



OKLAHOMA TRANSPORTATION CENTER

*ECONOMIC ENHANCEMENT THROUGH INFRASTRUCTURE STEWARDSHIP*

# LABORATORY MODELING OF ENERGY DISSIPATION IN BROKEN-BACK CULVERTS

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## SI (METRIC) CONVERSION FACTORS

Approximate Conversions to SI Units				
Symbol	When you know	Multiply by	To Find	Symbol
<b>LENGTH</b>				
in	inches	25.40	millimeters	mm
ft	feet	0.3048	meters	m
yd	yards	0.9144	meters	m
mi	miles	1.609	kilometers	km
<b>AREA</b>				
in <sup>2</sup>	square inches	645.2	square millimeters	mm <sup>2</sup>
ft <sup>2</sup>	square feet	0.0929	square meters	m <sup>2</sup>
yd <sup>2</sup>	square yards	0.8361	square meters	m <sup>2</sup>
ac	acres	0.4047	hectares	ha
mi <sup>2</sup>	square miles	2.590	square kilometers	km <sup>2</sup>
<b>VOLUME</b>				
fl oz	fluid ounces	29.57	milliliters	mL
gal	gallons	3.785	liters	L
ft <sup>3</sup>	cubic feet	0.0283	cubic meters	m <sup>3</sup>
yd <sup>3</sup>	cubic yards	0.7645	cubic meters	m <sup>3</sup>
<b>MASS</b>				
oz	ounces	28.35	grams	g
lb	pounds	0.4536	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams	Mg
<b>TEMPERATURE (exact)</b>				
°F	degrees Fahrenheit	(°F-32)/1.8	degrees Celsius	°C
<b>FORCE and PRESSURE or STRESS</b>				
lbf	poundforce	4.448	Newtons	N
lbf/in <sup>2</sup>	poundforce per square inch	6.895	kilopascals	kPa

Approximate Conversions from SI Units				
Symbol	When you know	Multiply by	To Find	Symbol
<b>LENGTH</b>				
mm	millimeters	0.0394	inches	in
m	meters	3.281	feet	ft
m	meters	1.094	yards	yd
km	kilometers	0.6214	miles	mi
<b>AREA</b>				
mm <sup>2</sup>	square millimeters	0.00155	square inches	in <sup>2</sup>
m <sup>2</sup>	square meters	10.764	square feet	ft <sup>2</sup>
m <sup>2</sup>	square meters	1.196	square yards	yd <sup>2</sup>
ha	hectares	2.471	acres	ac
km <sup>2</sup>	square kilometers	0.3861	square miles	mi <sup>2</sup>
<b>VOLUME</b>				
mL	milliliters	0.0338	fluid ounces	fl oz
L	liters	0.2642	gallons	gal
m <sup>3</sup>	cubic meters	35.315	cubic feet	ft <sup>3</sup>
m <sup>3</sup>	cubic meters	1.308	cubic yards	yd <sup>3</sup>
<b>MASS</b>				
g	grams	0.0353	ounces	oz
kg	kilograms	2.205	pounds	lb
Mg	megagrams	1.1023	short tons (2000 lb)	T
<b>TEMPERATURE (exact)</b>				
°C	degrees Celsius	9/5+32	degrees Fahrenheit	°F
<b>FORCE and PRESSURE or STRESS</b>				
N	Newtons	0.2248	poundforce	lbf
kPa	kilopascals	0.1450	poundforce per square inch	lbf/in <sup>2</sup>

# **LABORATORY MODELING OF ENERGY DISSIPATION IN BROKEN-BACK CULVERTS**

**Final Report**

**July 2009**

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## EXECUTIVE SUMMARY

This research investigates the reduction in scour downstream of a broken-back culvert by forming a hydraulic jump inside the culvert. A broken-back culvert is used in areas of high relief and steep topography as it has one or more breaks in profile slope. A broken-back culvert in the laboratory represents a 1 (vertical) to 2 (horizontal) slope after the upstream inlet and then continuing 100 feet at a 1 percent slope in the flat part of the culvert to the downstream outlet. The prototype for these experiments was either a two barrel 10-foot by 10-foot, or a two barrel 10-foot by 20-foot reinforced concrete culvert. The drop between inlet and outlet is selected as 24 feet. Three flow conditions were simulated, consisting of 0.8, 1.0 and 1.2 times the culvert depth ( $d$ ).

The Froude number ( $Fr$ ) of the hydraulic jump created in the flat part of the culvert ranges between 2.7 and 3.6. This  $Fr$  classifies the jump as an “oscillating jump”. Such a jump moves up and down in the barrel, its location changing over time. The jump in experiments began nearly at the toe by placing sills in the flat part. The optimal location was determined at a distance of 45 and 25 feet from the outlet face of the culvert in pressure flow conditions. The sills contain two small orifices at the bottom to allow the culvert to completely drain.

The impact of friction blocks was found to be minimal on the dissipation of energy. No friction blocks were used to further dissipate. The length of the culvert cannot be reduced as the pressure flow fills up the culvert barrels nearly completely.

For new culvert construction, the best option to maximize energy dissipation under open channel flow condition is to use one sill located 43 feet from the outlet. Again, frictional blocks were not effective in further reduction of energy. The maximum length of the culvert can be reduced by 15 feet to 44 feet. Such a scenario is important where right-of-way problems exist for culvert construction.

# INTRODUCTION

A recent research study conducted by the Oklahoma Transportation Center at Oklahoma State University indicated that there are 121 scour-critical culverts on the Interstate System (ISTAT), the National Highway System (NHS), and the State Transportation Program (STP) in Oklahoma (Tyagi, 2002). The average replacement cost of these culverts is about \$121M. A survey of culverts in Oklahoma indicates that the drop in flowline between upstream and downstream ends ranges between 6 and 24 feet. In this research, a drop of 24 feet was used in the laboratory model because it is the upper limit. Advantages of this research are to maximize the energy loss within the culvert, thus minimizing the scour around the culvert and decreasing the degradation downstream in the channel. This reduces the construction and rehabilitation costs of culverts in Oklahoma. The project is supported by the Bridge Division, Oklahoma Department of Transportation (ODOT).

The purpose of this project is to develop a methodology to analyze broken-back culverts in Oklahoma such that the energy is mostly dissipated within the culverts to minimize the degradation downstream. A broken-back culvert is used in areas of high relief and steep topography as it has one or more breaks in profile slope. The purpose of a culvert is to safely pass water underneath the roadways constructed in hilly topography or on the side of a relatively steep hill. The project investigates culverts with a vertical drop of 24 feet that may result in effective energy dissipation inside the culvert and consequently minimize the scour downstream of broken-back culverts. Culvert dimensions and hydraulic parameters for the scale model were provided by the Bridge Division, ODOT (personal communication with B. Rusch, 2007).

The research investigation includes the following tasks: 1) to obtain and review existing research currently available for characterizing the hydraulic jump in culverts; 2) to build a scale model to represent a prototype of a broken-back culvert 150 feet long, with two barrels of 10 X 10 feet, and a vertical drop of 24 feet; 3) to simulate different flow conditions for 0.8, 1.0 and 1.2 times the culvert depth ( $d$ ) in the scale model constructed in Task 2; 4) to evaluate the energy dissipation between upstream and downstream ends of the broken-back culvert with and without friction blocks of different

shapes; 5) to observe in physical experiments the efficiency of hydraulic jump with and without friction blocks between upstream and downstream ends of the culvert and the location of hydraulic jump from the toe of the drop in the culvert; and 6) to prepare a final report incorporating analysis of hydraulic jump and devices to create the jump and energy loss. These tasks are presented in the sections that follow.

# LITERATURE REVIEW

The literature search was performed for hydraulic jump and Acoustic Doppler Velocimeter and the results are discussed in the following sections.

## HYDRAULIC JUMP

The hydraulic jump is a natural phenomenon of a sudden rise in water level due to change from supercritical flow to subcritical flow, i.e., when there is a sudden decrease in velocity of the flow. This sudden change in the velocity causes the considerable turbulence and loss of energy. Consequently, the hydraulic jump has been recognized as an effective method for energy dissipation for many years. There have been many studies carried out to explain the characteristics of the hydraulic jump. Some of these studies are summarized in the following paragraphs.

Finnemore, et al. (2002) state that the characteristics of the hydraulic jump depend on Froude number ( $Fr$ ). The Froude number is the ratio between inertia force and gravity force. They added that in order for the hydraulic jump to occur, the flow must be supercritical, i.e. a jump can occur only when the Froude number is greater than 1.0. The hydraulic jump is classified according to its Froude number. When  $Fr$  is between 1.7 and 2.5, the flow is classified as a weak jump and will have a smooth rise in the water surface with less energy dissipation. A  $Fr$  between 2.5 and 4.5 results in an oscillating jump with 15-45% energy dissipation. A steady jump will occur when  $Fr$  ranges from 4.5 to 9.0. and results in energy dissipation from 45% to 70%. When  $Fr$  is above 9.0, a strong jump will occur with energy losses ranging from 70% to 85%.

Bhutto et al. (1989) provided analytical solutions for computing sequent depth and relative energy loss for free hydraulic jump in horizontal and sloping rectangular channels from their experimental studies. They used the ratio of jump length to jump depth and the Froude number to compute the length of free jump on a horizontal bed. Jump factor and shape factor were evaluated experimentally for free jump on a sloping bed. To check the efficiency of the equations, they made comparisons with previous

solutions by other researchers and found that the equations they derived could be used instead of equations by Ludin, Bakhmateff, Silvester and Chertoussove.

Gharanglk and Chaudhry (1991) present three models for the numerical simulation of hydraulic jumps in a rectangular channel while factoring in the considerable effect of nonhydrostatic pressure distribution. The one-dimensional Boussinesq equations are solved in time subject to appropriate boundary conditions to numerically simulate the hydraulic jump. The results were compared to experimental data which indicate that four-order models with or without Boussinesq terms give similar results for all Froude numbers tested. The Froude numbers ranged from 2.3 to 7.0. The MacCormack scheme and a dissipative two-four scheme was used to solve the governing equations subject to specified end conditions until a steady state was achieved.

A broken-back culvert is used in areas of high relief and steep topography as it has one or more breaks in profile slope. The purpose of a culvert is to safely pass water underneath the roadways constructed in hilly topography or on the side of a relatively steep hill. Hotchkiss and Donahoo (2001) report that the Broken-back Culvert Analysis Program (BCAP) is a simple but powerful analysis tool for the analysis of broken-back culverts and hydraulic jumps. This program is easy to understand, explain, and document, and is based on the energy equation and momentum equation for classical jumps. It is able to plot rating curves for the headwater, outlet depth and outlet velocity. They described a computer code capable of analyzing hydraulic jumps in the broken-back culvert.

Hotchkiss et al (2003) describe the available predictive tools for hydraulic jumps, the performance of the Broken-back Culvert Analysis Program (BCAP) in analyzing the hydraulics of a broken-back culvert, and the current applications and distribution of BCAP. They conducted tests on the broken-back culvert made up of Plexiglas<sup>®</sup> to assess the performance of BCAP in predicting headwater rating curves, the locations of hydraulic jumps, and the lengths of hydraulic jumps. They conclude that accounting for the losses within the jump because of friction in corrugated metal pipes and by more accurate prediction of the locations of hydraulic jumps may be improved by predictions of flow hydraulics within the culvert barrel.

The Utah Department of Transportation (UDOT) addresses aspects of broken-back culverts and hydraulic jumps in the state's *Manual of Instruction – Roadway Drainage (US Customary units), Culverts*. This manual illustrates steps for the design of broken-back culverts which include: to establish a flow-line profile, to size the culvert, to begin to calculate a supercritical profile, to complete profile calculations, and to consider hydraulic jump cautions. In Section F of Appendix 9 of the manual, aspects of hydraulic jumps in culverts are covered, including cause and effect, momentum friction, comparison of momentum and specific energy curves, and the potential occurrence of hydraulic jumps. The manual also takes into account the sequent depth of jump for rectangular conduits, circular conduits, and conduits of other shapes.

Larson, E. (2004), in her Master's thesis entitled *Energy Dissipation in Culverts by Forcing a Hydraulic Jump at the Outlet*, suggests forcing hydraulic jumps to reduce the outlet energy. She considered two design examples to create a hydraulic jump within a culvert barrel: (1) a rectangular weir placed on a flat apron and (2) a vertical drop along with a rectangular weir. These two designs were used to study the reduction in the energy of the flow at the outlet. From these experiments she found that both designs were effective in reduction of outlet velocity, momentum, and energy. These reductions would result in a decreased need for downstream scour mitigation.

Hotchkiss et al. (2005) proposed that by controlling the water at the outlet of a culvert, water scour around the culvert can be reduced. The effectiveness of a simple weir near the culvert outlet is compared to that of a culvert having a weir with a drop upstream in the culvert barrel. These two designs are intended to reduce the specific energy of the water at the outlet by inducing a hydraulic jump within the culvert barrel, without the aid of tailwater. The design procedure was proposed after studying the geometry and effectiveness of each jump type in energy reduction. In this research, they found the Froude number ranged from 2.6 to 6.0. It was determined that both forms of outlets are effective in reducing the velocity of water and hence the energy and momentum.

The *Hydraulic Design of Energy Dissipators for Culverts and Channels* (July, 2006), from the Federal Highway Administration, provided design information for analyzing and mitigating problems associated with the energy dissipation at culvert

outlets and in open channels. It recommends the use of the broken-back culvert design considering it as an internal energy dissipater. The proposed design for a broken-back culvert is limited to the following conditions: 1) the slope of the steep section must be less than or equal to 1.4:1 (V: H) and 2) the hydraulic jump must be completed within the culvert barrel.

According to this report, for situations where the runout section is too short and/or there is insufficient tailwater for a jump to be completed within the barrel, modifications may be made to the outlet that will induce a jump. The design procedure for stilling basins, streambed level dissipaters, riprap basins and aprons, drop structures and stilling wells is also given in this circular.

Pagliara et. al. (2008) analyzed the hydraulic jump that occurs in homogeneous and non homogeneous rough bed channels. They investigated the sequent flow depth and the length of the jump which are the influence parameters on the hydraulic jump. In this research, they drew on the general jump equation to analyze the jump phenomenon. In analyzing the rough bed data, they were able to formulate a representative equation to explain the phenomenon. The equations found in their study may be used to design stilling basins downstream of hydraulic structures.

Hotchkiss et al. (2008) analyzed the accuracy of the following seven programs on culvert hydraulics: HY-8, FishXing, Broken-back Culvert Analysis Program (BCAP), Hydraflow Express, CulvertMaster, Culvert, and Hydrologic Engineering Center River Analysis system (HEC-RAS). The software was tested on the accuracy of three calculations: headwater depths, flow control, and outlet velocities. The software comparison was made between software output values and hand calculations, not from laboratory experimental data. The hand calculations used were derived from laboratory experiments done by the National Bureau of Standards (NBS). Hotchkiss et al. concluded HEC-RAS is the most comprehensive program for both accuracy and features for culverts affected by upstream structures.

## **ACOUSTIC DOPPLER VELOCIMETER (ADV)**

Acoustic Doppler Velocimeter (ADV) is a sonar device which tracks suspended solids (particles) in a fluid medium to determine an instantaneous velocity of the particles in a sampling volume. In general, ADV devices have one transmitter head and between two and four receiver heads. Since their introduction in 1993, acoustic Doppler velocimeter have quickly become valuable tools for laboratory and field investigations of flow in rivers, canals, reservoirs, the oceans, around hydraulic structures and in laboratory scale models (Sontek, 2001).

Wahl (2000) discusses methods for filtering raw ADV data using a software application called WinADV. Wahl suggests that ADV data present unique requirements compared to traditional current-metering equipment, due to the types of data obtained, the analyses that are possible, and the need for filtering the data to ensure that any technical limitations of ADVs do not adversely affect the quality of the results. According to Wahl, the WinADV program is a valuable tool for filtering, analyzing, and processing data collected from ADVs. Further, this program can be used to analyze ADV files recorded using the real time data acquisition programs provided by ADV manufacturers.

Goring and Nikora (2002) formulated a new post processing method for despiking raw ADV data. The method combines three concepts, including:

1. That differentiation of the data enhances the high frequency portion of a signal which is desirable in sonar measurements.
2. That the expected maximum of a random series is given by the Universal threshold function.
3. That good data clusters are a dense cloud in phase space maps

These concepts are used to construct an ellipsoid in three-dimensional phase space, and points lying outside the ellipsoid are designated as spikes (bad data). The new method has superior performance over various other methods and has the added advantage of requiring no parameters. Several methods for replacing sequences of spurious data are presented. A polynomial fitted to good data on either side of the spike event, then interpolated across the event, is preferred by Goring and Nikora.

Mori et al. (2007) investigates measuring velocities in aerated flows using ADV techniques. ADV measurements are useful and powerful for measurements of mean and turbulent components of fluids in both hydraulic experimental facilities and fields. However, it is difficult to use the ADV in bubbly flows because air bubbles generate spike noise in the ADV velocity data. This study describes the validity of the ADV measurements in bubbly flows. The true three-dimensional phase space method is significantly useful to eliminating the spike noise of ADV recorded data in bubbly flow as compared to the classical low correlation method (Goring and Nikora, 2002). The results of the data analysis suggest the following:

1. There is no clear relationship between velocity and ADV's correlation/signal-to-noise ratio in bubbly flow;
2. Spike noise filtering methods based on low correlation and signal-to-noise ratio are not adequate for bubbly flow; and
3. The true 3D phase space method significantly removes spike noise of ADV velocity in comparison with the original 3D phase space method.

In addition the study found that ADV velocity measurements can be valid for 1 to 3% air void flows. The limitations of the ADV velocity measurements for high void fractions were not studied.

Chanson et al. (2008) investigated the use of ADVs to determine the velocity in turbulent open channel flow conditions in both laboratory and field experiments. They demonstrated that the ADV is a competent set of devices for steady and unsteady turbulent open channel flows. However, in order to accurately measure velocity, the ADV raw data must be processed and the unit must be calibrated to the suspended sediment concentrations. Accurately processing your ADV data requires practical knowledge and experience of the device's capabilities and limitations. Chanson concluded that turbulence properties should not be derived from unprocessed ADV signals and some despiking methods were not directly applicable to many field and laboratory applications.

# HYDRAULIC SIMILITUDE THEORY

Similarity between a hydraulic model and a prototype may be achieved in three basic forms: a) geometric similarity, b) kinematic similarity, and c) dynamic similarity (Chow, 1959).

## BROKEN-BACK CULVERT SIMILARITIES

a. Geometric similarity implies similarity of physical form. The model is a geometric reduction of the prototype and is accomplished by maintaining a fixed ratio for all homologous lengths between the physical quantities involved in geometric similarity: length (L), area (A), and volume (Vol). To keep the homologous lengths in the prototype (p) and the model (m) at a constant ratio (r), they may be expressed as,

$$\frac{L_p}{L_m} = L_r \quad (1)$$

An area (A), is the product of two homologous lengths; hence, the ratio of the homologous area is also a constant given as,

$$\frac{A_p}{A_m} = \frac{L_p^2}{L_m^2} = L_r^2 \quad (2)$$

A volume (Vol.) is the product of three homologous lengths; the ratio of the homologous volume can be represented as,

$$\frac{Vol_p}{Vol_m} = \frac{L_p^3}{L_m^3} = L_r^3 \quad (3)$$

b. Kinematic similarity implies similarity of motion. Kinematic similarity between the model and the prototype is attained if the homologous moving particles have the same

velocity ratio along geometrically similar paths. This similarity involves the scale of time and length. The ratio of times required for homologous particles to travel homologous distances in a model and prototype is given by,

$$\frac{T_p}{T_m} = T_r \quad (4)$$

The velocity (V) is defined as distance per unit time; thus, the ratio of velocities may be expressed as,

$$\frac{V_p}{V_m} = \frac{(L_p / T_p)}{(L_m / T_m)} = \frac{L_r}{T_r} \quad (5)$$

The distance (Q) is expressed as volume per unit time and may be given by,

$$\frac{Q_p}{Q_m} = \frac{(L_p^3 / T_p)}{(L_m^3 / T_m)} = \frac{L_r^3}{T_r} \quad (6)$$

c. Dynamic similarity implies similarity in forces involved in motion. In broken-back culverts, inertial force and gravitational (g) force are considered dominant forces in fluid motion. The Froude number is defined as,

$$F_r = \frac{\left[ V_p / (g_p L_p)^{1/2} \right]}{\left[ V_m / (g_m L_m)^{1/2} \right]} = 1 \quad (7)$$

As  $g_p$  and  $g_m$  are the same in a model and the prototype, these cancel in Equation 7.

$$\frac{V_r}{(L_r)^{1/2}} = 1 \quad (8)$$

$$V_r = \frac{V_p}{V_m} = (L_r)^{1/2}$$

$$V_p = V_m (L_r)^{1/2} \quad (9)$$

Using the three similarities, a variable of interest can be extrapolated from the model to the prototype broken-back culvert.

## LABORATORY MODEL

During the initial period of discussion regarding the construction of a scale model to represent a 150 feet long broken-back culvert with two barrels of 10 x 10 feet and a vertical drop of 24 feet, the research group visited the USDA Agricultural Research Service Hydraulic Engineering Research Laboratory in Stillwater, Oklahoma. This was the facility at which testing was done. The group visited with facility personnel and inspected the equipment that would be used to conduct tests. Physical dimensions of the flume that would be used were noted, as well as the flow capacity of the system.

Two scales were considered for the model. It was determined that a scale of 1:10 or 1:20 would allow for geometric similitude in a model that could easily be produced. The 1 to 20 scale was adopted due to space limitations at the testing facility, and in consideration of the potential need to expand the model depending on where the hydraulic jump occurred. If the hydraulic jump did not form within the model, the smaller scale would leave room to double the length of the culvert. In addition, a lower flow rate would be required during testing if a smaller scale were used.

Other considerations included the questions of what materials to use in building the model, and what construction methods would be best. The materials considered were wood and Plexiglas<sup>®</sup>. Plexiglas<sup>®</sup> was found preferable because it offered visibility as well as durability, and a surface which would more closely simulate the surface being modeled (Figures 1 through 3, and 6). The Manning's roughness value for Plexiglas<sup>®</sup> is 0.010 which is very close to the roughness of finished concrete at 0.012. The thickness of the Plexiglas<sup>®</sup> was decided based on weight, rigidity, workability, and ease at which the material would fit into scale. Half-inch Plexiglas<sup>®</sup> proved to be sturdy and was thick enough to allow connection hardware to be installed in the edges of the plates. This material also fit well into the proposed scale of 1 to 20 which equated one-half inch in the model to one foot in the prototype. The construction methods included constructing the model completely at the Oklahoma State University campus and moving it to the test facility, creating sections of the model at the university and assembling them at the test facility, or contracting with the testing facility to construct the model. It was decided to construct the model in sections and assemble them at the test facility.

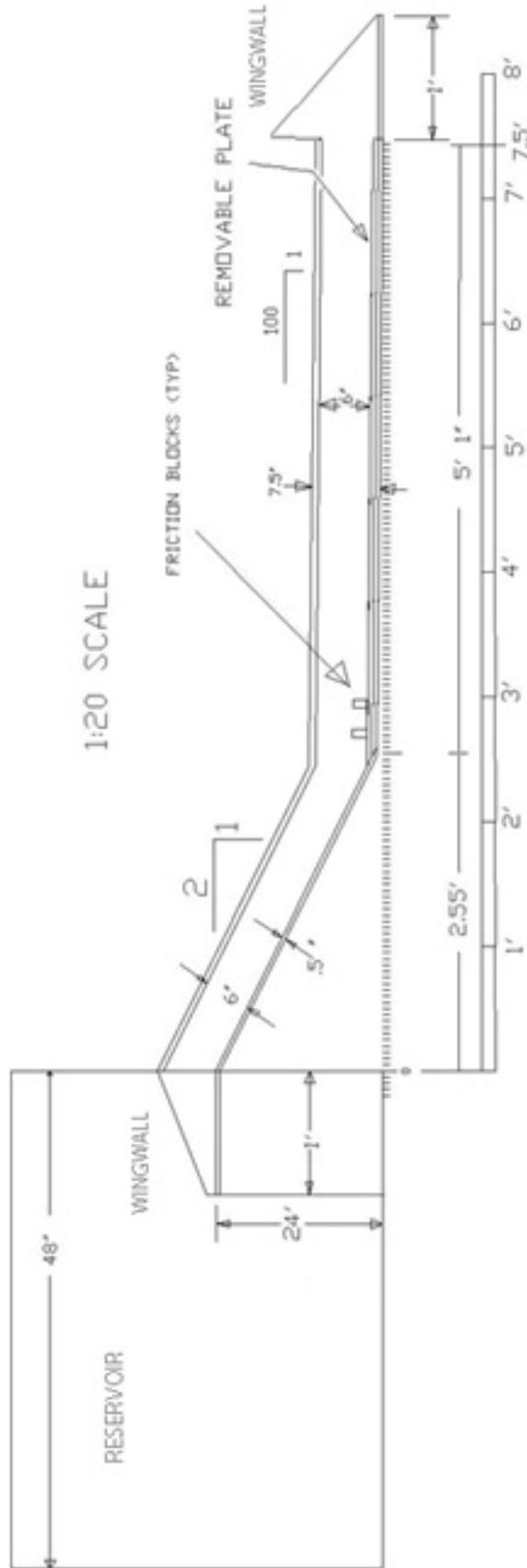
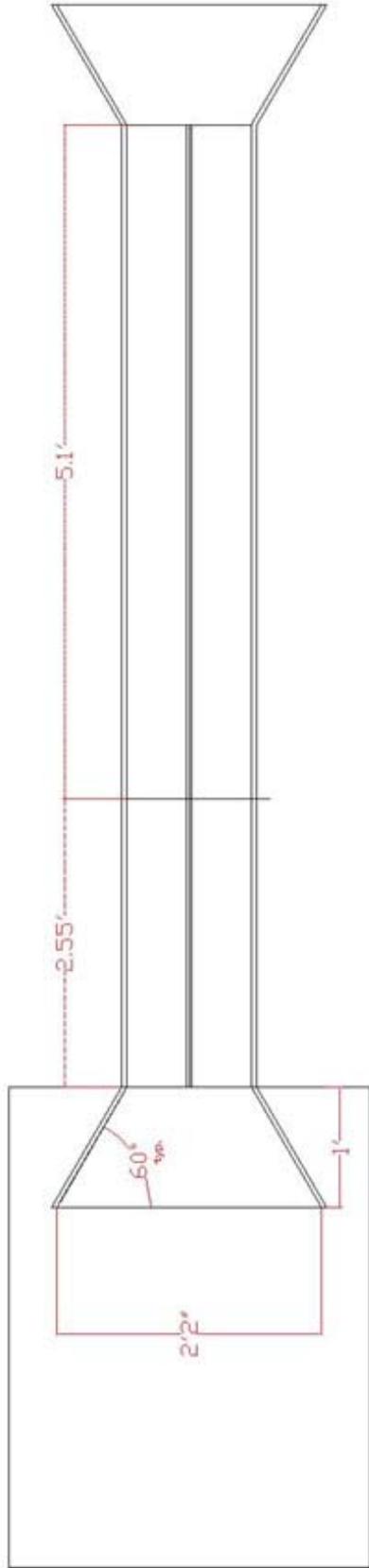
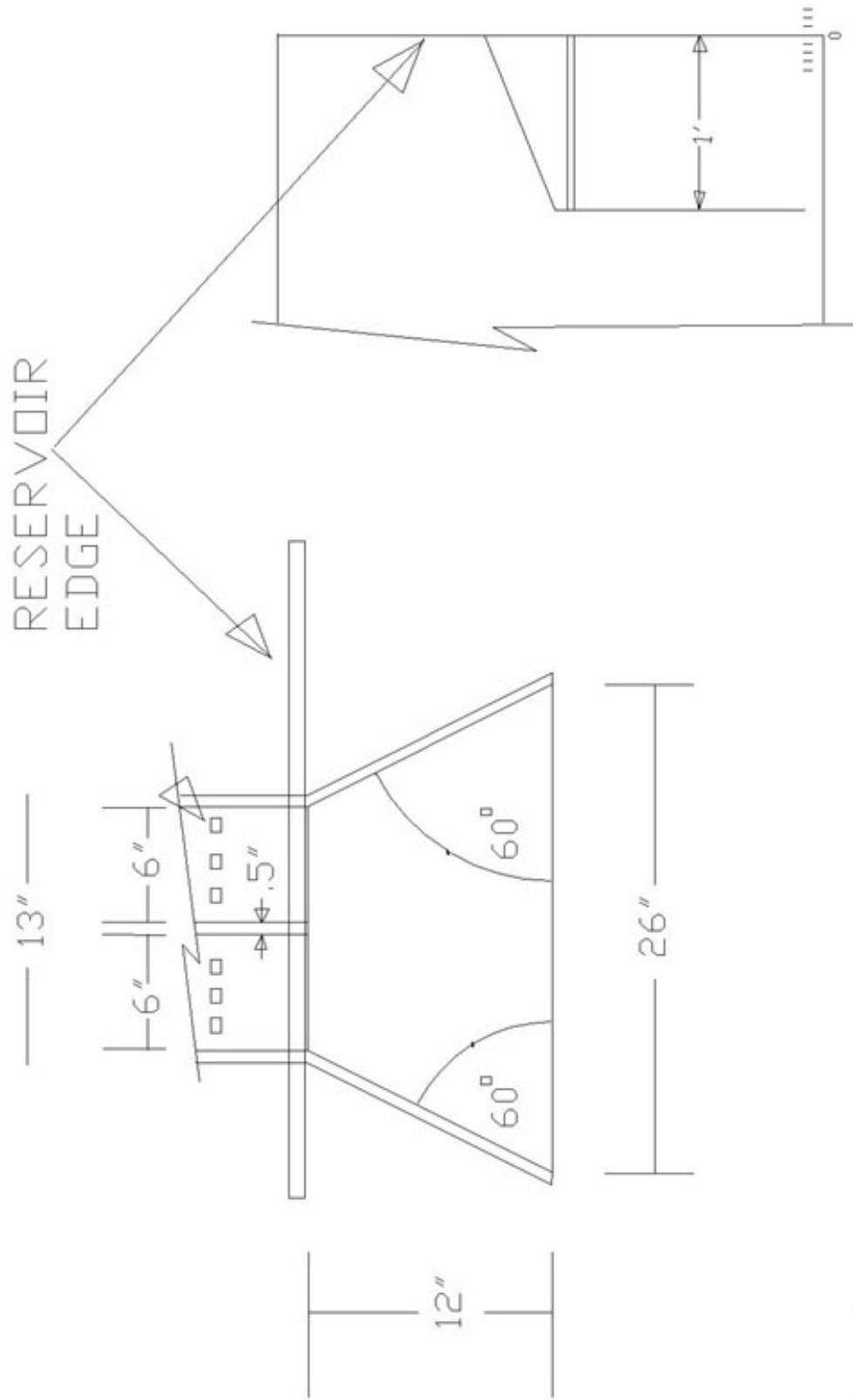


Figure 1. Profile view of model.



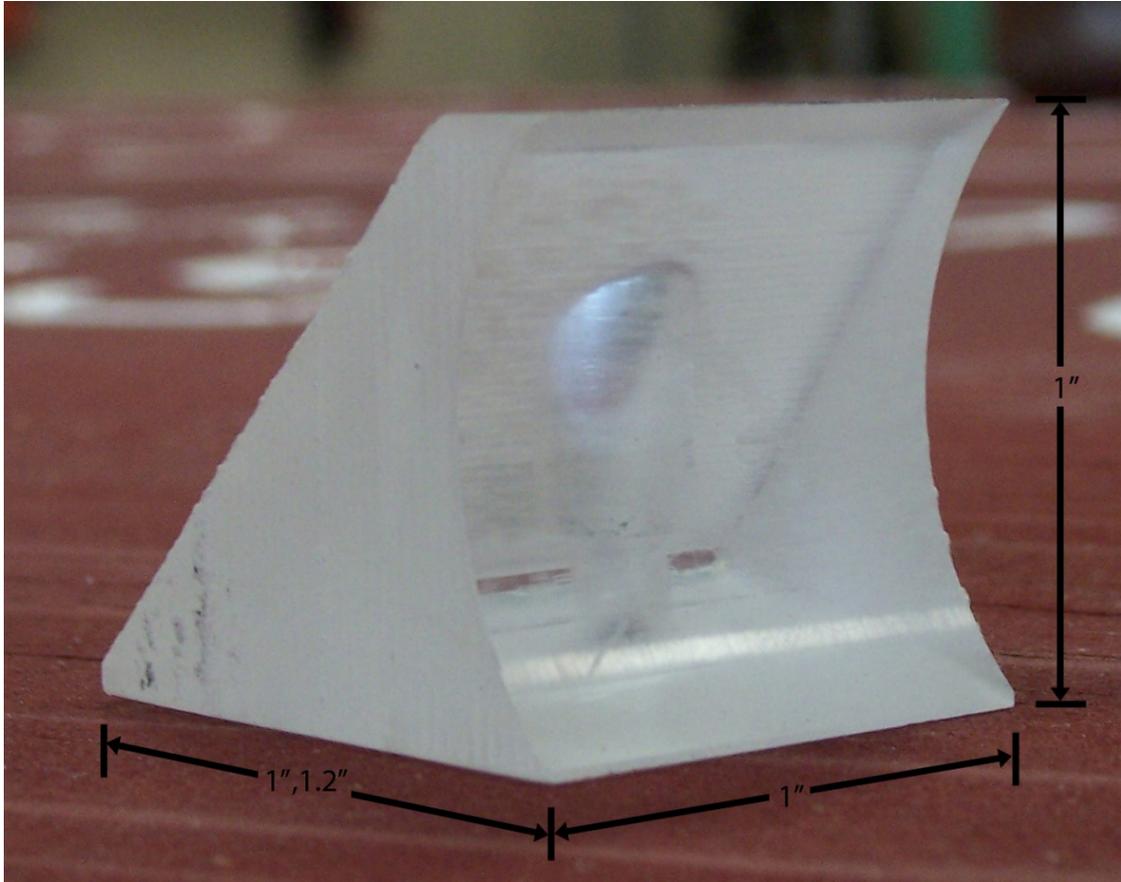
**Figure 2.** Plan view of model.



**Figure 3.** Inlet and outlet details.



**Figure 4.** Typical sill dimensions.



**Figure 5.** Example of friction block.

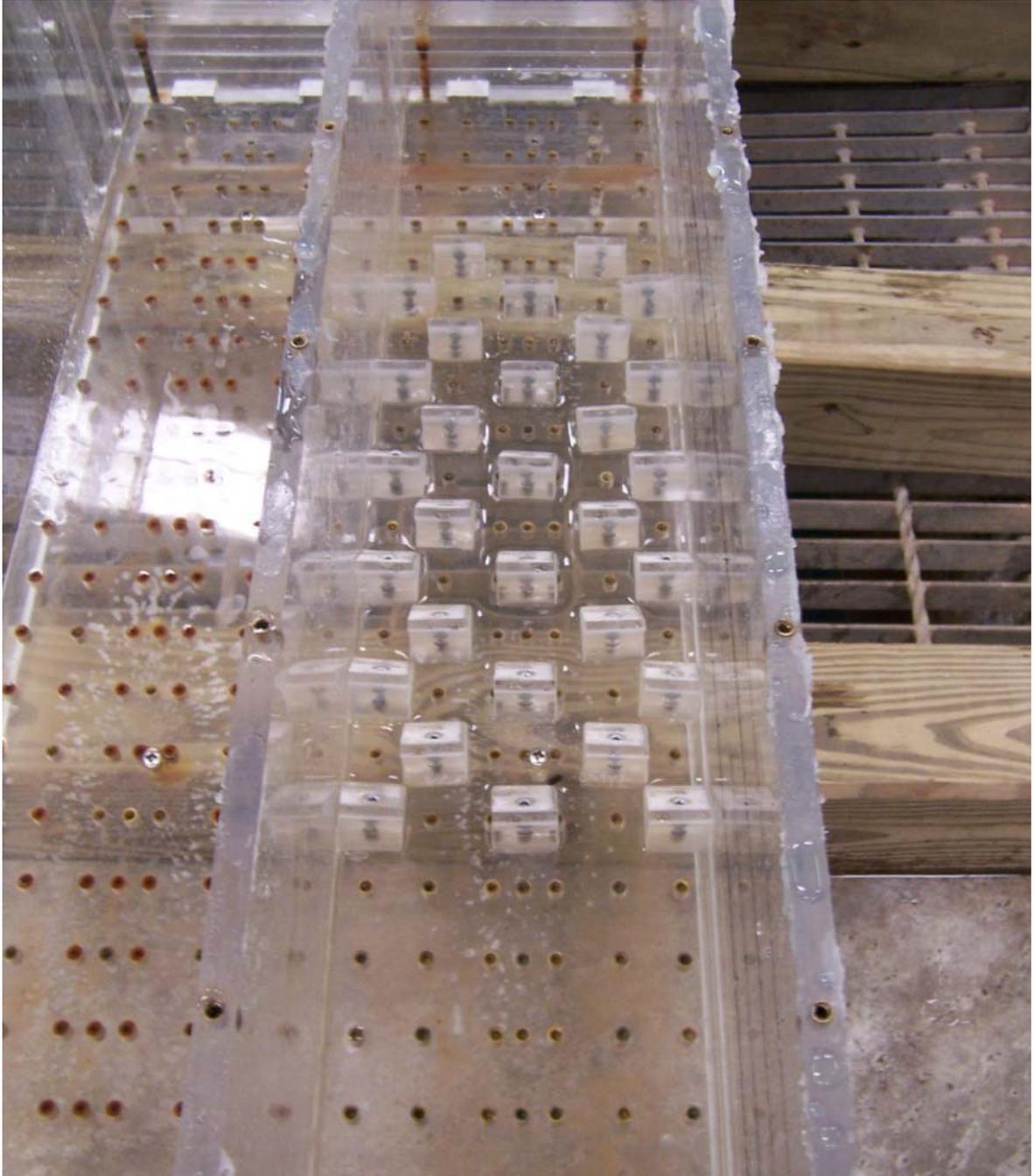


**Figure 6.** Broken-back culvert laboratory model.

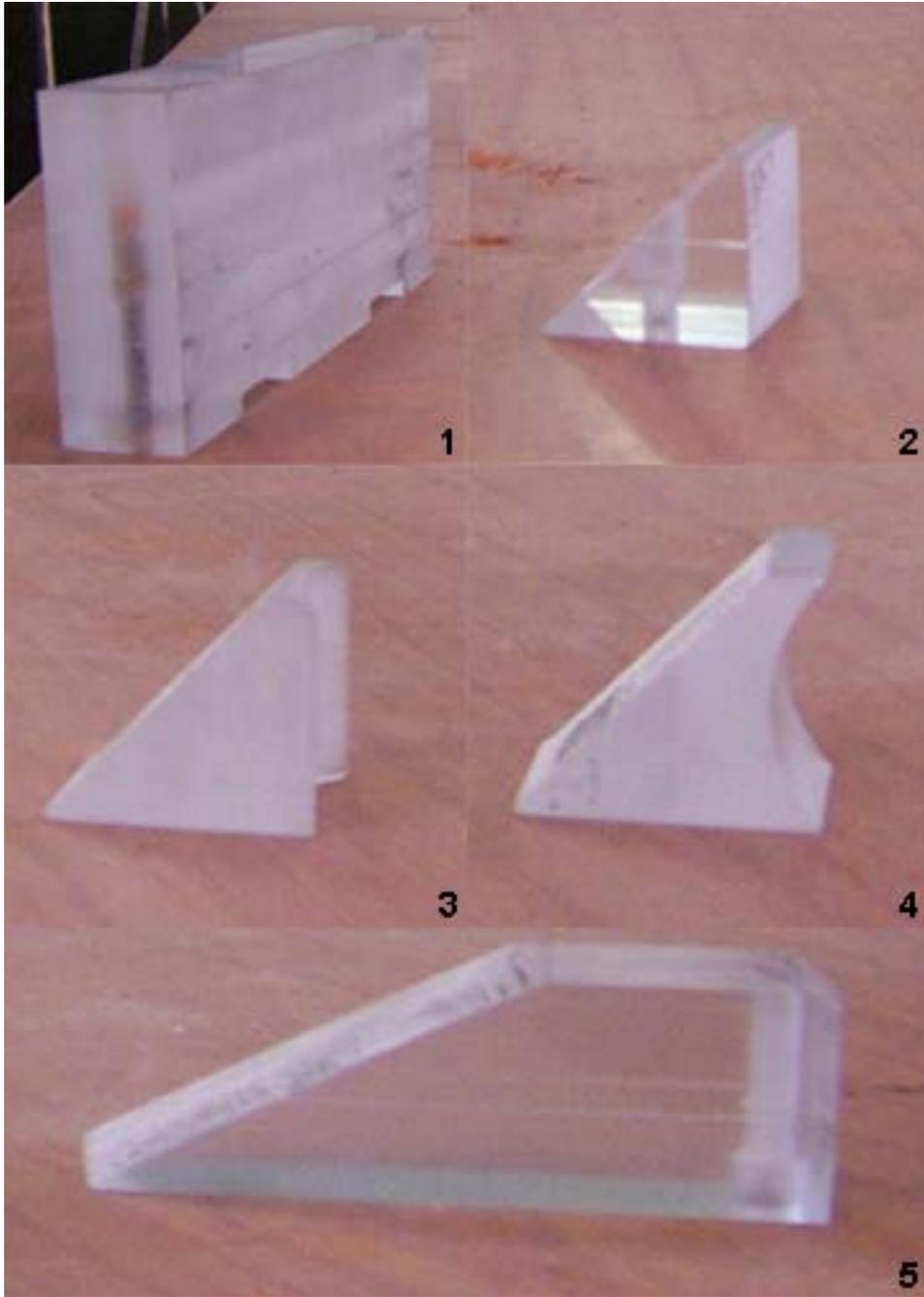
In addition to the Plexiglas<sup>®</sup> model of the culvert, a reservoir was constructed upstream of the model to collect and calm the fluid entering the model. The reservoir was constructed with plywood because it was not necessary to observe the behavior of the fluid at that stage (Figure 6). Within the reservoir, wing walls at an angle of 60 degrees were constructed to channel flow into the model opening. The base of the wing walls was constructed with plywood and the exposed wing wall models were formed with Plexiglas<sup>®</sup>. The same design was used for the outlet structure of the culvert.

The objective of the test was to determine the effect of sill and friction blocks on the hydraulic jump within the prototype, therefore the model was constructed so that different arrangements of friction blocks could be placed and observed within the model. Friction blocks were mounted in different arrangements on a sheet of Plexiglas<sup>®</sup> the same width as the barrels, and placed in the barrel. Three friction block shapes were selected: a regular flat faced, a semi-circular faced, and a c-shaped face blocks (Figure 8). Sills were located only on the horizontal portion of the model.

Additional sections of the culvert were constructed so that a qualitative analysis of a longer culvert could be conducted (Figure 9). To investigate the occurrence of a hydraulic jump without the aid of sills or friction blocks, only by extending the channel length, the horizontal channel was extended to 11.1 feet total. Also, two 3-foot sections were constructed and added to the model for several experiments. These sections served a dual purpose. During initial experimentation, it was observed that the original design was under pressure and that a theoretical hydraulic jump would occur above the confines of the existing culvert ceiling. These additional sections were inverted and mounted to the top of the original model making a culvert with 2 barrels with dimensions of 6 inches wide by 12 inches high and the original length of 62 inches (Figure 10). Access holes were cut into the bottom of these sections to allow for placement of a velocity meter when used as a cover for the expanded height.



**Figure 7.** Example of flat faced friction blocks arranged on model bottom.



**Figure 8.** Example of friction block shapes, sill, and chute blocks.  
(1. 3" Sill, 2. Regular flat-faced friction block, 3. Semi-circular friction block,  
4. C-shaped friction block, 5. Chute block)



**Figure 9.** Example of extended channel length under pressure condition.



**Figure 10.** Example of extended channel height to apply open channel condition.

## DATA COLLECTION

Many experiments were conducted to create energy dissipation within a broken-back culvert. Nearly 42 experiments were done for this model with variations in length, height, width, and energy dissipaters used. Each experiment tested three scenarios. They were run with upstream heads of 0.8d, 1.0d, and 1.2d with each depth denoted by A, B, or C, respectively. For example, 8A represents the 8<sup>th</sup> experiment run at 0.8d, 8B represents the 8<sup>th</sup> experiment run at 1.0d, and 8C represents the 8<sup>th</sup> experiment run at 1.2d. A SonTek 2D-side looking MicroADV sonar velocimeter was used to measure the velocity at the intake of the structure, after the hydraulic jump, and at the downstream end of the culvert. 2D-side looking denotes it has two receiver arms to give readings in the x and y planes. Also, a pitot tube was used to measure velocity at the toe before the hydraulic jump. The flow rates for all experiments were the same. For 0.8d, the flow rate was 0.77 cfs; for 1.0d, the flow rate was 1.25 cfs; and for 1.2d, the flow rate was 1.61 cfs. Also the velocity at the intake of the structure was the same for all experiments. For 0.8d, the velocity was 1.5 fps; for 1.0d, the velocity was 1.85 fps; and for 1.2d the velocity was 1.6 fps.

Experiments 1 through 20 and from 31 to 38 were run on a model with a culvert barrel 6 inches by 6 inches in area and a length of 5.1 feet which represented under pressure flow condition. The model was extended in length 6 feet for experiment 21 to a total length of 11.1 feet instead of 5.1 feet with the same 6 inches by 6 inches cross section and the same under pressure condition. For Experiments 22 through 30, and 39 through 42 the height of the culvert was raised to 12 inches with the original length of 5.1 feet and width of 6 inches which represented the open channel condition. Different configurations of chute blocks, friction blocks, and sills were used in the experiments. All results are shown in Table 1, and select experiment photos can be seen in Appendix A.

In these experiments, the length of the hydraulic jump ( $L$ ), the depth before the jump ( $Y_1$ ), the depth after the jump ( $Y_2$ ), the distance from the beginning of the hydraulic jump to the end of the culvert ( $X$ ), the depth of the water in the inclined channel ( $Y_s$ ), and the depth of the water downstream of the culvert ( $Y_{D/S}$ ) were

measured. All dimensions were measured by using a rule and point gage (Figure 13). The flowrate was measured by a two plate manometer between which measures the pressure difference in a fixed pipe opening size. As mentioned above, the velocity before the jump ( $V_1$ ) was measured by a pitot tube and the velocity at the inlet of structure as seen in Figure 9 ( $V_{u/s}$ ), the velocity after the jump ( $V_2$ ), and the velocity at downstream of culvert ( $V_{D/S}$ ) were all measured by ADV.

The procedure of the experiment is as follows:

1. Install energy dissipation (such as sills or friction blocks) in the model
2. Set point gage to the correct height in the reserve (for example, Experiment 1A means the head equal to  $0.8d$ ) (Figure 13)
3. Turn on pump in station
4. Adjust valve and coordinate the opening to obtain the amount of head for the experiment
5. Take the reading for flow rate (using a two plate manometer)
6. Run the model for 10 minute before taking measurements
7. Measure  $Y_s$ ,  $Y_1$ ,  $Y_2$ ,  $L$ ,  $X$ , and  $Y_{D/S}$
8. Measure velocities along the channel  $V_{u/s}$ ,  $V_1$ ,  $V_2$ , and  $V_{D/S}$
9. Post process the raw ADV data to determine final velocity values

Post processing the raw ADV data was essential to maintain data validity. A software program from the Bureau of Reclamation called WinADV was obtained to process the ADV data. The MicroADV was calibrated according to water temperature, salt content, and total suspended solids. The unit was calibrated to the manufacturer's specification for total suspended solid based on desired trace solution water content. At the end of each day of experiments, the reserve was drained to prevent mold growth in the reserve which could affect the suspended solid concentration of the water. If this change in sediment concentration were to occur, it could minimally affect velocity readings (Figure 12).

**Table 1.** Summary of collected hydraulic data.

No.	Condition	H. J.	No.	H	Date	Q	V <sub>u/s</sub>	Y <sub>s</sub>	Y <sub>toe</sub>	Y <sub>1</sub>	Y <sub>2</sub>	Y <sub>d/s</sub>	Fr <sub>1</sub>	V <sub>1</sub>	V <sub>2</sub>	V <sub>d/s</sub>	L	X	ΔE	THL	E <sub>2</sub> /E <sub>1</sub>			
1	5.1' horizontal channel without friction blocks	N	1a	.8d	1/22/2009	0.77	1.5	1.3	1.6	1.3	-	1.7	-	7.12	-	-	-	-	-	-	-	-		
			1b	1.0d	1/22/2009	1.25	1.85	2.0	2.0	1.6	-	2.1	-	9.37	-	-	-	-	-	-	-	-	-	
			1c	1.2d	1/22/2009	1.61	1.6	2.7	2.1	2.1	-	2.5	-	9.21	-	-	-	-	-	-	-	-	-	-
2	3 Chute Blocks for each channel	N	2a	.8d	1/23/2009	0.77	1.43	1.5	1.4	1.3	-	2.2	-	7.40	-	-	-	-	-	-	-	-	-	
			2b	1.0d	1/23/2009	1.25	1.75	2.0	2.0	2.0	-	2.0	-	7.49	-	-	-	-	-	-	-	-	-	-
			2c	1.2d	1/23/2009	1.61	1.6	2.5	2.4	2.0	-	2.5	-	9.67	-	-	-	-	-	-	-	-	-	-
3	3 Chute Blocks and 1" sill at 30.5" from the end	N	3a	.8d	1/23/2009	0.77	1.5	2.0	1.5	1.0	-	2.0	-	9.25	-	-	-	-	-	-	-	-	-	
			3b	1.0d	1/23/2009	1.25	1.85	2.5	2.0	1.5	-	2.5	-	9.99	-	-	-	-	-	-	-	-	-	-
			3c	1.2d	1/23/2009	1.61	1.6	2.5	2.5	2.0	-	2.5	-	9.67	-	-	-	-	-	-	-	-	-	-
4	3 Chute Blocks, 3 friction blocks, and 1" end sill	N	4a	.8d	1/23/2009	0.77	1.43	1.5	1.5	1.5	-	2.0	-	6.17	-	-	-	-	-	-	-	-	-	
			4b	1.0d	-	1.25	1.75	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
			4c	1.2d	-	1.61	1.6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	3 Chute Blocks and 1 row friction blocks (5) at 30.5" from end	N	5a	.8d	1/26/2009	0.77	1.5	2.0	1.5	1.5	-	2.0	-	6.17	-	-	-	-	-	-	-	-	-	
			5b	1.0d	1/26/2009	1.25	1.85	2.0	1.5	1.5	-	2.5	-	9.99	-	-	-	-	-	-	-	-	-	-
			5c	1.2d	1/26/2009	1.61	1.6	2.5	2.0	2.0	-	3.5	-	9.67	-	-	-	-	-	-	-	-	-	-
6	3 Chute Blocks, 4 rows friction blocks (5) 10" between them, and 1" end sill	N	6a	.8d	2/2/2009	0.77	1.43	1.5	1.6	1.6	-	3.0	-	5.78	-	-	-	-	-	-	-	-	-	
			6b	1.0d	2/2/2009	1.25	1.75	2.1	2.2	2.2	-	3.0	-	6.81	-	-	-	-	-	-	-	-	-	-
			6c	1.2d	2/2/2009	1.61	1.6	2.6	2.6	2.6	-	3.9	-	7.44	-	-	-	-	-	-	-	-	-	-
7	3 Chute Blocks, 4 rows friction blocks (5) 5" between them, and 1" end sill	N	7a	.8d	2/2/2009	0.77	1.5	1.5	1.8	2.1	-	2.1	-	4.40	-	-	-	-	-	-	-	-	-	
			7b	1.0d	2/2/2009	1.25	1.85	2.1	2.5	2.5	-	2.5	-	5.99	-	-	-	-	-	-	-	-	-	-
			7c	1.2d	2/2/2009	1.61	1.6	2.5	2.8	2.8	-	3.4	-	6.91	-	-	-	-	-	-	-	-	-	-

**Table 1. (continued)**

No.	Condition	H. J.	No.	H	Date	Q	V <sub>u/s</sub>	Y <sub>s</sub>	Y <sub>toe</sub>	Y <sub>1</sub>	Y <sub>2</sub>	Y <sub>d/s</sub>	Fr <sub>1</sub>	V <sub>1</sub>	V <sub>2</sub>	V <sub>d/s</sub>	L	X	ΔE	THL	E <sub>2</sub> /E <sub>1</sub>
8	3 Chute Blocks, 4 rows friction blocks (5) 5" between them, and 2" end sill	Y	8a	.8d	2/5/2009	0.77	1.43	1.5	2.5	2.6	5.5	5.5	1.35	3.56	-	-	8.5	16.5	-	-	0.95
			8b	1.0d	2/5/2009	1.25	1.75	2.0	2.5	2.8	<sup>6</sup> (U.P.)	<sup>6</sup> (U.P.)	-	5.35	-	-	11	22.5	-	-	0.91
			8c	1.2d	2/5/2009	1.61	1.6	2.5	2.9	2.9	6.0	6.0	-	6.67	-	-	20	55	-	-	0.84
9	3 Chute Blocks, 4 rows friction blocks (5) 5" between them, and 2" sill at 15" from end	N	9a	.8d	2/5/2009	0.77	1.5	1.5	2.1	3.0	-	3.0	-	3.08	-	-	-	-	-	-	-
		Y	9b	1.0d	2/5/2009	1.25	1.85	2.1	2.6	3.0	<sup>6</sup> (U.P.)	3.0	-	5.00	-	-	17	30.5	-	-	0.93
			9c	1.2d	2/5/2009	1.61	1.6	2.4	2.8	3.5	<sup>6</sup> (U.P.)	4.0	-	5.53	-	-	17	34.5	-	-	0.93
10	3 Chute Blocks, 4 rows friction blocks (5) 5" between them, and 2" sill at 30.5" from end	N	10a	.8d	2/6/2009	0.77	1.43	1.5	2.3	2.3	-	2.6	-	4.02	-	-	-	-	-	-	-
			10b	1.0d	2/6/2009	1.25	1.75	2.0	2.4	2.4	-	3.5	-	6.24	-	-	-	-	-	-	-
			10c	1.2d	2/6/2009	1.61	1.6	2.6	2.8	2.8	-	4.0	-	6.91	-	-	-	-	-	-	-
11	2" End Sill	N	11a	.8d	2/5/2009	0.77	1.5	1.4	1.8	1.5	-	<sup>6</sup> (U.P.)	-	6.17	-	-	-	-	-	-	-
		Y	11b	1.0d	2/5/2009	1.25	1.85	1.8	2.2	2.1	<sup>6</sup> (U.P.)	<sup>6</sup> (U.P.)	-	7.14	-	-	16	16	-	-	0.74
			11c	1.2d	2/5/2009	1.61	1.6	2.7	2.8	2.1	<sup>6</sup> (U.P.)	<sup>6</sup> (U.P.)	-	9.21	-	-	14	20	-	-	0.62
12	2" Sill at 15" from the end	Y	12a	.8d	2/5/2009	0.77	1.43	1.5	1.6	0.8	<sup>6</sup> (U.P.)	4.0	-	11.5 6	-	-	11	23	-	-	0.34
			12b	1.0d	2/5/2009	1.25	1.75	2.1	2.5	1.6	<sup>6</sup> (U.P.)	3.0	-	9.37	-	-	17	26	-	-	0.55
			12c	1.2d	2/5/2009	1.61	1.6	2.7	2.7	2.0	<sup>6</sup> (U.P.)	4.0	-	9.67	-	-	15	27	-	-	0.59
13	2" Sill at 30.5" from the end	N	13a	.8d	2/6/2009	0.77	1.5	1.5	1.5	1.5	-	2.3	-	6.17	-	-	-	-	-	-	-
		Y	13b	1.0d	2/6/2009	1.25	1.85	2.0	1.6	1.7	<sup>6</sup> (U.P.)	3.0	-	8.81	-	-	20	39	-	-	0.59
			13c	1.2d	2/6/2009	1.61	1.6	2.6	2.6	1.9	<sup>6</sup> (U.P.)	3.5	-	10.1 8	-	-	21	39	-	-	0.55
14	2.5" End Sill	Y	14a	.8d	2/28/2009	0.77	1.43	1.5	1.5	1.5	<sup>6</sup> (U.P.)	<sup>6</sup> (U.P.)	-	6.17	-	5.03	16	21	-	-	0.73
			14b	1.0d	2/28/2009	1.25	1.75	1.8	1.6	1.6	<sup>6</sup> (U.P.)	<sup>6</sup> (U.P.)	-	9.37	-	6.35	17	26	-	-	0.55
			14c	1.2d	2/28/2009	1.61	1.6	2.5	N	2.5	<sup>6</sup> (U.P.)	<sup>6</sup> (U.P.)	-	7.74	-	6.60	13	63	-	-	0.74

**Table 1. (continued)**

No.	Condition	H. J.	No.	H	Date	Q	V <sub>u/s</sub>	Y <sub>s</sub>	Y <sub>toe</sub>	Y <sub>1</sub>	Y <sub>2</sub>	Y <sub>d/s</sub>	Fr <sub>1</sub>	V <sub>1</sub>	V <sub>2</sub>	V <sub>d/s</sub>	L	X	ΔE	THL	E <sub>2</sub> /E <sub>1</sub>
15	3" End Sill	Y	15a	.8d	2/23/2009	0.77	1.5	1.5	1.7	1.7	<sup>6</sup> (U.P.)	<sup>6</sup> (U.P.)	-	5.44	-	-	18	48	-	-	0.81
			15b	1.0d	2/23/2009	1.25	1.85	2.4	N	2.4	<sup>6</sup> (U.P.)	<sup>6</sup> (U.P.)	-	6.24	-	-	14	68	-	-	0.83
			15c	1.2d	2/23/2009	1.61	1.6	3.2	N	3.2	<sup>6</sup> (U.P.)	<sup>6</sup> (U.P.)	-	6.04	-	-	20	81.5	-	-	0.89
16	3" Sill at 15" from the end	Y	16a	.8d	2/23/2009	0.77	1.43	1.5	1.2	1.2	<sup>6</sup> (U.P.)	4.0	-	7.71	-	-	17	33	-	-	0.58
			16b	1.0d	2/23/2009	1.25	1.75	1.5	1.5	1.7	<sup>6</sup> (U.P.)	4.3	-	8.81	-	-	17	36	-	-	0.59
			16c	1.2d	2/23/2009	1.61	1.6	2.7	N	2.7	<sup>6</sup> (U.P.)	4.5	-	7.16	-	-	17	68	-	-	0.79
17	2" sill @ 45" from the end and 3" sill @ 15" from the end	Y	17a	.8d	2/6/2009	0.77	1.5	1.6	N	1.6	<sup>6</sup> (U.P.)	3.5	-	5.78	-	-	17	67	-	-	0.77
			17b	1.0d	2/6/2009	1.25	1.85	2.5	N	2.5	<sup>6</sup> (U.P.)	4.0	-	5.99	-	-	19.5	71.5	-	-	0.85
			17c	1.2d	2/6/2009	1.61	1.6	3.3	N	3.3	<sup>6</sup> (U.P.)	5.0	-	5.86	-	-	19	77	-	-	0.90
18	2" sill @ 38" from the end and 3" sill @ 15" from the end	Y	18a	.8d	2/6/2009	0.77	1.43	1.8	N	1.8	<sup>6</sup> (U.P.)	3.0	-	5.29	-	-	15	61	-	-	0.83
			18b	1.0d	2/6/2009	1.25	1.75	2.5	N	2.5	<sup>6</sup> (U.P.)	3.5	-	5.99	-	-	13.5	68	-	-	0.85
			18c	1.2d	2/6/2009	1.61	1.6	3.2	N	3.2	<sup>6</sup> (U.P.)	5.0	-	6.04	-	-	17	71.5	-	-	0.89
19	2" sill @ 45" from the end and 3" sill @ 15" from the end	Y	19a	.8d	2/6/2009	0.77	1.5	1.5	1.6	1.6	<sup>6</sup> (U.P.)	3.5	-	5.97	-	-	14.5	45.5	-	-	0.75
			19b	1.0d	2/6/2009	1.25	1.85	2.2	N	2.2	<sup>6</sup> (U.P.)	4.0	-	6.81	-	-	14	60	-	-	0.77
			19c	1.2d	2/6/2009	1.61	1.6	2.8	N	2.8	<sup>6</sup> (U.P.)	4.5	-	6.91	-	-	13	66	-	-	0.82
20	2" sill @ 27" from the end and 3" sill @ 15" from the end	Y	20a	.8d	2/20/2009	0.77	1.5	1.5	1.5	1.5	<sup>6</sup> (U.P.)	3.0	-	6.17	-	3.09	18.00	43.00	-	14.84	0.73
			20b	1.0d	2/20/2009	1.25	1.85	2.0	1.8	1.8	<sup>6</sup> (U.P.)	3.5	-	8.33	-	4.16	20.00	55.00	-	14.31	0.63
			20c	1.2d	2/20/2009	1.61	1.6	2.6	N	2.5	<sup>6</sup> (U.P.)	4.0	-	7.74	-	3.98	18.00	62.00	-	15.13	0.74
21	11.1' horizontal channel without any friction blocks	N	21a	.8d	3/13/2009	0.77	1.5	1.5	1.5	1.5	-	1.5	-	6.17	-	4.2	-	-	-	14.83	-
			21b	1.0d	3/13/2009	1.25	1.85	2.0	2.0	1.8	-	1.9	-	8.33	-	7	-	-	-	10.01	-
			21c	1.2d	3/13/2009	1.61	1.6	2.4	2.2	-	-	2.3	-	8.41	-	7	-	-	-	10.65	-

**Table 1. (continued)**

No.	Condition	H. J.	No.	H	Date	Q	V <sub>u/s</sub>	Y <sub>s</sub>	Y <sub>toe</sub>	Y <sub>1</sub>	Y <sub>2</sub>	Y <sub>d/s</sub>	Fr <sub>1</sub>	V <sub>1</sub>	V <sub>2</sub>	V <sub>d/s</sub>	L	X	ΔE	THL	E <sub>2</sub> /E <sub>1</sub>			
22	1" End Sill with extended the channel height to 12"	N	22a	.8d	3/19/2009	0.77	1.43	1.6	1.7	2.0	-	-	-	4.63	-	-	-	-	-	-	-	-		
			22b	1.0d	3/19/2009	1.25	1.75	2.0	2.0	2.0	-	-	-	7.49	-	-	-	-	-	-	-	-	-	
			22c	1.2d	3/19/2009	1.61	1.6	2.4	2.5	2.5	-	-	-	7.74	-	-	-	-	-	-	-	-	-	-
23	2" End Sill with extended the channel height to 12"	N	23a	.8d	3/19/2009	0.77	1.5	1.7	1.7	2.0	-	3.0	-	4.63	-	-	-	-	-	-	-	-	-	
			23b	1.0d	3/19/2009	1.25	1.85	2.0	2.0	2.0	-	3.5	-	7.49	-	-	-	-	-	-	-	-	-	-
			23c	1.2d	3/19/2009	1.61	1.6	2.5	2.5	2.5	-	4.0	-	7.74	-	-	-	-	-	-	-	-	-	-
24	3" End Sill with extended the channel height to 12"	Y	24a	.8d	3/19/2009	0.77	1.43	1.5	1.5	1.2	7.0	7.0	4.46	7.71	1.00	1.0	17.0	25.0	5.81	12.39	0.56			
			24b	1.0d	3/19/2009	1.25	1.75	2.0	1.8	1.5	7.5	7.5	3.87	9.99	0.66	0.66	17.0	23.5	4.80	13.39	0.62			
			24c	1.2d	3/19/2009	1.61	1.6	2.5	2.5	2.0	8.0	8.0	3.16	9.67	0.88	0.88	17.0	17.0	3.38	13.93	0.72			
25	3.5" End Sill with extended the channel height to 12"	Y	25a	.8d	3/19/2009	0.77	1.5	1.5	1.5	1.0	7.5	7.5	5.65	9.25	0.75	0.75	18	30	9.15	12.01	0.46			
			25b	1.0d	3/19/2009	1.25	1.85	2.0	2.0	1.5	8.7	8.7	4.44	9.99	1.27	1.27	21	28	7.15	12.04	0.56			
			25c	1.2d	3/19/2009	1.61	1.6	2.5	2.5	2.0	9.5	9.5	3.70	9.67	0.75	0.75	22	27	5.55	12.47	0.65			
26	4" End Sill with extended the channel height to 12"	Y	26a	.8d	3/19/2009	0.77	1.5	1.5	1.5	1.2	8.5	8.0	4.30	7.71	1.60	3.78	20.00	34.00	9.53	8.96	0.58			
			26b	1.0d	3/19/2009	1.25	1.85	2.0	2.0	1.5	9.0	8.5	4.58	9.99	2.60	3.60	19.00	32.00	7.81	10.12	0.55			
			26c	1.2d	3/19/2009	1.61	1.6	2.8	2.5	2.2	10.0	9.5	3.55	8.79	2.60	4.50	20.00	31.00	5.39	8.80	0.66			
27	5" End Sill with extended the channel height to 12"	Y	27a	.8d	3/19/2009	0.77	1.5	1.5	-	1.5	9.0	9.0	4.58	6.17	1.24	1.24	19	64	7.81	10.33	0.55			
			27b	1.0d	3/19/2009	1.25	1.85	2.2	-	2.2	10.0	10.0	3.55	6.81	1.66	1.66	22	61	5.39	10.52	0.66			
			27c	1.2d	3/19/2009	1.61	1.6	2.5	2.3	2.3	11.5	11.5	3.87	8.41	2.00	2	24	59	7.36	9.83	0.62			
28	3.5" Sill @ 26" from the end with extended the channel height to 12"	Y	28a	.8d	3/20/2009	0.77	1.5	1.7	1.5	1.5	7.5	2.5	3.87	6.17	3.00	5.32	20.00	57.00	4.80	11.85	0.62			
			28b	1.0d	3/20/2009	1.25	1.85	2.1	2.5	2.0	8.5	3.2	3.34	7.49	3.50	5.40	21.00	58.00	4.04	12.40	0.69			
			28c	1.2d	3/20/2009	1.61	1.6	2.5	2.2	2.2	9.0	3.5	3.23	8.79	4.00	5.39	24.00	59.00	3.97	13.16	0.71			

**Table 1. (continued)**

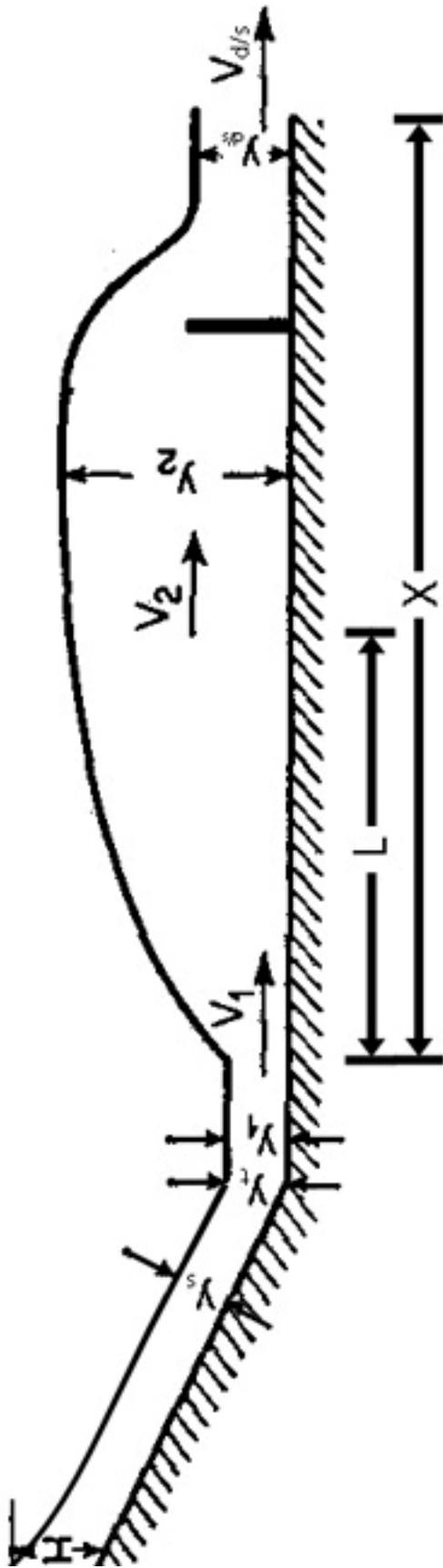
No.	Condition	H. J.	No.	H	Date	Q	V <sub>u/s</sub>	Y <sub>s</sub>	Y <sub>toe</sub>	Y <sub>1</sub>	Y <sub>2</sub>	Y <sub>d/s</sub>	Fr <sub>1</sub>	V <sub>1</sub>	V <sub>2</sub>	V <sub>d/s</sub>	L	X	ΔE	THL	E <sub>2</sub> /E <sub>1</sub>
29	3 rows friction blocks (3) 8" between them and 2.5" sill at 15" from end with extended the channel height to 12"	Y	29a	.8d	3/20/2009	0.77	1.5	1.5	1.5	1.5	6.0	2.5	3.16	6.17	-	2	11	27	2.53	16.37	0.72
			29b	1.0d	3/20/2009	1.25	1.85	2.0	2.0	2.0	7.0	3.0	2.81	7.49	-	2.46	12	20	2.23	16.91	0.77
			29c	1.2d	3/20/2009	1.61	1.6	2.5	2.5	2.5	7.0	3.2	2.31	7.74	-	3.4	13	20	1.30	16.72	0.86
30	3 rows friction blocks (3) 8" between them and 2.5" End sill with extended the channel height to 12"	Y	30a	.8d	3/20/2009	0.77	1.43	1.5	1.5	1.5	6.5	6.5	3.40	6.17	2.00	2	12	30	3.21	12.34	0.68
			30b	1.0d	3/20/2009	1.25	1.75	2.0	2.0	2.8	7.0	7.0	2.09	5.45	2.70	2.7	13	21	1.00	12.61	0.89
			30c	1.2d	3/20/2009	1.61	1.6	2.5	2.5	3.5	7.5	7.5	1.84	5.53	2.50	2.5	13	18	0.61	13.41	0.94
31	2" sill @ 27" from the end and 3" sill @ 15" from the end + 60 regular friction blocks	Y	31a	.8d	5/13/2009	0.77	1.5	1.5	-	1.5	6.0 (U.P.)	2.5	3.07	6.17	-	3.15	15	59	2.53	15.27	0.73
			31b	1.0d	5/13/2009	1.25	1.85	2.2	-	2.2	6.0 (U.P.)	3.0	2.80	6.81	-	2.74	15	61	1.04	16.64	0.77
			31c	1.2d	5/13/2009	1.61	1.6	3.0	-	3.0	6.0 (U.P.)	3.5	2.27	6.45	-	2.86	16	63	0.38	17.05	0.86
32	2" sill @ 27" from the end and 3" sill @ 15" from the end + 45 regular friction blocks	Y	32a	.8d	5/14/2009	0.77	1.5	1.6	1.8	1.8	6.0 (U.P.)	2.3	2.34	5.14	-	4.6	14	54	1.72	13.38	0.85
			32b	1.0d	5/14/2009	1.25	1.85	2.0	-	2.0	6.0 (U.P.)	3.0	3.23	7.49	-	4.82	14	61	1.33	13.71	0.71
			32c	1.2d	5/14/2009	1.61	1.6	3.0	-	3.0	6.0 (U.P.)	3.3	2.27	6.45	-	4.5	15	64	0.38	15.00	0.86
33	2" sill @ 27" from the end and 3" sill @ 15" from the end + 30 regular friction blocks	Y	33a	.8d	5/13/2009	0.77	1.5	1.6	1.6	1.6	6.0 (U.P.)	2.2	2.79	5.78	-	3.54	14	54	2.22	15.08	0.77
			33b	1.0d	5/13/2009	1.25	1.85	2.0	2.1	2.1	6.0 (U.P.)	2.5	3.01	7.14	-	4.01	16	59	1.18	15.54	0.74
			33c	1.2d	5/13/2009	1.61	1.6	3.0	-	3.0	6.0 (U.P.)	4.0	2.27	6.45	-	3.45	20	67	0.38	15.86	0.86
34	2" sill @ 27" from the end and 3" sill @ 15" from the end + 15 regular friction blocks @ 12.5 " from end	Y	34a	.8d	5/14/2009	0.77	1.5	1.6	1.6	1.6	6.0 (U.P.)	2.5	2.79	5.78	-	1.6	15	50	2.22	16.64	0.77
			34b	1.0d	5/14/2009	1.25	1.85	2.2	2.0	2.0	6.0 (U.P.)	2.8	3.23	7.49	-	3.97	16	55	1.33	15.30	0.71
			34c	1.2d	5/14/2009	1.61	1.6	2.8	-	2.8	6.0 (U.P.)	3.8	2.52	6.91	-	3.87	16	63	0.49	15.49	0.82
35	2" sill @ 27" from the end and 3" sill @ 15" from the end + 15 semi-circular friction blocks	Y	35a	.8d	5/26/2009	0.77	1.5	1.8	1.8	1.8	6.0 (U.P.)	3.0	2.34	5.14	-	5.59	16	50	1.72	12.00	0.85
			35b	1.0d	5/26/2009	1.25	1.85	2.0	2.2	2.0	6.0 (U.P.)	3.5	3.23	7.49	-	5.4	18	61	1.33	12.10	0.71
			35c	1.2d	5/26/2009	1.61	1.6	3.0	3.0	3.0	6.0 (U.P.)	4.2	2.27	6.45	-	4.71	20	67	0.38	13.74	0.86

**Table 1. (continued)**

No.	Condition	H. J.	No.	H	Date	Q	V <sub>u/s</sub>	Y <sub>s</sub>	Y <sub>toe</sub>	Y <sub>1</sub>	Y <sub>2</sub>	Y <sub>d/s</sub>	Fr <sub>1</sub>	V <sub>1</sub>	V <sub>2</sub>	V <sub>d/s</sub>	L	X	ΔE	THL	E <sub>2</sub> /E <sub>1</sub>
36	2" sill @ 27" from the end and 3" sill @ 15" from the end + 30 semi-circular friction blocks	Y	36a	.8d	5/26/2009	0.77	1.5	1.6	1.6	1.6	6.0 (U.P.)	2.5	2.79	5.78	-	3.70	16.0	50.0	2.22	14.57	0.77
			36b	1.0d	5/26/2009	1.25	1.85	2.0	2.0	2.0	6.0 (U.P.)	3.5	3.23	7.49	-	3.75	16.	49.0	1.33	14.92	0.71
			36c	1.2d	5/26/2009	1.61	1.6	2.5	2.5	2.5	6.0 (U.P.)	4.0	2.99	7.74	-	3.56	20.0	67.0	0.71	15.72	0.74
37	2" sill @ 27" from the end and 3" sill @ 15" from the end + 30 C-shaped friction blocks	Y	37a	.8d	5/27/2009	0.77	1.5	1.8	1.8	1.8	6.0 (U.P.)	3.0	2.34	5.14	-	4.72	11	50	1.72	12.47	0.85
			37b	1.0d	5/27/2009	1.25	1.85	2.2	2.2	2.2	6.0 (U.P.)	2.5	2.80	6.81	-	4.38	20	63	1.04	14.96	0.77
			37c	1.2d	5/27/2009	1.61	1.6	3.0	3.0	3.0	6.0 (U.P.)	3.8	2.27	6.45	-	3.64	21	68	0.38	15.81	0.86
38	2" sill @ 27" from the end and 3" sill @ 15" from the end + 15 C-shaped friction blocks	Y	38a	.8d	5/27/2009	0.77	1.5	2.0	2.0	2.0	6.0 (U.P.)	3.0	2.00	4.63	-	4.44	10.0	50.0	1.33	12.95	0.61
			38b	1.0d	5/27/2009	1.25	1.85	2.0	2.2	2.0	6.0 (U.P.)	3.5	3.23	7.49	-	4.44	17.0	63.0	1.33	13.86	0.57
			38c	1.2d	5/27/2009	1.61	1.6	3.0	3.0	3.0	6.0 (U.P.)	3.5	2.27	6.45	-	3.93	25.0	68.0	0.38	15.70	0.66
39	4" sill in the end + 15 C-shaped friction blocks	Y	39a	.8d	5/28/2009	0.77	1.5	2.0	2.0	2.0	8.6	2.50	3.38	2.09	3.86	19	60	4.18	8.64	0.82	0.69
			39b	1.0d	5/28/2009	1.25	1.85	2.0	2.6	2.0	9.0	8.4	3.52	7.49	2.46	3.86	13	48	4.76	9.86	0.67
			39c	1.2d	5/28/2009	1.61	1.6	3.0	2.6	3.0	10.0	9.5	2.69	6.45	3.01	4.16	18	48	2.86	9.35	0.79
40	3.5" Sill @ 26" from the end with extended the channel height to 12" with 15 regular FB	Y	40a	.8d	6/23/2009	0.77	1.50	1.6	-	1.6	8.0	2.2	3.87	5.78	3.17	5.82	20	61.5	5.12	11.11	0.62
			40b	1.0d	6/23/2009	1.25	1.85	2.1	-	2.1	9.0	3.0	3.37	7.14	3.73	6.2	21	61.5	4.35	10.88	0.69
			40c	1.2d	6/23/2009	1.61	1.60	2.2	-	2.3	10.0	3.2	3.41	8.41	2.97	6.15	24	51	4.96	11.83	0.68
41	3.5" Sill @ 26" from the end with extended the channel height to 12" with 15 Semi-circular FB	Y	41a	.8d	6/24/2009	0.77	1.50	1.6	-	1.6	8.0	2.0	3.87	5.78	1.18	5.82	20.0	62.0	5.12	11.31	0.62
			41b	1.0d	6/24/2009	1.25	1.85	2.1	-	2.1	9.0	2.9	3.37	7.14	1.45	6.41	21.0	62.0	4.35	10.48	0.69
			41c	1.2d	6/24/2009	1.61	1.60	2.2	2.2	2.3	9.5	3.2	3.26	8.41	2.43	7.00	24.0	51.0	4.27	9.75	0.71
42	3.5" Sill @ 26" from the end with extended the channel height to 12" with 15 C-shaped FB	Y	42a	.8d	6/24/2009	0.77	1.50	1.6	-	1.8	8.0	2.5	3.48	5.14	2.52	5.80	20.0	63.0	4.14	10.85	0.67
			42b	1.0d	6/25/2009	1.25	1.85	2.1	-	2.1	9.0	2.6	3.37	7.14	2.21	6.25	21.0	63.0	4.35	11.16	0.69
			42c	1.2d	6/25/2009	1.61	1.60	2.3	2.3	2.0	10.0	2.9	3.87	9.67	1.10	6.79	24.0	53.0	6.40	10.59	0.62

Table 1 variables key:

H. J.	= Hydraulic jump
Green Color	= Indicates the best scenario
H	= Head upstream of culvert, inches
Q	= Flow rate in cfs
$Y_s$	= Water depth at inclined channel, inch
$Y_{toe}$	= Water depth at toe of culvert, inch
$Y_1$	= Water depth before hydraulic jump in supercritical flow, inch
$Y_2$	= Water depth after hydraulic jump in subcritical flow, inch
$Y_{d/s}$	= Water depth at downstream of culvert, inch
$Fr_1$	= Froude Number in supercritical flow
$V_{u/s}$	= Velocity at upstream of culvert, fps
$V_1$	= Velocity before hydraulic jump in supercritical flow, fps
$V_2$	= Velocity after hydraulic jump in subcritical flow, fps
$V_{d/s}$	= Velocity downstream of culvert, fps
X	= Location of hydraulic jump from downstream end of culvert, inches
L	= Length of hydraulic jump, inch
$\Delta E$	= Energy loss due to hydraulic jump, inches
THL	= Total head loss for entire culvert, inches
E2/E1	= Efficiency of hydraulic jump
U.P.	= Under Pressure
N	= No hydraulic jump occurred
Y	= Hydraulic jump occurred



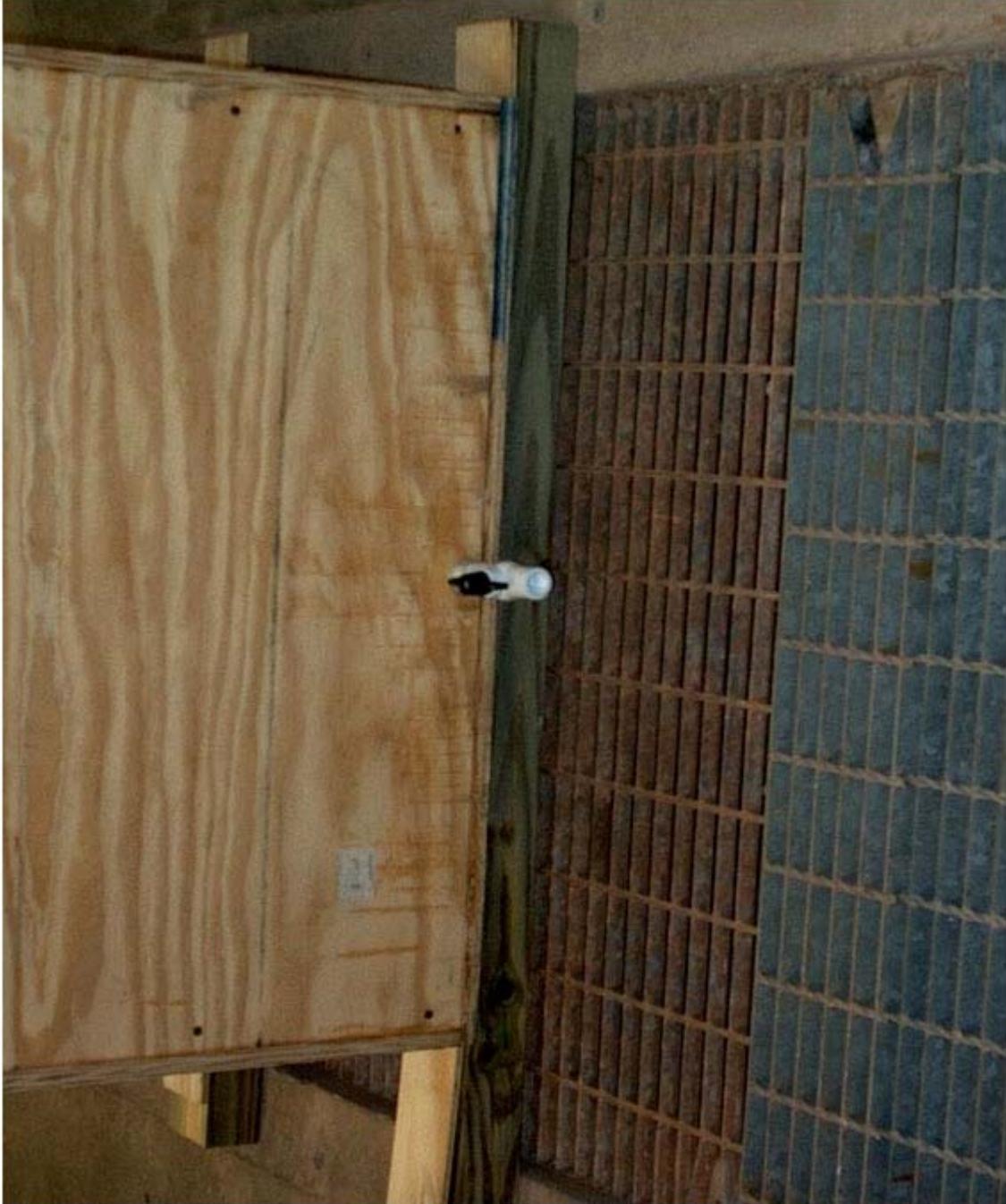
**Figure 11.** Drawing of hydraulic jump variables in a broken-back culvert.



**Figure 12.** Reservoir and channel inlet for culvert model.



**Figure 13.** Reservoir with point gage to measure water level.



**Figure 14.** Drain valve to drain water from reservoir.

## DATA ANALYSIS

Nine experiments were selected from forty-two experiments performed in the hydraulic laboratory. These experiments show the model runs without friction blocks, the effect of sill at the end of the model, and with friction blocks of different shapes as well as the sill. The friction blocks were comprised of three different shapes, including flat-faced friction blocks, semi-circular faced friction blocks, and C-shaped blocks. After the effectiveness was evaluated, the numbers of blocks were varied by 15, 30, 45, and 60 blocks.

Experiment 1 was run without any energy dissipation devices or sill in order to let us evaluate the hydraulic characteristics of the model, including the Froude number and supercritical flow conditions. This experiment is also an example of the current field practice to allow the kinetic energy of fluid to be transferred downstream without energy reduction. This experiment did not produce a hydraulic jump. The results can be found in Table 2, below.

**Table 2.** Hydraulic parameters for Experiment 1.

<b>Experiment 1A For 0.8d</b>	<b>Experiment 1B For 1.0d</b>	<b>Experiment 1C For 1.2d</b>
<b>Q = 0.77 cfs</b>	<b>Q = 1.25 cfs</b>	<b>Q = 1.61 cfs</b>
$Y_s = 1.3\text{in}$	$Y_s = 2.0\text{in}$	$Y_s = 2.7\text{in}$
$Y_t = 1.6\text{ in}$	$Y_t = 2.0\text{ in}$	$Y_t = 2.1\text{ in}$
$Y_1 = 1.3\text{ in}$	$Y_1 = 1.6\text{ in}$	$Y_1 = 2.1\text{ in}$
$Y_{d/s} = 1.7\text{ in}$	$Y_{d/s} = 2.1\text{ in}$	$Y_{d/s} = 2.5\text{ in}$
$V_1 = 7.12\text{ fps}$	$V_1 = 9.37\text{ fps}$	$V_1 = 9.21\text{ fps}$

The total head loss between upstream of structure and downstream of structure was calculated by applying the Bernoulli equation:

$$THL = \left( H + \frac{V_{u/s}^2}{2g} + Z \right) - \left( Y_{d/s} + \frac{V_{d/s}^2}{2g} \right) \tag{10}$$

Where

THL = Total head loss, inches

H = Water depth upstream of the culvert, inches

Z = Drop between upstream and downstream the model was 1.2 feet, representing a 24 foot drop in the prototype.

The loss of energy or energy dissipation in the jump was calculated by taking the difference between the specific energy before the jump and after the jump

$$\Delta E = E_1 - E_2 = \frac{(Y_2 - Y_1)^3}{4Y_1Y_2} \quad (11)$$

The efficiency of the jump was calculated by taking the ratio of the specific energy before and after the jump:

$$\frac{E_2}{E_1} = \frac{(8Fr_1^2 + 1)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2(2 + Fr_1^2)} \quad (12)$$

Where the downstream depth is known, the following equation was used to calculate the upstream supercritical flow Froude number (Fr) of the hydraulic jump:

$$Fr_1 = \sqrt{\frac{\left(\frac{2y_2}{y_1} + 1\right)^2 - 1}{8}} \quad (13)$$

If the downstream depth is unknown, the following equation was used to calculate the Froude number (Fr) of the hydraulic jump:

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} \quad (14)$$

Experiment 20 was run with a 3-inch sill located 15 inches from the downstream end of the culvert and a 2-inch sill located 27 inches from the downstream end of the culvert. Experiment 20 was chosen for two reasons: (1) a hydraulic jump formed inside the horizontal section of the model for all three flow conditions, and (2) it is an example of the field being under pressure due to the confined of the model. This experiment produced a hydraulic jump for all three conditions. The total head loss ranges between 14.3 inches to 15.1 inches. The results of this experiment are shown in Table 3.

**Table 3.** Hydraulic parameters for Experiment 20.

<b>Experiment 20A For 0.8d</b>	<b>Experiment 20B For 1.0d</b>	<b>Experiment 20C For 1.2d</b>
<b>Q = 0.77 cfs</b>	<b>Q = 1.25 cfs</b>	<b>Q = 1.61 cfs</b>
Y <sub>s</sub> = 1.5 in	Y <sub>s</sub> = 2 in	Y <sub>s</sub> = 2.6 in
Y <sub>t</sub> = 1.5 in	Y <sub>t</sub> = 1.8 in	Y <sub>t</sub> = None
Y <sub>1</sub> = 1.5 in	Y <sub>1</sub> = 1.8 in	Y <sub>1</sub> = 2.5 in
Y <sub>2</sub> = 6 in (under pressure)	Y <sub>2</sub> = 6 in (under pressure)	Y <sub>2</sub> = 6 in (under pressure)
Y <sub>d/s</sub> = 3 in	Y <sub>d/s</sub> = 3.5 in	Y <sub>d/s</sub> = 4 in
L = 18 in	L = 20 in	L = 18in
X = 43 in	X = 55 in	X = 62 in
V <sub>u/s</sub> = 1.5 fps	V <sub>u/s</sub> = 1.85 fps	V <sub>u/s</sub> = 1.6 fps
V <sub>1</sub> = 6.17 fps	V <sub>1</sub> = 8.33 fps	V <sub>1</sub> = 7.74 fps
V <sub>d/s</sub> = 3.09 fps	V <sub>d/s</sub> = 4.16 fps	V <sub>d/s</sub> = 3.98 fps
THL = 14.84 in.	THL = 14.31 in.	THL = 15.13 in.
E <sub>2</sub> /E <sub>1</sub> = 0.73	E <sub>2</sub> /E <sub>1</sub> = 0.63	E <sub>2</sub> /E <sub>1</sub> = 0.74

Experiment 26 was run with a 4-inch sill at the end of the culvert utilizing the increased culvert height of 12 inches. Experiment 26 illustrates the open channel flow condition, the fluid is at atmosphere pressure throughout the model, and the use of a single sill at the end to control the hydraulic jump. A hydraulic jump was observed in all three flow conditions. The results show that the Froude number values ranged from 3.7 to 4.5. This range of Froude number values is indicative of an Oscillating type of hydraulic jump. In an Oscillating jump, a cyclic jet of water enters the bottom of the jump and then rises to the water surface and back again with no periodicity in cycles. The energy dissipation due to hydraulic jump ranges between 5.8 inches to 6.3 inches and the total head loss for the whole culvert ranges between 8.8 inches to 10.1 inches. Additional results can be seen in Table 4.

**Table 4.** Hydraulic parameters for Experiment 26.

<b>Experiment 26A For 0.8d</b>	<b>Experiment 26B For 1.0d</b>	<b>Experiment 26C For 1.2d</b>
<b>Q = 0.77 cfs</b>	<b>Q = 1.25 cfs</b>	<b>Q = 1.61 cfs</b>
$Y_s = 1.5$ in	$Y_s = 2$ in	$Y_s = 2.8$ in
$Y_t = 1.5$ in	$Y_t = 2$ in	$Y_t = 2.5$ in
$Y_1 = 1.2$ in	$Y_1 = 1.5$ in	$Y_1 = 2.2$ in
$Y_2 = 8.5$ in	$Y_2 = 9$ in	$Y_2 = 10$ in
$Y_{d/s} = 8$ in	$Y_{d/s} = 8.5$ in	$Y_{d/s} = 9.5$ in
$L = 20$ in	$L = 19$ in	$L = 20$ in
$X = 34$ in	$X = 32$ in	$X = 31$ in
$V_{u/s} = 1.5$ fps	$V_{u/s} = 1.85$ fps	$V_{u/s} = 1.6$ fps
$V_1 = 7.71$ fps	$V_1 = 9.99$ fps	$V_1 = 8.79$ fps
$V_2 = 1.6$ fps	$V_2 = 2.6$ fps	$V_2 = 2.6$ fps
$Fr_1 = 5.35$	$Fr_1 = 4.58$	$Fr_1 = 3.55$
$V_{d/s} = 3.78$ fps	$V_{d/s} = 3.6$ fps	$V_{d/s} = 4.5$ fps
$\Delta E = 9.53$ inches	$\Delta E = 7.81$ inches	$\Delta E = 5.39$ inches
THL = 8.96 inches	THL = 10.12 inches	THL = 8.80 inches
$E_2/E_1 = 0.58$	$E_2/E_1 = 0.51$	$E_2/E_1 = 0.65$

Experiment 28 was run with a 3.5-inch sill 26 inches from the end of the culvert utilizing the increased culvert height of 12 inches. Experiment 28 was chosen to show a single sill located midway in the horizontal barrel under an open channel flow condition. A hydraulic jump was observed in all three flow conditions. The results show that the Froude number values ranged from 3.9 to 4.3. These ranges of Froude number values are indicative of an Oscillating type of hydraulic jump. The energy dissipation due to hydraulic jump ranges between 6.2 inches to 9.9 inches and the total head loss for the whole culvert ranges between 12.5 inches to 14.6 inches. Additional results can be seen in Table 5.

**Table 5.** Hydraulic parameters for Experiment 28.

<b>Experiment 28A For 0.8d</b>	<b>Experiment 28B For 1.0d</b>	<b>Experiment 28C For 1.2d</b>
<b>Q = 0.77 cfs</b>	<b>Q = 1.25 cfs</b>	<b>Q = 1.61 cfs</b>
$Y_s = 1.7$ in	$Y_s = 2.1$ in	$Y_s = 2.5$ in
$Y_t = 1.5$ in	$Y_t = 2.5$ in	$Y_t = 2.2$ in
$Y_1 = 1.5$ in	$Y_1 = 2$ in	$Y_1 = 2.2$ in
$Y_2 = 7.5$ in	$Y_2 = 8.5$ in	$Y_2 = 9$ in
$Y_{d/s} = 2.5$ in	$Y_{d/s} = 3.2$ in	$Y_{d/s} = 3.5$ in
$L = 20$ in	$L = 21$ in	$L = 24$ in
$X = 57$ in	$X = 58$ in	$X = 59$ in
$V_{u/s} = 1.5$ fps	$V_{u/s} = 1.85$ fps	$V_{u/s} = 1.6$ fps
$V_1 = 6.17$ fps	$V_1 = 7.49$ fps	$V_1 = 8.79$ fps
$V_2 = 3$ fps	$V_2 = 3.5$ fps	$V_2 = 4$ fps
$Fr_1 = 3.87$	$Fr_1 = 3.34$	$Fr_1 = 3.23$
$V_{d/s} = 5.32$ fps	$V_{d/s} = 5.40$ fps	$V_{d/s} = 5.39$ fps
$\Delta E = 4.80$ inches	$\Delta E = 4.04$ inches	$\Delta E = 3.97$ inches
THL = 11.85 inches	THL = 12.40 inches	THL = 13.16 inches
$E_2/E_1 = 0.73$	$E_2/E_1 = 0.71$	$E_2/E_1 = 0.65$

Experiment 34 was run with two sills, one 2-inch sill located 27 inches from the end of the culvert and a 3-inch sill located 15 inches from the end of the culvert. In addition, 15 flat faced friction blocks were placed in the horizontal portion of the channel in the pattern shown in Figure 7. Experiment 34 demonstrates the use of two sills to control the hydraulic jump under a pressure flow condition that is the fluid is excreting pressure against the top of the model. A hydraulic jump was observed in all three flow conditions. The results show that the Froude number values ranged from 2.5 to 3.2. These ranges of Froude number values are indicative of an Oscillating type of hydraulic jump. The energy dissipation due to hydraulic jump ranges between 0.7 inches to 2.2 inches and the total head loss for the whole culvert ranges between 14.6 inches to 15.7 inches. Additional results can be seen in Table 6.

**Table 6.** Hydraulic parameters for Experiment 34.

<b>Experiment 34A For 0.8d</b>	<b>Experiment 34B For 1.0d</b>	<b>Experiment 34C For 1.2d</b>
<b>Q = 0.77 cfs</b>	<b>Q = 1.25 cfs</b>	<b>Q = 1.61 cfs</b>
$Y_s = 1.6$ in	$Y_s = 2.2$ in	$Y_s = 2.8$ in
$Y_t = 1.6$ in	$Y_t = 2.0$ in	-
$Y_1 = 1.6$ in	$Y_1 = 2.0$ in	$Y_1 = 2.8$ in
$Y_2 = 6.0$ in (u.p.)	$Y_2 = 6.0$ in (u.p.)	$Y_2 = 6.0$ in (u.p.)
$Y_{d/s} = 2.5$ in	$Y_{d/s} = 2.8$ in	$Y_{d/s} = 3.8$ in
$L = 15$ in	$L = 16$ in	$L = 16$ in
$X = 50$ in	$X = 55$ in	$X = 63$ in
$V_{u/s} = 1.5$ fps	$V_{u/s} = 1.85$ fps	$V_{u/s} = 1.6$ fps
$V_1 = 5.78$ fps	$V_1 = 7.49$ fps	$V_1 = 6.91$ fps
-	-	-
$V_{d/s} = 1.6$ fps	$V_{d/s} = 3.97$ fps	$V_{d/s} = 3.87$ fps
THL = 16.64 inches	THL = 15.30 inches	THL = 15.49 inches

Experiment 36 was run with two sills, one 2-inch sill located 27 inches from the end of the culvert, and a 3 inch sill located 15 inches from the end of the culvert. In

addition, 30 Semi-circular faced friction blocks were placed in the horizontal portion of the channel in the pattern shown in Figure 7. Experiment 36 uses the same sill configuration as Experiment 34 with addition of semi-circular shaped friction block to investigate the further dissipation of energy. A hydraulic jump was observed in all three flow conditions. The results show that the Froude number values ranged from 2.8 to 3.2. These ranges of Froude number values are indicative of an Oscillating type of hydraulic jump. The energy dissipation due to hydraulic jump ranges between 0.5 inches to 2.2 inches and the total head loss for the whole culvert ranges between 15.3 inches to 16.6 inches. Additional results can be seen in Table 7.

**Table 7.** Hydraulic parameters for Experiment 36.

<b>Experiment 36A For 0.8d</b>	<b>Experiment 36B For 1.0d</b>	<b>Experiment 36C For 1.2d</b>
<b>Q = 0.77 cfs</b>	<b>Q = 1.25 cfs</b>	<b>Q = 1.61 cfs</b>
$Y_s = 1.6$ in	$Y_s = 2.0$ in	$Y_s = 2.5$ in
$Y_t = 1.6$ in	$Y_t = 2.0$ in	$Y_t = 2.5$ in
$Y_1 = 1.6$ in	$Y_1 = 2.0$ in	$Y_1 = 2.5$ in
$Y_2 = 6.0$ in (u.p.)	$Y_2 = 6.0$ in (u.p.)	$Y_2 = 6.0$ in (u.p.)
$Y_{d/s} = 2.5$ in	$Y_{d/s} = 3.5$ in	$Y_{d/s} = 4.0$ in
$L = 15$ in	$L = 16$ in	$L = 16$ in
$X = 50$ in	$X = 55$ in	$X = 63$ in
$V_{u/s} = 1.5$ fps	$V_{u/s} = 1.85$ fps	$V_{u/s} = 1.6$ fps
$V_1 = 5.78$ fps	$V_1 = 7.49$ fps	$V_1 = 7.74$ fps
-	-	-
$V_{d/s} = 3.70$ fps	$V_{d/s} = 3.75$ fps	$V_{d/s} = 3.56$ fps
THL = 14.57 inches	THL = 14.92 inches	THL = 15.72 inches

Experiment 38 was run with two sills, one 2-inch sill located 27 inches from the end of the culvert, and a 3-inch sill located 15 inches from the end of the culvert. In addition, 15 C-shaped faced friction blocks were placed in the horizontal portion of the channel in the pattern shown in Figure 7. Experiment 38 uses the same sill arrangement

as Experiment 34 with the addition of shaped friction blocks to investigate the effectiveness of further dissipating energy. A hydraulic jump was observed in all three flow conditions. The results show that the Froude number values ranged from 2.0 to 3.2. These ranges of Froude number values are indicative of either a Weak or Oscillating type of hydraulic jump. The energy dissipation due to hydraulic jump ranges between 0.4 inches to 1.3 inches and the total head loss for the whole culvert ranges between 13 inches to 15.7 inches. Additional results can be seen in Table 8.

**Table 8.** Hydraulic parameters for Experiment 38.

<b>Experiment 38A For 0.8d</b>	<b>Experiment 38B For 1.0d</b>	<b>Experiment 38C For 1.2d</b>
<b>Q = 0.77 cfs</b>	<b>Q = 1.25 cfs</b>	<b>Q = 1.61 cfs</b>
$Y_s = 2.0$ in	$Y_s = 2.0$ in	$Y_s = 2.0$ in
$Y_t = 2.0$ in	$Y_t = 2.2$ in	$Y_t = 3.0$ in
$Y_1 = 2.0$ in	$Y_1 = 2.0$ in	$Y_1 = 3.0$ in
$Y_2 = 6.0$ in (u.p.)	$Y_2 = 6.0$ in (u.p.)	$Y_2 = 6.0$ in (u.p.)
$Y_{d/s} = 3.0$ in	$Y_{d/s} = 3.5$ in	$Y_{d/s} = 3.5$ in
$L = 10$ in	$L = 16$ in	$L = 16$ in
$X = 50$ in	$X = 63$ in	$X = 68$ in
$V_{u/s} = 1.5$ fps	$V_{u/s} = 1.85$ fps	$V_{u/s} = 1.6$ fps
$V_1 = 4.63$ fps	$V_1 = 7.49$ fps	$V_1 = 6.45$ fps
-	-	-
$V_{d/s} = 4.44$ fps	$V_{d/s} = 3.75$ fps	$V_{d/s} = 3.56$ fps
$\Delta E = 1.33$ inches	$\Delta E = 1.33$ inches	$\Delta E = 0.38$ inches
THL = 12.95 inches	THL = 13.86 inches	THL = 15.70 inches
$E_2/E_1 = 0.61$	$E_2/E_1 = 0.57$	$E_2/E_1 = 0.66$

Experiment 40 was run with one 3.5-inch sill located 26 inches from the end of the culvert and utilized the increased culvert height of 12 inches. In addition, 15 flat faced friction blocks were placed in the horizontal portion of the channel in the pattern shown in Figure 7. Experiment 40 uses the same sill configuration as Experiment 34

with the addition of flat-faced friction blocks to further dissipate energy. A hydraulic jump was observed in all three flow conditions. The results show that the Froude number values ranged from 2.8 to 3.4. These ranges of Froude number values are indicative of an Oscillating type of hydraulic jump. The energy dissipation due to hydraulic jump ranges between 4.4 inches to 5.1 inches and the total head loss for the whole culvert ranges between 10.9 inches to 11.8 inches. Additional results can be seen in Table 9.

**Table 9.** Hydraulic parameters for Experiment 40.

<b>Experiment 40A For 0.8d</b>	<b>Experiment 40B For 1.0d</b>	<b>Experiment 40C For 1.2d</b>
<b>Q = 0.77 cfs</b>	<b>Q = 1.25 cfs</b>	<b>Q = 1.61 cfs</b>
$Y_s = 1.6$ in	$Y_s = 2.1$ in	$Y_s = 2.2$ in
*	*	*
$Y_1 = 1.6$ in	$Y_1 = 2.1$ in	$Y_1 = 2.3$ in
$Y_2 = 8.0$ in	$Y_2 = 9.0$ in	$Y_2 = 10.0$ in
$Y_{d/s} = 2.2$ in	$Y_{d/s} = 3.0$ in	$Y_{d/s} = 3.2$ in
$L = 20$ in	$L = 21$ in	$L = 24$ in
$X = 61.5$ in	$X = 61.5$ in	$X = 51$ in
$V_{u/s} = 1.5$ fps	$V_{u/s} = 1.85$ fps	$V_{u/s} = 1.6$ fps
$V_1 = 5.78$ fps	$V_1 = 7.14$ fps	$V_1 = 8.41$ fps
$Fr_1 = 3.87$	$Fr_1 = 3.37$	$Fr_1 = 3.41$
$V_{d/s} = 5.82$ fps	$V_{d/s} = 6.20$ fps	$V_{d/s} = 6.15$ fps
$\Delta E = 5.12$ inches	$\Delta E = 4.35$ inches	$\Delta E = 4.96$ inches
THL = 11.11 inches	THL = 10.88 inches	THL = 11.83 inches
$E_2/E_1 = 0.61$	$E_2/E_1 = 0.57$	$E_2/E_1 = 0.66$

\*Hydraulic jump formed on inclined part of culverts; unable to measure the flow depth at the toe.

# RESULTS

## PRESSURE CHANNEL FLOW CONDITION

After careful evaluation, two experiments were selected from the data analysis portion for a pressure flow condition. These experiments were selected by examining many factors, including their relatively low downstream velocities (3 to 4 fps), high total hydraulic head losses, and hydraulic jump efficiency. It was found that those experiments would be most applicable to modifying existing culverts with the addition of sills and/or friction blocks. Figure 15 shows characteristics of hydraulic jump for Experiment 20 in Table 10 and Figure 16 shows characteristics for Experiment 34 in Table 11. Figures 15 and 16 are also shown in Videos 1 and 2. Click on the video and wait one minute for it to play the experiment.

**Table 10.** Selected factors for Experiment 20.

<b>Experiment 20A For 0.8d</b>	<b>Experiment 20B For 1.0d</b>	<b>Experiment 20C For 1.2d</b>
$Y_2 = 6$ in (under pressure)	$Y_2 = 6$ in (under pressure)	$Y_2 = 6$ in (under pressure)
$V_{d/s} = 3.09$ fps	$V_{d/s} = 4.16$ fps	$V_{d/s} = 3.98$ fps
THL = 14.84 in.	THL = 14.31 in.	THL = 15.13 in.
$E_2/E_1 = 0.73$	$E_2/E_1 = 0.63$	$E_2/E_1 = 0.74$
Channel reduction = none	Channel reduction = none	Channel reduction = none

**Table 11.** Selected factors for Experiment 34.

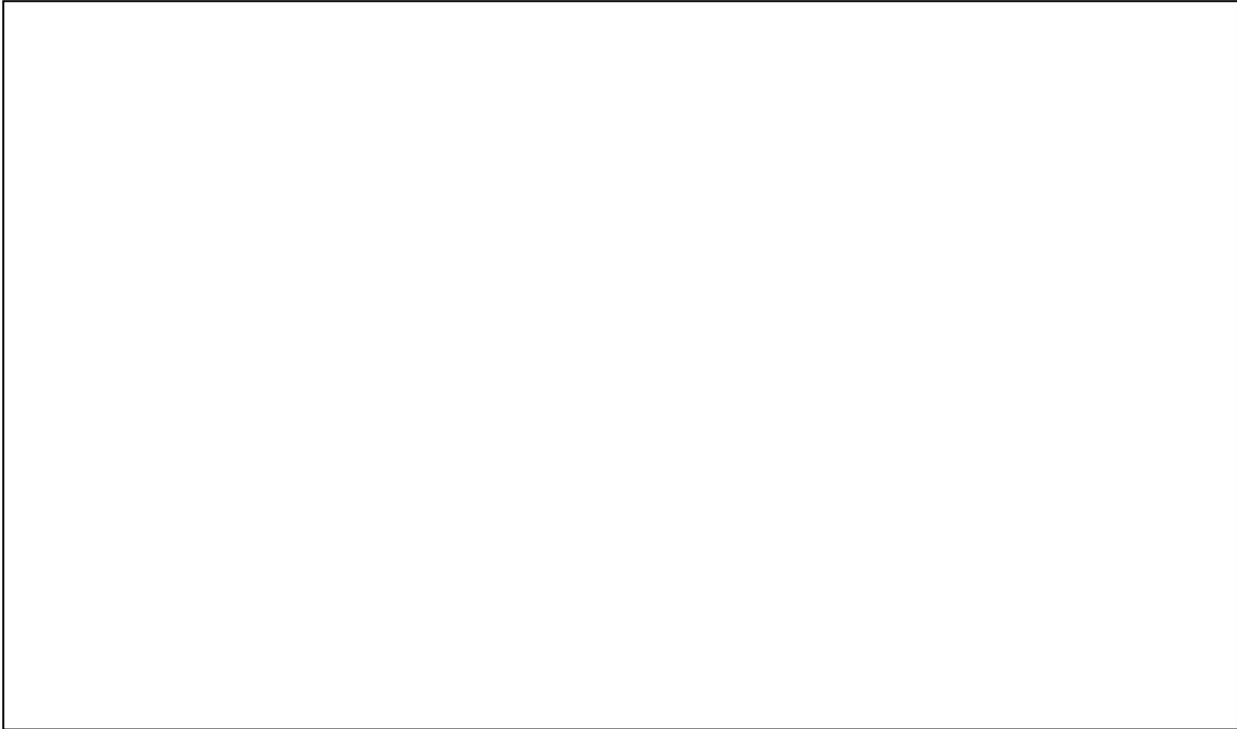
<b>Experiment 34A For 0.8d</b>	<b>Experiment 34B For 1.0d</b>	<b>Experiment 34C For 1.2d</b>
$Y_2 = 6$ in (under pressure)	$Y_2 = 6$ in (under pressure)	$Y_2 = 6$ in (under pressure)
$V_{d/s} = 1.6$ fps	$V_{d/s} = 3.97$ fps	$V_{d/s} = 3.87$ fps
THL = 16.64 inches	THL = 15.30 inches	THL = 15.49 inches
$E_2/E_1 = 0.77$	$E_2/E_1 = 0.71$	$E_2/E_1 = 0.82$
Channel reduction = none	Channel reduction = none	Channel reduction = none



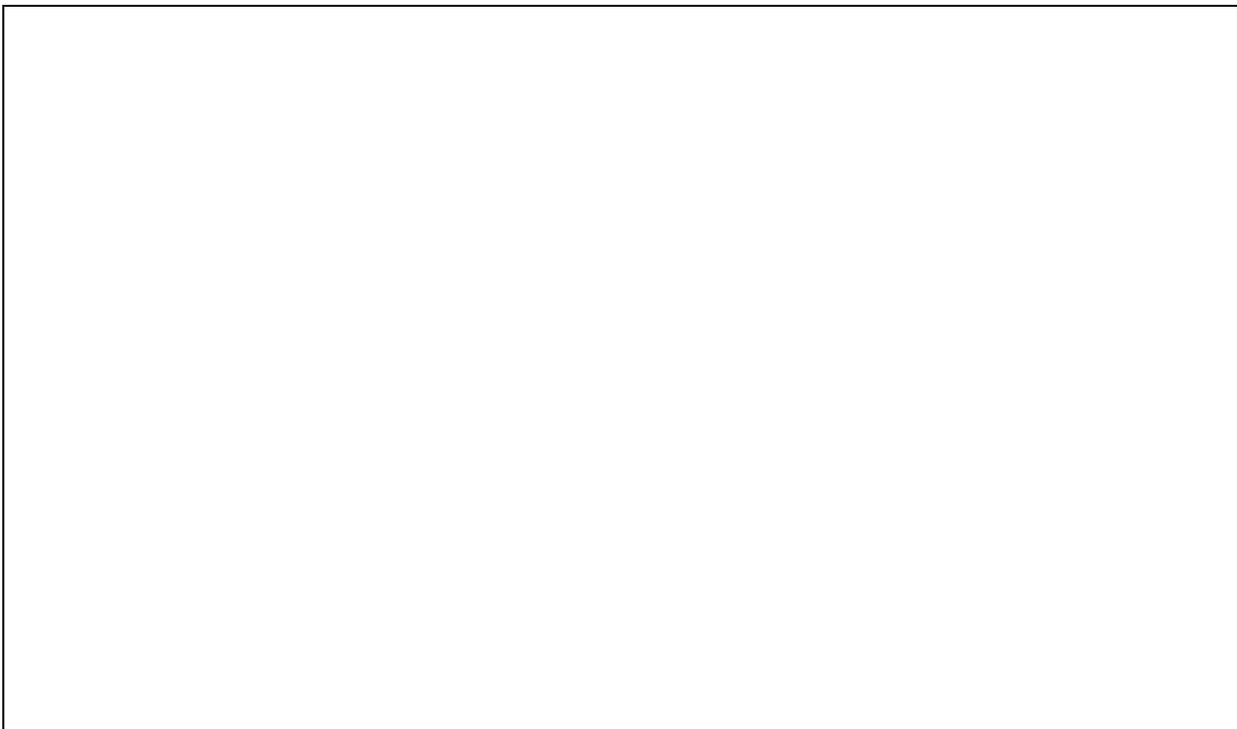
**Figure 15.** Characteristics of hydraulic jump for Experiment 20C under pressure flow condition.



Figure 16. Characteristics of hydraulic jump for Experiment 34C under pressure flow condition.



**Video 1.** Characteristics of hydraulic jump for Experiment 20C under pressure flow condition.



**Video 2.** Characteristics of hydraulic jump for Experiment 34C under pressure flow condition.

## OPEN CHANNEL FLOW CONDITION

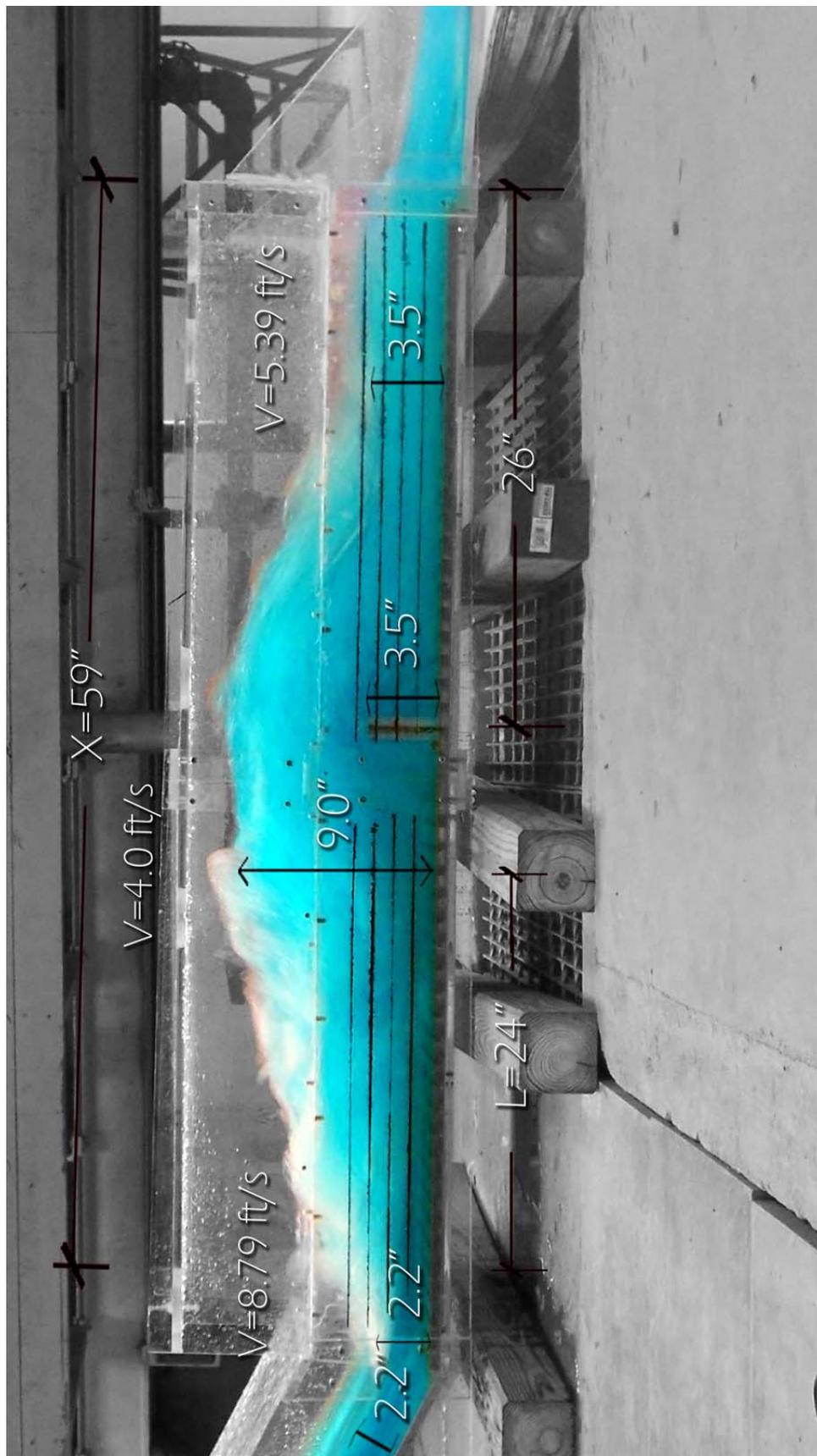
After careful evaluation, two experiments were selected from the data analysis portion for an open channel flow condition. These experiments were selected by examining many factors, including their relatively low downstream velocities, high total hydraulic head losses, acceptable hydraulic jump efficient, and possible reduction in channel length. Experiments 28 and 40 have the same sill arrangements, although Experiment 40 has friction blocks added to the horizontal channel barrel. It was found that these experiments would be most applicable to the new construction of culverts due to the increased ceiling height of the culvert. Channel could be reduced by reducing a section at the end of channel where the water surface profile is more uniform. Figure 17 shows characteristics of hydraulic jump for Experiment 28 in Table 12 and Figure 18 shows characteristics for Experiment 40 in Table 13. Figures 17 and 18 are also shown in Videos 3 and 4. Click on the video and wait one minute for it to play the experiment.

**Table 12.** Selected factors for Experiment 28.

<b>Experiment 28A For 0.8d</b>	<b>Experiment 28B For 1.0d</b>	<b>Experiment 28C For 1.2d</b>
$Y_2 = 7.5$ in	$Y_2 = 8.5$ in	$Y_2 = 9$ in
$V_{d/s} = 5.32$ fps	$V_{d/s} = 5.40$ fps	$V_{d/s} = 5.39$ fps
THL = 11.85 inches	THL = 12.40 inches	THL = 13.16 inches
$E_2/E_1 = 0.73$	$E_2/E_1 = 0.71$	$E_2/E_1 = 0.65$
Channel reduction = 13.5 inches	Channel reduction = 12.5 inches	Channel reduction = 9 inches

**Table 13.** Selected factors for Experiment 40.

<b>Experiment 40A For 0.8d</b>	<b>Experiment 40B For 1.0d</b>	<b>Experiment 40C For 1.2d</b>
$Y_2 = 8.0$ in	$Y_2 = 9.0$ in	$Y_2 = 10.0$ in
$V_{d/s} = 5.82$ fps	$V_{d/s} = 6.20$ fps	$V_{d/s} = 6.15$ fps
THL = 11.11 inches	THL = 10.88 inches	THL = 11.83 inches
$E_2/E_1 = 0.61$	$E_2/E_1 = 0.57$	$E_2/E_1 = 0.66$
Channel reduction = 14.5 inches	Channel reduction = 14 inches	Channel reduction = 12 inches



**Figure 17.** Characteristics of hydraulic jump for Experiment 28C under open channel flow condition.

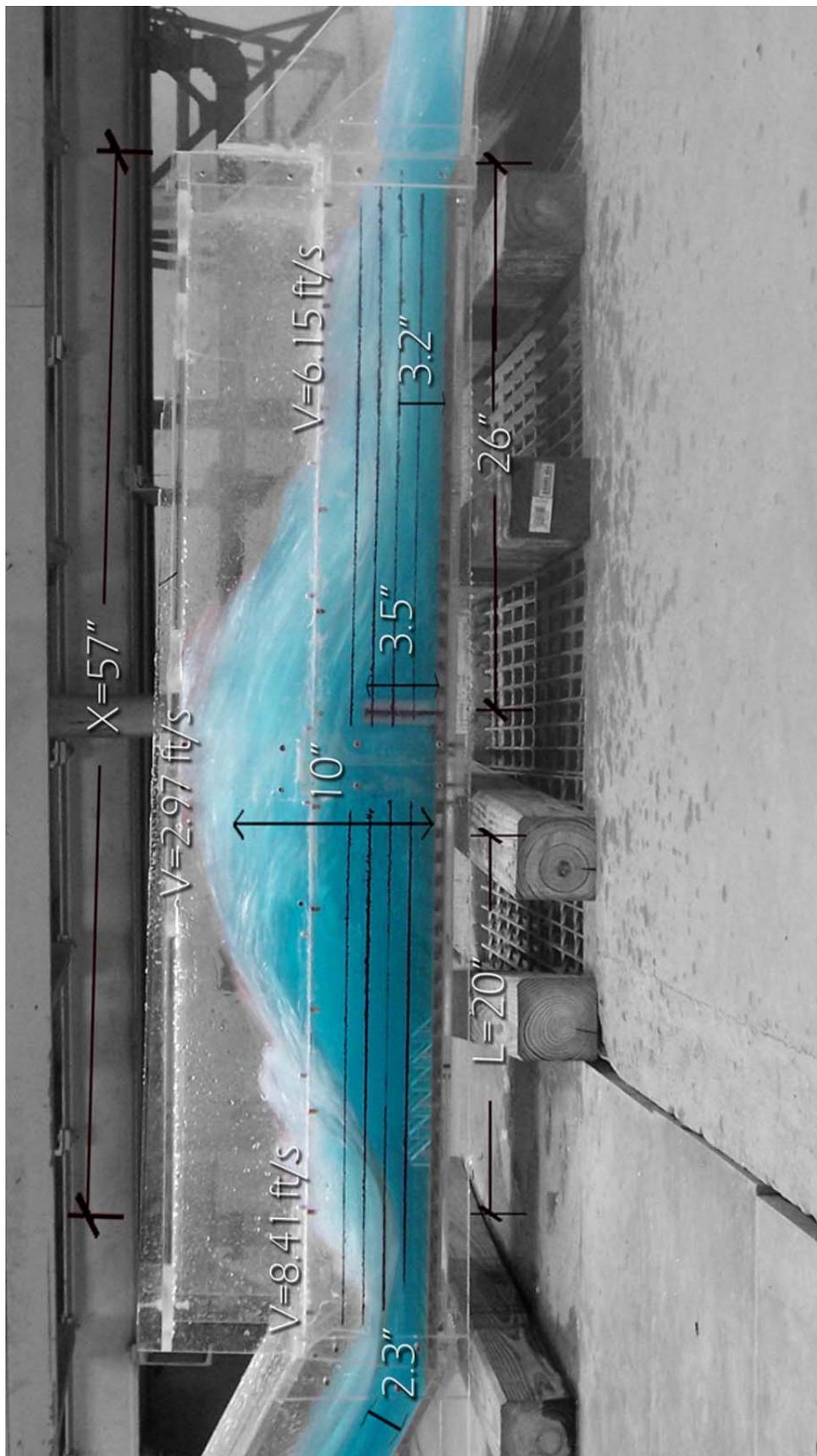
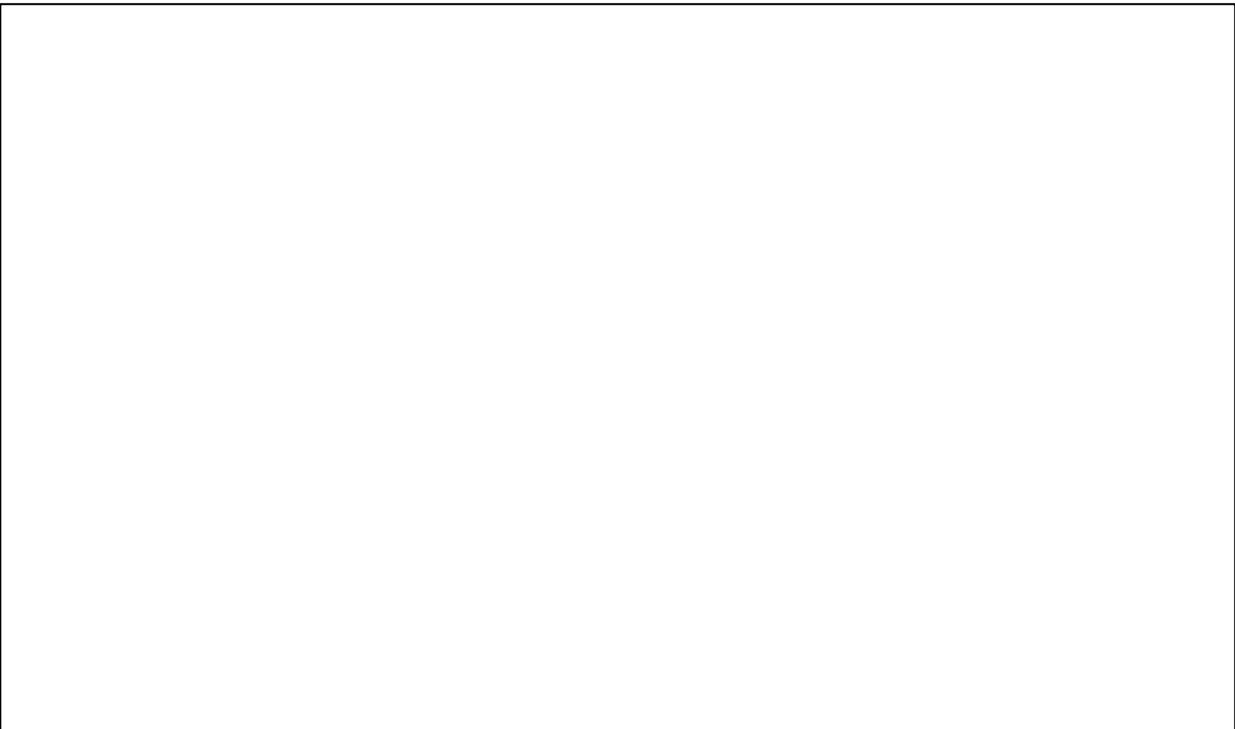


Figure 18. Characteristics of hydraulic jump for Experiment 40C under open channel flow condition.



**Video 3.** Characteristics of hydraulic jump for Experiment 28C under pressure flow condition.



**Video 4.** Characteristics of hydraulic jump for Experiment 40C under pressure flow condition.

# CONCLUSIONS

A scale model was constructed to represent a broken-back culvert. The idealized prototype contains a 1 (vertical) to 2 (horizontal) slope, continuing down to a 100 foot flat culvert with a 1 percent slope. The following dimensions are in terms of the prototype culvert. The following conclusions can be drawn based on the laboratory experiments:

1. For retrofitting an existing culvert, Experiment 20 is the best option for pressure flow condition. It consists of three flow conditions: 0.8, 1.0 and 1.2 times the upstream culvert depth. This scenario uses two sills with two small orifices at the bottom, so that water can be completely drained from the culvert. The sills are located at distances of 45 feet and 25 feet from the outlet face of the culvert.
2. Experiment 20 offers similar performance to friction block experiments without the additional cost.
3. For Experiment 20, no reduction in culvert length can be made due to the full flow at the end of the culvert, as can be seen in Table 10.
4. For new culvert construction, Experiment 28 is the best option for an open channel flow condition. This option includes one sill with two small orifices at the bottom for draining the culvert completely. The sill is located 43 feet from the end of the culvert. The height of the culvert should be 15 feet to allow open channel condition in the culvert.
5. Experiment 28 offers similar performance to friction block experiments without the additional cost.
6. Experiment 28 shows an opportunity to reduce the culvert length at the end in the range of 15 to 44 feet. The 15-foot reduction was determined by eliminating the downstream segment of the culvert where the water surface is no longer uniform after the jump. The 44-foot reduction results from removing a portion of the downstream culvert from the sill to the beginning of the downstream wing-wall section. This would result in a cost savings of \$150,000 to \$440,000 per culvert, assuming that the construction cost of a culvert is \$1,000,000 in Oklahoma. This option is important if there are problems with the right-of-way.

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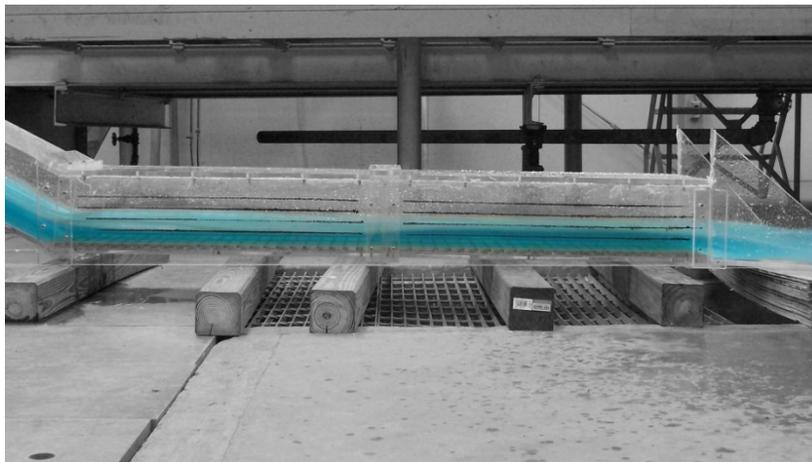
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# **APPENDIX A**

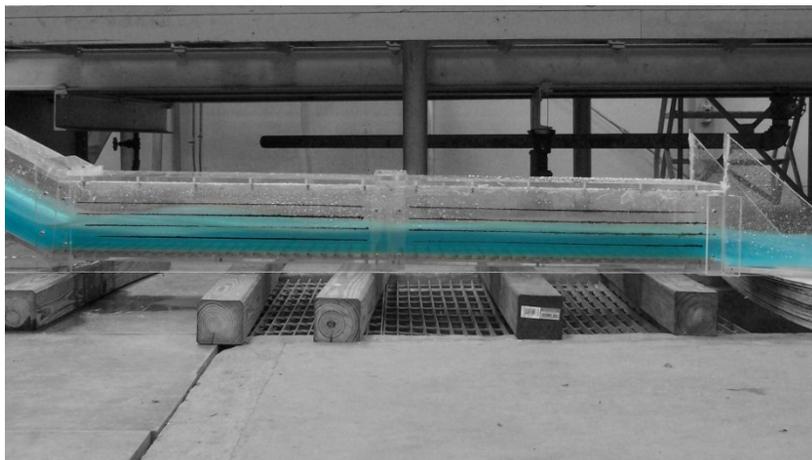
## **LABORATORY EXPERIMENTS FOR HYDRAULIC JUMP**



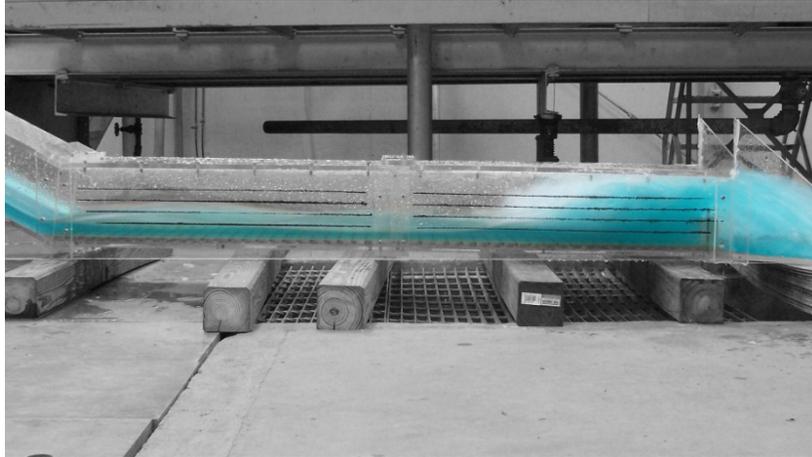
**Figure A1.** Experiment 1A.



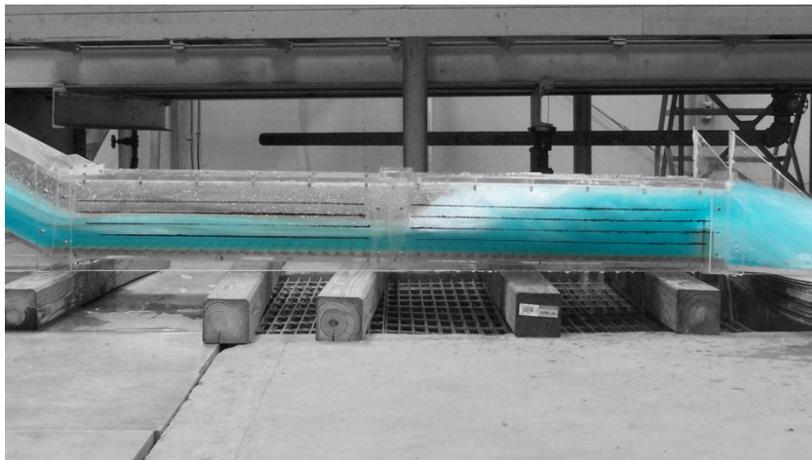
**Figure A2.** Experiment 1B



**Figure A3.** Experiment 1C



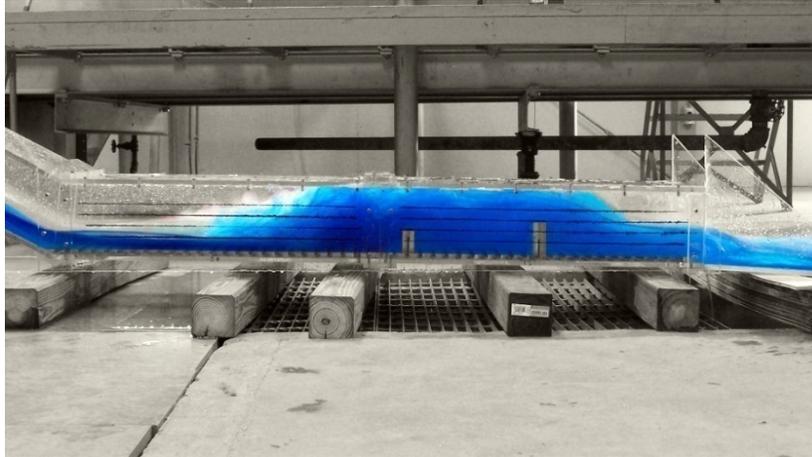
**Figure A4.** Experiment 14A



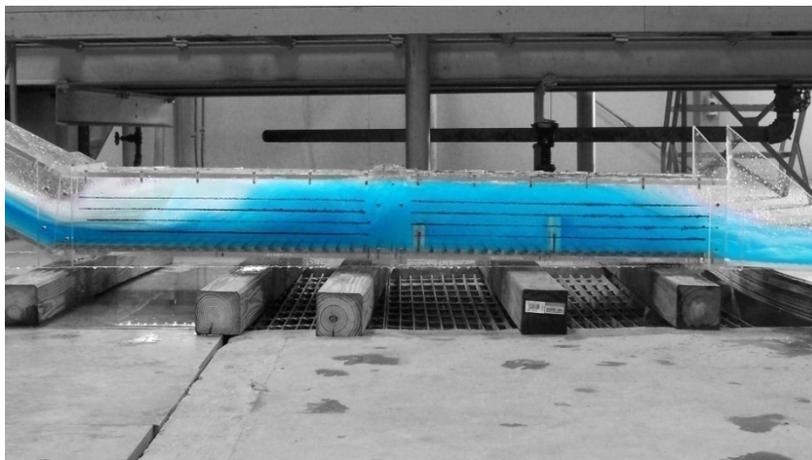
**Figure A5.** Experiment 14B



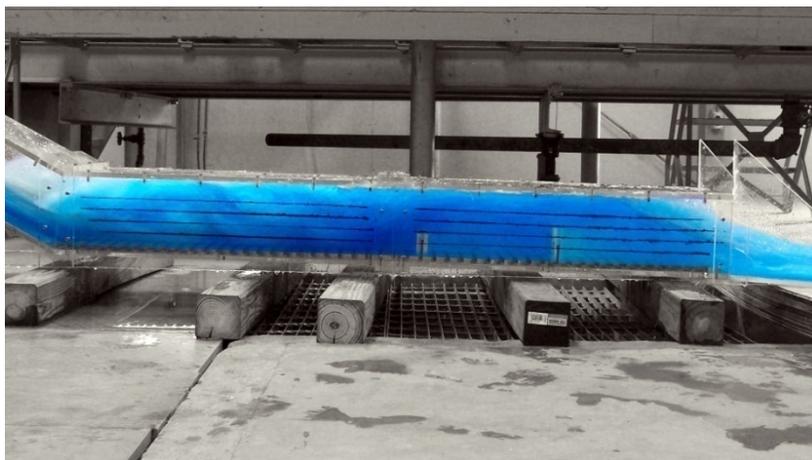
**Figure A6.** Experiment 14C



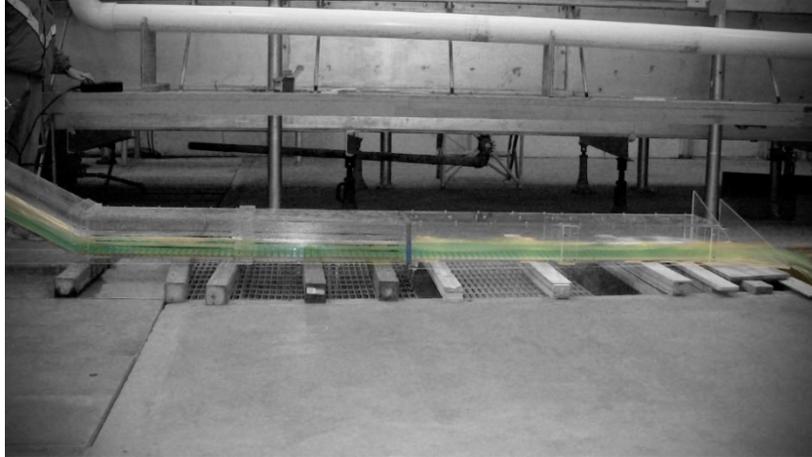
**Figure A7.** Experiment 20A



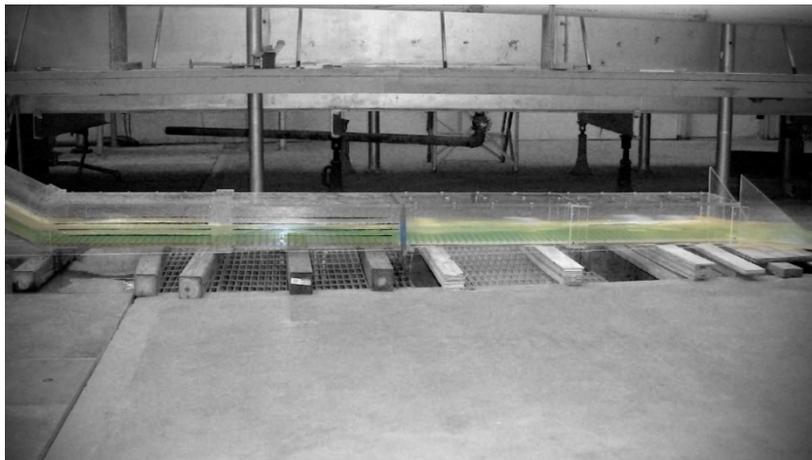
**Figure A8.** Experiment 20B



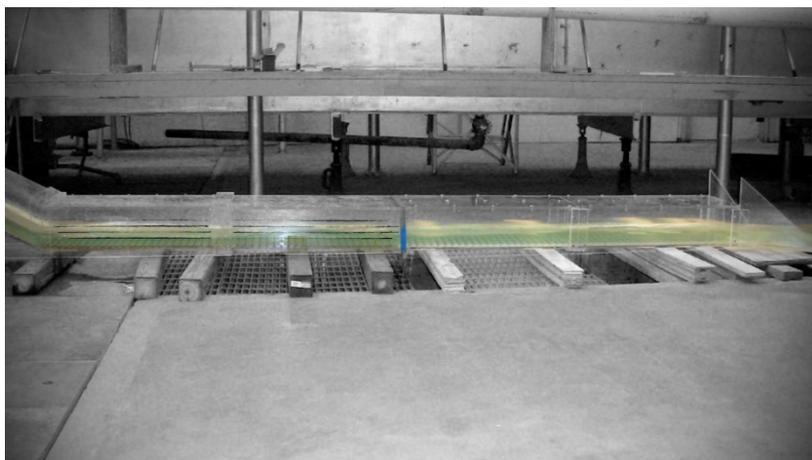
**Figure A9.** Experiment 20C



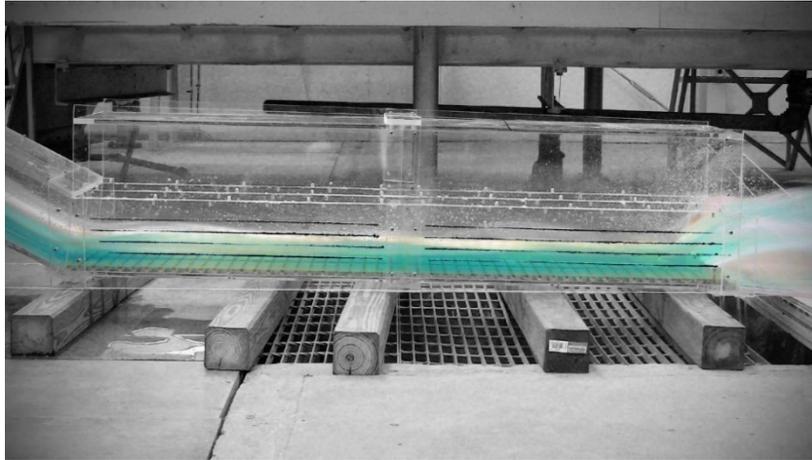
**Figure A10.** Experiment 21A



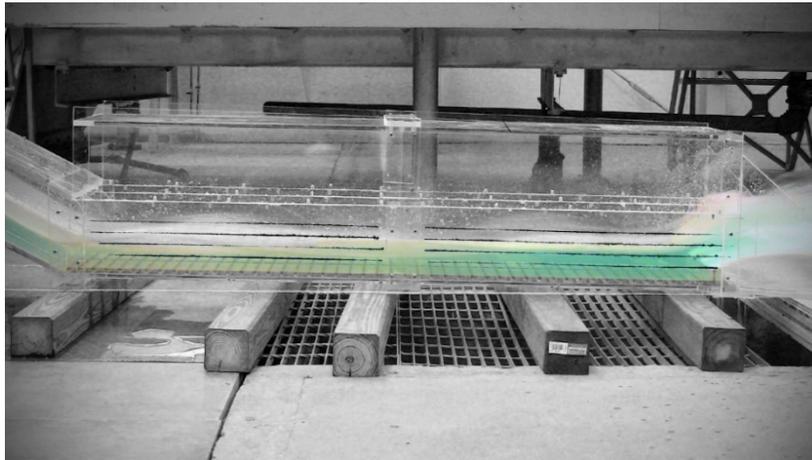
**Figure A11.** Experiment 21B



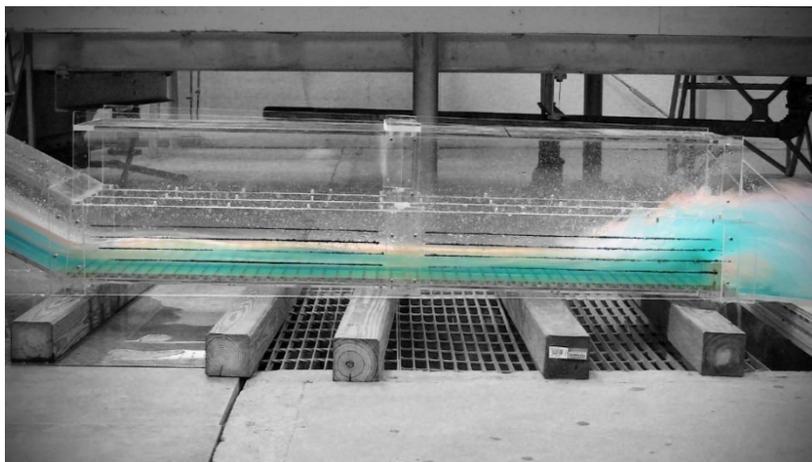
**Figure A12.** Experiment 21C



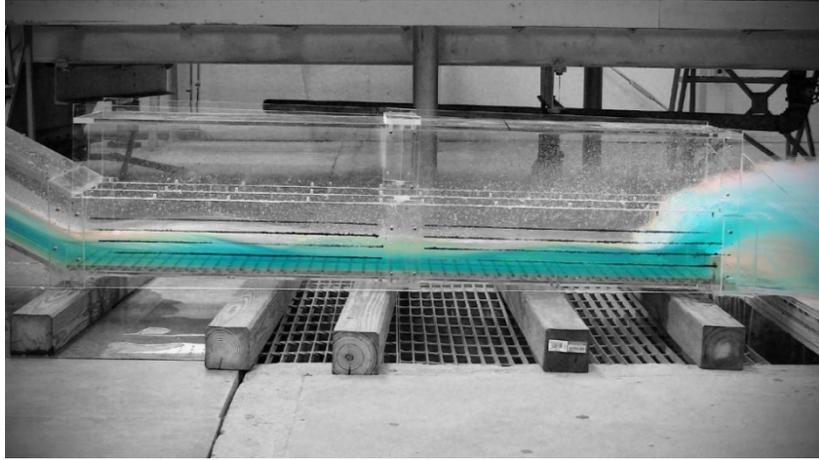
**Figure A13.** Experiment 22A



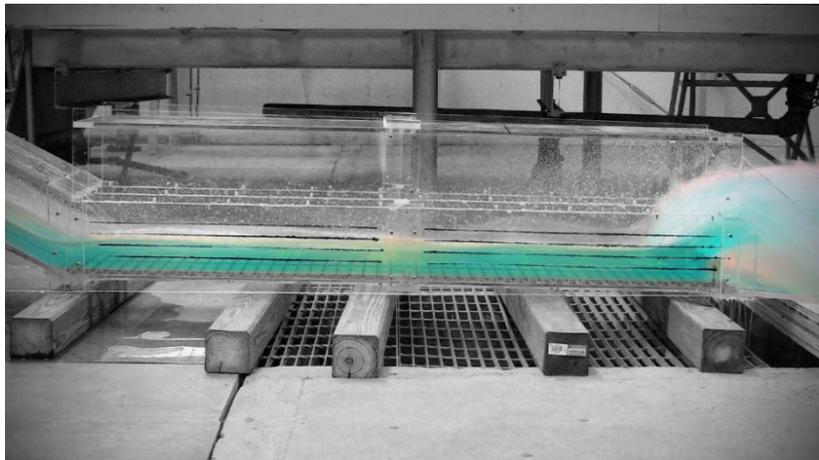
**Figure A14.** Experiment 22B



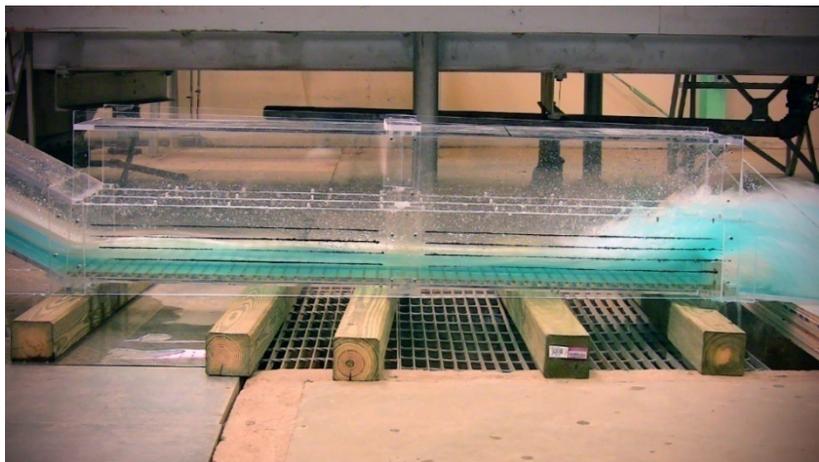
**Figure A15.** Experiment 22C



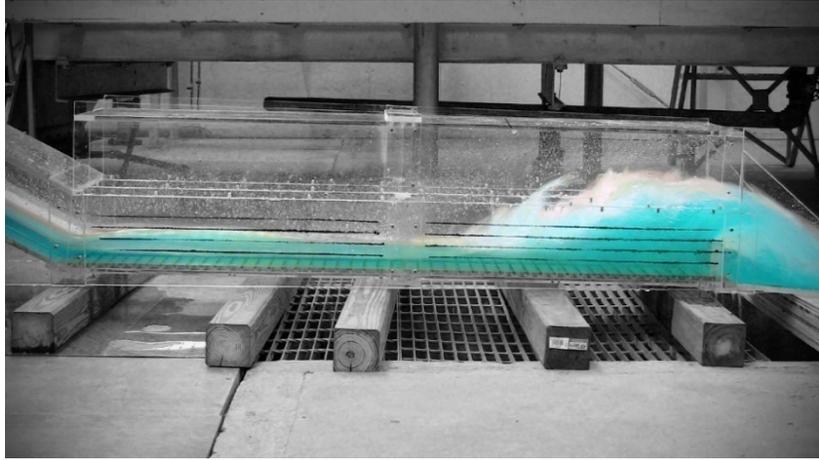
**Figure A16.** Experiment 23A



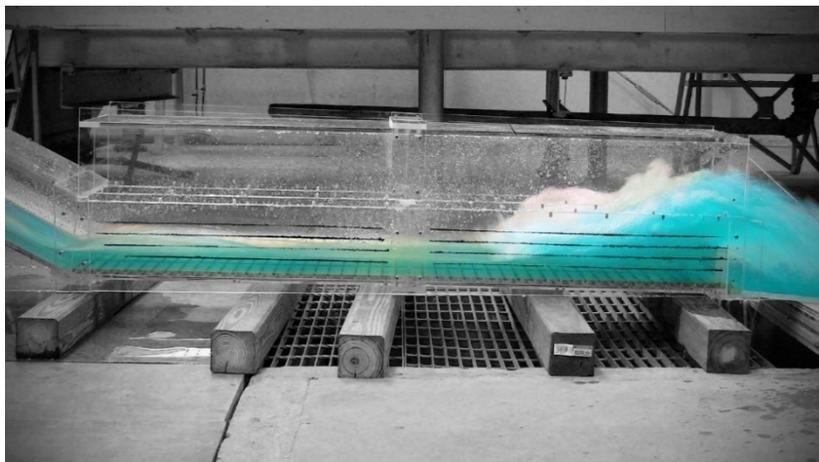
**Figure A17.** Experiment 23B



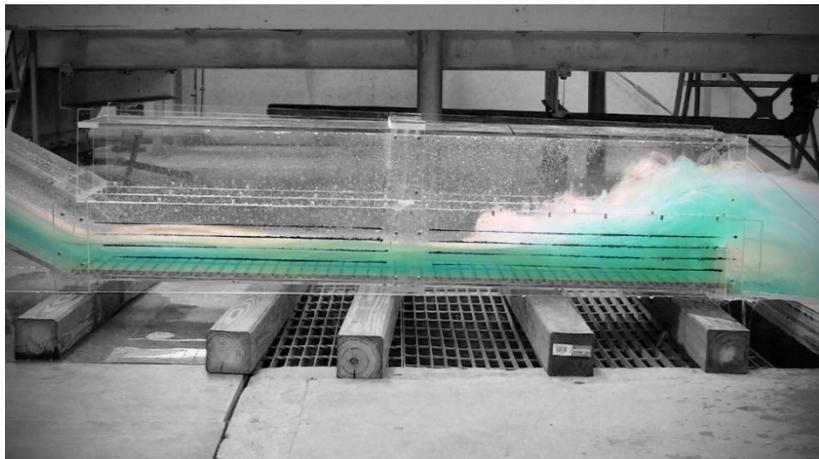
**Figure A18.** Experiment 23C



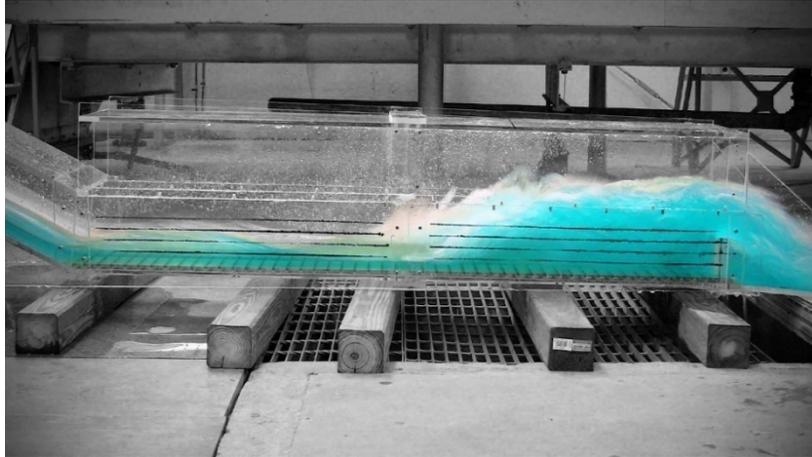
**Figure A19.** Experiment 24A



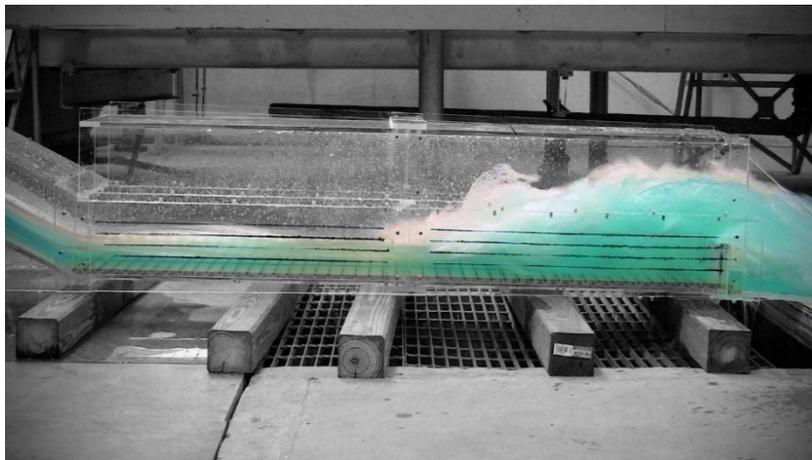
**Figure A20.** Experiment 24B



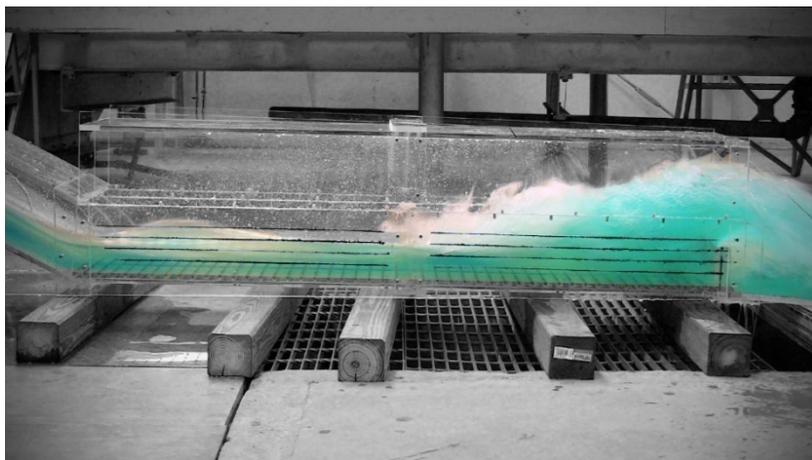
**Figure A21.** Experiment 24C



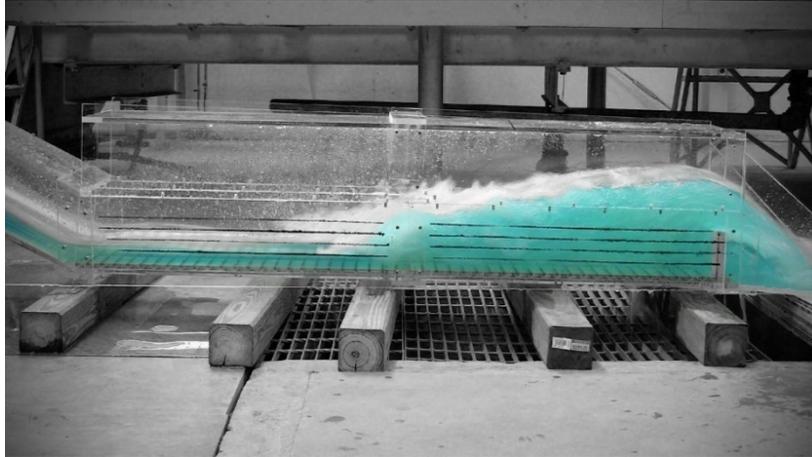
**Figure A22.** Experiment 25A



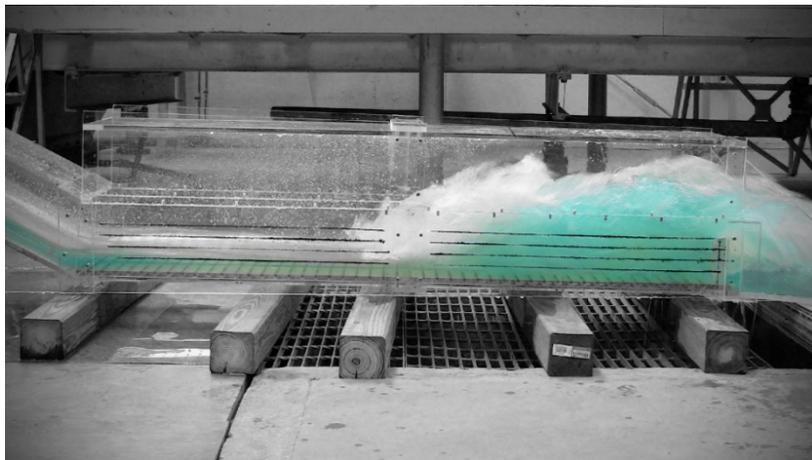
**Figure A23.** Experiment 25B



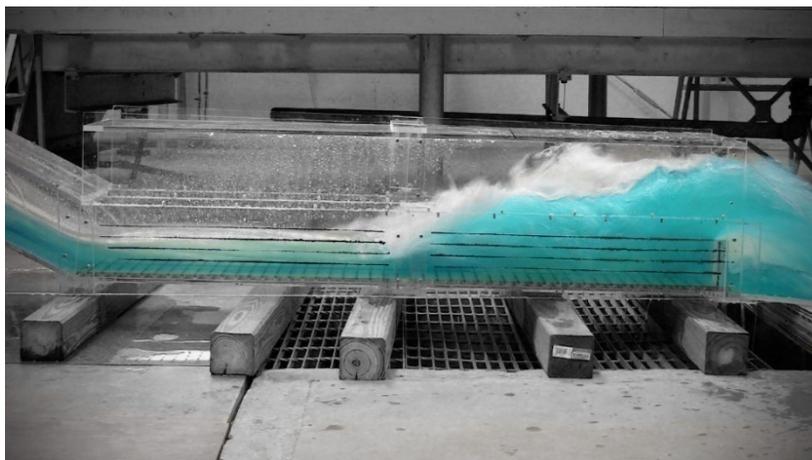
**Figure A24.** Experiment 25C



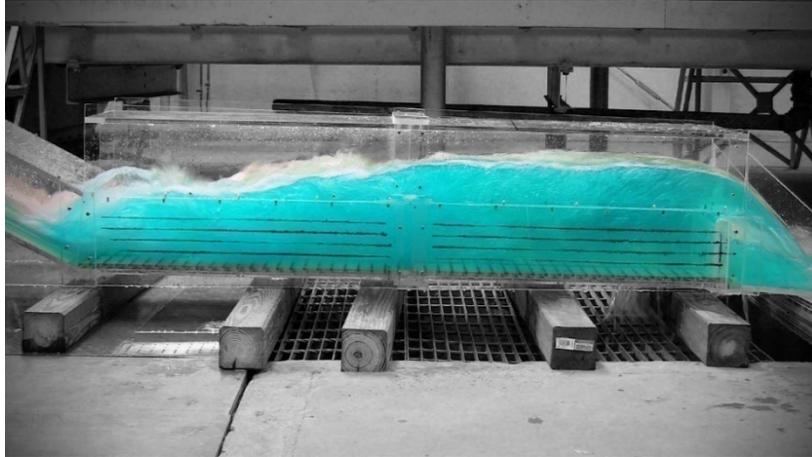
**Figure A25.** Experiment 26A



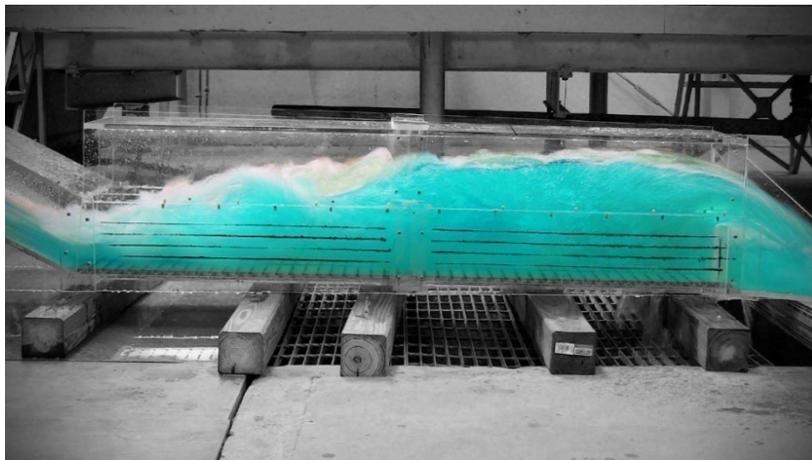
**Figure A26.** Experiment 26B



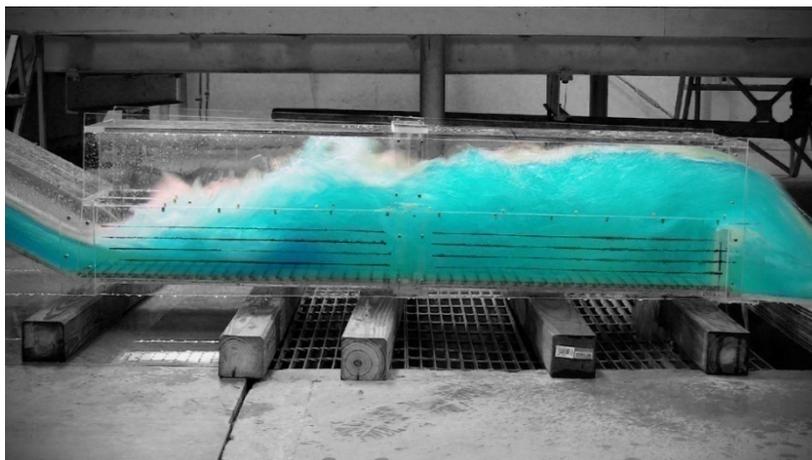
**Figure A27.** Experiment 26C



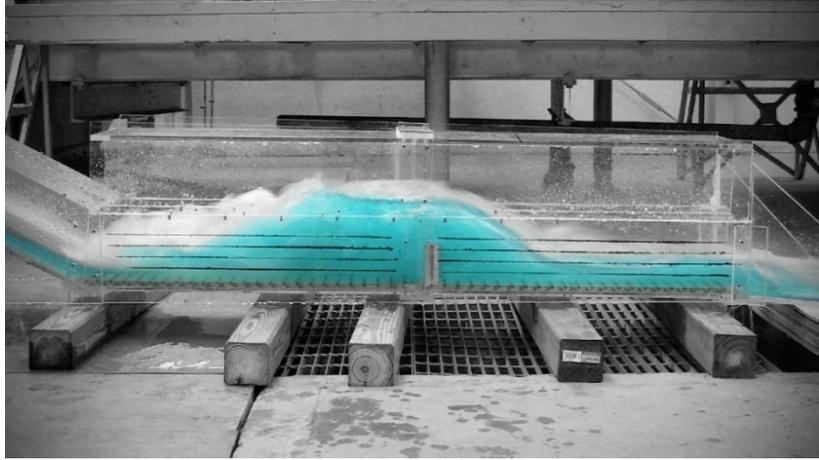
**Figure A28.** Experiment 27A



**Figure A29.** Experiment 27B



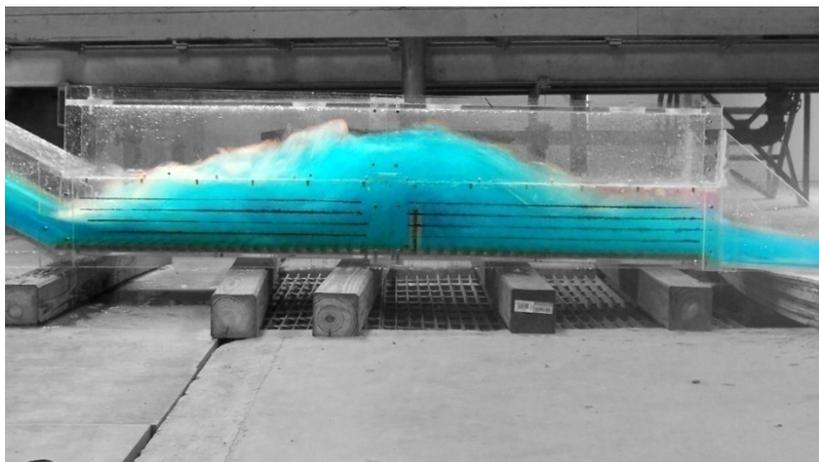
**Figure A30.** Experiment 27C



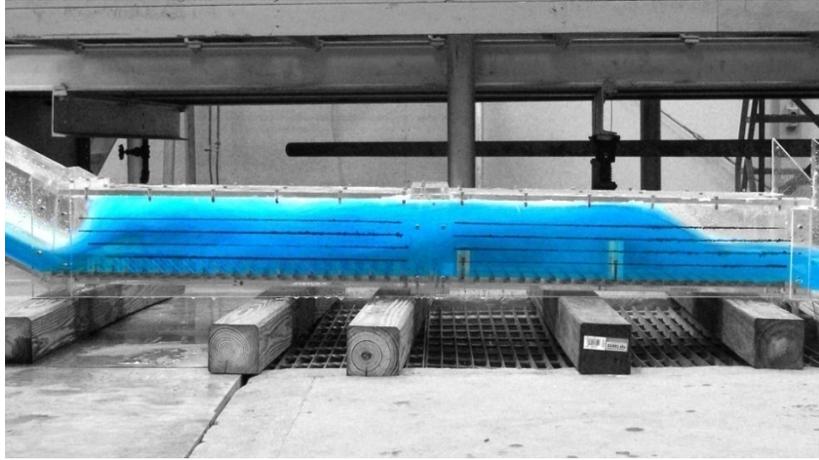
**Figure A31.** Experiment 28A



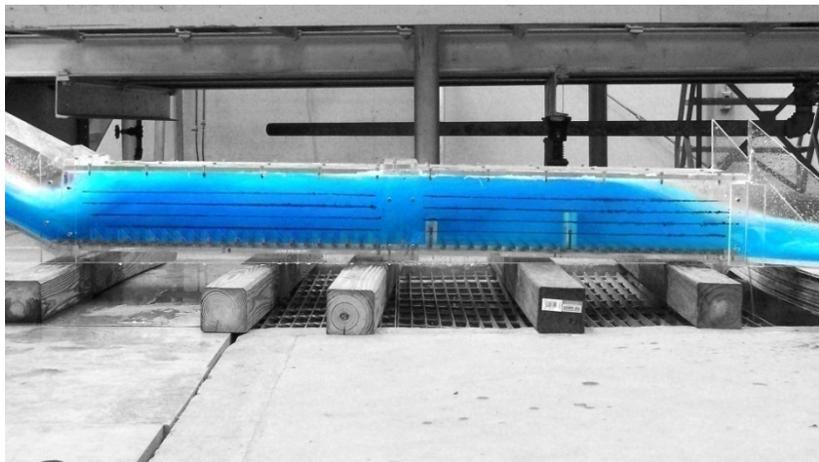
**Figure A32.** Experiment 28B



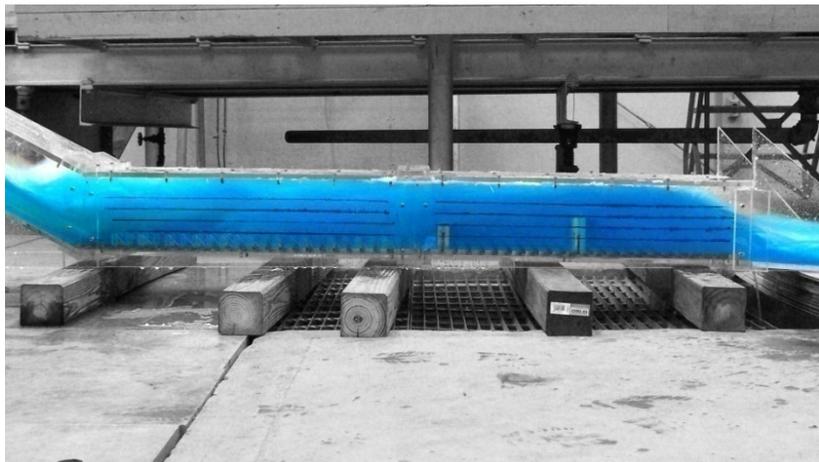
**Figure A33.** Experiment 28C



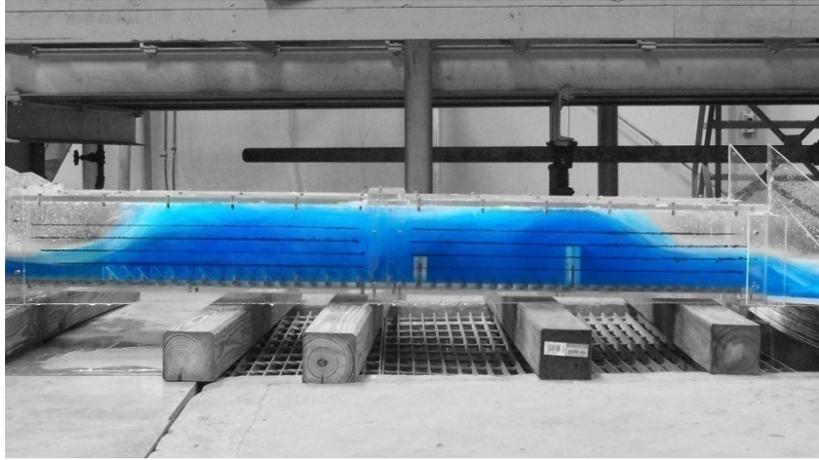
**Figure A34.** Experiment 31A



**Figure A35.** Experiment 31B



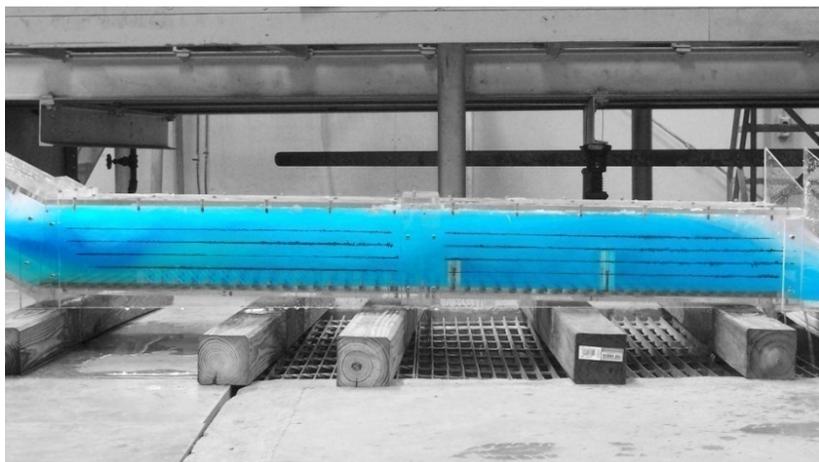
**Figure A36.** Experiment 31C



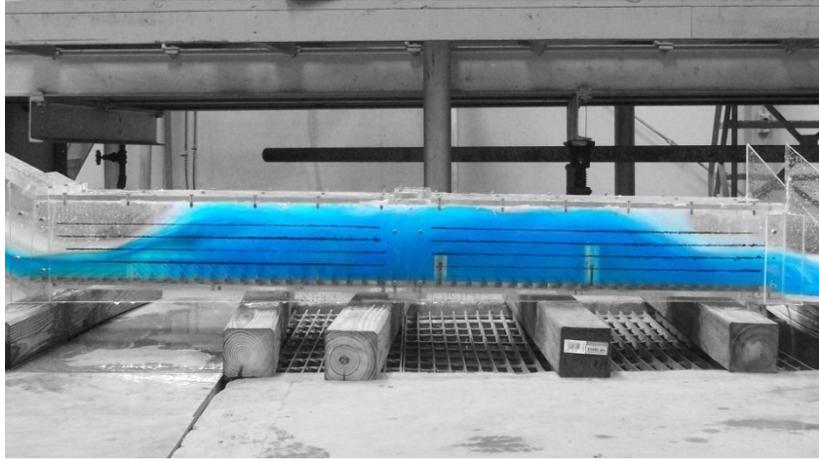
**Figure A37.** Experiment 32A



**Figure A38.** Experiment 32B



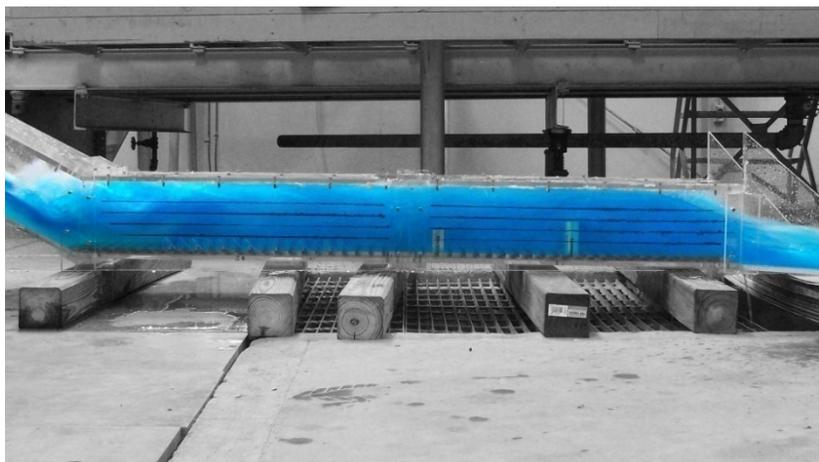
**Figure A39.** Experiment 32C



**Figure A40.** Experiment 33A



**Figure A41.** Experiment 33B



**Figure A42.** Experiment 33C



**Figure A43.** Experiment 34A



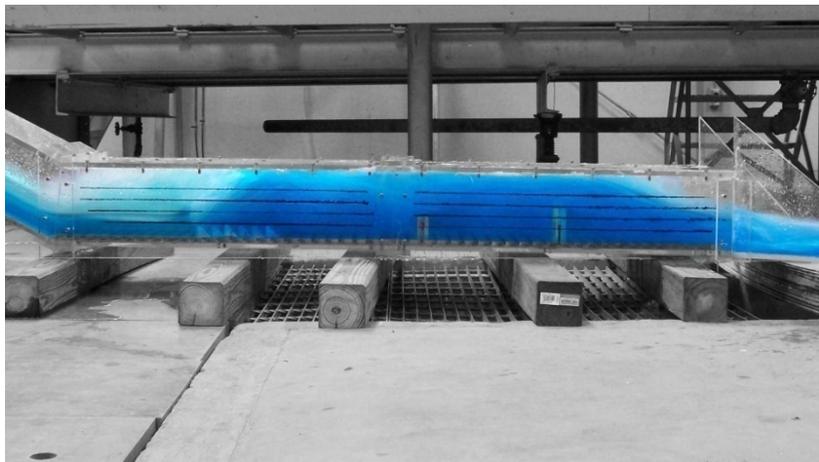
**Figure A44.** Experiment 34B



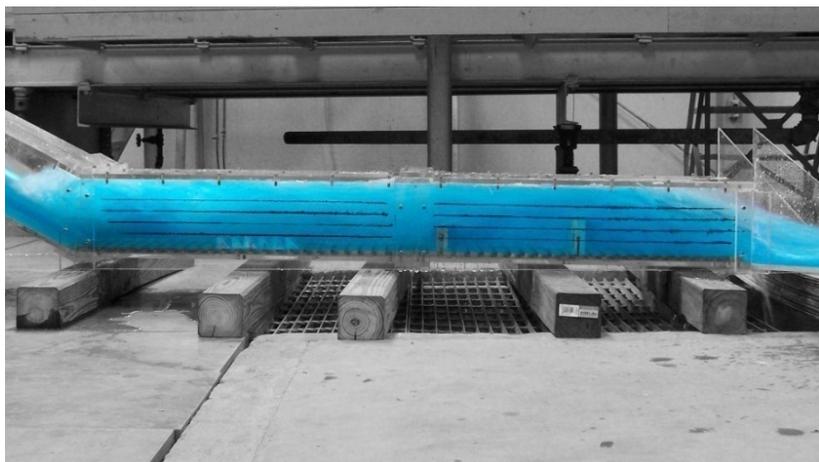
**Figure A45.** Experiment 34C



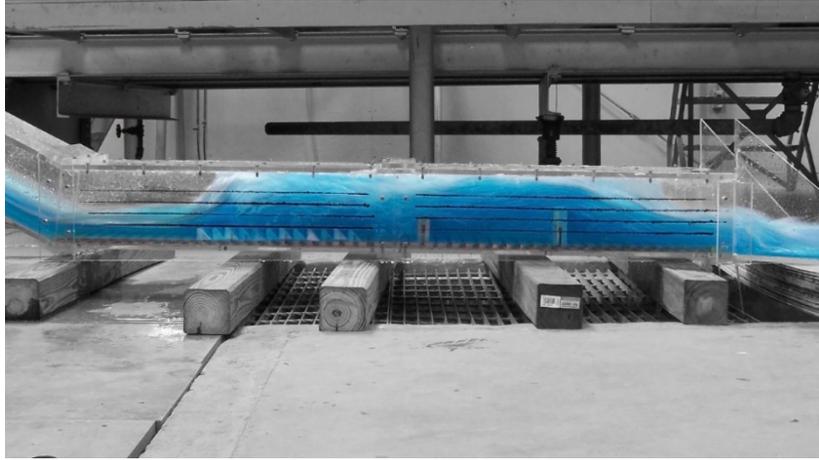
**Figure A46.** Experiment 35A



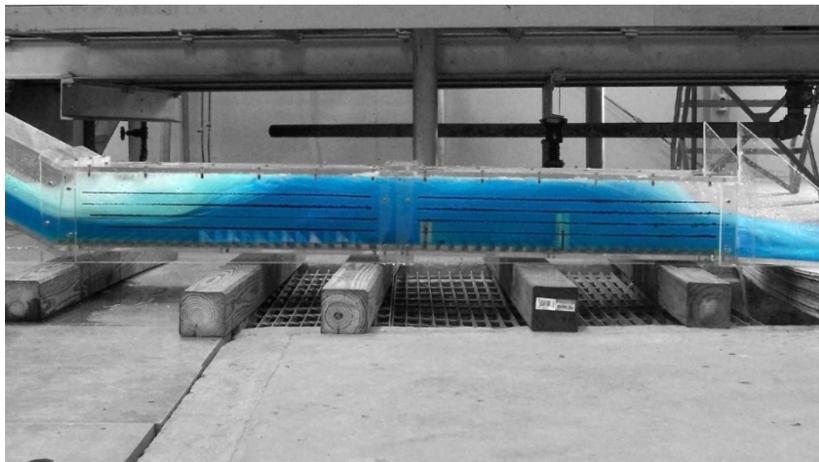
**Figure A47.** Experiment 35B



**Figure A48.** Experiment 35C



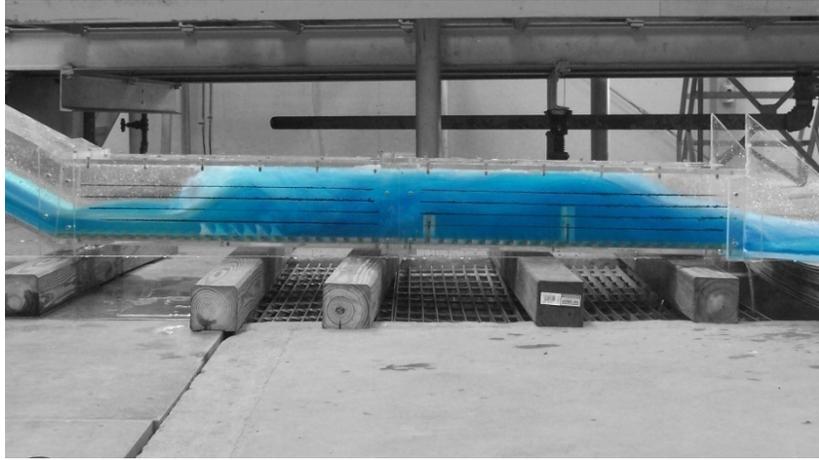
**Figure A49.** Experiment 36A



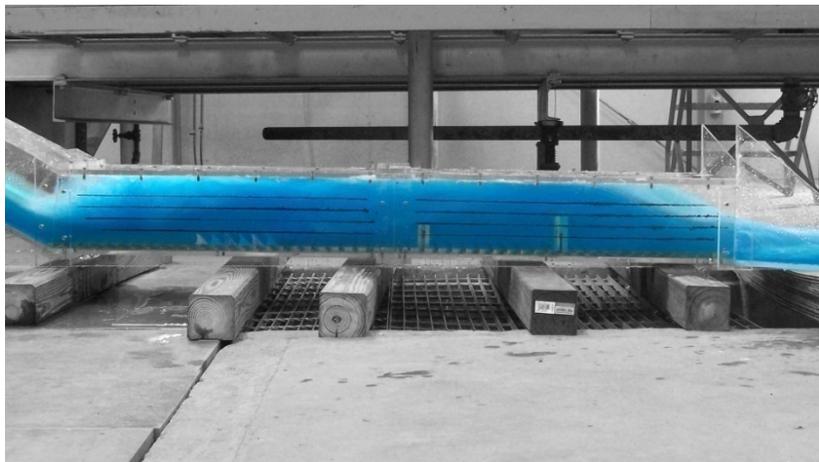
**Figure A50.** Experiment 36B



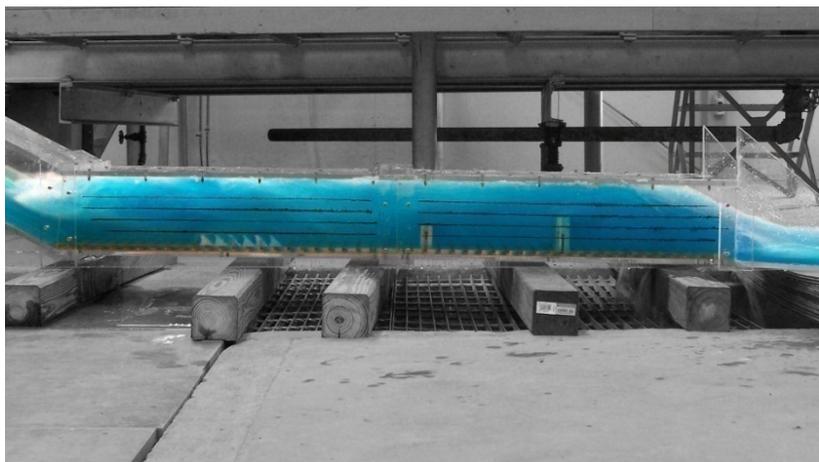
**Figure A51.** Experiment 36C



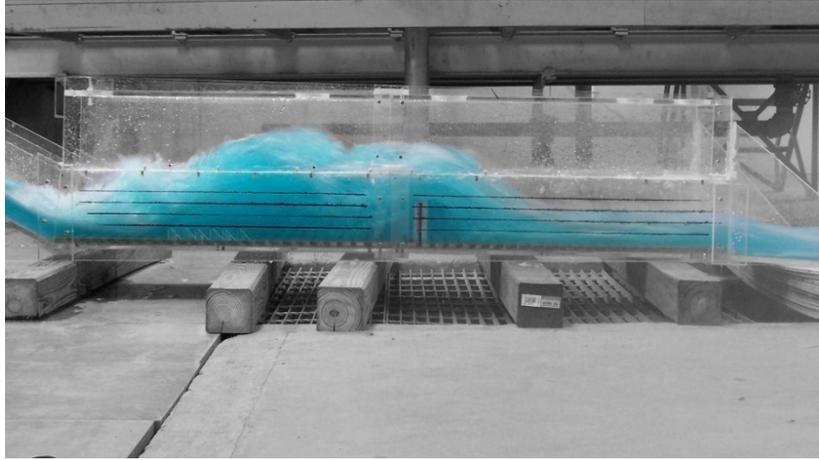
**Figure A52.** Experiment 38A



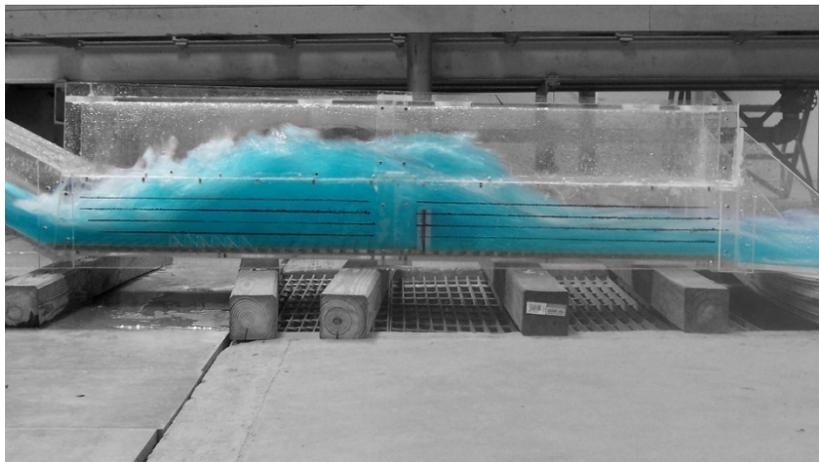
**Figure A53.** Experiment 38B



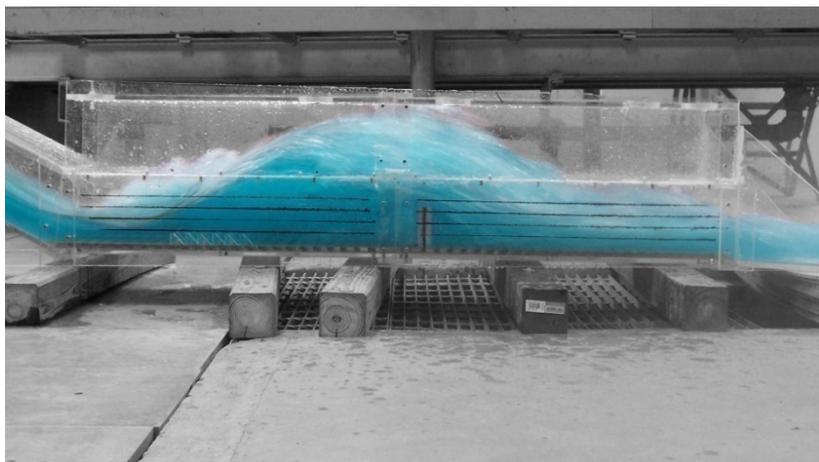
**Figure A54.** Experiment 38C



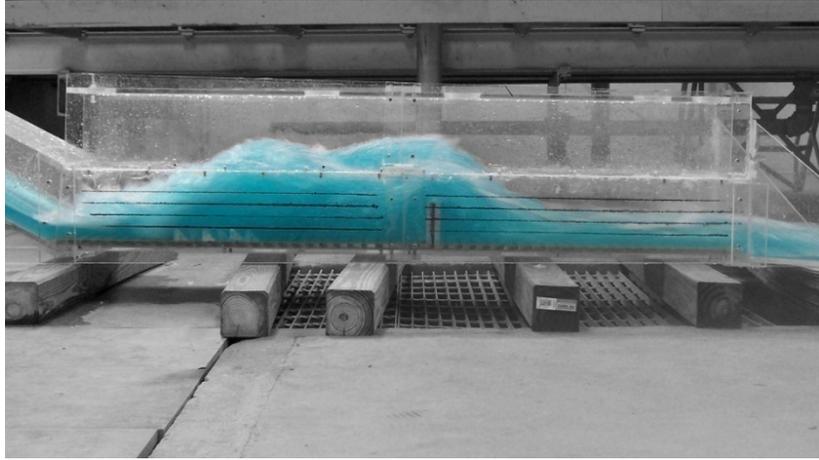
**Figure A55.** Experiment 40A



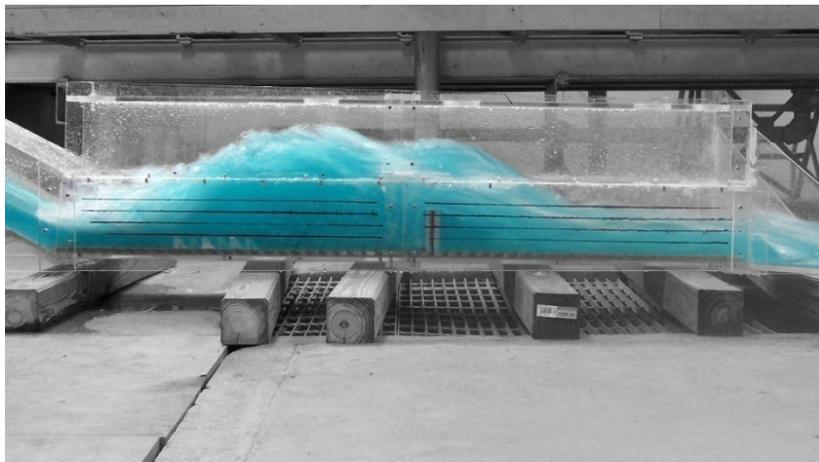
**Figure A56.** Experiment 40B



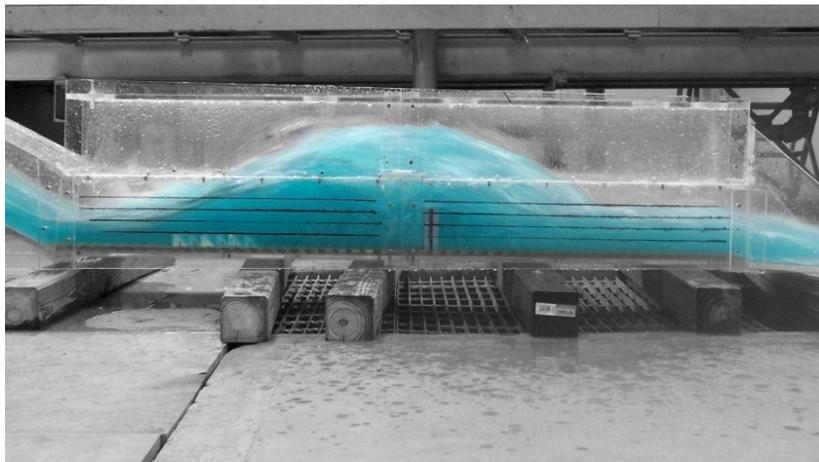
**Figure A57.** Experiment 40C



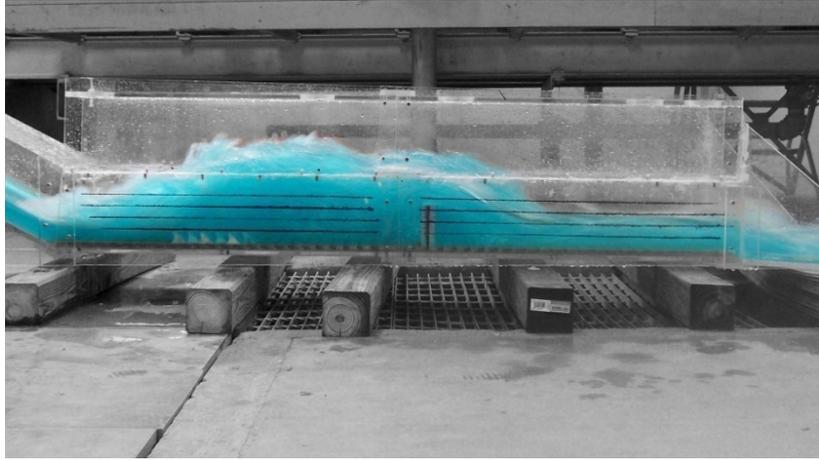
**Figure A58.** Experiment 41A



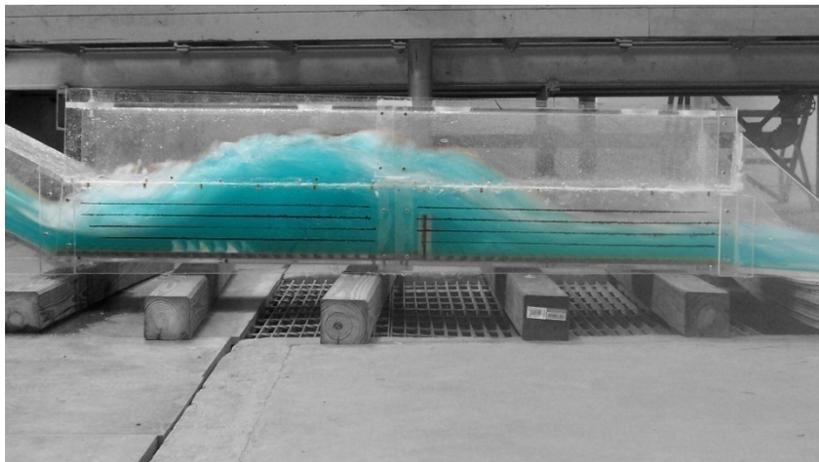
**Figure A59.** Experiment 41B



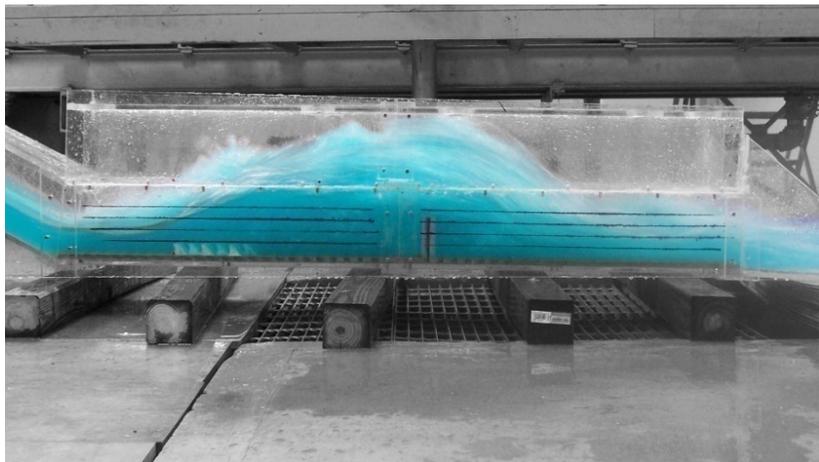
**Figure A60.** Experiment 41C



**Figure A61.** Experiment 42A



**Figure A62.** Experiment 42B



**Figure A63.** Experiment 42C