DEPARTMENT OF TRANSPORTATION

Remaining Service Life Asset Measure, Phase 2

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FEBRUARY 2022

Research Report
Final Report 2022-02

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16. Abstract (Limit: 250 words)
The main objectives of phase 2 of this project were to obtain relevant data to calculate the percent remaining service life interval (PRSI) and two additional metrics and to perform Markov chain analysis and dynamic programming to determine how much time and funding is required to bring the system to a stable configuration, which allows for more consistent planning.

First, relevant pavement management data was obtained from MnDOT and preliminary data analyses were performed. The prediction models and optimization process currently used by MnDOT were investigated and summarized.

Next, two additional metrics, Asset Sustainability Ratio and Deferred Preservation Liability, were calculated for MnDOT's network. Then details of the estimation process of state-to-state transition probabilities to be used in the Markov chain model were presented. To allow for site-specific variation, ordinal logistic regression models were incorporated in the Markov chain model.

The results were used in a dynamic programming optimization methodology to obtain baseline and optimal policies for different scenarios and a user-friendly excel spreadsheet tool was developed. Finally, a summary of the work performed followed by conclusions and recommendations was presented.

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REMAINING SERVICE LIFE ASSET MEASURE, PHASE 2

FINAL REPORT

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

EXECUTIVE SUMMARY

MnDOT has used the remaining service life (RSL) measure for pavement condition for more than a decade. However, it is not clear if this metric by itself can show the "true" condition of the system. A work plan was developed at the end of phase 1 of this project for a follow-up phase, in which the main objectives were to obtain relevant data to calculate the percent remaining service life interval (PRSI) and two additional metrics and to perform Markov chain analysis and dynamic programming to determine how much time and funding is required to bring the system to a stable configuration, which allows for more consistent planning.

First, a description of the data included in the Highway Pavement Management Application (HPMA) was performed, and the prediction models and pavement condition indices calculated as part of HPMA software were identified and discussed.

Then the research team used the HPMA data to estimate the Percent Remaining Service Interval (PRSI), the Asset Sustainability Ratio, and the Deferred Preservation Liability, as recommended in phase 1. The estimations were based on methods used by the Washington State Department of Transportation (WSDOT) to calculate these parameters for its pavement network. An example of using Percent Remaining Service Life, Asset Sustainability Ratio, and Deferred Preservation Liability to develop pavement performance predictions and demonstrate the effect of funding on pavement condition was presented.

The estimation of the state-to-state transition rates was described in detail in Chapter 4. After a brief introduction to Markov chains, a pavement performance metric was selected. The effect of repair activities and rate of repair were investigated and some examples of calculations were shown. Due to some inconsistencies in the data, some changes were made related to pavement type, surface and base thickness, and years since repair was performed. Using K-means cluster analysis of the performance model coefficients used by MnDOT, an optimal number of clusters for repair activities was determined and used in the analysis. An empirical model, ordinal logistic regression model, and a lean model were investigated and full Markov transition matrices were obtained for all conditions.

The logistic regression models bring major improvements to the Markov model: the enhanced Markov transition probability matrix allows for site-specific predictions and the comparison of several factors and how each of them influences the pavement performance and deterioration and provides an understanding of the interaction between severeal external factors, such as district location, repair history, functional class, base thickness, speed limit and pavement thickness. Using dynamic programming optimization, Markov Decision Process (MDP) calculations are performed to calculate an optimal maintenance policy and compare it with a baseline policy. A numerical example is provided to demonstrate how the Markov Transition Matrix can be used to model future pavement deterioration. A user-guide on how to use the proposed optimization spreadsheet tool for different situations is included.

The three new parameters proposed in this research can be immediately implemented. The Markov chain model and the optimization tool may require additional time for implementation. One critical step is to select a realistic value for the Ride Quality Index (RQI) monetary factor. The analysis can be further expanded to include other repair activities, since due to data limitations only three repair activities were considered as possible actions in the MDP.

CHAPTER 1: INTRODUCTION

1.1 BACKGROUND

Road networks in good condition are essential for safety, economic development, and quality of life. However, the rapid growth of population and traffic volume result in an accelerating wear of road pavements. In addition, transportation agencies and departments of transportation face budget constraints that can jeopardize their ability to fully address the maintenance needs of their assets.

This current scenario makes it crucial to optimize the allocation of available pavement management funds, which in turn requires accurate models that predict pavement performance and deterioration. Stochastic models based on Markov chains have been widely explored and lead naturally to applications of optimal decision theory. However, while Markov models are capable of capturing the uncertain behavior of pavements on average, their inability to allow for variations in site characteristics has constrained their usefulness.

This effort investigates the use of additional parameters that can help planners make decisions that are more informed and optimize the use of available funds, with the goal of having a more predictable evolution of the overall condition of the pavement network, which would allow for more consistent planning. It also investigates the enhancement of Markov transition probability matrices with ordinal logistic regression models.

1.2 OBJECTIVES

The main objectives of this research are to obtain relevant data to calculate the percent remaining service life interval (PRSI) for different categories of pavements, calculate two additional metrics, Asset Sustainability Ratio and Deferred Preservation Liability, and perform analyses to determine the optimal sequence of repair activities, which allows for more consistent planning.

1.3 ORGANIZATION OF THE REPORT

First, the pavement management data was obtained from MnDOT and preliminary data analyses were performed. The prediction models and optimization process currently used by MnDOT were investigated and summarized in Chapter 2. Next, in Chapter 3, two additional metrics, Asset Sustainability Ratio and Deferred Preservation Liability, were calculated for MnDOT’s network.

Chapter 4 describes the representation of pavement deterioration using Markov chains, and the estimation of the state-to-state transition probabilities from empirical data. Ordinal logistic regression models were used to allow for site-specific variation.

Chapter 5 describes the optimization methodology and provides a guide to the user-friendly Excel spreadsheet tool developed for this project. Chapter 6 consists of a summary of the work performed followed by conclusions and recommendations.

CHAPTER 2: OBTAINING PAVEMENT MANAGEMENT DATA REQUIRED FOR ANALYSES

In this chapter, the research team worked with the pavement management group in the Office of Materials and Road Research to better understand the capabilities and limitations of the current pavement management system, and to obtain the data required to perform the calculations and analyses required in chapters 3 and 4. The data was obtained at state level as well as district level, since the distribution of pavement conditions varies significantly among districts.

2.1 DATA ACQUISITION PROCESS

MnDOT collects pavement condition data every year on all State Highways and every two years on all County State Aid Highways. This results in driving 21,000 lane-miles of State Highway and 25,000 lane-miles of County Highways every year. (1)

The data collection is done by three Pathway Services Inc. Path Runner vans (figure 2.1) that measure pavement roughness, rutting, faulting, cracking and other pavement distresses. The vans also capture digital images of the right-of-way and pavement surface. (1)

Figure 2.1: MnDOT's Digital Inspection Vehicle (2)

2.1.1 Condition Indices

MnDOT calculates and uses four main condition indices: Ride Quality Index (RQI), Surface Rating (SR), Pavement Quality Index (PQI), and Remaining Service Life (RSL).

Ride Quality Index (RQI)

The Ride Quality Index (RQI) describes how smooth the pavement is (a higher value represents a smoother road). RQI is calculated based on information obtained from a panel of drivers that ride over different pavements and rate their condition from 0 to 5. The RQI is correlated to the International Roughness Index (IRI) that is calculated from the pavement's longitudinal profile measured with the front-mounted lasers on the digital inspection vehicle. IRI and RQI are correlated as shown in the charts in figure 2.2. RQI can be directly calculated from IRI using equation 1 for bituminous pavements, and equation 2 for concrete pavements.

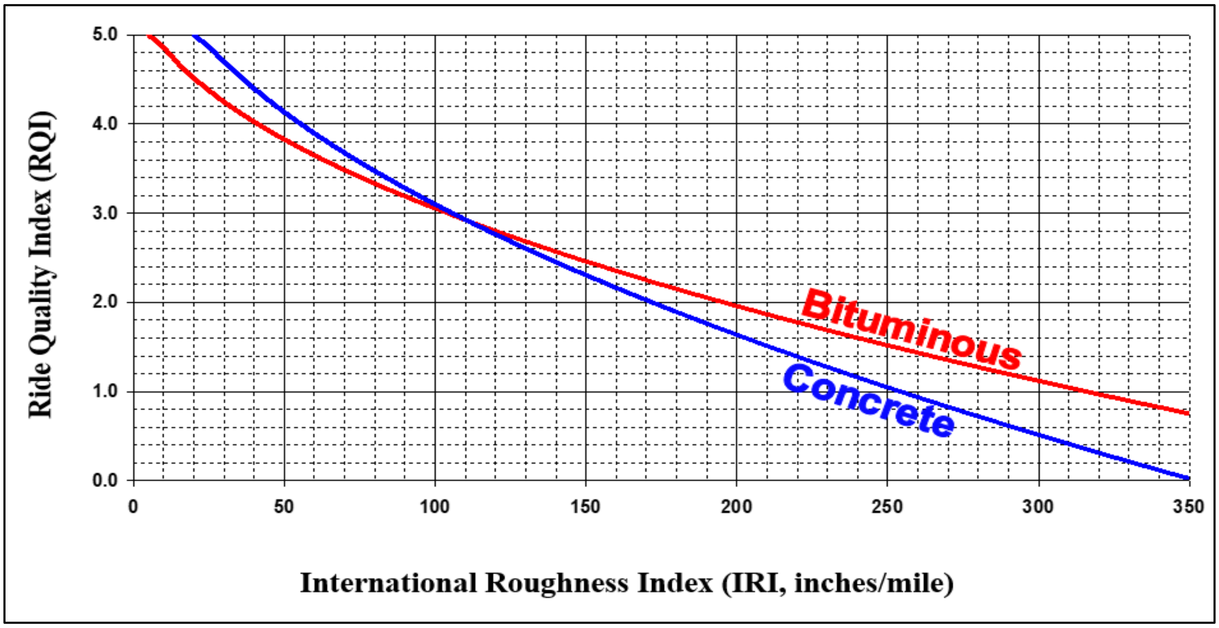

Figure 2.2: Graph for converting IRI to RQI (based on the 1997 rating panel) (2)

$$RQI = 5.697 - (0.264) * (\sqrt{IRI}) \quad [1]$$

$$RQI = 6.634 - (0.353) * (\sqrt{IRI}) \quad [2]$$

where IRI is given in inches/mile.

RQI is used to describe the pavement condition from very poor to very good. For example, pavements with RQI between 3.0 and 4.0 (Figure 2.3-a) have few or no visible signs of surface deterioration, while pavements with RQI between 1.0 and 2.0 (Figure 2.3-b) have significant deterioration to affect the speed of free-flow traffic.

Figure 2.3: Physical meaning of Ride Quality Index (RQI). a) RQI = 3.2 and b) RQI = 1.6

Surface Rating (SR)

The Surface Rating (SR) is used to quantify pavement distress. SR is estimated from the digital images captured by the inspection vehicle. The images are analyzed, and the road sections are rated from 0 to 4; a section that has a SR of 4 has no surface defects and a section that has a SR of 2.5 needs major rehabilitation. (3)

Pavement Quality Index (PQI)

The Pavement Quality Index (PQI) is calculated as the square root of the product of RQI and SR, as shown in equation 3. The PQI ranges from 0 to 4.5 and is used to gauge whether or not the state highway system meets the performance requirements of the Government Accounting Standards Board, Standard 34 (GASB 34). (3)

$$PQI = \sqrt{(RQI)(SR)} \qquad [3]$$

Remaining Service Life (RSL)

Remaining Service Life (RSL) is an estimation of the time (in years) until the next major rehabilitation of the pavement section. Using pavement deterioration curves, the time when a pavement section reaches an RQI of 2.5 is predicted and the RSL is simply calculated as the difference between the predicted and the present time. (3)

2.2 HIGHWAY PAVEMENT MANAGEMENT APPLICATION (HPMA)

As required by federal law, each state uses a risk-based pavement management system (PMS) that can take inputs related to the pavement network, analyze the data, and provide recommendations for treatment strategies. A key component in this process is the model used by the PMS to predict future pavement condition given historic conditions and future funding availability. MnDOT has used the Stantec Consulting software called Highway Pavement Management Application **(**HPMA) since the 1980's. The software stores all MnDOT's historical pavement data. For the purpose of this current study, HPMA was used to extract data on all pavement sections from 1995 to 2018. Data from before 1995 was not used due to a change in specifications caused by the implementation of Superpave mix design in the mid-1990s. Table 1 and figure 3.4 show the number of sections of each pavement type every year. Some inconsistencies are observed in table 2.1 and Figure 2.4 for years 2017 and 2018, compared to the previous years.

Table 2.1: Yearly distribution of sections by pavement type

	Pavement Type						
Years	Bituminous Aggregate Base	Bituminous Full Depth	Bituminous Over Bituminous	Bituminous Over Concrete	Concrete Doweled	Continuously Reinforced Concrete	Concrete Undoweled
1995	4857	27	396	1598	1391	2	2
1996	4479	13	336	1692	2007	38	
1997	4976	5	526	2078	1888	38	20
1998	4582	15	405	1983	1890	38	5
1999	4869	15	539	1939	1611	38	26
2000	4741		668	2180	1706	38	16
2001	8703		639	3431	2425	33	
2002	8744	2	713	3458	2375	34	
2003	9234	2	527	3561	2480	34	
2004	9115	2	638	3576	2472	34	
2005	8963	2	797	3559	2471	25	17
2006	9066	2	702	3635	2407	25	
2007	9183	4	586	3640	2395	25	4
2008	9140	15	633	3647	2385	25	
2009	9278	2	480	3636	2387	28	26
2010	8777	5	973	3676	2391	18	2
2011	9316	3	417	3692	2392	10	14
2012	8958	3	792	3627	2468	10	20
2013	8860	5	830	3660	2503	5	
2014	8897	2	774	3721	2465	5	3
2015	9013	1	687	3685	2469	5	2
2016	8187	6	1456	3638	2560	4	17
2017	1819	20	7857	3592	2469	4	126
2018	13153		12	10	2597	4	

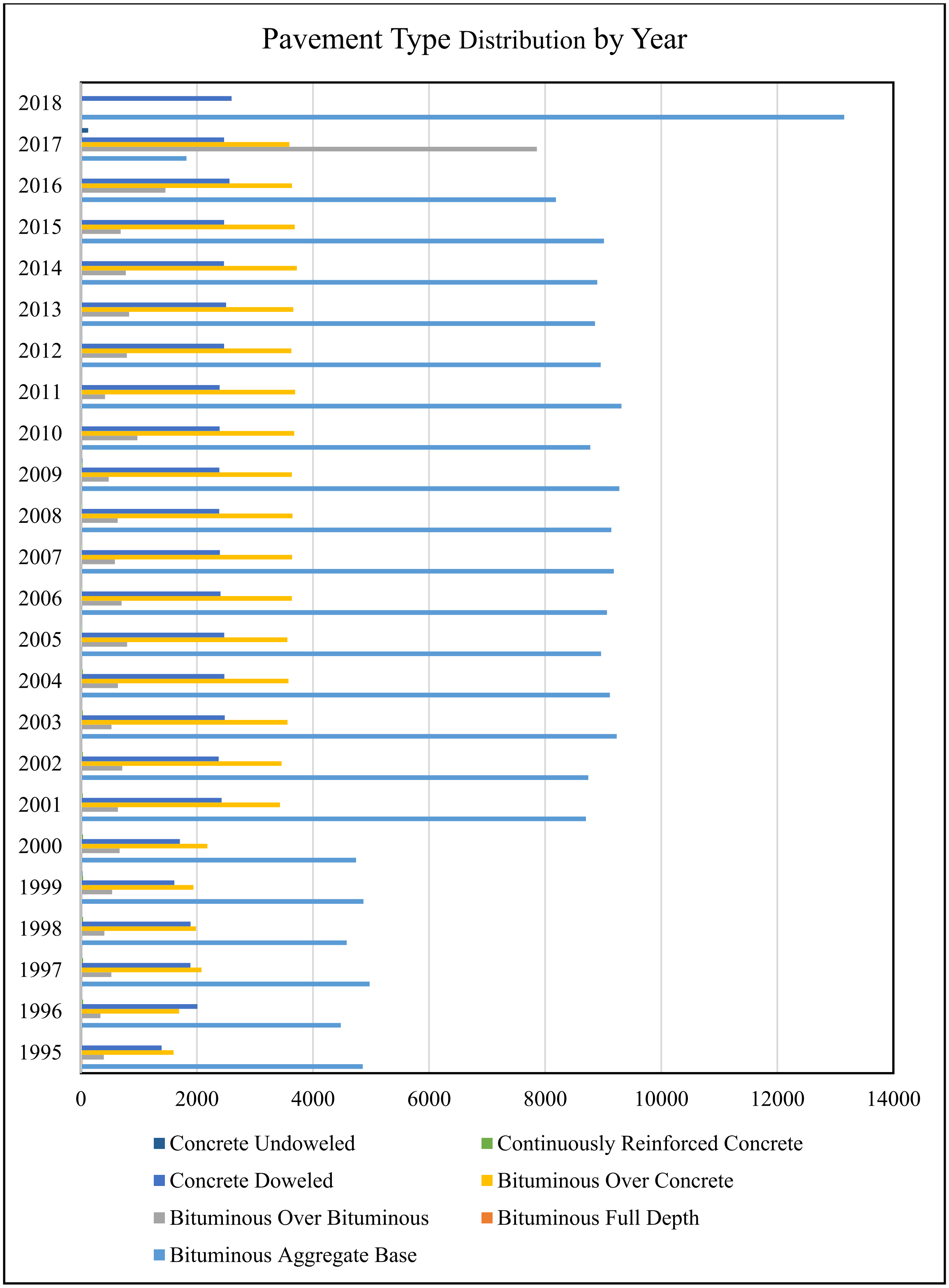

Figure 2.4: Yearly distribution of sections by pavement type

2.2.1 Prediction Models

The HPMA software has two types of prediction models: site specific and default. The default curves are generated through statewide average performances of similar pavements and rehabilitation types. Default curves are used whenever a section does not have enough data or has an unrealistic regression fit. The site-specific curves are the most accurate because they take in account differences in traffic, soils, construction etc. Figure 2.5 shows an example of a deterioration curve. (1)

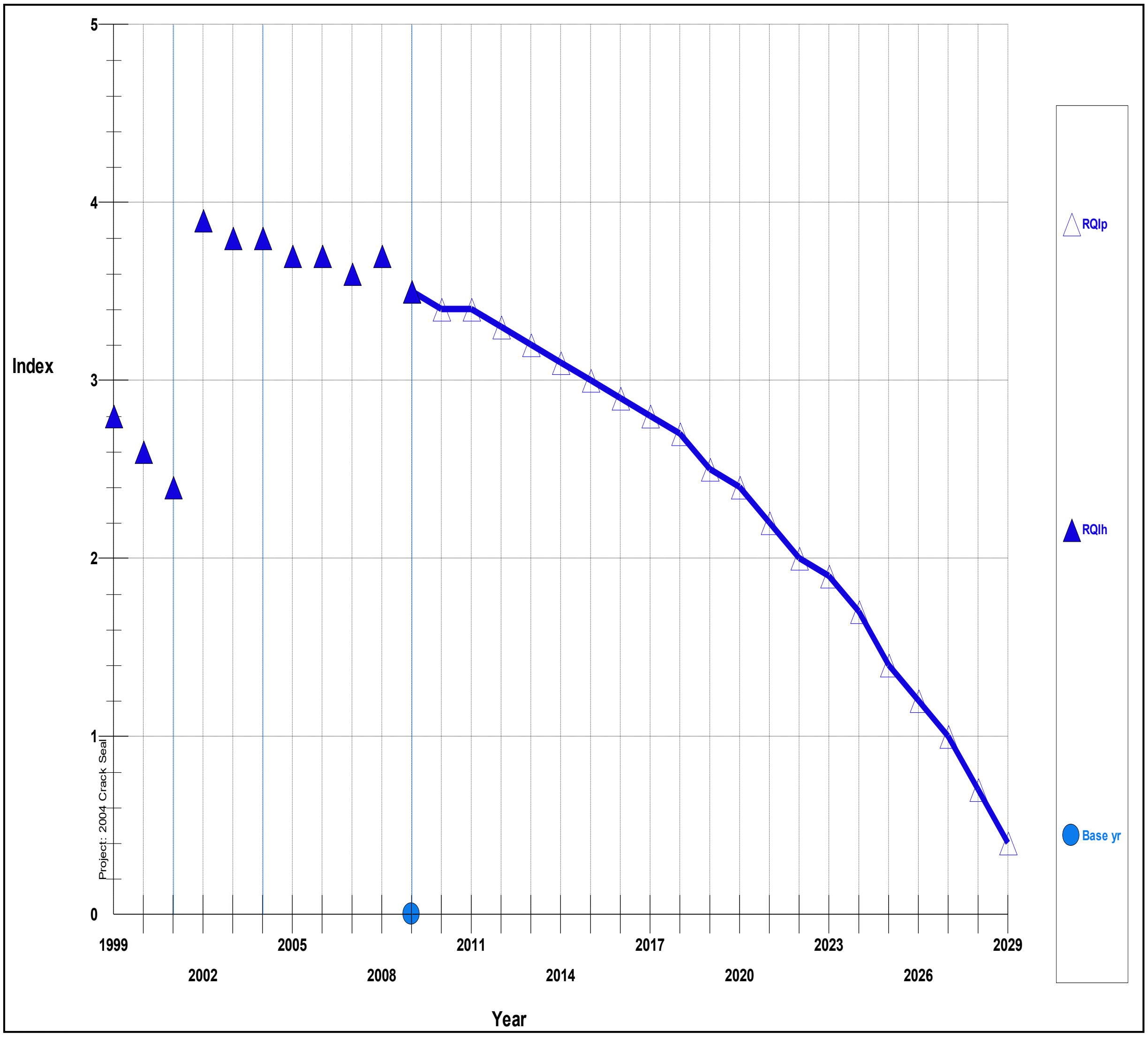

Figure 2.5: Example of deterioration curve (1)

HPMA software has complex decision trees that recommend different treatments that go from doing nothing to reconstructing, as shown in figure 2.6.

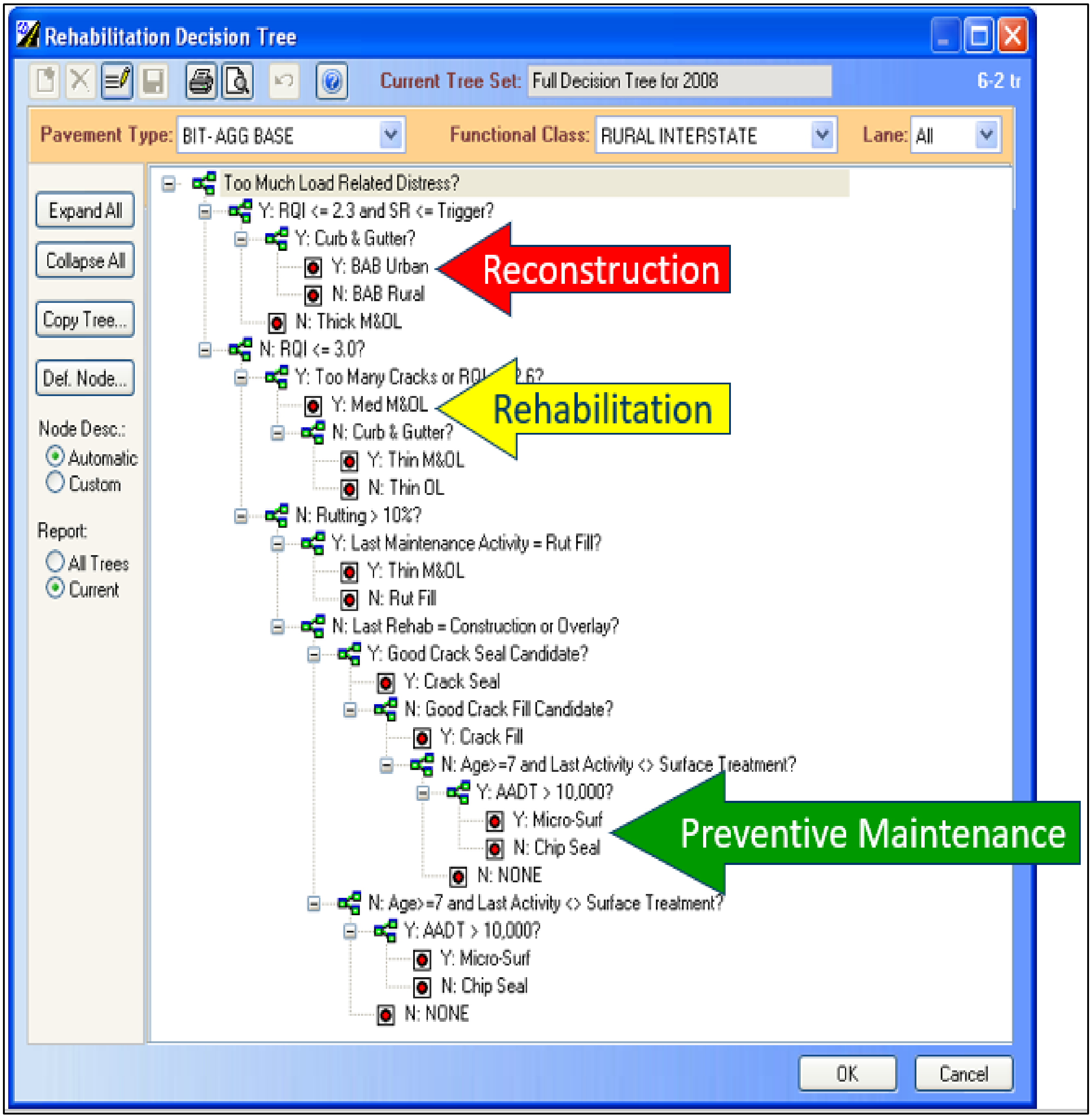

Figure 2.6: Rehabilitation decision tree (1)

The state of Minnesota has eight districts, as shown in figure 2.7. Tables 2.2 to 2.4 shows a summary of the top 3 most-used pavement types in each district, and the top 3 treatments (repair activities) most-used for each pavement type. The results are based on data collected from 1995 to 2018.

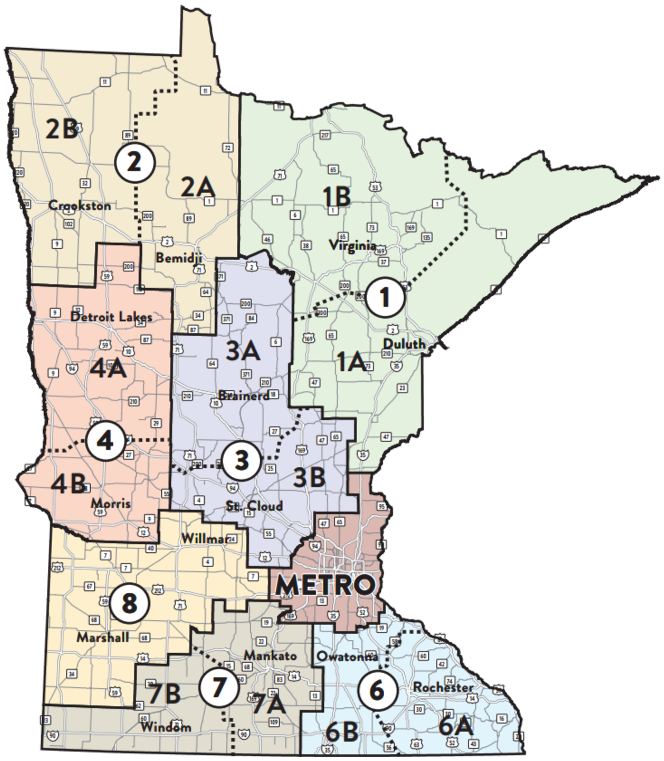

Figure 2.7: Map of districts in Minnesota (4)

Table 2.2: Top 3 most-used pavement types, and top 3 repair activities most used for each pavement type for district 1

District	Top 3 Most Used Pavement Types	Percent of Sections within the District	Top 3 Most Used Repair Activities by Pavement Type
1	Bituminous Aggregate Base	69.4	Maintenance Patching
			Medium Mill/Overlay
			BAB Construction – Rural
	Bituminous Over Concrete	13.3	Maintenance Patching
			Crack Seal
			Crack Fill
	Concrete Doweled	11.4	Maintenance Patching
			Unbonded Overlay
			Major CPR

Table 2.3: Top 3 most-used pavement types, and top 3 repair activities most used for each pavement type for districts 2, 3, 4 and Metro

District	Top 3 Most Used Pavement Types	Percent of Sections within the District	Top 3 Most Used Repair Activities by Pavement Type
2	Bituminous Aggregate Base	71.3	Crack Fill
			Chip Seal
			Medium Mill/Overlay
	Bituminous Over Concrete	15.4	Maintenance Patching
			Crack Fill
			Chip Seal
	Bituminous Over Bituminous	9.3	Chip Seal
			Maintenance Patching
			Medium Mill/Overlay
3	Bituminous Aggregate Base	68.5	Chip Seal
			Medium Mill/Overlay
			Micro-Surfacing
	Bituminous Over Concrete	10.8	Chip Seal
			Micro-Surfacing
			Medium Mill/Overlay
	Concrete Doweled	10.8	Maintenance Patching
			Unbonded Overlay
			Major CPR
4	Bituminous Aggregate Base	55.6	Chip Seal
			Crack Fill
			Maintenance Patching
	Bituminous Over Concrete	25.2	Maintenance Patching
			Chip Seal
			Medium Mill/Overlay
	Concrete Doweled	10.2	Maintenance Patching
			Unbonded Overlay
			CD Construction - Rural
Metro	Bituminous Aggregate Base	38.3	Medium Mill/Overlay
			Maintenance Patching
			BAB Construction – Rural
	Concrete Doweled	29.0	Maintenance Patching
			CD Construction - Rural
			Major CPR
	Bituminous Over Concrete	28.5	Medium Mill/Overlay
			Maintenance Patching
			Thin Mill/Overlay

Table 2.4: Top 3 most-used pavement types, and top 3 repair activities most used for each pavement type for districts 6 to 8

District	Top 3 Most Used Pavement Types	Percent of Sections within the District	Top 3 Most Used Repair Activities by Pavement Type
6	Bituminous Aggregate Base	45.2	Medium Mill/Overlay
			Chip Seal
			Maintenance Patching
	Bituminous Over Concrete	25.5	Maintenance Patching
			Medium Mill/Overlay
			Crack Seal
	Concrete Doweled	23.9	Maintenance Patching
			Thick Overlay
			Unbonded Overlay
7	Bituminous Aggregate Base	35.9	Maintenance Patching
			Chip Seal
			Crack Fill
	Bituminous Over Concrete	34.3	Maintenance Patching
			Crack Seal
			Medium Mill/Overlay
	Concrete Doweled	26.4	Maintenance Patching
			CD Construction - Rural
			Major CPR
8	Bituminous Aggregate Base	55.0	Chip Seal
			Maintenance Patching
			Thin OL
	Bituminous Over Concrete	22.5	Maintenance Patching
			Thin Overlay
			Chip Seal
	Concrete Doweled	14.8	Maintenance Patching
			CD Construction - Rural
			Whitetop (Doweled)

2.2.2 Marginal cost effectiveness optimization

The HPMA software contains a number of numerical tools that can be used to perform different types of analysis. One of them is a simple optimization process based on marginal cost effectiveness. First, the effectiveness of each fix is calculated using the area between the expected "do nothing" performance curve and the expected "post rehab" performance curve (based on the decision tree recommendation), as shown in figure 2.8. The PQI is used for determining the performance curves.

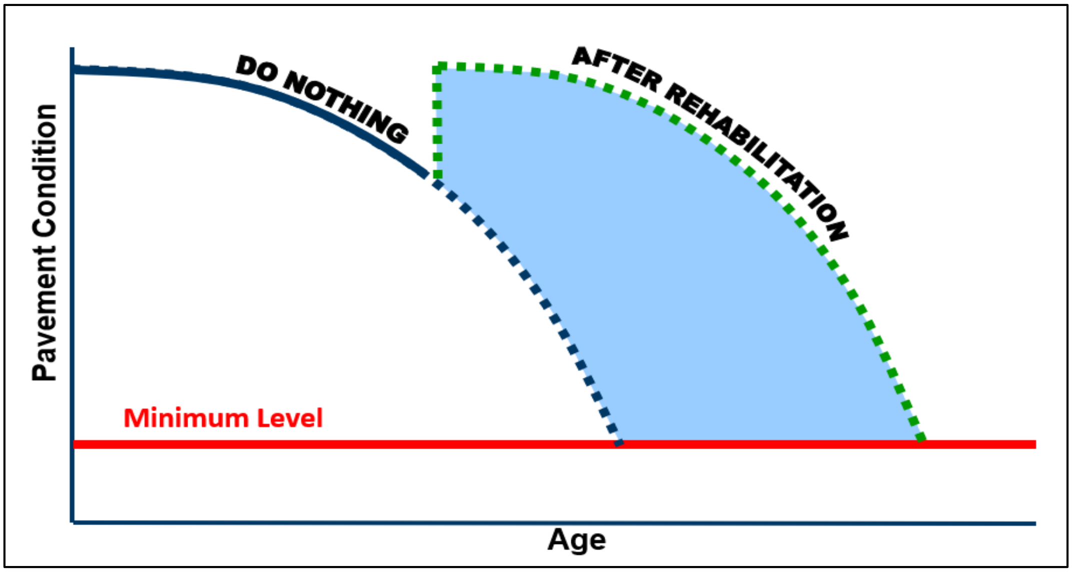

Figure 2.8: Estimated performance increase for a rehabilitation activity calculated as the area between the curves (1)

The area can also be calculated for a combination of multiple treatments, as shown in figure 2.9. This is necessary when a longer analysis period is used.

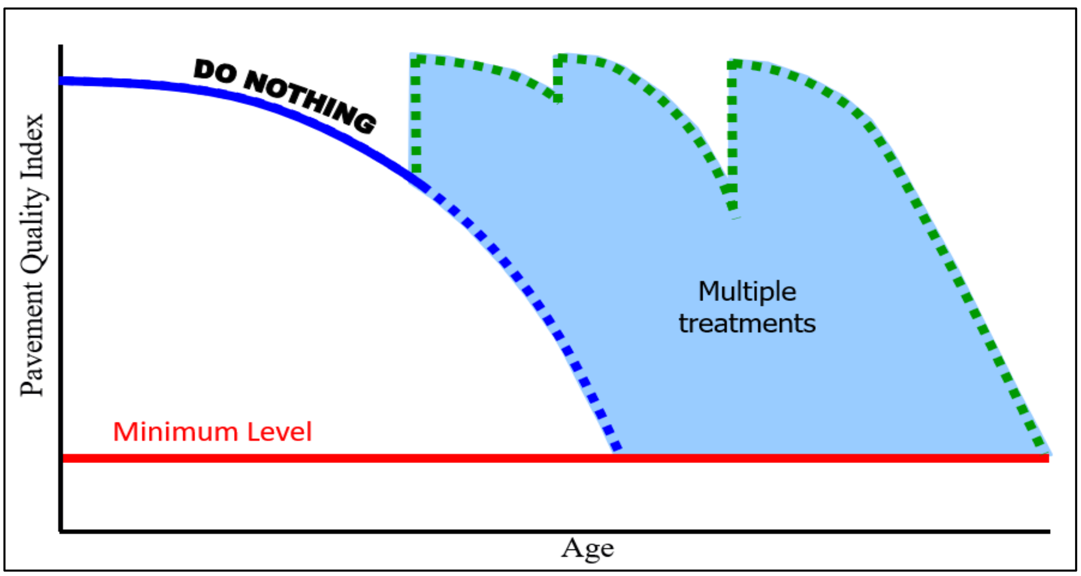

Figure 2.9: Estimated performance increase of combined maintenance/rehab activities (1)

The software also provides the cost associated with each recommended maintenance/rehab activity. An example is shown in figure 2.10. The cost is based on bid abstracts over a two-year period, and it is given in dollars per 12-foot lane-mile. The software takes into account inflation when dealing with multiyear analysis, as well as increased costs due to traffic control for projects in the Twin Cities metro district.

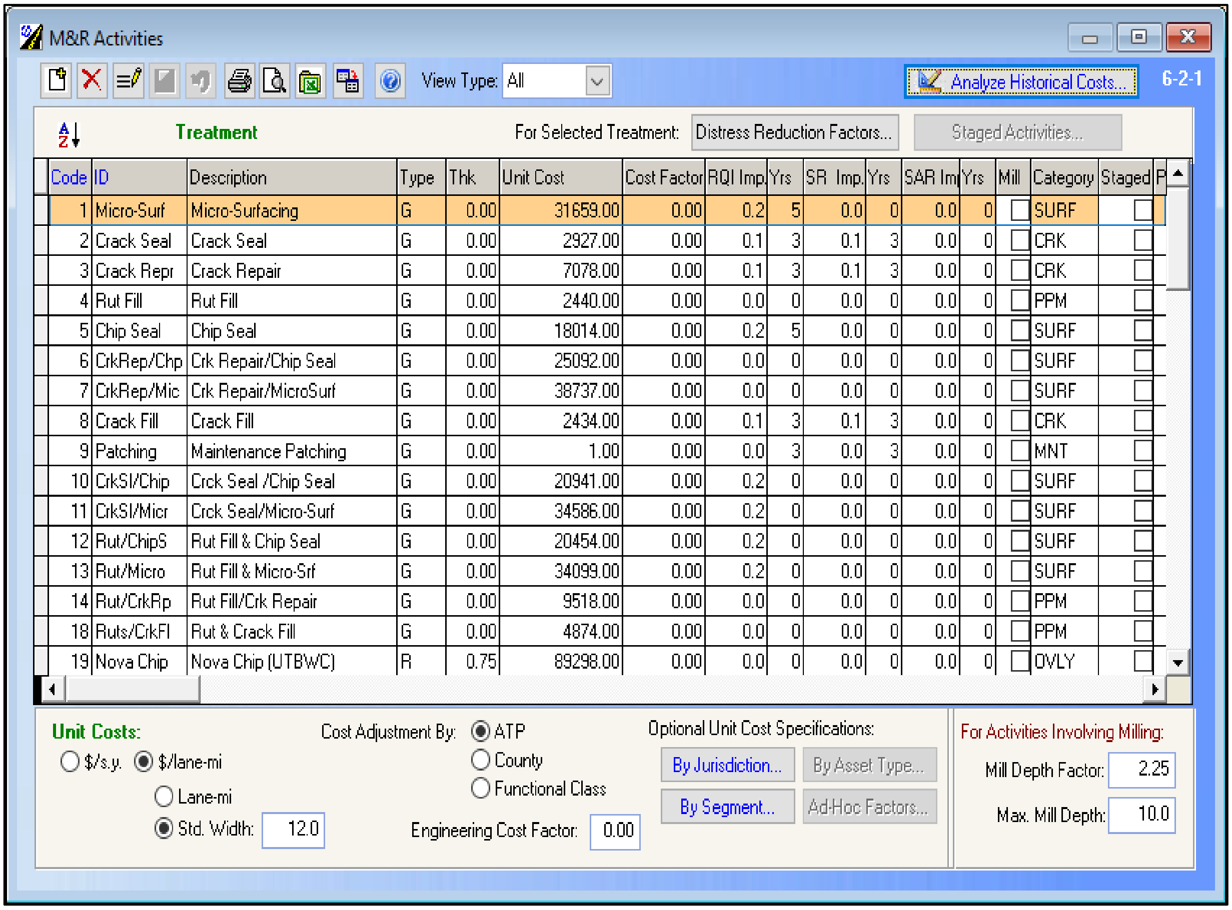

Figure 2.10: Maintenance activity costs

The effectiveness is calculated by multiplying the area between prediction curves by the length of the section and an effectiveness factor (five times the square root of the annual average daily traffic), as shown in equations 4 and 5. The equations are designed to give priority to longer sections and sections with higher traffic. (1)

$$\text{Effectiveness} = (\text{area between curves}) * (\text{lane miles}) * (\text{effectiveness factor}) \qquad [4]$$

$$\text{Effectiveness factor} = 5 * \sqrt{AADT} \qquad [5]$$

where AADT is the annual average daily traffic.

The Marginal cost effectiveness compares two possible actions to be taken in the segment. It is calculated by dividing the difference in the effectiveness of these two actions by the difference of their costs, as shown in equation 6.

$$MCE = \frac{E_r - E_s}{C_r - C_s} \quad [6]$$

The analysis results in a matrix with all possible treatments that is optimized by the marginal cost-effectiveness technique.

The feasible treatment for each analysis period is identified based on projected condition and established trigger levels. The cost-effectiveness (CE) of each combination of strategy is calculated as the ratio of effectiveness over cost. The highest CE is the best option.

If the marginal cost-effectiveness is negative, the comparative strategy is eliminated from future consideration; If not, it replaces the strategy selected in previous step. The process is repeated until no further selections can be made in any year of the analysis period. The final result is the highest cumulative effectiveness for a given budget. (1)

2.3 CONCLUSIONS

The HPMA software provides the information required to run the analyses proposed in chapter 3 and 4. It is possible that additional cost information may be necessary to calculate the Deferred Preservation Liability metric in chapter 3.

Further analysis of performance data for repeated maintenance/rehabilitation activities might be required to develop the Markov transition matrices proposed in chapter 4. It is important to determine if a maintenance/rehab activity creates a different rate of change, and once a road segment is treated it should move to a new performance chart. It is very possible that after a certain number of cycles, and for select maintenance/rehabilitation actions, the outcome of an activity is dependent on the history of what was done before on the section, rather than just the present condition. In other words, pavement maintenance/rehabilitation is not strictly a “memoryless” process. The research team will address this question in chapter 4 by evaluating other methods to preserve the benefits of the Markov modeling.

CHAPTER 3: CALCULATING ADDITIONAL PARAMETERS RECOMMENDED IN PHASE 1

In this chapter, the research team used the data from chapter 2 to estimate the Percent Remaining Service Interval (PRSI), the Asset Sustainability Ratio and the Deferred Preservation Liability, as recommended in phase 1. The estimates are based on the methods used by Washington State Department of Transportation (WSDOT) to calculate these parameters for their pavement network.

3.1 PERCENT REMAINING SERVICE INTERVAL (PRSI)

As indicated in Phase 1 of this project, using RSL does not offer a clear picture of the condition of the network, since some pavements have a design life of 30 years, others 20 years or less. For example, two given sections might have 9 years of remaining service life, but one was designed to last 10 years and the other to last 30, which means that, even though they have the same RSL, the first section is brand new, while the latter has passed two thirds of its design life (5).

By replacing it with a percent value that normalizes the RSL over different types of pavement, a more representative metric of the average aging condition of the network is obtained. For the example above, the first section would have a 9/10 = 90% remaining life, while the second one would have 9/30 = 30% remaining life (3).

Based on a recommendation from Federal Highway Administration (FHWA) to replace “life” with “interval”, the normalized value is identified as the Percent Remaining Service Interval (PRSI). This clarifies, when communicating the condition of assets with the general public, that pavements with zero RSL are not completely unusable, but rather have a poor ride and need to be rehabilitated (6).

Using HPMA data, the PRSI distribution of MnDOT’s network, as reported in 2018, was calculated and it is shown in Figure 3.1. It can be seen that almost 13% of sections have PRSI values between 0 to 5%. The network remaining service life, as an average percentage of original pavement life, was calculated to be 49.84% for 2018.

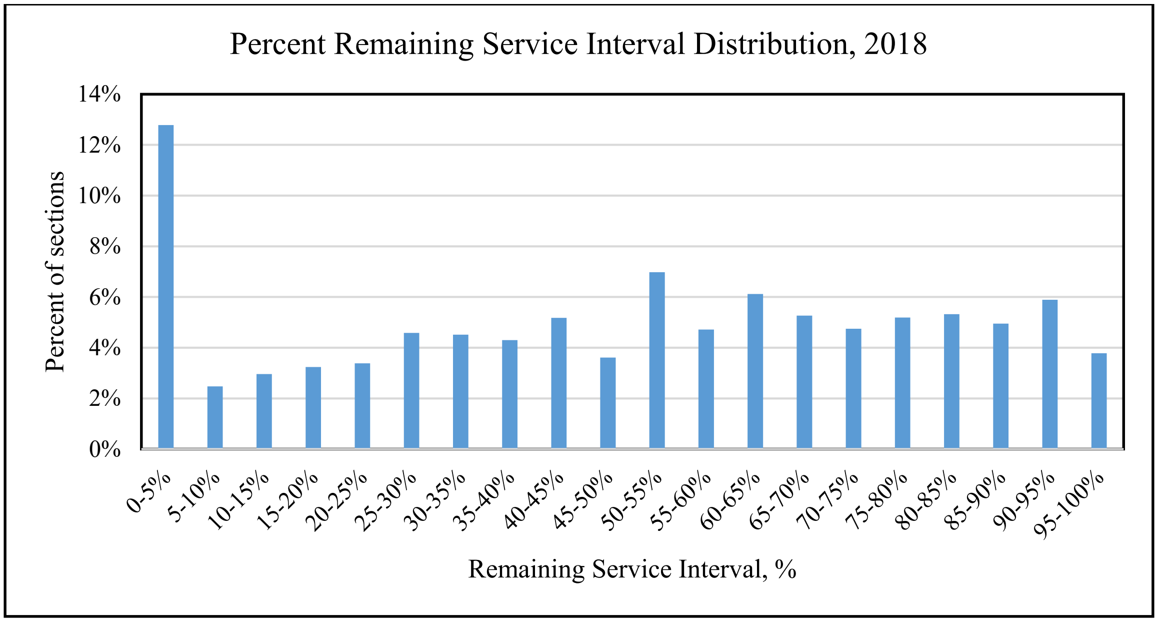

Figure 3.1: Percent Remaining Service Interval distribution for all MnDOT sections in 2018

For comparison purposes, WSDOT average Percent Remaining Service Life for the past several years is shown in Table 3.1. For 2018, MnDOT's value of 49.84% is slightly higher than the value of 46.9% reported by WSDOT.

Table 3.1: WSDOT remaining service life as a percentage of original pavement life (7, 8, 9, 10)

Year	Remaining service life Average percentage of original pavement life*
2018	46.9%
2017	47.4%
2016	48.6%
2015	47.1%
2014	46.9%
2013	46.1%
2012	47.3%
2011	47.8%

*WSDOT target range: 45% to 55%

The values shown in Table 3.1 were obtained from WSDOT past quarterly performance and accountability reports, called Gray Notebook, in which their annual pavement condition parameters can be found. Figures 3.2 and 3.3 show two examples.

WSDOT's pavement performance worsens from 2017 to 2018

2017 and 2018; Pavement annual performance measures

	PAVEMENT ANNUAL PERFORMANCE MEASURES[1,2]		2017	2018	Agency Target	Target[3]	Trend	Desired trend
Short term	**Percent of pavement in fair or better condition** Measured for asphalt and concrete pavement (chip seal data was collected but has not yet been processed). Condition is shown by lane miles and by vehicle miles traveled to reflect road use.	Lane Miles	91.8%	91.4%	90.0%	✓	↓	↑
		VMT[4]	91.5%	91.2%				
Long term	**Asset Sustainability Ratio[5]** Years of pavement service life added to the pavement network through rehabilitation in a given year divided by the service life consumed in that same year.		0.90	0.61	0.90 to 1.10	–	↓	↑
	Remaining Service Life[5] Average percentage of original total useful life remaining before rehabilitation or replacement is needed; average years remaining before rehabilitation or replacement is needed.		47.4% (7.7 yrs)	46.9% (7.6yrs)	45% to 55%	✓	↓	↑
	Deferred Preservation Liability (backlog) An estimate of the accumulated cost (in current dollars) to fund the backlog of past-due (deferred) pavement rehabilitation work.		$346 million	$420 million	$0	–	↑	↓

Data source: WSDOT Pavement Office.

Notes: **1** Calculations for all measures, excluding percent of pavement in fair or better condition, include all pavement types (asphalt, chip seal and concrete). **2** See p. 16 for additional discussion of long-term measures. **3** Check indicates target met, dash indicates target not met. **4** VMT = vehicle miles traveled. **5** Measure is weighted by vehicle miles traveled to better capture the typical road user's experience.

Figure 3.2: WSDOT performance indices for 2017 to 2018 (7)

WSDOT meets targets for short-term pavement conditions despite decline; long-term measures improve
2016 and 2017

	PAVEMENT ANNUAL PERFORMANCE MEASURES[1,2]		2016	2017	Agency Target	Target[3]	Trend	Desired trend
Short term	**Percent of pavement in fair or better condition** Measured for asphalt and concrete pavement (chip seal data was not collected in 2016 due to budget constraints; 2017 chip seal data was collected but has not yet been processed). Condition is shown by lane miles and by vehicle miles traveled to reflect road use.	Lane Miles	92.2%	91.8%	90.0%	✓	↓	↑
		VMT[4]	91.7%	91.5%				
Long term	**Asset Sustainability Ratio[5]** Years of pavement service life added to the pavement network through rehabilitation in a given year divided by the service life consumed in that same year.		0.68	0.90	0.90	✓	↑	↑
	Remaining Service Life[5] Average percentage of original total useful life remaining before rehabilitation or replacement is needed; average years remaining before rehabilitation or replacement is needed.		48.6% (7.6 yrs)	47.4%[6] (7.7 yrs)[6]	45% to 55%	✓	↓	↑
	Deferred Preservation Liability (backlog) An estimate of the accumulated cost (in current dollars) to fund the backlog of past-due (deferred) pavement rehabilitation work.		$330 million	$346 million	$0	–	↑	↓

Data source: WSDOT Pavement Office.

Notes: **1** Calculations for all measures, excluding percent of pavement in fair or better condition, include all pavement types (asphalt, chip seal and concrete). **2** See p. 11 for additional discussion of short- and long-term measures. **3** Check indicates target met, dash indicates target not met. **4** VMT = vehicle miles traveled. **5** Measure is weighted by vehicle miles traveled to better capture the typical road user's experience. **6** Between 2016 and 2017, WSDOT updated its RSL calculations to reflect the increased use of strategic maintenance (see p. 13). This update caused both the RSL in years and the estimates of total expected pavement lifetime to increase. As a result, the RSL percentage decreased.

Figure 3.3: WSDOT performance indices for 2016 to 2017 (8)

3.2 ASSET SUSTAINABILITY RATIO

The Asset Sustainability Ratio (ASR) is a parameter introduced by WSDOT in 2012 to measure the annual sustainability of investments in pavement asset protection, by quantifying how pavement replenishment is keeping up with pavement wear (8). The Asset Sustainability Ratio is calculated using equation 7:

$$ASR = \frac{\text{Annual Replenishment}}{\text{Annual Consumption}} \qquad [7]$$

In equation 7, the annual replenishment is calculated as a summation of average life added to the network with each rehabilitation activity performed. Depending on how the network ASR is weighted, the life addition corresponding to each rehabilitation activity is multiplied by either the lane miles that received the activity or by the Annual Vehicle Miles Traveled (AVMT), and the sum of all products the annual replenishment for that year. The annual consumption (or pavement wear) is the amount of life consumed and it is calculated as either a function of the number of lane miles consumed or of the AVMT consumed for the network. (13)

In the past years, MnDOT has started using ASR as an additional metric to the traditionally used RSL. Table 3.2 shows the amount of life added to the system for pavement repair activities frequently used by MnDOT (12).

Table 3.2: Life addition by pavement preservation category used by MnDOT (12)

Activity	Added Life (years)	Exception (years)	Activity	Added Life (years)	Exception (years)
Cold In-place Recycling	16		Rural Regrade	27	
Concrete Replacement	27		Rural Regrade (BIT)	27	
Crack/Seat/OL	19		Rural Regrade (CON)	27	
Hot In-place Recycle	11		Thick Mill/OL	17	15 (BOC)
Major CPR	15		Thick OL	15	
Major CPR/Grind	15		Thin Mill/OL	11	
Medium Mill/OL	15	13 (BOC)	Unbonded OL	34	
Medium OL	18	15 (CD)	Urban Regrade	27	
Micro-Mill/UTBWC	12		Urban Regrade (BIT)	27	
Micro-Surfacing	7		Urban Regrade (CON)	27	
Minor CPR	14		UTBWC	12	
Minor CPR/Grind	14		White-Topping	15	
Reclaim	24				

An example of calculating the annual replenishment is shown in Table 3.3, in which the values for 2017 were used. HPMA data was used to obtain the pavement miles that received preservation activities in 2017. The analyses were performed using lane-miles.

Table 3.3: Annual Replenishment Calculation for 2017 for MnDOT network

Activity	Life added per mile	2017	
		Number of miles that received repairs	Added Life (years)
BAB Constr - Rural	27	49.094	1325.5
BAB Constr - Urban	27	4.454	120.3
CD Constr - Rural	27	55.622	1501.8
CD Constr - Urban	27	1.68	45.4
CIR & Medium OL	16	43.704	699.3
Chip Seal	6	514.062	3084.4
Crack Fill		763.583	0.0
Crack Repair		6.528	0.0
Crack Seal		413.493	0.0
Crk Repair/MicroSurf		9.59	0.0
Major CPR/D.Grinding	15	26.669	400.0
Major CPR	15	5.169	77.5
Medium Mill/Overlay	15	645.816	9687.2
Medium Overlay	18	25.818	464.7
Micro-Surfacing	7	416.487	2915.4
Minor CPR	14	8.538	119.5
Nova Chip (UTBWC)	12	189.382	2272.6
Maintenance Patching		2065.421	0.0
Reclaim & Overlay	24	122.322	2935.7
Rut Fill		86.33	0.0
Spot Overlay (Maint)		162.154	0.0
Thick Mill/Overlay	17	26.076	443.3
Thin Mill/Overlay	11	345.521	3800.7
Thin Overlay		4.04	0.0
Unbonded	34	24.934	847.8
Unbonded (No Dowels)	34	1.994	67.8
Whitetop (Doweled)	15	23.674	355.1
Whitetop (Undoweled)	15	0.18	2.7
Total life added			**31167**

For the annual consumption, the total length of MnDOT's pavement network that did not receive repair activities, was used. The number of miles was multiplied by 1 year, which represents the yearly decrease in remaining service life. Please note that WSDOT calculates the amount of life lost every year as the total length of the network in the system (fixed and not fixed) multiplied by one year. Using equation 7, the Asset Sustainability Ratio for the last 12 years was calculated, and the results are shown in table 3.4. Again, lane-miles were used for both annual replenishment and annual consumption.

Table 3.4: Asset Sustainability Ratio of MnDOT's network for the last 10 years

Year	Annual Replenishment [years]	Annual Consumption [years]	ASR
2017	31167	22911	1.36
2016	27118	24096	1.13
2015	28347	23319	1.22
2014	31366	23704	1.32
2013	39430	23088	1.71
2012	40033	22409	1.79
2011	40033	25477	1.57
2010	27026	21525	1.26
2009	36648	24015	1.53
2008	20904	24580	0.85
2007	21807	25068	0.87
2006	26969	23876	1.13

3.2.1 Additional Comments on Asset Sustainability Ratio

ASR and PRSI (%RSL) are not completely independent metrics. For example, if for a given year, the ASR is 1, the annual replenishment equals the annual consumption. In other words, the remaining service life lost is equal to the life added back into the system, which also means that the average PRSI does not change. It is important to note that, while the average does not change, the distribution of PRSI changes. An ASR < 1 means the remaining service life lost is greater than the life added back into the system, in which case, the average PRSI for the system decreases. WSDOT's target value for ASR is 0.9. (5)

According to WSDOT, for a well-balanced network system, with an average PRSI (%RSL) of 50%, an ASR value of 1 corresponds to the Lowest Life-Cycle Cost (LLCC) condition for the system (11). If rehabilitation is done too early, a portion of the still available pavement life is wasted; if rehabilitation is done too late, the repairs will be very costly, as illustrated in figures 3.4 and 3.5 (5). WSDOT has an ASR target value of 0.9, which can be interpreted as an indication of a well-balanced system.

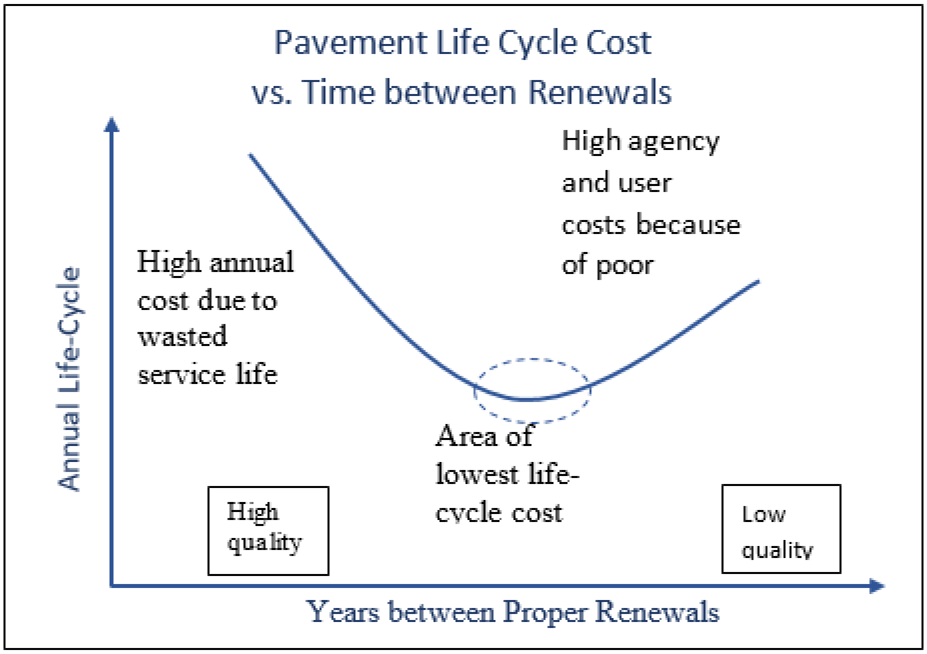

Figure 3.4: The concept of Lowest Life-Cycle Cost (5)

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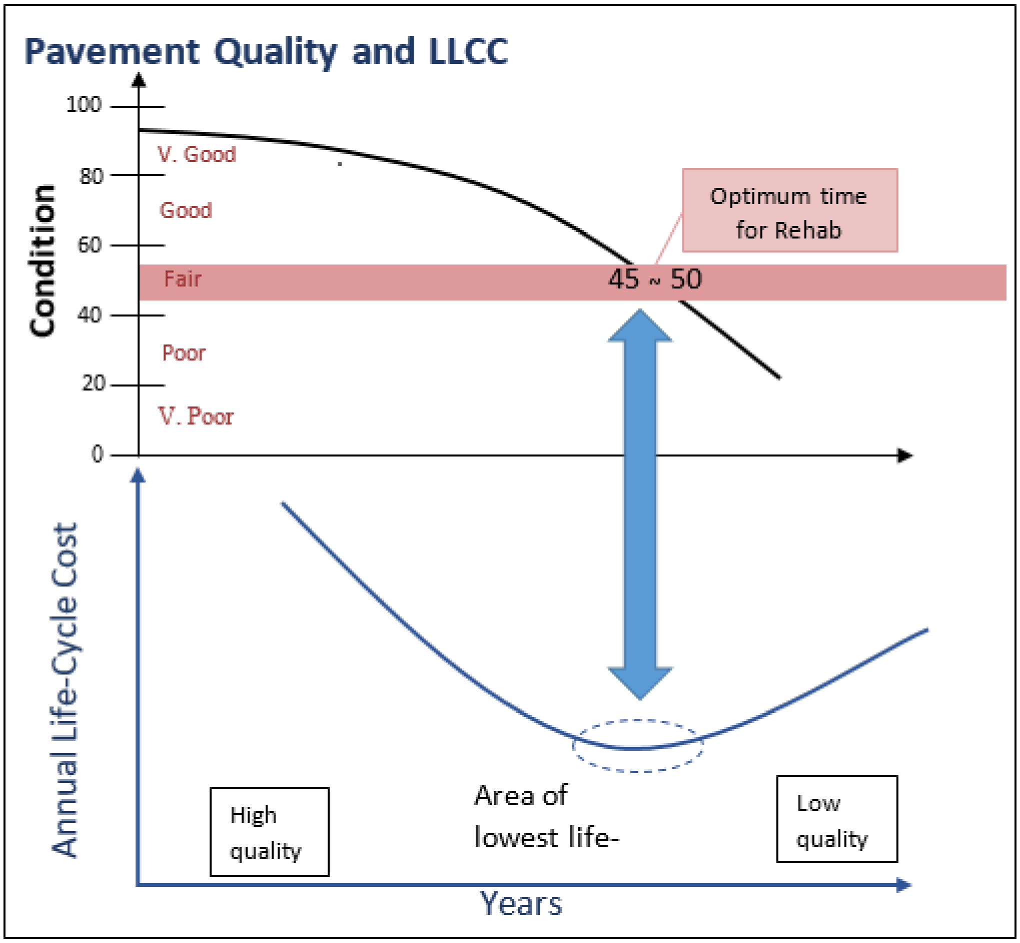

Figure 3.5: Minimal acceptable performance levels for lowest life-cycle cost (5)

As mentioned above, these assumptions are valid only for a well-balanced system. For a system in a worse average condition, it is obvious that more life needs to be added every year than what is annually consumed. For such a system, most likely the LLCC condition cannot be reached, and more substantial funds are needed every year, until the system reaches a well-balanced condition that does not change significantly from one year to another.

WSDOT chooses a fair condition of 45 to 50, on a 0 to 100 scale, as the optimum time for rehabilitation activities. For flexible pavements, as shown in Figure 3.6, the most cost-effective decision is resurfacing of the roadway at the optimum time. Most flexible pavement distresses are located at the surface and can be corrected through resurfacing activities. The resurface window (45 to 50% index value) was chosen because if resurfacing is done too early, remaining service life is wasted. If done too late, more costly rehabilitation or reconstruction are required.

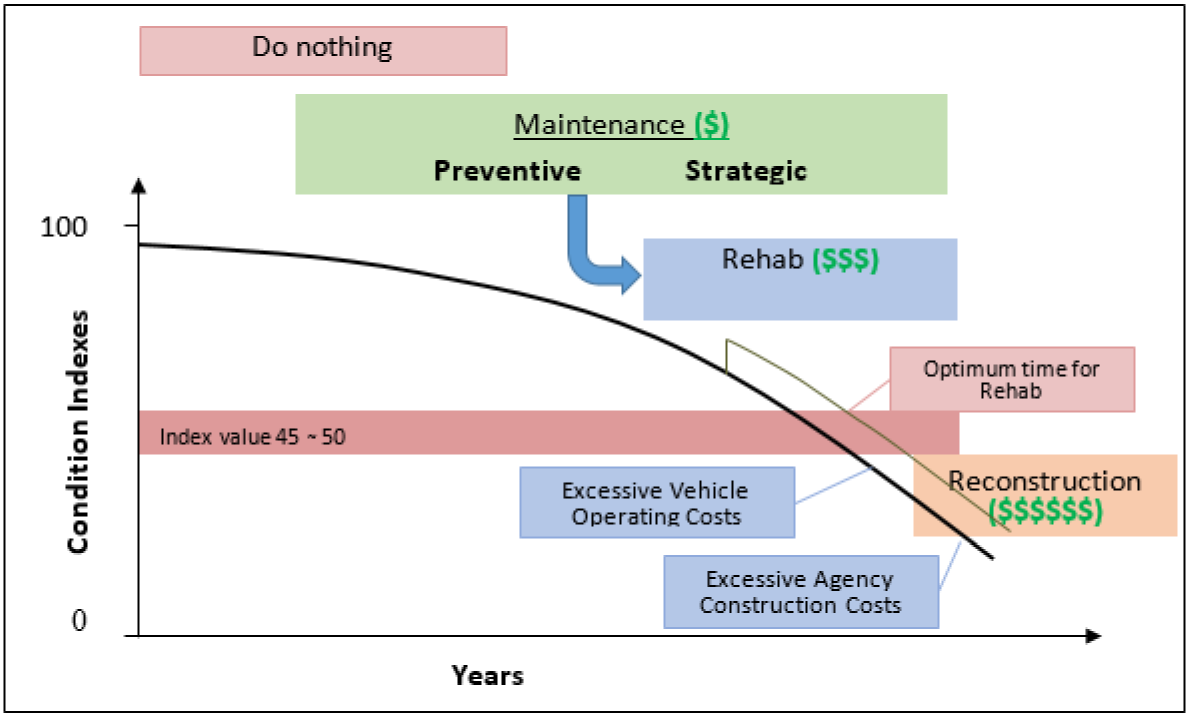

Figure 3.6: LLCC for asphalt and chip seal pavements (5)

Figure 3.7 shows a rigid pavement deterioration curve accompanied by action choices and their consequences. It is important to note that, differently from flexible structures, rigid pavements do not follow a cyclic model, such as the one shown in figure 3.8. In this case, the most cost-effective management consists of prolonging the pavement life before the inevitable reconstruction, while keeping minimum risk of catastrophic failure and acceptable performance (5). The main factors taken in consideration for concrete pavement management are the long period between reconstruction activities (50 or more years), the high capital cost, and the concrete durability in different regions. (5)

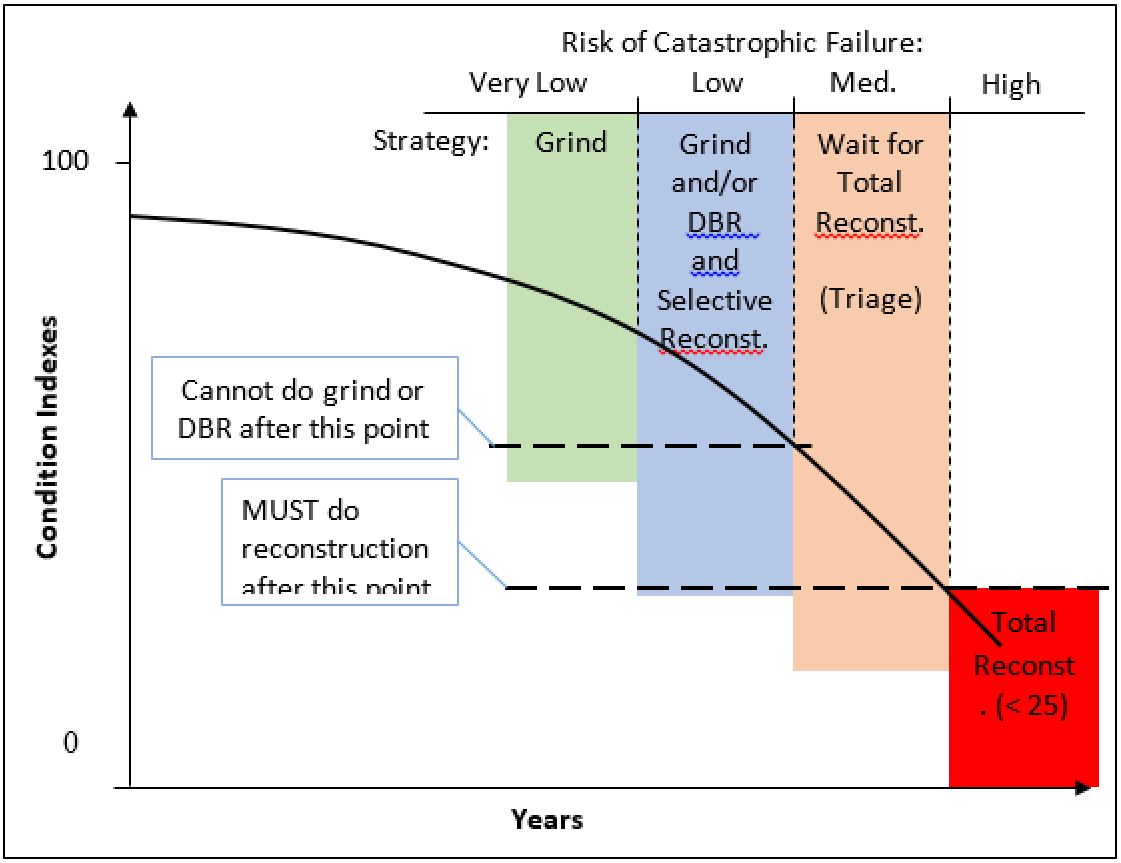

Figure 3.7: LLCC for concrete pavement (5)

3.3 DEFERRED PRESERVATION LIABILITY (DPL)

ASR and %RSL are good metrics to describe the overall condition of the system, but do not provide any information regarding the amount of funding needed to achieve target condition levels. For this reason, WSDOT has started using the Deferred Preservation Liability (DPL) metric, which estimates the funding required to address the cumulative backlog of deferred pavement rehabilitation. The estimate takes into account the higher cost of rehabilitation as pavement condition gets worse, and more extensive repairs are needed, and provides a cost estimate for the amount of preservation backlog carried by the system. (5) An alternative metric to the liability measure is the Cost of Inadequate Funding, which represents the additional cost of not funding 100% of what is considered the Lowest Life Cycle Cost (LLCC) funding. (5)

WSDOT reports DPL values every year in the Gray Notebook, as previously shown in figures 3.1 and 3.2 (7, 8). The numbers reported are based on:

- The distribution of Preservation Unit Due Years (due year is the year in which the pavement is due for rehabilitation);

- The average resurfacing costs for asphalt and chip seal;
- The average triage cost of concrete (diamond grinding and dowel bar retrofit) and
- The average cost of concrete reconstruction.

Due Years that are two or more years earlier than the year being reported, are used to accumulate liability. For example, for 2012, only Preservation Units due in 2010 and earlier were used (5).

Future liability can be estimated based on additional assumptions, such as predictions of funding level and funding distribution, as well as predictions for the pavement performance that corresponds to the predicted liability (5). As expected, for a well-balanced system, the DPL at the Lowest Life-Cycle Cost (LLCC) is $0. For systems in poor condition and under sustained underfunding circumstances, the accumulation of required reconstruction will cause the DPL to grow exponentially (11).

WSDOT calculates DPL by first identifying pavement sections with a due year of two or more years earlier than the current year. The sections are grouped by surface type, and concrete is further subdivided into sections identified as needing triage and sections identified as needing reconstruction. The total lengths of due sections for each pavement surface type are calculated and then are multiplied by the planning level unit costs shown in table 3.5 (13).

Table 3.5: Planning level unit costs for each pavement surface type used by WSDOT (10)

Surface Type	Cost per lane-mile
Chip seal	$50,000
Asphalt	$250,000
Concrete triage	$300,000
Concrete reconstruction	$2,500,000

Note that triaging aged concrete pavement is one of the goals of WSDOT's 30-year strategy. It aims to restore serviceability and repair structurally deficient panels. Triage temporarily preserves the existing pavement, while extending the service life by 10 to 15 years through minor to moderate rehabilitation techniques. Examples of triage are selective panel replacements, dowel bar retrofit and surface diamond grading. (14)

WSDOT does not assume a large preservation penalty (no additional rehabilitation or reconstruction penalties) for flexible pavement sections identified as far past due. The agency has used for many years an aggressive strategic maintenance approach, and the sections represent anomalies, and are not indicative of sections that need reconstruction or major rehabilitation.

As the WSDOT system continues to deteriorate, due to a large shortage of preservation funding appropriation, the management team plans to evaluate methodologies to estimate which asphalt/chip seal sections may need major rehabilitation or reconstruction due to deterioration and assign the backlog cost accordingly. (5, 13)

Figure 3.8 shows a 50-year comparison between WSDOT flexible and rigid pavement models. Flexible pavement structures are designed to carry the expected traffic loads for 50 years as long as periodic resurfacing is performed. The cost of $250,000 for asphalt accounts for these periodic surface renewals. According to WSDOT, flexible structures can be modeled "perpetually" if they are monitored and receive resurfacing at the right time. (5)

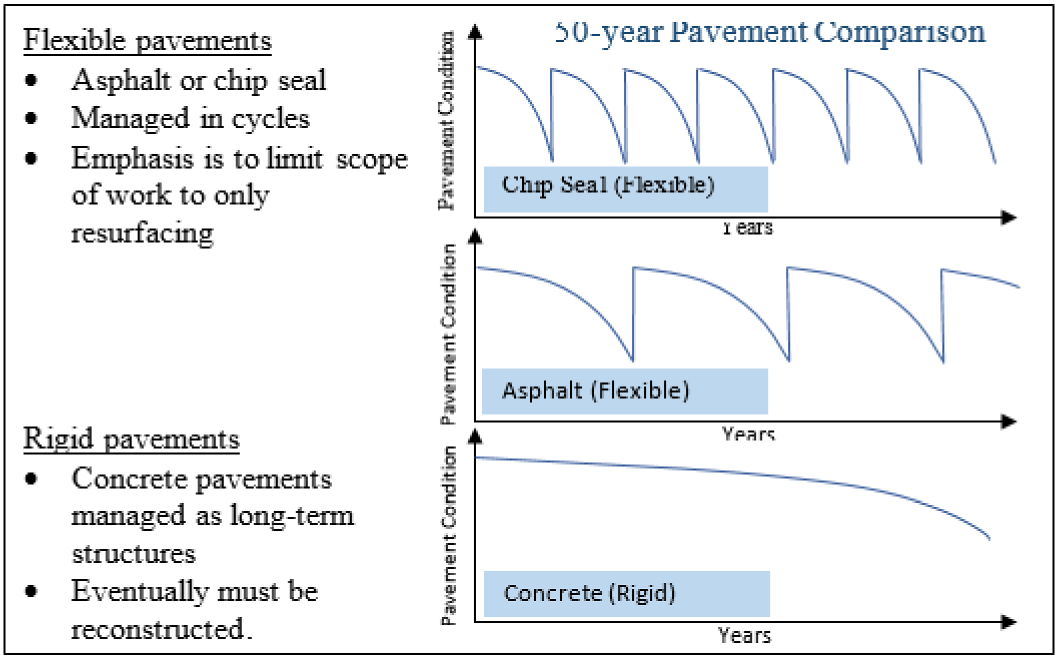

Figure 3.8: Flexible and rigid pavement models (5)

3.3.1 DPL Calculations for MnDOT Network

Following a similar approach, preliminary calculations of DPL values for MnDOT pavement sections were performed. The length in lane-miles of sections, by district, that have had zero Remaining Service Life for at least 2 years are summarizes in Table 3.6 and shown in Figure 3.9.

Table 3.6: Length in miles of sections that have had zero RSL for at least 2 years in Minnesota

	District								
Year	1	2	3	4	5	6	7	8	Total Network [miles]
2001	9.2	0	2.1	0	0.3	37.1	1.0	4.5	54.2
2002	70.5	89.5	10.0	25.0	139.4	109.8	27.1	17.0	488.3
2003	133.3	195.2	59.0	76.8	333.5	289.9	110.9	35.6	1234.3
2004	184.1	200.4	130.4	107.0	401.9	479.9	165.4	67.0	1736.1
2005	319.6	357.5	159.8	214.5	467.0	595.6	281.0	104.7	2499.7
2006	296.5	359.2	174.6	199.6	426.6	625.4	305.5	102.8	2490.3
2007	298.6	256.4	152.9	174.0	446.9	738.9	230.8	79.6	2378.1
2008	345.1	227.6	193.0	274.7	399.8	747.7	220.7	96.7	2505.4
2009	429.5	193.1	210.3	341.0	445.2	901.1	276.8	175.7	2972.7
2010	483.5	184.0	182.5	282.9	380.6	796.5	279.8	146.4	2736.2
2011	577.3	92.5	186.0	354.9	389.2	663.0	300.0	254.1	2816.9
2012	481.3	66.7	185.2	278.6	357.8	515.6	257.3	239.8	2382.2
2013	547.4	73.6	117.7	299.7	301.2	468.7	308.4	297.0	2413.8
2014	474.4	77.8	145.5	89.0	292.7	167.2	343.9	148.1	1738.6
2015	521.4	27.8	187.9	56.3	286.8	141.7	343.8	144.6	1710.3
2016	594.6	32.1	185.1	72.9	290.7	137.1	391.2	96.7	1800.4
2017	553.6	89.0	169.2	83.1	316.2	165.0	483.0	81.9	1940.8
2018	515.0	96.6	108.1	103.6	322.3	167.2	455.3	53.2	1821.3

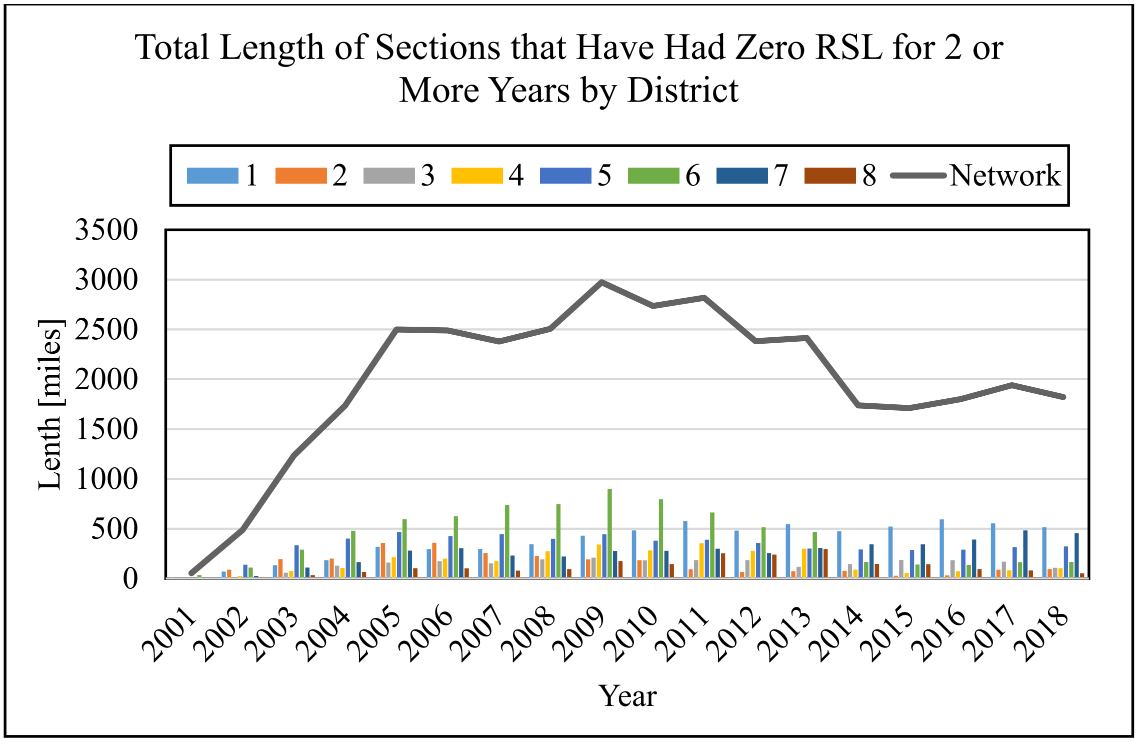

Figure 3.9: Total length of sections that have had zero RSL for at least 2 years in Minnesota

Preliminary costs were estimated using medium mill/overlay for flexible pavements, and thick overlay for rigid pavements. Medium mill/overlays add 15 years of life to the pavement network and costs $277,000.00 per lane-mile, while thick overlays generate a life addition of 17 years and costs $387,000.00 (9), as shown in table 3.7.

Table 3.7: Unit costs for new projects (9)

Pavement Type	Average project cost per lane-mile
Flexible	$277,000.00
Rigid	$387,000.00

The unit costs shown in table 3.7 are multiplied by the length of due sections shown in table 3.6 and the results represent the DPL values, which are summarized in table 8 and figure 3.10. The accuracy of values in table 3.8 can be improved if, rather than a state average value, estimated unit costs for each district are used. While the dollar values in tables 3.7 and 3.8 are only demonstrative they serve the purpose of illustrating the DPL.

Table 3.8: Length of sections with zero RSL for at least 2 years per surface type and DPL values in Minnesota

Year	Length [miles]	Deferred Preservation Liability [millions of dollars]

2001	54.2	$20
2002	488.3	$158
2003	1234.3	$395
2004	1736.1	$559
2005	2499.7	$815
2006	2490.3	$808
2007	2378.1	$770
2008	2505.4	$809
2009	2972.7	$961
2010	2736.2	$878
2011	2816.9	$896
2012	2382.2	$760
2013	2413.8	$760
2014	1738.6	$561
2015	1710.3	$547
2016	1800.4	$568
2017	1940.8	$604
2018	1821.3	$566

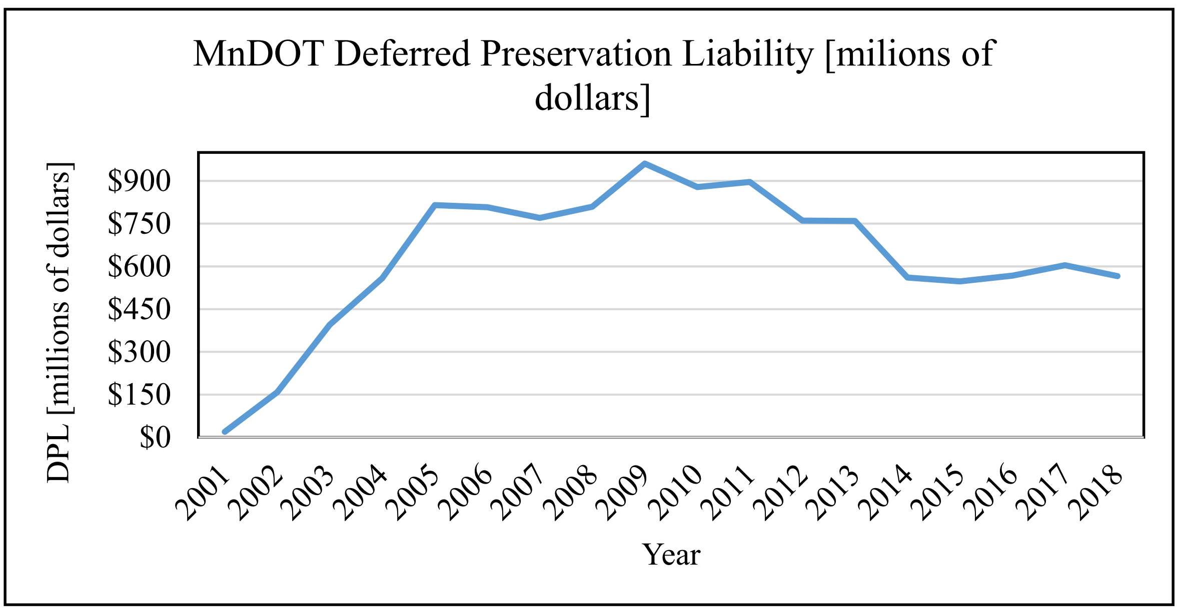

Figure 3.10: DPL values in Minnesota

3.4 APPLICATION OF PAVEMENT CONDITION PARAMETERS

An example of using Percent Remaining Service Life, Asset Sustainability Ratio and Deferred Preservation Liability to develop pavement performance predictions, and demonstrate the effect of funding on pavement condition, is shown in Figures 3.11 to 3.13 (15).

Using the three metrics, WSDOT investigated the effect of additional funding, provided by the 2015 Connecting Washington funding package. Please note that, on the right side of Figure 3.13, the grey color is used for projected values without additional funding, while on the right side of Figures 3.11 and 3.12, respectively, the grey color is used for projected values with additional funding. It is clearly shown that the additional funds would bring the system to a well-balanced condition, with an average ASR of 0.9 (close to 1), an average %RSL of 45 to 55, and an ideal DPL close to zero dollars.

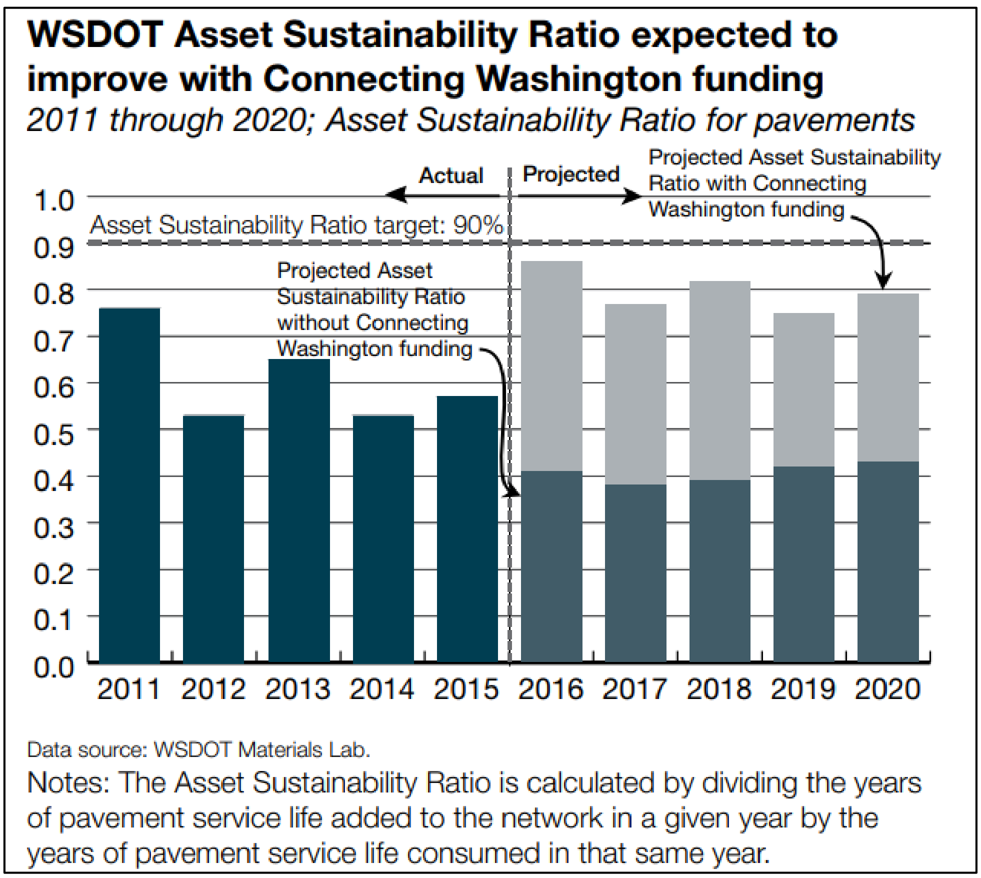

Figure 3.11: WSDOT ASR forecasting for different funding levels (15)

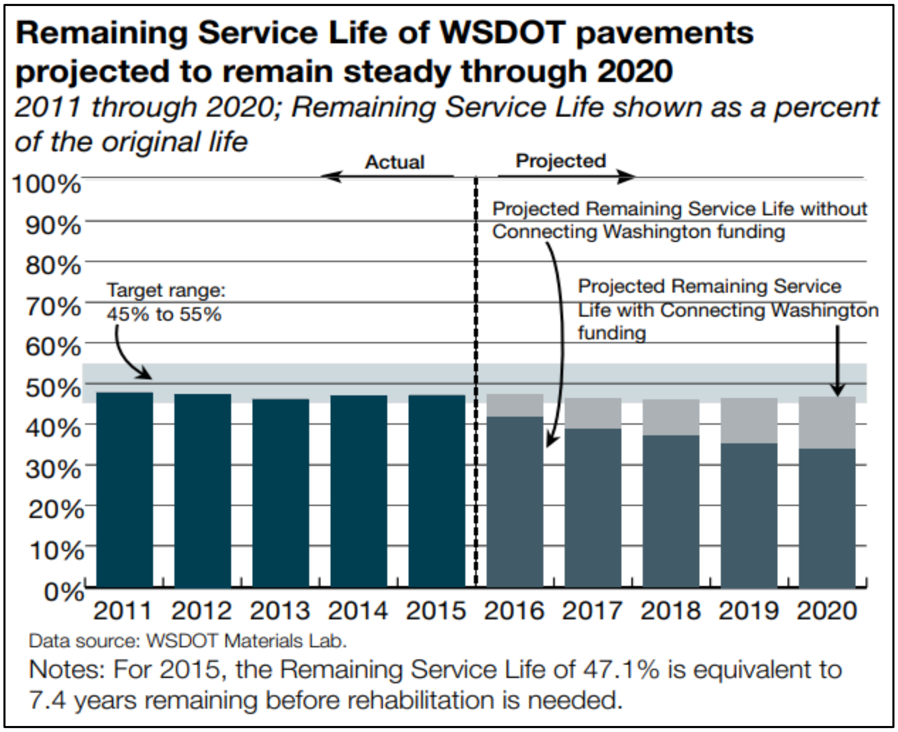

Figure 3.12: WSDOT RSL forecasting for different funding levels (15)

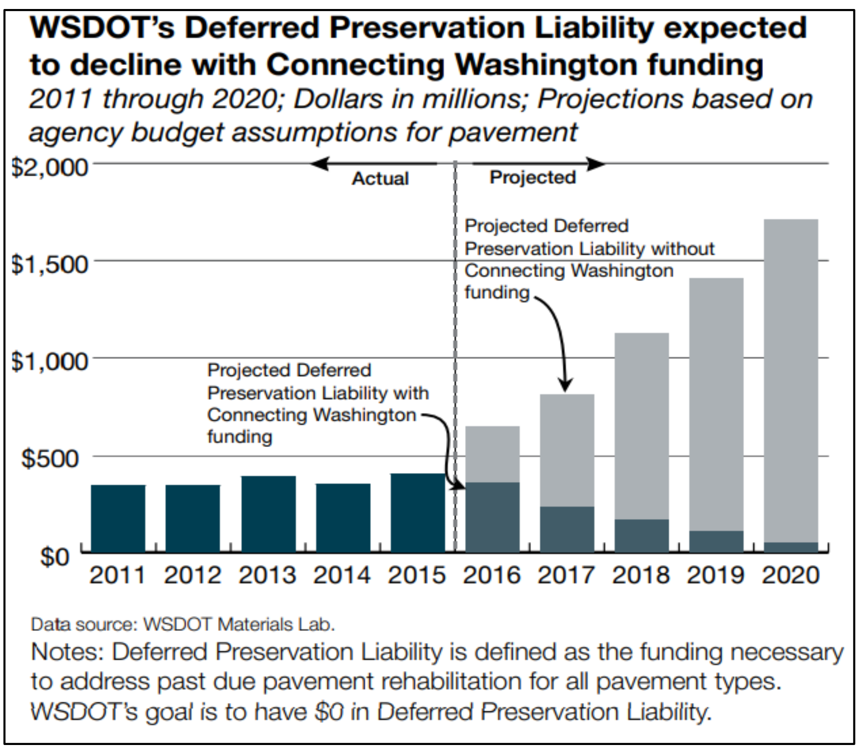

Figure 3.13: WSDOT DPL forecasting for different funding levels (15)

CHAPTER 4: ESTIMATING THE STATE-STATE TRANSITION RATES

A current challenge faced in pavement asset management is predicting pavement performance and deterioration rates. This prediction is essential because it provides information that allows forecasting repair demands and optimizing life-cycle costs. To address this critical need, many prediction models have been developed, mostly deterministic or empirical. (16, 17, 18)

In this chapter, we develop a Markovian model that can be used to model and predict the deterioration of pavements in Minnesota. The model is based on state-state deterioration probabilities that were estimated for pavement types that have the best available historical data.

4.1 SELECTION OF PAVEMENT PERFORMANCE METRIC

The first step in formulating a discrete-time model for pavement deterioration, is choosing the performance metric for the Markov matrix. After analyzing the metrics used by MnDOT to report pavement condition, RQI was selected as the base parameter to define the states in the Markov transition matrix. RQI was chosen because it takes into account customers' opinions and because it is used to obtain Remaining Service Life. The Remaining Service Life (RSL) is an estimate of the time until the next major rehabilitation of the pavement section. Using pavement deterioration curves, the time when a pavement section reaches an RQI of 2.5 is predicted and the RSL is simply calculated as the difference between the predicted and the present time. (19)

MnDOT's Stantec Consulting software called the Highway Pavement Management Application **(**HPMA) was used to extract the condition data of all pavement sections from 1995 to 2018 in the M-record format. Since Superpave came out in the mid-1990s, the data before 1995 was not used.

4.2 INTRODUCTION TO MARKOV CHAINS

The basic idea behind using Markov models in pavement management is that pavement condition can be characterized using a finite number of discrete states, and that the deterioration can be approximated by transitioning from the current state to the next one. An important factor is that the transition between states should be influenced only by the current state of the pavement.

An indexed collection of random variables $X = \{X_n, n < 0\}$ is a stochastic process, where Xn describes the system state at time n. A stochastic process is a Markov chain if $P\{X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \ldots, X_1 = i_1, X_0 = i_0\} = P\{X_{n+1} = j | X_n = i\}$. In other words, a stochastic process has the Markovian property if the conditional probability of a future event, given any past and present state, depends only upon the present state $X_n = i$ and is independent of the past states (21, 22). The conditional probabilities $P\{X_{n+1} = j | X_n = i\} = P_{ij}$ for a Markov chain are called (one-step) transition probabilities. This way, P_{ij} is the probability of jumping from state i to state j in one step. (21)

4.3 PRELIMINARY EMPIRICAL MARKOV MODEL

The states for the Markov transition matrix were derived based on the RQI and IRI thresholds adopted by MnDOT, as shown in table 4.1. Once the pavement degrades into very poor (enters state 5), it remains very poor unless it is repaired.

Table 4.1: States for Markov Transition based on RQI

State	RQI	IRI (in/mile)	Physical Meaning
1	4.1 - 5.0	36.6 - 7.0	Very Good
2	3.1 - 4.0	96.8 - 41.3	Good
3	2.1 - 3.0	185.6 - 104.4	Fair
4	1.1 - 2.0	303.2 - 196.1	Poor
5	0.0 - 1.0	465.7 - 316.5	Very Poor

After selecting the parameter to be used in the determination of the states, the transition matrix can be formulated. For pavement deterioration, the transition matrix should have the following format (17):

$$P = \begin{bmatrix} p(1) & 1-p(1) & 0 & 0 & 0 \\ 0 & p(2) & 1-p(2) & 0 & 0 \\ 0 & 0 & p(3) & 1-p(3) & 0 \\ 0 & 0 & 0 & p(4) & 1-p(4) \\ 0 & 0 & 0 & 0 & p(5) \end{bmatrix}$$

where *p(j)* is the probability of the pavement staying in state j, and *1-p(j)* is the probability of deteriorating to the next stage. (17)

The data exported from HPMA software was reorganized to show year, state, and maintenance activity. Then, the probability values p_i were calculated using equation 8:

$$p_i = \frac{T_{x_i}}{T_{n_i}} \quad [8]$$

where Tx_i is the sum of all pavement sections that transitioned from state *i* to state *i+1*, and Tn_i is the total number of pavement sections in state *i* that have not received any maintenace. The Transition Matrix computed using data from all pavement sections is shown below.

$$P = \begin{bmatrix} 0.565674 & 0.434326 & 0 & 0 & 0 \\ 0 & 0.895662 & 0.104338 & 0 & 0 \\ 0 & 0 & 0.936716 & 0.063284 & 0 \\ 0 & 0 & 0 & 0.910214 & 0.089786 \\ 0 & 0 & 0 & 0 & 1.000000 \end{bmatrix}$$

Table 4.2 shows the total number of sections recorded in the M-record format each year from 1995 to 2018 by pavement type. Bituminous Aggregate Base (BAB) sections had the most records and were selected for further analyses.

Table 4.2: Number of sections for each pavement type

	Pavement Type	Number of Records
BAB	Bituminous Aggregate Base	186910
BFD	Bituminous Full Depth	151
BOB	Bituminous Over Bituminous	22383
BOC	Bituminous Over Concrete	72914
CD	Concrete Doweled	54604
CRC	Continuously Reinforced Concrete	520
CU	Concrete Undoweled	300

4.3.1 Effect of Repair Activities and Rate of Repair

After estimating the transition matrix for each family of pavements, repair can be added to account for maintenance polices. Examples of commonly used repair techniques are crack sealing, crack filling, chip sealing, micro-surface, and joint sealing. The effect of repair policies should be analyzed individually, in other words, each repair policy should have its own recovery matrix (Q) that should look like the matrix below. The decision to apply a repair option at the end of each duty time generates policies. A policy can be defined as a rule for making decisions at each given duty time. (23)

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ q_{21} & 1 - q_{21} & 0 & 0 & 0 \\ q_{31} & q_{32} & 1 - \sum_{i=1}^{2} q_{3i} & 0 & 0 \\ q_{41} & q_{42} & q_{43} & 1 - \sum_{i=1}^{3} q_{4i} & 0 \\ q_{51} & q_{52} & q_{53} & q_{54} & 1 - \sum_{i=1}^{4} q_{5i} \end{bmatrix}$$

It is important to notice that only a certain percentage of pavement sections receive repair activities each year. Thus, it is important to estimate the rate of repair (R) for each repair activity, which describes what percentage of sections in each state is expected to receive a given repair.

4.3.2 Example of Calculations

To better demonstrate how the matrices were computed, we are showing an example of their calculation. The next matrices show the analysis of BAB pavement sections that were reclaimed.

P is the transition matrix with the total number of BAB sections in each state without any maintenance, showing natural deterioration:

$$P = \begin{bmatrix} 0.570992 & 0.429008 & 0 & 0 & 0 \\ 0 & 0.891424 & 0.108576 & 0 & 0 \\ 0 & 0 & 0.933736 & 0.066264 & 0 \\ 0 & 0 & 0 & 0.907232 & 0.092768 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Q shows the recovering effect of "Reclaim" activity:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.709220 & 0.290780 & 0 & 0 & 0 \\ 0.519802 & 0.477723 & 0.002475 & 0 & 0 \\ 0.454148 & 0.502183 & 0.043668 & 0 & 0 \\ 0.361111 & 0.527778 & 0 & 0.055556 & 0.055556 \end{bmatrix}$$

The rate of repair can be calculated as the proportion of sections that received reclamation. The matrices Pc and Qc show the number of sections in each state that were allowed to continue deteriorating or received reclamation, respectively.

$$
Pc = \begin{bmatrix} 3925 & 2949 & 0 & 0 & 0 \\ 0 & 71001 & 8648 & 0 & 0 \\ 0 & 0 & 35538 & 2522 & 0 \\ 0 & 0 & 0 & 3550 & 363 \\ 0 & 0 & 0 & 0 & 980 \end{bmatrix} \begin{matrix} =6874 \\ =79649 \\ =38060 \\ =3913 \\ =980 \end{matrix}
$$

$$
Qc = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 100 & 41 & 0 & 0 & 0 \\ 210 & 193 & 1 & 0 & 0 \\ 104 & 115 & 10 & 0 & 0 \\ 13 & 19 & 0 & 2 & 2 \end{bmatrix} \begin{matrix} =0 \\ =141 \\ =404 \\ =229 \\ =36 \end{matrix}
$$

The total counts from *Pc* and *Qc* are used to calculate the rate of repair applied in each state, and the results are shown in table 4.3.

Table 4.3: Rate of repair of reclamation for each state

State	RQI	Rate of Repair (r_i)
1	4.1 - 5.0	0
2	3.1 - 4.0	0.001767139
3	2.1 - 3.0	0.010503328
4	1.1 - 2.0	0.055287301
5	0.0 - 1.0	0.035433071

The total matrix T can be computed by combining the natural decay matrix P, the maintenance effect matrix Q and the rate of repair vector R. Table 4.3 illustrates the assembling of the parameters.

Table 4.4: Assembly of the total transition matrix for BAB pavements, including the effect of reclamation

		To State				
		1	2	3	4	5
Going from State	1	*p(1)*	*1-p(1)*	*0*	*0*	*0*
	2	$r_2 q_{21}$	*p(2)* - $r_2 q_{21}$	*1-p(2)*	*0*	*0*
	3	$r_3 q_{31}$	$r_3 q_{32}$	*p(3)* - $r_3 \sum_{i=1}^{2} q_{3i}$	*1-p(3)*	*0*
	4	$r_4 q_{41}$	$r_4 q_{42}$	$r_4 q_{43}$	*p(4)* - $r_4 \sum_{i=1}^{3} q_{4i}$	*1-p(4)*
	5	$r_5 q_{51}$	$r_5 q_{52}$	$r_5 q_{53}$	$r_5 q_{54}$	*1-* $r_5 \sum_{i=1}^{4} q_{5i}$

$$T = \begin{bmatrix} p(1) & 1-p(1) & 0 & 0 & 0 \\ r_2 q_{21} & p(2) - r_2 q_{21} & 1-p(2) & 0 & 0 \\ r_3 q_{31} & r_3 q_{32} & p(3) - r_3 \sum_{i=1}^{2} q_{3i} & 1-p(3) & 0 \\ r_4 q_{41} & r_4 q_{42} & r_4 q_{42} & p(4) - r_4 \sum_{i=1}^{3} q_{4i} & 1-p(4) \\ r_5 q_{51} & r_5 q_{52} & r_5 q_{53} & r_5 q_{54} & 1- r_5 \sum_{i=1}^{4} q_{5i} \end{bmatrix}$$

The total matrix T for BAB pavement sections, and repair type: “Reclaim”:

$$T = \begin{bmatrix} 0.570992 & 0.429008 & 0 & 0 & 0 \\ 0.001253 & 0.89017 & 0.108576 & 0 & 0 \\ 0.00546 & 0.005018 & 0.923259 & 0.066264 & 0 \\ 0.025109 & 0.027764 & 0.002414 & 0.851945 & 0.092768 \\ 0.012795 & 0.018701 & 0 & 0.001969 & 0.966535 \end{bmatrix}$$

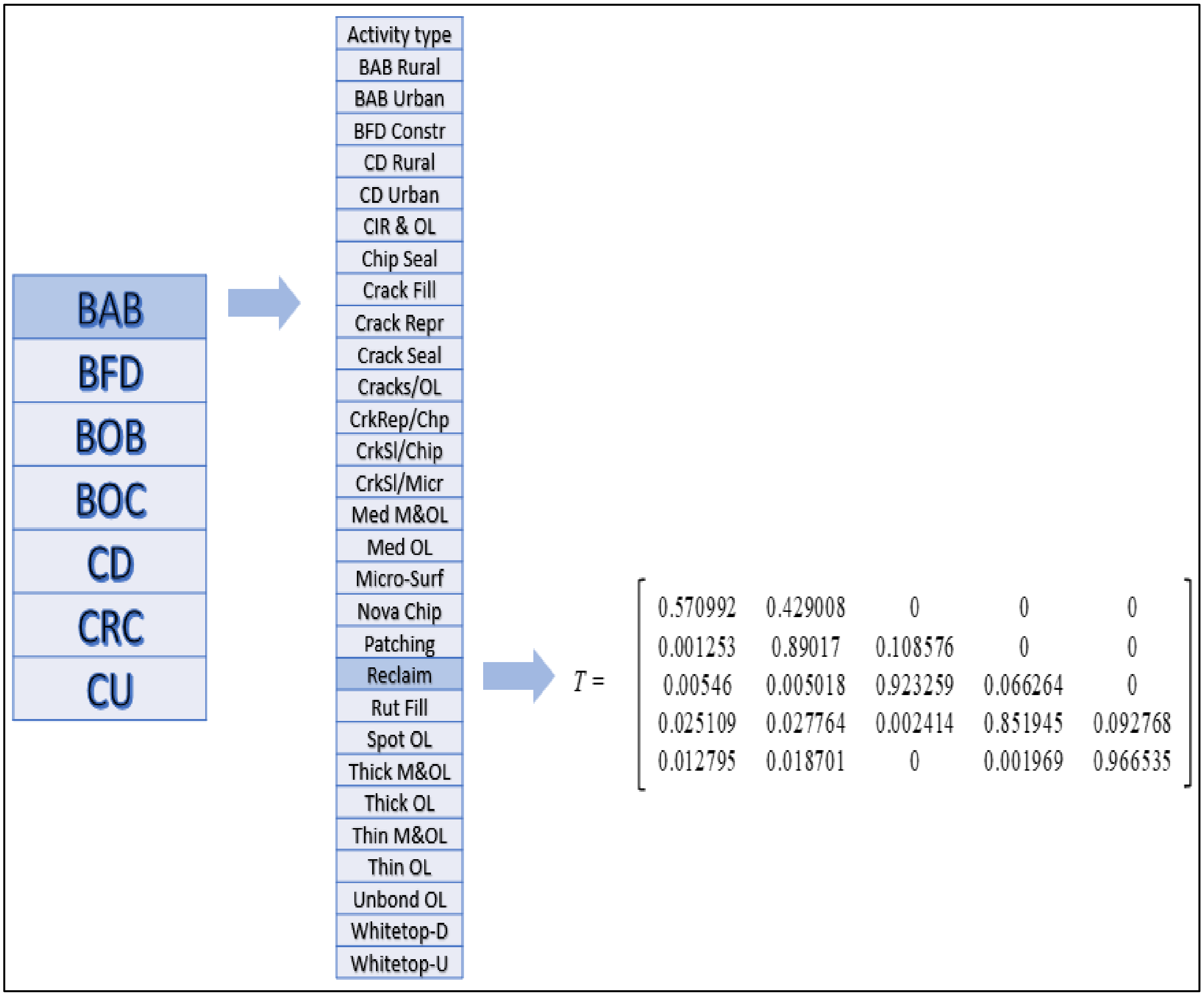

Figure 4.1: Total matrix T for BAB pavement sections and Reclamation repair

4.4 LONG RUN DISTRIBUTION

P_{ij}, previously defined as the probability of jumping from state *i* to state *j* in one step (one year), can be used to compute the n-step transition probability P_{ij}^{n}, defined as the probability that a process in state *i* will end in *j* after n steps (n years). This way, P_{ij}^{n} is defined as $P\{X_{n+k} = j | X_k = i\}$. (21)

Chapman-Kolmogorov equations are used to compute the n-step transition probability P_{ij}^{n}, using $P_{ij}^{n+m} = \sum_{k=0}^{\infty} P_{ik}^{n} P_{kj}^{m} = P\{X_{n+m} = j | X_0 = i\}$, where $P_{ik}^{n} P_{kj}^{m}$ represents the probability of going from state *i* to state *j* in *n+m* transition steps through a path that goes to state *k* at the n^{th} transition.

If $\boldsymbol{P}^{(n)}$ denotes the matrix of n-step transition probabilities P_{ij}^{n}, then the Chapman-Kolmogorov equation yields that $\boldsymbol{P}^{(n+m)} = \boldsymbol{P}^{(n)}\boldsymbol{P}^{(m)}$ and $\boldsymbol{P}^{(2)} = \boldsymbol{P}^{(1+1)} = \boldsymbol{P}.\boldsymbol{P} = \boldsymbol{P}^{2}$. Thus, $\boldsymbol{P}^{(n)} = \boldsymbol{P}^{(n-1+1)} = \boldsymbol{P}^{n-1}\boldsymbol{P} = \boldsymbol{P}^{n}$.

That is, the n-step transition probability matrix is obtained by n^{th} power of the transition matrix ***P***. (21, 22)

As n gets large, the probabilities stabilize, getting to a steady state, also known as long run probabilities, which represent the condition distribution of pavement sections in the long run. For example, the long run distribution of matrix T is shown in table 4.5.

Table 4.5: Long run distribution for BAB sections by executing only reclamation

State	1	2	3	4	5
Sections (%)	2.27	19.73	28.33	13.16	36.49

Table 5 shows the stationary distribution for BAB pavement sections in a scenario where the only repair activity used is reclamation. In the long run, 2.27% of the sections will be in state 1 (very good condition), and 36.49% will be in state 5 (very poor condition).

To improve the model accuracy, several (as many as possible) repair activities can be analyzed simultaneously. Figure 4.2 shows an example in which 2 repair activities are combined. The Final Transition Matrix for two repair activities should be assembled as shown in Table 4.6.

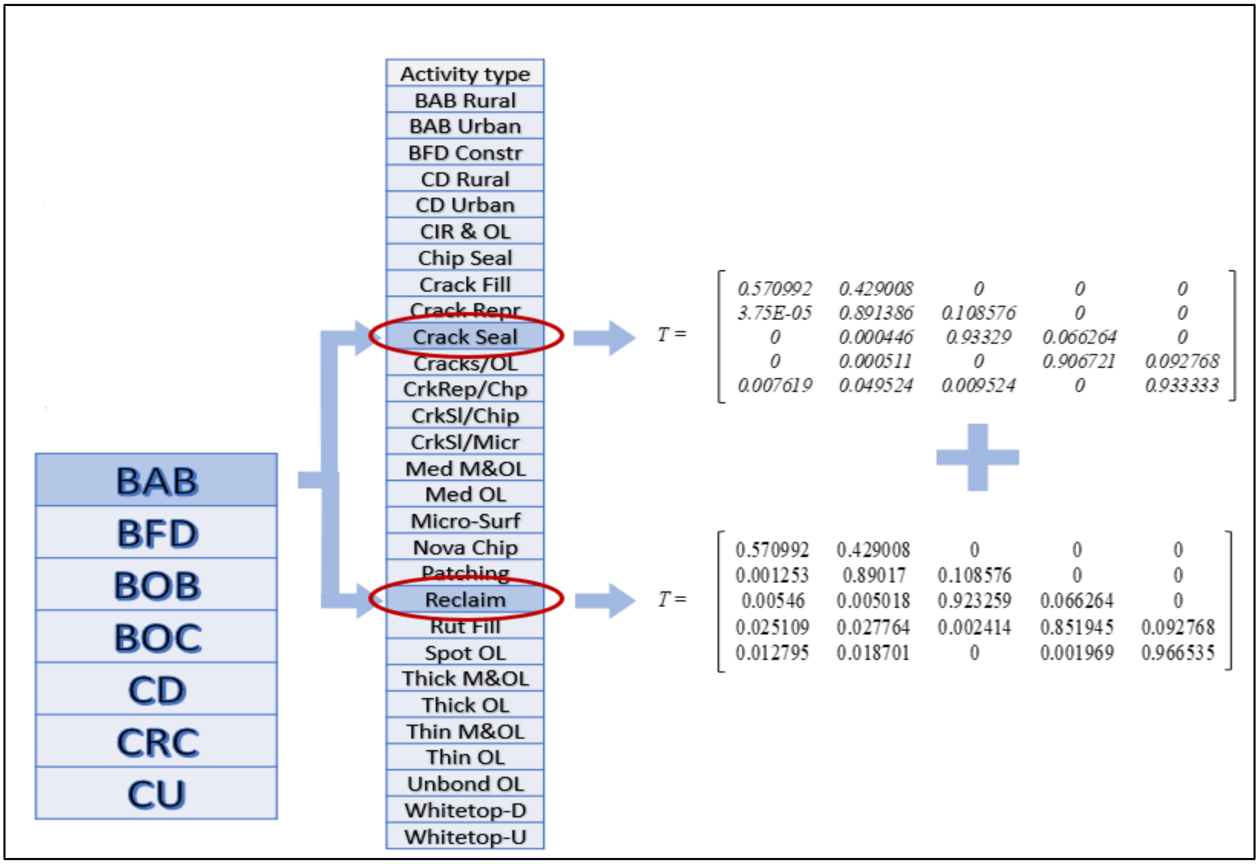

Figure 4.2: Example of combining the effect of Crack Seal and Reclamation

Table 4.6: Final Transition Matrix composition for two repair activities

		End State				
		1	**2**	**3**	**4**	**5**
Start State	**1**	*p(1)*	*1-p(1)*	*0*	*0*	*0*
	2	$r_{A2}q_{A21} + r_{B2}q_{B21}$	*p(2)* - $r_{A2}q_{21}$	*1-p(2)*	*0*	*0*
	3	$r_{A3}q_{A31} + r_{B3}q_{B31}$	$r_{A3}q_{A32} + r_{B3}q_{B32}$	*p(3)* $-\sum_{J=1}^{2}\sum_{i=1}^{2} r_{J3}q_{3i}$	*1-p(3)*	*0*
	4	$r_{A4}q_{A41} + r_{B4}q_{B41}$	$r_{A4}q_{A42} + r_{B4}q_{B42}$	$r_{A4}q_{A43} + r_{B4}q_{B43}$	*p(4)* $-\sum_{J=1}^{2}\sum_{i=1}^{3} r_{J4}q_{4i}$	*1-p(4)*
	5	$r_{A5}q_{A51} + r_{B5}q_{B51}$	$r_{A5}q_{A52} + r_{B5}q_{B52}$	$r_{A5}q_{A53} + r_{B5}q_{B53}$	$r_{A5}q_{A54} + r_{B5}q_{B54}$	*1-* $\sum_{J=1}^{2}\sum_{i=1}^{4} r_{J5}q_{5i}$

4.5 REVISED PAVEMENT TYPE

The preliminary analysis performed in chapter 2 revealed some inconsistencies in the pavement type recorded in the HPMA (Figure 4.3 and table 4.7). After contacting MnDOT, the research team received a revised dataset that was used to fix the analysis previously performed, as shown in Figure 4.4 and table 4.8.

Table 4.7: Yearly distribution of sections by pavement type shown in chapter 2

	Pavement Type							
Years	Bituminous Aggregate Base	Bituminous Full Depth	Bituminous Over Bituminous	Bituminous Over Concrete	Concrete Doweled	Continuously Reinforced Concrete	Concrete Undoweled	Total
2018	13153		12	10	2597	4		15776
2017	1819	20	7857	3592	2469	4	126	15887
2016	8187	6	1456	3638	2560	4	17	15868
2015	9013	1	687	3685	2469	5	2	15862
2014	8897	2	774	3721	2465	5	3	15867
2013	8860	5	830	3660	2503	5		15863
2012	8958	3	792	3627	2468	10	20	15878
2011	9316	3	417	3692	2392	10	14	15844
2010	8777	5	973	3676	2391	18	2	15842
2009	9278	2	480	3636	2387	28	26	15837
2008	9140	15	633	3647	2385	25		15845
2007	9183	4	586	3640	2395	25	4	15837
2006	9066	2	702	3635	2407	25		15837
2005	8963	2	797	3559	2471	25	17	15834
2004	9115	2	638	3576	2472	34		15837
2003	9234	2	527	3561	2480	34		15838
2002	8744	2	713	3458	2375	34		15326
2001	8703		639	3431	2425	33		15231
2000	4741		668	2180	1706	38	16	9349
1999	4869	15	539	1939	1611	38	26	9037
1998	4582	15	405	1983	1890	38	5	8918
1997	4976	5	526	2078	1888	38	20	9531
1996	4479	13	336	1692	2007	38		8565
1995	4857	27	396	1598	1391	2	2	8273

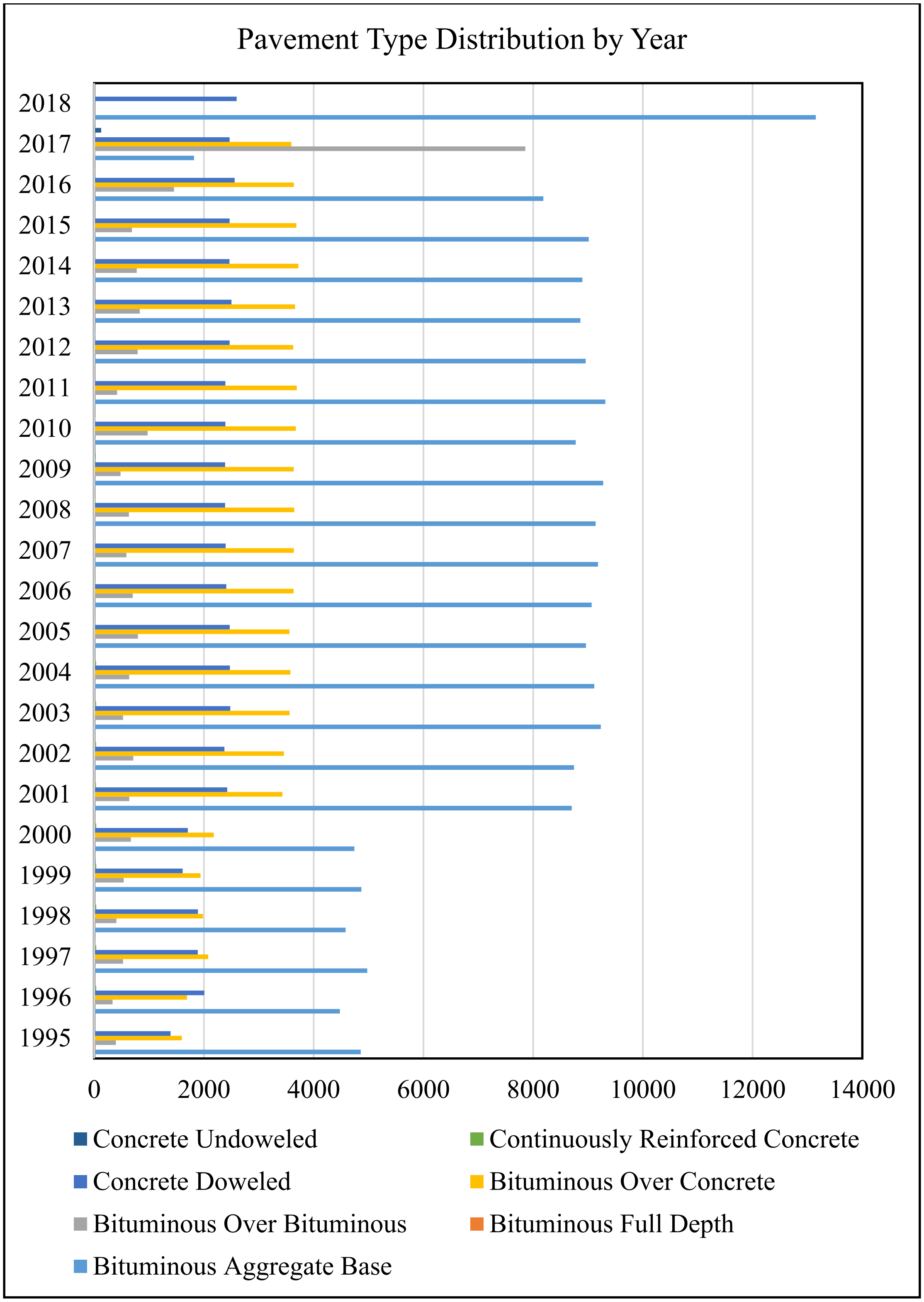

Figure 4.3: Yearly distribution of sections by pavement type shown in chapter 2

Table 4.8: Revised yearly distribution of sections by pavement type

	Pavement Type							
Years	Bituminous Aggregate Base	Bituminous Full Depth	Bituminous Over Bituminous	Bituminous Over Concrete	Concrete Doweled	Continuously Reinforced Concrete	Concrete Undoweled	Total
2018	1766	18	7788	3480	2436	4	136	15628
2017	1789	19	7790	3551	2460	4	125	15738
2016	1734	19	7809	3543	2416	4	136	15661
2015	1797	21	7747	3550	2331	4	126	15576
2014	1739	22	7786	3566	2295	5	126	15539
2013	1693	46	7735	3501	2310	5	123	15413
2012	1682	54	7711	3490	2293	8	125	15363
2011	1633	73	7754	3522	2217	8	119	15326
2010	1497	72	7883	3529	2176	16	122	15295
2009	1429	80	7931	3486	2119	16	147	15208
2008	1355	120	7944	3422	2119	12	143	15115
2007	1297	130	7931	3418	2091	12	143	15022
2006	1233	131	7946	3397	2061	12	143	14923
2005	1140	157	7939	3295	2118	12	144	14805
2004	1057	171	7930	3277	2076	12	154	14677
2003	1023	196	7815	3246	2078	12	154	14524
2002	971	192	7775	3169	2039	12	155	14313
2001	872	210	7700	3129	2010	12	168	14101
2000	514	136	4475	2055	1425	12	142	8759
1999	484	155	4419	1784	1330	15	84	8271
1998	383	203	4063	1908	1516	15	133	8221
1997	412	200	4510	1921	1571	15	93	8722
1996	345	264	3802	1634	1605	15	136	7801
1995	398	243	4184	1394	1116		83	7418

The revised analysis showed that the most used pavement type is Bituminous over Bituminous (BOB), and not Bituminous over Aggregate Base (BAB).

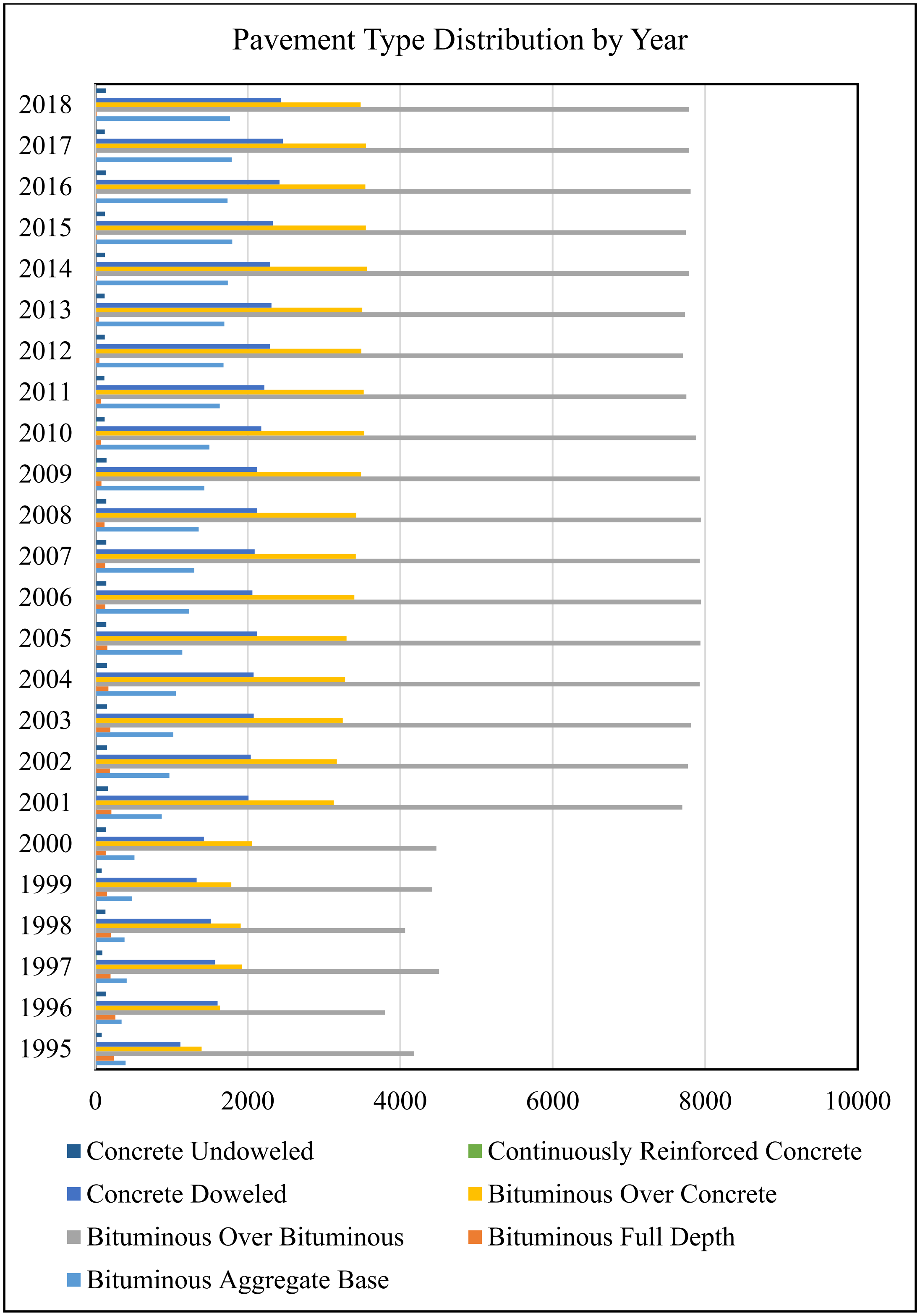

Figure 4.4: Revised yearly distribution of sections by pavement type.

Following the results from table 4.8 and Figure 4.4, bituminous over bituminous (BOB), the most significant pavement type, was selected for the statistical analysis. The HPMA software was used to extract the historical data for the past 23 years (from 1995 to 2018).

4.6 REVISED SURFACE AND BASE THICKNESS, AND YEARS

During the second TAP meeting, it was brought to our attention that the pavement thickness data we were initially given were inaccurate. After receiving the revised thickness, we mapped it into our dataset and repeated the analyses. It was also revealed that between the years 2000 and 2001, MnDOT changed the way of collecting data. This change resulted in a great increase in the number of sections recorded, as shown in figure 4.4. Because of that, we updated our datafile to include observations from 2001 to 2018 and repeated the analyses.

4.7 AGGREGATING PAVEMENT SECTIONS BASED ON THE LAST REPAIR ACTIVITY THEY HAVE RECEIVED USING K-MEANS CLUSTER ANALYSIS OF COEFFICIENTS USED BY MNDOT

During the first TAP meeting, the panel raised the concern that pavement sections that receive different repair activities deteriorate at different rates, and that the repair history should be considered during the analysis. In order to address that concern and create a homogeneous group of pavement sections, they were aggregated according to the repair activity they have last received.

Based on the coefficients of the deterioration models used by MnDOT, shown in Table 4.9, K-means clustering, an algorithm that groups observations into clusters based on their characteristics, was used to find subgroups of repair activities for which activities result in similar pavement behavior.

Table 4.9: Coefficients received from MnDOT

INDEX	PAVEMENT	ACTIVITY	Coeff. A	Coeff. B	Coeff. C	Coeff. O
RQI	BOB	Chip Seal	4.544	6.798	1.185	3.32
RQI	BOB	Micro-Surfacing	7.367	13.252	1.322	3.33
RQI	BOB	Rut Fill	9.871	25.283	1.490	3.06
RQI	BOB	Crck Seal /Chip Seal	71.743	75.073	1.019	3.36
RQI	BOB	Crck Seal/Micro-Surf	7.367	13.252	1.322	3.33
RQI	BOB	Rut Fill & Chip Seal	71.743	75.073	1.019	3.36
RQI	BOB	Rut Fill & Micro-Srf	7.367	13.252	1.322	3.33
RQI	BOB	Rut Fill/Crk Repair	9.871	25.283	1.490	3.06
RQI	BOB	Thin Overlay	40.471	43.176	1.028	3.67
RQI	BOB	Medium Overlay	35.848	38.283	1.026	3.90
RQI	BOB	Thick Overlay	11.456	14.871	1.100	3.76
RQI	BOB	Thin Mill/Overlay	39.812	43.691	1.039	3.39
RQI	BOB	Medium Mill/Overlay	11.920	14.874	1.094	3.77
RQI	BOB	Thick Mill/Overlay	35.576	39.673	1.042	3.75
RQI	BOB	Crack Repr/Med OL	35.848	38.283	1.026	3.90
RQI	BOB	CIR & Medium OL	75.214	79.989	1.024	3.98
RQI	BOB	Full Mill & Thick OL	35.576	39.673	1.042	3.75
RQI	BOB	Crk Repair/Chip Seal	71.743	75.073	1.019	3.36
RQI	BOB	Crk Repair/MicroSurf	7.367	13.252	1.322	3.33
RQI	BOB	Nova Chip (UTBWC)	40.471	43.176	1.028	3.67
RQI	BOB	Full Mill & Replace	35.576	39.673	1.042	3.75
RQI	BOB	Rut & Crack Fill	9.871	25.283	1.490	3.06

The first step is to standardize the coefficients so that the variables have mean zero and standard deviation one. This ensures that all variables have the same effect on the clustering results. The standardized coefficients are shown in table 4.10.

Table 4.10: Standardized coefficients for each repair activity

	Coefficient A	Coefficient B	Coefficient C	Coefficient O
Chip Seal	-1.0822	-1.2910	0.1501	-0.6564
Micro-Surfacing	-0.9656	-1.0075	0.9315	-0.6216
Rut Fill	-0.8623	-0.4791	1.8897	-1.5612
Crck Seal /Chip Seal	1.6922	1.7076	-0.7967	-0.5172
Crck Seal/Micro-Surf	-0.9656	-1.0075	0.9315	-0.6216
Rut Fill & Chip Seal	1.6922	1.7076	-0.7967	-0.5172
Rut Fill & Micro-Srf	-0.9656	-1.0075	0.9315	-0.6216
Rut Fill/Crk Repair	-0.8623	-0.4791	1.8897	-1.5612
Thin Overlay	0.4011	0.3067	-0.7454	0.5615
Medium Overlay	0.2103	0.0918	-0.7568	1.3619
Thick Overlay	-0.7968	-0.9364	-0.3347	0.8747
Thin Mill/Overlay	0.3739	0.3293	-0.6826	-0.4128
Medium Mill/Overlay	-0.7777	-0.9363	-0.3689	0.9095
Thick Mill/Overlay	0.1990	0.1529	-0.6655	0.8399
Crack Repr/Med OL	0.2103	0.0918	-0.7568	1.3619
CIR & Medium OL	1.8355	1.9235	-0.7682	1.6402
Full Mill & Thick OL	0.1990	0.1529	-0.6655	0.8399
Crk Repair/Chip Seal	1.6922	1.7076	-0.7967	-0.5172
Crk Repair/MicroSurf	-0.9656	-1.0075	0.9315	-0.6216
Nova Chip (UTBWC)	0.4011	0.3067	-0.7454	0.5615
Full Mill & Replace	0.1990	0.1529	-0.6655	0.8399
Rut & Crack Fill	-0.8623	-0.4791	1.8897	-1.5612

4.7.1 Clustering Distance Measures

A distance matrix was computed to illustrate the similarity or dissimilarity in pairwise comparisons between activities. The Euclidean distance method was used, which is defined in equation 9. (25)

$$d(p,q) = d(q,p) = \sqrt{\sum_{i=1}^{n}(q_i - p_i)^2} \quad [9]$$

where *p* and *q* are vectors of length *n*.

The results are plotted in figure 4.5.

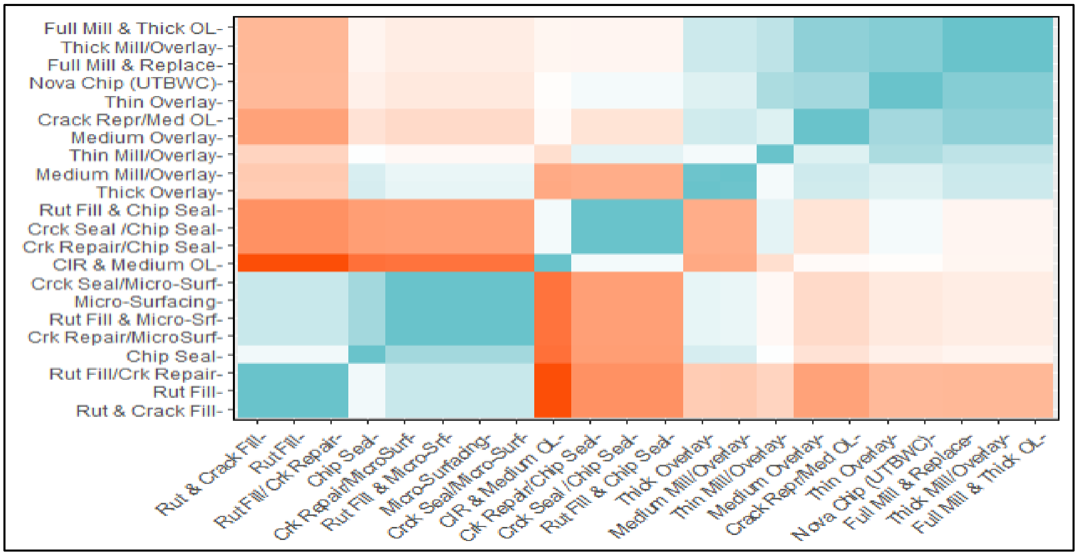

Figure 4.5: Euclidean distance matrix

The distance matrix for the pairwise comparison between activities is shown in figure 4.7. The matrix shows which activities have large dissimilarities (red) versus those that appear to be similar (blue).

4.7.2 K-means Clustering

K-means clustering was computed using the Hartigan-Wong algorithm (25), which minimizes the within-cluster variation, and defines it as the sum of squared Euclidean distances between points and the corresponding mean (centroid).

The k-cluster algorithm consists of specifying the number of clusters k, then randomly selecting k data points as the initial clusters means. Each data point is then assigned to the closest centroid based on the Euclidean distance between point and centroid. The new mean values of all the data points in a cluster is calculated and the cluster centroid is updated. Finally, the total within sum of squares is iteratively minimized.

The total within-cluster sum of squares is defined in equation 10. (25)

$$total\ within\ ss = \sum_{k=1}^{k} W(C_k) = \sum_{k=1}^{k} \sum_{x_i \epsilon C_k} (x_i - \mu_k)^2 \qquad [10]$$

where x_i is a data point that belongs to the cluster C_k and μ_k is the mean value of the points in the cluster C_k.

The k-means results were plotted and are shown in figure 4.6. (Note: because there are more than two variables (dimensions) in the data, principal component analysis (PCA) was performed and the first two principal components that explain most of the variance were used to plot the data points). The plots show simulations with several different numbers of clusters, k (from 2 to 7).

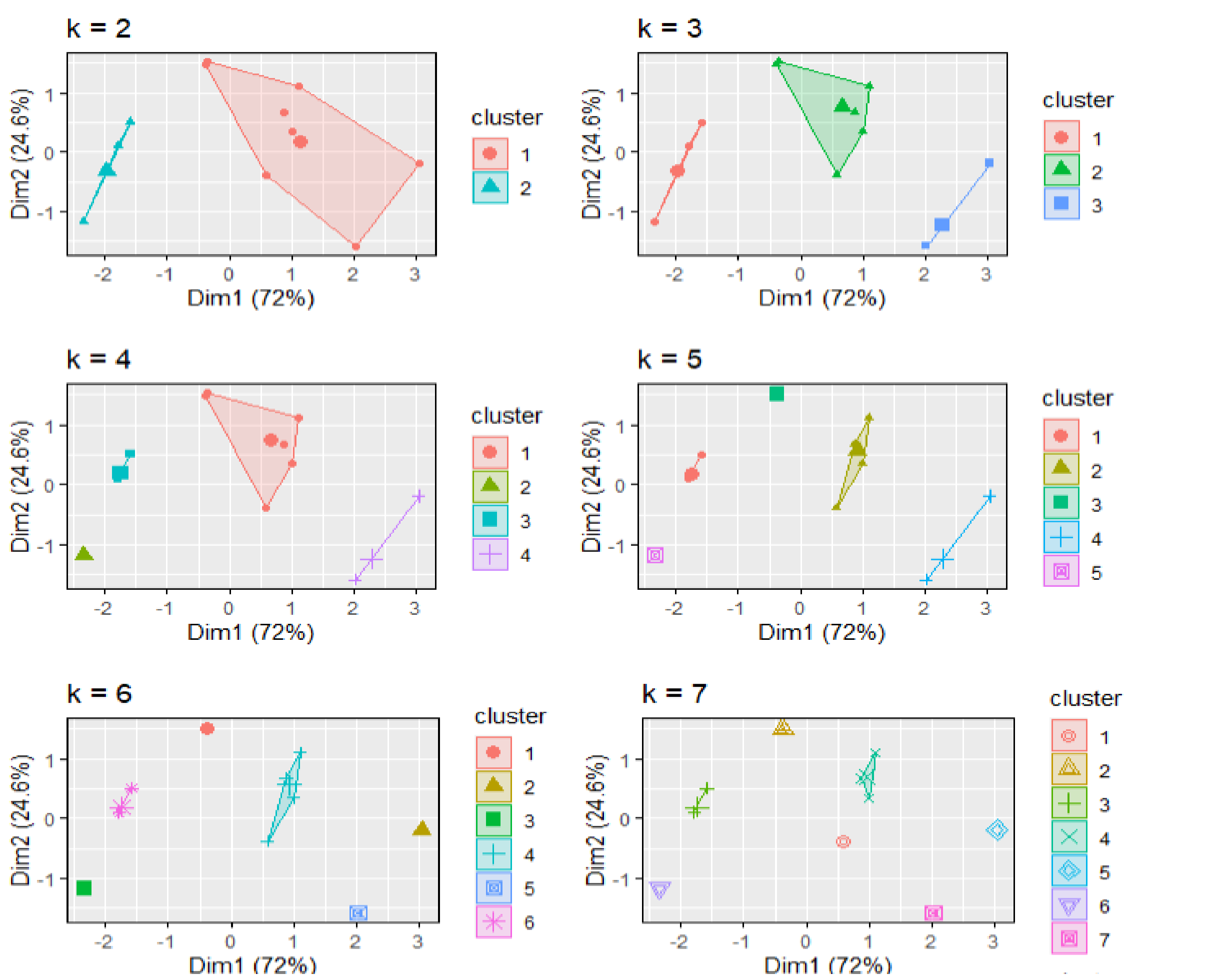

Figure 4.6: k-means results for k varying from 2 to 7

4.7.3 Determining the Optimal Number of Clusters, k

The optimal number of clusters was investigated through the Elbow Method, which is defined in equation 11. (25)

$$minimize\left(\sum_{k=1}^{k} W(C_k)\right) \qquad [11]$$

where C_k is the k^{th} cluster and W(C_k) is the within-cluster variation.

The within groups sum of squares was plotted against the number of clusters as shown in figure 4.7. The optimal number of clusters is determined in the plot, where a bend similar to an elbow on the arm is located.

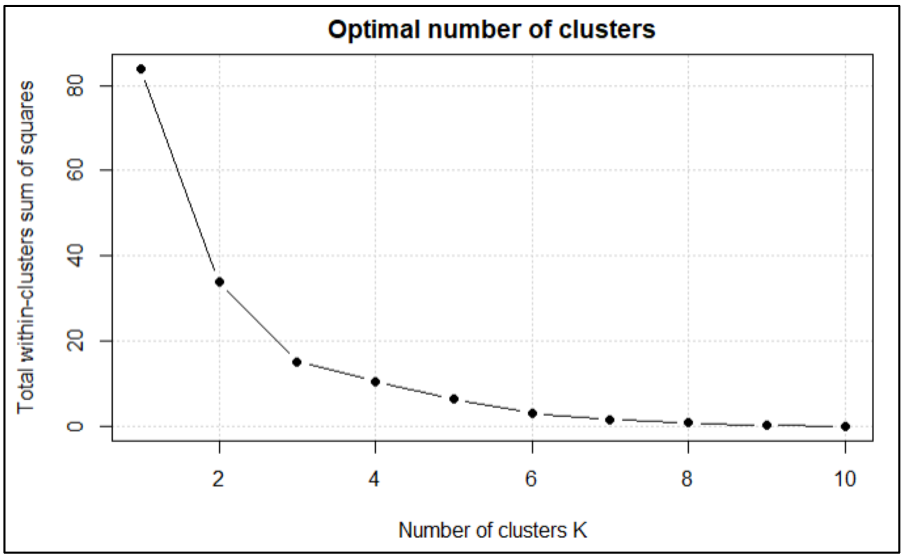

Figure 4.7: Number of clusters versus total within-clusters sum of squares

The plot in figure 4.7 suggests that the optimal number of clusters is 5. As a consequence, the repair activities were divided into 5 clusters, as shown in figure 4.8. Table 4.11 shows the activities distribution.

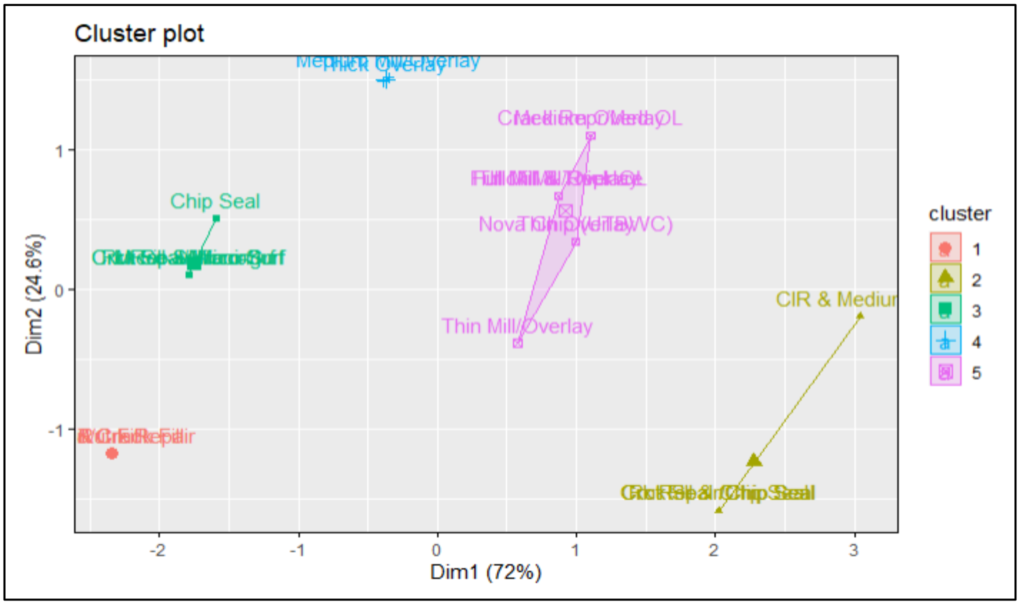

Figure 4.8: Visual representation of 5 clusters

Table 4.11: Repair activities present in each cluster

Activity	Cluster	Activity	Cluster
Rut Fill	1	Crk Repair/MicroSurf	3
Rut Fill/Crk Repair	1	Thick Overlay	4
Rut & Crack Fill	1	Medium Mill/Overlay	4
Crck Seal /Chip Seal	2	Thin Overlay	5
Rut Fill & Chip Seal	2	Medium Overlay	5
CIR & Medium OL	2	Thin Mill/Overlay	5
Crk Repair/Chip Seal	2	Thick Mill/Overlay	5
Chip Seal	3	Crack Repr/Med OL	5
Micro-Surfacing	3	Full Mill & Thick OL	5
Crck Seal/Micro-Surf	3	Nova Chip (UTBWC)	5
Rut Fill & Micro-Srf	3	Full Mill & Replace	5

The cluster distribution was added to the initial data and used to create a summary of the coefficients as shown in table 4.12.

Table 4.12: Descriptive statistics

Cluster	Coefficient A	Coefficient B	Coefficient C	Coefficient O
1	9.871	25.283	1.490	3.060
2	72.611	76.302	1.020	3.515
3	6.802	11.961	1.295	3.328
4	11.688	14.873	1.097	3.765
5	37.397	40.704	1.034	3.723

4.7.4 Analysis Excluding Preventive Maintenance Activities

After input from MnDOT, the analyses were repeated excluding the Preventive Maintenance activities. The new set of activities taken in account is shown in table 4.13.

Table 4.13: Coefficients received from MnDOT, excluding Preventive Maintenance

INDEX	PAVEMENT	ACTIVITY	Coefficient A	Coefficient B	Coefficient C	Coefficient O
RQI	BIT OVER BIT	Thin Overlay	40.471	43.176	1.028	3.67
RQI	BIT OVER BIT	Medium Overlay	35.848	38.283	1.026	3.9
RQI	BIT OVER BIT	Thick Overlay	11.456	14.871	1.1	3.76
RQI	BIT OVER BIT	Thin Mill/Overlay	39.812	43.691	1.039	3.39
RQI	BIT OVER BIT	Medium Mill/Overlay	11.92	14.874	1.094	3.77
RQI	BIT OVER BIT	Thick Mill/Overlay	35.576	39.673	1.042	3.75
RQI	BIT OVER BIT	Crack Repr/Med OL	35.848	38.283	1.026	3.9
RQI	BIT OVER BIT	CIR & Medium OL	75.214	79.989	1.024	3.98
RQI	BIT OVER BIT	Full Mill & Thick OL	35.576	39.673	1.042	3.75
RQI	BIT OVER BIT	Nova Chip (UTBWC)	40.471	43.176	1.028	3.67
RQI	BIT OVER BIT	Full Mill & Replace	35.576	39.673	1.042	3.75

The new Euclidean distance is plotted in figure 4.9.

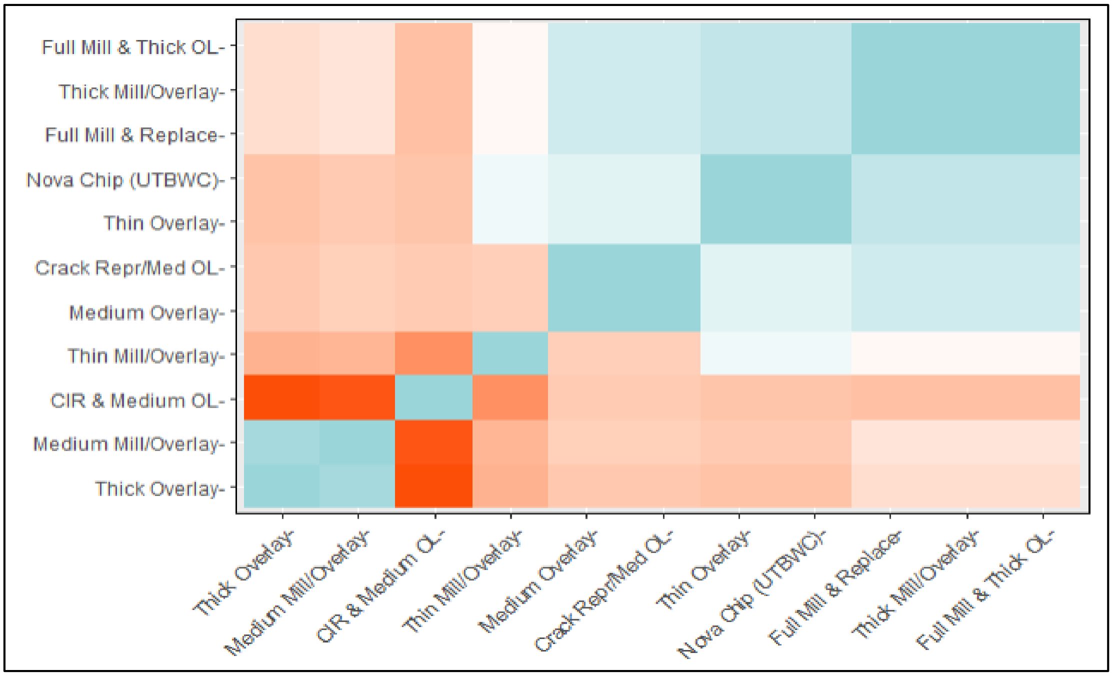

Figure 4.9: Euclidean distance matrix

Figure 4.10 shows k-means cluster results simulations for different number of clusters.

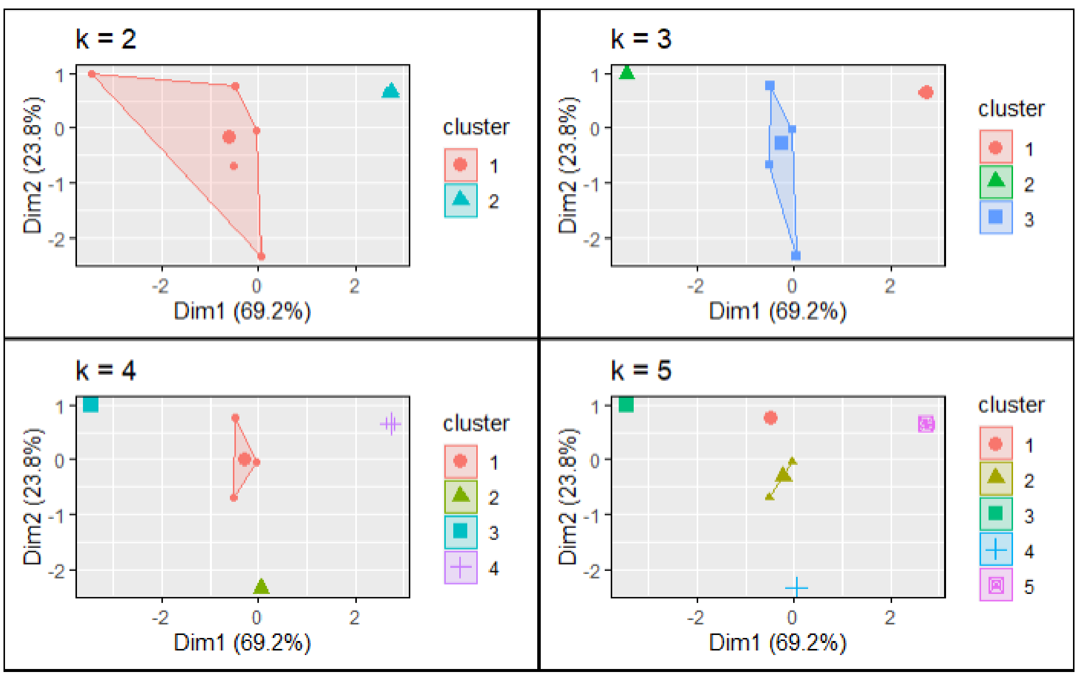

Figure 4.10: k-means results for k varying from 2 to 5

Based on the future needs of the project, the repair activities were divided into 2 clusters, as shown in figure 4.11. Table 4.14 shows the activities distribution.

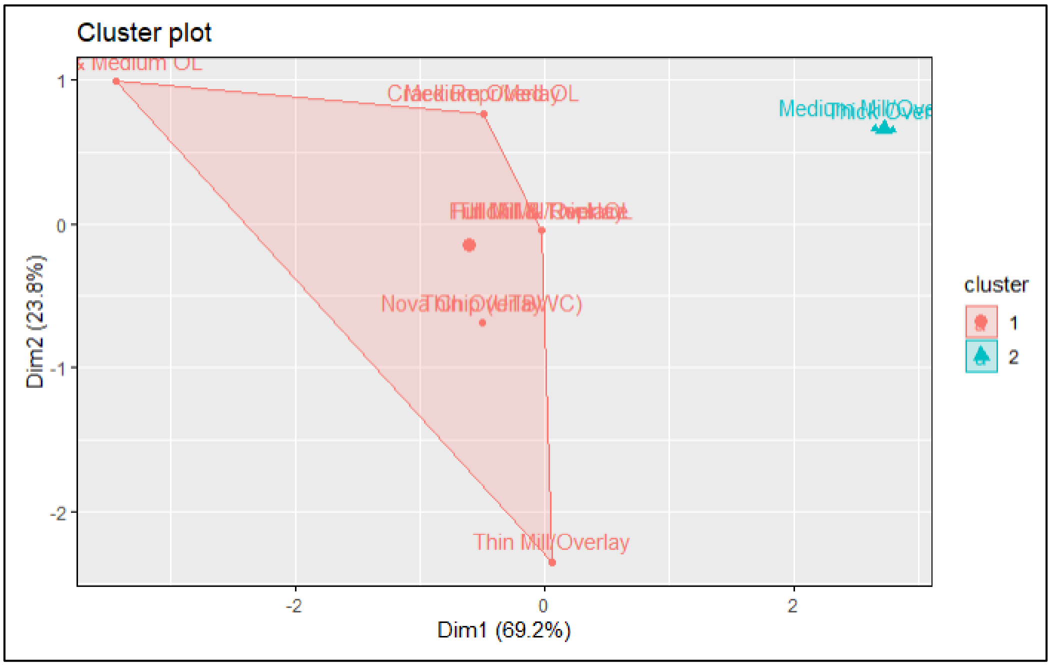

Figure 4.11: Visual representation of 2 clusters

Table 4.14: Cluster distribution from the k-means analysis

Activity	Cluster
CIR & Medium OL	1
Thick Overlay	2
Medium Mill/Overlay	2
Thick Mill/Overlay	1
Full Mill & Thick OL	1
Nova Chip (UTBWC)	1
Full Mill & Replace	1
Thin Overlay	1
Medium Overlay	1
Crack Repr/Med OL	1
Thin Mill/Overlay	1

An assessment of the numbers of records revealed that cluster 1 has more records than cluster 2, thus cluster 1 was selected for the next analyses.

4.8 REARRANGEMENT OF CONDITION STATES

The time that each section spent in a condition state before deteriorating to the next ones was called lifetime. An analysis on the time sections spend in each state, using the state definitions described above, revealed these have a Poisson distribution, which constitutes a semi-Markovian behavior, instead of Markovian.

A semi-Markov model would bring increasing analytics complexity and affect the future optimization computations. To avoid a semi-Markov model, the states were rearranged so that the lifetimes were approximately geometrically distributed. After several trials, the distribution in table 4.15 proved to fix the problem and the analysis proceeded to consider a Markov model.

Table 4.15: RQI ranges used to create condition states

State	RQI	Physical Meaning
1	5.0 - 4.2	Very Good
2	4.1 - 4.0	Very Good
3	3.9	Good
4	3.8	Good
5	3.7	Good
6	3.6	Good
7	3.5	Good
8	3.4	Good
9	3.3	Good
10	3.2	Good
11	3.1	Good
12	3.0	Good
13	2.9 - 2.8	Fair
14	2.7 - 2.6	Fair
15	2.5 - 2.4	Fair
16	2.3 - 2.2	Fair
17	2.1 - 2.0	Fair

Sections with RQI lower than 2.5 are attributed zero remaining service life and sections with RQI lower than 2.0 are considered in poor condition. Thus, the new arrangement of states only considered sections with RQI greater or equal to 2.0.

Figures 4.12 and 4.13 show the distribution of the time sections spent in each state. As shown in the plots, the lifetimes have approximately geometric distributions, which implies Markovian behavior.

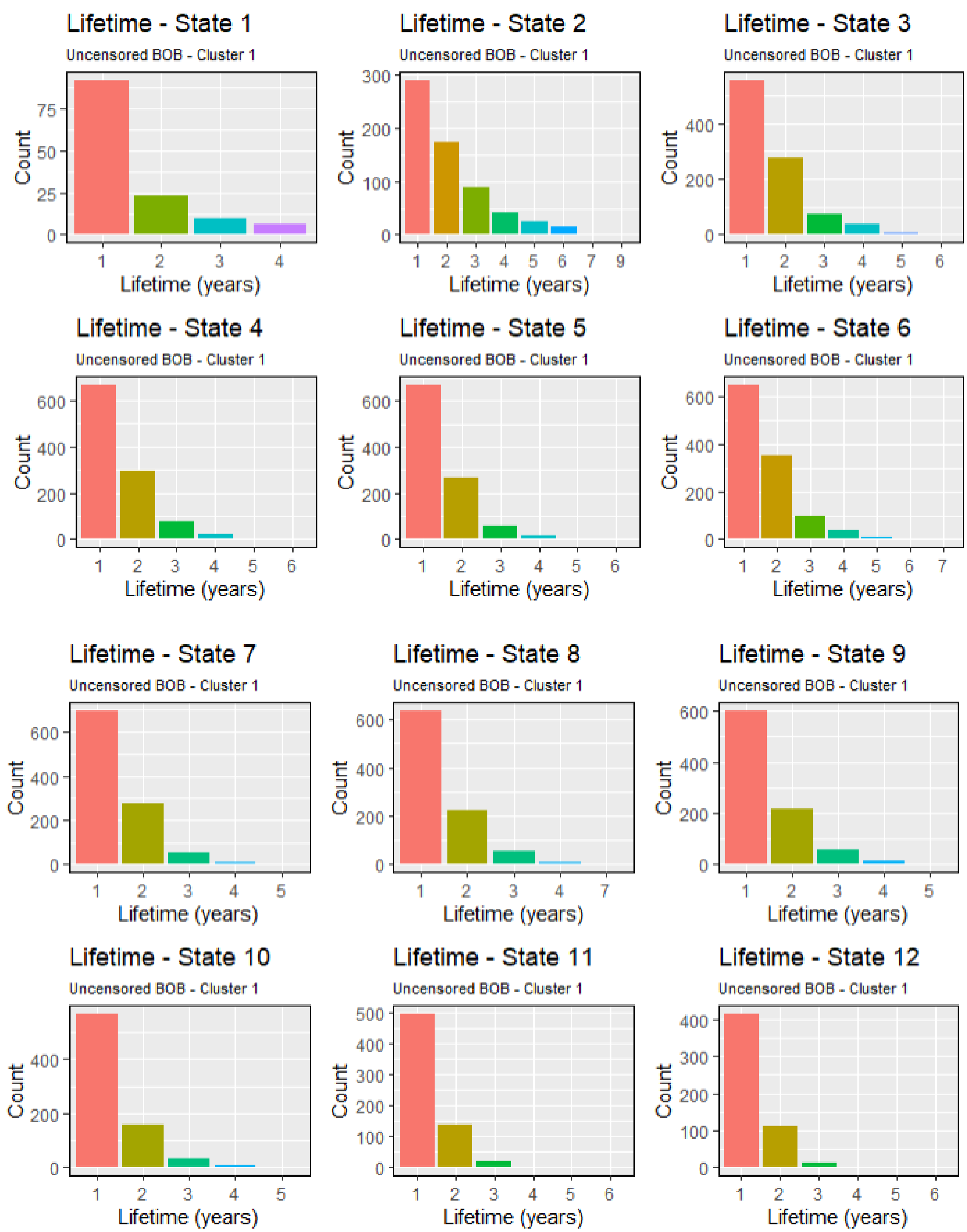

Figure 4.12: Distribution of time sections spent in states 1 to 12

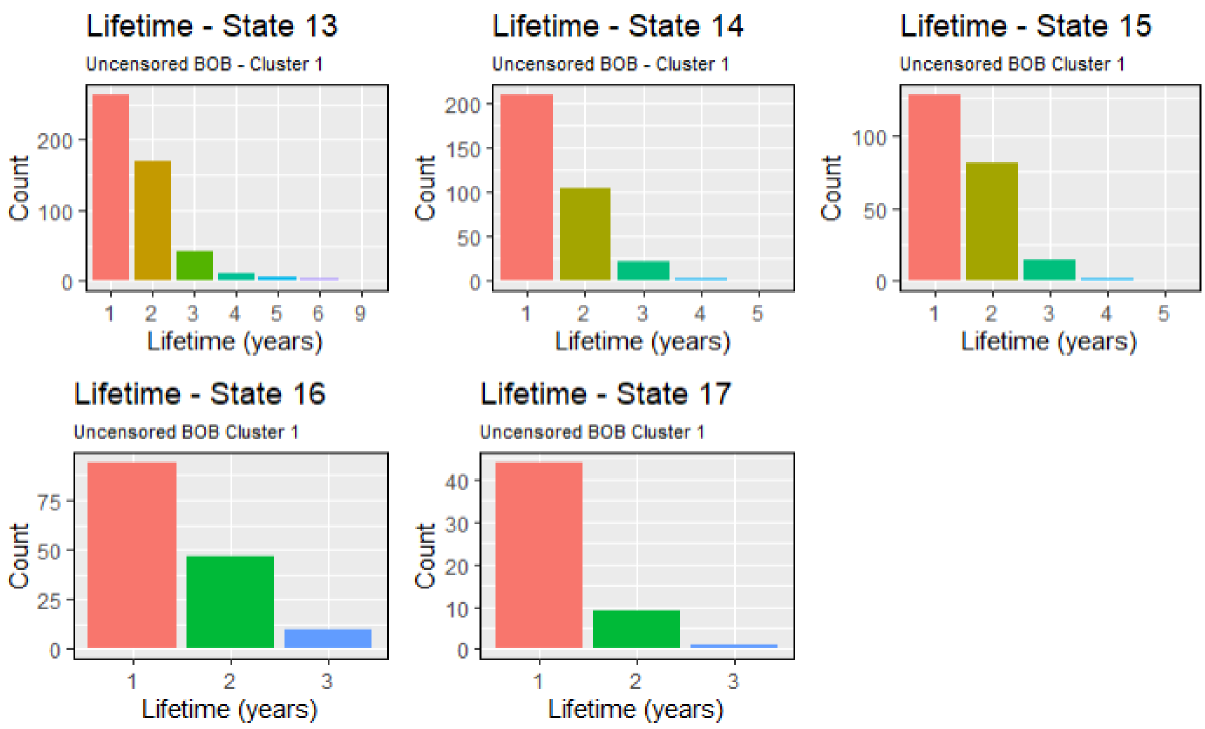

Figure 4.13: Distribution of time sections spent in states 13 to 17

4.9 NEW DATA PREPARATION

Previous analysis revealed, in each state, several sections that never deteriorated to a following condition state (never transitioned). This happened because such sections received a major repair activity that improved their condition and moved them to a better condition state before they deteriorated to a worse state. It does not imply that sections received major repair too early or too late; it only shows that major repair was performed and that needs to be taken into consideration. Such sections do not accurately represent natural pavement deterioration and can cause misclassification problems in the model. Thus, they were removed from the analysis. To avoid data loss and bias, these sections were removed only in the state they received major repair.

Figure 4.14 illustrates this data filtering by showing the behavior of a section over time. The y-axis contains the condition state, where lower state number represents better RQI condition (Markov states tend to start at either zero or one and transition to higher numbers). In the example below, a pavement section was in state 3 in the year 2005 and deteriorated until it arrived in state 13 in 2013. Between the years 2013 and 2014, it received a major repair activity (Nova Chip UTBWC), which improved the pavement condition and brought the sections to state 2. Since the model is developed individually for each state, state 13 never got a transition. The filtering algorithm removed this section from the data used to develop the model for state 13, but kept it in the data used for the other states.

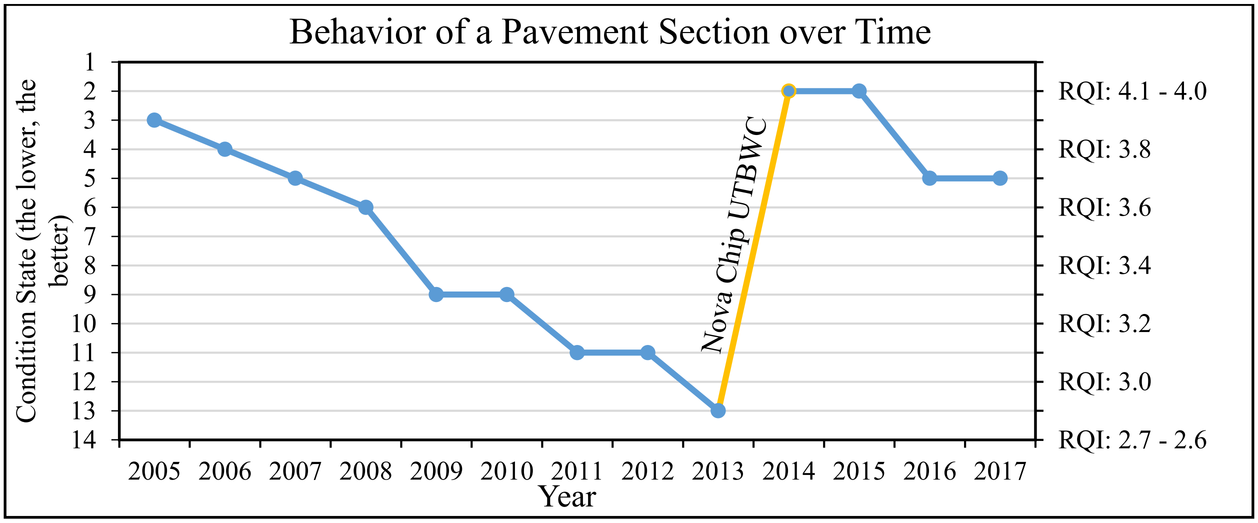

Figure 4.14: Condition behavior of a pavement section

Sections that showed condition improvement in years that they did not receive any repair were considered outliers and were also filtered out of the dataset. Again, this filtering was performed individually for each condition state.

4.10 EMPIRICAL MODEL

For comparison purposes, the simpler empirical model, that did not allow transitions to vary with site differences as performed in the preliminary analysis, was repeated using the 17 states format and allowing a section to transition to any of the following states.

The probability values p_{ij} were calculated as shown in equation 12.

$$p_{ij} = \frac{T_{x_j}}{T_{n_i}} \qquad [12]$$

where Tx_j is the sum of all pavement sections that transitioned from state *i* to state *j*, and Tn_i is the total number of pavement sections in state *i*. The results are summarized in table 4.16.

Table 4.16: Empirical Markov Probability Matrix

Start State		End State 1 (RQI: 5.0-4.2)	2 (RQI: 4.1-4.0)	3 (RQI: 3.9)	4 (RQI: 3.8)	5 (RQI: 3.7)	6 (RQI: 3.6)	7 (RQI: 3.5)	8 (RQI: 3.4)	9 (RQI: 3.3)	10 (RQI: 3.2)	11 (RQI: 3.1)	12 (RQI: 3.0)	13 (RQI: 2.9-2.8)	14 (RQI: 2.7-2.6)	15 (RQI: 2.5-2.4)	16 (RQI: 2.3-2.2)	17 (RQI: 2.1-2.0)
1	(RQI: 5.0-4.2)	0.321	0.542	0.089	0.032	0.005	0.005	0	0	0.005	0	0	0	0	0	0	0	0
2	(RQI: 4.1-4.0)	0	0.516	0.374	0.076	0.02	0.009	0.003	0.002	0	0.001	0	0	0	0	0	0	0
3	(RQI: 3.9)	0	0	0.386	0.405	0.155	0.043	0.005	0.002	0.002	0.001	0.001	0.001	0	0	0	0	0
4	(RQI: 3.8)	0	0	0	0.338	0.399	0.21	0.039	0.006	0.005	0.001	0	0.001	0	0	0	0	0
5	(RQI: 3.7)	0	0	0	0	0.312	0.488	0.151	0.032	0.013	0.003	0.001	0	0	0	0	0	0
6	(RQI: 3.6)	0	0	0	0	0	0.389	0.381	0.169	0.044	0.011	0.002	0.003	0.001	0.001	0	0	0
7	(RQI: 3.5)	0	0	0	0	0	0	0.292	0.398	0.24	0.047	0.015	0.005	0.003	0	0	0.001	0
8	(RQI: 3.4)	0	0	0	0	0	0	0	0.288	0.428	0.201	0.058	0.017	0.006	0.001	0	0	0
9	(RQI: 3.3)	0	0	0	0	0	0	0	0	0.299	0.393	0.225	0.059	0.021	0.002	0.001	0	0
10	(RQI: 3.2)	0	0	0	0	0	0	0	0	0	0.254	0.42	0.23	0.088	0.006	0.003	0	0
11	(RQI: 3.1)	0	0	0	0	0	0	0	0	0	0	0.24	0.426	0.292	0.035	0.006	0.002	0
12	(RQI: 3.0)	0	0	0	0	0	0	0	0	0	0	0	0.233	0.614	0.132	0.015	0.006	0
13	(RQI: 2.9-2.8)	0	0	0	0	0	0	0	0	0	0	0	0	0.401	0.477	0.108	0.012	0.001
14	(RQI: 2.7-2.6)	0	0	0	0	0	0	0	0	0	0	0	0	0	0.318	0.546	0.13	0.006
15	(RQI: 2.5-2.4)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.367	0.494	0.139
16	(RQI: 2.3-2.2)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.438	0.562
17	(RQI: 2.1-2.0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

4.11 ORDINAL LOGISTIC REGRESSION MODEL

During the first TAP meeting, a concern was raised regarding the possibility of external factors influencing the pavement deterioration. To address that, an enhancement to the Markov transition matrix was proposed, which allows the probabilities to be functions of those external factors.

After considering binomial and multinomial logistic regression approaches, the ordinal logistic regression was found to model the effect of external factors most accurately on pavement deterioration while accounting for the ordered nature of our Markov states. The deterioration is understood as ordered because the classes in the response variable can be understood to have a certain order (sections are expected to move from state one to seventeen), which can be translated as sections going from a higher (good) condition do a lower (poor) condition.

Each year, a pavement section either remains in its condition state or deteriorates to one of the lower (worse) states. When deteriorating, sections sometimes go to the immediate next state, and sometimes they skip the immediate next state and move a next one, which means the response is a categorical variable with multiple response levels.

Ordinal logistic regression can accommodate a categorical response variable with more than two levels (26). The response variable is the condition of a pavement section, and the model classes are the yearly transition numbers, such that:

0 = section stays the same,

1 = section moves to the next following condition state,

2 = section moves to the second followind state,

3 = section moves to the third following state …

The external factors analyzed were the county and district where the road sections were located, their functional class, speed limit, asphalt concrete thickness, base thickness, and the last repair activity the sections received. Initially, annual average daily traffic (AADTA) and annual equivalent single axle load (ESAL) were also considered but we were informed that these were not accurate, so they were removed from the analyses.

The *polr* (proportional odds logistic regression) command from the MASS R package was used to estimate the ordinal logistic regression model. The *polr* ordinal logistic regression is parameterized as shown in equation 13. Equations 14 and 15 give the transition probability from state *i* to state *j*.

$$logit(P(Y \leq j)) = \beta_{j0} - \eta_1 x_1 - \cdots - \eta_p x_p \quad [13]$$

$$P(Y \leq j) = \frac{\exp(logit(P(Y \leq j))}{(1 + \exp\left(logit(P(Y \leq j))\right))} \quad [14]$$

$$P_{ij} = P(Y = j) = P(Y \leq j) - P(Y \leq (j-1)) \quad [15]$$

where *Y* is the ordinal outcome with *J* categories, $P(Y \leq j)$ is the cumulative probability of *Y* less than or equal to the category *j*, P_{ij} is the probability that the section will transition from state *i* to state *j*, β_{j0} is the intercept and η_1 …η_p are the coefficients associated with the predictors x_1 … x_p.

Table 4.17 shows the coefficients and p-values for a model using all predictor variables. The p-values were calculated by comparing the t-values against the standard normal distribution.

Table 4.17: Ordinal logistic regression coefficients and p-values for a model using all predictors

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT 2	-0.208	0.168	-1.238	0.216	
DISTRICT 3	0.267	0.222	1.205	0.228	
DISTRICT 4	0.028	0.174	0.160	0.873	
DISTRICT 5	-0.442	0.221	-2.004	0.045	*
DISTRICT 6	0.180	0.260	0.693	0.488	
DISTRICT 7	-0.067	0.246	-0.271	0.787	
DISTRICT 8	0.623	0.189	3.291	0.001	**
LAST.ACTIV Crack Repr/Med OL	0.739	0.692	1.067	0.286	
LAST.ACTIV Medium Overlay	0.361	0.262	1.375	0.169	
LAST.ACTIV Nova Chip (UTBWC)	1.271	0.550	2.313	0.021	*
LAST.ACTIV Thick Mill/Overlay	0.167	0.272	0.617	0.538	
LAST.ACTIV Thin Mill/Overlay	0.781	0.266	2.938	0.003	**
LAST.ACTIV Thin Overlay	0.520	0.247	2.108	0.035	*
SPEED.LIMIT	0.000	0.005	0.097	0.923	
FLEXIBLE.THK	-0.011	0.019	-0.585	0.558	
BASE.THK	-0.001	0.006	-0.162	0.871	
0|1	-0.264	0.385	-0.685	0.493	
1|2	1.483	0.387	3.834	0.000	***
2|3	3.383	0.400	8.458	0.000	***
3|4	4.805	0.442	10.858	0.000	***
4|5	5.459	0.489	11.163	0.000	***
5|6	6.764	0.694	9.748	0.000	***
6|8	7.865	1.072	7.339	0.000	***

Signif. codes for p-values: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual Deviance: 3925.808

AIC: 3971.808

In table 4.17, the predictors district location and last activity are categorical variables. Thus, one of their categories was taken as reference and are not displayed in the table. The reference categories are district 1 and CIR & medium overlay, respectively. Since they were taken as reference, they should receive 1 for their coefficients. In addition, the metro district is shown as district 5.

4.11.1 Lean Model

The predictors SPEED.LIMIT, FLEXIBLE.THK, and BASE.THK were removed from the model and the model was re-estimated. The resulting coefficients are shown in table 4.18.

Table 4.18: Ordinal logistic regression coefficients and p-values for a model using districts and last repair activities as predictors

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT 2	-0.197	0.166	-1.184	0.236	
DISTRICT 3	0.285	0.218	1.310	0.190	
DISTRICT 4	0.038	0.172	0.217	0.828	
DISTRICT 5	-0.461	0.215	-2.148	0.032	*
DISTRICT 6	0.186	0.255	0.730	0.466	
DISTRICT 7	-0.066	0.245	-0.268	0.789	
DISTRICT 8	0.606	0.187	3.250	0.001	**
LAST.ACTIV Crack Repr/Med OL	0.742	0.686	1.082	0.279	
LAST.ACTIV Medium Overlay	0.359	0.257	1.396	0.163	
LAST.ACTIV Nova Chip (UTBWC)	1.248	0.539	2.315	0.021	*
LAST.ACTIV Thick Mill/Overlay	0.154	0.266	0.579	0.563	
LAST.ACTIV Thin Mill/Overlay	0.786	0.261	3.016	0.003	**
LAST.ACTIV Thin Overlay	0.523	0.242	2.166	0.030	*
0|1	-0.183	0.273	-0.671	0.502	
1|2	1.563	0.276	5.657	0.000	***
2|3	3.463	0.294	11.769	0.000	***
3|4	4.885	0.350	13.968	0.000	***
4|5	5.540	0.407	13.612	0.000	***
5|6	6.845	0.639	10.716	0.000	***
6|8	7.945	1.037	7.664	0.000	***

Signif. codes for p-values: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual Deviance: 3926.173

AIC: 3966.173

Using equations 13 to 15, the estimated model can be written, such as:

$$logit\big(P(Y \leq 0)\big) = -0.183 - (1)D1 - (-0.197)D2 - (0.285)D3 - (0.038)D4 - (-0.461)D5 - (0.186)D6 - (-0.066)D7 - (0.606)D8 - (1)LA.CIR\&MO - (0.742)LA.CRMO - (0.359)LA.MO - (1.248)LA.NC - (0.154)LA.ThkMO - (0.786)LA.ThnMO - (0.523)LA.ThnO$$

4.11.2 Goodness-of-fit test

The Hosmer-Lemeshow test was used to evaluate the goodness of fit of the ordinal logistic regression model. It was originally developed to evaluate binary logistic regression models, and then adapted to multinomial and ordinal logistic regression models (27, 28, 29). The null hypothesis, H_0, is that the model results in a good data fit, while the alternative hypothesis, H_a, assumes the fit from the model is poor.

The observations are grouped into several groups (g) according to the model-predicted response probabilities. It is recommended to use g = 10, but any number can be used, keeping in mind that a too large number will cause the contingency table to be sparsely populated, while a too small number might result in a poor test due to heterogeneity within groups. (28)

A *g x c* contingency table is constructed containing the observed and estimated frequencies for each group. The goodness of fit test is obtained from the Pearson chi-squared statistic from the table, as shown in equation 16. The reference distribution is chi-squared with *(g - 2) (c - 1) + (c - 2)* degrees of freedom. (28, 29)

$$C_g = \sum_{k=1}^{g}\sum_{j=1}^{c}(O_{kj} - E_{kj})^2/E_{kj} \qquad [16]$$

where O_{kj} and E_{kj} denote the sums of the observed and estimated frequencies in each group for each response category, respectively.

An ordinal logistic regression model was developed using 100% of the data for BOB sections that received one of the activities listed in the cluster number 1 as their last repair activities, in state condition 4. The model coefficients are shown in table 4.19.

Table 4.19: Ordinal Logistic Regression Model

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT 2	-0.197	0.166	-1.184	0.236	
DISTRICT 3	0.285	0.218	1.310	0.190	
DISTRICT 4	0.038	0.172	0.217	0.828	
DISTRICT 5	-0.461	0.215	-2.148	0.032	*
DISTRICT 6	0.186	0.255	0.730	0.466	
DISTRICT 7	-0.066	0.245	-0.268	0.789	
DISTRICT 8	0.606	0.187	3.250	0.001	**
LAST.ACTIV Crack Repr/Med OL	0.742	0.686	1.082	0.279	
LAST.ACTIV Medium Overlay	0.359	0.257	1.396	0.163	
LAST.ACTIV Nova Chip (UTBWC)	1.248	0.539	2.315	0.021	*
LAST.ACTIV Thick Mill/Overlay	0.154	0.266	0.579	0.563	
LAST.ACTIV Thin Mill/Overlay	0.786	0.261	3.016	0.003	**
LAST.ACTIV Thin Overlay	0.523	0.242	2.166	0.030	*
0|1	-0.183	0.273	-0.671	0.502	
1|2	1.563	0.276	5.657	0.000	***
2|3	3.463	0.294	11.769	0.000	***
3|4	4.885	0.350	13.968	0.000	***
4|5	5.540	0.407	13.612	0.000	***
5|6	6.845	0.639	10.716	0.000	***
6|8	7.945	1.037	7.664	0.000	***

Signif. codes for p-values: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual Deviance: 3926.173

AIC: 3966.173

Then, the contingency table of frequencies was calculated with the expected and observed values. The table was divided into tables 4.20 and 4.21, expected and observed values, respectively.

Table 4.20: Expected values in the contingency table of frequencies

Group (g)	Decile Range	Class							
		0	1	2	3	4	5	6	8
1	[-0.582,159]	76.59	57.11	20.91	3.31	0.52	0.41	0.10	0.05
2	(159,317]	65.48	61.29	25.61	4.22	0.67	0.53	0.13	0.07
3	(317,476]	60.40	62.85	28.35	4.81	0.76	0.61	0.15	0.08
4	(476,634]	59.28	63.17	28.97	4.94	0.78	0.62	0.16	0.08
5	(634,792]	57.32	64.11	30.53	5.28	0.84	0.67	0.17	0.08
6	(792,950]	51.63	64.62	33.69	6.04	0.96	0.77	0.19	0.10
7	(950,1.11e+03]	50.45	64.74	34.49	6.23	1.00	0.80	0.20	0.10
8	(1.11e+03,1.27e+03]	46.11	64.85	37.60	7.05	1.14	0.91	0.23	0.11
9	(1.27e+03,1.42e+03]	36.91	63.44	45.11	9.32	1.53	1.23	0.31	0.15
10	(1.42e+03,1.58e+03]	32.66	62.13	49.52	10.87	1.81	1.46	0.37	0.18

Table 4.21: Observed values in the contingency table of frequencies

Group (g)	Decile Range	Class							
		0	1	2	3	4	5	6	8
1	[-0.582,159]	72	63	19	5	0	0	0	0
2	(159,317]	64	61	30	2	0	1	0	0
3	(317,476]	60	73	22	1	1	1	0	0
4	(476,634]	59	60	31	8	0	0	0	0
5	(634,792]	61	63	29	6	0	0	0	0
6	(792,950]	61	55	33	5	2	1	1	0
7	(950,1.11e+03]	46	69	34	7	1	0	0	1
8	(1.11e+03,1.27e+03]	51	57	40	8	1	1	0	0
9	(1.27e+03,1.42e+03]	39	65	41	6	3	3	1	0
10	(1.42e+03,1.58e+03]	25	62	55	14	2	1	0	0

From table 4.21, it is possible to see that some cells in the expected contingency table of frequencies are smaller than 1, which causes the chi-square approximation to be dubious (28, 29). This was possibly caused by the small number of observations in classes 4 to 8.

To fix this problem, the smallest categories in the contingency table of frequencies were combined and the chi-squared test was then calculated. Tables 4.22 and 4.23 show the revised expected and observed frequencies of the contingency table after classes 4 to 8 were aggregated.

Table 4.22: Revised expected contingency table of frequencies for the model shown in table 4.19

Group (g)	Decile Range	Class				
		0	1	2	3	4 - 8
1	[-0.582,159]	76.59	57.11	20.91	3.31	1.08
2	(159,317]	65.48	61.29	25.61	4.22	1.39
3	(317,476]	60.40	62.85	28.35	4.81	1.59
4	(476,634]	59.28	63.17	28.97	4.94	1.64
5	(634,792]	57.32	64.11	30.53	5.28	1.76
6	(792,950]	51.63	64.62	33.69	6.04	2.02
7	(950,1.11e+03]	50.45	64.74	34.49	6.23	2.09
8	(1.11e+03,1.27e+03]	46.11	64.85	37.60	7.05	2.38
9	(1.27e+03,1.42e+03]	36.91	63.44	45.11	9.32	3.22
10	(1.42e+03,1.58e+03]	32.66	62.13	49.52	10.87	3.81

Table 4.23: Revised observed contingency table of frequencies for the model shown in table 4.19

Group (g)	Decile Range	Class				
		0	1	2	3	4 - 8
1	[-0.582,159]	72	63	19	5	0
2	(159,317]	64	61	30	2	1
3	(317,476]	60	73	22	1	2
4	(476,634]	59	60	31	8	0
5	(634,792]	61	63	29	6	0
6	(792,950]	61	55	33	5	4
7	(950,1.11e+03]	46	69	34	7	2
8	(1.11e+03,1.27e+03]	51	57	40	8	2
9	(1.27e+03,1.42e+03]	39	65	41	6	7
10	(1.42e+03,1.58e+03]	25	62	55	14	3

Equation 16 was used to calculate the Chi-squared statistics. Table 4.24 summarizes the chi-squared results and the p-value

Table 4.24: Test results and p-value

g	c	Chi-squared	Degrees of Freedom	p-value
10	5	34.7512	35	0.4801

The p-value of 0.4801 is non-significant at the significance level of 0.05. Thus, we fail to reject the null hypothesis that the model provides an acceptable fit.

4.11.3 Modeling All States

The analyses performed for state 4 were repeated for the other states and the results were used to fill in the Markov transition matrix. The model coefficients for all states can be found in tables 25 to 40. All models were evaluated with the Hosmer-Lemeshow test, which resulted in p-values larger than 0.05. The large p-values show that the test failed to reject the null hypothesis that the models for all datasets fit the data well.

Table 4.25 displays the model coefficients for the dataset of state 1. The Hosmer-Lemeshow test was performed and, with a p-value of 0.727, it failed to reject the null hypothesis that the model fits the data well.

Table 4.25: Best Fitted Model for State 1

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT2	-2.993	0.603	-4.965	0.000	***
DISTRICT3	-1.555	0.654	-2.377	0.017	*
DISTRICT4	-3.184	0.869	-3.663	0.000	***
DISTRICT5	0.079	0.839	0.094	0.925	
DISTRICT6	-2.678	0.707	-3.787	0.000	***
DISTRICT7	0.284	0.753	0.377	0.706	
DISTRICT8	-2.814	0.779	-3.611	0.000	***
LAST.ACTIV Medium Overlay	1.107	0.851	1.301	0.193	
LAST.ACTIV Nova Chip (UTBWC)	-3.298	1.269	-2.598	0.009	**
LAST.ACTIV Thick Mill/Overlay	-2.855	0.980	-2.912	0.004	**
LAST.ACTIV Thin Mill/Overlay	-0.455	0.917	-0.496	0.620	
LAST.ACTIV Thin Overlay	0.744	0.778	0.957	0.339	
0|1	-3.170	0.918	-3.454	0.001	**
1|2	0.474	0.875	0.542	0.588	
2|3	2.073	0.902	2.299	0.022	*
3|4	3.302	1.014	3.255	0.001	**
4|5	3.720	1.093	3.403	0.001	**
5|8	4.414	1.302	3.390	0.001	**

Signif. codes for p-values: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual Deviance: 353.4411

AIC: 389.4411

In table 4.25, both categorical variables, district and last activity have one of their categories taken as reference, and thus they are not shown in the table. These reference categories are district number one and CIR & medium overlay, respectively. Since they were taken as reference, their coefficients should be taken as 1.

Table 4.26: Best Fitted Model for State 2

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT2	-0.822	0.194	-4.243	0.000	***
DISTRICT3	0.316	0.308	1.029	0.304	
DISTRICT4	-0.075	0.237	-0.316	0.752	
DISTRICT5	-0.266	0.357	-0.746	0.456	
DISTRICT6	-0.643	0.286	-2.246	0.025	*
DISTRICT7	1.843	0.374	4.932	0.000	***
DISTRICT8	-0.138	0.227	-0.608	0.543	
LAST.ACTIV Medium Overlay	0.524	0.263	1.992	0.046	*
LAST.ACTIV Nova Chip (UTBWC)	0.737	0.534	1.381	0.167	
LAST.ACTIV Thick Mill/Overlay	-0.188	0.279	-0.673	0.501	
LAST.ACTIV Thin Mill/Overlay	1.035	0.275	3.756	0.000	***
LAST.ACTIV Thin Overlay	1.093	0.257	4.246	0.000	***
FUNCTIONAL.CLASS RURAL MAJOR COLL	0.785	0.441	1.781	0.075	.
FUNCTIONAL.CLASS RURAL MINOR ART	0.404	0.376	1.073	0.283	
FUNCTIONAL.CLASS RURAL PRIN ART	0.743	0.369	2.012	0.044	**
FUNCTIONAL.CLASS URBAN INTERSTATE	0.258	0.535	0.482	0.630	
FUNCTIONAL.CLASS URBAN MINOR ART	1.183	1.597	0.741	0.459	
FUNCTIONAL.CLASS URBAN PRIN ART	0.313	0.488	0.642	0.521	
FUNCTIONAL.CLASS URBAN PRIN ART FRWY	-0.132	0.707	-0.187	0.852	
BASE.THK	-0.018	0.008	-2.236	0.025	*
0|1	0.643	0.440	1.462	0.144	
1|2	2.861	0.448	6.386	0.000	***
2|3	4.319	0.471	9.174	0.000	***
3|4	5.641	0.542	10.404	0.000	***

Signif. codes for p-values: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual Deviance: 2511.076

AIC: 2559.076

Similar to the model for state 1, the model for the data in state 2, shown in table 4.26, have categorical variables and one of the categories was taken as reference. District number 1 was the reference category for district, CIR & Medium Overlay was the reference for the Last Activity performed and the reference for functional class was Rural Interstate. All these were included in the model equations shown in the next topic and received 1 for their coefficients.

The Hosmer-Lemeshow test for state 2 resulted in a chi-squared of 46.68 in 26 degrees of freedom, which gave a p-value of 0.008. At the 0.05 significance level, we rejected the null hypothesis that the model fits the data well. Out of the 17 states, state 2 was the only one with this result. Using a significance level of 0.05, one would expect that, out of 17 tests, 17(.05) = 0.85 of these to be significant by chance alone, and the question is where or not this could apply to state 2. The expected and observed transition frequencies for state 2 are shown in Tables 4.27 and 4.28, respectively, and after comparing these the team decided that the model still produced an acceptable fit.

Table 4.27: Expected contingency table of frequencies

g	cutyhats	y_X0	y_X1	y_X2	X3 and X4
1	[-0.285,130]	94.20	29.78	3.81	1.20
2	(130,258]	83.75	38.08	5.43	1.75
3	(258,386]	78.78	41.06	6.16	2.00
4	(386,515]	75.11	44.54	7.04	2.31
5	(515,644]	71.81	46.15	7.56	2.49
6	(644,772]	68.93	48.87	8.40	2.79
7	(772,900]	59.59	54.17	10.61	3.63
8	(900,1.03e+03]	54.77	57.64	12.30	4.29
9	(1.03e+03,1.16e+03]	48.08	60.27	14.46	5.19
10	(1.16e+03,1.29e+03]	32.98	62.81	23.03	10.18

Table 4.28: Observed contingency table of frequencies

g	cutyhats	y_0	y_1	y_2	y_3 and y_4
1	[-0.285,130]	90	29	7	3
2	(130,258]	82	44	3	0
3	(258,386]	70	56	1	1
4	(386,515]	83	37	6	3
5	(515,644]	82	38	5	3
6	(644,772]	72	42	15	0
7	(772,900]	62	44	15	7
8	(900,1.03e+03]	46	66	13	4
9	(1.03e+03,1.16e+03]	46	65	12	5
10	(1.16e+03,1.29e+03]	35	62	22	10

Table 4.29 shows the coefficients for the best model for the data in state 3. The Hosmer-Lemeshow test resulted in a 25.38 Chi-squared, 26 degrees of freedom and a p-value of 0.497, which shows we failed to reject the null hypothesis that this model is a good fit for the data.

Table 4.29: Best Fitted Model for State 3

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT2	-0.338	0.165	-2.045	0.041	*
DISTRICT3	-0.113	0.215	-0.524	0.600	
DISTRICT4	0.269	0.176	1.529	0.126	
DISTRICT5	-0.469	0.221	-2.124	0.034	*
DISTRICT6	-0.096	0.237	-0.406	0.685	
DISTRICT7	1.432	0.276	5.182	0.000	***
DISTRICT8	0.295	0.190	1.553	0.120	
LAST.ACTIV Medium Overlay	0.218	0.225	0.969	0.333	
LAST.ACTIV Nova Chip (UTBWC)	1.524	0.423	3.606	0.000	***
LAST.ACTIV Thick Mill/Overlay	-0.059	0.233	-0.254	0.799	
LAST.ACTIV Thin Mill/Overlay	1.007	0.234	4.306	0.000	***
LAST.ACTIV Thin Overlay	0.624	0.214	2.922	0.003	**
0|1	-0.045	0.243	-0.186	0.852	
1|2	1.854	0.249	7.455	0.000	***
2|3	3.423	0.267	12.809	0.000	***
3|4	5.039	0.340	14.832	0.000	***
4|5	5.633	0.400	14.089	0.000	***
5|6	6.147	0.476	12.914	0.000	***
6|7	6.841	0.627	10.910	0.000	***
7|8	7.247	0.748	9.686	0.000	***
8|9	7.941	1.029	7.713	0.000	***

Signif. codes for p-values: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual Deviance: 3684.239

AIC: 3726.239

This process of developing and testing an ordinal logit model was repeated for the datasets of all other states. For conciseness, the tables with their coefficients were displayed in Appendix A.

The dataset for state 16 only has two response levels (a section can either stay the same, or it can move to state 17). The ordinal logistic regression requires 3 or more level, so a binomial logistic regression was used, which revealed no statistically significant predictor. Thus, the likelihood method was applied, and the probabilities were calculated using equation 8. The resulting constant probabilities are shown in table 4.30.

Table 4.30: Constant probabilities for state 16

Total sections	Sections that transitioned to the next state	Prob of transitioning	Prob of remaining in state 16
218	123	0.564	0.436

4.12 FULL MARKOV TRANSITION MATRICES

The coefficients from the ordinal logistic regression models were used into the model equations 13, 14 and 15 to generate the probabilities in the Markov transition matrix. The form for this matrix is illustrated in table 4.31.

Table 4.31: Markov Probability Matrix Enhanced by Ordinal Logistic Regression

		End State																
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Start State	1	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}			P_{19}								
	2		P_{22}	P_{23}	P_{24}	P_{25}	P_{26}											
	3			P_{33}	P_{34}	P_{35}	P_{35}	P_{37}	P_{38}	P_{39}	$P_{3\text{-}10}$	$P_{3\text{-}11}$	$P_{3\text{-}12}$					
	4				P_{44}	P_{45}	P_{46}	P_{47}	P_{48}	P_{49}	$P_{4\text{-}10}$		$P_{4\text{-}12}$					
	5					P_{55}	P_{56}	P_{57}	P_{58}	P_{59}	$P_{5\text{-}10}$	$P_{5\text{-}11}$						
	6						P_{66}	P_{67}	P_{68}	P_{69}	$P_{6\text{-}10}$	$P_{6\text{-}11}$	$P_{6\text{-}12}$	$P_{6\text{-}13}$	$P_{6\text{-}14}$			
	7							P_{77}	P_{78}	P_{79}	$P_{7\text{-}10}$	$P_{7\text{-}11}$						
	8								P_{88}	P_{89}	$P_{8\text{-}10}$	$P_{8\text{-}11}$	$P_{8\text{-}12}$	$P_{8\text{-}13}$				
	9									P_{99}	$P_{9\text{-}10}$	$P_{9\text{-}11}$	$P_{9\text{-}12}$	$P_{9\text{-}13}$				
	10										$P_{10\text{-}10}$	$P_{10\text{-}11}$	$P_{10\text{-}12}$	$P_{10\text{-}13}$	$P_{10\text{-}14}$	$P_{10\text{-}15}$		
	11											$P_{11\text{-}11}$	$P_{11\text{-}12}$	$P_{11\text{-}13}$	$P_{11\text{-}14}$	$P_{11\text{-}15}$	$P_{11\text{-}16}$	
	12												$P_{12\text{-}12}$	$P_{12\text{-}13}$	$P_{12\text{-}14}$	$P_{12\text{-}15}$	$P_{12\text{-}16}$	
	13													$P_{13\text{-}13}$	$P_{13\text{-}14}$	$P_{13\text{-}15}$	$P_{13\text{-}16}$	$P_{13\text{-}17}$
	14														$P_{14\text{-}14}$	$P_{14\text{-}15}$	$P_{14\text{-}16}$	
	15															$P_{15\text{-}15}$	$P_{15\text{-}16}$	$P_{15\text{-}17}$
	16																0.436	0.564
	17																	1

The values for each cell can be calculated using the model equations in Appendix B.

4.12.1 4.11.1 Application of the Markov Transition Matrix

The Markov Transition Matrix can be used to model future pavement deterioration and to investigate the importance of several factors. For example, it can be used to test and compare different districts. As a demonstration, a group of 100 dummy sections was created, Group A, and the sections in this group were given the following characteristics.

- Characteristics of section in group A:
  - District 2
  - Last Activity Performed: Thin Overlay
  - Functional Class: Rural Minor Arterial
  - Base Thickness: 8 inches
  - Speed Limit: 55 mph
  - Flexible Thickness: 8 inches

The 100 sections in Group A were placed in initial condition state 3 (RQI = 3.9), as shown in figure 4.15.

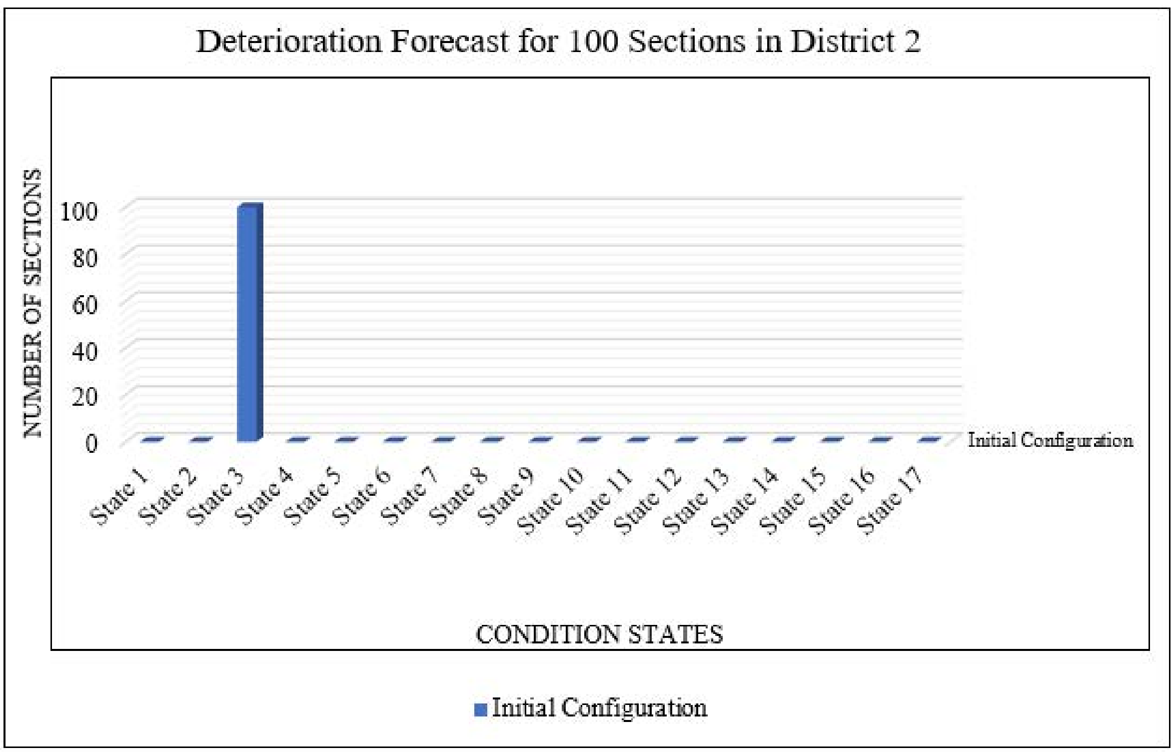

Figure 4.15: Initial configuration of the 100 sections placed in state condition 3 (RQI = 3.9)

Next, the equations and format from table 31 were used to create the Markov transition matrix for the 100 sections in Group A. This matrix is displayed in table 4.32.

Table 4.32: Markov Transition Matrix for sections in Group A

		End State																
		1 (RQI: 5.0-4.2)	2 (RQI: 4.1-4.0)	3 (RQI: 3.9)	4 (RQI: 3.8)	5 (RQI: 3.7)	6 (RQI: 3.6)	7 (RQI: 3.5)	8 (RQI: 3.4)	9 (RQI: 3.3)	10 (RQI: 3.2)	11 (RQI: 3.1)	12 (RQI: 3.0)	13 (RQI: 2.9-2.8)	14 (RQI: 2.7-2.6)	15 (RQI: 2.5-2.4)	16 (RQI: 2.3-2.2)	17 (RQI: 2.1-2.0)
Start State	1	(RQI: 5.0-4.2)	0.285	0.654	0.049	0.009	0.001	0.001	0	0	0.001	0	0	0	0	0	0	0	0
	2	(RQI: 4.1-4.0)	0	0.456	0.429	0.086	0.021	0.008	0	0	0	0	0	0	0	0	0	0	0
	3	(RQI: 3.9)	0	0	0.418	0.410	0.131	0.033	0.004	0.002	0.001	0.0005	0.0005	0.0005	0	0	0	0	0
	4	(RQI: 3.8)	0	0	0	0.375	0.400	0.183	0.031	0.005	0.004	0.001	0	0.0005	0	0	0	0	0
	5	(RQI: 3.7)	0	0	0	0	0.324	0.492	0.141	0.029	0.011	0.002	0.001	0	0	0	0	0	0
	6	(RQI: 3.6)	0	0	0	0	0	0.448	0.373	0.135	0.032	0.008	0.001	0.002	0.001	0.0004	0	0	0
	7	(RQI: 3.5)	0	0	0	0	0	0	0.344	0.409	0.202	0.034	0.010	0	0	0	0	0	0
	8	(RQI: 3.4)	0	0	0	0	0	0	0	0.319	0.440	0.177	0.047	0.013	0.005	0	0	0	0
	9	(RQI: 3.3)	0	0	0	0	0	0	0	0	0.316	0.397	0.216	0.053	0.018	0	0	0	0
	10	(RQI: 3.2)	0	0	0	0	0	0	0	0	0	0.311	0.435	0.186	0.062	0.004	0.002	0	0
	11	(RQI: 3.1)	0	0	0	0	0	0	0	0	0	0	0.279	0.442	0.247	0.026	0.004	0.002	0
	12	(RQI: 3.0)	0	0	0	0	0	0	0	0	0	0	0	0.227	0.622	0.130	0.015	0.006	0
	13	(RQI: 2.9-2.8)	0	0	0	0	0	0	0	0	0	0	0	0	0.422	0.474	0.094	0.010	0.001
	14	(RQI: 2.7-2.6)	0	0	0	0	0	0	0	0	0	0	0	0	0	0.282	0.570	0.148	0
	15	(RQI: 2.5-2.4)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.410	0.481	0.110
	16	(RQI: 2.3-2.2)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.436	0.564
	17	(RQI: 2.1-2.0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.000

The initial state vector is defined as *x0* and shows all 100 sections starting in condition state 3:

$$x_0 = [\,0 \quad 0 \quad 100 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\,]$$

From the transition matrix, P, and the state vector x0, the future condition of the 100 sections can be estimated using equations 17 to 20. Equation 17 gives the distribution of the condition of the sections after the first year. (21)

$$x_1 = x_0 * P \qquad [17]$$

Similarly, equation 18 gives the condition distribution after two years, and so on.

$$x_2 = x_1 * P = x_0 \; * P^2 \qquad [18]$$

$$x_3 = x_2 * P = x_0 * P^3 \qquad [19]$$

…

$$x_{10} = x_9 * P = x_0 * P^{10} \qquad [20]$$

Figure 4.16 shows the forecasted annual condition distribution for the 100 sections for the first 5 years.

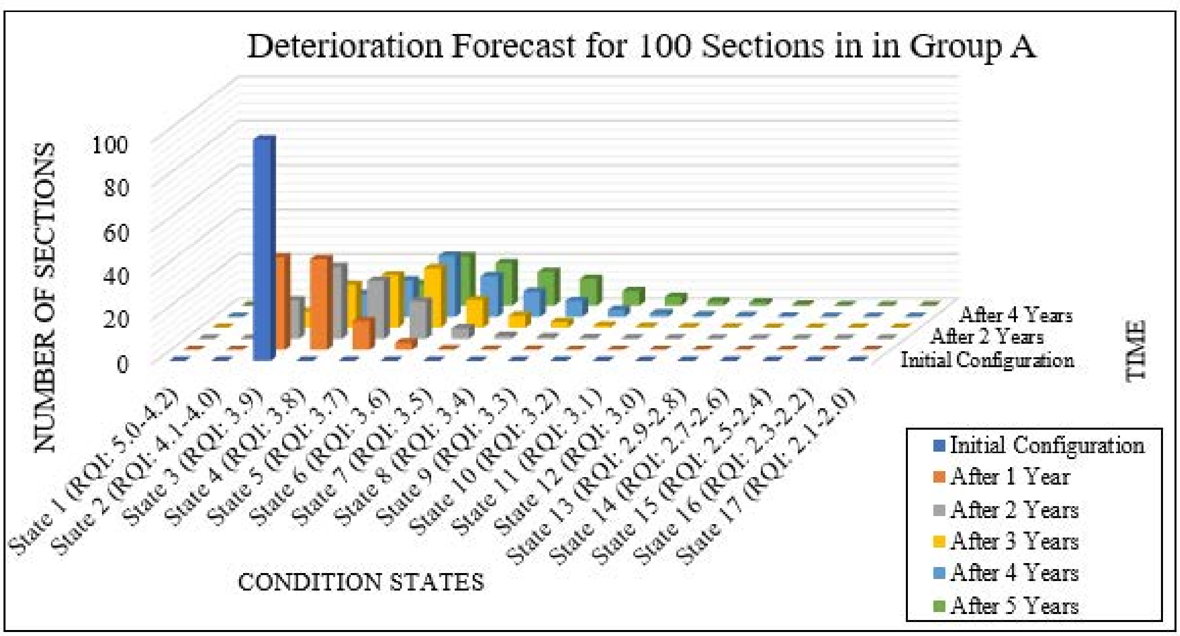

Figure 4.16: 5-year deterioration forecast for 100 sections in group A

Figure 4.17 includes the condition forecast after 10, 15 and 20 years. It is possible to see that after 20 years, the majority of the sections reached the worst state.

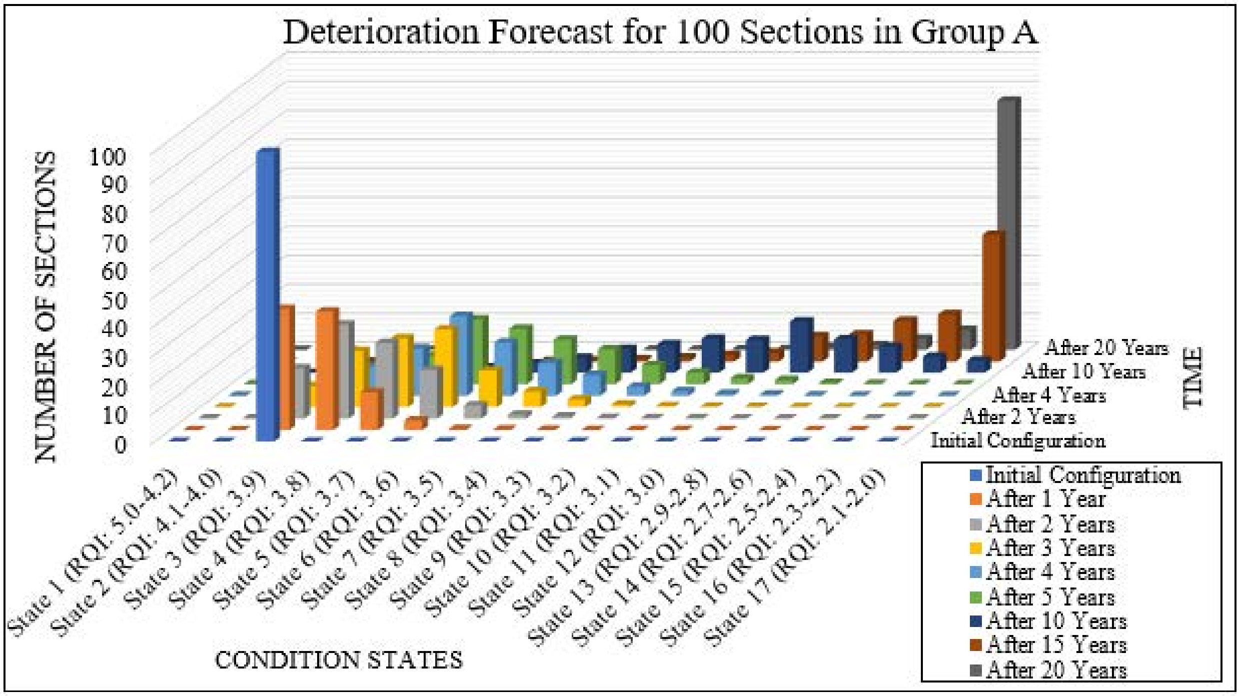

Figure 4.17: 20-year deterioration forecast for 100 sections in group A

To compare the rate of deterioration for different districts, another Group, B, was created with 100 dummy sections. These sections received the same characteristics as the ones in Group A, except for the district. While sections in Group A were placed in district 2, sections in Group B were located in district 7.

- Characteristics of section in group B:
  - District 7
  - Last Activity Performed: Thin Overlay
  - Functional Class: Rural Minor Arterial
  - Base Thickness: 8 inches
  - Speed Limit: 55 mph
  - Flexible Thickness: 8 inches

Using the sections' characteristics and the ordinal logistic regression equations, the Markov transition matrix was calculated and shown in table 4.33.

Table 4.33: Markov Transition Matrix for sections in Group B

			End State																
			1 (RQI: 5.0-4.2)	2 (RQI: 4.1-4.0)	3 (RQI: 3.9)	4 (RQI: 3.8)	5 (RQI: 3.7)	6 (RQI: 3.6)	7 (RQI: 3.5)	8 (RQI: 3.4)	9 (RQI: 3.3)	10 (RQI: 3.2)	11 (RQI: 3.1)	12 (RQI: 3.0)	13 (RQI: 2.9-2.8)	14 (RQI: 2.7-2.6)	15 (RQI: 2.5-2.4)	16 (RQI: 2.3-2.2)	17 (RQI: 2.1-2.0)
Start State	1	(RQI: 5.0-4.2)	0.015	0.350	0.375	0.167	0.030	0.031	0	0	0.033	0	0	0	0	0	0	0	0
	2	(RQI: 4.1-4.0)	0	0.055	0.294	0.348	0.199	0.104	0	0	0	0	0	0	0	0	0	0	0
	3	(RQI: 3.9)	0	0	0.109	0.341	0.347	0.155	0.021	0.011	0.008	0.003	0.003	0.003	0	0	0	0	0
	4	(RQI: 3.8)	0	0	0	0.345	0.406	0.201	0.035	0.006	0.004	0.001	0	0.001	0	0	0	0	0
	5	(RQI: 3.7)	0	0	0	0	0.201	0.499	0.221	0.052	0.021	0.005	0.001	0	0	0	0	0	0
	6	(RQI: 3.6)	0	0	0	0	0	0.312	0.407	0.204	0.056	0.014	0.002	0.003	0.001	0.001	0	0	0
	7	(RQI: 3.5)	0	0	0	0	0	0	0.294	0.414	0.236	0.042	0.013	0	0	0	0	0	0
	8	(RQI: 3.4)	0	0	0	0	0	0	0	0.277	0.443	0.202	0.056	0.016	0.006	0	0	0	0
	9	(RQI: 3.3)	0	0	0	0	0	0	0	0	0.264	0.394	0.251	0.066	0.023	0	0	0	0
	10	(RQI: 3.2)	0	0	0	0	0	0	0	0	0	0.244	0.434	0.229	0.085	0.006	0.003	0	0
	11	(RQI: 3.1)	0	0	0	0	0	0	0	0	0	0	0.280	0.442	0.246	0.026	0.004	0.002	0
	12	(RQI: 3.0)	0	0	0	0	0	0	0	0	0	0	0	0.186	0.628	0.159	0.019	0.007	0
	13	(RQI: 2.9-2.8)	0	0	0	0	0	0	0	0	0	0	0	0	0.334	0.521	0.129	0.015	0.001
	14	(RQI: 2.7-2.6)	0	0	0	0	0	0	0	0	0	0	0	0	0	0.349	0.538	0.113	0
	15	(RQI: 2.5-2.4)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.410	0.481	0.110
	16	(RQI: 2.3-2.2)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.436	0.564
	17	(RQI: 2.1-2.0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.000

The sections were also placed in condition state 3 (RQI = 3.9). Using the transition matrix, the state vector, and equations 17 to 20, the future deterioration of the 100 sections in Group B was predicted and illustrated in figure 4.18.

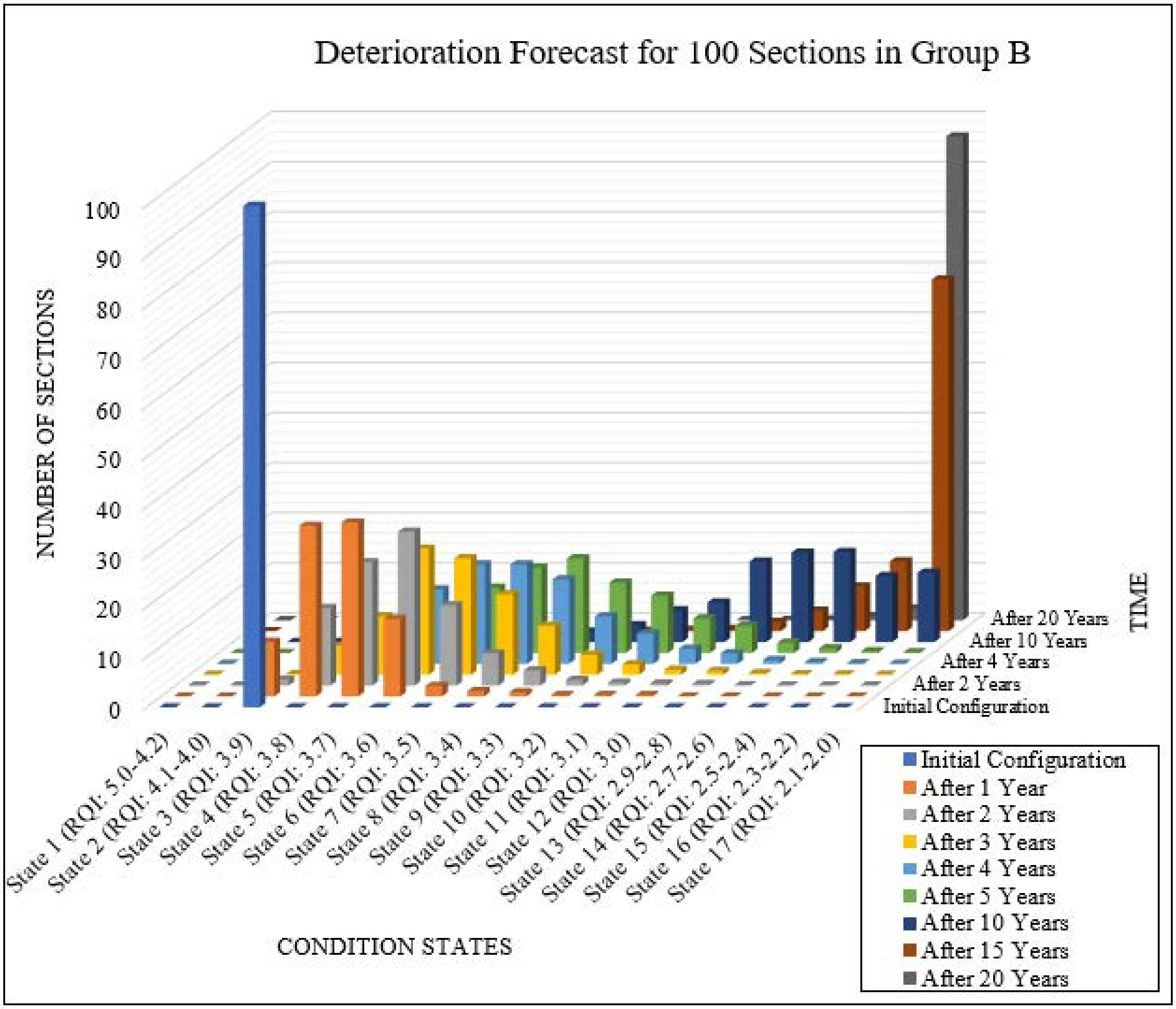

Figure 4.18: 20-year deterioration forecast for 100 sections in group B

The deterioration of the two groups, A and B, shown in figures 4.17 and 4.18 can be compared side by side to illustrate the difference between the rates of deterioration of districts 2 and 7. Figures 4.19 to 4.22 show the 1, 5, 10 and 20-year deterioration forecast, respectively. It is possible to see that sections in district 7 are expected to deteriorate faster than sections in district 2.

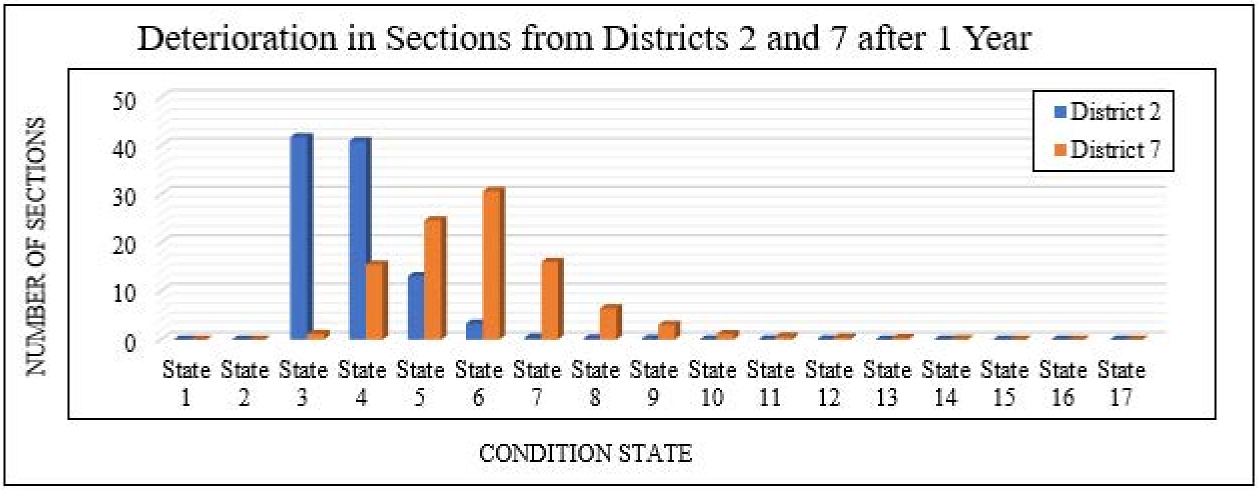

Figure 4.19: Comparison of deterioration forecast for sections in districts 2 and 7 after one year

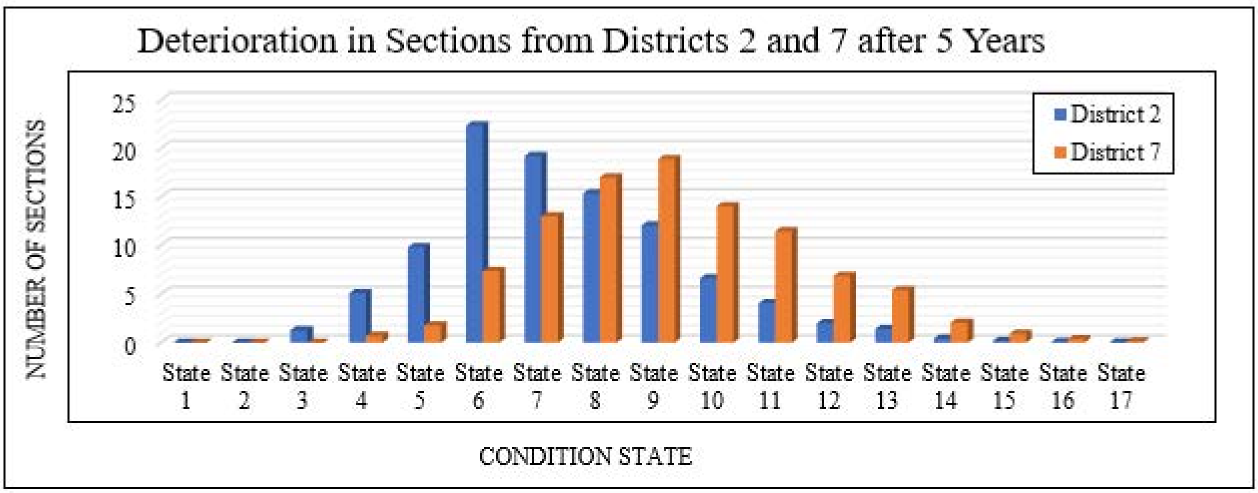

Figure 4.20: Comparison of deterioration forecast for sections in districts 2 and 7 after five years

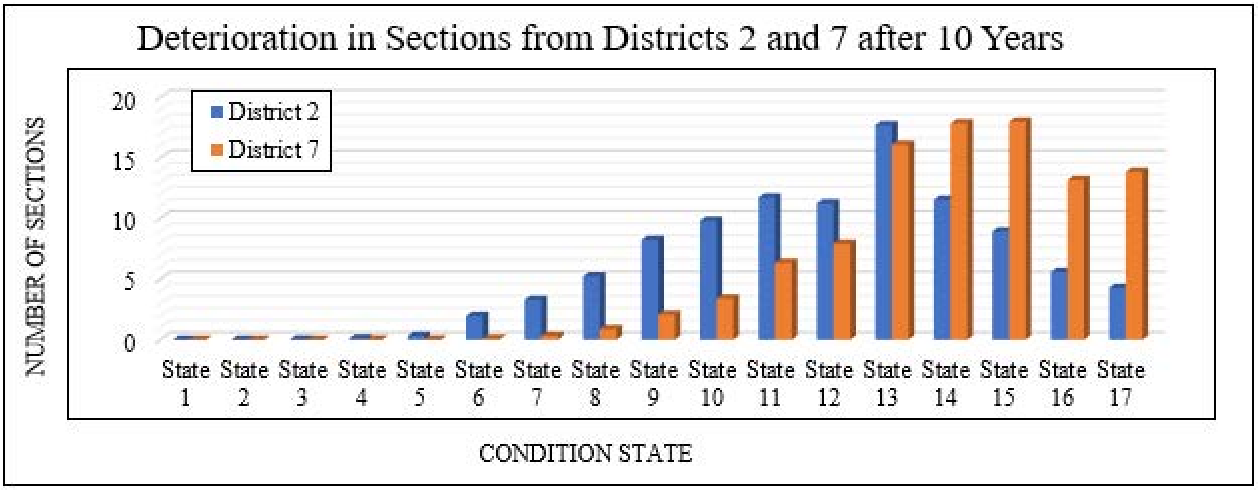

Figure 4.21: Comparison of deterioration forecast for sections in districts 2 and 7 after ten years

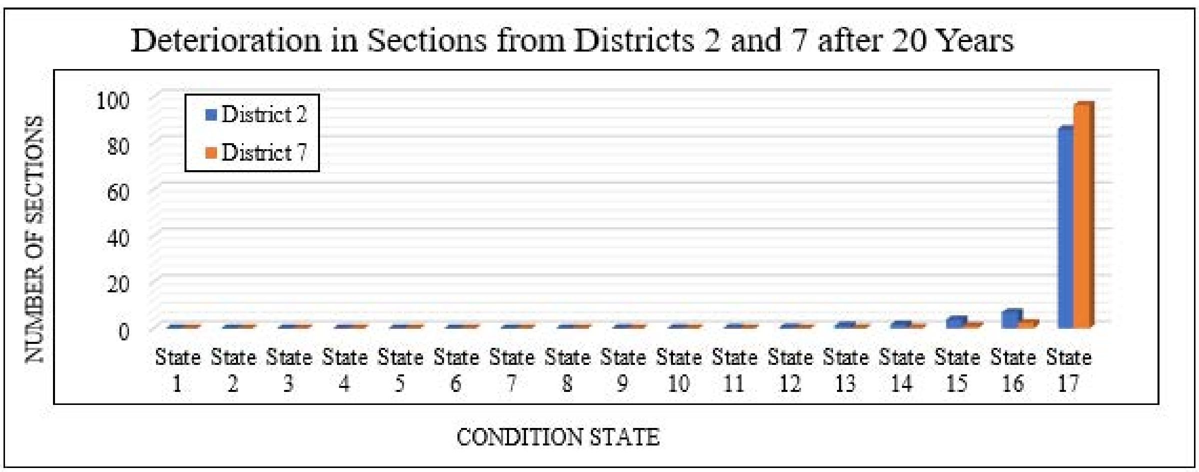

Figure 4.22: Comparison of deterioration forecast for sections in districts 2 and 7 after twenty years

This process can be repeated for all districts. Figure 4.23 illustrates how sections placed in each of the 8 districts are expected to deteriorate 5 years after being placed in state condition 3 (RQI = 3.9).

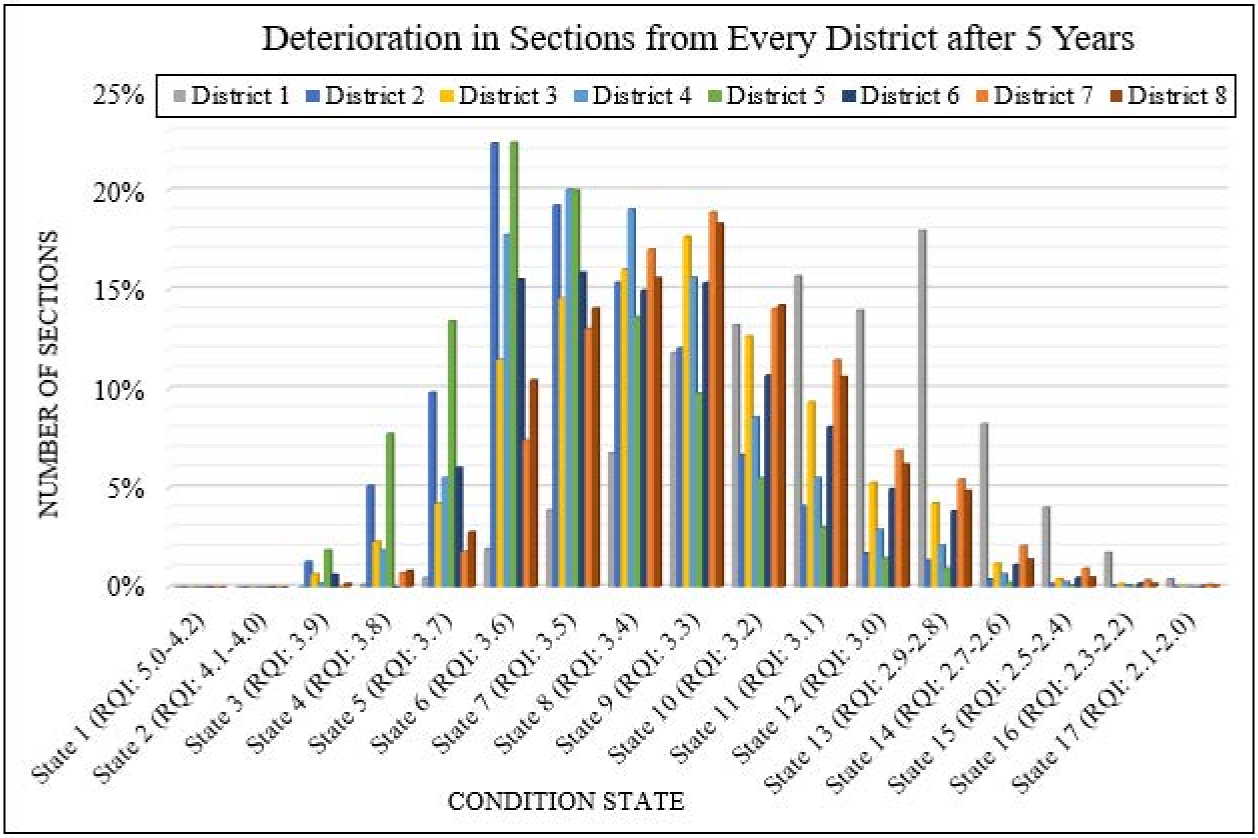

Figure 4.23: Comparison of deterioration forecast for sections in all districts after five years

The same methodology can be used to investigate how the pavement deteriorates after receiving different repair activities. For demonstrations purposes, figure 4.24 shows the forecasted deterioration of sections with the same characteristics, but that have received different repair activities.

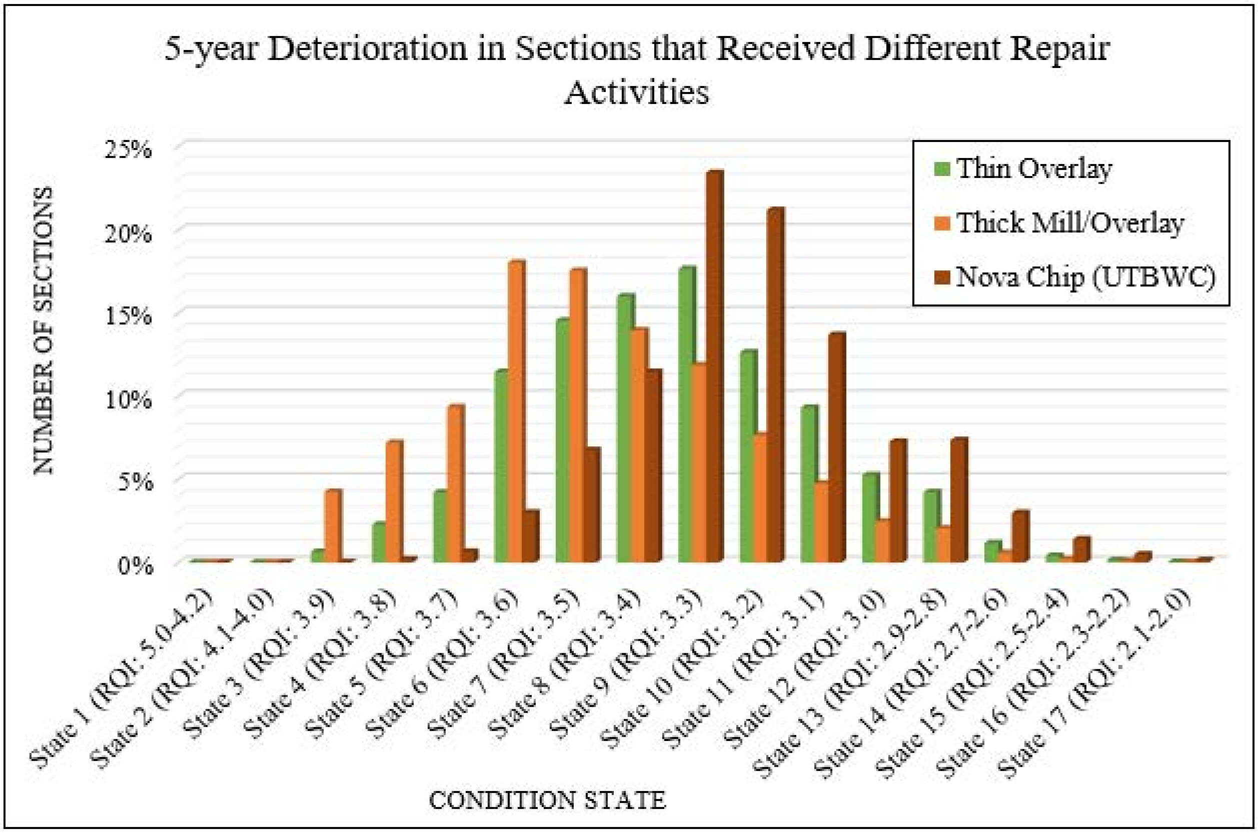

Figure 4.24: Comparison of deterioration forecast for sections that received different repair activities

4.12.2 Summary Measures

Mean Time Spent in Transient States

It is also possible to estimate how long a section would spend in each of the transient states. For example, let's consider a section with the following characteristics. We can estimate how long it would take to go from state 2 (average RQI of 4.05 – very good condition) to state 13 (average RQI of 2.85 - fair condition).

- Characteristics of section:
  - District 1
  - Last Activity Performed: Thin Overlay
  - Functional Class: Rural Minor Arterial
  - Base Thickness: 8 inches
  - Speed Limit: 55 mph
  - Flexible Thickness: 8 inches

After calculating the Markov probability matrix **P** for a section with the given characteristics, let $\mathbf{P_T}$ be the part of **P** formed by the probabilities from the transient states into transient states. Differently from the Markov probability matrix, some of the row sums in $\mathbf{P_T}$ are expected to be less than one (21):

$\mathbf{P_T}$ =

0.007	0.212	0.362	0.245	0.052	0.057	0	0	0.065	0	0	0	0	0	0
0	0.119	0.435	0.288	0.110	0.047	0	0	0	0	0	0	0	0	0
0	0	0.159	0.399	0.301	0.110	0.014	0.007	0.005	0.002	0.002	0.002	0	0	0
0	0	0	0.154	0.356	0.364	0.092	0.016	0.013	0.003	0	0.002	0	0	0
0	0	0	0	0.109	0.423	0.319	0.095	0.042	0.009	0.002	0	0	0	0
0	0	0	0	0	0.128	0.326	0.342	0.141	0.040	0.006	0.011	0.004	0.002	0
0	0	0	0	0	0	0.096	0.285	0.430	0.138	0.050	0	0	0	0
0	0	0	0	0	0	0	0.090	0.309	0.355	0.168	0.057	0.022	0	0
0	0	0	0	0	0	0	0	0.098	0.271	0.385	0.172	0.073	0	0
0	0	0	0	0	0	0	0	0	0.089	0.300	0.358	0.226	0.018	0.009
0	0	0	0	0	0	0	0	0	0	0.077	0.281	0.507	0.106	0.019
0	0	0	0	0	0	0	0	0	0	0	0.100	0.580	0.267	0.039
0	0	0	0	0	0	0	0	0	0	0	0	0.200	0.546	0.223
0	0	0	0	0	0	0	0	0	0	0	0	0	0.137	0.563
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.410

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

For transient states *i* and *j*, s_{ij} represents the expected number of time periods (number of years for our problem) that the Markov chain is expected to be in state *j*, given it started in state *i*. This relationship is shown in equation 21. (21)

$$s_{ij} = \delta_{i,j} + \sum_{k} P_{ik} s_{kj} = \delta_{i,j} + \sum_{k=1}^{t} P_{ik}\, s_{kj} \qquad [21]$$

where $\delta_{i,j}$ is 1 when *i=j* and 0 otherwise.

Equation 16 can be written in matrix notation, resulting in equation 22. (21)

$$\mathbf{S} = \mathbf{I} + \mathbf{P_T S} = (\mathbf{I} - \mathbf{P_T})^{-1} \qquad [22]$$

where **S** denotes the matrix of s_{ij} values, and **I** is an identity matrix.

S =

1.007	0.242	0.559	0.637	0.533	0.674	0.504	0.482	0.621	0.488	0.540	0.522	0.933	0.829	1.201
0	1.136	0.588	0.663	0.604	0.706	0.544	0.515	0.588	0.499	0.538	0.522	0.933	0.829	1.201
0	0	1.188	0.560	0.625	0.687	0.543	0.513	0.587	0.498	0.538	0.522	0.933	0.829	1.201
0	0	0	1.182	0.473	0.723	0.547	0.513	0.589	0.499	0.537	0.522	0.933	0.829	1.201
0	0	0	0	1.123	0.545	0.592	0.507	0.594	0.500	0.542	0.520	0.933	0.829	1.202
0	0	0	0	0	1.147	0.413	0.560	0.569	0.501	0.532	0.523	0.932	0.830	1.201
0	0	0	0	0	0	1.106	0.347	0.647	0.496	0.554	0.516	0.934	0.829	1.202

0	0	0	0	0	0	0	1.099	0.377	0.540	0.532	0.522	0.934	0.829	1.202
0	0	0	0	0	0	0	0	1.109	0.330	0.571	0.521	0.935	0.829	1.201
0	0	0	0	0	0	0	0	0	1.098	0.357	0.548	0.934	0.827	1.205
0	0	0	0	0	0	0	0	0	0	1.084	0.339	0.933	0.828	1.199
0	0	0	0	0	0	0	0	0	0	0	1.111	0.806	0.853	1.189
0	0	0	0	0	0	0	0	0	0	0	0	1.251	0.790	1.224
0	0	0	0	0	0	0	0	0	0	0	0	0	1.158	1.104
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.694
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The time this section is expected to spend between states 2 to 13 is given by:

$$Time_{2\ to\ 13} = S_{2,2} + S_{2,3} + S_{2,4} + S_{2,5} + S_{2,6} + S_{2,7} + S_{2,8} + S_{2,9} + S_{2,10} + S_{2,11} + S_{2,12} + S_{2,13} = 7.834$$

Thus, a section with the listed characteristics is expected to go from state 2 to state 13 in 7.8 years. In other words, in 7.8 years, the section's RQI will deteriorate from 4.1 (very good) to 2.9 (fair).

We can go further and calculate the time until this particular section reaches zero remaining service life. Zero RSL happens when the section reaches 2.5 RQI. From table 15, 2.5 RQI is in state 15. This way,

$$Time_{2\ to\ 15} = S_{2,2} + S_{2,3} + S_{2,4} + S_{2,5} + S_{2,6} + S_{2,7} + S_{2,8} + S_{2,9} + S_{2,10} + S_{2,11} + S_{2,12} + S_{2,13} + S_{2,14} + S_{2,15} = 9.864$$

This way, a section with the described characteristics is expected to go from RQI 4.1 to RQI 2.5 and reach zero remaining service life in 9.9 years.

As shown in these sections, the enhancement of the Markov transition matrix with ordinal logistic regression models can be used to analyze how the patterns of pavement deterioration are affected by district location, last activity performed, functional class, base thickness, speed limit and pavement thickness.

Another advantage of using the ordinal logistic regression models is the possibility of performing site specific predictions. As shown in the examples, the model forecasts the behavior of specific pavement sections, resulting in higher accuracy predictions.

In chapter 5, the effects of different repair activities will be incorporated into the Markov transition matrix and ordinal logistic regression models. The final Markov probability matrix will be used to determine the sequence of maintenance policies that optimally reach a desired target distribution from the current network condition.

CHAPTER 5: DYNAMIC PROGRAMING OPTIMIZATION

In this chapter we explain the methodology used in Markov Decision Process and describe how it will be combined to the state-to-state transition probabilities derived in chapter 4 to create a user-friendly optimization tool. The tool will be used to determine the optimal repair policies for site specific pavement sections.

5.1 BACKGROUND AND STRATEGY

Markov Decision Process (S, A, T, R)

Markov Decision Process (MDP) is a mathematical framework used for modeling recursive decision making, where the immediate and long-term rewards are taking in consideration. MDP is a 4-tuple model: states, actions, transition probabilities and the rewards. (30, 31)

- States, S: The Markov states represent how the world works. They can be XY coordinates, condition, grid locations, levels etc., and will be affected by the actions.
- Actions, A: The actions are the possible decision that can be made in each state.
- Transition Probabilities, T: The transition probabilities determine how the states will be affected by each action.

T: S x A x S x {0,1, … ,H} → [0,1], T_t (s,a,s') = P(s_{t+1} = s' | s_t = s, a_t =a)

- Reward, R: The immediate value resulting from performing an action in each of the states.

R: S x A x S x {0, 1, … , H} → < R_t (s,a,s') = reward for (s_{t+1} = s', s_t = s, a_t =a)

- Horizon, H: Horizon over which the agent will act.
- Policies: Policies determine which set of actions should be taken. The optimal policy is the one that maximizes the total reward. (31)

The optimal policy π^* in M =⟨S,A,P,R,H,s_0⟩can be derived from the related Bellman optimality equation, which describes the reward for selecting the actions that yield the highest expected reward, as shown in equations 23 and 24 (32)

$$V^*(s) = max_{a \in A} Q^*(a,s) \quad [23]$$

$$Q^*(a,s) = R(a,s) + \sum_{s' \in S} P(s'|a,s) \cdot V^*(s') \quad [24]$$

where $V^*(s)$ and $Q^*(a,s)$ represent the optimal state-value and action-value functions, respectively.

If the transition probabilities *P(s'|a,s)* and *R(a,s)* are known, the problem can be solved using dynamic programing along with the Bellman equations. If they are unknown, reinforcement learning can be used to solve the problem.

5.2 ILLUSTRATIVE EXAMPLE

We developed a simple illustrative example to help introduce the MDP methodology (figure 5.1). In this example, there are three pavement condition states and three possible actions:

S (set of states) = 3 {Good, Fair, Poor}

A (set of actions) = 3 {a_1: do nothing, a_2: minimal repair, a_3: major repair}

T (transition probabilities) =

$$P_1 = \begin{matrix} 0.3 & 0.7 & 0 \\ 0 & 0.6 & 0.4 \\ 0 & 0 & 1 \end{matrix} \qquad P_2 = \begin{matrix} 0.6 & 0.4 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.7 & 0.3 \end{matrix} \qquad P_3 = \begin{matrix} 0.9 & 0.1 & 0 \\ 0.9 & 0.1 & 0 \\ 0.9 & 0.1 & 0 \end{matrix}$$

R (rewards) =

$$R_1 = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \qquad R_2 = \begin{matrix} -1 \\ -1 \\ -1 \end{matrix} \qquad R_1 = \begin{matrix} -5 \\ -5 \\ -5 \end{matrix}$$

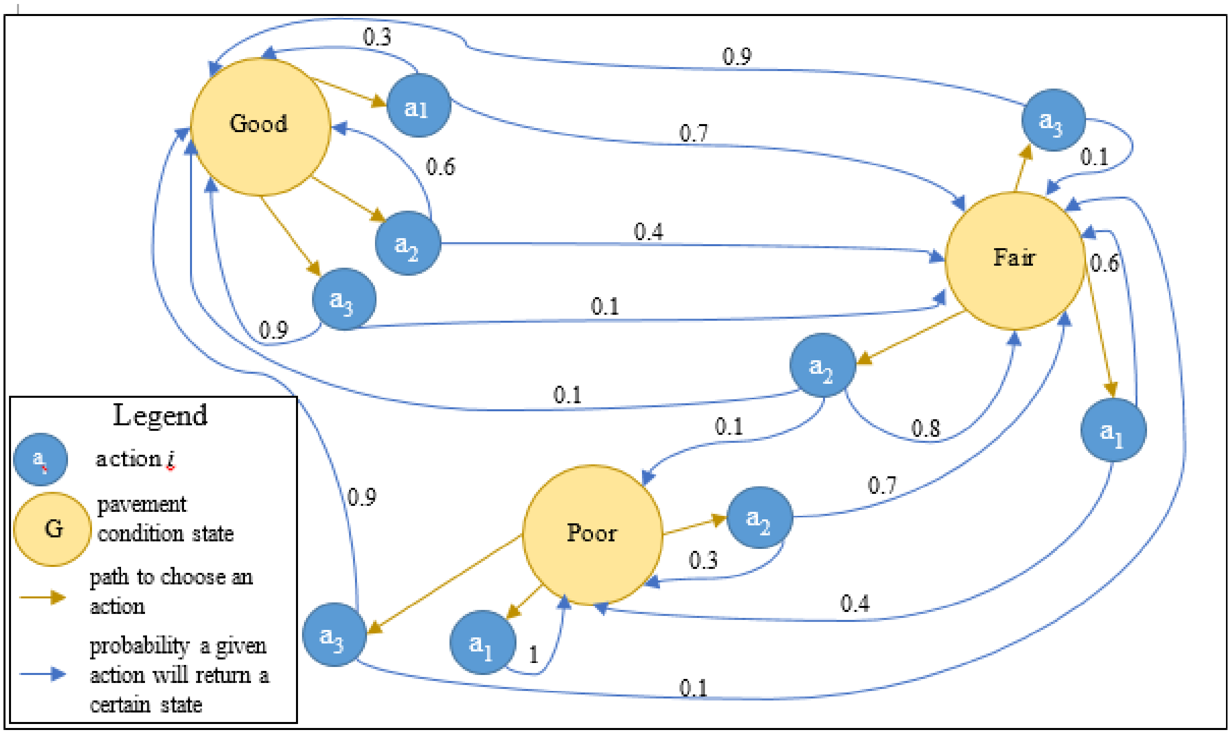

Figure 5.1: MDP example with three states (Good, Fair and Poor) and three actions (a1, a2 and a3)

Different scenarios were created to show the effect of several possible polices. For instance, figure 5.2 shows the 5-year behavior of sections that started in state 'Good' and did not receive any repair. The black node at year 0 shows that all sections started in good condition. Then, the transition probabilities were used to forecast the deterioration of the sections, where darker nodes indicate higher probability of landing in a given condition state. It can be seen that, if no repair is performed, most sections will be in poor condition by year 5.

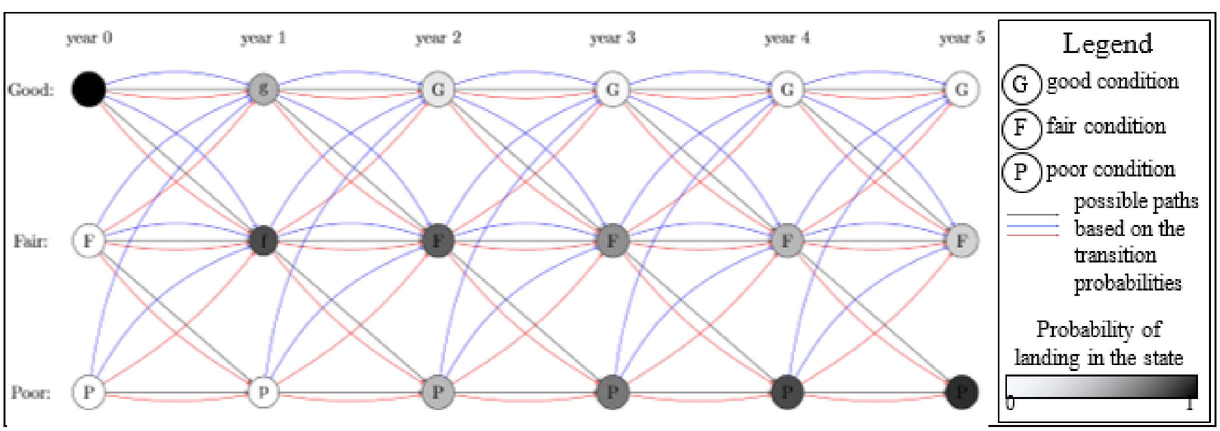

Figure 5.2: 5-year distribution for Policy A: do nothing

Figures 5.3 and 5.4 show the pavement condition distribution over 5 years if policies B (Action 2 for sections in state 'Fair') and C (Action 3 for sections in state 'Poor') were applied, respectively.

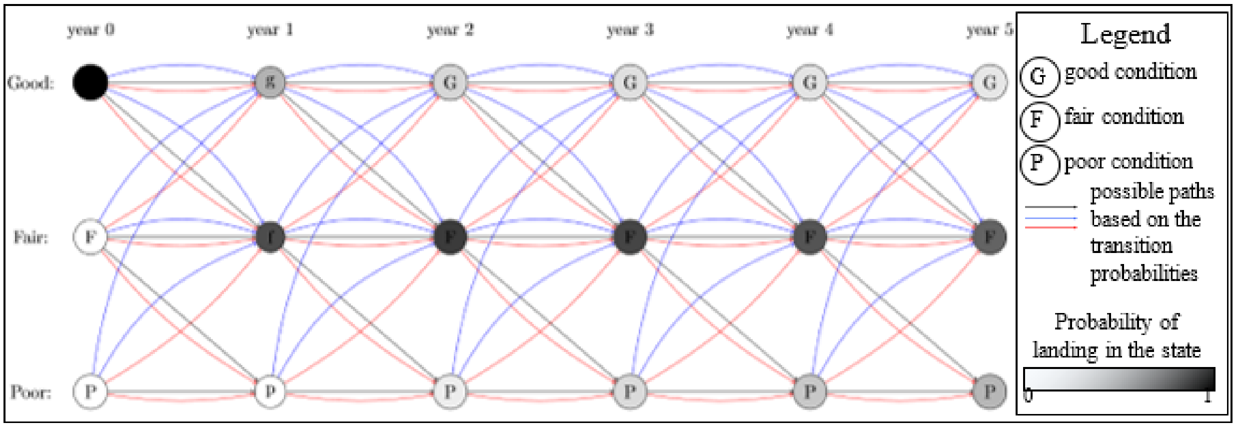

Figure 5.3: 5-year distribution for Policy B: minimal repair if in state 'Fair'

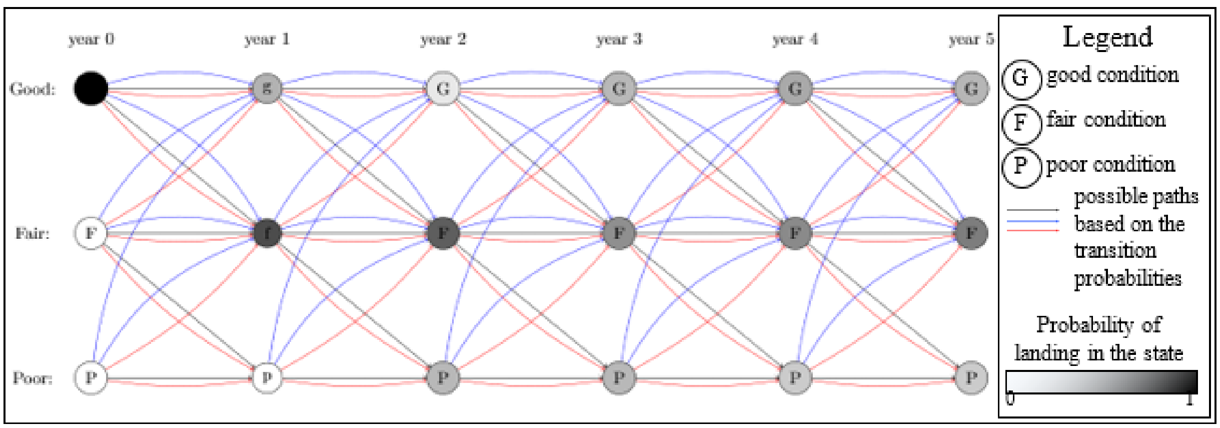

Figure 5.4: 5-year distribution for Policy C: major repair if in state 'Poor'

Figures 5.3 and 5.4 show the effect of different levels of repair, applied to sections in different conditions. Differently from the scenario in figure 5.2, the repairs prevented the sections from landing mostly in poor condition at year 5.

A fourth scenario can be created, where the optimal repair policies are chosen. Initially, The Bellman optimality equation, introduced in equations 23 and 24, can be used to calculate the action-value function, as shown in table 5.1.

Table 5.1: MDP calculation – optimal action-value functions

Condition	Action	Q_t4	Q_t3	Q_t2	Q_t1	Q_t0
Good	1	8.60	8.18	8.05	8.02	8.00
	2	8.20	7.36	7.11	7.03	7.01
	3	4.80	3.54	3.16	3.05	3.01
Fair	1	4.80	3.84	3.07	2.71	2.67
	2	6.40	5.12	4.10	3.61	3.56
	3	4.80	4.48	4.51	4.45	4.45
Poor	1	0.00	4.80	6.04	6.41	6.52
	2	4.60	6.04	6.41	6.52	6.56
	3	4.80	4.80	4.80	4.80	4.80

The optimal policy D can be defined from the maximum action-value function results. The pavement condition distribution following the optimal policy for the next 5 years is illustrated in figure 5.5. It can be seen from figure 5.5 that most sections landed in fair condition at year 5.

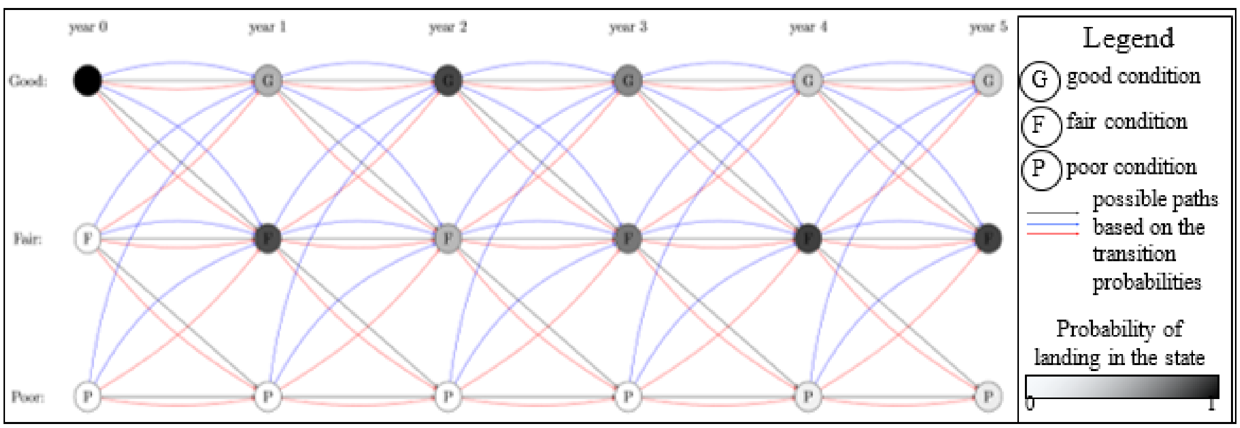

Figure 5.5: 5-year distribution for Policy D: Optimal Solution to MDP

5.3 PROBLEM FORMALIZATION AND ANALYSES

The same methodology was applied to the project data, using the results from chapter 4.

First, we define the States (S). The states for the Markov transition matrix were derived in chapter 4 based on the RQI values, as shown in table 5.2.

Table 5.2: States for Markov Transition Probability based on RQI

State	RQI	Physical Meaning
1	5.0 - 4.2	Very Good
2	4.1 - 4.0	Very Good
3	3.9	Good
4	3.8	Good
5	3.7	Good
6	3.6	Good
7	3.5	Good
8	3.4	Good
9	3.3	Good
10	3.2	Good
11	3.1	Good
12	3.0	Good
13	2.9 - 2.8	Fair
14	2.7 - 2.6	Fair
15	2.5 - 2.4	Fair
16	2.3 - 2.2	Fair
17	2.1 - 2.0	Fair

Then, we define the Actions (A). An assessment of activities performed in the sections classified in cluster 1 (defined in chapter 4) was performed to find the number of records for each repair activity from 2001 to 2018, and the three most common ones were selected: thin mill and overlay, thin overlay and thick mill and overlay. The results are shown in table 5.3.

Table 5.3: Assessment of repair activities performed

Repair Activity	Number of times the activity was performed
CIR & Medium OL	58
Crack Repr/Med OL	18
Full Mill & Thick OL	1
Medium Overlay	184
Nova Chip	194
Thick Mill/Overlay	362
Thin Mill/Overlay	1183
Thin Overlay	1160

The option of doing nothing was added to the three activities to create a group of four possible actions, as shown in table 5.4.

Table 5.4: Four possible actions

Action 1	Do Nothing
Action 2	Thin Mill and Overlay
Action 3	Thin Overlay
Action 4	Thick Mill and Overlay

Next, we compute the Transition Probabilities (T). The transition probabilities determine how the states will be affected by each action. Thus, each action should have its own transition probability matrix that shows how the pavement sections behave in the case of a certain action being adopted.

From chapter 4, the transition probabilities for choosing action 1 "do nothing" are simply the decay matrix computed from the ordinal logistic regression models, as shown in table 5.5. The transition probability matrices for actions 2, 3 and 4 (shown in tables C4 to C6, in the appendix C) were computed by adding the natural decay probability, shown in table 5.5 to the condition improvement that each repair activity has on the pavement condition (shown in tables C1 to C3) and the applied rate of repair.

Table 5.5: Markov Probability Matrix Enhanced by Ordinal Logistic Regression

		End State																
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Start State	1	P_{1-1}	P_{1-2}	P_{1-3}	P_{1-4}	P_{1-5}	P_{1-6}			P_{1-9}								
	2		P_{2-2}	P_{2-3}	P_{2-4}	P_{2-5}	P_{2-6}											
	3			P_{3-3}	P_{3-4}	P_{3-5}	P_{3-5}	P_{3-7}	P_{3-8}	P_{3-9}	P_{3-10}	P_{3-11}	P_{3-12}					
	4				P_{4-4}	P_{4-5}	P_{4-6}	P_{4-7}	P_{4-8}	P_{4-9}	P_{4-10}		P_{4-12}					
	5					P_{5-5}	P_{5-6}	P_{5-7}	P_{5-8}	P_{5-9}	P_{5-10}	P_{5-11}						
	6						P_{6-6}	P_{6-7}	P_{6-8}	P_{6-9}	P_{6-10}	P_{6-11}	P_{6-12}	P_{6-13}	P_{6-14}			
	7							P_{7-7}	P_{7-8}	P_{7-9}	P_{7-10}	P_{7-11}						
	8								P_{8-8}	P_{8-9}	P_{8-10}	P_{8-11}	P_{8-12}	P_{8-13}				
	9									P_{9-9}	P_{9-10}	P_{9-11}	P_{9-12}	P_{9-13}				
	10										P_{10-10}	P_{10-11}	P_{10-12}	P_{10-13}	P_{10-14}	P_{10-15}		
	11											P_{11-11}	P_{11-12}	P_{11-13}	P_{11-14}	P_{11-15}	P_{11-16}	
	12												P_{12-12}	P_{12-13}	P_{12-14}	P_{12-15}	P_{12-16}	
	13													P_{13-13}	P_{13-14}	P_{13-15}	P_{13-16}	P_{13-17}
	14														P_{14-14}	P_{14-15}	P_{14-16}	
	15															P_{15-15}	P_{15-16}	P_{15-17}
	16																0.436	0.564
	17																	1

*The values for each cell can be calculated using the model equations in Appendix B derived in Chapter 4.

In order to augment the decay matrix and the repair matrix, it is necessary to determine the pavement characteristics that will be used in the equations from the logit models, as shown in table 5.5. For this example, the following section characteristics were selected:

- District Location: District 2
- Functional Class: Rural Minor Arterial
- Last Activity Performed: Thin Overlay
- Speed Limit: 55 mph
- Base Thickness: 8 in

- Surface Thickness: 8 in

Finally, we calculate the Reward (R). First, we define the cost, and then we define the benefit.

The cost represents the immediate value resulting from performing an action in each of the states. In our problem, the immediate reward is the cost of performing a given repair activity. Based on the information from the HPMA software, the costs of the repair activities were adopted as shown in table 5.6.

Table 5.6: Cost associated with each repair activity from HPMA

Action	Cost [dollars/12-foot lane-mile]
Do Nothing	$ 0.00
Thin Mill and Overlay	$ 105,826.00
Thick Mill and Overlay	$ 211,550.00
Thin Overlay	$ 66,852.00

For the benefit, as shown in tables A1 to A3, each repair activity results in a specific condition improvement. This improvement can be quantified as the increase in RQI caused by any action at any condition state *i*, as shown in equation 25.

$$RQI\ improvement(s_i|a) = \sum_{j=1}^{17} P_{i,j} * (RQI(i) - RQI(j)) \qquad [25]$$

In order to compare cost and benefit, they should have comparable units. Thus, the RQI improvement should be multiplied by a factor γ, which gives RQI a monetary value. γ can be understood as the dollar value associated with an increase of RQI by one unit.

The optimization process seeks to maximize the difference between the benefit of performing a repair activity and the cost the activity requires, as shown in equation 26.

$$Objective = \max(benefit - cost) = \max(\gamma * RQI_{improvement} - cost) \qquad [26]$$

where γ is the conversion factor that gives RQI a monetary value. γ is used to give the same unit to the cost of performing a repair activity and the improvement observed as a result of that repair activity.

In addition, we need to specify the Horizon and the Policies used to determine which set of actions should be taken. The optimal policy is the one that maximizes the total reward.

A Finite Horizon MDP is used, since the process is expected to run for a finite time period (finite number of decision epochs). The optimal policy for an MDP with finite horizon is a time-dependent policy under the expected rewards. Since the transition probabilities are known from chapter 4, the optimal policy can be calculated using dynamic programing (33). Dynamic programming technique uses the sum of the present cost and the expected future cost to rank the decisions, assuming optimal decision making for the succeeding stages. This way, dynamic programming accounts for the balance of desired low present cost and undesired high future costs. (34)

The adapted Bellman optimality equation can be used to find the optimal policy that maximizes the action-value function, as shown in equations 27 and 28.

$$V^*(s) = \begin{cases} 0 & \text{if } s \text{ is terminal,} \\ max_{a \in A} Q^*(a,s) & \text{otherwise} \end{cases} \qquad [27]$$

$$B_{s \to s'} = \gamma * RQI_{improvement}(s \to s')$$

$$Q^*(a,s) = R(a,s) + \sum\nolimits_{s' \in S} P(s'|a,s) \cdot (V^*(s') + B_{s \to s'}) \qquad [28]$$

where $V^*(s)$ and $Q^*(a,s)$ represent the optimal state-value and action-value functions, respectively.

The finite horizon MDP adopted in this project does not include a discount factor, neither for the MDP formulation, nor for the future value of money. The calculated benefit value from the tool is simply the cumulative benefits minus costs and should not be interpreted as a net present value.

Adopting the RQI monetary factor as γ = $100,000.00, the optimal policies can be determined from the maximum action value function values, as shown in table 5.7.

Table 5.7: Optimal actions for the next 10 years, using γ = $100,000.00

Condition State	RQI Range	Optimal Action to Take										
		Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
1	RQI: 5.0-4.2	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
2	RQI: 4.1-4.0	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
3	RQI: 3.9	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
4	RQI: 3.8	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
5	RQI: 3.7	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
6	RQI: 3.6	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
7	RQI: 3.5	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
8	RQI: 3.4	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
9	RQI: 3.3	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
10	RQI: 3.2	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
11	RQI: 3.1	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
12	RQI: 3.0	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
13	RQI: 2.9-2.8	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
14	RQI: 2.7-2.6	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
15	RQI: 2.5-2.4	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
16	RQI: 2.3-2.2	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
17	RQI: 2.1-2.0	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay

The optimal policies shown in table 5.7 are based on the Markov transition probabilities, the cost of each repair activity and the RQI monetary conversion factor. The RQI monetary factor has a significant effect, when defining the optimal policies. The sensitivity of decisions to this factor can be investigated by comparing the results from table 5.7 to the results obtained when a different RQI monetary factor is used. For example, table 5.8 shows a summary of optimal repair actions estimated for pavement sections containing the same parameters as the ones used in table 5.7, and RQI monetary factor of $1,000,000.00.

Table 5.8: Optimal actions for the next 10 years, using γ = $1,000,000.00

Condition State	RQI Range	Optimal Action to Take										
		Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
1	RQI: 5.0-4.2	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay
2	RQI: 4.1-4.0	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
3	RQI: 3.9	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
4	RQI: 3.8	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
5	RQI: 3.7	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
6	RQI: 3.6	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay
7	RQI: 3.5	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay
8	RQI: 3.4	3 - Thin Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay	2 - Thin Mill and Overlay
9	RQI: 3.3	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
10	RQI: 3.2	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
11	RQI: 3.1	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	3 - Thin Overlay
12	RQI: 3.0	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
13	RQI: 2.9-2.8	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay
14	RQI: 2.7-2.6	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay
15	RQI: 2.5-2.4	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay
16	RQI: 2.3-2.2	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay
17	RQI: 2.1-2.0	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay	4 - Thick Mill and Overlay

Table 5.8 shows highly conservative repair policies, caused by a high RQI monetary factor. In this case, the cost of performing a repair activity was minimal compared to its resulting benefit. The difference between tables 5.7 and 5.8 reflects the importance of selecting a balance value for the RQI monetary conversion factor.

5.4 THE OPTIMIZATION TOOL

A user-friendly excel spreadsheet tool was created to determine the optimal repair policies for site-specific pavement sections. The excel spreadsheet allows for easy calibration of the most influencing input factors, such as RQI monetary value, the cost of repair activities, and the pavement characteristics. As more accurate values are obtained, they can be easily replaced in the template.

5.4.1 Input Parameters

Pavement Characteristics: the ordinal logistic regression models developed in chapter 4 allow for site-specific predictions of pavement performance and deterioration. This way, the pavement characteristics should be entered in first tab of the excel template. The pavement characteristics input parameters are District, Functional Class, Last Repair Activity Performed, Speed Limit, Base Thickness and Surface Thickness. The analyses were performed on the most common pavement type, which has the most data available, Bituminous over Bituminous (BOB). As a result, the excel tool, as it stands, should be used to analyze BOB pavement sections.

RQI Monetary Converter: a conversion factor was created to give a monetary value to the RQI improvement observed after a repair activity is performed. This is necessary to give comparable units to the cost of performing a repair activity and the improvement observed as a result of that repair activity. It can be understood as the dollar value associated with an increase of RQI by one unit per lane mile. An initial value of $100,000.00 was suggested, but it can be calibrated for more accurate results. Based in input from TAP members, the excel template can be easily changed to accommodate different monetary values corresponding to different RQI values.

Cost of Repair Activities: based on the information from the HPMA software, the costs of the repair activities were adopted as previously shown in table 5.6. Preventive activities can be added in the future, as more data becomes available. The repair costs can be adjusted to account for inflation and to reflect the costs of specific districts. It is important to note that only BOB pavement sections were considered in the analyses.

Figure 5.6 shows a screenshot of the excel tool, where the input parameters can be entered.

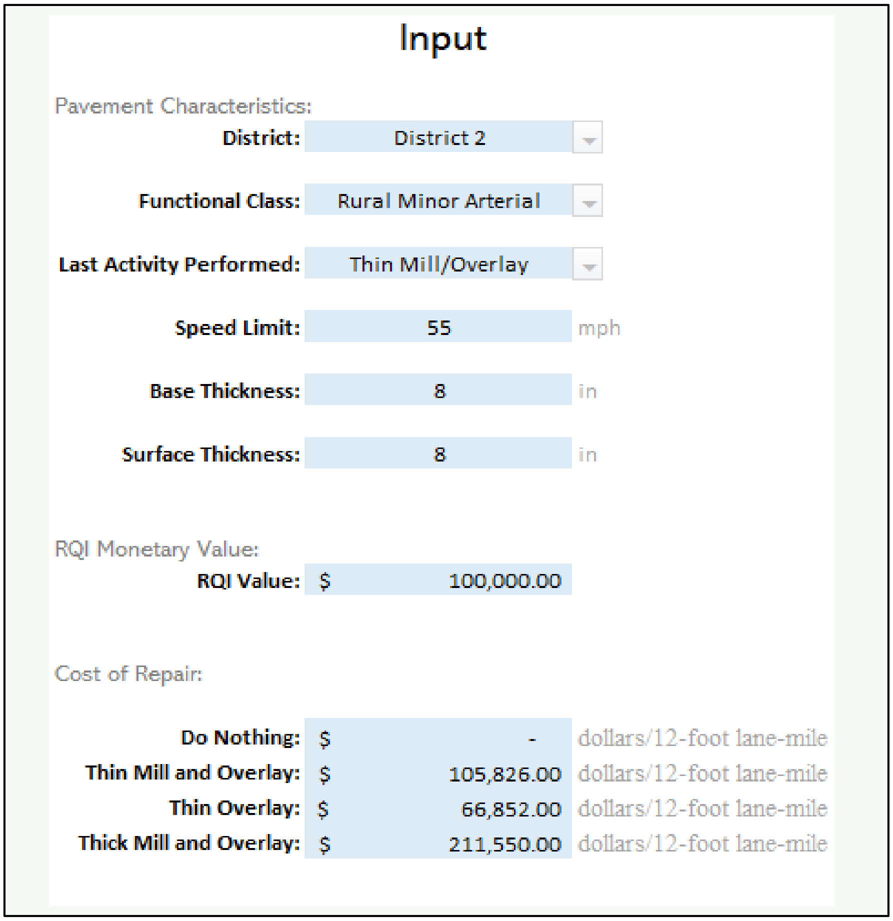

Figure 5.6: First sheet of the excel tool, devoted to the input parameters

After the input parameters are entered, the benefit of each repair activity is calculated by multiplying the improvement in RQI observed after the activity was performed and the RQI monetary factor. Figure 5.7 shows the table that contains the Benefit of each repair activity by condition state. The benefit of performing the selected repair activities is higher for pavements in worse conditions because they experience greater condition improvement.

Condition State	Average RQI	2 - Thin Mill and Overlay		
		Cost	RQI Improvement	Benefit
1	4.6	$ 105,826.00	0	$ -
2	4.05	$ 105,826.00	0.55	$ 55,000.00
3	3.9	$ 105,826.00	0.15	$ 15,000.00
4	3.8	$ 105,826.00	0.525	$ 52,500.00
5	3.7	$ 105,826.00	0.430769231	$ 43,076.92
6	3.6	$ 105,826.00	0.494444444	$ 49,444.44
7	3.5	$ 105,826.00	0.587037037	$ 58,703.70
8	3.4	$ 105,826.00	0.645945946	$ 64,594.59
9	3.3	$ 105,826.00	0.695098039	$ 69,509.80
10	3.2	$ 105,826.00	0.719565217	$ 71,956.52
11	3.1	$ 105,826.00	0.772674419	$ 77,267.44
12	3	$ 105,826.00	0.808163265	$ 80,816.33
13	2.85	$ 105,826.00	0.923969072	$ 92,396.91
14	2.65	$ 105,826.00	1.036746988	$ 103,674.70
15	2.45	$ 105,826.00	1.254577465	$ 125,457.75
16	2.25	$ 105,826.00	1.401754386	$ 140,175.44
17	2.05	$ 105,826.00	1.475806452	$ 147,580.65

Figure 5.7: Table in the excel tool with the results from cost, RQI improvement and benefit of each repair activity, by condition state

The RQI improvement observed for state *s*, after performing action *a*, was calculated as shown in equation 29.

$$RQI\ improvement(s_i|a) = \sum_{j=1}^{17} P_{i,j} * (RQI(i) - RQI(j)) \qquad [29]$$

where $P_{i,j}$ are the transition probabilities that show the improvement in pavement condition observed after action *a* is performed, which was estimated empirically, from the data, in chapter 4. Figure 5.8 shows an illustration of the transition probabilities observed after repair, as shown in the excel tool.

		Effect of repair matrix for Thin Mill and Overlay																
		End State																
		1 (RQI: 5.0-4.2)	2 (RQI: 4.1-4.0)	3 (RQI: 3.9)	4 (RQI: 3.8)	5 (RQI: 3.7)	6 (RQI: 3.6)	7 (RQI: 3.5)	8 (RQI: 3.4)	9 (RQI: 3.3)	10 (RQI: 3.2)	11 (RQI: 3.1)	12 (RQI: 3.0)	13 (RQI: 2.9-2.8)	14 (RQI: 2.7-2.6)	15 (RQI: 2.5-2.4)	16 (RQI: 2.3-2.2)	17 (RQI: 2.1-2.0)
Start State 1	(RQI: 5.0-4.2)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	(RQI: 4.1-4.0)	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	(RQI: 3.9)	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	(RQI: 3.8)	0.5	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	(RQI: 3.7)	0.307692	0.307692	0.076923	0.307692	0	0	0	0	0	0	0	0	0	0	0	0	0
6	(RQI: 3.6)	0.166667	0.666667	0.055556	0	0.111111	0	0	0	0	0	0	0	0	0	0	0	0
7	(RQI: 3.5)	0.185185	0.481481	0.185185	0.148148	0	0	0	0	0	0	0	0	0	0	0	0	0
8	(RQI: 3.4)	0.162162	0.540541	0.108108	0.054054	0.054054	0	0.081081	0	0	0	0	0	0	0	0	0	0
9	(RQI: 3.3)	0.137255	0.45098	0.117647	0.078431	0.117647	0.039216	0.039216	0.019608	0	0	0	0	0	0	0	0	0
10	(RQI: 3.2)	0.101449	0.362319	0.217391	0.028986	0.086957	0.072464	0.043478	0.057971	0.028986	0	0	0	0	0	0	0	0
11	(RQI: 3.1)	0.05814	0.267442	0.22093	0.209302	0.05814	0.093023	0.046512	0.011628	0.011628	0.023256	0	0	0	0	0	0	0
[illegible]	[illegible]	[illegible]	[illegible]	0.163265	0.163265	0.112245	0.091837	0.112245	0.05102	0	0	0.030612	0	0	0	0	0	0
13	(RQI: 2.9-2.8)	0.041237	0.170103	0.164948	0.159794	0.164948	0.123711	0.051546	0.06701	0.020619	0.020619	0.005155	0.010309	0	0	0	0	0
14	(RQI: 2.7-2.6)	0.036145	0.084337	0.13253	0.174699	0.162651	0.162651	0.060241	0.054217	0.060241	0.018072	0.018072	0	0.036145	0	0	0	0
15	(RQI: 2.5-2.4)	0.056338	0.056338	0.147887	0.183099	0.15493	0.140845	0.091549	0.049296	0.014085	0.056338	0.028169	0.014085	0.007042	0	0	0	0
16	(RQI: 2.3-2.2)	0.078947	0.078947	0.070175	0.061404	0.166667	0.157895	0.122807	0.078947	0.061404	0.035088	0.035088	0.04386	0	0.008772	0	0	0
17	(RQI: 2.1-2.0)	0.010753	0.086022	0.075269	0.075269	0.11828	0.11828	0.182796	0.096774	0.064516	0.053763	0.010753	0.043011	0.032258	0.010753	0	0.021505	0

Figure 5.8: Transition probabilities matrix showing the pavement condition recovery observed after Thin Mill and Overlay was performed

The matrix in figure 5.8 shows the probability that the pavement will recover from a state to another state after receiving the repair activity. For example, as shown in the highlighted cells of figure 5.9, the probability that a pavement section in state 12 (RQI of 3.0) will improve to state condition 1 is 0.05102. Similarly, the probability that this pavement will improve to state 2 is 0.22449.

		1 (RQI: 5.0-4.2)	2 (RQI: 4.1-4.0)	3 (RQI: 3.9)	4 (RQI: 3.8)	5 (RQI: 3.7)	6 (RQI: 3.6)	7 (RQI:
Start State 1	(RQI: 5.0-4.2)	0	0	0	0	0	0	
2	(RQI: 4.1-4.0)	1	0	0	0	0	0	
3	(RQI: 3.9)	0	1	0	0	0	0	
4	(RQI: 3.8)	0.5	0.5	0	0	0	0	
5	(RQI: 3.7)	0.307692	0.307692	0.076923	0.307692	0	0	
6	(RQI: 3.6)	0.166667	0.666667	0.055556	0	0.111111	0	
7	(RQI: 3.5)	0.185185	0.481481	0.185185	0.148148	0	0	
8	(RQI: 3.4)	0.162162	0.540541	0.108108	0.054054	0.054054	0	0.08
9	(RQI: 3.3)	0.137255	0.45098	0.117647	0.078431	0.117647	0.039216	0.03
10	(RQI: 3.2)	0.101449	0.362319	0.217391	0.028986	0.086957	0.072464	0.04
11	(RQI: 3.1)	0.05814	0.267442	0.22093	0.209302	0.05814	0.093023	0.04
[illegible]	[illegible]	[illegible]	[illegible]	0.163265	0.163265	0.112245	0.091837	0.11
13	(RQI: 2.9-2.8)	0.041237	0.170103	0.164948	0.159794	0.164948	0.123711	0.05

Figure 5.9: Zoomed in portion of the matrix shown in figure 5.8

The first tab in the excel tool also contains the transition probability matrix for natural decay (in figure 5.10), which shows how the pavement deteriorates naturally, and was derived from the ordinal logistic regression models in chapter 4. The probabilities are conditioned to the input parameters entered in the input section, as previously shown in figure 5.6.

			Natural decay Matrix																
			End State																
			1 (RQI: 5.0-4.2)	2 (RQI: 4.1-4.0)	3 (RQI: 3.9)	4 (RQI: 3.8)	5 (RQI: 3.7)	6 (RQI: 3.6)	7 (RQI: 3.5)	8 (RQI: 3.4)	9 (RQI: 3.3)	10 (RQI: 3.2)	11 (RQI: 3.1)	12 (RQI: 3.0)	13 (RQI: 2.9-2.8)	14 (RQI: 2.7-2.6)	15 (RQI: 2.5-2.4)	16 (RQI: 2.3-2.2)	17 (RQI: 2.1-2.0)
Start State	1	(RQI: 5.0-4.2)	0.569056	0.411527	0.015431	0.002816	0.000399	0.000385	0	0	0.000385	0	0	0	0	0	0	0	0
	2	(RQI: 4.1-4.0)	0	0.470534	0.420369	0.081391	0.020167	0.00754	0	0	0	0	0	0	0	0	0	0	0
	3	(RQI: 3.9)	0	0	0.328716	0.43713	0.174293	0.047368	0.005557	0.002776	0.002078	0.000694	0.000694	0.000694	0	0	0	0	0
	4	(RQI: 3.8)	0	0	0	0.316047	0.409869	0.22063	0.040014	0.006413	0.005111	0.001277	0	0.000638	0	0	0	0	0
	5	(RQI: 3.7)	0	0	0	0	0.317562	0.494277	0.144011	0.029333	0.011739	0.002463	0.000616	0	0	0	0	0	0
	6	(RQI: 3.6)	0	0	0	0	0	0.436593	0.377526	0.139395	0.033832	0.008181	0.001221	0.002034	0.000812	0.000406	0	0	0
	7	(RQI: 3.5)	0	0	0	0	0	0	0.272297	0.412952	0.253121	0.046972	0.014658	0	0	0	0	0	0
	8	(RQI: 3.4)	0	0	0	0	0	0	0	0.259994	0.442667	0.213401	0.060576	0.017144	0.006218	0	0	0	0
	9	(RQI: 3.3)	0	0	0	0	0	0	0	0	0.281507	0.396363	0.239186	0.061445	0.021499	0	0	0	0
	10	(RQI: 3.2)	0	0	0	0	0	0	0	0	0	0.228289	0.430073	0.240896	0.091594	0.006103	0.003046	0	0
	11	(RQI: 3.1)	0	0	0	0	0	0	0	0	0	0	0.209159	0.428911	0.315177	0.037908	0.006318	0.002527	0
	12	(RQI: 3.0)	0	0	0	0	0	0	0	0	0	0	0	0.176535	0.627861	0.167381	0.02066	0.007562	0
	13	(RQI: 2.9-2.8)	0	0	0	0	0	0	0	0	0	0	0	0	0.373788	0.501112	0.111646	0.012235	0.001218
	14	(RQI: 2.7-2.6)	0	0	0	0	0	0	0	0	0	0	0	0	0	0.202781	0.586234	0.210985	0
	15	(RQI: 2.5-2.4)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.409508	0.480713	0.109779
	16	(RQI: 2.3-2.2)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.436	0.564
	17	(RQI: 2.1-2.0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Figure 5.10: Transition probabilities matrix showing the pavement natural deterioration

The transition probabilities matrices for condition recovery and natural decay, illustrated in figures 5.8 and 5.10, were combined to the rate of repair to create the full transition matrix, shown in figure 5.11. Some other tables in the input sheet are destined to the multiplications involved in the augmentation of the matrices and should not be modified.

			Augmented matrix for decay and Thin Mill and Overlay																
			End State																
			1 (RQI: 5.0-4.2)	2 (RQI: 4.1-4.0)	3 (RQI: 3.9)	4 (RQI: 3.8)	5 (RQI: 3.7)	6 (RQI: 3.6)	7 (RQI: 3.5)	8 (RQI: 3.4)	9 (RQI: 3.3)	10 (RQI: 3.2)	11 (RQI: 3.1)	12 (RQI: 3.0)	13 (RQI: 2.9-2.8)	14 (RQI: 2.7-2.6)	15 (RQI: 2.5-2.4)	16 (RQI: 2.3-2.2)	17 (RQI: 2.1-2.0)
Start State	1	(RQI: 5.0-4.2)	0.5691	0.4115	0.0154	0.0028	0.0004	0.0004	0.0000	0.0000	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	(RQI: 4.1-4.0)	0.0041	0.4665	0.4204	0.0814	0.0202	0.0075	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	(RQI: 3.9)	0.0000	0.0011	0.3276	0.4371	0.1743	0.0474	0.0056	0.0028	0.0021	0.0007	0.0007	0.0007	0.0000	0.0000	0.0000	0.0000	0.0000
	4	(RQI: 3.8)	0.0003	0.0003	0.0000	0.3155	0.4099	0.2206	0.0400	0.0064	0.0051	0.0013	0.0000	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000
	5	(RQI: 3.7)	0.0010	0.0010	0.0003	0.0010	0.3142	0.4943	0.1440	0.0293	0.0117	0.0025	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6	(RQI: 3.6)	0.0006	0.0023	0.0002	0.0000	0.0004	0.4331	0.3775	0.1394	0.0338	0.0082	0.0012	0.0020	0.0008	0.0004	0.0000	0.0000	0.0000
	7	(RQI: 3.5)	0.0012	0.0031	0.0012	0.0009	0.0000	0.0000	0.2659	0.4130	0.2531	0.0470	0.0147	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	8	(RQI: 3.4)	0.0015	0.0051	0.0010	0.0005	0.0005	0.0000	0.0008	0.2506	0.4427	0.2134	0.0606	0.0171	0.0062	0.0000	0.0000	0.0000	0.0000
	9	(RQI: 3.3)	0.0017	0.0056	0.0015	0.0010	0.0015	0.0005	0.0005	0.0002	0.2691	0.3964	0.2392	0.0614	0.0215	0.0000	0.0000	0.0000	0.0000
	10	(RQI: 3.2)	0.0019	0.0069	0.0041	0.0006	0.0017	0.0014	0.0008	0.0011	0.0006	0.2093	0.4301	0.2409	0.0916	0.0061	0.0030	0.0000	0.0000
	11	(RQI: 3.1)	0.0014	0.0066	0.0054	0.0052	0.0014	0.0023	0.0011	0.0003	0.0003	0.0006	0.1845	0.4289	0.3152	0.0379	0.0063	0.0025	0.0000
	12	(RQI: 3.0)	0.0016	0.0070	0.0051	0.0051	0.0035	0.0028	0.0035	0.0016	0.0000	0.0000	0.0009	0.1455	0.6279	0.1674	0.0207	0.0076	0.0000
	13	(RQI: 2.9-2.8)	0.0016	0.0068	0.0066	0.0064	0.0066	0.0049	0.0021	0.0027	0.0008	0.0008	0.0002	0.0004	0.3338	0.5011	0.1116	0.0122	0.0012
	14	(RQI: 2.7-2.6)	0.0017	0.0039	0.0062	0.0081	0.0076	0.0076	0.0028	0.0025	0.0028	0.0008	0.0008	0.0000	0.0017	0.1562	0.5862	0.2110	0.0000
	15	(RQI: 2.5-2.4)	0.0030	0.0030	0.0079	0.0097	0.0082	0.0075	0.0049	0.0026	0.0007	0.0030	0.0015	0.0007	0.0004	0.0000	0.3563	0.4807	0.1098
	16	(RQI: 2.3-2.2)	0.0053	0.0053	0.0047	0.0042	0.0113	0.0107	0.0083	0.0053	0.0042	0.0024	0.0024	0.0030	0.0000	0.0006	0.0000	0.3683	0.5640
	17	(RQI: 2.1-2.0)	0.0012	0.0096	0.0084	0.0084	0.0132	0.0132	0.0204	0.0108	0.0072	0.0060	0.0012	0.0048	0.0036	0.0012	0.0000	0.0024	0.8882

Figure 5.11: Augmented transition probabilities matrix

5.4.2 MDP Calculations

The second sheet in the excel tool is dedicated to the Markov Decision Process (MDP) calculations. The optimal policy was calculated through dynamic programming. The adapted Bellman optimality equation, in equation 30, was used to find the policy that maximizes the action-value function, *Q* (a,s)*.

$$Q^*(a,s) = C(a,s) + \sum_{s\prime \in S} P(s'|a,s) \cdot (V^*(s') + B_{s \to s'}) \qquad [30]$$

where *C(a,s)* is the cost of performing repair action *a* at state *s*, *P(s'|a,s)* is the probability that a pavement section will transition to state *s'* given it is in state *s* and received repair activity *a*, *V*(s)* is the optimal state-value function shown in equation 31 and $B_{s \to s'|a}$ is the benefit gained from improving a pavement condition from state *s* to *s'* after performing repair activity *a*, as shown in equation 32.

$$V^*(s) = \begin{cases} 0 & \text{if } s \text{ is terminal,} \\ max_{a \in A} Q^*(a,s) & \text{otherwise} \end{cases} \qquad [31]$$

$$B_{s \to s'|a} = \gamma * RQI_{improvement}(s \to s'|a) \qquad [32]$$

where γ is the monetary convertor for RQI and $RQI_{improvement}(s \to s'|a)$ was derived from the HPMA data and shows the RQI improvement after performing repair activity *a.*

Figure 5.12 shows an illustration of the excel tool. The action-value is calculated for all repair actions in each state and the maximum values are selected.

State Condition	Action	Q(t=10)	Q(t=9)	Q(t=8)	Q(t=7)	Q(t=6)	Q(t=5)	Q(t=4)	Q(t=3)	Q(t=2)	Q(t=1)	Q(t=0)	Net Benefit (m)
1	1 - Do Nothing	0.0	2.8	12.2	51.1	202.6	722.9	2182.3	5514.5	11927.2	22711.8	39017.4	39017.4
	2 - Thin Mill and Overlay	-82749.7	-82746.8	-82737.5	-82698.6	-82547.1	-82026.8	-80567.4	-77235.2	-70822.5	-60037.9	-43732.3	
	3 - Thin Overlay	-65610.8	-65607.9	-65598.6	-65559.7	-65408.2	-64887.9	-63428.5	-60096.3	-53683.6	-42899.0	-26593.4	
	4 - Thick Mill and Overlay	-211479.0	-211476.1	-211466.8	-211427.9	-211276.4	-210756.1	-209296.7	-205964.5	-199551.8	-188767.2	-172461.6	
2	1 - Do Nothing	0.0	0.0	39.7	256.3	1094.6	3504.6	8888.5	18831.2	34772.7	57726.6	88091.5	88091.5
	2 - Thin Mill and Overlay	-68350.9	-68350.9	-68311.2	-68094.8	-67257.1	-64849.9	-59473.7	-49547.1	-33632.5	-10717.5	19597.8	
	3 - Thin Overlay	-33134.4	-33134.4	-33094.7	-32878.2	-32039.8	-29629.8	-24245.9	-14303.3	1638.3	24592.1	54957.1	
	4 - Thick Mill and Overlay	-210030.3	-210030.3	-209990.6	-209774.0	-208935.6	-206525.6	-201141.7	-191199.1	-175257.6	-152303.7	-121938.7	
3	1 - Do Nothing	0.0	63.5	311.9	1255.8	4125.5	10689.7	22781.2	41880.9	68839.4	103731.2	145858.1	145858.1
	2 - Thin Mill and Overlay	-67240.8	-67177.3	-66928.9	-65985.3	-63116.5	-56554.5	-44467.7	-25375.7	1572.4	36451.7	78565.0	
	3 - Thin Overlay	-2869.2	-2805.7	-2557.5	-1614.7	1251.3	7804.4	19871.2	38927.9	65823.0	100632.7	142663.9	
	4 - Thick Mill and Overlay	-200283.1	-200219.6	-199971.1	-199027.3	-196157.6	-189593.3	-177501.8	-158402.2	-131443.7	-96551.9	-54425.0	
4	1 - Do Nothing	0.0	77.2	589.7	2775.2	8611.3	20144.3	38991.1	66065.4	101435.1	144325.3	193294.4	193294.4
	2 - Thin Mill and Overlay	-57420.1	-57342.9	-56830.5	-54645.2	-48810.2	-37280.0	-18438.6	8627.4	43986.2	86863.4	135818.6	
	3 - Thin Overlay	-10765.3	-10688.1	-10175.8	-7992.1	-2163.2	9350.8	28160.9	55177.5	90468.8	133263.7	182127.5	
	4 - Thick Mill and Overlay	-190924.1	-190846.9	-190334.4	-188149.0	-182313.2	-170781.0	-151935.5	-124863.0	-89495.4	-46607.5	2359.4	

Figure 5.12: Illustration of the results displayed in the MPD Calculations sheet

5.4.3 Optimal Policies

The optimal policy was defined by selecting the repair activities that resulted on the highest value for the action-value function, *Q* (a,s)*. The third sheet in the excel tool shows a summary of the optimal policies for each state (figure 5.13) based on the inputted pavement characteristics, cost of repairs and RQI monetary conversion factor.

Condition State	RQI Range	Optimal Action to Take										
		Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
1	RQI: 5.0-4.2	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
2	RQI: 4.1-4.0	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
3	RQI: 3.9	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
4	RQI: 3.8	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
5	RQI: 3.7	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
6	RQI: 3.6	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
7	RQI: 3.5	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
8	RQI: 3.4	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing	1 - Do Nothing
9	RQI: 3.3	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
10	RQI: 3.2	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
11	RQI: 3.1	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
12	RQI: 3.0	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
13	RQI: 2.9-2.8	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
14	RQI: 2.7-2.6	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
15	RQI: 2.5-2.4	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
16	RQI: 2.3-2.2	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay
17	RQI: 2.1-2.0	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay	3 - Thin Overlay

Figure 5.13: Optimal policy for each state over the course of 10 years

The total net benefit is a suggested metric that quantifies the gain of a certain policy. It is calculated using the terminal action-value function, *Q* (a,s)*, as shown in equation 30. It takes into account the immediate cost of performing an activity and the resulting gain on the pavement condition and considers an initial uniform distribution of pavement condition across all 17 states. Figure 5.14 shows the gain obtained from choosing the optimal policy, in dollars per 12-foot lane-mile over 10 years.

Total net benefit (m) from optimal policy = $430,787.90 (dollars/12-foot lane-mile over 10 years)

* Values are subject to certain assumptions

Figure 5.14: Total net benefit from the optimal policy

5.4.4 Baseline Policy

The last sheet in the excel tool provides a platform for policy comparison. The net benefit can be calculated for any desired policy, which can be compared to the net benefit of the optimal policy. A summary of net benefits (figure 5.15) is displayed on the top of the page for easy comparison between policies. An alternative policy can be entered using the dropdown options (shown in figure 5.16). Please note that, for comparison purposes, only these final values in Figure 5.15 should be used. The values shown in Figure 5.16 represent partial calculations and, for a given state condition, comparing the value in the column for the alternative policy versus the value in the column for optimal policy is not meaningful.

Total net benefit (m) from optimal policy =	$430,787.90	(dollars/12-foot lane-mile over 10 years)
Total net benefit (m) from alternative policy =	$66,298.20	(dollars/12-foot lane-mile over 10 years)
Gain from optimal policy =	$364,489.71	(dollars/12-foot lane-mile over 10 years)

* Values are subject to certain assumptions

Figure 5.15: Comparison between the optimal policy and an alternative policy using the net benefit

Input Alternative Policy	Results		
	Net Benefit for Alternative Policy (dollars/12-foot lane-mile over 10 years)	Net Benefit for Optimal Policy (dollars/12-foot lane-mile over 10 years)	Gain from Optimal Policy (dollars/12-foot lane-mile over 10 years)
State 1 (RQI: 5.0-4.2) 1 - Do Nothing	-$30,223.45	$39,017.39	$69,240.84
State 2 (RQI: 4.1-4.0) 1 - Do Nothing	-$44,859.76	$88,091.53	$132,951.29
State 3 (RQI: 3.9) 1 - Do Nothing	-$47,112.72	$145,858.05	$192,970.78
State 4 (RQI: 3.8) 1 - Do Nothing	-$41,614.65	$193,294.41	$234,909.05
State 5 (RQI: 3.7) 1 - Do Nothing	-$32,134.47	$240,794.46	$272,928.93
State 6 (RQI: 3.6) 1 - Do Nothing	-$17,499.25	$296,663.68	$314,162.93
State 7 (RQI: 3.5) 1 - Do Nothing	$4,496.15	$376,254.12	$371,757.97
State 8 (RQI: 3.4) 1 - Do Nothing	$24,280.31	$428,403.96	$404,123.65
State 9 (RQI: 3.3) 2 - Thin Mill and Overlay	$22,078.42	$479,566.24	$457,487.82

Figure 5.16: Input of an alternative policy

It is important to note that the final values are subject to the input parameters entered in the input sheet. The RQI monetary conversion factor can be calibrated to better represent the value of improving RQI by one unit. Additional repair activities can also be added to the analyses as more data becomes available.

5.4.5 Step-by-step Quick Guide

In the first sheet, use the dropdown menus to select the desired district location, functional class and last activity performed, as shown in figure 5.17.

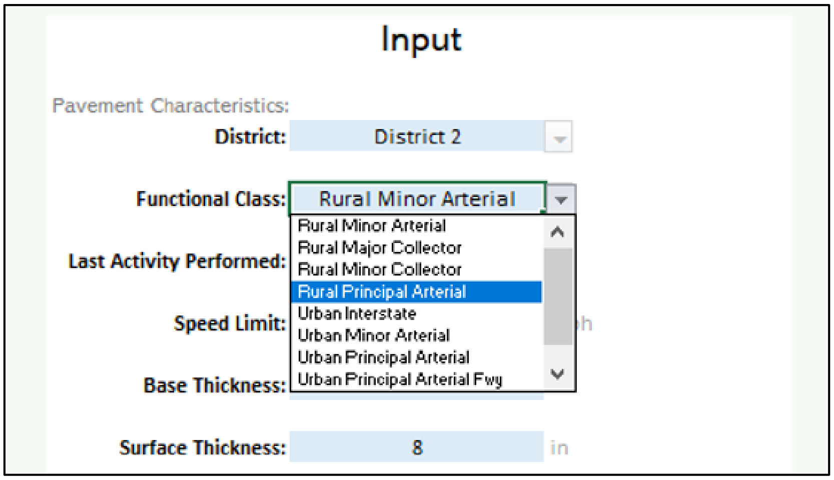

Figure 5.17: Dropdown menus for input parameters

Specify the speed limit, base thickness and surface thickness (keep in mind the model is designed for flexible pavements).

Enter a monetary conversion factor for RQI and the cost of each repair activity (cost can be adjusted to account for inflation and to reflect values for specific districts).

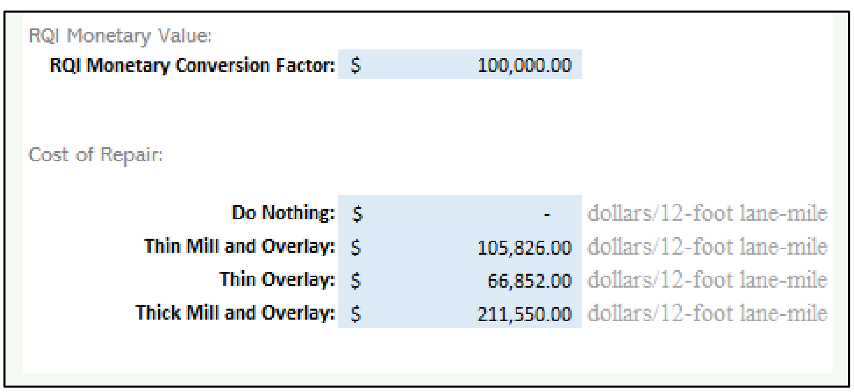

Figure 5.18: Area destined to cost of repair activities and RQI monetary conversion factor

Move to the 'Optimal Policies' sheet to see the optimal repair activity for each condition state over the course of 10 years.

To compare the optimal policy to an alternative policy, move to the last sheet and use the dropdown menus to enter the desired repair activity for each state, as shown in figure 5.19.

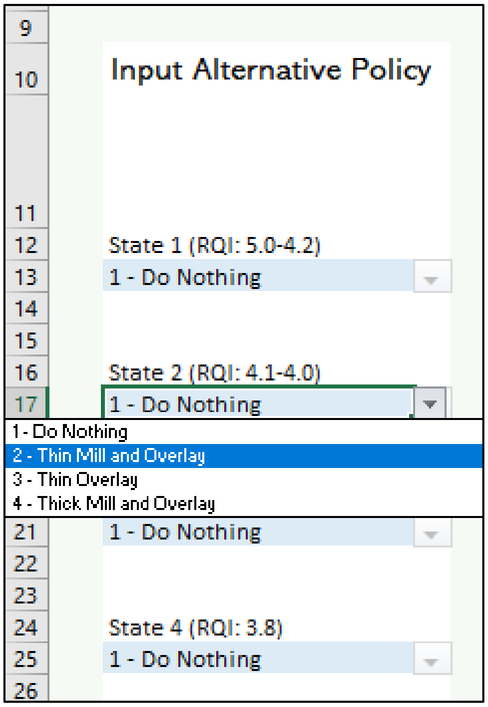

Figure 5.19: Dropdown menus where alternative policy can be entered

The final net benefit for all states can be find on the top of the page.

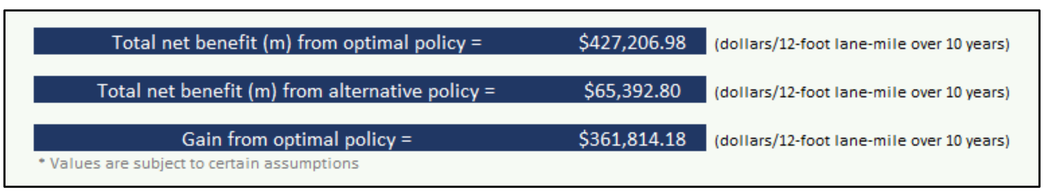

Figure 5.20: Final net benefit for optimal and alternative policies

As previously mentioned, the finite horizon MDP adopted in this project does not include a discount factor, neither for the MDP formulation, nor for the future value of money. The calculated benefit value from the tool is simply the cumulative benefits minus costs and should not be interpreted as a net present value.

CHAPTER 6: SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The project started with an overview of the data acquisition process used by MnDOT to gather pavement condition information. An overview of MnDOT's pavement management program, including the current prediction models and optimization methodology was also presented. Next, the available data were used to estimate three pavement condition parameters: the Percent Remaining Service Interval (PRSI), the Asset Sustainability Ratio, and the Deferred Preservation Liability. The parameters, used by Washington State Department of Transportation (WSDOT), were recommended in the first phase of this project. An overview of how each parameter was estimated and an example of how they can be used to demonstrate the effect of funding on pavement condition were detailed.

MnDOT's pavement condition data were then used to develop a Markov chain model to predict the deterioration of pavements in Minnesota. The Ride Quality Index (RQI) was selected as the condition parameter to formulate the Markov chain model, and a homogeneous group of pavement sections was created by selecting the best represented pavement type and clustering the pavement sections based on their repair history. The model characterized the pavement condition using a finite set of discrete states and considered that the pavement transitions among the states have a random component that depends only on the current state.

Some concerns were raised that external factors could possibly influence pavement deterioration. Therefore, several external factors were investigated and added to the dataset so they could be considered in the modeling process. Ordinal logistic regression models were developed, in which the site-specific features and external factors were used as model predictors and the pavement transition between the condition states was used as the dependent variable. After being tested by the Hosmer-Lemeshow test, the logistic regression models were used to develop the transition probabilities and fill in the Markov transition probability matrix. The Markov transition matrix enhanced with the ordinal logistic regression models was used to predict pavement performance and deterioration.

The addition of logistic regression made possible the forecasting of pavement behavior based on its site-specific features. Several examples were used to demonstrate the model's capability to forecast the behavior of specific pavement sections. Examples were also used to demonstrate the estimation of the total expected time until a specific pavement section was expected to require major repair intervention. Another advantage of combining ordinal logistic regression with Markov chain was the possibility to analyze how the patterns of pavement deterioration were affected by several external factors, such as district location, last activity performed, functional class, base thickness, speed limit and pavement thickness. Examples showed that this feature can be used to analyze the pavement behavior based on the district where it is located, allowing for district comparisons. It can also be used to investigate how different repair activities can affect long-term pavement performance.

The effects of different repair activities were incorporated into the Markov transition matrix and ordinal logistic regression models. The final Markov probability matrix was used in a dynamic programming optimization model to determine the optimal sequence of maintenance and repair policies that maximizes the net benefits of maintenance and repair actions. A user-friendly spreadsheet tool was developed to allow users to enter the characteristics of the payment sections and obtain the corresponding optimal policy. The tool allows the comparison of optimal policy with alternative policies in terms of net benefit. The finite horizon MDP adopted in this project does not include a discount factor. The calculated benefit value from the tool is simply the cumulative benefits minus costs and should not be interpreted as a net present value.

6.1 RECOMMENDATION

While the demonstrated procedure and results shown in Chapter 5 give a good estimation for the optimal policies, the results can get more accurate as input values are calibrated. One of the main input values to be calibrated is the RQI monetary factor. The RQI monetary factor is very significant, since it determines how effective it is to pay the cost of a given repair activity by increasing or decreasing its benefit value. The precision of the RQI monetary factor can be increased by answering the question: What is the most appropriate dollar amount approximation of the improvement achieved when RQI is raised by one unit?

Additional repair activities can be added to the Markov Decision Process as more data become available. Due to data limitations, only three repair activities were considered as possible actions in the MDP. While three repair activities were sufficient to demonstrate the model’s capability, a more accurate picture of the pavement network can be achieved by adding more repairs, including preventive maintenance activities.

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APPENDIX A – ORDINAL LOGISTIC REGRESSION COEFFICIENTS FOR STATES 4 TO 15

Table A1: Best Fitted Model for State 4

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT 2	-0.197	0.166	-1.184	0.236	
DISTRICT 3	0.285	0.218	1.310	0.190	
DISTRICT 4	0.038	0.172	0.217	0.828	
DISTRICT 5	-0.461	0.215	-2.148	0.032	*
DISTRICT 6	0.186	0.255	0.730	0.466	
DISTRICT 7	-0.066	0.245	-0.268	0.789	
DISTRICT 8	0.606	0.187	3.250	0.001	**
LAST.ACTIV Crack Repr/Med OL	0.742	0.686	1.082	0.279	
LAST.ACTIV Medium Overlay	0.359	0.257	1.396	0.163	
LAST.ACTIV Nova Chip (UTBWC)	1.248	0.539	2.315	0.021	*
LAST.ACTIV Thick Mill/Overlay	0.154	0.266	0.579	0.563	
LAST.ACTIV Thin Mill/Overlay	0.786	0.261	3.016	0.003	**
LAST.ACTIV Thin Overlay	0.523	0.242	2.166	0.030	*
0|1	-0.183	0.273	-0.671	0.502	
1|2	1.563	0.276	5.657	0.000	***
2|3	3.463	0.294	11.769	0.000	***
3|4	4.885	0.350	13.968	0.000	***
4|5	5.540	0.407	13.612	0.000	***
5|6	6.845	0.639	10.716	0.000	***
6|8	7.945	1.037	7.664	0.000	***

Signif. codes for p-values: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual Deviance: 3926.173

AIC: 3966.173

Table A2: Best Fitted Model for State 5

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT2	-0.362	0.183	-1.979	0.048	*
DISTRICT3	0.433	0.246	1.766	0.077	.
DISTRICT4	-0.211	0.193	-1.094	0.274	
DISTRICT5	-0.640	0.227	-2.825	0.005	**
DISTRICT6	-0.110	0.254	-0.431	0.667	
DISTRICT7	0.280	0.248	1.129	0.259	
DISTRICT8	0.147	0.203	0.722	0.470	
LAST.ACTIV Crack Repr/Med OL	-1.739	0.667	-2.606	0.009	**
LAST.ACTIV Medium Overlay	-0.304	0.301	-1.010	0.313	
LAST.ACTIV Nova Chip (UTBWC)	0.413	0.555	0.744	0.457	
LAST.ACTIV Thick Mill/Overlay	-0.422	0.319	-1.322	0.186	
LAST.ACTIV Thin Mill/Overlay	-0.008	0.298	-0.028	0.978	
LAST.ACTIV Thin Overlay	-0.037	0.284	-0.130	0.897	
SPEED.LIMIT	0.020	0.006	3.632	0.000	***
0|1	-0.035	0.395	-0.089	0.929	
1|2	2.192	0.399	5.490	0.000	***
2|3	3.805	0.413	9.213	0.000	***
3|4	4.927	0.446	11.059	0.000	***
4|5	6.510	0.597	10.895	0.000	***
5|6	8.122	1.076	7.551	0.000	***

Signif. codes for p-values: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual Deviance: 3415.607

AIC: 3455.607

Table A3: Best Fitted Model for State 6

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT2	-0.708	0.159	-4.457	0.000	***
DISTRICT3	0.063	0.212	0.298	0.765	
DISTRICT4	-0.572	0.165	-3.474	0.001	**
DISTRICT5	-0.627	0.203	-3.087	0.002	**
DISTRICT6	-0.236	0.218	-1.084	0.278	
DISTRICT7	-0.125	0.210	-0.595	0.552	
DISTRICT8	-0.086	0.183	-0.472	0.637	
LAST.ACTIV Crack Repr/Med OL	-0.231	0.528	-0.439	0.661	
LAST.ACTIV Medium Overlay	-0.609	0.268	-2.273	0.023	*
LAST.ACTIV Nova Chip (UTBWC)	0.161	0.488	0.329	0.742	
LAST.ACTIV Thick Mill/Overlay	-0.800	0.286	-2.792	0.005	**
LAST.ACTIV Thin Mill/Overlay	-0.406	0.266	-1.526	0.127	
LAST.ACTIV Thin Overlay	-0.452	0.255	-1.776	0.076	.
0|1	-1.369	0.287	-4.775	0.000	***
1|2	0.363	0.285	1.275	0.202	
2|3	1.907	0.294	6.492	0.000	***
3|4	3.243	0.331	9.799	0.000	***
4|5	4.291	0.410	10.465	0.000	***
5|6	4.611	0.450	10.254	0.000	***
6|7	5.595	0.641	8.731	0.000	***
7|8	6.694	1.038	6.450	0.000	***

Signif. codes for p-values: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual Deviance: 4583.43

AIC: 4625.43

Table A4: Best Fitted Model for State 7

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT2	-0.602	0.183	-3.289	0.001	**
DISTRICT3	0.075	0.237	0.318	0.750	
DISTRICT4	-0.659	0.188	-3.498	0.000	***
DISTRICT5	-1.029	0.225	-4.580	0.000	***
DISTRICT6	0.078	0.256	0.307	0.759	
DISTRICT7	-0.371	0.218	-1.703	0.089	.
DISTRICT8	0.036	0.207	0.174	0.862	
LAST.ACTIV Crack Repr/Med OL	0.801	0.673	1.191	0.234	
LAST.ACTIV Medium Overlay	-0.816	0.325	-2.514	0.012	*
LAST.ACTIV Nova Chip (UTBWC)	-0.447	0.588	-0.760	0.447	
LAST.ACTIV Thick Mill/Overlay	-1.182	0.347	-3.406	0.001	**
LAST.ACTIV Thin Mill/Overlay	-0.340	0.324	-1.049	0.294	
LAST.ACTIV Thin Overlay	-0.679	0.314	-2.164	0.030	*
BASE.THK	0.011	0.007	1.628	0.104	
0|1	-1.837	0.368	-4.994	0.000	***
1|2	-0.076	0.364	-0.208	0.835	
2|3	1.869	0.373	5.006	0.000	***
3|4	3.354	0.417	8.047	0.000	***

Signif. codes for p-values: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual Deviance: 3636.284

AIC: 3672.284

Even though Base Thickness was not statistically significant, as shown in table A4, the Hosmer-Lemeshow test revealed its presence improved the model fit.

Table A5: Best Fitted Model for State 8

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT2	-0.556	0.191	-2.914	0.004	**
DISTRICT3	0.044	0.271	0.164	0.869	
DISTRICT4	-0.759	0.192	-3.949	0.000	***
DISTRICT5	-0.533	0.309	-1.723	0.085	.
DISTRICT6	0.231	0.266	0.867	0.386	
DISTRICT7	-0.359	0.218	-1.645	0.100	
DISTRICT8	-0.260	0.220	-1.184	0.236	
LAST.ACTIV Crack Repr/Med OL	-1.026	0.676	-1.518	0.129	
LAST.ACTIV Medium Overlay	-0.975	0.343	-2.838	0.005	**
LAST.ACTIV Nova Chip (UTBWC)	-0.690	0.669	-1.032	0.302	
LAST.ACTIV Thick Mill/Overlay	-0.967	0.360	-2.688	0.007	**
LAST.ACTIV Thin Mill/Overlay	-0.742	0.340	-2.181	0.029	*
LAST.ACTIV Thin Overlay	-1.028	0.334	-3.083	0.002	**
FUNCTIONAL.CLASS RURAL MAJOR COLL	-2.051	1.137	-1.805	0.071	.
FUNCTIONAL.CLASS RURAL MINOR ART	-2.031	1.128	-1.801	0.072	.
FUNCTIONAL.CLASS RURAL MINOR COLL	-1.760	1.269	-1.387	0.165	
FUNCTIONAL.CLASS RURAL PRIN ART	-2.263	1.131	-2.002	0.045	*
FUNCTIONAL.CLASS URBAN INTERSTATE	-2.048	1.258	-1.628	0.104	
FUNCTIONAL.CLASS URBAN MINOR ART	-3.329	1.252	-2.659	0.008	**
FUNCTIONAL.CLASS URBAN PRIN ART	-2.521	1.167	-2.160	0.031	*
FUNCTIONAL.CLASS URBAN PRIN ART FRWY	-2.664	1.213	-2.196	0.028	*
0|1	-4.375	1.170	-3.740	0.000	***
1|2	-2.469	1.167	-2.116	0.034	*
2|3	-0.939	1.163	-0.807	0.420	
3|4	0.404	1.170	0.345	0.730	
4|5	1.745	1.208	1.444	0.149	

Signif. codes for p-values: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual Deviance: 3309.53

AIC: 3361.53

The best model for the data in state 8 includes the sections' functional class as one of the predictors. Since it is a categorical variable, one of the categories was taken as a reference. In this case, Rural Interstate was taken as the reference, thus its coefficient will be taken as 1 in the model equations.

Table A6: Best Fitted Model for State 9

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT2	-0.444	0.189	-2.352	0.019	*
DISTRICT3	-0.056	0.258	-0.218	0.827	
DISTRICT4	-0.544	0.181	-3.011	0.003	**
DISTRICT5	-0.654	0.234	-2.796	0.005	**
DISTRICT6	0.116	0.252	0.460	0.645	
DISTRICT7	-0.194	0.200	-0.970	0.332	
DISTRICT8	-0.160	0.210	-0.762	0.446	
LAST.ACTIV Crack Repr/Med OL	-0.739	0.608	-1.214	0.225	
LAST.ACTIV Medium Overlay	-0.322	0.362	-0.889	0.374	
LAST.ACTIV Nova Chip (UTBWC)	-0.783	1.346	-0.582	0.561	
LAST.ACTIV Thick Mill/Overlay	-0.159	0.397	-0.402	0.688	
LAST.ACTIV Thin Mill/Overlay	0.197	0.360	0.549	0.583	
LAST.ACTIV Thin Overlay	0.033	0.346	0.094	0.925	
0|1	-1.184	0.375	-3.161	0.002	**
1|2	0.497	0.373	1.334	0.182	
2|3	2.156	0.382	5.640	0.000	***
3|4	3.571	0.418	8.542	0.000	***

Signif. codes for p-values: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual Deviance: 3281.093

AIC: 3315.093

Table A6 shows the model coefficients for the data in state 9. Even though no last activity was statistically significant, the Hosmer-Lemeshow test revealed that keeping it as a predictor improves the fit considerably.

Table A7: Best Fitted Model for State 10

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT2	-0.529	0.210	-2.520	0.012	*
DISTRICT3	-0.110	0.276	-0.397	0.692	
DISTRICT4	-0.357	0.201	-1.775	0.076	.
DISTRICT5	-1.002	0.282	-3.553	0.000	***
DISTRICT6	0.242	0.259	0.934	0.350	
DISTRICT7	-0.193	0.206	-0.934	0.350	
DISTRICT8	-0.144	0.250	-0.575	0.565	
LAST.ACTIV Crack Repr/Med OL	-0.738	0.594	-1.241	0.215	
LAST.ACTIV Medium Overlay	-0.895	0.328	-2.731	0.006	**
LAST.ACTIV Nova Chip (UTBWC)	-1.310	0.841	-1.558	0.119	
LAST.ACTIV Thick Mill/Overlay	-0.902	0.390	-2.311	0.021	*
LAST.ACTIV Thin Mill/Overlay	-0.330	0.330	-1.003	0.316	
LAST.ACTIV Thin Overlay	-0.751	0.310	-2.423	0.015	*
0|1	-2.077	0.349	-5.958	0.000	***
1|2	-0.203	0.341	-0.595	0.552	
2|3	1.330	0.348	3.820	0.000	***
3|4	3.826	0.472	8.109	0.000	***
4|5	4.932	0.667	7.395	0.000	***

Signif. codes for p-values: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual Deviance: 2612.32

AIC: 2648.32

Table A8: Best Fitted Model for State 11

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT2	-0.527	0.223	-2.366	0.018	*
DISTRICT3	-0.142	0.276	-0.514	0.607	
DISTRICT4	-0.626	0.206	-3.033	0.002	**
DISTRICT5	-0.873	0.284	-3.075	0.002	**
DISTRICT6	-0.095	0.290	-0.329	0.742	
DISTRICT7	-0.532	0.219	-2.428	0.015	*
DISTRICT8	-0.233	0.283	-0.824	0.410	
LAST.ACTIV Crack Repr/Med OL	-0.919	0.729	-1.260	0.208	
LAST.ACTIV Medium Overlay	-0.829	0.378	-2.190	0.028	*
LAST.ACTIV Thick Mill/Overlay	-0.275	0.437	-0.629	0.529	
LAST.ACTIV Thin Mill/Overlay	-0.325	0.373	-0.870	0.384	
LAST.ACTIV Thin Overlay	-0.705	0.354	-1.993	0.046	*
0|1	-2.182	0.388	-5.622	0.000	***
1|2	-0.285	0.380	-0.749	0.454	
2|3	2.163	0.404	5.354	0.000	***
3|4	3.867	0.528	7.318	0.000	***
4|5	5.126	0.798	6.426	0.000	***

Signif. codes for p-values: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual Deviance: 2090.805

AIC: 2124.805

Table A9: Best Fitted Model for State 12

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT2	0.023	0.269	0.084	0.933	
DISTRICT3	0.357	0.350	1.020	0.308	
DISTRICT4	-0.019	0.254	-0.074	0.941	
DISTRICT5	-0.882	0.329	-2.684	0.007	**
DISTRICT6	0.112	0.296	0.378	0.706	
DISTRICT7	0.274	0.243	1.126	0.260	
DISTRICT8	0.030	0.349	0.085	0.932	
LAST.ACTIV Crack Repr/Med OL	-0.964	0.753	-1.280	0.200	
LAST.ACTIV Medium Overlay	-0.530	0.520	-1.020	0.308	
LAST.ACTIV Thick Mill/Overlay	-0.508	0.612	-0.830	0.407	
LAST.ACTIV Thin Mill/Overlay	0.068	0.517	0.132	0.895	
LAST.ACTIV Thin Overlay	-0.247	0.497	-0.497	0.619	
0|1	-1.449	0.523	-2.773	0.006	**
1|2	1.505	0.523	2.878	0.004	**
2|3	3.630	0.576	6.306	0.000	***
3|4	4.968	0.717	6.925	0.000	***

Signif. codes for p-values: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual Deviance: 1399.357

AIC: 1431.357

Similar to the model for some other states, the last activity performed was not statistically significant, but its presence as a predictor was proven to improve the fit.

Table A10: Best Fitted Model for State 13

Predictor	Coefficients	Std. Error	t-value	p-value	
DISTRICT2	-0.068	0.260	-0.263	0.793	
DISTRICT3	-0.283	0.260	-1.087	0.277	
DISTRICT4	-0.197	0.245	-0.803	0.422	
DISTRICT5	-0.762	0.312	-2.438	0.015	*
DISTRICT6	0.350	0.261	1.343	0.179	
DISTRICT7	0.306	0.204	1.501	0.133	
DISTRICT8	-0.126	0.375	-0.336	0.737	
LAST.ACTIV Crack Repr/Med OL	-0.313	0.871	-0.359	0.719	
LAST.ACTIV Medium Overlay	-1.451	0.689	-2.107	0.035	*
LAST.ACTIV Thick Mill/Overlay	-1.009	0.757	-1.333	0.183	
LAST.ACTIV Thin Mill/Overlay	-1.045	0.689	-1.518	0.129	
LAST.ACTIV Thin Overlay	-1.245	0.679	-1.834	0.067	.
FLEXIBLE.THK	-0.070	0.029	-2.423	0.015	*
0|1	-2.189	0.724	-3.023	0.003	**
1|2	0.272	0.719	0.378	0.706	
2|3	2.622	0.770	3.403	0.001	**
3|4	5.036	1.226	4.108	0.000	***

Signif. codes for p-values: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual Deviance: 1634.442

AIC: 1668.442

Table A11: Best Fitted Model for State 14

Predictor	Coefficients	Std. Error	t-value	-value	
DISTRICT2	0.090	0.439	0.204	0.838	
DISTRICT3	0.025	0.364	0.068	0.946	
DISTRICT4	-0.480	0.340	-1.414	0.157	
DISTRICT5	-0.838	0.441	-1.899	0.058	.
DISTRICT6	0.424	0.370	1.146	0.252	
DISTRICT7	-0.220	0.231	-0.951	0.342	
DISTRICT8	-0.343	0.487	-0.705	0.481	
LAST.ACTIV Thick Mill/Overlay	0.255	0.526	0.485	0.628	
LAST.ACTIV Thin Mill/Overlay	0.609	0.264	2.307	0.021	*
LAST.ACTIV Thin Overlay	0.174	0.219	0.795	0.427	
0|1	-0.670	0.208	-3.218	0.001	**
1|2	2.018	0.232	8.687	0.000	***

Signif. codes for p-values: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual Deviance: 898.6207

AIC: 922.6207

The reference for the last activity performed in the model shown in table A11 is Medium Overlay. Thus, its coefficient was taken as 1 in the model equations.

Table A12: Best Fitted Model for State 15

Predictor	Coefficients	Std. Error	t-value	p-value	
FUNCTIONAL.CLASS RURAL MINOR ART	-0.820	0.276	-2.974	0.003	**
FUNCTIONAL.CLASS RURAL MINOR COLL	-0.294	0.701	-0.420	0.675	
FUNCTIONAL.CLASS RURAL PRIN ART	-0.166	0.438	-0.379	0.705	
FUNCTIONAL.CLASS URBAN MINOR ART	-1.391	0.453	-3.071	0.002	**
FUNCTIONAL.CLASS URBAN PRIN ART	-0.576	0.673	-0.855	0.392	
0|1	-1.186	0.251	-4.716	0.000	***
1|2	1.273	0.253	5.026	0.000	***

Signif. codes for p-values: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual Deviance: 670.4195

AIC: 684.4195

The reference category for functional class for the model shown in table A12 is Rural Major Collector. Thus, its coefficient was taken as 1 in the model equations.

APPENDIX B – ORDINAL LOGIT MODEL EQUATIONS USED TO GET THE PROBABILITIES FOR THE MARKOV PROBABILITY MATRIX

P_{11} = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit\left(P(Y \leq 0)\right)\right))}$$

$$\begin{aligned} logit\left(P(Y \leq 0)\right) &= -3.17 - (1)D1 - (-2.993)D2 - (-1.555)D3 - (-3.184)D4 - (0.079)D5 \\ &- (-2.678)D6 - (0.284)D7 - (-2.814)D8 - (1)LA.CIR\&MO - LA.MO(1.107) \\ &- LA.NC(-3.298) - LA.ThkMO(-2.855) - LA.ThnMO(-0.455) \\ &- LA.ThnO(0.744) \end{aligned}$$

P_{12} = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit\left(P(Y \leq 1)\right)\right))}$$

$$\begin{aligned} logit\left(P(Y \leq 1)\right) &= 0.474 - (1)D1 - (-2.993)D2 - (-1.555)D3 - (-3.184)D4 - (0.079)D5 \\ &- (-2.678)D6 - (0.284)D7 - (-2.814)D8 - (1)LA.CIR\&MO - LA.MO(1.107) \\ &- LA.NC(-3.298) - LA.ThkMO(-2.855) - LA.ThnMO(-0.455) \\ &- LA.ThnO(0.744) \end{aligned}$$

P_{13} = P(Y=2) = P(Y=<2) - P(Y=<1)

$$P(Y \leq 2) = \frac{\exp(logit(P(Y \leq 2))}{(1 + \exp\left(logit\left(P(Y \leq 2)\right)\right))}$$

$$\begin{aligned} logit\left(P(Y \leq 2)\right) &= 2.073 - (1)D1 - (-2.993)D2 - (-1.555)D3 - (-3.184)D4 - (0.079)D5 \\ &- (-2.678)D6 - (0.284)D7 - (-2.814)D8 - (1)LA.CIR\&MO - LA.MO(1.107) \\ &- LA.NC(-3.298) - LA.ThkMO(-2.855) - LA.ThnMO(-0.455) \\ &- LA.ThnO(0.744) \end{aligned}$$

P_{14} = P(Y=3) = P(Y=<3) - P(Y=<2)

$$P(Y \leq 3) = \frac{\exp(logit(P(Y \leq 3))}{(1 + \exp\left(logit\left(P(Y \leq 3)\right)\right))}$$

$$logit(P(Y \leq 3)) = 3.302 - (1)D1 - (-2.993)D2 - (-1.555)D3 - (-3.184)D4 - (0.079)D5 - (-2.678)D6 - (0.284)D7 - (-2.814)D8 - (1)LA.CIR\&MO - LA.MO(1.107) - LA.NC(-3.298) - LA.ThkMO(-2.855) - LA.ThnMO(-0.455) - LA.ThnO(0.744)$$

P_{15} = P(Y=4) = P(Y=<4) - P(Y=<3)

$$P(Y \leq 4) = \frac{\exp(logit(P(Y \leq 4))}{(1 + \exp\left(logit(P(Y \leq 4))\right))}$$

$$logit(P(Y \leq 4)) = 3.720 - (1)D1 - (-2.993)D2 - (-1.555)D3 - (-3.184)D4 - (0.079)D5 - (-2.678)D6 - (0.284)D7 - (-2.814)D8 - (1)LA.CIR\&MO - LA.MO(1.107) - LA.NC(-3.298) - LA.ThkMO(-2.855) - LA.ThnMO(-0.455) - LA.ThnO(0.744)$$

P_{16} = P(Y=5) = P(Y=<8) - P(Y=<4)

$$P(Y \leq 5) = \frac{\exp(logit(P(Y \leq 5))}{(1 + \exp\left(logit(P(Y \leq 5))\right))}$$

$$logit(P(Y \leq 5)) = 4.414 - (1)D1 - (-2.993)D2 - (-1.555)D3 - (-3.184)D4 - (0.079)D5 - (-2.678)D6 - (0.284)D7 - (-2.814)D8 - (1)LA.CIR\&MO - LA.MO(1.107) - LA.NC(-3.298) - LA.ThkMO(-2.855) - LA.ThnMO(-0.455) - LA.ThnO(0.744)$$

P_{19} = P(Y=8) = 1 - P(Y=<5)

P_{22} = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit(P(Y \leq 0))\right))}$$

$$logit(P(Y \le 0)) = 0.643 - (1)D1 - (-0.822)D2 - (0.316)D3 - (-0.075)D4 - (-0.266)D5 - (-0.643)D6 - (1.843)D7 - (-0.138)D8 - (1)LA.CIR\&MO - (0.524)LA.MO - (0.737)LA.NC - (-0.188)LA.ThkMO - (1.035)LA.ThnMO - (1.093)LA.ThnO - (1)FC.RI - (0.785)FC.RMjC - (0.404)FC.RMnA - (0.743)FC.RPA - (0.258)FC.UI - (1.183)FC.UMnA - (0.313)FC.UPA - (-0.132)FC.UPAF - 0.018(BT)$$

P_{23} = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \le 1) = \frac{\exp(logit(P(Y \le 1))}{(1 + \exp\left(logit(P(Y \le 1))\right))}$$

$$logit(P(Y \le 1)) = 2.861 - (1)D1 - (-0.822)D2 - (0.316)D3 - (-0.075)D4 - (-0.266)D5 - (-0.643)D6 - (1.843)D7 - (-0.138)D8 - (1)LA.CIR\&MO - (0.524)LA.MO - (0.737)LA.NC - (-0.188)LA.ThkMO - (1.035)LA.ThnMO - (1.093)LA.ThnO - (1)FC.RI - (0.785)FC.RMjC - (0.404)FC.RMnA - (0.743)FC.RPA - (0.258)FC.UI - (1.183)FC.UMnA - (0.313)FC.UPA - (-0.132)FC.UPAF - 0.018(BT)$$

P_{24} = P(Y=2) = P(Y=<2) - P(Y=<1)

$$P(Y \le 2) = \frac{\exp(logit(P(Y \le 2))}{(1 + \exp\left(logit(P(Y \le 2))\right))}$$

$$logit(P(Y \le 2)) = 4.319 - (1)D1 - (-0.822)D2 - (0.316)D3 - (-0.075)D4 - (-0.266)D5 - (-0.643)D6 - (1.843)D7 - (-0.138)D8 - (1)LA.CIR\&MO - (0.524)LA.MO - (0.737)LA.NC - (-0.188)LA.ThkMO - (1.035)LA.ThnMO - (1.093)LA.ThnO - (1)FC.RI - (0.785)FC.RMjC - (0.404)FC.RMnA - (0.743)FC.RPA - (0.258)FC.UI - (1.183)FC.UMnA - (0.313)FC.UPA - (-0.132)FC.UPAF - 0.018(BT)$$

P_{25} = P(Y=3) = P(Y=<3) - P(Y=<2)

$$P(Y \leq 3) = \frac{\exp(logit(P(Y \leq 3))}{(1 + \exp\left(logit\left(P(Y \leq 3)\right)\right))}$$

$$\begin{aligned} logit\left(P(Y \leq 3)\right) \\ &= 5.641 - (1)D1 - (-0.822)D2 - (0.316)D3 - (-0.075)D4 - (-0.266)D5 \\ &- (-0.643)D6 - (1.843)D7 - (-0.138)D8 - (1)LA.CIR\&MO - (0.524)LA.MO \\ &- (0.737)LA.NC - (-0.188)LA.ThkMO - (1.035)LA.ThnMO - (1.093)LA.ThnO \\ &- (1)FC.RI - (0.785)FC.RMjC - (0.404)FC.RMnA - (0.743)FC.RPA \\ &- (0.258)FC.UI - (1.183)FC.UMnA - (0.313)FC.UPA - (-0.132)FC.UPAF \\ &- 0.018(BT) \end{aligned}$$

P_{26} = P(Y=4) = 1 - P(Y=<3)

P_{33} = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit\left(P(Y \leq 0)\right)\right))}$$

$$\begin{aligned} logit\left(P(Y \leq 0)\right) \\ &= -0.045 - (1)D1 - (-0.338)D2 - (-0.113)D3 - (0.269)D4 - (-0.469)D5 \\ &- (-0.096)D6 - (1.432)D7 - (0.295)D8 - (1)LA.CIR\&MO - (0.218)LA.MO \\ &- (1.524)LA.NC - (-0.059)LA.ThkMO - (1.007)LA.ThnMO - (0.624)LA.ThnO \end{aligned}$$

P_{34} = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit\left(P(Y \leq 1)\right)\right))}$$

$$\begin{aligned} logit\left(P(Y \leq 1)\right) \\ &= 1.854 - (1)D1 - (-0.338)D2 - (-0.113)D3 - (0.269)D4 - (-0.469)D5 \\ &- (-0.096)D6 - (1.432)D7 - (0.295)D8 - (1)LA.CIR\&MO - (0.218)LA.MO \\ &- (1.524)LA.NC - (-0.059)LA.ThkMO - (1.007)LA.ThnMO - (0.624)LA.ThnO \end{aligned}$$

P_{35} = P(Y=2) = P(Y=<2) - P(Y=<1)

$$P(Y \leq 2) = \frac{\exp(logit(P(Y \leq 2))}{(1 + \exp\left(logit\big(P(Y \leq 2)\big)\right))}$$

$$logit\big(P(Y \leq 2)\big)$$
$$= 3.423 - (1)D1 - (-0.338)D2 - (-0.113)D3 - (0.269)D4 - (-0.469)D5 - (-0.096)D6 - (1.432)D7 - (0.295)D8 - (1)LA.CIR\&MO - (0.218)LA.MO - (1.524)LA.NC - (-0.059)LA.ThkMO - (1.007)LA.ThnMO - (0.624)LA.ThnO$$

P$_{36}$ = P(Y=3) = P(Y=<3) - P(Y=<2)

$$P(Y \leq 3) = \frac{\exp(logit(P(Y \leq 3))}{(1 + \exp\left(logit\big(P(Y \leq 3)\big)\right))}$$

$$logit\big(P(Y \leq 3)\big)$$
$$= 5.039 - (1)D1 - (-0.338)D2 - (-0.113)D3 - (0.269)D4 - (-0.469)D5 - (-0.096)D6 - (1.432)D7 - (0.295)D8 - (1)LA.CIR\&MO - (0.218)LA.MO - (1.524)LA.NC - (-0.059)LA.ThkMO - (1.007)LA.ThnMO - (0.624)LA.ThnO$$

P$_{37}$ = P(Y=4) = P(Y=<4) - P(Y=<3)

$$P(Y \leq 4) = \frac{\exp(logit(P(Y \leq 4))}{(1 + \exp\left(logit\big(P(Y \leq 4)\big)\right))}$$

$$logit\big(P(Y \leq 4)\big)$$
$$= 5.633 - (1)D1 - (-0.338)D2 - (-0.113)D3 - (0.269)D4 - (-0.469)D5 - (-0.096)D6 - (1.432)D7 - (0.295)D8 - (1)LA.CIR\&MO - (0.218)LA.MO - (1.524)LA.NC - (-0.059)LA.ThkMO - (1.007)LA.ThnMO - (0.624)LA.ThnO$$

P$_{38}$ = P(Y=5) = P(Y=<5) - P(Y=<4)

$$P(Y \leq 5) = \frac{\exp(logit(P(Y \leq 5))}{(1 + \exp\left(logit\big(P(Y \leq 5)\big)\right))}$$

$$logit\big(P(Y \leq 5)\big)$$
$$= 6.147 - (1)D1 - (-0.338)D2 - (-0.113)D3 - (0.269)D4 - (-0.469)D5 - (-0.096)D6 - (1.432)D7 - (0.295)D8 - (1)LA.CIR\&MO - (0.218)LA.MO - (1.524)LA.NC - (-0.059)LA.ThkMO - (1.007)LA.ThnMO - (0.624)LA.ThnO$$

P$_{39}$ = P(Y=6) = P(Y=<6) - P(Y=<5)

$$P(Y \leq 6) = \frac{\exp(logit(P(Y \leq 6))}{(1 + \exp\left(logit\left(P(Y \leq 6)\right)\right))}$$

$$logit\left(P(Y \leq 6)\right) = 6.841 - (1)D1 - (-0.338)D2 - (-0.113)D3 - (0.269)D4 - (-0.469)D5 - (-0.096)D6 - (1.432)D7 - (0.295)D8 - (1)LA.CIR\&MO - (0.218)LA.MO - (1.524)LA.NC - (-0.059)LA.ThkMO - (1.007)LA.ThnMO - (0.624)LA.ThnO$$

P3-10 = P(Y=7) = P(Y=<7) - P(Y=<6)

$$P(Y \leq 7) = \frac{\exp(logit(P(Y \leq 7))}{(1 + \exp\left(logit\left(P(Y \leq 7)\right)\right))}$$

$$logit\left(P(Y \leq 7)\right) = 7.247 - (1)D1 - (-0.338)D2 - (-0.113)D3 - (0.269)D4 - (-0.469)D5 - (-0.096)D6 - (1.432)D7 - (0.295)D8 - (1)LA.CIR\&MO - (0.218)LA.MO - (1.524)LA.NC - (-0.059)LA.ThkMO - (1.007)LA.ThnMO - (0.624)LA.ThnO$$

P3-11 = P(Y=8) = P(Y=<8) - P(Y=<7)

$$P(Y \leq 8) = \frac{\exp(logit(P(Y \leq 8))}{(1 + \exp\left(logit\left(P(Y \leq 8)\right)\right))}$$

$$logit\left(P(Y \leq 8)\right) = 7.941 - (1)D1 - (-0.338)D2 - (-0.113)D3 - (0.269)D4 - (-0.469)D5 - (-0.096)D6 - (1.432)D7 - (0.295)D8 - (1)LA.CIR\&MO - (0.218)LA.MO - (1.524)LA.NC - (-0.059)LA.ThkMO - (1.007)LA.ThnMO - (0.624)LA.ThnO$$

P3-12 = P(Y=9) = 1 - P(Y=<8)

P44 = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit\left(P(Y \leq 0)\right)\right))}$$

$$logit(P(Y \leq 0)) = -0.183 - (1)D1 - (-0.197)D2 - (0.285)D3 - (0.038)D4 - (-0.461)D5 - (0.186)D6 - (-0.066)D7 - (0.606)D8 - (1)LA.CIR\&MO - (0.742)LA.CRMO - (0.359)LA.MO - (1.248)LA.NC - (0.154)LA.ThkMO - (0.786)LA.ThnMO - (0.523)LA.ThnO$$

P_{45} = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit(P(Y \leq 1))\right))}$$

$$logit(P(Y \leq 1)) = 1.563 - (1)D1 - (-0.197)D2 - (0.285)D3 - (0.038)D4 - (-0.461)D5 - (0.186)D6 - (-0.066)D7 - (0.606)D8 - (1)LA.CIR\&MO - (0.742)LA.CRMO - (0.359)LA.MO - (1.248)LA.NC - (0.154)LA.ThkMO - (0.786)LA.ThnMO - (0.523)LA.ThnO$$

P_{46} = P(Y=2) = P(Y=<2) - P(Y=<1)

$$P(Y \leq 2) = \frac{\exp(logit(P(Y \leq 2))}{(1 + \exp\left(logit(P(Y \leq 2))\right))}$$

$$logit(P(Y \leq 2)) = 3.463 - (1)D1 - (-0.197)D2 - (0.285)D3 - (0.038)D4 - (-0.461)D5 - (0.186)D6 - (-0.066)D7 - (0.606)D8 - (1)LA.CIR\&MO - (0.742)LA.CRMO - (0.359)LA.MO - (1.248)LA.NC - (0.154)LA.ThkMO - (0.786)LA.ThnMO - (0.523)LA.ThnO$$

P_{47} = P(Y=3) = P(Y=<3) - P(Y=<2)

$$P(Y \leq 3) = \frac{\exp(logit(P(Y \leq 3))}{(1 + \exp\left(logit(P(Y \leq 3))\right))}$$

$$logit(P(Y \leq 3)) = 4.885 - (1)D1 - (-0.197)D2 - (0.285)D3 - (0.038)D4 - (-0.461)D5 - (0.186)D6 - (-0.066)D7 - (0.606)D8 - (1)LA.CIR\&MO - (0.742)LA.CRMO - (0.359)LA.MO - (1.248)LA.NC - (0.154)LA.ThkMO - (0.786)LA.ThnMO - (0.523)LA.ThnO$$

P_{48} = P(Y=4) = P(Y=<4) - P(Y=<3)

$$P(Y \leq 4) = \frac{\exp(logit(P(Y \leq 4))}{(1 + \exp\left(logit\big(P(Y \leq 4)\big)\right))}$$

$$logit\big(P(Y \leq 4)\big) = 5.540 - (1)D1 - (-0.197)D2 - (0.285)D3 - (0.038)D4 - (-0.461)D5 - (0.186)D6 - (-0.066)D7 - (0.606)D8 - (1)LA.CIR\&MO - (0.742)LA.CRMO - (0.359)LA.MO - (1.248)LA.NC - (0.154)LA.ThkMO - (0.786)LA.ThnMO - (0.523)LA.ThnO$$

P_{49} = P(Y=5) = P(Y=<5) - P(Y=<4)

$$P(Y \leq 5) = \frac{\exp(logit(P(Y \leq 5))}{(1 + \exp\left(logit\big(P(Y \leq 5)\big)\right))}$$

$$logit\big(P(Y \leq 5)\big) = 6.845 - (1)D1 - (-0.197)D2 - (0.285)D3 - (0.038)D4 - (-0.461)D5 - (0.186)D6 - (-0.066)D7 - (0.606)D8 - (1)LA.CIR\&MO - (0.742)LA.CRMO - (0.359)LA.MO - (1.248)LA.NC - (0.154)LA.ThkMO - (0.786)LA.ThnMO - (0.523)LA.ThnO$$

$P_{4\text{-}10}$ = P(Y=6) = P(Y=<6) - P(Y=<5)

$$P(Y \leq 6) = \frac{\exp(logit(P(Y \leq 6))}{(1 + \exp\left(logit\big(P(Y \leq 6)\big)\right))}$$

$$logit\big(P(Y \leq 6)\big) = 7.945 - (1)D1 - (-0.197)D2 - (0.285)D3 - (0.038)D4 - (-0.461)D5 - (0.186)D6 - (-0.066)D7 - (0.606)D8 - (1)LA.CIR\&MO - (0.742)LA.CRMO - (0.359)LA.MO - (1.248)LA.NC - (0.154)LA.ThkMO - (0.786)LA.ThnMO - (0.523)LA.ThnO$$

$P_{4\text{-}12}$ = P(Y=8) = 1 - P(Y=<6)

$\mathbf{P_{55}}$ = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit\left(P(Y \leq 0)\right)\right))}$$

$$logit\left(P(Y \leq 0)\right) = -0.035 - (1)D1 - (-0.362)D2 - (0.433)D3 - (-0.211)D4 - (-0.640)D5 - (-0.110)D6 - (0.280)D7 - (0.147)D8 - (-1.739)LA.CRMO - (-0.304)LA.MO - (0.413)LA.NC - (-0.422)LA.ThkMO - (-0.008)LA.ThnMO - (-0.037)LA.ThnO - (0.020)SL$$

$\mathbf{P_{56}}$ = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit\left(P(Y \leq 1)\right)\right))}$$

$$logit\left(P(Y \leq 1)\right) = 2.192 - (1)D1 - (-0.362)D2 - (0.433)D3 - (-0.211)D4 - (-0.640)D5 - (-0.110)D6 - (0.280)D7 - (0.147)D8 - (1)LA.CIR\&MO - (-1.739)LA.CRMO - (-0.304)LA.MO - (0.413)LA.NC - (-0.422)LA.ThkMO - (-0.008)LA.ThnMO - (-0.037)LA.ThnO - (0.020)SL$$

$\mathbf{P_{57}}$ = P(Y=2) = P(Y=<2) - P(Y=<1)

$$P(Y \leq 2) = \frac{\exp(logit(P(Y \leq 2))}{(1 + \exp\left(logit\left(P(Y \leq 2)\right)\right))}$$

$$logit\left(P(Y \leq 2)\right) = 3.805 - (1)D1 - (-0.362)D2 - (0.433)D3 - (-0.211)D4 - (-0.640)D5 - (-0.110)D6 - (0.280)D7 - (0.147)D8 - (1)LA.CIR\&MO - (-1.739)LA.CRMO - (-0.304)LA.MO - (0.413)LA.NC - (-0.422)LA.ThkMO - (-0.008)LA.ThnMO - (-0.037)LA.ThnO - (0.020)SL$$

P$_{58}$ = P(Y=3) = P(Y=<3) - P(Y=<2)

$$P(Y \leq 3) = \frac{\exp(logit(P(Y \leq 3))}{(1 + \exp\left(logit\left(P(Y \leq 3)\right)\right))}$$

$$logit\left(P(Y \leq 3)\right)$$
$$= 4.927 - (1)D1 - (-0.362)D2 - (0.433)D3 - (-0.211)D4 - (-0.640)D5 - (-0.110)D6 - (0.280)D7 - (0.147)D8 - (1)LA.CIR\&MO - (-1.739)LA.CRMO - (-0.304)LA.MO - (0.413)LA.NC - (-0.422)LA.ThkMO - (-0.008)LA.ThnMO - (-0.037)LA.ThnO - (0.020)SL$$

P$_{59}$ = P(Y=4) = P(Y=<4) - P(Y=<3)

$$P(Y \leq 4) = \frac{\exp(logit(P(Y \leq 4))}{(1 + \exp\left(logit\left(P(Y \leq 4)\right)\right))}$$

$$logit\left(P(Y \leq 4)\right)$$
$$= 6.510 - (1)D1 - (-0.362)D2 - (0.433)D3 - (-0.211)D4 - (-0.640)D5 - (-0.110)D6 - (0.280)D7 - (0.147)D8 - (1)LA.CIR\&MO - (-1.739)LA.CRMO - (-0.304)LA.MO - (0.413)LA.NC - (-0.422)LA.ThkMO - (-0.008)LA.ThnMO - (-0.037)LA.ThnO - (0.020)SL$$

P$_{5\text{-}10}$ = P(Y=5) = P(Y=<5) - P(Y=<4)

$$P(Y \leq 5) = \frac{\exp(logit(P(Y \leq 5))}{(1 + \exp\left(logit\left(P(Y \leq 5)\right)\right))}$$

$$logit\left(P(Y \leq 5)\right)$$
$$= 8.122 - (1)D1 - (-0.362)D2 - (0.433)D3 - (-0.211)D4 - (-0.640)D5 - (-0.110)D6 - (0.280)D7 - (0.147)D8 - (1)LA.CIR\&MO - (-1.739)LA.CRMO - (-0.304)LA.MO - (0.413)LA.NC - (-0.422)LA.ThkMO - (-0.008)LA.ThnMO - (-0.037)LA.ThnO - (0.020)SL$$

P$_{5\text{-}11}$ = P(Y=6) = 1 - P(Y=<5)

$\mathbf{P_{66}}$ = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit\left(P(Y \leq 0)\right)\right))}$$

$$\begin{aligned} logit\left(P(Y \leq 0)\right) \\ &= -1.369 - (1)D1 - (-0.708)D2 - (0.036)D3 - (-0.572)D4 - (-0.627)D5 \\ &- (-0.236)D6 - (-0.125)D7 - (-0.086)D8 - (-0.231)LA.CRMO \\ &- (-0.609)LA.MO - (0.161)LA.NC - (-0.800)LA.ThkMO - (-0.406)LA.ThnMO \\ &- (-0.452)LA.ThnO \end{aligned}$$

$\mathbf{P_{67}}$ = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit\left(P(Y \leq 1)\right)\right))}$$

$$\begin{aligned} logit\left(P(Y \leq 1)\right) \\ &= 0.363 - (1)D1 - (-0.708)D2 - (0.036)D3 - (-0.572)D4 - (-0.627)D5 \\ &- (-0.236)D6 - (-0.125)D7 - (-0.086)D8 - (1)LA.CIR\&MO \\ &- (-0.231)LA.CRMO - (-0.609)LA.MO - (0.161)LA.NC - (-0.800)LA.ThkMO \\ &- (-0.406)LA.ThnMO - (-0.452)LA.ThnO \end{aligned}$$

$\mathbf{P_{68}}$ = P(Y=2) = P(Y=<2) - P(Y=<1)

$$P(Y \leq 2) = \frac{\exp(logit(P(Y \leq 2))}{(1 + \exp\left(logit\left(P(Y \leq 2)\right)\right))}$$

$$\begin{aligned} logit\left(P(Y \leq 2)\right) \\ &= 1.907 - (1)D1 - (-0.708)D2 - (0.036)D3 - (-0.572)D4 - (-0.627)D5 \\ &- (-0.236)D6 - (-0.125)D7 - (-0.086)D8 - (1)LA.CIR\&MO \\ &- (-0.231)LA.CRMO - (-0.609)LA.MO - (0.161)LA.NC - (-0.800)LA.ThkMO \\ &- (-0.406)LA.ThnMO - (-0.452)LA.ThnO \end{aligned}$$

$\mathbf{P_{69}}$ = P(Y=3) = P(Y=<3) - P(Y=<2)

$$P(Y \leq 3) = \frac{\exp(logit(P(Y \leq 3))}{(1 + \exp\left(logit\left(P(Y \leq 3)\right)\right))}$$

$$\begin{aligned} logit\left(P(Y \leq 3)\right) \\ &= 3.243 - (1)D1 - (-0.708)D2 - (0.036)D3 - (-0.572)D4 - (-0.627)D5 \\ &- (-0.236)D6 - (-0.125)D7 - (-0.086)D8 - (1)LA.CIR\&MO \\ &- (-0.231)LA.CRMO - (-0.609)LA.MO - (0.161)LA.NC - (-0.800)LA.ThkMO \\ &- (-0.406)LA.ThnMO - (-0.452)LA.ThnO \end{aligned}$$

$\mathbf{P_{6\text{-}10}}$ = P(Y=4) = P(Y=<4) - P(Y=<3)

$$P(Y \leq 4) = \frac{\exp(logit(P(Y \leq 4))}{(1 + \exp\left(logit\left(P(Y \leq 4)\right)\right))}$$

$$\begin{aligned} logit\left(P(Y \leq 4)\right) \\ &= 4.291 - (1)D1 - (-0.708)D2 - (0.036)D3 - (-0.572)D4 - (-0.627)D5 \\ &- (-0.236)D6 - (-0.125)D7 - (-0.086)D8 - (1)LA.CIR\&MO \\ &- (-0.231)LA.CRMO - (-0.609)LA.MO - (0.161)LA.NC - (-0.800)LA.ThkMO \\ &- (-0.406)LA.ThnMO - (-0.452)LA.ThnO \end{aligned}$$

$\mathbf{P_{6\text{-}11}}$ = P(Y=5) = P(Y=<5) - P(Y=<4)

$$P(Y \leq 5) = \frac{\exp(logit(P(Y \leq 5))}{(1 + \exp\left(logit\left(P(Y \leq 5)\right)\right))}$$

$$\begin{aligned} logit\left(P(Y \leq 5)\right) \\ &= 4.611 - (1)D1 - (-0.708)D2 - (0.036)D3 - (-0.572)D4 - (-0.627)D5 \\ &- (-0.236)D6 - (-0.125)D7 - (-0.086)D8 - (1)LA.CIR\&MO \\ &- (-0.231)LA.CRMO - (-0.609)LA.MO - (0.161)LA.NC - (-0.800)LA.ThkMO \\ &- (-0.406)LA.ThnMO - (-0.452)LA.ThnO \end{aligned}$$

$\mathbf{P_{6\text{-}12}}$ = P(Y=6) = P(Y=<6) - P(Y=<5)

$$P(Y \leq 6) = \frac{\exp(logit(P(Y \leq 6))}{(1 + \exp\left(logit\left(P(Y \leq 6)\right)\right))}$$

$$logit(P(Y \leq 6))$$
$$= 5.595 - (1)D1 - (-0.708)D2 - (0.036)D3 - (-0.572)D4 - (-0.627)D5 - (-0.236)D6 - (-0.125)D7 - (-0.086)D8 - (1)LA.CIR\&MO - (-0.231)LA.CRMO - (-0.609)LA.MO - (0.161)LA.NC - (-0.800)LA.ThkMO - (-0.406)LA.ThnMO - (-0.452)LA.ThnO$$

$P_{6\text{-}13}$ = P(Y=7) = P(Y=<7) - P(Y=<6)

$$P(Y \leq 7) = \frac{\exp(logit(P(Y \leq 7))}{(1 + \exp\left(logit(P(Y \leq 7))\right))}$$

$$logit(P(Y \leq 7))$$
$$= 6.694 - (1)D1 - (-0.708)D2 - (0.036)D3 - (-0.572)D4 - (-0.627)D5 - (-0.236)D6 - (-0.125)D7 - (-0.086)D8 - (1)LA.CIR\&MO - (-0.231)LA.CRMO - (-0.609)LA.MO - (0.161)LA.NC - (-0.800)LA.ThkMO - (-0.406)LA.ThnMO - (-0.452)LA.ThnO$$

$P_{6\text{-}14}$= P(Y=8) = 1 - P(Y=<7)

P_{77} = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit(P(Y \leq 0))\right))}$$

$$logit(P(Y \leq 0))$$
$$= -1.837 - (1)D1 - (-0.602)D2 - (0.075)D3 - (-0.659)D4 - (-1.029)D5 - (0.078)D6 - (-0.371)D7 - (0.036)D8 - (1)LA.CIR\&MO - (0.801)LA.CRMO - (-0.816)LA.MO - (-0.447)LA.NC - (-1.182)LA.ThkMO - (-0.340)LA.ThnMO - (-0.679)LA.ThnO - (0.011)BT$$

P_{78} = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit(P(Y \leq 1))\right))}$$

$$logit(P(Y \leq 1)) = -0.076 - (1)D1 - (-0.602)D2 - (0.075)D3 - (-0.659)D4 - (-1.029)D5 - (0.078)D6 - (-0.371)D7 - (0.036)D8 - (1)LA.CIR\&MO - (0.801)LA.CRMO - (-0.816)LA.MO - (-0.447)LA.NC - (-1.182)LA.ThkMO - (-0.340)LA.ThnMO - (-0.679)LA.ThnO - (0.011)BT$$

P_{79} = P(Y=2) = P(Y=<2) - P(Y=<1)

$$P(Y \leq 2) = \frac{\exp(logit(P(Y \leq 2))}{(1 + \exp\left(logit(P(Y \leq 2))\right))}$$

$$logit(P(Y \leq 2)) = 1.869 - (1)D1 - (-0.602)D2 - (0.075)D3 - (-0.659)D4 - (-1.029)D5 - (0.078)D6 - (-0.371)D7 - (0.036)D8 - (1)LA.CIR\&MO - (0.801)LA.CRMO - (-0.816)LA.MO - (-0.447)LA.NC - (-1.182)LA.ThkMO - (-0.340)LA.ThnMO - (-0.679)LA.ThnO - (0.011)BT$$

$P_{7\text{-}10}$ = P(Y=3) = P(Y=<3) - P(Y=<2)

$$P(Y \leq 3) = \frac{\exp(logit(P(Y \leq 3))}{(1 + \exp\left(logit(P(Y \leq 3))\right))}$$

$$logit(P(Y \leq 3)) = 3.354 - (1)D1 - (-0.602)D2 - (0.075)D3 - (-0.659)D4 - (-1.029)D5 - (0.078)D6 - (-0.371)D7 - (0.036)D8 - (1)LA.CIR\&MO - (0.801)LA.CRMO - (-0.816)LA.MO - (-0.447)LA.NC - (-1.182)LA.ThkMO - (-0.340)LA.ThnMO - (-0.679)LA.ThnO - (0.011)BT$$

$P_{7\text{-}11}$ = P(Y=4) = 1- P(Y=<3)

P_{88} = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit(P(Y \leq 0))\right))}$$

$$logit(P(Y \leq 0)) = -4.375 - (1)D1 - (-0.556)D2 - (0.044)D3 - (-0.759)D4 - (-0.533)D5 - (0.231)D6 - (-0.359)D7 - (0.231)D8 - (1)LA.CIR\&MO - (-1.026)LA.CRMO - (-0.975)LA.MO - (-0.690)LA.NC - (-0.967)LA.ThkMO - (-0.742)LA.ThnMO - (-1.028)LA.ThnO - (-2.051)FC.RMjC - (-2.031)FC.RMnA - (-1.760)FC.RMnC - (-2.263)FC.RPA - (-2.048)FC.UI - (-3.329)FC.UMnA - (-2.521)FC.UPA - (-2.664)FC.UPAF$$

P_{89} = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit(P(Y \leq 1))\right))}$$

$$logit(P(Y \leq 1)) = -2.469 - (1)D1 - (-0.556)D2 - (0.044)D3 - (-0.759)D4 - (-0.533)D5 - (0.231)D6 - (-0.359)D7 - (0.231)D8 - (1)LA.CIR\&MO - (-1.026)LA.CRMO - (-0.975)LA.MO - (-0.690)LA.NC - (-0.967)LA.ThkMO - (-0.742)LA.ThnMO - (-1.028)LA.ThnO - (1)FC.RI - (-2.051)FC.RMjC - (-2.031)FC.RMnA - (-1.760)FC.RMnC - (-2.263)FC.RPA - (-2.048)FC.UI - (-3.329)FC.UMnA - (-2.521)FC.UPA - (-2.664)FC.UPAF$$

$P_{8\text{-}10}$ = P(Y=2) = P(Y=<2) - P(Y=<1)

$$P(Y \leq 2) = \frac{\exp(logit(P(Y \leq 2))}{(1 + \exp\left(logit(P(Y \leq 2))\right))}$$

$$logit(P(Y \leq 2)) = -0.939 - (1)D1 - (-0.556)D2 - (0.044)D3 - (-0.759)D4 - (-0.533)D5 - (0.231)D6 - (-0.359)D7 - (0.231)D8 - (1)LA.CIR\&MO - (-1.026)LA.CRMO - (-0.975)LA.MO - (-0.690)LA.NC - (-0.967)LA.ThkMO - (-0.742)LA.ThnMO - (-1.028)LA.ThnO - (1)FC.RI - (-2.051)FC.RMjC - (-2.031)FC.RMnA - (-1.760)FC.RMnC - (-2.263)FC.RPA - (-2.048)FC.UI - (-3.329)FC.UMnA - (-2.521)FC.UPA - (-2.664)FC.UPAF$$

$P_{8\text{-}11}$ = P(Y=3) = P(Y=<3) - P(Y=<2)

$$P(Y \leq 3) = \frac{\exp(logit(P(Y \leq 3))}{(1 + \exp\left(logit\big(P(Y \leq 3)\big)\right))}$$

$$\begin{aligned} logit\big(P(Y \leq 3)\big) \\ &= 0.404 - (1)D1 - (-0.556)D2 - (0.044)D3 - (-0.759)D4 - (-0.533)D5 \\ &- (0.231)D6 - (-0.359)D7 - (0.231)D8 - (1)LA.CIR\&MO - (-1.026)LA.CRMO \\ &- (-0.975)LA.MO - (-0.690)LA.NC - (-0.967)LA.ThkMO \\ &- (-0.742)LA.ThnMO - (-1.028)LA.ThnO - (1)FC.RI - (-2.051)FC.RMjC \\ &- (-2.031)FC.RMnA - (-1.760)FC.RMnC - (-2.263)FC.RPA - (-2.048)FC.UI \\ &- (-3.329)FC.UMnA - (-2.521)FC.UPA - (-2.664)FC.UPAF \end{aligned}$$

$P_{8\text{-}12}$ = P(Y=4) = P(Y=<4) - P(Y=<3)

$$P(Y \leq 4) = \frac{\exp(logit(P(Y \leq 4))}{(1 + \exp\left(logit\big(P(Y \leq 4)\big)\right))}$$

$$\begin{aligned} logit\big(P(Y \leq 4)\big) \\ &= 1.745 - (1)D1 - (-0.556)D2 - (0.044)D3 - (-0.759)D4 - (-0.533)D5 \\ &- (0.231)D6 - (-0.359)D7 - (0.231)D8 - (1)LA.CIR\&MO - (-1.026)LA.CRMO \\ &- (-0.975)LA.MO - (-0.690)LA.NC - (-0.967)LA.ThkMO \\ &- (-0.742)LA.ThnMO - (-1.028)LA.ThnO - (1)FC.RI - (-2.051)FC.RMjC \\ &- (-2.031)FC.RMnA - (-1.760)FC.RMnC - (-2.263)FC.RPA - (-2.048)FC.UI \\ &- (-3.329)FC.UMnA - (-2.521)FC.UPA - (-2.664)FC.UPAF \end{aligned}$$

$P_{8\text{-}13}$ = P(Y=5) = 1 - P(Y=<4)

P_{99} = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit\big(P(Y \leq 0)\big)\right))}$$

$$\begin{aligned} logit\big(P(Y \leq 0)\big) \\ &= -1.184 - (1)D1 - (-0.444)D2 - (-0.056)D3 - (-0.544)D4 - (-0.654)D5 \\ &- (0.116)D6 - (-0.194)D7 - (-0.160)D8 - (1)LA.CIR\&MO - (-0.739)LA.CRMO \\ &- (-0.322)LA.MO - (-0.783)LA.NC - (-0.159)LA.ThkMO - (0.197)LA.ThnMO \\ &- (0.033)LA.ThnO \end{aligned}$$

P$_{9\text{-}10}$ = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit\left(P(Y \leq 1)\right)\right))}$$

$$logit\left(P(Y \leq 1)\right) = 0.497 - (1)D1 - (-0.444)D2 - (-0.056)D3 - (-0.544)D4 - (-0.654)D5 - (0.116)D6 - (-0.194)D7 - (-0.160)D8 - (1)LA.CIR\&MO - (-0.739)LA.CRMO - (-0.322)LA.MO - (-0.783)LA.NC - (-0.159)LA.ThkMO - (0.197)LA.ThnMO - (0.033)LA.ThnO$$

P$_{9\text{-}11}$ = P(Y=2) = P(Y=<2) - P(Y=<1)

$$P(Y \leq 2) = \frac{\exp(logit(P(Y \leq 2))}{(1 + \exp\left(logit\left(P(Y \leq 2)\right)\right))}$$

$$logit\left(P(Y \leq 2)\right) = 2.156 - (1)D1 - (-0.444)D2 - (-0.056)D3 - (-0.544)D4 - (-0.654)D5 - (0.116)D6 - (-0.194)D7 - (-0.160)D8 - (1)LA.CIR\&MO - (-0.739)LA.CRMO - (-0.322)LA.MO - (-0.783)LA.NC - (-0.159)LA.ThkMO - (0.197)LA.ThnMO - (0.033)LA.ThnO$$

P$_{9\text{-}12}$ = P(Y=3) = P(Y=<3) - P(Y=<2)

$$P(Y \leq 3) = \frac{\exp(logit(P(Y \leq 3))}{(1 + \exp\left(logit\left(P(Y \leq 3)\right)\right))}$$

$$logit\left(P(Y \leq 3)\right) = 3.571 - (1)D1 - (-0.444)D2 - (-0.056)D3 - (-0.544)D4 - (-0.654)D5 - (0.116)D6 - (-0.194)D7 - (-0.160)D8 - (1)LA.CIR\&MO - (-0.739)LA.CRMO - (-0.322)LA.MO - (-0.783)LA.NC - (-0.159)LA.ThkMO - (0.197)LA.ThnMO - (0.033)LA.ThnO$$

P$_{9\text{-}13}$ = P(Y=4) = 1 - P(Y=<3)

P$_{10\text{-}10}$ = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit\left(P(Y \leq 0)\right)\right))}$$

$$logit\left(P(Y \leq 0)\right) = -2.077 - (1)D1 - (-0.529)D2 - (-0.110)D3 - (-0.357)D4 - (-1.002)D5 - (0.242)D6 - (-0.193)D7 - (-0.144)D8 - (1)LA.CIR\&MO - (-0.738)LA.CRMO - (-0.895)LA.MO - (-1.310)LA.NC - (-0.902)LA.ThkMO - (-0.330)LA.ThnMO - (-0.751)LA.ThnO$$

P$_{10\text{-}11}$ = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit\left(P(Y \leq 1)\right)\right))}$$

$$logit\left(P(Y \leq 1)\right) = -0.203 - (1)D1 - (-0.529)D2 - (-0.110)D3 - (-0.357)D4 - (-1.002)D5 - (0.242)D6 - (-0.193)D7 - (-0.144)D8 - (1)LA.CIR\&MO - (-0.738)LA.CRMO - (-0.895)LA.MO - (-1.310)LA.NC - (-0.902)LA.ThkMO - (-0.330)LA.ThnMO - (-0.751)LA.ThnO$$

P$_{10\text{-}12}$ = P(Y=2) = P(Y=<2) - P(Y=<1)

$$P(Y \leq 2) = \frac{\exp(logit(P(Y \leq 2))}{(1 + \exp\left(logit\left(P(Y \leq 2)\right)\right))}$$

$$logit\left(P(Y \leq 2)\right) = 1.330 - (1)D1 - (-0.529)D2 - (-0.110)D3 - (-0.357)D4 - (-1.002)D5 - (0.242)D6 - (-0.193)D7 - (-0.144)D8 - (1)LA.CIR\&MO - (-0.738)LA.CRMO - (-0.895)LA.MO - (-1.310)LA.NC - (-0.902)LA.ThkMO - (-0.330)LA.ThnMO - (-0.751)LA.ThnO$$

P$_{10\text{-}13}$ = P(Y=3) = P(Y=<3) - P(Y=<2)

$$P(Y \leq 3) = \frac{\exp(logit(P(Y \leq 3))}{(1 + \exp\left(logit\left(P(Y \leq 3)\right)\right))}$$

$$logit(P(Y \leq 3))$$
$$= 3.826 - (1)D1 - (-0.529)D2 - (-0.110)D3 - (-0.357)D4 - (-1.002)D5 - (0.242)D6 - (-0.193)D7 - (-0.144)D8 - (1)LA.CIR\&MO - (-0.738)LA.CRMO - (-0.895)LA.MO - (-1.310)LA.NC - (-0.902)LA.ThkMO - (-0.330)LA.ThnMO - (-0.751)LA.ThnO$$

P10-14 = P(Y=4) = P(Y=<4) - P(Y=<3)

$$P(Y \leq 4) = \frac{\exp(logit(P(Y \leq 4))}{(1 + \exp\left(logit(P(Y \leq 4))\right))}$$

$$logit(P(Y \leq 4))$$
$$= 4.932 - (1)D1 - (-0.529)D2 - (-0.110)D3 - (-0.357)D4 - (-1.002)D5 - (0.242)D6 - (-0.193)D7 - (-0.144)D8 - (1)LA.CIR\&MO - (-0.738)LA.CRMO - (-0.895)LA.MO - (-1.310)LA.NC - (-0.902)LA.ThkMO - (-0.330)LA.ThnMO - (-0.751)LA.ThnO$$

P10-15 = P(Y=5) = 1 - P(Y=<4)

P11-11 = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit(P(Y \leq 0))\right))}$$

$$logit(P(Y \leq 0))$$
$$= -2.182 - (1)D1 - (-0.527)D2 - (-0.142)D3 - (-0.626)D4 - (-0.872)D5 - (-0.095)D6 - (-0.532)D7 - (-0.233)D8 - (1)LA.CIR\&MO - (-0.919)LA.CRMO - (-0.829)LA.MO - (-0.275)LA.ThkMO - (-0.325)LA.ThnMO - (-0.705)LA.ThnO$$

P11-12 = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit(P(Y \leq 1))\right))}$$

$$logit(P(Y \leq 1)) = -0.285 - (1)D1 - (-0.527)D2 - (-0.142)D3 - (-0.626)D4 - (-0.872)D5 - (-0.095)D6 - (-0.532)D7 - (-0.233)D8 - (1)LA.CIR\&MO - (-0.919)LA.CRMO - (-0.829)LA.MO - (-0.275)LA.ThkMO - (-0.325)LA.ThnMO - (-0.705)LA.ThnO$$

P11-13 = P(Y=2) = P(Y=<2) - P(Y=<1)

$$P(Y \leq 2) = \frac{\exp(logit(P(Y \leq 2))}{(1 + \exp\left(logit(P(Y \leq 2))\right))}$$

$$logit(P(Y \leq 2)) = 2.163 - (1)D1 - (-0.527)D2 - (-0.142)D3 - (-0.626)D4 - (-0.872)D5 - (-0.095)D6 - (-0.532)D7 - (-0.233)D8 - (1)LA.CIR\&MO - (-0.919)LA.CRMO - (-0.829)LA.MO - (-0.275)LA.ThkMO - (-0.325)LA.ThnMO - (-0.705)LA.ThnO$$

P11-14 = P(Y=3) = P(Y=<3) - P(Y=<2)

$$P(Y \leq 3) = \frac{\exp(logit(P(Y \leq 3))}{(1 + \exp\left(logit(P(Y \leq 3))\right))}$$

$$logit(P(Y \leq 3)) = 3.867 - (1)D1 - (-0.527)D2 - (-0.142)D3 - (-0.626)D4 - (-0.872)D5 - (-0.095)D6 - (-0.532)D7 - (-0.233)D8 - (1)LA.CIR\&MO - (-0.919)LA.CRMO - (-0.829)LA.MO - (-0.275)LA.ThkMO - (-0.325)LA.ThnMO - (-0.705)LA.ThnO$$

P11-15 = P(Y=4) = P(Y=<4) - P(Y=<3)

$$P(Y \leq 4) = \frac{\exp(logit(P(Y \leq 4))}{(1 + \exp\left(logit(P(Y \leq 4))\right))}$$

$$logit(P(Y \leq 4)) = 5.126 - (1)D1 - (-0.527)D2 - (-0.142)D3 - (-0.626)D4 - (-0.872)D5 - (-0.095)D6 - (-0.532)D7 - (-0.233)D8 - (1)LA.CIR\&MO - (-0.919)LA.CRMO - (-0.829)LA.MO - (-0.275)LA.ThkMO - (-0.325)LA.ThnMO - (-0.705)LA.ThnO$$

P$_{11\text{-}16}$ = P(Y=5) = 1 - P(Y=<4)

P$_{12\text{-}12}$ = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit(P(Y \leq 0))\right))}$$

$$logit(P(Y \leq 0)) = -1.449 - (1)D1 - (0.023)D2 - (0.357)D3 - (-0.019)D4 - (-0.882)D5 - (0.112)D6 - (0.274)D7 - (0.030)D8 - (1)LA.CIR\&MO - (-0.964)LA.CRMO - (-0.530)LA.MO - (-0.508)LA.ThkMO - (0.068)LA.ThnMO - (-0.247)LA.ThnO$$

P$_{12\text{-}13}$ = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit(P(Y \leq 1))\right))}$$

$$logit(P(Y \leq 1)) = 1.505 - (1)D1 - (0.023)D2 - (0.357)D3 - (-0.019)D4 - (-0.882)D5 - (0.112)D6 - (0.274)D7 - (0.030)D8 - (1)LA.CIR\&MO - (-0.964)LA.CRMO - (-0.530)LA.MO - (-0.508)LA.ThkMO - (0.068)LA.ThnMO - (-0.247)LA.ThnO$$

P$_{12\text{-}14}$ = P(Y=2) = P(Y=<2) - P(Y=<1)

$$P(Y \leq 2) = \frac{\exp(logit(P(Y \leq 2))}{(1 + \exp\left(logit(P(Y \leq 2))\right))}$$

$$logit(P(Y \leq 2)) = 3.630 - (1)D1 - (0.023)D2 - (0.357)D3 - (-0.019)D4 - (-0.882)D5 - (0.112)D6 - (0.274)D7 - (0.030)D8 - (1)LA.CIR\&MO - (-0.964)LA.CRMO - (-0.530)LA.MO - (-0.508)LA.ThkMO - (0.068)LA.ThnMO - (-0.247)LA.ThnO$$

$P_{12\text{-}15}$ = P(Y=3) = P(Y=<3) - P(Y=<2)

$$P(Y \leq 3) = \frac{\exp(logit(P(Y \leq 3))}{(1 + \exp\left(logit(P(Y \leq 3))\right))}$$

$$logit(P(Y \leq 3)) = 4.968 - (1)D1 - (0.023)D2 - (0.357)D3 - (-0.019)D4 - (-0.882)D5 - (0.112)D6 - (0.274)D7 - (0.030)D8 - (1)LA.CIR\&MO - (-0.964)LA.CRMO - (-0.530)LA.MO - (-0.508)LA.ThkMO - (0.068)LA.ThnMO - (-0.247)LA.ThnO$$

$P_{12\text{-}16}$ = P(Y=4) = 1 - P(Y=<3)

$P_{13\text{-}13}$ = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit(P(Y \leq 0))\right))}$$

$$logit(P(Y \leq 0)) = -2.189 - (1)D1 - (-0.068)D2 - (-0.283)D3 - (-0.197)D4 - (-0.762)D5 - (0.350)D6 - (0.306)D7 - (-0.126)D8 - (1)LA.CIR\&MO - (-0.313)LA.CRMO - (-1.451)LA.MO - (-1.009)LA.ThkMO - (-1.045)LA.ThnMO - (-1.245)LA.ThnO - (-0.070)FT$$

$P_{13\text{-}14}$ = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit(P(Y \leq 1))\right))}$$

$$logit(P(Y \leq 1)) = 0.272 - (1)D1 - (-0.068)D2 - (-0.283)D3 - (-0.197)D4 - (-0.762)D5 - (0.350)D6 - (0.306)D7 - (-0.126)D8 - (1)LA.CIR\&MO - (-0.313)LA.CRMO - (-1.451)LA.MO - (-1.009)LA.ThkMO - (-1.045)LA.ThnMO - (-1.245)LA.ThnO - (-0.070)FT$$

$\mathbf{P_{13\text{-}15}}$ = P(Y=2) = P(Y=<2) - P(Y=<1)

$$P(Y \leq 2) = \frac{\exp(logit(P(Y \leq 2))}{(1 + \exp\left(logit(P(Y \leq 2))\right))}$$

$$logit(P(Y \leq 2)) = 2.622 - (1)D1 - (-0.068)D2 - (-0.283)D3 - (-0.197)D4 - (-0.762)D5 - (0.350)D6 - (0.306)D7 - (-0.126)D8 - (1)LA.CIR\&MO - (-0.313)LA.CRMO - (-1.451)LA.MO - (-1.009)LA.ThkMO - (-1.045)LA.ThnMO - (-1.245)LA.ThnO - (-0.070)FT$$

$\mathbf{P_{13\text{-}16}}$ = P(Y=3) = P(Y=<3) - P(Y=<2)

$$P(Y \leq 3) = \frac{\exp(logit(P(Y \leq 3))}{(1 + \exp\left(logit(P(Y \leq 3))\right))}$$

$$logit(P(Y \leq 3)) = 5.036 - (1)D1 - (-0.068)D2 - (-0.283)D3 - (-0.197)D4 - (-0.762)D5 - (0.350)D6 - (0.306)D7 - (-0.126)D8 - (1)LA.CIR\&MO - (-0.313)LA.CRMO - (-1.451)LA.MO - (-1.009)LA.ThkMO - (-1.045)LA.ThnMO - (-1.245)LA.ThnO - (-0.070)FT$$

$\mathbf{P_{13\text{-}17}}$ = P(Y=4) = 1 - P(Y=<3)

$\mathbf{P_{14\text{-}14}}$ = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit(P(Y \leq 0))\right))}$$

$$logit(P(Y \leq 0)) = -0.670 - (1)D1 - (0.090)D2 - (0.025)D3 - (-0.480)D4 - (-0.838)D5 - (0.424)D6 - (-0.220)D7 - (-0.343)D8 - (1)LA.MO - (0.255)LA.ThkMO - (0.609)LA.ThnMO - (0.174)LA.ThnO$$

P$_{14\text{-}15}$ = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit(P(Y \leq 1))\right))}$$

$$logit(P(Y \leq 1)) = 2.018 - (1)D1 - (0.090)D2 - (0.025)D3 - (-0.480)D4 - (-0.838)D5 - (0.424)D6 - (-0.220)D7 - (-0.343)D8 - (1)LA.MO - (0.255)LA.ThkMO - (0.609)LA.ThnMO - (0.174)LA.ThnO$$

P$_{14\text{-}16}$ = P(Y=2) = 1 - P(Y=<1)

P$_{15\text{-}15}$ = P(Y=0) = P(Y=<0)

$$P(Y \leq 0) = \frac{\exp(logit(P(Y \leq 0))}{(1 + \exp\left(logit(P(Y \leq 0))\right))}$$

$$logit(P(Y \leq 0)) = -1.186 - (1)FC.RMjC - (-0.820)FC.RMnA - (-0.294)FC.RMnC - (-0.166)FC.RPA - (-1.391)FC.UMnA - (-0.576)FC.UPA$$

P$_{15\text{-}16}$ = P(Y=1) = P(Y=<1) - P(Y=0)

$$P(Y \leq 1) = \frac{\exp(logit(P(Y \leq 1))}{(1 + \exp\left(logit(P(Y \leq 1))\right))}$$

$$logit(P(Y \leq 1)) = 1.273 - (1)FC.RMjC - (-0.820)FC.RMnA - (-0.294)FC.RMnC - (-0.166)FC.RPA - (-1.391)FC.UMnA - (-0.576)FC.UPA$$

$\mathbf{P_{15-17}}$ = P(Y=2) = 1 - P(Y=<1)

Where:

- FT: Flexible surface thickness
- BT: Base thickness
- SL: Speed limit
- AADTA: Average annual daily traffic
- *Di* stands for the district *i* and should be replaced with 0 or 1 (1 for true and 0 for false),
- FC.xxx: stands for the functional class of the section being analyzed and should be replaced with 0 or 1 (1 for true and 0 for false), in which:
    - FC.RMnA: Functional Class Rural Minor Arterial
    - FC.RMjC: Functional Class Rural Major Collector
    - FC.RMnC: Functional Class Rural Minor Collector
    - FC.RPA: Functional Class Rural Principal Arterial
    - FC.UI: Functional Class Urban Interstate
    - FC.UMnA: Functional Class Urban Minor Arterial
    - FC.UPA: Functional Class Urban Principal Arterial
    - FC.UPAF: Functional Class Urban Principal Arterial Freeway
    - FC.RI: Functional Class Rural Interstate
- LA.xxx: stands for the last activity performed in the section and should be replaced with 0 or 1 (1 for true and 0 for false), in which:
    - LA.CIR&MO: CIR & Medium Overlay
    - LA.CRMO: Last Activity Crack Repr/Med OL
    - LA.MO: Last Activity Medium Overlay
    - LA.NC: Last Activity Nova Chip (UTBWC)
    - LA.ThkMO: Last Activity Thick Mill/Overlay
    - LA.ThnMO: Last Activity Thin Mill/Overlay
    - LA.ThnO: Last Activity Thin Ove

APPENDIX C – MARKOV TRANSITION PROBABILITY MATRICES

Table C1: Effect of repair matrix for Thin Mill and Overlay

			Effect of repair matrix for Thin Mill and Overlay																
			End State																
			1 (RQI: 5.0-4.2)	2 (RQI: 4.1-4.0)	3 (RQI: 3.9)	4 (RQI: 3.8)	5 (RQI: 3.7)	6 (RQI: 3.6)	7 (RQI: 3.5)	8 (RQI: 3.4)	9 (RQI: 3.3)	10 (RQI: 3.2)	11 (RQI: 3.1)	12 (RQI: 3.0)	13 (RQI: 2.9-2.8)	14 (RQI: 2.7-2.6)	15 (RQI: 2.5-2.4)	16 (RQI: 2.3-2.2)	17 (RQI: 2.1-2.0)
Start State	1	(RQI: 5.0-4.2)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2	(RQI: 4.1-4.0)	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	(RQI: 3.9)	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4	(RQI: 3.8)	0.5	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	(RQI: 3.7)	0.3077	0.3077	0.0769	0.3077	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	(RQI: 3.6)	0.1667	0.6667	0.0556	0	0.1111	0	0	0	0	0	0	0	0	0	0	0	0
	7	(RQI: 3.5)	0.1852	0.4815	0.1852	0.1481	0	0	0	0	0	0	0	0	0	0	0	0	0
	8	(RQI: 3.4)	0.1622	0.5405	0.1081	0.0541	0.0541	0	0.0811	0	0	0	0	0	0	0	0	0	0
	9	(RQI: 3.3)	0.1373	0.451	0.1176	0.0784	0.1176	0.0392	0.0392	0.0196	0	0	0	0	0	0	0	0	0
	10	(RQI: 3.2)	0.1014	0.3623	0.2174	0.029	0.087	0.0725	0.0435	0.058	0.029	0	0	0	0	0	0	0	0
	11	(RQI: 3.1)	0.0581	0.2674	0.2209	0.2093	0.0581	0.093	0.0465	0.0116	0.0116	0.0233	0	0	0	0	0	0	0
	12	(RQI: 3.0)	0.051	0.2245	0.1633	0.1633	0.1122	0.0918	0.1122	0.051	0	0	0.0306	0	0	0	0	0	0
	13	(RQI: 2.9-2.8)	0.0412	0.1701	0.1649	0.1598	0.1649	0.1237	0.0515	0.067	0.0206	0.0206	0.0052	0.0103	0	0	0	0	0
	14	(RQI: 2.7-2.6)	0.0361	0.0843	0.1325	0.1747	0.1627	0.1627	0.0602	0.0542	0.0602	0.0181	0.0181	0	0.0361	0	0	0	0
	15	(RQI: 2.5-2.4)	0.0563	0.0563	0.1479	0.1831	0.1549	0.1408	0.0915	0.0493	0.0141	0.0563	0.0282	0.0141	0.007	0	0	0	0
	16	(RQI: 2.3-2.2)	0.0789	0.0789	0.0702	0.0614	0.1667	0.1579	0.1228	0.0789	0.0614	0.0351	0.0351	0.0439	0	0.0088	0	0	0
	17	(RQI: 2.1-2.0)	0.0108	0.086	0.0753	0.0753	0.1183	0.1183	0.1828	0.0968	0.0645	0.0538	0.0108	0.043	0.0323	0.0108	0	0.0215	0

Table C2: Effect of repair matrix for Thin Overlay

			Effect of repair matrix for Thin Overlay																
			End State																
			1 (RQI: 5.0-4.2)	2 (RQI: 4.1-4.0)	3 (RQI: 3.9)	4 (RQI: 3.8)	5 (RQI: 3.7)	6 (RQI: 3.6)	7 (RQI: 3.5)	8 (RQI: 3.4)	9 (RQI: 3.3)	10 (RQI: 3.2)	11 (RQI: 3.1)	12 (RQI: 3.0)	13 (RQI: 2.9-2.8)	14 (RQI: 2.7-2.6)	15 (RQI: 2.5-2.4)	16 (RQI: 2.3-2.2)	17 (RQI: 2.1-2.0)
Start State	1	(RQI: 5.0-4.2)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2	(RQI: 4.1-4.0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	(RQI: 3.9)	0.8667	0.1333	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4	(RQI: 3.8)	0.9231	0	0.0769	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	(RQI: 3.7)	0.2941	0.4118	0.1176	0.1765	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	(RQI: 3.6)	0.2500	0.4500	0.1500	0.1000	0.0500	0	0	0	0	0	0	0	0	0	0	0	0
	7	(RQI: 3.5)	0.1250	0.3750	0.1875	0.1250	0.1875	0	0	0	0	0	0	0	0	0	0	0	0
	8	(RQI: 3.4)	0	0.3636	0.2273	0.2273	0.0455	0.1364	0	0	0	0	0	0	0	0	0	0	0
	9	(RQI: 3.3)	0.0968	0.4516	0.1613	0.0645	0.1613	0.0323	0.0323	0	0	0	0	0	0	0	0	0	0
	10	(RQI: 3.2)	0.0339	0.4068	0.2712	0.0847	0.0847	0.0678	0.0169	0.0339	0	0	0	0	0	0	0	0	0
	11	(RQI: 3.1)	0.0794	0.3333	0.2540	0.1429	0.0476	0.0794	0.0317	0.0317	0	0	0	0	0	0	0	0	0
	12	(RQI: 3.0)	0.0200	0.2800	0.2900	0.1900	0.0800	0.0700	0.0100	0.0300	0.0200	0.0100	0	0	0	0	0	0	0
	13	(RQI: 2.9-2.8)	0.0099	0.2709	0.2463	0.1527	0.1478	0.0640	0.0394	0.0296	0.0099	0.0099	0.0049	0.0148	0	0	0	0	0
	14	(RQI: 2.7-2.6)	0	0.2010	0.2732	0.1804	0.1392	0.1186	0.0361	0.0155	0.0155	0.0155	0	0	0.0052	0	0	0	0
	15	(RQI: 2.5-2.4)	0	0.1703	0.2033	0.1978	0.1593	0.1264	0.0659	0.0330	0.0000	0.0165	0.0055	0.0110	0	0.0110	0	0	0
	16	(RQI: 2.3-2.2)	0.0085	0.1538	0.1624	0.2308	0.1282	0.1624	0.0342	0.0427	0.0342	0.0171	0	0	0	0	0.0256	0	0
	17	(RQI: 2.1-2.0)	0	0.1351	0.2432	0.1081	0.1081	0.1757	0.0541	0.0676	0.0676	0.0270	0	0	0	0.0135	0	0	0

Table C3: Effect of repair matrix for Thick Mill and Overlay

			Effect of repair matrix for Thick Mill and Overlay																
			End State																
			1 (RQI: 5.0-4.2)	2 (RQI: 4.1-4.0)	3 (RQI: 3.9)	4 (RQI: 3.8)	5 (RQI: 3.7)	6 (RQI: 3.6)	7 (RQI: 3.5)	8 (RQI: 3.4)	9 (RQI: 3.3)	10 (RQI: 3.2)	11 (RQI: 3.1)	12 (RQI: 3.0)	13 (RQI: 2.9-2.8)	14 (RQI: 2.7-2.6)	15 (RQI: 2.5-2.4)	16 (RQI: 2.3-2.2)	17 (RQI: 2.1-2.0)
Start State	1	(RQI: 5.0-4.2)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2	(RQI: 4.1-4.0)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	(RQI: 3.9)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4	(RQI: 3.8)	0	0	1.0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	(RQI: 3.7)	0	1.0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	(RQI: 3.6)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	(RQI: 3.5)	0	1.0000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	8	(RQI: 3.4)	0.2000	0.4000	0.2000	0	0	0	0.2000	0	0	0	0	0	0	0	0	0	0
	9	(RQI: 3.3)	0.0909	0.6364	0.1818	0.0909	0	0	0	0	0	0	0	0	0	0	0	0	0
	10	(RQI: 3.2)	0	0.3750	0.5000	0	0	0.1250	0	0	0	0	0	0	0	0	0	0	0
	11	(RQI: 3.1)	0.0526	0.2105	0.5263	0.1579	0.0526	0	0	0	0	0	0	0	0	0	0	0	0
	12	(RQI: 3.0)	0	0.3333	0.1333	0.1667	0	0.1333	0.0333	0.1000	0.1000	0	0	0	0	0	0	0	0
	13	(RQI: 2.9-2.8)	0.1091	0.4364	0.2545	0.0909	0.0364	0.0182	0.0182	0	0.0182	0.0182	0	0	0	0	0	0	0
	14	(RQI: 2.7-2.6)	0.0635	0.3810	0.2222	0.1429	0.0794	0.0317	0.0159	0.0317	0	0.0159	0.0159	0	0	0	0	0	0
	15	(RQI: 2.5-2.4)	0.1515	0.4242	0.1818	0.0455	0.1212	0.0303	0	0.0152	0.0152	0.0152	0	0	0	0	0	0	0
	16	(RQI: 2.3-2.2)	0.1569	0.3529	0.2549	0.0980	0.0588	0.0196	0.0196	0.0196	0	0	0.0196	0	0	0	0	0	0
	17	(RQI: 2.1-2.0)	0.1042	0.2083	0.2917	0.1875	0.0833	0.0417	0	0.0208	0.0208	0	0.0208	0	0	0.0208	0	0	0

Table C4: Augmented matrix for decay and Thin Mill and Overlay

			Augmented matrix for decay and Thin Mill and Overlay																
			End State																
			1 (RQI: 5.0-4.2)	2 (RQI: 4.1-4.0)	3 (RQI: 3.9)	4 (RQI: 3.8)	5 (RQI: 3.7)	6 (RQI: 3.6)	7 (RQI: 3.5)	8 (RQI: 3.4)	9 (RQI: 3.3)	10 (RQI: 3.2)	11 (RQI: 3.1)	12 (RQI: 3.0)	13 (RQI: 2.9-2.8)	14 (RQI: 2.7-2.6)	15 (RQI: 2.5-2.4)	16 (RQI: 2.3-2.2)	17 (RQI: 2.1-2.0)
Start State	1	(RQI: 5.0-4.2)	0.2848	0.6536	0.0485	0.0092	0.0013	0.0013	0.0000	0.0000	0.0013	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	(RQI: 4.1-4.0)	0.0041	0.4520	0.4290	0.0856	0.0213	0.0080	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	(RQI: 3.9)	0.0000	0.0011	0.4169	0.4095	0.1309	0.0331	0.0038	0.0019	0.0014	0.0005	0.0005	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
	4	(RQI: 3.8)	0.0003	0.0003	0.0000	0.3749	0.3996	0.1834	0.0312	0.0050	0.0039	0.0010	0.0000	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
	5	(RQI: 3.7)	0.0010	0.0010	0.0003	0.0010	0.3206	0.4923	0.1408	0.0285	0.0114	0.0024	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6	(RQI: 3.6)	0.0006	0.0023	0.0002	0.0000	0.0004	0.4445	0.3730	0.1345	0.0324	0.0078	0.0012	0.0019	0.0008	0.0004	0.0000	0.0000	0.0000
	7	(RQI: 3.5)	0.0012	0.0031	0.0012	0.0009	0.0000	0.0000	0.3380	0.4091	0.2019	0.0342	0.0105	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	8	(RQI: 3.4)	0.0015	0.0051	0.0010	0.0005	0.0005	0.0000	0.0008	0.3092	0.4401	0.1768	0.0468	0.0130	0.0047	0.0000	0.0000	0.0000	0.0000
	9	(RQI: 3.3)	0.0017	0.0056	0.0015	0.0010	0.0015	0.0005	0.0005	0.0002	0.3034	0.3968	0.2161	0.0530	0.0183	0.0000	0.0000	0.0000	0.0000
	10	(RQI: 3.2)	0.0019	0.0069	0.0041	0.0006	0.0017	0.0014	0.0008	0.0011	0.0006	0.2917	0.4353	0.1856	0.0625	0.0040	0.0020	0.0000	0.0000
	11	(RQI: 3.1)	0.0014	0.0066	0.0054	0.0052	0.0014	0.0023	0.0011	0.0003	0.0003	0.0006	0.2542	0.4416	0.2470	0.0264	0.0043	0.0017	0.0000
	12	(RQI: 3.0)	0.0016	0.0070	0.0051	0.0051	0.0035	0.0028	0.0035	0.0016	0.0000	0.0000	0.0009	0.1961	0.6222	0.1300	0.0152	0.0055	0.0000
	13	(RQI: 2.9-2.8)	0.0016	0.0068	0.0066	0.0064	0.0066	0.0049	0.0021	0.0027	0.0008	0.0008	0.0002	0.0004	0.3817	0.4735	0.0938	0.0100	0.0010
	14	(RQI: 2.7-2.6)	0.0017	0.0039	0.0062	0.0081	0.0076	0.0076	0.0028	0.0025	0.0028	0.0008	0.0008	0.0000	0.0017	0.2356	0.5703	0.1475	0.0000
	15	(RQI: 2.5-2.4)	0.0030	0.0030	0.0079	0.0097	0.0082	0.0075	0.0049	0.0026	0.0007	0.0030	0.0015	0.0007	0.0004	0.0000	0.3563	0.4807	0.1098
	16	(RQI: 2.3-2.2)	0.0053	0.0053	0.0047	0.0042	0.0113	0.0107	0.0083	0.0053	0.0042	0.0024	0.0024	0.0030	0.0000	0.0006	0.0000	0.3683	0.5640
	17	(RQI: 2.1-2.0)	0.0012	0.0096	0.0084	0.0084	0.0132	0.0132	0.0204	0.0108	0.0072	0.0060	0.0012	0.0048	0.0036	0.0012	0.0000	0.0024	0.8882

Table C5: Augmented matrix for decay and Thin Overlay

			Augmented matrix for decay and Thin Overlay																
			End State																
			1 (RQI: 5.0-4.2)	2 (RQI: 4.1-4.0)	3 (RQI: 3.9)	4 (RQI: 3.8)	5 (RQI: 3.7)	6 (RQI: 3.6)	7 (RQI: 3.5)	8 (RQI: 3.4)	9 (RQI: 3.3)	10 (RQI: 3.2)	11 (RQI: 3.1)	12 (RQI: 3.0)	13 (RQI: 2.9-2.8)	14 (RQI: 2.7-2.6)	15 (RQI: 2.5-2.4)	16 (RQI: 2.3-2.2)	17 (RQI: 2.1-2.0)
Start State	1	(RQI: 5.0-4.2)	0.2848	0.6536	0.0485	0.0092	0.0013	0.0013	0.0000	0.0000	0.0013	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	(RQI: 4.1-4.0)	0.0000	0.4561	0.4290	0.0856	0.0213	0.0080	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	(RQI: 3.9)	0.0037	0.0006	0.4137	0.4095	0.1309	0.0331	0.0038	0.0019	0.0014	0.0005	0.0005	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
	4	(RQI: 3.8)	0.0032	0.0000	0.0003	0.3719	0.3996	0.1834	0.0312	0.0050	0.0039	0.0010	0.0000	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
	5	(RQI: 3.7)	0.0013	0.0018	0.0005	0.0008	0.3195	0.4923	0.1408	0.0285	0.0114	0.0024	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6	(RQI: 3.6)	0.0010	0.0017	0.0006	0.0004	0.0002	0.4441	0.3730	0.1345	0.0324	0.0078	0.0012	0.0019	0.0008	0.0004	0.0000	0.0000	0.0000
	7	(RQI: 3.5)	0.0005	0.0014	0.0007	0.0005	0.0007	0.0000	0.3406	0.4091	0.2019	0.0342	0.0105	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	8	(RQI: 3.4)	0.0000	0.0020	0.0013	0.0013	0.0003	0.0008	0.0000	0.3131	0.4401	0.1768	0.0468	0.0130	0.0047	0.0000	0.0000	0.0000	0.0000
	9	(RQI: 3.3)	0.0007	0.0034	0.0012	0.0005	0.0012	0.0002	0.0002	0.0000	0.3083	0.3968	0.2161	0.0530	0.0183	0.0000	0.0000	0.0000	0.0000
	10	(RQI: 3.2)	0.0006	0.0066	0.0044	0.0014	0.0014	0.0011	0.0003	0.0006	0.0000	0.2944	0.4353	0.1856	0.0625	0.0040	0.0020	0.0000	0.0000
	11	(RQI: 3.1)	0.0014	0.0060	0.0046	0.0026	0.0009	0.0014	0.0006	0.0006	0.0000	0.0000	0.2608	0.4416	0.2470	0.0264	0.0043	0.0017	0.0000
	12	(RQI: 3.0)	0.0006	0.0089	0.0092	0.0060	0.0025	0.0022	0.0003	0.0009	0.0006	0.0003	0.0000	0.1954	0.6222	0.1300	0.0152	0.0055	0.0000
	13	(RQI: 2.9-2.8)	0.0004	0.0113	0.0103	0.0064	0.0062	0.0027	0.0016	0.0012	0.0004	0.0004	0.0002	0.0006	0.3798	0.4735	0.0938	0.0100	0.0010
	14	(RQI: 2.7-2.6)	0.0000	0.0109	0.0149	0.0098	0.0076	0.0064	0.0020	0.0008	0.0008	0.0008	0.0000	0.0000	0.0003	0.2277	0.5703	0.1475	0.0000
	15	(RQI: 2.5-2.4)	0.0000	0.0116	0.0139	0.0135	0.0109	0.0086	0.0045	0.0022	0.0000	0.0011	0.0004	0.0007	0.0000	0.0007	0.3414	0.4807	0.1098
	16	(RQI: 2.3-2.2)	0.0006	0.0107	0.0113	0.0160	0.0089	0.0113	0.0024	0.0030	0.0024	0.0012	0.0000	0.0000	0.0000	0.0000	0.0018	0.3666	0.5640
	17	(RQI: 2.1-2.0)	0.0000	0.0120	0.0216	0.0096	0.0096	0.0156	0.0048	0.0060	0.0060	0.0024	0.0000	0.0000	0.0000	0.0012	0.0000	0.0000	0.9111

Table C6: Augmented matrix for decay and Thich Mill and Overlay

			Augmented matrix for decay and Thin Overlay																
			End State																
			1 (RQI: 5.0-4.2)	2 (RQI: 4.1-4.0)	3 (RQI: 3.9)	4 (RQI: 3.8)	5 (RQI: 3.7)	6 (RQI: 3.6)	7 (RQI: 3.5)	8 (RQI: 3.4)	9 (RQI: 3.3)	10 (RQI: 3.2)	11 (RQI: 3.1)	12 (RQI: 3.0)	13 (RQI: 2.9-2.8)	14 (RQI: 2.7-2.6)	15 (RQI: 2.5-2.4)	16 (RQI: 2.3-2.2)	17 (RQI: 2.1-2.0)
Start State	1	(RQI: 5.0-4.2)	0.2848	0.6536	0.0485	0.0092	0.0013	0.0013	0.0000	0.0000	0.0013	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	(RQI: 4.1-4.0)	0.0000	0.4561	0.4290	0.0856	0.0213	0.0080	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	(RQI: 3.9)	0.0037	0.0006	0.4137	0.4095	0.1309	0.0331	0.0038	0.0019	0.0014	0.0005	0.0005	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
	4	(RQI: 3.8)	0.0032	0.0000	0.0003	0.3719	0.3996	0.1834	0.0312	0.0050	0.0039	0.0010	0.0000	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
	5	(RQI: 3.7)	0.0013	0.0018	0.0005	0.0008	0.3195	0.4923	0.1408	0.0285	0.0114	0.0024	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6	(RQI: 3.6)	0.0010	0.0017	0.0006	0.0004	0.0002	0.4441	0.3730	0.1345	0.0324	0.0078	0.0012	0.0019	0.0008	0.0004	0.0000	0.0000	0.0000
	7	(RQI: 3.5)	0.0005	0.0014	0.0007	0.0005	0.0007	0.0000	0.3406	0.4091	0.2019	0.0342	0.0105	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	8	(RQI: 3.4)	0.0000	0.0020	0.0013	0.0013	0.0003	0.0008	0.0000	0.3131	0.4401	0.1768	0.0468	0.0130	0.0047	0.0000	0.0000	0.0000	0.0000
	9	(RQI: 3.3)	0.0007	0.0034	0.0012	0.0005	0.0012	0.0002	0.0002	0.0000	0.3083	0.3968	0.2161	0.0530	0.0183	0.0000	0.0000	0.0000	0.0000
	10	(RQI: 3.2)	0.0006	0.0066	0.0044	0.0014	0.0014	0.0011	0.0003	0.0006	0.0000	0.2944	0.4353	0.1856	0.0625	0.0040	0.0020	0.0000	0.0000
	11	(RQI: 3.1)	0.0014	0.0060	0.0046	0.0026	0.0009	0.0014	0.0006	0.0006	0.0000	0.0000	0.2608	0.4416	0.2470	0.0264	0.0043	0.0017	0.0000
	12	(RQI: 3.0)	0.0006	0.0089	0.0092	0.0060	0.0025	0.0022	0.0003	0.0009	0.0006	0.0003	0.0000	0.1954	0.6222	0.1300	0.0152	0.0055	0.0000
	13	(RQI: 2.9-2.8)	0.0004	0.0113	0.0103	0.0064	0.0062	0.0027	0.0016	0.0012	0.0004	0.0004	0.0002	0.0006	0.3798	0.4735	0.0938	0.0100	0.0010
	14	(RQI: 2.7-2.6)	0.0000	0.0109	0.0149	0.0098	0.0076	0.0064	0.0020	0.0008	0.0008	0.0008	0.0000	0.0000	0.0003	0.2277	0.5703	0.1475	0.0000
	15	(RQI: 2.5-2.4)	0.0000	0.0116	0.0139	0.0135	0.0109	0.0086	0.0045	0.0022	0.0000	0.0011	0.0004	0.0007	0.0000	0.0007	0.3414	0.4807	0.1098
	16	(RQI: 2.3-2.2)	0.0006	0.0107	0.0113	0.0160	0.0089	0.0113	0.0024	0.0030	0.0024	0.0012	0.0000	0.0000	0.0000	0.0000	0.0018	0.3666	0.5640
	17	(RQI: 2.1-2.0)	0.0000	0.0120	0.0216	0.0096	0.0096	0.0156	0.0048	0.0060	0.0060	0.0024	0.0000	0.0000	0.0000	0.0012	0.0000	0.0000	0.9111