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TESTING A SLAB IN STUDY OF PAVEMENT STRESSES

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D. M. BEACH, *Editor*

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The reports of research published in this magazine are necessarily qualified by the conditions of the tests from which the data are obtained. Whenever it is deemed possible to do so, generalizations are drawn from the results of the tests; and, unless this is done, the conclusions formulated must be considered as specifically pertinent only to described conditions.

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THE STRUCTURAL DESIGN OF CONCRETE PAVEMENTS

BY THE DIVISION OF TESTS, PUBLIC ROADS ADMINISTRATION

Reported by L. W. TELLER, Principal Engineer of Tests
and
EARL C. SUTHERLAND, Senior Highway Engineer

PART 5.—AN EXPERIMENTAL STUDY OF THE WESTERGAARD ANALYSIS OF STRESS CONDITIONS IN CONCRETE PAVEMENT SLABS OF UNIFORM THICKNESS

THIS IS the last of a series of reports of an extensive investigation, undertaken by the Public Roads Administration in 1930 with the general objective of developing information that would be of assistance in better understanding the structural action of concrete pavement slabs. Much of the work has been described in four previous reports published in this same journal, as follows:

PART 1.—A Description of the Investigation, vol. 16, No. 8, October 1935.

PART 2.—Observed Effects of Variations in Temperature and Moisture on the Size, Shape and Stress Resistance of Concrete Pavement Slabs, vol. 16, No. 9, November 1935.

PART 3.—A Study of Concrete Pavement Cross Sections, vol. 16, No. 10, December 1935.

PART 4.—A Study of the Structural Action of Several Types of Transverse and Longitudinal Joint Designs, vol. 17, Nos. 7 and 8, September and October 1936.

Since concrete is a material with recognized elastic properties, engineers concerned with the design of concrete pavement have long searched for a theory that would adequately express the relations between applied forces and the resultant stresses in pavement slabs of this material.

A reliable general theory of slab stresses would serve at least three important purposes. In the first place it would enable the designer to determine the thickness and the form that the slab should have to function without failure under specified conditions of loading and support; second, it would make possible accurate estimates of the loads which might be imposed with safety on existing pavements; and third, it would provide a useful tool for judging the relative effects of vehicle loads of various magnitudes in studies of the costs of providing facilities for vehicles of different types and weights.

From time to time, over a period of many years, theoretical treatments of the load-stress relation in concrete pavements have been offered (*9, 4, 10, 1*).[1] None of the early analyses was general in scope. Rather, each was concerned with some special situation that was assumed to be critical and for which rather broad assumptions were sometimes proposed. The principal weakness consistently seemed to lie in the assumptions that were made regarding the conditions of support, although other assumptions open to serious question were sometimes present.

ELASTIC ACTION OF SUBGRADE AN IMPORTANT FACTOR IN WESTERGAARD ANALYSIS

It was not until the original Westergaard analysis was published that a rational theory of general application became available (*23*). In this treatment, it is assumed that the slab acts as a homogeneous, isotropic, elastic solid in equilibrium and that the reactions of the soil are vertical only and are proportional to the deflections of the slab. The relations between applied loads and critical stresses are then developed on the basis of elastic theory for the three cases of a wheel load applied on the surface of the slab at a free corner, at an interior point and at a free edge (at some distance from a corner) respectively.

In the first paper of this series (*18*) mention was made of the Westergaard analysis and it was stated that one of the objects of the investigation being described was to study the elements and relationships of the analysis by means of load tests on full-size pavement slabs of constant thickness together with such collateral tests as might be found necessary. The investigation was planned in 1929, the sections constructed in 1930 and a considerable amount of testing under load was done during 1931 and 1932. These early tests were referred to by Westergaard in his supplementary paper (*25*) in which he says "The tests suggest certain adaptations of the theory. There will be needed some restatements of analytical results and some supplementing and modification of the theory".

The extension of the original analysis that is contained in this second paper permitted a more comprehensive study to be made and, as a result, much additional information has been developed. The present paper contains a description of the work that was done on this part of the general project, with a presentation of the data obtained and a discussion of their significance.

If one studies the Westergaard analysis he is at once impressed by the importance of the assumption which relates to subgrade support. In the original theory (*23*) it was assumed that the reactions of the subgrade are vertical only and are proportional to the deflections of the slab, the reaction per unit of area at a given point being the product of the deflection at that point and a coefficient of subgrade stiffness, k, which was termed the modulus of subgrade reaction. This modulus is normally expressed in pounds per square inch per inch of deflection (or pounds per inch cube).

[1] Numbers in italics refer to bibliography at end of article. The above references are in chronological order.

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In the supplementary theory (*25*) Westergaard proposed a new coefficient, K, defined by the relation

$$K=kl$$

in which l is a linear dimension, termed the radius of relative stiffness, which appeared in the original theory.

The reason for the proposed new coefficient was an expectation that K would be less dependent on the stiffness of the pavement slab than k. Westergaard stated that "the truth may lie between the two extreme cases of a constant k and a constant K."

To make practical use of the analysis one must be able to assign a value to the modulus of subgrade reaction for the particular soil structure with which he is concerned. At the time the investigation was undertaken no determinations of the value of such a soil coefficient had been made, so there was no background of experience in testing that would indicate either the probable range in values of the coefficient or a procedure by which values might be obtained. Therefore, it was necessary to devise a test procedure that would indicate how the soil of the subgrade beneath the test sections behaved when subjected to pressure intensities and vertical deformations of the same order as occur under pavement slabs in service.

Because of its importance as a part of the study of the Westergaard analysis and because of its current general interest, this study of the elastic action of the subgrade is reported in some detail in this paper, being presented as a separate section, preceding the discussion of the results of the studies of the other relationships expressed in the analysis.

DETERMINATION OF THE MODULUS OF SUBGRADE REACTION

The ideal subgrade assumed by Westergaard is perfectly elastic, has uniform elastic properties at all points and its vertical deformation varies as a linear function of the vertical pressure exerted on its surface. Such a subgrade probably does not exist and the problem becomes one of determining by some test procedure, how nearly the soil under a given pavement approaches the ideal and what stiffness coefficient, if any, can reasonably be assigned to it for the purpose of applying the method of analysis to a particular problem.

While it is a fact that stress values as computed with the Westergaard equations are not particularly sensitive to variations in the value of the subgrade modulus k, if comparisons are to be made between computed stresses and those determined experimentally, the value of the coefficient k used in the theoretical computations must be determined with at least a fair degree of accuracy before dependable values for the computed stresses can be had.

The elastic deflections of a concrete pavement slab of usual design under the action of normal highway loadings are quite small, probably of the order of 0.05 inch or less, depending upon the position of the load, the details of the slab design and other factors that influence deflection. The area over which deflections occur is, on the other hand, relatively large, as may be seen by referring to load-deflection data presented in Part 3 of this series (*20*). In developing a test to be applied to the soil in place for determining the modulus of subgrade reaction, it seemed only reasonable to study the behavior of the soil when subjected to deformations of the same general order of magnitude as would obtain under a pavement slab deflected by a motor vehicle wheel load and, at the same time, to deform the soil over a relatively large area.

THREE METHODS OF MEASURING LOAD SUSTAINING ABILITY OF SOIL DISCUSSED

There appear to be at least three methods or procedures by which the load sustaining ability of the soil can be measured under field conditions. These may be described briefly, as follows:

1. Load-displacement tests in which loads are applied at the center of rigid circular plates of relatively small size, the pressure intensity on the soil being uniform over the entire area of the plate. In these tests the applied load, the mean vertical plate displacement and usually the time intervals are measured.

2. Load-displacement or load-deflection tests in which the load is applied at the center of slightly flexible rectangular or circular plates of relatively large dimensions. In this case some bending of the plate (or slab) occurs and the pressure intensity under the plate is not uniform throughout the area of its contact with the soil. The load, the vertical displacement of various points throughout the area of the plate and possibly time intervals are measured.

3. Load-deflection tests on full-size pavement slabs in which the load-deflection data are obtained by measurement and used in the Westergaard deflection formulas to provide a value for the soil stiffness coefficient or "modulus of subgrade reaction."

If all three methods were equally satisfactory the first procedure offers the practical advantage of requiring test loads of lesser magnitude with a corresponding reduction in the size of the equipment. Also the number of measurements is less since no plate deflections are involved. Its use is complicated, however, by two conditions. The first is that the ability of a soil to sustain a given unit pressure varies within limits with the area over which the pressure is applied to the soil. This variation may be quite marked and this makes it necessary to determine the effect of size of plate in order to avoid error from using a bearing plate that is too small. The second complication is that the supporting ability of a soil varies with its moisture state and it is necessary, therefore, to take special precautions to insure that the soil on which the bearing plate is placed is in the same physical state and moisture condition as that which will obtain or does obtain under the pavement to be considered.

The second procedure has a certain theoretical appeal but offers considerable practical difficulty as a method of test. In this procedure the plate is deflected by the centrally applied load much as the pavement slab deflects under the action of a wheel load. The shape of the deflected plate must be determined precisely and its vertical displacement measured in order to be able to estimate accurately the volumetric displacement of the soil that is effected by the application of the test load on the plate. The modulus of subgrade reaction is then computed by dividing the load (in pounds) by the volume of displaced soil (in cubic inches). Some use has been made of this procedure in England (*12*). The possibilities of the method should be more thoroughly explored.

When using the third method, the procedure is to apply test loads at the free corner, free edge or interior point of a pavement slab of uniform thickness and of normal size. If the elastic modulus of the concrete in the slab is known, for the moisture and other conditions that obtain, it is possible to determine the value of the effective modulus of subgrade reaction from the maximum slab deflection under the applied load by means of the deflection formulas given by Westergaard in his supplementary paper (*25*). This method for determining the soil stiffness coefficient will be discussed further in a later part of this report in connection with the presentation of deflection data obtained in such tests. The remaining part of this section of the report will be devoted primarily to a discussion of work done with the first method, i. e., load-displacement tests with rigid plates of relatively small size.

Load-displacement tests with rigid plates have been made many times in the past in studies of the bearing capacity of soil for foundations. The data obtained in such tests are not applicable to the problem of pavement support, however, because of the conditions surrounding the tests and the extent to which the soil deformations were carried.

LOAD-DISPLACEMENT TESTS WITH RIGID PLATES

When a rigid plate, supported by soil, is subjected to a vertical force or load applied at its center, the soil deforms and the plate moves downward. This downward movement of the plate under load has been variously termed deflection, displacement, penetration, settlement or subsidence, and tests which make use of this action have been termed accordingly load-deflection—or load-subsidence tests. In this discussion the terms displacement and load-displacement tests will be used.

The load-displacement tests which were a part of this general investigation comprise four series, as follows:

Series 1.—Five circular bearing plates and one square bearing plate were used. The diameters of the circular plates were 8, 12, 16, 20, and 36 inches respectively and the square plate had 48-inch sides. The magnitudes of the loads applied to each plate were such as would cause displacements within the range 0.01 to 0.05 inch, this being the approximate range of concrete pavement slab deflections under the action of legal maximum wheel loads.

The tests of this series were divided into two parts. In the first part, all plates were placed successively, in the descending order of size, at the same location on the subgrade while in the second part of the series the test with each plate was made at a different location but within the same general area.

Series 2.—Eleven circular plates, having diameters of 2, 4, 6, 8, 12, 16, 20, 26, 36, 54 and 84 inches respectively, were used. As in series 1, the applied loads were such as to cause displacements within the range 0.01 to 0.05 inch. In series 2 the tests with each size of plate were made at a separate location on the subgrade.

Series 3.—These tests were the same as those of series 2 except that the maximum plate displacement was increased to approximately 0.25 inch, and a somewhat different loading procedure was followed.

Series 4.—Only the 54-inch diameter plate was used. The plate remained at one location and the displacements were kept within the range 0.01 to 0.05 inch as in series 2. The loads were applied, however, in June and in January.

The first tests made in this study were those of series 1. They were intended to explore the effect of size of bearing plate on the load-displacement relation and also to compare data obtained in a series of tests at one location with those from tests that were identical except that each test of the series was made at a different location in the same general area. Obviously it would be preferable to have each test made in an area undisturbed by previous loading, provided there was sufficient general uniformity in the soil structure to eliminate the possibility of local variations in structure affecting certain tests (particularly those in which small bearing plates were used).

The tests of series 2 were designed to extend considerably the data from series 1 on the effect of size of the load-displacement relation.

The tests of series 3 were to provide information on the comparative supporting ability of the soil when subjected to deformations greater than those which usually occur under rigid pavements. These data extended the range of the information obtained and permitted certain comparisons with other data to be made.

Series 4 was simply a study of the effect of the annual change in the physical state of the soil directly under the slab on its ability to support load. For this reason only the 54-inch diameter bearing plate was used and the displacements were limited to the 0.01 to .005 inch range.

METHODS OF TESTING DESCRIBED

All of the soil tests of the four series were made on a part of the originally prepared subgrade that was reserved for this purpose. The soil was described as a uniform brown silt loam (classification A–4) and detailed information concerning it is contained in the first report of this series (*18*).

When making tests such as those described it is important that the soil on which the bearing plates are placed be in the same physical state as that under the pavement to which the tests are to be related. An effort was made to accomplish this by casting the larger bearing plates (which were of concrete) on the subgrade several months in advance of the first scheduled tests and by casting, at the same time, a number of concrete slabs, each 4 feet square, at those locations where tests with the smaller bearing plates were to be made later. By this means the soil was given the same protection and the same opportunity for moisture equilibrium was afforded as obtained with the test pavement itself.

The larger plates of concrete were left in place and loads were applied at the scheduled time. The smaller plates were of steel and with these the procedure was different. At the proper time the small concrete protecting slab was removed and waterproof paper spread over the area where it had been. A thin layer of portland cement mortar or of plaster of Paris was then spread over the waterproof membrane and the bearing plate was bedded in this mortar. This procedure gave a uniform contact between the soil and the bearing plate yet prevented moisture from the mortar from entering the soil. Figure 1 shows one of these smaller

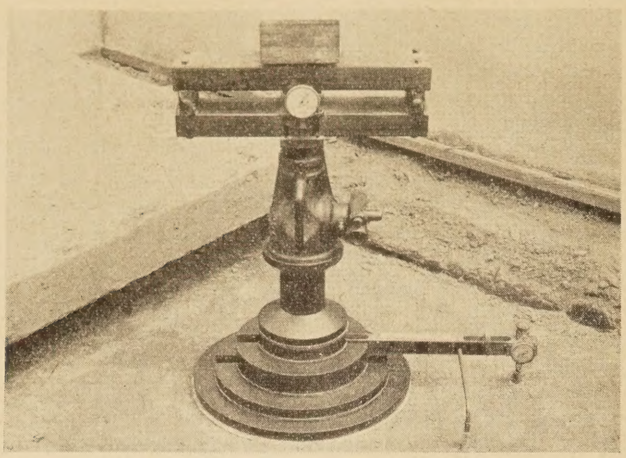


FIGURE 1.—DETERMINING THE MODULUS OF SUBGRADE REACTION WITH A SMALL BEARING PLATE. THE SMALL CONCRETE SLAB THAT COVERED THE SUBGRADE HAS BEEN TURNED BACK. DISPLACEMENT IS MEASURED WITH A CLINOMETER.

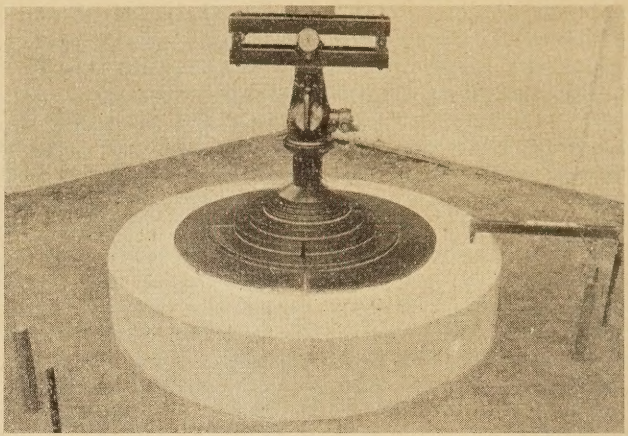


FIGURE 2.—DETERMINING THE MODULUS OF SUBGRADE REACTION WITH A LARGE BEARING PLATE. THE BEARING PLATE WAS CAST ON THE SUBGRADE SOME TIME BEFORE TESTING. DISPLACEMENT IS MEASURED WITH A CLINOMETER.

plates in place while figure 2 shows a test with one of the larger plates.

The vertical loading force was applied by a jack reacting against a large horizontal cylindrical tank mounted on a dolly frame and filled with water. The magnitude of the force was determined with a dynamometer. All of this equipment is described in the first report (*18*). The vertical movements of the bearing plate were measured either with dial micrometers supported on a bridge or by a series of clinometer measurements in the manner described in the first report (*18*). When using the clinometer it was necessary to place the reference point at some distance from the bearing plate and to carry the level line over a series of intermediate points to the bearing plate so that soil movements in the vicinity of the bearing plate would not cause errors in the displacement data. With the smaller bearing plates the mean displacement was obtained from a single point in the center of the plate while for the large plates the displacements of three points symmetrically spaced along the perimeter were measured and averaged. All of the plates were sufficiently rigid to prevent appreciable bending as used.

In the tests of series 1, 2 and 4 an effort was made to deform the soil in a manner simliar to that which might be expected under a concrete pavement. A comparatively large number of loadings were applied in each test and the magnitudes of the plate displacements were kept within the normal load-deflection limits of a concrete pavement slab.

For each size of bearing plate a series of 3 to 5 ascending load values was selected, such that the series would give a good spread of displacement values and the maximum would not produce a displacement greater than the desired limit. With the smallest load value selected for a plate of a given size, the load was applied and removed several times. The number of applications was not constant but was determined by the character of the data, it being desired to reach a condition such that each succeeding application of a given load would produce the same vertical displacement of the bearing plate. This might be termed a state of approximate elastic equilibrium. The number of loadings required to develop this condition, with the soil on which tests were made, usually varied from about 5 to 10. When a satisfactory load-displacement relation had been determined for the lowest load value, the procedure was repeated with the next higher load value and so on until the displacement limit was reached. As stated previously, when the tests with plates of various diameters were to be made at one location, the procedure was to test with the largest plate first then with the next largest and so on.

When a load is applied to a bearing plate a displacement of the bearing plate begins and, under some conditions, may continue for a long time before a state of complete equilibrium is reached. Similarly when the load is removed a certain amount of elastic recovery occurs and this too may continue for some time. As a practical matter it is not possible to continue each test cycle until the last vestiges of either displacement or recovery have disappeared before proceeding with the next loading cycle. After some experimentation it was decided that for the soil condition, load intensities and sizes of bearing plates in this investigation, a condition of essentially complete equilibrium would be reached if each load was maintained for 5 minutes after reaching its full magnitude and if after the complete removal of that load 5 minutes elapsed before the application of the next load.

The procedure followed in the tests of series 3 was somewhat different, as mentioned previously. As the data from this series were, in part, to be compared with data developed in tests of series 2, the tests of series 3 were made immediately after the completion of those of series 2 so that no change in the condition of the subgrade soil could occur between the two series. In order to obtain load-deformation data for the soil in question which might be compared directly with those obtained by other agencies for other soils, the soil deformation limit was increased to approximately 0.25 inch for this series and the loading procedure was modified to conform more closely to that followed in the tests made by others. In the tests of series 3 only one load of each magnitude was applied. This load was left on for 5 minutes, removed completely and a period of 5 minutes allowed to elapse before the next larger load was applied. The maximum displacement limit of approximately 0.25 inch applied only to the smaller sizes of bearing plates since the maximum reaction possible with the loading equipment used was only about 50,000 pounds and this was insufficient to cause a dis-

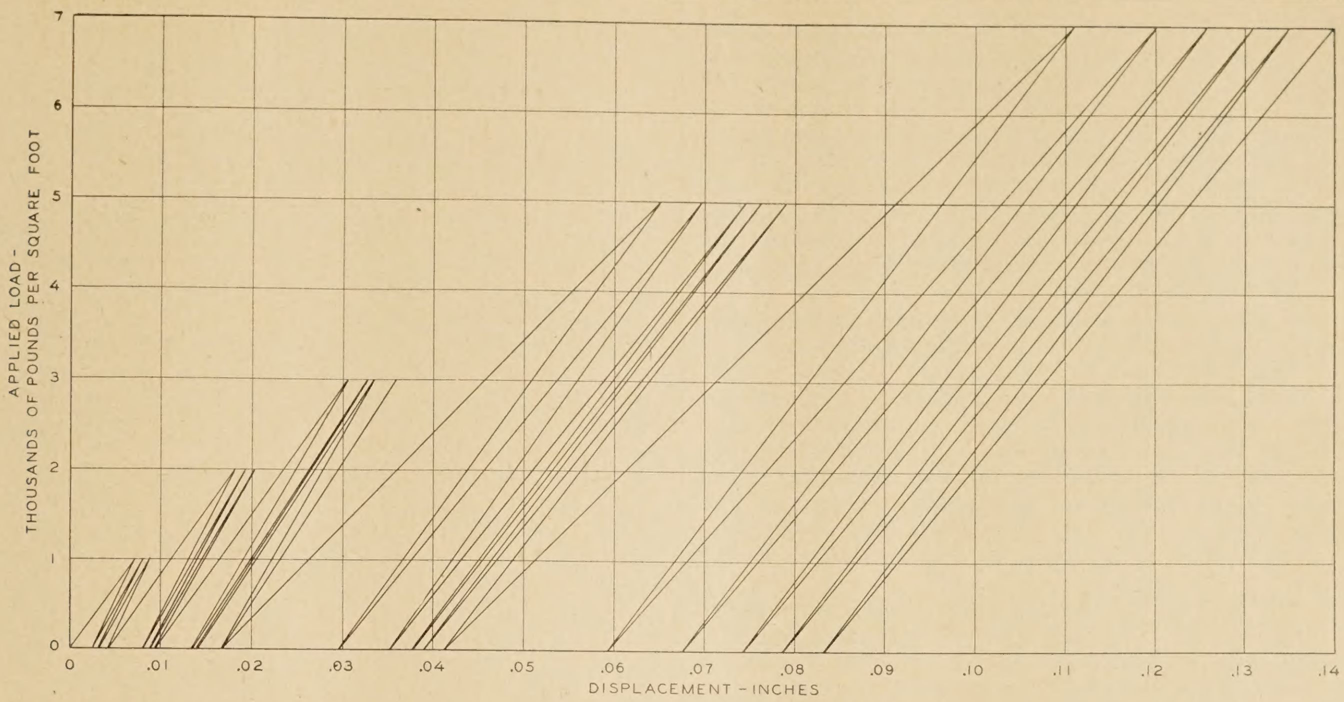


FIGURE 3.—TYPICAL LOAD-DISPLACEMENT DATA FOR A 26-INCH DIAMETER BEARING PLATE PLOTTED WITH RESPECT TO THE INITIAL POSITION.

placement of the desired magnitude with the larger plates.

Information regarding the moisture condition of the soil was obtained from samples taken from under a 4- by 4-foot concrete slab immediately adjacent to the point at which the bearing test was being made. The soil samples were taken just before and just after the bearing test. The moisture content was determined by weighing, drying, and reweighing.

SOILS TESTED SHOW HIGH PERCENTAGE OF RECOVERY AFTER REMOVAL OF TEST LOAD

When a bearing plate is subjected to a sequence of loads in the manner described for series 1, 2, and 4, data of the type shown in figure 3 result. This graph shows a log of the progressive displacements of a 26-inch diameter plate caused by a series of loadings of five magnitudes, the load of each magnitude being repeated several times, as described earlier in the report. The mean plate displacements are shown throughout both with respect to the original plate position and to the position just before the particular load was applied. The graph shows also the magnitude of the recovery during the 5 minute period following the removal of each load.

The data obtained in these load-displacement tests showed that, for a given plate size and load intensity, the magnitude of the displacement usually decreased somewhat with each load application until several loads have been applied, after which the load-displacement relation remained fairly stable. As soon as the pressure intensity was increased by the application of a greater force to the plate, the same repetition of loadings was necessary to again bring about the condition of approximate elastic equilibrium. Data of this type and extent were obtained for each of the various sizes of bearing plate listed earlier.

It is apparent in the graph that for the conditions of soil, pressure intensity, soil deformation, and time which obtained in this test, the action was never completely elastic. This is evidenced by the residual deformation after each loading. A study was made of the data to determine the percentage of recovery that followed the complete removal of the load and the result of this study is summarized in table 1.

TABLE 1.—*Average recovery of soil after removal of test load*

Displacement	Diameter of bearing plate in inches—			
	4	20	54	84
Inches	*Percent*	*Percent*	*Percent*	*Percent*
0.010	93	90	93	89
.020	90	90	95	97
.050	90	91	95	99

The percentages of deformation recovered upon removal of the load, as shown in this table, are values determined by averaging the movements after preliminary loadings had developed a state of approximate elastic stability.

The data indicate that the percentage of recovery was rather high in all cases, that it was greater with large plates than with small ones and that it tended to be greater for the larger plate displacements than for the smaller ones.

EFFECT OF SIZE OF BEARING AREA STUDIED

Since the recovery is not complete, there is a cumulative residual or permanent displacement as the test proceeds. For the soil on which these tests were made, the magnitude of this cumulative deformation appears to vary with the number of load applications and with the pressure intensity used.

This gradual settlement of the bearing plates under the succession of test loads raised the interesting question as to what effect such a yielding of the soil might have on the load-strain relation of a concrete pavement slab. To obtain information on this point, tests were made on a full-size pavement slab on the same subgrade that was used for the soil bearing tests. In the tests on the pavement slab a load was applied a relatively large number of times at the corner, the edge and the interior and the critical strain measured for each load application. It was found that at the corner there was a progressive increase in the strain caused by a given load, but at a diminishing rate, until there had been about 70 repetitions of the load. After this there was apparently no increase in the critical strain. The maximum increase in strain from the initial to the final loading was approximately 10 percent. Repetition of load at the free edge and at an interior point caused no appreciable change in the critical strains. The data from these load strain tests are presented in graph form in a later section of this report.

It will be recalled that in series 1 tests were made with various sizes of bearing plates, both at a common location and at individual locations. The tests with each size of plate provided data of the type shown in figure 3. The data from all of the individual tests of series 1 are summarized in figure 4 which shows load-displacement relations for each of the several sizes of bearing plate, both for the tests at a common location and for those at individual locations. The displacement values in this figure are with respect to the position of the plate immediately before the application of the particular load, being in each case the average of 2 or 3 determinations made after the subgrade had attained a condition of approximate elastic equilibrium for the particular displacement. No tests were made at the common location with the 36-inch round plate and the 48-inch square plate.

The data obtained in this series of tests afforded two significant comparisons. They showed first the important effect of plate area on the pressure intensity required to produce a given plate displacement on the soil in question. A similar effect has been observed by other investigators in tests where much larger plate displacements were used. This effect will be discussed in more detail later in this report. The second comparison was between the tests with a series of plate sizes at one location and a similar series at individual locations. For this comparison the data show that for a given plate size (within the 8- to 20-inch diameter range) essentially the same load intensity-plate displacement relation obtained. It was considered better procedure, however, to make the tests with each plate size at individual locations in the subsequent program of series 2.

In figure 5a and 5b the same data are arranged in a different manner. In figure 5a are shown the relations between pressure intensity and the diameter of the bearing plate for three magnitudes of plate displacement, 0.01, 0.02, and 0.05 inch, respectively. This graph brings out forcibly the importance of both the area and the degree of the displacement in determining the load supporting ability of the soil. The shape of the curves, shown in this graph, is generally similar to that found in other tests, both in this country and in Europe, in which loads were applied to rigid plates of various sizes that were supported by soils not strictly granular in character (*3*, *5*, *6*, *8*, *2*). Generally speaking, the other tests cited differed from those described in this report in two important particulars. In the first place, the soil deformations were carried considerably farther than in the present study and, in the second place, the maximum dimension of the bearing plate, i. e., the diameter of the circular plate or the side of a square plate was, with one exception, less than 42 inches.

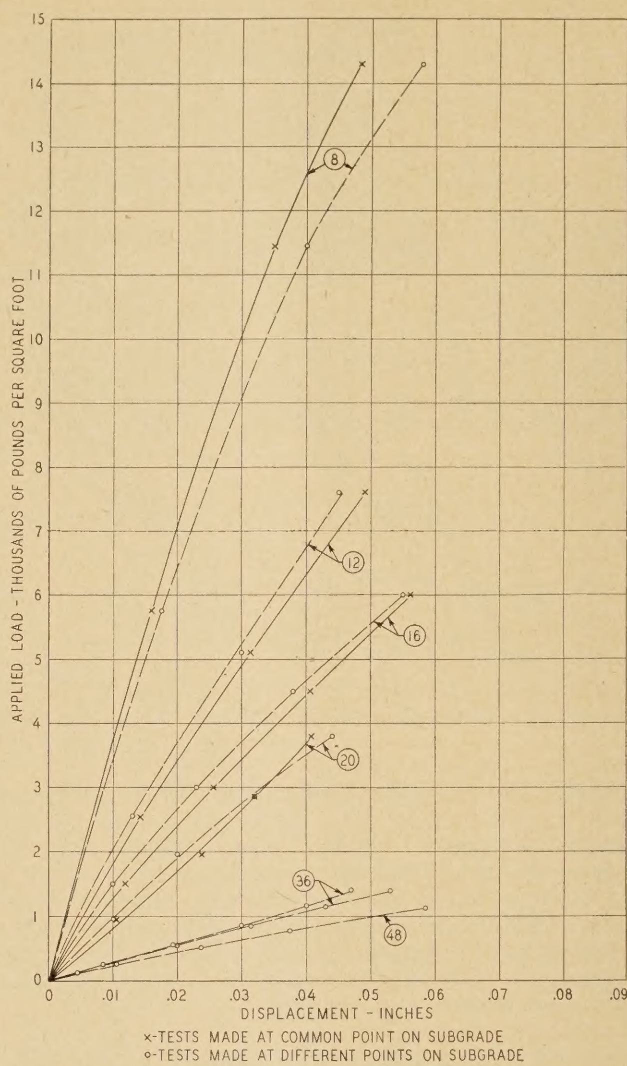


FIGURE 4.—LOAD-DISPLACEMENT RELATIONS FOR THE SEVERAL BEARING AREAS OF THE FIRST SERIES OF TESTS. FIGURES IN CIRCLES INDICATE THE DIAMETER OF THE BEARING PLATE IN INCHES.

In figure 5b the data from series 1 are again shown but, in this case, with the pressure intensity related to the perimeter-area ratio, an inverse function of the plate size, in the manner suggested by Housel (*6*). Three curves, one for each of the three magnitudes of plate displacement used in the tests, are given in the graph.

In both figures 5a and 5b the data obtained with the 48-inch square plate are included, being plotted on the basis of area, the shape factor being ignored. Thus there may be reason to question the accuracy of the points which are plotted on the 54-inch diameter and 0.083-inch^{-1} perimeter-area ratio ordinates respectively in the two parts of this particular figure. Data obtained in the subsequent series of tests support the relations shown in figures 5a and 5b, however.

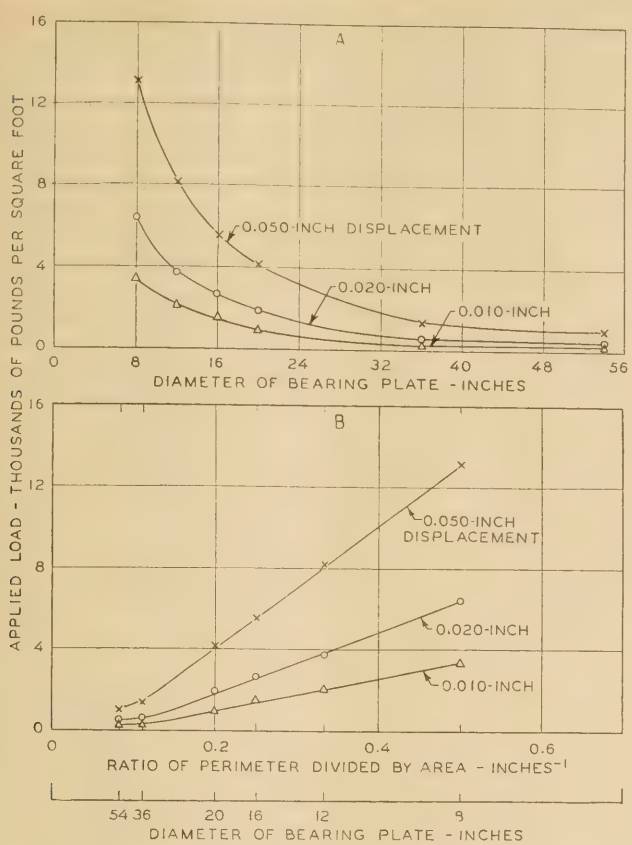


FIGURE 5.—EFFECT OF BEARING PLATE SIZE ON THE LOAD REQUIRED TO PRODUCE A GIVEN PLATE DISPLACEMENT. DATA FROM SERIES 1.

It is of interest to note that, when the data are plotted in the manner shown in figure 5b, an essentially straight line relation exists between the pressure intensity and the ratio of perimeter to area for plates of 20-inch diameter and less while for the larger plates used in this series, it appears that the linear relation does not hold. The use of the inverse function of plate size $\left(\frac{2}{\text{radius}}\right)$ for the abscissas results in such a convergence of this scale near the origin that it is difficult to determine at what point on these curves the departure from linearity begins but it appears to be in the vicinity of 30 inches on the diameter of bearing plate scale.

SMALL PLATES FOUND UNSUITABLE FOR USE IN DETERMINING RELATION BETWEEN PRESSURE INTENSITY AND PLATE DISPLACEMENT

It will be recalled that, in series 2 and series 3, tests were made with eleven sizes of circular bearing plates ranging from 2 to 84 inches in diameter. In series 2 the loading procedure and displacement ranges were the same as in series 1, while in series 3 the loading procedure was different and the displacement range was increased consistently.

In figures 6 and 7 are summarized the load displacement data obtained in the tests of series 2 and series 3 respectively. These graphs correspond to figure 4 which contains similar data from the tests of series 1. Attention is called to the difference in the horizontal scales used in the two figures, this being necessary because of the difference in the displacement range of the two series of tests. It will be noted that in series 3 the displacements are limited to values less than the maximum in the case of the large bearing plates. With the large areas, the available load reaction of approximately 50,000 pounds limited the pressure intensity (and the plate displacement) which could be obtained.

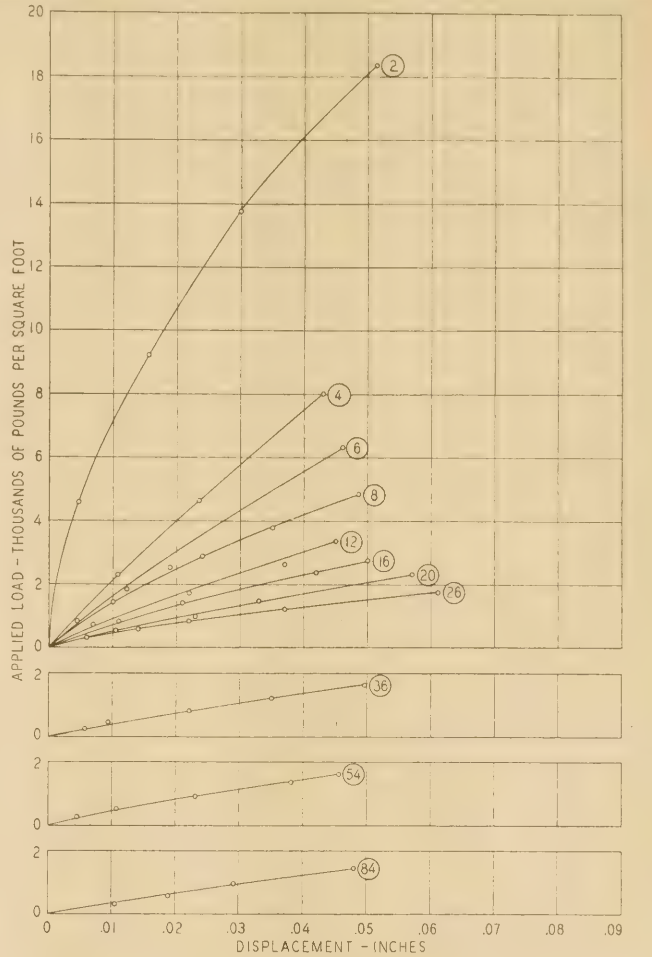


FIGURE 6.—LOAD-DISPLACEMENT RELATIONS FOR THE SEVERAL BEARING AREAS OF THE SECOND SERIES OF TESTS. FIGURES IN CIRCLES INDICATE THE DIAMETER OF THE BEARING PLATE IN INCHES.

Figure 8 shows the load-displacement data from the tests of series 2, plotted in the same manner as was used in figure 5a for series 1 in order to bring out the effect of the size of the bearing plate on the ability of the soil to resist deformation. As in the corresponding earlier graph the relation is shown for each of the three displacement magnitudes, 0.01, 0.02 and 0.05 inch respectively.

Figure 9 is similar to figure 8 but contains data from the tests of series 3. Because of the larger displacements in the tests of series 3, the action of the soil was somewhat different and this difference was most marked when the 4-, 6-, and 8-inch diameter bearing plates were employed. Generally speaking, however, the relations between plate size and the pressure intensity necessary to produce a given displacement were similar to those developed by the tests in which the displacements were limited to the small values of 0.01, 0.02 and 0.05 inch, as shown in figures 5a and 8.

The data consistently show that, for the conditions that existed in the present tests, the effect of plate size on the pressure intensity-plate displacement rela-

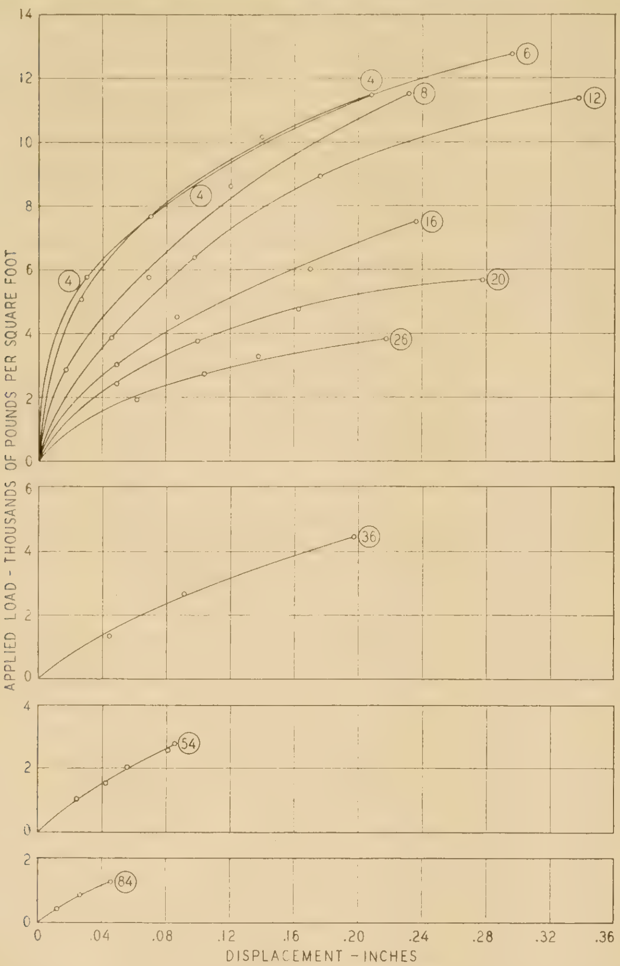


FIGURE 7.—LOAD-DISPLACEMENT RELATIONS FOR THE SEVERAL BEARING AREAS OF THE THIRD SERIES OF TESTS. FIGURES IN CIRCLES INDICATE THE DIAMETER OF THE BEARING PLATE IN INCHES.

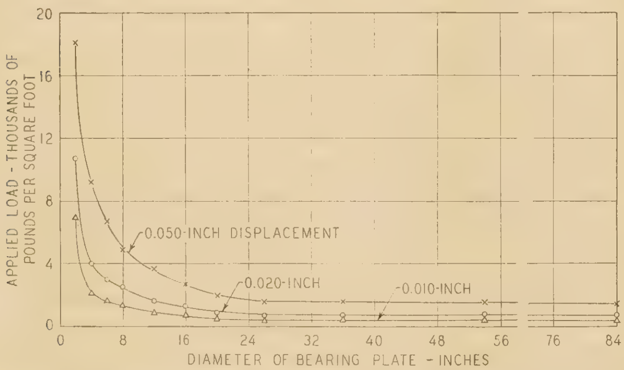


FIGURE 8.—EFFECT OF BEARING PLATE SIZE ON THE LOAD REQUIRED TO PRODUCE A GIVEN PLATE DISPLACEMENT. DATA FROM SERIES 2.

tion is not important for plate diameters of about 26 inches or larger, but for plates of less than the 26-inch diameter there is an effect which becomes increasingly large as the plate size diminishes.

Certain earlier investigators, (*3*) and (*5*), concluded that, for soils of the cohesive type, a given pressure intensity applied to bearing plates of various sizes might be expected to produce vertical soil deformations which would be directly proportional to the square root of the areas of the plates. An analysis of the present data indicates that the relation just stated applies quite well for bearing plates having diameters of 26 inches or less, but that for larger plates it does not apply, the magnitude of the divergence increasing rapidly as the size of the plate is increased. For a plate 84 inches in diameter the measured displacement was approximately one half of that which would be predicted if the above-mentioned relation held true.

It should not be assumed that the relations found in the bearing tests at Arlington are applicable generally. Further tests are needed on other soils in place, particularly tests that include large bearing areas, before any attempt can be made to generalize on the relations. It should be remembered also that the studies at Arlington were made primarily to determine the behavior of soil undergoing relatively small deformations and the magnitude of the deformation is shown to be an important variable in the test.

The data obtained are important, however, in showing the size of plates which must be considered in studies of the soil condition of the Arlington pavement research and furthermore because they indicate the necessity for a knowledge of the effect of plate size when making bearing tests with other soil conditions.

SEASONAL VARIATION IN SUBGRADE SUPPORT STUDIED

It will be recalled that the modulus of subgrade reaction is a stiffness coefficient which expresses the resistance of the soil structure to deformation under load in pounds per square inch of pressure per inch of deformation (in the direction of the loading force). Certain pertinent facts have been brought out. It has been shown that, (1) the soil structure is imperfectly elastic; (2) the elastic behavior of the soil is affected by its moisture state; (3) the load resistance of the soil structure, i. e., the pressure intensity required to produce a certain deformation, depends upon the magnitude of the deformation and, within certain limits, upon the area over which the pressure is applied to the soil structure. It is evident that these conditions place limitations on the manner in which the stiffness coefficient can be determined and on the extent to which it can be applied. However, it is believed that, for soils of cohesive character at least, it is possible to obtain approximate but usable values for the coefficient from load-displacement tests with rigid bearing plates on the soil in place, provided certain precautions are taken to minimize the effect of the disturbing influences mentioned above. A study of the data suggests the nature of the precautions to be taken.

If values of the stiffness coefficient k are calculated from load-deformation data, such as those shown in figure 3, it will be found that the value of the coefficient varies with the size of the plate used in the test and with the magnitude of the soil deformation. This has been mentioned previously and is indicated by the relations shown in figures 8 and 9. An analysis of the data obtained in the more comprehensive tests of series 2 was made to determine the extent and characteristics of the variations in the value of the coefficient for the conditions that obtained in this series of tests. Values were calculated for each size of bearing plate used and for soil deformations of 0.01, 0.02, and 0.05 inch for each plate size. The variation in the value of the coefficient with plate size and with the magnitude of the plate displacement or soil deformation is shown in figure 10. From this figure it appears that when making tests to determine the value of the soil stiffness coefficient k

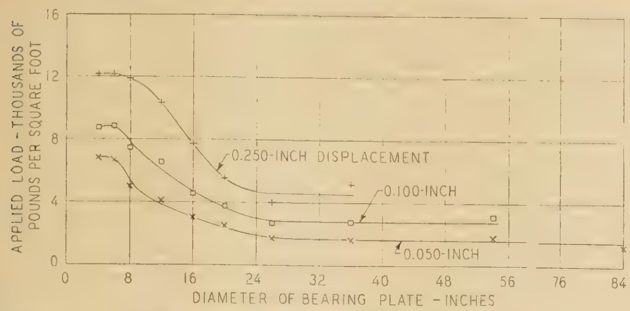


Figure 9.—Effect of Bearing Plate Size on the Load Required to Produce a Given Plate Displacement. Data From Series 3.

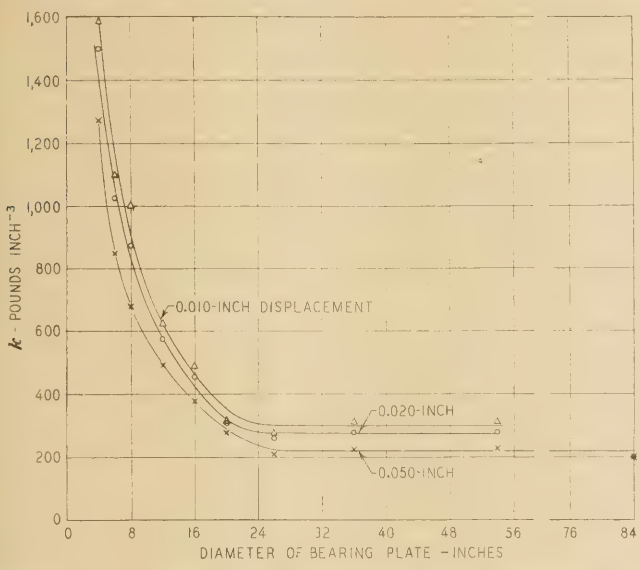


Figure 10.—Values of k Determined From Bearing Tests Made on Circular Plates of Various Diameters. Data From Series 2.

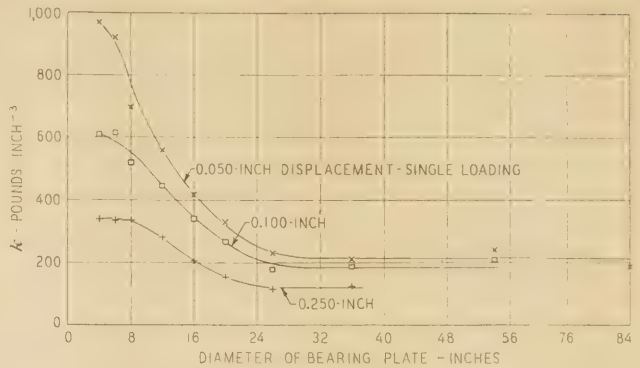


Figure 11.—Values of k Determined From Bearing Tests Made on Circular Plates of Various Diameters. Data From Series 3.

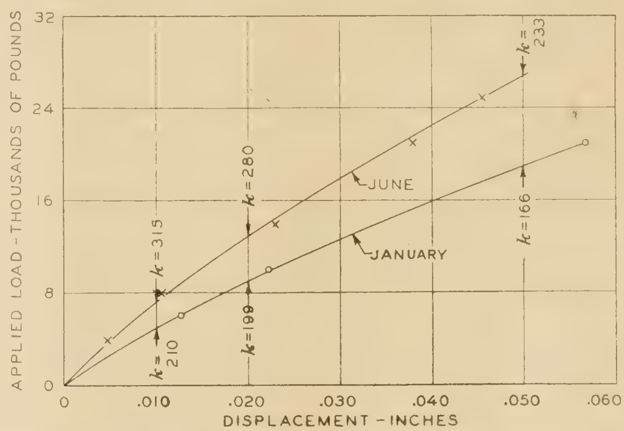


Figure 12.—Effect of Seasonal Moisture Condition and of Soil Deformation on the Load Supporting Ability of the Subgrade. Data From Series 4.

it is necessary to limit the deformation to a magnitude within the range of pavement deflection and that it is of great importance to use a bearing plate of adequate size. For the conditions of the tests of series 2 the minimum plate diameter is indicated as being about 26½ inches. It should not be concluded, however, that this size of bearing plate would necessarily be adequate for other soil conditions. There is, in fact, evidence that it would not be. This point will be referred to again later.

A similar analysis was made of the data obtained in the tests of series 3. It will be recalled that in this series larger displacements were included and only one load of each magnitude was applied. The values of the coefficient for the various conditions of the tests of series 3 are shown in figure 11.

A comparison of figure 11 with figure 10 shows that the general indications, mentioned above, as to the effect on the value of the stiffness coefficient of the magnitude of soil deformation and of the size of the bearing plate are supported by the data from the tests of series 3 as well as those of series 2. Furthermore, in the one case where the two series are most nearly comparable, i. e., the tests with a limiting soil deformation of 0.05 inch, the agreement between the values obtained in series 2 and series 3 is rather good.

The shapes of the curves shown in figure 11 are different from those in figure 10, particularly in the region of the small bearing plates. This is attributed to the difference in test procedure used in the two series of tests as mentioned in connection with the discussion of figures 8 and 9.

The tests of series 2 and series 3, from which the data shown in figures 10 and 11 were obtained, were made during the summer. The moisture content of the soil was determined to a depth of about 12 inches and was found to be quite uniform, averaging 17 percent.

Because the moisture content of the soil beneath the pavement was known to vary from summer to winter, load-displacement tests with the 54-inch diameter bearing plate were made under both normal summer and normal winter conditions. This made possible some study of the effect of seasonal moisture change on the value of the soil stiffness coefficient.

In these tests, which were designated series 4, the bearing plate remained in position on the subgrade for some time before the test so that moisture conditions were stabilized at the time of test. For the summer condition the moisture content of the soil was uniformly 17 percent to a depth of about 12 inches while for the winter condition it was about 25 percent in the 0- to 6-inch zone and about 19 percent in the 6- to 12-inch zone.

The loading procedure in series 4 was the same as in series 2.

Load-displacement data from the tests of series 4 are shown in figure 12 for both the summer and the winter condition. On the graph are shown also values of the modulus k calculated from data obtained in the bearing plate tests for soil deformations of 0.01, 0.02 and 0.05 inch, respectively. It is apparent from this figure that the soil structure in question was much more resistant to load deformation under the conditions which prevailed when the summer tests were made than it was at the time of the winter tests, the increase in stiffness from winter to summer being roughly 40 to 50 percent. Undoubtedly the magnitude of the seasonal change in subgrade support will vary with the characteristics of the soil material and with the temperature and moisture changes that occur. Under favorable conditions the variation may be much smaller than that cited. It is obviously important, however, to make sure that the soil at the site of the bearing test is of the same character and density and is in the same moisture state as the soil supporting the pavement under consideration. The important influence which the magnitude of the soil deformation has on the value of the modulus k is further emphasized in this graph.

CAREFUL PROCEDURE NECESSARY IN DETERMINING MODULUS OF SUBGRADE REACTION FROM LOAD-DISPLACEMENT TESTS

The experiments with rigid bearing plates have led to the conclusion that approximate but usable values for the modulus of subgrade reaction as used in the Westergaard equations can be obtained by means of load-displacement tests with rigid bearing plates. The criterion by which the usability of the modulus values determined in this manner is judged has been a comparison of calculated and measured strains in full-size pavement slabs. Since the modulus of subgrade reaction k is but one of several coefficients that must be determined before the comparison can be made the discussion of this comparison will appear in the more general section to follow.

The studies have indicated that there are certain limitations which must be recognized, certain precautions which must be taken and certain procedures which are desirable when determining values for the modulus of subgrade reaction by means of load-displacement tests with rigid bearing plates. These will be mentioned briefly under appropriate headings.

Condition of the Soil at the Site of the Test.—It is obvious that if the data are to reflect the supporting properties of the subgrade, the soil at the site of the test should be truly representative of that subgrade. Not only should the soil materials be the same, but the structure, density and moisture state must likewise be duplicated. It appears that this can be accomplished best by giving the soil to be tested the same compaction as that given the subgrade, by covering the soil to be tested with a concrete slab and by permitting sufficient time to elapse for the development of a stable condition of soil mositure before testing. Unless these precautions are taken, there is no assurance that the soil structure tested was representative of that under the pavement. In the case of an existing pavement these conditions can be satisfied by removing a small section of the pavement and making the bearing plate test on the subgrade itself. Both procedures have been used in the Arlington tests and both are believed to be satisfactory.

Bearing Plate.—Certain physical characteristics of the bearing plate must be considered. It was shown that, within limits, the area of the plate had a very marked effect on the value of the modulus k as determined by the bearing plate test and furthermore that the value which was obtained with data from tests with relatively large plates could not be predicted from similar data obtained in tests with small plates. Also there is evidence that the minimum size of plate that will give satisfactory data depends upon the soil structure being tested. It is important, therefore, to determine by tests with plates of several sizes the minimum size that will be satisfactory for a given soil condition. If this is not possible an expedient would be to test with a relatively large plate, a 48- to 60-inch diameter being suggested by the data at present available.

The rigidity of the plate is apparently an important factor. The use of slightly flexible plates may be feasible but the degree of flexibility is another influence to be considered and sufficient data are not yet available to permit comparisons to be made of the relative value of rigid and slightly flexible bearing plates. Until further studies are made of the use of slightly flexible plates it is believed that rigid plates should be used exclusively, a rigid plate being arbitrarily defined as one which, under the conditions of the load-displacement test, does not deflect or "dish" from perimeter to midpoint by more than 0.004 inch. (NOTE.—The 54-inch diameter bearing plate used in these tests was made of concrete and was 12 inches in thickness. Under a 15,000-pound total applied load, the maximum measured deflection of a diameter of this disc was 0.0034 inch.)

The effect of the shape of the bearing plate is another element which has not been studied adequately. Until more information is available it seems advisable to make all bearing tests with circular plates.

Range of Plate Displacement.—It has been shown that the magnitude of the soil deformation (or plate displacement) had an important effect on the soil stiffness coefficient in the tests at Arlington. How great this effect might be with soil structures radically different from the subgrade under the experimental pavement is not known. It is believed, however, that the maximum displacement of the bearing plate in tests to determine the modulus of subgrade reaction k should not exceed the average deflection of the pavement slab under the expected wheel load. In the absence of measured values, it is probably safe to assume a value within the range 0.02–0.03 inch for this purpose.

Bedding the Bearing Plate.—Because of the small maximum displacement it is especially important that the bearing plate be carefully bedded on the soil under test. The procedure followed in the tests previously described appears to be one way by which a satisfactory initial contact can be obtained.

Loading Procedure.—It seems desirable to have the soil structure in as nearly an elastic condition as possible when the modulus of subgrade reaction is determined and it appears that in the tests at Arlington this end was substantially attained by the procedure adopted. It will be recalled that in this procedure a sequence of loads of equal magnitude was applied to the bearing plate until the soil structure reached a condition such that each successive load produced essentially the same plate displacement and elastic recovery in a given time interval.

Other Tests.—In conjunction with a subsequent program of studies of the structural action of certain types of transverse joints, there was afforded an opportunity to make tests that provide limited comparisons between values of the modulus of subgrade reaction as determined by pavement slab deflections and values obtained by bearing plate tests for a somewhat different soil structure. These comparisons will be discussed later in the more general section of this report.

LOAD-DEFLECTION AND LOAD-STRESS RELATIONS FOR PAVEMENT SLABS OF UNIFORM THICKNESS

WESTERGAARD EQUATIONS FOR DEFLECTION AND STRESS

The Westergaard equations for deflection and for stress in concrete pavement slabs are developed for three cases of load position, as follows:

Case I in which "a wheel load acts close to a rectangular corner of a large panel of the slab" (*23*), referred to as a corner loading.

Case II in which "the wheel load is at a considerable distance from the edges," referred to as an interior loading.

Case III in which "the wheel load is at the edge but at a considerable distance from any corner," referred to as an edge loading.

In the original analysis it was assumed that the slab acts as a homogeneous, isotropic, elastic solid in equilibrium and that the reactions of the subgrade are vertical only and are proportional to the deflection of the slab.

On the basis of these assumptions the following equations for maximum deflection and for critical stress were presented:

Maximum deflections:

$$z_c=\left(1.1-0.88\,\frac{a_1}{l}\right)\frac{P}{kl^2} \qquad (1)$$

$$z_i=\frac{P}{8kl^2} \qquad (2)$$

$$z_e=\frac{1}{\sqrt{6}}\,(1+0.4\,\mu)\,\frac{P}{kl^2} \qquad (3)$$

Critical stresses:

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{Eh^3}{12(1-\mu^2)k}\right)^{-0.15}(a\sqrt{2})^{0.6}\right] \qquad (4)$$

$$\sigma_i=0.3162\frac{P}{h^2}[\log_{10}(h^3)-4\log_{10}(\sqrt{1.6a^2+h^2}-0.675h)-\log_{10}k+6.478] \qquad (5)$$

$$\sigma_e=0.572\frac{P}{h^2}[\log_{10}(h^3)-4\log_{10}(\sqrt{1.6a^2+h^2}-0.675h)-\log_{10}k+5.767] \qquad (6)$$

In the above equations, the following notation is used:

z_c, z_i, z_e, maximum deflection for corner, interior, and edge loadings respectively.

σ_c, σ_i, σ_e, maximum tensile stress for corner, interior, and edge loadings respectively.

P=applied load, in pounds.

h=thickness of slab, in inches.

a=radius, in inches of the circular area (cases I and II) or the semicircular area (case III) over which the load P is assumed to be uniformly distributed.

$a_1=a\sqrt{2}$ in the case of the corner loading, the distance from the extreme slab corner to the center of the area of load application when the slab edges are tangent to that area.

$l=\sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}}$ a dimension, termed the radius of relative stiffness, measured in inches.

k=modulus of subgrade reaction, in pounds inch^{-3}.

E=modulus of elasticity of the concrete, in pounds inch^{-2}.

μ=Poisson's ratio for concrete.

It will be noted that the elastic constants E and μ do not appear in equations (5) and (6). In the original presentation these equations were limited to the case where E=3,000,000 lbs. in.$^{-2}$ and μ=0.15. Subsequently Westergaard generalized these equations and transcribed equation (4). These equations as restated are:

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.15}(a\sqrt{2})^{0.6}\right] \qquad (7)$$

$$\sigma_i=0.275(1+\mu)\frac{P}{h^2}\log_{10}\left(\frac{Eh^3}{kb^4}\right) \qquad (8)$$

$$\sigma_e=0.529(1+0.54\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)-0.71\right] \qquad (9)$$

the term b being an equivalent radius dependent upon a and h and expressed in inches.

In this subsequent paper (*25*) Westergaard proposed a new coefficient, K, defined by the relation

$$K=kl \text{ (measured in lbs. in.}^{-2}\text{)}.$$

This new coefficient K, like k, is a measure of the resistance of the subgrade. The reason given for the proposal was the expectation that K would be less dependent on the stiffness of the slab than is k and Westergaard stated his belief that "the truth may lie between the two extremes of a constant k and a constant K."

Westergaard in this subsequent discussion introduced another coefficient D which he called the deflection modulus of the pavement. This was defined as

$$D=kl^2=Kl$$

He restated equations (7), (8) and (9) in terms of the new coefficient of subgrade stiffness and gave a new equation for the maximum stress for the interior case of loading based upon the conception that, for this case, the reactions of the subgrade will be more closely concentrated around the load than are the deflections. The modified equation for maximum stress for the interior case of loading is

$$\sigma''_i=0.275(1+\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)-54.54\left(\frac{l}{L}\right)^2 Z\right] \qquad (10)$$

in which,

L is the maximum radial distance from the center of load application within which a redistribution of subgrade reactions is made, in inches.

Z is a ratio of reduction of the maximum deflections which Westergaard states may be expected to vary between 0 and 0.39. If Z=0, equation (10) reduces to equation (8).

It is with the equations given above that the discussion which follows will be concerned. For a more complete explanation of the terms used, reference to the Westergaard papers (*23*) and (*25*) is suggested.

EXPERIMENTAL PROCEDURE DESCRIBED

In the original planning of the investigation at Arlington four sections of uniform thickness were provided. Each test section was 20 feet wide by 40 feet long divided into four equal quadrants by a longitudinal joint and a transverse joint. The thicknesses of the four sections of uniform depth were 6, 7, 8 and 9 inches respectively. Except in the case of a few special studies, the test sections were protected in order to maintain them in a uniform moisture condition and to prevent warping from variations in temperature while load testing was in progress. The method used for this protection is described in detail in the first of this series of reports (*18*).

It was stated in the first report that the program of tests for these sections of uniform thickness was planned in such a way that each of the factors which theoretically might influence the load-stress relation could be examined experimentally and the observed effects compared with those predicted by the theory. Thus originally it was planned to study the effect on the load-stress relation of slab thickness, of load position and of size of bearing area. As the work proceeded the study was extended considerably beyond the limits originally contemplated and an examination was made of various assumptions used in the development of the equations and of the several coefficients that appear in them.

In general, loads were applied to the test sections at the three positions assumed in the development of the Westergaard analysis, i. e., the corner, an interior point and a free edge. These positions are shown at E, H, and A on the plan of a typical test section shown in figure 13. The positions of the strain gages installed for each case of loading are shown by the short solid lines in quadrant 1 while the dash lines in quadrant 2 show where the slab deflections were measured.

In certain studies of the structural action of slab corners to be described later, strains were measured in four directions in the corner area. The positions of the strain gages used in this part of the investigation are shown in quadrant 3.

It will be noted that for case I, with the load acting at E, both the strains and the deflections were measured along the bisector of the corner angle E–P. For case II, with the load acting at H, strains were measured in two directions immediately under the load and deflections were measured along the line I–H. For case III, with the load acting at A, the strains were measured on the slab edge 1 inch below the top surface and 1 inch above the bottom surface directly opposite the center of the load A while the deflections were measured along the line E–A.

The apparatus used for obtaining and measuring test loads, the various plates used in applying these loads to the pavement, and the instruments used in

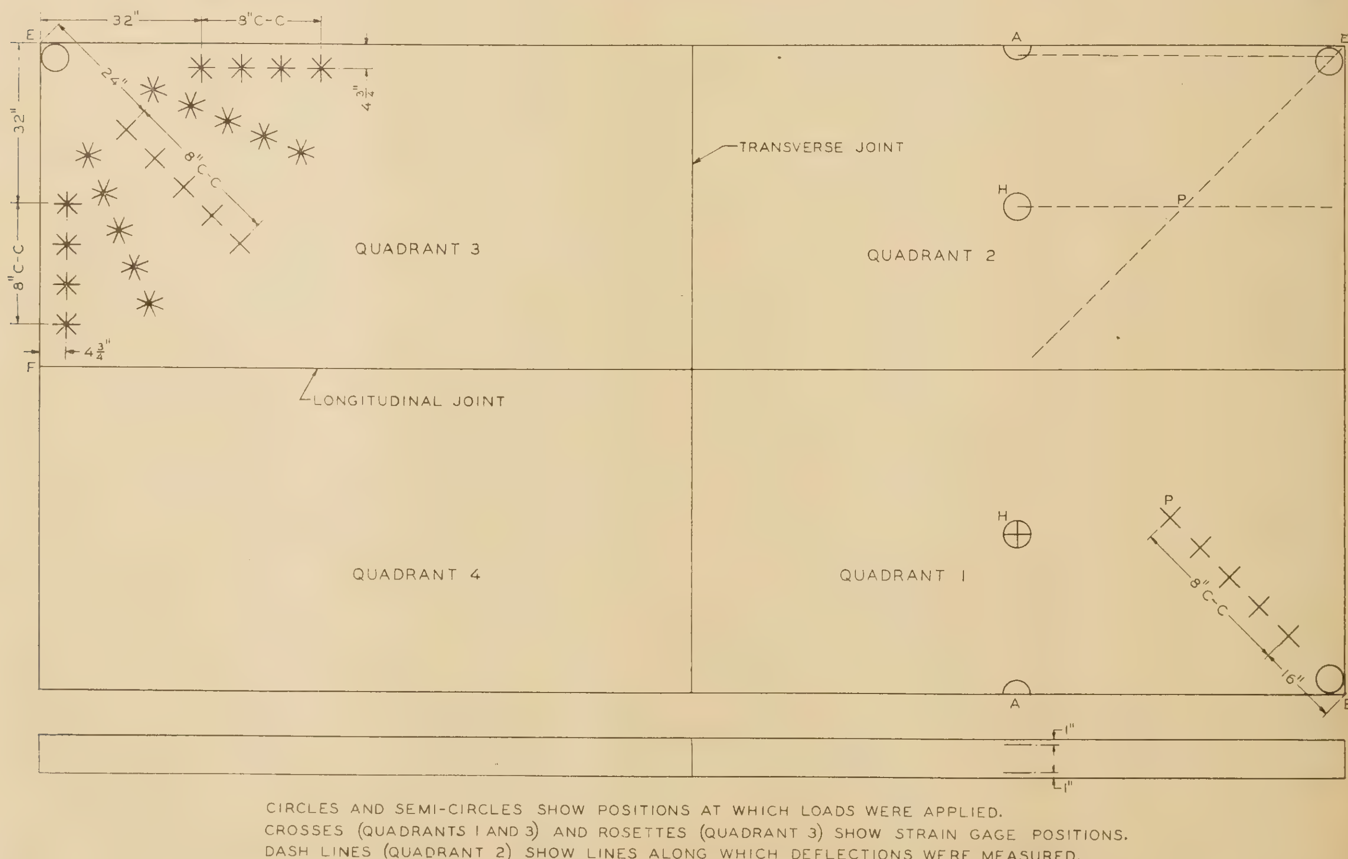


FIGURE 13.—PLAN AND ELEVATION OF A TYPICAL 20- BY 40-FOOT TEST SECTION.

the strain and deflection measurements were completely described in the first report of this series (*18*) and the details will not be repeated here.

The bearing plates used for the corner and interior cases of loading were circular. For the case of edge loading Westergaard assumed the loaded area to be semicircular in shape and in the major part of the tests being reported semicircular plates were used for loads applied at the edge. In certain of the edge tests, however, circular plates were used and in such cases the fact is noted either in the text or on the figures.

The loading tests on sections of uniform thickness considered in this report can be roughly divided into four groups. First, there were a number of what may be called auxiliary or collateral tests designed to settle questions of instrumentation or to supply information needed for the proper execution of the main program or for the interpretation of the data obtained in it. Second, there were certain preliminary or exploratory tests on the experimental sections themselves designed to develop satisfactory methods for obtaining the desired information. Third, there was the main program which comprised the determination of the load-deflection and load-strain relations for the four uniform thickness test sections under the conditions prescribed for the tests together with the determination of values for the several coefficients that appear in the theoretical equations. Fourth, there were a few supplementary studies made because, with test sections available and a testing technique developed, it seemed desirable to obtain data that would throw light on certain problems which were not a part of the original program. If these objectives are kept in mind, it is believed that the purpose for which given tests were made will be clear in the subsequent discussion.

METHOD OF STRESS DETERMINATION OUTLINED

In general the method of arriving at experimental stress values was the same as that described in the earlier reports of this investigation. However, there were some unusual conditions involved in the program being described which made special studies of the strain measuring technique desirable.

It will be recalled that the method used generally throughout this investigation was to measure strains with a temperature compensated recording type of gage (*17*) installed between metal points set in the surface of the pavement slab. From the strains recorded by these gages, stress values were obtained by means of the equations

$$\sigma_x=\frac{E}{1-\mu^2}(e_x+\mu e_y) \qquad (11)$$

$$\sigma_y=\frac{E}{1-\mu^2}(e_y+\mu e_x) \qquad (12)$$

in which,

σ_x is the stress in the direction of the x-axis.
σ_y is the stress in the direction of the y-axis.
e_x is the unit deformation in the direction of the x-axis caused by stress in that direction.
e_y is the unit deformation in the direction of the y-axis caused by stress in that direction.
E is the modulus of elasticity of the concrete.
μ is Poisson's ratio for the concrete.

In applying these equations to measured strains, values for E were determined experimentally because of the importance of the term in the equations. Poisson's ratio, on the other hand, was assumed and the value 0.15 was used in all computations in which the term entered. There were two reasons for this, first, such experimental values of the ratio as are available lie generally between 0.10 and 0.20 with the majority in the upper half of the range and second, small changes in the value of the ratio have an unimportant effect on the computed stress values.

It is obvious that, in any comparison of theoretical stresses with values determined experimentally, it is essential that the strain measuring technique be such as to yield representative values. In this particular investigation the question of gage length was of unusual importance because stress values are the basis for the principal comparison and the test conditions are such as to produce rather abrupt stress variations in the area where strain measurements were to be made. For theoretical reasons, therefore, a short gage length is indicated. Experience has shown, however, that because concrete in small masses may lack the homogeneity that is assumed, the use of too short a gage length is likely to cause trouble by giving strain values that are not representative. A short gage mounted directly over a large piece of aggregate may show quite a different strain indication than would a similar gage mounted between two such pieces because the modulus of elasticity of the stone is different from that of the mortar. A fairly long gage tends to average these deformations and to indicate unit strains which, when combined with the "average" modulus of elasticity, will give stress values that are nearly correct.

In this investigation a collateral study was made to determine the length of gage necessary to give representative strain values in the tests that were scheduled. It was found that a gage length of about 6 inches was sufficient to average out local effects of aggregate in the concrete being used. This was evidenced by the fact that when this gage length was used a given load applied at homologous points on a given test slab always produced essentially the same maximum strain.

Another study was made to determine the effect of gage length on indicated unit strains when the measurements were being made within the area over which the tests loads were applied. For this study a special gage having an effective gage length of 2.2 inches was constructed and this gage was used to determine strain variation under the bearing plates of 6-, 8-, 12-, 16- and 20-inch diameters. Figure 14 shows typical data obtained in these tests. The curve marked A shows the maximum unit strain values recorded in a 2.2-inch gage length at the middle of a diameter of the bearing area for a given applied load and each size of bearing area listed above. Curve B shows, for the same load and bearing areas, unit strain values which are averaged from three 2.2-inch gage lengths arranged end to end along one diameter of the bearing area. These average unit strains, therefore, are those which would be shown by a single 6.6-inch gage centrally located along the same diameter. The location of the gage lengths with respect to the bearing area is shown in the legend of figure 14.

Because of possible local aggregate effects with a gage of this length the actual procedure was to leave the gage in one position and to shift the position of the loaded areas with respect to the gage length. The data indicate that for the conditions of these tests, the stress variation along a diameter of a bearing area was not sufficient to show different unit strains for gage lengths

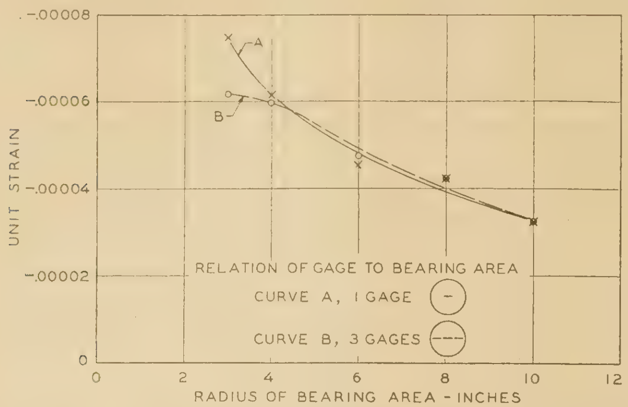


Figure 14.—Comparison of Average Unit Strains Measured Within Bearing Areas of Different Sizes. The Strains Were Measured Over A Distance of 2.2 Inches for Curve A and Over a Distance of 6.6 Inches for Curve B.

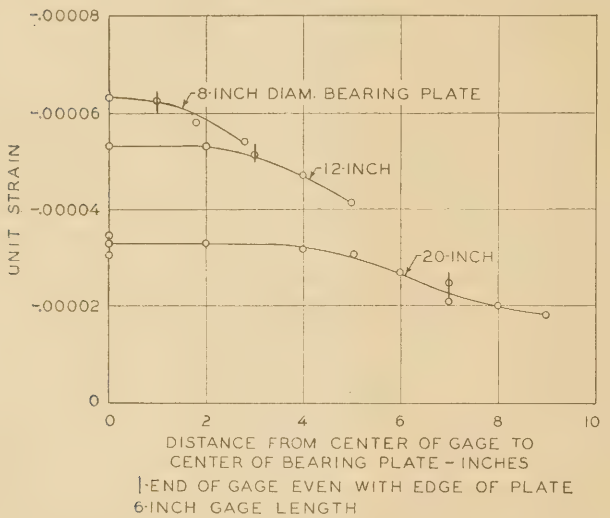


Figure 15.—Variation in the Magnitude of Indicated Unit Strain Under Bearing Plates of Three Sizes.

of 2.2 inches and 6.6 inches respectively so long as the plate diameter was 8 inches or greater and the gage was positioned along a diameter of the area.

In this same group of tests some study was made of the strain variations within the bearing areas when a gage length of about 6 inches was employed. Bearing areas of 8-, 12- and 20-inch diameters were used and the tests were made by shifting the bearing area with respect to the gage length in the manner previously described. However, in these tests, the strain determinations were made over the entire length of the diameter in each case. The data obtained are shown in figure 15. It appears from these data that the maximum unit strain occurs under the center of the bearing area, as would be expected, and that for bearing areas of these sizes there is no appreciable reduction in the strain for some distance toward the edge of the bearing area. In other words the strain variation curve is rather flat in this region. The reduction in strain magnitude near the edge of the bearing area is evident in these data.

In tests such as those just described the precision with which unit strain values are measured is a very important matter. In the published description of the strain gage used so extensively in this general investigation (*17*) it was stated that the accuracy of the gage is sufficient to permit the determination of stress in concrete to within 20 or 25 pounds per square inch, where dependence is placed upon a single observation. Subsequent extensive use of these gages has shown this to be a very conservative estimate. Stress determination is affected by precision in determining the effective modulus of elasticity as well as by precision in measuring strains in the material so that any lack of precision in stress determination may be the combined result of several influences. In this general investigation every effort was made to keep the gages functioning at maximum efficiency and usually dependence was not placed on a single observation unless that observation was supported by other data. In order to find out what consistency could be expected in the load-strain measurements when the same operation was repeated, an 8,000-pound load was applied 80 times at the edge of the 7-inch uniform-thickness test section and the strain recorded for each application. Of the 80 individual determinations only one differed from the average by more than 3½ percent, and some idea of the consistency of the combined load and strain measuring operation may be had from the following figures.

Edge loading, 8,000-pound load, 7-inch slab thickness

Average unit strain	0.00006675
Standard deviation	.00000146
Coefficient of variation 2.18.	

DETERMINATIONS MADE OF MODULUS OF ELASTICITY OF THE CONCRETE

Two methods were used for studying the values of the modulus of elasticity of the concrete, first, the testing of laboratory specimens in compression or bending and, second, by deflection measurements on the test sections themselves.

Stress determinations from measured strains require a knowledge of the value of the modulus and since the exposed pavement sections were tested over a long period of time it was necessary to know also how much the value varied from summer to winter. It was shown in the second report of this series (*19*) that there is an annual change in the moisture state of the concrete in the test sections sufficient to cause rather large volume changes from summer to winter. It is well established that changes in moisture state cause changes in the stiffness of concrete. It seemed probable, therefore, that this change in moisture content from its lowest value in summer to a higher value in winter was such as to cause changes in the modulus of elasticity that would have to be taken into account.

When the test sections were originally laid, extra slabs were provided from which laboratory test specimens could be taken. From these slabs cores were drilled and prisms for flexure testing were sawed and used in laboratory studies of the modulus of elasticity. It was found by flexure tests on prisms that those dried for 12 months by storage in the normal atmosphere of the laboratory had an average modulus of elasticity of 4,500,000 pounds per square inch while a comparable group that had been immersed in water at laboratory temperature had an average modulus of 6,000,000 pounds per square inch.

An attempt was made to determine the moisture content of the concrete in the test pavement in order that some comparison might be made between it and the mositure content of the concrete in the test prisms as an indirect measure of the modulus of elasticity of the concrete in the pavement sections. Fragments broken from the extra slabs on the subgrade were weighed and dried. The effort was not entirely satisfactory, however, because of variation in moisture content from top to bottom of the pavement and because of difficulty in getting a representative sample for the moisture determination. As nearly as could be determined the moisture content of the concrete that had been immersed for 10 months was slightly greater than that of the pavement during the wet winter months. It seems reasonable to conclude, therefore, that the modulus of elasticity of the pavement concrete in winter would be somewhat less than 6,000,000 pounds per square inch. While it was not possible to establish a reliable comparison between the moisture content of the air dry prisms and that of the pavement during the dry summer condition it is believed that the difference in the corresponding modulus values would not be large and that the value of 4,500,000 pounds per square inch determined by the laboratory tests might be used in conjunction with the strains measured during the summer tests.

Values for the modulus of elasticity determined from slab deflections will be discussed later and comparisons will be made between them and the values determined from the flexure tests of prisms.

POSITION AND DIRECTION OF CRITICAL STRESSES INVESTIGATED

Before attempting to make direct comparisons between computed and observed stresses it was considered desirable to make certain exploratory tests on the test sections to determine the position and direction of the critical stress for each position of loading, to ascertain the effect of repetition of load on stress magnitude and to establish the relation between load magnitude and stress magnitude for each case.

In the discussion of the comparisons of computed and observed deformations which is presented later it was found convenient to discuss the cases of corner, interior and edge loading separately and in that order. Because of this fact, the discussion of the work that comprised the preliminary load tests has been arranged in the same order.

A study was made of the strain variations in the corner area of a typical section for case I (the corner loading) in order to determine the location and direction of maximum strain. Strains were measured in two directions at various positions along the bisector of the corner angle and in four directions at comparable positions along two other lines radiating from the slab corner and making an angle of 22.5° with the corner bisector. The maximum strain at each position along the three radial lines for the particular test conditions that obtained is shown in figure 16. Along the bisector of the corner angle the direction of the maximum stress was known to be parallel to the bisector but along the other two radial lines the direction of the maximum strain was found to make an angle of about 15° with the radial line in each case. This matter of the direction of stresses in the corner area will be discussed much more fully later in the report.

The strains in the vicinity of the slab edges are not shown in figure 16 but it has been well established that for a corner loading on a concrete pavement slab of uniform thickness the strains along the edges that form the corner are definitely of less magnitude than those along and in proximity to the corner bisector. The data shown in figure 16 are typical of those obtained in

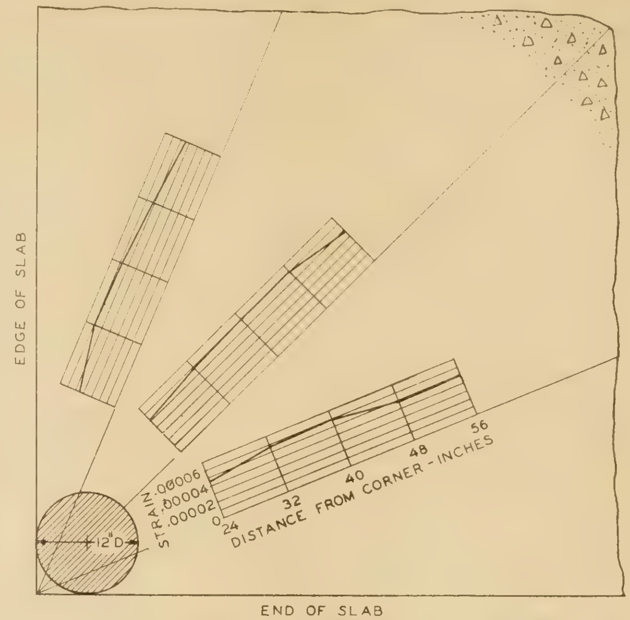


Figure 16.—Variation in Magnitude of Maximum Strains—7-Inch Uniform-Thickness Section—Symmetrical Corner Loading.

a number of similar corner tests and these data consistently indicated that the strain measured along the corner bisector at the proper location would be at least as large as, if not larger than any other strain to be found in the corner region. This being true it is only necessary to locate the position along the bisector at which the radial strain reaches a maximum and at this position measure the radial strain and the strain at right angles to it. The combination of these two strains determines the critical stress for the case of the corner loading.

In his analysis of this case Westergaard gives an equation for finding the distance from the corner along the corner bisector at which the maximum stress theoretically develops. This equation is

$$x_1 = 2\sqrt{a_1 l}$$

in which

x_1 is the distance from the corner to the point of maximum stress measured along the bisector of the corner angle.

a_1 and l are as previously defined.

The distance x_1 was determined experimentally for each of the slab thicknesses, for a range in loads, after various numbers of load applications and for different conditions of temperature warping.

In table 2 the experimentally determined values of this dimension are given for the four slab thicknesses, a range of loads and after various numbers of loadings had been applied. The tests covered a considerable period of time so that the soil moisture was not the same at the time the different sections were tested. For example, when the 6-inch and 7-inch sections were tested the subgrade soil was in a relatively dry state and a slight separation between the lower surface of the slab was noted after the application of a number of loads. In contrast, the subgrade was in a wet condition after severe freezing and thawing when the comparable tests were made on the 8-inch and 9-inch sections.

TABLE 2.—*Experimentally determined values of the distance, x_1, along the bisector of the corner angle at which the maximum stress was found to occur. These values are averages from the stress curves of the four quadrants. Loads were applied on 12-inch diameter bearing plates*

Slab thickness	Load	Distance, x_1		
		After first load	After 12 load applications	After 35 load applications
Inches	*Pounds*	*Inches*	*Inches*	*Inches*
6	4,000	34	35	35
	5,000	34	33	35
	6,000	33	33	33
7	4,000	35	34	38
	5,000	35	34	37
	7,000	33	33	34
	9,000	33	34	33
8	3,000	33	43	41
	6,000	33	33	39
	8,000	33	33	32
	10,000	33	33	31
9	3,000	46	48	46
	6,000	41	47	43
	8,000	41	40	42
	10,000	41	39	40

The data of table 2 indicate a tendency for the distance x_1 to increase, for the low loads, with the number of load applications. For the higher loads the distance remained fairly constant as the loads were repeated. There seems also to be a tendency for the distance to decrease as the magnitude of the load is increased, other conditions remaining the same.

In table 3 are shown experimentally determined values for x_1 for three conditions of temperature warping. Each value in this table was from an average stress curve, each point of which was the average of eight separate stress determinations. The values in table 3 tend to decrease as the degree of contact between the slab corners and the subgrade is increased. This same effect was noted in the data of table 2 as the magnitude of the load was increased.

TABLE 3.—*Experimentally determined values of the distance, x_1, along the bisector of the corner angle at which the maximum stress was found to occur. These values are averages from the stress curves of the four quadrants for each condition of warping. Loads were applied on 12-inch diameter bearing plates*

Slab thickness	Distance, x_1		
	Edges warped down	Slab flat	Edges warped up
Inches	*Inches*	*Inches*	*Inches*
6	31	36	37
7	30	34	36
9	31	33	38

With the possible exception of those for the 9-inch slab the experimentally determined values for x_1 were found to be in good agreement with computed values. This is especially true for the condition of the unwarped or the downward warped slab corners. When the corner was warped upward an increase in the value of the distance x_1 was noted.

In figure 17 average values of x_1 from a considerable number of tests with the pavement in an unwarped condition are compared with values computed with the equation mentioned above. It will be noted that the comparisons include three sizes of bearing area and four slab thicknesses.

Generally there is good agreement between the computed and observed values of this dimension. While there is some tendency for the experimentally determined values to be greater than the computed values, particularly with the larger bearing areas, the difference is not great and when the flat character of the stress variation curve along the corner bisector is considered, it is apparent that small errors in determining the value of x_1 would be of very little importance. It is concluded, therefore, that the theoretical equation for computing this distance is sufficiently accurate for all practical purposes.

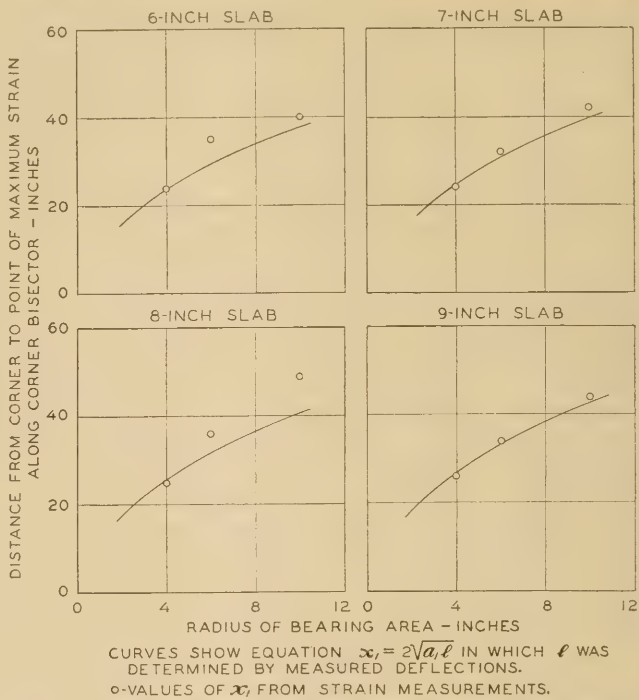


FIGURE 17.—COMPARISON OF THE OBSERVED AND THEORETICAL DISTANCES FROM THE CORNER TO THE POINT OF MAXIMUM TENSILE STRAIN ALONG THE BISECTOR OF THE CORNER ANGLE.

EFFECTS OF REPEATED CORNER LOADING STUDIED

The behavior of a slab corner under repeated loading was first studied in a series of tests on the 7-inch uniform-depth test section made during the late fall when the subgrade soil was in a normal state. A load of 8,000 pounds was applied 80 times to the free corner of each of the four quadrants of the test section. Each load was maintained at full value for 1 minute and after the removal of the load 1 minute was allowed to elapse before the next loading was begun. The loads were applied in groups of ten and the maximum strains for each group of 10 loadings were averaged. The data obtained are shown in figure 18. Since, in this graph the data from all four quadrants are combined, each point on the curve is an average of 10 tests in each of four quadrants or 40 observed values.

The data show a distinct tendency for the strain to increase with repetition of the test load, the average strain for the last group of ten loadings being approximately 10 percent greater than the average strain for the first group. The rate of increase is not constant, decreasing as the number of load applications increased. The increase after 60 repetitions of the load was very small.

Following these experiments it was decided to investigate the relation for all four thicknesses of pavement. In carrying out this subsequent program the procedure followed with each test section was the same and comprised the following schedule:

1. Two applications of a test load of each of three or four different magnitudes were made on each of the four free corners of each of the four sections. For each load application the maximum deflection and the maximum strain were measured.

2. A series of 12 loads of the maximum magnitude used in (1), above, was applied but no measurements of deflection or strain were made.

3. The same loadings and measurements made under (1), above, were repeated.

4. A series of 50 to 70 loadings of the maximum magnitude used in (1), (above), was applied, no measurements of deflection or strain being made.

5. The same loadings and measurements made under (1), above, were again repeated.

The load-deflection and load-strain relations developed during the tests of this schedule are shown in figure 19 while the strain variation along the bisector of the corner angle, for the maximum load magnitude is shown for each slab thickness in figure 20. Each point shown in these two figures is an average of two measurements in each of four quadrants or eight observations in all.

The tests extended over a winter period so that the subgrade was not in the same condition when all of the sections were tested.

The 7-inch section was tested in early December a short time after the data shown in figure 18 were obtained. The earlier series of tests may have affected the soil under the slab corners. The 6-inch section was tested in early January. Although the soil beneath the pavement was not frozen when the tests were made it probably had been frozen slightly a short time before. The tests on the 8-inch section were made during March shortly after the subgrade had thawed after having been frozen to a depth of about 12 inches. During the time that the soil was frozen the test section was heaved upward approximately 1 inch but at the time of load testing it had resumed approximately its original position. The soil was in a wet condition but there is reason to believe that the pavement was in good contact with the soil over its entire area when the load tests were made. The 9-inch section was tested about the first of April. The subgrade had been frozen

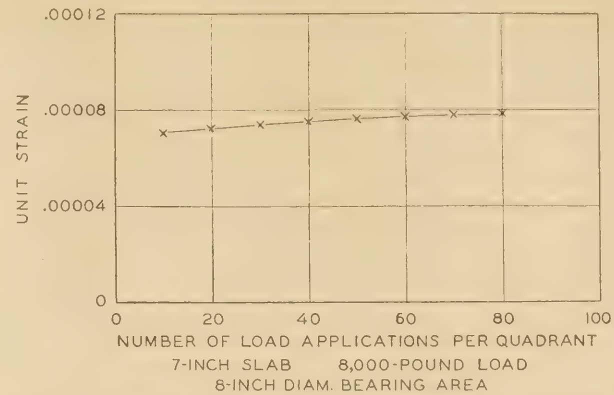


FIGURE 18.—EFFECT ON STRAIN OF REPEATED LOADS APPLIED AT THE CORNER OF A CONCRETE PAVEMENT.

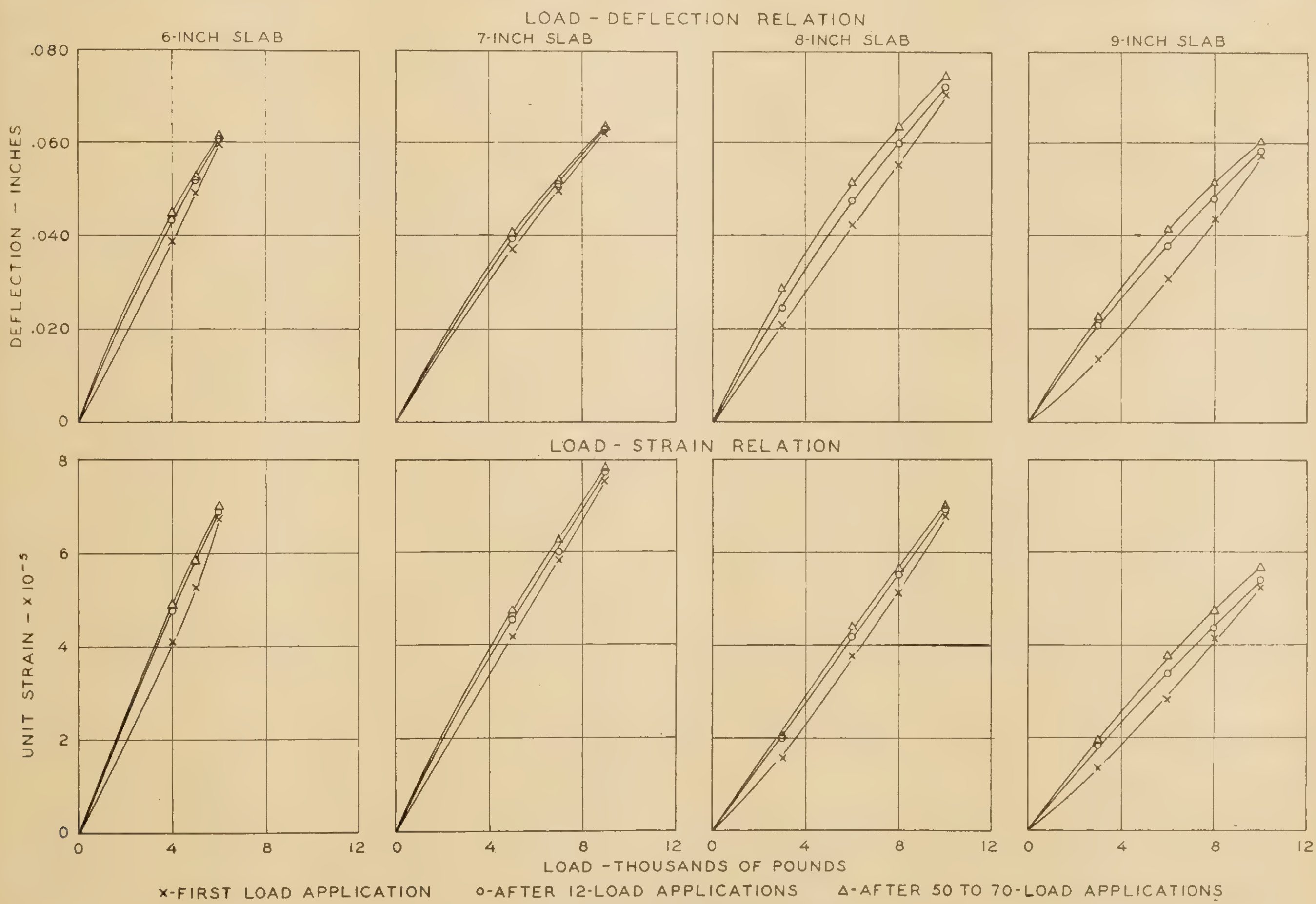


FIGURE 19.—EFFECT OF REPEATED LOADING ON THE LOAD-DEFLECTION AND LOAD-STRAIN RELATIONS AT A FREE CORNER.

as described above and, in addition, the section had been inundated for about 30 hours by flood backwater. The soil was, therefore, quite soft at the time the tests were made on this section. The swelling of the soil from moisture probably improved the uniformity of its contact with the lower surface of the pavement, however.

It is indicated by the data of figure 19 that the deflections and the stresses caused by a given load increased with the repetition of the load for all thicknesses in the same manner as found earlier with the 7-inch section. It will be noted also that when the load magnitude was greatest the effect of repeated loading on deflection and stress was least. The data consistently indicate that the change in deflection and strain under repeated loading is the result of an adjustment of the conditions of soil support under the slab corner as the repetition of loading proceeds. This adjustment continues until a condition of equilibrium is established.

The lack of linearity in the load-deflection and load-strain data for these slab corners is further evidence that as the slab corner is deflected the conditions of soil reaction under it change.

Similar tests to determine the load-deflection and load-strain characteristics of the interior region of the test sections were made and representative data from these are shown in figures 21, 22 and 23.

Figure 21 is a typical strain variation graph for an interior loading. Each point on the curves is the average of two strain measurements in each of four quadrants, eight observations in all. The radial strains were measured along a radius from the center of the loaded area and at distances from that center as shown. The tangential strains were measured at the same positions but in a direction normal to the radius. The shape of the curves under the bearing plate was drawn in accordance with data discussed earlier in this report and shown in figure 15.

These data indicate that directly under the bearing area the two strains have essentially the same magnitude but at points not within the bearing area the tangential strain is greater than the radial strain. The radial strains become zero at a distance from the center of the bearing area approximately equal to l while the tangential strains approach zero at a distance of about $1.6l$. The data in figure 21 were obtained on the section of 9-inch thickness but tests on the sections of other thicknesses showed the relation to be essentially the same when expressed in terms of l. This means that the critical stress from a given wheel load will not be increased by the presence of another equal load so long as the two loads are separated by a distance of $1.6l$ or more. For most highway pavements this would correspond to a distance of 40 to 60 inches.

It is interesting to note how closely this is in accord with experimental strain data obtained in studies of the effect of 6-wheel trucks on concrete pavements made by the Public Roads Administration some 17 years ago (*16*).

PRELIMINARY TESTS YIELDED SIGNIFICANT RESULTS

Repeated load tests were made at the interior position (case II) following the same procedure as that used for the corner case. Typical data, obtained in the tests on the 7-inch thickness section, are shown in figure 22. The data from the tests at the interior consistently show that repeated loading causes no increase in the magnitude of the critical strain, within the limits of the tests of this program.

A large number of tests were made in studies of the load-deflection and load-strain relations for the interior loading. Typical data from these tests are given in figure 23. Both the load-deflection and the load-strain relation were consistently linear for the interior loading, within deflection and strain limits such as those shown in this figure.

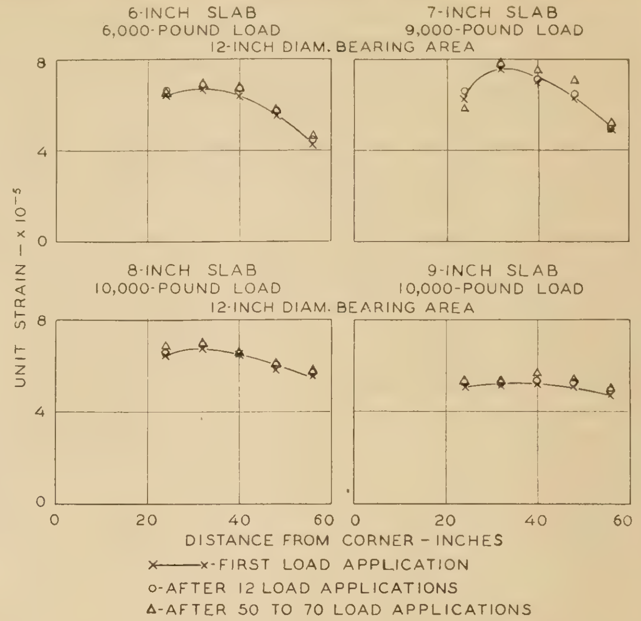


FIGURE 20.—EFFECT OF REPEATED LOADING ON THE STRAIN DISTRIBUTION ALONG THE BISECTOR OF THE CORNER ANGLE.

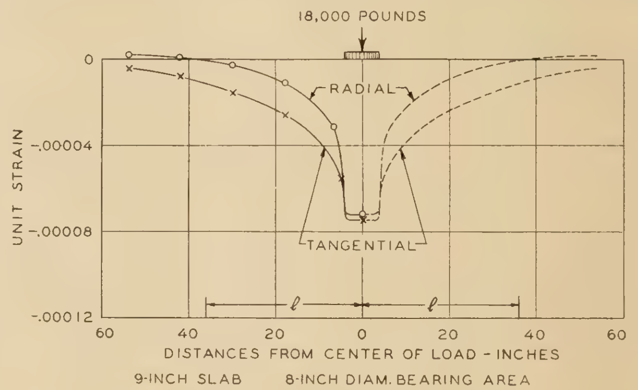


FIGURE 21.—VARIATION IN TANGENTIAL AND RADIAL STRAINS IN THE VICINITY OF A LOADED AREA AT THE INTERIOR OF A CONCRETE PAVEMENT SLAB.

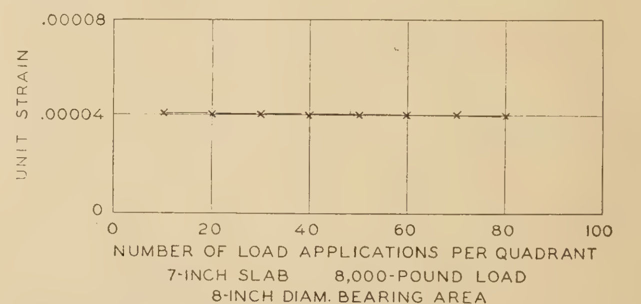


FIGURE 22.—EFFECT ON STRAIN OF REPEATED LOADS APPLIED AT THE INTERIOR OF A CONCRETE PAVEMENT.

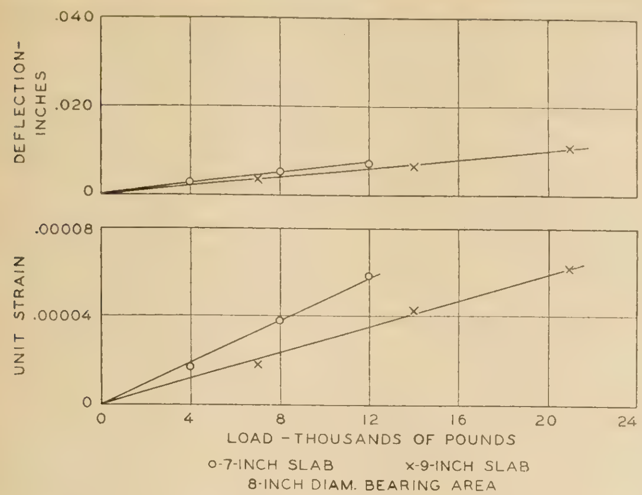


FIGURE 23.—TYPICAL LOAD-DEFLECTION AND LOAD-STRAIN RELATIONS AT THE INTERIOR OF A CONCRETE PAVEMENT.

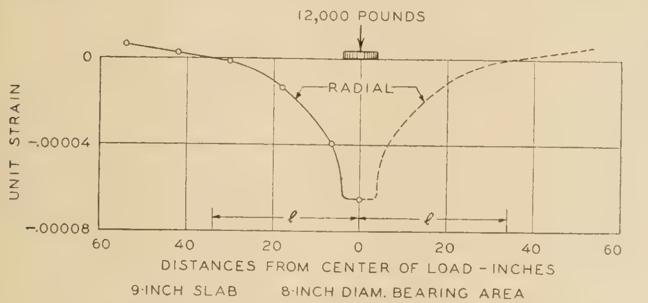


FIGURE 24.—VARIATION IN RADIAL STRAINS ALONG THE EDGE OF A PAVEMENT FOR A LOAD APPLIED ON A CIRCULAR AREA AT THE EDGE.

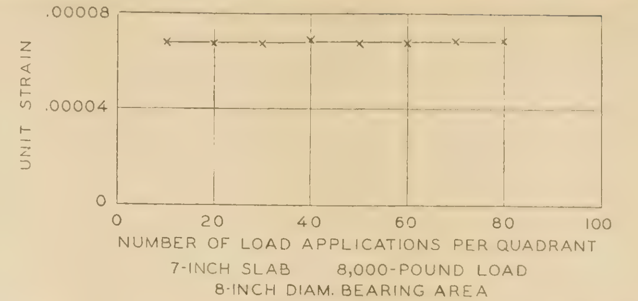


FIGURE 25.—EFFECT ON STRAIN OF REPEATED LOADS APPLIED AT THE EDGE OF A CONCRETE PAVEMENT.

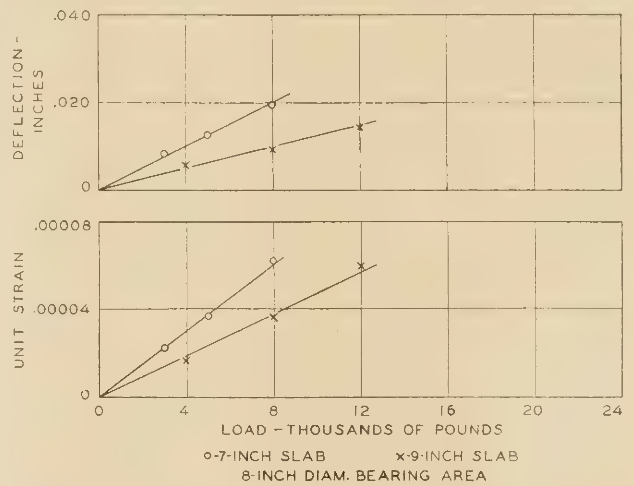


FIGURE 26.—TYPICAL LOAD-DEFLECTION AND LOAD-STRAIN RELATIONS AT THE EDGE OF A CONCRETE PAVEMENT.

At the free edge, tests were made to determine the shape of the strain variation curve along a line through the center of the bearing area and parallel to the edge of the pavement; to determine the effect of repeated loads; and, to determine the load-deflection and load-strain relations using circular bearing areas centered 6 inches from the slab edge. Typical data from these tests are shown in figures 24, 25 and 26.

The strain variation curve (fig. 24) shows radial strains only since tangential strains at the points of measurement shown are negligible. Each point shown on the curve is an average of eight observations, two in each of the four quadrants of the test section. The measurements of strain were made on the upper surface of the slab, hence the maximum strain shown is a compression in the upper surface of the pavement. It will be noted that the distance from the center of the loaded area to the point where the strain changes from compression to tension is approximately equal to l. The tensile strain in the upper surface of the pavement is much smaller than that in the bottom surface directly under the area of load application. It is this maximum tensile deformation in the bottom of the slab directly under the load that is the critical strain. Numerous tests have shown that the maximum compressive strain measured under the center of a circular bearing plate of 8- to 12-inch diameter when the plate is tangent to the slab edge is essentially equal to the maximum tensile strain measured at the bottom of the slab edge. In other words the maximum strain shown in the variation diagram may be considered as equal to the critical strain.

The data from repeated loading are given in figure 25. The procedure was the same as that followed for case I (corner) and case II (interior). It is indicated that the critical strain for the edge loading is not increased by a repetition of load application. In this respect it is like the interior case. This difference in behavior between the corner on the one hand and the interior and edge on the other is attributed to two factors, first, there is more deflection and hence more soil deformation when the free corner of a slab of uniform thickness is loaded and, second, for corner loading there is less confinement for the soil than is present with either the edge or interior loadings.

In figure 26 are given some typical load-deflection and load-strain data for the edge loading. It was found that these relations were linear within the limits of the tests.

To summarize, the preliminary load testing program gave data as to the strain distribution which located the position of the critical strain for each of the three cases of loading considered in the Westergaard analysis. It verified the equation for locating the position of maximum stress for corner loads as given by Westergaard. The application of repeated loads was found to affect the magnitude of the critical strain to a measurable extent with corner loading but not for the interior or the edge loadings. Finally the preliminary load testing program showed that the load-deflection and load-strain relations were nearly linear for corner loading and linear for the interior and edge loadings, within the limits of the tests.

LOAD-DEFLECTION DATA AND THEIR SIGNIFICANCE

The methods used for determining the deflected shape of the test sections under various load and other conditions were described and discussed in the earlier reports of this series.

The principal use made of the deflection data in the present study is for evaluating the constants that appear in the various equations of the Westergaard analysis.

In connection with the presentation of his generalized equations (*25*) Westergaard suggests methods for determining the value of the quantities D, l, k, K, E, L and Z from observed values of deflections and stresses. These will be discussed briefly.

The deflection modulus, D, was defined as being equal to kl^2 and values for it may be obtained by substituting the observed maximum deflection value in the appropriate equation for maximum deflection (1), (2) or (3).

The value of l is obtained by adjusting the scales (both horizontal and vertical) of a theoretical deflection diagram, in which the ordinates are deflection units and the abscissas are distances expressed in terms of l, until the theoretical elastic curve coincides as nearly as possible with the observed shape of the deflected slab.

For the corner loading the theoretical deflection diagram may be constructed, once D is determined, by means of the generalized equation for corner deflection

$$z=\frac{P}{D}\left(1.1e^{-\frac{x}{l}}-\frac{a_1}{l}0.88e^{-\frac{2x}{l}}\right) \quad \text{(13)}$$

z being the deflection at any distance x from the slab corner, measured along the bisector of the corner angle, while e is the Naperian base. Values of l are assumed until coincidence is obtained.

For the interior and edge cases, the theoretical deflection diagrams were constructed by means of coefficients of deflection obtained from the diagrams given by Westergaard in the original analysis (*23*) (figures 4 and 8). Values of l were assumed as in the case of the corner loading until the best degree of coincidence was attained.

With values for D and l determined in the manner just described k, K, and E may be evaluated by means of the following equations:

$$k=\frac{D}{l^2} \quad \text{(14)}$$

$$K=\frac{D}{l} \quad \text{(15)}$$

$$E=\frac{12(1-\mu^2)Dl^2}{h^3} \quad \text{(16)}$$

There appears to be no direct method for the determination of values for the quantities L and Z from deflection data. Westergaard suggests that these terms be evaluated from the theoretical equations for stress. Discussion of this question will accordingly be deferred until after the introduction of the stress data. Except where stated otherwise, the value of Z will be taken as zero, however.

Observed deflection data for each of the four test sections of uniform thickness for the three cases of corner, interior and edge loading are shown in figures 27, 28, and 29, respectively. On these diagrams the observed shape of the deflected slab is shown by curves drawn through crosses. Each cross is the average of 12 observed values, three measurements in each of four quadrants. The small circles are computed values which indicate the position of the matched theoretical diagram constructed for the purpose of determing l.

Values of the several coefficients, derived by the methods just described from the deflection data shown in figures 27 to 29, inclusive, are given in table 4.

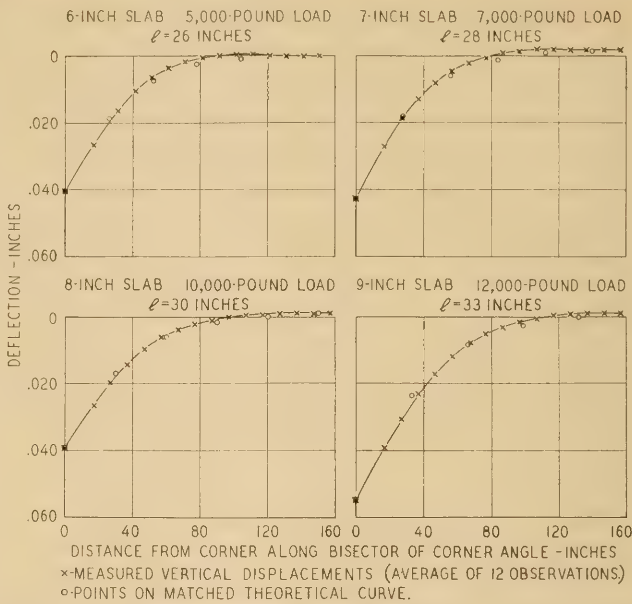


FIGURE 27.—DEFLECTION OF THE BISECTOR OF THE CORNER ANGLE USED FOR EVALUATING l.

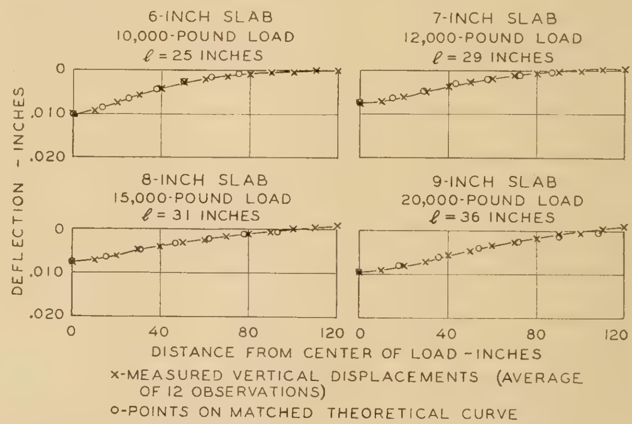


FIGURE 28.—DEFLECTION OF SLAB FROM INTERIOR LOADING USED FOR EVALUATING l.

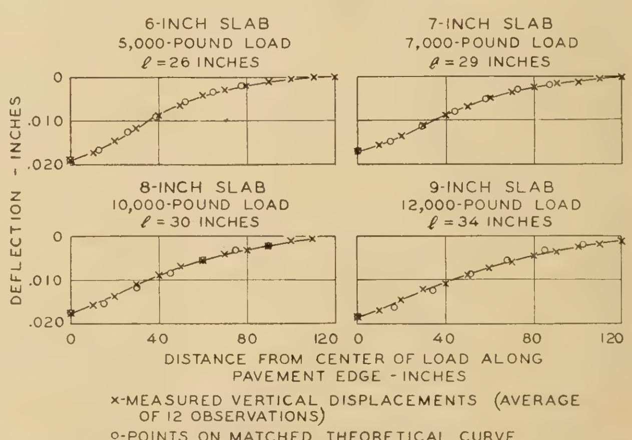


FIGURE 29.—DEFLECTIONS ALONG FREE EDGE OF PAVEMENT USED FOR EVALUATING l.

TABLE 4.—*Values for various coefficients, used in the Westergaard equations, determined from measured deflections*

Position of load	Time of testing	Slab thickness	l	k	K	D	E
		Inches	*Inches*	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-1}$	*Lbs. in.*$^{-2}$
Corner	Late summer	6	26	143	3, 708	96, 400	3, 540, 000
	Winter	7	28	161	4, 515	126, 400	3, 390, 000
	Winter	8	30	227	6, 825	204, 700	4, 220, 000
	Late fall	9	33	168	5, 535	182, 600	3, 200, 000
Interior	Late summer	6	25	195	4, 880	122, 000	4, 140, 000
	Winter	7	29	238	6, 895	200, 000	5, 750, 000
	Summer	7	28	222	6, 230	174, 400	4, 670, 000
	Winter	8	31	260	8, 065	250, 000	5, 500, 000
	Late fall	9	36	203	7, 315	263, 200	5, 490, 000
	Summer	9	33	220	7, 290	240, 500	4, 210, 000
Edge	Late summer	6	26	171	4, 440	115, 400	4, 235, 000
	Winter	7	29	212	6, 145	178, 200	5, 125, 000
	Winter	8	30	279	8, 365	251, 000	5, 175, 000
	Late fall	9	34	243	8, 260	280, 800	5, 220, 000

The deflection tests on the four sections were not made at the same time but covered a period of about 4 months, as follows:

Test section:	*Month tested*
6-inch	September.
7-inch	December.
8-inch	November.
9-inch	October.

During the period when a particular section was under test it was insulated against air temperature fluctuations with a layer of straw and was protected from precipitation and direct sunlight with a temporary canvas shelter. However, during the 4-month period between late summer and winter there are important changes in temperature, evaporation rate, ground moisture, etc., and it seems reasonable that some effects of this seasonal change are present in the data. If so, the effect would be most noticeable in comparisons between different test sections.

For a given slab thickness, values of the radius of relative stiffness, l, are in good agreement for the three cases of loading. For conditions that are comparable there is rather good agreement also between the values of modulus of subgrade reaction, k, as determined by pavement deflection, for the interior and edge loadings but the value for the corner loading is consistently lower. This is believed to be the result of incomplete contact between the corner area and the subgrade and is in accord with the evidence of the strain data previously discussed. It will be noted that the values of k, as determined from the deflections of the 6-inch section of uniform thickness are somewhat lower than as determined by the deflection data from the three other test sections.

SEASONAL CHANGES AFFECT CERTAIN COEFFICIENTS

In the earlier discussion of the modulus of subgrade reaction, k, in the first section of this report, the data from the bearing plate tests of series 4 showed that for a given plate displacement the value of k was appreciably greater in summer than in winter. For example, at a displacement of 0.02 inch, the summer value was 280 lbs. in.$^{-3}$ and the winter value was 199 lbs. in^{-3}. This same trend is not evident in the limited data on the values of k, determined from the deflections of the interior of the 7-inch and 9-inch sections shown in table 4. It will be noted that the value of k is nearly the same for summer conditions as it is for late fall or winter conditions in the two comparisons available in this table. It may be recalled that in the study of seasonal effects on the value of k with the bearing plate tests, the plate was left continuously in place on the subgrade. While the plate used was relatively large, it actually was much smaller than a full-size pavement slab. It is conceivable, therefore, that a larger change in soil moisture may have occurred under the bearing plate than was possible under the test section and that this larger change in soil moisture might account for the larger variation in the value of k as determined from the bearing plate test.

The general level of values of k from the bearing plate tests is in reasonably good agreement with that determined from pavement deflections.

The coefficients D and K being dependent on l and k are affected directly by any conditions that influence the value of either l or k. The values of the modulus of elasticity for the concrete, E, as determined from the slab deflections are in the same general range as the values that were obtained from the tests of the laboratory specimens. Values obtained from corner deflections are distinctly lower than those determined by deflection data from interior or edge loading. This is believed to be a direct reflection of the imperfect subgrade support that obtained under the unwarped slab corners and of certain rotational movements that will be discussed presently. For this reason the value of E, determined from corner deflections, is considered a less reliable index of slab stiffness than those determined from interior or edge deflection data. It is noted also that there is a tendency for the values determined from the deflections of the 6-inch section to be slightly lower than those for the other three sections. A comparison of the data obtained in summer with those obtained in winter in tests at the interior of the 7-inch section show a much higher value of E for the winter condition. The same trend is evident in the tests made in summer and in late fall at the interior of the 9-inch section.

Since the value of the modulus of elasticity of the concrete varies with the moisture state of the material and since the moisture state is known to vary but no good measure of its range or rate of variation was available, an uncertainty as to the exact value of the modulus E was always present in any consideration of data that required its use, particularly where long time periods were involved in the testing.

Assuming that values of E=5,500,000 lbs. in.$^{-2}$ and k=200 lbs. in.$^{-3}$ were fairly representative of the winter condition and that values of E=4,500,000 lbs. in.$^{-2}$ and k=280 lbs. in.$^{-3}$ are equally representative of the summer condition of the test sections, values of the coefficients l, D and K were computed for each of the four thicknesses of test section. These computed values are given in table 5. A comparison can be made between these computed values and certain of the values derived from the measured deflections which were shown in table 4. In making such comparisons it is well to bear in mind, first, that the computed values are only as good as the selected values of E and k on which the computations are based, and second, that the computations presuppose the ideal subgrade reaction to obtain. The agreement between the computed values and those derived from the measured deflections is, in general, better for the winter observations than for the summer and better for the interior and edge loadings than for the corner. For the winter condition several comparisons are available and for the interior and edge cases the agreement may be considered good. For the corner case the values of D and K derived from measured deflections tend to be lower than the computed values.

TABLE 5.—*Computed values of coefficients used in the Westergaard equations*

WINTER CONDITION

[E=5,500,000 lbs. in.$^{-2}$, k=200 lbs. in.$^{-3}$]

Slab thickness	l	D	K
Inches	*Inches*	*Lbs. in.$^{-1}$*	*Lbs. in.$^{-2}$*
6	26.7	142,300	5,340
7	29.9	179,300	5,990
8	33.1	219,100	6,620
9	36.2	261,500	7,230

SUMMER CONDITION

[E=4,500,000 lbs. in.$^{-2}$, k=280 lbs. in.$^{-3}$]

Slab thickness	l	D	K
6	23.3	152,300	6,530
7	26.2	191,900	7,330
8	28.9	234,500	8,100
9	31.6	279,800	8,850

For the summer condition only two comparisons are available, the interior case for the 7-inch and 9-inch sections. In these the values of D and K derived from the measured deflections tend to be 10 to 15 percent lower than the corresponding computed values of table 5. There is generally good agreement in the values of l for the four pavement thicknesses and the three cases of loading.

Further studies of the corner case were made in an effort to establish the reason for the differences between the computed values of the coefficients and those obtained from the measured corner deflections. Tests were made with the 7-inch and 9-inch sections, loads being applied when the corners were flat, warped up and warped down. Measurements were made not only of the deflection but also of the rotational movement of the slab about the longitudinal joint as the corner loads were applied. These rotational movements were measured by placing two recording strain gages on the slab end spanning the joint, one gage near the top and one near the bottom surface of the pavement.

The vertical displacement of the free corner resulting from rotational movement at the longitudinal joint was calculated from the movements recorded by the two strain gages. This displacement is an apparent deflection of the slab corner that must be subtracted from the measured maximum deflection at the corner if the displacement caused by slab flexure alone is to be obtained. Displacement values calculated in this manner for the 7,000-pound load on the corner of the 7-inch test section and the 10,000-pound load on the 9-inch test section are shown in table 6. It will be noted that the values are about the same for the two slab thicknesses but that they vary considerably with the degree of subgrade support afforded the loaded corner, being approximately three times as great for the upward warped corner as for the same corner brought into better contact with the subgrade by downward warping.

The measured total vertical displacements along the bisectors of the corner angle for the 7-inch and 9-inch sections, for the three conditions of temperature warping are shown in figure 30. Each value on the apparent deflection curves in this figure is the total displacement from a fixed datum. The calculated movement of the corner resulting from rotation of the slab about the longitudinal joint is indicated as a correction to be subtracted from the measured total downward movement of the corner point. Because of the rotational movement of the slab about the longitudinal joint it is apparent that the measured load-deflection relation for the corner case cannot be expected to compare directly with the theoretical relation. As nearly as could be determined in this investigation, the measured maximum corner deflection for the downward warped condition when corrected for slab rotation is in reasonably good agreement with the theoretical value. The measured maximum values for the other two conditions of warping are approximately equal after correction and both are approximately 25 percent greater than the corrected value for the downward warped condition.

TABLE 6.—*Displacements caused by rotational movement of the slabs about the longitudinal joint*

Condition of edges of slab	Corner displacement	
	7-in. slab	9-in. slab
	Inches	*Inches*
Warped down	.006	.008
Flat	.015	.016
Warped up	.021	.020

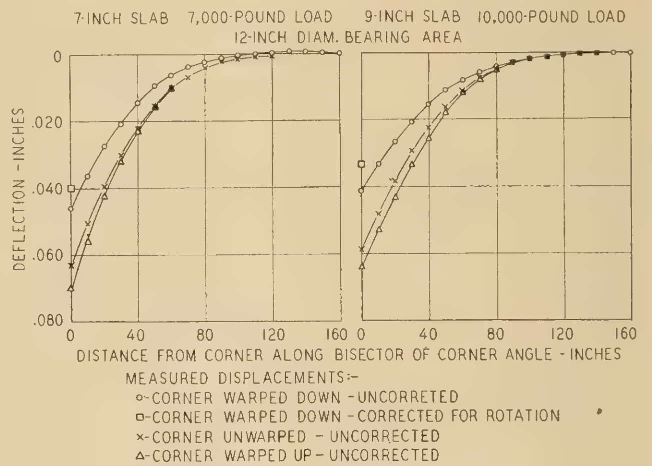


FIGURE 30.—DEFLECTION OF THE BISECTOR OF THE CORNER ANGLE FOR THREE CONDITIONS OF TEMPERATURE WARPING.

It is believed that this special study of the deflection of the corners of the 7-inch and 9-inch test sections indicates that, because the measured apparent deflections of the slab corner contain displacements from causes other than flexure, such measurements are not suitable for use in determining the value of the several coefficients previously discussed. One possible exception is the value of the radius of relative stiffness, l, which is determined more by the coincidence of curve shapes than by absolute deflection magnitude. Because of this, values of l determined from corner loadings are in good agreement with values determined from interior and edge loadings for all four slab thicknesses.

LOAD-STRESS RELATIONS ANALYZED

One of the early steps in the testing program was a study of the effect of the size of the bearing area over which the load was distributed on the amount of strain produced by a given load for test sections of various thicknesses and for the three cases of loading considered by Westergaard.

In testing the corners bearing plates having radii, a, of 3, 4, 6, 8, and 10 inches were used. In testing the the interior and edge positions, however, the 6-inch

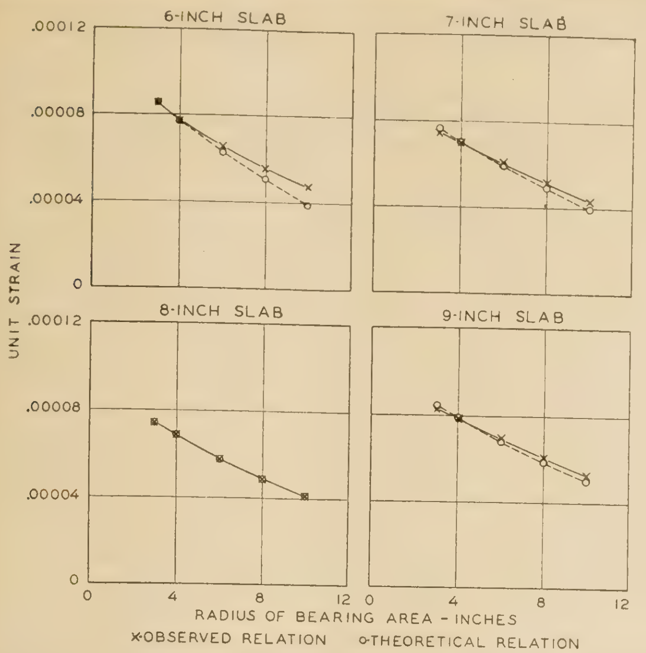


FIGURE 31.—EFFECT OF SIZE OF BEARING AREA ON STRAIN—CORNER LOADING.

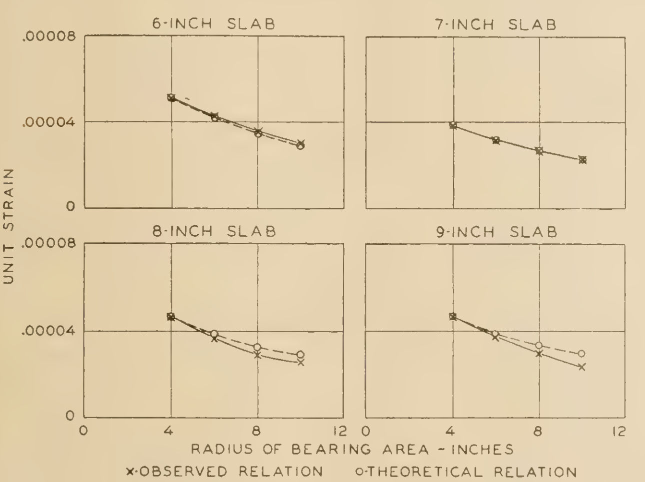


FIGURE 32.—EFFECT OF SIZE OF BEARING AREA ON STRAIN—INTERIOR LOADING.

diameter plate ($a=3$ inches) was omitted because of having to measure strains under the bearing plate with a gage length exceeding the plate diameter. The measured critical strains for each size of bearing area and each slab thickness are shown for the corner, interior and edge loadings in figures 31, 32 and 33, respectively. In order to compare the observed effect of size of bearing area with that indicated by theory, strain values were computed for each value of the radius, a, and these theoretical relations are shown by the broken line curves in the three figures. For purposes of comparison the two curves in each graph were made to coincide for the case of the bearing area with 4-inch radius. For this reason they do not indicate the relative magnitude of observed and computed strains for a given test condition.

In the case of the corner loading (fig. 31) the observed effect of size of bearing area on strain is almost exactly the same as that indicated by theory. The two curves coincide exactly in the comparison for the 8-inch slab while for the other three sections the observed influence of the size of bearing area is only slightly less than the theoretical.

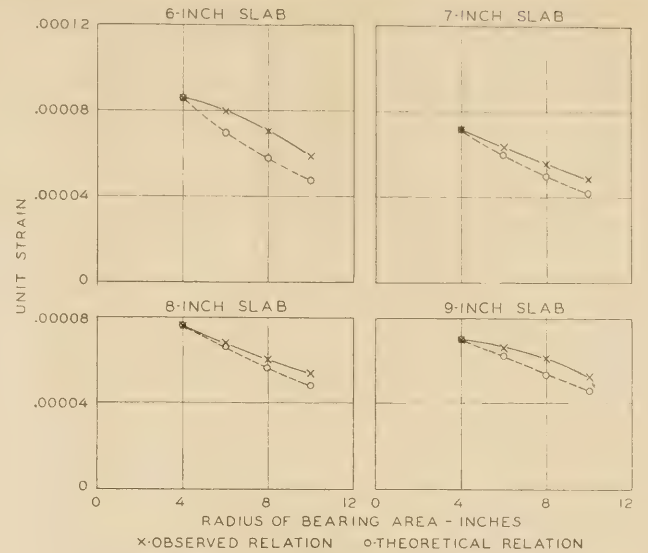


FIGURE 33.—EFFECT OF SIZE OF BEARING AREA ON STRAIN—EDGE LOADING.

For loads applied at the interior (fig. 32) the agreement between the observed and theoretical relations is very close in all cases. Such divergences as appear are so small as to be without significance in interpreting the results.

The comparison between the observed and theoretical relations for the edge loading (fig. 33) is the only one in which a significant difference exists. It will be recalled that in this edge loading (case III) a semicircular bearing area was used in the original analysis. In the test program at Arlington a semicircular bearing plate was used and the effort was made to center the load over the center of area of the plate so as to have uniform load distribution to the slab surface. On the 7-inch and 8-inch test sections the curve showing the observed relation has essentially the same shape as the theoretical curve, although lying somewhat flatter with respect to the x-axis. The curves showing the observed relation for the 6-inch and 9-inch sections on the other hand are of somewhat different shape, being concave with respect to the x-axis instead of convex. The apparent divergence between the observed and theoretical curves for the 6-inch and 9-inch sections is emphasized by the fact that it is the data for the smallest bearing area which appear to be out of line and it so happened that it was at this point that the curves were made to coincide, principally because the 8-inch diameter bearing area was used extensively in the load-strain studies. Even with the discrepancies that have been pointed out for the edge loading there is fair agreement between the experimental data and the theory.

As a general statement it is believed that the observed effect of size of bearing area on strain is so similar to the theoretical relation as to indicate that the latter can be used with confidence in its accuracy.

In subsequent tests where the size of bearing area entered as a variable only the 8-, 12- and 20-inch diameter plates were used.

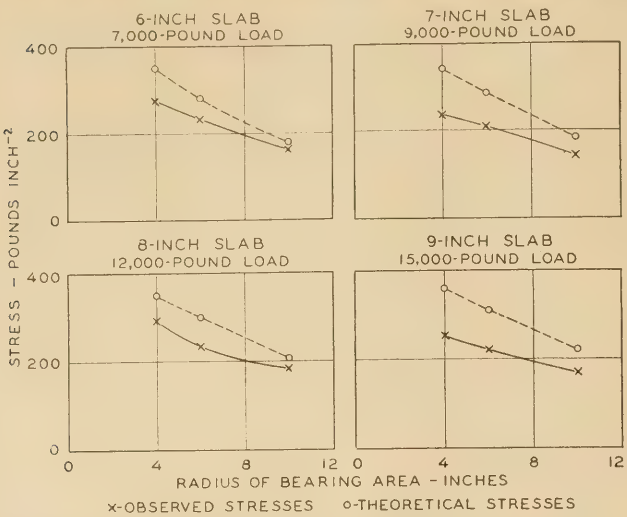


FIGURE 34.—COMPARISON OF OBSERVED AND THEORETICAL STRESSES FOR CORNER LOADING USING COEFFICIENTS DETERMINED FROM DEFLECTIONS MEASURED AT THE CORNER.

LOAD-STRESS RELATION FOR CORNER LOADING

The first direct comparison between observed and theoretical stresses is shown in figure 34. The term observed stresses used hereafter refers to the critical stress values derived from measured strains, in the manner described earlier in the report. The observed stress values in figure 34 are compared with theoretical values computed with coefficients obtained from corner deflections measured at the same time as the strain measurements. Each observed stress value in this and subsequent figures is based on an average of eight measured strains (2 tests in each quadrant of the test section).

It will be observed in figure 34 that, in every case, the observed stress value is smaller than the theoretical value. This difference is due in large part to the fact that the apparent modulus of elasticity of the concrete, as determined from corner deflection data, is much smaller than the value of the modulus of elasticity determined by other methods. This was brought out in table 4 and the attendant discussion.

When values of the modulus of elasticity obtained from the measured deflections of the interior and edge were averaged and applied to the corner strains for a given test section a better agreement between observed stresses and theoretical stresses resulted. This comparison is shown in figure 35. It is believed that this is a better comparison than that of figure 34 because all evidence indicates that the modulus of elasticity determined from deflection data obtained in tests at the interior and edge is approximately correct. In figure 35 the agreement between the observed and theoretical stress values is very close for the 6-inch and 8-inch sections while for the 7-inch and 9-inch sections the observed stresses are higher than the theoretical stresses by 14 and 18 percent, respectively. The observed influence of the size of bearing area is essentially that indicated by theory.

Some further tests were made to discover, if possible, the reason for the difference between the agreement for the 6- and 8-inch sections and that for the 7- and 9-inch sections as shown in figure 35. The first of these tests were made on the 7- and 9-inch sections during the summer under the condition of maximum downward

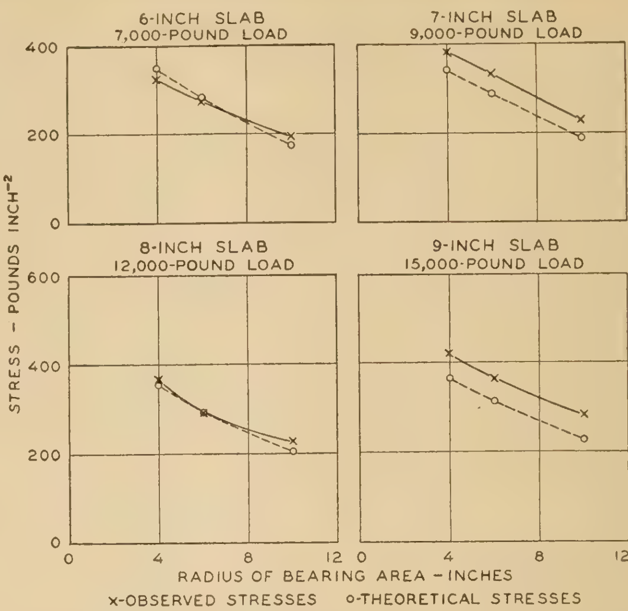


FIGURE 35.—COMPARISON OF OBSERVED AND THEORETICAL STRESSES FOR CORNER LOADING USING AVERAGE COEFFICIENTS DETERMINED FROM DEFLECTIONS MEASURED AT THE INTERIOR AND EDGE.

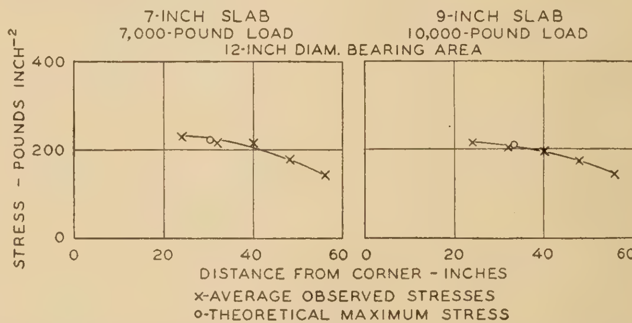


FIGURE 36.—COMPARISON OF THEORETICAL MAXIMUM STRESS FOR CORNER LOADING WITH OBSERVED STRESSES ALONG THE BISECTOR OF THE CORNER ANGLE WITH THE CORNER WARPED DOWNWARD.

warping of the slab corners. It is reasonable to assume, therefore, that the pavement corner was in good contact with the subgrade at the time of test. While only one size of bearing plate was used, tests were made on all four quadrants of the test sections and the tests were repeated in two different years. The data may, therefore, be considered representative.

The average observed stresses along the corner bisector for both the 7-inch and the 9-inch test sections, together with the theoretical maximum stress, are shown in figure 36. Each observed value is an average based on 16 strain measurements. The agreement between the observed and theoretical stress values from these tests indicates that, if the conditions are such that the corner is receiving full subgrade support, values of critical stress computed with the Westergaard equation for critical stress for corner loading (case I) can be used with confidence. When full subgrade support does not exist the computed stresses will be too low.

Further study was given to the effect of the condition of subgrade support on critical stress for corner loading in a series of tests on the 6-inch, 7-inch, and 9-inch sections. In this study loads were applied to the slab corners when those corners were warped upward, unwarped and warped downward. The slabs were as-

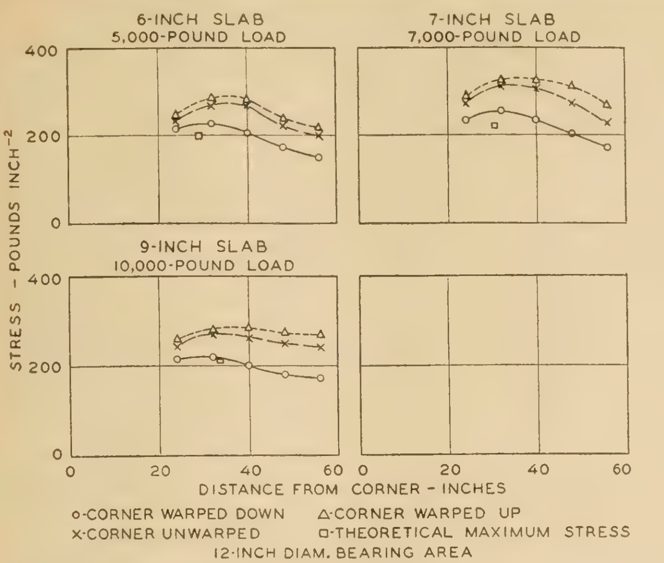


Figure 37.—Average Observed Stresses Along the Bisector of the Corner Angle for Three Conditions of Warping.

sumed to be flat when the temperature of the upper surface and of the lower surface were the same. The 8-inch section was not included because it was necessary to keep the program within practical limits. Conditions for maximum upward warping occur only at certain times in the night or early morning hours in winter and with only three sections included the program extended over nearly one entire winter before the desired data could be obtained. The data from this study are shown in figure 37. Each observed stress value is the result of eight separate strain measurements. The maximum theoretical corner stress is shown on each graph.

Table 7 shows the observed maximum stresses for each condition of warping and for each slab thickness, expressed as a percentage of the theoretical stress. It will be noted that the observed stresses are slightly greater than the theoretical stresses for the condition of downward warping. It is quite possible that even when the corners were warped downward full contact with the subgrade was not established. The observed stresses for the flat and upward warped conditions, i. e., for the conditions of least subgrade support, are higher than the theoretical values by 30 to 50 percent. Thus it is evident that where the assumed condition of full subgrade support does not obtain some correction must be applied to the stress values computed by the theoretical equation for maximum stress from corner loading. Westergaard analyzed the load-deflection and the load-stress relations for slab corners that are not fully supported by introducing a modified condition of subgrade reaction that was assumed to represent the support given a slab corner when that corner was warped upward (*26*).

This analysis provides corrections to be applied to values computed with the equations of the original analysis. In order to compute the corrections it is necessary to determine experimentally certain quantities that appear in the correction equations. A serious effort was made to determine the quantities and the corrections in connection with the investigation being described but for various reasons it was not

Table 7.—*Comparison of theoretical corner stresses with maximum observed stresses for three conditions of warping*

Slab thickness	Observed stresses expressed as percentages of theoretical values		
	Corner warped downward	No warping	Corner warped upward
Inches	*Percent*	*Percent*	*Percent*
6	114	137	144
7	116	141	149
9	105	132	138

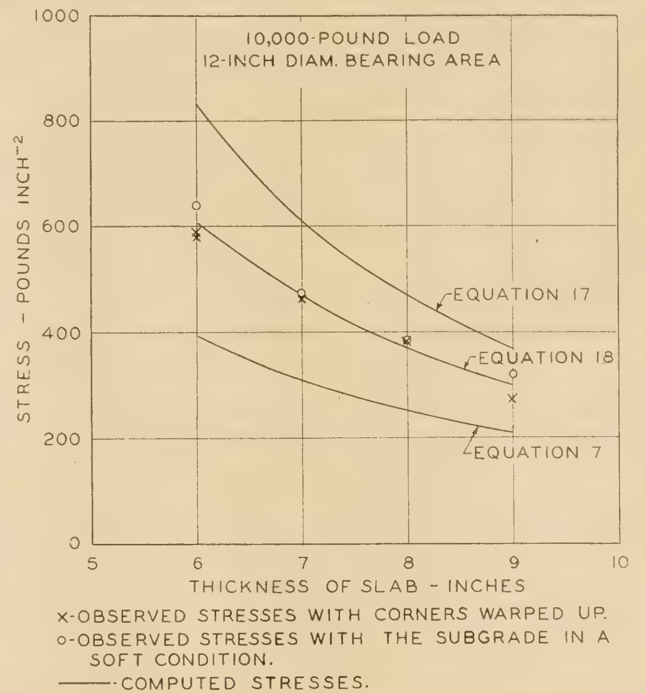


Figure 38.—Comparison of Observed Maximum Corner Stresses With Those Computed by Three Stress Equations for Four Slab Thicknesses.

possible to do so in a satisfactory manner. As a general statement it may be said that the corrections obtained in the manner suggested were too small, at least for the conditions of this investigation.

OBSERVED STRESSES COMPARED WITH VALUES COMPUTED BY VARIOUS STRESS EQUATIONS

In figures 38 and 39 comparisons are made between observed maximum corner stresses and those computed with three different equations which will be described presently. The comparison in figure 38 is for four slab thicknesses, the load and size of loaded area being constant while those for figure 39 are for different sizes of loaded area, with a constant load and slab thickness. As in most of the tests, each observed stress value is the result of eight individual strain measurements. The stress values in figure 38 were either for the condition of upward warping (crosses) or for a very soft subgrade condition (circles) hence are representative of a low corner support condition. Those in figure 39 are for the upward warped condition only. The values of E and k used were selected as representative for the conditions that obtained at the time of the tests.

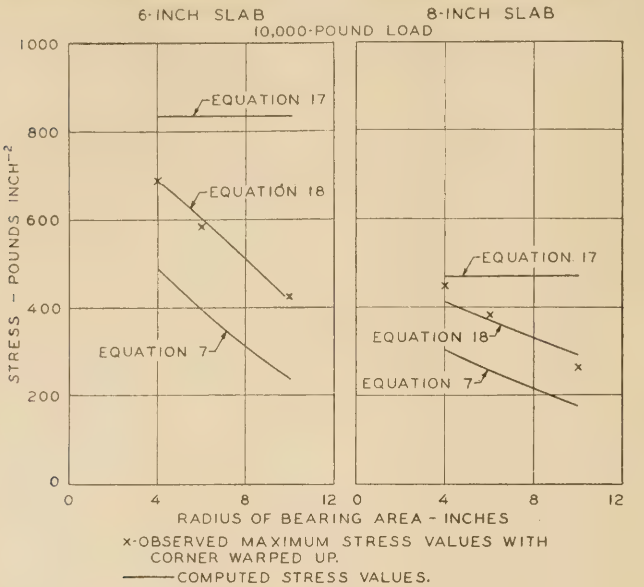


FIGURE 39.—COMPARISON OF MAXIMUM OBSERVED CORNER STRESS VALUES WITH THOSE COMPUTED BY THREE STRESS EQUATIONS TO SHOW EFFECT OF SIZE OF BEARING AREA ON STRESS.

The three equations referred to above for computing the maximum corner stress values shown in figures 38 and 39 are

$$\sigma_c=\frac{3P}{h^2} \quad \text{(17)}$$

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.15}(a\sqrt{2})^{0.6}\right] \quad \text{(7)}$$

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.3}(a\sqrt{2})^{1.2}\right] \quad \text{(18)}$$

Equation 17 was suggested by Goldbeck (*4*) in 1919 as an approximate formula for computing the stress in concrete slabs. In various forms it came to be known as the "corner formula." It assumes that the slab corner receives no support from the subgrade and that the load is applied at a point at the extreme corner of the slab. In general, stress values computed with this equation are very much higher than those observed in this investigation. In figure 38 the difference will be observed to vary with the pavement thickness, the computed stress being 38, 30, 22, and 24 percent greater respectively than the average observed maximum stress for the corners of the 6-, 7-, 8-, and 9-inch test sections. Since in equation 17 the load is assumed to be concentrated at a point it is obvious that the computed stresses would not be affected by the size of the loaded area. This is apparent in the comparisons of figure 39. For the 6-inch section the computed stresses are greater than those observed for the bearing areas with radii of 4, 6, and 10 inches by 21, 43, and 97 percent respectively. For the 8-inch section the percentages are 4, 23, and 79 respectively for the same three sizes of bearing area.

Equation 7 is the original Westergaard equation for maximum stress for a load applied on the pavement corner over a circular area of radius, a, with full subgrade support measured by the reaction modulus k. (NOTE.—It is apparent that equation 17 is a special case of equation 7 for the condition of $k=0$ with a infinitely small).

The data presented in figures 38 and 39 support those shown previously in demonstrating that the maximum stresses for corner loading observed in this investigation are considerably greater than the theoretical maxima. If the comparative data from these three figures are averaged it is found that the theoretical values are about one-third less than the observed values and as evidenced by the graphs the difference is fairly constant for various pavement thicknesses and sizes of bearing area.

Equation 18 is an empirical equation developed by modification of the exponents in the Westergaard equation in such a manner as to cause the computed values to coincide more nearly with the observed data for the case of the incompletely supported slab corners shown in figures 38 and 39. The equation thus has no theoretical foundation. It was published earlier in slightly different form (*7*) but the comparisons with observed data were not presented at that time. The data in figure 38 show that for the poorly supported corners to which the observed stress values apply, stress values computed by the empirical equation are in good agreement. The data shown in this figure were obtained in tests using a 12-inch diameter bearing plate. Comparisons with test data obtained with bearing plates of other sizes are shown in figure 39 and the agreement between the observed and computed stress values is generally good.

In addition to the comparisons just shown, it is of interest to note that in a recent report of a laboratory investigation of stress conditions in the corner region of concrete slabs at the Iowa Engineering Experiment Station (*13*) it is stated that the critical corner stresses determined experimentally in that investigation are in general agreement with values calculated by the empirical stress equation developed from the data obtained at Arlington (equation 18).

To summarize, it has been shown that for the corner loading (case I) the conditions of soil support for which the Westergaard equation for maximum stress was developed existed only for short periods of extreme downward warping and that for these periods only was there agreement between the observed stresses and those computed by the Westergaard equation. For the rest of the time the corners were not fully supported and the computed stresses were much lower than the observed stresses.

An empirical equation has been developed that seems to fit the observed data reasonably well and, for conditions of poor corner support, its use is suggested, at least until more comprehensive information is available. It is emphasized, however, that the equation is without theoretical foundation and may not be applicable to all conditions.

LOAD-STRESS RELATION FOR INTERIOR LOADING

A direct comparison between observed and computed stresses for interior loading (case II) is shown in figure 40. The values of observed stress shown in this graph are the critical values for each size of loaded area as determined from strain measurements in the manner previously described and each is the average of eight individual observations. The theoretical stress values in this figure were computed with the equation of the original Westergaard analysis, as restated in generalized

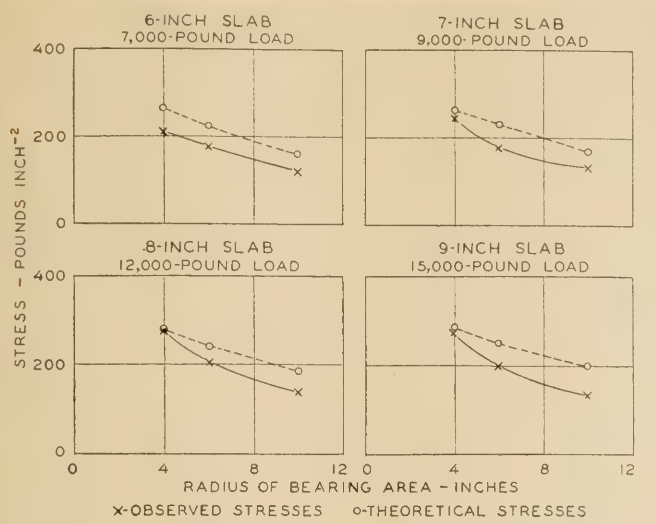


FIGURE 40.—COMPARISON OF OBSERVED AND THEORETICAL STRESSES FOR INTERIOR CASE OF LOADING $Z=0$.

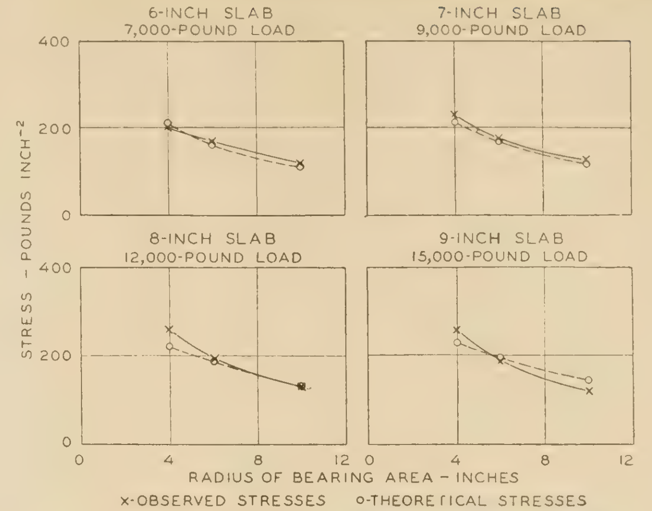


FIGURE 41.—COMPARISON OF OBSERVED AND THEORETICAL STRESSES FOR INTERIOR CASE OF LOADING $Z = 0.05$ AND $L = 1.75l$.

form (equation 8), the coefficients being determined from deflection measurements of the pavement slab made at the time the strains were measured.

In this comparison the theoretical values are appreciably higher than the observed stresses. The opinion has been expressed by Westergaard that for interior loading the subgrade reactions may be more closely concentrated around the load than are the deflections and that for a given subgrade this redistribution of subgrade reactions results in more support than was contemplated in the original analysis. Based on this conception his "supplementary theory" for interior loading (*25*) was developed. In this supplementary analysis the equation for maximum load stress was modified to the form shown as equation 10 earlier in this discussion. Two new quantities, Z and L, appeared in this equation. Values for these must be determined from experimental data. Westergaard suggests that these constants be established by adjusting their values until fair agreement exists between observed and computed deflection and stresses.

In the present investigation for the subgrade as originally constructed this procedure resulted in the following values: $Z=0.05$ and $L=1.75l$, for the four sections of uniform thickness.

The values of the various coefficients determined from deflections using a value of $Z=0.05$ are shown in table 8. These were developed from the same deflection data as were used for those of table 4 but differ slightly because of the influence of the quantity Z. The values of the modulus of elasticity E, shown in table 8, are in good agreement with those determined by other methods. The values of k, shown in table 8 for the late fall and winter conditions, are in good agreement with the values determined by the bearing tests with rigid plates. For the summer condition the agreement is not as good and a possible explanation for this was offered earlier in the report.

A direct comparison between observed maximum stresses for the interior loading and maximum stress values computed by the Westergaard equation based on a redistribution of subgrade reactions is given in figure 41. Except for the value of L, which as stated above was found to be equal to $1.75l$, the coefficients used in the computation were those given in table 8.

TABLE 8.—*Coefficients determined from the deflections for the interior case of loading $Z=0.05$*

Time of testing	Slab thickness	l	k	K	D	E
	Inches	*Inches*	*Lbs. in.$^{-3}$*	*Lbs. in.$^{-2}$*	*Lbs. in.$^{-1}$*	*Lbs. in.$^{-2}$*
Late summer	6	25	185	4, 640	116, 000	3, 930, 000
Winter	7	29	226	6, 550	190, 000	5, 460, 000
Summer	7	28	211	5, 920	165, 700	4, 430, 000
Winter	8	31	247	7, 640	237, 000	5, 220, 000
Late fall	9	36	193	6, 940	250, 000	5, 220, 000
Summer	9	33	210	6, 920	228, 500	4, 000, 000

In general, there is good agreement between the observed and the computed values for all sizes of bearing areas and for all thicknesses of pavement even though only one value of Z and one of L were used in the computation.

So far there has been little or no opportunity to study experimentally the range of variation of these constants to be expected with various concretes and subgrade conditions. Also while the range in size of loaded areas for which data are shown are probably adequate for the usual conditions of highway loading the areas of contact of some airplane tires are quite outside the range. In this connection one of the supplementary studies which is described at the end of this report was made to determine the effect of large bearing areas and of bearing areas of other than circular shape on the load-stress relation and experimental data obtained with larger areas for interior loading are presented in connection with the description of that work.

LOAD-STRESS RELATION FOR EDGE LOADING

A direct comparison between observed stresses and computed stresses for the edge loading is shown in figure 42. The observed stresses are based on measured critical strains while the computed values were obtained with the Westergaard equation for maximum stress for the edge loading, case III, given earlier in this report as equation 9. The coefficients used in the computations were those obtained from deflections made at the time of the strain measurements and are shown in table 4.

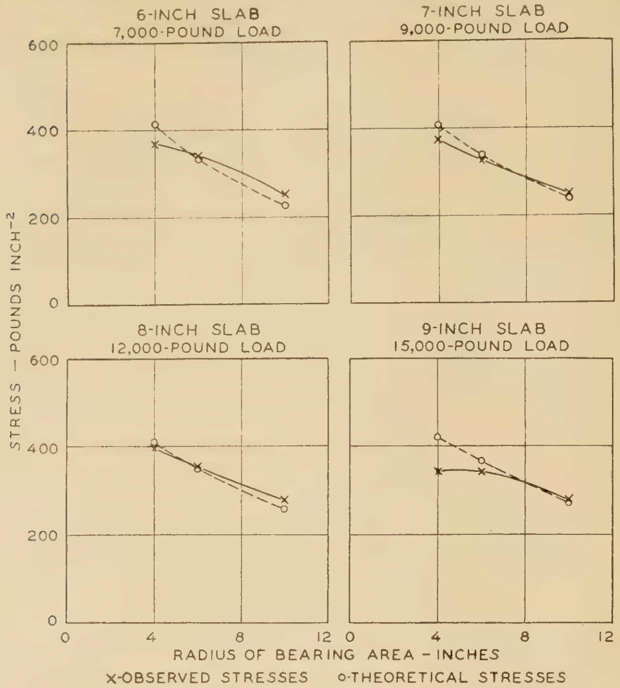


FIGURE 42.—COMPARISON OF OBSERVED AND THEORETICAL STRESSES FOR THE EDGE CASE OF LOADING.

It is apparent from these data that theoretically the effect of size of bearing area is somewhat more pronounced than that observed in these tests although for the 7-inch and 8-inch sections the difference is quite small and the general agreement between the observed and the computed values is quite good. In the case of the 6-inch and 9-inch sections, the agreement is good for the 12-inch and 20-inch diameter bearing areas but for the 8-inch diameter plate the observed stress is definitely lower for both test sections than theory indicates it should be. It will be recalled that this same anomaly was apparent in the strain data shown in figure 33. Just why the stress values for the 8-inch semicircular plate should be low on these sections is not clear. The tests made on the several quadrants of the test sections show no more than the usual spread between individual values so that the average value is not adversely affected by erratic data. Furthermore, repeat tests made 2 years later on the same sections showed essentially the same load-strain values. Even with these values included the general agreement between the observed and computed values may be considered good and it seems reasonable to conclude that the Westergaard equation for maximum stress for edge loading gives an accurate indication when the assumed condition for the loaded area exists and the slab is in an unwarped condition.

Some additional study of the case of an edge load was made to determine the effect of upward warping on the load-stress relation. Some of the tests were made during the late fall with the soil of the subgrade in a moderately wet condition while others were made when the subgrade was soft from thawing after having been frozen to a depth of several inches. The observed stress values from these tests are shown in figures 43 and 44 together with relations computed by two equations for edge stress. The first is the theoretical equation developed by Westergaard for edge loading (case III) the generalized form of which has been

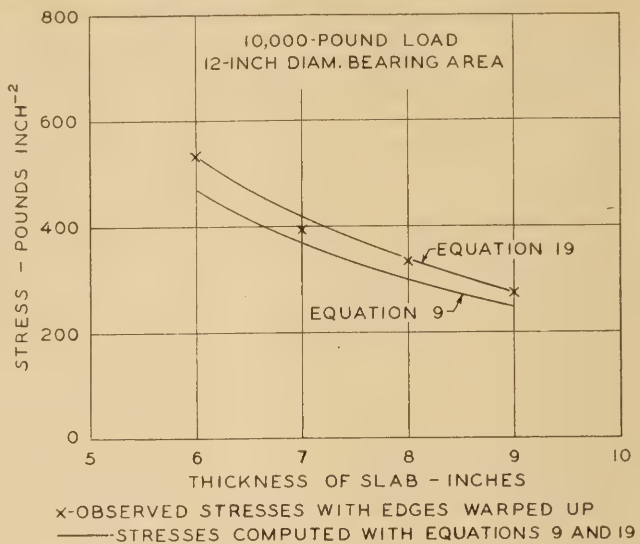


FIGURE 43.—COMPARISON OF OBSERVED MAXIMUM EDGE STRESSES AND EDGE STRESSES COMPUTED BY THEORETICAL EQUATION 9 AND EMPIRICAL EQUATION 19 FOR FOUR SLAB THICKNESSES.

given as equation 9. The second equation is as follows

$$\sigma_e = 0.529(1+0.54\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right) + \log_{10}\left(\frac{b}{1-\mu^2}\right) - 1.0792\right] \qquad (19)$$

and is an empirical equation developed to fit the observed stress values obtained in load tests on the upward warped slab edges for conditions of subgrade support such as prevailed when these particular tests were made. The same empirical equation, in slightly different form, has been published previously (7) but the data from which it was developed were not presented at that time.

Figure 43 shows the effect of slab thickness on the maximum stress from edge loading while in figure 44 the effect on stress magnitude of the size of the loaded area is brought out. As in the graphs shown previously each observed stress value is an average derived from eight individual strain measurements, two on each of the four quadrants of the test section. From the data in this figure it is apparent that the observed stresses, for the conditions of the test, exceed the values computed by the Westergaard equation for maximum edge stress by about 10 percent. In making this comparison it is well to keep in mind that upward warping of slab edges develops only when the bottom surface of the slab is warmer than the upper surface. Appreciable upward warping occurs relatively infrequently in the region where these tests were made. The tests for which the data are shown in figure 43 were made before sunrise on winter mornings when the cycle of temperature changes happened to be favorable. At the time of year during which the observed data were obtained the supporting value of the subgrade is apt to be lessened by a high moisture content. In view of the critical conditions represented it is rather surprising that the observed stress values do not exceed those computed by equation 9 by a greater percentage.

The observed stress values are in good agreement with the values computed by the empirical equation for maximum edge stress, equation 19, for all four pavement thicknesses represented in the investigation.

In figure 44 the observed data indicate that, when the slab edge is warped up the effect of the size of loaded area, as measured by the radius, a, is slightly less pronounced than theory indicates. The values are higher than those computed by the theoretical equation (equation 9) also, as would be expected in the light of the preceding discussion. Stress values computed by the empirical equation (equation 19) agree closely with the observed stresses for the 12-inch and 20-inch diameter bearing areas but are somewhat higher than the stresses observed in the tests with the 8-inch diameter area, the average difference being of the order of 10 percent.

The empirical equation was derived by adjusting the theoretical relation expressed by the Westergaard equation for maximum edge stress until the empirical relation fitted the observed data as closely as possible. The empirical equation is, therefore, specific rather than general in its application. However, it yields stress values that are somewhat higher, for given conditions, than those computed with the theoretical equation, a difference that would be expected to exist generally when the edge of the slab is warped upward. Therefore, if this extreme condition is being considered, the use of the empirical equation is suggested, at least until such time as more comprehensive data are available.

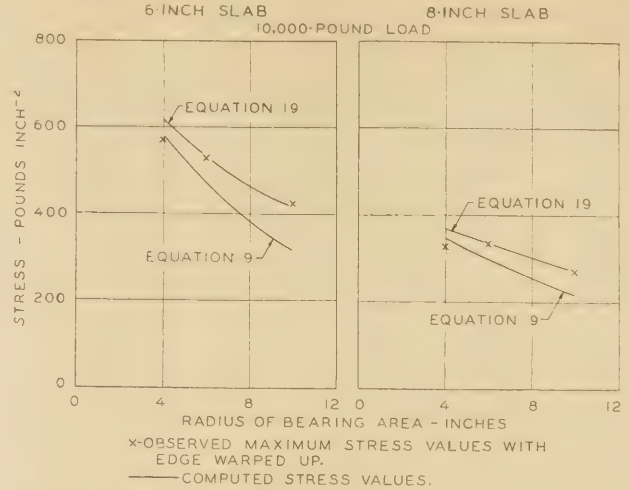


FIGURE 44.—COMPARISON OF OBSERVED MAXIMUM EDGE STRESSES AND EDGE STRESSES COMPUTED BY THEORETICAL EQUATION 9 AND EMPIRICAL EQUATION 19 FOR THREE SIZES OF BEARING AREA.

RESULTS OF INVESTIGATION REVIEWED

Before presenting the discussion of certain supplementary studies of pavement stresses that were made as a part of this investigation it is desirable to review briefly and to discuss the general aspects of the material which has been presented up to this point.

The Westergaard analyses of the load-deflection and load-stress relations in concrete pavement slabs of uniform thickness, resting on an elastic subgrade, have been studied experimentally. Comparisons have been made between computed and observed deflections and between computed and observed stresses on full-size pavement sections of 6-, 7-, 8- and 9-inch uniform thicknesses. Studies have been made of methods for determining the various coefficients and other quantities that appear in the Westergaard equations.

As a result of this study the general conclusion is drawn that the Westergaard analysis expresses quite accurately the relations between load and deflection and between load and critical stress for various thicknesses of pavement and for various sizes of bearing area provided the slab is in full contact with the subgrade. The subgrade stiffness coefficient and other quantities must of course be determined for the particular conditions that exist.

It has been found possible to determine usable values of the modulus of subgrade reaction, k, by means of load-displacement tests with rigid bearing plates. In making such tests it is necessary to use plates of adequate size and to limit the displacement to a value that approximates the pavement deflection under load.

The experimental determination of the other quantities that appear in the equations has been found to be feasible although the facility with which the determinations may be made and the precision of the values obtained is not the same for all quantities. An uncertain factor is the effect of moisture variation in the concrete of the pavement. Moisture variation affects the stiffness of the concrete and if a moisture differential exists it may affect the state of stress through restrained warping. The influence of variation in concrete stiffness is relatively unimportant so far as the use of the theoretical equations is concerned but has a direct effect when measured strains are being converted into stresses. No satisfactory means has yet been found for indicating the moisture conditions that exist in a concrete pavement slab in place on the subgrade.

Unless the values for the quantities that appear in the stress equations have been determined with great care too great reliance should not be placed on the absolute value of computed deflections or stresses.

For conditions where the slab corners or edges are not in full contact with the subgrade because of temperature warping or other cause, empirical equations have been developed that fit the data obtained in tests reported herein. These equations are not offered as a replacement for the theoretical equations of Westergaard but it is thought they may prove useful in certain studies where incomplete subgrade support exists or is assumed.

USE OF THE EQUATIONS

In the second report of this series (*19*) it was shown that the stresses caused by restrained temperature warping can be large and that the magnitude of these stresses depends not only on the temperature differential between the upper and lower surfaces of the pavement but also on the length and thickness of the pavement unit or slab. The greatest temperature differentials develop during the afternoon of clear days in late spring and early summer, the temperature of the upper surface being as much as 20° to 30° Fahrenheit or more above that of the lower surface at such times (for the vicinity of Washington, D. C.). This condition causes the edges of the slab to tend to warp downward and, as this tendency is restrained, a tensile stress is developed in the bottom of the slab. When the upper surface of the pavement is at a temperature lower than that of the bottom the tendency for an upward warping of the slab edges is created. This condition may develop during the night and early morning hours from time to time throughout the year. The magnitude of the maximum temperature differential developed at night usually does not exceed about one-third of that which may develop during the day. The restraint to the upward warping of the slab edges caused by the weight of the slab or by the joint construction produces tensile stress in the upper surface of the slab.

These stresses which develop as a result of restraint to warping combine with the maximum stresses developed by wheel loads. An adequate slab design, therefore, is one which will satisfactorily withstand the most critical combination of load and warping stresses. It has been shown *(19)* that the most effective means for controlling the magnitude of the temperature warping stresses in concrete pavements is by limiting the length and width of the panels or slab units and maximum values of the order of 10 to 15 feet were suggested. Even with slab units limited to dimensions of 10 or 15 feet, however, the stresses from restrained temperature warping are not eliminated. Their magnitude is simply controlled to a reasonable maximum.

This brief discussion of temperature warping has been included because it is believed that it is a subject which must be considered in most computations of pavement slab stresses. For a more thorough discussion the reader is referred to published reports *(24, 19, 7)*.

For computing the maximum stress for the corner loading Westergaard developed the generalized theoretical equation given earlier in the report and for convenience repeated here.

Theoretical Equations (Case I)

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.15}(a\sqrt{2})^{0.6}\right] \quad (7)$$

This can be restated in terms of l as follows:

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right]$$

These equations give the most accurate indication of maximum load stress when the pavement corner is in full contact with the subgrade. In this investigation the condition was attained only when the corner was warped downward. If these equations are used for computing load stress the condition of corner warping due to temperature would be such as to create a moderate compressive stress in the upper surface of the slab in the region where the load would create the maximum tensile stress. Thus the combined stress would tend to be slightly lower than the load stress alone.

When the slab corner is not in complete bearing on the subgrade, due to upward warping, the theoretical equation will give load-stress values somewhat lower than those that will be developed. For this condition the Arlington experiments indicate the empirical equation, repeated below, will give computed values that are more nearly in accord with those observed.

Empirical Equations (Case I)

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.3}(a\sqrt{2})^{1.2}\right] \quad (18)$$

or expressed in terms of l

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{a\sqrt{2}}{l}\right)^{1.2}\right]$$

As a general rule the most critical condition for the corner loading is at night when the corner tends to warp upward. The subgrade support is least effective at that time and any warping stress present in the corner is additive to the load stress.

The case of interior loading is covered by the original Westergaard stress equation in generalized form, given as equation 8 earlier in the paper, or by the equation which was developed for the modified condition of subgrade support in the supplementary paper *(25)* and given in this report as equation 10.

Theoretical Equations (Case II)

$$\sigma_i''=0.275(1+\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)-54.54\left(\frac{l}{L}\right)^2 Z\right] \quad (10)$$

If $\mu=0.15$ and $Z=0$ this equation can be simply expressed in terms of l, as follows:

$$\sigma_i=0.316\frac{P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right)+1.069\right]$$

The load stresses at the interior of a slab are not appreciably affected by the condition of warping of the slab and the same equation may be used for either upward or downward warping. Temperature warping stresses are highest in the interior region of the slab, however, and the combined stress value may vary widely from night to day.

There is little information at the present time to indicate what values of Z and L should be used in the general equation for various conditions of pavement and subgrade stiffness. It will be recalled that the data presented in figure 41 showed good agreement between observed and computed stress values when $Z=0.05$ and $L=1.75l$. These values were determined from the deflections and stresses observed on all of the four different thicknesses of pavement and there was no apparent difference in the value of either coefficient for the various thicknesses of slab. As stated previously, however, there is no information to indicate what values of the coefficients should be applied to other concretes and other subgrades. When dealing with conditions that are not known the values assigned to the coefficients Z and L should be such as to result in conservative load stresses. For such use the values $Z=0.2$ and $L=5l$ have been suggested *(7)* pending the development of more comprehensive data.

The stresses resulting from edge loading, like those from corner loading are affected by the degree of warping present at the time the load is applied. The theoretical equation for edge stress as given in generalized form by Westergaard appeared as equation 9 earlier in the report, as follows:

Theoretical Equations (Case III)

$$\sigma_e=0.529(1+0.54\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)-0.71\right] \quad (9)$$

If $\mu=0.15$ it can be expressed simply in terms of l:

$$\sigma_e=0.572\frac{P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right)+0.360\right]$$

In the Arlington tests it was found that under critical conditions of subgrade support and upward warping of the slab edges, the values computed by this equation were somewhat low and an empirical equation was developed which fitted the observed stress values more closely.

Empirical Equations (Case III)

$$\sigma_e=0.529(1+0.54\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)+\log_{10}\left(\frac{b}{1-\mu^2}\right)-1.079\right] \quad (19)$$

This equation also can be expressed in terms of l, when $\mu=0.15$, as follows:

$$\sigma_e=0.572\frac{P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right)+\log_{10}b\right]$$

The critical stress from edge loading is a tension in the bottom edge of the pavement directly under the loaded area in a direction parallel to the pavement edge. Temperature warping stress in this direction can at times be high in the region of the slab edge and rather wide variations in combined stress from night to day may occur. The most critical condition for edge loading is that which occurs when the stress created during downward warping of the pavement edge (during the day) is combined with load stress. For this case the load stress would be computed with the theoretical equation (equation 9).

TABLE 9.—*Stresses computed by the Westergaard equation for the case of a corner loading and full subgrade support*

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.15}(a\sqrt{2})^{0.6}\right] \quad (7)$$

[P=10,000 pounds. μ=0.15]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress																			
		E=3,000,000 a in inches —					E=4,000,000 a in inches —					E=5,000,000 a in inches —					E=6,000,000 a in inches —				
		2	4	6	8	10	2	4	6	8	10	2	4	6	8	10	2	4	6	8	10
Inches	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$
6	50	641	541	461	390	327	649	553	476	409	348	655	563	488	423	364	660	570	497	434	377
	100	619	509	420	342	271	628	523	437	362	295	635	533	450	378	313	641	540	461	390	327
	200	596	474	375	288	210	606	489	394	311	236	613	500	408	328	257	619	509	420	342	271
	300	581	451	346	254	171	592	467	366	278	199	600	479	382	296	219	606	489	394	311	236
7	50	480	412	357	309	265	486	420	367	321	280	490	427	376	331	291	493	432	382	339	299
	100	466	390	329	275	227	472	399	341	289	243	476	406	350	300	255	479	412	357	309	265
	200	450	366	298	238	185	456	376	311	254	203	462	384	321	266	216	465	390	329	275	228
	300	439	350	278	215	158	447	361	292	232	177	452	369	303	244	191	453	376	311	254	204
8	50	373	325	285	250	218	377	330	292	259	229	380	335	298	266	237	383	339	303	271	243
	100	363	309	265	226	191	367	315	273	236	203	371	320	279	244	211	373	324	285	250	218
	200	352	291	242	199	161	356	298	251	211	174	360	304	259	219	183	363	309	264	226	191
	300	344	280	228	182	141	349	288	238	194	155	353	294	245	203	165	356	298	252	211	174
9	50	299	262	233	206	183	302	267	238	213	191	304	270	242	218	197	305	273	246	223	201
	100	291	250	217	188	162	294	255	224	196	171	297	259	228	202	178	298	262	232	206	183
	200	282	237	201	168	139	286	243	208	177	149	289	247	213	183	156	290	250	217	188	162
	300	277	229	190	156	125	281	235	197	165	135	284	239	203	172	143	285	243	208	177	149

TABLE 10.—*Stresses computed by a modified equation for the case of corner loading and deficient subgrade support*

[NOTE: This is a purely empirical modification of the Westergaard corner equation, developed on the basis of experimental stress determinations, for the case of a slab corner which does not receive full subgrade support.]

$$\sigma_c=\frac{3P}{h^2}\left[1-\left(\frac{12(1-\mu^2)k}{Eh^3}\right)^{0.3}(a\sqrt{2})^{1.2}\right] \quad (18)$$

[P=10,000 pounds. μ=0.15]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress																			
		E=3,000,000 a in inches—					E=4,000,000 a in inches—					E=5,000,000 a in inches—					E=6,000,000 a in inches—				
		2	4	6	8	10	2	4	6	8	10	2	4	6	8	10	2	4	6	8	10
Inches	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$
6	50	789	731	667	598	525	792	739	680	617	551	795	745	690	631	569	797	750	698	642	583
	100	778	707	628	543	454	783	718	645	567	486	786	725	657	585	508	789	731	667	598	525
	200	766	678	581	476	367	771	691	601	506	405	775	700	616	527	433	778	707	628	543	454
	300	757	658	548	430	306	763	672	571	463	350	768	683	588	487	381	771	691	601	506	405
7	50	584	547	506	462	415	586	552	514	474	432	588	556	521	483	443	589	559	526	490	452
	100	577	532	481	427	370	580	538	492	442	390	582	543	500	453	404	584	547	506	462	415
	200	569	513	451	384	314	573	521	464	403	339	575	527	474	416	356	577	532	481	427	370
	300	563	500	430	354	275	567	509	445	376	303	570	516	456	391	323	573	521	464	403	338
8	50	449	424	396	367	335	451	428	402	375	346	452	431	407	381	354	453	433	410	386	360
	100	445	414	380	343	304	447	419	387	353	318	448	422	392	361	328	449	424	396	367	335
	200	439	401	359	314	266	442	407	368	327	283	444	411	375	336	295	445	414	380	343	304
	300	436	393	345	294	240	438	399	355	308	259	440	403	362	319	272	442	407	368	327	283
9	50	357	339	319	298	275	358	341	323	304	283	359	343	326	308	289	359	345	329	311	293
	100	353	331	307	281	253	355	335	312	288	263	356	337	316	294	270	357	339	319	298	275
	200	350	322	292	260	226	351	326	299	269	238	353	329	303	276	247	353	331	307	281	253
	300	347	316	282	246	208	349	321	290	256	221	350	324	295	264	231	351	326	299	269	238

TABLES OF COMPUTED STRESSES PREPARED TO SHOW EFFECT OF VARIABLES IN EQUATIONS

In order to show the manner in which the various quantities that appear in the equations for maximum stress affect the computed values tables 9 to 16 have been prepared. Tables 9 and 10 apply to corner loading, case I, and show stress values for full and partial subgrade support respectively. Tables 11 and 12 show stress values for the interior loading, case II, for two assumptions regarding the distribution of the subgrade reactions within the deflected area. The stress values of table 11 are based on the original Westergaard assumptions (Z being 0) while those of table 12 were computed for values of Z and L determined experimentally in this investigation. Tables 13 and 14 give computed stress values for edge loading, case III, for a fully supported edge and for one that has warped upward and has only partial subgrade support. The equations used for computing the stress values in tables 10 and 14 are empirical, developed to fit the experimental data of this investigation. The stress values of table 12 although computed with a theoretical equation are based on constants determined experimentally for the conditions that existed at Arlington. The stress values of these three tables (10, 12, and 14) may be limited in their application, therefore.

Tables 15 and 16, computed by equations 8 and 9 respectively, show the effect of varying the value of several of the coefficients on the computed stress magnitude for interior loading, case II, and edge loading, case III.

TABLE 11.—*Stresses computed by the Westergaard equation for the case of interior loading for the case $Z=0$*

$$\sigma_i=0.275(1+\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)\right] \qquad (8)$$

[P=10,000 pounds. μ=0.15]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress																			
		E=3,000,000 a in inches —					E=4,000,000 a in inches —					E=5,000,000 a in inches —					E=6,000,000 a in inches —				
		2	4	6	8	10	2	4	6	8	10	2	4	6	8	10	2	4	6	8	10
Inches	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$
6	50	487	421	361	313	274	498	432	372	324	285	507	441	381	333	294	514	448	388	340	301
	100	461	395	335	287	248	472	406	346	298	259	480	414	354	306	267	487	421	361	313	274
	200	435	368	308	260	221	446	379	319	271	232	454	388	328	280	241	461	395	335	287	248
	300	419	353	293	245	206	430	364	304	256	217	439	372	312	264	225	446	379	319	271	232
7	50	360	319	279	245	217	368	327	287	253	225	374	333	293	259	231	379	339	298	264	236
	100	341	300	260	226	197	349	308	268	234	206	355	314	274	240	212	360	319	279	245	217
	200	321	280	240	206	178	329	288	248	214	186	335	295	254	220	192	341	300	260	226	198
	300	310	269	229	195	167	318	277	237	203	175	324	283	243	209	181	329	288	248	214	186
8	50	276	250	222	197	176	282	256	228	203	182	287	261	233	208	187	291	265	237	212	191
	100	261	235	207	182	161	267	241	213	188	168	272	246	218	193	172	276	250	222	197	176
	200	246	220	192	167	146	252	226	198	174	153	257	231	203	178	157	261	235	207	182	161
	300	238	211	183	159	138	244	217	189	165	144	249	222	194	170	149	252	226	198	174	153
9	50	218	200	180	162	146	223	205	185	167	151	227	209	189	171	155	230	212	192	174	158
	100	206	188	169	150	134	211	193	173	155	139	215	197	177	159	143	218	200	180	162	146
	200	194	177	157	138	123	199	182	162	143	127	203	185	166	147	131	206	188	169	150	134
	300	188	170	150	132	116	192	175	155	137	121	196	179	159	140	124	199	182	162	143	127

TABLE 12.—*Stresses computed by the Westergaard equation for the case of interior loading*

$$\sigma_i''=0.275(1+\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)-54.54\left(\frac{l}{L}\right)^2Z\right] \qquad (10)$$

[P=10,000 pounds. μ=0.15. Z=0.05. L=1.75l]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress																			
		E=3,000,000 a in inches—					E=4,000,000 a in inches—					E=5,000,000 a in inches—					E=6,000,000 a in inches—				
		2	4	6	8	10	2	4	6	8	10	2	4	6	8	10	2	4	6	8	10
Inches	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$
6	50	409	343	283	235	196	420	354	294	246	207	429	362	303	254	216	436	369	310	261	222
	100	383	317	257	209	170	394	327	268	219	181	402	336	276	228	189	409	343	283	235	196
	200	357	290	230	182	143	367	301	241	193	154	376	310	250	201	163	383	317	257	208	170
	300	341	275	215	166	128	352	286	226	177	139	360	294	234	186	147	367	301	241	193	154
7	50	303	262	222	188	160	311	270	229	196	168	317	276	236	202	174	322	281	241	207	179
	100	284	243	203	169	140	291	250	210	176	148	297	257	216	182	154	303	262	221	188	159
	200	264	223	183	149	121	272	231	191	157	129	278	237	197	163	135	283	242	202	168	140
	300	252	211	171	137	109	260	220	179	145	117	267	226	186	152	124	272	231	191	157	129
8	50	232	206	178	153	132	238	212	184	159	138	243	217	189	164	143	247	221	193	168	147
	100	217	191	163	138	117	223	197	169	144	124	228	202	174	149	128	232	206	178	153	132
	200	202	176	148	123	102	208	182	154	130	109	213	187	159	134	113	217	191	163	138	117
	300	194	167	139	115	94	200	173	145	121	100	205	178	150	126	105	208	182	154	130	109
9	50	183	165	145	127	111	188	170	150	132	116	192	174	154	136	120	195	177	157	139	123
	100	171	153	134	115	99	176	159	139	120	104	180	162	142	124	108	183	165	146	127	111
	200	159	142	122	103	88	165	147	127	109	93	168	151	131	112	96	171	154	134	115	99
	300	153	135	115	97	81	158	140	120	102	86	161	144	124	105	90	165	147	127	109	93

TABLE 13.—*Stresses computed by the Westergaard equation for the case of edge loading and full subgrade support*

$$\sigma_e=0.529(1+0.54\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)-0.71\right] \quad (9)$$

[P=10,000 pounds. μ=0.15]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress																			
		E=3,000,000 a in inches—					E=4,000,000 a in inches—					E=5,000,000 a in inches—					E=6,000,000 a in inches—				
		2	4	6	8	10	2	4	6	8	10	2	4	6	8	10	2	4	6	8	10
Inches	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$
6	50	769	649	541	453	383	789	669	561	473	403	804	684	576	489	419	817	697	589	501	431
	100	721	601	493	406	335	741	621	513	426	355	756	637	528	441	371	769	649	541	454	383
	200	673	553	445	358	287	693	573	465	378	307	708	589	480	393	323	721	601	493	406	336
	300	645	525	417	330	260	665	545	437	350	279	680	561	452	365	295	693	573	465	378	307
7	50	568	494	422	360	309	583	509	436	375	324	594	520	448	386	335	603	530	457	395	345
	100	533	459	386	325	274	548	474	401	340	289	559	485	412	351	300	568	494	422	360	310
	200	498	424	351	290	239	513	439	366	305	254	524	450	377	316	265	533	459	386	325	274
	300	477	404	331	269	219	492	418	345	284	233	503	429	357	295	245	513	439	366	305	254
8	50	436	388	337	293	255	447	399	349	304	266	456	408	357	313	275	463	415	365	320	282
	100	409	361	311	266	228	420	372	322	277	239	429	381	331	286	248	436	388	338	293	255
	200	382	334	284	239	201	393	345	295	250	213	402	354	304	259	221	409	361	311	266	228
	300	366	319	268	223	186	377	330	279	235	197	386	338	288	243	205	393	345	295	250	213
9	50	344	312	276	243	214	353	321	285	252	223	360	328	292	259	230	365	333	297	264	235
	100	323	291	255	222	193	332	300	264	230	202	338	306	270	237	208	344	312	276	243	214
	200	301	269	233	200	171	310	278	242	209	180	317	285	249	216	187	323	291	255	222	193
	300	289	257	221	188	159	298	266	230	197	168	305	273	237	204	175	310	278	242	209	180

TABLE 14.—*Stresses computed by a modified equation for the case of edge loading and deficient subgrade support*

[NOTE: This is a purely empirical modi fication of the Westergaard edge equation, developed on the basis of experimental stress determinations, for the case of a slab edge which does not receive full subgrade support.]

$$\sigma_e=0.529(1+0.54\mu)\frac{P}{h^2}\left[\log_{10}\left(\frac{Eh^3}{kb^4}\right)+\log_{10}\left(\frac{b}{1-\mu^2}\right)-1.0792\right] \quad (19)$$

[P=10,000 pounds. μ=0.15]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress																			
		E=3,000,000 a in inches—					E=4,000,000 a in inches—					E=5,000,000 a in inches—					E=6,000,000 a in inches—				
		2	4	6	8	10	2	4	6	8	10	2	4	6	8	10	2	4	6	8	10
Inches	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$
6	50	774	684	603	537	485	793	704	622	557	504	809	719	638	572	520	821	732	650	585	532
	100	726	636	555	489	437	746	656	575	509	457	761	671	590	525	472	774	684	603	537	485
	200	678	588	507	442	389	698	608	527	461	409	713	623	542	477	424	726	636	555	489	437
	300	650	560	479	414	361	670	580	499	433	381	685	595	514	449	396	698	608	527	461	409
7	50	577	521	467	421	383	591	536	481	435	397	603	547	493	447	409	612	556	502	456	418
	100	542	486	432	386	348	556	501	446	400	362	567	512	458	412	374	577	521	467	421	383
	200	506	451	397	351	313	521	466	411	365	327	532	477	422	376	338	542	486	432	386	348
	300	486	431	376	330	292	500	445	391	345	307	512	456	402	356	318	521	466	411	365	327
8	50	446	410	372	339	311	457	421	384	350	322	466	430	392	359	330	473	437	399	366	338
	100	419	383	346	312	284	430	395	357	323	295	439	403	365	332	304	446	410	372	339	311
	200	392	356	319	285	257	403	368	330	296	268	412	376	338	305	277	419	383	346	312	284
	300	376	341	303	269	241	388	352	314	281	252	396	361	323	289	261	403	368	330	296	268
9	50	355	331	304	279	257	364	340	313	288	266	371	347	320	295	273	376	352	325	300	279
	100	334	310	283	258	236	342	318	292	267	245	349	325	298	273	252	355	331	304	279	257
	200	312	288	261	237	215	321	297	270	245	224	328	304	277	252	231	334	310	283	258	236
	300	300	276	249	224	202	309	285	258	233	211	316	292	265	240	218	321	297	270	245	224

As will be noted the computed stress values in table 11 are for a value of $Z=0$ (and $L=0$). The values may be corrected for other values of Z and L, rather simply, by adding a stress correction, σ_i', computed with the following equation:

$$\sigma_i'=-\frac{15(1+\mu)ZP}{h^2}\left(\frac{l}{L}\right)^2$$

The stress values in tables 9 to 14, inclusive, were computed for a value of $\mu=0.15$. The maximum stress for corner loading is affected very little by changes in μ within the normal range of variation of this ratio. The effect of changes in the value of μ for the interior and edge cases of loading is shown in tables 15 and 16 respectively. The effect is not large and if it is desired to obtain computed stress values for intermediate values of the ratio, direct interpolation is sufficiently accurate for all practical purposes.

TABLE 15.—*Stresses computed for the case of interior loading to show the effects of variations in the values of h, k, a, E and μ*

[P=10,000 pounds. Z=0]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress: E=3,000,000, a in inches— 2	6	10	E=5,000,000, a in inches— 2	6	10	E=6,000,000, a in inches— 2	6	10
Inches	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$
μ=0.05										
6	100	421	306	226	439	324	244	445	330	250
	200	397	282	202	415	299	220	421	306	226
	300	383	268	188	400	285	206	407	292	212
8	100	238	189	147	248	199	157	252	202	161
	200	225	175	134	235	185	144	238	189	147
	300	217	167	126	227	177	136	230	181	139
9	100	188	154	123	196	162	131	199	165	133
	200	178	143	112	185	151	120	188	154	123
	300	171	137	106	179	145	114	182	148	116
μ=0.15										
6	100	461	335	248	480	354	267	487	361	274
	200	435	308	221	454	328	241	461	335	248
	300	419	293	206	439	312	225	446	319	232
8	100	261	207	161	272	218	172	276	222	176
	200	246	192	146	257	203	157	261	207	161
	300	238	183	138	249	194	149	252	198	153
9	100	206	169	134	215	177	143	218	180	146
	200	194	157	123	203	166	131	206	169	134
	300	188	150	116	196	159	124	199	162	127
μ=0.25										
6	100	501	364	269	522	385	291	530	393	298
	200	472	335	241	493	356	262	501	364	269
	300	455	318	224	477	340	245	484	347	253
8	100	284	225	175	296	237	187	300	241	191
	200	268	209	159	280	221	171	284	225	175
	300	258	199	150	270	211	162	274	215	166
9	100	224	183	146	234	193	155	237	196	159
	200	211	170	133	221	180	143	224	183	146
	300	204	163	126	213	172	135	217	176	138

TABLE 16.—*Stresses computed for the case of edge loading to show the effects of variations in the values of h, k, a, E and μ*

[P=10,000 pounds]

Thickness of slab, h	Modulus of subgrade reaction, k	Maximum load stress: E=3,000,000, a in inches— 2	6	10	E=5,000,000, a in inches— 2	6	10	E=6,000,000, a in inches— 2	6	10
Inches	*Lbs. in.*$^{-3}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$	*Lbs. in.*$^{-2}$
μ=0.05										
6	100	685	468	319	719	502	352	730	514	364
	200	640	423	273	673	456	307	685	468	319
	300	613	396	247	646	430	280	658	442	292
8	100	389	295	217	407	314	236	414	321	242
	200	363	270	191	382	288	210	389	295	217
	300	348	255	176	367	274	195	374	280	202
9	100	307	242	183	322	257	198	327	262	203
	200	286	222	163	301	237	178	307	242	183
	300	275	210	151	290	225	166	295	230	171
μ=0.15										
6	100	721	493	335	756	528	371	769	541	383
	200	673	445	287	708	480	323	721	493	336
	300	645	417	260	680	452	295	693	465	307
8	100	409	311	228	429	331	248	436	338	255
	200	382	284	201	402	304	221	409	311	228
	300	366	268	186	386	288	205	393	295	213
9	100	323	255	193	338	270	208	344	276	214
	200	301	233	171	317	249	187	323	255	193
	300	289	221	159	305	237	175	310	242	180
μ=0.25										
6	100	757	518	352	794	555	389	807	568	402
	200	707	467	302	744	504	339	757	518	352
	300	677	438	273	714	475	310	728	488	323
8	100	429	326	240	450	347	261	458	354	268
	200	401	298	211	422	319	232	429	326	240
	300	385	281	195	405	302	216	413	310	223
9	100	339	268	202	355	284	219	361	290	225
	200	317	245	180	333	262	196	339	268	202
	300	304	232	167	320	249	183	326	254	189

SUPPLEMENTARY STUDIES

Mention was made early in this report of certain supplementary studies that were not a part of the originally scheduled program but which, because of circumstances, it seemed desirable to make and to report in conjunction with the work already described in this report. The supplementary studies comprise two distinct investigations; one, of the deflection and stress conditions in the vicinity of loaded slab corners and, two, of the stress reductions effected when the area over which the load is applied is considerably larger than any used in the tests described previously.

THE EFFECT OF LOADS ON SLAB CORNERS STUDIED

In the original manuscript of his paper "Theory of Stresses in Concrete Roads" Westergaard lists certain questions relating to the structural action of slab corners suggesting them as desirable subjects for further investigation. Among these were the following:

1. The influence of some nonuniformity in the distribution of the bending moment over the section of width $2x$; this influence may be expected to have the greater relative importance, the larger the value of the distance a_1, from the corner to the load.
2. The influence of twisting due to an eccentric position of the load, with the load closer to one edge than to the other.
3. The influence of an uneven thickness of slab, especially in the case of a thickened edge.

A fourth question, concerning impact loads, was mentioned but will not be discussed here. Certain data obtained in this investigation have a bearing on the three items that are listed, however.

The shape assumed by a slab corner when deflected by a load applied at or near the corner was determined by measuring the vertical displacements along the two edges of the slab corner and along the bisector of the corner angle.

In figure 45 deflection contours are shown for the 1st and the 60th application of a test load of 9,000 pounds on the corner of a slab of 7-inch uniform thickness, the loaded area being 8 inches in diameter and so placed that the slab edges were tangent to it. These data indicate that, while repeated loading caused a slight increase in the magnitude of the deflection at a given point, it produced no important change in the shape of the deflected corner area.[2]

Deflection contours for a similar symmetrical corner loading but of somewhat lesser magnitude are shown, for the 7-inch uniform-thickness and the 9–7–9-inch sections, in figure 46. It will be observed that within a distance of about 80 inches from the slab corner the

[2] In this and in subsequent similar graphs the deflection contours are the result of averaging the measurements on the four free corners of the test section.

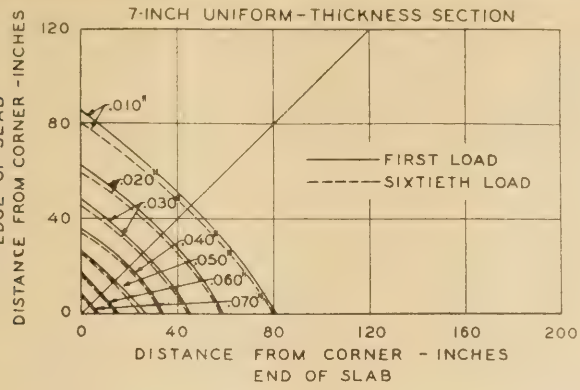


FIGURE 45.—EFFECT OF REPEATED LOADING ON THE DEFLECTED SHAPE OF A SLAB CORNER—SYMMETRICAL LOADING.

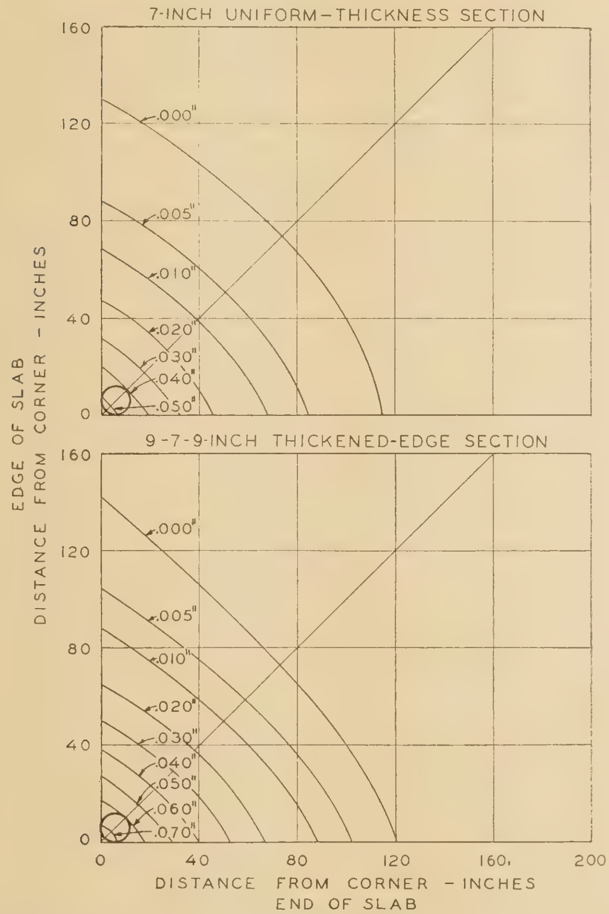


FIGURE 46.—DEFLECTION CONTOURS FOR SYMMETRICAL CORNER LOADING.

deflections appear to be symmetrical on either side of the corner bisector for both the uniform-thickness and the thickened-edge cross sections. Beyond this distance the deflection of the slab end is slightly less than that of the side edge. The difference is small but appears in the data for both sections. It is believed to be due to the difference in the panel dimensions. Along the side edge there is a continuous structure 240 inches in length while along the end edge are two units each 120 inches in length hinged together at the longitudinal joint. It was stated earlier in the report that when the free corner is loaded there is a slight rotation about the longitudinal

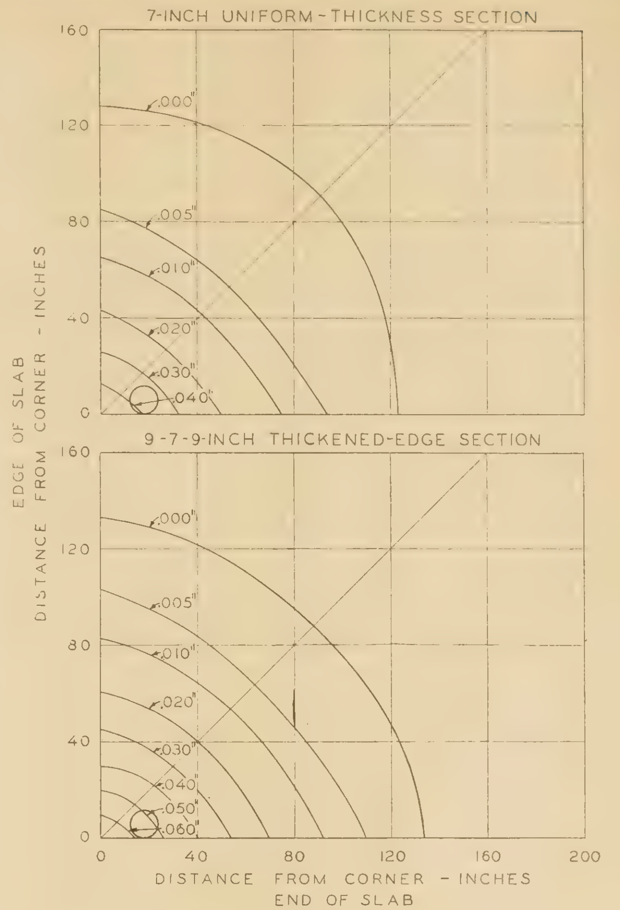


FIGURE 47.—DEFLECTION CONTOURS FOR ECCENTRIC CORNER LOADING.

joint. Obviously there is no corresponding movement along the side edge of the slab.

In figure 47 are deflection contours for the same two corners under the action of a load of the same magnitude but displaced 18 inches toward the center line of the pavement. Such an eccentricity of load would tend to create a twisting moment in the slab corner. It is apparent from the data that with this loading the deflection, at a given distance from the corner, is greater at the end than at the side of the slab. A comparison of the data in figure 46 with those for the same test section in figure 47 shows that the maximum deflection for the given load is greater when that load is applied on the slab corner, which is as would be expected.

In order to determine both the direction and magnitude of the stresses at various locations in the corner region, measurements of strain were made in either three or four directions at each location. From these "rosette" strain measurements the direction and magnitude of the principal strains may be obtained by a method described by Osgood and Strum (*11*).

The location of the various gage lines used in this study were shown in figure 13 (quadrant 3). Where strains are measured in four directions instead of three, the fourth strain value serves as a useful check.

The installation of each strain rosette necessitated the drilling of eight small holes into which the gage points were cemented. Since these were installed in the tension face of the slab the tendency would be to weaken somewhat the flexural resistance of the corner and to cause some variation in relative strength at different distances from the corner. Because of this possibility certain of the tests were made with fewer installations than are shown in figure 13.

MAXIMUM STRAIN UNDER ECCENTRIC LOADING LOCATED

It will be noted that the strains were measured along the two slab edges and/or along three rays 22½° apart which quadrisect the corner angle, the center ray being the bisector of that angle. These rays will be spoken of as the corner bisector and the 22½° rays in the subsequent discussion. The positions of the loaded area in these special corner tests were described in connection with the deflection data just presented.

For the symmetrically placed corner loading on the 7-inch and 9-inch uniform-thickness sections, strains were measured with rosette installations along the 22½° rays and in two directions at intervals along the corner rays. No strains were measured along the slab bisector. For the same position of loading in testing the edges. 8-inch uniform-thickness and 9–6–9-inch thickened-edge sections, strains were measured with rosette installations along both slab edges and the two 22½° rays and in two directions at intervals along the corner bisector. For this position of load, the direction of the principal strain in the region near the bisector of the corner angle was known to be parallel to that bisector.

For the eccentric corner loading, in which the position of the loaded area was moved along the end edge of the slab toward the pavement center line, for a distance of 18 inches, rosette installations were used along the two edges, the two 22½° rays and along the corner bisector as well.

The data obtained in these load-strain studies are shown in figures 48 to 66 inclusive.

Four of the test sections had loads applied symmetrically with respect to the slab corner, two had the load applied eccentrically. This made six combinations of load position and test section. For each of these six combinations there are three graphs.[3] The first graph in each group shows the direction of maximum strain for each of the 15 or more locations at which strains were measured. The second graph in each group shows the magnitudes of both the maximum strain and the minimum strain at each of these gage locations. The third graph of each group shows how the magnitude of the maximum strain varies with distance from the slab corner along the various rays on which gages were installed.

The slab corners used for the symmetrical loading were those of the 7-, 8-, and 9-inch uniform-thickness and the 9–6–9-inch thickened-edge sections. Those used for the eccentric loading were of the 8-inch uniform-thickness and the 9–6–9-inch thickened-edge sections.

The six graphs showing the direction of the maximum strains for the six slab-load combinations are figures 48, 51, 54, 57, 60, and 63, respectively.

The six graphs that show the magnitude of the maximum strains and the magnitude of the minimum strains (or those in the perpendicular direction at each gage location) are figures 49, 52, 55, 58, 61, and 64, respectively.

[3] In these figures and the attendant discussion maximum strain denotes the maximum strain for the given gage location and minimum strain is the strain measured at the same location in a direction perpendicular to that of the maximum strain. The maximum strains are approximately radial and the minimum strains approximately tangential in direction with respect to the center of load application.

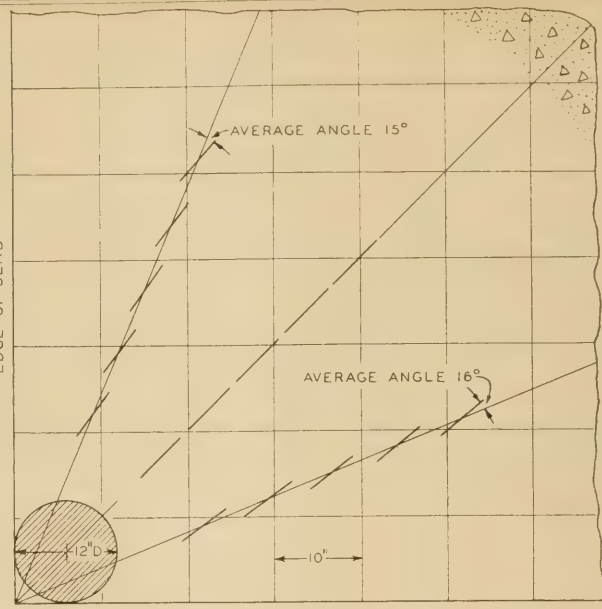


FIGURE 48.—DIRECTION OF MAXIMUM STRAINS—7-INCH UNIFORM-THICKNESS SECTION—SYMMETRICAL CORNER LOADING.

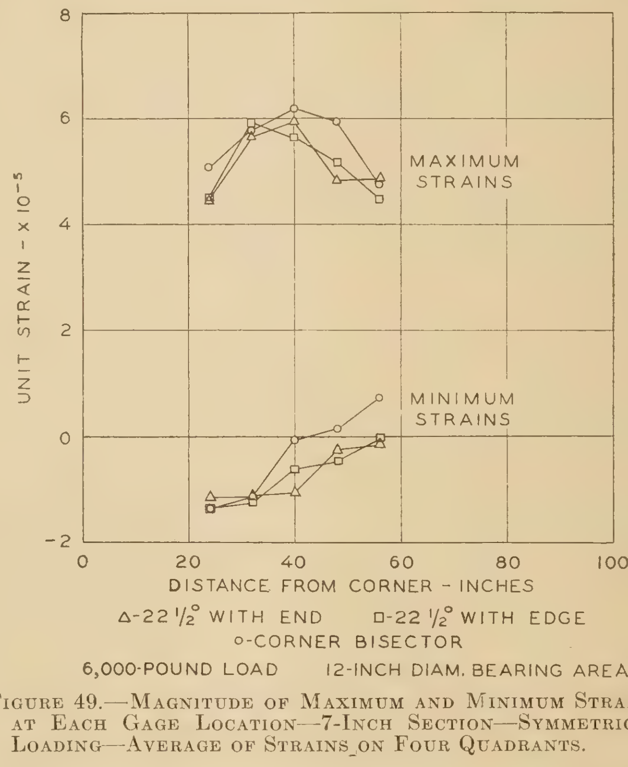


FIGURE 49.—MAGNITUDE OF MAXIMUM AND MINIMUM STRAINS AT EACH GAGE LOCATION—7-INCH SECTION—SYMMETRICAL LOADING—AVERAGE OF STRAINS ON FOUR QUADRANTS.

The six graphs showing the strain variation for various points along the rays on which the gages were installed are figures 50, 53, 56, 59, 62, and 65, respectively.

All of these figures are largely self-explanatory.

Referring to figures 48, 51, and 54 which show the direction of maximum strains for the symmetrical corner loading on the three sections of uniform thickness, it is apparent that while the maximum strain is along the central ray (the corner bisector) the maximum strains along the two 22½° rays are not parallel to the respective rays but have a direction which makes an angle of approximately 15° with these rays. The strains along the edges of the slab corner were not measured in the

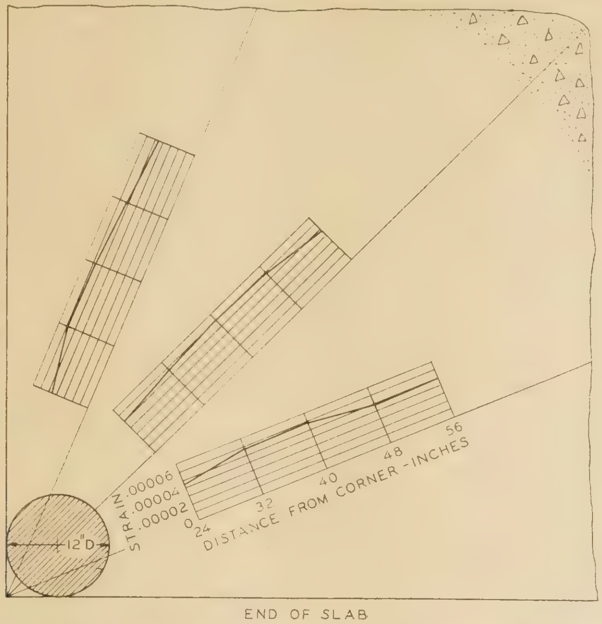


FIGURE 50.—VARIATION IN MAGNITUDE OF MAXIMUM STRAINS—7-INCH UNIFORM-THICKNESS SECTION—SYMMETRICAL CORNER LOADING.

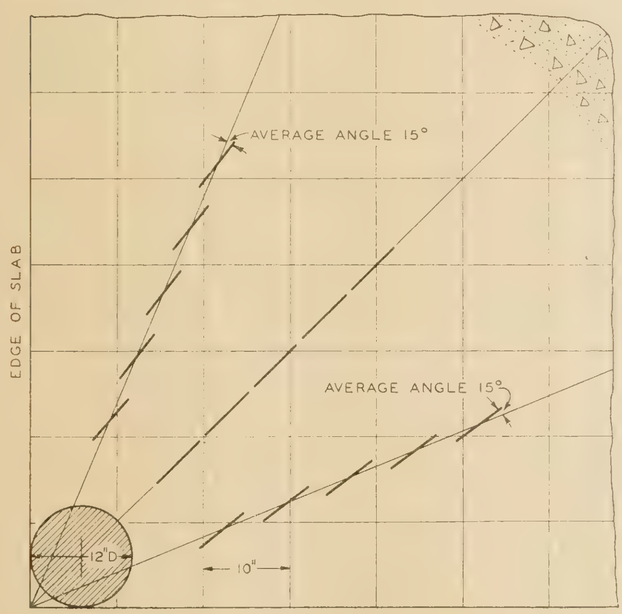


FIGURE 51.—DIRECTION OF MAXIMUM STRAINS—9-INCH UNIFORM-THICKNESS SECTION—SYMMETRICAL CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

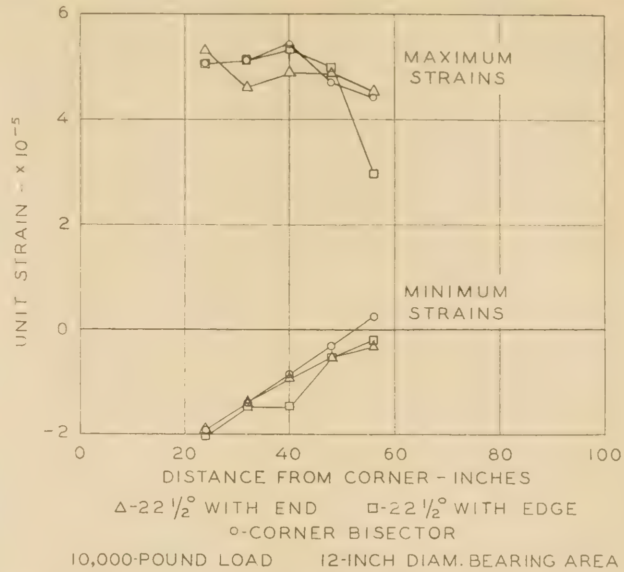


FIGURE 52.—MAGNITUDE OF MAXIMUM AND MINIMUM STRAINS AT EACH GAGE LOCATION—9-INCH SECTION—SYMMETRICAL LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

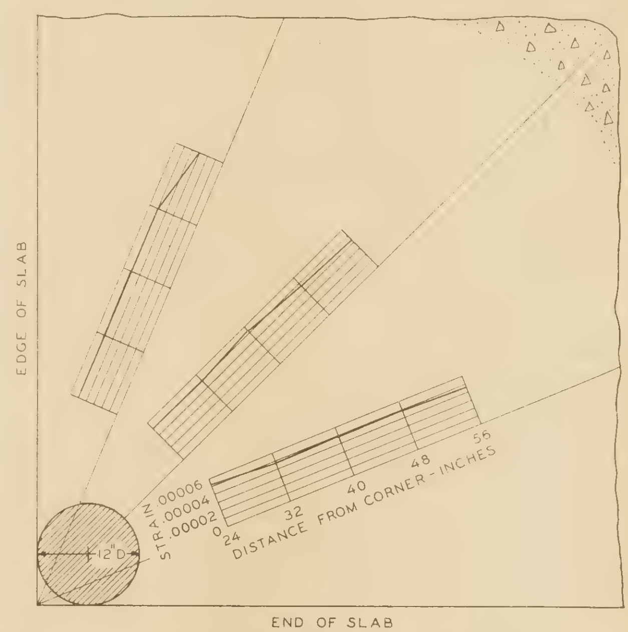


FIGURE 53.—VARIATION IN MAGNITUDE OF MAXIMUM STRAINS—9-INCH UNIFORM-THICKNESS SECTION—SYMMETRICAL CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

cases of the 7-inch and 9-inch sections, figures 48 and 51. These strains were measured, however, in testing the corner of the 8-inch uniform-thickness section and, from figure 54, it is evident that the maximum strain makes an angle of approximately 28° with the slab edge.

For the symmetrical loading the directions of the maximum strains in the corner area of the 9–6–9-inch thickened-edge section were found to be essentially the same as for the uniform-thickness section (see figure 57). Apparently the thickening of the edge of this section was so distributed that it did not appreciably affect the stress trajectories.

The direction of the maximum strains changed appreciably, however, when the eccentric load, previously described, was used. The effect of this eccentricity can be seen by comparing figure 54 with figure 60 and figure 57 with figure 63. Because of the variation in the angle along some of the rays the range in the angle between the direction of the maximum stress and the ray is indicated by the small figures beside the rays in figures 57, 60, and 63.

SECTION OF MAXIMUM MOMENT NOT A STRAIGHT LINE

It was stated earlier that strains were not measured along the two slab edges in the cases of the 7-inch and 9-inch test sections and there was reason to believe that the installation of a large number of gage points in the tension face of the slab might affect the test data.

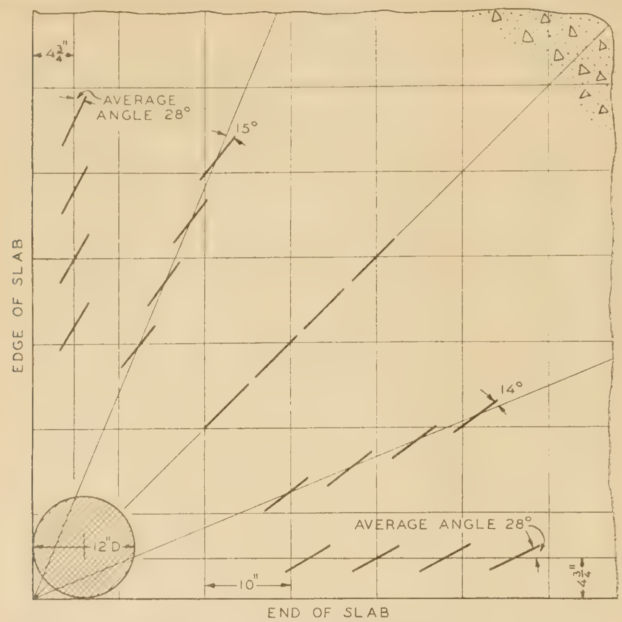


FIGURE 54.—DIRECTION OF MAXIMUM STRAINS—8-INCH UNIFORM-THICKNESS SECTION—SYMMETRICAL CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

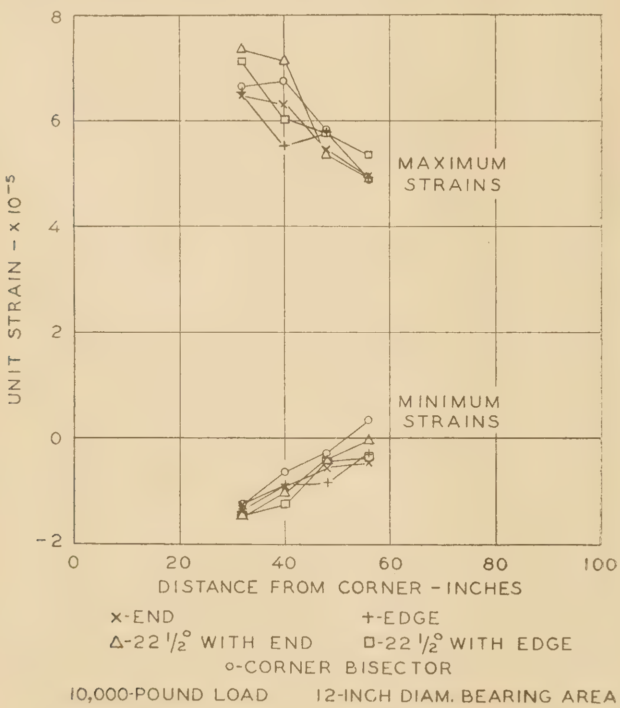


FIGURE 55.—MAGNITUDE OF MAXIMUM AND MINIMUM STRAINS AT EACH GAGE LOCATION—8-INCH SECTION—SYMMETRICAL LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

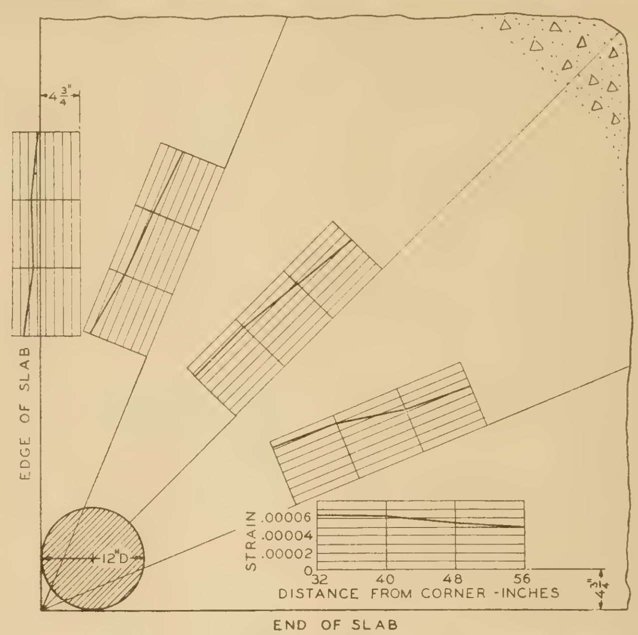


FIGURE 56.—VARIATION IN MAGNITUDE OF MAXIMUM STRAINS—8-INCH UNIFORM-THICKNESS SECTION—SYMMETRICAL CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

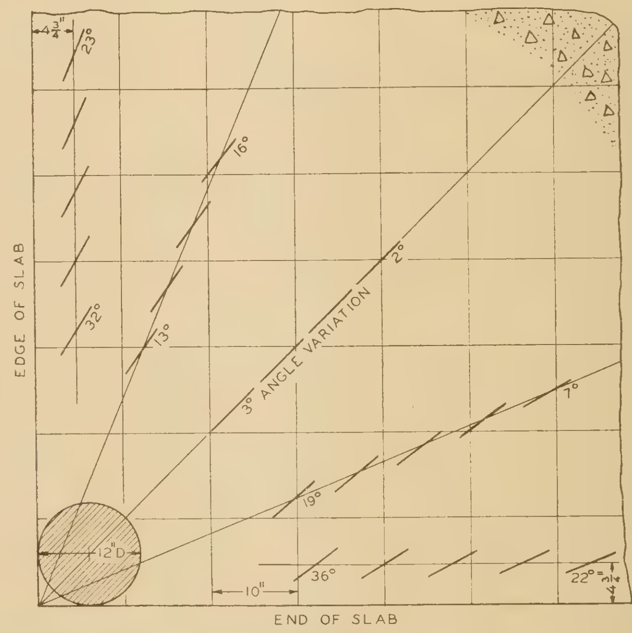


FIGURE 57.—DIRECTION OF MAXIMUM STRAINS—9-6-9-INCH THICKENED-EDGE SECTION—SYMMETRICAL CORNER LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

There is evidence to this effect in the set of graphs showing the strain variations along the several rays. The data for the 7-inch and 9-inch corners, figures 49, 50, 52, and 53, indicate the maximum measured strain to be on the corner bisector and at a distance of about 40 inches from the corner, for the conditions of the tests, while the maximum strain values on the two 22½° rays were of slightly lesser magnitude and at approximately the same distance from the slab corner. When, however, the strain gage points were installed along the two slab edges as well as on the rays mentioned it was found that the entire section of maximum strain was appreciably nearer the slab corner and the maximum strain no longer appeared on the corner bisector.

It is concluded, therefore, that the strain data for the 7-inch and 9-inch sections give a somewhat more reliable indication of the strain distribution in the corner of a section of uniform thickness than those obtained with the 8-inch-section and shown in figures 55 and 56.

Similar data for the eccentric loading applied to the 8-inch uniform-thickness and the 9–6–9–inch thickened-edge sections are shown in figures 61 and 62 and in figures 64 and 65, respectively.

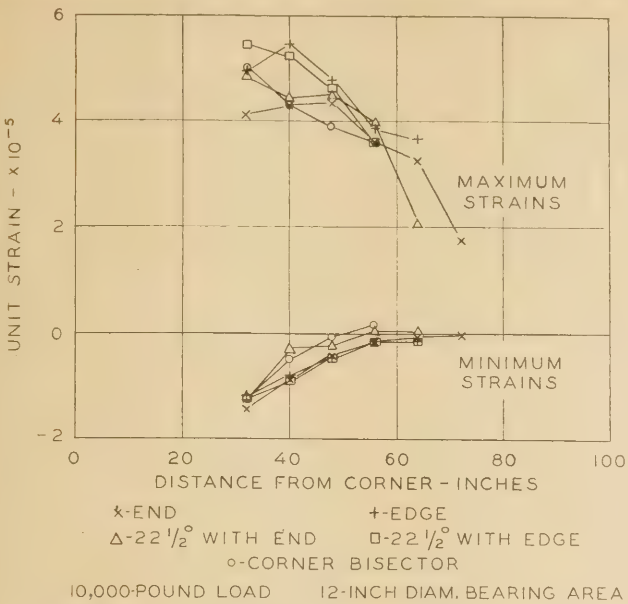


Figure 58.—Magnitude of Maximum and Minimum Strains at Each Gage Location—9–6–9-Inch Section—Symmetrical Loading—Average of Strains on Three Quadrants Only.

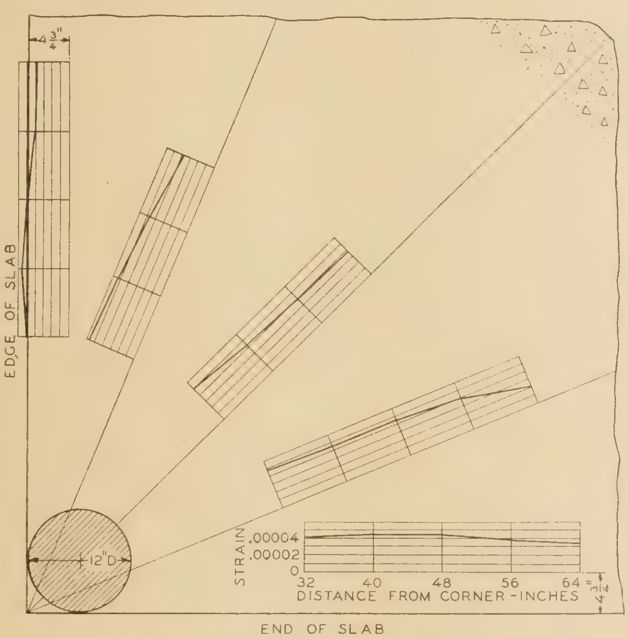


Figure 59.—Variation in Magnitude of Maximum Strains—9–6–9-Inch Thickened-Edge Section—Symmetrical Corner Loading—Average of Strains on Three Quadrants Only.

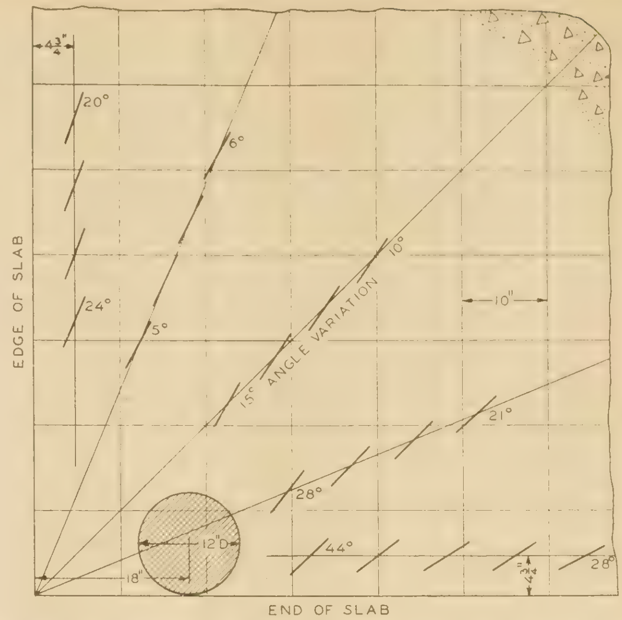


Figure 60.—Direction of Maximum Strains—8-Inch Uniform-Thickness Section—Eccentric Corner Loading—Average of Strains on Three Quadrants Only.

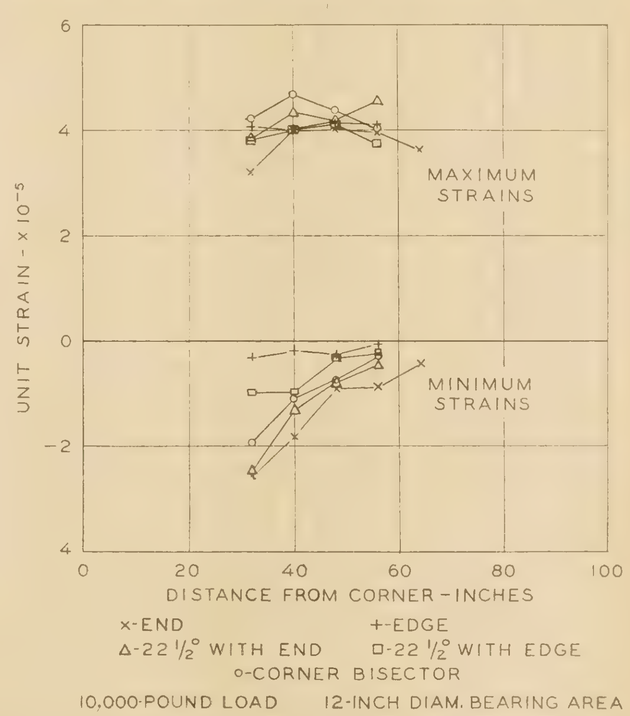


Figure 61.—Magnitude of Maximum and Minimum Strains at Each Gage Location—8-Inch Section—Eccentric Loading—Average of Strains on Three Quadrants Only.

For a given test section there is a striking difference in the stress magnitudes resulting from the symmetrical and eccentric load positions used in this investigation. The maximum tensile strain caused by the symmetrical loading was approximately 50 percent greater than that caused by the eccentric loading and this applies to both the 8-inch uniform and the 9–6–9-inch test sections. Compressive strains, on the other hand, were higher for the eccentric loading.

It is apparent from the strain data which have been presented that there is a relatively large area across the slab corner within which the magnitude of the maximum stress, as previously defined, does not vary appreciably. This is in concordance with data obtained by Spangler and Lightburn (*14*) in carefully controlled laboratory studies of the strain distribution in the corner of a concrete slab.

In his original analysis of pavement slab stresses Westergaard states that in the case of corner loading when the distance x_1 (from the corner to the section of maximum moment) is not too large, the bending moment will be approximately uniformly distributed over the width, $2x_1$, of the cross section.

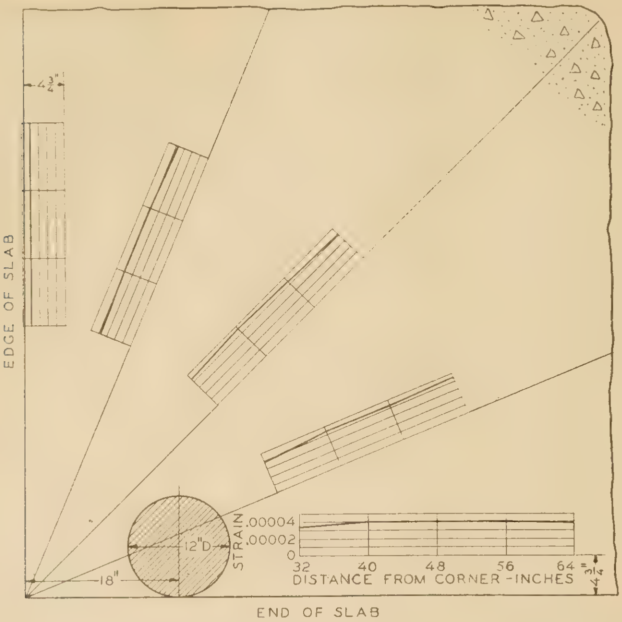


Figure 62.—Variation in Magnitude of Maxiumm Strains—8-Inch Uniform-Thickness Section—Eccentric Corner Loading—Average of Strains on Three Quadrants Only.

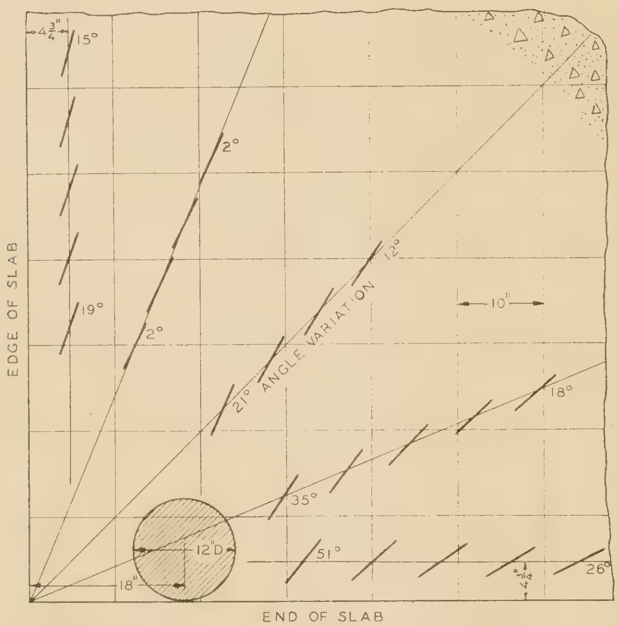


Figure 63.—Direction of Maximum Strains—9–6–9-Inch Thickened-Edge Section—Eccentric Corner Loading—Average of Strains on Three Quadrants Only.

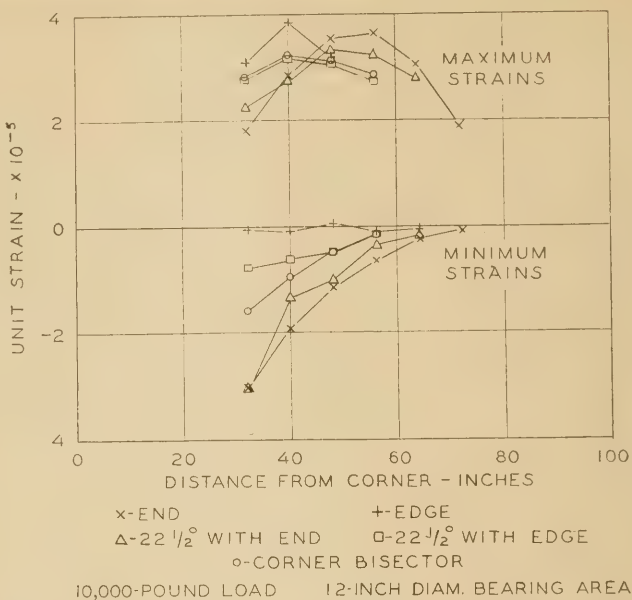


Figure 64.—Magnitude of Maximum and Minimum Strains at Each Gage Location—9–6–9-Inch Section—Eccentric Loading—Average of Strains on Three Quadrants Only.

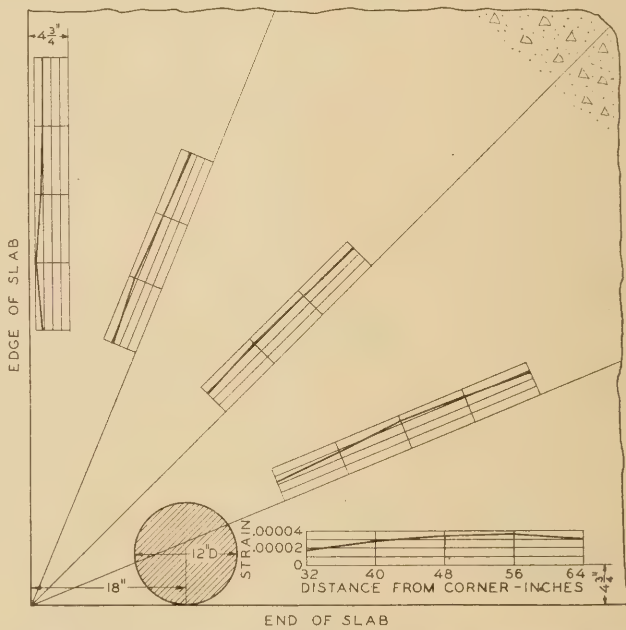


Figure 65.—Variation in Magnitude of Maximum Strains—9–6–9-Inch Thickened-Edge Section—Eccentric Corner Loading—Average of Strains on Three Quadrants Only.

Both observation of corner failures in the field and failures deliberately produced by corner loading during tests (*15, 22, 14*) have shown a tendency for the line of fracture resulting from a corner loading to be curved rather than straight although the curvature usually is not pronounced. The strain data that have been presented indicate that for the conditions of these tests the section of maximum moment is not a straight line normal to the bisector of the corner angle but follows a curved path similar to the deflection contours. In figure 66 are shown data obtained from the tests of the 8-inch uniform-thickness section. In this figure maximum strain values, as determined from strains measured in four directions, have been resolved to show the component in the direction parallel to the corner bisector. These are the strain values on the straight line section across the corner perpendicular to the corner bisector. In the figure the section shown is at a calculated distance, $x_1 = 32.2$ inches, from the corner. In addition to the plotted values of the strains observed on the five lines radiating from the slab corner the value of the average normal strain is shown by the horizontal dash line. It is apparent that in this test the component of the maximum strain observed on one of the 22½° rays was approximately 10 percent greater than the average

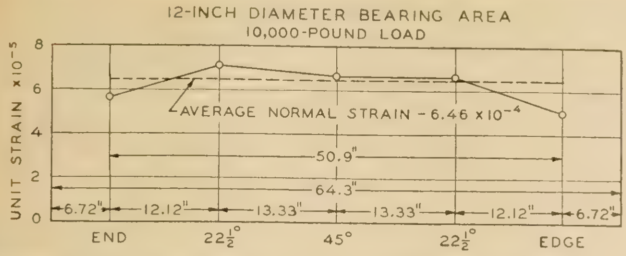


FIGURE 66.—DISTRIBUTION OF OBSERVED STRAINS ACROSS PLANE SECTION PERPENDICULAR TO CORNER BISECTOR AT A COMPUTED DISTANCE, X_1, FROM THE CORNER—8-INCH SECTION—SYMMETRICAL LOADING—AVERAGE OF STRAINS ON THREE QUADRANTS ONLY.

strain normal to the section. The variation in strain across the assumed plane section is evident.

Since strains were not measured along the slab edges at the corners of the 7-inch and 9-inch sections it is not possible to analyze the data from those sections in the manner shown in figure 66.

LOAD TESTS MADE WITH LARGE CONTACT AREAS

Originally the Westergaard analysis of the stresses in concrete pavement slabs (*23, 25*) was a discussion of the effects of such loadings as were being encountered in highway practice and, for purposes of simplicity, it was assumed that the load was applied to the pavement over areas that were either circular or semicircular in shape.

In planning the program of tests previously discussed in this report, provision was made for a range ofcircular and semicircular areas of contact sufficient to represent the tire contact areas normally met with in highway service.

Subsequently Westergaard supplemented his original analysis with two papers which discussed respectively the effect of (1) larger wheel loads on correspondingly larger loaded areas (*27*) and (2) loaded areas that are of other than circular shape (*28*).

In the first paper (*27*) the analysis for the case of interior loading is reexamined to determine its suitability for determining stresses when the areas of contact are much larger than those found in highway service. It was concluded by the author that the analysis is applicable to the conditions of loading found in airport runway service except that under some conditions certain small corrections are desirable. The means for making these corrections are given in the paper. Only the interior case of loading is discussed.

In the second paper (*28*) the effect of shape of loaded area is one of the subjects discussed. This is a valuable addition to the analysis since it furnishes for the first time the means for estimating the importance of variations in shape of loaded area which are known to exist. Both of these subjects are of great current interest because of their direct bearing on the design of concrete pavement slabs for airport service.

The original program of the investigation being reported contained no provision for tests with very large areas of contact since it was intended to cover the conditions of highway service. However, before it became necessary to vacate the test site there was an opportunity to make a limited number of tests that yielded data which afford significant comparisons between theory and observed performance for concrete pavements loaded over relatively large areas of both circular and elliptical shape.

The concrete pavement available for the load tests with the large contact areas was one which had been constructed in 1940 for use in another investigation. It was 20 feet wide divided at the center by a tongue and groove joint with ⅝-inch diameter tie-bars at 60-inch intervals. The sections were 30 feet in length between transverse expansion joints and were divided into 15-foot slab lengths with contraction joints of the plane of weakness type. The slab units were thus 10 by 15 feet. The pavement was of 8-inch uniform depth.

The concrete used in the pavement was mixed in the following proportions (dry weight):

	Pounds
Cement	94
Fine aggregate (sand)	200
Coarse aggregate (siliceous gravel)	330

The concrete had a 2½-inch slump at the time of placing. At 28 days the average crushing strength was 4,600 pounds per square inch and the average modulus of rupture was 550 pounds per square inch. The average modulus of elasticity at the time of the actual load testing on the pavement was 4,800,000 pounds per square inch.

These test sections had been laid on the site of some of the sections of the original investigation (*18*). However, the grade had been changed slightly and the character of the subgrade immediately under the new sections had been altered by mixing sand with the original silty loam to a depth of several inches and recompacting at optimum moisture to maximum density. This resulted in an average dry density of 136 pounds per cubic foot within the treated depth. The effect of this treatment will be discussed later.

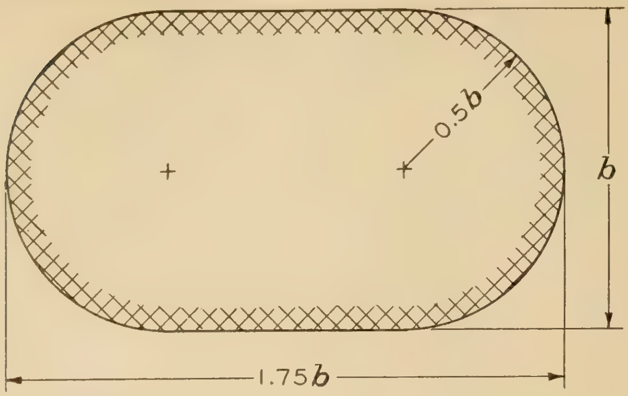
The equipment used for applying the loads was essentially the same as that described in the first report of this series except that for the larger loads a hydraulic jack and test gage were substituted for the mechanical jack and steel beam dynamometer. The hydraulic jack and test gage were calibrated prior to use in the tests.

Loads were applied to the pavement through rigid bearing plates that were either circular or elliptical in shape. The elliptical plates were of the proportions shown in figure 67. This is a conventionalized average obtained from actual tire impressions which seem to conform reasonably well to the contact areas of large-size, low-pressure pneumatic tires at recommended inflation pressure and capacity load. Dimensional data for both the circular and the elliptical areas of contact are given in table 17.

TABLE 17.—*Dimensions of contact areas*

Circular contact areas		Elliptical contact areas		
Diameter	Area	Length of major axis	Length of minor axis	Area
Inches	*Sq. ins.*	*Inches*	*Inches*	*Sq. ins.*
8	50	12	6.86	72
12	113	24	13.71	289
20	314	36	20.57	649
25	491	48	27.43	1,155
30	707	--------	-------	-------
36	1,018	--------	-------	-------

Considerable care was taken in seating the bearing plates to insure uniform distribution of the load over the area of contact. A thin layer of plaster of Paris was first placed on the concrete test slab and formed to



$A = 1.535b^2$

FIGURE 67.—PROPORTIONS OF ELLIPTICALLY SHAPED BEARING AREAS.

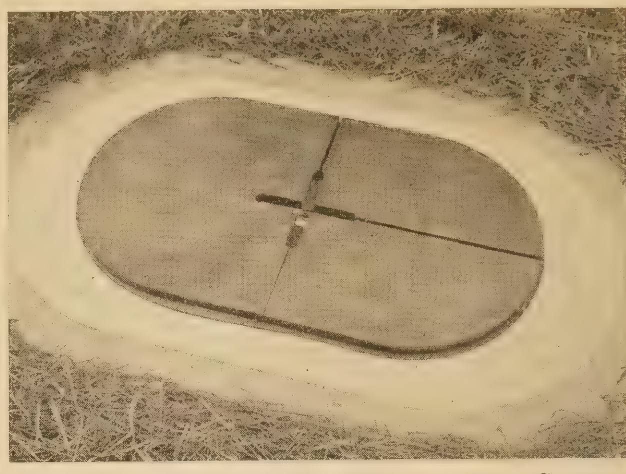


FIGURE 69.—ELLIPTICAL PLYWOOD BEARING PLATE SEATED ON THE PAVEMENT.

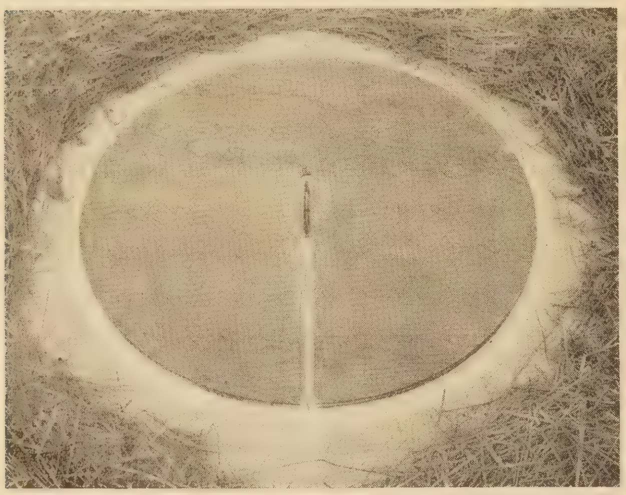


FIGURE 68.—CIRCULAR PLYWOOD BEARING PLATE SEATED ON THE PAVEMENT.

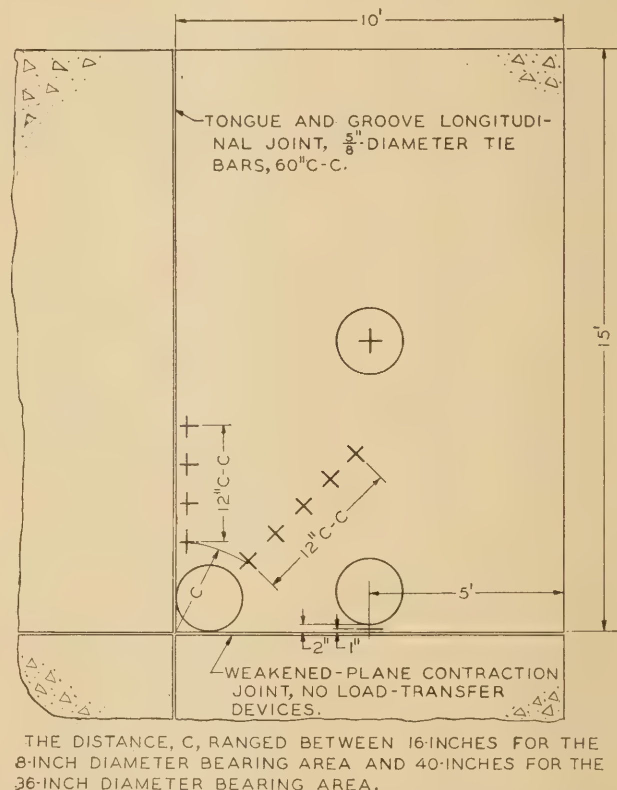


FIGURE 70.—LOCATIONS AT WHICH LOADS WERE APPLIED AND THE POSITIONS AT WHICH GAGES WERE PLACED TO MEASURE THE CRITICAL STRAINS.

a smooth plane surface. When this had hardened, a sponge rubber mat ½ inch in thickness was placed on the plaster surface. A plate of plywood, 1 inch in thickness, cut to the exact shape of the desired area of contact was then laid on the rubber mat and on this plywood plate an assembly of stiff plates was built up in such a manner as to insure rigidity. These back-up plates were of steel and of concrete and were bedded individually in plaster of Paris. Typical circular and elliptical plywood plates are shown in figures 68 and 69 respectively, while the complete assembly for a test with a circular area is shown on the cover page.

TEST PROGRAM PLANNED TO YIELD DATA FOR AIRPORT DESIGN

Strains were measured with the recording type of strain gage used throughout the investigation except that for these tests the glass slide on which the movement of the stylus is traced was fitted into a small carriage so arranged that it could slide longitudinally on the gage body. This carriage was moved by means of a fine wire attached to it and extending from the gage position to the edge of the loaded area in a groove provided in the plywood plate. The strain gage, groove, and wire can be seen in figures 68 and 69. This arrangement permitted a series of strain measurements to be made without disturbing the bearing plate assembly.

Because of a limit on the time during which the pavement sections were available it was necessary to restrict the study to a comparatively small number of tests. It was decided, therefore, to apply loads at the locations that seemed most important from the standpoint of airport pavement service. The locations selected were:

1. An interior corner, i. e., one formed by the intersection of a transverse contraction joint and the longitudinal joint.
2. An interior point, as far as possible from slab edges.
3. A transverse contraction joint edge at some distance from a slab corner.

These locations together with the strain gage positions used with each are shown in figure 70.

No tests were made at free edges or free corners and no tests were made along the longitudinal joint edge because it had been found in earlier tests (*21*) that the type of joint in these slabs is highly effective in controlling the critical stress caused by a load acting in the region along the joint.

A study was made of the strain distribution within these large bearing areas for the interior and edge loadings and it was found that strains measured at the positions shown would be as high as any within the area. In the case of the corner loading where strains were being measured outside the bearing area it was necessary to make preliminary tests to locate the position at which the strain along the bisector of the corner angle would be a maximum for each of the several bearing plate sizes. In this connection it may be recalled that the studies of joint behavior described in the fourth report of this series (*21*) showed that at interior slab corners, where corner support is obtained from adjoining panels, the critical stress may develop directly under the loaded area or at some point along the edges of the loaded panel rather than along the bisector of the corner angle as is the case with a free corner.

The protection given the test section and other details of procedure were the same as those already described for the main part of the investigation.

SUBGRADE TESTS GIVE SURPRISING RESULTS

Since the character of the subgrade immediately beneath the pavement of the test section had been altered in the manner previously described it was necessary to determine a value for the effective modulus of subgrade reaction before any comparisons could be made between theoretical and observed stresses. These subgrade tests developed data that are of considerable interest.

It was found in the determination of values for k on the original subgrade that a rigid circular plate 30 inches or more in diameter was of sufficient size to eliminate the effect of plate size. Because of this finding the first tests on the modified subgrade were made with a rigid circular plate of 36-inch diameter in the manner described in the first section of this report. The data obtained indicated a value of approximately 400 lbs. in.$^{-3}$ for the modulus of subgrade reaction and, contrary to the data from the earlier tests (see fig. 12), showed about the same values for plate displacements of 0.01, 0.02, and 0.05 inch. While it appeared reasonable to expect that the treatment given the subgrade soil would cause an increase in its resistance to deformation under load, it had not been anticipated that the rather marked effect of the magnitude of the plate displacement on the modulus of subgrade reaction which had been evident in the bearing tests on the original subgrade would be so greatly reduced in the tests on the modified subgrade.

The value of the modulus, k, as determined from load deflection tests on the pavement on one quadrant of the test section was found to be 285 lbs. in.$^{-3}$, considerably lower than that indicated by the bearing tests with the 36-inch diameter rigid plate but essentially the same as that found for the original unmodified subgrade under similar summer conditions.

This rather surprising result has several interesting implications. It is indicated that in modifying the character of the upper layer of the subgrade, in the manner previously described, its load supporting ability within a given deformation limit was considerably increased so long as the given unit load was applied over a relatively small area. When the loaded area was relatively large, as with the slab deflection test, on the other hand, the influence of the strengthened upper layer on the load support offered by the subgrade as a whole tended to disappear. This suggests that there is a relation between the size of a loaded area on a pavement and the strengthening effect to be expected from a given base course beneath that pavement.

Unfortunately there was no opportunity to study this matter thoroughly in connection with the present investigation. It was possible, however, to make determinations of the value of k from slab deflection tests on other quadrants of the test section and also to make one determination with a rigid bearing plate of greater size. This latter test was made with a 54-inch diameter plate on the subgrade in an opening cut through the pavement in one quadrant of the test section. This is shown in figure 71. The data obtained from the bearing test indicated a value of $k = 315$ lbs. in.$^{-3}$ at a displacement of 0.02 inch, about 10 percent greater than the value obtained from the slab deflection in this particular quadrant. This difference in the values of the modulus of subgrade reaction would have a negligible effect on the magnitude of computed stresses for the conditions that obtained in this study.

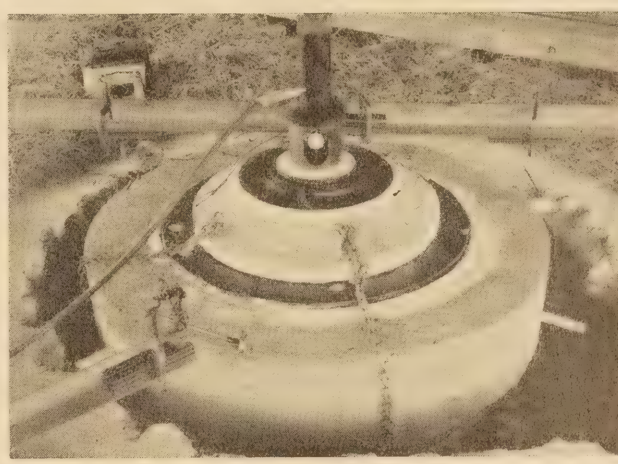


FIGURE 71.—DETERMINING THE MODULUS OF SUBGRADE REACTION WITH A LARGE BEARING PLATE PLACED ON THE SUBGRADE THROUGH AN OPENING CUT IN THE PAVEMENT.

It was not possible to make bearing tests with the 54-inch diameter rigid plate through the pavement on each quadrant of the test section. An average value for k determined in this manner is, therefore, not available.

The average value obtained from the data obtained in the load-deflection tests on the pavement in three of the four quadrants of the test section was 250 lbs. in.$^{-3}$. This value was used in computing certain of the stress values that will be shown in one of the subsequent graphs.

In order to obtain data that would permit strain data to be handled more flexibly in the analysis, a study was made of the linearity of the load-strain relation for a range of areas of contact which included both those used in the general investigation and the larger areas used in this supplementary study. These tests were made for interior loading only. Data obtained with circular contact areas with diameters ranging from 8 to

36 inches are shown in figure 72. The maximum load magnitude was sufficient in each case to produce a stress of at least 300 pounds per square inch. Each value shown in this figure is the average of eight observations, two in each of the four quadrants of the test section.

From these data it can be concluded that the load-stress relation is linear within the stress range shown for all the contact areas used.

UNCERTAIN NATURE OF EDGE SUPPORT AT FRACTURED FACES DEMONSTRATED

In figure 73 there is shown a comparison of stress values computed by means of the Westergaard equation with stress values derived from the strains observed in the loading tests, for each of the six sizes of circular contact area included in the tests.

For concrete slabs lying on a subgrade and acted upon by vertical static loads applied in the interior region, both theory and test show that the maximum flexural stress is closely proportional to the applied load. This fact makes it possible to extend load-stress data beyond the range of observed values without the risk of appreciable error. In deriving the observed stress values of figure 73 the data were extended somewhat beyond the observed range in the cases of the two smaller bearing areas in order to obtain stress values corresponding to the assumed load of 20,000 pounds. While these two stress values may be slightly above the usual working stress range for concrete in flexure the procedure is sound and the comparison between the theoretical and observed values valid throughout the stress range shown.

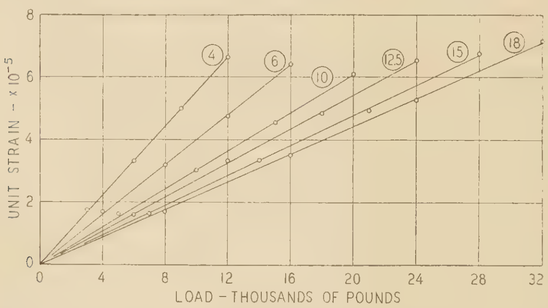


Figure 72.—Load-Strain Relation for a Range of Size of Loaded Circular Areas—Interior Case of Loading—Figures in Circles Show Radii of Loaded Areas.

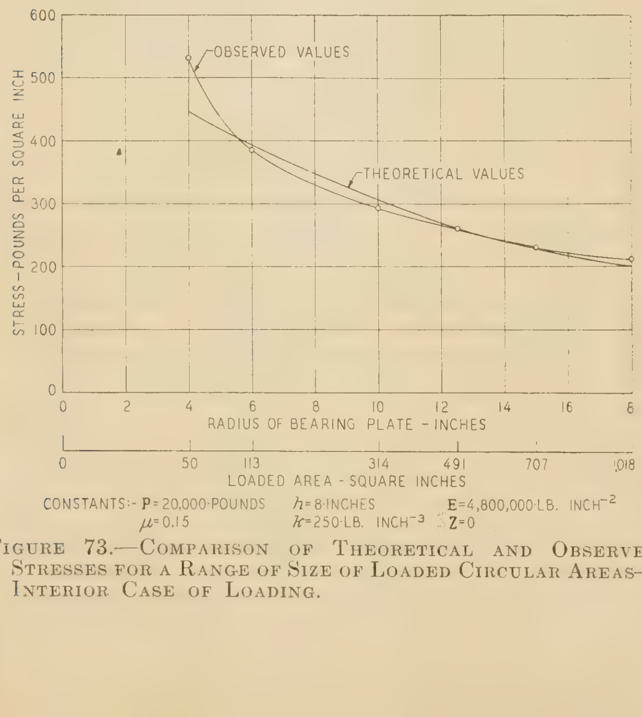


Figure 73.—Comparison of Theoretical and Observed Stresses for a Range of Size of Loaded Circular Areas—Interior Case of Loading.

It will be noted in the legend that, for calculating the theoretical stresses, it was assumed the quantity Z (termed the ratio of reduction) equals 0. When this value is assumed the general agreement between the theoretical stress values and those obtained from the measured strains is remarkably close and it is evident that the observed effect of size of loaded area on the stress developed by a given load is essentially the same as the theoretical relation. The one exception is the value obtained with the 8-inch diameter area where the observed value exceeds the theoretical by approximately 19 percent. The reason for this difference is not known. Referring back to similar data from the tests on the original pavement sections, shown in figure 40, it will be observed that the same tendency is evident where the 8-inch diameter area was used on the 8-inch and 9-inch sections of uniform thickness. In this connection it is pointed out that a contact area of this size is usually associated with a wheel load of about 3,500 pounds which, with pavement slabs of usual thicknesses, would cause a stress of less than 100 pounds per square inch. Thus the disparity occurs under conditions that are relatively unimportant.

In the computations for the theoretical stress values of figure 73 a value of $k=250$ lbs. in.$^{-3}$ was used since this represented an average from tests on three quadrants. While this value may appear somewhat low in view of the data from the bearing tests, it is worth noting that the use of a value of $k=300$ lbs. in.$^{-3}$ would cause a decrease in the computed stress values of only about 3 percent.

From these data it may be concluded that for the conditions of these tests there is good agreement between the observed stresses and those computed by means of the Westergaard equation for bearing areas of circular shape over a range of diameters from 8 to 36 inches.

Tests were made also with loaded areas of elliptical shape. On the basis of strain distribution studies the gages were located at the center of both the major and minor axes of the area as shown in figure 69. In figure 74 the stress values derived from the strains measured along each axis are shown for all of the elliptical areas listed in table 17, i. e., for a range of areas from 72 to 1,155 square inches. The observed maximum stresses for corresponding circular areas are shown on the same graph. The graph affords several useful comparisons. In the first place it is apparent that the stress along the major axis of the elliptical area is less than that along the minor axis, the difference being as much as 20 percent for the largest area. The maximum stress observed under each corresponding circular area was somewhat greater than that found along the major axis and slightly less than that found along the minor axis of the elliptical area.

In the past, for purposes of stress computation for highway loadings, it has often been assumed that the actual tire contact area could be represented by the circle of equivalent area. It is interesting to note that for areas such as are found in highway service, the data indicate the error resulting from this assumption would be small, probably 5 percent or less. For the larger areas the percentage would be slightly more.

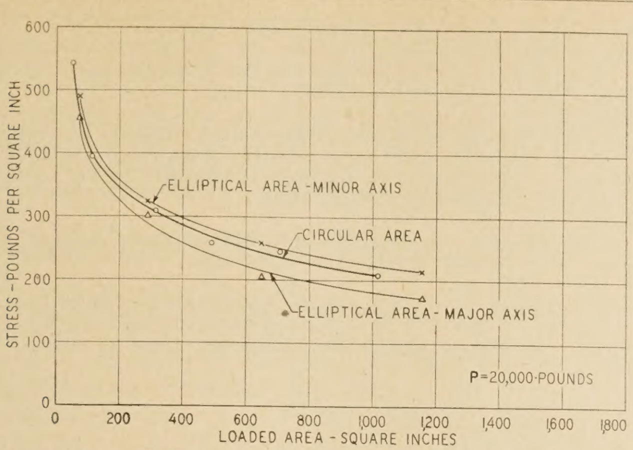


FIGURE 74.—COMPARISON BETWEEN MAXIMUM STRESSES OBSERVED UNDER CIRCULAR AND ELLIPTICAL BEARING PLATES OF EQUAL AREA—INTERIOR CASE OF LOADING.

It was mentioned earlier that Westergaard has, in a recent paper, extended his analysis to include loaded areas of other than circular shape (*28*). In a discussion of a part of this paper by the authors (*28*) comparisons were made between observed stresses and those computed by the equations developed by Westergaard for elliptically shaped loaded areas. It was shown that there was good agreement between the observed data and the theoretical relations.

Figure 75 shows observed stress values for a load of 20,000 pounds applied on circular contact areas ranging from 8 to 36 inches in diameter placed at each of three load positions shown in figure 70. When the load was applied at the mid-point of the slab end, i. e., along the weakened plane contraction joint midway between the slab edge and the longitudinal joint, tests were made with the contraction joint open and with it closed by a horizontal force of 60,000 pounds (over the 10-foot lane width) applied at the ends of the test section. When the load was applied at the interior corner the contraction joint was open.

The strain gages for the interior corner loading were located along the bisector of the corner angle at the position to measure the maximum strain developed in this direction. It is evident from the relatively low stress values that the slab corner is being strongly supported along the longitudinal joint and does not act as a free corner. Rather it approaches the condition of a free edge and the maximum stress for this case would be within the loaded area and more nearly equal in magnitude to that found in the test at the mid-point of the slab end, with the joint open.

The stress values for the interior loading serve as a reference in analyzing the other stress values.

The stresses observed for the load applied at the mid-point of the slab end bear a normal relation to those for the interior loading so long as the contraction joint is open. When, however, the joint is closed by pressure applied at the opposite ends of the test section, it is observed that the stresses for this edge loading are reduced appreciably for all sizes of bearing area and that for the larger areas the stresses become essentially equal to those found in the tests at the interior of the slab. For convenience the stresses, expressed as the percentages by which they exceed the stress for interior loading, are shown at three points along the graph. This comparison shows very clearly the uncertain nature of the edge support furnished by the fractured

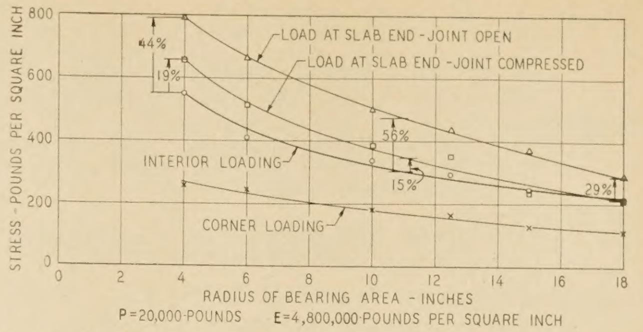


FIGURE 75.—EFFECT OF VARIATION IN POSITION OF LOAD AND SIZE OF BEARING AREA ON THE STRESS CAUSED BY A GIVEN LOAD.

faces alone in a joint of the plane of weakness type, and indicates the need for edge strengthening if a balanced design is to be obtained.

CONCLUSION

In the presentation and discussion of the data obtained in this rather extended study, an effort has been made to simplify the presentation by dividing the material into sections, each more or less complete in itself, with occasional summary statements in which the significant developments are pointed out and such conclusions as seem warranted are given. These detailed statements need not be repeated here.

The broad purpose of all of the studies described in this report has been to compare the observed structural behavior of pavement slabs of uniform thickness supported by a subgrade and subjected to vertically applied static loads with the behavior indicated by the Westergaard analysis (*23, 25, 27, 28*). On this point it is concluded that, within the limits of the investigation and so long as the basic conditions assumed for the analysis are approximated, the Westergaard theory describes quite accurately the action of the pavement.

There is need for further experimental study of some of the quantities that appear in the analysis, however. It is particularly desirable that information be developed which will permit the equations to be applied more readily and with greater certainty to problems that require the computation of values of absolute stress.

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