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Public Roads

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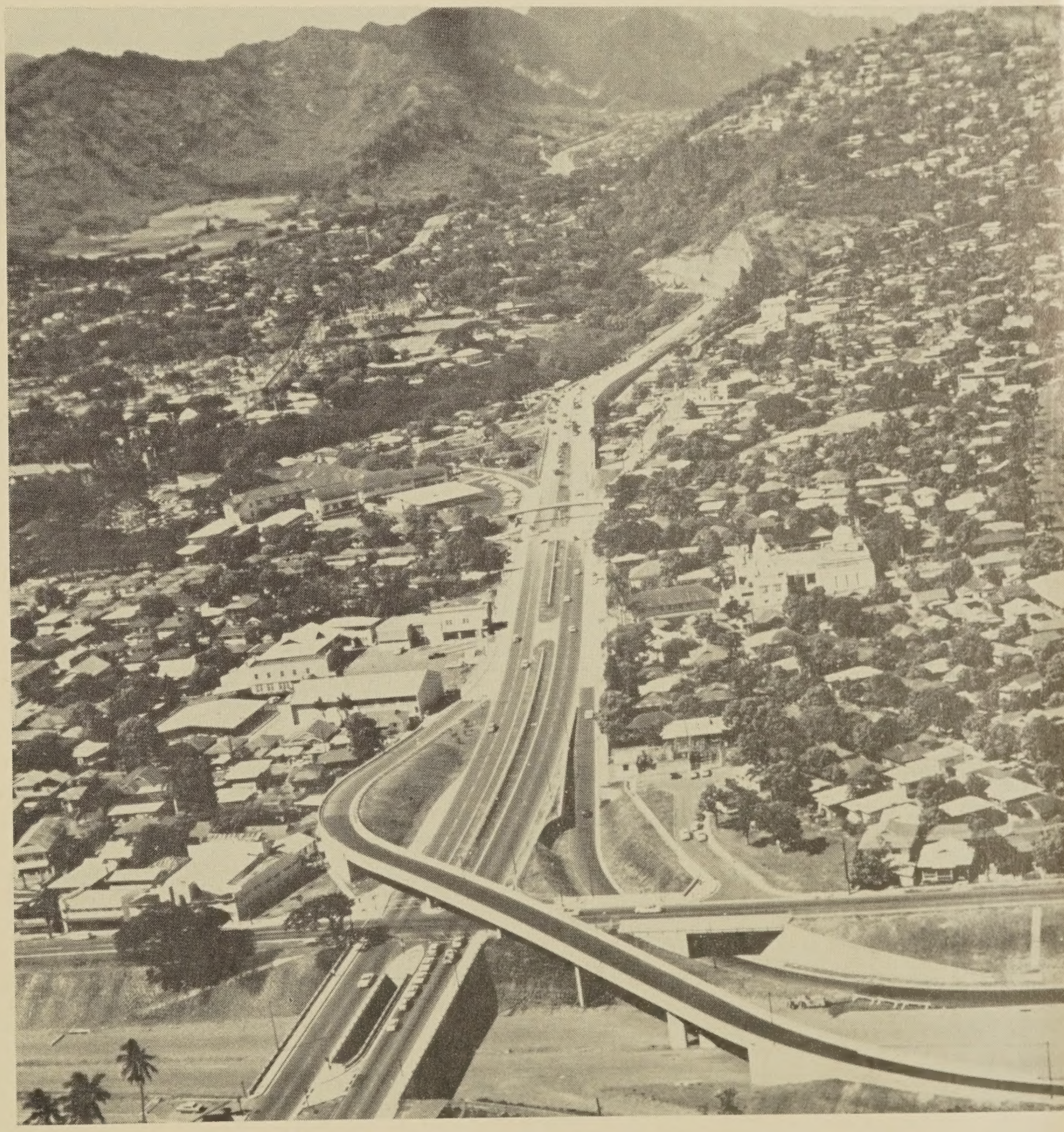

Pali Highway, Honolulu, Hawaii

Pali Highway is Federal-Aid Primary Route 61 and runs across the island of Oahu from the harbor in Honolulu to connect with FAS 630 at Kailua. The connection with Lunalilo Freeway, Hawaii Interstate Highway 1, and the construction on this highway is shown in the foreground.

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Perceptual Basis of Vehicular Guidance

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by DONALD A. GORDON,[1] Research Psychologist,
Traffic Systems Research Division

INTRODUCTION

Vehicular guidance is related to the driver's visual environ-ıent—described in basic terms of position and movement—in the ›ur parts of this article: "Generalized Equations, the Driver's Ioving Visual Environment"; "Static and Dynamic Visual Fields ı Vehicular Guidance"; "Motion Parallax and Perceptual Iypothesis Testing"; and "Perceptual Mechanisms in Vehicular :uidance." As the visual environment is an organized spatial ntity, hence it may be considered a field having static and dy-amic aspects. The positional field includes the angular coordi-ates of spatial points around the eye position. The velocity field ıcludes the vectors of angular motion around the driver's eyes, s he moves on his path. These vectors vary with the speed and irection of the driver's motion. Inasmuch as the location of the ectors is determined by the positional field, this field might be ılled the positional-velocity field. The acceleration field is efined in terms of angular acceleration vectors rather than velocity. This field might be called the positional-acceleration field.

In Part 1, equations are presented on the general organization of visual space around the eye of the moving driver. Derivations included on the effects of rectilinear motion, and horizontally and vertically curved motions, and combinations of these motions. In Part 2, the principles applying to the perception of the positional, velocity, and acceleration fields under rectilinear motion are discussed. In Part 3, the concept of motion parallax, widely accepted as a cue to depth, is examined. It was concluded that terrain movements on a circular path could not be interpreted in any consistent manner to show the distance of seen objects. A perceptual hypothesis principle is proposed to explain the major contributions of observer motion to space perception. In Part 4, the driver's perceptions are analyzed in the basic vehicular maneuvers of steering, perceptual anticipation, and car following.

Findings and Conclusions

The perceptual problems in vehicular guid-nce were considered in the context of the ositional, velocity, and acceleration fields round the moving vehicle. These are very eneral and persistent aspects of the driver's isual environment. The equations govern-ıg these fields, and the fields themselves, 'ere considered for features and regularities hat might explain human spatial perception. 'he following conclusions were made from the nalyses.

art 2

- The interpretive scaling of visual angle is key factor in interpreting perspective and in erceiving size, distance, and motion per-eption.
- Simple and obvious features of the visual nvironment, which have often been ignored ı explanations of space perception, are be-:eved to provide important aids for vehicular uidance. The roadway ahead of the vehicle, or example, may be used to obtain the scale f the terrain and objects in it.
- The driver may see his vehicle, or some art of the environment as reference for mo-ion. If the foreground is visually fixated, a urious illusion of motion is seen. The back-;round seems to rotate forward and around he foreground. This velocity parallax curl is based upon the difference in velocity vectors between foreground and background.
- Roadway boundaries and lane markings are used in alining the moving vehicle with the road. This conclusion challenges the often quoted statement that the focus of expansion is the cue for the direction of sensed locomotion.
- Angular acceleration increases as the square of vehicular speed. The consequences of this relation for the perception of vehicular speed are indicated.
- The pattern of the angular acceleration field does not resemble any familiar pattern of visual experience. It therefore seems that angular acceleration is not directly sensed. By extension, it is doubtful that higher derivatives of motion are seen as such.

Part 3

Helmholtz's formulation of the motion parallax cue to distance fails when the observer follows a curved path. Angular velocity on the ground plan does not decrease systematically with distance; rather it shows an asymmetrical pattern, which would lead to an erroneous interpretation under the rules of linear motion parallax. It has been suggested that observer motion aids space perception by providing a test of prior perceptual interpretations or hypotheses under changed perspective.

Part 4

- When the vehicle is alined with a straight or regularly curved highway, the road assumes a steady state appearance. The borders and lane markers remain almost stationary in the driver's field of view. The driver's problem in lateral guidance, car following, and other maneuvers may be to maintain an acceptable steady state condition and to null deviations from the steady condition by utilizing visual feedback information.
- If the moving vehicle is misalined laterally with the road, the entire field moves as a unit. No one part of the road borders or lane markers is essential for steering.
- The extent of lateral misalinement is indicated by the rate and extent of slewing and sideslipping of the road borders and lane markers. The driver's perceptual response is based upon an integration of these and other items of information.
- The driver's anticipation requirements must be considered in the design of road features such as curves, signal lights and signs. These requirements have not been extensively studied.
- The driver's ability to estimate the time required to reach an object ahead may be based upon his estimate of perceived distance.
- The visual stimulus to car following on a curvilinear path is discussed. It is concluded that need exists for empirical validation studies of car following theory.
- Guidance theories based upon characteristics of the velocity field, such as the motion parallax cue to depth, the center of expansion, and null locus indicators of alinement, fail on curved roads. Features of the velocity field shift with the vehicle's curved path of motion.

[1] The author acknowledges the advice and support pro-'ided by Dr. Richard M. Michaels, now Science Advisor,)ffice of Research and Development, during the research.

Part 1—Generalized Equations—the Driver's Moving Visual Environment

Introduction

TO ANALYZE DRIVING, the moving visual environment to which the driver is reacting must be described. Equations describing the environment of linear translation at constant speed have been developed by other researchers (*1*, *2*).[2] This article presents a general procedure applicable to horizontal and vertical curvature, separately or in combination, with fixed or varying speeds of vehicles motion. To describe the moving environment, the position and induced movement of objects expressed in Cartesian coordinates (x, y, z, dx/dt, dy/dt, dz/dt) must be translated into the spherical coordinates (θ, ϕ, $d\theta/dtd\phi/dt$, $d^2\theta/dt^2$, $d^2\phi/dt^2$) of the observer's visual world.

Visual Coordinates

The coordinate system is shown in figure 1. The eye is at the origin. Distance ahead of the eye is represented by x, to the side by y, and up and down by z. Distance from the eye to a terrain or other point is represented by ρ whose projection on the xy plane is r. It is not essential that the ground surface be parallel to the momentary path. Thus, the ground surface relative to a landing aircraft can be specified in x, y, and z coordinates, and can be dealt with.

Angular position

Angular position can be determined by use of the following equations.

$$\theta=\arctan y/x \text{ rad.} \qquad (1)$$

$$\phi=\arcsin z/\rho \text{ rad.} \qquad (2)$$

Therefore,

$$r=(x^2+y^2)^{1/2} \qquad (3)$$

$$\rho=(x^2+y^2+z^2)^{1/2} \qquad (4)$$

Angular velocity

To determine angular velocity the following procedure can be used.

$$\frac{d\theta}{dt}=\frac{1}{r^2}\left(-y\,\frac{dx}{dt}+x\,\frac{dy}{dt}\right) \text{ rad./sec.} \qquad (5)$$

$$\frac{d\phi}{dt}=\frac{1}{r}\left(\frac{-zx}{\rho^2}\frac{dx}{dt}-\frac{zy}{\rho^2}\frac{dy}{dt}+\frac{r^2}{\rho^2}\frac{dz}{dt}\right) \text{ rad./sec.} \qquad (6)$$

Angular acceleration

The equations for angular acceleration are, as follows.

$$\frac{d^2\theta}{dt^2}=\frac{2xy}{r^4}\left[\left(\frac{dx}{dt}\right)^2-\left(\frac{dy}{dt}\right)^2\right]-\frac{1}{r^2}\left(y\,\frac{d^2x}{dt^2}-x\,\frac{d^2y}{dt^2}\right)+2\left[\frac{y^2-x^2}{r^4}\left(\frac{dx}{dt}\frac{dy}{dt}\right)\right] \text{ rad./sec./sec.} \qquad (7)$$

$$\frac{d^2\phi}{dt^2}=\frac{-z}{\rho^4r^3}\Big[(\rho^2y^2-2r^2x^2)\left(\frac{dx}{dt}\right)^2+(\rho^2x^2-2r^2y^2)\left(\frac{dy}{dt}\right)^2\Big]-\frac{2rz}{\rho^4}\left(\frac{dz}{dt}\right)^2+\left(\frac{z^2-r^2}{\rho^4r}\right)\left(2x\,\frac{dx}{dt}\frac{dz}{dt}+2y\,\frac{dy}{dt}\frac{dz}{dt}\right)+\frac{2xyz}{\rho^4r^3}\,(2r^2+\rho^2)\,\frac{dx}{dt}\frac{dy}{dt}-\frac{zx}{r\rho^2}\frac{d^2x}{dt^2}-\frac{zy}{r\rho^2}\frac{d^2y}{dt^2}+\frac{r}{\rho^2}\frac{d^2z}{dt^2} \text{ rad./sec./sec.} \qquad (8)$$

Rectilinear Motion

When the eye moves with constant velocity in a straight line, the environment translates in x at a rate of $-dx/dt$ (negative speed of forward motion of the eye). The terms dy/dt and dz/dt equal zero. The equations of velocity and acceleration in spherical coordinates reduce to:

$$\frac{d\theta}{dt}=\frac{-y}{r^2}\,\frac{dx}{dt} \text{ rad./sec.} \qquad (9)$$

$$\frac{d\phi}{dt}=\frac{-zx}{\rho^2r}\,\frac{dx}{dt} \text{ rad./sec.} \qquad (10)$$

$$\frac{d^2\theta}{dt^2}=\frac{2xy}{r^4}\left(\frac{dx}{dt}\right)^2 \text{ rad./sec./sec.} \qquad (11)$$

$$\frac{d^2\phi}{dt^2}=-z\left[\frac{\rho^2y^2-2r^2x^2}{\rho^4r^3}\right]\left(\frac{dx}{dt}\right)^2 \text{ rad./sec./sec.} \qquad (12)$$

Horizontal Curvature

On a horizontally curved path, the environment shows dx/dt and dy/dt components, while dz/dt equals zero. It is convenient to describe the rotary movement relative to an origin distance, R distance in y from the observer's eye, as shown in figure 2. For point P, in figure 2, the following expressions apply.

β=positive angle of rotation of O_1P
$x_i=x$
$y_i=R-y$
$r_i=[(x_i)^2+(y_i)^2]^{1/2}$

Then,

$$\overrightarrow{PQ}=\text{velocity vector of } O_1P=r_i\,\frac{d\beta}{dt} \qquad (13)$$

$$\frac{dx_i}{dt}=x_i \text{ component of } \overrightarrow{PQ}=r_i\sin\beta\,\frac{d\beta}{dt}=-y_i\,\frac{d\beta}{dt}=(y-R)\,\frac{d\beta}{dt} \text{ ft./sec.} \qquad (14)$$

$$\frac{dy_i}{dt}=y_i \text{ component of } \overrightarrow{PQ}=r_i\cos\beta\,\frac{d\beta}{dt}=x_i\,\frac{d\beta}{dt}=x\,\frac{d\beta}{dt} \text{ ft./sec.} \qquad (15)$$

$$\frac{d^2x_i}{dt^2}=-x\left(\frac{d\beta}{dt}\right)^2+(y-R)\,\frac{d^2\beta}{dt^2} \text{ ft./sec./sec.} \qquad (16)$$

$$\frac{d^2y_i}{dt^2}=(y-R)\left(\frac{d\beta}{dt}\right)^2+x\,\frac{d^2\beta}{dt^2} \text{ ft./sec./sec.} \qquad (1$$

Substitution of dx_i/dt and $-dy_i/dt$ f dx/dt and dy/dt in equation (5) produc equation (18), which gives angular vel ci at the driver's eye related to position a angular speed around origin, O_1.

$$\frac{d\theta}{dt}=\left[\frac{-y}{r^2}\,(y-R)-\frac{x}{r^2}\,(x)\right]\frac{d\beta}{dt} \text{ rad./sec.} \qquad (1$$

This reduces to

$$\frac{d\theta}{dt}=\left(\frac{-y}{r^2}+\frac{1}{R}\right)\frac{dx}{dt} \text{ rad./sec.} \qquad (1$$

On a curve to the right

$$\frac{d\theta}{dt}=\left(\frac{-y}{r^2}-\frac{1}{R}\right)\frac{dx}{dt} \text{ rad./sec.} \qquad (2$$

Similar substitution in equation (6) produ

$$\frac{d\phi}{dt}=\frac{-z}{r\rho^2}\,[x(y-R)-xy]\,\frac{d\beta}{dt}=\frac{-xz}{r\rho^2}\,\frac{dx}{dt} \text{ rad./sec.}$$

Angular Acceleration

Angular acceleration at the driver's e related to position and angular accelerat around origin, O_1, can be derived from equati (7) and (8) by substitution of dx_i/dt, $-dy_i$ d^2x_i/dt^2, $-d^2y_i/dt^2$ for dx/dt, dy/dt, d^2x d^2y/dt^2 and by setting $d^2\beta/dt^2$ equal to zero.

$$\frac{d^2\theta}{dt^2}=\left(\frac{2xyR^2}{r^4}-\frac{xR}{r^2}\right)\left(\frac{d\beta}{dt}\right)^2=\left(\frac{2xy}{r^4}-\frac{y}{Rr^2}\right)\left(\frac{dx}{dt}\right)^2 \text{ rad./sec./sec.}$$

On a curve to the right

$$\frac{d^2\theta}{dt^2}=\left(\frac{2xy}{r^4}+\frac{x}{Rr^2}\right)\left(\frac{dx}{dt}\right)^2 \text{ rad./sec./sec.}$$

$$\frac{d^2\phi}{dt^2}=\left[\frac{-z}{\rho^4r^3}\,(\rho^2y^2-2r^2x^2)+\frac{zy}{\rho^2rR}\right]\left(\frac{dx}{dt}\right)^2 \text{ rad./sec./sec.}$$

On a curve to the right

$$\frac{d^2\phi}{dt^2}=\left[-\frac{z}{\rho^4r^3}\,(\rho^2y^2-2r^2x^2)-\frac{zy}{\rho^2rR}\right]\left(\frac{dx}{dt}\right)^2 \text{ rad./sec./sec.}$$

Vertical Curvature

The analysis of vertical curvature fig. 3) follows that of horizontal curva Environmental movement is resolved dx/dt and dz/dt components; dy/dt mover is absent, and hence equal to zero. In fi 3, the eye is at the origin, A feet above ground. The center of rotation is at a tance R above the ground (below for a co curvature). There are two derivations: cave (upward) and convex (downward).

[2] Italic numbers in parentheses indicate the references listed on page 67.

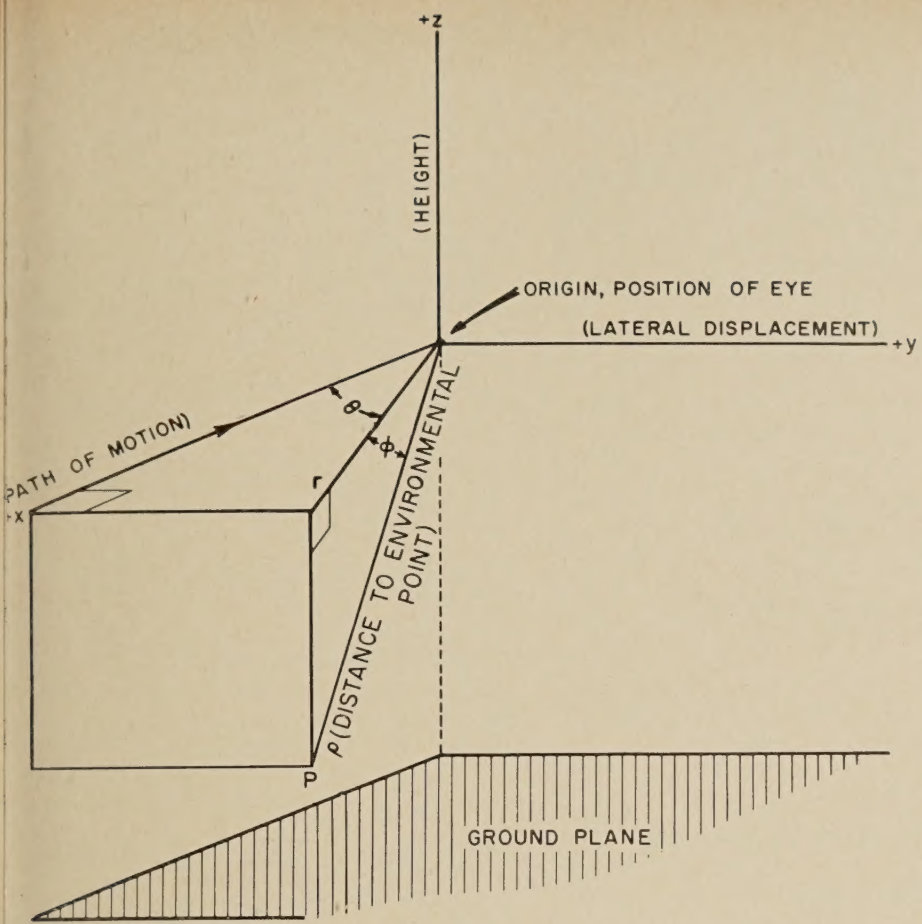


Figure 1.—Basic coordinate relationships.

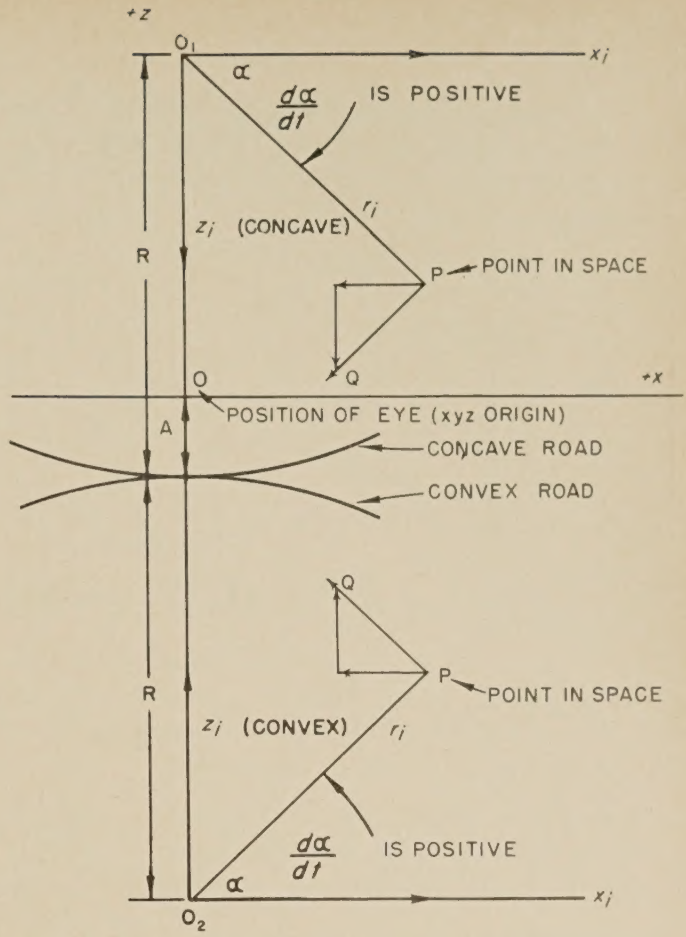


Figure 3.—Concave and convex vertical curvature.

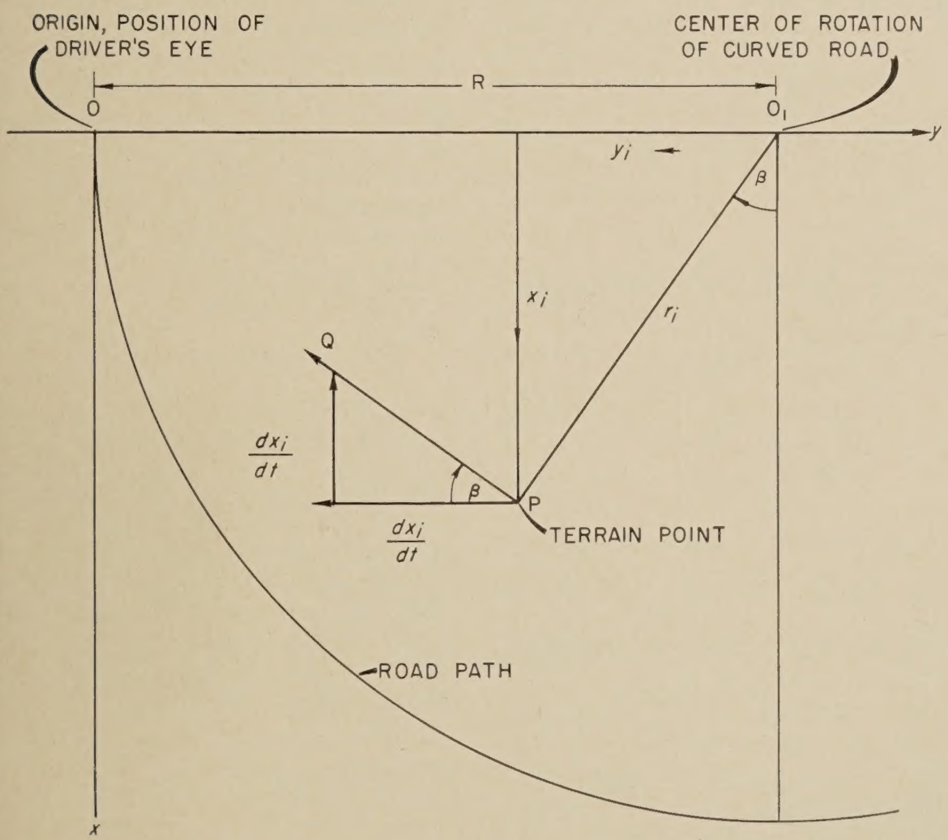


Figure 2.—Horizontal curvature.

Concave curvature

For environmental point P, when the curvature is concave,

α=positive angle rotation of O_1P

$x_i=x$

$z_i=R-z-A$

$$r_i=[(x_i)^2+(z_i)^2]^{1/2}$$

$$\overrightarrow{PQ}=\text{velocity vector of } \overrightarrow{O_1P}=r_i\,\frac{d\alpha}{dt}$$

$$\frac{dx_i}{dt}=x_i \text{ component of } \overrightarrow{PQ}=-r_i \sin\alpha\,\frac{d\alpha}{dt}$$

$$=-z_i\,\frac{d\alpha}{dt}=-(R-z-A)\,\frac{d\alpha}{dt}\text{ ft./sec.}$$

$$\frac{dz_i}{dt}=z_i \text{ component of } \overrightarrow{PQ}=r_i \cos\alpha\,\frac{d\alpha}{dt}$$

$$=x_i\,\frac{d\alpha}{dt}=x\,\frac{d\alpha}{dt}\text{ ft./sec.}$$

$$\frac{d^2x_i}{dt^2}=-x\left(\frac{d\alpha}{dt}\right)^2-(R-z-A)\,\frac{d^2\alpha}{dt^2}\text{ ft./sec./sec.}$$

R and A are positive.

$$\frac{d^2z_i}{dt^2}=\frac{d}{dt}\left(r_i\cos\alpha\,\frac{d\alpha}{dt}\right)$$

$$=-(R-z-A)\left(\frac{d\alpha}{dt}\right)^2+x\,\frac{d^2\alpha}{dt^2}\text{ ft./sec./sec.}\quad(26)$$

To determine angular velocity for concave curvature use equations (1) and (2). When dx_i/dt and $-dz_i/dt$ are substituted for dx/dt and dz/dt and when dy/dt is set equal to zero, equations (5) and (6) reduce to

$$\frac{d\theta}{dt}=\left\{-\frac{y}{r^2}\left[1-\frac{(z+A)}{R}\right]\right\}\frac{dx}{dt}\text{ rad./sec.}\quad(27)$$

$$\frac{d\phi}{dt}=\left[\frac{-zx}{r\rho^2}\left(1-\frac{(\rho^2+zA)}{-zR}\right)\right]\frac{dx}{dt}\text{ rad./sec.}\quad(28)$$

Note that

$$\frac{d\alpha}{dt}=-\frac{1}{R}\,\frac{dx}{dt}\quad(29)$$

Angular accelerations for concave curvature related to rectilinear coordinates of motion can be derived from equations (7) and (8) by substitution and by setting $d\alpha^2/dt^2$ equal to zero.

$$\frac{d^2\theta}{dt^2}=\left[\frac{2xy}{r^4}+\frac{2xy}{R^2r^4}\left(z^2+A^2-2Rz-2AR+2Az+\frac{r^2}{2}\right)\right]\left(\frac{dx}{dt}\right)^2\text{ rad./sec./sec.}\quad(30)$$

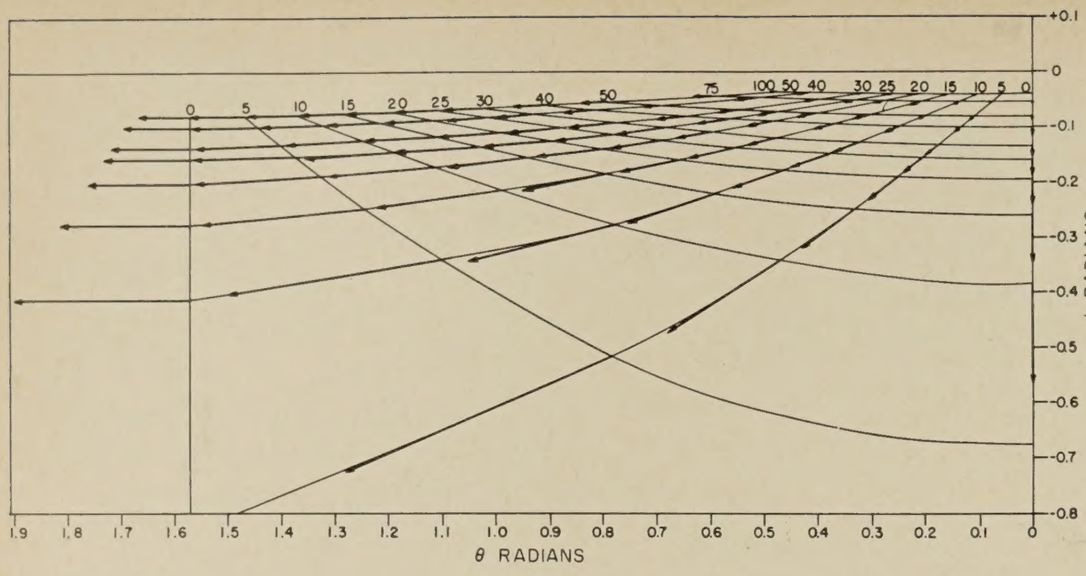


Figure 4.—Vector field of linear velocity shown on horizontal plane.

$$\frac{d^2\phi}{dt^2}=\left\{\frac{-z}{\rho^4r^3}(\rho^2y^2-2r^2x^2)+\frac{1}{R^2}\left[\frac{Ax^2z}{\rho^4r}(2A-4R)+\frac{y^2z}{\rho^2r^3}(Rz+2AR-A^2-2Az)-Rx^2r^2\right]+\frac{A}{\rho^2r}(x^2-y^2)+\frac{y^2}{r^3}(R-z)\right\}\left(\frac{dx}{dt}\right)^2 \text{ rad./sec./sec.} \quad (31)$$

Convex curvature

For environmental point P, when the curvature is convex,

α=positive angle of rotation of O_2P

$x_i=x$

$z_i=R+z+A$

$r_i=[(x_i)^2+(z_i)^2]^{1/2}$

$\overrightarrow{PQ}$=velocity vector of $\overrightarrow{O_2P}=r_i\frac{d\alpha}{dt}$

$$\frac{dx_i}{dt}=x_i \text{ component of } \overrightarrow{PQ}=-r_i\sin\alpha\frac{d\alpha}{dt}=-z_i\frac{d\alpha}{dt}=-(R+z+A)\frac{d\alpha}{dt}$$

$$\frac{dz_i}{dt}=z_i \text{ component of } \overrightarrow{PQ}=r_i\cos\alpha\frac{d\alpha}{dt}=x_i\frac{d\alpha}{dt}=x_i\frac{d\alpha}{dt}$$

Therefore,

$$\frac{d^2x_i}{dt^2}=\frac{d}{dt}\left(-r_i\sin\alpha\frac{d\alpha}{dt}\right)=-r_i\cos\alpha\left(\frac{d\alpha}{dt}\right)^2-r_i\sin\alpha\left(\frac{d^2\alpha}{dt^2}\right)=-x\left(\frac{d\alpha}{dt}\right)^2-(R+z+A)\frac{d^2\alpha}{dt^2} \text{ ft./sec./sec.} \quad (32)$$

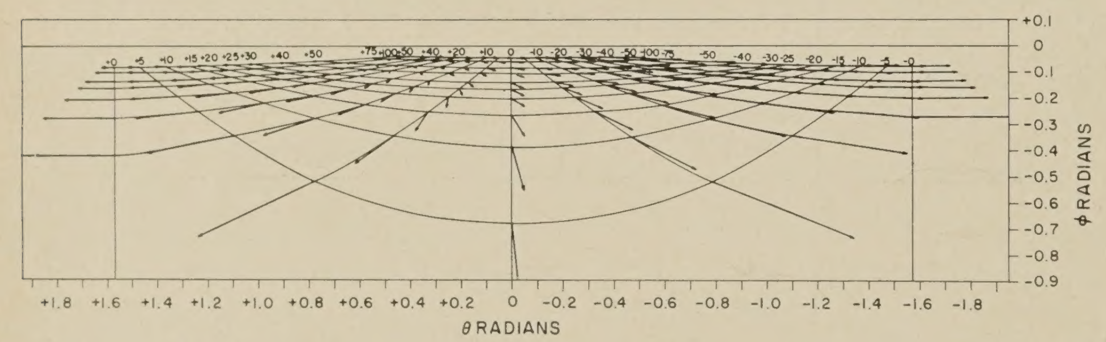


Figure 5.—Vector velocity field of horizontal curved motion shown on horizontal plane.

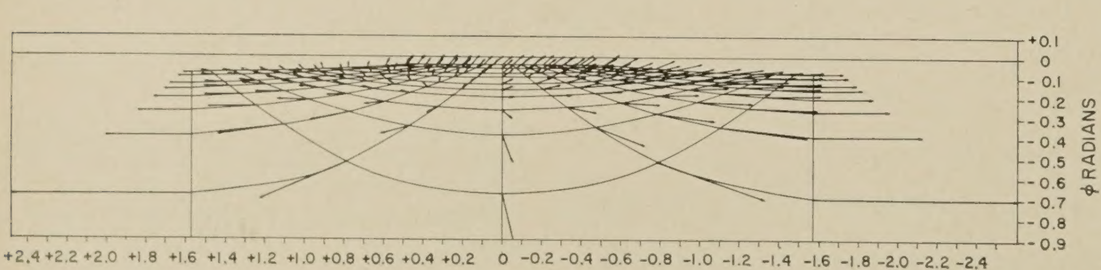


Figure 6.—Vector velocity field of horizontal and vertical curved motion shown on horizontal plane.

$$\frac{d^2z_i}{dt^2}=\frac{d}{dt}\left(r_i\cos\alpha\frac{d\alpha}{dt}\right)=-r_i\sin\alpha\left(\frac{d\alpha}{dt}\right)^2+r_i\cos\alpha\left(\frac{d^2\alpha}{dt^2}\right)=-(R+z+A)\left(\frac{d\alpha}{dt}\right)^2+x\frac{d^2\alpha}{dt^2} \text{ ft./sec./sec.} \quad (33$$

Angular position, velocity, and acceleratio

The derivations for angular velocity an acceleration for convex curvature parall those for concave curvature.

$$\frac{d\theta}{dt}=\left\{\frac{-y}{r^2}\left[1+\left(\frac{z+A}{R}\right)\right]\right\}\frac{dx}{dt} \text{ rad./sec.} \quad (34$$

$$\frac{d\phi}{dt}=\left\{\frac{-z}{r\rho^2}\left[1+\left(\frac{\rho^2+zA}{zR}\right)\right]\right\}\frac{dx}{dt} \text{ rad./sec.} \quad (3$$

$$\frac{d^2\theta}{dt^2}=\left[\frac{2xy}{r^4}+\frac{2xy}{r^4R^2}\left(z^2+A^2+2Rz+2AR+2Az+\frac{r^2}{z}\right)\right]\left(\frac{dx}{dt}\right)^2 \text{ rad./sec./sec.} \quad (3$$

$$\frac{d^2\phi}{dt^2}=\left\{\frac{-z}{\rho^4r^3}(\rho^2y^2-2r^2x^2)+\frac{1}{R^2}\left[\frac{Ax^2z}{\rho^4r}(2A+4R)+\frac{y^2z}{\rho^2r^3}(-Rz-2AR-A^2-2Az)+Rx^2r^2+\frac{A}{\rho^2r}(x^2-y^2)\frac{-y^2}{r^3}(R+z)\right]\right\}\left(\frac{dx}{dt}\right)^2 \text{ rad./sec./sec.} \quad (3$$

Complex curves

On complex curves, the angular velocity the sum of horizontal and vertical curvatu contributions. Acceleration components al summate. Angular position is unaffected motion of the vehicle. For example, t angular velocity formulas for a left conv curve can be developed from combinations previous equations. R_1 is the horizon radius of curvature and R_2 is the verti radius.

$$\frac{d\theta}{dt}=\left[\frac{-y}{r^2}+\frac{1}{R_1}-\frac{y}{r^2}\frac{(z+A)}{R_2}\right]\frac{dx}{dt} \text{ rad./sec.} \quad (3$$

$$\frac{d\phi}{dt}=\left[\frac{-yz}{r\rho^2}-\left(\frac{\rho^2+zA}{zR_2}\right)\right]\frac{dx}{dt} \text{ rad./sec.} \quad ($$

If the motion of the vehicle varies velocity, an additional term is required in angular acceleration equations—positional velocity equations are unaffected by accele tion. In the linear situation, the additio terms for angular acceleration can be deri from equations (6) and (7), where the d^2x terms represent the presumably known ac eration.

$$\frac{d^2\theta}{dt^2}=\frac{2xy}{r^4}\left(\frac{dx}{dt}\right)^2-\frac{y}{r^2}\frac{d^2x}{dt^2} \text{ rad./sec./sec.}$$

$$\frac{d^2\phi}{dt^2}=\frac{-z}{\rho^4r^3}(\rho^2y^2-2r^2x^2)\left(\frac{dx}{dt}\right)^2-\frac{zx}{r\rho^2}\frac{d^2x}{dt^2} \text{ rad./sec./sec.}$$

For defining angular acceleration for cur motion, the additional acceleration te needed include d^2x_i/dt^2, d^2y_i/dt^2, and d^2z_i. In horizontal curvature, the x_i componen celeration has been defined in equation 6) as,

$$\frac{d^2x_i}{dt^2}=-x\left(\frac{d\beta}{dt}\right)^2+(y-R)\,\frac{d^2\beta}{dt^2}\ \text{ft./sec./sec.} \qquad (16)$$

Where the horizontal curved motion is iiform, the $d^2\beta/dt^2$ component of acceleration zero. If the motion accelerates, this term not zero, but is evaluated as

$$(y-R)\,\frac{d^2\beta}{dt^2}=\left(1-\frac{y}{R}\right)\frac{d^2x}{dt^2} \qquad (42)$$

volving the relationship of

$$d^2\beta/dt^2=\frac{-d^2x/dt^2}{R} \qquad (43)$$

here d^2x/dt^2 is the acceleration of the eye or ehicle. Similarly, the acceleration terms f other motions can be developed. Velocity nd acceleration fields having implications for ie theory of human space perception are resented in figures 4 through 7. Additional iformation on the applications of velocity nd acceleration fields is given in references and *2*.

In figure 4 the vector field of linear velocity shown from a position 4 feet above the lane. The field is bilaterally symmetrical nd the rear mirrors the front, with the vectors eversed. As shown, the velocity vectors are een as being tangential, and sometimes coin-ident, with the flow lines of angular positions. 'he vector velocity field of horizontal curved iotion on the horizontal plane is also shown rom a position 4 feet above the plane. The adius of curvature is 100 feet to the left of he eye (origin). The field is asymmetrical nd has a null point at the center of curvature, which approaches a value of $1/R\ dx/dt$ at infinite distance.

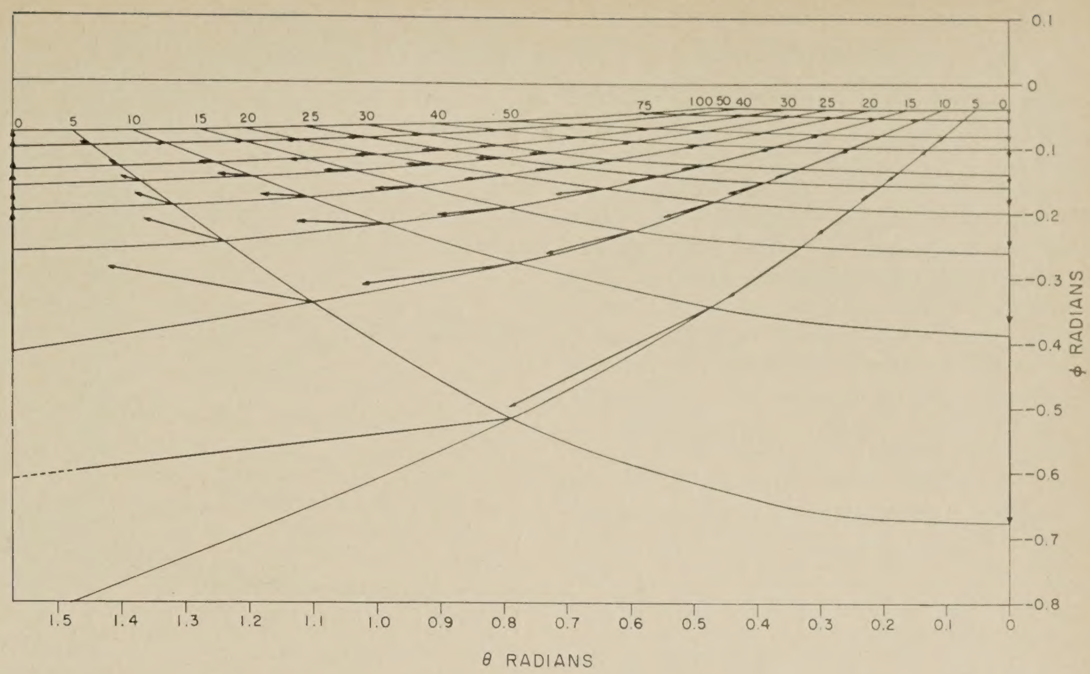


Figure 7.—Vector field of linear acceleration on horizontal plane.

The vector velocity field for both horizontal and vertical curved motion is shown on the horizontal plane in figure 6. As in figures 4 and 5 the field is shown from a position 4 feet above the plane. The center of horizontal curvature is viewed 100 feet to the left of the origin, the center of vertical curvature is 100 feet below the eye. The field shown in figure 6 and in figure 5 have implications for perceptual theories that relate space perception to features of the velocity field, such as the motion parallax cue to distance. Under curved motion, features of the velocity field are too unstable to provide a basis for the perception of distance or orientation.

As in figures 4 through 6, the vector field of linear acceleration on the horizontal plane is shown in figure 7 from a position 4 feet above the plane. The basic psychological question raised by this field is whether acceleration information is immediately sensed as a change of velocity. The lack of resemblance of this field to any familiar pattern of visual experience provides evidence that angular acceleration is not directly sensed.

Part 2—Static and Dynamic Visual Fields in Vehicular Guidance

Research Approach

THE IMPORTANCE for understanding driving, of analyzing the visual input, can readily be recognized. The investigation of visual inputs reported in this article was begun with the study of the positional, velocity, and acceleration fields around the moving vehicle. These fields are general and persistent aspects of the visual environment. The velocity and acceleration fields, which present time varying aspects of the environment, are of particular interest as they provide information not available in static viewing.

The many cues available in spatial perception have been discussed by previous investigators (*3*, *4*, and *5*) and the terrain characteristics that may orient the human in his spatial environment have also been described (*6*). These studies indicate methods that might be employed in vehicular guidance; but research also should be aimed at identifying the inputs that the driver actually uses. Other researchers have concerned themselves with human errors in space perception such as the systematic overestimation of size in distant vision (*7*) and the hyperbolic distortion shown in the judgment of space in certain reduction situations (*8*). In the analyses reported here, the characteristic of the driver's judgment of space to be explained is its accuracy rather than its incompatability with physical space.

In the preceding Part 1, the mathematical description of the moving ground plane from the driver's point of view was developed. The environment seen by the driver involves a perspective transformation of ground position, velocity, and acceleration. The positional field includes the angular coordinates from eye position of points in the driver's environment; this field includes the vectors of angular motion around the driver's eye as he moves along the road. The acceleration field presents vectors of angular acceleration rather than velocity.

The problems considered in this part concern the use made by the driver of the positional, velocity, and acceleration fields. To affect driving, these characteristics have to be registered by the driver and the driver's sensitivity to them influences their utility. The analysis covers the condition of steady state driving, where the vehicle moves rectilinearly with constant velocity.

Equations of Position and Motion

The coordinate system is illustrated in figure 1. The driver's eye is at the origin and the road is considered to be on an infinite plane at some distance in z below his eye. Distance ahead of the eye is represented by x, to the side by y. Distance from the eye to any point of the field is represented by ρ, whose projection on the xy plane is r.

The equations for ground position and motion were derived previously. Equations (1) through (8) describe angular position velocity and acceleration under rectilinear, uniform, vehicular motion. These equations apply to the induced motion of the ground, and all other environmental points, viewed from a moving vehicle. The analysis of ground motion into separate azimuth ($d\theta/dt$) and declination ($d\phi/dt$) components seemed more appropriate than the development of an equation for total angular velocity (*1*, *2*). In some situations, the driver reacts differently to these components of motion (*9*).

Positional Field and Vehicular Guidance

The angular coordinates (θ, ϕ) of the ground plane from 0 to 50 feet to the left, and from 0 to 100 feet in front of the driver are shown in figure 8. As the driver sits on the left of the vehicle, the window areas and road views are asymmetrical. Although the left side is shown in figure 8, the flow lines also apply to the right side of the vehicle, if the appropriate window area is superimposed, and to the rear areas as well. The driver's eye is placed at a representative height of 4 feet above the ground. The shaded area at the right of the figure is the automobile cab and hood, which partially cuts off the view of the road. The blind areas at 0.65 and 0.9 radians are the roof supports.

The equirectangular projection shown in figure 8 is one of many possible ways of representing a three-dimensional environment on a flat surface. The figure is distorted; at the zenith and nadir of actual space, 360° of azimuth are reduced to a point, whereas in the figure they occupy the same extent as on the horizontal meridian. A rectification in the θ dimension may be achieved by curving the page through 90° and viewing it from a point close to the center of curvature. The ϕ dimension is not rectified, but it need not be as it covers a limited half-radian range.

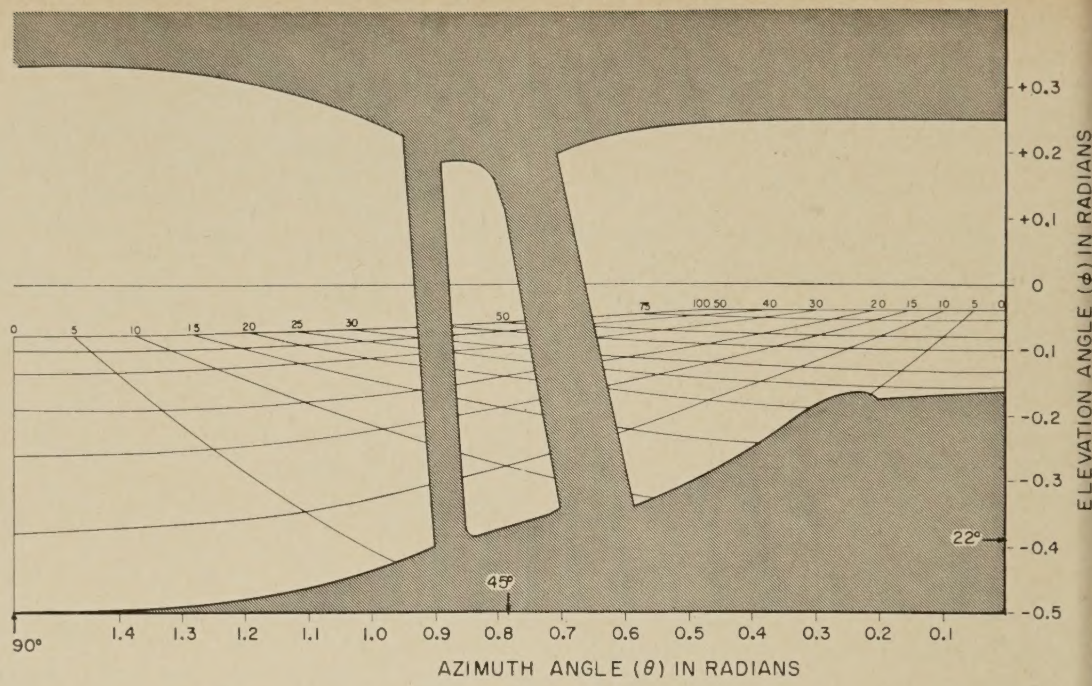


Figure 8.—Positional field through windshield and side windows of car.

Linear Perspective and Interpretive Scale

Linear perspective, the diminution of angular size with distance, is related to the positional field. The angular scale of the positional field may be expressed in terms such as $\Delta\theta/\Delta x$; in this instance it indicates the change in θ angle associated with a small change in x. If $\Delta \ell$ is a small change in x, y, and z, that is $\sqrt{\Delta x^2+\Delta y^2+\Delta z^2}$, and $\Delta\alpha$ is the change in angle associated with $\Delta \ell$, then angular scale is $\Delta\alpha/\Delta \ell$ in radians per foot or equivalent units. Thus, angular scale expresses the angular effects underlying perspective and establishes a relation between linear perspective and the positional field.

Interpretive scale may be defined as the inverse of the angular scaling effects underlying perspective. If angular scale is $\alpha/\Delta \ell$, then the corresponding expression of interpretive scale is $\Delta \ell/\Delta\alpha$ in feet per radians or equivalent units. Applied to a road map, interpretive scale represents the miles per inch required to interpret map lengths rather than the inches per mile used to draft the map. The perception of interpretive scale enables the driver to calibrate visual angle, in terms of length, on the road. Scaling may be explicitly in terms of feet and inches, but more commonly it involves distance exemplified by the driver's estimation of the space to a road point. The driver's conception of scale may be designated by primes ($\Delta \ell'/\Delta\alpha'$) to denote that a subjective estimate is implied.

As scaling involves the inverse of the angular effects underlying linear perspective, scaling would be expected to show up somewhere in space perception. Interpretive scaling is a key factor when the observer is making quantitative judgments of size, distance, or motion. In size perception, the observer estimates scale and then evaluates the visual extent of interest in this scale. Gibson describes the process as follows:

". . . with fixed monocular stimulation the size of an object is given by the size of the elements of texture or structure in the adjacent optical array. . . . Size is perceived relative to the size scale of the place where the object is seen." (*10*)

Scaling also applies to distance judgments. Gogel finds that judgments of the relative distance of objects can be explained in terms of the ratio, familiar size divided by retinal size (*11*). This ratio is equivalent to interpretive scale. Gogel, referring to the evidence of other experiments, states that the ratio underlies the judged distance between objects of similar shape but different size (*3, 12, 13*), different shapes (*14, 15*), and familiar objects of different sizes and shapes (*11, 14*). If the scale of one object appears smaller than that of another, it will be judged as being closer to the driver; if it is seen as equal, they will be judged as being equidistant from the driver; and if the scale of one object appears to be larger than another, it will be judged as being farther away from the driver. In the experiments discussed here, where background cues were minimized, judgments of relative distance were based upon relative scale.

Often distance judgments are made between widely separated objects. If the separation is large, the scale changes along the path and the judgment would be expected to take the following form, where the $\Delta_{\ell'}$ is a convenient length between the objects.

$$\text{Judged distance} = \sum_{\ell'=1}^{\ell_1=n} (\text{scale of } \Delta_{\ell'})\Delta_{\ell'} \quad (44)$$

It may be predicted that the observer woul summate in situations where the scale change A similar approach holds for judging distance from the eye, except that scale is learne through experience and does not have to b repeatedly resolved.

It seems reasonable to believe that inte pretive scale, once resolved, would be applie to size, distance, and motion judgments in situation. Thus, the scale of a familiar-size object may be applied to distance and spee judgments in the same setting. Speed jud ments may depend upon interpretive scal the angular movement of objects seen by stationary eye is converted to speed by t same rule that governs angle and length, b differentials are involved instead of tin derivatives. For example the following equ tion would be used rather than equation (9

$$\Delta\theta=\frac{-y}{r^2}\,\Delta x \quad (4$$

Thus, Δx is converted into $\Delta\theta$ by the same fun tion of y and r^2 as dx/dt is converted to $d\theta/$ The same approach holds for $\Delta\phi$. As angul stimulus is the vectorial sum of $\Delta\theta$ and components, and angular movement stimul the sum of $d\theta/dt$ and $d\phi/dt$ components motion, the communality of angle and angul velocity effects is demonstrated. The sim larity of interpretive scales required for t correct perception of length and speed also demonstrated.

The relationship between length and d tance judgments is discussed in referen under the size-distance invariance hypothe (*16, 17, 18, 19*). The hypothesis has be stated as, "A visual angle of given size det mines a unique ratio of apparent size apparent distance." (*20*) As the hypothe describes a perceptual relationship, it must shown valid on the basis of perceptual rat than geometrical evidence. Thus, if size a distance judgments are based upon the sa interpretive scale, a rational basis would ovided for an invariance hypothesis. In ne circumstances, the scaling underlying e and distance judgments differs. The scale a film projection may be established by niliar objects included in the scene, which uld not directly indicate the scale of eye tance. The same principle is true in size lgments of geological specimens. A hamr included in a page illustration may provide size scale, but distance from the eye would t be shown thereby. The moon illusion is puzzling example in which the seen size and tance are paradoxically unrelated (*21*). e relation of linear and velocity scales has ceived less research attention. Experiments J. F. Brown may be interpreted as inditing that linear scale is actually applied in ed judgments (*22*).

The author finds it tempting to think that nt and shape perception could also be plained in terms of interpretive scale. nt could be dealt with as the seen ratio of ontparallel scale to surface scale perpenular to this line. If the ratio of these les were one-half, the surface would be 60° the line of sight, that is, cosine 60° is onelf. This explanation of slant perception objectionable because it seems more complex an the judgment explained. The author nds to believe that slant and the perception shape are basic perceptions that precede ther than follow scaling judgments. Hower, if an observer participating in a shape nstancy experiment makes estimates of the mensions of simple rectangles or ellipses ojected at a slant to the eye, the responses ay be considered to involve relative scaling.

The question remains of how scale is obined. In theory, a decoding could be nply achieved as every x and y point on the ound plane has a unique θ and ϕ representtion. A mechanical robot could steer own a level road and could avoid obstacles it were able to sense θ and ϕ and if the road ere correctly coded for it. Such a twomensional approach has an appealing simicity but the nature of the visual world is ree-dimensional. Scale is clearly shown by miliar sized objects, particularly those havg parallel sides. The quantity $\Delta\ell'$ is then e known length of the familiar object; $\Delta\alpha'$ its seen angular extent. An example is ovided by the space perceptions of the otorist. An obvious key to the terrain conguration is the shape of the road ahead. he roadway is of almost constant width, it nverges rapidly in perspective, and it prodes a ready scale for converting visual angle linear extent, which may be applied to her objects. Four road configurations are own in figure 9.

In part 4 of figure 9, the projection of the ad boundaries shows Δz above $L_1 = L_2$. If e borders of the road are straight, the road self has no vertical curvature. If the orders are concave, the road surface is conave. If the borders are convex, the road is so. On an uphill, the road may rise above e height; downhill it will be below eye level, d the road may in fact be overlapped and dden. Light and shadow, texture, occular ljustments, intersections of surfaces, and overlapping of contours may enhance these perceptions. The roadway is a particularly convenient scale since it is always available, has known characteristics, and provides continual feedback to the driver on the accuracy of his perceptions. However, other familiar objects such as vehicles, pedestrians, houses, fence posts, sidewalks, and crosswalks are usually in the field of view and may also be used to calibrate visual angle.

Angular Velocity Field and Vehicular Guidance

Velocity vectors along the flat ground plane have been plotted in figure 10. The ground area covered has been superimposed on figure 8; the magnitude and direction of the ground flow is shown by the length and direction of the vectors in the figure. The angular velocity field appears to fit almost exactly over the positional field. The resemblance is the result of the approximate equivalence of $d\theta/dx$ and $d\phi/dx$ vectors to $\Delta\theta/\Delta x$ and $\Delta\phi/\Delta x$ angular extents. As the eye moves in x, velocity vectors give the effect of equally sized rods parallel to the x axis. On the ground plane these vectors fall on perspective lines. As shown in figure 10, $d\theta/dx$ is zero along the median plane, approaches zero at $\theta = \pi/2$, and both $d\theta/dx$ and $d\phi/dx$ are zero at the vanishing point at eye level directly ahead. Objects off the path change their $d\phi/dx$ velocity component to predominating $d\theta/dx$ as they are approached.

Perception of motion

The psychological basis of motion perception is discussed in references (*4*, *5*). At slow rates, motion is inferred from a change in position. A familiar example of this type of motion perception is the minute hand of a watch. Motion is interpreted by comparison of the positions assumed by the hand with the passing of time. At more rapid rates, motion is directly perceived. This perception is exemplified by the motion of the second hand of a watch, which actually is seen to move. At still more rapid rates, motion appears as a blur. Similar judgments and perceptions enable the driver to register impressions of motion related to the velocity field around him.

Basic reference for object motion

Just as the positional field provides a scale for object size, the velocity field might provide the background for the seen movement of objects. However, the driver may see the ground as a still reference or he may conceive that other cars are moving in reference to his own vehicle. For example, the driver reacts promptly when a vehicle approaching his moving car has no apparent sidewise velocity vector. This is visual warning of an impending collision. This relation was used by

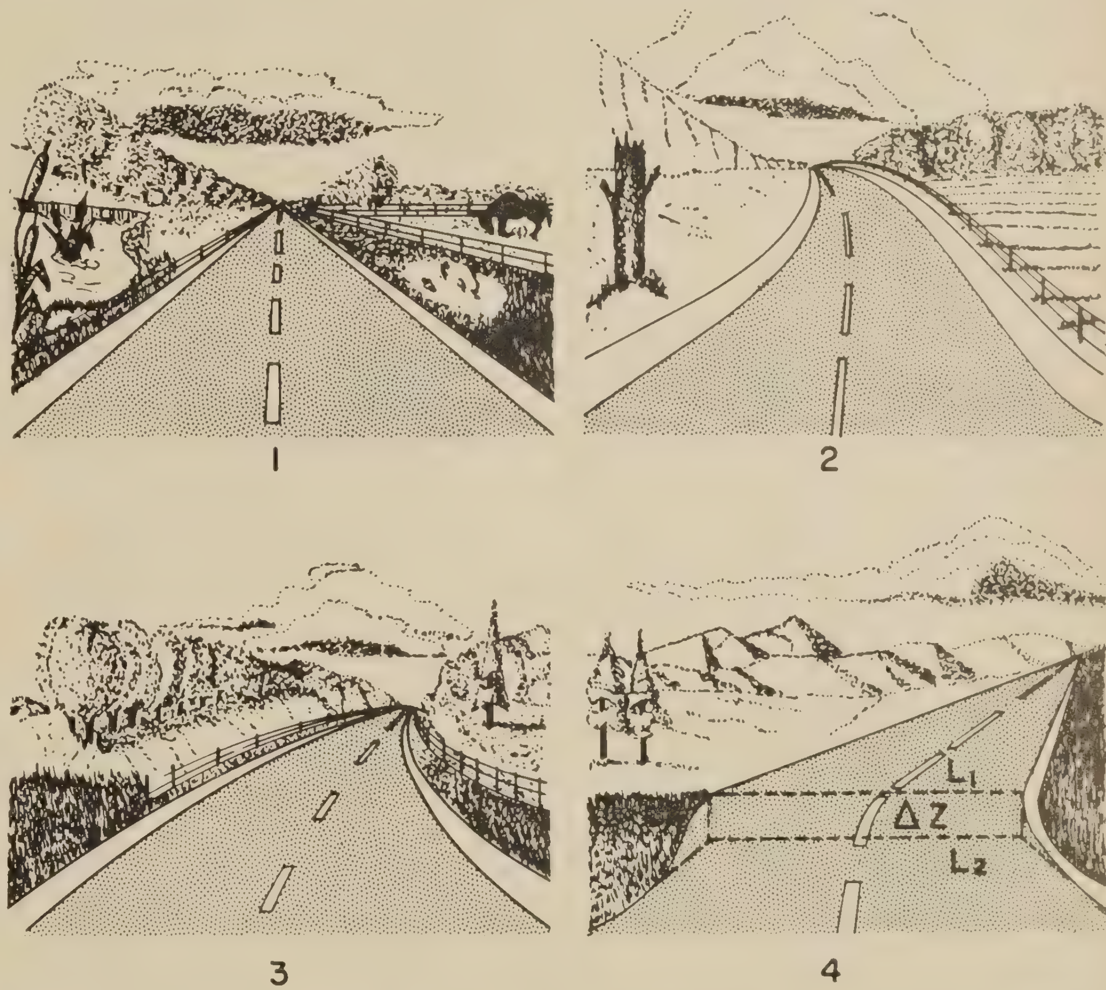


Figure 9.—Road configurations. Width of road provides linear scale; convergence pattern shows vertical curvature; in part 4, projection of road boundaries shows Δz above $L_1 = L_2$.

Michaels and Cozan to explain vehicular avoidance reactions (9). An analogous situation occurs when the resultant vector is directed toward the median plane. The intruding vehicle will cross a driver's path and may collide. The driver uses his own vehicle as reference when he reacts to such situations.

The velocity field affects the driver's sensitivity to motion. The relatively small angular motions ahead of the vehicle favor the perception of moving objects, particularly in the θ direction. The fast movement of the field at $\theta=\pi/2$ inhibits perception of object motion. If the vehicle moves very rapidly, the resolution of independent motion is impaired by blurring. Angular velocities of several hundred degrees per second are generated at the side of a rapidly moving vehicle and the driver's vision is reduced even if he follows object movement with his head and eyes (*23, 24, 25*).

Perception of speed and direction of vehicular motion

The velocity field provides information on the speed and direction of the vehicle's forward motion. Although observed motion is the most direct indicator of vehicular motion, it is but one of its accompaniments. Vehicular speed is also indicated by direct speedometer readings, the pull of the steering wheel, the gear in use, the response to the accelerator, the pitch of the engine and tire squeal, the roughness of the ride, and centrifugal force when a curve is rounded. Apparently the visual appearance of motion cannot be claimed to be the only or even the most useful input for the estimation of speed.

The direction of vehicular motion is indicated by the flow characteristics of the velocity field. On a straight path there is no $d\theta/dt$ component in the median plane ahead of the driver. This lack of motion may be used by the driver, along with information of postu
and the position of objects in the windshield,
indicate the direction of the vehicle's motic

The importance of expansion patterns
an indication of the vehicle's direction
movement has been pointed out by Jam
Gibson (*6*), as follows:

"When an observer approaches a surfa
instead of moving parallel to it, a modificati
of its deformation is introduced in that t
focus of expansion is no longer on the horiz
of that surface but at a particular spot on it
the point of collision with the surface. T
rule is that all deformation in a forward visu
field radiates from this point. Crud
speaking, the environmental scene expa
as we move into it, and the focus of expansi
provides us with a point of aim for our locon
tion. An object in our line of travel, regard
as a patch of color, enlarges as we approa
It is not difficult to understand, therefc
why this expansion should be a stimulus
sensed locomotion as well as a stimulus
sensing the lay of the land. The behav
involved in steering an automobile,
instance, has usually been misunderstoo
It is less a matter of aligning the car with
road than it is a matter of keeping the focus
expansion in the direction one must go."

Although the direction of vehicular mot
is related to the focus of expansion, the fo
itself is not an effective cue. The focus
expansion of a flat horizontal plane lies
the vanishing point in the sky or it
occupy points on trees or buildings if the r
is curved. Generally, it is difficult, if
impossible, for the driver to locate the fo
of expansion (figs. 5, 6, and 10) and, contr
to Gibson, the borders and lane markings
used in vehicular guidance. When the veh
is off course, these lines have a lateral co
ponent of movement.

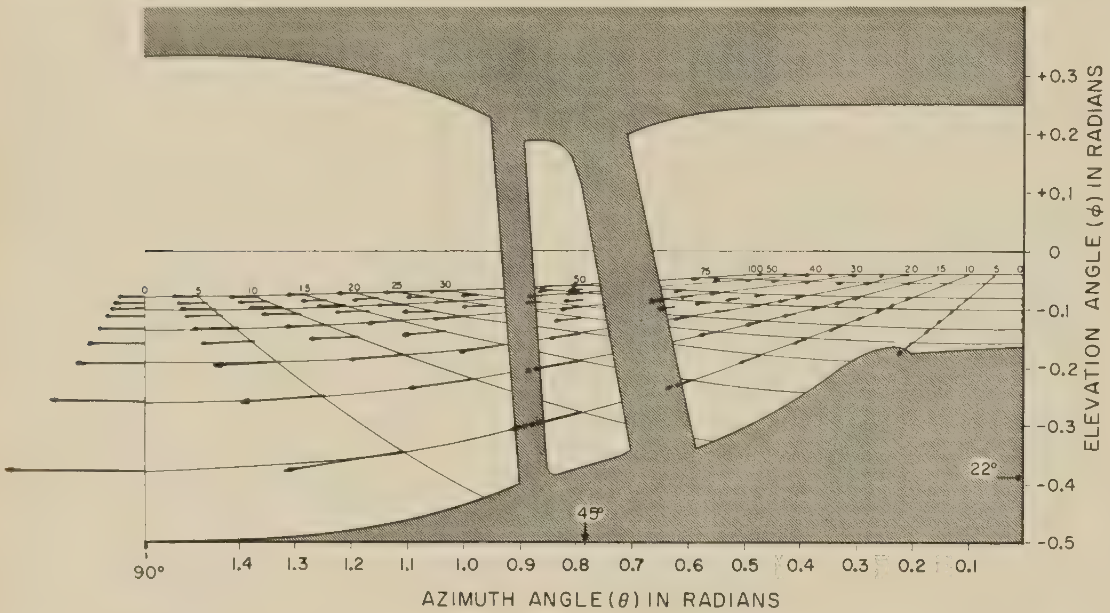


Figure 10.—Velocity vectors on basic ground plane. Direction and amplitude of angular velocity is indicated by direction and length of vectors.

Acceleration Field

The projection of the acceleration field on
the ground plane is shown in figure 11.
vectors on the field represent the differe
in successive velocity vectors, divided by
time, as time approaches zero. The
area is the same as in figures 8 and 10,
the vector scale is 10 times as large. A
shown in figure 11, there is no azimu
component ahead of the eye under rectili
motion, and $d^2\phi/dx^2$ is directed toward the
in these positions. Vectors are directed a
from the eye at $\theta=\pi/2$ where $d\theta/dx$
through a maximum. At angles betwe
$\theta=0$ and $\theta=\pi/2$, the vectors shift from
approaching to a receding direction and
generally largest close to the eye.

Perceptual Problems

The major perceptual problem of the
acceleration input is whether it is dir
sensed. The human can distinguish acce
ations, but it is not certain that they
detected as such. They may possibl
inferred from successive impressions of c
ing rates. Gottsdanker and Frick and J
hard showed that group performance is

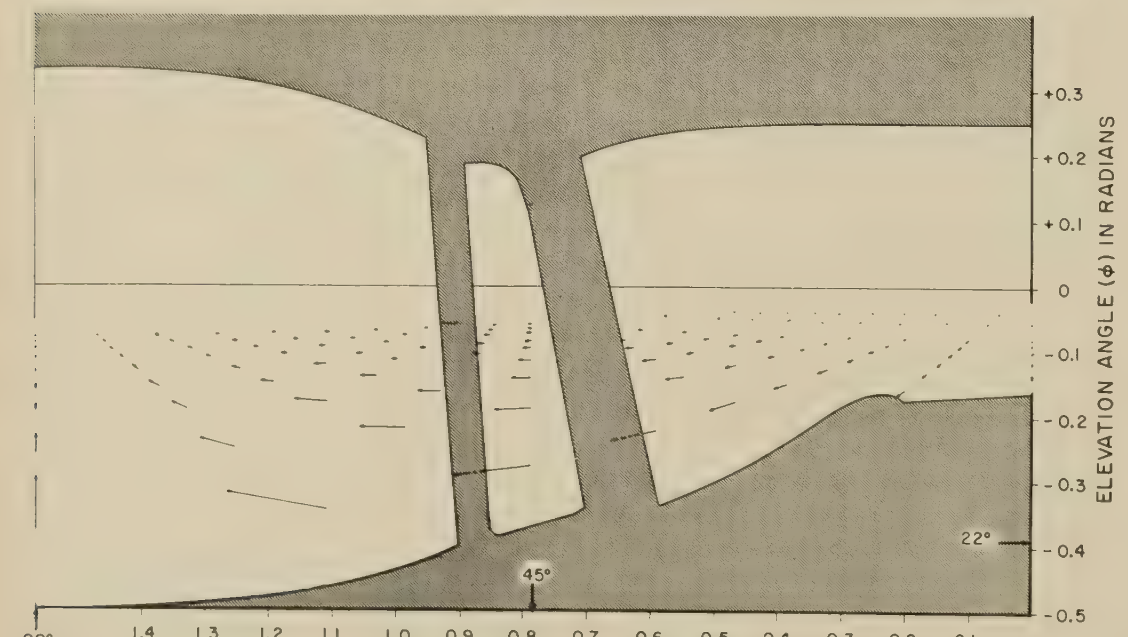


Figure 11.—Acceleration vectors on basic ground plane.

early ordered in terms of a threshold based upon total change in velocity than in terms of direct sensory impression of acceleration (3, 27).

The vector field shown in figure 11 provides evidence on the sensing of acceleration. The field appears unnatural and there is no characteristic of experience that we can associate with the pattern of vectors shown. This differs from the psychophysical correspondence between light wave length and hue, physical energy, brightness, and so on. As the observer does not directly or precisely register accelerations, the acceleration field and the gradients within it cannot be considered a primary visual input. The same conclusion probably holds, by extension, to higher velocity derivatives. This is not to suggest that accelerations may not be perceived as changes in velocity.

Acceleration varies as the square of speed, as shown in equations (11) and (12). If speed of the moving eye is doubled, angular acceleration is quadrupled. The same condition holds for the eye on a curved trajectory. This relation leads to the paradoxical situation that the angular acceleration is more sensitive to speed than is angular velocity. It would be expected that the appearance of the environment would change markedly as linear speed is increased and that there would be acceleratory indications of velocity. The visual appearance of increased velocity on a roadway may be a sharp swoop of objects and road features as they change from a ϕ to a θ direction. The imperfections of lane markers and road edges may show acceleratory jitters. These acceleratory effects, however, have not been systematically verified.

Part 3—Motion Parallax and Perceptual Hypothesis Testing

Introduction

WHEN A PERSON is driving a car or walking, objects seem to pass on either side; distant objects seem to move more slowly than those close by. The relative motion of seen objects, formally called motion parallax, might be expected to provide an indication of distance. Discussions of depth perception, such as in introductory psychology textbooks, mention motion parallax as a classical cue to distance, along with visual angle, interposition, linear perspective, aerial perspective, and shadows (*28*, *29*, *30*, *31*).

Motion Parallax as Distance Indicator

The first thorough discussion of motion parallax as an indicator of distance is given by H. von Helmholtz in his volume of *Physiological Optics* (1866). According to translation, Helmholtz wrote (*5*):

"In walking along, the objects that are at rest by the wayside stay behind us; that is, they appear to glide past us in our field of view in the opposite direction to that in which we are advancing. More distant objects do the same way, only more slowly, while very remote bodies like the stars maintain their permanent positions in the field of view, provided the direction of the head and body keep the same directions. Evidently, under these circumstances, the apparent angular velocities of objects in the field of view will be inversely proportional to their real distances away; and, consequently, safe conclusions can be drawn as to the real distance of the body from its apparent angular velocity.

"Moreover, in this case there is a relative displacement of objects at different distances with respect to each other. Those that are farther off as compared with those that are nearer seem to be advancing with the observer, whereas those that are near seem to be coming toward him; and the result is we have a very distinct apperception of the fact that they are unequally far from us. Suppose, for instance, that a person is standing still in a thick woods, where it is impossible for him to distinguish, except vaguely and roughly, in the mass of foliage and branches all around him what belongs to one tree and what to another, or how far apart the separate trees are, etc. But the moment he begins to move forward, everything disentangles itself, and immediately he gets an apperception of the material contents of the woods and their relations to each other in space, just as if he were looking at a good stereoscopic view of it."

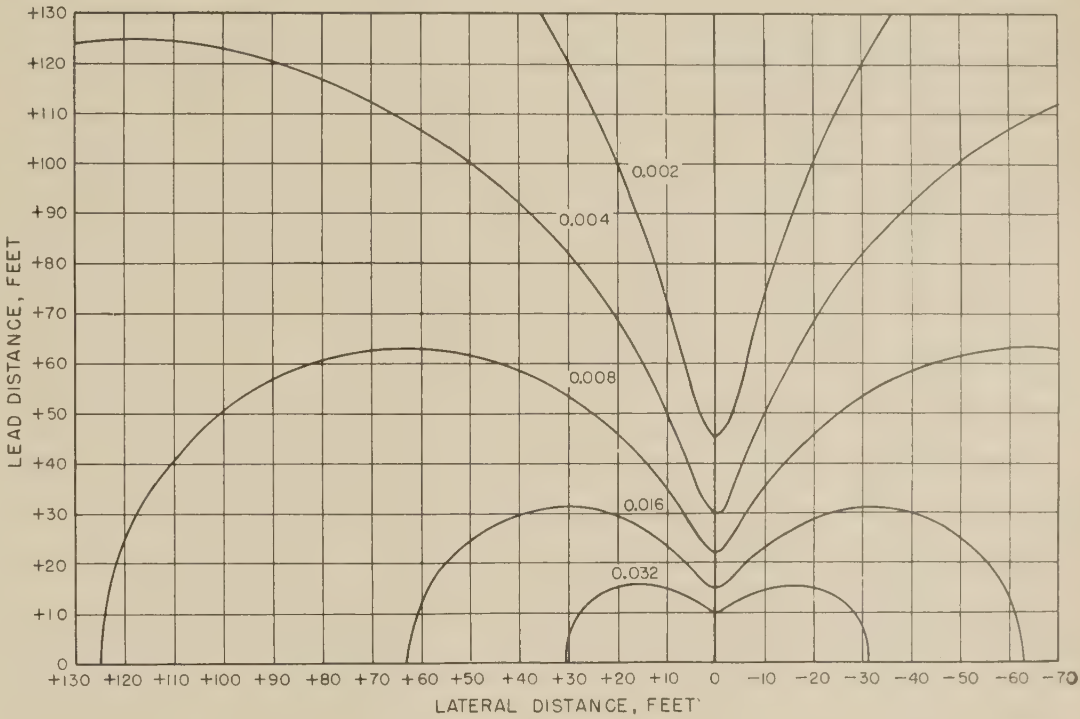


Figure 12.—Isoangular-velocity curves under linear motion. Angular velocity in radians per second of each contour can be obtained by multiplying value shown for each curve by vehicular speed in feet per second.

Helmholtz's statement that ". . . safe conclusions can be drawn as to the real distance of the body from its apparent angular velocity", can be questioned. When the observer follows a curved path, angular motion of ground points becomes an unreliable indicator of distance. The basic geometry of the situation is altered, so that motion of the ground or other points does not decrease regularly along a sight line. This is shown by the velocity field of curvilinear motion illustrated in figure 5 and by a comparison of isoangular-velocity curves of linear and curvilinear observer motions shown in figures 12 and 13. The curves are based upon equations (9), (10), (19), and (20) derived for object movement under linear and curvilinear observer movements. Isoangular-velocity curves involve a fixed sum of azimuthal and elevation velocity components, which add vectorially as

$$\sqrt{\left(\frac{d\theta}{dt}\right)^2+\left(\frac{d\phi}{dt}\right)^2} \qquad (46)$$

An infinite number of specific combinations of $d\theta/dt$ and $d\phi/dt$ may produce a given sum. Two combinations making up the sum 0.001 are $d\theta/dt=0.000707$ $d\phi/dt=0.000707$, and $d\theta/dt=0.0008$ $d\phi/dt=0.0006$. Each combination when substituted in equations (9) and (10) or (19) and (20) can be solved for x and y and hence yield a point for plotting on an isoangular-velocity curve. In practice, the solving of these equations was done on an electronic computer.

Complications

The vector field of horizontally curved observer motion depicted in figure 5 shows a number of features that complicate a motion

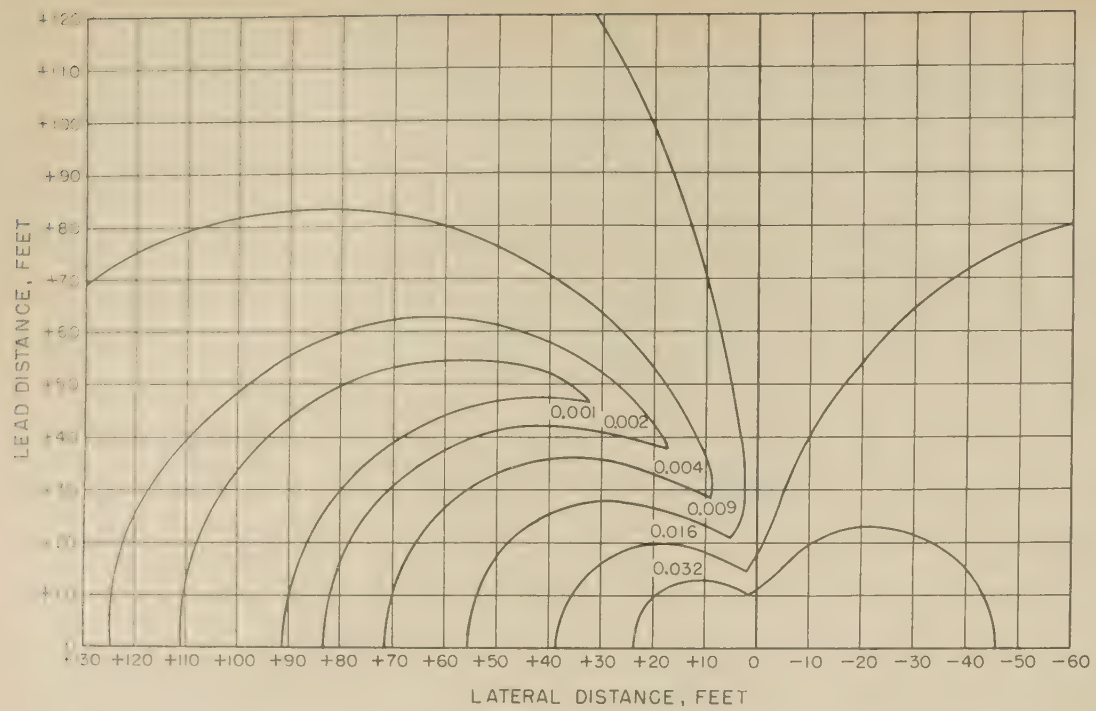


Figure 13.—Isoangular-velocity curves under curvilinear motion. Center of curvature is 100 feet to left of origin. Angular velocity in radians per second for each contour can be obtained by multiplying value shown for each curve by vehicular speed in feet per second.

parallax interpretation. As distance from the eye increases along an azimuth line, the direction of ground motion changes as well as magnitude. Motion on 0.3 radian azimuth line is to the right at far distances and to the left at close points. The interpretation of distance from these motions alone would be difficult. A somewhat similar objection has been previously raised by Gibson who considered linear motion and noted that angular velocity relates to distance only along sight lines off the vanishing point directly ahead (*1*).

Difficulties in using motion-parallax under curved motion are shown in the isoangular-velocity plots. These curves show the locus of terrain points of equal angular velocity, regardless of direction of motion. Thus, linear isoangular-velocity curves decrease fairly systematically with the reciprocal of distance on each azimuth. The curvilinear isoangular-velocity pattern is different. The functions are asymmetrical and approach a limiting value of $1/R\ dx/dt$, where R is the radius of curvature and dx/dt the speed of the motion. Angular movement, as shown in figure 14, is seen to reverse itself on the sight line through the center of rotation at $x=0$, $y=100$ feet. An interpretation in accord with the motion parallax approach would place the stationary center of rotation at infinite distance from the eye. Positions beyond the center of rotation on the same azimuth line increase in angular velocity and hence would be interpreted as decreasing in distance. Evidently, distance is not related to ground motion in the manner proposed by Helmholtz, when the vehicle is following a curved path.

On a straight trajectory, distances at right angles to the line of movement where angular velocity is high are not noticeably easier to estimate than those ahead where angular velocity is minimal. On the contrary, illusory movements of the terrain are seen to the side, which depend upon the observer's visual fixation position. If the foreground is viewed from a car, the background seems to move and rotate forward around it. If the background is fixated, the foreground seems to turn. This illusion of rotation, which may be called motion parallax curl, is based on differences in angular

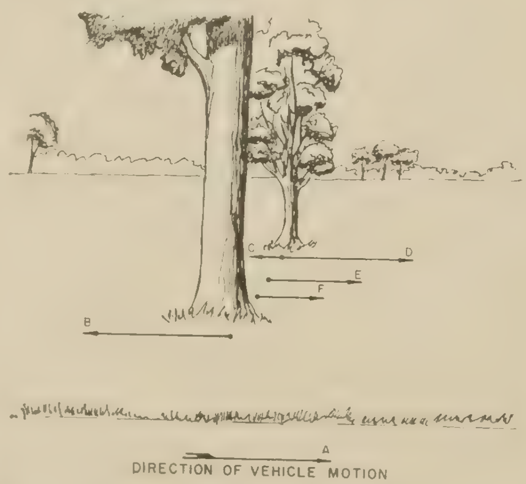


Figure 14.—Motion parallax curl. Illusion shown at right angles to vehicle's line of movement. Vehicle moving in direction A, inducing vectors B in foreground tree and vectors C in background tree. If foreground tree is fixated, background moves to right with velocity D. Ground positions between trees have linearly decreasing velocity vectors that produce appearance of rotation.

velocity between the foreground and backgroun as shown in figure 14. The point of reference of seen movement is ambiguous and is no necessarily the vehicle itself, as a motio parallax formulation would require. Vo Kries mentions this illusion: He believes tha the reference point of motion is the stationar point of fixation, according to a note on pag 372 of reference *5*. The magnitude of th illusion is given by the following derive formula:

$$V=\left[\frac{(r_1-r_2)}{r_1r_2}\sin\theta\right]\frac{dx}{dt} \qquad (47$$

Where,

V = angular velocity of curl, rad./sec.
θ = angle of object from line of motion
r_1 = distance of foreground fixation
r_2 = distance of background object
dx/dt = vehicle's speed, ft./sec.

The illusion increases with the distanc between the foreground and backgroun velocity of the vehicle, and lateral angle. I decreases with remoteness of both foregroun and background points from the eye. parallax curl is also seen when tree limb telephone wires, and other objects above th horizontal are viewed. However, this form the illusion seldom is noticed.

Experimental evidence lends support to th geometrical analysis. The context of motio must be revealed to the observer if he is to se depth—differential motion alone is not suffi cient to produce the experience (*32*). Thus the movement of a pattern produced by shadow caster gives the appearance of varyin depth only if the pattern is seen as a surface If it is seen as a moving surface, a more ac curate judgment of slant is given than if th surface is stationary (*33, 34*). If the observe surmises that he is viewing objects at differen distances, as when he sees luminous patches i a dark room, he can estimate relative but no absolute object distance by moving his hea (*34, 35*). If the arrangement of a shado caster is described to an observer, he ca make relatively accurate depth judgments moving objects even though movement itse may not be experienced (*34*). These result show that it is difficult, if not impossible, t estimate distance from object movement alon

Perceptual Hypothesis Testing

Examples exist in the literature, which indi cate that motion does facilitate space percep tion. The effect is not through motio parallax, but by what may be called per ceptual hypothesis testing. Other facilitativ effects of motion are the kinetic depth effec (*36*) and minute image movements (*37* These phenomena are not closely related t motion parallax so are not considered herein When the observer alters his viewing positio perspective relationships change in a uniqu manner dictated by the structure of the fiel and the movement made. This change per mits previous hypotheses concerning the func tional nature of objects and surfaces to b verified or amended. Perceptual hypothesi testing is also related to the Transactiona heory of Perception (*38, 39*) and to Brunsick's Probabilistic Functionalism (*40*). As own by Ames (*38*), Gibson (*6*), Gogel (*41*), d others, a static view may lead to multiple even illusory perceptions. But the faulty rception will not pass the test of redundant ewing under movement; it is not supported the perspective changes that occur. ypothesis testing, involving observer motion, o may involve testing by coherency. This inciple is illustrated by Helmholtz's tree ample. Under observer movement, a single e among others shows a coherent movement that differs in speed and extent from that of other trees differently placed. The induced movements serve to resolve the scene into spatially separate components. This simple coherency principle aids space perception, regardless of the observer's path of motion or fixation position. Coherency testing is related to Wertheimer's "law of common fate," according to which points moving simultaneously in the same direction are readily seen as a group (*42*).

In certain limited and special situations, observers have been reported to use motion parallax information to tell distance. Such cases include among others the animal studies by Wallace, Pumphrey and Grinnell, and Gibson and Walk, which have been summarized by Shinkman (*43*). Experiments involving infants placed on an optical cliff have also been interpreted to indicate the importance of motion parallax (*44*). The author questions whether these situations really involved the deduction of distance from angular movement of two or more separated objects. Much of what has been interpreted as the operation of motion parallax may perhaps be explained as hypothesis testing involving change of perspective.

Part 4—Perceptual Mechanisms in Vehicular Guidance

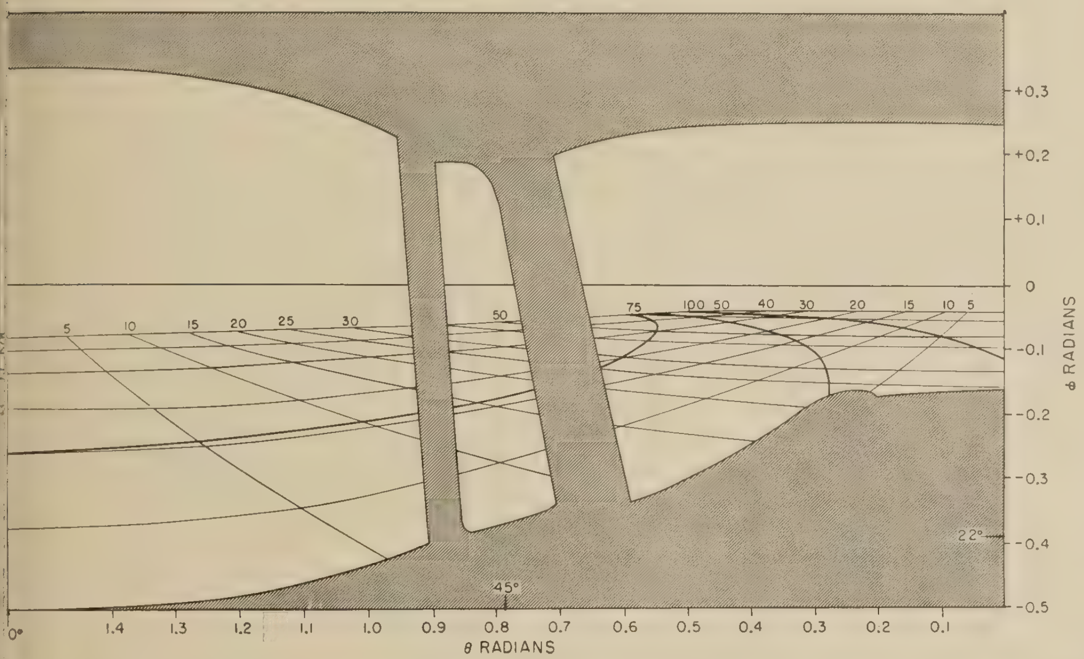


Figure 15.—Positional field through windshield and side window on a curved road. Left and right road borders and center lane divider are shown.

Introduction

IN THIS PART of the article curvilinear vehicular motion is discussed in addition to ctilinear motion, and the results are applied the basic maneuvers of lateral guidance teering), anticipation, and car following.

It is important that the curvilinear motion the vehicle be studied. Visual studies of rcraft landing (*45, 46, 2*), car following (*7, 48*), and spatial orientation in motion ve been exclusively on rectilinear motion. he aircraft or automobile often follows a rved path and, even on a supposedly raight path, the actual path of the vehicle ll be slightly curved. Rectilinear motion ay, in fact, be considered as a special currence of curvilinear motion, where the dius of curvature becomes infinite. Several cepted guidance theories, plausible enough r application to driving on straight roads, il to apply generally to the curvilinear tuation.

Position and Motion

The derivation of angular position and the motion of ground points under curvilinear movement of the eye are discussed in the first part of this article. The coordinate system is shown in figures 1 and 2. In these figures, the driver's eye is placed at the coordinate origin, distance ahead is represented by x, to the side by y, and up and down by z. Distance from the eye to any point of the field is represented by ρ, whose projection on the xy plane is r. Equations (1), (2), (5), (6), (7), and (8) hold for horizontally curved, constant speed, vehicular motion. On curves to the right the equations are:

$$\frac{d\theta}{dt}=\left(\frac{-y}{r^2}-\frac{1}{R}\right)\frac{dx}{dt}\ \text{rad./sec.} \qquad (48)$$

$$\frac{d\phi}{dt}=\frac{-xz}{r\rho^2}\frac{dx}{dt}\ \text{rad./sec.} \qquad (49)$$

$$\frac{d^2\theta}{dt}=\left(\frac{2xy}{r^4}+\frac{x}{Rr^2}\right)\left(\frac{dx}{dt}\right)^2\ \text{rad./sec./sec.} \qquad (50)$$

$$\frac{d^2\phi}{dt^2}=\left[\frac{-z}{\rho^4 r^3}(\rho^2y^2-2r^2x^2)\ \frac{-zy}{\rho^2 rR}\right]\left(\frac{dx}{dt}\right)^2\ \text{rad./sec./sec.} \qquad (51)$$

The positional equations (1) and (2) are identical to those for linear translation, thus indicating that perspective is unaffected by the path of motion. The declination angular velocity, equation (6), also is unchanged by circular motion. But azimuthal angular velocity, $d\theta/dt=\left[(-y/r^2)+\frac{1}{R}\right]\frac{dx}{dt}$, differs from the linear, $d\theta/dt=-y/r^2\ dx/dt$ by the constant $1/R\ dx/dt$, which is equal to angular velocity, $d\beta/dt$. The velocity effects of circular field movement therefore involve the simple addition of translational and rotational effects. The expressions for $d\theta/dt$ and $d^2\theta/dt^2$ are independent of viewing height, z. In contrast, expressions for $d\phi/dt$ and $d^2\phi/dt^2$ do involve viewing height. These equations of position and motion have broad applicability. They govern the angular position, velocity, and acceleration of any point in x, y, and z. These equations also may be modified to describe the seen movement of vehicles, pedestrians, and other objects that are in motion.

Positional Field

The positional field is illustrated in figure 15. The driver's eye is placed at a representative height of 4 feet above the ground. The vehicle is shown on a sharp curve of 100-foot radius. As shown by figure 15 a road of constant curvature assumes a steady state appearance up to the break in curvature. The existence of a steady state is important in describing driving. Lateral guidance and car following could consequently be considered to involve the maintainance, through visual feedback, of an acceptable steady state condition and the nulling of deviations from it.

The positional field shown in figure 15 is a graphing of a mathematical relationship; yet it is readily interpreted as the view from inside a car. Perspective lines on the positional field provide sufficient information to elicit a perception of the ground plane, even

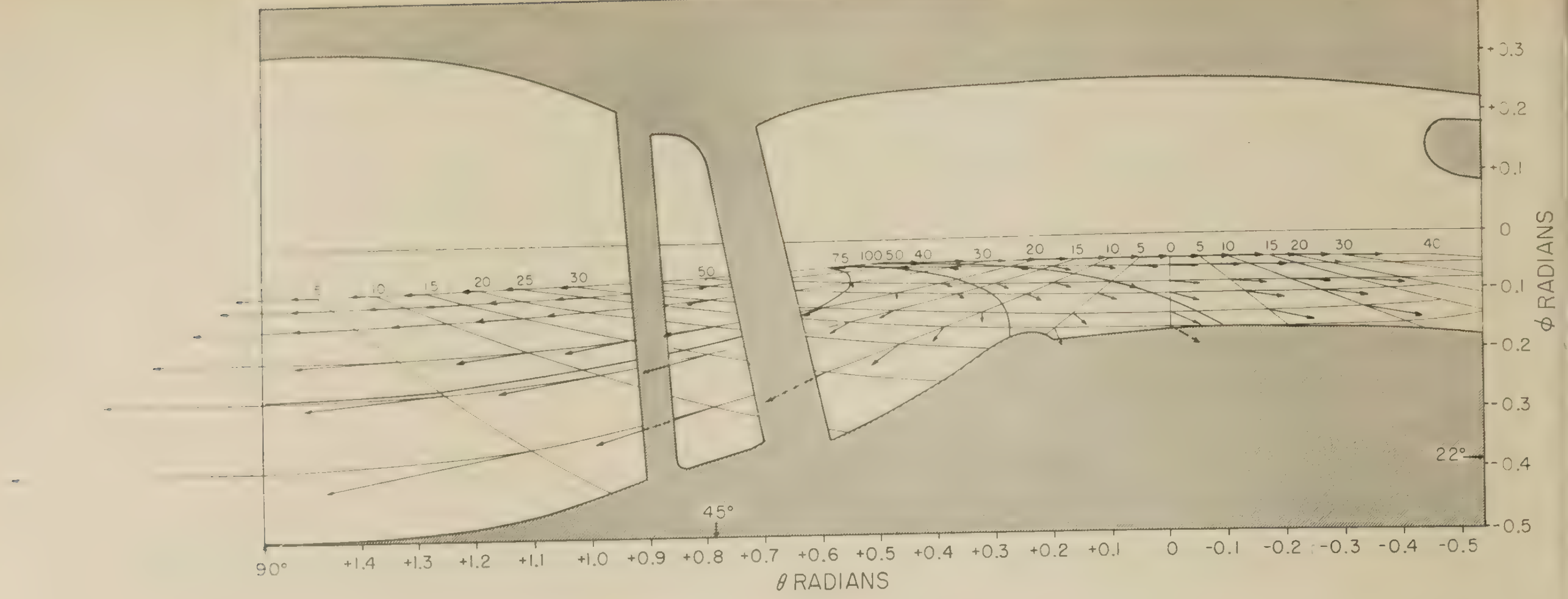


Figure 16.—Velocity vectors on basic ground plane. Direction and amplitude of angular velocity is indicated by direction and length of vectors.

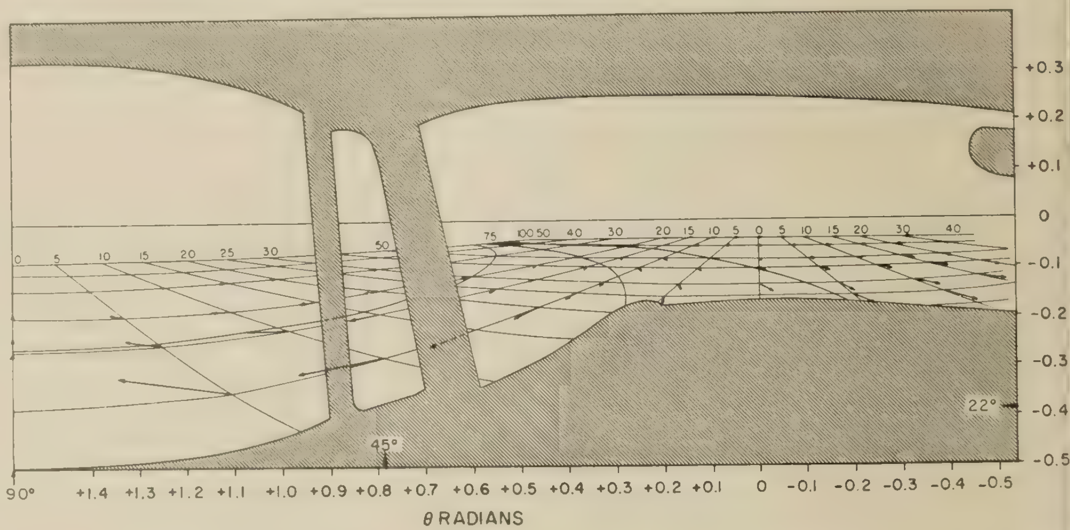


Figure 17.—Acceleration vectors on basic ground plane. Direction and amplitude angular acceleration is indicated by direction and length of vectors.

though texture, binocular disparity, road objects, motion, and other information are absent. Previously, it was suggested that the interpretive scaling of visual angle is a fundamental process in length, distance, and motion estimation, and that the configuration of the road might enable the driver to obtain this scale. Interpretive scaling also applies in curvilinear motion, as the positional field is not affected by the vehicle's path of motion.

Velocity Field

The velocity field on the basic ground plane is plotted in figure 16. The magnitude and direction of the ground flow is shown by the length and direction of the vectors. If the vehicle accelerates or decelerates, the vector pattern remains unchanged, but individual vectors will change proportionally in magnitude.

The curvilinear velocity field resembles the linear field to the side of the vehicle where the vectorial effects of simple translation are themselves large. The field differs at distant points, where rotational effects overshadow those of translation. The field is asymmetrical: angular rotation is added to azimuthal velocity on the side opposite the center of curvature and subtracted from the near side. The velocity vectors are tangent to the road boundaries and lane markings, a condition necessary for the existence of a steady state. The only stationary point on the field is the center of circular curvature located R feet to the side of the vehicle.

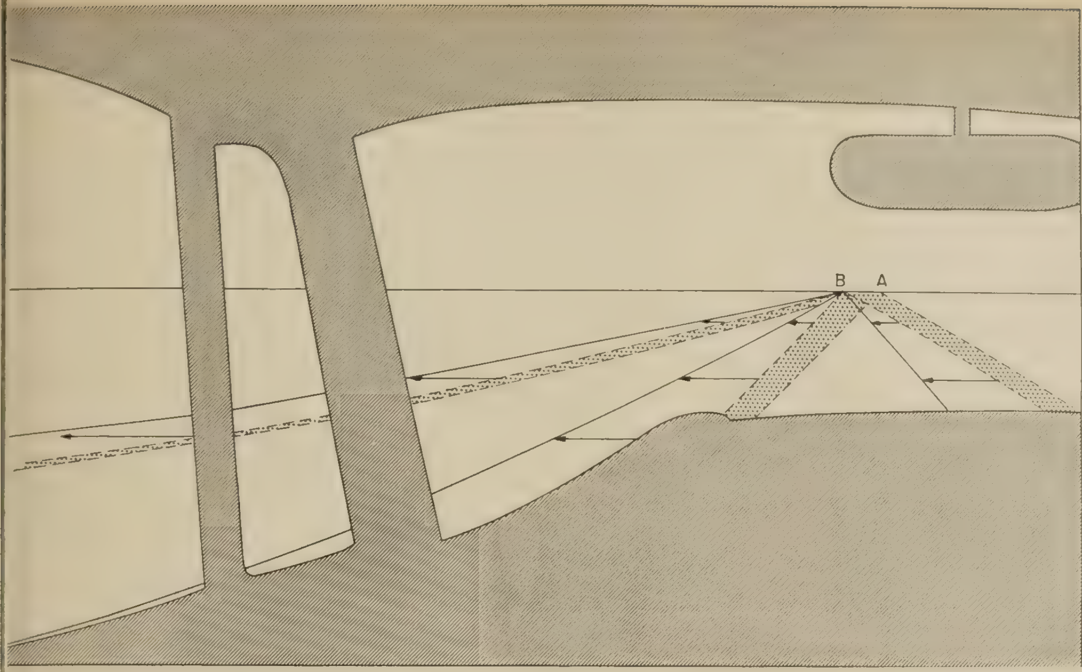
The velocity field is perceived as a positional field in motion. The moving observer does not see a confusing field of independent motions, such as might be visualized by examining separate points of figure 16. Rather, the velocity vectors are perceptually organized, or inherent, in a unitary conception of a terrain in motion. The velocity vectors are a function of the speed and direction of the observer's motion, and unlike the visual angle stimulus, do not reveal the scale of object length.

A velocity projection such as shown in figure 16 is generated by an infinite number of stimulus conditions. If distance to the surface is multiplied by a constant, and speed as well, the vectorial pattern is unchanged. Distance to the surface must be specified to permit a judgment to be made of the speed of motion. The perception of movement implies a perceived relationship between the eye and moving environment. If references are absent, as in a film record of a featureless terrain of sand or brush, the direction of motion is difficult to determine. Experimental evidence suggests that the driver guides himself by reference to the road edges and the center stripe (*49*).

Acceleration Field

The angular acceleration field is presented in vector form in figure 17. The field represents differences in successive velocity vectors, divided by time, as time approaches ze The cab area and ground of figure 17 are same as in figure 16, but the vector scale is times as large. The direction of the f vectors resembles the direction of perspec flow lines at large θ angles, but the directio otherwise rather irregular. The field dif from the acceleration field of linear mot particularly at $\theta=0$, where the vectors directed toward negative θ rather than strai toward the observer.

The acceleration field associated with cu linear vehicular motion relates to probl previously considered in relation to rectili motion. Analogously, the peculiarities of acceleration field pattern, particularly $\theta=\pi/2$ provide evidence that angular ac eration is not directly sensed. The equat for curvilinear acceleration also contai $(dx/dt)^2$ term, indicating the sensitivity acceleration to speed and raising the expe tion that there would be accelerative ind tions of velocity.



igure 18.—Lateral guidance on a straight road. Initial slewing is shown by shaded area and sideslip by arrows.

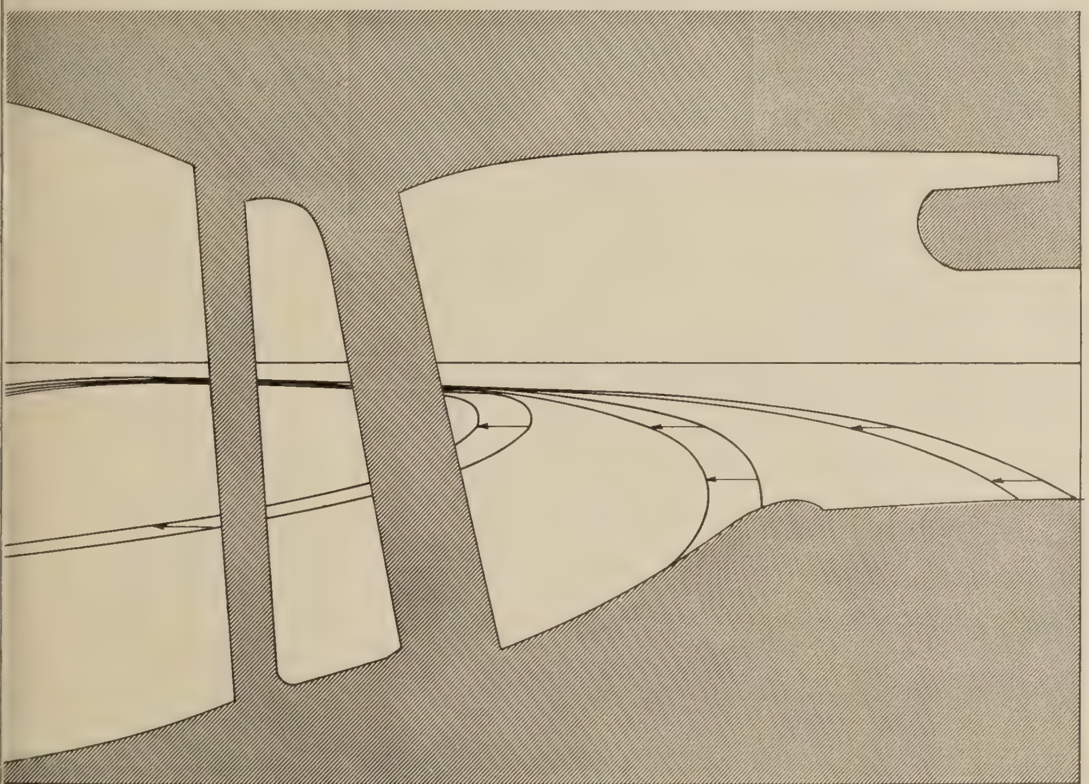


Figure 19.—Lateral guidance on curved road. Sideslip is shown by arrows.

Vehicular Guidance Modes

The previous discussion now may be aplied to explain the vehicular maneuvers of teral guidance (steering), perceptual anticiation, and car following. These guidance aneuvers have been the subject of other search and are accepted as important driving ctivities (*47, 48, 50, 51, 52, 53*).

Steering

When the moving vehicle is alined with the ighway, each point on the road border and lane marker falls on the angular position previously occupied by another point of the border, and the road assumes a steady state appearance. When the steering wheel is turned, the road borders and lane marker appear to move in the opposite direction. All parts move; no one part is essential for tracking. In fact, the driver may assume a somewhat unlocalized surveillance of the road, which would facilitate seeing a steady state and also reduce the involuntary rapid movement of the eyeball, which is technically referred to as nystagmus. The driver may become aware of the misalinement of the car by slewing shifts in direction, and by sideslipping, sidewise movements. These effects are illustrated by the perspective diagrams of figures 18 and 19. A 2½° shift in angle on a straight road is shown in figure 18. The initial 2½° slew exceeds the human visual acuity threshold, which is about a minute of visual angle (*5*). If the movement is executed in less than a second, the movement also exceeds the threshold for visual motion, which is about 2 minutes of arc per second (*5, 54, 55*). Sidewise movement to the road borders is perceptible either as a movement or change in position.

The visual effects of misalinement on a curved road are illustrated in figure 19. At 30 m.p.h. velocity, the tire will hit the shoulder of the 100-foot radius road in 1½ seconds. Average slewing rate is 10° per second over the 15° shift in angle. Evidently, amount or rate of slewing, or amount or rate of sideslipping, may provide the driver with feedback from the maneuver. Even if road movement is as small as 3½ inches (1/12 the values of the illustration) sideslipping and slewing on a straight road would exceed human visual position and movement thresholds. On the basis of human perception theory, it is difficult to determine which of the four combinations of slew, sideslip, rate, and amplitude the driver perceives. The driver responds to a total situation, not to isolated or ranked cues. If the vehicle changed heading but did not sideslip, the driver would rightly conclude that it was slipping on ice. Other visual inputs would confirm the impression. If the road moved sidewise without a change in heading, the driver would conclude that his vehicle was entering a curve. The perceived situation is a hypothesis that best reconciles all inputs.

Questioned

Several theorists have emphasized local characteristics of the velocity field as aids in lateral guidance. Gibson claims that the center of expansion is the prime cue used by the driver to determine his direction of movement, as quoted in part 2 (*6*). A difficulty with this theory is the indiscernibility of the center of expansion against the sky background. Additional difficulties are posed by curvilinear motion. On a curved trajectory, the entire field moves and the center of expansion vanishes, as shown in figure 16. The only still point on the field is the center of rotation that lies far to the side of the driver's line of regard.

A similar difficulty applies to the theory that the null $d\theta/dt$ locus may serve as an indicator of the vehicle's path of motion. The null $d\theta/dt$ locus is made up of all the vectors aimed directly toward the vehicle. This locus coincides with the future path when the car moves rectilinearly as shown in figure 20. In the curvilinear situation, the pattern becomes a semicircle between the eye and the center of rotation that does not coincide with the vehicle's path. Another cue, based on the velocity field, that motion parallax is an indication of depth, also fails when the vehicle follows a curved path. Evidently, the features of the velocity field are path dependent and are too unstable to provide guidance alinement on curved paths.

Perceptual Anticipation

Perceptual anticipation is of central importance to the driving task. The design of highways must permit the driver to anticipate ahead. If the driver's view is limited by fog, rain, or lack of illumination, he slows down or stops; and if forced to maintain speed, he feels uncomfortable. Perceptual anticipation is illustrated in figure 21 by Taragin's data on driver behavior on curved roads (*56*).

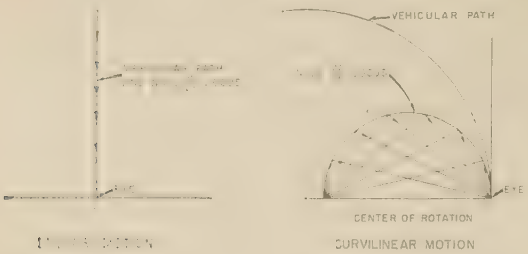


Figure 20.—The null dθ/dt locus in linear and curvilinear motion.

He reported that most drivers adjust their speed on the approach to a curve and do not change it appreciably after entering the curve, as illustrated in figure 21. Operating speed was related closely to the degree of curvature and only slightly to sight distance on the curve. Anticipation effects also have been shown in the experimental data on lateral guidance (*9*) and in studies of driver behavior on vertical curves (*57*).

The driver must anticipate at least one reaction time ahead if he is to meet the current situation. Cumming (*50*) states this relationship as:

"In a tracking task . . . the reaction time is about 0.3 seconds; i.e., there is a time lag of 0.3 seconds between a signal and the corresponding action. This leads to the strategy, in the development of a tracking skill, of taking a preview of the required path and programing ahead to compensate for reaction time . . . . As the skill develops it becomes possible to extract the 'constants' to allow for smoothness and coordination and so be able to program further ahead than half a second.

"It is this ability to view the process in an unhurried way on broader time scale, with longer ballistic actions between instructions and without monitoring, that signifies a 'skill' in the generally understood sense."

The driver must also provide time for shifting his mental gears, for adjusting his planning, and for coping with emergency highway conditions that might arise. Warning signs and signals should be placed well in advance of the hazardous situation they refer to. The driver's visual fixation distance has been related to anticipation requirements. Wohl (*53*) believes that fixation position may be predicted by the following equation.

$$D = \tau V \qquad (52)$$

Where,

D = Sight distance ahead
τ = Driver's response time lag
V = Vehicle velocity

The equation implies that the driver looks ahead a distance equal to vehicular velocity multiplied by human response time lag. If the driver did not look at least this far ahead, he could not respond appropriately. However, even casual observation of driving behavior indicates that the operator does not view a fixed distance ahead. Rather, he looks far ahead, returns to middle distance, and, seemingly in disregard of anticipation requirements, he may check his alinement with the road and nearby vehicles. His behavior is more variable than the equation would demand. It seems easier to accept the following rearrangement of Wohl's equation.

$$\tau_A = \frac{D}{V} \qquad (53)$$

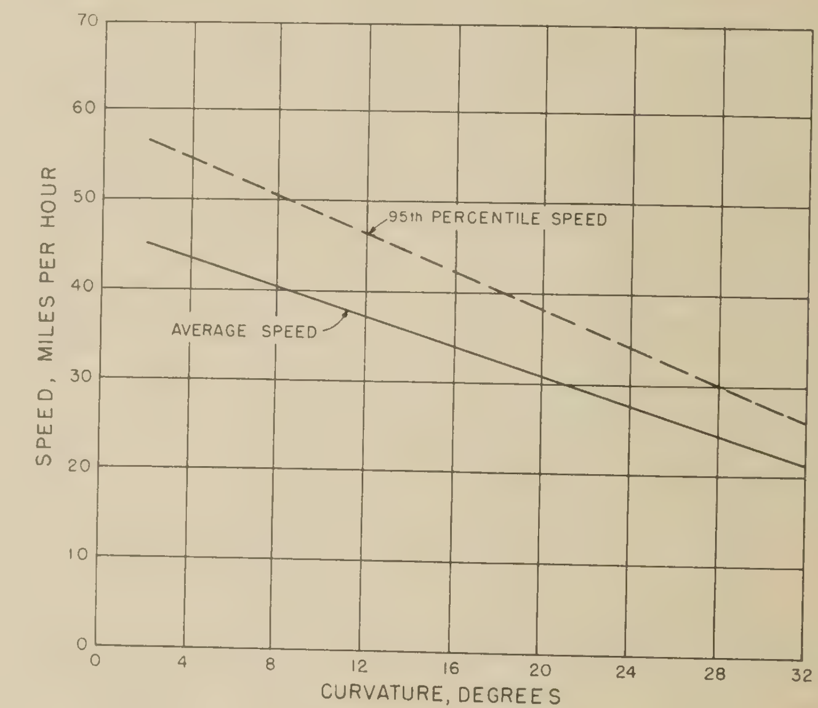


Figure 21.—Relation between speed and horizontal curvature. Speed on the curve decreases as the sharpness of curvature increases.

If the vehicle's velocity is fairly uniform, seen distance to a point ahead is linearly convertible to anticipation time. It is unfortunate that so little is known about the important factor of anticipation in driving. Exploratory studies are needed to determine optimal or minimal anticipation distances applicable to road signs, barriers, curves, and so on. The studies should be carried out at different vehicular speeds, under different visibility conditions, and with different car dynamics.

Car Following

In traffic, the driver's acceleratory and braking responses are made primarily in response to movements of the vehicle ahe[ad]. Car following responses may therefore be [a] basic element of complex traffic states, s[uch] as highway shock waves and instabili[ties] causing accidents (*47*).

Car following has been intensively stu[died] by Herman and his coworkers (*47*). On [the] basis of experimental observation on a [test] track and in New York City tunnels, [the] following equation has been derived for [the] car following process

$$(d^2x/dt^2)_{n+1} = \alpha_0 \frac{(dx/dt)_n - (dx/dt)_{n+1}}{x_n - x_{n+1}}$$

Where,

$(d^2x/dt^2)_{n+1}$ is the acceleration of the foll[ow]ing car.
$\alpha_0 \pi 8_0$ is a constant related to speed.
$(dx/dt)_n - (dx/dt)_{n+1}$ is the difference in sp[eed] between cars.
$x_n - x_{n+1}$ is the headway distance betw[een] cars.

Herman instructed his subjects to ". . . follow at a minimum safe distance." Th[ese] instructions do not include all possible dri[ver] intentions but they may cover the importa[nt] situations of driving in single lanes, on bridg[es] and in tunnels. A perceptual basis for H[er]man's equation has been suggested [by] Michaels (*51*), in terms of the driver's seein[g] change in the angular subtense of the vehi[cle] ahead. If the following driver overtakes fr[om] a long distance, he can detect absolute chang[es] in size of the vehicle ahead, but not the actu[al] motion of the lead vehicle. When separati[on] distances become sufficiently short, he will [be] able to detect angular velocity by expansion [of] the lead car's horizontal angle, whose rate [of] change is as follows.

$$\frac{d\theta}{dt}=K\frac{(dx/dt)_n-(dx/dt)_{n+1}}{(x_n-x_{n+1})^2} \qquad (55)$$

here,

$d\theta/dt$ is the angular velocity of expansion of the lead car, width in rad./sec.

K is the width of lead car, feet.

close relation is shown in the form of erman's equation and the angular expansion uation, which suggests that Michaels has ovided a perceptual basis for car following. Michaels' explanation implies a close relan between the driver's perception and his sponse. Such automatic reaction may be an equate model for Herman's experimental uation, which involved close monitoring of eed, but it is unlikely that the driver in affic gives immediate response to his perptions. Field observations by Perchonik d Seguin (*58*) indicated that the factor of adway distance was effective only at short hicle spacing and that relative speed exerted aximum influence for vehicle separations in e 50- to 100-foot range. The factors in erman's equations had no significant effects longer headways (*49*).

As the driver is observed, he is not conually adjusting his velocity relative to the r ahead. Rather, he avoids coming danrously close to that car and also avoids ifting so far behind that the gap will be led by a car in an adjacent lane. If this proach is valid, then speed, traffic density, d the position of cars in adjacent lanes would fect car following, as well as the relative eed and position suggested by Herman. hese opposing suggestions as to the basis of r following should receive a field test aluation.

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New Publications

Updated versions of two publications providing information on Federal Aid for Highways have recently been issued by the Bureau of Public Roads, U.S. Department of Commerce. *America's Lifelines—Federal Aid for Highways* and *Federal Laws, Regulations, and Other Material Relating to Highways* may be purchased from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402, prepaid. A brief description of each publication and its purchase price is given in the following paragraphs.

America's Lifelines—Federal Aid for Highways

America's Lifelines—Federal Aid for Highways (1966), 20 cents a copy, is a red-white-and-blue brochure presenting interesting and informative facts concerning the different Federal-aid highway programs and how they are implemented. Individual sections contain information on use of highways, the Federal role in financing highways, the National System of Interstate and Defense Highways (popularly called the Interstate System), the Federal-aid ABC program, and related programs such as highway planning, safety, a[illegible] research.

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Highway Research and Development Studies, Using Federal-Aid Research and Planning Funds (May 1965). 75 cents.

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Highway Transportation Criteria in Zoning Law *and* Police Power and Planning Controls for Arterial Streets (1960). 35 cents.

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Manual for Highway Severance Damage Studies (1961). $1.00.

Manual on Uniform Traffic Control Devices for Streets and Highways (1961). $2.00.
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Traffic Assignment and Distribution for Small Urban Areas (1965). $1.00.

Traffic Assignment Manual (1964). $1.50.

Traffic Safety Services, Directory of National Organizations (1963). 15 cents.

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