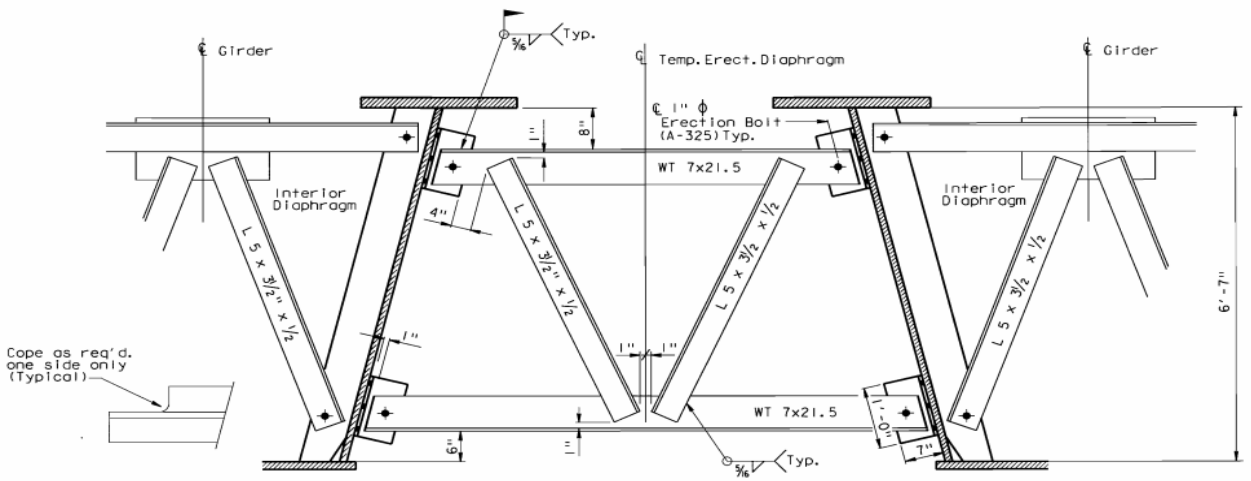
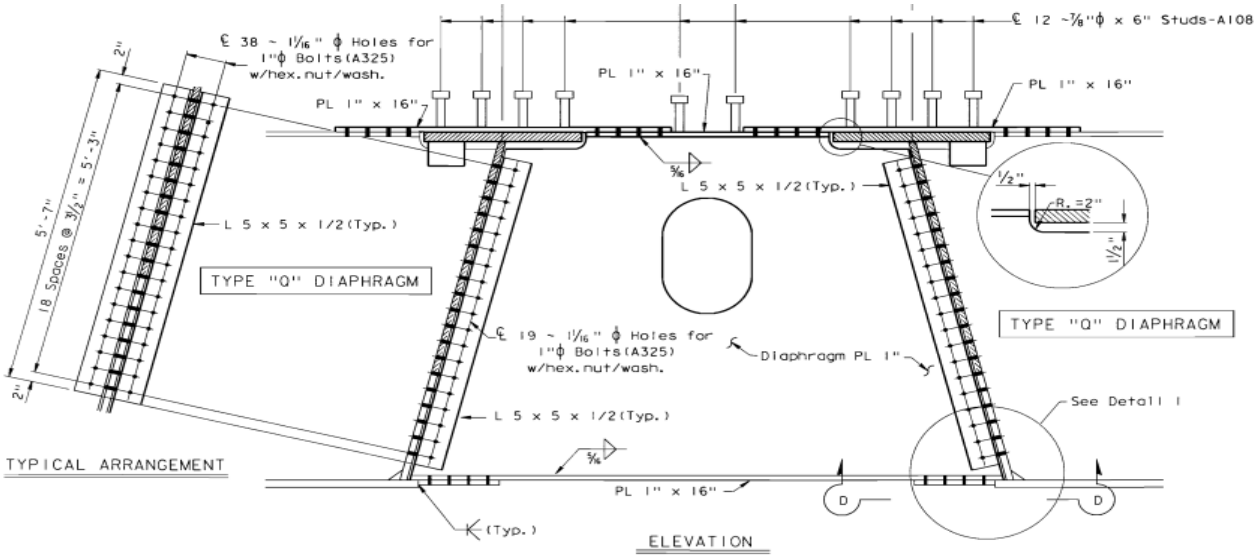
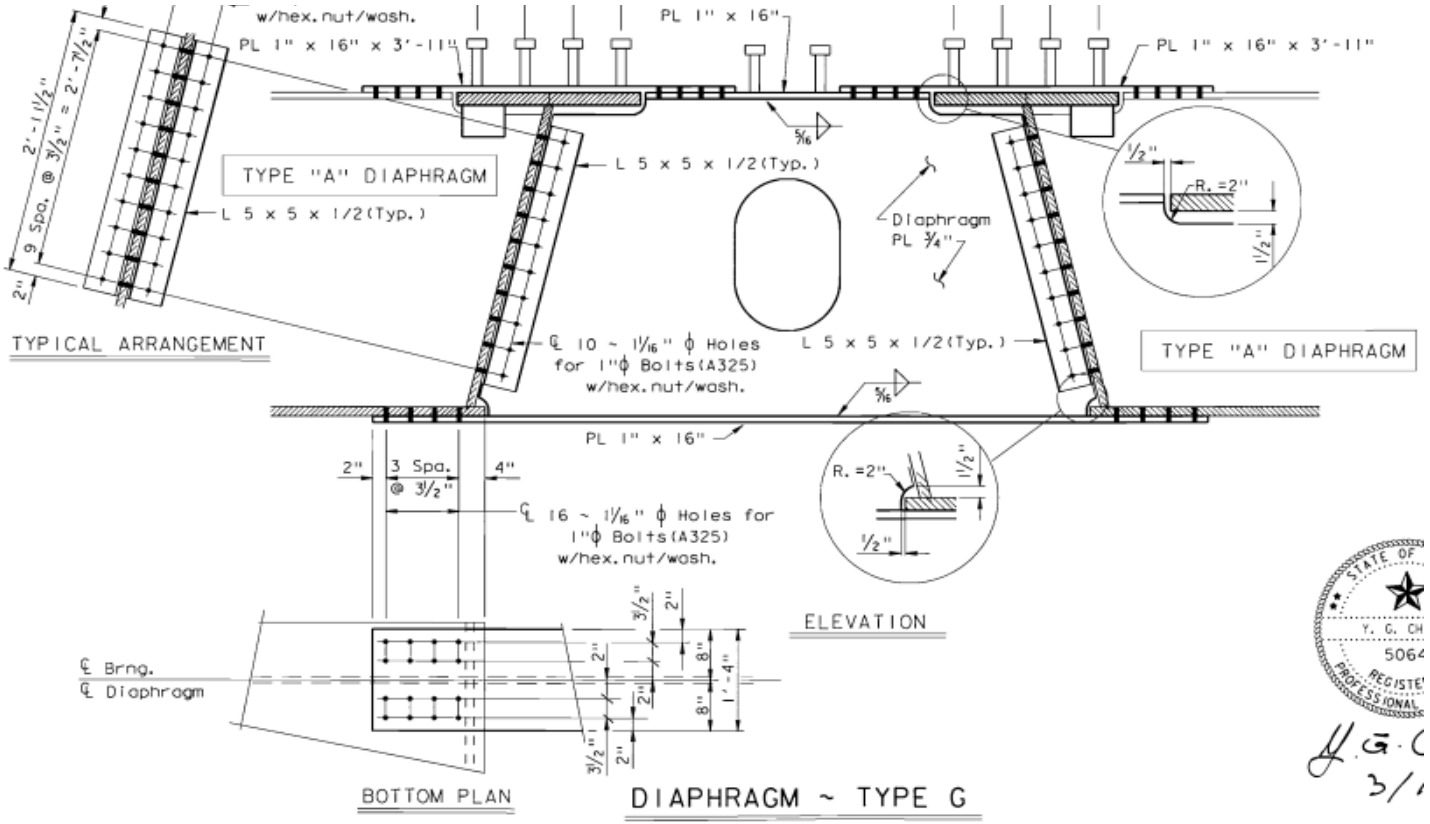
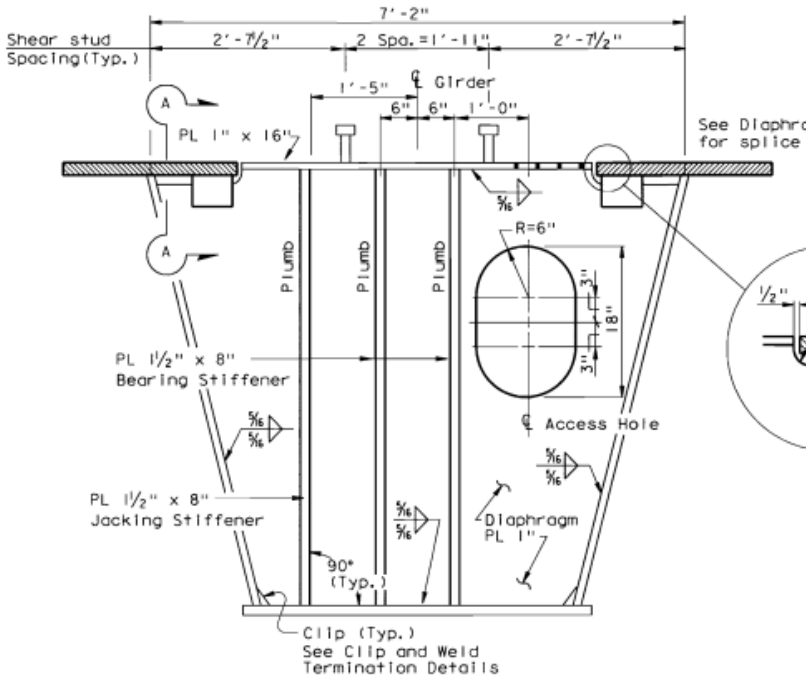


DIAPHRAGM ~ TYPE F (Temporary)

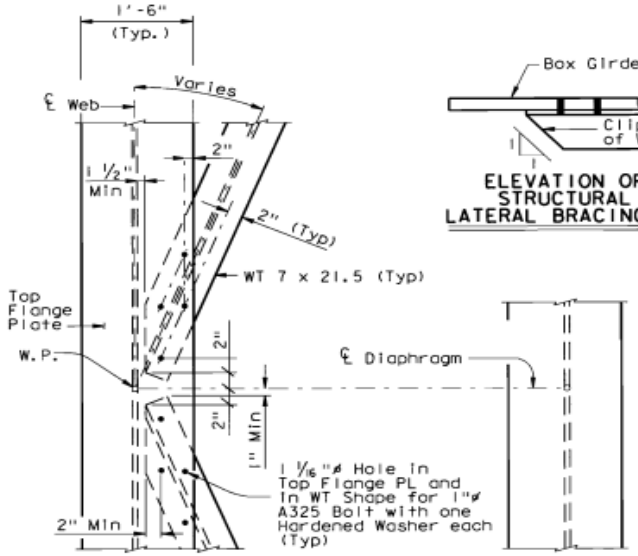


DIAPHRAGM ~ TYPE U

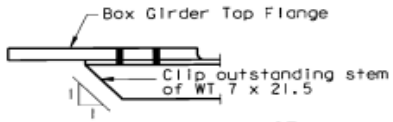

w/hex.nut/wash.
PL 1" x 16" x 3'-11"
PL 1" x 16"
PL 1" x 16" x 3'-11"
2'-1½"
9 Spa. @ 3½" = 2'-7½"
2"
TYPE "A" DIAPHRAGM
L 5 x 5 x 1/2(Typ.)
5/16
L 5 x 5 x 1/2(Typ.)
Diaphragm
PL ¾"
½"
R. =2"
1½"
TYPICAL ARRANGEMENT
℄ 10 ~ 1 1/16" ϕ Holes
for 1"ϕ Bolts(A325)
w/hex.nut/wash.
L 5 x 5 x 1/2(Typ.)
TYPE "A" DIAPHRAGM
5/16
PL 1" x 16"
2"
3 Spa.
@ 3½"
4"
℄ 16 ~ 1 1/16" ϕ Holes for
1"ϕ Bolts(A325)
w/hex.nut/wash.
R. =2"
1½"
½"
ELEVATION
℄ Brng.
℄ Diaphragm
3½"
2"
2"
8"
1'-4"
2"
8"
2"
3½"
BOTTOM PLAN
DIAPHRAGM ~ TYPE G



DIAPHRAGM ~ TYPE Q



TYPICAL PLAN VIEW OF LATERAL BRACING TYPE "I I"

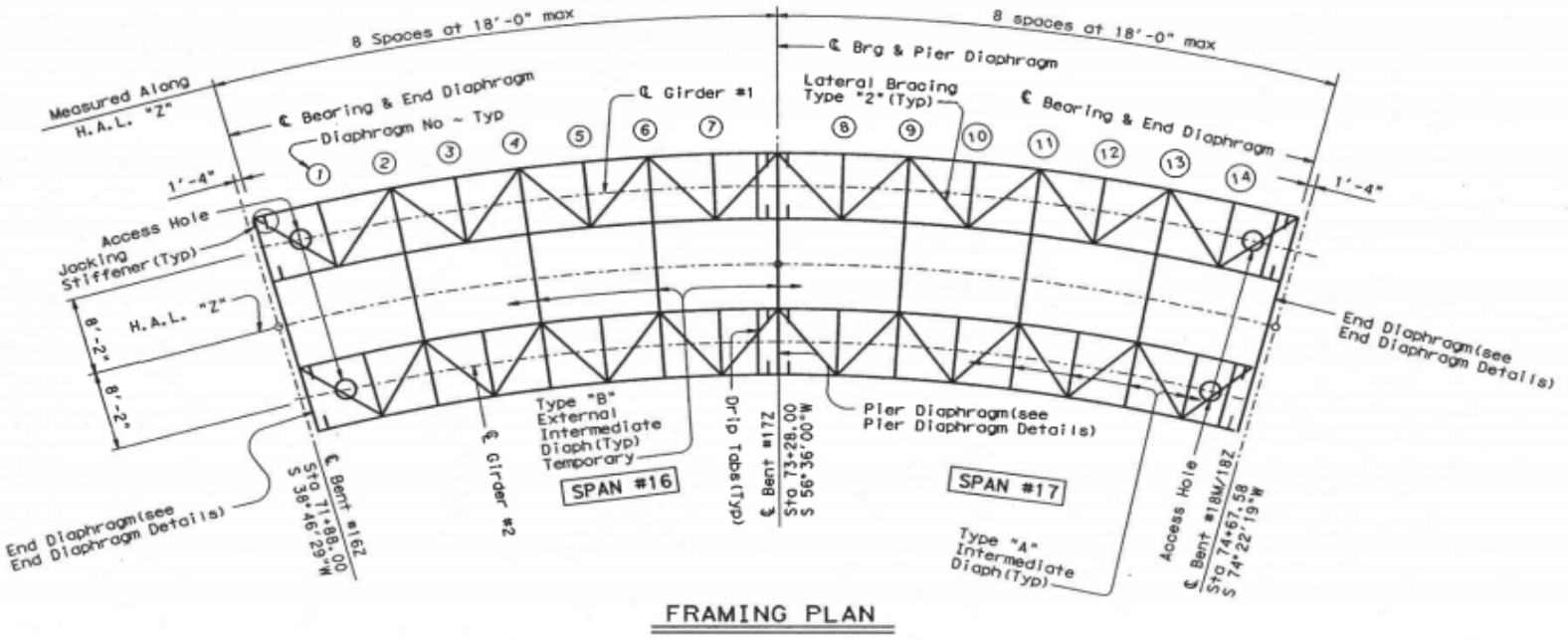


ELEVATION OF END OF STRUCTURAL TEE OF LATERAL BRACING TYPE "I I"

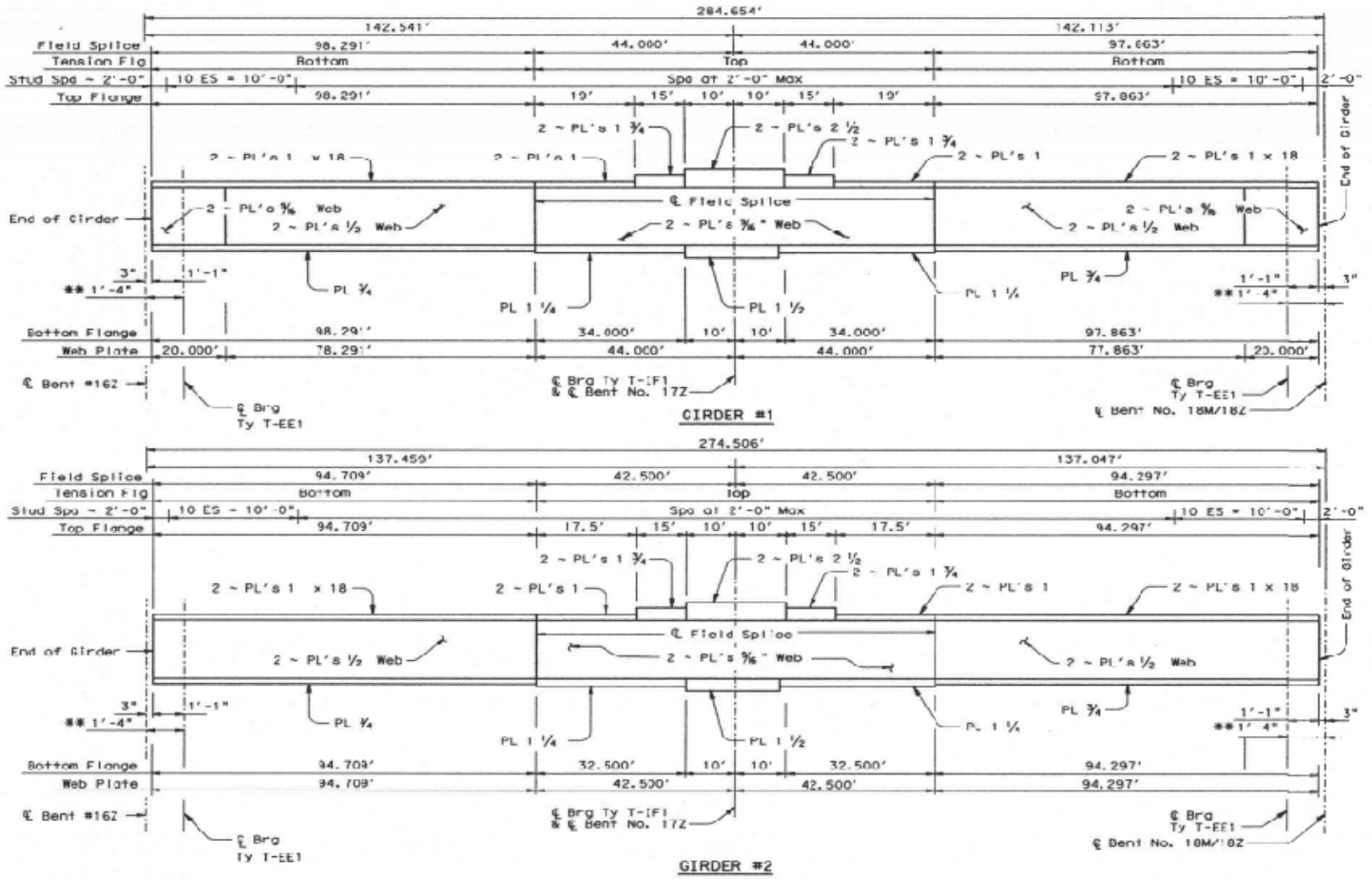
Installed in shop except at locations spanning a field splice, which shall be installed in the field.

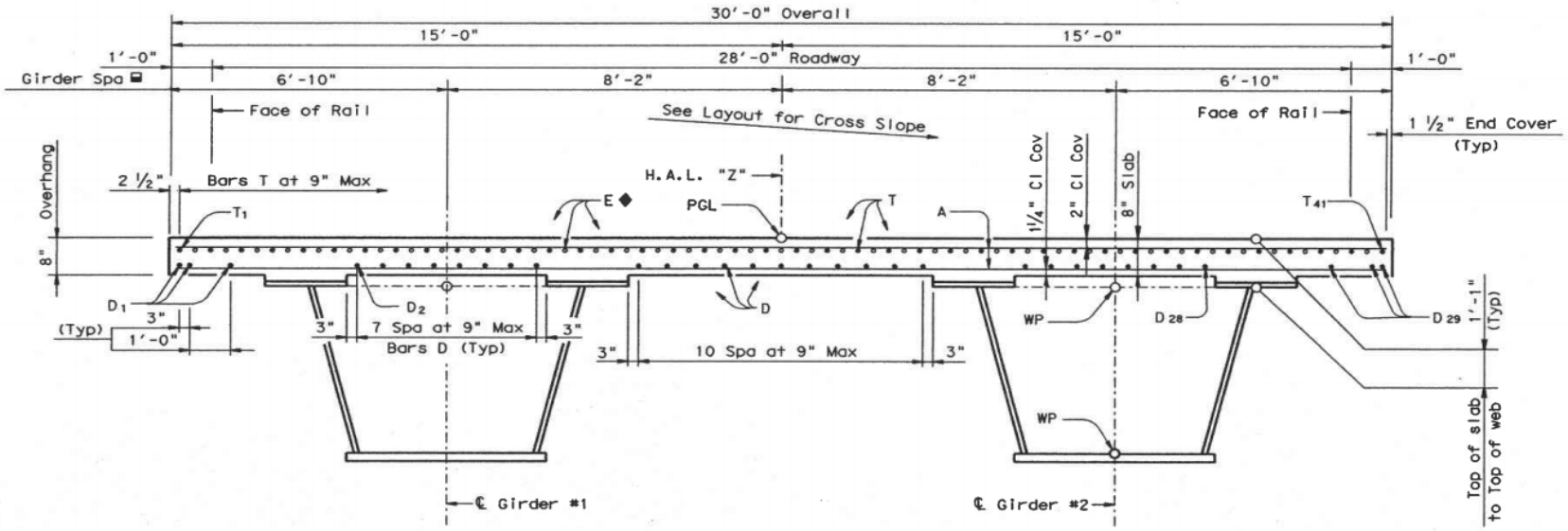
APPENDIX B. STRUCTURAL DRAWINGS—BRIDGE 5

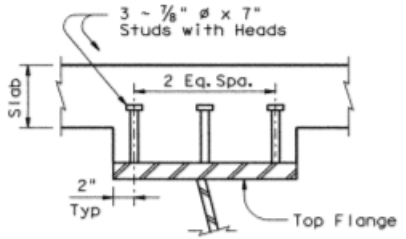
BRIDGE 5: 14-227-0-0015-13-452



FRAMING PLAN

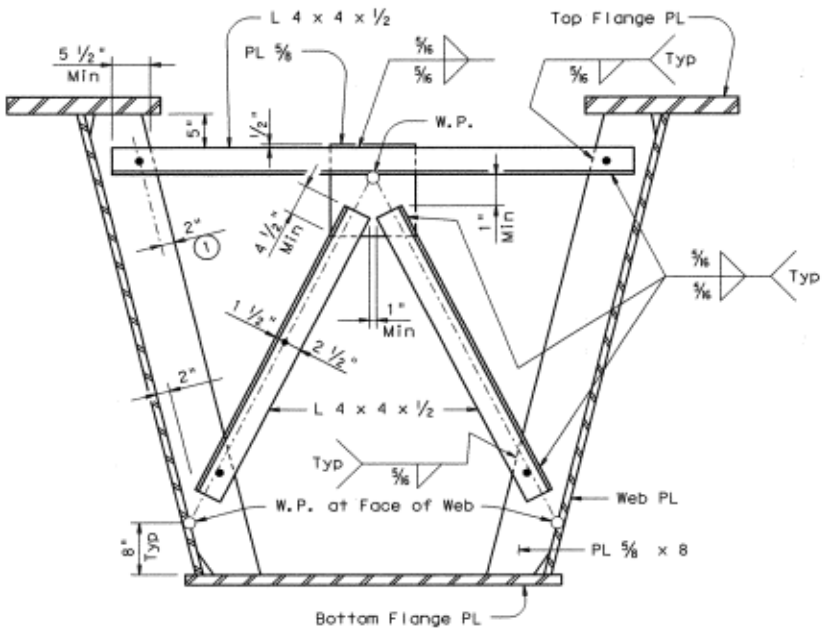

284.654'
142.541'
142.113'
Field Splice
Tension Flg
Stud Spa - 2'-0"
Top Flange
98.291'
44.000'
44.000'
97.863'
Bottom
Top
Bottom
10 ES = 10'-0"
Spa at 2'-0" Max
2'-0"
19'
15'
10'
10'
15'
19'
2 - PL's 1 3/4
2 - PL's 2 1/2
2 - PL's 1 x 18
2 - PL's 1
End of Girder
2 - PL's 5/8 Web
2 - PL's 1/2 Web
℄ Field Splice
2 - PL's 5/8" Web
PL 3/4
PL 1 1/4
PL 1 1/2
3"
** 1'-4"
1'-1"
Bottom Flange
Web Plate
98.291'
34.000'
97.863'
20.000'
78.291'
44.000'
77.863'
℄ Bent #16Z
℄ Brg Ty T-EE1
℄ Brg Ty T-EF1 & ℄ Bent No. 17Z
℄ Brg Ty T-EE1 ℄ Bent No. 18M/18Z
GIRDER #1
274.506'
137.459'
137.047'
94.709'
42.500'
94.297'
17.5'
32.500'
GIRDER #2





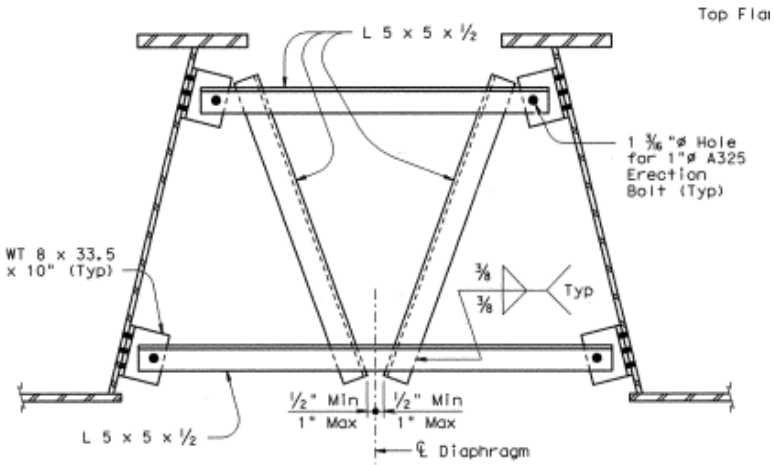
SHEAR CONNECTOR STUD DETAIL

Studs shall be Electric arc end-welded to the flanges with complete fusion. (See Span details for spacing along girder.)

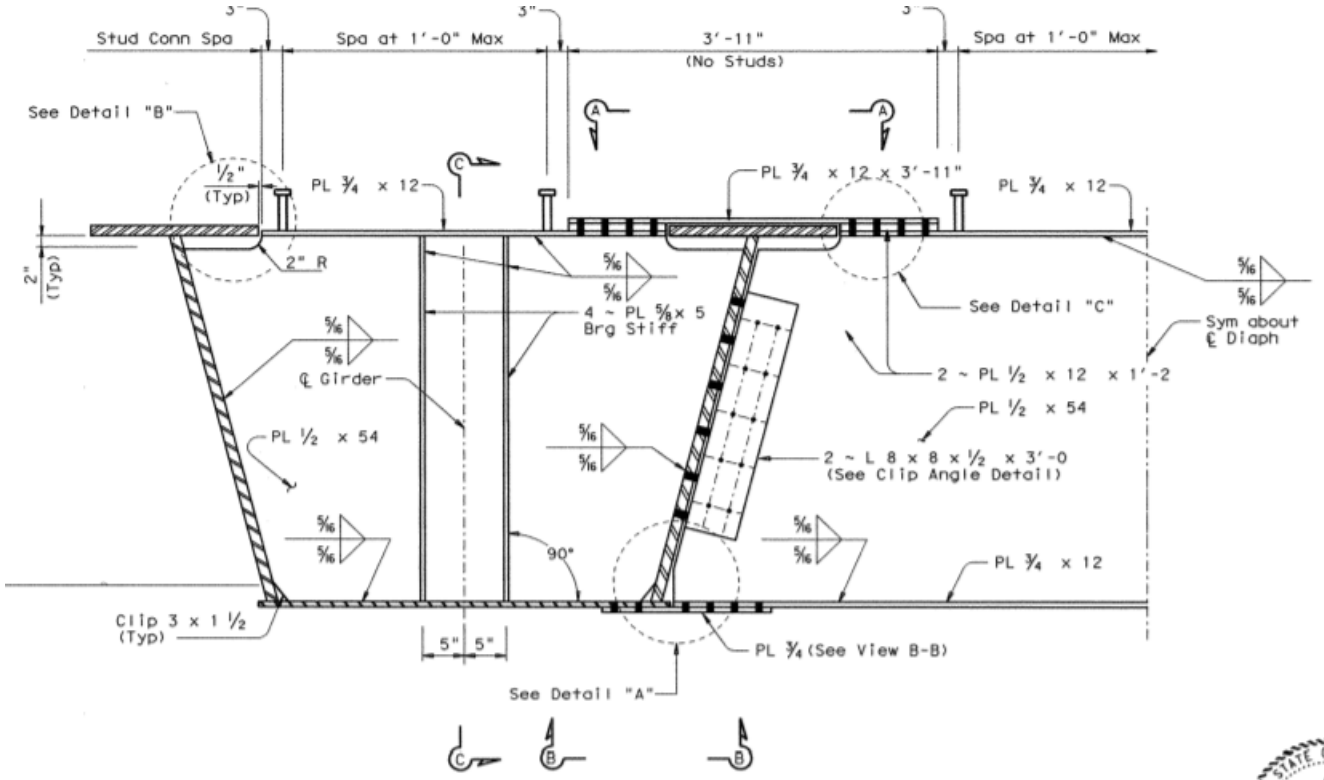


TYPICAL SECTION AT TYPE "A"
INTERNAL INTERMEDIATE DIAPHRAGM

To be fully installed in the shop.



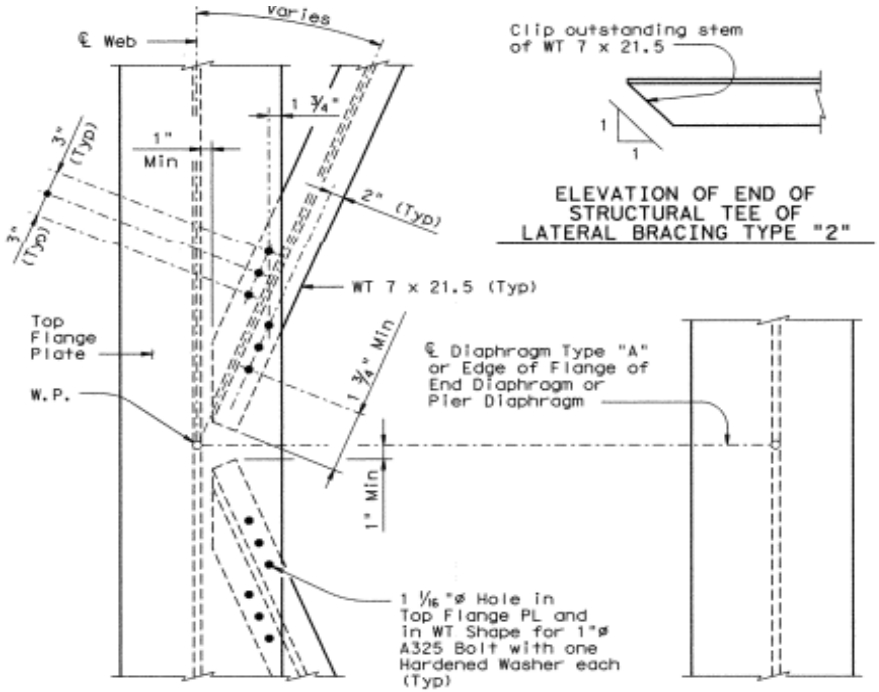
TYPICAL SECTION AT TYPE "B" EXTERNAL
INTERMEDIATE DIAPHRAGM (TEMPORARY)



HALF ELEVATION

END DIAPHRAGM DETAILS

(At Bent Nos 16Z & 18M/18Z)



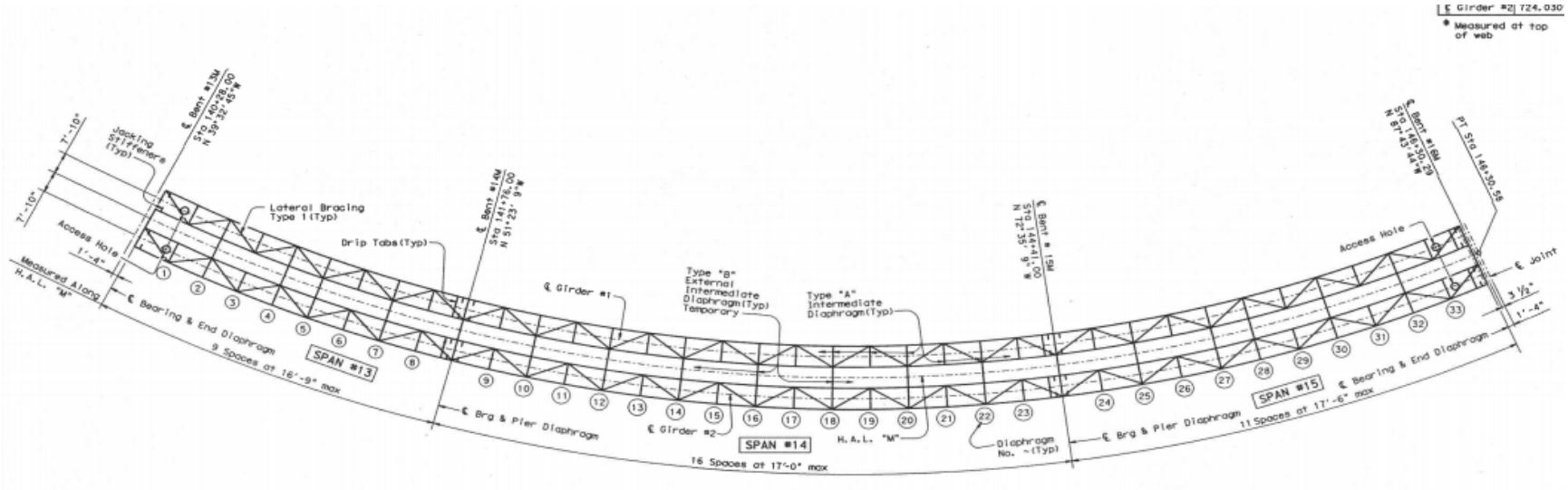
ELEVATION OF END OF STRUCTURAL TEE OF LATERAL BRACING TYPE "2"

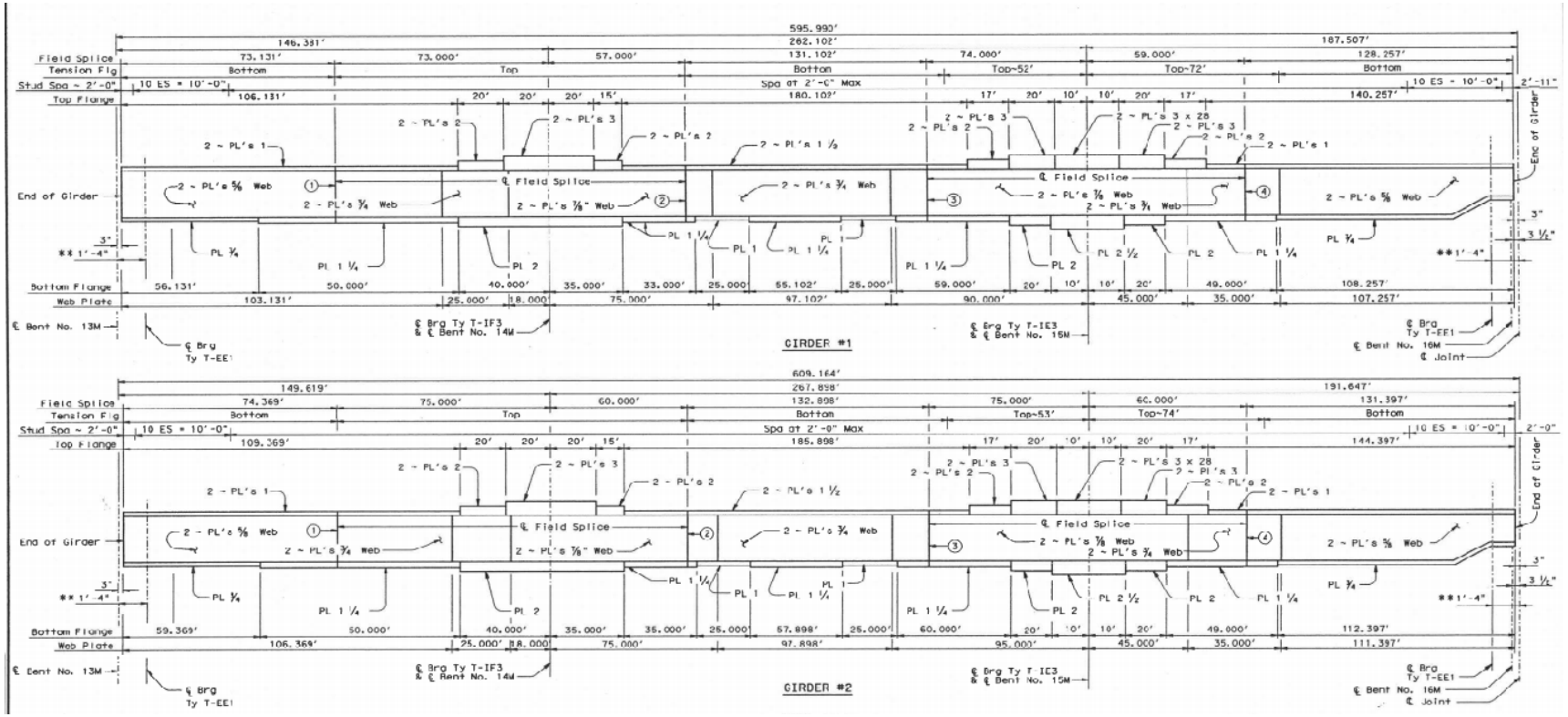
TYPICAL PLAN VIEW OF LATERAL BRACING TYPE "2"

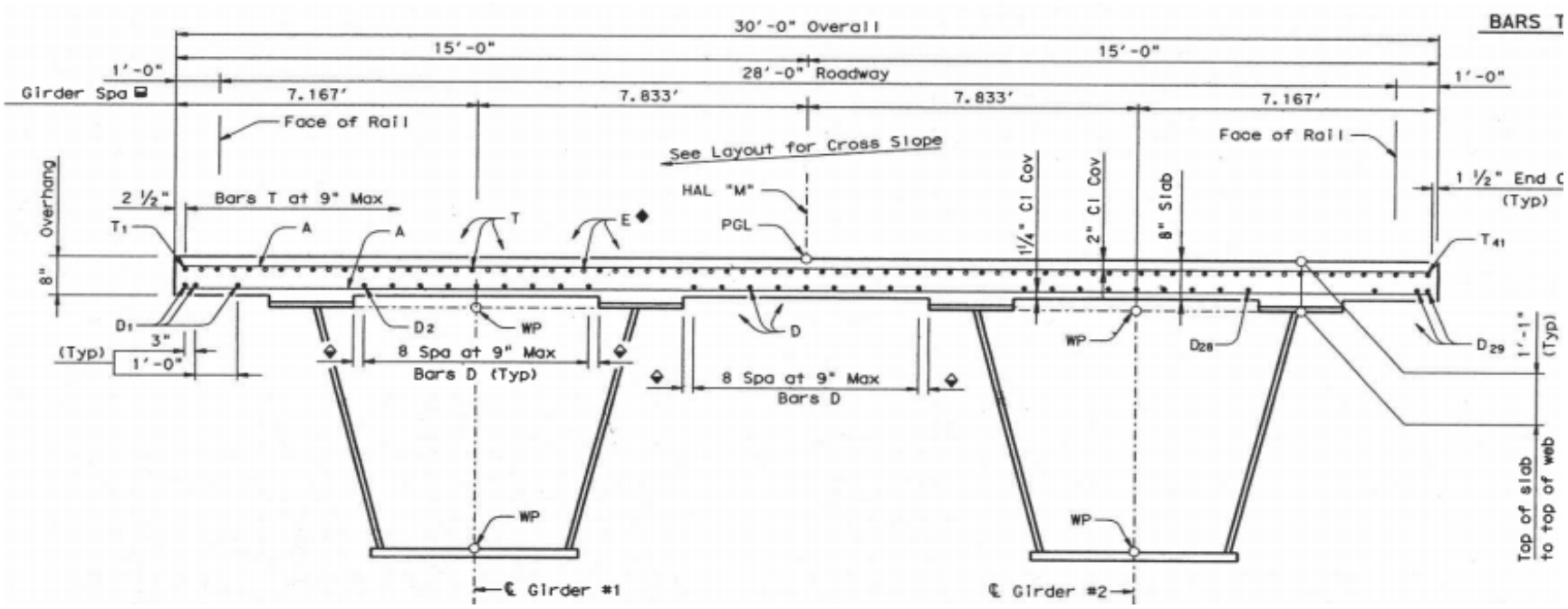
Installed in shop except at locations spanning a field splice, which shall be installed in the field.

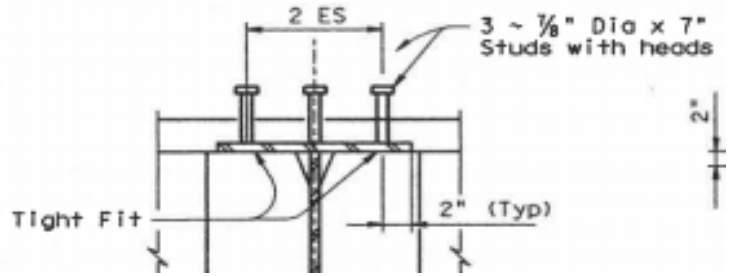
APPENDIX C. STRUCTURAL DRAWINGS—BRIDGE 10

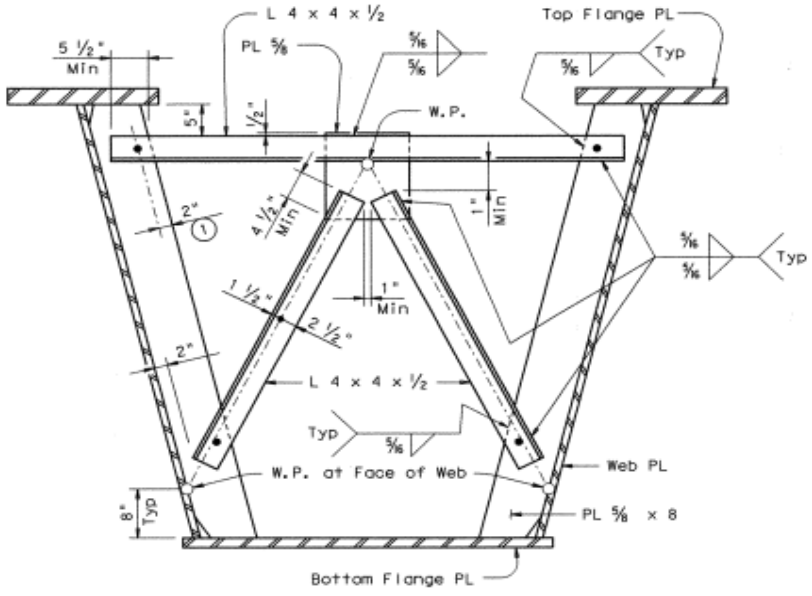
BRIDGE 10: 14-227-0-0015-13-450




595.990'
146.381'
262.102'
187.507'
Field Splice
Tension Flg
Stud Spa ~ 2'-0"
Top Flange
Bottom
Top
Bottom
Top-52'
Top-72'
Bottom
73.131'
73.000'
57.000'
131.102'
74.000'
59.000'
128.257'
Spa at 2'-0" Max
10 ES = 10'-0"
106.131'
180.103'
140.257'
End of Girder
Field Splice
Bottom Flange
Web Plate
56.131'
50.000'
40.000'
35.000'
33.000'
25.000'
55.102'
26.000'
59.000'
49.000'
108.257'
103.131'
25.000'
18.000'
75.000'
97.102'
90.000'
45.000'
35.000'
107.257'
℄ Bent No. 13M
℄ Brg Ty T-EE1
℄ Brg Ty T-IF3 & ℄ Bent No. 14M
℄ Brg Ty T-IE3 & ℄ Bent No. 15M
℄ Brg Ty T-EE1
℄ Bent No. 16M
℄ Joint
GIRDER #1
609.164'
149.619'
267.898'
191.647'
74.369'
75.000'
60.000'
132.898'
75.000'
60.000'
131.397'
Top-53'
Top-74'
109.369'
185.898'
144.397'
59.369'
50.000'
40.000'
35.000'
35.000'
26.000'
67.898'
26.000'
60.000'
49.000'
112.397'
106.369'
25.000'
18.000'
75.000'
97.898'
95.000'
45.000'
35.000'
111.397'
GIRDER #2

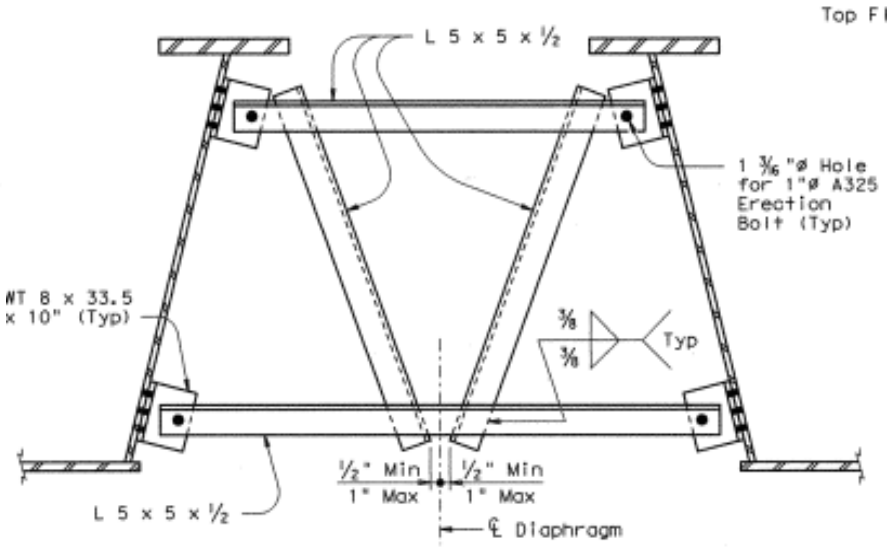

BARS T
30'-0" Overall
15'-0"
15'-0"
1'-0"
28'-0" Roadway
1'-0"
Girder Spa
7.167'
7.833'
7.833'
7.167'
Face of Rail
See Layout for Cross Slope
Face of Rail
1 ½" End C
(Typ)
Overhang
2 ½"
Bars T at 9" Max
HAL "M"
PGL
1¼" Cl Cov
2" Cl Cov
8" Slab
T1
A
A
T
E
T41
8"
D1
3"
(Typ)
1'-0"
D2
WP
8 Spa at 9" Max
Bars D (Typ)
D
8 Spa at 9" Max
Bars D
WP
D28
D29
1'-1"
(Typ)
Top of slab
to top of web
WP
WP
℄ Girder #1
℄ Girder #2


2 ES
3 ~ ⅞" Dia x 7"
Studs with heads
2"
Tight Fit
2" (Typ)

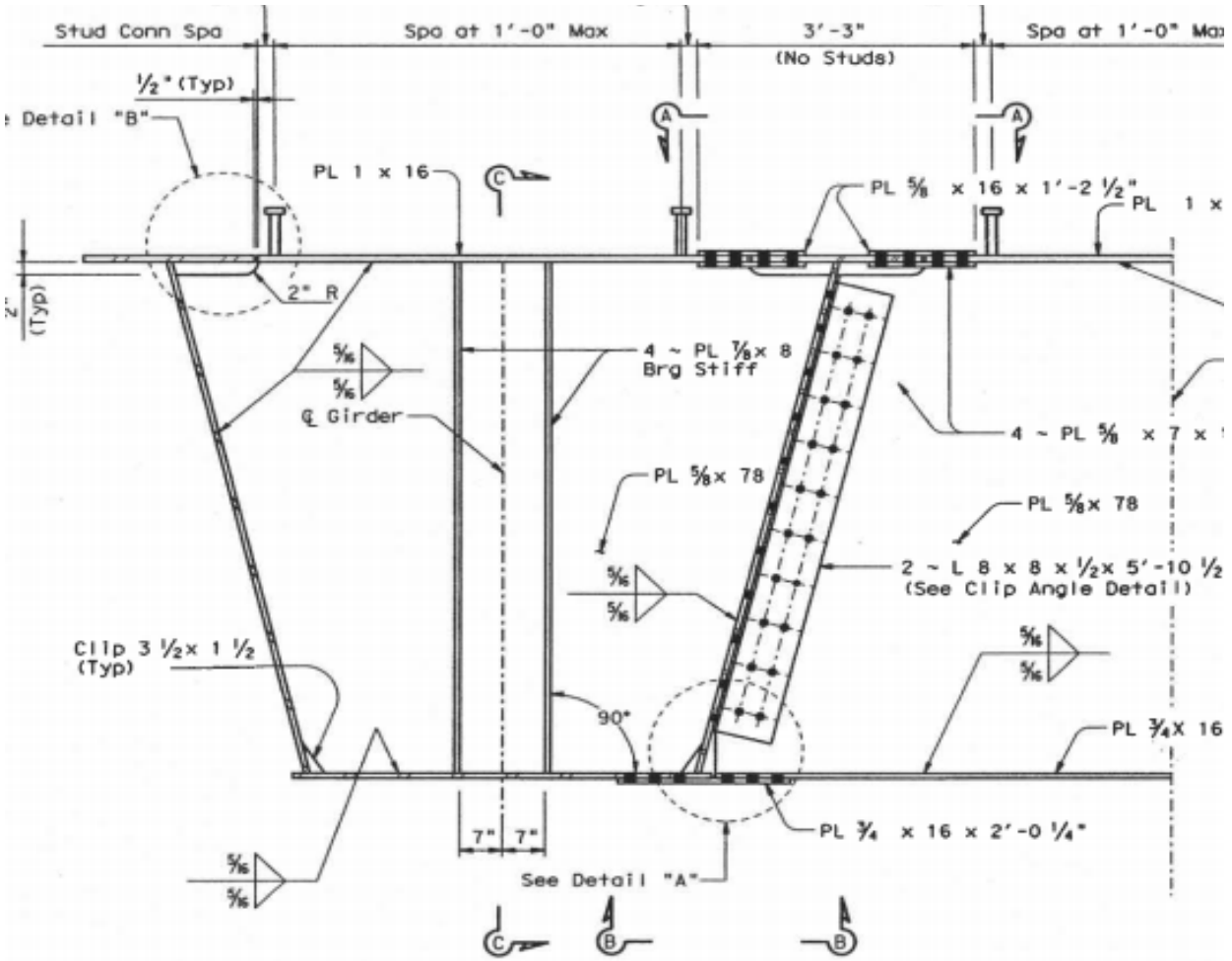


TYPICAL SECTION AT TYPE "A"
INTERNAL INTERMEDIATE DIAPHRAGM

To be fully installed in the shop.



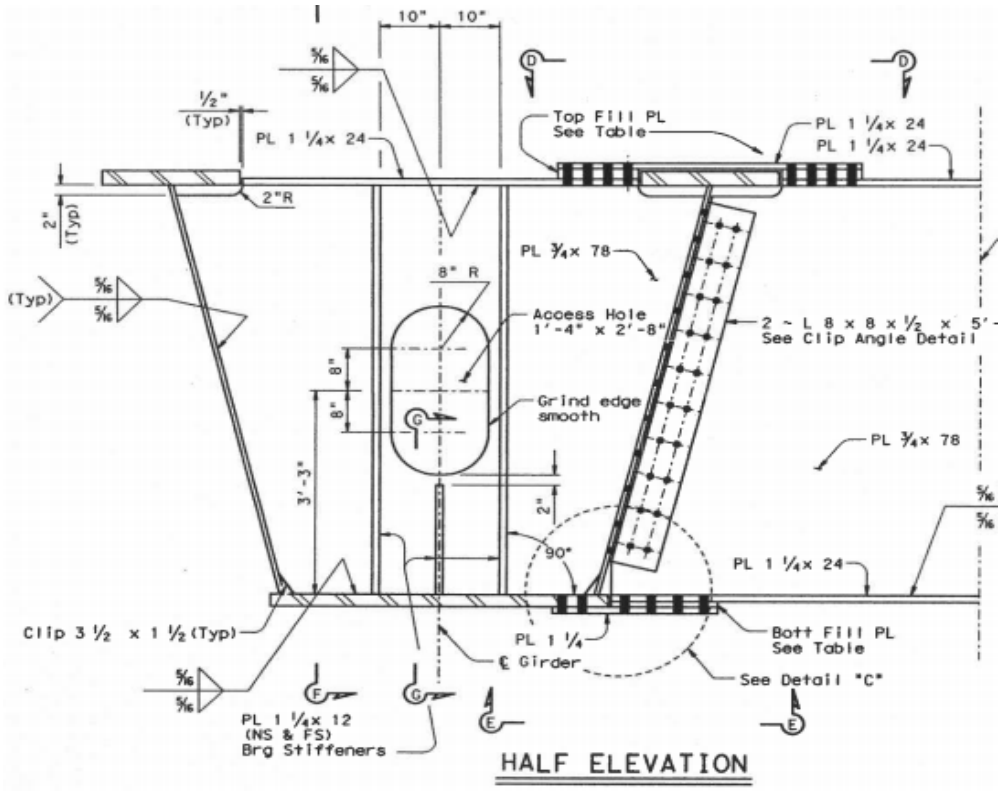
TYPICAL SECTION AT TYPE "B" EXTERNAL
INTERMEDIATE DIAPHRAGM (TEMPORARY)



HALF ELEVATION

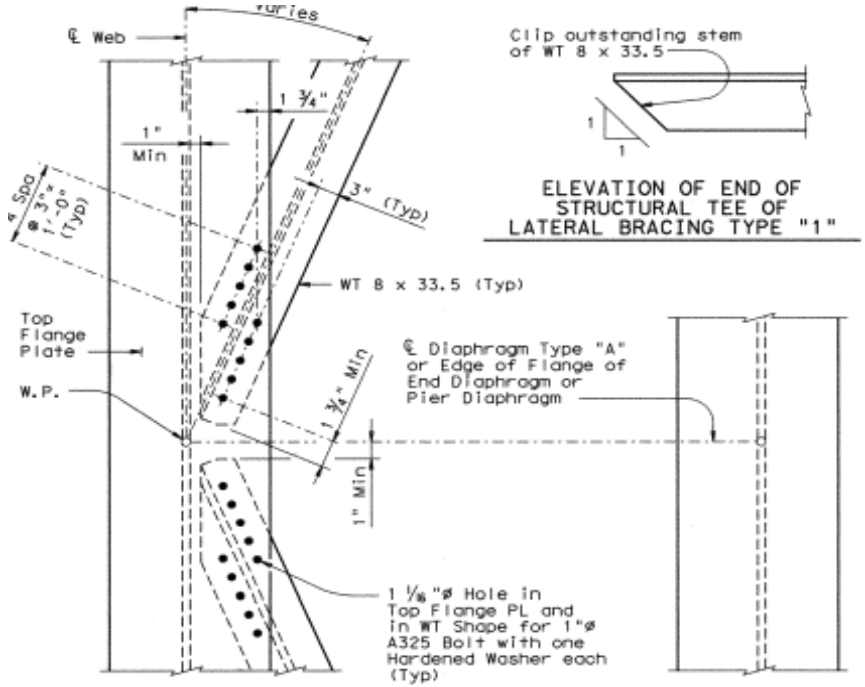
END DIAPHRAGM DETAILS

(At Bent No. 13M)



PIER DIAPHRAGM DETAILS

(At Bent Nos. 14M & 15M)



TYPICAL PLAN VIEW OF LATERAL BRACING TYPE "1"

ELEVATION OF END OF STRUCTURAL TEE OF LATERAL BRACING TYPE "1"

Installed in shop except at locations spanning a field splice, which shall be installed in the field.