Transportation Consortium of South-Central States

Tran-SET

Solving Emerging Transportation Resiliency, Sustainability, and Economic Challenges through the Use of Innovative Materials and Construction Methods: From Research to Implementation

Resource Guide for State DOT's Maintenance Equipment Fleet Management Decisions

Project No. 20OPOSU037

Lead University: Oklahoma State University

Final Report

August 2021

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7. Author(s)

PI: Yongwei Shan https://orcid.org/0000-0001-5918-042X
Co-PI: Samir Ahmed https://orcid.org/0000-0001-9530-7213
GRA: Lei Qiao https://orcid.org/0000-0002-1205-5610

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16. Abstract

This research created a guide for state Departments of Transportation fleet management on utilizing equipment fleet management system data to make informed fleet management decisions. The research team took the historical equipment fleet data from the Oklahoma Department of Transportation and developed workflows, algorithms, and examples to demonstrate the use of historical equipment data for equipment decisions. Specifically, the research team demonstrated 1) the use of life cycle analysis and dynamic programming models for equipment replacement decisions; 2) developed algorithms to calculate equipment rental rates that can be used by the Department to forecast and allocate equipment operational budget among field districts and central office, 3) developed a procedure to make own-rent/lease decisions based on historical equipment management data. Using the two class codes of equipment (Class Code 5355 – 2 Yd. front-end loaders and Class Code 5385 – 1/2-ton fleetside pickup trucks) as examples, the research team presented the result of the equipment replacement models using both life cycle analysis and dynamic programming approaches and discussed the difference of those two methods. In addition, the impact of model input parameters (specifically depreciation cost estimation using both straight line and double declining balance depreciation calculation methods) on equipment replacement outcomes is discussed. The framework for deciding between on renting or owning for the two class codes of equipment was developed. Also, the equipment rental rates for the most frequently used equipment by ODOT were updated per class code.

17. Key Words

Equipment Management, Life Cycle, Dynamic Programming, Replacement, Economic Decisions

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SI* (MODERN METRIC) CONVERSION FACTORS

APPROXIMATE CONVERSIONS TO SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
		LENGTH		
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
		AREA		
in^2	square inches	645.2	square millimeters	mm^2
ft^2	square feet	0.093	square meters	m^2
yd^2	square yard	0.836	square meters	m^2
ac	acres	0.405	hectares	ha
mi^2	square miles	2.59	square kilometers	km^2
		VOLUME		
fl oz	fluid ounces	29.57	milliliters	mL
gal	gallons	3.785	liters	L
ft^3	cubic feet	0.028	cubic meters	m^3
yd^3	cubic yards	0.765	cubic meters	m^3
		NOTE: volumes greater than 1000 L shall be shown in m^3		
		MASS		
oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")
		TEMPERATURE (exact degrees)		
^oF	Fahrenheit	5 (F-32)/9 or (F-32)/1.8	Celsius	^oC
		ILLUMINATION		
fc	foot-candles	10.76	lux	lx
fl	foot-Lamberts	3.426	candela/m^2	cd/m^2
		FORCE and PRESSURE or STRESS		
lbf	poundforce	4.45	newtons	N
lbf/in^2	poundforce per square inch	6.89	kilopascals	kPa

APPROXIMATE CONVERSIONS FROM SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
		LENGTH		
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
		AREA		
mm^2	square millimeters	0.0016	square inches	in^2
m^2	square meters	10.764	square feet	ft^2
m^2	square meters	1.195	square yards	yd^2
ha	hectares	2.47	acres	ac
km^2	square kilometers	0.386	square miles	mi^2
		VOLUME		
mL	milliliters	0.034	fluid ounces	fl oz
L	liters	0.264	gallons	gal
m^3	cubic meters	35.314	cubic feet	ft^3
m^3	cubic meters	1.307	cubic yards	yd^3
		MASS		
g	grams	0.035	ounces	oz
kg	kilograms	2.202	pounds	lb
Mg (or "t")	megagrams (or "metric ton")	1.103	short tons (2000 lb)	T
		TEMPERATURE (exact degrees)		
^oC	Celsius	1.8C+32	Fahrenheit	^oF
		ILLUMINATION		
lx	lux	0.0929	foot-candles	fc
cd/m^2	candela/m^2	0.2919	foot-Lamberts	fl
		FORCE and PRESSURE or STRESS		
N	newtons	0.225	poundforce	lbf
kPa	kilopascals	0.145	poundforce per square inch	lbf/in^2

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ACRONYMS, ABBREVIATIONS, AND SYMBOLS

DDB	Double Declining Balance
DOT	Department of Transportation
DP	Dynamic Programming
DPH	Dollars Per Hour
DPM	Dollars Per Mile
FHWA	Federal Highway Administration
HPG	Hours Per Gallon
LCCA	Life Cycle Cost Analysis
MPG	Miles Per Gallon
ODOT	Oklahoma Department of Transportation
SL	Straight Line
SQL	Structured Query Language

EXECUTIVE SUMMARY

The state Departments of Transportation (DOT) in the U.S. typically possess a big fleet of vehicles and equipment. The equipment management constantly looks into opportunities to reduce cost and improve the efficiency of equipment utilization. The Oklahoma Department of Transportation (ODOT) has approximately 4,300 pieces of equipment, with equipment purchase years ranging from 1964 to the present. A lot of the equipment has already exceeded its useful life and many others are running under suboptimal conditions, which could increase operating costs due to equipment aging and deterioration. Equipment replacement decisions could play a very important role in managing these costs. However, currently, the ODOT lacks decision support tools, and the decisions are purely dependent on fleet managers' experience. Other than owning equipment as the single means for equipment sourcing, state DOTs may also need to examine the possibility of renting or leasing to augment their existing fleet so that the best economic decisions can be made. The ODOT is currently using equipment "rental rates" (the sum of depreciation cost and operating cost expressed as mileage and hourly rates) for forecasting and allocating equipment budget among the eight field offices and central offices. Outdated rates may subject ODOT to inaccurate budget forecasts and allocation.

This research directly addresses the need of ODOT. The overarching goal of this study is to provide a guide for state DOTs like ODOT to strategically use equipment data recorded in their equipment fleet management systems to make optimal economic decisions. Specifically, this research targets the calculation of equipment rental rates, equipment replacement decision models, and own-rent/lease recommendations. Procedures and SQL queries to calculate the equipment rental rates for the most frequently used equipment per class code were developed and the rental rates per equipment class code were updated. Advanced data analytics of life cycle cost analysis, exploratory data analysis, and dynamic programming models were applied to inform equipment replacement policies and rent/leasing strategies for specific class codes of equipment.

Two classes of equipment from Class Code 5355 (2 yd. front-end loaders) and 5385 (1/2 ton fleetside pickup trucks) were selected to demonstrate various examples in this research. The life cycle cost analysis showed that there is a similar rental rate versus age pattern for both class codes and cost rates were in a decreasing trend for both classes in the life cycle cost analysis. With the application of replacement strategies suggested by the dynamic programming model, the average cumulative total cost over the study life span could potentially be reduced by an amount between $ 6,000 and $ 8, 500 for each piece of equipment (suggested for replacement) in Class Code 5355 and Class Code 5385 when benchmarking with original equipment decision plan. Based on the different depreciation calculation methods used along with the DP approach, the decision recommendations can change dramatically. It was found that, using the double-declining balance (DBB) depreciation approach, the number of equipment that needs replacement decreases significantly. It was also found that, in comparison to the rental quotes from various online sources, 10% (7/70) of current equipment in class code 5355 and less than 1% (12 out of 449) of current equipment in class code 5385 are suggested for renting rather than owning.

1. INTRODUCTION

Strategies for highway maintenance and repair activities across the state include using contractors or in-house personnel combined with equipment sourced through either purchase, lease, or rent. State DOTs tend to use their in-house personnel and own equipment. As a result, they typically possess a big fleet of vehicles and equipment. Equipment ownership cost and operating costs are the two major categories of costs used to determine the lifecycle cost of a piece of equipment. Douglas (*1*) organized the most common methods for estimating ownership costs and operating costs, including the Associated General Contractors of America (AGC) method (*2*), the Corps of Engineers method (*3*), Peurifoy and Schexnayder method (*4*), Federal Emergency Management Agency (FEMA), and Cost Recovery Rental Rate Blue Book. However, many assumptions are made in these methods, and it could be impossible to provide accurate equipment costs. Using the data recorded by fleet management systems tends to yield more accurate results for equipment ownership and operating costs since real equipment data are used and fewer assumptions are needed. As more state DOTs adopt computerized equipment management systems, fleet managers should be able to estimate the ownership and operating costs based on accurate data so that better economic decisions can be made. The data record by the fleet management systems reflects how individual DOTs use and maintain their equipment fleet. The decision analysis performed on the more accurate data would afford agencies better solutions, such as optimal replacement schedules and own-rent/lease decisions. Moreover, the budget forecast for equipment fleet can be better determined.

The Oklahoma Department of Transportation (ODOT) utilizes "rental rates" as the primary metric in its equipment budget. The rental rate is the sum of equipment depreciation costs and operating costs per unit of usage in terms of hours or miles. An earlier study by the research team indicates that the rates have not been updated since Fiscal Year 2010. Furthermore, there is no established best management practice for analyzing and adjusting equipment rental rates for reporting and budget forecasting. This creates uncertainty and inaccuracies.

Moreover, ODOT has approximately 4,300 pieces of equipment, with equipment purchase years ranging from 1964 to the present. A lot of the equipment has already exceeded its useful life. Running equipment under suboptimal conditions increases operating costs due to equipment aging and deterioration. The default equipment useful life specified by ODOT is subjective and lacks scientific reasoning. Equipment replacement decisions are purely dependent on fleet managers' experience. Furthermore, the ODOT primarily buys equipment. When it comes to equipment sourcing, strategies include own, rent, and lease. ODOT may miss the opportunity of investigating other equipment sourcing alternatives.

2. OBJECTIVES

This research directly addresses the need of ODOT. The overarching goal of this research effort is to help ODOT strategically improve its equipment management practices using the data recorded in its equipment fleet management system. The system has a common feature of tracking equipment inventory, equipment repair and maintenance records, work orders, fuel records, and equipment usage. However, built-in advanced data analytics for decision-making is still lacking. The specific objectives of this project are to:

- Assist ODOT in calculating ownership and operating costs of the selected types of equipment.
- Develop models for equipment management decisions (including replacement and own or rent/lease decisions).
- Introduce ODOT management to state-of-the-art data analytical techniques and practices for equipment management.

3. LITERATURE REVIEW

The following subsections present the literature review and background of the equipment life cycle cost analysis, methods for estimating equipment ownership and operating costs, equipment fleet management systems, equipment replacement models, and own-rent/lease decisions.

3.1. Life Cycle Cost Analysis

Life cycle cost analysis (LCCA) models have been traditionally used as the basis for equipment management decisions (*5*). Previous studies (*5-7*) defined equipment life cycle cost (LCC) as the sum of equipment ownership costs and operating costs. Ownership costs are often called fixed costs, which occur regardless of equipment operation while operating costs are variable costs that are incurred when the equipment is used (*8*). The total ownership cost should consider initial capital cost, depreciation, investment cost, insurance cost, taxes, and storage cost and is mapped to a unit cost either in an hourly or a mileage cost. Operating costs vary with the capacity of equipment, operating hours, and operating conditions and may be computed by the sum of maintenance and repair cost, tire cost, consumable cost, fuel cost, lubricating oil cost, mobilization and demobilization cost, equipment operator cost, and special item cost (*8*). As shown in Fig.1, the unit ownership cost tends to decrease over age while the unit operating cost tends to increase due to increased repair and maintenance costs as well as reduced fuel efficiency because of equipment aging. The goal of the LCCA is to find the lowest cost point throughout the life cycle of the equipment. Then, the economic life of the equipment can be determined.

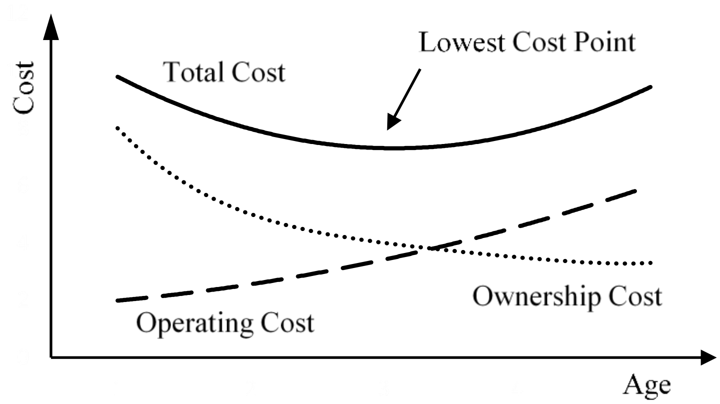


Figure 1. Conceptual graph of LCCA analysis

3.2. Methods for Estimating Equipment Ownership and Operating Costs

There are various methods used for estimating ownership and operating costs. Douglas (*1*) organized the most common methods for estimating ownership costs and operating costs, including the Associated General Contractors of America (AGC) method (AGC), the Corps of engineer method, and the Peurifoy and Schexnayder method. Table 1 shows the list of the common methods and the included elements for ownership cost and operating cost for each method. In general, the rates calculated by the AGC method are the highest while the equipment costs computed by the Corps of Engineers method are the lowest (*8*) because each method may have its own formula and estimation principles as well as the cost items included in the calculation.

Table 1. Methods for calculating equipment ownership and operating costs

Method	Ownership costs	Operating costs
Caterpillar method	Depreciation, interest, insurance, and taxes	Fuel, filter, oil, and grease (FOG) costs, tires, repairs, special items, operator's wages
Corps of Engineer method	Depreciation, facilities capital cost of money (FCCM) - Exclude: license, tax, storage, and insurance cost	Fuel, filter, oil, grease, servicing the equipment, repair and maintenance, and tire wear and tire repair
Associated General Contractors of America (AGC) method	Depreciation, interest, insurance, and taxes - Same as Caterpillar method but an incremental replacement cost is considered additionally	Field and shop repairs, overhaul, and replacement of tires and tracks, etc. - Exclude: FOG costs, operator's wages
Peurifoy and Schexnayder method	Depreciation, interest	Maintenance, tire, fuel, and the FOG costs - Exclude: operator wages

3.2.1 Caterpillar Method

The ownership cost of the caterpillar method is the sum of depreciation cost, interest, insurance cost, and taxes. Depreciation cost can be calculated by the straight-line method. In this method, the interest of capital for purchase equipment would be considered. Meanwhile, the operating cost of the method considered fuel cost, filter, oil, and grease (FOG) costs, tire cost, repairs cost, special items, and Operator's wages. The operating cost can be obtained from the caterpillar performance handbook (*9*) except tire cost, repairs cost, and wages of the operator. The tire cost can be estimated by its historical data and the wages can be estimated by referring to the local wages and fringe benefits.

3.2.2 Corps of Engineers Method

The Corps of Engineers method is the most sophisticated method that considers both economic and geographical conditions. The method calculates the hourly rate of construction equipment based on a 40-hour work week (*10*). Using this method, the US Army Corps of Engineers calculate the hourly rate and hourly standby rate and continuously present the results on a pamphlet titled 'Construction Equipment Ownership and Operating Expense Schedule' from June 1999 to present. In this pamphlet, the hourly rate is also the sum of ownership cost and operating cost and the rate is based on the 40-hour work week. The publication defines ownership costs as the sum of depreciation and facilities capital cost of money (FCCM). On the other hand, the publication defines operating costs as the sum of five elements, including (1) fuel cost, (2) filters, oil and grease (FOG) cost, (3) repairs cost, (4) tire wear cost, and (5) tire repair cost.

3.2.3 Associated General Contractors of America (AGC) Method

The AGC method can be used by equipment owners to determine the capital recovery of the equipment investment. The ownership cost of the AGC method is very similar to other methods and considers purchases price, sales tax, shipping, assembly cost, and salvage value (assumed 10% of acquisition costs). The operating costs include maintenance and repair costs that are estimated per the percentage of acquisition costs. The rental rates calculated by the AGC methods are expressed in hourly cost and the rates are classified according to the engine size (*11*).

3.2.4. Peurifoy and Schexnayder method

Peurifoy and Schexnayder's method is a widely used approach for equipment economic decisions. (*5*) used the Peurifoy and Schexnayder method to construct a life-cycle cost analysis (LCCA) model and used a stochastic approach to calculate the life cycle cost and the economic life of the equipment. The operating costs in Peurifoy and Schexnayder method include fuel, maintenance, filter, oil, grease costs, tire, and tire repair cost.

All those methods mentioned above have certain formulas to estimate the costs. For detailed calculations, please refer to the cited publications.

3.3. Equipment Management Systems Used by DOTs

Managing a big fleet of equipment can be a complex task for state DOTs. To facilitate the efficiency of equipment inventory management and decision-making, many equipment management software programs have been developed and adopted by various DOTs. Table 2 shows a summary of equipment management tools for fleet management, their basic functions, and representative state DOT clients. Most of the tools have a common feature of tracking equipment inventory, equipment repair and maintenance records, work orders, fuel records, and equipment usage. However, built-in advanced data analytics for decision making is still lacking.

Table 2. Summary of fleet management software tools used by representative state DOTs

Software	Developer/year	Description	DOT Client
Fleet and equipment manager of AgileAssets®	AgileAssets	- Estimate the depreciation, LCC, and replacement of equipment - Fuel, inventory, repair management - Record the history of vehicle usage, maintenance, labor Requirements, and used costs for parts.	Oklahoma DOT; Illinois DOT; Colorado DOT; Louisiana DOTD
IMS, Fleet Maintenance Pro	Innovative Maintenance Systems/ 1994	- Track and manage fleet inventory - Track repair records and work orders - Provide daily inspection checklist - Create customizable reports	Minnesota DOT
RTA Fleet Management	Ron Turley/ 1979	- Track the equipment, performance, vehicle use, and labor. - Determine the maintenance, repair necessary time	Minnesota DOT

Software	Developer/year	Description	DOT Client
FleetFocus	AssetWorks/1984	- Equipment life cycle management (budgeting, acquisition, capital improvement, campaigns, and disposal management) - Track various functions of vehicles and equipment - Estimate repair, preventive maintenance, operating cost of vehicles, equipment	New Jersey DOT; New York DOT; Ohio DOT; Oklahoma DOT; Oregon DOT; Virginia DOT; Washington DOT

Although the methods mentioned in Subsection 3.2 could be used for estimating equipment ownership and operating costs, more assumptions are often needed. Using the data recorded by fleet management systems for equipment ownership and operating costs estimating tends to yield more accurate results since real-world data is used; thus, fewer assumptions are needed. The data recorded by the fleet management systems reflects how individual state DOTs use and maintain their equipment fleet. The decision analysis performed on the more accurate data would afford agencies better solutions, such as optimal replacement schedules and own-rent/lease decisions. Moreover, the budget forecast can be better determined.

3.4. Equipment Replacement Decisions

Fleet managers always face difficulties in decisions on when the best time is to replace certain equipment. Many factors could be considered in replacing certain types of equipment, such as ages, mileages, running hours, operating costs, and even indirect costs of labor and supportive services. In reality, fleet managers tend to use their own experience to make replacement decisions based on some general rules (e.g. age > 10 or mileage > 150,000 for a 1/2-ton pickup truck).

However, a better decision could be approached by considering the life cycle costs of equipment and finding the minimum cost time of the cycle (e.g. the economic life). With comprehensive fleet inventory and usage records, life cycle cost analysis can serve as an important indicator for managers to decide the proper time of replacement.

In addition to LCCA, equipment replacement decisions can be assisted with mathematical models that involve a series of optimal calculations in costs. Equipment replacement models have been primarily researched in the industrial engineering field. Different economic models, including opportunity cost models, operation and maintenance costs equilibrium models, profitability models, and replacement cost models (*12-16*) have been developed. The goal of the replacement analysis is to optimize the cost or utility function. In terms of optimization, various operations research techniques, including integer programming (*17*), dynamic programming (*18*), decision trees (*19*), simulation techniques (*20*), Markovian models (*21; 22*), and partially observable models (*23*) have made significant contributions to this field. All methods come with their nuances, which require different model inputs.

Among the reviewed methods, dynamic programming is promising since it provides a systematic procedure to determine optimal replacement choices for a series of interrelated decisions. Both deterministic dynamic programming (DDP) and stochastic dynamic programming (SDP) have also been applied in equipment replacement optimization with consideration of vehicles' annual utilization and maintenance costs (*24-26*).

3.5. Own and Rent/Lease Decisions

Other than purchasing, state DOTs may use rent or lease to augment their existing fleet. There are also different approaches developed in other fields to help with buy-rent-or-lease decisions. For example, Johnson and Lewellen (*27*) developed a financial model for analyzing lease-or-buy decisions and illustrate the model with an equipment example. Hargreaves (*28*) developed a financial model comparing the economics of owning versus renting houses. Both studies admitted that lease-or-buy decisions are not purely economic decisions. Other non-economic factors can influence the final decision.

As evidenced by previous studies, various analysis models do exist. However, an effort is required to glean those various models, re-examine the models, and fit them into ODOT's current fleet management system and available equipment data, so that optimal equipment decisions can be achieved.

3.6. Literature Review Summary

Life cycle cost analysis is a traditional method used for equipment economic analysis, such as determining equipment rental rate (the sum of equipment ownership cost and operating cost) expressed either in dollars per mile or dollars per hour and determining the optimal replacement age. In order to estimate ownership and operating costs, different methods have been reviewed, including the Caterpillar method, Corps of Engineer method, Associated General Contractors of America (AGC) method, and Peurifoy and Schexnayder method. Caterpillar method considers straight-line depreciation that includes the interest of capital for equipment purchase while operating cost takes into account fuel cost, maintenance and repair cost, tire cost, special items cost, and operator wages. Corps of Engineers method assumes a 40-hour workweek to calculate hourly costs. The ownership cost is the sum of depreciation cost and Facilities Capital Cost of Money (FCCM). Operating costs consist of fuel cost, maintenance and repair cost, tire wear, and repair cost. AGC method is primarily used to determine the capital recovery of the investment using estimated ownership and operating costs as inputs. Peurifoy and Schexnayder Method is also widely used for equipment economic decisions. In calculating ownership cost, the method considers initial cost, depreciation, investment cost, insurance, taxes, and storage cost. Operating cost considers fuel cost, maintenance and repair cost, tire and tire repair cost.

Due to the adoption of computerized equipment management systems by state DOTs, the calculation of ownership and operating costs can be based on historical data collected by the equipment management system. The review of the equipment management systems used by various DOTs reveals that various software applications have been used by different DOTs to track equipment inventory, performance, fueling records, equipment usage, repair and maintenance activities, etc. The data can be better utilized to make equipment management decisions.

In addition to life cycle cost analysis, other advanced mathematical models (such as integer programming, dynamic programming, decision trees, simulation techniques, and Markovian models have been used for equipment replacement decisions. Among those methods, dynamic programming is promising since it provides a systematic procedure to determine optimal replacement choices for a series of interrelated decisions.

Own and rent/lease decisions have been traditionally studied based on economic analysis. In addition to economic analysis, other non-economic factors, such as the frequency of equipment usage, purchase lead time, and mobilization cost, should be considered in the decision.

4. METHODOLOGY

In this section, the research team presents the dataset from ODOT's equipment fleet management database, data processing in MySQL, the developed SQL queries to compute equipment rental rates (ownership and operating costs), life cycle cost analysis, and dynamic programming models and model parameter estimates, and exploratory data analysis in detail.

4.1. Equipment Management Database and Datasets

ODOT has been using the services of Agile Assets since 2010 and the system was intended to help the Department reduce costs and increase the return on its asset investments using smart programming and work management. The system helps ODOT track, coordinate, summarize, and report all activities throughout the asset life. Prior to the use of computerized fleet management tools, field personnel used to track equipment activities with paper records that might have never been made to the office from the job site or might have been mixed with the other job site logs. The fleet management system made it easy and quick for ODOT to enter and retrieve required entries. It also helps to ensure complete and accurate routine maintenance, which could lead to longer equipment life and less downtime.

ODOT provided the entire dataset including records on equipment fleet inventory as well as operation, maintenance, and repair activities. The entire dataset was exported from Agile Assets into Excel spreadsheets provided by the ODOT maintenance division. At the time of the study, the dataset covered data records from Oct. 2010 to Sept. 2020. Since the data obtained from ODOT was an export from a relational database, multiple data tables were used to capture different aspects of information related to the equipment fleet. The research team did not have the access to ODOT's equipment fleet management system since it is a proprietary application. To facilitate the information query for the estimation of replacement model parameters, a relational database was recreated in an open-source database platform, MySQL Workbench (Figure 2). Python Jupiter Notebook was the programming front-end interface used in this study to develop computation algorithms and SQL queries to interact with the backend MySQL database.

A detailed description of the tables exported from ODOT's equipment fleet management system can be found below.

- Equipment_Class_Code – The table presents the basic information about the classification of equipment. The class code classifies all the equipment based on equipment type and equipment size. A group of similar equipment shares the same class code.
- Equipment_Inventory – The inventory data table provides the basic information about every individual piece of equipment in ODOT's current inventory. Each piece of equipment is assigned with a unique Equipment ID.
- Equipment_Fueling – This data table provides information about the fuel purchase activities associated with individual pieces of equipment. Fuel consumption and fuel cost are major components of the operating cost of equipment. The equipment fueling data table associates all the fuel records with equipment IDs.
- COMDATA_Fueling – COMDATA Fueling Data contains the record of all the purchases charged to the COMDATA card, including both fuel and non-fuel purchases. The data records are associated with equipment IDs.

- Setup_Project – This data table consists of records on maintenance repair activities and costs performed on all the equipment. All of the activities are associated with equipment IDs.
- Work Orders_Equipment_DC – This table shows miles driven or hours operated during the operation of the equipment to perform regular and maintenance work.
- Work_Orders – This table shows all costs incurred and miles driven or hours operated during the operation of the equipment to perform maintenance work in the field. Different from Work Orders_Equipment_DC, this table also includes costs not involving equipment operations.

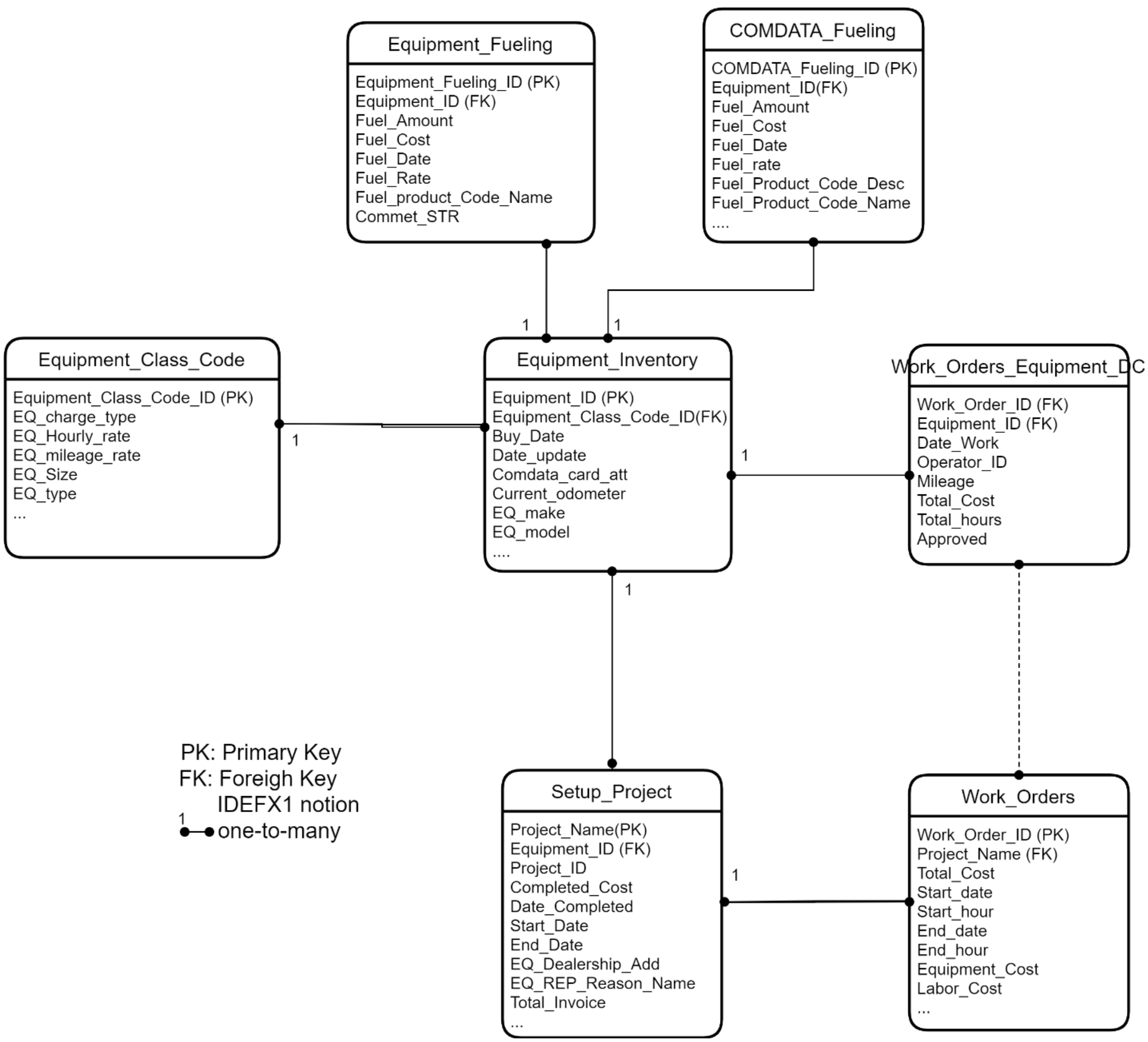


Figure 2. Entities-Relational Diagram (ERD) of ODOT equipment management database

This study focused on the equipment bought since 2010 since the complete history of its operations and management including fueling, maintenance, and repairs that occurred during the life cycle are well recorded in the Agile Assets system, which can facilitate later analysis. A query was created to select all the equipment bought since 2010 which is ranked based on quantity and shown in Table 3. The most frequently bought equipment includes ½ ton fleetside pickup trucks with a total quantity of 543 followed by gas-powered weed eaters with a total quantity of 527.

Table 3. The count of equipment in each class bought since 2010

No.	Equip. Class Code ID	Equip. Count	Equipment Size/Description	Equipment Type
1	5385	543	1/2 Ton Fleetside	Pickup
2	5115	527	Gas Powered	Weed Eater
3	5363	368	One Way	Snow Plow
4	5435	367	41000 GVW-Diesel	Truck
5	5136	261	Single Spinner	Spreader-Heavy Duty
6	5442	255	3/4 Ton	Crew Cab Pickup
7	5375	237	85 H.P. Diesel	Wheel Tractor
8	5261	218	15' Rotary -	Mowing Attachment
9	5118	216	Gasoline Powered	Blower/Vacuum
10	5486	198	Approx. 5 H.P.	Chain Saw
11	5488	180	Approx. 4 H.P.	Chain Saw
12	5117	104	Gasoline Powered	Hedge Trimmer/Pruner
13	5349	100	For Tractor/Skid Steer	Attachment - Front End Loader
14	6499	86	Gasoline Engine	Chemical Induction System
15	5355	80	2 Yd.	Front End Loader
16	5102	68	5 Hp - 10 Hp	Air Compressor
17	5189	68	Self Propelled	Power Sweeper
18	5238	64	150 H.P.	Motor Grader
19	5218	60	Solar Power	Traffic Warning System
20	5395	57	Fullsize	Pickup
21	6497	57	1/2 Ton	Crew Cab Pickup
22	5443	54	1 Ton	Crew Cab Pickup
23	5444	54	2 Wheel	Trailer
24	5135	50	Single Spinner	Spreader-Heavy Duty
25	5434	49	24000 Gvw-Diesel	Truck
26	5226	48	Trailer Mounted	Attenuator
27	5180	47	Truck Mounted	Chemical Applicator
28	5116	45	Gas Powered	Edge Trimmer
29	5183	43	Gasoline Engine	Power Washer
30	5185	39	Solar/Battery Powered	Radar/Speed Monitor
31	5319	37	9 Wheel	Roller
32	5164	35	60 Lb. - 26 Inch	Paving Breaker
33	5260	33	Trailer Mounted	Brush Chipper
34	5123	31	92 Net H.P.	Backhoe-Loader-Tractor Unit

No.	Equip. Class Code ID	Equip. Count	Equipment Size/Description	Equipment Type
35	5357	31	1/3 Cu. Yd. Cap.	Skid Steer Loader
36	5113	29	Gas Powered	Edge Trimmer
37	5098	28	185 Cfm	Air Compressor
38	5394	28	1 Ton, Dual Rear	Pickup
39	5195	26	1750 Watts - 4 Hp And	Generator
40	5214	26	Tractor Mounted	Excavator
41	5251	26	50 Inch Cut	Mower - Rotary
42	5089	25	Four Door Sedan-Mid Size	Auto - White Color
43	5197	25	5000 Watts-10 Hp	Generator
44	5248	24	60 Inch	Mower - Rotary
45	6292	23	Building Backup	Generator
46	5166	20	Vibro-Plate 3 Hp	Compactor
47	5176	20	Hyd. Drainage	Power Washer
48	5386	20	3/4 Ton Fleetside	Pickup
49	6387	20	25-50 Gallon, Electric	Herbicide Spot Sprayer

4.2. Methods for Equipment Rental Rate Calculation

There are various methods used for estimating ownership and operating costs. Each method may have its own formula and estimation principles. On this project, the data recorded by fleet management systems were used to obtain more accurate results for equipment ownership and operating costs. For this study, the ownership cost only includes equipment depreciation cost. Storage and insurance were not considered in this study since ODOT has its own storage yard and the equipment is self-insured. The operating cost mainly includes fueling, maintenance, and repair costs. The operator's cost was not considered in this study. The time value of money is considered in this study because ODOT does not seek profit and ODOT does take loans to purchase equipment.

The following section describes the data processing flow charts and the SQL procedures involved in calculating equipment rental rates. The rental rate (the sum of equipment depreciation rate and operating cost) is expressed in dollars per hour or mile (Equation 1) based on the charge type of the equipment. Depreciation could be one of the most important parts of the equipment cost. It is a fixed cost. However, depending on calculation methods, it could be very different for each year. The research team used two common methods, the Straight-Line method (SL) and the Double Declining-Balance method (DDB) (Equation 2 and Equation 3). The SL method gives an equal amount of depreciation in each year of useful life while DDB generates very high initial depreciation in the first year of the equipment useful life, which then decreases in a factor toward the end of the equipment useful life. The operating cost of the equipment is the sum al maintenance and repair cost and fueling cost normalized by mileage or hours (Equation 4).

$$\text{Rental Rate } \left(\frac{\$}{\text{hour}} \text{ or } \frac{\$}{\text{mile}}\right) = \text{Depreciation Rate} + \text{Operating Cost} \qquad [1]$$

$$\text{Depreciation rate (SL)} = \frac{\sum_{i=1}^{n}[(\text{Purchase price} - \text{Sold price or Salvage value}) * \text{SLDP}]}{\text{miles driven/hours used}} \qquad [2]$$

$$\text{Depreciation rate (DDB)} = \frac{\sum_{i=1}^{n} 2 * \text{SLDP} * \text{BV}i}{\text{miles driven/hours used}} \qquad [3]$$

where:

SLDP = straight-line depreciation percent (%);

BVi = book value at the beginning of the age I ($); and

n = ages (1, 2, …,10) (year).

$$\text{Operating Cost} = \frac{\text{Maintenance and repair cost + Fueling cost}}{\text{miles driven/hours used}} \quad [4]$$

Both equipment depreciation calculation methods require the original price, useful life, and salvage value of the equipment, and the information can be found in “equipment_inventory” and “equipment_class_code” tables. A procedure was developed (named as “load_dp”, see Query 1 in Appendices) to calculate annual depreciation for each piece of equipment through a loop function in MySQL. Figure 3 shows the flow chart on the depreciation calculation process.

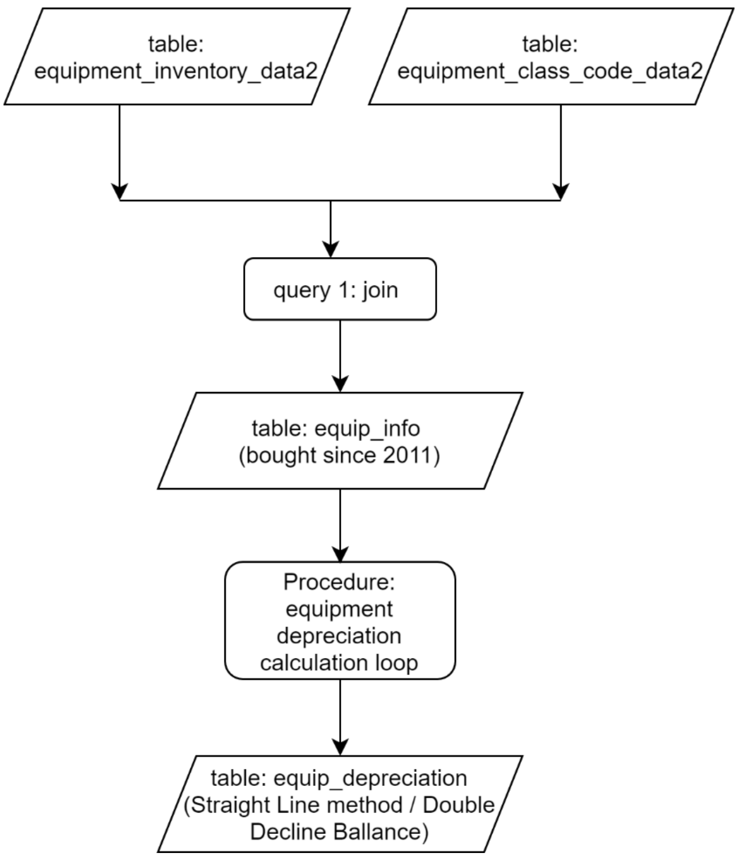


Figure 3. Flowchart for calculating depreciation rate ($/year)

The annual fueling cost and maintenance and repair cost were calculated in time series in accordance with depreciation values that were created previously. The tables of Comdata_fueling, Equipment_fueling, and Setup_project were involved in this process. The table Comdate_fueling is primarily composed of fueling cost records but some maintenance costs (such as spare parts change, oil change, ties related cost) were also included. Based on the fueling rate (if greater than $4/gallon), this part of the cost was separated and classified into the maintenance cost category. The table Equipment_fueling only contains fueling activity records, such as fueling amount and cost. The table of setup_project provides the maintenance/repair cost that was further divided into sub-classes of equipment cost, parts cost, and labor cost, etc. A procedure was developed (named as “dp_to_all_costs”, see Query 2 in Appendices) to calculate the annual total cost for each piece of equipment in MySQL. Figure 4 shows the flowchart on the total cost calculation process.

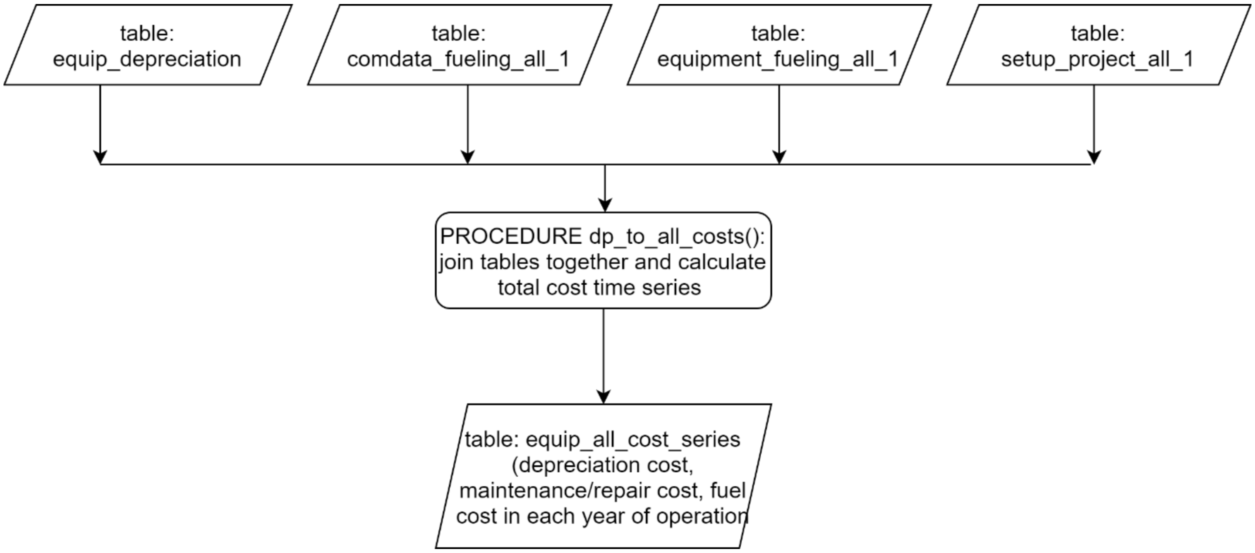


Figure 4. Flowchart for generation of annual total cost time series ($/year)

Equipment rental rate charged by miles was calculated as dollars per mile (DPM), which relies on the table, equip_all_cost_series, containing all types of cost obtained in the previous step and the table containing basic information of equipment (such as odometer). Miles per gallon (MPG) was calculated for each piece of equipment based on its odometer and total fueling amount, which can be used to obtain annual mileage in turns based on the annual fueling amount. Therefore, DPM at each year or cumulatively for the whole life can be calculated once the total annual cost and mileage were calculated. Figure 5 is the flow process on obtaining the MPG and DPM at both annual and whole-life scales for the equipment charged by miles. A procedure called “dollar_per_mile” (Query 3 in Appendices) was developed to facilitate this process.

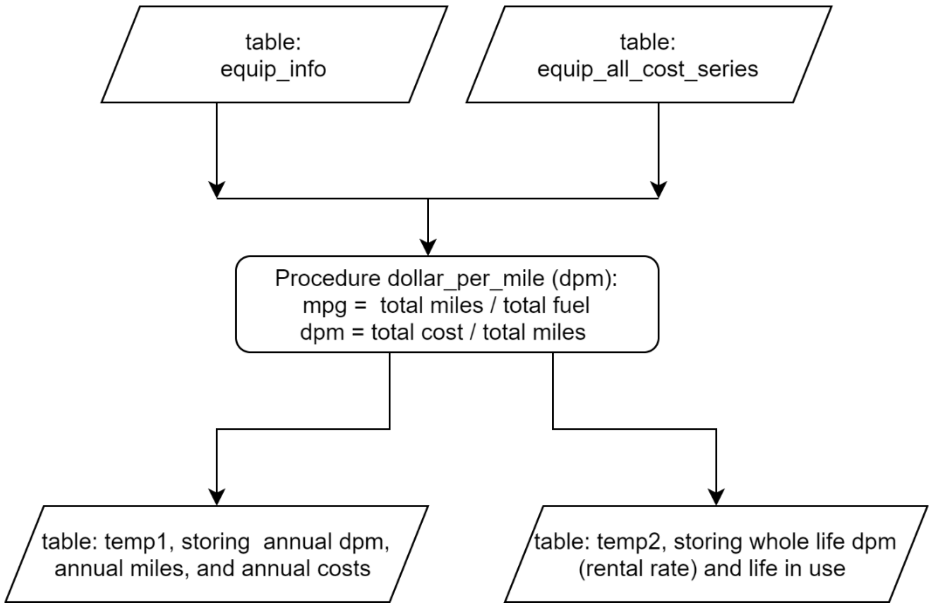


Figure 5. Flowchart for calculating the rate of dollars per mile at both annual and whole-life scales for equipment charged by mile

Similarly, equipment cost charged on hours was calculated as dollars per hour (DPH). The table, equip_all_cost_series, containing all types of cost obtained in the previous step was needed; however, an additional table, work_orders_equipment_dc, was also needed for the total operating hours of equipment. Hours per gallon (HPG) was calculated for each piece of equipment based on its total work hours and fueling amount, which was used in turn to obtain annual operating hours based on the annual fueling amount. Therefore, DPH for both each year and cumulatively for the whole life can be found once the annual cost and mileage were calculated. Figure 6 is the flowchart on obtaining both HPG and DPH at both annual and whole-life scales for the equipment charged by hours. A procedure called "dollar_per_hour" (Query 4 in Appendices) was developed to facilitate this calculation process.

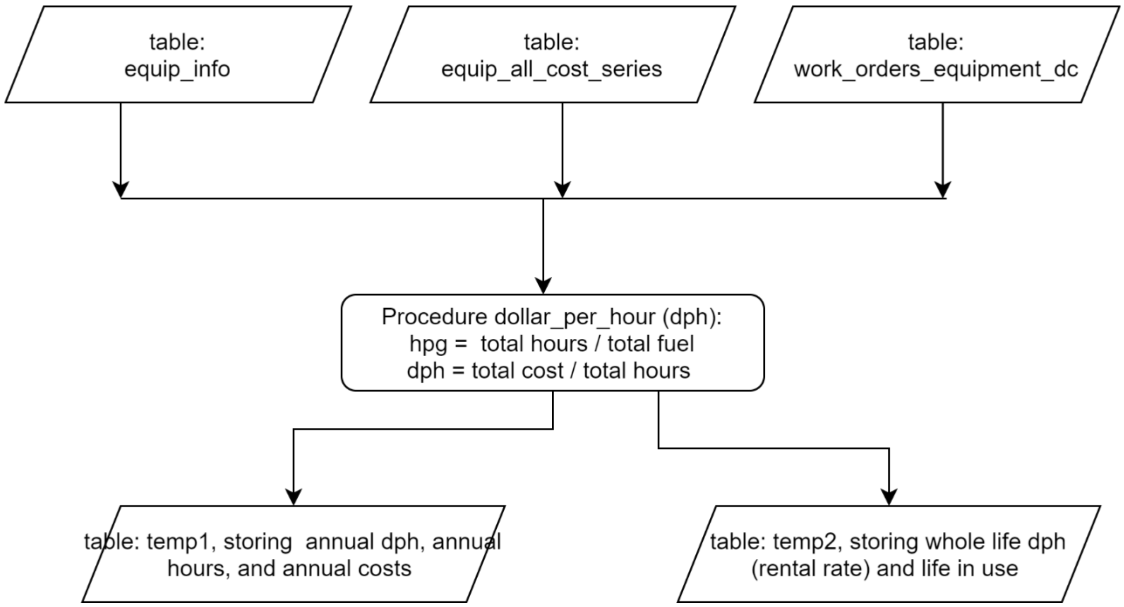


Figure 6. Flowchart for calculating the rate of dollars per hour at both annual and whole-life scales for equipment charged by hour

As mentioned earlier, the equipment charged by DPM and DPH can be processed by the procedures "dollar_per_mile" and "dollar_per_hour", respectively. One more procedure (named as "class_code_cost", Query 5 in Appendices) was created to process all the equipment (the ones bought since 2010) in a loop where charge type acted as the control to divert the process either to procedure "dollar_per_mile" or "dollar_per_hour" until all the equipment was processed. Four tables in two sets were finally generated, two tables for mile-based equipment and another two for hour-based equipment. One set of tables is for time series of annual cost and rental rates (DPM and DPH) and the other set is the average rental rates over the entire life cycle and other information, such as current ages and average work miles or hours each year. The detailed process is shown in Figure 7.

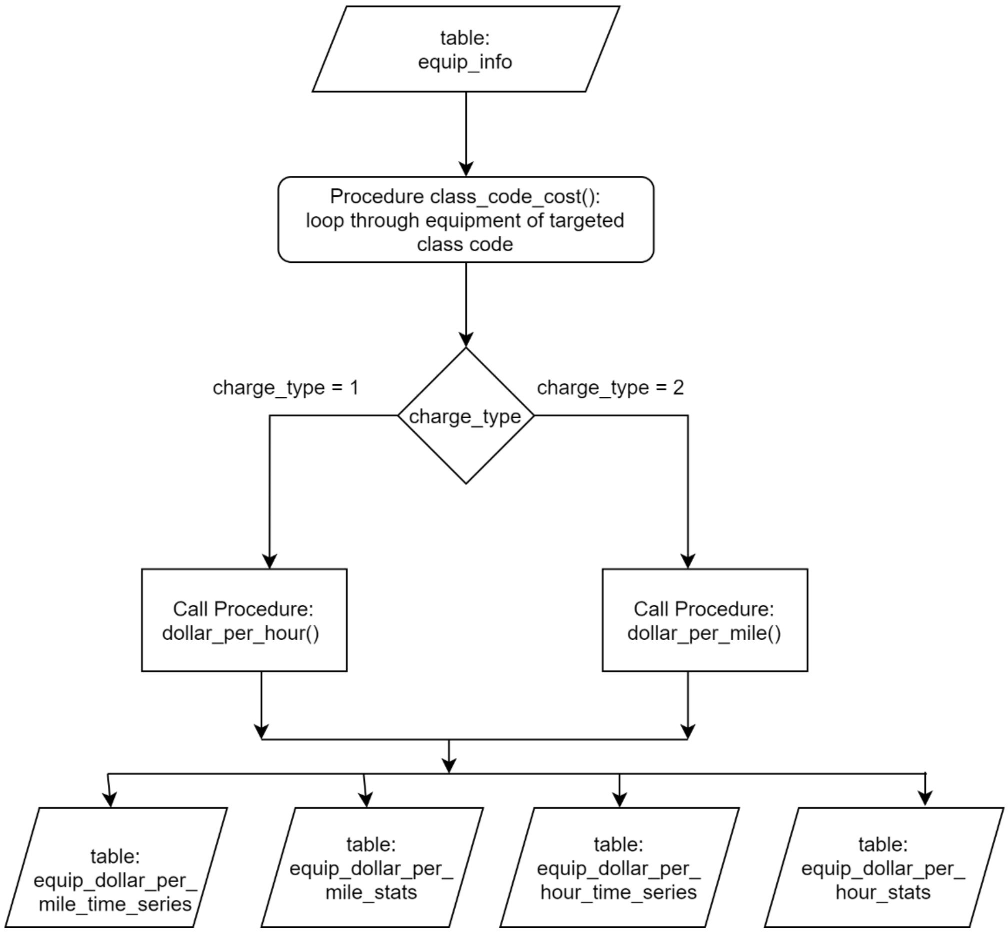


Figure 7. Flowchart for calculating rental rates (dpm and dph) and time series for all the equipment in each class code

4.3. LCCA

Based on the queries and tables created in Section 4.2, the research team was able to perform an LCCA of any equipment. The main purpose of the LCCA for this study was to examine the trend of equipment rental rate (DPM or DPH) over time so that equipment economical life can be determined.

4.4. Dynamic Programing Models

Figure 8 describes the different possible "keep-replace" decision scenarios for a piece of equipment over a 3-year and a 4-year decision span. The "keep-replace" decision is assumed to be made at each stage. The numbers in the circles represent the current age of the equipment at the stage. For example, for a 3-year decision span, at stage 1, if the equipment is kept for another year, the equipment's age turns to 2 at Stage 2. Whereas, if the equipment is replaced, the equipment's age turns to 1 at Stage 2. Therefore, at the end of Year 2 (Stage 2), the equipment could have two states in terms of age, either 1-year-old or 2-year-old. In this particular case, the number of states is equivalent to the stage number when the stage number is greater or equal to one. For each "keep" and "replace" decision made, the following year would have different costs associated with it. From the beginning state to the ending states, there are different paths (arcs) linking the beginning state to each of the ending states. Summing the cost up along with a path would yield the total cost for a series of decisions. The goal for the equipment decision is to find the decision path that has the lowest total costs over the decision span under consideration. As the decision life span grows, the number of paths can grow significantly. Computing the total costs of all paths through exhaustive enumeration can be almost intractable and computationally inefficient for larger decision networks (*29*).

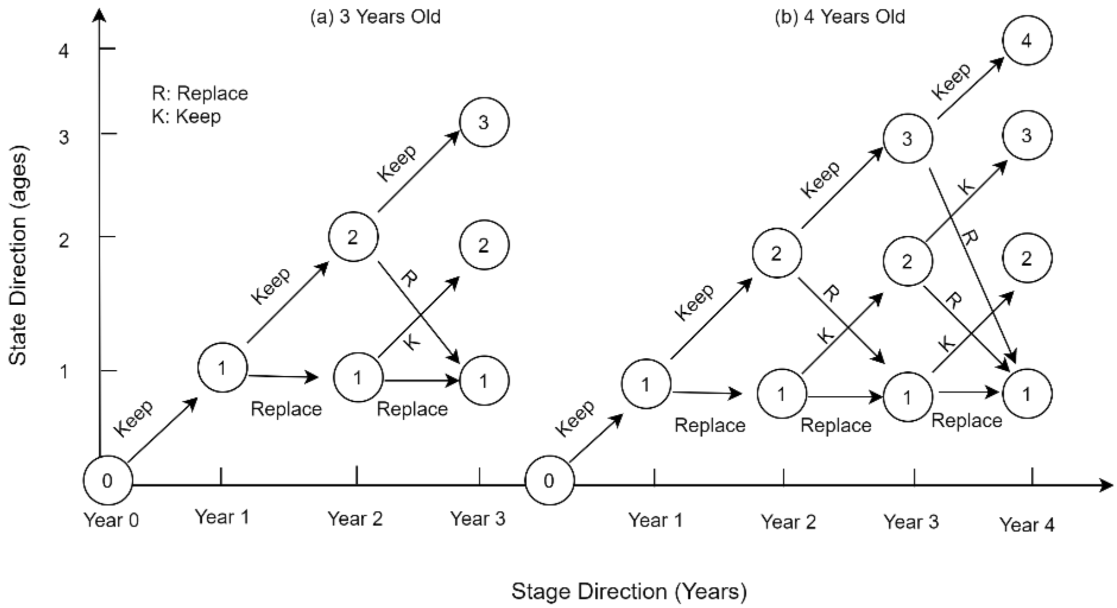


Figure 8. The network diagrams of all possible decisions for 3-year and 4-year spans

4.4.1 Dynamic Programing - Minimize the Recursive Function

The optimal "keep-replace" decision problem described above can be solved using the dynamic programming approach (DP). DP approaches large problems by finding the optimal solution for a smaller subset of the problem, then gradually looking for an optimal solution for the enlarged one based on the preceding one until the entire large problem is solved (*29*). DP is used for equipment replacement decisions due to its high efficiency in the optimization of equipment total cost over a time horizon (*29*). The computing cost is relatively low even if the fleet size is on the order of thousands. The objective of the DP for this study is to find out the optimal "keep-replace" decision based on the equipment's historical operation as well as maintenance repair records. In other words, the research team wanted to find out the optimal alternatives other than keeping the equipment until its useful life. The optimal cost was also benchmarked against the original decision (keep until the useful life).

The basic parameters for constructing the dynamic programming models for this study include 1) decision stage, 2) states under each decision stage, 3) decision, and 4) costs associated with each decision at each decision stage. For this study, the number of decision stages corresponds to the number of years that the equipment experienced. The state represents the age of the equipment at the decision stage. For example, for a piece of 10-year-old equipment, there are 10 decision stages, and there are eight possible age states at the 8^{th} decision stage. For each state, there are two decisions to make, either to keep or replace. The cost associated with each decision at each decision stage includes the annual fueling, depreciation, and maintenance and repair costs, which can be obtained from the historical records in the equipment management system using the SQL procedures created in Section 4.2.

A backward iterative solution (beginning by finding the optimal decision from the last stage) was used to find the optimal decision series that would be made over the historical life span of the equipment to reduce total cost in the consideration of depreciation cost, maintenance/repair cost, and fueling cost. A recursive function was developed for Stage n given the optimal decision for Stage n+1, described in Eq. 5 and Eq. 6. The dynamic programing used in this study was a deterministic model, which means the state at the next stage is completely determined by the state and decision made at the current stage (*29*). The goal of a DP is to find the optimal decision variable ("keep" or "replace") that yields the lowest total cost at Stage n given the optimal costs for stages n+1 onward and the immediate cost at Stage n.

$$f_n(S_n) = cost_n(S_n) + f^*_{n+1}(\{S_n + 1, if\ Q_n = \text{"Keep"}; 1, if\ Q_n = \text{"Replace"}\}) \quad [5]$$

Where:

$f_n(S_n)$ = total cost of at Stage S_n ($);

$f^*_{n+1}(\{S_{n+1})$ = the optimal solution among all states at Stage n+1 ($);

$S_n = states\ of\ equipment\ at\ stage\ n; corresponding\ to\ the\ ages\ of\ equipment$;

$S_0 = 0;\ S_1 = 1; S_2 = \{1,2\}; S_3 = \{1,2,3\}; \ldots; S_{10} = \{1,2,3, \ldots ,10\}$;

$Q_n = decisions\ made\ at\ stage\ n\ (\text{"}Keep\text{"}\ or\ \text{"}Replace\text{"}\ equipment)$;

$cost_n: total\ cost\ at\ stage\ n\ (including\ depreciation, fueling\ cost, and\ maintenance\ cost)$ ($); and

n: decision stage, ranging from zero to the current age of the equipment.

$$cost_n(S_n) = depreciation\ (S_n, n) + maintenance\_cost(S_n) + fueling\_cost(n) \quad [6]$$

Where:

$depreciation\ (S_n, n) =$ the depreciation cost at the stage n, depending on the stage and the equipment ages S_n ($);

$maintenance\_cost\ (S_n, n) =$ maintenance and repair cost, depending on the ages of equipment S_n ($); *and*

fueling_cost = fueling cost at stage n, depending only on the stage.

To facilitate the problem solving, the research team created a DP function with Python programming language (see Appendix).

4.4.2 Model Parameter Estimates

To demonstrate the use of DP for equipment replacement, two class codes of equipment were selected from ODOT's current fleet: Class Code 5355 (2 yd. diesel engine front-end loader) and Class Code 5385 (½ ton fleetside pickup trucks). The useful life specified by the Department for both types of equipment was 10 years. In order to properly construct the DP model, the cost items, such as depreciate cost and maintenance cost should be properly estimated. All other costs were excluded since they would not differ from the keep-replace decisions. The cost items were all estimated on an annual basis.

4.4.2.1. Depreciation

The equipment depreciation is dependent on the purchase price and salvage value as well as specified useful life. The purchase price of the equipment tends to increase over time. In order to properly estimate the purchase prices over time, historical purchase prices of the equipment with the same class code were used to fit a linear regression line. Then, equipment purchase prices at different time points can be estimated using the linear regression line. Fig. 3 depicts the scatter plot of the equipment's original purchase prices and the fitted lines for the class code 5355 (2 Yd. front-end loader) and 5385 (½ ton fleetside pickup). The original price for Class Code 5355 increased from $78,000 in 2011 to $130,000 in 2017. The original price for Class Code 5385 increased from $19,000 in 2011 to $41,000 in 2018. The price increases were mostly caused by upgrades for the new models of the equipment. Based on the literature, two methods of depreciation calculation are often used: straight line (SL) and double-declining balance (DDB) depreciation methods as shown in Eq. 7 and Eq. 8. The default salvage values specified by the Department were used for the straight-line depreciation calculation. For a 10-year useful life, the SL depreciation rate is 10%. The depreciation for DDB is 20% of the book value at the beginning of the year under consideration. Both methods were adopted to examine how depreciation methods affect decision outcomes.

$$\text{Depreciation Cost (SL)} = (\text{Purchase price} - \text{Salvage value}) \times \text{SLDP} \qquad [7]$$

$$\text{Depreciation Cost (DDB)} = \text{BVi} \times 2 \times \text{SLDP} \qquad [8]$$

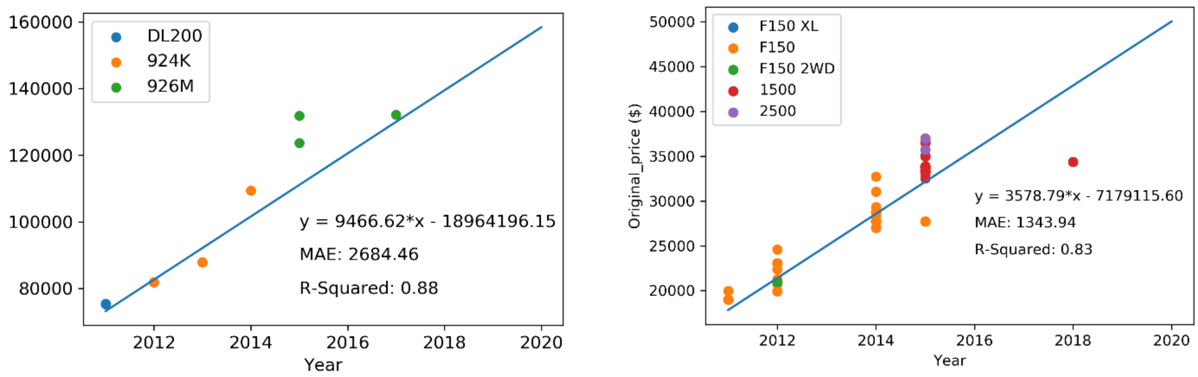
Where:

SLDP = straight-line depreciation percent (10% for the useful life of 10 years)

BVi = book value at the beginning of the age i (i =1, 2, 3,…10)

SLDP: = straight-line depreciation percent (10% for the useful life of 10 years)

BVi = book value at the beginning of the age i (i =1, 2, 3,…10)



(a) Class Code 5355 (2 yd. front-end loader) (b) Class Code 5385 (1/2 ton fleetside pickup)

Figure 9. Scatter plot and fitted regression line of the original purchase prices for equipment Class Code 5355 and Class Code 5385

4.4.2.2. Maintenance/Repair Cost and Fueling Cost

For each piece of equipment, the annual maintenance/repair costs were assumed to follow the same pattern as historical records with respect to equipment age. For example, if the equipment is bought at the beginning of Year 1 and replaced at the beginning of Year 3, the maintenance/repair costs for Year 3 would be the same as historical maintenance/repair costs that occurred in Age 1 (Year 1). However, if the equipment continues to be kept for Year 3, the maintenance/repair costs would be the same as the historical maintenance/repair costs that occurred in Age 3 (Year 3).

For each piece of equipment, the annual fueling costs completely followed a historical pattern with respect to the year that the equipment was operating over the equipment's life span. The fueling costs were independent of equipment age. In other words, it is assumed that tasks performed by equipment are kept the same whether the equipment is replaced or not. Whether to include the fueling cost in the model would not change the outcome of the optimal equipment decision because the fueling costs would be the same for both the optimal solution and original decisions. However, the authors still kept the annual fueling cost as model input in this study.

4.4.3. Dynamic Programing Model Construction

To show the process of seeking the optimal equipment decision using DP, a half-ton fleetside pickup with Equipment ID 1081414 under the Class Code of 5385 was selected. The equipment was bought in July 2015, but it did not have any operation records. At the time of this study, the records for 2020 were not complete. Therefore, only the records that occurred between 2016 and 2019 were used. The scenarios for both SL and DDB depreciation calculation methods were included.

4.4.3.1. DP Model for A Pickup Truck using SL Depreciation Calculation

In this scenario, an SL depreciation calculation was used. The cost profile for the original equipment decision policy is shown in Table 4.

Table 4. Cost profile of the original decision policy for a ½ ton pickup (Eq. ID 1081414) using SL depreciation

Year	Age	Fueling Cost, $	M/R Cost, $	Depre. (SL) $	Original Annual TC, $
2016	1	409.76	202.51	3,218.90	3,831.17
2017	2	504.25	338.34	3,218.90	4,061.49
2018	3	897.24	1,086.38	3,218.90	5,202.52
2019	4	1,700.66	2,329.40	3,218.90	7,248.96
				Total:	20,344.14

As mentioned above, this study assumed that maintenance/repair costs depend only on the age of equipment. Fueling costs depend only on the stage at which the equipment is operating. These assumptions can maximally mimic equipment maintenance/repair and operation scenarios. Figure 10 describes the decision network for the piece of equipment selected and the annual total cost (the sum of annual fueling, maintenance/repair, and depreciation costs) associated with each decision at each stage is assigned to each immediate arc.

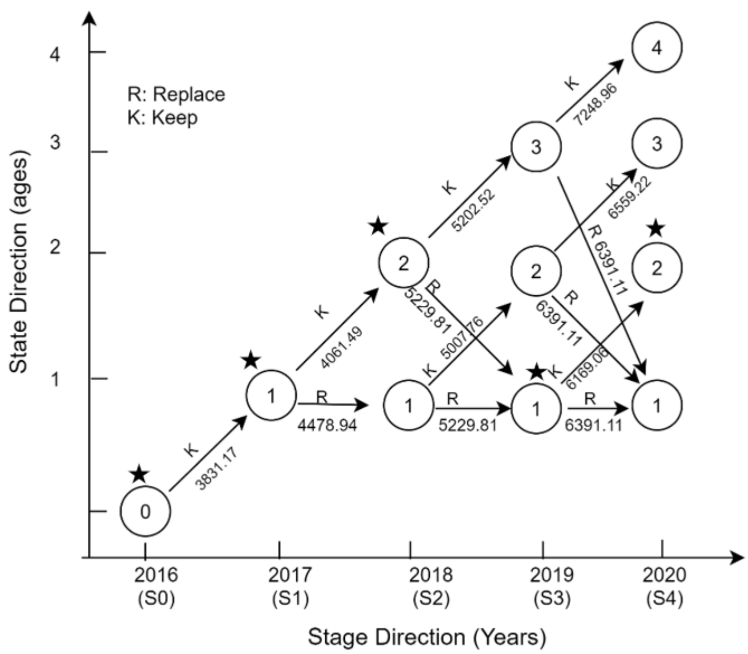


Figure 10. Decision network with costs on arcs

Using the backward recursive function, the problem-solving procedures are presented below (Tables 5 to 7).

The calculation starts from the last, n=4 (Stage 4): $f_4^* \, (S4 = 1; 2; 3; or\ 4) = 0$. Then, move backward and perform the calculation at Stage 3 (Table 2) until Stage 0 is reached.

Table 5. Optimal solutions for states at n=3 (Stage 3)

Stage 3 (States)	Stage 4 (States)	Solution at Stage 3: $f_3(S_3)$	$f_3^*\ (S_3)$	S_4^*
3	4	7,248.96	-	-
3	1	6,391.11	✓	1
2	3	6,559.22	-	-
2	1	6,391.11	✓	1
1	2	6,169.06	✓	2
1	1	6,391.11	-	-

Note: S_4^* denotes the states at Stage 4 that contribute to the optimal solution for all states at Stage 3.
$f_3(S_3)$ denotes all solutions at all states at Stage 3, and $f_3^*\ (S_3)$ denotes the optimal solutions at Stage 3.

Table 6. Optimal solutions for states at n=2 (Stage 2)

Stage 2 (States)	Stage 3 (States)	Solution at Stage 2: $f_2(S_2)$	$f_2^*\ (S_2)$	S_3^*
2	3	11,593.63	-	-
2	1	11,398.87	✓	1
1	2	11,398.87	✓	2
1	1	11,398.87	✓	1

Table 7. Optimal solutions for states at n=1 (Stage 1)

Stage 1 (States)	Stage 2 (States)	Solution at Stage 1: $f_1(S_1)$	$f_1^*\ (S_1)$	S_2^*
1	2	15,460.36	✓	2
1	1	15,877.81	-	-

Therefore, n=0 (Stage 0): $f_0(S_0) = 15{,}460.36 + 3{,}831.17 = 19{,}291.53$

Based on the above-described procedures, the shortest path can be found. The decision nodes on the shortest path are marked with stars (Figure. 10). The completed cost profile for the optimized replacement strategy is presented in Table 8. Compared with the original plan (Table 4), the optimized solution for the equipment over a 4-year life span can save about $1,053.

Table 8. Optimized costs profile suggested by DP model using SL depreciation for a ½ ton pickup (Eq. ID 1081414)

Stage	Year	State	Decision	Fueling Cost $	M/R $	Depre. $	Optimized Annual TC $
0	2016	0	Keep	409.76	202.51	3,218.90	3,831.17
1	2017	1	keep	504.25	338.34	3,218.90	4,061.49
2	2018	2	Replace	897.24	202.51	4,130.06	5,229.81
3	2019	1	Keep	1,700.66	338.34	4,130.06	6,169.06
4	2020	2	-	-	-	-	-
						Total	19,291.53

4.4.3.2. DP Solution for the Same Equipment using DDB Depreciation Calculation

To find out the optimal solution for the same equipment using a DDB depreciation method, a different depreciation cost profile was used. The cost profile for the original equipment decision considering the DDB depreciation method is shown in Table 6.

Table 9. Cost profile of the original decision policy for a ½ ton pickup (Eq. ID 1081414) using DDB depreciation

Years	Age	Fueling Cost, $	M/R, $	Depre., $	Original Annual TC, $
2016	1	409.76	202.51	5,403.36	6,015.63
2017	2	504.25	338.34	4,322.69	5,165.28
2018	3	897.24	1,086.38	3,458.15	5,441.77
2019	4	1,700.66	2,329.40	2,766.52	6,796.58
				Total	23,419.26

Using the same DP approach, it turned out that the original plan was the optimal solution. No replacement was suggested when using the DDB depreciation calculation method. The difference in the decisions between the two methods lies in the difference in maintenance/repair costs as well as the difference in depreciation costs between two consecutive years. For the original plan, the M/R costs increased by 748.03 from 2017 to 2018 and by 1243.02 from 2018 to 2019, respectively. Using the SL depreciation method, despite the slight increase in depreciation costs, the replacement can avoid big M/R costs incurred in 2018 and 2019. However, for the DDB method, keeping the equipment throughout the study life span is the most optimal solution because the DDB methods tend to depreciate the equipment more in the first couple of years. Despite the increase in M/R costs in 2018 and 2019, it still would not justify the replacement to avoid increased M/R costs.

The DP approach was applied to all the equipment under Class Codes of 5355 (2 Yd. front-end loader) and 5385 (1/2 ton fleetside pickup trucks) purchased between 2011 and 2018 using both SL and DDB depreciation methods. The results will be presented in Section 5.

4.5. Own-Rent/Lease Decision Metrics

Exploratory data analysis has been widely used by the data science field to analyze and investigate data sets and summarize their main characteristics for pattern recognition, anomalies identification, and hypothesis tests. Target variables and predictive variables are correlated or examined for their relationships. In this study, rental rates of dollar per hour/mile are target variables and predictive variables are total cost, utilization, and other information related to the equipment. Then, the best predictive variable can be selected as the benchmark metric.

When considering own, rent, or lease a piece of equipment, the economic factor might not be the only factor to consider. In the literature, one can easily identify the pros and cons associated with each of the equipment souring methods. In this study, the research team only evaluated the alternatives from the perspective of economics. Based on the available market equipment rental rates identified by the research team, threshold values associated with the selected metrics can be established. The computed threshold value can be used as a metric to make proper own-rent/lease decisions.

5. ANALYSIS AND FINDINGS

In this section, the results of the life cycle cost analysis, DP replacement models, the updated rental rates, and the own-or-rent/lease recommendations will be presented. Other than the calculation for the rental rates, the rest of the analyses was performed on the two types of frequently used equipment under Class Codes 5355 (2 Yd. front-end loader) and 5385 (Fleetside Pickup truck). The front-end loaders are charged by hours and the pickup trucks are charged by miles.

5.1. Life Cycle Cost Analysis (LCCA)

When performing LCCA, including the relationship between rental rate and equipment age was performed on equipment Class Code 5355 and 5285

5.1.1. Equipment Class Code 5355

There are 70 pieces of equipment in Class Code 5355 with ages varying from 4 to 10 as of the Year 2020, which allow a continuous life cycle analysis from age 1 to age 10 with consideration of cost variability from the different equipment pieces. Equipment rental rate ($/hour) after the first year of use has a huge variability, ranging from more than $700/hour to less than $20/hour irrespective of the deprecation methods used (Figure 11). The variability reduces very quickly starting from the second year and it becomes very stable after 5 years of use using the straight line (SL) method so that there is no clear decreasing trend ($p = 0.29$ for ages 5 -10). However, the double declining balance (DDB) method has a clear decreasing trend in the cost rate for the whole life span of the equipment ($p < 0.05$). This phenomenon happens because a large portion of deprecation occurs in the early life of equipment while minimal depreciation is ascribed to later years in the DDB method. Figure 12 describes the trend of the mean rental rates over time. Again, a continuously decreasing trend shows for the DDB method but not for the SL method. However, both methods produced an equal mean in the rental rate for the equipment pieces that were 10 years old in Class Code 5355.

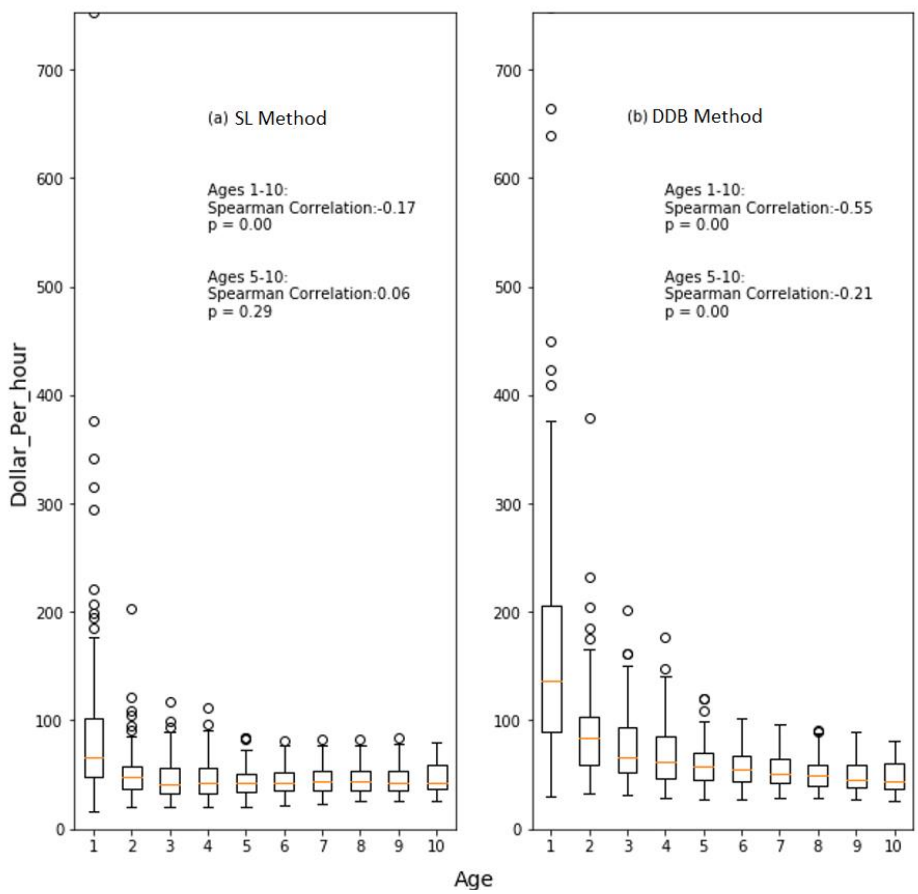


Figure 11. The boxplots of rental rates over time for equipment Class Code 5355

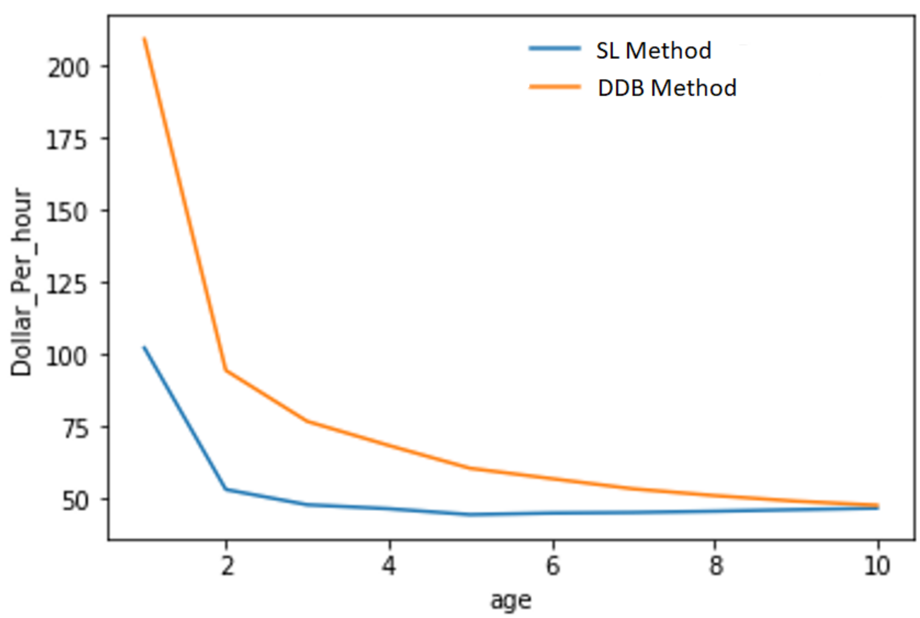


Figure 12. The mean rental rates over time for equipment Class Code 5355

The LCCA suggests that the rental rate of the equipment in Class Code 5355 decreases very fast in the first five years of life span with both SL and DDB depreciation methods. It keeps decreasing from age 5 to age 10 in the DDB while mostly keeping constant in the Straight-Line method. According to the current dataset, no matter which depreciation method is considered, it is better to keep the equipment until the end of the useful life for an economic purpose. The front-end loaders examined in this study were specified by ODOT with a 10-year useful life. Since this study only considered the equipment purchased after 2010, the number of front-end loaders that were 10 years old was still limited. The rental rates should be kept updated so that more data points can be accumulated. A similar analysis should be revisited in the future to see if the same trend holds.

5.1.2. Equipment Class Code 5385

There are 543 (449 with valid data) pieces of equipment in Class Code 8385 charged by dollars per mile, with ages varying from 2 to 10 as of the Year 2020, which allows a continuous life cycle analysis of all equipment pieces from Age 1 to Age 10 with consideration of cost variability from the different equipment pieces. The rental rate ($/mile) is usually the largest in the first year with huge variability (Figure 13), which ranges from more than $25/mile to less than $1/mile irrespective of the deprecation methods of Straight-Line (SL) or Double Declining Balance (DBB). This variability primarily comes from the recorded equipment usage variability in the first year because the equipment was purchased at different months of the year. This variability reduces very quickly starting from the second year and it becomes very stable from the eighth year for both the SL and DBB methods. Both depreciation methods have a clear decreasing trend in the cost for the whole life span of the equipment ($p < 0.01$). The means of the cost rate in each age group are different but change in a similar pattern with a clear decreasing trend for the two methods (Figure 14). The two methods produced an equal mean in the rental rate at the age of 10 for this equipment class, which is close to $0.5/mile (Figure 14).

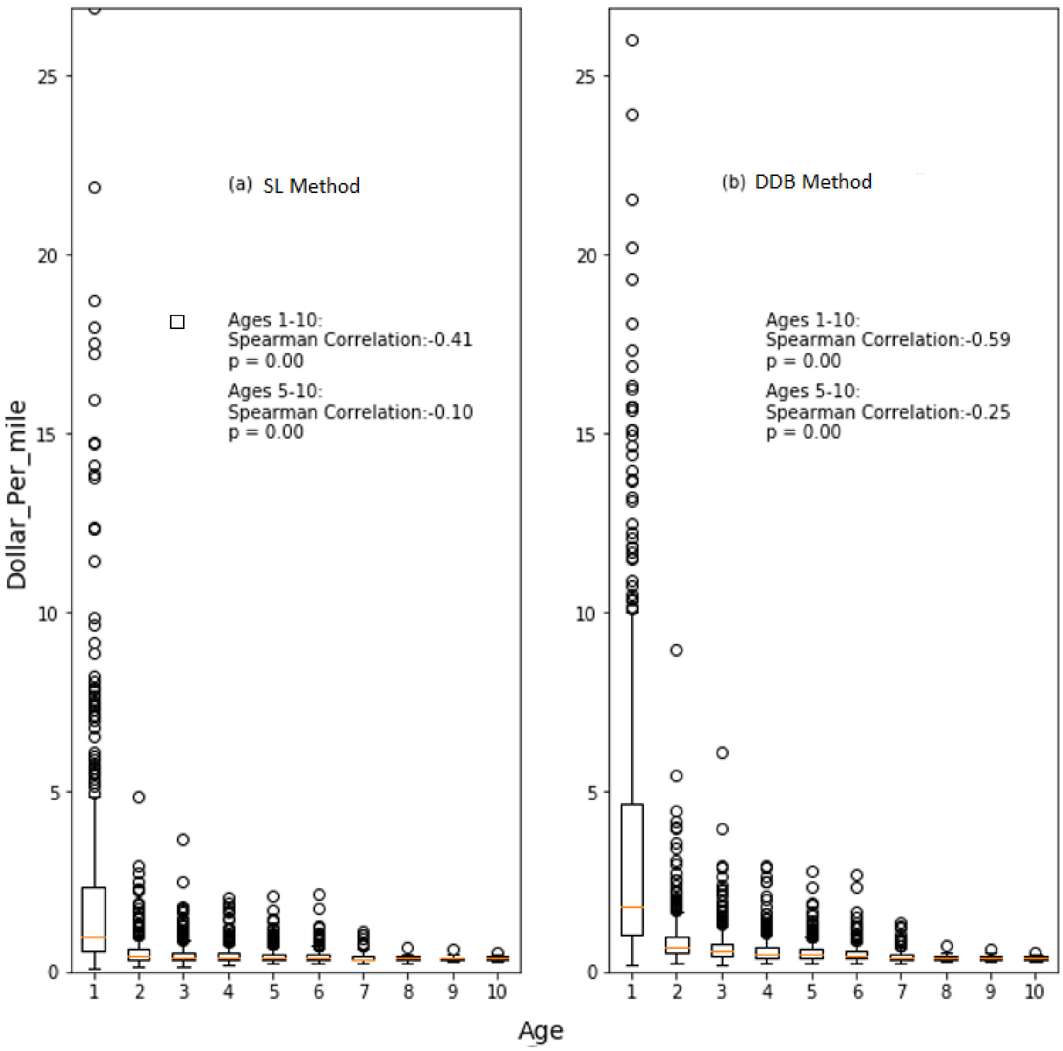


Figure 13. The boxplots of rental rates over time for equipment Class Code 5385

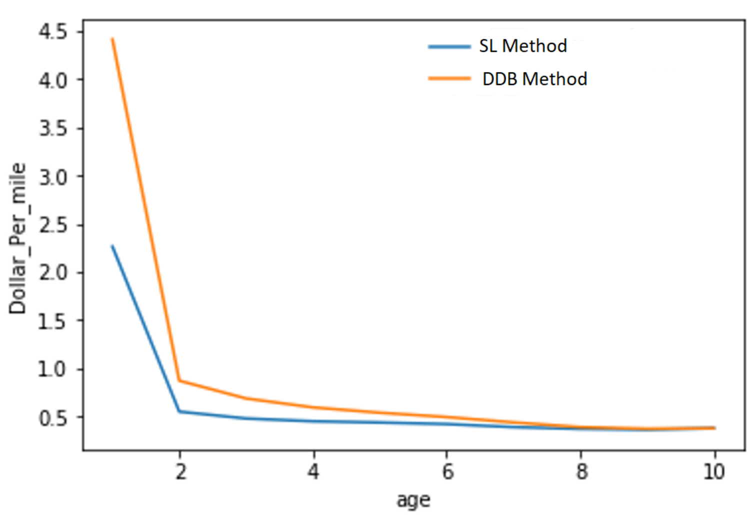


Figure 14. The mean rental rates over time for equipment Class Code 5385

The LCCA suggests that the rental rate of the equipment in Class Code 5385 decreases very fast after one year of using both SL and DDB depreciation methods. It keeps decreasing from Age 2 to Age 10 for both SL and DDB methods. Regardless of the depreciation methods used, the mean rental rate at Age 10 is the same. According to ODOT's current usage practice of the equipment, keeping the equipment for Class Code 5385 until the end of the specified useful life –10 years is still an economical decision.

5.2. Dynamic Programming Modeling Results

The following section presents the results of the DP applied to all equipment under the Class Codes of 5355 (2 Yd. front-end loader) and 5385 (1/2 ton fleetside pickup trucks) purchased between 2011 and 2018 using both SL and DDB depreciation methods. The summary statistics of the results are presented and discussed.

5.2.1. Replacement Suggestions for Class Code 5355 Using SL Depreciation

Using the SL depreciation method, the DP modeling resulted in replacement strategies for 12 out of 70 pieces of equipment under Class Code 5355 (Fig. 15). The other 58 pieces of equipment would not need any replacement strategies since they are cost-optimal under current management activities. The majority of the front-end loaders purchased in earlier years (e.g., 2011, 2012, and 2013) did not need a replacement. However, the results show that equipment bought in more recent years (e.g., 2014, 2015, and 2017) all need a replacement at a certain time point in order to reduce the cumulative annual cost that occurred on the equipment. By examining the historical records of those individual cases, a big jump in maintenance/repair costs was observed in the mid-year of the study life span.

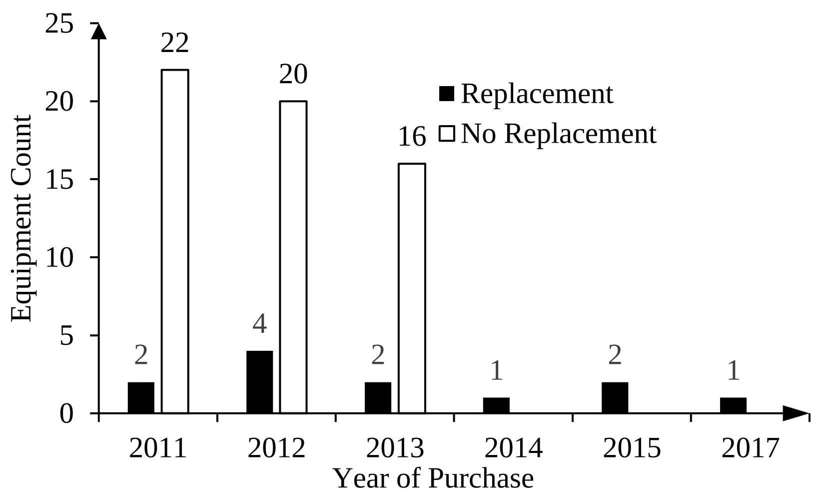


Figure 15. Equipment count with replacement suggestions vs. no replacement suggestions per purchase year for Class Code 5355 (2 Yd. front-end loader) using SL depreciation

With the application of replacement strategies suggested by the DP approach using the SL depreciation, the average cumulative total cost for each piece of equipment could be reduced roughly by $7,000 within the study period between 2011 and 2019 among the 12 front-end loaders suggested for replacement at a time point under Class Code 5355 (Fig. 16a). Six loaders were recommended for replacement at the end of the first year to avoid high costs in maintenance and repairs in later years (Figure. 16b).

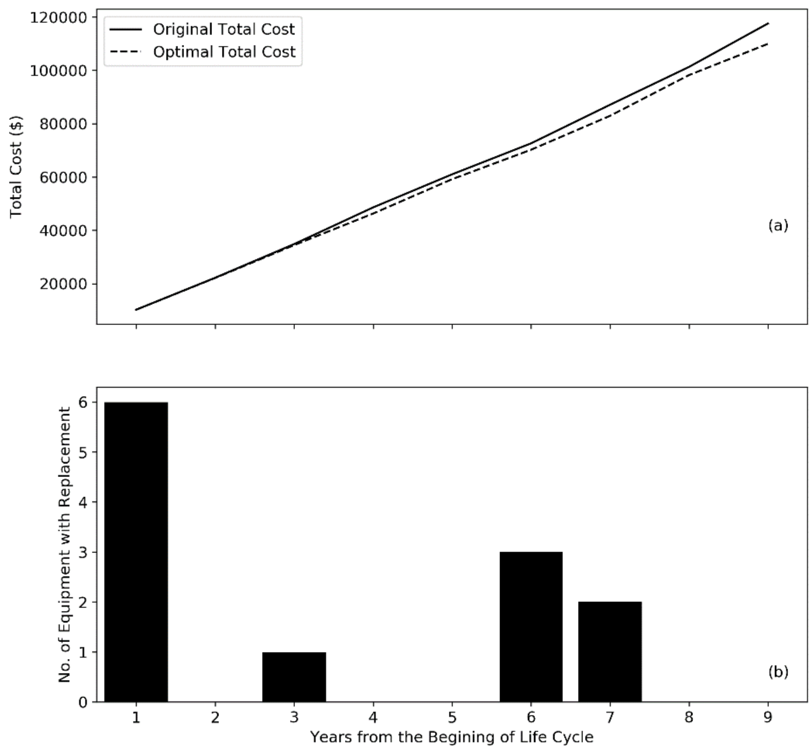


Figure 16. a) Average cumulative cost of optimized vs. original decision policies over the study life span for 2 yd. front-end loaders (Class Code 5355) recommended for replacement by DP using SL depreciation calculation; b) Age distribution of the equipment's first replacement.

5.2.2. Replacement Suggestions for Class Code 5355 Using DDB Depreciation

With the DP modeling using the DDB depreciation calculation method, there was no recommendation for a replacement for any of the 70 2-yd. front-end loaders (Figure 17).

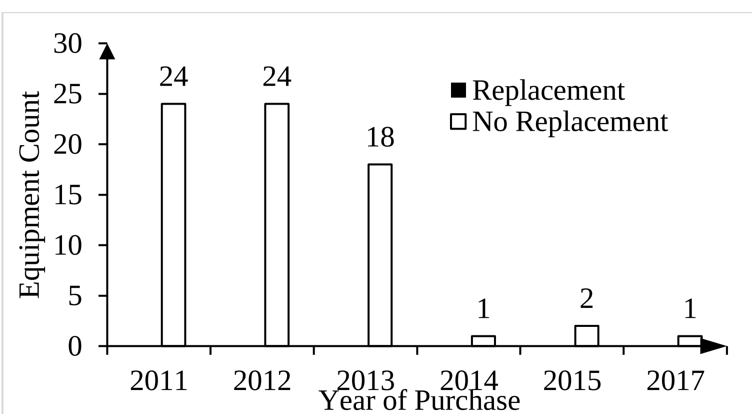


Figure 17. Equipment count with replacement suggestions vs. no replacement suggestions per purchase year for Class Code 5355 (2 Yd. front-end loader) using DDB depreciation

5.2.3. Replacement Suggestions for Class Code 5385 Using SL Depreciation

The DP modeling using the SL depreciation approach resulted in a replacement recommendation strategy for 123 (out of 449) pieces of equipment to reduce the cumulative cost over the life span for Class Code 5385 (Figure 18). The rest 326 pieces of equipment would not need any replacement since their costs are optimal under current management activities.

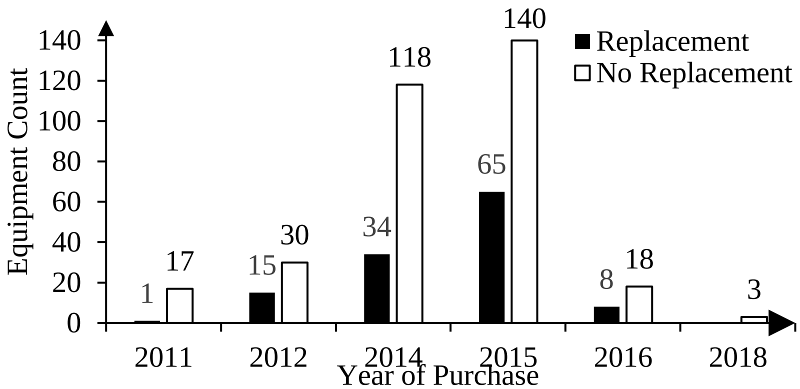


Figure 18. Equipment count with replacement suggestions vs. no replacement suggestions per purchase year for Class code 5385 (1/2 ton fleetside pickup) using SL depreciation

With the recommendation strategy suggested by the DP using the SL depreciation approach, the average cumulative cost for each pickup over the nine years could be reduced roughly by $ 8,500 among the 123 trucks under Class Code 5385 recommended for replacement at a certain time point (Fig 19 a). More than half of the 123 trucks were replaced at the end of their first year after purchase.

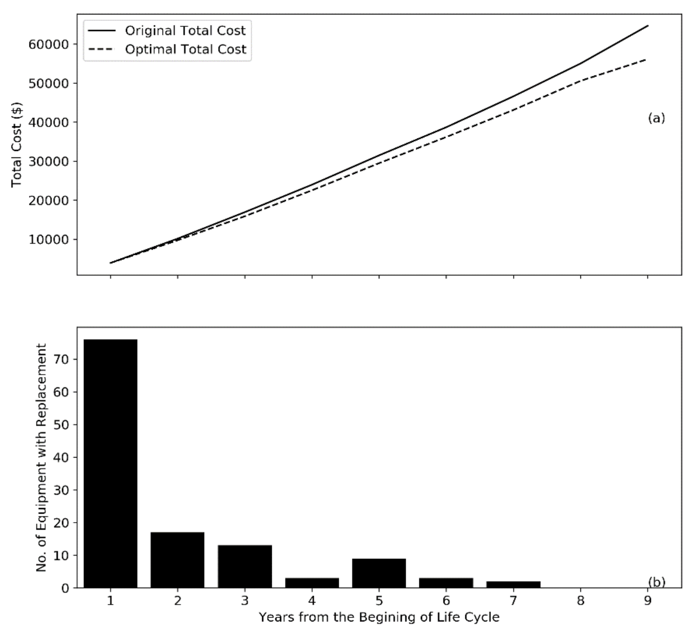


Figure 19. a) Average cumulative cost of optimized vs. original decision policies over the study life span for ½ ton fleetside pickup (Class Code 5385) recommended for replacement by DP using SL depreciation calculation; b) Age distribution of equipment's first replacement

5.2.4. Replacement Suggestions for Class Code 5385 Using DDB Depreciation

The trend is very similar to the result of Class Code 5355 using DP with the DDB depreciation calculation method. The number of equipment that needed a replacement was significantly reduced from 123 to 8 (Fig. 10). All the pickup trucks that needed a replacement were purchased in 2014 and 2015. On average, the cumulative cost for each piece of equipment (out of the eight pickup trucks) being considered for replacement can save approximately $6,000 over the six-year study period (Fig. 11a). All eight pieces of equipment were recommended for replacement when they became one year old (Fig. 11b.) to avoid expensive repairs when they became 4 or 5 years old.

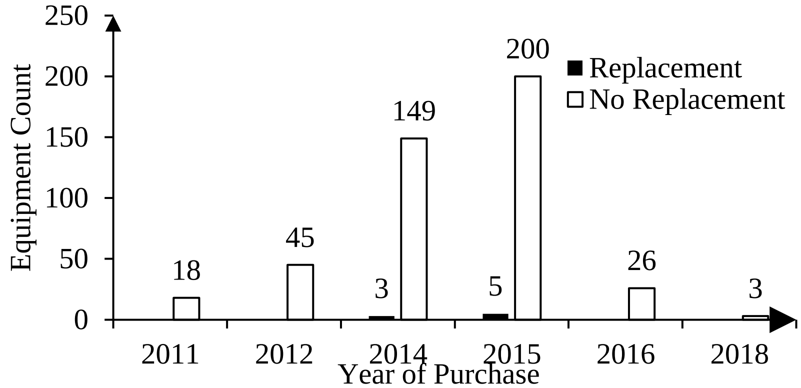


Figure 20. Equipment count with replacement suggestions vs. no replacement suggestions per purchase year for Class Code 5385 (1/2 ton fleetside pickup) using DDB depreciation

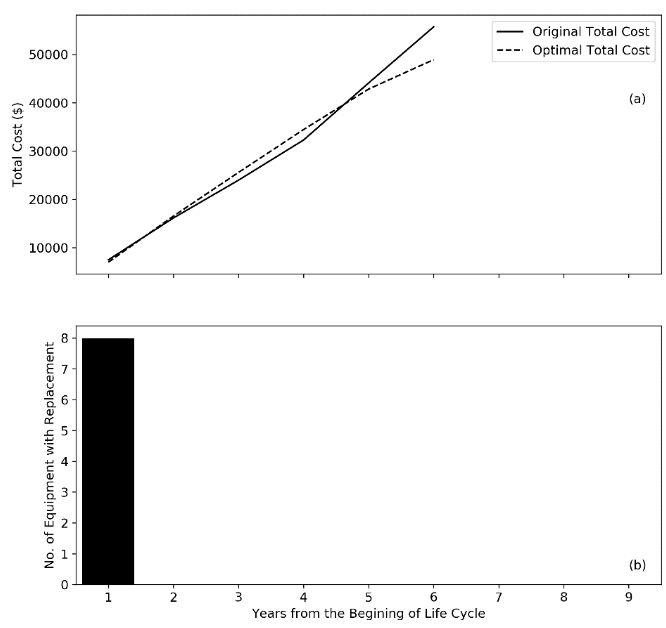


Figure 21. a) average cumulative cost of optimized vs. original decision policies over the study life span for ½ ton fleetside pickup (Class Code 5385) recommended for replacement by DP using DDB depreciation calculation; b) Age distribution of equipment's first replacement

5.2.5. Comparison of LCCA and DP Replacement Result

The life cycle cost analysis typically evaluates the equipment cost rate (expressed in dollar per mile or hour) change over the life cycle to find the lowest cost point. Then, the equipment is suggested for replacement when reaching the lowest cost point. However, the DP approach for equipment cost optimization takes a different approach. It seeks optimal cumulative cost solutions through a wider range of "Keep-Replace" alternatives over a specified equipment life span.

The DP approach also indicates that suggestions for replacement occur when a significant amount of maintenance/repair costs are incurred for some specific equipment pieces in their mid-year life span. The DP approach shows that despite the increase in the purchase price for newer equipment, it is still worth upgrading in order to avoid large maintenance later. It is also possible to replace the equipment earlier in its life to reach the optimal cost in a life span.

Based on the different depreciation calculation methods used along with the DP approach, the decision recommendations can change dramatically. Using the DDB depreciation approach, the number of equipment that needs replacement decreases significantly. It was noted that, among the front-end loaders, no replacement was recommended using the DDB depreciation calculation method. For the SL depreciation calculation method, 12 out of 70 2-yd. front-end loaders were recommended for a replacement during the study life span. The same trend was observed on ½ ton fleetside pickup trucks. Under the SL depreciation, 27.4% (123/449) of the pickup trucks were recommended for a replacement while only 1.8% (8/449) of the pick trucks were recommended for a replacement. By comparing the replacement outcomes from both depreciation methods, one can conclude that proper estimation of depreciation is very critical to the decision outcome.

There are also limitations to the DP models presented in this research. In this study, the maintenance/repair costs were assumed to repeat their own history. However, the occurrence of repairs can have a stochastic nature. Better models to estimate maintenance/repair costs as DP model input would be recommended for future research. In addition, this study used two common depreciation calculation methods – both SL and DDB depreciation approaches – to estimate the annual depreciation as model input. When using the DP model, the practitioners are advised to use better sources of book value information that reflects the true book value of the used equipment market to improve the accuracy of annual depreciation estimation so that better decisions can be made. In addition, budget constraints were not considered for upgrading equipment in this study.

5.3. Summary of Updated Rental Rates

Tables 10 and 11 present the rental rates at the class code level updated for the most frequently used equipment charged by hours and miles, respectively. Per ODOT's practice, the SL deprecation method was used for the rental rate calculation presented in this subsection. These rates were computed based on equipment's depreciation and operating costs incurred since the purchase date. However, equipment without key records (such as original price, working hours, etc.) missing was excluded in the calculation process. Therefore, the rental rates for the class codes with a large sample size (e.g. the class code 5385 with 450 pieces) would have higher accuracy than those class codes with a smaller sample size (usually less than 10 pieces). Most of the rates are slightly different from ODOT's previous report; however, the differences are comparable for the class codes with larger enough sample sizes. For instance, the updated rental rate for Class Code 5355 is $47. 3/hour ($44.6/hour in the previous report); for Class Code 5385, the rate is $0.41/mile which is estimated as $0.35/mile in the previous report.

Table 10. Rental rates for equipment charged by hours using SL depreciation method

Class Code_Id	Spec Number	Description	Deprec. Rate	Operation Cost	Rental Rate	Equipment Count
5095	EQ 03-07	Welder	$10.03	$18.19	$28.22	3
5096	EQ 04-02	Air Compressor	$26.25	$12.65	$38.90	3
5098	EQ 04-04	Air Compressor	$31.90	$57.02	$88.92	13
5101	EQ 04-08	Air Compressor	$53.70	$1.69	$55.40	3
5102	EQ 04-09	Air Compressor	$21.15	$10.46	$31.61	14
5104	EQ 06-02	Asphalt Distributor	$181.87	$18.88	$200.74	6
5105	EQ 06-03	Asphalt Distributor	$294.83	$34.55	$329.38	3
5121	EQ 11-03	Backhoe-Loader-Tractor Unit	$45.86	$7.87	$53.74	3
5123	EQ 11-05	Backhoe-Loader-Tractor Unit	$41.63	$10.25	$51.88	21
5135	EQ 15-06	Spreader-Heavy Duty	$16.88	$6.45	$23.33	14
5136	EQ 15-07	Spreader-Heavy Duty	$30.82	$15.40	$46.22	123
5189	EQ 30-13	Power Sweeper	$50.99	$20.02	$71.01	26
5237	EQ 42-13	Motor Grader	$69.81	$32.54	$102.34	18
5238	EQ 42-14	Motor Grader	$57.76	$21.22	$78.98	60
5259	EQ 46-01	Brush Chipper	$66.51	$9.17	$75.68	3
5260	EQ 46-02	Brush Chipper	$0.73	$5.84	$6.57	3
5261	EQ 47-06	Mowing Attachment	$7.79	$6.57	$14.36	53
5266	EQ 47-11	Mowing Attachment	$13.05	$10.29	$23.34	11
5293	EQ 58-03	Derrick Unit	$113.91	$0.46	$114.37	4
5319	EQ 60-16	Roller	$149.75	$13.07	$162.82	8
5355	EQ 77-02	Front End Loader	$34.05	$13.26	$47.31	70
5357	EQ 77-04	Skid Steer Loader	$35.54	$19.34	$54.88	24
5363	EQ 78-01	Snowplow	$31.66	$9.34	$41.00	144
5375	EQ 82-10	Wheel Tractor	$15.60	$17.65	$33.25	84
5378	EQ 82-13	All-Terrain Vehicle	$2.94	$5.13	$8.07	3

Table 11. Rental rates for equipment charged by miles using SL depreciation method

Class Code ID	Description	Spec Number	Size	Deprec. Rate	Operation Cost	Rental Rate	Equipment Count
5086	Auto - Factory Color	Eq 01-02	Four Door Sedan-Mid Size	$0.17	$0.16	$0.33	8
5089	Auto - White Color	Eq 02-02	Four Door Sedan-Mid Size	$0.36	$0.14	$0.51	25
5090	Auto - White Color	Eq 02-03	Four Door Sedan-Compact	$0.57	$0.19	$0.77	8
5385	Pickup	Eq 84-01	1/2 Ton Fleetside	$0.23	$0.18	$0.41	450
5386	Pickup	Eq 84-02	3/4 Ton Fleetside	$0.17	$0.27	$0.44	15
5394	Pickup	Eq 84-16	1 Ton, Dual Rear	$0.42	$0.40	$0.82	26
5395	Pickup	Eq 84-17	Fullsize	$0.18	$0.22	$0.40	53
5399	Pickup	Eq 84-22	15,000 GVW	$0.45	$0.52	$0.97	3
5401	Van-Mini	Eq 85-04	4900 GVW	$0.28	$0.24	$0.52	4
5407	Van	Eq 85-13	8500 GVW	$0.11	$0.39	$0.49	9
5419	Truck - Maintenance	Eq 86-23	2 Ton W/Steel Flat Bed (86-B-6)	$0.37	$0.56	$0.93	3
5420	Truck	Eq 86-25	24000 GVW - Diesel	$0.25	$0.43	$0.67	3
5428	Truck - Tractor	Eq 86-40	3 Ton - Diesel - Haul	$0.74	$0.98	$1.72	10
5429	Truck - Diesel-Haul	Eq 86-41	3 Ton Diesel	$0.80	$0.95	$1.75	3
5430	Truck	Eq 86-42	41000 GVW - Diesel	$1.38	$2.38	$3.76	3
5433	Truck	Eq 86-46	27,500 GVW-Mid Range	$0.29	$0.95	$1.24	4
5434	Truck	Eq 86-47	24000 GVW -Diesel	$0.64	$0.86	$1.50	39
5435	Truck	Eq 86-48	41000 GVW -Diesel	$0.66	$0.83	$1.49	235
5442	Crew Cab Pickup	Eq 88-01	3/4 Ton	$0.20	$0.29	$0.49	196
5443	Crew Cab Pickup	Eq 88-02	1 Ton	$0.27	$0.40	$0.67	49
6497	Crew Cab Pickup	Eq 88-03	1/2 Ton	$0.23	$0.20	$0.43	36

5.4. Own-Rent/Lease Decisions

This section presents an example of the established metric for own-rent/lease decisions for equipment from Class Codes 5355 and 5385 from the economic perspective. The suggestions for ODOT are discussed. In addition, non-economic related factors are also discussed.

5.4.1. Own-Rent/Lease Economic Decisions

The equipment "rental rate" described earlier is a function of equipment utilization (mileage or operation hours). If a piece of equipment is below a typical level of utilization, the calculated rental rate would be higher than normal rental rates. If that is the case, keeping the equipment would not be economical. Instead, lease or rent might be a good alternative. Using the method described in Section 4.5, effective measures to determine costly equipment were identified. Again, equipment in Class Codes 5355 and 5385 was used as examples to demonstrate the procedure. It should be noted that the rental rate calculated for two equipment classes used the SL depreciation method as it has been used by ODOT for generating its equipment rental rates. Through exploratory analysis, it was found that the average annual hours/miles that the equipment operated is a good predictor for the rental rates.

Figure 22a shows the cumulative density function of the rental rate from equipment in Class Code 5355 and the strong correlation between average annual hours and cost rates is shown in Figure 22b. In order to have an apple-to-apple comparison with rental companies' quotes, the fueling cost was excluded from the rental rate calculation.

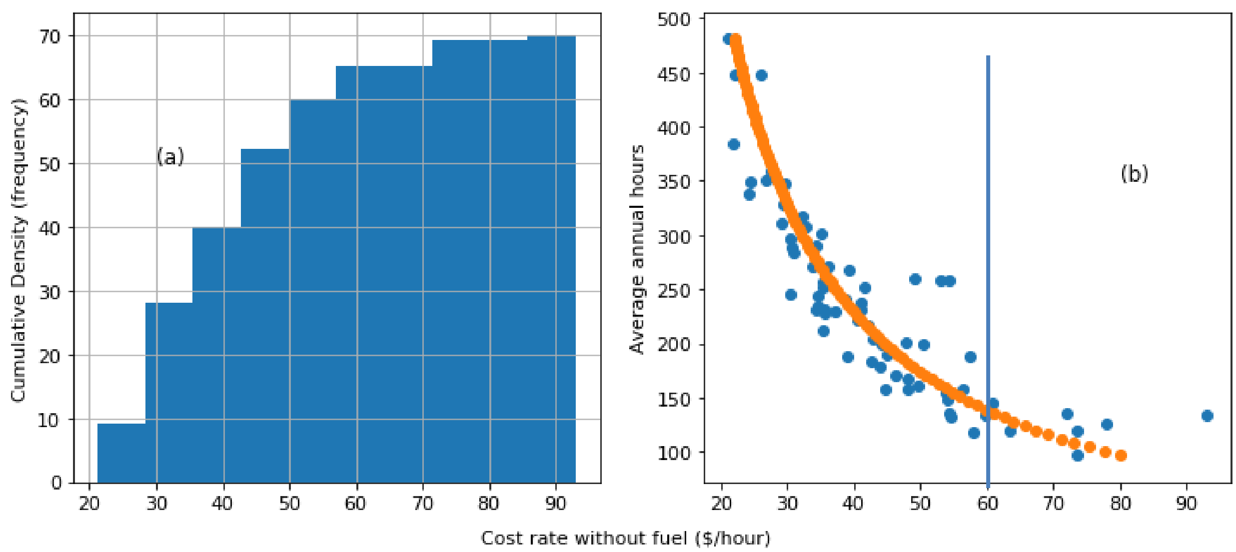


Figure 22. a) The cumulative density function of rental rates; b) The relationship between average annual operating hours and rental rate (excluding fueling cost) for equipment in Class Code 5355 (2 Yd. front-end loader)
Note: The blue line represents the market rental quote referenced online sources

The research team visited three online equipment rental companies' (DOZE, BingRentz, and Ada Sales and Rental), quotes of similar equipment on hourly, daily, weekly, and months were referenced. To be on the conservative side, a front-end loader market rental quote of around $60/hour was used in this study. According to Figure 22b, the rate of $60/hour corresponds to 150 hours per year. Therefore, when the equipment's annual operating hour is less than 150 hours, the rental rate for owning the equipment without fueling could be greater than $60/hour. In other words, renting would be a better alternative. By examining the average annual operating hours of the equipment Class Code 5355, 10% of the front-end loaders in Class Code 5355 could be considered for renting.

Similarly, the cumulative density function of the rental rates (excluding fueling costs) from equipment for class code 5385 and the relationship between average annual mileage and rental rates are shown in Figure 23. The research team referenced the rental quote of an 8 ft pickup truck from U-haul (0.59 mile + $19.95/day). If a 125-mile mileage is assumed, the referenced market rental rate would be $0.75/mile. Using Figure 23b, $0.75/mile corresponds to 5,000 miles per year. When a pickup truck usage is less than 5000 miles per year, the cost rate without fueling could be greater than $0.75/mile, and renting would be considered as a more economical alternative. Among the pickup trucks under Class Code 5385 purchased since 2010, less than 1% (12 out of 449) of current equipment could be considered for renting.

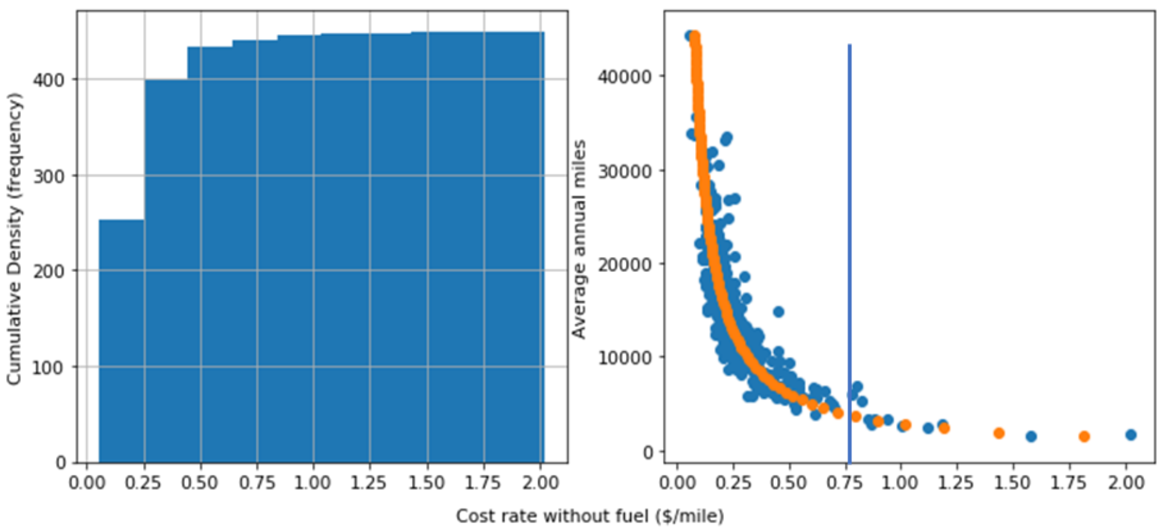


Figure 23. a) The cumulative density function of rental rates; b) The relationship between average annual operating hours and rental rate (excluding fueling cost) for equipment in Class Code 5385 (1/2 ton pickup trucks)
Note: the blue line represents the rental quote from U-Haul for 8 ft pickup truck

5.4.2. Recommendations for ODOT

Owning equipment is the primary equipment sourcing approach adopted by ODOT. However, idling equipment more than necessary could drive up the equipment “rental rate”. By using the procedures developed in this section, the Department can establish similar metrics for each class code. They can be used as a primary screening process to identify candidates that rental or leasing could be a better fit. The Department can periodically report the statistics of the established metrics for each piece of equipment. Then, anomaly equipment can be identified, and further actions can be recommended. Not necessarily, all underutilized equipment should be suggested for rental. For example, if a piece of equipment located in a field office is constantly underused compared with other similar equipment and the underutilization is the result of a long-standing mechanical issue, then proper measures should be identified and taken. If the equipment is beyond repair, proper disposal measures will be taken.

5.4.3 Non-economic Factors Related to Rent/Lease

Economics should not be the only criterion for own-rent/lease decisions. Rent/lease also comes with some advantages. Under the following circumstances, the rent/lease should be considered:

- The equipment is only for short-term use;
- The equipment is urgently needed, and the purchasing may take a long lead time;
- The mobilization distance is too long, and the equipment can be rented nearby; and
- The equipment owner has not made up his/her mind in purchasing a particular brand and model. Rent/lease can provide the flexibility to try out different manufactures and models before finalizing the purchase decision.

6. CONCLUSIONS

Using the data recorded in ODOT's equipment fleet management system, this research directly addresses the need of ODOT to strategically improve its equipment management practices. Particularly, LCCA, dynamic programming, and machine learning techniques such as exploratory data analysis, data imputation, important measures, and anomalies detection were used to generate equipment rental rates at the class code level, suggest equipment replacement decisions, and develop a framework for making own-rent/lease decisions.

Throughout the research, the LCCA, DP, and own-rent/lease models were applied to two class codes of equipment 5355 (2 yd. front-end loader) and 5385 (1/2 ton pickup trucks) Life cycle economic analysis was used to determine the best time to replace equipment in its life cycle at the class code level. Dynamic programing models, specifically deterministic dynamic programming, were developed to determine the best replacement policy for individual pieces of equipment. In addition, an own-rent/lease strategy comparison was also carried out so that manager could better decide if it is better to own or rent equipment pieces for these two class codes. The results and recommendations for equipment management practices are as follows:

1. The LCCA shows that the rental rate of equipment Class 5355 decreases very fast in the first five years of life span with both the SL and DDB depreciation methods. The rental rate keeps decreasing from age 5 to age 10 with the DDB method but keeps relatively flat in using the SL method. The rental rates using two methods all converge at the same rate at Age 10. This suggests that it is economic to keep the equipment until the end of useful life (10 years) specified by ODOT.

2. The LCCA for Class Code 5385 suggests that the rental rate of this class keeps decreasing over time no matter which depreciation methods were used. A similar strategy would be proposed for Class Code 5385 that it is economical to keep the equipment until the end of useful life (10 years) specified by ODOT.

3. The LCCA looks for minimal equipment rental rate over the entire life cycle of the equipment. However, DP seeks the optimal solution through a series of "keep-replace" scenarios over the study period. The dynamic programming approach was applied to all individual equipment under the two class codes. Using the DDB depreciation approach, the number of equipment that needs replacement decreases significantly. It was noted that, among the front-end loaders, no replacement was recommended using the DDB depreciation calculation method. For the SL depreciation calculation method, 12 out of 70 2-yd. front-end loaders were recommended for a replacement during the study life span. The same trend was observed on ½ ton fleetside pickup trucks. Under the SL depreciation, 27.4% (123/449) of the pickup trucks were recommended for a replacement while only 1.8% (8/449) of the pick trucks were recommended for a replacement using the DDB depreciation. Therefore, proper estimation of the depreciation cost is the key to find optimal replacement strategies.

4. For the two class codes of equipment studied, the average annual mileage/operating hours is a good predictor of equipment rental rate. The average annual mileage/operating hours can be used as the most important factor to identify anomaly equipment with an exceptionally high rental rate. Proper measures, such as renting or leasing, should be

examined. However, economic factors should not always be the only factor to evaluate when considering own-rent/lease alternatives. Other non-economic factors should be evaluated as well.

Although detailed replacement and own-rent/lease studies focused on equipment in Class Codes 5355 and 5385, the rest equipment classes were also investigated, and their rental rates (including both ownership cost and operation cost) were calculated. By using the approaches demonstrated in this study, ODOT can perform similar studies for the rest of the equipment classes in order to provide comprehensive management decisions for the entire fleet.

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APPENDICES

Query 1: Procedure "load_dp" for depreciation rate calculation

```
CREATE DEFINER=`root`@`localhost` PROCEDURE `Load_dp`()
begin
DECLARE counter,odo_yr,odo_mon,odo_day INT DEFAULT 1;
DECLARE dt,eq,byr,ul,ov,q,actv int DEFAULT 1;
declare ddb,bv,slv double default 0;
select count(*) from equip_info into dt;
loop1: WHILE counter <= dt DO
        select EQUIPMENT_ID,Buy_Year, if(USEFUL_LIFE <> 0,
USEFUL_LIFE,0),ORIGINAL_VALUE,Actual_Value,year(ODOMETER_DATE),
month(ODOMETER_DATE),day(ODOMETER_DATE) from equip_info where RowID =
counter into eq, byr,ul,ov,actv, odo_yr,odo_mon,odo_day;
        if (ul=0) then
          set counter = counter + 1;
          iterate loop1;
                    end if;
        set q = byr+ul-1;
        set bv = ov;
         while byr <= q and byr<=odo_yr do
                                        set ddb = bv*2/ul;
              set bv = bv-ddb;
              set slv = actv/ul;
              if byr=odo_yr
              then set ddb = ddb*(odo_mon*30+odo_day)/365; # adjust last year depreciation value
if sold early
                  set slv = actv/ul*(odo_mon*30+odo_day)/365;
              end if;
                                  insert into
equip_depreciation(EQUIPMENT_ID,YEAR_DP,Deprec1,Deprec2)
              values(eq,byr,slv,ddb);
              set byr = byr + 1;
                      end while;

       SET counter = counter + 1;

END WHILE loop1;
End
```

Query 2: Procedure "dp_to_all_costs" for operation costs time-series calculation according to ownership costs

```
CREATE DEFINER=`root`@`localhost` PROCEDURE `dp_to_all_costs`()
begin
### create a table to store annual cost time series
drop table if exists equip_all_cost_series;
Create table equip_all_cost_series
### first import depreciation table
select tab1.*, fueling1_cost,
fueling2_cost,maint1_cost,maint2_cost,fueling1_amount,fueling2_amount,
ifnull(Deprec1,0)+ifnull(fueling1_cost,0)+ifnull(fueling2_cost,0)+ifnull(maint1_cost,0)+ifnull(
maint2_cost,0) as
total_cost1,ifnull(Deprec2,0)+ifnull(fueling1_cost,0)+ifnull(fueling2_cost,0)+ifnull(maint1_cost,
0)+ifnull(maint2_cost,0) as total_cost2, ifnull(fueling1_amount,0)+ifnull(fueling2_amount,0) as
total_fuel from
(select * from equip_depreciation) as tab1
### join in comdata fueling tabel - fueling only part
left join
(select  EQUIPMENT_ID,sum(FUEL_AMOUNT) as fueling1_amount, sum(FUEL_COST) as
fueling1_cost, FUEL_PRODUCT_CODE_NAME,
FUEL_PRODUCT_CODE_DESC,Year(if(length(FUEL_DATE) = length('29-OCT-14
00:00:00'),str_to_date(FUEL_DATE,'%d-%b-%y %H:%i:%s'),str_to_date(FUEL_DATE,'%Y-%
m-%d %H:%i:%s'))) as Year1
from comdata_fueling_all_1
where FUEL_RATE < 4 and if(length(FUEL_DATE) = length('29-OCT-14
00:00:00'),str_to_date(FUEL_DATE,'%d-%b-%y %H:%i:%s'),str_to_date(FUEL_DATE,'%Y-%
m-%d %H:%i:%s')) <= ODOMETER_DATE
group by EQUIPMENT_ID,Year1) as fueling1
on tab1.EQUIPMENT_ID=fueling1.EQUIPMENT_ID AND tab1.YEAR_DP = fueling1.Year1
### join in equipment_fueling table
left join
(select  EQUIPMENT_ID,sum(FUEL_AMOUNT) as fueling2_amount, sum(FUEL_COST) as
fueling2_cost,Year(if(length(FUEL_DATE) = length('29-OCT-14
00:00:00'),str_to_date(FUEL_DATE,'%d-%b-%y %H:%i:%s'),str_to_date(FUEL_DATE,'%Y-%
m-%d %H:%i:%s'))) as Year1
from equipment_fueling_all_1
where if(length(FUEL_DATE) = length('29-OCT-14
00:00:00'),str_to_date(FUEL_DATE,'%d-%b-%y %H:%i:%s'),str_to_date(FUEL_DATE,'%Y-%
m-%d %H:%i:%s')) <= ODOMETER_DATE
group by EQUIPMENT_ID,Year1) as fueling2
on tab1.EQUIPMENT_ID = fueling2.EQUIPMENT_ID and tab1.YEAR_DP = fueling2.Year1
### join in maintenance data from setup_project table
left join
(select  EQUIPMENT_ID,sum(COMPLETED_COST) as maint1_cost,
Year(if(length(DATE_COMPLETED) = length('29-OCT-14
```

```
00:00:00'),str_to_date(DATE_COMPLETED,'%d-%b-%y %H:%i:%s'),str_to_date(DATE_CO
MPLETED,'%Y-%m-%d %H:%i:%s'))) as Year1
from setup_project_all_1
where if(length(DATE_COMPLETED) = length('29-OCT-14
00:00:00'),str_to_date(DATE_COMPLETED,'%d-%b-%y %H:%i:%s'),str_to_date(DATE_CO
MPLETED,'%Y-%m-%d %H:%i:%s')) <= ODOMETER_DATE
group by EQUIPMENT_ID,Year1) as maint1
on tab1.EQUIPMENT_ID = maint1.EQUIPMENT_ID and tab1.YEAR_DP = maint1.Year1
### join in maintenance data from comdata_fueling table
left join
(select  EQUIPMENT_ID,sum(FUEL_COST) as maint2_cost, Year(if(length(FUEL_DATE) =
length('29-OCT-14
00:00:00'),str_to_date(FUEL_DATE,'%d-%b-%y %H:%i:%s'),str_to_date(FUEL_DATE,'%Y-%
m-%d %H:%i:%s'))) as Year1
from comdata_fueling_all_1
where FUEL_RATE >= 4 and if(length(FUEL_DATE) = length('29-OCT-14
00:00:00'),str_to_date(FUEL_DATE,'%d-%b-%y %H:%i:%s'),str_to_date(FUEL_DATE,'%Y-%
m-%d %H:%i:%s')) <= ODOMETER_DATE
group by EQUIPMENT_ID,Year1) as maint2
on tab1.EQUIPMENT_ID = maint2.EQUIPMENT_ID AND tab1.YEAR_DP = maint2.Year1
order by EQUIPMENT_ID, YEAR_DP;

end
```

Query 3: Procedure "dollar_per_mile" for cost rate calculation of mile based equipment

```
CREATE DEFINER=`root`@`localhost` PROCEDURE `dollar_per_mile`(in equip_id int)
begin
declare t_odo,Class_Code int default 0;
declare t_fuel,mpg double default 0;
SELECT sum(total_fuel) FROM `odot-database3`.equip_all_cost_series where
EQUIPMENT_ID=equip_id into t_fuel;
Select CURRENT_ODOMETER,EQUIPMENT_CLASS_CODE_ID from equip_info where
EQUIPMENT_ID=equip_id into t_odo,Class_Code;
set mpg = t_odo/t_fuel;
set @msum1 :=0;
set @csum1 :=0;
set @msum2 :=0;
set @csum2 :=0;
# create a temporary table to store cost time series
drop table if exists temp1;
create temporary table temp1
SELECT *, mpg, Class_Code, total_fuel*mpg as annual_miles , (@csum1 := @csum1 +
total_cost1)/(@msum1 := @msum1 + total_fuel*mpg) as dollar_per_mile1_cum, (@csum2 :=
@csum2 + total_cost2)/(@msum2 := @msum2 + total_fuel*mpg) as dollar_per_mile2_cum
FROM `odot-database3`.equip_all_cost_series where EQUIPMENT_ID=equip_id;
# creat a temporary table to store annual average of the time series
#drop table if exists temp2;
#create temporary table temp2
#SELECT EQUIPMENT_ID,Class_Code, sum(total_cost1)/sum(annual_miles) as
rental_rate1,sum(total_cost2)/sum(annual_miles) as rental_rate2, mpg as mile_per_hour,
avg(annual_miles),count(*) as Current_age FROM `odot-database3`.temp1;
End
```

Query 4: Procedure “dollar_per_hour” for cost rate calculation of hourly based equipment

```
CREATE DEFINER=`root`@`localhost` PROCEDURE `dollar_per_hour`(in equip_id int)
begin
declare t_hours,Class_Code int default 0;
declare t_fuel,hpg double default 0;
declare odo_date text;
SELECT sum(total_fuel) FROM `odot-database3`.equip_all_cost_series where
EQUIPMENT_ID=equip_id into t_fuel;
Select ODOMETER_DATE,EQUIPMENT_CLASS_CODE_ID from equip_info where
EQUIPMENT_ID = equip_id into odo_date,Class_Code;
Select sum(TOTAL_HOURS) from work_orders_equipment_dc where
EQUIPMENT_ID=equip_id and DATE_WORK<=odo_date into t_hours;
set hpg = t_hours/t_fuel;
set @msum1 :=0;
set @csum1 :=0;
set @msum2 :=0;
set @csum2 :=0;
# create a temporary table to store cost time series
drop table if exists temp1;
create table temp1
select a.*,b.annual_hours,(@csum1 := @csum1 + a.total_cost1)/(@msum1 := @msum1 +
b.annual_hours) as dollar_per_hour1_cum, (@csum2 := @csum2 + a.total_cost2)/(@msum2 :=
@msum2 + b.annual_hours) as dollar_per_hour2_cum,total_fuel*hpg as annual_hours_on_feul
from
(SELECT *, hpg as hour_per_gallon, Class_Code FROM `odot-database3`.equip_all_cost_series
where EQUIPMENT_ID=equip_id) as a
left join
(Select Year(DATE_WORK) as Year1, sum(TOTAL_HOURS) as annual_hours from
work_orders_equipment_dc where EQUIPMENT_ID=equip_id and DATE_WORK<=odo_date
group by Year1) as b
on a.YEAR_DP = b.Year1;
# creat a temporary table to store annual average of the time series
#drop table if exists temp2;
#create table temp2
#SELECT EQUIPMENT_ID, Class_Code, sum(total_cost1)/sum(annual_hours) as
rental_rate1,sum(total_cost2)/sum(annual_hours) as rental_rate2,hpg as hour_per_gallon,
avg(annual_hours),count(*) as Current_age FROM `odot-database3`.temp1;
End
```

Query 5: Procedure "class_code_cost", a loop call to calculate cost rates for each piece of equipment based on their charge types (either miles or hourly based)

```
CREATE DEFINER=`root`@`localhost` PROCEDURE `class_code_cost`( in charge_types int)
begin
declare equip_id,counter,len_of_table, equip_charge_type int default 1;
SELECT count(*) FROM `odot-database3`.equip_info into len_of_table;
drop table if exists equip_dollar_per_mile_time_series;
if charge_types =2 then
create table equip_dollar_per_mile_time_series
(EQUIPMENT_ID int ,
YEAR_DP int,
Deprec1 double,
Deprec2 double ,
fueling1_cost double ,
fueling2_cost double ,
maint1_cost double ,
maint2_cost double ,
fueling1_amount double ,
fueling2_amount double,
total_cost1 double ,
total_cost2 double ,
total_fuel double ,
mile_per_gallon double ,
Class_Code int,
annual_miles double ,
dollar_per_mile1 double ,
dollar_per_mile2 double);
#drop table if exists equip_dollar_per_mile_stats;
#create table equip_dollar_per_mile_stats
#(EQUIPMENT_ID int ,
#Class_Code int,
#Rental_Rate1 double,
 #Rental_Rate2 double,
 #Mile_per_gallon double,
 #Avg_annual_miles double,
 #Current_Life int);
else
drop table if exists equip_dollar_per_hour_time_series;
create table equip_dollar_per_hour_time_series
(EQUIPMENT_ID int ,
YEAR_DP int,
Deprec1 double,
Deprec2 double ,
fueling1_cost double ,
fueling2_cost double ,
```

```
maint1_cost double ,
maint2_cost double ,
fueling1_amount double,
fueling2_amount double,
total_cost1 double ,
total_cost2 double ,
total_fuel double ,
hour_per_gallon double ,
Class_Code int,
annual_hours double ,
dollar_per_hour1 double ,
dollar_per_hour2 double,
annual_hours_on_fuel double);
#drop table if exists equip_dollar_per_hour_stats;
#create table equip_dollar_per_hour_stats
#(EQUIPMENT_ID int ,
#Class_Code int,
#Rental_Rate1 double,
#Rental_Rate2 double,
#Hour_per_gallon double,
#Avg_annual_hours double,
#Current_Life int);
end if;
loop1: while counter <= len_of_table do
        SELECT EQUIPMENT_ID,EQ_CHARGE_TYPE FROM `odot-database3`.equip_info
where RowID=counter into equip_id, equip_charge_type;
    if equip_charge_type = 2 and  charge_types =2 then
    call dollar_per_mile(equip_id);
    insert into equip_dollar_per_mile_time_series
    select * from temp1;
        #insert into equip_dollar_per_mile_stats
    #select * from temp2;
    end if;
    if equip_charge_type = 1 and  charge_types =1 then
        call dollar_per_hour(equip_id);
    insert into equip_dollar_per_hour_time_series
    select * from temp1;
    #insert into equip_dollar_per_hour_stats
    #select * from temp2;
    end if;
    set counter = counter +1;
end while loop1;
end
```

Python Programming Code for Dynamic Programming Function

```
#loading packages
import pandas as pd
import numpy as np

# define a dynamic programing function so that it can be called multiple times
def dynamic_pro_equip(inventory):
    # define the stages
    stages=inventory.age.values
    # definie the states in each stage
    options=range(len(stages)+2)

states=[options[1:2],options[1:3],options[1:4],options[1:5],options[1:6],options[1:7],options[1:8]
,options[1:9],options[1:10],options[1:11],options[1:12]]
    fsn = []
    # add one more stage to the final stage and initialize the f(Sn) tables for each stage
    for n in range(len(stages)+1):
        temp = pd.DataFrame(columns = ['Sn','Keep','Replace','f_star','Q_star'])
        temp.Sn = states[n]
        fsn.append(temp)
    fsn[len(stages)].f_star = 0
    # initiale decision matrix
    decisions=['Keep','Replace']
    # calculate f(Sn) for each stage
    for n in range(len(stages))[::-1]:
        # for each state
        for j in list(range(len(fsn[n].Sn))):
            # for each decision
            for k in decisions:
                if k == 'Keep':
                    # current stage cost is composed of depreciation (dependent on state and equip
bought year), maintanence cost (dependdent on state), and fueling cost
                    #depre = inventory.loc[inventory.age==n+2-
stages[j],'Depreciation_boughtyear'].values # SL
                    depre = inventory.loc[inventory.age==n+2-
stages[j],'Depreciation_boughtyear'].values*10*(4/5)**(stages[j]-1)*(1/5) # DDB
                    fsn[n].loc[j,k] = depre +
inventory.loc[inventory.age==stages[j],'total_maint_cost'].values +
inventory.loc[inventory.age==n+1].total_fueling_cost.values +
fsn[n+1][fsn[n+1].Sn==fsn[n].Sn[j]+1]['f_star'].values[0]
                if k == 'Replace':
                    #depre = inventory.loc[inventory.age==n+2-
stages[j],'Depreciation_boughtyear'].values
                    depre = inventory.loc[inventory.age==n+2-
stages[j],'Depreciation_boughtyear'].values*10*(4/5)**(stages[j]-1)*(1/5)
```

```
                fsn[n].loc[j,k] = depre +
inventory.loc[inventory.age==stages[j],'total_maint_cost'].values +
inventory.loc[inventory.age==n+1].total_fueling_cost.values +
fsn[n+1][fsn[n+1].Sn==1]['f_star'].values[0]
            fsn[n].loc[j,'f_star'] = fsn[n].loc[j,['Keep','Replace']].min()
            x = [decisions[m] for m in range(len(decisions)) if
fsn[n].loc[j,decisions[m]]==fsn[n].loc[j,'f_star']]
            y = len(x)
            fsn[n].loc[j,'Q_star'] = x[0]# if costs of both options are equal, choose the "Keep" option
    #sort-out the best decision at each stage and calculate optimal annual cost
    optimal_decisions = pd.DataFrame(columns = ['Sn','f_star','Q_star'])
    for i in range(len(fsn)-1):
        temp = fsn[i][['Sn','f_star','Q_star']]
        if i ==0:
            optimal_decisions = pd.concat([optimal_decisions,temp])
            opv = optimal_decisions.loc[i,'Q_star']
        else:
            if opv =='Keep':
                temp = fsn[i][fsn[i].Sn==optimal_decisions.loc[i-1,'Sn']+1][['Sn','f_star','Q_star']]
                optimal_decisions = pd.concat([optimal_decisions,temp])
                optimal_decisions.reset_index(drop=True,inplace=True)
            else:
                temp = fsn[i][fsn[i].Sn==1][['Sn','f_star','Q_star']]
                optimal_decisions = pd.concat([optimal_decisions,temp])
                optimal_decisions.reset_index(drop=True,inplace=True)
            opv = optimal_decisions.loc[i,'Q_star']
    opt_cost=[optimal_decisions.f_star[i]-optimal_decisions.f_star[i+1] for i in
range(len(optimal_decisions)-1)]
    opt_cost.append(optimal_decisions.f_star.iloc[-1])
    optimal_decisions['total_cost_optimal']=opt_cost
    return pd.concat([inventory.reset_index(),optimal_decisions],axis=1)
```