Modeling Resilience and Impact in Multi-Modal Transportation Networks

KASH BARKER, Ph.D.

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In this report, we propose an alternative, though complementary, view of multi-modal transportation network component importance relative to (i) their commodity flows and (ii) the interdependent, multi-regional, multi-industry impact of those commodity flows (as a primary role of the multi-modal network is as a connector of multiple industries). In this project, we (i) modeled a multi-commodity network flow representation of a multi-modal transportation network, (ii) related disruptions in the flow of commodities on this network to interdependent multi-industry and multi-regional losses, (iii) characterized transportation network components to multi-regional, multi-industry economic vulnerability, and (iv) developed a means to measure the effectiveness of preparedness decisions to reduce the impacts of transportation network disruptions. A primary contribution of this work is the relating network vulnerability and interdependent impact to individual network components – *an important perspective not currently available in the literature*. That is, what role does a particular multi-modal link or node play in linking multiple industries and multiple regions together? And how valuable is this complementary view in measuring component importance? Several reasons make this work important to the Oklahoma region, including: Oklahoma’s central role in the transport of goods via three interstates, railways, and connection to the Mississippi River Navigation System via two inland waterway ports; and as elements of the multi-modal network are prone to disruption and delay, understanding the role of individual links in the role of regional economic productivity is important.

Vulnerability, Resilience, Economic impact, Commodity flows, Multi-regional, Planning

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*Si is the symbol for the International System of Units. Appropriate rounding should be made to comply with Section 4 of ASTM E380.*
MODELING RESILIENCE AND IMPACT IN MULTI-MODAL TRANSPORTATION NETWORKS

Final Report
November 2019

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# TABLE OF CONTENTS

1. Introduction .................................................................................................................. 1
2. Methodological Background.......................................................................................... 2
3. Multi-industry Vulnerability-based Component Importance Measures ....................... 5
   3.1. Step 1. Baseline Network Flow .............................................................................. 5
   3.2. Step 2. Network Disruption ............................................................................... 6
   3.3. Step 3. Multi-industry Impact ............................................................................. 8
   3.4. Step 4. Vulnerability Analysis and Component Importance .................................. 11
   3.5. Illustrative Example ............................................................................................ 12
   3.6. Discussion ........................................................................................................... 21
4. Adaptive Capacity Planning in a Freight Transportation Network ................................... 22
   4.1. Step 1: Freight Movement and Disruption ............................................................ 23
   4.2. Step 2: Multi-industry Impact ............................................................................ 25
   4.3. Step 3: Planning for Adaptive Capacity ............................................................... 29
   4.4. Illustrative Example ............................................................................................ 33
   4.5. Discussion ........................................................................................................... 40
5. Conclusions ................................................................................................................... 41
6. References ..................................................................................................................... 43
LIST OF FIGURES

Figure 1. System performance, \( \varphi(t) \), trajectory following a disruptive event............... 2
Figure 2. The performance components of static resilience........................................ 4
Figure 3. Four step approach to assessing transportation component importance with multi-industry impacts........................................................................................................... 5
Figure 4. Representations of (a) spatial location of multi-modal nodes in Oklahoma and surrounding states, and (b) the connected transportation network. .................................................. 13
Figure 5. Economic inoperability across six most important industries within the state of Oklahoma. .......................................................................................................................... 19
Figure 6. Network component importance measures across the Oklahoma economy. 20
Figure 7. Network component importance measures focusing on particular Oklahoma industries. ................................................................................................................................................. 21
Figure 8. Three-step approach to planning for adaptive capacity with multi-industry impacts. 23
Figure 9. Graphical depiction of inoperability across six most important industries within the state of Oklahoma.................................................................................................................................................................................. 36
Figure 10. Interdependent economic losses in Oklahoma due to network component removal. 37
Figure 11. Economic inoperability caused by the disruption devising a contingent rerouting. 38
Figure 12. Total economic loss across all industries in Oklahoma, contingent rerouting versus no action. .................................................................................................................. 39
Figure 13. Effects of contingent rerouting on different industries. ................................. 40
LIST OF TABLES

Table 1. Spatial location of multi-modal nodes in Oklahoma and surrounding states. .. 15
Table 2. Names and NAICS codes for main industries using the network. ................. 15
Table 3. Combined annual demands/supplies at supply/demand nodes connecting through the network (in thousand tons). .................................................................................................................. 16
Table 4. Link capacities among the origin/destination nodes in the illustrative network (in thousand tons). ........................................................................................................................................... 16
Table 5. Commodities remaining at suppliers with the removal of network components (in thousand tons). ........................................................................................................................................... 17
Table 6. Interdependent industry inoperability resulting from network component removal. 18
Table 7. Economic losses (in 100 million USD) across the six most important industries within the state of Oklahoma. .............................................................................................................. 19
Table 8. Combined monthly demands/supplies at supply/demand nodes connecting through the network (in tons). ............................................................................................................................. 34
Table 9. Tons of remaining commodities at suppliers with the removal of network components. 35
Table 10. Industry inoperability across six most important industries within the state of Oklahoma. ........................................................................................................................................... 35
Table 11. Economic losses across the six most important industries within the state of Oklahoma. ........................................................................................................................................... 37
Table 12. Economic inoperability caused by the disruption after devising a contingent rerouting strategy ........................................................................................................................................... 38
Table 13. Economic losses, in million USD, within the state of Oklahoma after planning for adaptive capacity. ........................................................................................................................................... 39
EXECUTIVE SUMMARY

Recent US planning documents focus on transportation network preparedness, emphasizing “securing and managing flows of people and goods” along transportation networks. Presidential Policy Directive 21 states that critical infrastructure “must be secure and able to withstand and rapidly recover from all hazards.” This combination of the abilities to (i) withstand the effects of a disruption (or its lack of vulnerability) and (ii) recover timely (or its recoverability) is often referred to as resilience. This project addresses the vulnerability dimension of resilience from the perspective of the value of commodities that flow along a multi-modal transportation network.

The importance of individual components in the multi-modal transportation network (e.g., highways, railways, inland waterways) is often determined according to how “busy” they are, via measures such as traffic throughput or congestion. As such, during planning for improvements (e.g., capacity expansion) or determining maintenance priorities, high traffic areas tend to rank highly. In this report, we propose an alternative, though complementary, view of multi-modal transportation network component importance relative to (i) their commodity flows and (ii) the interdependent, multi-regional, multi-industry impact of those commodity flows (as a primary role of the multi-modal network is as a connector of multiple industries). In this project, we (i) modeled a multi-commodity network flow representation of a multi-modal transportation network, (ii) related disruptions in the flow of commodities on this network to interdependent multi-industry and multi-regional losses, (iii) characterized transportation network components to multi-regional, multi-industry economic vulnerability, and (iv) developed a means to measure the effectiveness of preparedness decisions to reduce the impacts of transportation network disruptions. A primary contribution of this work is the relating network vulnerability and interdependent impact to individual network components – an important perspective not currently available in the literature. That is, what role does a particular multi-modal link or node play in linking multiple industries and multiple regions together? And how valuable is this complementary view in measuring component importance?

Several reasons make this work important to the Oklahoma region, including: Oklahoma’s central role in the transport of goods via three interstates, railways, and connection to the Mississippi River Navigation System via two inland waterway ports; and as elements of the multi-modal network are prone to disruption and delay (and will only continue to do so as long-term climate related degradation can lead to more efficient disruption under smaller scale disasters), understanding the role of individual links in the role of regional economic productivity is important.

It is recommended that the Oklahoma Department of Transportation consider the models provided in this report to supplement and complement existing approaches when considering investments to reduce vulnerability in components of the multi-modal transportation infrastructure in the state. The methods proposed in this report are illustrated with data-driven studies from Oklahoma, though they are at levels of granularity that may not be directly conducive to investments about specific components (e.g., a particular bridge).
1. INTRODUCTION

In response to the growing vulnerability of critical infrastructure given their exposure to natural hazards, malevolent attacks, and the challenges of aging, the Presidential Policy Directive on Critical Infrastructure Security and Resilience (PPD-21)\(^1\) was established to focus national efforts to enhance the critical infrastructure network resilience.

*The Nation’s critical infrastructure provides the essential services that underpin American society. Proactive and coordinated efforts are necessary to strengthen and maintain secure, functioning, and resilient critical infrastructure – including assets, networks, and systems – that are vital to public confidence and the Nation's safety, prosperity, and well-being.*

- Presidential Policy Directive/PPD-21: Critical Infrastructure Security and Resilience\(^1\)

Among the critical infrastructures defined by the US government are transportation networks, which are vital to a society and support many economic activities including commerce and tourism. Disruptions triggered by natural hazards, human-made events, or common failures can severely compromise a region’s ability to move people and commodities, consequently leading to irreversible economic losses as well as public safety concerns. Many recent large-scale examples highlight the growing need to deal with disruptions: Hurricane Sandy that affected multiple infrastructure networks, including downed power lines and massive flooding on New York and New Jersey roadways and one million cubic yards of debris that impeded transportation networks\(^2\); the August 2003 US electric power blackout that caused transportation network disruptions\(^3\); and Hurricane Isabel that adversely impacted the transportation system of the Hampton Roads, VA region in 2003 and overwhelmed emergency response\(^4\). The current state of disrepair of the US transportation network (e.g., roads given an American Society of Civil Engineers Infrastructure Report Card\(^5\) grade of D, bridges a C+, inland waterways a D-) could make the network especially vulnerable to a disruptive event. The situation is no better for the state of Oklahoma, where bridges in particular received a lower letter grade of D+, which by definition is interpreted as a “poorly performing” infrastructure\(^6\). Recent US planning documents focus on transportation network preparedness\(^7\)\(^-\)\(^9\), emphasizing “securing and managing flows of people and goods” along transportation networks\(^10\).

The physical freight transportation network of the US, the largest in the world, consists of four million miles of public roads, 140,000 miles of railroad tracks, 11,000 miles of navigable waterways, and a network of airports with the combined ability of shipping almost 68,000 tons of cargo per year\(^11\). Furthermore, the same document highlights the importance of the US transportation network in facilitating the convenient movement of resources among suppliers, manufacturers, wholesalers, and customers, with more than 300 million people and 7.5 million organizations across 3.8 million square miles being served. The vital role the freight network plays in transporting raw materials and final products between manufacturers and consumers highlights its position in commerce. The functionality of this network is threatened by disruptive events that can disable the capacity of the network to enable flows of commodities and cause an
interruption of economic productivity across multiple industries. That is, the ultimate usefulness of understanding transportation network disruptions is not just a descriptor of physical damage, but of economic interruption due to infrastructure inoperability. As such, discussions of transportation network vulnerability should account for multi-industry impacts.

This work focuses on the freight transportation network, particularly on its role of enabling the flow of commodities and facilitating economic productivity, and thus a methodological approach to measure network vulnerability in the context of multi-industry impacts is sought. That is, this work seeks to answer: if a transportation node or link is disrupted, what is the effect on local industries? This research addresses (i) measuring the vulnerability of a multimodal freight transportation network with multi-industry impacts in mind, and (ii) using this vulnerability analysis to develop a measure of importance for each network component.

2. METHODOLOGICAL BACKGROUND

Presidential Policy Directive 21 states that critical infrastructure “must be secure and able to withstand and rapidly recover from all hazards”[1]. This combination of the ability to (i) withstand the effects of a disruption and (ii) recover timely from the disruption is often referred to as resilience [13]. Figure 1 highlights these two dimensions of resilience: vulnerability and recoverability [14],[15]. The network service function $\varphi(t)$ describes the behavior or performance of the network at time $t$ (e.g., $\varphi(t)$ could describe traffic or commodity flow in a transportation network). The vulnerability dimension of resilience is the focus of this work.

![Figure 1. System performance, $\varphi(t)$, trajectory following a disruptive event.](image)

Similarly, Vugrin and Camphouse[16] suggest that the resilience capacity of a system is a function of three components: (i) absorptive capacity, or the ability of a system to absorb or withstand a disruption with essentially no change in performance, (ii) adaptive...
capacity, or a short-term means to quickly regain a desired performance, and (iii) restorative capacity, or the long-term repair of physical damage. Vugrin and Camphouse\[16\] pose absorptive, adaptive, and restorative capacities as first, second, and third “lines of defense,” where the next is engaged if the previous fails. In a transportation network context, (i) absorptive capacity may describe the physical characteristics of, say, a bridge to withstand the shock of an earthquake, (ii) adaptive capacity may include alternate paths in the network that could be engaged quickly to work around damaged areas, and (iii) restorative capacity may describe the long-term bridge reconstruction activities required to restore the transportation network. Relative to Figure 1, the collection of absorptive and adaptive capacities may reduce vulnerability, while restorative capacity would improve recoverability.

Faturechi and Miller-Hooks\[17\] thoroughly review the literature on transportation system performance considering disruptions to physical infrastructure. Defining a four-phase disaster life cycle as (i) mitigation, (ii) preparedness, (iii) response, and (iv) recovery, they suggest that most work focuses on assessing the transportation system’s ability to deal with disruption consequences, with less work assessing strategies to manage the system after the disruption. In a freight transportation network, vulnerability is considered to be a problem of interrupted serviceability or accessibility of network components, leading to reduced system functionality\[18\],[19]. O’Kelly\[20\] classifies network vulnerability into link vulnerability, or the reduction of a network’s capability after losing a link, and nodal vulnerability, or the extent to which a node plays a critical role in the operation of the whole network. From the network interdiction literature, where network components (nodes or arcs) are disabled intentionally, there are three approaches to evaluate network vulnerability\[21\]: (i) scenario-specific evaluation, where the potential consequences of a specific disruptive scenario or set of scenarios is evaluated (e.g., studying the impact of losing a bridge, a road segment, or a hub on network performance\[22\]-\[25\]), (ii) strategy-specific assessment, where vulnerability is assessed with respect to a hypothesized sequence or strategy of disruptions targeting components perceived to be important (e.g., [26]-[28]), and (iii) mathematical modeling assessment (e.g., [29],[30]), using game-theoretical techniques to find worst-case scenarios. In our work, to analyze network vulnerability and define a measure of importance for network components, a scenario-specific approach is taken by analyzing the proportional effect on the flow of commodities given the removal of one node/link at time.

Most work in network vulnerability focuses on network behavior after a disruption in terms of graph theoretic measures, such as average shortest distance, network diameter, average edge betweenness, and cluster efficiency (e.g., [31]-[35]), which describe what is commonly referred to as structural vulnerability. This is different from functional vulnerability, where operational characteristics (e.g., network flow) of different components are taken into consideration\[36\]. To capture the functional aspects of network vulnerability, a measure of importance for network components was introduced by Nagurney and Qiang\[37\]-\[39\] based on network performance/efficiency considering demands, costs, and flows, as well as behavior of the users of the network. Following the lead of Nagurney and Qiang, the emphasis of this paper deals with describing
network vulnerability with respect to flow along the network, a more tangible approach than focusing solely on topological features of the network and amenable to an analysis of multi-industry economic impacts. That is, \( \varphi(t) \) is used to describe the flow along the transportation network. Further literature describing network component importance based on flow measures is sparse\(^{40},^{41} \), and, to the authors’ knowledge, the methodology proposed in this paper for pinpointing the contribution and importance of individual transportation network components to multi-industry economic impacts is an area that has not been previously pursued in the literature. This work considers network vulnerability as a relative drop in the commodity flows along the network after the removal of a particular node or link. And a drop in the flow of commodities would generate subsequent impacts on multiple industries relying on those commodities. While several approaches have been proposed to capture interdependencies among infrastructure and industries\(^{42-45} \), this work makes use of an economic input-output model extension that quantifies the propagation of multi-industry inoperability (the extent to which industry output will not be produced) caused by perturbations in supply and/or demand.

While most definitions of resilience recognize the time-dependent nature of withstanding and recovering from a disruption, Rose\(^{46} \) defined static resilience as “the ability of an entity or system to maintain function when shocked.” This is depicted in Figure 2, where \( \%\Delta Y_{\text{max}} \) represents the maximum percentage change given the worst-case level of performance following a disruptive event, and \( \%\Delta Y \) represents the actual percentage change in the performance of the system (assuming the implementation of a mitigation strategy)\(^{47} \). The original application of static resilience, as well as several subsequent studies (e.g., [47]-[52]), deal with economic disruption. Mathematically, static resilience is measured in terms of the maximum potential drop in system performance and the estimated performance drop, as shown in Eq. (1). This quantitative approach is used in this study to define a performance measure for post-disaster rerouting, though we prefer the term adaptive capacity rather than static resilience.

![Figure 2. The performance components of static resilience.](image)
3. MULTI-INDUSTRY VULNERABILITY-BASED COMPONENT IMPORTANCE MEASURES

Despite the excellent use of network-based models in representing interdependencies which consider various aspects of network vulnerability\cite{42,53,54}, there exists a need to integrate parts of these models with multi-industry impacts to address freight transportation functionality as enabling the flow of commodities and facilitating economic productivity. This need is addressed with a four-step methodology, as illustrated in Figure 3, which then culminates in a transportation network component importance measure. This section has been published by Darayi et al.\cite{55}.

\[
\text{static resilience} = \frac{\% \Delta D Y^\text{max} - \% \Delta D Y}{\% \Delta D Y^\text{max}}
\]  

(1)

Figure 3. Four step approach to assessing transportation component importance with multi-industry impacts.

3.1. STEP 1. BASELINE NETWORK FLOW

Freight transportation planning models have been classified at strategic, tactical, and operational levels\cite{56}. At the strategic level, long-term decisions include the design of the physical transportation network and the location of main facilities (e.g., rail yards, multi-modal platforms). At the tactical level, medium-term decisions are made, such as the design of the service network (i.e., route choice and type of service to operate, aggregate scheduling). The operational level includes shorter-term decisions, including crew or container scheduling. It is at the operational level of planning that routing of different types of commodities in an existing multi-modal transportation network is sought. The multi-modal freight transportation network of interest in this work will be modeled with a typical multi-commodity network flow problem. Multi-commodity network flow (MCNF) problems, which minimize the cost of the flow of multiple commodities
across a capacitated network of supply and demand nodes, arise in a wide variety of applications, including telecommunications\textsuperscript{[57]}, warehousing\textsuperscript{[58]}, and multi-modal transportation networks\textsuperscript{[59],[60]}, among others.

To study the vulnerability of a multi-modal freight transportation network, which serves as a facilitator of interacting industries, the topology of the network and corresponding supply and demand nodes must be extracted. The conventional MCNF problem for a network, $G(N, L)$ with a set of nodes, $N$, a set of links, $L$, and a number $K$ of commodities, is formulated in model M1. The flow of commodity $k$ on link $(i, j)$ is represented with $f_{ij}^k$, and the cost of shipment for commodity $k$ on link $(i, j)$ is $w_{ij}^k$. The capacity of link $(i, j)$ is represented with $u_{ij}$, and the supply/demand of commodity $k$ at node $i$ is represented with $b_{ik}$, defining the “bundle” and “mass balance” constraints in model M1, respectively. Note that $b_{ik}$ is positive for supply nodes, negative for demand nodes, and zero for transshipment (or intermediate) nodes. The capacity of each link is considered as a shared constraint for all commodities flowing on the link.

\[
\min \sum_{(i, j) \in L} \sum_k w_{ij}^k f_{ij}^k \\
\text{s.t.} \sum_k f_{ij}^k \leq u_{ij} \quad \forall (i, j) \in L \\
\sum_{(i, j) \in L} f_{ij}^k - \sum_{(j, i) \in L} f_{ji}^k = b_{ik} \quad \forall i \in N, k = 1, ..., K \\
f_{ij}^k \geq 0, \quad \forall (i, j) \in L, k = 1, ..., K
\] (M1)

In fact, a generic MCNF model provides a means to formulate the supply-demand network in which a multi-modal freight transportation network connects industries and enables trading relationships and interactions. From a tactical point of view, the integration of (i) business economic sectors and (ii) their supply capabilities or demand requirements together with (iii) the structure of the transportation network can result in a minimum cost MCNF model that can route the commodities from suppliers to the demand nodes via $f_{ij}^k$, collectively representing the flow of commodities on the links of a baseline (undisrupted) network.

3.2. STEP 2. NETWORK DISRUPTION

A common theme in the analysis and evaluation of network vulnerability is interdiction\textsuperscript{[21],[61],[62]}, in which scenario-based removal of network components is assumed to represent the effects of a disruptive event. The consequences of a targeted attack, accident, or natural disaster are simulated as disruptions in the flow of valuable goods or services through the network caused by disabling network components. The network is analyzed to determine how vulnerable it is to a disruption, and which nodes or links, if lost, result in the most damage to network performance. Further, the temporal and spatial scales at which analysis is conducted, as well as the duration of the disruptive event, affect the disruption analysis.
Approaches to interdict a network differ based on how disruption scenarios are assessed and understood. A disruption scenario is defined by the set of network components that are impacted, the degree to which they are disabled, and the operating conditions (e.g., network activity and link/node capacities) of the network prior to the disruption regardless of the initiating event that causes the disruption. In extreme cases, an affected facility may be rendered completely inoperable by a disruption (e.g., losing a road completely due to a bridge collapse as in the case of the I-35 Mississippi River bridge failure in 2007). In other instances, a disruption may impact network activity to a lesser degree given that only some of the functionality of a facility may be lost (e.g., an accident blocking a single lane of an interstate highway segment). The identification of disruption scenarios enables an impact assessment. Impacts can range from those directly associated with network operation, such as connectivity, flow, or capacity reduction, to more complex associations, such as the economic impacts affecting the production and consumption of flows[63].

The flexibility in defining scenario-specific disruptions based on historical data or other desired analysis makes it appropriate for network vulnerability studies. In particular, it provides opportunities for understanding a component’s role and importance within a network. For example, one might be interested in the impact of the closure of a bridge or a road segment on network performance (e.g., the flow of commodities, the topological behavior of a post-disruption network, the multi-industry economic impacts). A deterministic scenario-specific approach[21], where the potential ramifications of the removal of a particular network component is evaluated, is often used to quantify network component importance measures (e.g., [39], [64]). Stochasticity could be introduced to capture uncertainty in disruptive scenarios (e.g., [54], [22], [65]).

This step evaluates the effect of losing a network component on freight flow through the network and resulting consequences on supply/demand nodes. Hence, a disruptive scenario is defined as the removal of a particular network component. An optimization formulation is developed to reroute commodity flows through the residual network, pursuing the maximum flow throughout the network and capturing failure in the form of remaining commodities at supply nodes and unmet demands at demand nodes, as formulated in model M2. Intuitively, a decision maker would likely desire to reroute commodities to take advantage of the remaining capacity of the residual network. Note the difference in perspective in the post-disruption MCNF developed here: prior to the disruption, model M1 minimizes the cost of transporting commodities along the capacitated network, where model M2 maximizes the flow to meet as much demand as possible given the interrupted network with updated sets of links $L'$ and nodes $N'$. To capture undelivered commodities remaining with the suppliers or unsatisfied demand at demand nodes, a slack variable $S_i^k$ is defined. The magnitude of $S_i^k$ is positive, and multiplier $\gamma_i$ takes on a negative value for the set of demand nodes (after disruption) $N'_-$, a positive value for supply nodes (after disruption) $N'_+$, and zero for transshipment nodes (after disruption) $N'_0$. The objective function maximizes the sum of commodity-specific flows, where $f_{ij}^{nk}$ represents the flow of commodity $k$ across link $(i,j)$ which remains in the updated set of links, $L'$. Slack variable $S_i^k$ will be used in the next step to calculate inoperability among multiple industries. Here it is assumed that each type of
commodity represents an industry, and interdependent inoperability propagated through the entire regional economy caused by unsatisfactory levels of demands/supplies will be pursued in the next section.

\[
\begin{align*}
\text{max} & \sum_{(i,j) \in L'} \sum_k f_{ij}^{t_k} \\
\text{s.t.} & \sum_k f_{ij}^{t_k} \leq u_{ij} \forall (i,j) \in L' \\
& \sum_{(i,j) \in L'} f_{ij}^{t_k} - \sum_{(i,j) \in L'} f_{ji}^{t_k} + \gamma_i S_i^k = b_i^k \quad \forall i \in N', k = 1,\ldots,K
\end{align*}
\] (M2)

3.3. STEP 3. MULTI-INDUSTRY IMPACT

To represent the multi-industry impacts of unmet demands at demand nodes and remaining commodities at the suppliers' side in the MCNF, an extension of the input-output model is used. The input-output (I-O) model, for which Wassily Leontief\cite{66} won a Nobel Prize, has been widely accepted as a useful model for analyzing the interdependent connections among industries\cite{67}. Under a static equilibrium, the total output of the industry \( s \) is distributed to other industries and also satisfies external (consumer) demand. This equilibrium condition is described with \( x_k = \sum_{r=1}^n z_{kr} + c_k \), where \( x_k \) is the total output of industry \( k \), \( z_{kr} \) is the flow of commodities produced by industry \( k \) and used as input to production in industry \( r \), and \( c_k \) is the external demand for industry \( k \). The flow of commodities \( z_{kr} \) is assumed to be proportional to the output of industry \( r \) \((r \in \{1,\ldots,K\} \text{ and } r \neq k)\), expressed as \( z_{kr} = a_{kr} x_r \). Further, it is assumed that each industry produces a sole commodity, such that industry \( k \) produces commodity \( k \). The common form of the Leontief input-output model is expressed in Eq. (2), where \( x \) is the vector of industry production outputs, \( A \) is an industry-by-industry matrix of interdependency coefficients, \( a_{kr} \), and \( c \) is a vector of final demands. The model shows that total production is made up of industry-to-industry intermediate production, \( Ax \), and production to satisfy final demand, \( c \). Terms \( a_{kr}, x_r, \) and \( c_k \) are measured in monetary units.

\[
x = Ax + c \Rightarrow x = [I - A]^{-1}c
\] (2)

Despite of the I-O model’s assumption of a linear relationship of commodity flows among industries, the extensive usage of I-O models is due in part to the availability data describing the parameters of the I-O model in a number of countries\cite{68,69}. This includes a data collection effort by the US Bureau of Economic Analysis (BEA), which maintains input-output tables at different levels of aggregation\cite{70}. Extending the capability of the I-O model, Santos and Haimes\cite{71} propose the Inoperability Input-Output Model (IIM) to represent the propagation of inoperability, or the proportional
extent to which industries are unproductive after a change in demand or a forced change in demand due to a lack of supply. The use of the IIM can model inoperability in an economic setting, or in a set of interdependent infrastructures. The IIM and some extensions have been deployed in a number of contexts, including analyses of infrastructure disruptions, workforce losses, and supply chain risk, among others. Furthermore, the IIM has been used in multi-industry vulnerability studies (e.g., Yu et al. developed a multi-perspective approach for vulnerability decomposition with the aim of prioritizing key economic sectors in the aftermath of disruptive events).

Instead of describing the connections between the interdependent industries in terms of commodity flow dollars, the IIM illustrates how normalized production losses propagate through interconnected industries, providing a different perspective from the traditional I-O model. The IIM is provided in Eq. (3), describing the relationships among \( K \) industries, resulting in matrices of size \( K \times K \) and vectors of length \( K \).

\[
q = A^* q + c^* \Rightarrow q = [I - A^*]^{-1} c^*
\]  

(3)

Vector \( q \) is a vector of industry inoperability describing the proportional extent to which as-planned productivity or functionality is not realized following a disruptive event. Inoperability for industry \( k \) is defined in Eq. (4), where as-planned total output is represented with \( \hat{x}_k \) and degraded total output resulting from a disruption is represented with \( \bar{x}_k \). An inoperability of 0 suggests that an industry is operating at normal production levels, while an inoperability of 1 suggests that the industry has become completely inoperable.

\[
q_k = (\bar{x}_k - \hat{x}_k) / \hat{x}_k \Leftrightarrow q = [\text{diag}(\hat{x})]^{-1}(\bar{x} - \hat{x})
\]  

(4)

Normalized interdependency matrix \( A^* \) is a normalized form of the original \( A \) matrix describing the extent of interdependence among a set of industries. As stated by Eq. (5), the row elements of \( A^* \) indicate the proportion of additional inoperability that are contributed by a column industry to the row industry.

\[
a^*_{rk} = a_{rk} (\bar{x}_k / \hat{x}_r) \Leftrightarrow A^* = [\text{diag}(\hat{x})]^{-1} A [\text{diag}(\hat{x})]
\]  

(5)

Eq. (6) provides the calculation of \( c^* \), a vector of normalized demand reduction. The elements of \( c^* \) represent the difference in as-planned demand \( \hat{c}_k \) and perturbed demand \( \bar{c}_k \) divided by as-planned production, quantifying the reduced final demand for industry \( k \) as a proportion of total as-planned output.

\[
c^*_k = (\bar{c}_k - \hat{c}_k) / \hat{x}_k \Leftrightarrow c^* = [\text{diag}(\hat{x})]^{-1}(\bar{c} - \hat{c})
\]  

(6)

For the traditional economic loss metric, losses can be calculated by multiplying each industry’s production level in monetary units by its inoperability level: for industry \( k \), \( Q_k = x_k q_k \), or for the entire economy of industries, \( Q = x^T q \). As such, planning decisions
can be made with respect to inoperability or economic impact at the industry level, or with respect to economic impact across multiple industries.

When a disruption within the transportation network results in remaining commodities at supply nodes and/or unmet demand at demand nodes, inoperability propagates throughout industries in a region. Without loss of generality, each node within the network is considered to be either a supplier or a consumer of a particular commodity. Each commodity belongs to an industry in the economy as defined by the North American Industry Classification System (NAICS).

In industry $k$ the amount of import is $\sum_{i \in (N_\cap N_k)} -b_i^k$, where $N_k$ represents the set of nodes which are producers/consumers of commodity output from industry $k$, located in a geographical area of interest (e.g., a business economic area, county, state, entire country). Note again that it is assumed that industry $k$ produces commodity $k$. The amount of import contributes to the total output, $\hat{x}_k$, and final demand, $\hat{c}_k$, of industry $k$ as shown in Eq (8). Thus, unmet demand, $\sum_{i \in (N_\cap N_k)} \Delta b_i^k$, in Eq (9) results in the loss of output, $\Delta \hat{x}_k$, and final demand, $\Delta \hat{c}_k$, representing $(\hat{c}_k - \hat{c}_k)$.

\[
\sum_{i \in (N_\cap N_k)} -b_i^k = \hat{x}_k + \hat{c}_k \quad k \in \{1, ..., K\} \tag{7}
\]

\[
\sum_{i \in (N_\cap N_k)} -\Delta b_i^k = \Delta \hat{x}_k + \Delta \hat{c}_k \quad k \in \{1, ..., K\} \tag{8}
\]

Therefore, for industry $k$, unmet demands cause an inoperability, $q_k$, and demand perturbation, $c_k^*$, which are modeled in Eq. (10) and (11), respectively, as adapted from Pant et al.[78],[79]. Inoperability is a measure of the loss of production in industry $k$ as a proportion of its original production level, and demand perturbation is a measure of the change in demand as a proportion of the original production level in industry $k$.

\[
q_k = \frac{\Delta \hat{x}_k}{\hat{x}_k} = \frac{\hat{x}_k - \bar{x}_k}{\hat{x}_k} \tag{9}
\]

\[
c_k^* = \frac{\Delta \hat{c}_k}{\hat{x}_k} \tag{10}
\]

\[
\sum_{i \in (N_\cap N_k)} -\Delta b_i = \sum_{i \in (N_\cap N_k)} S_i^k \quad k \in \{1, ..., K\} \tag{11}
\]

Considering Eqs. (8)-(12), for the industries experiencing difficulties only in importing their required commodities, there exists a demand perturbation, as modeled in Eq. (13).

\[
c_k^* = \frac{\sum_{i \in (N_\cap N_k)} S_i^k}{\hat{x}_k} - q_k \quad k \in \{1, ..., K\} \tag{12}
\]

For the industries experiencing difficulties only in exporting commodities, the total amount of remaining commodities at supply nodes relating to that particular industry, $S_i^k$,
will be considered as a perturbation in demand. Hence, the demand perturbation for exporting industry $k$ is modeled with Eq. (14).

$$c_k^* = \frac{\sum_{i \in (N_{+\cap N_k}) S_i^k}{\bar{x}_k}}{k \in \{1, \ldots, K\}}$$ (13)

Eqs. (13) and (14) combined with the IIM in Eq. (4) form a complete solvable system that quantifies the inoperability and demand perturbations for the entire economy of interconnected industries. For simplicity, the demand perturbations in Eqs. (13) and (14) assume failure in only demand nodes or in supply nodes within a particular industry, whereas in actual situations, some industries would likely consist of both demand and supply nodes. Therefore, the total demand perturbation for industry $k$ in the case of having both importing and exporting roles is given in Eq. (15).

$$c_k^* = \frac{\sum_{i \in (N_{+\cap N_k}) S_i^k}{\bar{x}_k}}{k \in \{1, \ldots, K\}} + \frac{\sum_{i \in (N_{-\cap N_k}) S_i^k}{\bar{x}_k}}{k \in \{1, \ldots, K\}} - q_k$$ (14)

Based on the exporting or importing nature of the nodes representing each industry, either Eq. (13), Eq. (14), or Eq. (15) captures the perturbation vector, $c^*$, which parameterizes the interdependency model in Eq. (4). As such, $q_k$ can then be calculated to measure the proportional extent to which as-planned productivity or functionality is not realized following a transportation network disruption that results in unmet demand or commodities remaining with suppliers.

3.4. STEP 4. VULNERABILITY ANALYSIS AND COMPONENT IMPORTANCE

Network vulnerability analysis emerged from the network reliability literature, which is often interested in the probability of a desired network performance\cite{85} or the consequences of the failure of a network component regardless of the probability of failure\cite{86,87}. This second perspective enables the calculation of component importance measures, a long-studied area in reliability engineering\cite{88}, wherein network components that impact the performance of the network are identified.

Step 4 develops scenario-specific component importance measures based on vulnerability. The consideration of the economic impacts of a disruption of transportation network components enhances the literature on transportation network vulnerability, which have traditionally focused on flow or topological aspects of the network (e.g., connectivity and accessibility) as metrics for network performance\cite{89,91}. As such, when we define our new importance measure, we consider the ultimate role of a freight transportation network as a facilitator of economic productivity. Such impact is calculated for different disruptive scenarios, $e^p$, where $p$ represents the component removed from the network (as displayed in Figure 1). As a result, this work advances the study of network vulnerability from the perspective of network performance in terms of commodity-driven multi-industry impact rather than graph theoretic or flow importance measures. Malfunction of a freight transportation network serving a regional economy -- comprised of interdependent industries -- causes failure in the form of a delayed
shipment of commodities at supply nodes and/or unmet demands at demand nodes. Here, the interdependent effect of the failure in multiple industries is captured by the IIM as described in Section 3.3. And finally, vulnerability is defined as the magnitude of this failure in terms of multi-industry economic impact, given the occurrence of a particular disruptive event, $e^p$. Note, of course, that network vulnerability is highly dependent upon the type and extent of $e^p$, which assumes complete removal of component $p$ (though a proportional reduction could also be explored).

Two vulnerability measures, stated in Eqs. (15) and (16), are proposed. For network topology $G$, fixed demand/supply vector $b$, and disrupted (removed) component $p$, vulnerability is measured as the relative network efficiency, or the multi-industry economic loss $Q(G - p, b)$, after $p$ is removed from the network, $G$. $Q_{\text{max}}$ is the maximum multi-industry loss caused by a shutdown in the entire network (i.e., a removal of all nodes). As such, the vulnerability measure in Eq. (15) quantifies the proportional economy-wide impact of a loss of component $p$ relative to a loss in all components. This measure lies on $[0,1]$, where 0 means that losing component $p$ has no effect on the total economy and 1 means that the loss of component $p$ is as disruptive as having a shutdown of the entire network.

$$\eta_p(G, b) = \frac{Q(G - p, b)}{Q_{\text{max}}} \quad (15)$$

Eq. (16) similarly provides the economic vulnerability experienced by a particular industry $k$ due to lost component $p$, providing an industry-by-industry perspective to the importance of vulnerable transportation network elements. $Q_{\text{max}}^k$ is the maximum loss in a particular industry $k$ caused by a shutdown in the entire network, capturing indirect economic loss effect on each industry based on the IIM model.

$$\eta_k^p(G, b) = \frac{Q_k(G - p, b)}{Q_{\text{max}}^k} \quad (16)$$

Thus, Eqs. (15) and (16) provide economy-wide and industry-specific vulnerability measures for the disruption of component $p$ in the multi-modal transportation network. Naturally, certain industries may be more impacted by certain network components than others, which is an important consideration (e.g., a particular industry may be more critical to a state or regional economy than another).

3.5. ILLUSTRATIVE EXAMPLE

The proposed transportation network vulnerability analysis methodology and component importance measures, found as a result of the four-step process in Section 3, is illustrated with a case study based on a portion of a multi-modal freight transportation network within the state of Oklahoma and surrounding states whose industries trade with Oklahoma industries. Oklahoma plays a strong role in the transport of goods via a multi-modal transportation network consisting of three important interstate highways, as
well as railways and inland waterways that connect to the Mississippi River Navigation System via two ports.

Figure 4 highlights a supply-demand network in which supply nodes are all within the state of Oklahoma: the three important business economic areas of Oklahoma City (node 1), the Port of Catoosa in Tulsa (node 2), and the Port of Muskogee (node 3)[92]. Demand nodes consist of states external to Oklahoma that are the most important states to interact with Oklahoma industries: Texas, Louisiana, Arkansas, and Illinois. The effects of a disruption within the network on exporting industries within the state are sought, as are the consequences in the entire Oklahoma economy, hence the four importing states are considered as four combined demand nodes connecting to Oklahoma’s multi-modal freight transportation network.

![Figure 4](image_url)

*Figure 4. Representations of (a) spatial location of multi-modal nodes in Oklahoma and surrounding states, and (b) the connected transportation network.*

The nodes of the network are discussed in brief in Table 1. The Oklahoma City business economic area is connected to the north-south corridor through I-35 and east-west corridor through I-40 and I-44. In addition to the truck way facilities, Burlington Northern Santa Fe (BNSF) railroad has an intermodal rail-truck facility in Oklahoma City near the junction of I-35 and I-40. The Port of Catoosa, the largest inland port in the United States in terms of area, is located near the city of Tulsa, adjacent to I-44, US 169, and rail lines. Industries listed in Table 2 are almost the port’s largest exporters in terms of commodity flows. The Port of Muskogee is connected to the freight transportation network through Highway 165 and a rail marshalling yard. Supply nodes in Figure 4 include Oklahoma City (node 1), the Port of Catoosa (node 2), and the Port of
Muskogee (node 3). Demand nodes include Texas city, TX (node 4), New Orleans, LA (node 5), Little Rock, AR (node 6), and Chicago, IL (node 7). Node 8 represents the intermodal terminal that facilitates the movement of commodities from industries in the industrial park of Port of Catoosa to their out-of-state customers using railroad company, BNSF. Links (1,7) and (1,4) are part of the North America railroad which connects Oklahoma City, OK, with Chicago, IL, and with Texas City, TX, respectively. The Port of Catoosa is connected to the North America railroad through a local railroad represented by link (2,8). Links (2,5), (2,4), (2,6), and (2,7) are part of the inland waterway network navigated by McClellan–Kerr Arkansas River Navigation System and connect Port of Catoosa with the Port of New Orleans, the Port of Texas City, the Port of Little Rock, and the Port of Chicago, respectively. The Port of Muskogee is connected to the Port of Little Rock, the Port of Texas City, and the Port of New Orleans through the same inland waterway network represented by links (3,6), (3,4), and (3,5), respectively, and it is linked to the North America railroad through a local railroad depicted with link (3,8). Node 9 is an intermediate node that connects the Oklahoma City business economic area to the U.S. interstate highways to the north and south through I-35 and to the east through I-44. Node 10 is Fort Smith, AR which is a connecting point on I-40 to link Oklahoma City and Tulsa, OK to Little Rock, AR. Node 11 represents an intermediate node that connects the Port of Catoosa industrial park to interstate highway I-44, link (9,4) connects Oklahoma City to Texas City, TX, using interstate highways I-35 and I-45, and link (9,11) is part of interstate highway I-44 which connects Oklahoma City to Tulsa.

In total, there are 62 industries operating in Oklahoma as identified by NAICS, suggesting that the \( A^* \) matrix regionalized for Oklahoma is \( 62 \times 62 \). In the proposed supply-demand network, six industries, listed in Table 2, are considered to be industries that primarily export commodities to out-of-state customers according to high trade figures\(^\text{[93]}\). In the developed illustrative MCNF example, each commodity belongs to an industry as defined by NAICS economic sectors, and each node within the network is considered to be either a supplier or a customer of a particular commodity.

Table 3 lists the combined estimated annual supply and demand in tons for the associated industries and states, compiled from different databases\(^\text{[93]-[97]}\).

Baseline network flow in the supply-demand network is calculated with model M1, where the cost vector is computed based on the transportation mode and the mileage of the distances between nodes. The cost per ton-mile for a barge is estimated at $0.97, compared to $2.53 for rail, and $5.35 for trucking\(^\text{[98]}\). The capacity of each link, shown in Table 4, representing the availability of transportation facilities, is estimated from historical data as a shared constraint for all commodities flowing on the link\(^\text{[99]}\). The baseline flow resulted in no remaining commodities at supply nodes and no unsatisfied demand at demand nodes, suggesting that supply nodes send out all the commodities and demand nodes satisfy all their demands. Based on Table 3, the total supply of commodity \( k \) is assumed to be equal to the total demand of the same commodity within the entire supply-demand network as depicted in Figure 4.
Table 1. Spatial location of multi-modal nodes in Oklahoma and surrounding states.

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 1</td>
<td>Oklahoma City, a supply node for multiple industries</td>
</tr>
<tr>
<td>Node 2</td>
<td>Port of Catoosa, a supply node for multiple industries</td>
</tr>
<tr>
<td>Node 3</td>
<td>Port of Muskogee, a supply node for multiple industries</td>
</tr>
<tr>
<td>Node 4</td>
<td>Port of Texas City, a demand node for multiple industries</td>
</tr>
<tr>
<td>Node 5</td>
<td>Port of New Orleans, a demand node for multiple industries</td>
</tr>
<tr>
<td>Node 6</td>
<td>Port of Little Rock, a demand node for multiple industries</td>
</tr>
<tr>
<td>Node 7</td>
<td>Port of Chicago, a demand node for multiple industries</td>
</tr>
<tr>
<td>Node 8</td>
<td>Intermodal terminal, Tulsa, OK</td>
</tr>
<tr>
<td>Node 9</td>
<td>Transshipment node that connects the Oklahoma City, OK, business economic area to the north and south through I-35 and to the east through I-44</td>
</tr>
<tr>
<td>Node 10</td>
<td>Transshipment node in Fort Smith, AR, that is a connecting point on I-40 to link Oklahoma City and Tulsa, OK to Little Rock, AR</td>
</tr>
<tr>
<td>Node 11</td>
<td>Transshipment node that connects the Tulsa Port of Catoosa industrial park to I-44</td>
</tr>
<tr>
<td>Link (1,7)</td>
<td>Part of the North America railroad that connects Oklahoma City, OK with Chicago, IL</td>
</tr>
<tr>
<td>Link (8,7) and (8,4)</td>
<td>Part of the North America railroad which connects Tulsa, OK, with Chicago, IL and Texas City, TX, respectively</td>
</tr>
<tr>
<td>Link (2,8)</td>
<td>A local railroad connecting Port of Catoosa to the North America railroad</td>
</tr>
<tr>
<td>Link (1,4)</td>
<td>Part of the North America railroad that connects Oklahoma City, OK with Texas City, TX</td>
</tr>
<tr>
<td>Links (2,5), (2,4), (2,6), and (2,7)</td>
<td>Part of the inland waterway network navigated by McClellan–Kerr Arkansas River Navigation System and connect Port of Catoosa with the Port of New Orleans, the Port of Texas City, the Port of Little Rock, and the Port of Chicago, respectively</td>
</tr>
<tr>
<td>Links (3,6), (3,4), and (3,5)</td>
<td>Part of the inland waterway network navigated by McClellan–Kerr Arkansas River Navigation System and connect the Port of Muskogee to the Port of Little Rock, the Port of Texas City, and the Port of New Orleans, respectively</td>
</tr>
<tr>
<td>Link (9,4)</td>
<td>The roadway that connects Oklahoma City to Texas City, TX using interstate highways I-35 and I-45</td>
</tr>
<tr>
<td>Link (9,11)</td>
<td>Part of interstate highway I-44 that connects Oklahoma City to Tulsa.</td>
</tr>
</tbody>
</table>

Table 2. Names and NAICS codes for main industries using the network.

<table>
<thead>
<tr>
<th>Industry name</th>
<th>NAICS code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food and beverage and tobacco products</td>
<td>311</td>
</tr>
<tr>
<td>Petroleum and coal products</td>
<td>324</td>
</tr>
<tr>
<td>Chemical products</td>
<td>325</td>
</tr>
<tr>
<td>Nonmetallic mineral products</td>
<td>327</td>
</tr>
<tr>
<td>Machinery</td>
<td>333</td>
</tr>
<tr>
<td>Miscellaneous manufacturing</td>
<td>339</td>
</tr>
</tbody>
</table>
Table 3. Combined annual demands/supplies at supply/demand nodes connecting through the network (in thousand tons).

<table>
<thead>
<tr>
<th>Supply nodes in OK</th>
<th>Food and beverage</th>
<th>Petroleum and coal</th>
<th>Chemical products</th>
<th>Nonmetallic mineral</th>
<th>Machinery mfg.</th>
<th>Misc. mfg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oklahoma City</td>
<td>4351</td>
<td>0</td>
<td>3606</td>
<td>2198</td>
<td>285</td>
<td>1419</td>
</tr>
<tr>
<td>Port of Catoosa</td>
<td>603</td>
<td>5459</td>
<td>3416</td>
<td>303</td>
<td>30</td>
<td>5</td>
</tr>
<tr>
<td>Port of Muskogee</td>
<td>0</td>
<td>408</td>
<td>0</td>
<td>383</td>
<td>0</td>
<td>361</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand nodes outside of OK</th>
<th>Food and beverage</th>
<th>Petroleum and coal</th>
<th>Chemical products</th>
<th>Nonmetallic mineral</th>
<th>Machinery mfg.</th>
<th>Misc. mfg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Texas City, TX</td>
<td>1167</td>
<td>3804</td>
<td>2448</td>
<td>0</td>
<td>310</td>
<td>362</td>
</tr>
<tr>
<td>New Orleans, LA</td>
<td>604</td>
<td>221</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Little Rock, AR</td>
<td>3183</td>
<td>1842</td>
<td>4574</td>
<td>492</td>
<td>2</td>
<td>654</td>
</tr>
<tr>
<td>Chicago, IL</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2392</td>
<td>0</td>
<td>769</td>
</tr>
</tbody>
</table>

Table 4. Link capacities among the origin/destination nodes in the illustrative network (in thousand tons).

<table>
<thead>
<tr>
<th>Nodes</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2800</td>
<td></td>
<td>2900</td>
<td></td>
<td>1700</td>
<td></td>
<td>6200</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>650</td>
<td>750</td>
<td>500</td>
<td>3400</td>
<td></td>
<td>3700</td>
<td>1350</td>
</tr>
<tr>
<td>3</td>
<td>355</td>
<td>185</td>
<td>3010</td>
<td></td>
<td>300</td>
<td></td>
<td>290</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>3800</td>
<td></td>
<td>300</td>
<td></td>
<td></td>
<td></td>
<td>1700</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1800</td>
<td></td>
<td>1200</td>
<td>1600</td>
<td>2000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Considering a disruptive scenario as the removal of a particular network component, the supply-demand network might experience a failure in satisfying demands in the interrupted network. The network components that were considered for disruption include: (i) three transshipment nodes within the state of Oklahoma, which have a vital role in connecting segments of high volume-freight-traffic interstate highways, (ii) some segments of the North America Railroad, (iii) a local railroad which connects industrial parks to the North America Railroad, and (iv) parts of waterway system (described in Table 1). Discussed previously in Section 3.2, a decision maker would likely desire to reroute commodities to take advantage of the remaining capacity of the residual network, as shown in model M2, by maximizing the flow to meet as much demand as possible given the interrupted network. Failure in the form of undelivered commodities remaining with the suppliers, or unsatisfied demand at demand nodes, represented by $S_k^i$, affect industry output and inoperability propagates through many of the interconnected industries. In the illustrative example, all the supply nodes are within the state of Oklahoma and the four demand nodes are located outside of Oklahoma. Table 5 reports $\sum_{\ell \in (N_s \cap N_k)} S_k^i$, the sum of the slack (remaining supply) by industry type at the supply nodes when different network components are disrupted.

In the case of any disruption within the network resulting in the loss of exports, there is a demand perturbation in the industries using the network, as calculated in Eq. (14). Assuming the only losses in the state economy are due to the loss of exports, the interdependent cascade of the demand perturbations causes losses to all the other
state industries, as captured in \( Q \). It is further assumed that industries not using the transportation network have zero demand perturbations, though could suffer from interdependent inoperability.

Table 5. Commodities remaining at suppliers with the removal of network components (in thousand tons).

<table>
<thead>
<tr>
<th>Removed component</th>
<th>Food and beverage</th>
<th>Petroleum and coal</th>
<th>Chemical products</th>
<th>Nonmetallic mineral</th>
<th>Machinery mfg.</th>
<th>Misc. mfg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 9</td>
<td>291</td>
<td>0</td>
<td>803</td>
<td>0</td>
<td>25</td>
<td>2</td>
</tr>
<tr>
<td>Node 8</td>
<td>478</td>
<td>2508</td>
<td>623</td>
<td>0</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Node 11</td>
<td>34</td>
<td>143</td>
<td>1067</td>
<td>0</td>
<td>25</td>
<td>7</td>
</tr>
<tr>
<td>Link (1,7)</td>
<td>290</td>
<td>0</td>
<td>0</td>
<td>2000</td>
<td>0</td>
<td>770</td>
</tr>
<tr>
<td>Link (9,11)</td>
<td>189</td>
<td>108</td>
<td>823</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Link (2,5)</td>
<td>504</td>
<td>37</td>
<td>0</td>
<td>0</td>
<td>28</td>
<td>0</td>
</tr>
<tr>
<td>Link (8,4)</td>
<td>367</td>
<td>2108</td>
<td>923</td>
<td>0</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Link (2,8)</td>
<td>189</td>
<td>2308</td>
<td>823</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Link (2,4)</td>
<td>0</td>
<td>105</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Link (3,8)</td>
<td>0</td>
<td>0</td>
<td>210</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Link (8,7)</td>
<td>0</td>
<td>0</td>
<td>251</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

As network component importance rankings are ultimately calculated on a relative basis, inoperability is calculated in terms of annual impact, as it is assumed that annual industry production accumulates consistently across the year (i.e., neither production nor interdependency relationships vary day-to-day, week-to-week, month-to-month). A smaller time horizon could be considered as a proportion of a year if a particular disruptive event is modeled (e.g., a two-week closure of port facilities\(^{78}\)).

Using the remaining commodities left at supply nodes, shown in Table 5, demand perturbations were calculated with Eq. (14). In the example, the industries in Oklahoma experience difficulties in exporting commodities, individually for each of the 11 disrupted network components. The resulting industry inoperability, \( q^k \), for each disrupted component is found, as shown in Table 6 and plotted in Figure 5. Results show that most industries are vulnerable to disruptions that affect the functionality of either rail transportation or interstate highways but less susceptible to disruptions to the inland waterway which has a smaller share (less than 5%\(^{99}\)) in outbound freight movement in Oklahoma. As such, perhaps the external capacity in rail and truck freight transport suggest that they could serve as alternative transportation modes during a disruption, though more costly. A disruption that affects the functionality of the intermodal terminals would cause the most significant drop in the productivity of most industries. Examples of this include (i) node 8, which facilitates trade between industries located in the business economic area in Port of Catoosa, OK with their customers in Chicago, IL and Texas City, TX through the North America railroad, and (ii) nodes 9 and 11, important transshipment nodes that connect the three important business economic areas within the state of Oklahoma to their customers through interstate highways.
From a single industry point of view, it is shown that the productivity of the *Petroleum and coal* (324) industry is mostly vulnerable to its accessibility to the North America railroad through the intermodal terminal (node 8) in Tulsa, OK. In general, most disruption scenarios may affect the productivity of the *Chemical products* (325) industry, either by a local disruption that interrupts the access of the business economic area at the Port of Catoosa through a local railroad (e.g., link (2,8) to the intermodal terminal at node 8) or a state-wide disruption that affects Oklahoma’s major trucking corridors (e.g., interstate highways I-35, I-44, and I-40). Also shown is that parts of the transportation network that are less important for most industries may be quite important to the productivity of a particular industry (e.g., the *Nonmetallic and mineral products* (327) industry is influenced by the malfunction of the local railroad which connects the Port of Muskogee to the North America railroad (link (3,8)) though all the other five industries are much less vulnerable to this link). Understanding these inoperability-related vulnerabilities could motivate further studies to guide investments in alternative transportation modes. The inoperability values in Table 6 may appear to be negligible at first, but these numbers are significant when linked to the concept of failure probability in the reliability or quality engineering literature (i.e., the maximum allowable failure probability for a six-sigma compliant system is 3.4E-06).

**Table 6. Interdependent industry inoperability resulting from network component removal.**

<table>
<thead>
<tr>
<th>Removed component</th>
<th>Food and beverage</th>
<th>Petroleum and coal</th>
<th>Chemical products</th>
<th>Nonmetallic mineral</th>
<th>Machinery mfg.</th>
<th>Misc. mfg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 9</td>
<td>2.98E-04</td>
<td>7.15E-06</td>
<td>4.37E-04</td>
<td>1.21E-05</td>
<td>1.81E-04</td>
<td>2.64E-05</td>
</tr>
<tr>
<td>Node 8</td>
<td>4.93E-04</td>
<td>1.11E-03</td>
<td>4.25E-04</td>
<td>4.03E-05</td>
<td>1.82E-05</td>
<td>9.44E-05</td>
</tr>
<tr>
<td>Node 11</td>
<td>3.61E-05</td>
<td>6.83E-05</td>
<td>5.71E-04</td>
<td>8.27E-06</td>
<td>1.80E-04</td>
<td>2.37E-05</td>
</tr>
<tr>
<td>Link (1,7)</td>
<td>3.02E-04</td>
<td>1.11E-05</td>
<td>2.86E-05</td>
<td>9.12E-04</td>
<td>7.71E-06</td>
<td>5.56E-04</td>
</tr>
<tr>
<td>Link (9,11)</td>
<td>1.94E-04</td>
<td>5.20E-05</td>
<td>4.43E-04</td>
<td>6.54E-06</td>
<td>2.37E-06</td>
<td>1.42E-05</td>
</tr>
<tr>
<td>Link (2,5)</td>
<td>5.16E-04</td>
<td>2.28E-05</td>
<td>2.43E-05</td>
<td>1.69E-05</td>
<td>2.05E-04</td>
<td>3.36E-05</td>
</tr>
<tr>
<td>Link (8,4)</td>
<td>3.80E-04</td>
<td>9.33E-04</td>
<td>5.66E-04</td>
<td>3.34E-05</td>
<td>1.53E-05</td>
<td>8.01E-05</td>
</tr>
<tr>
<td>Link (2,8)</td>
<td>1.98E-04</td>
<td>1.02E-03</td>
<td>5.14E-04</td>
<td>3.15E-05</td>
<td>1.53E-05</td>
<td>7.61E-05</td>
</tr>
<tr>
<td>Link (2,4)</td>
<td>1.61E-07</td>
<td>4.63E-05</td>
<td>3.39E-06</td>
<td>1.20E-06</td>
<td>6.19E-07</td>
<td>2.79E-06</td>
</tr>
<tr>
<td>Link (3,8)</td>
<td>1.41E-07</td>
<td>7.11E-07</td>
<td>2.15E-06</td>
<td>1.06E-04</td>
<td>3.80E-05</td>
<td>2.84E-06</td>
</tr>
<tr>
<td>Link (8,7)</td>
<td>7.10E-08</td>
<td>4.90E-07</td>
<td>1.36E-06</td>
<td>1.26E-04</td>
<td>2.30E-07</td>
<td>9.62E-07</td>
</tr>
</tbody>
</table>

In addition to inoperability, the complementary perspective of economic losses in Table 7 can supplement the analysis. The *Petroleum and coal products* (324) industry is a high dollar industry in Oklahoma, and this industry would be significantly impacted by a disruption that affects the functionality of rail transportation (e.g., a local railroad such as link (2,8), part of the level-one railroad that connects Oklahoma to the North America railroad such as link (8,4), or intermodal terminal facilities such as node 8). A second prominent industry is the *Food, beverage, and tobacco products* (311) industry, and several transportation components contribute to the dollar volume of production in this industry, especially a part of the inland waterway network that connects Port of Catoosa with the Port of New Orleans, LA (link (2,5)), and part of the North America railroad that connects Tulsa, OK with Texas City, TX (link (8,4)). In fact, the intermodal terminal (node 8) which facilitates freight transport at the Port of Catoosa is a prominent
component in the dollar volume of several exporting industries in Oklahoma. In general, rail transportation and major trucking corridors have a high impact on the economy of most industries, though less important components (e.g., part of the inland waterway such as links (2,4) or (2,5)) may still have a large impact on a particular industry (e.g., Miscellaneous manufacturing (339) and Petroleum and coal products (324) by millions of dollars).

The component importance measures, quantifying the proportional economy-wide impact of a loss of component $p$ relative to a loss of the whole network, are calculated with Eq. (15) and are depicted in Figure 6. This measure lies on $[0,1]$, where $\eta_p = 0$ means that the removal of component $p$ doesn’t affect the whole economy, and $\eta_p = 1$.

![Figure 5. Economic inoperability across six most important industries within the state of Oklahoma.](image)

**Table 7. Economic losses (in 100 million USD) across the six most important industries within the state of Oklahoma.**

<table>
<thead>
<tr>
<th>Removed component</th>
<th>Food and beverage</th>
<th>Petroleum and coal</th>
<th>Chemical products</th>
<th>Nonmetallic mineral</th>
<th>Machinery mfg.</th>
<th>Misc. mfg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 9</td>
<td>1.627</td>
<td>0.090</td>
<td>0.594</td>
<td>0.025</td>
<td>1.308</td>
<td>0.267</td>
</tr>
<tr>
<td>Node 8</td>
<td>2.688</td>
<td>13.974</td>
<td>0.578</td>
<td>0.082</td>
<td>0.132</td>
<td>0.954</td>
</tr>
<tr>
<td>Node 11</td>
<td>0.197</td>
<td>0.860</td>
<td>0.777</td>
<td>0.017</td>
<td>1.304</td>
<td>0.240</td>
</tr>
<tr>
<td>Link (1,7)</td>
<td>1.647</td>
<td>0.140</td>
<td>0.039</td>
<td>1.854</td>
<td>0.056</td>
<td>5.622</td>
</tr>
<tr>
<td>Link (9,11)</td>
<td>1.059</td>
<td>0.655</td>
<td>0.602</td>
<td>0.013</td>
<td>0.017</td>
<td>0.143</td>
</tr>
<tr>
<td>Link (2,5)</td>
<td>2.813</td>
<td>0.287</td>
<td>0.033</td>
<td>0.034</td>
<td>1.484</td>
<td>0.340</td>
</tr>
<tr>
<td>Link (8,4)</td>
<td>2.072</td>
<td>11.757</td>
<td>0.769</td>
<td>0.068</td>
<td>0.111</td>
<td>0.810</td>
</tr>
<tr>
<td>Link (2,8)</td>
<td>1.079</td>
<td>12.837</td>
<td>0.698</td>
<td>0.064</td>
<td>0.111</td>
<td>0.769</td>
</tr>
<tr>
<td>Link (2,4)</td>
<td>0.001</td>
<td>0.583</td>
<td>0.005</td>
<td>0.002</td>
<td>0.004</td>
<td>0.028</td>
</tr>
<tr>
<td>Link (3,8)</td>
<td>0.001</td>
<td>0.009</td>
<td>0.003</td>
<td>0.216</td>
<td>0.275</td>
<td>0.029</td>
</tr>
<tr>
<td>Link (8,7)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.002</td>
<td>0.256</td>
<td>0.002</td>
<td>0.010</td>
</tr>
</tbody>
</table>
means that the particular component removal shuts down the whole economy. As shown in Figure 6, the most important components relate to the rail transportation and major trucking corridors. A main component of the rail freight transport, node 8 is the intermodal terminal facilitates the movement of commodities in the industrial park of Port of Catoosa to out-of-state customers and is the most important component in the analyzed transportation network. This facility is followed by link (8,4), a portion of railroad that connects Oklahoma to Texas City, TX, link (2,8), a local railroad that connects the Port of Catoosa to the North America railroad intermodal terminal, and link (1,7), a portion of railroad that connects Oklahoma City, OK to Chicago, IL. This suggests a further attention to the functionality of the facilities of the most important components within the network to avoid any malfunction, or in the case of any disaster which deactivates multiple components of the network, there should be priorities to recover the most important components. The framework proposed here could be used to evaluate alternative transportation modes for shipping commodities after a disruption or to guide planning for transportation investments to reduce vulnerability, and thus multi-industry impacts.

![Network component importance measures across the Oklahoma economy](image)

**Figure 6. Network component importance measures across the Oklahoma economy.**

Figure 7 emphasizes component importance to individual industries, quantifying the proportional impact of a loss of component $p$ on a particular industry relative to the impact of a loss in all components on that industry, as calculated with Eq. (16). This measure lies on $[0,1]$, where $\eta_p^k = 0$ means that the removal of component $p$ does not affect industry $k$, while $\eta_p^k = 1$ suggests that the particular component removal completely shuts down industry $k$. As it is shown in Figure 7, any failure that results in disconnection of link (2,5), the inland waterway connecting the Port of Catoosa to the Port of New Orleans, would have the largest impact on Food, beverages, and tobacco products (311) relative to other industries. The intermodal terminal (node 8), which connects the Port of Catoosa to its out-of-state customers through the North America railroad, has the largest impact on Petroleum and coal products (324). It also demonstrates that the malfunction of local railroads (e.g., link (2,8)) may have a high
impact on the productivity of Petroleum and coal products (324). In addition, the Nonmetallic and mineral products (327) industry, primarily located in the Oklahoma City business economic area, is highly vulnerable to the functionality of the part of the North America railroad that connects Oklahoma City, OK with Chicago, IL. Note that some components are important from the perspective of a particular industry though perhaps not the entire economy, such as link (1,7), which suggests lower priority in Figure 6 but is quite impactful for the Nonmetallic and minerals products (327) industry. Figure 6 also suggests that the Petroleum and coal products (324) industry can be impacted by the disruption of several network components, more so than any other industry.

A four-step approach (i) calculates baseline (undisrupted) multi-commodity flow according to minimum cost, (ii) measures slack at supply and demand nodes, in the form of undelivered supply and unmet demand, when individual components are removed (one-at-a-time) from the network according to a maximum flow perspective, (iii) relates slack in the network to perturbations and inoperability among interdependent

3.6. DISCUSSION
Transportation network vulnerability studies have largely attempted to quantify the reduction in system functionality, following a disruption, as (i) topological properties of the network, and (ii) flow importance measures. These structural and flow-related measures ignore a larger role that the transportation network plays in facilitating economic productivity. This work offers a broader perspective on freight transportation network vulnerability analysis with a means to measure importance of network components considering economic impacts of degradation of transportation network. In particular, this study considers a multi-modal freight network consisting of highway, railway, and waterway transportation, and implements the proposed vulnerability analysis framework to understand and rank the criticality of multi-modal transportation nodes and links.
industries relying on commodities flowing along the network, and (iv) quantifies the importance of each component from industry-specific and overall regional economy perspectives. The primary contribution of this approach is the integration of the multi-commodity network flow representation of the multi-modal transportation network with the interdependent, multi-industry economic model and a framework to measure a transportation network component importance considering its multi-industry impact.

This approach is illustrated with a stylized case study of a multi-modal transportation network in the state of Oklahoma, where supply nodes are located within the state and demand nodes are located outside of the state. Results of the case study suggest that the Petroleum and coal products industry is particularly susceptible to disruptions in several components, and certain components can impact multiple industries. Also, analysis shows that the economy of the state and most industries are primarily vulnerable to the malfunction of the parts of the railway that connect the state to the North America railroad and major trucking corridors including interstate highways I-35, I-44, and I-40. While the application pursued in this study focused primarily on the state of Oklahoma, the base model can be applied to other freight transportation networks to identify the critical nodes/links that can instigate the largest vulnerability across interdependent sectors that uniquely vary from region to region. Hence, the proposed model and its future applications could provide significant value to homeland security preparedness planning.

Furthermore, the vulnerability analysis perspective proposed in this study can be implemented to highlight priorities in maintaining certain network components (to reduce common-cause failure), or in rerouting of commodity flows after a disruption. There also exists an opportunity to extend the base approach discussed in this work to analyze network completion strategies where capacity enhancement (e.g., link capacity) and additional transportation facilities (e.g., added links/nodes) could harden the network around the most vulnerable components. Further, longer term transportation infrastructure design plans could be informed by this kind of analysis.

4. ADAPTIVE CAPACITY PLANNING IN A FREIGHT TRANSPORTATION NETWORK

A disruption within a freight transportation network affects its vital role in transporting raw materials among manufacturers and final products between manufacturers and consumers. Such a disruption in the flow of commodities leads to economic losses across multiple industries. To devise an adaptive capacity strategy (i.e., post-disruption rerouting) to lessen total economic losses following a disruption, we propose an optimization framework that integrates (i) a multi-commodity network flow model of freight movement, (ii) a risk-based interdependency model of multi-industry impacts, and (iii) an objective function that addresses adaptive capacity with a measure of static economic resilience\[^{47,49,52}\]. The proposed optimization model is developed following a three-step approach, illustrated in Figure 8. This section is based on Darayi et al.\[^{100}\].
4.1. STEP 1: FREIGHT MOVEMENT AND DISRUPTION
To model a supply-demand network for a set of business economic areas consisting of different industries interacting with their suppliers and customers located outside of their region through a multi-modal freight transportation system, a typical multi-commodity network flow (MCNF) model (e.g., [58]) is used. The goal of this model is to facilitate the commodity flows between suppliers and consumers through a capacitated transportation network while minimizing the cost of transportation. Planning decisions in a multi-modal freight transportation network is made at strategic, tactical, and operational levels [56]. It is assumed that (i) strategic decisions determine general development policies and define the operating strategies of the system over relatively long time horizons (e.g., the location of the physical transportation network, the location of main facilities such as rail yards or multi-modal platforms [59]), (ii) tactical plans deal mostly with medium-term decisions (e.g., route choice and type of service to operate, aggregate scheduling [101]), and (iii) operational level decisions are made when real or near real-time response is required (e.g., crew or container scheduling [102]). In this work, when a disruption interrupts the movement of commodities through the network, a tactical contingent rerouting plan is sought, for the period of disruption, to maintain the functionality of the supply-demand network as much as possible.

The topology of the multi-modal freight transportation network, as well as corresponding supply and demand nodes, must be extracted to model and analyze the behavior of the network before and after disruption. The transportation network is considered to be a facilitator of $K$ interacting industries, where multiple supply and demand nodes of commodity $k$ could represent a particular industry. Based on a conventional MCNF model, the network is defined on directed graph $G = (N, L)$, where $N$ is a set of nodes, each of which could be home to either suppliers or consumers of multiple commodities, and $L$ is a set of links connecting nodes. Let $K$ denote the number of commodities in a
network instance, each representing an industry. Let \( f_{ij}^k \geq 0 \) denote the decision variable associated with the flow quantity of commodity \( k \in \{1, \ldots, K\} \) on link \((i, j) \in L\). Let parameter \( w_{ij}^k \) denote the associated per-unit transportation cost. The costs differ based on link properties such as length and transportation mode (e.g., waterway, railway, highway). Let parameter \( u_{ij} \) denote the total flow capacity of link \((i, j) \in L\). That is, the capacity of each link is a shared or “bundle” constraint for all commodities flowing on the link. The supply/demand requirement of commodity \( k \) at node \( i \in N \) is denoted by parameter \( b_i^k \). If \( b_i^k \) is positive, then node \( i \) is a supply node of commodity \( k \). Similarly, if \( b_i^k \) is negative, then node \( i \) is a demand node for commodity \( k \). If \( b_i^k \) is zero, the node \( i \) is a transshipment node with respect to commodity \( k \). The mathematical formulation for the MCNF problem is provided in Eq. (17). Without loss of generality, each node within the network can be home to either suppliers or consumers of multiple commodities. The set of nodes then can be partitioned into three mutually exclusive sets: \( N = (N_-, N_+, N_0) \) where \( N_- \) denotes the set of nodes representing nodes which are home to consumers, \( N_+ \) denotes which are home to suppliers, and \( N_0 \) denotes all transshipment nodes. Each commodity belongs to an industry in the economy as defined by the North American Industry Classification System (NAICS).

\[
\begin{align*}
\min & \quad \sum_{(i,j) \in L} \sum_k w_{ij}^k f_{ij}^k \\
\text{s.t.} & \quad \sum_k f_{ij}^k \leq u_{ij} \quad \forall (i, j) \in L \\
& \quad \sum_{(i,j) \in L} f_{ij}^k - \sum_{(j,i) \in L} f_{ji}^k = b_i^k \quad \forall (i,j) \in L, k = 1, \ldots, K \\
& \quad f_{ij}^k \geq 0, \forall (i,j) \in L, k = 1, \ldots, K
\end{align*}
\]

From a tactical point of view, integrating (i) industries and (ii) their supply capabilities or demand requirements together with (iii) the structure of the transportation network, can result in a minimum cost MCNF model that can route commodities from suppliers to demand nodes via \( f_{ij}^k \), collectively representing the flow of commodities on the links of a baseline (undisrupted) network.

Natural hazards, human-made events, or common failures could threaten the functionality of the network components and consequently interrupt commodity flows. A scenario-based removal of network components known as interdiction\textsuperscript{[21]} is a common theme in modeling and analysis of supply-demand network disruption. The consequences of a hazards, attacks, or failures are simulated as disruptions in the flow of valuable goods or services through the network caused by disabling network components. The functionality of the network is analyzed to determine how vulnerable it is to interdiction, and which nodes or links, if lost, result in the most damage to network performance. Interdiction analyses encompass a wide range of possible disruptions that may vary with respect to spatial scales, correlation of disruptive events, sequence of failures, and event duration.
A disruption scenario is defined as the set of network components that are impacted, the degree to which they are disabled, and the operating conditions (e.g., network activity, link/node capacities) of the network prior to the disruption regardless of the initiating event that causes the disruption. Different approaches to model a transportation network disruption have been offered (e.g., losing a bridge, a road segment, or a hub), with most approaches considering one component being affected. A disrupted network component may be rendered completely inoperable by a disruption (e.g., losing a road completely due to a bridge collapse), or its functionality may drop to a lower level (e.g., an accident blocking a single lane of an interstate highway segment). Simulating the disruption scenario enables the evaluation of the impact of the failure. Impacts can be considered as the direct associated failures in network operability (e.g., flow or capacity reduction) or consequential failures (e.g., the economic impacts affecting the production and consumption of flows). It takes time to recover affected network components (e.g., after Hurricane Katrina, it took up to six months in southern regions to recover highway networks, whereas northeast regions recovered much more quickly; after an I-40 bridge collapsed in Oklahoma following a barge collision in 2002, traffic was rerouted for nearly two months while crews rebuilt the infrastructure). As such, devising an efficient and effective contingent rerouting strategy immediately after extreme events would assist the economic productivity of the disrupted region.

In the case of any disruption modeled as the removal of a network component or a set of components (or a drop in functionality of the network modeled as reduction of link capacities, \(u'_{ij}\)), the consequences are sought by deducting the commodity flows on the affected links from the baseline flow, as calculated in Eq. (1). Let \(G' = (N', L')\) represent the network after disruption with updated sets of links, \(L'\) and nodes, \(N'\). The sets \(N^L, N^+_i, N^0_i\) denote the post disruption sets of nodes associated with home of consumers, home of suppliers, and transshipment nodes, respectively. The quantity of commodity \(k\) at node \(i\) that is either undelivered and remaining with the suppliers, or unsatisfied demand of consumers, is reflected in the slack variable \(S^k_i\). This slack variable will be used subsequently to drive the calculation of inoperability among multiple industries. It is assumed that each type of commodity represents the output of a lone industry, and interdependent inoperability propagated through a set of industries caused by unsatisfactory demands/supplies will be modeled in the next section.

4.2. STEP 2: MULTI-INDUSTRY IMPACT

In this work, we use an extension of the input-output economic model, for which Wassily Leontief won a Nobel Prize, to capture the multi-industry impacts of unmet demands at demand nodes and remaining commodities at supply nodes as the result of a disruption to components of the transportation network. The input-output (I-O) model is a widely accepted model for analyzing the interdependent connections among industries, and the use of the I-O enterprise for studying disruptions was among the 10 Most Important Accomplishments in Risk Analysis: 1980-2010.

Under a static equilibrium, the total output of industry (or economic sector) \(k\) is distributed to other industries and also satisfies external (consumption) demand. Under
a proportionality assumption, this equilibrium condition is described with \( x_k = \sum_{r=1}^{K} z_{kr} + c_k \), where \( x_k \) is the total output of industry \( k \), \( z_{kr} \) is the input of industry \( k \) to the production of industry \( r \) (intermediate consumption), and \( c_k \) is the external (final) consumption for industry \( k \)'s output. The intermediate consumption, \( z_{kr} \), is assumed to be proportional to the output of industry \( r \) (\( r \in \{1, ..., K\} \) and \( r \neq k \)), expressed as \( z_{kr} = a_{kr} x_r \). In the common form of the Leontief I-O model, industry production is modeled as \( x = Ax + c \), where \( x \) is the vector of industry production outputs, \( A \) is an industry-by-industry matrix of interdependency coefficients, \( a_{kr} \) (proportion of industry \( k \)'s input to \( r \), with respect to total production of industry \( r \)), and \( c \) is a vector of final consumption. The model shows that total production is made up of industry-to-industry intermediate production, \( Ax \), and production to satisfy final consumption, \( c \).

The availability of data describing the parameters of the I-O model in the US through the Bureau of Economic Analysis (BEA)\(^{[70]} \), as well as a number of other countries\(^{[68]} \), justifies the extensive use of I-O models. To model the propagation of inoperability, or the proportional extent to which industries are unproductive after a change in final consumption or a forced change in final consumption due to a lack of supply, Santos and Haimes\(^{[71]} \) propose the Inoperability Input-Output Model (IIM), extending the capability of the I-O model to model not only economic interdependency but interdependency in broader infrastructure sectors. This risk-based model is defined from two metrics\(^{[106]} ,^{[107]} \): (i) inoperability \( q_k \) and (ii) final consumption perturbation \( c_k^* \), which are defined in Eqs. (19) and (21), respectively. Providing a different perspective from the traditional I-O model, the IIM shows how normalized production losses propagate through interconnected industries with a normalized interdependency matrix \( A^* \).

Describing the relationships among \( K \) industries, resulting in matrices of size \( K \times K \) and vectors of length \( K \), Eq. (18) formulates the propagation of the inoperability in a group of interconnected industries.

\[
q = A^* q + c^* \Rightarrow q = [I - A^*]^{-1} c^* \quad (18)
\]

Vector \( q \) is a vector of industry inoperability describing the proportional extent to which as-planned productivity or functionality is not realized following a disruptive event. Inoperability for industry \( k \) is defined in Eq. (19), where as-planned total output is represented with \( \tilde{x}_k \) and degraded total output resulting from a disruption is represented with \( \bar{x}_k \). An inoperability of 0 suggests that an industry is operating at normal production levels, while an inoperability of 1 represents the situation in which an industry is completely inoperable.

\[
q_k = (\tilde{x}_k - \bar{x}_k) / \tilde{x}_k \iff q = [\text{diag}(\tilde{x})]^{-1} (\bar{x} - \tilde{x}) \quad (19)
\]

A normalized form of the original \( A \) matrix describing the extent of interdependence among a set of industries or sectors is defined as \( A^* \). The row elements of \( A^* \) indicate the proportion of additional inoperability that are contributed by a column industry to the row industry, shown in Eq. (20).

\[
a_{r k}^* = a_{rk} (\tilde{x}_r / \tilde{x}_k) \iff A^* = [\text{diag}(\tilde{x})]^{-1} A [\text{diag}(\tilde{x})] \quad (20)
\]
The calculation of $c^*$, a vector of normalized final consumption reduction is provided in Eq. (21), where the elements of $c^*$ represent the difference in as-planned final consumption $\hat{c}_k$ and perturbed final consumption $\check{c}_k$ divided by as-planned production, quantifying the reduced final consumption for industry $k$ as a proportion of total as-planned output.

$$c_k^* = (\hat{c}_k - \check{c}_k)/\hat{x}_k \iff c^* = [\text{diag}(\hat{x})]^{-1}(\hat{c} - \check{c}) \quad (21)$$

In addition to industry inoperability, a traditional economic loss metric can be calculated by multiplying each industry’s production level, $x_k$, in dollars, by its inoperability level: for industry $k$, $Q_k = x_k q_k$. Such a measure can also be expressed for the collection of $K$ industries, $Q = x^T q$. As such, decisions to plan for adaptive capacity can be made with respect to economic impact across multiple industries.

The freight transportation network provides a platform for commodity flows between industries. Since the IIM models how demand-related risk in a given industry propagates to other industries due to their interdependent productivity, the multi-industry impact of a disruption to a freight transportation network can be studied when network losses are related to final consumption reduction and inoperability terms as shown in subsequent subsections. The demand-reduction IIM proposed by Santos and Haimes\cite{71} has been successfully employed to study multi-industry impacts of perturbations in supply and demand (e.g., \cite{78},\cite{108}-\cite{110}). However, some (e.g., \cite{111},\cite{112}) have questioned the usefulness (and theoretical plausibility) of supply-driven models developed from concepts by Ghosh\cite{113}. Leung et al.\cite{114} integrated a supply-side price IIM and output-side IIM to address initiating perturbations related to input factors (value added) and to industry output levels, though some aspects of this model may be impractical for integration with supply-demand networks as applied in our proposed approach (though may be effective in modeling disruptions to manufacturing systems, as noted by Kelly\cite{111}). Here, we translate a disruption in the form of remaining commodities at supply nodes and/or unmet demand at demand nodes into the two IIM metrics of inoperability and final consumption perturbation, based on a demand-reduction IIM implemented by Pant et al.\cite{78} in modeling supply and demand perturbation caused by a port closure. Pant et al.\cite{78} considered commodities remaining at suppliers after a disruption to calculate the final consumption perturbation. And the authors considered unmet demands to calculate a “forced” demand reduction, assuming that a disruption decreases the supply of a commodity for a demand node while the final external consumption remains virtually unaffected. In such a case, the demand nodes temporarily sacrifice their internal need for that commodity until it returns to its as-planned supply level, and a surrogate to supply reduction is calculated from the combination of “forced” internal consumption and an output inoperability.

In the following subsections, $N^\alpha$ represents the set of nodes within the area of interest $\alpha$, and $N^{\bar{\alpha}}$ represents the set of nodes outside of the area of interest, such that $N = N^\alpha \cup N^{\bar{\alpha}}$. We formulate the economic consequences of a failure within a particular area of interest (e.g., a business economic area, county, state, entire country). As such, the
failure in the form of remaining commodities at suppliers and unmet demand at consumers are captured only in the nodes within the area of interest and each of the economic parameters (i.e., \( x, c, c^*, q \)) are indicators of the industries specific to the region of interest. To simplify the notation, superscript \( \alpha \) is not included for these economic metrics to avoid unnecessary indices.

Transportation facilities operate as facilitators of commodity flows across business economic areas. For a supplier of commodity \( k \) located in node \( i \), any transportation network disruption that perturbs its desired export will be considered to be a reduction in final consumption. As modeled in Eq. (22), final consumption for industry \( k \) includes commodities consumed by industry \( k \) itself internally, \( \hat{c}_k^{\text{int}} \), and the amount of external consumption that is exported through the network, \( \hat{c}_k^G \). It is assumed that the disruption results in losses of commodity flows only through the network, while industry production activities unrelated to the network experience no direct failure but might be affected indirectly by a disruption within the network (due to an interdependent loss of economic productivity). When industry \( k \) has difficulty only in exporting commodities, it experiences commodities remaining at supply nodes in the region of interest totaling \( \sum_{i \in (N^\alpha \cap \mathbb{N}_k')} S_i^k \), where \( N^\alpha_+ \) represents the set of nodes that are home to suppliers in the region of interest \( \alpha \) after the disruption, as shown in Eq. (23). As such, the final consumption perturbation for industries that experience difficulties only in exporting commodities is modeled as the amount of slack divided by as-planned industry output in Eq. (24). Note that the supply-demand network may consist of suppliers and consumers located outside of the region of interest, yet failures to these suppliers and consumers are not accounted for in this model.

\[
\hat{c}_k = (\hat{c}_k)^{\text{int}} + (\hat{c}_k)^G \quad k \in \{1, \ldots, n\} \tag{22}
\]

\[
\hat{c}_k - \hat{c}_k = \sum_{i \in (N^\alpha \cap \mathbb{N}_k')} S_i^k \quad k \in \{1, \ldots, n\} \tag{23}
\]

\[
c_k^* = \frac{\sum_{i \in (N^\alpha \cap \mathbb{N}_k')} S_i^k}{\hat{x}_k} \quad k \in \{1, \ldots, n\} \tag{24}
\]

As discussed by Pant et al.\[78\], the amount of import (input) of industry \( k \) at demand nodes in the supply-demand network defined as \( \sum_{i \in (N^\alpha \cap \mathbb{N}_k')} -b_i^k \) contributes toward the production activity and the internal consumption of industry \( k \). Thus, when industry \( k \) has difficulty only in importing commodities, it experiences unmet demands in the region of interest totaling \( \sum_{i \in (N^\alpha \cap \mathbb{N}_k')} S_i^k \). This results in the loss of output, \( \Delta \hat{x}_k \), representing \( \hat{x}_k - \hat{x}_k \), and final internal consumption, \( \Delta (\hat{c}_k)^{\text{int}} \). Here, \( N^\alpha_- \) represents the set of nodes after disruption located in the geographical area of interest \( \alpha \) that are home to consumers of commodity \( k \).

\[
\sum_{i \in (N^\alpha \cap \mathbb{N}_k')} S_i^k = \Delta \hat{x}_k + \Delta (\hat{c}_k)^{\text{int}} \quad k \in \{1, \ldots, n\} \tag{25}
\]
Therefore, for industry \( k \), unmet demand causes an inoperability, \( q_k \), measured as the loss of production in industry \( k \) as a proportion of its original production level, as shown in Eq. (4) with \( \frac{\Delta \hat{x}_k}{\hat{x}_k} \). Also, internal consumption failure, as shown in Eq. (22), causes a final consumption perturbation, \( c^*_k \), and is modeled as a measure of the change in the final consumption as a proportion of the original production level in industry \( k \), as shown in Eq. (6) with \( \frac{\Delta \hat{c}_k}{\hat{x}_k} \). The approach to formulate failure in the form of unmet demand is adapted from the port disruption work of Pant et al.\cite{78,79}, and the transportation network vulnerability formulation of Darayi et al.\cite{55}, in which a slack variable \( S^k_{i} \) is defined to capture unsatisfied demand at demand nodes (or undelivered commodities remaining with the suppliers), shown in Eq. (26). For the industries experiencing difficulties only in importing their required commodities, there exists a final consumption perturbation, as modeled in Eq. (27).

\[
\frac{\Delta \hat{c}_k}{\hat{x}_k} = \frac{\sum_{i \in (N^k \cap N^{{\alpha}_k})} S^k_{i} - \Delta \hat{x}_k}{\hat{x}_k} \quad k \in \{1, \ldots, n\} \tag{26}
\]

\[
c^*_k = \frac{\sum_{i \in (N^k \cap N^{{\alpha}_k})} S^k_{i}}{\hat{x}_k} - q_k \quad k \in \{1, \ldots, n\} \tag{27}
\]

Eqs. (24) and (27) combined with the IIM in Eq. (3) form a complete solvable system that quantifies the inoperability and final consumption perturbations for the collection of \( K \) interconnected industries. For simplicity, the demand perturbations in Eqs. (24) and (27) assume failure in either only demand nodes or only supply nodes within a particular industry, whereas in actual situations, some industries would likely consist of both supply and demand nodes. Therefore, the total final consumption perturbation for industry \( k \), in the case of having both importing (demand) and exporting (supply) roles, is given in Eq. (28).

\[
c^*_k = \frac{\sum_{i \in (N^k \cap N^{{\alpha}_k})} S^k_{i}}{\hat{x}_k} + \frac{\sum_{i \in (N^k \cap N^{{\alpha}_k})} S^k_{i}}{\hat{x}_k} - q_k \quad k \in \{1, \ldots, n\} \tag{28}
\]

Any of Eqs. (24), (27), or (28) captures the perturbation vector \( c^* \) that parameterizes the interdependency model in Eq. (3) based on the exporting or importing nature of the nodes belonging to each industry. Thus, \( q \) can then be calculated to measure the proportional extent to which as-planned productivity or functionality is not realized following a transportation network disruption that results in unmet demand or commodities remaining with suppliers, and a contingent rerouting strategy can be devised during the period of disruption to lessen the multi-industry impact of the disruption.

4.3. **STEP 3: PLANNING FOR ADAPTIVE CAPACITY**

Adaptive capacity is considered to be the extent to which a freight transportation network is capable of facilitating economic productivity by the (short-term) rerouting of
commodities through the residual network to reduce remaining commodities at suppliers and unsatisfied demand at consumers. Inoperability in industry $k$ is calculated with Eq. (3), and economic losses for industry $k$ can be found by multiplying the proportional inoperability by expected production level in monetary units, $Q_k = x_k q_k$. Economic losses for the entire set of industries is calculated with $Q = x^T q$. As such, inoperability or economic impact at the industry level, or total economic impact at the across all industries, can be used to valuate strategies for strengthening adaptive capacity. Proposed in Eqs. (29) and (15) are two such metrics motivated by Eq. (1).

When planning emphasis is placed on a particular industry (i.e., rerouting freight in the transportation network to reduce the impact to industry $k$), Eq. (29) is proposed to valuate a strategy to strengthen adaptive capacity. Term $\mathcal{R}_e^k$ is a proportional measure involving (i) the economic loss, $Q_e^k$, experienced by a particular industry $k$ following disruptive event $e$ when no adaptive capacity planning is taken and (ii) the economic loss, $Q_R^k$, in industry $k$ when a strategy is taken to avoid the maximum economic loss in that particular industry.

$$\mathcal{R}_e^k = \frac{Q_e^k - Q_R^k}{Q_e^k} \quad (29)$$

For a perspective that spans all industries, Eq. (30) provides a similar proportional metric, where $Q_e$ is the multi-industry economic loss caused by disruption $e$ in the baseline case, and $Q_R$ is the multi-industry loss when a rerouting strategy is taken to avoid the maximum economic loss.

$$\mathcal{R}_e^R = \frac{Q_e - Q_R}{Q_e} \quad (30)$$

Assuming a multi-industry perspective and considering a hypothetical decision maker interested in limiting economic losses across multiple industries, Eq. (30) serves as the objective function in the following optimization framework that integrates the multi-commodity network flow model from Section 2.1 and the Inoperability Input-Output Model from Section 2.2. Following a particular disruption $e$ that affects a particular set of transportation links, the proposed model in Eqs. (31)-(39) seeks to optimally reroute the flow of commodities through the residual network such that a measure of static economic resilience is minimized. Here, it is assumed that the result of the model provides decision makers with a rerouting strategy across different modes. The period of disruption is assumed to be sufficiently long enough to employ intermodal container scheduling models (e.g., [102],[115]) to devise operational-level plans based on the resulted contingent rerouting strategy in the simplified static supply-demand network.

Notation employed in the problem formulation is summarized as follows, noting that network variables (e.g., the sets of links and nodes) with a prime as superscript are related to the network after disruption, referred to as the residual network.

**Parameters**
Based on this notation, planning for adaptive capacity by rerouting the flow of commodities through the residual network is formulated as follows.

\[
\max \sum_{k} \frac{R_{e}^{k}}{\mu_{k}} \quad \text{s.t.} \quad \sum_{k=1}^{K} f_{ij}^{ik} \leq u_{ij}' \quad \forall (i, j) \in L'
\]

\[
\sum_{(i,j) \in L'} f_{ij}^{ik} - \sum_{(j,i) \in L'} f_{ji}^{ik} + \gamma_i S_i^{k} = b_i^{ik} \quad k = 1, \ldots, K
\]

\[
\gamma_i = \begin{cases} 
-1 & \text{for } i \in N_-
+1 & \text{for } i \in N_+
0 & \text{for } i \in N_0
\end{cases}
\]

\[
c_i^{*} = \frac{\sum_{i \in (N_{+}^{\alpha} \cap N_{k}^{\alpha})} S_i^{k}}{\hat{x}_k} + \frac{\sum_{i \in (N_{-}^{\alpha} \cap N_{k}^{\alpha})} S_i^{k}}{\hat{x}_k} - \mu_{k} q_{k} \quad k = 1, \ldots, K
\]

\[
\frac{1}{M} \sum_{i \in (N_{-}^{\alpha} \cap N_{k}^{\alpha})} S_i^{k} \leq \mu_{k} \leq M \sum_{i \in (N_{+}^{\alpha} \cap N_{k}^{\alpha})} S_i^{k} \quad k = 1, \ldots, K
\]
\[
\begin{bmatrix}
q_1 \\
\vdots \\
q_K
\end{bmatrix}
= 
\begin{bmatrix}
a_{11}^* & \cdots & a_{1K}^* \\
\vdots & \ddots & \vdots \\
a_{K1}^* & \cdots & a_{KK}^*
\end{bmatrix}
\begin{bmatrix}
q_1 \\
\vdots \\
q_K
\end{bmatrix}
+ 
\begin{bmatrix}
c_{1}^* \\
\vdots \\
c_{K}^*
\end{bmatrix}
\] (37)

\[Q_R = \sum_{k=1}^{K} x_k q_k\] (38)

\[f_{ij}^{lk} \geq 0 \ \forall \ (i, j) \in L', k = 1, \ldots, K\]
\[\mu_k \in \{0, 1\}, \ k = 1, \ldots, K\] (39)

The formulation implements the idea of planning for adaptive capacity in a disrupted transportation network where the residual active network is presented by \(G' = (N', L')\), with updated sets of links, \(L'\), and nodes, \(N'\). The bundle constraint in Eq. (32) ties together the commodities by restricting the total flow of all the commodities on each link \((i, j)\) to at most \(u_{ij}'\), the capacity of that particular link after disruption. \(f_{ij}^{lk}\) represents the flow of commodity \(k\) across link \((i, j)\) which remains in the updated set of links, \(L'\). Eq. (33) represents mass balances on each node, where \(b_{i}^{lk}\) captures demand/supply at each node in the residual network. A slack variable \(S_{i}^{lk}\) is defined to capture undelivered commodities remaining with the suppliers, or unsatisfied demand at demand nodes. The magnitude of \(S_{i}^{lk}\) is positive, and multiplier \(\gamma_{i}\) takes on a negative value for set of demand nodes (after disruption) \(N'\), a positive value for supply nodes (after disruption) \(N'_+\), and zero for transshipment nodes (after disruption) \(N'_0\), as shown in Eq. (34). Eqs. (35)-(37) are constraints that translate remaining commodities at supply nodes and unsatisfied demand at demand nodes (in the geographical area of interest, \(\alpha\)) into multi-industry inoperability. Here, \(c_{k}^{*}\) transfers remaining commodities of type \(k\) at the supplier and/or unsatisfied demands, \(S_{i}^{lk}\), into a final consumption reduction from Eq.(28) with respect to the total output of that particular commodity, representing the total output of industry \(k\), \(x_{k}\). Considering \(N_{k}'\) as set of nodes related to industry \(k\) (in the residual network), which either supply or demand commodity \(k\), in Eq. (35), \(q_k\) is added to capture the consequences of unsatisfied demand at nodes within the region on the inoperability of that industry, reasoning that any disruption leading to unsatisfied demands has an impact on the output of that particular industry which needs to be taken care of in the total interdependent inoperability. As the network might connect industries within the region of interest into their suppliers or customers out of the geographical area of interest, it is desired to consider the effect of failure in terms of remaining commodities at suppliers in the region of interest represented by \(N_{k+\alpha}'\), and unmet demand at demand nodes within the region of interest represented by \(N_{-\alpha}'\). A binary coefficient, \(\mu_{k}\), in Eq. (35) takes on value 0 when there are no unsatisfied demands at demand nodes within the region under study and 1 when there is at least one demand node with unsatisfied needs. Eq. (36) requires that \(\mu_{k}\) be binary, defining a sufficiently large \(M\). Eq. (37) implements the IIM to capture the adverse effect of the disruption in terms of remaining commodities at supply nodes and unsatisfied demand at demand nodes. The multi-industry economic impacts of the failure devising a rerouting strategy are captured in Eq. (38) with total economic loss \(Q_{R}\). And the objective function is the proportional economic saving, parametrized based on Eq. (30) in which \(Q_{e}\), maximum economic loss experienced by the whole economy in the case of
a disruption when no mitigating strategy is taken, is already calculated based on Section 4.1. and 4.2. The proposed approach benefits from the flexibility, scalability, and efficiency of the base MCNF paradigm with respect to optimization\cite{58}, as practiced in modeling interdependencies in critical infrastructure networks (e.g., [53]).

4.4. ILLUSTRATIVE EXAMPLE
A multi-modal freight transportation network, consisting of three important interstate highways, railways, and inland waterways that connect to the Mississippi River Navigation System via two ports, plays an important role in transporting commodities produced in the business economic areas within the state of Oklahoma to consumers in neighboring states. A portion of this multi-modal freight transportation network is illustrated on a case study to implement the proposed model to improve adaptive capacity with a post-disruption rerouting strategy. A scenario-based disruption defined as the removal of a particular network component is considered in the illustrative example. Customers in surrounding states are considered to be four combined demand nodes connecting to Oklahoma’s multi-modal freight transportation network. The multi-industry impact of the disruption within the economy of the state of Oklahoma guides the rerouting of commodities throughout the residual network as an adaptive (short-term) strategy. This illustrative network is adapted from Darayi et al.\cite{55} and was also discussed in Section 3.5.

Recall that Figure 4 depicts a supply-demand network considering supply nodes as the three important business economic areas within the state of Oklahoma, consisting of Oklahoma City (node 1), the Port of Catoosa in Tulsa (node 2), and the Port of Muskogee (node 3). Customers (demand nodes) in the most important states interacting with Oklahoma industries are Texas, Louisiana, Arkansas, and Illinois\cite{92}.

The multi-modal freight transportation network, which enables the commodity flows from suppliers within the state of Oklahoma to the out of state consumers, were discussed in brief in Table 1. The network consists of a part of interstate highways I-35, which connects Oklahoma to the north-south corridor, and I-40 and I-44, which enable trade through the east-west corridor. Part of US highways 169 and 165 within Oklahoma connects the Port of Catoosa and the Port of Muskogee to the interstate highway network. In addition to the truck way facilities, an intermodal rail-truck facility in Oklahoma City near the junction of I-35 and I-40, and the one in Tulsa, OK, which run by Burlington Northern Santa Fe (BNSF) railroad are considered in developing the network, as well as part of the inland waterway network navigated by McClellan–Kerr Arkansas River Navigation System which connects the Port of Catoosa and the Port of Muskogee to the Port of New Orleans, LA (node 5), the Port of Chicago, IL (node 7), the Port of Little Rock, AR (node 6), and the Port of Texas City, TX (node 4).

As defined by NAICS, 62 industries operate in Oklahoma, therefore the $A^*$ matrix regionalized for Oklahoma is $62 \times 62$. Six industries are considered to be industries that primarily export commodities to out-of-state customers, listed back in Table 2. Discussed previously, it is assumed that each commodity belongs to an industry as defined by NAICS economic sectors, and each node within the network is considered to
be home to either suppliers or consumers of multiple commodities. A list of monthly supply and demand is presented in Table 8 (assuming constant monthly demand, or annual demand divided by 12).

Table 8. Combined monthly demands/supplies at supply/demand nodes connecting through the network (in tons).

<table>
<thead>
<tr>
<th>Supply nodes in OK</th>
<th>Industry</th>
<th>311</th>
<th>324</th>
<th>325</th>
<th>327</th>
<th>333</th>
<th>339</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oklahoma City</td>
<td></td>
<td>362526</td>
<td>0</td>
<td>300501</td>
<td>183188</td>
<td>23790</td>
<td>118242</td>
</tr>
<tr>
<td>Port of Catoosa</td>
<td></td>
<td>50244</td>
<td>454911</td>
<td>284685</td>
<td>25268</td>
<td>2470</td>
<td>424</td>
</tr>
<tr>
<td>Port of Muskogee</td>
<td></td>
<td>0</td>
<td>33962</td>
<td>0</td>
<td>31886</td>
<td>0</td>
<td>30021</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand nodes outside of OK</th>
<th></th>
<th>311</th>
<th>324</th>
<th>325</th>
<th>327</th>
<th>333</th>
<th>339</th>
</tr>
</thead>
<tbody>
<tr>
<td>TX</td>
<td></td>
<td>97281</td>
<td>316905</td>
<td>204006</td>
<td>0</td>
<td>25838</td>
<td>30154</td>
</tr>
<tr>
<td>LA</td>
<td></td>
<td>50244</td>
<td>18449</td>
<td>0</td>
<td>0</td>
<td>267</td>
<td>0</td>
</tr>
<tr>
<td>AR</td>
<td></td>
<td>265245</td>
<td>153518</td>
<td>381180</td>
<td>41038</td>
<td>156</td>
<td>54494</td>
</tr>
<tr>
<td>IL</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>199304</td>
<td>0</td>
<td>64039</td>
</tr>
</tbody>
</table>

To parametrize the MCNF model in Eq. (17), the cost vector is computed based on the transportation mode and the mileage of the distances between nodes: the per ton-mile for a barge is estimated at $0.97, compared to $2.53 for rail, and $5.35 for trucking\(^{98}\). The monthly capacity of each link is estimated from historical data as a shared constraint for all commodities flowing on the link\(^{99}\), representing the availability of transportation facilities. Assuming that the total supply of commodity \(k\) is equal to the total demand of the same commodity throughout the network, a baseline flow resulted in no remaining commodities at supply nodes and no unsatisfied demand at demand nodes when there is no disruption to the functionality of the network.

In the illustrative example, disruption scenarios are defined as the removal of a single network component at a time. It is assumed that a disruption, or the removal of a particular network component, lasts for a period of one month. Assuming that annual industry production accumulates consistently across the year (i.e., neither production nor interdependency relationships vary day-to-day, week-to-week, month-to-month), a smaller month-long time horizon is considered here as an appropriate proportion of a year to calculate the particular disruptive event cascading effect (e.g., a two-week closure of port facilities\(^{78}\)). Shown in Table 9, three transshipment nodes within the state of Oklahoma, some segments of high volume-freight-traffic interstate highways, some segments of the North America Railroad, a local railroad which connects industrial parks to the North America Railroad, and parts of waterway system, were individually removed from the network to define the disruption scenarios. Focusing on the economy of the state of Oklahoma, and considering supply nodes within the state interacting with demand nodes in surrounding states, undelivered commodities remaining with suppliers or unsatisfied demand at demand nodes, as represented by \(S^k\), affect industry output and result in propagated inoperability through many of the interconnected industries. In the illustrative example, all the supply nodes are within the state of Oklahoma and the four demand nodes are located outside of Oklahoma. Table 9 reports \(\sum_{i \in (N^d_i \cap N^s_k)} S^k_i\), the
sum of the slack (remaining supply) by commodity at the supply nodes when different network components are disrupted, omitting the flow on the disrupted component from the baseline flow within the network. As shown in Table 9, the Petroleum and coal industry (324) is directly vulnerable in all disruption scenarios except for the loss of link (1,7), while the Food and beverage and tobacco industry (311) would be affected only by the loss of link (2,5).

Table 9. Tons of remaining commodities at suppliers with the removal of network components.

<table>
<thead>
<tr>
<th>Removed component</th>
<th>Sum of remaining commodities at supply nodes (tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>311</td>
</tr>
<tr>
<td>Node 9</td>
<td>0</td>
</tr>
<tr>
<td>Node 8</td>
<td>0</td>
</tr>
<tr>
<td>Node 11</td>
<td>0</td>
</tr>
<tr>
<td>Link (1,7)</td>
<td>0</td>
</tr>
<tr>
<td>Link (9,11)</td>
<td>0</td>
</tr>
<tr>
<td>Link (2,5)</td>
<td>50244</td>
</tr>
<tr>
<td>Link (8,4)</td>
<td>0</td>
</tr>
<tr>
<td>Link (2,8)</td>
<td>14793</td>
</tr>
</tbody>
</table>

As all the demand nodes are located outside of Oklahoma, failure in the form of the inability of suppliers to export commodities is modeled as a demand perturbation as calculated in Eq. (29). Other industries within the state will be affected by the interdependent effect of this failure, as captured by \( q^k \) in Eq. (19), representing the extent to which an industry output will not be produced. And the effect of the disruption on the economy of the state is captured by \( Q \), assuming that industries not using the transportation network have not experienced any demand perturbation.

Given the remaining commodities left at supply nodes, shown in Table 9, demand perturbation is calculated with Eq. (21). Resulting industry inoperability, \( q^k \), is provided in Table 10 and depicted in Figure 9. The Petroleum and coal industry (324) is most vulnerable to the removal of the link (2,8), link (2,4), or node 8. The removal of these components also affect the operability of the Nonmetallic minerals industry (327), though to a lesser extent than the removal of link (1,7). The productivity of the Chemical products industry (325) is highly dependent on the connectivity of Tulsa and Oklahoma City through I-44, as represented by link (9,11), as well as transshipment nodes 9 and 11.

Table 10. Industry inoperability across six most important industries within the state of Oklahoma.

<table>
<thead>
<tr>
<th>Removed component</th>
<th>Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>311</td>
</tr>
<tr>
<td>Node 9</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>Node 8</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>Node 11</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>Link (1,7)</td>
<td>0.00E+00</td>
</tr>
</tbody>
</table>
Considering each industry’s production level in monetary value and calculating total impact of the disruption across the state’s industries with Q, Table 11 and Figure 10 provide the supplementary analysis which elaborates the magnitude of loss (in million USD) experienced by different industries regarding the total economic loss. The interconnected nature of the industries within a region affect productivity of the other 56 industries operating in Oklahoma though individually to a much lesser extent than the six industries directly affected. Many industries are vulnerable to any sort of disruption affecting the operability of node 8, the intermodal terminal facilities at the Port of Catoosa, or either of the links connecting it to nodes 2 or 4, the port itself and the state of Texas, respectively. The Petroleum and coal products industry (324) is a high dollar industry in Oklahoma affected the most by the disruption scenarios, though less vulnerable to disruptions that remove links (2,5) or (1,7) from service.
Table 11. Economic losses across the six most important industries within the state of Oklahoma.

<table>
<thead>
<tr>
<th>Removed component</th>
<th>Industry</th>
<th>Total multi-industry impact</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>311</td>
<td>324</td>
</tr>
<tr>
<td>Node 9</td>
<td>0.12</td>
<td>11.04</td>
</tr>
<tr>
<td>Node 8</td>
<td>0.24</td>
<td>146.24</td>
</tr>
<tr>
<td>Node 11</td>
<td>0.04</td>
<td>10.80</td>
</tr>
<tr>
<td>Link (1,7)</td>
<td>0.23</td>
<td>0.92</td>
</tr>
<tr>
<td>Link (9,11)</td>
<td>0.04</td>
<td>10.80</td>
</tr>
<tr>
<td>Link (2,5)</td>
<td>28.04</td>
<td>2.70</td>
</tr>
<tr>
<td>Link (8,4)</td>
<td>0.24</td>
<td>146.20</td>
</tr>
<tr>
<td>Link (2,8)</td>
<td>2.12</td>
<td>146.29</td>
</tr>
</tbody>
</table>

Figure 10. Interdependent economic losses in Oklahoma due to network component removal.

During the month-long period of disruption, the efficacy of contingency rerouting through the residual network is determined according to its reduction in economic productivity of Oklahoma. Respectively, Table 12 and Table 13 report interdependent economic inoperability experienced by the six most important industries in Oklahoma and the consequential multi-industry economic losses following the contingency rerouting strategy devised from the model developed in Eqs. (31)-(39) to minimize $\xi_e$ and $\eta_p$. $\xi_e$ is defined as a measure to lessen the maximum potential drop in the regional economy, lies on $[0,1]$, where $\xi_e = 0$ means that under a disruption scenario $e$, there is no way to avoid the maximum possible loss in the economy of the region by rerouting the supply-demand network, and $\eta_p = 1$ means that under a disruption scenario $e$, it is possible to maintain the full productivity of the regional economy by rerouting commodity flows through the residual network. Comparing the inoperability caused by the removal of the network component with and without devising a contingent rerouting strategy during the period of disruption, shown in Figure 11 and Figure 12 respectively, shows that the
proposed model to plan for adaptive capacity tries to facilitate the trades in high dollar industries like *Petroleum and coal products* (324) and *Miscellaneous manufacturing* (339), while having less impact on *Chemical products* (325) or *Food and beverage and tobacco* (311) industries.

**Table 12. Economic inoperability caused by the disruption after devising a contingent rerouting strategy.**

<table>
<thead>
<tr>
<th>Removed component</th>
<th>311</th>
<th>324</th>
<th>325</th>
<th>327</th>
<th>333</th>
<th>339</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 9</td>
<td>2.00E-04</td>
<td>0.00E+00</td>
<td>4.80E-03</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>Node 8</td>
<td>1.10E-03</td>
<td>7.20E-03</td>
<td>5.00E-03</td>
<td>8.00E-04</td>
<td>1.00E-04</td>
<td>5.00E-04</td>
</tr>
<tr>
<td>Node 11</td>
<td>2.00E-04</td>
<td>0.00E+00</td>
<td>4.70E-03</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>Link (1,7)</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
<td>1.00E-04</td>
<td>7.50E-03</td>
<td>0.00E+00</td>
<td>1.00E-04</td>
</tr>
<tr>
<td>Link (9,11)</td>
<td>2.00E-04</td>
<td>0.00E+00</td>
<td>3.60E-03</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>Link (2,5)</td>
<td>0.00E+00</td>
<td>1.00E-04</td>
<td>1.50E-03</td>
<td>0.00E+00</td>
<td>2.00E-04</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>Link (8,4)</td>
<td>1.10E-03</td>
<td>7.20E-03</td>
<td>5.00E-03</td>
<td>2.00E-04</td>
<td>1.00E-04</td>
<td>5.00E-04</td>
</tr>
<tr>
<td>Link (2,8)</td>
<td>1.50E-03</td>
<td>7.00E-03</td>
<td>5.20E-03</td>
<td>2.00E-04</td>
<td>1.00E-04</td>
<td>5.00E-04</td>
</tr>
</tbody>
</table>

**Figure 11. Economic inoperability caused by the disruption devising a contingent rerouting.**

Figure 11 depicts how contingent rerouting would affect the maximum loss across multiple Oklahoma industries following the removal of the particular components. And, as listed in Table 13, this strategy could lessen the vulnerability of the whole system with respect to the removal of particular components like link (2,5) as part of the inland waterway network. It is also inferred that industries in Oklahoma are most vulnerable to disruptions that cause inoperability in (i) node 8, the intermodal terminal facilitates the movement of commodities in the industrial park of Port of Catoosa to out-of-state customers, (ii) link (8,4), a portion of railroad that connects Oklahoma to Texas City, TX,
or (iii) link (2,8), a local railroad that connects the Port of Catoosa to the North America railroad intermodal terminal, as even rerouting cannot sufficiently enhance the performance the collective industries, as measured by $\mathcal{R}^\mathcal{E}$, by more than 36%. As shown in Table 13, the maximum possible loss resulting from the removal of a network component will be avoided with a contingent rerouting strategy, as in some cases system performance improved up to 85%.

<table>
<thead>
<tr>
<th>Removed component</th>
<th>Industry</th>
<th>Total multi-industry impact</th>
<th>$\mathcal{R}^\mathcal{E}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 9</td>
<td>311</td>
<td>0.85</td>
<td>6.57 0.03 0.05 0.42 3.00</td>
</tr>
<tr>
<td>Node 8</td>
<td>324</td>
<td>5.98</td>
<td>90.15 1.66 0.78 5.17 100.68</td>
</tr>
<tr>
<td>Node 11</td>
<td>325</td>
<td>0.85</td>
<td>90.15 0.40 0.03 0.05 0.41 2.92</td>
</tr>
<tr>
<td>Link (1,7)</td>
<td>327</td>
<td>0.02</td>
<td>0.37 0.11 15.26 0.10 0.58 7.33</td>
</tr>
<tr>
<td>Link (9,11)</td>
<td>333</td>
<td>0.84</td>
<td>4.83 0.02 0.04 0.32 2.36</td>
</tr>
<tr>
<td>Link (2,5)</td>
<td>339</td>
<td>0.02</td>
<td>1.82 2.06 0.02 1.43 0.30 3.28</td>
</tr>
<tr>
<td>Link (8,4)</td>
<td>Others</td>
<td>8.42</td>
<td>87.77 7.09 0.45 0.78 5.21 99.48</td>
</tr>
<tr>
<td>Link (2,8)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 13. Economic losses, in million USD, within the state of Oklahoma after planning for adaptive capacity.

![Figure 12. Total economic loss across all industries in Oklahoma, contingent rerouting versus no action.](image)

As a contingent rerouting strategy is sought considering total economic impact, priorities given to high-dollar industries and those with the highest interdependent impacts across industries. Though Figure 12 shows the absolute benefit of implementing the adaptive capacity planning strategy in the case of different disruption scenarios, there might be cases in which the rerouting strategy results in losses to particular industries. Figure 13
shows how contingent rerouting strategies affect different industries (in the form of box plots generated across the eight disruption scenarios). For example, the rerouting strategies taken following the eight different disruption scenarios would lessen the economic loss in Petroleum and coal products (324) industries by $25.46 million, on average, and at least $0.55 million, in the case of losing link (1.7). Overall, the Chemical products (325) and Food and beverage and tobacco (311) industries are most adversely impacted, as shown in Figure 13, because optimal contingency rerouting tends not to benefit these industries in favor of the larger economy, as shown in Figure 12.

![Figure 13. Effects of contingent rerouting on different industries.](image)

4.5. DISCUSSION

With regard to the three components of resilience capacity identified by Vugrin and Camphouse\cite{16}, most freight transportation network resilience studies focus on pre-disruption prevention investments via absorptive capacity or post-disaster network restoration strategies via restorative capacity. And such is typically done by defining system performance as a measure related to the serviceability of the system (e.g., travel time/distance, flow, throughput) or a topological measure related to the network structure (e.g. centrality, connectivity, betweenness). This work, however, emphasizes adaptive capacity in the form of contingent rerouting strategies to manage the supply-demand network after a disruptive event to lessen the total economic impact.

More specifically, this work proposes an optimization formulation to accommodate the flow through the residual network and maintain the productivity of the economy of the desired region by (i) integrating a multi-commodity network flow model, representing a multi-modal freight transportation network, with a risk-based economic interdependency model, to capture the propagation of the failure in a group of interconnected industries, and (ii) defining a measure of adaptive capacity to valuate rerouting strategies. The formulation provides a means to consider the final role of a freight transportation network as the facilitator within the economy in planning for adaptive capacity after a disruption.
Part of a multi-modal freight transportation network connecting Oklahoma to surrounding states has been considered to develop a stylized case study in which supply nodes are located in the state of Oklahoma and demand nodes are located in surrounding states. We address the efficacy of implementing the adaptive capacity planning formulation in Oklahoma when a scenario-based disruption disables a particular network component for a month. Results suggest a successful avoidance of maximum potential loss in high dollar industries such as Petroleum and coal products (324) and Miscellaneous manufacturing (339), and a consequent static resilience in the economy of the state, as the average maximum loss could be avoided by more than 50%. Though a proportion of the total economic impact has been considered to seek adaptive planning strategies in this study, further work should embed larger social and community impacts in the problem formulation.

This initial formulation can be further improved by accounting for the real-world intermodal container planning considerations and other dynamic issues. Complementary models to plan for system resilience as a function of absorptive and restorative capacity, as well as the adaptive capacity-focused formulation proposed here, could more effectively highlight the tradeoffs among different resilience capacity planning perspectives.

5. CONCLUSIONS
This report discussed new approaches that provide a new perspective on understanding the effects of disruptions to components in a multi-modal transportation network: relating multi-industry and multi-regional impacts to specific components in the network. The vulnerability analysis perspective proposed in this study can be implemented to highlight priorities in maintaining certain network components (to reduce common-cause failure) or in rerouting of commodity flows after a disruption. An adaptive capacity formulation is provided to reduce (multi-industry and multi-regional) vulnerability in the short term after a disruption.

This approach is illustrated with a stylized case study of a multi-modal transportation network in the state of Oklahoma, where supply nodes are located within the state and demand nodes are located outside of the state. Results of the case study suggest that the Petroleum and coal products industry is particularly susceptible to disruptions in several components, and certain components can impact multiple industries. Also, analysis shows that the economy of the state and most industries are primarily vulnerable to the malfunction of the parts of the railway that connect the state to the North America railroad and major trucking corridors including interstate highways I-35, I-44, and I-40. While the application pursued in this study focused primarily on the state of Oklahoma, the base model can be applied to other freight transportation networks to identify the critical nodes/links that can instigate the largest vulnerability across interdependent sectors that uniquely vary from region to region. Hence, the proposed model and its future applications could provide significant value to homeland security preparedness planning.
Further, in implementing the adaptive capacity planning formulation in Oklahoma, results suggest a successful avoidance of maximum potential loss in high dollar industries such as *Petroleum and coal products* (324) and *Miscellaneous manufacturing* (339), and a consequent static resilience in the economy of the state, as the average maximum loss could be avoided by more than 50%. Though a proportion of the total economic impact has been considered to seek adaptive planning strategies in this study, further work should embed larger social and community impacts in the problem formulation.

This kind of analysis enables the ability to plan more effectively for disruptions. This work, *which represents unexplored topics in modeling disruptions in multi-modal transportation*, can be impactful in modeling the efficacy of planning scenarios, including (i) rerouting strategies and engaging new network components in the short-term, and (ii) adding capacity to components in the long-term.
6. REFERENCES


[7] The House Committee on Transportation and Infrastructure, 2013. *Improving the Nation’s Freight Transportation System*.


[99] Oklahoma Department of Transportation. 2013. *Freight and goods movement*.


