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16. Abstract

In the transit business, planners often face a difficult decision when having to choose what type of operating policy to put in place in a given service area. In fact, the decision is not straightforward, mainly because the demand for the service is often unknown beforehand and it will depend on the established system itself. This is especially true for feeder lines, one of the most often used types of flexible transit services connecting a service area to a major transit network through a transfer point. They often switch operations from/to a demand responsive to/from a fixed-route policy. In designing and operating such systems, the identification of the condition justifying the operating switch is often hard to properly evaluate.

In this research, we propose an analytical modeling and solution of the problem to assist decision makers and operators in their choice. By employing continuous approximations, we derive handy but powerful closed-form expression to estimate the critical demand densities, representing the switching point between the competing operating policies.

Based on the results of one-vehicle and two-vehicle operations for various scenarios and their comparison to simulation generated values, we verify the validity of our analytical modeling approach. Estimated critical demand densities for the one-vehicle case and a service area with L=2 and W=0.5 range from 14 to 30 customers/hr/mile^2.

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Performance Assessment and Comparison between Fixed and Flexible Transit Services for Different Urban Settings and Demand Distributions

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Research Report SWUTC/08/473700-00090-1

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ABSTRACT

In the transit business, planners often face a difficult decision when having to choose what type of operating policy to put in place in a given service area. In fact, the decision is not straightforward, mainly because the demand for the service is often unknown beforehand and it will depend on the established system itself. This is especially true for feeder lines, one of the most often used types of flexible transit services connecting a service area to a major transit network through a transfer point. They often switch operations from/to a demand responsive to/from a fixed-route policy. In designing and operating such systems, the identification of the condition justifying the operating switch is often hard to properly evaluate.

In this research, we propose an analytical modeling and solution of the problem to assist decision makers and operators in their choice. By employing continuous approximations, we derive handy but powerful closed-form expression to estimate the critical demand densities, representing the switching point between the competing operating policies.

Based on the results of one-vehicle and two-vehicle operations for various scenarios and their comparison to simulation generated values, we verify the validity of our analytical modeling approach. Estimated critical demand densities for the one-vehicle case and a service area with L=2 and W=0.5 range from 14 to 30 customers/hr/mile^2.

EXECUTIVE SUMMARY

Traditionally, transit services have been divided in two broad categories: fixed route (FRT) and demand responsive (DRT). The typical cost efficiency of FRT system is due to the predetermined schedule, the large loading capacity of the vehicles and the consolidation of many passenger trips onto a single vehicle (ridesharing). However, the general public considers them to be more and more inconvenient because of their lack of flexibility. DRT systems instead provide much of the desired flexibility with a door to door type of service, but they are generally much more costly to deploy and, therefore, largely limited to specialized operations such as taxicabs, shuttle vans or dial a ride services, other than paratransit services.

In the last decades, modern urban areas, especially within residential communities, are experiencing a steady decrease in their population density as a consequence of urban sprawl, one of the most evident phenomena of our time. This increasing "dispersion" of population causes conventional fixed-route transit systems serving those areas to become progressively more inefficient and relegated to a marginal role, since they are designed to serve few established routes and they heavily rely on concentrated demand. Therefore, an increasingly larger portion of the growing population relies almost exclusively on private automobiles for their transportation needs, causing modern urban areas to suffer from severe congestion and pollution problems. Hence, transit agencies are facing a growing demand for improved and extended services.

The broad and fairly new category of "flexible" transit services includes all types of hybrid services that combine pure demand responsive and fixed route features. Among the most used ones are feeder lines, also know as Demand Responsive Connector (DRC), connecting residential communities to a major transit network. Most of them switch their operations between a demand responsive and a fixed-route policy, depending on the demand. When designing and operating such systems, planners need to decide what type of operations, between FRT or DRC, would be the most appropriate and/or what conditions would justify a "switch" from FRT to DRC (or vice versa). The identification of the condition justifying the operating switch is often hard to properly evaluate.

In this research we propose an analytical modeling and solution of the problem to assist transit agencies' decision makers and operators in their choice. Utility functions for the fixed-route and demand responsive operating policy are derived and equalized to determine the critical demand density, representing the condition for the switch. For the one-vehicle case we derived closed-form expressions, function of the parameters of each scenario, such as the geometry of the service area, the vehicle speed and especially the weights assigned to each term contributing to the utility function: walking time, waiting time and riding time. Weights' assessments are left to the decision makers, which might select them depending on the circumstances and the changing conditions of each scenario.

Analytical results compared to simulation outcomes show a good match and a validation of our methodological approach. Estimated critical demand densities for the one-vehicle case and a service area with 2-mile length and 0.5-mile width range from 14 to 30 customers/hr/mile^2 slightly underestimating the simulated values, as predicted, however, by our approximation

procedure. Similar results are obtained for the two-vehicle case. We also performed sensitivities over different L/W ratios, different overall sizes of the service area and over different demand distributions.

With this research we suggest and encourage transit planners to employ this methodological approach in selecting the proper operating policy for feeders. In addition we provide them with a handy but powerful approximate closed-form analytical expression to estimate the critical demand density, which would justify the switch from/to one operating policy to/from the other, for a large range of possible scenarios. Because of the increasing interest in this kind of services and the growing need of properly identify ways to improve their performance, we feel that this paper provides a significant contribution to the transportation field.

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CHAPTER 1. INTRODUCTION

In the last decades, modern urban areas, especially within residential communities, are experiencing a steady decrease in their population density as a consequence of urban sprawl, one of the most evident phenomena of our time. In the US, from 1960 to 2000, the density dropped 15% despite an average overall population growth of 86% (www.demographia.com). In the majority of the rest of the world this trend is even more evident. This increasing "dispersion" of population causes conventional fixed-route transit systems serving those areas to become progressively more inefficient and relegated to a marginal role, since they are designed to serve few established routes and they heavily rely on concentrated demand.

Traditionally, transit services have been divided in two broad categories: fixed-route (FRT) and demand responsive (DRT). The typical cost efficiency of FRT system is due to the predetermined schedule, the large loading capacity of the vehicles and the consolidation of many passenger trips onto a single vehicle (ridesharing). However, the general public considers them to be more and more inconvenient because of their lack of flexibility, since either the locations of pick-up and/or drop-off points or the service's schedule do not match the individual rider's desires. Therefore, an increasingly larger portion of the growing population relies almost exclusively on private automobiles for their transportation needs, causing modern urban areas to suffer from severe congestion and pollution problems.

DRT systems instead provide much of the desired flexibility with a door-to-door type of service, but they are generally much more costly to deploy and, therefore, largely limited to specialized operations such as taxicabs, shuttle vans or dial-a-ride services, other than paratransit services (mandated under the ADA). Hence, transit agencies are facing a growing demand for improved and extended services.

The broad and fairly new category of "flexible" transit services includes all types of hybrid services that combine pure demand responsive and fixed-route features. These services have established stop locations and/or established schedules, combined with some degree of demand responsive operation. Their characteristics have, in several cases, efficiently responded to some of the needs and wants of both the customers and the transit agency as well. However, their use has been quite limited in practice so far, as opposed to regular FRT systems.

The Demand Responsive Connector (DRC), also know as "feeder" transit line, is one of them. A survey conducted by Koffman (2004) for a Transit Cooperative Research Program (TCRP) project found that the DRC has been operated in quite a few cities and is one of the most often used types of flexible transit service, especially within low density residential areas. The service operates in a demand responsive fashion within a service area and move passengers from/to a transfer point that connects to a major fixed-route transit network (see Figure 4), thus closing the gap perceived as the most critical by the majority of the potential transit users.

In most cases, the service operates as a FRT service during daytime and switches to a DRC type of service during evenings, nights or early morning, when the demand is lower. When designing and operating such systems, planners need to decide what type of operations, between

FRT or DRC, would be the most appropriate and/or what conditions would justify a "switch" from FRT to DRC (or vice versa). The decision is not straightforward, mainly because the demand for the service is often unknown beforehand and it will depend on the established service itself. In addition, even assuming a known demand, it is not clear what the best type of service would be. This is because the service quality provided to customers is not easy to assess and might depend on external conditions, such as safety, weather, time of the day; plus, the balance between operating costs and service quality is also frequently hard to evaluate.

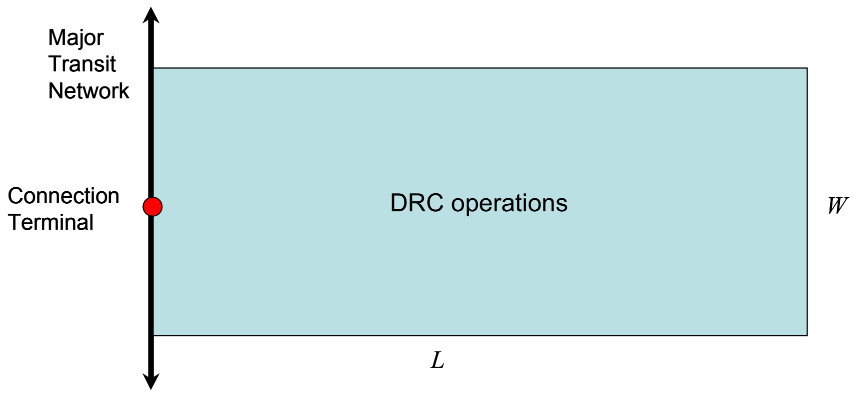

Figure 4 - Demand Responsive Connector (DRC)

With the ultimate goal of improving the efficiency and performance of this type of services, we present a methodology to assist decision makers in their choice by providing an analytical modeling and solution of the problem, with the use of continuous approximations. As noted by Daganzo (1991), the main purpose of this type of approach is to obtain reasonable solutions with as little information as possible. Hall (1986) also pointed out that these approximate models are easier for humans to comprehend; we would add that they may provide handy but powerful tools to help solving many complicated decision problems. In particular, in this research, we develop relationships to assess the service quality of the two competing operating policies (FRT and DRC) and derive the "critical demand densities", representing the point where the two services could reasonably be considered equivalent and where a switch from one type of service to the other would be desirable.

CHAPTER 2. LITERAURE REVIEW

Flexible transit services merge the flexibility of demand responsive transit systems with the low cost operability of fixed-route systems. Koffman's survey (Koffman, 2004) study shows that flexible transit services have been used in several cities. We here present a review of the work performed on them.

Research on flexible transit services is quite limited and in particular, to our knowledge, there is no research performed on decision methodologies to select the best operating practice for feeders like the DRC. A specific work on the DRC itself, which is the focus of our research, has been conducted by Cayford and Yim (2004). Authors surveyed the customers' demand for DRC for the city of Millbrae. They also designed and implemented an automated system used for the DRC services. The service uses an automated phone-in-system for reservations, computerized dispatching over a wireless communication channel to the bus driver and an automated callback system for customer notifications.

Some flexible transit services involve checkpoints. Daganzo (1984) describes a flexible system in which the pick-up and drop-off points are concentrated at centralized locations called checkpoints. A related system has been investigated by Quadrifoglio. The Mobility Allowance Shuttle Transit (MAST) system allows buses to deviate from the fixed path so that customers within the service area may be picked up or dropped off at their desired locations. According to Koffman (2004), this type of service is also often used and is also known as "Route Deviation". Quadrifoglio et al. (2006) developed bounds on the maximum longitudinal velocity to evaluate the performance and help the design of MAST services by employing continuous approximations. Quadrifoglio et al. (2007) developed an insertion heuristic for scheduling MAST services by using control parameters, which properly regulate the consumption of the slack time. Finally, Quadrifoglio et al. (2008b) formulated the scheduling of the MAST services as a mixed integer programming with added logic constraints. Experiments showed that the developed inequalities achieved 90% reduction of the CPU time for some instances.

Some other examples of work on flexible systems are the following ones. Cortés and Jayakrishnan (2002) proposed and simulated one type of flexible transit called High-Coverage Point-to-Point Transit (HCPPT), which requires the availability of a large number of transit vehicles. Pagès et al. (2006) identified the problem called real-time mass transport vehicle routing problem and developed a global solution algorithm. The mass transport network design problem was formulated and solved by the developed algorithm. Aldaihani et al. (2004) developed an analytical model that aids decision makers in designing a hybrid grid network that integrates a flexible demand responsive service with a fixed-route service. Their model is to determine the optimal number of zones in an area, where each zone is served by a number of on-demand vehicles. Khattak and Yim (2004) explored the demand for a consumer oriented personalized DRT (PDRT) service in the San Francisco Bay Area. About 60% of those surveyed were willing to consider PDRT as an option, about 12% reported that they were ''very likely'' to use PDRT. Many were willing to pay for the service and highly valued the flexibility in scheduling the service.

Although research on the DRC and flexible transit services is quite limited, the purely DRT systems have been extensively investigated. Savelsbergh and Sol (1995), Desaulniers et al. (2000) and Cordeau and Laporte (2003) provide comprehensive reviews on the proposed methodologies and solutions to deal with these very difficult problems. We summarize a few more papers describing different issues and problem solving approaches to the purely demand responsive services.

Sandlin and Anderson (2004) presented a procedure for calculating a serviceability index (SI) for DRT operators based on regional socioeconomic conditions and internal operation data. The SI can be used to evaluate and compare DRT operation. Palmer et al. (2004) studied the DRT system consisting of dial-a-ride programs that transit agencies use for point to point pick up and delivery of the elderly and handicapped. Their results of a nationwide survey involving 62 transit agencies show that the use of paratransit computer aided dispatching (CAD) system and agency service delivery provide a productivity benefit. Diana et al. (2006) studied the problem of determining the number of vehicles needed to provide a DRT service with a predetermined quality for the user in terms of waiting time at the stops and maximum allowed detour. Quadrifoglio et al. (2008a) used simulation methods to investigate the effect of using a zoning vs. a no zoning strategy and time window settings on performance measures such as total trip miles, deadhead miles and fleet size. They identified quasi linear relationships between the performance measures and the independent variable, either the time-window size or the zoning policy. Dessouky et al. (2003) demonstrated through simulation that it is possible to reduce environmental impact substantially, while increasing operating costs and service delays only slightly for the joint optimization of cost, service, and life cycle environmental consequences in vehicle routing and scheduling of a DRT system.

In this research we aim to investigate and establish the conditions which would justify the implementation of a demand responsive operating policy for the feeder transit services as opposed to a traditional fixed route one. To our knowledge, this work is the first to develop a methodology for solving this problem. In this research, we also utilize continuous approximations as part of our methodology. There is a significant body of work in the literature on continuous approximation models for transportation systems. Most of the work has been developed to provide decision support tools for strategic planning in the design process. Langevin et al. (1996) provide a detailed overview of the research performed in the field. They concentrate primarily on freight distribution systems, while in this paper we focus on public transport; but most of the issues of interest are common to both fields. Szplett (1984) provides a review of the research performed on continuous models specifically for public transport.

CHAPTER 3. SYSTEM DESCRIPTION

3.1 Service Area and Demand

The service area is a representation of a residential community located on the side of and connected to a main road where the major fixed-route transit service network would be in service and is modeled as a rectangle of width W and length L (see Figure 4). The terminal connecting to the major fixed-route transit network is located in the middle (at width $W/2$) on the left edge of the area. The temporal distribution of the demand is assumed to be a Poisson process with exponentially distributed interarrival times and average rate λ. We assume that α% of the customers need to be transferred from the service area to the connection terminal ("pick-up" customers) and (1-α)% of them vice versa ("drop-off" customers). The customers' location, either for a pick-up or for a drop-off, has a uniform distribution within the service area. While assuming a temporal Poisson distribution for pick-up customers is very realistic, the drop-off customers would instead reasonably show up in groups according to the arrival of the vehicles serving the outside FRT network. However, with the additional assumption that the number of transit lines passing by the connection terminal is high enough and/or the headways between vehicles are low enough, a Poisson distribution for the arrivals is still a reasonable assumption. The analysis performed in this paper can be updated and refined in future research with a more refined temporal distribution for the customers' arrivals, but we would not expect a substantial alteration of our results.

3.2 Competing transit policies

We consider two competing operating policies (FRT and DRC) of the transit service. For each one of them we will analyze the one-vehicle case and the two-vehicle case. In all our considered scenarios we assume an average speed of the vehicles of v_b miles/hr and a dwelling time at each stop of s sec. We also assume that the same type of vehicle(s) is used in all cases.

FRT Policy

The FRT operating policy offers continuous service with the vehicle moving back and forth along the route between stop 1 (the connection terminal) and stop N located in the middle of the service area (see

Figure 5). There are N-2 stations between 1 and N and the distance between adjacent stations is a constant d miles. The pick-up customers show up at random within the service area, walk to the nearest station and wait for the bus. The drop-off customers show up and wait at the terminal, take a ride and then walk to their final destination at random within the service area.

In the one-vehicle case, there is only a single bus performing the operations. In the two-vehicle case we assume that the two buses begin their operations at the same time leaving from stop 1 and N respectively. At any point in time during the operations, the vehicle moving left-to-right performs the drop-off operations (transferring customers from terminal 1 to their stops closest to their final destination) and the vehicle moving right-to-left performs the pick-up operations (transferring customers from their stops closest to their final destination to terminal 1).

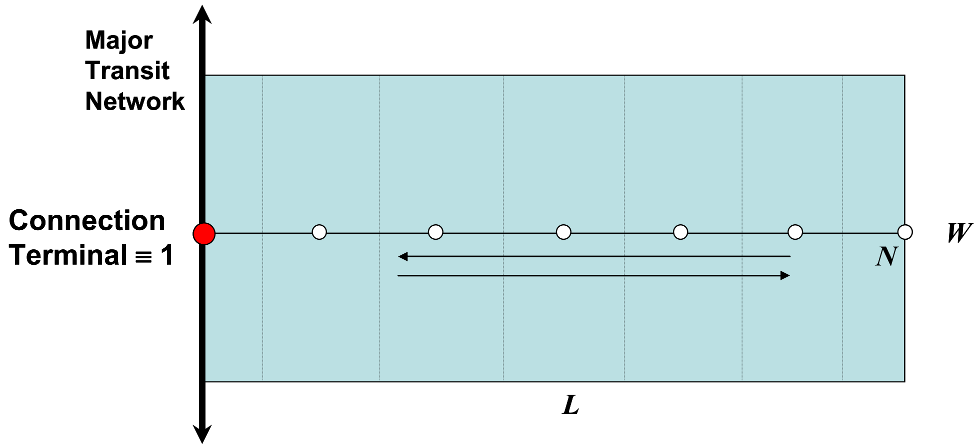

Figure 5 - FRT Policy

DRC Policy

The DRC policy provides a shared-ride demand responsive terminal-to-door (and vice versa) service to customers, by picking them up and dropping them off at their desired locations. The vehicle begins and ends each of its trips from the terminal. We assume that pick-up customers are able to notify their presence by means of a phone or internet booking service. Immediately before the beginning of each trip, waiting customers (both pick-up and drop-off ones) are scheduled and the route for the trip in the service area is constructed. There is no planned idle time in between trips. To schedule the requests we assume that the schedule is calculated by an insertion algorithm attempting to minimize the total distance traveled by the vehicle. An insertion heuristic approach is adopted because they are widely used in practice to solve transportation scheduling problems, as they often provide very good solutions compared to optimality, they are computationally fast and they can easily handle complicating constraints (Campbell and Savelsbergh, 2004). Rectilinear movements are assumed and often chosen instead of Euclidean ones, since they better estimate distances traveled in real road networks and generally provide good approximations (see Quadrifoglio et al., 2008a).

For the two-vehicle case, as assumed for the FRT policy, one bus performs the pick-up operations (many-to-one) and the other bus performs the drop-off operations (one-to-many). For both vehicles, an insertion heuristic algorithm is again adopted to perform the scheduling task. Both vehicles begin service at the same time and perform continuous operations with no planned idle time in between trips.

3.3 Performance Measures

The performance of a transit system can roughly be considered as a combination of operating costs and service quality. The relative weights to be assign to each one of those two categories is a disputed matter and can differ between public transportation agencies and privately owned ones. However, in this paper, we may assume the operating costs to be equivalent for the two competing transit services. The assumption is reasonable in our comparisons, because the vehicle is assumed to be the same and run continuously during the operations for both service policies at the same average speed v_b and the demand served is also the same. Other than possible negligible differences, we do not see a major disparity of the operating costs between the two cases which would cause our assumption to be unreasonable.

Thus, the comparison between the two services can be performed by considering only the service quality provided to customers. If we disregard other possible sources of noise that could influence customers' perceptions and opinions, the service quality can be considered as a combination of the following performance measures:

- $E(T_{wk})$: expected value of walking time of the passengers needed to/from their closest bus stop from/to their destination.
- $E(T_{wt})$: expected value of waiting time of the passengers from their ready time to their pick-up time (subtracting the possible walking time).
- $E(T_{rd})$: expected value of ride time of the passengers from pick-up to drop-off.

Generally, needed transfers between vehicles to complete a trip are a major service quality factor as well, but there are none in this case. Thus, the service quality provided to customers is represented by the utility function U defined as the weighed sum of the above terms:

$$U = w_{wk}\times E(T_{wk}) + w_{wt}\times E(T_{wt}) + w_{rd}\times E(T_{rd}). \qquad (1)$$

Lower values of U indicate a better level of service. The assessments of the weights (w_{wk}, w_{wt}, and w_{rd}) are generally difficult to make, they are dependent upon several factors, they are not unique for all cases and they can change dynamically depending on the circumstances. For example: the walking time could be considered more or less acceptable (thus, with a different relative weight), depending on the safety or the weather conditions of a certain area and/or the profile of the customers. However, the weight assignment is not the scope of this paper. We wish to provide decision makers with tools which will help them decide the proper service policy, once they have selected the proper weights for their scenario. A more detailed discussion for the weights can be found in two recent studies, Wardman (2004) and Guo and Wilson (2004).

In the next chapters we will focus on the analytical computation of U for both competing policies, so we can compare them and select the one with the lower value.

CHAPTER 4. ANALYTICAL MODELING OF ONE-VEHICLE CASE

4.1 FRT

In this section we calculate the expected values of the three performance measures $E(T_{wk})$, $E(T_{wt})$, $E(T_{rd})$ for the one-vehicle scenario when a FRT operating policy is adopted.

Assuming that customers would walk to the nearest bus stop with a rectilinear path, the expected value of the walking time $E(T_{wk})$ is

$$E\left(T_{wk}\right)=\frac{1}{4v_{wk}}\left(\frac{L}{N-1}+W\right), \tag{2}$$

where v_{wk} is the average walking speed and N is number of FRT bus stations, including the connection terminal 1 (see
Figure 5).

Since the bus dwelling time at each station is s, the cycle time of the journey beginning at terminal 1 and back is

$$C=\frac{2L}{v_b}+2\left(N-1\right)s\,. \tag{3}$$

The derivation of the expected values for the waiting time and riding time depends upon the relationship between the values assumed for the weights w_{wt} and w_{rd}. As mentioned, our scope is not to assess the weights, but to provide analytical tools given their assumed values.

A $w_{wt} < w_{rd}$ (case 1) would mean that customers would spend their time waiting rather than being on the vehicle. This is a reasonable assumption if the waiting location is a comfortable one, like at home or at nicely built connection terminal. The expected value of the waiting time for pick-up customers, drop-off customers and all customers are:

$$E\left(T_{wt-1}^{p}\right)=\left[\frac{1}{2}-\frac{1}{4\left(N-1\right)}\right]C\,, \tag{4}$$

$$E\left(T_{wt-1}^{d}\right)=\left[\frac{1}{2}-\frac{1}{4\left(N-1\right)}\right]C\,, \tag{5}$$

$$E\left(T_{wt-1}\right)=\alpha E\left(T_{wt-1}^{p}\right)+\left(1-\alpha\right)E\left(T_{wt-1}^{d}\right)=\left[1-\frac{1}{2\left(N-1\right)}\right]\left[\frac{L}{v_b}+\left(N-1\right)s\right]. \tag{6}$$

The expected value of the ride time for pick-up customers, drop-off customers and all customers are instead:

$$E\left(T_{rd-1}^{p}\right)=\frac{C}{4}, \tag{7}$$

$$E\left(T_{rd-1}^{d}\right)=\frac{C}{4}, \tag{8}$$

$$E\left(T_{rd-1}\right)=\alpha E\left(T_{rd-1}^{p}\right)+(1-\alpha)E\left(T_{rd-1}^{d}\right)=\frac{1}{2}\left[\frac{L}{v_b}+(N-1)s\right]. \tag{9}$$

A $w_{wt} > w_{rd}$ (case 2) would instead mean that customers would spend their time onboard rather than waiting. This could be the case when most of the waiting occurs at possibly unsafe locations, maybe at night and/or with adverse weather conditions. Equations (4) to (9) are then recalculated by employing conditional probability (for brevity we skip the mathematical passages, which can be found in the appendix):

$$E\left(T_{wt-2}^{p}\right)=\left[\frac{1}{3}-\frac{1}{4(N-1)}+\frac{1}{6(N-1)^2}\right]C, \tag{4a}$$

$$E\left(T_{wt-2}^{d}\right)=E\left(T_{wt-1}^{d}\right)=\left[\frac{1}{2}-\frac{1}{4(N-1)}\right]C, \tag{5a}$$

$$\begin{aligned} E\left(T_{wt-2}\right)&=\alpha E\left(T_{wt-2}^{p}\right)+(1-\alpha)E\left(T_{wt-2}^{d}\right)=\\ &=\left[\frac{\alpha}{3}\left(\frac{1}{(N-1)^2}-1\right)-\frac{1}{2(N-1)}+1\right]\left[\frac{L}{v_b}+(N-1)s\right], \end{aligned} \tag{6a}$$

$$E\left(T_{rd-2}^{p}\right)=\left[\frac{5}{12}-\frac{1}{6(N-1)^2}\right]C, \tag{7a}$$

$$E\left(T_{rd-2}^{d}\right)=E\left(T_{rd-1}^{d}\right)=\frac{C}{4}, \tag{8a}$$

$$E\left(T_{rd-2}\right)=\alpha E\left(T_{rd-2}^{p}\right)+(1-\alpha)E\left(T_{rd-2}^{d}\right)=\left[\frac{\alpha}{3}\left(1-\frac{1}{(N-1)^2}\right)+\frac{1}{2}\right]\left[\frac{L}{v_b}+(N-1)s\right]. \tag{9a}$$

4.2 DRC

The calculation of the expected values of the performance measures for the demand responsive operating policy is not straightforward, due to the fact that at each cycle, the vehicle performs a different tour, to serve the demand uniformly but randomly distributed across the service area. However, it is possible to provide good estimates by following a methodology similar to the one adopted in Quadrifoglio et al. (2006). In this paper, authors proved that the distance traveled by a vehicle traveling along a corridor to serve uniformly distributed demand scheduled with an insertion heuristic algorithm (attempting to minimize the total distance traveled) is upper bounded and closely approximated (especially for lower densities) by the distance traveled by the vehicle following a rectilinear "no-backtracking policy", which forbids backwards movements with respect to the current forward direction and, therefore, serves the customers in order of their horizontal coordinate. In our case, since the vehicle is performing a cycle to/from the terminal stop 1, we assume that a half of the customers are served first in a no-backtracking policy left-to-right and the remaining half are served in a no-backtracking policy right-to-left.

Let n be the number of customers served per cycle by the DRC vehicle. Since their spatial distribution is assumed to be uniform, if x_i is the horizontal coordinate within the service area ($0 \le x_i \le$ L) of customer i (with i = 1, …, n), the expected value of the maximum horizontal distance that the vehicle will need to travel can be derived as follows:

$$E\left[\max\left(x_i\right)\middle|i=1,\ldots,n\right]=\int_0^L\left\{P\left[\max\left(x_i\right)\middle|i=1,\ldots,n\right]\ge t\right\}dt=$$
$$=\int_0^L\left\{1-P\left[\max\left(x_i\right)\middle|i=1,\ldots,n\right]\le t\right\}dt=\int_0^L\left\{1-\prod_{i=1}^n\left[P\left(x_i\right)\le t\right]\right\}dt= \tag{10}$$
$$=\int_0^L\left(1-t^n\right)dt=L\frac{n}{n+1}.$$

If we denote with y and y' the random variables indicating the vertical distance between any pair of customers uniformly distributed within the service area and the vertical distance between the terminal 1 (located at W/2) and the first and last customer in the schedule (see Figure 6), it is known that:

$$E\left(y\right)=\frac{W}{3}, \tag{11}$$

$$E\left(y'\right)=\frac{W}{4}. \tag{12}$$

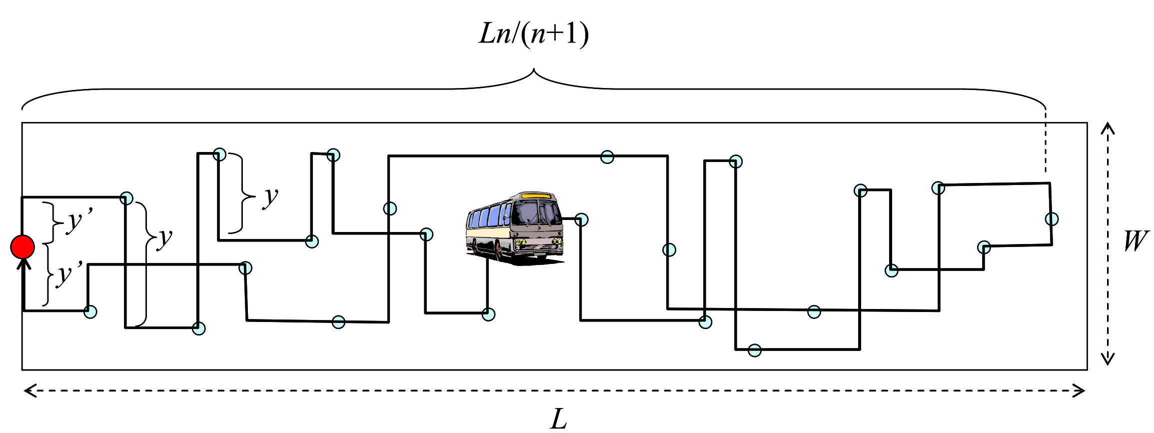

Figure 6 – No-backtracking policy

Thus, if D represents the total rectilinear distance per cycle for a no-backtracking policy, C is the cycle time and λ is the average customer demand (in customers/hr), the following relationships hold:

$$D = 2L\frac{n}{n+1} + 2\frac{W}{4} + (n-1)\frac{W}{3} = 2L\frac{n}{n+1} + \frac{W}{2} + (n-1)\frac{W}{3}, \tag{13}$$

$$C = \frac{D}{v_b} + (n-1)s, \tag{14}$$

$$n = \lambda C. \tag{15}$$

Drop-off customers will need to wait an average of $E\left(T_{wt}^{d}\right) = C/2$, since they will show up and wait at the connection terminal 1 uniformly from time 0 to C of the previous cycle. They will also ride an average of $E\left(T_{rd}^{d}\right) = C/2$, since they can be dropped off uniformly anytime from time 0 to C of their cycle.

Pick-up customers will instead need to wait $E\left(T_{wt}^{p}\right) = C/2 + C/2 = C$, since they will wait an average of $C/2$ from their show up time to the end of the previous cycle and an additional average of $C/2$, waiting for the vehicle to reach them. They will also ride an average of $E\left(T_{rd}^{p}\right) = C/2$, as the drop-off customers.

Thus, the expected values of the total waiting time and riding time are

$$E\left(T_{wt}\right) = \alpha E\left(T_{wt}^{p}\right) + (1-\alpha)E\left(T_{wt}^{d}\right) = (1+\alpha)\frac{C}{2}, \tag{16}$$

$$E\left(T_{rd}\right) = \alpha E\left(T_{rd}^{p}\right) + (1-\alpha)E\left(T_{rd}^{d}\right) = \frac{C}{2}. \tag{17}$$

In order to derive C, we need to solve the system of equations composed by (13), (14) and (15). In doing so we obtain the following quadratic equation:

$$aC^2 + bC + c = 0, \tag{18}$$

where:

$$a = \lambda\left[\lambda\left(\frac{W}{3} + sv_b\right) - v_b\right], \tag{19}$$

$$b = \lambda\left(\frac{W}{2} + 2L + 2sv_b\right) - v_b, \tag{20}$$

$$c = \frac{W}{6} + sv_b. \tag{21}$$

Two obvious conditions should be satisfied: $C > 0$ and $b^2 - 4ac \geq 0$. However, a closed-form expression for C is not easy to derive.

On the other hand, in Equation (13) we could reasonably assume that

$$\frac{n}{n+1} \cong 1, \tag{22}$$

thus overestimating D by a factor of $\frac{2L}{n+1}$, which becomes increasingly negligible with increasing n and becomes zero for $n \to \infty$. The approximate cycle time $\tilde{C}$ so obtained would be an upper bound of the actual cycle time C and thus still an upper bound of the actual cycle time obtainable by an insertion heuristic. After rearranging (13) with the above approximation and combining it with (14) and (15), we are able to obtain a closed-form expression for the approximate cycle time

$$\tilde{C} = \frac{sv_b + W/6 + 2L}{v_b - \lambda\left(W/3 + sv_b\right)}. \tag{23}$$

The approximate values $E\left(\tilde{T}_{wt}\right)$ and $E\left(\tilde{T}_{rd}\right)$ can be obtained by substituting C with $\tilde{C}$ in (16) and (17). $E(T_{wk})$ and $E\left(\tilde{T}_{wk}\right)$ are zero, since the DRC offer a door-to-door service and no walking is necessary.

4.3 Critical demand

For case 1 ($w_{wt} < w_{rd}$), we obtain the utility function for the FRT policy by substituting (2), (6) and (9) in (1); similarly, by substituting (16) and (17) in (1) we obtain the utility function for the DRC policy. We can now equate these two expressions and solve for λ. The resulting value λ_c represents the critical demand rate at which the two services would be equivalent in terms of service quality provided to customers.

C does not have a closed-form expression and so does not λ_c, but solutions can be obtained with numerical methods. However, if we use $\tilde{C}$, a closed-form expression for the approximation of λ_c can be derived and is

$$\tilde{\lambda}_c = \frac{1}{\frac{W}{3v_b} + s} - \frac{\left[(1+\alpha)w_{wt} + w_{rd}\right]\left[\frac{6v_b s + W + 12L}{4\left(3v_b s + W\right)}\right]}{\frac{w_{wk}}{4v_{wk}}\left(\frac{L}{N-1} + W\right) + \left[\frac{L}{v_b} + s\left(N-1\right)\right]\left(\frac{w_{rd}}{2} + \frac{w_{wt}}{\frac{2N-2}{2N-3}}\right)}. \tag{24}$$

An analogous equation is similarly calculated for case 2 ($w_{wt} > w_{rd}$) and is:

$$\tilde{\lambda}_c = \frac{1}{\frac{W}{3v_b}+s} - \frac{\left[(1+\alpha)w_{wt}+w_{rd}\right]\left[\frac{6v_b s + W + 12L}{4(3v_b s + W)}\right]}{\frac{w_{wk}}{4v_{wk}}\left(\frac{L}{N-1}+W\right)+\left[\frac{L}{v_b}+s(N-1)\right]\left[\frac{\alpha(w_{wt}-w_{rd})}{\frac{3(N-1)^2}{N(2-N)}}+\frac{w_{rd}}{2}+\frac{w_{wt}}{\frac{2N-2}{2N-3}}\right]}. \quad (24a)$$

Finally, the critical demand density (customers/hr/mile^2) is defined as

$$\rho_c = \frac{\lambda_c}{WL}, \quad (25)$$

and its approximation is $\tilde{\rho}_c = \frac{\tilde{\lambda}_c}{WL}$.

CHAPTER 5. ANALYTICAL MODELING OF TWO-VEHICLE CASE

5.1 FRT

For the two-vehicle FRT case, the expected value of customer walking time $E(T_{wk})$ is the same as the one-vehicle case and represented by Equation (2).

Assuming that the two vehicles have the same average speed v_b, the 1st vehicle starts from the terminal 1, and the 2nd vehicle starts from the bus station at stop N. The cycle time is still represented by Equation (3).

For this two-vehicle case we will reasonably assume that pick-up customers will always wait for the first bus going right-to-left. The expected value of the waiting time is $C/4$ for all customers, except customers walking directly to/from the terminal 1 (which are $\frac{1}{2(N-1)}$ in proportion to the total, on average) and would not ride the bus nor spend time waiting. Thus the expected value of the waiting time for pick-up customers, drop-off customers and all customers are

$$E\left(T_{wt}^{p}\right)=\left[1-\frac{1}{2(N-1)}\right]\frac{C}{4}, \tag{26}$$

$$E\left(T_{wt}^{d}\right)=\left[1-\frac{1}{2(N-1)}\right]\frac{C}{4}, \tag{27}$$

$$E\left(T_{wt}\right)=\alpha E\left(T_{wt}^{p}\right)+(1-\alpha)E\left(T_{wt}^{d}\right)=\left[\frac{1}{2}-\frac{1}{4(N-1)}\right]\frac{L}{v_b}+\left(N-\frac{3}{2}\right)\frac{s}{2}. \tag{28}$$

The expected value of the riding time for pick-up customers, drop-off customers and all customers are

$$E\left(T_{rd}^{p}\right)=\frac{C}{4}, \tag{29}$$

$$E\left(T_{rd}^{d}\right)=\frac{C}{4}, \tag{30}$$

$$E\left(T_{rd}\right)=\alpha E\left(T_{rd}^{p}\right)+(1-\alpha)E\left(T_{rd}^{d}\right)=\frac{C}{4}=\frac{L}{2v_b}+(N-1)\frac{s}{2}. \tag{31}$$

5.2 DRC

Let $\lambda_p = \alpha\lambda$ and $\lambda_d = (1-\alpha)\lambda$ be the demand rates of pick-up and drop-off customers respectively. The two vehicles are assumed to perform independent operations, one serving the pick-up customers (many-to-one) and the other one serving the drop-off customers (one-to-many), both following the same insertion scheduling policy adopted for the one-vehicle case. Average cycle times C_p and C_d for no-backtracking policies can be calculated with Equation (13), (14) and (15), replacing λ with λ_p and λ_d respectively. As for the one-vehicle case, a quadratic equation needs to be solved and closed-form expressions for C_p and C_d are not easy to derive. However, $n_p = \lambda_p C_p = \alpha\lambda C_p$ and $n_d = \lambda_d C_d = (1-\alpha)\lambda C_d$ are the expected number of customers for each cycle, respectively for the pick-up vehicle and the drop-off vehicle, and by adopting the same approximation assumed for the one-vehicle case, that is $\frac{n_p}{n_p+1} \cong 1$ and $\frac{n_d}{n_d+1} \cong 1$, we are able again to obtain approximate but closed-form expressions for C_p and C_d:

$$\tilde{C}_p = \frac{sv_b + W/6 + 2L}{v_b - \alpha\lambda\left(W/3 + sv_b\right)}, \tag{32}$$

$$\tilde{C}_d = \frac{sv_b + W/6 + 2L}{v_b - (1-\alpha)\lambda\left(W/3 + sv_b\right)}. \tag{33}$$

The expected values of customer waiting and riding time are

$$E\left(T_{wt}\right) = \alpha C_p + \left(1-\alpha\right)\frac{C_d}{2}, \tag{34}$$

$$E\left(T_{rd}\right) = \alpha\frac{C_p}{2} + \left(1-\alpha\right)\frac{C_d}{2}. \tag{35}$$

The approximate values $E\left(\tilde{T}_{wt}\right)$ and $E\left(\tilde{T}_{rd}\right)$ can be obtained by substituting C_p and C_d with $\tilde{C}_p$ and $\tilde{C}_d$ in (34) and (35).

5.3 Critical demand

By substituting (2), (28) and (31) in (1) we obtain the utility function for the 2-vehicle FRT policy; similarly, by substituting (34) and (35) in (1) we obtain the utility function for the 2-vehicle DRC policy. We can now equate the two expressions and solve for λ. The resulting value λ_c represents the critical demand rate at which the two services would be equivalent in terms of service quality provided to customers.

Unfortunately, there is no closed-form expression for λ_c for both the rigorous and approximate cases, but numerical and graphical solutions can be obtained.

CHAPTER 6. SIMULATION DEVELOPMENT

The analytical modeling of DRC in the previous two chapters uses the approximation of non-backtracking policy. Without this approximation the analytical derivation of the terms of the utility function for the DRC performance is very difficult because of the embedded vehicle routing problem. We use a simulation model to replicate the operations of the insertion heuristic algorithm. We also simulate the FRT operations to verify the analytical formulas.

The insertion heuristic algorithm is described below.

Let C_1, C_2, C_3,…, C_n, denote n customers and $T_1 < T_2 < T_3 < \ldots < T_n$ be their show-up times. The insertion algorithm creates the customer sequence choosing the minimum additional distance at each insertion step in an $O(n^2)$ fashion, as follows:

(1) Insert C_1: AC_1A is the only possible route.

(2) Insert C_2: Possible routes include AC_2C_1A, and AC_1C_2A; find the route R_2 with the minimum DRC running distance among the two possible routes. Suppose R_2 is Route AC_1C_2A.

(3) Insert C_3: Possible routes include $AC_3C_1C_2A$, $AC_1C_3C_2A$, and $AC_1C_2C_3A$; find the route R_3 with the minimum DRC running distance among the three possible routes.

(4) …

(n) Insert C_n: Suppose the route R_{n-1} is generated by inserting C_{n-1}; insert C_n to the route R_{n-1}; find the route R_n with the minimum DRC running distance among the n possible routes.

The algorithm complexity is polynomial $O(n^2)$, which can be solved almost instantaneously by any modern PC. It is not the scope of this report to provide an assessment of our heuristic, which could be done by comparing its performance against optimality, obtained by solving the related Vehicle Routing Problem, notoriously a NP-Hard problem. An example of this kind of appraisal for insertion algorithms can be found in Quadrifoglio et al. (2007).

The simulation was developed with MATLAB (The MathWorks, Inc., 2007) software on a Pentium® 4 2.80GHz CPU and 1.00GB RAM. We performed 30 replications of 100 DRC cycles, scheduling 2,000 customers each. While the actual operations will not last for so long, the long simulation time is needed to generate stable values of the means of the output parameters. The simulations include one-vehicle and two-vehicle operations of FRT and DRC. The means of the output parameters (customer walking time, customer ride time and customer waiting time) will be used in the result analysis in the next chapter.

CHAPTER 7. RESULT ANALYSIS

In this chapter we provide numerical results to validate our analytical modeling (rigorous and approximate) vs. simulation.

7.1 Values of Parameters

To represent a residential area, the values of the parameters assumed for our analyses are as follows:

- FRT bus station distance $d = 0.25$ mile.
- pedestrian walking speed $v_{wk} = 2$ mile/hr.
- bus running speed $v_b = 20$ mile/hr.
- bus dwell time at each station or customer location $s = 30$ second.
- The service area $L \times W = 1$ mile^2. However, we considered three different L/W ratios: with the length L equal to 4, 2, 1 mile and the width W to 0.25, 0.5, 1 mile respectively.
- We considered a range of different customer demand densities: from 0 up to 90 customers/mile^2/hr.
- We assume $\alpha = 0.5$, meaning that 50% of the demand are pick-up customers and 50% are drop-off customers (sensitivity results over the α value can be found in Section 7.5 by simulation of the One-Vehicle Case with 20 mile^2 service area).
- We assume the weights $w_{rd} = 1$ and $w_{wt} = 2$ (case 1); we also analyze the results for $w_{rd} = 2$ and $w_{wt} = 1$ (case 2). As mentioned, the value of w_{wk} is the most susceptible to variation, due to weather and changing safety conditions; therefore, we consider $w_{wk} = 3$ as a "base case", but we also perform sensitivity analyses.

7.2 One-Vehicle Case for L=2/W=0.5

We calculated the utility function values for the FRT policy using Equations (1) for different demand densities and four different values for w_{wk} (2, 3, 4 and 5). To compute the three terms in (1), Equations (2), (6) and (9) have been used for case 1 ($w_{wt} = 1 < w_{rd} = 2$) and Equations (2), (6a) and (9a) for case 2 ($w_{wt} = 2 > w_{rd} = 1$). Simulation runs were also performed to validate our FRT analytical modeling, which is perfectly matched and will not be explicitly shown here for brevity.

We calculated the utility function values for the no-backtracking DRC policy using Equation (1) for different demand densities. The rigorous analytical values of the three terms in (1) were computed with Equations (16) and (17) and by solving Equation (18) by numerical methods. The approximate analytical values of the three terms in (1) were instead computed with Equations (16), (17) and (23). To compare and evaluate our analytical results we performed simulation runs where the DRC vehicle serves the demand following a schedule calculated with an insertion heuristic algorithm attempting to minimize the vehicle's total travel distance in each cycle.

Figure 7 and Figure 8 graphically show the computed utility function values.

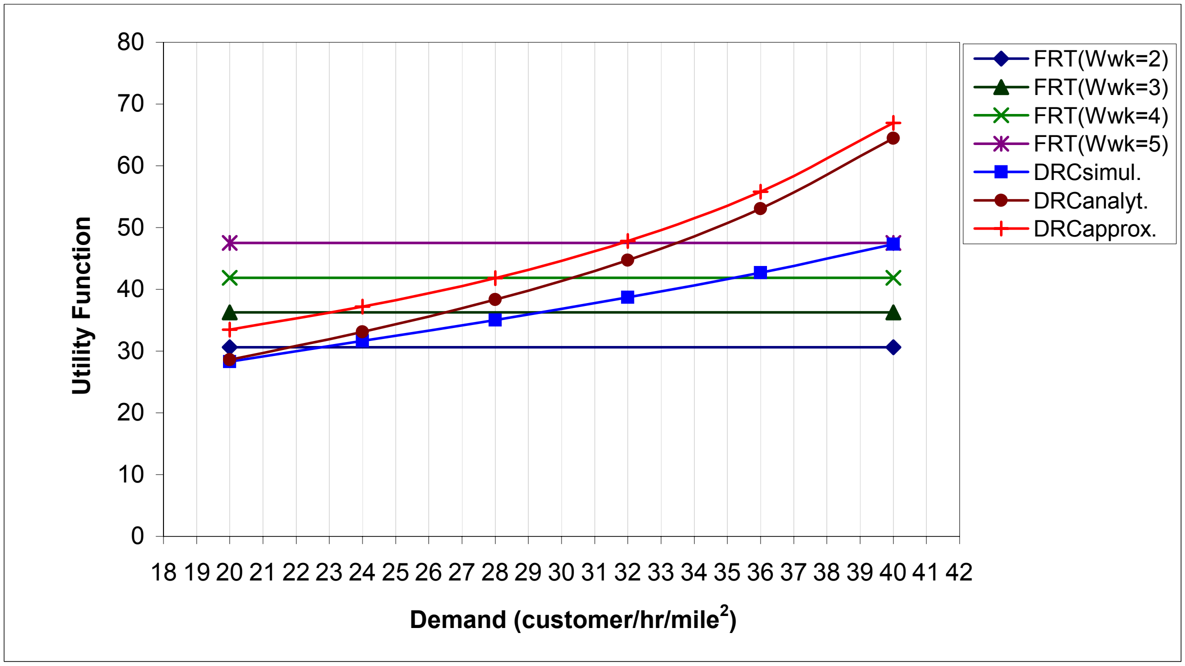

Figure 7 - Utility Functions for the One-Vehicle Case with *L*=2, *W*=0.5, w_{wt}=1 and w_{rd}=2

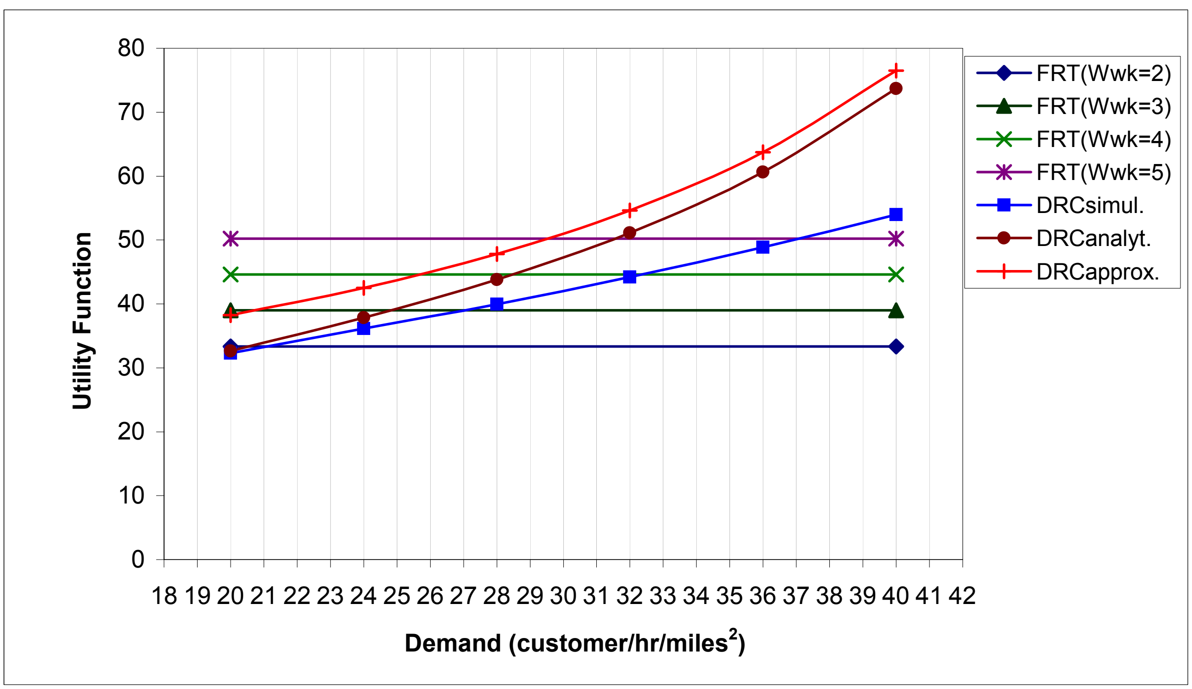

Figure 8 - Utility Functions for the One-Vehicle Case with *L*=2, *W*=0.5, w_{wt}=2 and w_{rd}=1

FRT utility functions have four different flat values as the weight w_{wk} changes from 2 to 5, since they do not depend on the demand. While DRC utility functions (rigorous analytical, approximate analytical and simulation) increase with the demand and do not depend on w_{wk} since there is no walking. While we did not assume any capacity constraint in developing our methodology, in all our simulated cases we observed a maximum loading capacity of 15 passengers within our considered range of demand densities. Thus, all our scenarios could have been performed comfortably by a 20-seat van (for example). Clearly, for higher demand densities, capacity constraints must be taken in consideration, as well as alternative scheduling policies, especially for the DRC.

From the above chart the following observation can be made with regards to the DRC curves.

- The rigorous analytical values are upper bounds for the corresponding simulated values. This is expected, since the no-backtracking policy provides an upper bound of the insertion heuristic algorithm in terms of the distance traveled and consequently in terms of the utility function as well. However, the error is reasonably small (in the range of 1%-30% for the considered scenarios) and gets smaller with lower demand densities, as also expected, confirming the good approximations provided by the no-backtracking policy.
- The approximate analytical values are an upper bound for the corresponding rigorous values, since our approximate models overestimate the total distance traveled and the gap gets smaller with increasing demand densities, as expected, because of assumption (22).
- In general, the three curves are fairly close to each other, especially for lower densities, which would allow using the developed approximate but handy analytical formula to estimate the actual utility function values.

The intersections between the DRC curves and the FRT curves represent the critical demand densities at which the FRT policy and DRC policy have the same utility function values and thus equal performance. For demand densities lower than the critical one, the DRC would be the preferred choice and vice versa. Equation (24) and (24a) provide closed-form expression for these critical demand densities for the approximate case ($n \cong n+1$). The critical demand densities are listed in Table 4 and shown in Figure 9.

Table 4 - Critical customer demands (cust/hr/mile^2) for *L*=2, *W*=0.5; One-Vehicle Case

Weights	Case	$w_{wk} = 2$	$w_{wk} = 3$	$w_{wk} = 4$	$w_{wk} = 5$
w_{wt}=1 w_{rd}=2	Simulation	22.8	29.2	35.2	40
	Analytical	21.9	26.5	30.3	33.5
	Approx.	16.3	23.1	28	31.8
w_{wt}=2 w_{rd}=1	Simulation	21.1	26.9	32.4	37.1
	Analytical	20.1	24.8	28.5	31.6
	Approx.	14.1	20.8	25.7	29.5

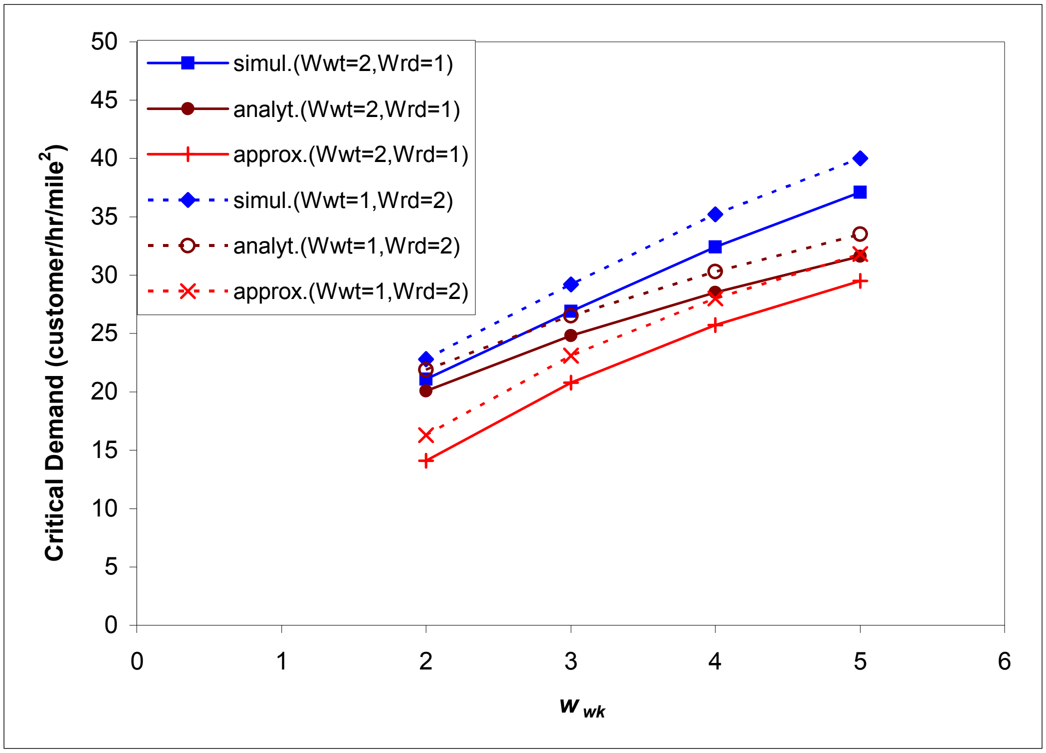

Figure 9 – Critical Demand Densities for *L*=2, *W*=0.5; One-Vehicle Case

The above results show that approximate analytical values for the critical demand densities underestimate the rigorous analytical and simulated ones. This would mean that the critical "switching point" from DRC to FRT predicted by Equations (24) and (24a) would be slightly anticipated with increasing demand (and vice versa).

As an illustrative example, consider the scenario where estimated values for the weights are $w_{wk} = 4$, $w_{wt} = 1$, $w_{rd} = 2$. The approximate value of the critical demand density given by Equation (24) is 28 customers/hr/mile^2. As soon as the demand is expected to drop below this value a switch from a FRT to DRC operating policy would be desirable to maximize the service quality provided to customers. While this procedure clearly has intrinsic approximations built in it, it certainly provides a good justifiable estimate and is better than guessing.

7.3 Effect of *L*/*W* Ratio

In addition to the *L*=2, *W*=0.5 scenario, we produced the critical customer demand densities, shown in Table 5 and Figure 10, for *L*=4, *W*=0.25 and *L*=1, *W*=1 scenarios to analyze the effects of various *L*/*W* ratios. Here we show case 2 ($w_{wt} = 2 > w_{rd} = 1$); however, case 1 is comparable.

Table 5 - Critical demand densities for various *L/W* ratios (One-Vehicle Case, w_{wt}=2, w_{rd}=1)

L/W	Case	w_{wk} =2	w_{wk} =3	w_{wk} =4	w_{wk} =5
1/1	Simulation	26.5	34.9	41.1	45.8
	Analytical	23.3	26.8	29.1	30.8
	Approx.	20.8	25.5	28.3	30.2
2/0.5	Simulation	21.1	26.9	32.4	37.1
	Analytical	20.1	24.8	28.5	31.6
	Approx.	14.1	20.8	25.7	29.5
4/0.25	Simulation	15	18.2	21.2	24.4
	Analytical	15	17.8	20.5	23
	Approx.	5.6	10.5	14.8	18.6

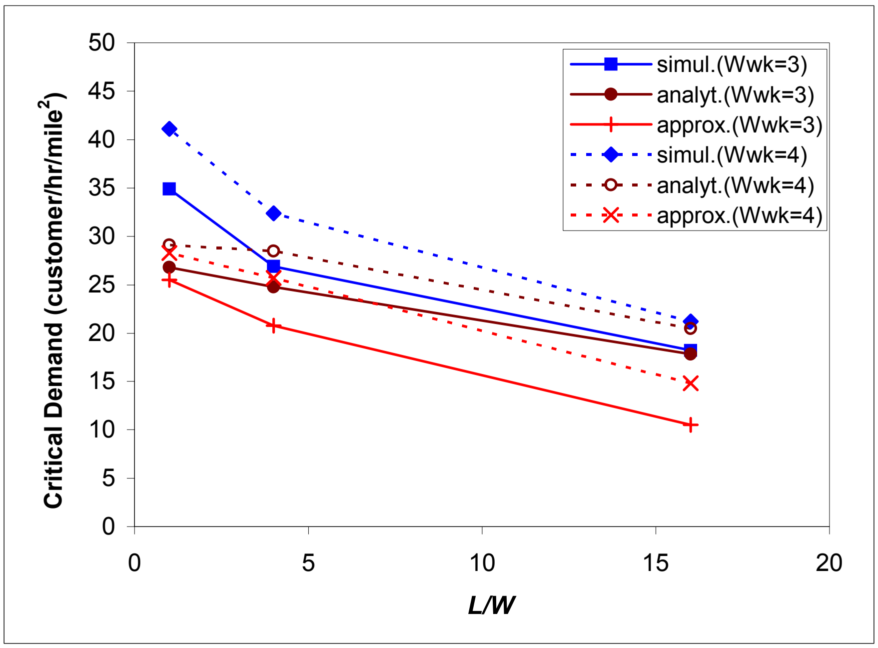

Figure 10 – Effects of Various *L/W* Ratios

The critical demand densities decrease with the increase of *L*/*W* ratio. We note that for larger *L*/*W* ratio and lower w_{wk} value, such as *L*=4, *W*=0.25 and w_{wk}=2, the approximated values may have significant difference from the simulated one. However, for such scenario the analytical critical demand is very close to the simulated one. Therefore for this situation (large *L*/*W* ratio) the analytical formulas should be adopted instead of the approximation. We also note that for lower *L*/*W* ratio and larger w_{wk} value, such as *L*=1, *W*=1 and w_{wk}=5, the approximated and analytical critical demands are very close but the difference from the simulated one is 34%. This is reasonable because the no-backtracking policy is not as effective for low *L*/*W* ratio (such as 1) as for large *L*/*W* ratio.

7.4 Two-Vehicle Case

We briefly present the results obtained for the two-vehicle case. For the FRT policy the utility function values were computed with Equations (1), (28) and (31). For the DRC policy the rigorous analytical values of utility function were computed with Equations (1) and (34) and (35), deriving the cycle times C_p and C_d with numerical techniques; the approximate values were computed with Equations (1) and (32)-(35). As for the one-vehicle case, we developed simulations to compute the utility function values for two-vehicle DRC policy. Figure 11 show the computed utility function values for L=2, w_{wt}=2 and w_{rd}=1 (case 2).

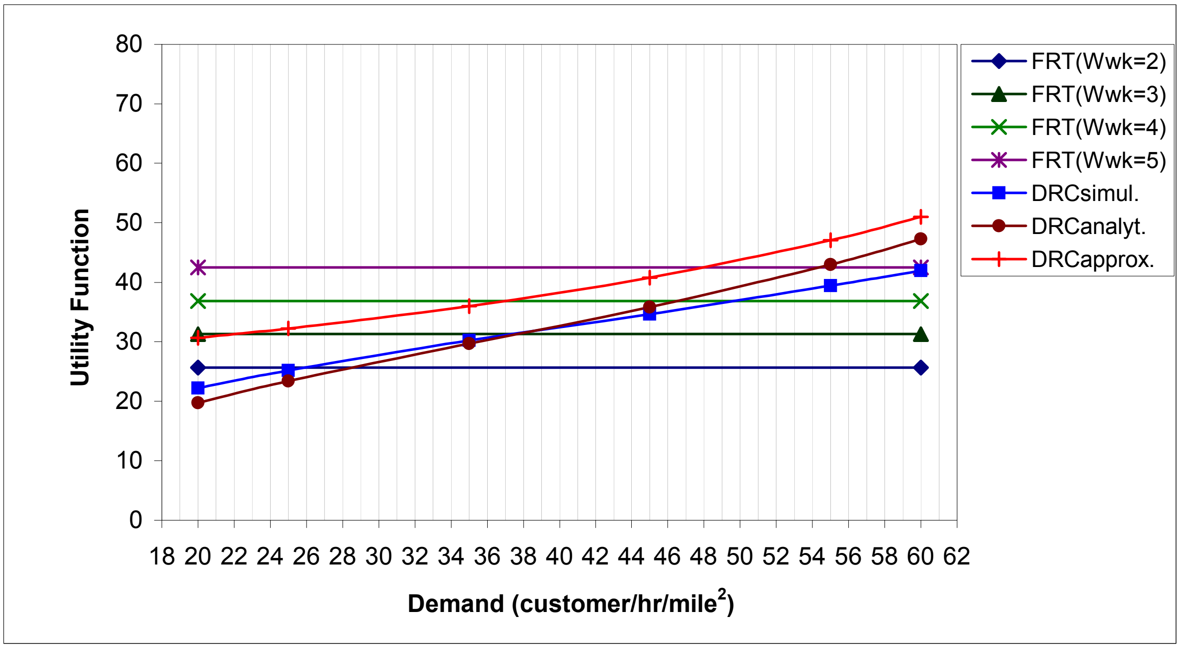

Figure 11 - Values of Utility Function for *L*=2, *W*=0.5; Two-Vehicle Case

As for the one-vehicle case, the approximate values provide an upper bound to the analytical and simulated values. Note that at demand density of about 37 customer/hr/mile^2, the analytical and the simulated curves cross. The cross points between the DRC curves and the FRT curves show the critical demand densities which are listed in Table 6.

Table 6 - Critical demand densities for *L*=2, *W*=0.5; Two-Vehicle Case

Case	w_{wk}=2	w_{wk}=3	w_{wk}=4	w_{wk}=5
Simulation	26	37.6	49.8	60.2
Analytical	28.4	37.7	46.8	54.4
Approx.	0.6	22.1	37	48

While the approximate values for higher values of w_{wk} provide reasonable estimates, the approximate value for w_{wk}=2 is too small and not valid. This is because the assumptions $n_p \cong n_p + 1$ and $n_d \cong n_d + 1$ are less acceptable for low demands. However, we note that the approximated formula have less significance for the two-vehicle case, since they do not have handy closed-from expressions as for the one-vehicle case. Thus, we recommend considering only the rigorous analytical formula for decision making purposes, also in light of the fact noting that their estimates are remarkably close to the simulated ones, validating our analytical modeling.

7.5 Effects of α Values with Simulation of One-Vehicle Case

In this section we present the simulation results of one-vehicle case for various α values. The parameter values we used are different from the sections above and may represent a larger service area such as a rural community:

- FRT bus station distance d =1 mile
- pedestrian walking speed v_{wk} =2 miles/hr
- bus running speed v_b =30 miles/hour
- bus dwell time at each station or customer location s =30 sec
- The service area is $L \times W$ = 20 mile^2. However, we considered three different W/L ratios: with the length L equal to 5, 10, 20 miles and the width W to 4, 2, 1 miles respectively.
- We consider α = 0.5 as a "base case", meaning that 50% of the demand are pick-up customers and 50% are drop-off customers. The number of pick-up customers may be not equal to the number of drop-off customers, such as in morning or afternoon peak hours. We investigate the effects of various α values.
- On the basis of two recent studies, Wardman (2004) and Guo and Wilson (2004), we assume w_{rd} = 1 and w_{wt} = 2. As mentioned, the value of w_{wk} is the most susceptible to variation, due to weather and changing safety conditions; therefore, we consider w_{wk} = 3 as a "base case", but we also perform sensitivity analyses.

With Equations (2), (6a) and (9a), the FRT performances are calculated. The results with α = 0.5 are listed in Table 1.

Table 4 - Analytical Results of FRT performance for 20 mile^2 area; One-Vehicle Case

MOP (min)	L=5 miles W=4 miles	L=10 miles W=2 miles	L=20 miles W=1 mile
E(T_{wk})	37.50	22.50	15
E(T_{wt})	9.25	19.625	40.4375
E(T_{rd})	8.25	16.625	33.3125

For the designed scenarios, the simulations generated performances for both FRT and DRC. With α = 0.5, E(T_{wk}), E(T_{wt}) and E(T_{rd}) are listed in Tables 5, 6 and 7. E(T_{wk}) for DRC is zero, since it serves customers at their desired locations.

Table 5 - Simulation Results for L=5 miles, W=4 miles and α = 0.5; One-Vehicle Case

MOP (min) \ Demand (cust/ml^2/hr)		0.3	0.375	0.5	0.75	1	1.2	1.5	1.7
FRT	E(T_{wk})	37.48	37.54	37.50	37.44	37.54	37.47	37.47	37.43
	E(T_{wt})	9.19	9.20	9.21	9.24	9.27	9.26	9.28	9.25
	E(T_{rd})	8.24	8.29	8.29	8.28	8.29	8.23	8.24	8.27
DRC	E(T_{wt})	18.82	21.58	25.54	34.26	42.83	53.89	70.53	83.93
	E(T_{rd})	12.87	14.21	16.49	22.50	28.83	35.72	46.59	56.05

Table 6 - Simulation Results for L=10 miles, W=2 miles and α = 0.5; One-Vehicle Case

MOP (min) \ Demand (cust/ml^2/hr)		0.3	0.375	0.5	0.75	1	1.2	1.5
FRT	E(T_{wk})	22.51	22.53	22.45	22.48	22.46	22.49	22.50
	E(T_{wt})	19.58	19.64	19.57	19.61	19.59	19.58	19.60
	E(T_{rd})	16.59	16.58	16.67	16.68	16.61	16.68	16.57
DRC	E(T_{wt})	29.87	33.26	36.83	43.75	52.37	59.37	74.35
	E(T_{rd})	19.76	21.94	24.43	28.73	34.97	39.08	49.62

Table 7 - Simulation Results for L=20 miles, W=1 mile and α = 0.5; One-Vehicle Case

MOP (min) \ Demand (cust/ml^2/hr)		0.15	0.3	0.375	0.5	0.75	1	1.2	1.5
FRT	E(T_{wk})	15.00	15.05	14.99	15.04	15.00	14.97	15.00	15.02
	E(T_{wt})	40.44	40.48	40.31	40.57	40.38	40.32	40.56	40.45
	E(T_{rd})	33.31	33.26	33.30	33.34	33.44	33.37	33.16	33.37
DRC	E(T_{wt})	48.65	60.77	62.71	68.05	75.79	84.64	90.57	103.84
	E(T_{rd})	32.57	39.09	41.10	44.49	49.57	56.11	60.46	69.61

From Tables 4-7, we can make the following observations:

- By comparing Table 4 with Tables 5, 6 and 7, it is possible to verify the validity of the analytical values obtained by Equations with the simulation results.
- The MOPs of FRT are independent of the demand. This is expected because the bus capacity is assumed to be sufficiently large not to be a binding constraint. We verified that the maximum passenger load in any segment, considering all the performed simulations, does not exceed the value 60. However, in most cases much smaller capacity vehicles are needed to serve all the demand. For brevity, we are not

providing the maximum required capacity for each case, since this is not the primary scope of this research.

- For FRT, $E(T_{wk})$ decreases with the decrease of service area width W. This is also expected, since narrower service areas would result in shorter walking distances to the closest stop.
- For FRT, $E(T_{wt})$ and $E(T_{rd})$ increase with the increase of service area length L, because the FRT cycle time increases.
- For DRC, $E(T_{wt})$ and $E(T_{rd})$ increase with the increase of customer demand, since the DRC trip time is proportional to the number of customers served each time.
- For DRC, $E(T_{wt})$ and $E(T_{rd})$ increase with the increase of L, since a narrower area is less compact, leading to longer trips.

Combining the MPOs with the assumed weights, it is possible to calculate the utility function U for both services and identify the best type of service for each scenario. While U for FRT is independent of the demand, the U for DRC is a monotonic increasing function of the demand. It is already established that lower demand densities are more suitable for DRC types of services and vice versa, but the identification of a "switching point" between the two services is not always so obvious. Thus, we label the customer demand as "critical" when the FRT and DRC have the same utility function value. For demands lower than the critical demand, the DRC service is better than the FRT service. The FRT service is better than the DRC service for demands greater than the critical demand.

Figures 4, 5 and 6 show the utility function values of FRT and DRC for various demands and w_1 with $\alpha = 0.5$. Critical demands are drawn from these figures, and listed in Table 5, where we also list them for other values for α. For example, in Figure 4, the DRC curve intersects the FRT ($w_1 = 5$) curve at the demand value 0.87 customer/mile^2/hour which is the critical demand. These critical demands in Table 5 are quantitative references for planners to make the decision of feeding with a FRT or DRC operating policy.

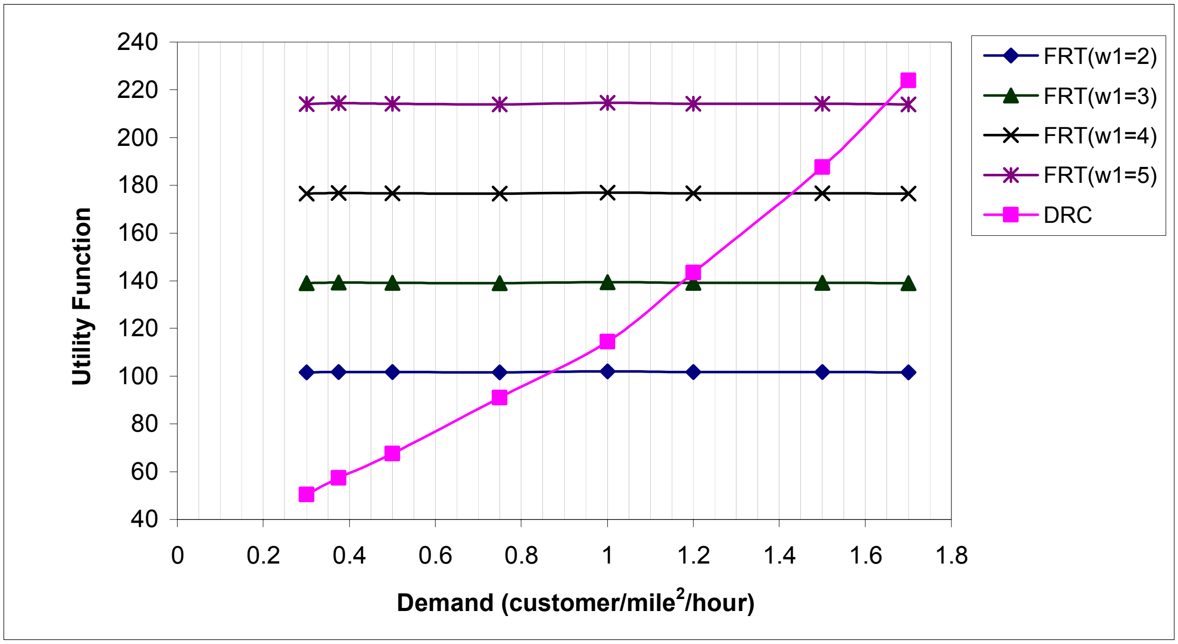

Figure 9 - Utility function for the One-Vehicle Case with L = 5, W = 4 and α = 0.5

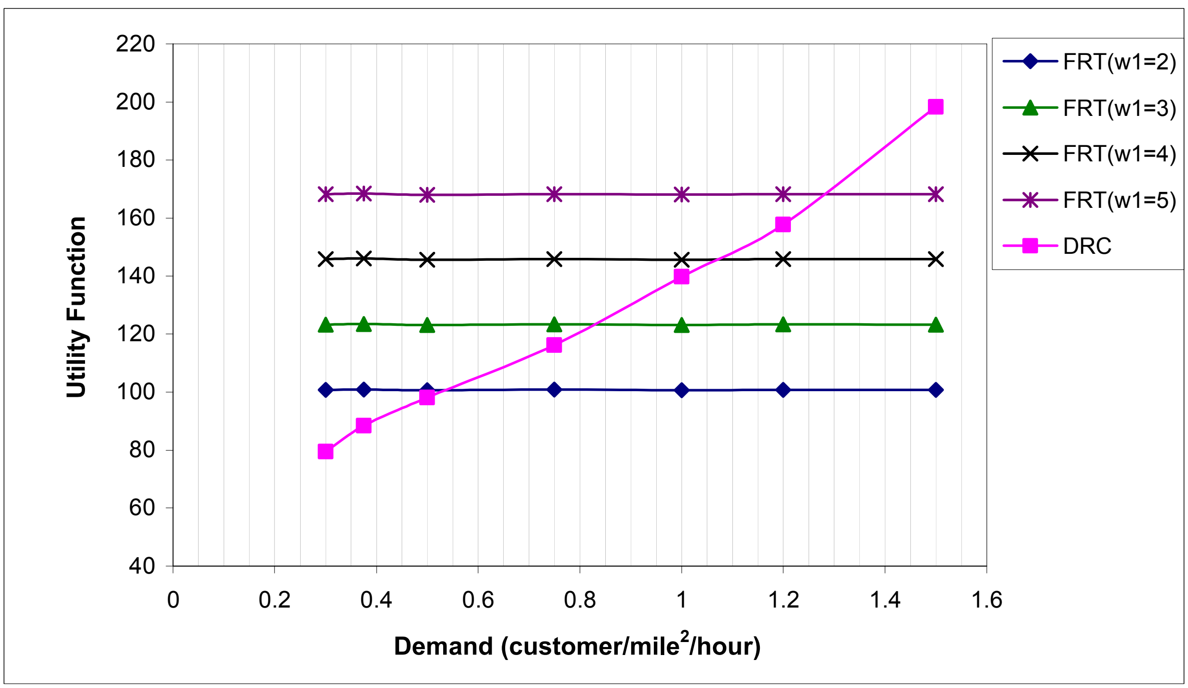

Figure 10 - Utility function for the One-Vehicle Case with L = 10, W = 2 and α = 0.5

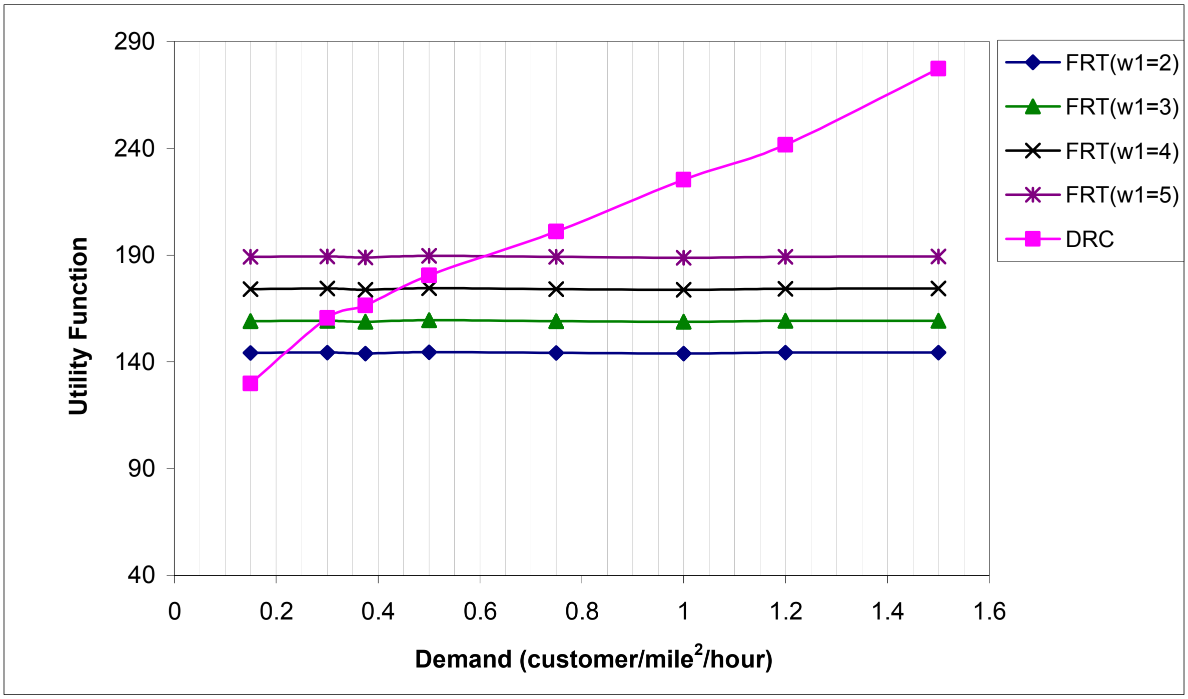

Figure 11 - Utility function for the One-Vehicle Case with L = 20, W = 1 and α = 0.5

Table 8 - Critical demands (cust/hr/mile^2) for various w_{wk}, α and L/W ratios; One-Vehicle Case

L/W	α	w_1=2	w_1=3	w_1=4	w_1=5
1.25	0.1	1.12	1.43	1.68	1.91
	0.3	0.98	1.31	1.55	1.76
	0.5	0.87	1.17	1.43	1.65
	0.7	0.78	1.09	1.33	1.56
	0.9	0.69	0.99	1.23	1.44
5	0.1	0.91	1.2	1.43	1.61
	0.3	0.72	1.02	1.25	1.45
	0.5	0.54	0.83	1.07	1.28
	0.7	0.44	0.72	0.96	1.16
	0.9	0.31	0.53	0.77	0.99
20	0.1	0.6	0.83	1.03	1.22
	0.3	0.33	0.51	0.7	0.89
	0.5	0.22	0.29	0.44	0.61
	0.7	0.16	0.23	0.32	0.45
	0.9	0.11	0.13	0.18	0.26

Figure 12 shows the critical demands. With the increase of weight w_{wk}, the critical values increase. That is, the DRC service is more preferred when planners give larger values of weight for customer walking time. Figure 12 also shows that the critical values decrease with the increase of L/W ratios.

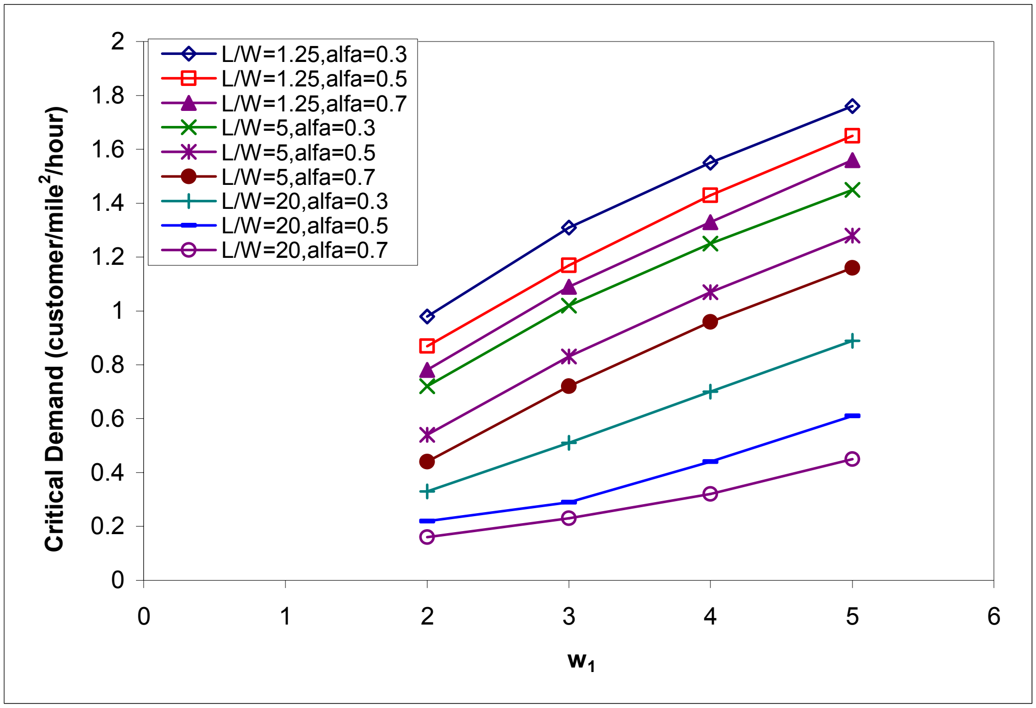

Figure 12 - Critical demands for various *L/W* ratios and α values; One-Vehicle Case

The numbers of pick-up and drop-off customers may significantly vary throughout the day, such as in morning or afternoon peak hours. In Table 8 and Figure 12 we included the results for different values of α and analyze the effects. We observe from Equations (2), (6a) and (9a) that when α increases the expected value of the customer walking time remains constant, the expected value of the customer ride time increases and the expected value of the customer waiting time decreases for FRT service. On the other hand, from the simulation results for DRC service, it is found that the expected value of the customer ride time slightly changes and the expected value of the customer waiting time increases significantly when α increases. The combination of these effects causes the derived critical customer demands to be larger when the α values are smaller. That is, the DRC service is more preferred when there are more drop-off customers than pick-up customers (such as in afternoon peak hours) and vice versa.

CHAPTER 8. CONCLUSIONS

A proper design and operations of feeder transit services within the modern sprawled residential areas are becoming increasingly more important to enhance the performance of the public transportation system network. Feeders are generally operated with a demand responsive policy which might be converted to a traditional fixed-route one for higher demand. In this research we investigated the conditions that would justify the switch from/to the two policies and we provide an analytical modeling framework of the decision problem, developing rigorous and approximate but handy analytical formulas to help decision makers and operators in their choice. To our knowledge, there is no specific research performed in developing decision tools for this purpose.

Utility functions for the fixed-route and demand responsive operating policy are derived and equalized to determine the critical demand density, representing the condition for the switch. For the one-vehicle case we derived closed-form expressions, function of the parameters of each scenario, such as the geometry of the service area, the vehicle speed and especially the weights assigned to each term contributing to the utility function: walking time, waiting time and riding time. Weights' assessments are left to the decision makers, which might select them depending on the circumstances and the changing conditions of each scenario.

Analytical results compared to simulation outcomes show a good match and a validation of our methodological approach. Estimated critical demand densities for the one-vehicle case and a service area with L=2 and W=0.5 range from 14 to 30 customers/hr/mile^2 slightly underestimating the simulated values, as predicted, however, by our approximation procedure. Similar results are obtained for the two-vehicle case. We also performed sensitivities over different L/W ratios, different area sizes and different demand distributions.

We would also add that our approach and solution to the problem should not be limited to urban residential areas, but can be applied to rural transit scenarios, where service areas defined by W and L could be much larger.

In conclusion, with this research we suggest and encourage transit planners to employ this methodological approach in selecting the proper operating policy for feeders. In addition, we provide them with a handy but powerful approximate closed-form analytical expression to estimate the critical demand density, which would justify the switch from/to one operating policy to/from the other, for a large range of possible scenarios.

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APPENDIX DERIVATION OF EQUATION (7a)

For pick-up customers, let X denotes the nearest bus station to customers, $x \in \{1, 2, ..., N\}$. Let Y denotes the ride direction of pick-up customers at the bus station, $y \in \{1, -1\}$. $Y = 1$ for direction leaving the terminal, and $Y = -1$ for direction approaching the terminal. The Probability Mass Function (pmf) of X is

$$f_X(x) = \begin{cases} \dfrac{1}{2(N-1)} & \text{for } x = 1 \\ \dfrac{1}{N-1} & \text{for } x = 2, ..., N-1 \\ \dfrac{1}{2(N-1)} & \text{for } x = N-1 \end{cases}.$$

The conditional pmf $f(y|x) = P(Y = y|X = x)$ is

$$f(1|x) = \frac{x-1}{N-1},\ f(-1|x) = \frac{N-x}{N-1},\ \text{for } x = 1, 2, ..., N-1,$$
$$f(1|N) = 0,\ f(-1|N) = 1.$$

The ride time of pick-up customers $T^p_{rd-2} = g(X, Y)$. Assuming $w_{rd} < w_{wt}$ then we have

$$g(x, y) = \begin{cases} 0 & \text{for } x = 1 \\ T_c - \dfrac{(x-1)T_c}{2(N-1)} & \text{for } y = 1;\ x = 2, ..., N \\ \dfrac{(x-1)T_c}{2(N-1)} & \text{for } y = -1;\ x = 2, ..., N \end{cases}.$$

Therefore,

$$E(T^p_{rd-2}) = \sum_{x,y} g(x, y) f(x, y) = \sum_{x,y} g(x, y) f(y|x) f_X(x) = \left[\frac{5}{12} - \frac{1}{6(N-1)^2}\right] T_c.$$