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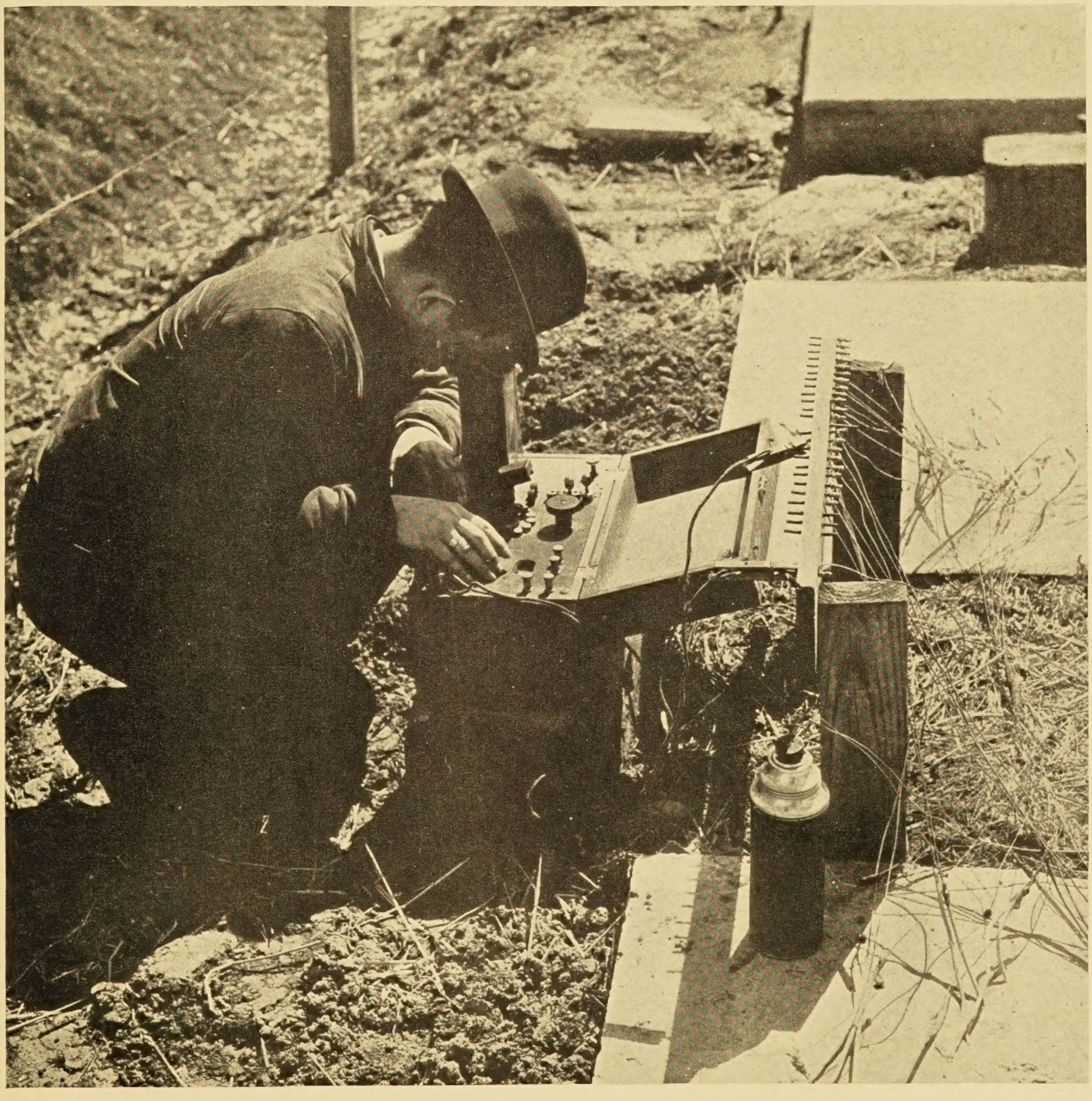

DETERMINING PAVEMENT TEMPERATURE USING EMBEDDED THERMOCOUPLES

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The reports of research published in this magazine are necessarily qualified by the conditions of the tests from which the data are obtained. Whenever it is deemed possible to do so, generalizations are drawn from the results of the tests; and, unless this is done, the conclusions formulated must be considered as specifically pertinent only to described conditions.

In This Issue

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THE STRUCTURAL DESIGN OF CONCRETE PAVEMENTS

BY THE DIVISION OF TESTS, BUREAU OF PUBLIC ROADS

Reported by **L. W. TELLER**, Senior Engineer of Tests, and **EARL C. SUTHERLAND**, Associate Highway Engineer

PART 2.—OBSERVED EFFECTS OF VARIATIONS IN TEMPERATURE AND MOISTURE ON THE SIZE, SHAPE, AND STRESS RESISTANCE OF CONCRETE PAVEMENT SLABS

IT HAS been known for more than half a century that when the temperature or the moisture content of portland cement concrete is changed a corresponding change in volume occurs. Since the pioneer work of Bauschinger [1] in this field many other research workers have contributed to the store of information concerning the extent of these changes and the factors that affect them.

As concrete began to be generally used as a paving material engineers gradually came to appreciate that, under some conditions at least, these volume changes were of such magnitude that provision for them was necessary in the pavement design.

Because of the relative physical proportions of a pavement slab, it was but natural that the first manifestation of volume change to be noticed was expansion and contraction in a longitudinal direction. More than 20 years ago the Bureau studied the expansion and contraction of concrete pavements [2] in an effort to discover the laws that govern such movements.

RESULTS OF EARLY INVESTIGATIONS INADEQUATE FOR PAVEMENT DESIGN

The early investigations were of value, first, for the information that they developed, and, second, because each one served to arouse more wide-spread interest in the subject among highway engineers. As a result of this interest new researches were begun and it was soon discovered that differences in temperature between the upper and lower surfaces of a pavement slab would cause it to warp to a measurable degree,[3] and further, that a similar warping could be brought about by creating a moisture differential between the two surfaces of a concrete slab.[4]

These discoveries were important because the exposure conditions to which pavements are subjected are severe; and expansion, contraction, and warping may be expected daily throughout the life of a concrete pavement slab. If the slab were completely free to move and had no weight it would change in size and shape without restraint and no stress would result. Because of its weight and intimate contact with the subgrade, however, restraint in some degree is always present and every attempt of the slab to change either its size or shape develops stress within the structure.

Other early tests indicated the approximate amount of resistance encountered when the pavement slab expands or contracts longitudinally along the subgrade,[5] and the development of this information made it possible to estimate approximately the tensile or compressive stresses induced in the concrete by this form of restraint.[6]

More recently a careful analysis has been made of the theoretical stress conditions resulting from slab warping caused by temperature differences within the concrete and assumed conditions of restraint.[7] This analysis supplies the means for estimating the stresses that occur under certain conditions of temperature warping.

EXTENSIVE PROGRAM OF STUDY OF MOISTURE AND TEMPERATURE EFFECTS UNDERTAKEN

It will be noted that each of the researches referred to has contributed some information that represents a distinct step toward a better understanding of the reasons for observed slab behavior. However, the engineer who undertakes to apply the information obtained by these previous studies to the design of a pavement finds that there are numerous gaps and uncertainties in the data. When the pavement-design project at Arlington was being planned, it was decided to include a study of the extent of the moisture and temperature changes that occur in pavements in this locality and to determine, as fully as possible, the effect of those changes on the pavement slab. The rather extensive program of observations outlined in part 1 of this series was developed with this object in mind.

The observations that have been made may be grouped according to purpose as follows:

1. A study of the extent of the temperature changes that occur in the various parts of concrete pavement slabs.
2. A study of the longitudinal expansion and contraction of pavement slabs caused by temperature changes and changes in moisture content.
3. A study of the resistance offered by the subgrade to horizontal slab displacement and of the stresses developed in the slab by this resistance.
4. A study of the warping of concrete pavement slabs resulting from variations in temperature and of the stress conditions that result from warping.

The tests and observations reported in this paper were made on the 10 full-size pavement slabs and, in general, the data cover periods of time of from 1 to 3 years. The complete description of the test sections and of the methods of test employed has already been given in part 1 of this series.[8]

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[1] Tests of Different Portland Cements, by J. Bauschinger, Mitt. Mech. Tech. Lab., Tech. Hochschule (Munich) v. 8, 1879.

[2] The Expansion and Contraction of Concrete and Concrete Roads, by A. T. Goldbeck and F. H. Jackson, Bull. 532, U. S. Department of Agriculture, October 1917.

[3] The Bates Experimental Road, by Clifford Older, and Highway Researches and What the Results Indicate, by A. T. Goldbeck, papers in the Proc. American Road Builders' Association, 1922.

[4] Effect of Moisture on Concrete, by W. K. Hatt, PUBLIC ROADS, vol. 6, no. 1, March 1925.

[5] Friction Tests of Concrete on Various Subbases, by A. T. Goldbeck, PUBLIC ROADS, vol. 5, no. 5, July 1924.

[6] The Interrelationship of Longitudinal Steel and Transverse Cracks in Concrete Pavements, by A. T. Goldbeck. PUBLIC ROADS, vol. 6, no. 6, August 1925.

[7] Analysis of Stresses in Concrete Roads Caused by Variations in Temperature, by H. M. Westergaard. PUBLIC ROADS, vol. 8, no. 3, May 1927.

[8] The Structural Design of Concrete Pavements, Part 1, by L. W. Teller and Earl C. Sutherland, PUBLIC ROADS, vol. 16, no. 8, October 1935.

TEST PROCEDURE DESCRIBED

Temperature measurements.—The temperature measurements were made in large part on two small slabs that were constructed especially for this purpose some months after the 10 test sections were constructed. These slabs, shown on the cover page, are 4 feet square, one being 6 and the other 9 inches in thickness. They were constructed on the subgrade adjacent to the test sections and the materials and proportions used in the concrete were exactly the same as were used for the large slabs.

The resistance thermometers originally installed in the large slabs were not entirely satisfactory for several reasons, so when the slabs for temperature measurements were built copper-constantan thermocouples were installed. The thermocouples were located at the centers of the slab areas and at 1-inch intervals throughout the depth, and also at both upper and lower slab surfaces. Two thermocouples were also placed in the subgrade beneath each slab, one about one-fourth inch and the other about 2 inches below the surface of the subgrade.

With these installations it was possible to determine quite completely the temperature conditions that existed throughout the depth of the slab. From these data one may estimate with considerable accuracy both the average temperature of the slab and the temperature differential between its upper and lower surfaces. In estimating the temperature conditions for the various pavement sections from the data obtained from the thermocouples, two assumptions were made: First, that the temperatures in the pavement slab were the same as those found in the slab of equal thickness on which temperature measurements were made; and second, that the temperature of slabs having thicknesses intermediate between 6 and 9 inches could be predicted by a straight-line interpolation between the temperatures measured in the 6- and 9-inch slabs.

The average temperature of the slab was obtained by averaging the temperatures measured at the several points throughout its depth. The temperature differential was estimated by drawing a mean curve for the data which were plotted to show the variation in temperature through the slab depth, and from this mean curve taking the indicated temperatures for the upper and lower surfaces of the pavement.

The temperature measurements were usually made in connection with measurements of slab expansion or of slab warping. They cover a period of approximately 2 years and, while not continuous, were planned so as to develop full information concerning both the daily and yearly temperature cycles that occurred during this time.

During the latter part of the investigation it became desirable to obtain data that would show the relation between the temperature differentials in thickened-edge cross sections and those in sections of constant depth. No means for obtaining these data were provided in the original construction and it was necessary to drill small holes to the proper points in the slab cross section, insert thermocouples in the bottom of these holes, and then backfill with cement-sand mortar. Thermocouples were placed in this manner near the upper and lower surfaces of the slab and at the third points of the slab depth, and this arrangement made it possible to estimate very closely the differential in temperature that existed at the several positions on the test sections at which the measurements were made.

Two types of information were desired: First, a knowledge of how the temperature differential varies along the different types of cross section; and second, a comparison of the differential in the edge of a 9-6-9 thickened-edge section with those which occur under the same conditions in the edges of constant-thickness sections of different depths. For the first purpose, the thermocouple installations were made at distances of 2, 18, and 36 inches from the free edge, while for the second only the installation at 2 inches from the edge was used. In each case the thermocouple units were placed at the various depths as described above.

Moisture measurements.—Since moisture variations within the concrete are known to cause physical changes in the size and shape of the pavement slab that are similar to those caused by temperature variations, it was desired to obtain quantitative data concerning the changes. Such data are difficult to obtain, however, because there is no reliable method for determining the moisture content of concrete except by weighing and this is usually not feasible.

In this investigation the moisture content of the concrete was determined by weighing and drying fragments broken from the short slabs originally provided for the purpose of furnishing samples. No means was found for determining differences in moisture content at various depths in the slabs so that no data were obtained to show the nature of the moisture gradient at various seasons of the year. However, measurements were made of the seasonal warping presumably caused by such a moisture gradient. Measurements were also made from which it was possible to determine the direct longitudinal expansion and contraction caused by factors other than temperature change, and from these data it was found possible to estimate the extent and period of the yearly variation in slab length attributable to moisture change alone.

The method of measuring the warping of the slabs under the action of these seasonal moisture changes is described later.

Measurements of the expansion and contraction of the pavement sections.—Practically all of the slab-expansion data were obtained by measurements on the 6-inch constant-thickness section. This section was separated from the other nine by a space of several feet so that there was no possibility of its being affected by the movements of the other slabs. Fixed reference points of the type shown in figure 1 were constructed at each end of the 40-foot section, and the horizontal movement of the slab ends with respect to these reference points was measured with the special micrometer described in the previous paper. The movement at the transverse joint was measured at the same time. The frame of the micrometer is made of soft steel and its length changes considerably with temperature changes. Corrections for this were made by referring all measurements to an invar standard, the length of which changes less than 0.001 inch for a temperature change of 100° F.

The procedure followed was first to place the micrometer and reference bar out of doors and allow them to reach a condition of temperature stability. The measurements at the slab ends were then made and any change in the length of the micrometer, as indicated by the reference reading on the standard bar, applied as a correction to the measured slab displacement.

The expansion and contraction measurements were made once or twice each month over a period of about 4 years. Each set of observations was started in the early hours of the morning, before the minimum slab length was attained, and was continued until late in the evening, long after the maximum expansion had occurred. Measurements were made at 2-hour intervals during this time.

This daily cycle of observations and the related temperature data made possible the determination of the maximum and minimum slab lengths, the range in average slab temperatures that caused the length changes, and from these the length change per degree of temperature change for the slab as a whole.

The series of observations obtained over the yearly periods made possible the determination of not only the annual cycle of length change resulting from temperature but also those resulting from other causes, such as variation in moisture content.

In addition to the measurements at the ends of the 6-inch constant-thickness slab, some data were obtained from telemeters (resistance strain gages) that were installed at the center of one 10 by 20-foot panel in each of two other test sections at the time of construction. These instruments were placed in a longitudinal direction and in the neutral plane of the slab. They were in a position to receive direct longitudinal deformation but not deformation caused by bending of the slab. Each telemeter is provided with an electrical resistance thermometer housed within its shell with which the temperature within the telemeter can be measured at the same time that the deformations are determined.

The schedule of observations with the telemeters was the same as that followed in the measurement of the movement of the slab ends. The telemeter observations covered a period of about 2 years and coincided with the period of the micrometer measurements for only about 8 months, as shown later in figure 14. Failure of the resistance thermometer elements caused the abandonment of the telemeter observations.

Measurements of subgrade resistance to horizontal slab movement.—The measurement of the resistance offered by the subgrade to the horizontal displacement of the pavement slab was determined in tests made with small slabs cast upon the subgrade for this particular purpose, as described in part 1 of this series of papers. (See pt. 1, fig. 20.)

These slabs, each 4 feet square, were moved horizontally with a powerful, smoothly operating mechanical jack. The force required to cause the movement and the amount of the displacement were determined.

Before the tests were begun, a preliminary investigation was made to find a desirable test procedure. As a a result of this preliminary work it was decided:

1. That the slabs should be moved at a very slow rate, one comparable with that which obtains in a pavement during the daily cycle of temperature variation.
2. That the slabs should be moved alternately in opposite directions.
3. That the magnitude of the displacement used in the tests should be comparable to the movements that occur at the ends of pavement slabs of moderate length under the influence of daily temperature variations.

This procedure subjected the subgrade under the small test slabs to the same manipulation that it would receive under the end of a pavement slab.

In a number of the tests the horizontal displacement of the subgrade at various depths was measured as the slab was displaced. In order to measure the movement of the subgrade, small recesses were cut into the earth just in front of the slab and micrometer dials were placed in these recesses. The dials were supported on a rigid member that was supported in turn by the subgrade at some distance from the slab. The stems of the dials bore against small wooden plugs inserted in the vertical face of the subgrade at the desired depths. The dials were usually placed at three points across the front or leading edge of the slab.

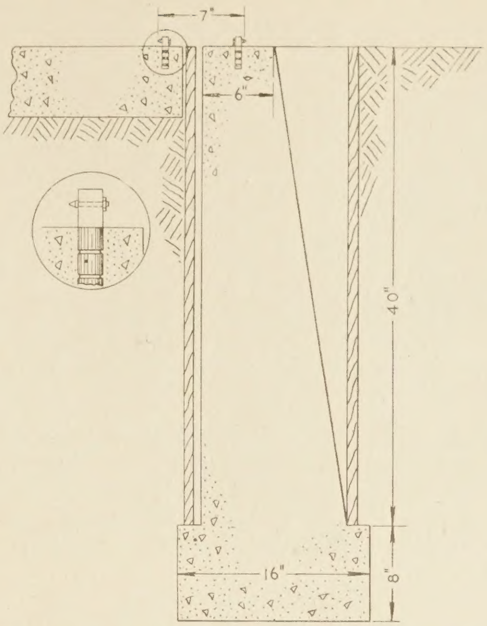

FIGURE 1.—REFERENCE POINT FOR MEASURING CHANGE IN LENGTH OF CONCRETE SLAB.

A few tests were made to determine the influence of slab thickness (or weight) on the resistance offered to horizontal displacement. For these tests four small slabs of 2-, 4-, 6-, and 8-inch thicknesses were constructed on the regular subgrade. The method followed in testing with these slabs was the same as that used with the 6-inch slabs previously described.

Warping measurements.—Observations have shown that under usual conditions of exposure there is seldom a time when the temperature of a pavement slab is uniform throughout. Either the air is warmer than the subgrade or vice versa, and heat is being conducted by the slab either to the subgrade or away from it. Because concrete is a relatively poor conductor of heat, the transfer of heat through a pavement of ordinary thickness takes time, and a differential in temperature is created within the various parts of the structure. Each element of the concrete attempts to adjust its volume to its own particular temperature.

As previously stated, if the slab were completely free and without weight it would assume a distorted shape without resistance and no stresses would result. Actually, almost no part of the slab is free to adjust itself and, while measurable distortion occurs, restraint caused by the slab weight and by the subgrade reactions is always present. It is apparent that the warping of a pavement is not a simple phenomenon and the distortion that is measured is the result of a very complex system of forces.

Warping has two important effects on the structural action of a pavement slab. In the first place, the distortion that occurs alters the condition of subgrade support and thus affects the magnitude of the stress that will be produced by a given wheel load. In the

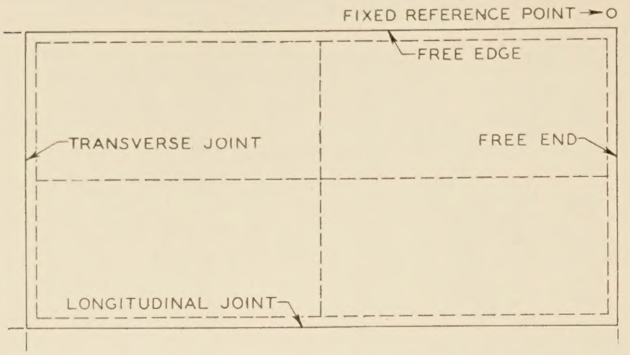

Figure 2.—Plan of 10- by 20-Foot Section Showing the Lines Along Which Deflections Were Measured.

second place, because of the weight of the slab, the warping in itself causes important stresses within the slab structure. Both of these actions place limitations on the maximum wheel load that may be carried by the pavement and, for this reason, any information concerning them is of value.

The studies of warping that were a part of this investigation were divided into three groups, as follows:

1. Measurement of the extent and characteristics of the actual warping that occurs in pavement slabs of conventional designs.
2. Determination of the magnitude of the effect of warping on the critical stresses caused by a given load applied at different points on the pavement slab.
3. Determination of the magnitude and distribution of the stresses caused by restraint to warping in the various parts of the pavement slab.

In order to determine the changes in shape that occur when the pavement warps, it is necessary to measure the vertical displacement of a great many points on its surface. In this investigation the measurements were confined to one quadrant or panel of the test section, and in this quadrant the shapes of the two principal axes and of all of the boundaries were determined. The lines along which the shape determinations were made are shown in figure 2. It will be noted that there is one free edge and one free end to the panel, and that the other edge and end are attached to the other panels of the test section by the longitudinal and transverse joints.

Clinometer points were set at 10-inch intervals along the lines shown in figure 2, and the shape of the slab was determined by measuring successively the slopes of these intervals with the 10-inch clinometer. The measurements were started from a bench mark or reference point located on the shoulder near the pavement, and were carried forward along the line of clinometer points exactly as a line of levels is run.

The measurements of the temperature warping were usually made on days when a large variation in pavement temperature occurred, three complete sets of observations being made. The first measurements were made in the morning at a time when the observed temperatures in the upper and lower surfaces of the slab were equal. Under these conditions the slab was assumed to be flat and all subsequent measured distortions were referred to this plane as a base. The second set of measurements was made during the early afternoon, at the time when the temperature of the upper surface of the pavement was at or near the maximum and the slab was consequently warped downward at the edges. The final measurements were made early the following morning when the temperature of the upper surface of the pavement had reached its minimum value and the maximum upward movement of the edges of the pavement slab had occurred. The three sets of observations were sufficient to determine quite completely the shape of the slab under the conditions of maximum upward and downward temperature warping.

Some additional measurements were made for the purpose of establishing the relative movements of various parts of the slab, in tests that covered the full 24-hour cycle of temperature changes. Because of the frequency of the observations and of the time required to make one set, the measurements were limited to determinations of the transverse curvature of the slab panel along the free end, the center line, and the transverse joint. The vertical displacement at a few points was also measured with micrometer dials supported on long steel stakes driven into the subgrade. Most of these measurements were made at slab corners, and a few at the center of the edge. The measurements were made for the purpose of obtaining additional data on the movement of slab corners during the daily temperature cycle and for comparing, hour by hour, the displacements for slabs of various thicknesses.

Measurements of the warping caused by seasonal changes other than temperature were made with the clinometer in much the same manner as the other warping measurements. These measurements were made only on the 9-inch constant-thickness section. They were made at intervals of about 1-month throughout the year at times when the thermocouples indicated that no temperature differential was present. Because of the relatively short period of time during which this condition usually obtained it was not possible to extend the clinometer measurements over the entire slab. For this reason the observations were limited to what were considered to be the most important regions.

Measurements were started from a bench mark and carried across the two ends and along the longitudinal center line of one of the panels. Since it was necessary to maintain the same datum throughout the year, special bench marks, the details of which are shown in figure 3, were installed near the edges of the slab on which these measurements were made. It will be noted that an electrical resistance thermometer is provided in the air space between the casing wall and the standard that carries the reference point. This permits a correction to be made for variations in the length of the standard resulting from seasonal temperature variations.

Measurements of the effect of temperature warping on load stresses.—The effect of the distortion of the pavement caused by temperature warping on the magnitude of the stresses produced by given wheel loads was studied by determining the critical stresses in the concrete for a given applied load with the slab flat and again in the condition of maximum upward and maximum downward warping. The loads were applied at the corner, the edge, and the interior of each slab upon which these tests were made.

The technique of applying the load and of measuring the strains was essentially the same as that followed in the other tests. The loads were applied to the slab through a bearing block 8 inches in diameter. Since a warped slab was a requirement of the test, it was necessary to arrange the loading tank so as to avoid shading the concrete and thus interfering with normal warping. This was done by moving the tank over the end of the adjoining section and placing a heavy I-beam

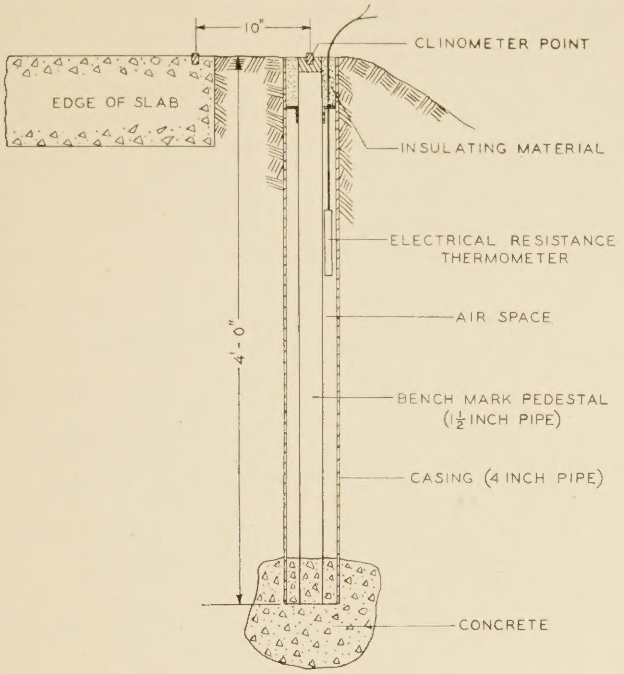

Figure 3.—Details of Clinometer Bench Mark.

beneath and bearing against the tank with one end resting on a fixed support and the other end furnishing a reaction for the jack applying the load. Extension of the jack tended to raise the end of the **I**-beam and to lift the tank, the **I**-beam acting as a simple lever. The force exerted downward by the jack supplied the load to the test section. The equipment used for making these tests is shown in figure 4.

STRESSES CAUSED BY RESTRAINED TEMPERATURE WARPING DETERMINED

The magnitudes of the stresses created in the various parts of the pavement slab by restrained temperature warping were determined by comparing the deformations measured in the concrete at the points in question with the deformation at a point where there was little or no restraint.

Theoretically, at the edge of a pavement, there should be no warping stress in a direction perpendicular to the edge.[9] Practically, it is not possible to measure these strains at the edge, and in the test section the gages for this determination were placed with their centers 6 inches from the edge. A preliminary study showed that for slabs of these dimensions the stresses at the gage positions were all very small and that the least restraint exists at the mid-point of the free edge in the direction perpendicular to the edge. A strain gage placed at this point, perpendicular to the edge, will record most nearly the full deformation or volume change that the temperature change demands, while similar gages at other places will record different deformations, deformations that are modified by the magnitude of the restraint present at each gage position.

In the discussion that follows, measurements made with a gage in the position just described were considered as resulting from unrestrained warping and were used as a base for determining the amount of restraint existing at other parts of the pavement.

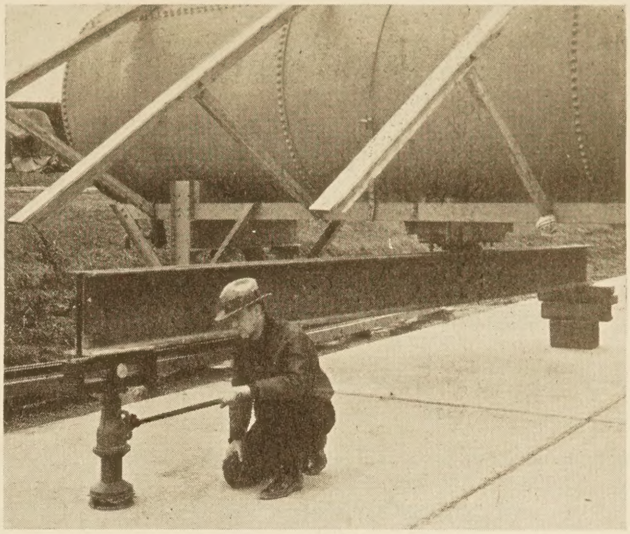

Figure 4.—Method of Applying Load to Slabs That Had Been Allowed to Warp.

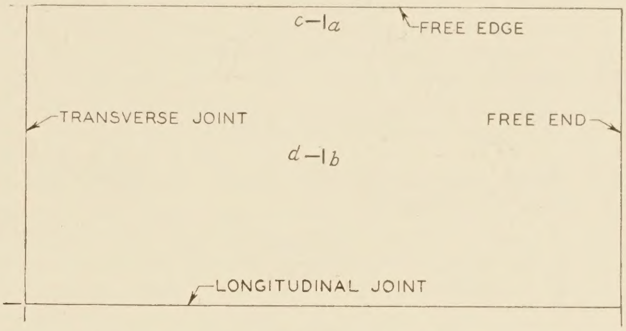

Figure 5.—Plan of 10- by 20-Foot Section Showing Location of the four Typical Strain Gages Referred to in the Description of the Method of Determining Restrained Warping Stress.

For example, in the plan of one of the 10- by 20-foot slabs shown in figure 5, four strain gage positions have been indicated by the letters *a*, *b*, *c*, and *d*. The direction of the line denoting the gage position shows the direction in which the deformation was measured. The deformation measured at *a* was considered to be the full deformation of unrestrained warping and the values measured at the other points were considered to be deformations that were modified by the restraint that existed. Any rise or fall of the temperature of the slab as a whole will cause equal volume changes at all of the gage positions and will not affect the determination of warping stresses to a measurable degree.

The four typical gage positions shown in figure 5 will be used to illustrate the method that was followed in making this study.

Since, at most points on the pavement slab, the concrete is stressed in more than one direction, consideration must be given to the effects of these other stresses on the deformations which are measured with the strain gage before the measured deformations can be converted into stresses. If an isotropic material is subjected to a force acting in a given direction, a certain unit deformation and a corresponding stress in the direction of this force will result. In addition, another deformation of lesser magnitude, of opposite sense and having a line of action perpendicular to that of the applied

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[9] See Analysis of Stresses in Concrete Pavements Due to Variations in Temperature, by H. M. Westergaard, Proc. Sixth Annual Meeting, Highway Research Board, 1927. Also see PUBLIC ROADS, vol. 8, no. 3, May 1927.

force, will be produced. The relation between the magnitudes of these two deformations is constant for a given material and is known as "Poisson's ratio." This constant provides the means for determining the effect of combined stresses.

If σ_x represents the unit stress in the direction x, then the unit deformation caused by this stress would be measured by the expression $\frac{\sigma_x}{E}$, in which E is the modulus of elasticity of the material.

Similarly, if σ_y represents the unit stress in the direction y (perpendicular to direction x), the unit deformation in the direction y would be expressed by $\frac{\sigma_y}{E}$. Also a deformation in the direction x would be induced, the magnitude of which would be expressed by $\mu\frac{\sigma_y}{E}$, μ being Poissons' ratio for the material, and this deformation would be opposite in sense to that produced by σ_x.

In case the stresses σ_x and σ_y are acting simultaneously at a point, there will be in each direction a direct and an induced deformation that must be combined. If e represents the unit deformation caused by stress, then

$$e_x=\frac{\sigma_x}{E}-\mu\frac{\sigma_y}{E}$$

and similarly

$$e_y=\frac{\sigma_y}{E}-\mu\frac{\sigma_x}{E}$$

From these equations, the following expressions for unit stress are obtained:

$$\sigma_x=\frac{(e_x+\mu e_y)E}{1-\mu^2} \qquad (1)$$

and

$$\sigma_y=\frac{(e_y+\mu e_x)E}{1-\mu^2} \qquad (2)$$[10]

DEFORMATIONS MADE UP OF THREE COMPONENTS

In this investigation the stresses caused by temperature warping were determined by comparing the deformations measured at various points on the slab surface over the period during which the temperature differential was changed and warping developed. The strain gages were generally installed at a time when no temperature differential existed and then allowed to remain until the maximum day or night temperature differential was observed. For these conditions the deformation recorded by each gage was the sum of three components combined algebraically. These component deformations are:

1. A change in length caused by the uniform change in the temperature of the slab as a whole. This change in length extends uniformly through the entire depth of the slab, creates no warping, and affects all strain gages equally.
2. The change in length of the upper surface of the pavement with respect to that of the lower surface, caused by the temperature differential created by the change in air temperature during the day. The differential in length caused by this temperature condition causes warping which, if the slab were weightless, would occur freely and would be unaccompanied by stress.
3. A deformation caused entirely by the bending stresses produced by the efforts of the slab to accommodate itself to the shape demanded by the temperature differential against the resistance of its own weight.

Since the measured deformations are not the result of combined stress alone it is necessary to modify somewhat equations (1) and (2) in order to adapt them to this method of determining the stresses produced by warping. In the following paragraphs the modified stress formulas are developed for each of the four gage positions that were shown in figure 5. The subscripts indicate the particular gage position that is being referred to.

δ = the unit change in the length of the concrete from all causes as actually measured by the strain gage.
ϵ = the unit change in the length of the concrete as measured by the strain gage but with a correction applied for the effect of the stress in a direction perpendicular to that being considered.
e = the unit change in length of the concrete caused by stress (unit strain).
μ = Poisson's ratio.

Referring again to the four gage positions shown in figure 5:

$$\epsilon_a=\delta_a-\mu e_c$$
$$\epsilon_b=\delta_b-\mu e_d$$
$$\epsilon_c=\delta_c$$
$$\epsilon_d=\delta_d-\mu e_b$$

Assuming that the concrete at point a is completely free to deform and using this deformation as a base, the strains at the four positions are:

$$e_a=0$$
$$e_b=\epsilon_a-\epsilon_b$$
$$e_c=\epsilon_a-\epsilon_c$$
$$e_d=\epsilon_a-\epsilon_d$$

If the proper values of ϵ are substituted in these formulas the strains in the concrete can be determined. These formulas multiplied by E, the modulus of elasticity of the concrete, are formulas for stress, expressed in terms of the measured deformations and the elastic constants of the material. They have the following form:

$$\sigma_a=0$$

$$\sigma_b=\frac{\delta_a-\delta_b+\mu(\delta_c-\delta_d)}{1-\mu^2}E$$

$$\sigma_c=\frac{\delta_a-\delta_c}{1+\mu}E$$

$$\sigma_d=\frac{\delta_a-\delta_d+\mu(\delta_c-\delta_b)}{1-\mu^2}E$$

Using the method described the stresses resulting from temperature warping during the day were determined for all of the gage positions shown in figure 6 for the 6- and 9-inch constant-thickness sections. For the conditions of warping that develop at night the stresses determined were those along the lines A–C and D–E and for the 6-inch constant-thickness section only.

The deformations in the concrete were measured with the recording-strain gage that has been previously described.[11] Because of the nature of the tests it was necessary to expose the strain gages to a wide variation in air temperature, varying from complete shade and

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[10] For a discussion of the action of combined stresses see Strength of Materials by S. Timoshenko, pt. I, p. 58

[11] See An Improved Recording Strain Gage, PUBLIC ROADS, vol. 14, no. 10, December 1933.

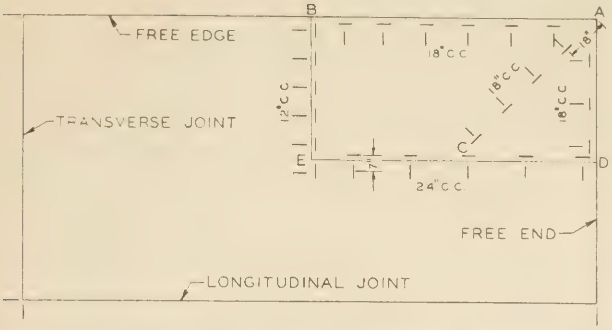

Figure 6.—Plan of 10- by 20-Foot Section Showing Location of All Strain Gages Used in Determining the Stresses Caused by Restrained Warping.

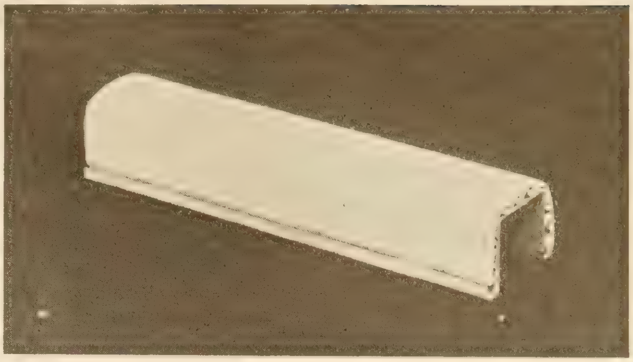

Figure 7.—Small Protective Cover Used Over the Strain Gages in Determining Stresses Caused by Restrained Warping.

low temperature to full exposure to the sun's rays and high temperature, during the period that the deformations were developing. The gages are, by their design, compensated for temperature changes to an extent that makes the effect of ordinary temperature changes negligible. Also, since all gages in the test were exposed to the same temperature conditions, theoretically the comparison of deformations used to determine the stresses would be unaffected by temperature change.

In order to insure the greatest precision possible in these measurements, it was thought advisable to give all of the gages some measure of protection from the extreme temperatures to which they were exposed. Figure 7 shows a close-up view of one of the ventilated covers used for this purpose. These covers were made of sheet metal covered with corrugated paper board as insulation. The cover was painted white on the outside to minimize heat absorption and the ends were left open to allow the air to circulate around the gage. The shelters were made as small as possible in order that the shade afforded the gages would cover no appreciable portion of the slab.

The tests to determine the stresses produced by the restrained temperature warping were made during the spring and summer months for the day condition and during the fall months for the night condition, because the temperature data showed that the maximum temperature differential for each condition occurred at these respective seasons of the year.

STRESSES CAUSED BY RESTRAINED MOISTURE WARPING DIFFICULT TO OBTAIN

At the present time there are two chief obstacles that prevent the development of information concerning the stresses caused by restrained moisture warping comparable in scope to that developed on the subject of restrained temperature warping. The first of these, the inability to determine with any precision the character of the moisture distribution in the concrete, has already been mentioned. The second is the necessity of a very long period over which the strain observations must be continuous. In this investigation it has not yet been possible to make a determination of the stresses resulting from moisture warping. However, later in this report there is some discussion of the subject based on certain observations that have been made.

DATA PRESENTED AND DISCUSSED

To give an idea of the weather conditions that prevail in the area where the tests were made, certain pertinent meteorological data covering the period from November 1, 1931, to November 1, 1932, are given in figure 8. This figure shows the daily and annual temperature variations, the monthly precipitation, and the monthly average relative humidity for a period that is believed to be typical. It will be noted that there is comparatively little freezing weather during the year, so little in fact that the earth beneath concrete pavement sections is rarely frozen. It was not possible to study the structural behavior of the sections for the condition of a frozen subgrade as extensively as was desired. The daily and annual temperature variations are large, however, and during the period of the tests a wide range of temperatures was encountered.

The temperature of the concrete in the pavement was measured on a number of days during the year for the purpose of studying the daily temperature variations of the concrete at different seasons. In some cases these observations were made at 1 or 2-hour intervals for the complete 24-hour period; in other cases they were started at about 4 a. m. and continued until late in the evening. From the data obtained it is possible to find the critical temperature conditions for each of the days and, inasmuch as the observations were made on days when large changes occurred, it is also possible to form an accurate idea of the occurrence of critical temperature conditions throughout the year. Table 1 contains typical data obtained in this manner from the 6- and 9-inch temperature slabs. This table shows the average temperature of the concrete when it was at the minimum and the maximum values for the day on which the observations were made. It also shows the actual measured temperatures in the upper and lower surfaces of the two slabs.

The tabulated difference between these two temperatures may not be the effective temperature differential, since the temperature gradient between the two surfaces may or may not be uniform. This point will be discussed more fully later.

Several interesting facts are brought out in this table. It is shown that the average slab temperature varies about 75° F. during the year. This figure is important because it is the value that controls the magnitude of the annual change in length of the pavement caused by temperature changes. The maximum surface temperature recorded during this period was 112.5° F. The data show that the temperature of the 6-inch slab is, as a whole, much more responsive to variations in air temperature than is the 9-inch slab, yet the actual temperature differential between the upper and lower surfaces is always greater for the thicker slab. Under certain maximum conditions the value of the tem-

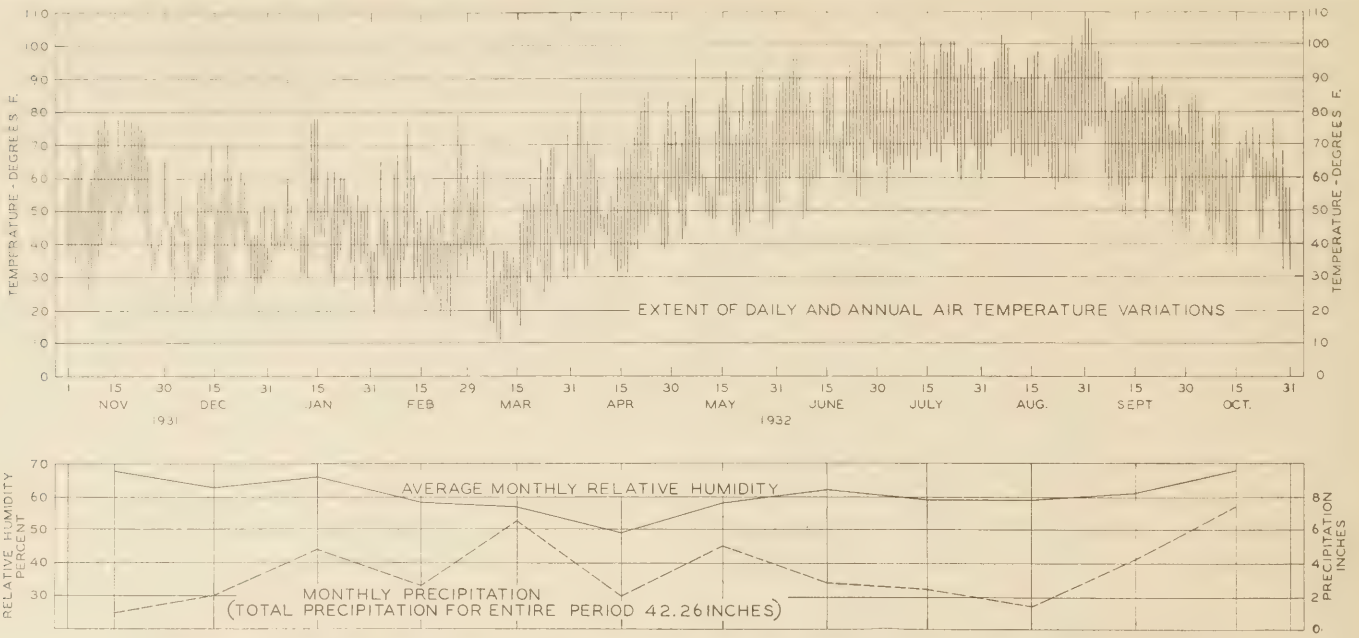

FIGURE 8.—ANNUAL VARIATIONS IN AIR TEMPERATURE, RELATIVE HUMIDITY, AND PRECIPITATION.

TABLE 1.—*Observed temperatures in concrete pavements (degrees F.)*

Date	Average temperature of concrete				Maximum temperature values											
					Night						Day					
	6-inch slab		9-inch slab		6-inch slab			9-inch slab			6-inch slab			9-inch slab		
	Minimum	Maximum	Minimum	Maximum	Bottom	Top	Difference	Bottom	Top	Difference	Bottom	Top	Difference	Bottom	Top	Difference
1931																
Nov. 24		65.1		62.1							59.2	70.0	10.8	55.6	69.1	13.5
Nov. 25	52.2		53.0		56.3	52.3	4.0	57.2	52.0	5.2						
1932																
Feb. 1	26.6	41.2	29.4	40.8	31.1	24.4	6.7	36.1	24.6	11.5	36.7	44.9	8.2	35.8	45.3	9.5
Apr. 14		67.6		63.6							59.7	81.0	21.3	50.0	81.0	31.0
Apr. 15	39.9		42.3		43.0	36.5	6.5	48.9	39.7	9.2	56.8	76.1	19.3	51.6	73.9	22.3
June 8	65.5	91.9	63.5	84.0	70.3	63.9	6.4	68.9	61.3	7.6	80.2	102.7	22.5	69.3	96.1	26.8
July 13	75.4	102.7	76.7	96.8	79.9	75.4	4.5	81.3	75.6	5.7	90.9	112.5	21.6	82.8	111.7	28.9
Aug. 5	70.7	98.4	72.0	93.7	73.8	68.4	5.4	75.4	68.0	7.4	81.3	101.7	20.4	80.2	105.8	25.6
Sept. 1	77.5	100.2			78.8	76.1	2.7				90.7	106.3	15.6			
Oct. 11	60.3	70.0			61.0	59.7	1.3				64.8	76.5	11.7			
Nov. 4	37.4	56.7			40.5	34.9	5.6				46.4	58.3	11.9			
1933																
Jan. 3	28.5	40.6			31.1	27.0	4.1				32.0	40.5	8.5			
Feb. 24	37.0	58.8			39.4	35.1	4.3				49.5	65.1	15.6			
Apr. 13	42.8	69.4			45.9	39.9	6.0				56.8	81.1	24.3			
May 19	59.7	85.6			63.0	56.7	6.3				74.5	96.1	21.6			
June 2	60.1	86.7			63.0	57.4	5.6				72.7	94.1	21.4			
Aug. 15	69.8	92.8			73.2	67.5	5.7				80.6	99.3	18.7			

perature differential in the 9-inch slab is nearly 50 percent greater than the corresponding value in the 6-inch slab. The maximum differential observed at any time in the 6-inch section was 24.3° F., while in the 9-inch slab it was 31° F.

RANGE OF DAILY PAVEMENT TEMPERATURES DETERMINED

From the data that are summarized in table 1, four individual cycles of observations were selected for graphical presentation to illustrate the range and character of the daily and annual variations in temperature that occur in concrete pavements in this locality. Each of the four cycles chosen is typical for the season in which it occurred although, as mentioned before, the observations were made on days when there were large variations in the temperature of the pavement for that particular season. In figures 9, 10, and 11 these data are presented in different ways, each graph being arranged to bring out the significance of the data with respect to some particular point.

The character of the daily variation of temperature throughout the depth of the 6- and 9-inch slabs is shown in figure 9 by means of gradient curves taken periodically throughout the day. It is apparent that the variations are much greater during the day when the sun is shining than at night, and also that the greatest variations occur during the warm seasons of the year. Both of these effects are caused by the absorption of heat from the sun's rays, and the more intense the sunlight the greater will be the effect.

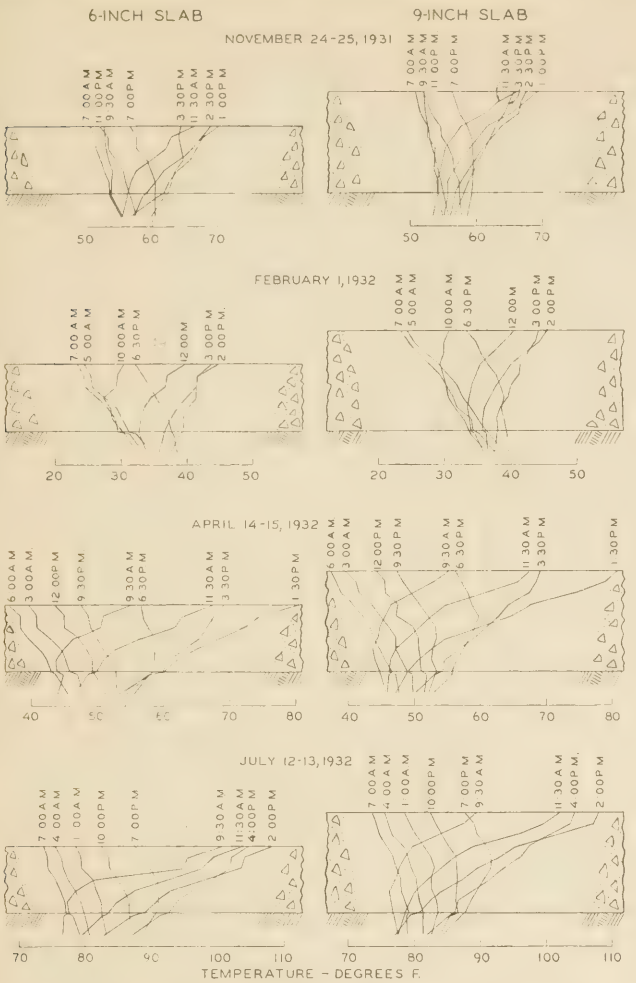

FIGURE 9.—TYPICAL DAILY AND SEASONAL TEMPERATURE VARIATIONS IN CONCRETE PAVEMENT SECTIONS.

Figure 9 shows why the difference between the temperatures of the two surfaces of a pavement is not necessarily the effective temperature differential. It is probable that an average line drawn through each of the gradient curves shown in this figure would give a better approximation of the effective differential. This is the method that has been used for determining the values of the differential given in this report. It should be noted that in the early morning and in the afternoon when the maximum temperature differentials occur, there is approximately a straight-line gradient in temperature between the upper and lower surfaces of the concrete. These are the two times of the day that are most important in the determination of stresses caused by warping.

It is not unusual at certain seasons of the year to find that the absorption of the heat from the sun has caused the temperature in the upper surface of the slab to be from 10° to 20° F. higher than the air temperature. The effect is greatest when the angle of incidence of the sun's rays to the pavement surface is greatest. In figure 10 the relation between the air temperature, the average slab temperature, and the temperatures of the two surfaces are shown for typical days at four different times during the year.

As previously explained, the average pavement temperature is obtained by averaging the values obtained from all of the thermocouples throughout the slab depth. For this reason the average temperature may not be equal to the mean of the upper and lower surface temperatures. These curves show the rapidity with which the temperature of the upper surface changes during the day and the extent to which it rises above air temperature during certain parts of the year.

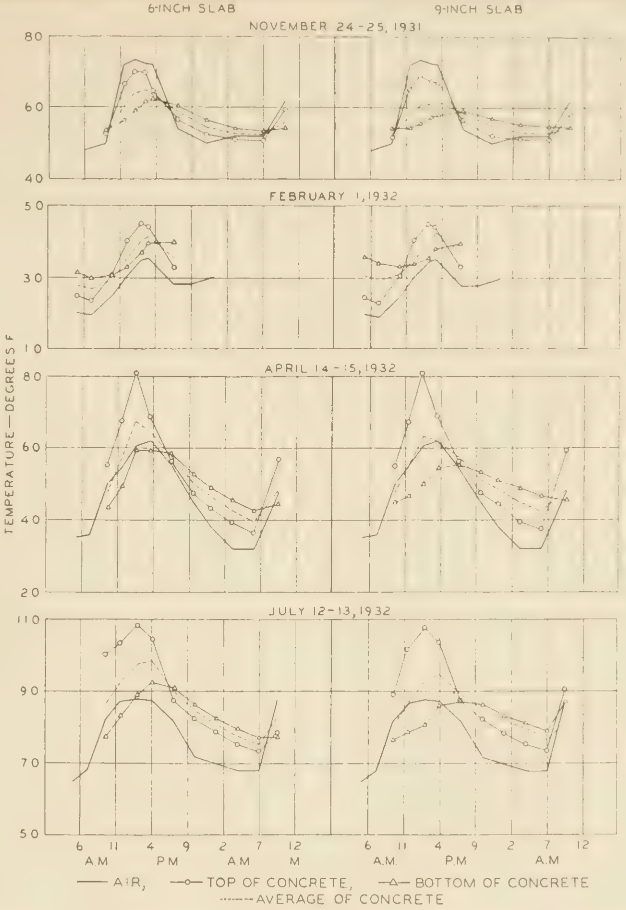

FIGURE 10.—RELATION OF AIR TEMPERATURE TO PAVEMENT TEMPERATURE.

FACTORS AFFECTING PAVEMENT TEMPERATURE DISCUSSED

During the warmer seasons of the year even the average temperature of the concrete rises above the air temperature for considerable periods of time. The temperature of the concrete on any one day is controlled not only by the air temperature on that day but also by several other factors such as the angle of incidence of the sun's rays, the previous temperature conditions, particularly as they affect the temperature of the subgrade, the moisture conditions, and the humidity of the atmosphere.

An example of the effect of these other factors is found in the data in figure 10 for February 1, 1932. It will be observed that the temperatures of both surfaces of the slab are higher than the air temperature on this date. An examination of the temperature data for the period showed that for several days preceding February 1 higher temperatures had prevailed. The heat absorbed by the concrete and the subgrade during the warmer period had not been dissipated when the observations shown in the figure were made.

The observed differences in temperature between the upper and lower surfaces of the 6-inch and 9-inch

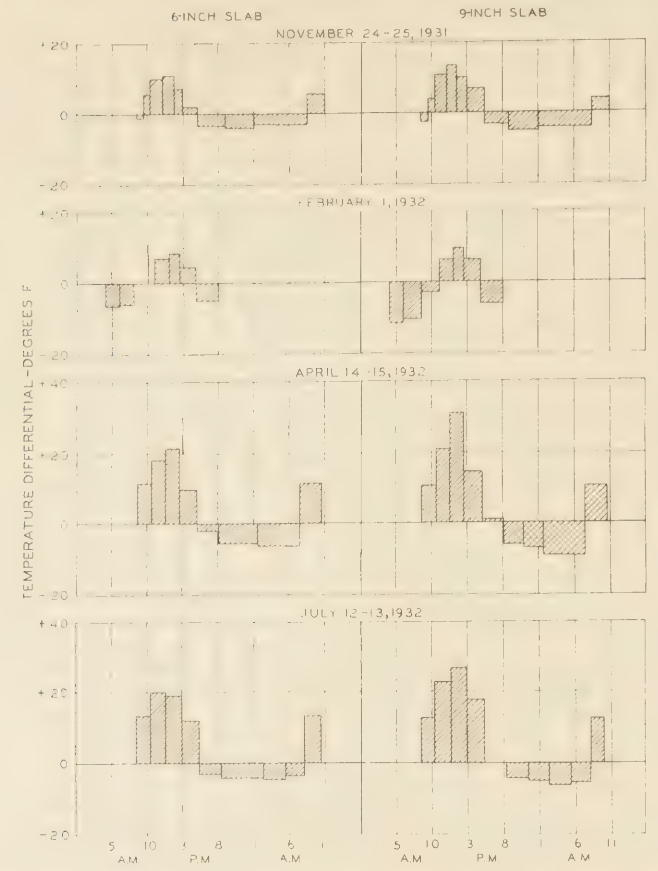

Figure 11.—Typical Daily and Seasonal Variations of Temperature Differentials in Concrete Pavement Sections.

Table 2.—*Observed maximum temperature differentials between the upper and lower surfaces of concrete pavement sections*

Date, 1934	Maximum temperature differential				
	At edge of constant-thickness sections		Thickened-edge section, 9-6-9 inch		
	6-inch slab	9-inch slab	Edge	18 inches from edge	36 inches from edge
	°F.	*°F.*	*°F.*	*°F.*	*°F.*
Apr. 3			27	25	21
Apr. 5			21	21	20
Apr. 6		20	18	17	15
Apr. 8		25	25	25	22
Apr. 10	15	23	26	24	21
Apr. 15	17	26	25	25	22
Apr. 18	18	27	27	25	22
Apr. 21	18	24	22	21	20
Apr. 22	14	24	21	20	19
Apr. 24	21	31	29	27	20
Apr. 29	15	24	25	23	21
May 1	18	28	28	25	25
May 5	23	32	33	31	27
May 7	24	31	30	30	26
May 8	20	27	28	24	22
May 9	18	25	24	22	20
May 13	21	29	31	28	24
May 14	16	24	27	26	24
May 18	21	30	30	28	25
May 19	20	26	29	27	24
May 20	23	33	32	30	28
May 21	20	31	32	29	26
May 24	19	25	25	23	21
May 28			23	25	21
June 1	21	30	30	28	26
June 3	16	29	28	28	22
June 4	19	27	27	27	23

slabs, for the 4 typical days, are shown in figure 11. The positive values indicate differences when the upper surface of the pavement is warmer than the lower surface, while the negative values apply to the opposite condition. The principal purpose of this chart is to assist in tracing the variations, during the day and during the year, of the temperature differences that cause warping.

It will be noted that during the spring and summer months when the daily changes in the temperature of the concrete are large, the difference between the maximum positive temperature differentials in the 6- and 9-inch constant-thickness slabs is approximately proportional to the difference in slab depths. During the late fall and winter months, however, when the daily changes in the temperature of the concrete are much smaller, the difference between the positive temperature differentials is much smaller. The maximum difference between the positive temperature differentials of the two slabs appears to be in the spring. At this time, although the intensity of the sunlight is very great, the subgrade is still relatively cold and the subgrade under the 9-inch slab warms up more slowly than that under the 6-inch slab. The negative temperature differentials are so small and vary so much from day to day that it is difficult to find any direct relation between these temperature differentials in slabs of different thicknesses and at different times of the year.

Mention was made previously of thermocouples that were installed especially to determine the effect of a thickened-edge cross section on the magnitude of the temperature differentials developed for various conditions of air and subgrade temperature.

Table 2 contains data obtained from measurements made on three of the test sections during a 2-month period in the spring of 1934. Each temperature differential shown is the maximum that occurred during the particular day, and the observations were made when there were large temperature variations for the season of the year. The second and third columns of the table show the observed differentials at the edges of the 6-inch and 9-inch constant-thickness sections, respectively, while the last three columns contain comparative data at three points along the cross section of a representative thickened-edge section.

A comparison of the values in the second with those in the fourth column shows the relation between the temperature differential at the edge of a 6-inch slab that is not thickened at the edge and that of one that is thickened to 9 inches. A similar comparison of the data in the third and fourth columns shows the relation between the differential of temperature in the 9-inch edge of a constant-thickness slab and that in the 9-inch edge of a thickened-edge cross section. These comparisons show that the temperature differential that develops at the edge of a 9–6–9 thickened-edge design is approximately equal to that in the edge of a 9-inch constant-thickness section and is approximately 45 percent greater than that in the edge of a 6-inch constant-thickness section. It is indicated by these data, therefore, that increasing the edge thickness of a pavement may be expected to result in a proportionate increase in the temperature differential in the edge region of the slab.

The data in the last three columns show the temperature differentials observed at distances of 2, 18, and 36 inches from the free edge of the 9–6–9 section. The data obtained at a point 36 inches from the edge probably represent very closely those that would be found throughout the 6-inch interior portion of a slab of this design. Comparing the values in the fourth column with those in the sixth, it is indicated that the differential in the edge of this section averaged, for the period of the measurements, about 20 percent greater than that at a point 36 inches from the edge. At a point 18 inches from the edge the increase was approximately 13 percent. The edge depth is 50 percent greater than the depth 36 inches from the edge and, of course, the increase in depth at 18 inches from the edge is one-half of this or 25 percent. Hence, it appears that the increase in the temperature differential near the edges over that in the interior of this thickened-edge design is less than would be expected in view of the relation of center depth to edge depth. It is believed that this is due to the stabilizing influence of the earth shoulder along the vertical face of the slab edge that acts to reduce somewhat the temperature differential. This would apply to both thickened-edge and constant-thickness sections, although the result will probably be less as the edge thickness of the sections is reduced.

EXPANSION AND CONTRACTION OF PAVEMENT SLABS MEASURED

At the same time that the temperature determinations were made the change in length of one of the 40-foot test sections was measured. In figure 12 the variations in length of this section are shown, together with the simultaneous variations in the average temperature of the concrete. These data are plotted to a common base for the same four cycles considered in the discussion of the temperature data. It will be noted that there is a very close phase relation between the temperature and expansion curves, there being little or no lag even at those times of the day when the temperature is changing most rapidly. The lag of the average pavement temperature with respect to air temperature has already been shown in figure 10.

These data show that for a given average temperature the pavement does not always have the same length. For example, at an average temperature of 60° F. in November the change in length, with respect to a certain base, was −0.0105 inch, while in April it was +0.017. This indicates that in the 5 months between November and April the length of the slab has increased 0.0275 inch from some cause other than temperature changes.

In the vicinity of Washington, D. C., the mean monthly precipitation was averaged for the period covered by the observations (1931 to 1934, inclusive) and it was found to be 4.6 inches for the summer months (April to September, inclusive) and 3.1 inches for the winter months (October to March, inclusive). Thus the precipitation was greatest for the period when the slabs were found to be shortest for a given temperature. However, evaporation measurements made in this locality a number of years ago by the Weather Bureau show that during the summer months the loss from a free-water surface averages more than 6 inches per month, while during the winter the loss through evaporation is very small.

This suggests that beginning in the late fall there should be a progressive increase in the moisture content of the soil that continues until the spring temperature rise begins, after which there should be a corresponding progressive decrease. Such soil-moisture determinations as were made confirmed this idea. This being true, it seems reasonable that during the summer months lowered moisture content in the subgrade and a relatively high evaporation rate reduce the moisture content of the concrete and that the opposite conditions in the winter will increase the moisture content of the concrete and thus account for the observed volume changes.

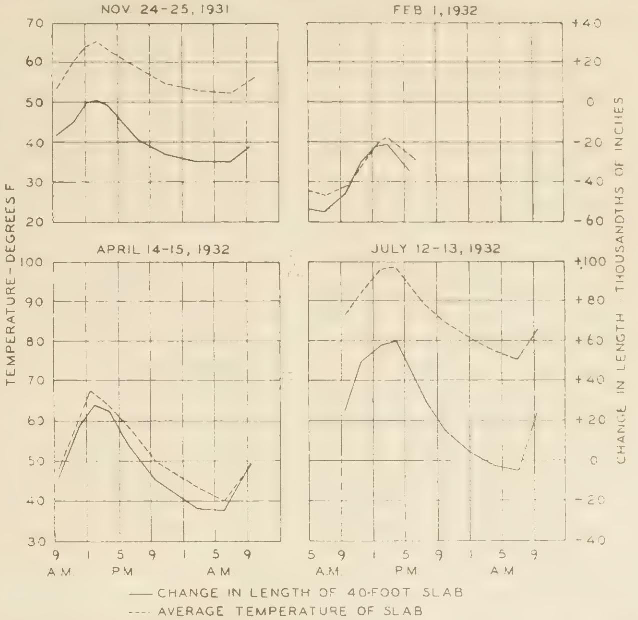

FIGURE 12.—VARIATIONS IN SLAB LENGTH COMPARED WITH VARIATIONS IN AVERAGE CONCRETE TEMPERATURE.

In order to separate the changes in length resulting from these two different causes so that the magnitude of each could be determined, the data obtained during the various daily cycles of observations were grouped on a single sheet and all referred to a common base for comparison in the manner shown in figure 13. The changes in slab length measured on any given day are plotted against the corresponding average concrete temperatures. The plotted points pertaining to a day's observations are averaged with a straight line, the slope of which is the coefficient of thermal expansion for the slab as a whole, as indicated by those particular data. Since all of the data are plotted to a common base, the spread horizontally between these daily average lines is, when taken at a common temperature, a measure of the expansion resulting from causes other than temperature changes.

Figure 13 contains a few typical data plotted in this manner for the purpose of demonstrating the method of analysis. In this graph the coefficient of thermal expansion of the concrete, as determined in the laboratory, is shown by the slope of the dash line through the center. It will be noted that the lines showing the daily averages appear to converge slightly toward the dash line. Throughout the data this convergence varies systematically with the season, indicating that the coefficient of thermal expansion determined from the daily observations of slab expansion undergoes a small annual variation. The cause of this was not determined but it seems likely that the coefficient of thermal expansion effective in the slabs varies slightly with the moisture content of the concrete.

It is possible also that warping in the slab introduces a small error in the determination of the slab length, and such an error, if present, would tend to vary systematically in the same manner as does the extent of the warping. The possibility of subgrade resistance variation being a factor was also considered but calculations indicate that the variations in subgrade resistance caused by variations in subgrade moisture would produce an effect of negligible magnitude.

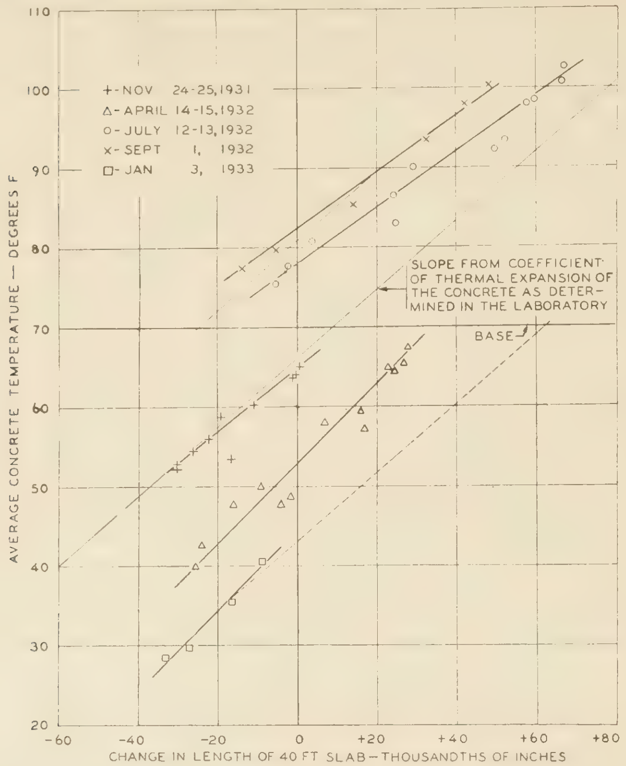

Figure 13.—Relations Between Average Concrete Temperature and Change in Length of Pavement Slab for Various Periods During the Year.

In order to eliminate, to the fullest extent possible, any error caused by this slight convergence, the determination of spread was made by extending auxiliary dotted lines from the center of gravity of each day's observations and parallel to the dash line previously mentioned to an intersection on a common temperature line, chosen arbitrarily at 70° F. in this instance. The distance between intersections is a measure of the length change attributed to moisture.

CHANGES IN PAVEMENT LENGTH CAUSED BY VARIATIONS IN MOISTURE CONTENT DETERMINED

In figure 13 data from only five sets of observations are shown. Actually, observations were made on a great many days and all of the data were analyzed in this way. From this analysis the variations in the length of the 40-foot test section resulting from variations in moisture content were obtained at frequent intervals during a 5-year period. The change in length resulting from variations in moisture content alone during this time is shown in figure 14, plotted with respect to the original length at the time that the concrete took its initial set. The measurements of length change were made on one section with the embedded telemeter during the period between September 1930 and August 1932, and with the micrometer on another section during the period between November 1931 and September 1935. During the 8 months between November 1931 and August 1932, data from both methods are available and there is very close agreement between them.

This graph indicates that there is an annual cyclic variation in length caused by variations in moisture content, the sections being longest (for a given temperature) during the winter and shortest during the summer. The magnitude of this length change is appreciable, corresponding to that which would be caused by a temperature change of 20° to 40° F. for the different years during which the measurements were made. In a 40-foot section this variation in length amounts to about 0.05 to 0.10 inch.

It will be noted that there is a considerable variation in the extent of this length change for the different years, the smallest change occurring in 1930–31 and the largest thus far observed in 1932–33. It is believed that the variation in the magnitude of the length change from year to year is the result of the particular precipitation and evaporation conditions that happened to prevail. The precipitation record is as follows:

	Inches
1930	21. 7
1931	33. 5
1932	49. 5
1933	49. 1
1934	51. 1

Unfortunately, there is not a similar record of the annual evaporation at the site of the tests. Nor was it practicable to determine the annual variation in the moisture content of the concrete over this period. As previously stated, there was no dependable method available for determining the moisture content of the concrete in the pavement. Using fragments broken from the specimen slabs and determining the moisture content by drying in the laboratory, it was found that the concrete contained 3.5 percent moisture during the summer and 3.8 percent during the winter. These values should be considered as nothing more than an indication of the variation of the moisture content of the pavement.

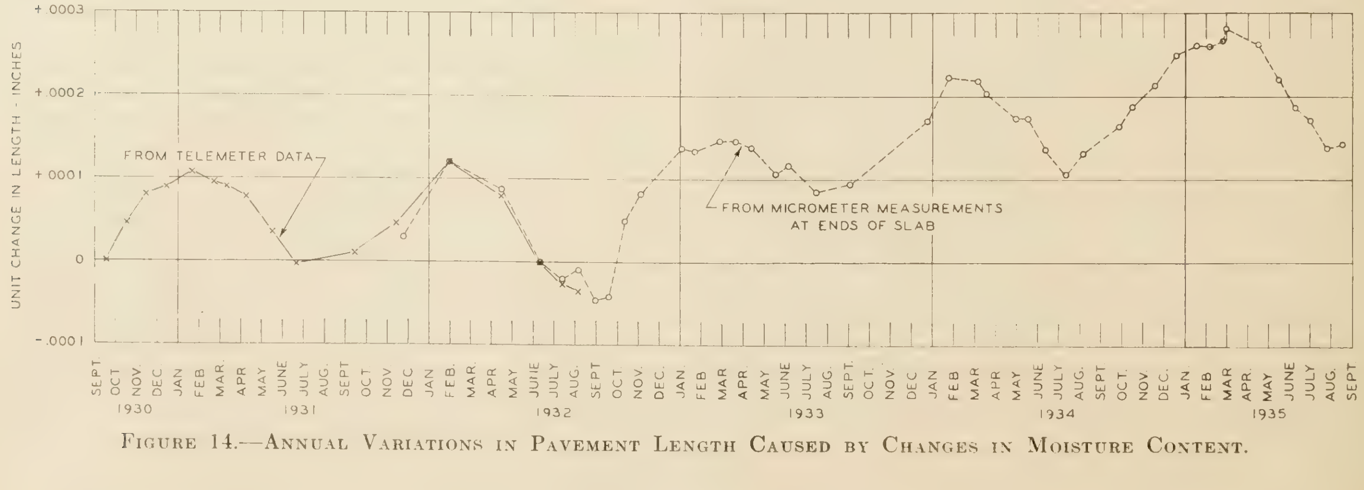

Figure 14.—Annual Variations in Pavement Length Caused by Changes in Moisture Content.

The data presented in figure 14 show a definite progressive increase in the length of the pavement that has become more marked during the fourth and fifth years. The slab, during the summer of 1934, was longer than in the summer of 1931 by 1 part in 10,000. This may seem to be a small amount but it represents a length gain of 6.3 inches in a mile of pavement. Looking at it another way, if the slab ends were completely restrained such a length gain might develop a compressive stress of several hundred pounds per square inch.

Although this tendency for concrete to "grow" in the presence of moisture has long been known and has been the subject of much speculation and experiment, the phenomenon is not well understood and the ultimate extent of the growth cannot be predicted for given materials and conditions of exposure with the information that has thus far been developed.

COEFFICIENTS OF THERMAL EXPANSION FOR CONCRETE DETERMINED BY BOTH LABORATORY AND FIELD TESTS

In figure 13 the variation in slab length was compared with the variation in average concrete temperature for typical cycles of daily observation. Such data can be used to determine the coefficient of thermal expansion that is effective for the slab as a whole. In figure 15 this has been done. The graph was constructed by plotting the maximum change in length for each daily cycle against the corresponding change in the average temperature of the slab. Each daily cycle of observations thus supplied one point for the figure. Through these points a straight line was drawn to average the data. The slope of this average line is the coefficient of thermal expansion that affects the slab as it lies on the subgrade. The value of the coefficient as obtained by the method just described is 0.0000047 inch per inch per degree F. from the telemeter data, and 0.0000049 inch per inch per degree F. from the micrometer measurements at the ends of the slab. These values compare with one of 0.0000048 determined for the same concrete in the laboratory.

The method used in the laboratory determinations was described in part 1 of this series of papers. It consists, briefly, of the casting of a 12 by 24-inch cylindrical specimen in a moisture-tight copper container with a telemeter or recording strain gage of the electrical resistance type embedded in the center of the specimen. As previously noted, each telemeter contains a resistance thermometer, thus permitting simultaneous observations of deformations and temperatures.

After the concrete had hardened and cooled, the sealed cylinder was placed in water baths at 32° F. and 110° F. alternately a number of times, remaining in each until complete temperature equilibrium was attained. Since loss of moisture is prevented by the copper jacket, the difference in length measured by the telemeter under these conditions is the result of temperature change and from it the value of the coefficient of thermal expansion was obtained. The method was later used for determining the thermal coefficient for the concrete in connection with the elaborate test program on the Rogue River bridge.[12]

The close agreement between the coefficients of thermal expansion for the pavement and those obtained with unrestrained concrete in the laboratory indicates that the stresses in the pavement caused by the resistance of the subgrade to horizontal slab movement must be very small.

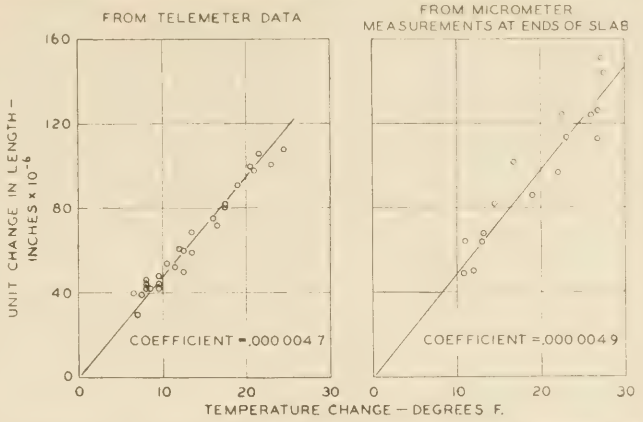

Figure 15.—Relations Between Temperature Change and Expansion for Concrete Pavement Slab.

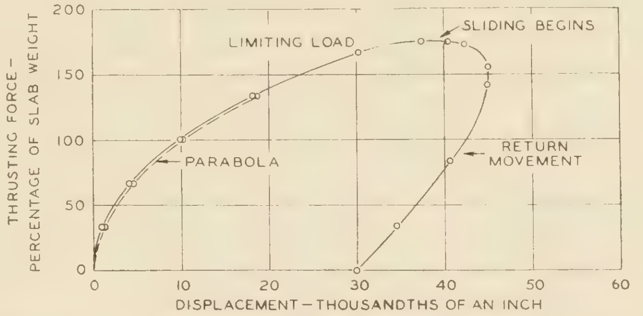

Figure 16.—Typical Force-Displacement Curve for Horizontal Slab Movement.

RESISTANCE OF SUBGRADE TO HORIZONTAL SLAB DISPLACEMENT MEASURED

It will be recalled that in the earlier mention of these tests it was stated that the procedure adopted as a result of the preliminary study was one that would subject the subgrade to the same manipulation, as nearly as possible, as that which it receives under a pavement.

The test slabs were moved a distance of approximately 0.040 inch during a period of approximately 6 hours and they were moved in opposite directions on alternate days. The displacement of the slab was measured immediately after the application of each increment of thrust and again just before the next increment was applied.

Figure 16 shows typical data resulting from one of these tests. In this instance the horizontal force, or thrust, was applied at the rate of 50 pounds every 10 minutes until a total of 2,100 pounds caused visible sliding to begin. As soon as this point was reached the reduction of the force was started and continued at the rate of 100 pounds every 10 minutes until all horizontal thrust had been removed.

The curve shows the force-displacement data for the entire test. As the increments of force were applied the successive increments of displacement increased in magnitude in a ratio that closely approximates a parabola as shown by the dotted line adjacent to the curve. During this period there was no visible evidence of the concrete sliding on the subgrade. As the force being applied reached a value of approximately 150

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[12] Application of the Freyssinet Method of Concrete Arch Construction, by Gemeny and McCullough. Tech. Bul. No. 2, Oregon State Highway Department, 1933.

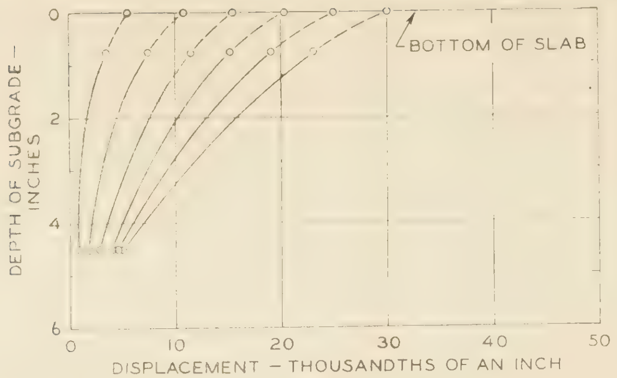


FIGURE 17.—MOVEMENT OF SUBGRADE COMPARED WITH SLAB DISPLACEMENT.

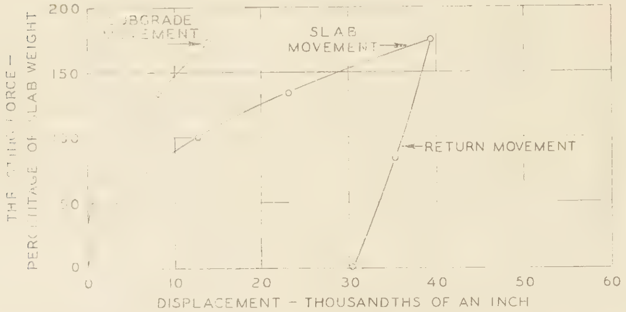


FIGURE 18.—RELATIVE MOVEMENTS OF PAVEMENT AND SUBGRADE FOR A 6-INCH SLAB.

percent of the weight of the test slab there is evidence that a condition of actual sliding is impending.

In this test free sliding occurred with a thrust equal to 175 percent of the slab weight and a force greater than this could not be applied. Reduction of this force started a movement of the slab in the opposite or return direction, and in this test it will be observed that complete removal of the thrust caused a recovery of about one third of the total displacement. This recovery is believed to result from elastic deformation of the subgrade caused by the adherence of the earth to the bottom of the slab. When the slab moves there is an actual movement of the subgrade with it.

The character of the horizontal movement that occurs in the subgrade when there is a displacement of the slab is indicated by the test data in figure 17. In these tests micrometer dials measured the horizontal soil movement at depths of three-fourths inch and 4½ inches as the slab was being moved. The particular test for which the data are shown was one in which a slot or groove 5 inches deep had been cut vertically into the subgrade just ahead of the leading edge of the slab. The presence of this groove probably affected somewhat the magnitude of the displacements produced by a given thrust.

Figure 18 shows the average data obtained from a number of tests in which the "bending" of the subgrade was measured without the disturbing influence of the groove just mentioned. The subgrade displacement in this figure is the average of measurements made at both sides and the center of the leading edge of the slab and at a depth of three-fourths inch. In will be observed that at this depth the subgrade movement is about 30 percent of the slab displacement. The return movement of the slab after the release of the thrust is about 25 percent. In many cases the percentage of return was greater than this, ranging up to about 40 percent of the total displacement.

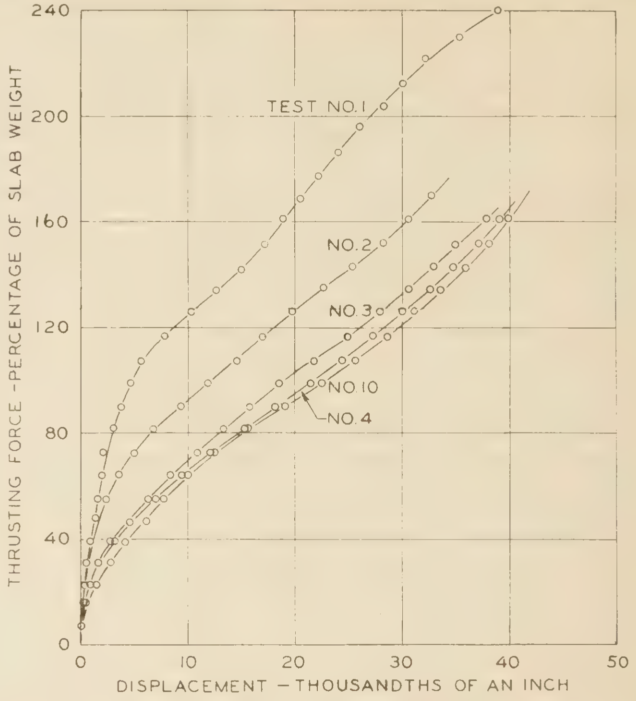


FIGURE 19.—FORCE-DISPLACEMENT CURVES FOR REPEATED TESTS ON 6-INCH SLAB.

A few tests were made to determine how long the subgrade would maintain this elastic resistance that caused it to move the slab back toward its original position, and it was found that after the horizontal thrust had been kept at a constant value for 8 days and then released, the return movement was practically as great as if the slab had been displaced but momentarily. This indicates that the subgrade tested had a high degree of elastic action for small displacements.

Tests were made in which the slab was displaced a given amount several times, in exactly the same manner each time. The data obtained are shown in figure 19 and it appears that with each successive application there is a reduction in the amount of thrust required to produce a given movement. A condition of practical stability seems to have been reached, however, after a comparatively small number of movements. These data indicate that, for a given subgrade, the resistance to slab movement may be greater for the first movements of the newly constructed pavement than it is later when the concrete has expanded and contracted a number of times.

EFFECT OF SLAB WEIGHT ON RESISTANCE TO HORIZONTAL DISPLACEMENT INVESTIGATED

As previously mentioned, some effort was made to determine the effect of slab weight on the resistance to horizontal displacement by means of tests with slabs of 2, 4, 6, and 8-inch thicknesses.

In making these tests the procedure was first to move the slabs forward and backward through a distance of 0.040 inch several times until it appeared that the subgrade resistance had become stabilized for displacements of this magnitude. The test slabs were then moved through distances of 0.070 and 0.100 inch in the same manner. There was a tendency for all of the slabs to continue to slide under the action of

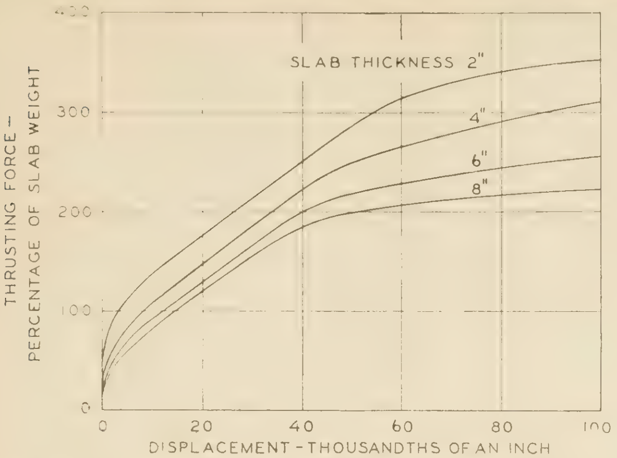


FIGURE 20.—FORCE-DISPLACEMENT CURVES FOR 2-, 4-, 6-, AND 8-INCH SLABS.

thrusting forces that would produce the 0.100 inch displacement, and these may be considered as the maximum horizontal resisting forces that could be developed.

Figure 20 shows the relation between thrusting force and displacement for each thickness of slab. The data for displacements of 0.040 inch or less were obtained during the tests in which the maximum displacement was 0.040 inch, and the data for displacements of more than 0.040 inch were obtained from the tests in which the slabs were moved either 0.070 or 0.100 inch.

It is apparent from this figure that the forces necessary to move the slabs of different thicknesses do not bear a constant relation to the respective slab weights for the subgrade in question. In this connection it is well to bear in mind that the total displacement of the slab may be composed of two parts: First, an elastic or semielastic displacement of the soil particles with no sliding of the slab as such; and second, an actual slipping of the slab over the soil surface. The first action necessarily begins as soon as slab displacement starts. Whether or not the second action follows depends upon the nature of the soil and the magnitude of the displacement.

The relation between thrusting force and slab thickness for displacements of several magnitudes is shown in a different manner in figure 21. Again it is apparent that the magnitude of the thrusting forces required is not directly proportional to the respective slab thicknesses (or weights). If it were, the sheaf of curves in this figure would all be straight lines passing through the origin of the graph. To illustrate the variation in another way, if all of the resisting forces were summed up in a coefficient to be applied to the weight of the slab, the value of this coefficient, instead of being constant, would vary with slab weight and displacement as given in table 3.

TABLE 3.—*Variations of subgrade coefficients of resistance to displacement with slab weight*

Slab thickness	Slab weight	Coefficients of resistance to displacement for displacements of—					
		0.01 inch	0.02 inch	0.03 inch	0.04 inch	0.07 inch	0.10 inch
Inches	*Lbs. per sq. in.*						
8	0.67	0.8	1.2	1.5	1.8	2.1	2.2
6	.50	.9	1.3	1.6	2.0	2.4	2.5
4	.33	1.1	1.5	1.8	2.2	2.8	3.1
2	.17	1.3	1.7	2.1	2.5	3.3	3.5

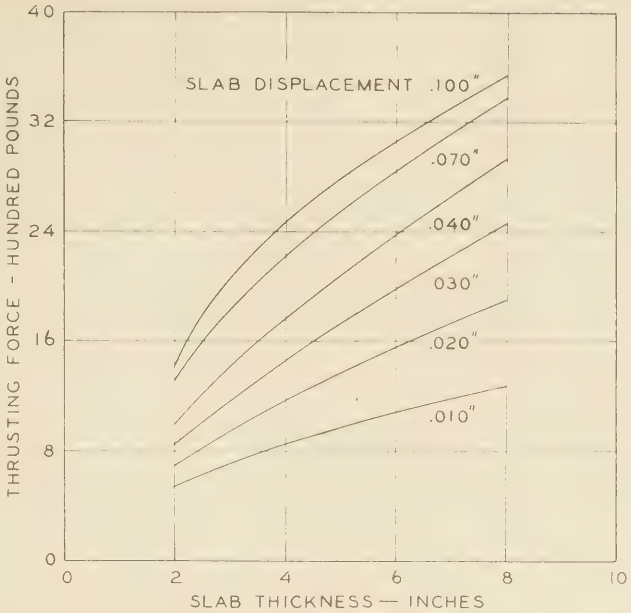


FIGURE 21.—RELATIONS BETWEEN SLAB THICKNESS AND RESISTANCE TO DISPLACEMENT.

The data indicate that a weightless slab in intimate contact with the subgrade might, under certain conditions of soil and moisture, develop a very considerable resistance to horizontal displacement.

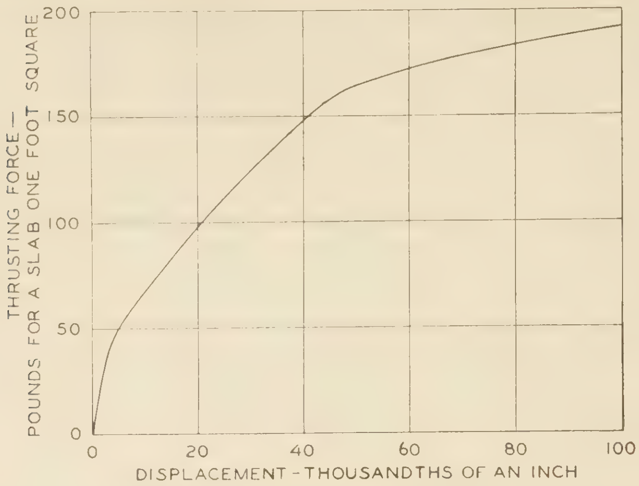
From the results of these studies it seems reasonable to conclude that the resistance offered by the subgrade to the horizontal movement of a pavement slab is composed of two elements: (1) A resistance caused by an elastic or semielastic deformation within the soil; and (2) a resistance that approximates closely that of simple sliding friction. The first of these appears to be independent of slab weight, while the second varies directly with slab weight. It seems quite probable that the relative magnitudes of these two components will vary with different subgrade soils, although no data on this point have been obtained.

METHOD OF CALCULATING PAVEMENT STRESSES DEVELOPED BY SUBGRADE RESISTANCE ILLUSTRATED

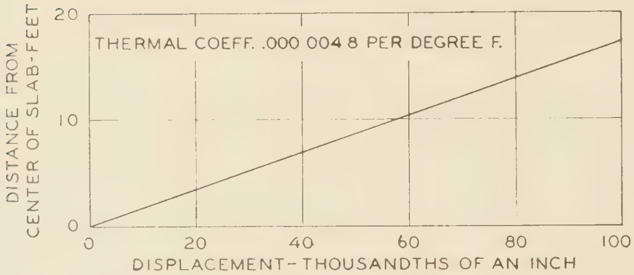
It will be apparent from the data and discussion which have been presented that, in any consideration of pavement stresses developed by this subgrade resistance, account should be taken of the extent of the displacement of each part of the slab. A suggested method for utilizing the data for this purpose is outlined in the following example:

In figure 22A the force-displacement curve for the 6-inch slab is repeated from figure 20. In this figure the vertical scale applies to a unit area of one square foot. This scale was chosen simply as a matter of convenience for use in the example. In figure 22B is a curve showing the displacement, resulting from thermal expansion, of the various parts of the slab with respect to its center point, based on the measured thermal coefficient of the concrete and a temperature change of 100° F. Inasmuch as the observed average temperature of the test pavement underwent an annual change of only 75° F., this figure may be too high but it serves the purpose of illustration as well as a lower one.

After having determined the forces necessary to move the unit slab through various displacements and having determined the displacements to which the



A - FORCE - DISPLACEMENT RELATION FOR A 6 - INCH SLAB OF UNIT AREA



B. - DISPLACEMENT AT VARIOUS DISTANCES FROM SLAB CENTER DUE TO A TEMPERATURE CHANGE OF 100° F.

Figure 22.—Relations Between Displacement and Thrusting Force, and Between Displacement and Distance from Center of Slab, for Concrete Pavement Slabs.

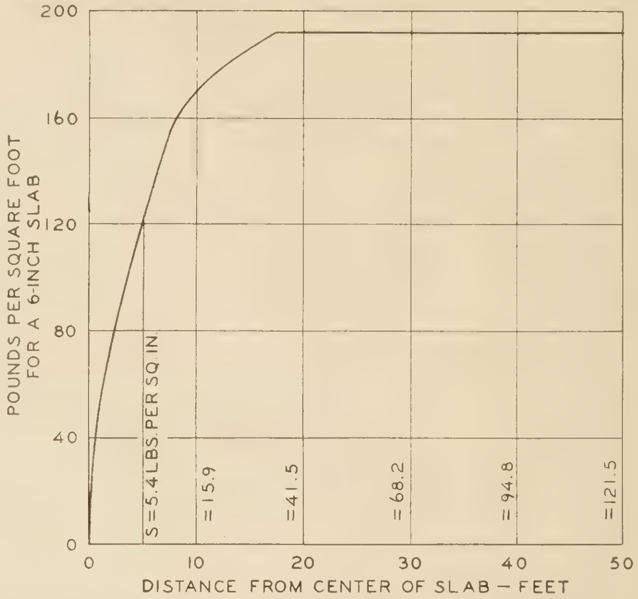


Figure 23.—Variations in Force Needed to Move Unit Areas at Various Points Along the Pavement Slab. The Resultant Stresses are Shown at Intervals.

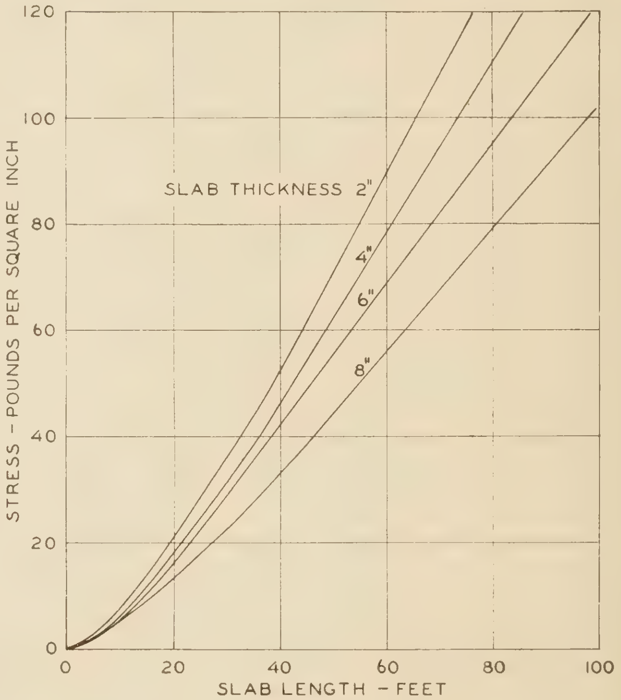


Figure 24.—Effect of Slab Length and Slab Weight on the Stresses Caused by Expansion or Contraction Resulting from a Temperature Change of 100° F.

various unit areas of the pavement may be moved by thermal expansion, the next step is to combine these data on the basis of equal displacements in order to determine the forces of resistance effective at each point throughout the length of the slab. The procedure is as follows:

Assuming a strip of pavement 1 foot in width and of appreciable length, consideration was given to successive sections 1 foot apart beginning at the mid-section of the strip (the section at which no displacement occurs during expansion and contraction). From figure 22B the displacement to be expected at each of the successive sections, caused by the assumed change in temperature, was determined. Then for each displacement the corresponding thrusting force was obtained for each section from the data given in figure 22A. The values of force developed in this way and plotted at the proper distances from the center of the slab determine the curve shown in figure 23. The total force necessary to move a strip of pavement of any given length may be determined from the area under the curve in this figure. The unit stress values to be expected are shown at intervals along the diagram, at the points where they apply.

The stresses caused by subgrade resistance were computed in this manner for the several slab thicknesses from the force-displacement curves shown in figure 20 and the resulting stress distribution diagrams are given in figure 24. These diagrams are applicable only to the particular subgrade conditions that obtained in these tests. It will be observed that the unit stresses developed by the large temperature change that was assumed for the example are small for all moderate slab lengths and also that the unit stresses for a given slab length decrease with increase in slab thickness, in accordance with the theory of subgrade resistance previously discussed.

It is interesting to compare the unit stresses obtained in the above analysis with those which would be obtained by assuming that all of the subgrade resistance developed as the result of simple sliding friction. Referring back to figure 20, it is found that for the 6-inch slab the thrust required to cause sliding was about 250 percent of the slab weight. In other words, a coefficient of friction of 2.5 might be assumed to apply. For a 6-inch slab 50 feet in length the application of this flat coefficient indicates a maximum unit stress of 62.5 pounds. This compares with a unit stress of 56 pounds by the more exact method. The difference is small but it should be remembered that this difference is caused by the elastic action of the soil and will, therefore, vary with the type and condition of the subgrade.

It seems probable that the stresses determined by the two methods would agree best for granular soils, and that for tenacious clays the values obtained by the assumption of a flat coefficient might be considerably in error. For slabs of average length, however, the stresses developed by this type of subgrade resistance are so small that, for average subgrade conditions, stresses computed by the simpler method are probably sufficiently accurate. In the tests at Arlington it was found that there was a tendency for the resistance to horizontal slab displacement to increase as the moisture content of the subgrade increased. Since the coefficient values given in the preceding tabulation were based upon data obtained at a time when the moisture content of the soil was high, it is probable that the values given approximate a maximum for the subgrade in question.

During the period of the observations there was no extended period during which the soil beneath the concrete was frozen, and it was not possible to make a study of the effect of a frozen subgrade upon the stresses being discussed. Temperature measurements in the concrete and subgrade and measurements of length changes during periods of cold weather showed that the changes in average concrete temperature at these times were relatively small. The daily air temperature range is much smaller in winter than in summer because of the decreased intensity of the sunlight. It seems quite possible, therefore, that even if the subgrade is frozen to the slab, the thermal changes in the pavement during such periods will be so small that the stresses in the pavement will not be increased to any important degree by the frozen subgrade.

TEMPERATURE WARPING DISCUSSED

Some difficulty was experienced in determining the shape of a warped slab, as approximately 1½ hours were required to make a complete set of clinometer measurements, and the temperature conditions that produced the warping rarely held constant for this length of time. This was especially true in the early morning when the maximum upward movement at the edges occurred. The data of most value are those made on days when, during the period of the actual measurement, the least change in temperature differential occurred. The occasions were rare when practically constant temperature conditions prevailed during the periods of measurement of both the flat and warped slab. Some measurements were obtained under these conditions, however, and it is believed that these data show very well the shape of the warped surface and the relative movements of its various parts.

Figures 25 and 26 show data obtained on one panel of each of the 6- and 9-inch uniform-thickness sections, respectively, and figure 27 shows similar data from one panel of the 9–7–9 thickened-edge section.

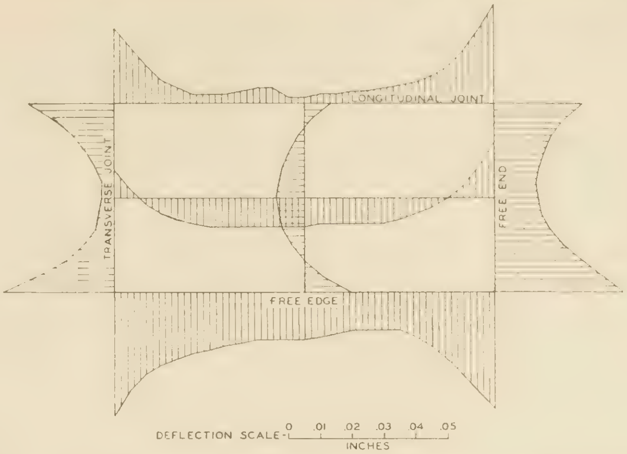


Figure 25.—Deflections Caused by Temperature Warping of a 10-by-20-Foot Panel of a 6-Inch Uniform-Thickness Pavement from a 22° F. Temperature Differential. On April 20, 1931: Slab Flat at 8:30 p. m.; Edges Warped Down at 1:30 p. m.

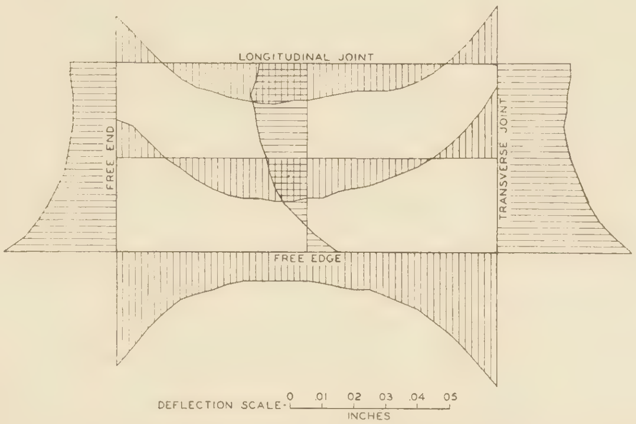


Figure 26.—Deflections Caused by Temperature Warping of a 10- by 20-foot Panel of a 9-inch Uniform-Thickness Pavement from a 23° F. Temperature Differential. On April 8, 1931; Slab Flat at 10:30 p. m.; Edges Warped Down at 2:30 p. m.

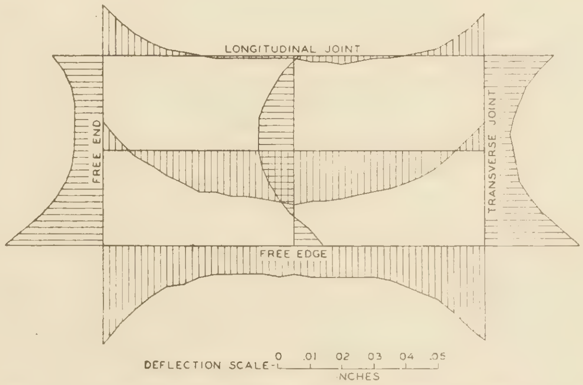


Figure 27.—Deflections Caused by Temperature Warping of a 10- by 20-foot Panel of a 9–7–9 Pavement from a 20° F. Temperature Differential. On April 9, 1931; Slab Flat at 9:30 p. m.; Edges Warped Down at 2 p. m.

The restraining influence of the attachment to other panels at the longitudinal and transverse joints is evident in all of these diagrams. In the 9-inch uniform-thickness section the longitudinal joint is of the weakened-plane type. At the time of the warping measurements this plane had not cracked, with the result that the section acted almost as a full 20-foot-width slab. The formed groove in the upper surface of the pavement undoubtedly had some effect on the shape of the warped cross section even though the concrete below it had not cracked.

It will be noted that there are slight discrepancies in the deflections recorded at the common corner points when the measurements were made along the different edges of the panel. These are caused by the slight changes in the temperature differential during the period of the measurements.

All of the observations were made during the month of April when there is probably as much warping as at any time during the year, and the data shown are for the condition of downward movement of the edges. The data for the different sections are not directly comparable because there were some small differences in the differential at the time the various measurements were made, as noted on the diagrams.

These diagrams convey a very clear picture of the movements that occur daily in all concrete pavement slabs. In the transverse direction the panels appear to warp quite freely (except in the case of the unbroken longitudinal joint mentioned above). In the longitudinal direction the tendency for the weight of the slab to force the central area to lie flat is evident, being most noticeable if a comparison is made of the central portion of the longitudinal axes in the diagrams for the 6- and 9-inch sections shown in figures 25 and 26, respectively. This is as would be expected since the curvature should be some function of the ratio of length to thickness. The effect of this restraint on the stresses caused by warping will be apparent in data that are presented later.

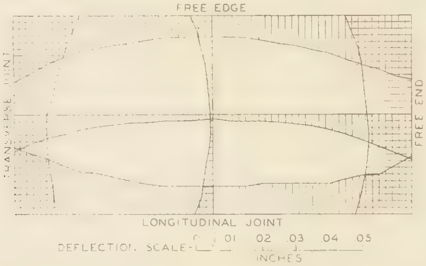


Figure 28.—Deflections Caused by Temperature Warping of a 10- by 20-foot Panel of a 9–6.3–9 Pavement from a 5° F. Temperature Differential. (American Association of State Highway Officials Cross Section). On May 27–28, 1931; Slab Flat at 8 p. m.; Edges Warped up at 5 a. m.

Figure 28 shows data obtained from a thickened-edge section of a different type at a time when the upper surface was at a lower temperature than the lower surface, with a consequent upward movement of the edges. As would be expected from the data on temperature differentials previously presented, the magnitude of the warping in this direction is always much less, since the temperature differential is less than that which occurs when the temperature conditions are reversed. The relation between the magnitude of the temperature differentials and that of the edge movements in the two directions is not a direct one.

In figures 25, 26, and 27 the downward edge movement shown was caused by differentials of about 20° F. The upward movement shown in figure 28, which is about half of that shown in the preceding three figures. is the result of a temperature differential of only 5° F. If the warping diagrams are examined it will be observed that the interior of the panel was actually raised above the flat position as the edges warped downward but that there is little or no depression of the central area as the edges are warped upward. It seems quite probable that the redistribution of subgrade reactions which must attend these changes in shape would account for the difference in the freedom of warping in the two directions that has been noted.

Figure 29.—Measuring the Pressure Between the Pavement Slab and the Subgrade.

RELATIONS BETWEEN TEMPERATURE WARPING AND SUBGRADE PRESSURE DETERMINED

Some data on the variation in intensity of subgrade pressure resulting from temperature warping were obtained with the pressure cells before the installation ceased functioning. The pressures were measured at several points across the transverse axis of one panel of the 9-inch uniform-thickness section (see fig. 29). The weakened-plane longitudinal joint in this section had not broken at the time and the slab was acting practically as one of a full 20-foot width.

Figure 30 shows the measured unit pressures at five different times during the day. The relation between the warping of the slab and the distribution of reactions is readily apparent. In the morning with the edges of the slab warped upward, the greatest pressure measured was in the interior, the pressure toward the edge being at its minimum at this time. During the day, as the upper surface of the slab expands, there is a complete relief of the high pressure in the interior region and a development of a maximum reaction near the edge, exceeding the maximum previously developed in the interior by at least 50 percent.

Both the warping data and the subgrade pressure data suggest the possibility of an actual lifting of the central area from the subgrade as the edges warp downward. While it is possible that this may actually occur, it seems more probable that the vertical movements measured were all within the range of elastic deformation of this particular subgrade soil. Sub-

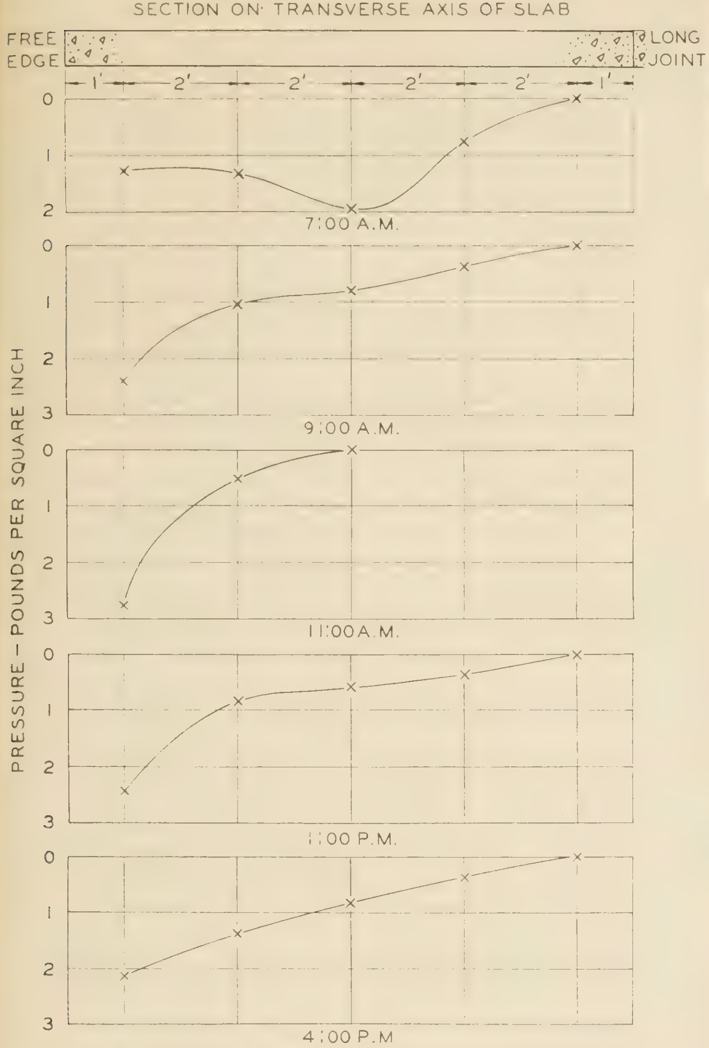


Figure 30.—Effect of Warping upon the Pressure of the Slab Against the Subgrade for a 9-inch Slab.

grade bearing tests indicate that the changes of unit pressure shown by the pressure cells might be expected to cause soil deformations approximating in magnitude the vertical movements of the interior of the slab during the daily cycle of warping.

The daily variations in shape of the transverse sections of a pavement slab are well illustrated by the data in figure 31. These data furnish additional evidence on the relative magnitude of the upward and downward temperature warping and show that the maximum warping occurs during the warmest part of the day. Measurements taken at many different times throughout the year show this to be true for all normal days irrespective of the season. This is as would be expected in view of the temperature data that have been obtained.

The daily movements of certain points at the edges of some of the sections measured with micrometer dials are presented in figures 32 and 33. The object of the measurements shown in figure 32 was to compare the movements of sections having different thicknesses and cross sections under exactly the same conditions of air temperature variation. The vertical movement of the corner of a 9-inch uniform-thickness section is 150 percent of that of one 6 inches thick. In other

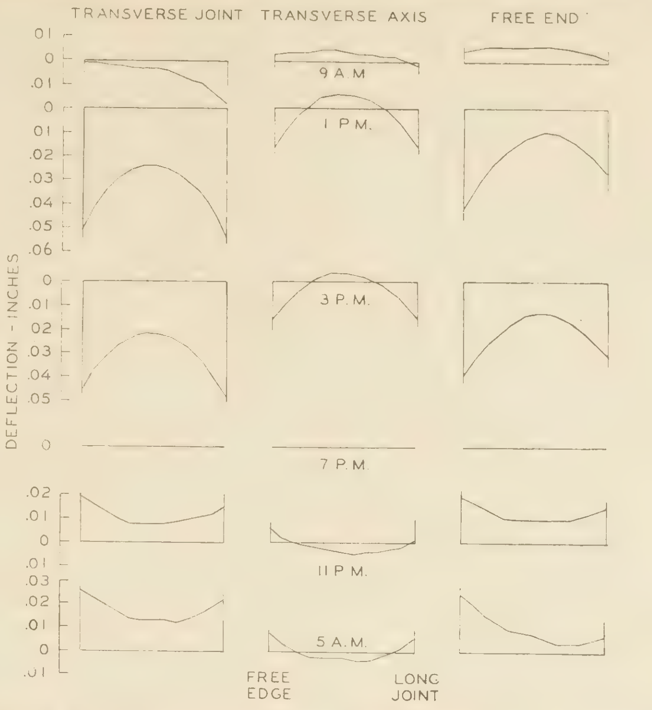


Figure 31.—Warping of Transverse Elements of 6-inch Uniform-Thickness Section. May 27–28, 1931.

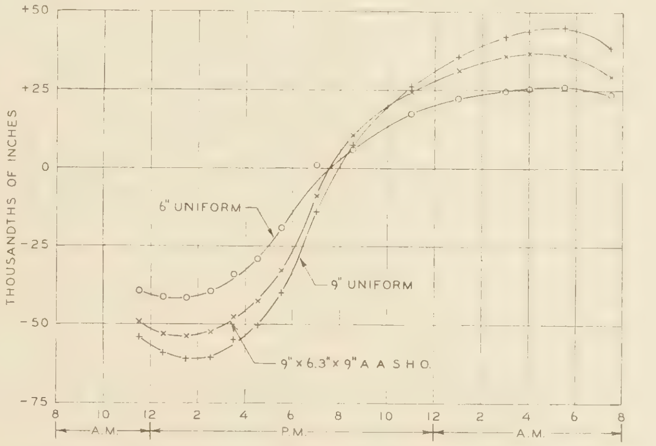


Figure 32.—Warping of Corners of Three Test Sections, May 27–28, 1931.

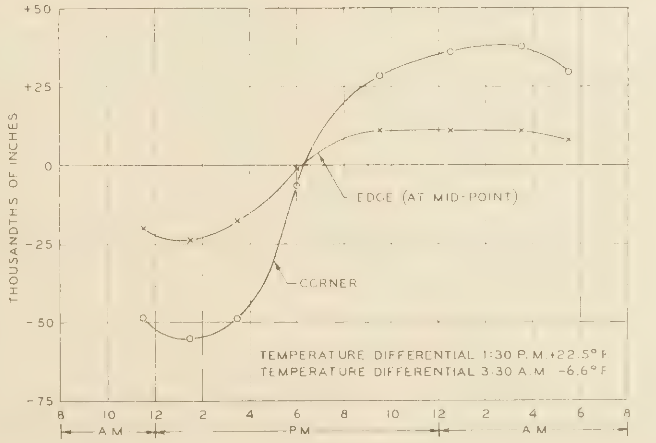


Figure 33.—Warping of Edge and Corner of 6-inch Uniform Thickness Slab, June 8–9, 1932.

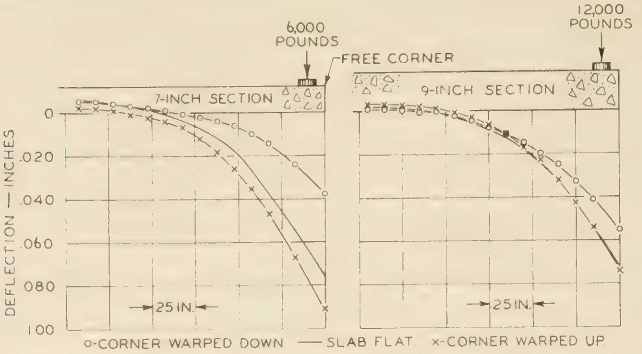


FIGURE 34.—EFFECT OF TEMPERATURE WARPING ON THE CORNER DEFLECTIONS CAUSED BY AN APPLIED LOAD (DEFLECTIONS MEASURED ALONG THE DIAGONAL).

words, direct proportionality apparently exists between the thickness of the section and the magnitude of the corner movement.

Another important indication is that the 9-inch edge of a typical thickened-edge section does not cause warping movements as great as would be found in a section that was uniformly 9 inches thick. In figure 33 the vertical movements of the free corner and the mid-point of the free edge are compared. The maximum temperature differentials observed during this series of observations are noted in the figure and it is of interest to compare these with the vertical movements that they produce at the two points at which the measurements were made. The temperature differential that caused the upward movement of the edges and corners is approximately 30 percent of that which caused downward movement, yet at the corner the upward movement is 70 percent and at the mid-point of the free edge 46 percent of the downward movement. These relations are in general accord with the data previously presented in connection with the discussion of the warping data for the entire slab panel.

EFFECT OF APPLIED LOADS ON WARPED PAVEMENTS INVESTIGATED

It is logical that any redistribution of subgrade reactions, such as those occurring when a pavement warps, must result in a change in the deflections and the stresses that will be produced by a given applied load. As mentioned earlier in the discussion of the investigation, a study was made of the effect of slab shape on the deflections and stresses caused by applied loads. Figure 34 shows the elastic curves of the diagonal at the slab corner under the applied loads noted, for three conditions of warping as they occurred during a single 24-hour period. It is apparent from these curves that the 7-inch slab is affected to a greater degree than the 9-inch slab. Downward warping of the corner reduced the deflection resulting from load by about 50 percent for the 7-inch section and only 25 percent for the 9-inch section. It is believed that this difference is due to the fact that the thinner the slab the more dependent it is on the conditions of local subgrade support.

The data indicate that upward warping has but little effect on the extent of the corner deflection produced by a given load. In the case of the 9-inch slab there is no increase, while for the 7-inch section a slight increase is noted. This condition could be caused by the lack of complete contact with the subgrade with the slab in the flat position, a condition which might easily obtain at the corner of a pavement slab. Complete contact would then be developed only by a downward deflection of the corner.

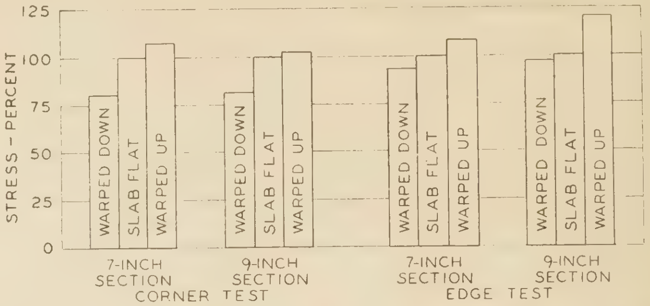


FIGURE 35.—EFFECT OF TEMPERATURE WARPING ON THE MAGNITUDE OF THE STRESS CAUSED BY AN APPLIED LOAD.

Some idea of the effect of the condition of warping on the magnitude of the stress that a given load will cause may be had from figure 35. This chart shows the variations in the critical stresses at the corner and edge of two of the sections, referred to the stress produced by the given load on the unwarped slabs as a base. These data were obtained at the same time and under the same conditions as the deflection data given in the preceding figure.

There is a reduction of approximately 20 percent in the critical stress for the corner loading when the corner is warped downward. Since the maximum working stress was about 300 pounds per square inch, this reduction amounted to approximately 60 pounds per square inch. There is also a slight increase in the critical stress if the load is applied at a time when the corner is warped upward.

For loads applied at the edges, it appears that downward warping results in but a slight reduction in the stress produced by a load while upward warping will cause increases that may amount to as much as 20 percent.

Reference to the figures which show the shape of the warped panel suggests a reason for the effects that have just been noted. When the temperature conditions are such that the edges of the pavement warp downward, the longitudinal curvature of the panel is such that the mid-point of the edge tends to move upward at the same time that the transverse curvature is forcing it downward. The result is that this point is not displaced downward to nearly the same degree as is the corner of the slab. So far as subgrade support is concerned the situation is probably but little better than it is for the flat slab condition.

Similar tests were made at the interior of both the 7-inch and 9-inch slabs and it was found that at this point the condition of slab warping has a negligible effect upon the magnitude of the critical stress produced by a given applied load.

These tests have shown quite definitely that even extreme conditions of temperature warping produce variations in the critical stresses caused by applied loads that are considerably smaller than has been generally supposed.

MOISTURE WARPING DISCUSSED

As stated earlier, clinometer measurements on certain critical regions of the 9-inch constant-thickness slab were made periodically over the year at times when no temperature differential could be detected in the concrete. The curves obtained from these measure-

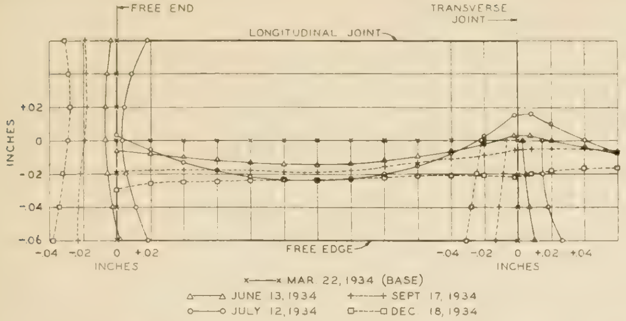


Figure 36.—Variation in Slab Shape at Various Times During the Year from Causes Other Than Temperature Changes, 9-inch Section. Upward Movements Shown as Positive, and Downward Movements Shown as Negative.

ments show the variation in shape of certain parts of the pavement slab at various times during the year from causes other than temperature changes.

Figure 36 shows typical data obtained from these observations of the 9-inch section. While actually a considerable number of sets of such data were obtained, in the figure only a few sets are shown for the sake of clarity. Since no means was available for determining the moisture gradient of the slab, it was not possible to predict with certainty the time when moisture conditions would be such as to cause it to be in a flat condition. In the presentation of the data in figure 36 the slab was assumed to be flat at the time of the March 22 observation on the basis of reasoning that follows.

The measurements of longitudinal expansion and contraction shown in figure 14 indicated that the expansion from moisture reaches a maximum during the winter months (January to March). For the year during which the moisture warping measurements were made (1934), the observed expansion was a maximum in January and by March it had dropped off slightly. This is a period during which the subgrade moisture content reaches a maximum value and during which the rate of evaporation is very low. It seems logical to conclude, therefore, that the moisture content of the concrete would be both high and most nearly constant during these months and that the moisture gradient that causes warping would be a minimum.

If a comparison is made between the curves in figure 36 showing the shape of the longitiudinal center line on March 22 and on December 18, it will be noted that on these dates the shapes are essentially the same, although some vertical movement of the slab as a whole had occurred. This tends to substantiate a conclusion that the slab is warped but little by moisture during the midwinter months. However, it should be remembered that the data shown in figure 36 are referred to the March observations as a base and that the curves indicate the changes in slab shape that occurred between March 22 and the other dates listed in the figure. If the slab was in an unwarped condition at the time of the March measurements, as was assumed, obviously the curves in this figure would indicate the true upward and downward warping of the slab.

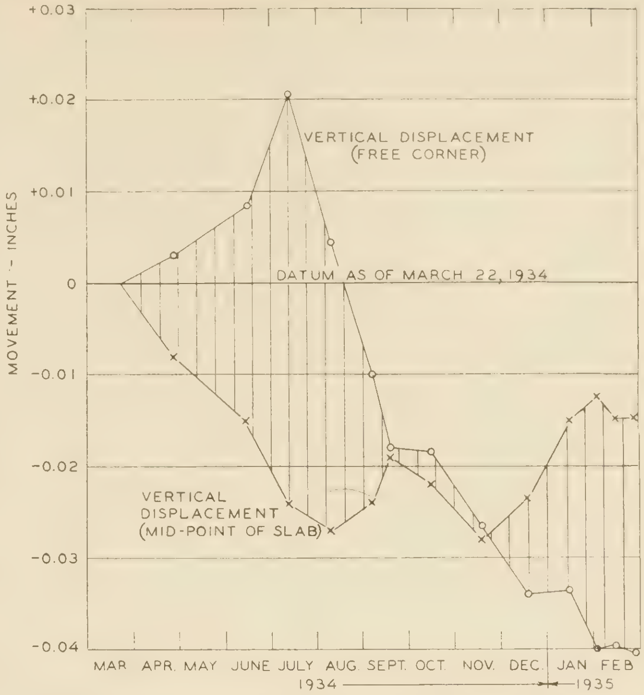


Figure 37.—Seasonal Variation of Warping Caused by Moisture Change, 10- by 20-foot Panel of 9-inch Thickness. The Distance Between Curves (Shaded Area) Shows the Extent of Moisture Warping.

Figure 37 shows the effect of moisture on the warping of a free corner and on the vertical displacement of the mid-point of one of the 10- by 20-foot panels, over a period of approximately 1 year. This graph was constructed from the same data and based upon the same measurements as the previous figure, although more observations were included. In determining the warping at the free corner, it was necessary to make corrections for any vertical displacements of the slab as a whole, as well as for any tilting of the slab that might have occurred. In making the first correction, it was assumed that the vertical displacement of the geometrical center of the panel with respect to the bench mark or reference point at the edge of the slab represented fairly the vertical displacement that took place over the slab as a whole. The correction for tilting was determined by averaging the displacements of three corners of the panel to establish a plane for each observation.

Referring to figure 37, it will be noted that after the middle of July a sudden reversal in the direction of the moisture warping occurred. This is believed to be caused by a marked change in weather conditions that took place at about this time. During August and September of this particular year the precipitation was much above normal (over 17 inches for September alone) and there was an unusually high percentage of hazy and cloudy weather. Normally it would be expected that this change in the direction of moisture warping would occur later, possibly in late August or early September. In this connection it is interesting to compare the relation between moisture warping and time, as shown in this graph, with that between moisture expansion and time as shown in figure 14 and to note the close correlation that exists.

DIFFICULTY ENCOUNTERED IN DETERMINING STRESSES CAUSED BY MOISTURE WARPING

It will be observed from the graph that during July the loss of moisture from the upper surface had caused the free corner to be warped upward approximately 0.045 inch with respect to the mid-point of the panel. From this, an estimate of the stress developed by moisture warping might be attempted, but this is not warranted for the following reasons:

1. The true shape of the warped slab was not determined.
2. Plastic flow undoubtedly enters as a factor and its importance is unknown.
3. Settlement of the slab into the subgrade alters the degree of restraint that exists.

The curve showing the movement of the center of the panel indicates that, as the seasonal warping takes place, the slab settles into the subgrade. This seems probable when one considers that the condition of warping from moisture change develops slowly over a long period of time and that the development is most active during the spring months when the soil of the subgrade contains a considerable amount of moisture. If this is the case, it probably has an important influence upon the amount of restraint that the slab encounters when it warps from moisture changes. If there were complete settlement of the slab into the subgrade so that the subgrade conformed completely to the warped shape, then both edges and interior would have full subgrade support and no restraint would be developed by the weight of the slab.

The extent to which the subgrade adapts itself to the slab as moisture warping develops is no doubt largely dependent upon the type and physical condition of the subgrade material, but it is reasonable to believe that, because of the time element and its effect on both subgrade behavior and on plastic yielding of the concrete itself, the restraint and therefore the stresses developed by moisture warping are not as great as the magnitude of the curvature might lead one to suspect.

The data indicate that the curvature caused by moisture is principally an upward warping of the edges caused by a moisture loss from the upper surface of the pavement. The downward warping of the edges, resulting from a condition in which the moisture content in the upper part of the pavement exceeds that in the lower part, seems to be considerably smaller for the conditions of these tests.

Thus it appears from the data that, at those times when high stresses are developed by temperature warping, as for example, an afternoon in midsummer, the effect of moisture is to cause curvature such that any stresses developed by it will tend to relieve rather than aggravate the stresses caused by the restraint to temperature warping.

STRESSES CAUSED BY RESTRAINED TEMPERATURE WARPING DETERMINED

It is believed that one of the most important results of the entire investigation has been the development for the first time of reasonably reliable experimental data showing the magnitude and distribution of the stresses caused by the restrained temperature warping of typical pavement sections. These data, obtained by the methods that were described at the beginning of this paper, are presented in various ways in the figures that follow.

It has been shown that under normal conditions the temperature differential that causes warping is much larger during the day than during the night, and that usually the daily maximum occurs in the early afternoon. Since it was desired principally to determine the magnitude of the warping stresses for the condition of average maximum temperature differential, the greater portion of the measurements were made during the daytime with the upper surface of the pavement at a higher temperature than the lower surface. The warping stresses occurring under this condition are more important also because tension is developed in the bottom of the slab. A sufficient number of night observations was made, however, to give a clear indication of the magnitude and relative importance of stresses developed at night.

The manner in which the stresses produced by restrained temperature warping during the day vary along the two principal axes of two of the test section panels is indicated by the curves in figures 38 and 39. The data in figure 38 apply to the transverse axes, while those in figure 39 apply to the longitudinal axes of the two slabs. At each point along each axis the stresses in both the transverse and longitudinal directions were determined. The values shown in these figures are averages of several sets of measurements made on selected days during the summer and fall.

The data pertaining to the transverse axis were all obtained during the summer but some of those pertaining to the longitudinal axis were obtained during the fall when the temperature differentials were not as large as during the summer. The values shown in figure 38 probably represent the largest that will occur with any frequency in the locality where the tests were made. The maximum values observed at any time were approximately 15 percent greater than the averages shown in this figure.

During a part of the tests the slabs used for the temperature measurements were unavoidably shaded so that complete data on the temperature differentials causing these stresses are not available. From the temperature data obtained it is estimated that the average temperature differentials causing the stresses shown in figure 38 were approximately 18° F. for the 6-inch and 23° F. for the 9-inch test sections.

In the absence of data on sections less than 20 feet in length, it is not possible to predict accurately what the maximum warping stresses on shorter sections would be. However, up to the present time very few concrete pavements have been laid in which the length of the slab units was less than 20 feet. For slab lengths greater than 20 feet, it is believed the data indicate that, for a 6-inch slab thickness, the maximum warping stress will not exceed that developed in the 20-foot slab, while in a 9-inch pavement the maximum warping stress may be somewhat greater than that shown. These conclusions are based on the shape of the longitudinal axis of the two warped slabs as determined with the clinometer.

The relative magnitudes of the stresses from restrained temperature warping at several points near the corners of the 6-inch and 9-inch uniform thickness sections are shown by the stress diagrams in figure 40. The stresses were determined along the lines A—B, A—C, and A—D of figure 6. Along the free edges of the slab the stress in the direction perpendicular to the edge is in all cases negligible and, for this reason, only stresses in the direction parallel to the edge are shown for the lines A—B and A—D (fig. 6). Along the diagonal at the free corner (line A—C, fig. 6), measurable stresses are found in both directions and the stresses perpendicular to and parallel to the diagonal are shown in figure 40.

It is interesting to note that the variation in the magnitude of the stresses measured perpendicular to the diagonal is similar to that of the stresses in the direction parallel to the diagonal. There seems to be

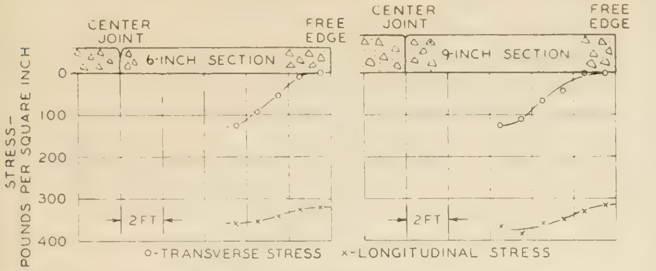


FIGURE 38.—MEASURED WARPING STRESSES CAUSED BY RESTRAINED TEMPERATURE WARPING ALONG THE TRANSVERSE AXIS OF TWO TEST SECTION PANELS.

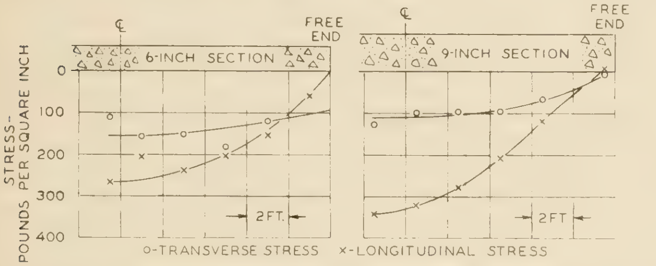


FIGURE 39.—MEASURED WARPING STRESSES CAUSED BY RESTRAINED TEMPERATURE WARPING ALONG THE LONGITUDINAL AXIS OF TWO TEST SECTION PANELS.

a consistent lack of uniformity in the variation in stress along the diagonal that might reasonably be attributed to a buckling action across the corner as this portion of the slab attempts to respond to two conflicting sets of forces.

The maximum stress along the free edge as shown in figure 40 is smaller than that shown in figure 38. This is because the stresses at the corner were determined for somewhat smaller temperature differentials.

Figure 41 shows stresses caused by restrained temperature warping at the corner and along the longitudinal axis of the 6-inch constant-thickness section under normal night conditions, i. e., with the upper surface at a lower temperature than the lower surface. The stress values shown are the averages of several sets of observations made on nights when, for night conditions, relatively large temperature differentials developed. The observations were made on 3 of the 4 panels of the test section.

It will be observed that the stresses vary in much the same manner as was found in the daytime measurements, although their magnitude is but about one-fourth as great. This is as would be expected as the observed temperature differentials were in approximately the same ratio. Under night conditions, the stresses developed are tensile stresses in the upper surface and compressive stresses in the lower surface of the pavement. They are, therefore, opposite in sense to the stresses caused by applied loads except for the case of a load applied on the corner of the slab.

STRESSES GREATEST IN LONGITUDINAL DIRECTION

In order to present a general picture of the critical stress conditions that result from restrained temperature warping, the stress diagrams shown in figures 42 to 45, inclusive, were prepared, utilizing the average measured stress curves to determine the shape of the variation curves and adjusting the stress magnitudes to a common value at the interior points of the slab. Figures 42 and 43 show the stresses parallel to the longitudinal axes of the slabs except at the diagonal at the free corner where the stresses are parallel to the diagonal. Figures 44 and 45 are similar diagrams for the stresses perpendicular to the longitudinal axes or perpendicular to the diagonal.

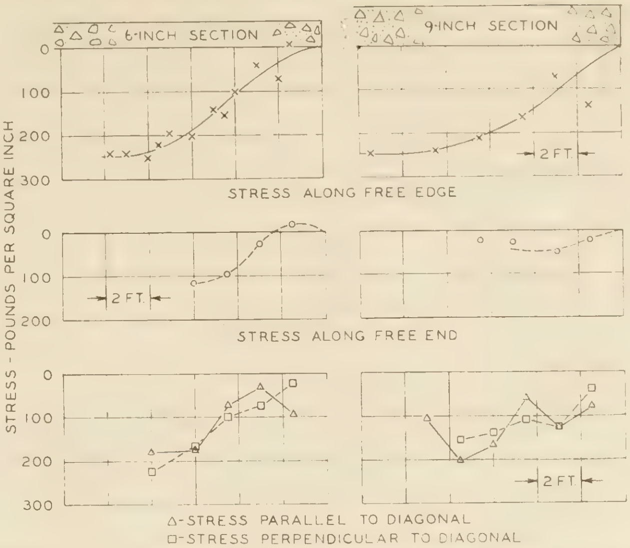


FIGURE 40.—MEASURED WARPING STRESSES CAUSED BY RESTRAINED TEMPERATURE WARPING NEAR THE SLAB CORNERS.

It was found by numerous measurements that stresses of the magnitudes indicated in these 4 diagrams may be expected to occur frequently in the daytime during the spring and summer months in the locality of Washington, D. C. As previously stated, stresses somewhat exceeding these were found occasionally on days of extreme temperature changes.

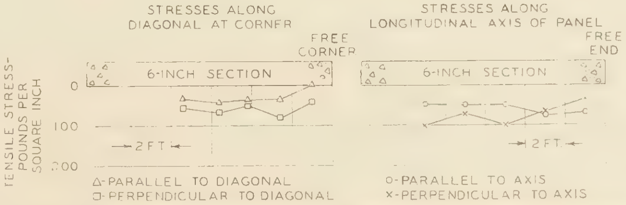


FIGURE 41.—AVERAGE MAXIMUM TENSILE STRESSES OBSERVED IN THE UPPER SURFACE OF THE 6-INCH SECTION CAUSED BY RESTRAINED TEMPERATURE WARPING AT NIGHT.

THEORETICAL AND MEASURED STRESSES COMPARED

Reference has been made to the analysis, by H. M. Westergaard, of the stresses caused by restrained temperature warping from the standpoint of theoretical mechanics. Figure 46 shows theoretical warping stresses computed for the transverse section of a slab of infinite length but finite width and utilizing elastic constants known to apply to the materials in the Arlington tests. The temperature differentials of 18° F. and 25° F. for the 6-inch and 9-inch slabs, respectively, are reasonable in the light of the temperature data obtained in this investigation. Both values are considerably higher than that assumed by Dr. Westergaard in the examples given in his analysis.

Since the Westergaard analysis is based upon the assumption of a slab of infinite length, it is perhaps not permissible to make direct comparisons between the theoretical stresses and those determined experimentally. However, since the length of the experimental slabs is twice the width, it is believed that the sections, particularly the thinner ones, will behave in a

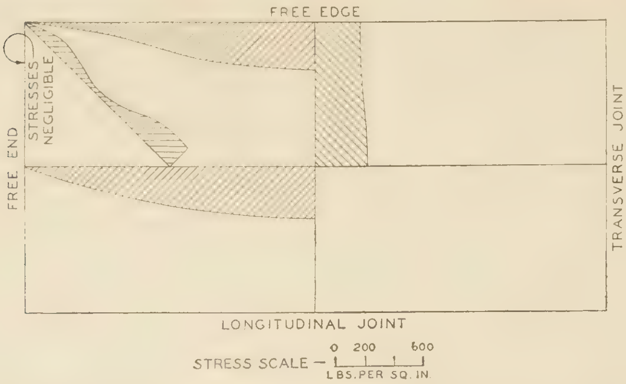


Figure 42.—Variations in Compressive Stress in the Upper Surface of 10- by 20-foot Panel of a 6-inch Pavement. Stresses Measured in a Longitudinal Direction or Parallel to Diagonal. Stresses are Caused by Restrained Temperature Warping.

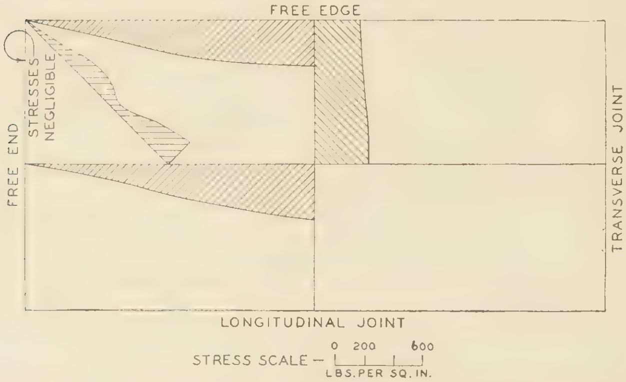


Figure 43.—Variations in Compressive Stress in the Upper Surface of 10- by 20-foot Panel of a 9-inch Pavement. Stresses Measured in a Longitudinal Direction or Parallel to Diagonal. Stresses are Caused by Restrained Temperature Warping.

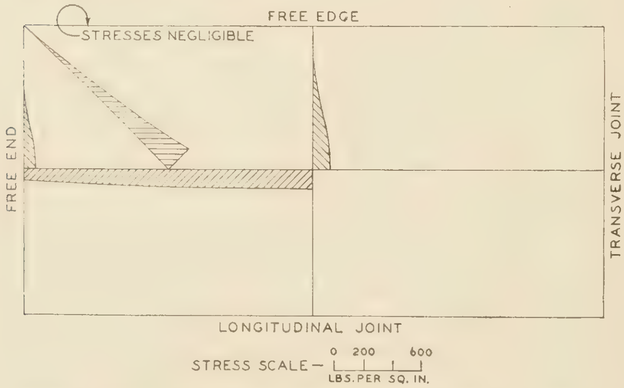


Figure 44.—Variations in Compressive Stress in the Upper Surface of 10- by 20-foot Panel of a 6-inch Pavement. Stresses Measured in a Transverse Direction or Perpendicular to Diagonal. Stresses are Caused by Restrained Temperature Warping.

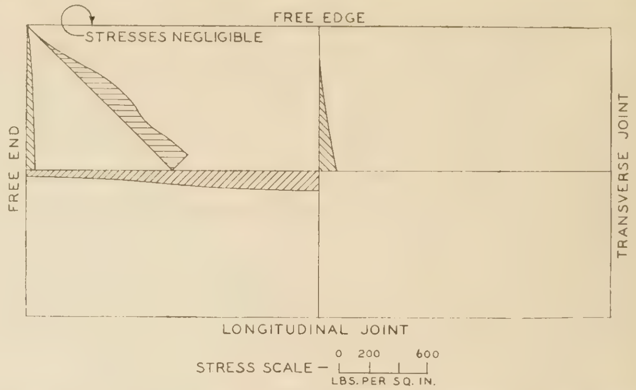


Figure 45.—Variations in Compressive Stress in the Upper Surface of 10- by 20-foot Panel of a 9-inch Pavement. Stresses Measured in a Transverse Direction or Perpendicular to Diagonal. Stresses are Caused by Restrained Temperature Warping.

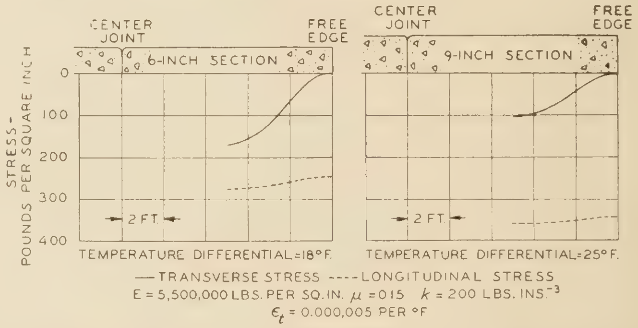


Figure 46.—Theoretical Stresses Caused by Restrained Temperature Warping at a Transverse Axis of a Slab of Infinite Length.

manner approximating that of longer slabs, and that the stresses in a transverse section near the center are sufficiently like those in the longer slab to make a comparison with the theory of value.

If the curves in figure 46 are compared with the experimental curves shown in figure 38, it will be noted that the shapes of the theoretical and measured stress curves are very similar, and also that the magnitude of the theoretical stresses are of the same order as the values found by measurement. In this connection it should be pointed out again that the average temperature differentials were estimated and may not be exact. From this comparison it seems reasonable to conclude that warping stresses properly calculated with the theoretical formulas will give a fair estimate of the critical warping stresses in pavement slabs 20 feet or more in length.

There is one noticeable difference, however, between the theoretical and measured stress relations. The measured stresses have practically the same magnitudes for both the 6-inch and 9-inch slabs in spite of the 5° F. difference in the temperature differentials. The stresses computed by the theory, on the other hand, reflect this difference in higher stresses for the 9-inch thickness. It is reasonable that the greater temperature differential in the thicker slab should produce higher stresses. Had the length of the 9-inch test section been greater than 20 feet, it is believed that the measured stresses would have been higher and thus more in accord with the theory.

It has been shown previously that, when warped, the 6-inch section tends to remain flat along the central portion of the longitudinal axis, while the 9-inch section showed continued curvature in this region. If the 9-inch slab had been longer it too would have been flattened in the mid-portion of its long axis and this undoubtedly would have increased somewhat the value of the measured stresses.

MERITS OF THICKENED-EDGE DESIGN INVESTIGATED

Data presented earlier in this paper show that when the edge thickness of a paving slab is increased, for the purpose of increasing the load-carrying capacity, there is certain to be a corresponding increase in the temperature differential that develops in this portion of the pavement. For example, it was shown that the temperature differentials observed at the edge of a 9–6–9 section were about 45 percent greater than those observed in the edge of a 6-inch constant-thickness section and approximately the same as those in the edge of a 9-inch constant-thickness section. Since an increase in the temperature differential at any part of a slab that is restrained from warping causes a corresponding increase in the warping stresses at that point, the effect just mentioned is an important consideration in the design of concrete pavements.

The effect of increasing the thickness of the edges of a pavement slab on the magnitude of the critical warping stresses can be illustrated by considering two long and relatively narrow pavement slabs both of the same interior thickness. One is of constant thickness while the other is of a conventional thickened-edge cross section with an edge depth 50 percent greater than the interior depth. The temperature differentials and therefore the warping stresses that develop in the interior of the two slabs will be approximately the same since the slabs have the same thickness in this region and are both sufficiently long to develop complete restraint. At the free edges, however, the differential in temperature for the thickened-edge cross section will be, according to the data obtained in this investigation, some 45 percent greater than in the slab of constant thickness.

The results of the increased temperature differential at the edge of the thickened-edge section will be an increase in the stresses caused by restraint to warping in the edge region of the slab. Since the slabs are relatively narrow in a transverse direction they are relatively free to warp and the warping stresses will be small. Thickening the edge will therefore have but little effect upon the transverse warping stresses. In the direction parallel to the edge of the pavement, the magnitude of the warping stresses varies with the degree of restraint that obtains at the particular point under consideration. The degree of restraint varies, in turn, with the distance from the free end of the slab and with the slab depth. At the extreme end no restraint exists as the slab can warp freely at this point. The rate at which restraint develops with the distance from the extreme end will depend upon the depth of the slab, as will the distance to the point where complete restraint is obtained.

The two slabs considered in the example were long enough to develop complete restraint and consequently maximum warping stresses in their mid-length, and in this region it was found that the temperature differential at the edge of the thickened-edge section was about 45 percent greater than in the edge of the slab having a constant thickness equal to that of the interior of the thickened-edge section. Since, theoretically, there is a direct relation between the magnitude of the stress resulting from completely restrained warping and the temperature differential that exists, this 45 percent increase in the temperature differential would lead one to expect a corresponding increase in the warping stress.

The relations between slab length, slab depth, and restraint to temperature warping were not determined by this investigation. To study this problem thoroughly, a range of slab lengths in each of several thicknesses would have to be constructed and the critical warping stresses determined for each. The data obtained would make it possible to determine what lengths of slab are sufficiently free to warp to make relatively unimportant the increase in warping stresses that results from the increased edge thickness. All of the slabs cast for this study of concrete pavement design were of the same length and only limited data bearing upon the relations could be obtained. These data are presented later in this paper.

WARPING STRESSES IN THICKENED-EDGE AND CONSTANT-THICKNESS SECTIONS COMPARED

Some measurements were made to determine the warping stresses at the edge of a 9–6–9 test section in comparison with those at the same point in the 6-inch and 9-inch constant-thickness sections. The data obtained are given in table 4.

TABLE 4.—*Observed longitudinal warping stresses at the edge of three 20-foot pavement slabs*

Date, 1934	Temperature differential at edge			Observed longitudinal warping stresses			Increase in stress, 9–6–9 slab over—	
	6-inch slab	9-inch slab	9–6–9 slab	6-inch slab	9-inch slab	9–6–9 slab	6-inch slab	9-inch slab
	° F.	*° F.*	*° F.*	*Lb. per sq. in.*	*Lb. per sq. in.*	*Lb. per sq. in.*	*Percent*	*Percent*
Apr. 18	18	27	27	220	--------	316	44	--------
Apr. 22	14	24	21	186	--------	218	17	--------
Apr. 24	21	31	29	195	--------	291	49	--------
May 9	18	25	24	209	191	245	17	28
May 18	21	30	30	--------	298	380	--------	28
May 19	20	26	29	252	306	380	51	24
May 20	23	33	32	--------	302	361	--------	20
May 21	20	31	32	320	329	409	28	24
May 22	--------	--------	--------	322	252	347	8	38
May 24	19	25	25	266	213	282	6	32
June 2	--------	--------	--------	229	251	336	47	34
June 3	--------	--------	--------	281	273	377	34	38
Average	--------	--------	--------	--------	--------	--------	30	30

These data were obtained from simultaneous measurements at the mid-length of the three 20-foot test slabs—the 6-inch constant-thickness slab (sec. no. 10), the 9-inch constant-thickness slab (sec. no. 6) and the 9–6–9 thickened-edge slab (sec. no. 5). The observations extended over a considerable period of time and it is thought that in spite of some inconsistencies, the data give a very good indication of the relative magnitude of the longitudinal warping stresses in slabs of these thicknesses and this length. The temperature differentials are typical of the highest average values that may be expected to occur frequently in the locality where the tests were made.

It will be noted that the stresses measured at the edge of the 9–6–9 slab are in every instance higher than those measured at the edge of either of the constant-thickness slabs. At times this difference is as much as 100 pounds per square inch for the 6-inch and 80 pounds per square inch for the 9-inch constant-thickness slabs. The last two columns of table 4 show these differences, expressed as a percentage of the stress in the constant-thickness slab. There is a considerable variation in these values for the comparison with the 6-inch slab, the reason for which is not known.

The average increase of warping stress of the thickened-edge section over the 6-inch constant-thickness section is approximately 75 pounds per square inch, or

30 percent. The average difference in corresponding temperature differentials is 47 percent. These data indicate, therefore, that the increase in stress is not as great as might be expected in view of the measured increase in temperature differentials. This may be the result of the thickened-edge slab warping slightly more at the point where the stresses were determined than does the 6-inch slab, because of the difference in the length-depth ratio, and is thus able partially to offset the effect of the increased temperature differential. From these tests it might be concluded that the full effect of increased temperature differential resulting from the increased depth of the slab edge of this 9–6–9 cross-section does not develop in a slab length of 20 feet.

It may seem surprising that the longitudinal warping stress in the thickened-edge section is consistently greater than that in a constant-thickness slab of the same edge depth, particularly in view of the fact that the measured temperature differentials were approximately the same in the two slabs. The explanation is believed to be that in slabs of the same length the 9-inch constant-thickness slab would be expected to warp more freely than the 9–6–9 thickened-edge design.

The data resulting from loading tests that are to be presented in a subsequent report of this series show that the 50-percent increase in edge thickness of the 9–6–9 as compared with the 6-inch constant-thickness slab resulted in a reduction of approximately 28 percent in the critical stress caused by load applied at the edge of the pavement. For example, the reduction in the critical load stress effected by the thickened edge for a 7,000-pound load applied at the edge position amounted to approximately 100 pounds per square inch in these tests.

Figure 39 shows that the average warping stress at the edge of the 6-inch constant-thickness section is approximately 320 pounds per square inch during the summer. It has just been shown that the edge thickening under consideration caused an increase of approximately 30 percent in these stresses. The edge thickening, therefore, causes an increase of approximately 90 pounds per square inch, which is practically equal to the reduction in load stress accomplished by the increased edge thickness. This indicates that, on the basis of the combined load and temperature stresses, at times when the warping stresses are high, the load-carrying capacity of the 20-foot, 9–6–9 section is not increased by the edge thickening. For much shorter slabs this would not be true because the warping stresses would be low, but for slabs of greater length it is indicated that at times when the warping stresses are high, the load-carrying capacity of the edge of a pavement slab may actually be reduced by thickening the slab edge.

IMPORTANCE OF REDUCING WARPING STRESSES DISCUSSED

The data that have just been presented clearly indicate that the stresses arising from restrained temperature warping equal in importance those caused by the heaviest wheel loads. The stresses from this cause are actually large enough to cause failure in concrete of low flexural strength, and since the direction of the stresses is such that they become added to the critical stresses caused by wheel loads, there is little doubt but that warping stress is primarily responsible for much of the cracking in concrete pavements. It must be concluded also that so long as the slabs are of considerable length originally, a thickened-edge design will not reduce the amount of transverse cracking that will occur. This conclusion is in agreement with the observations made on this point in the extensive pavement survey conducted several years ago by the Highway Research Board.[13]

It is evident that either the magnitude of the warping stresses must be reduced by building smaller slab units or by some other means not yet proposed, or the amount of stress resistance available for supporting wheel loads will be greatly curtailed.

The most practical means at present available for reducing warping stresses is through the construction of shorter slabs. As previously mentioned, some data that indicate the possibilities of reducing warping stresses through a decrease in slab length were obtained in this investigation. In the first place, a comparison of the relative magnitudes of the stresses created by warping at the center of the slab in the transverse and longitudinal directions shows that the stress in the direction of the 10-foot dimension is approximately one-third of that in the 20-foot dimension.

Some other data upon this point were obtained during the spring of 1934 when a transverse crack developed at the center of one quadrant of one of the thickened-edge sections (sec. no. 4). This afforded an opportunity to measure the longitudinal warping stress in both the 10- and 20-foot lengths of the same slab and to make a comparison of their magnitudes. The longitudinal stresses were determined at two positions in the mid-lengths of each slab, at one point 6 inches from the edge and at one 5 feet from the edge (center of the panel).

The measurements were made on suitable days during the months of April, May, and June 1934, and all of the data obtained are presented in table 5.

TABLE 5.—*Observed longitudinal warping stresses in 10- and 20-foot slab lengths of sec. no. 4*

Date, 1934	Maximum air temperature	Maximum longitudinal warping stress				Reduction in stress from decreased slab length	
		Interior		Edge			
		20-foot length	10-foot length	20-foot length	10-foot length	Interior	Edge
	°F.	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Lbs. per sq. in.*	*Percent*	*Percent*
Apr. 26	68	307	132			57	
May 1	74	376	142			62	
May 2	71			121	68		[1] 44
May 13	83	287	81			72	
May 14	90			278	21		92
May 28	83	429	151			65	
June 1	90			354	46		87
June 11	94			285	38		87
June 14	88			313	20		94
June 15	94			252	19		92
June 21	101	451	132			71	
June 22	96	414	130			69	
June 25	97			283	51		82
Average						66	89

[1] Not included in the average.

It is apparent again in these data that the longitudinal warping stress in the 10-foot slab length is consistently much smaller than the corresponding stress in the 20-foot slab. This is especially true for the stresses measured near the edges of the two slabs. The average reduction in the critical warping stress caused by the decreased slab length is approximately 66 percent in the interior of the slab and 89 percent at the point near the free edge. The greater reduction

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[13] Economic Value of Reinforcement in Concrete Roads, by C. A. Hogentogler Proc. Fifth Annual Meeting, Highway Research Board, pt. II.

noted in the stresses near the edge of the slab may be a natural condition or it may be caused by the fact that the two 10-foot sections of the slab were restrained from free warping to a certain degree at the longitudinal joint. This restraint results from the fact that the two 10-foot lengths of this section were attached to an uncracked half of the same section by the tongue-and-groove longitudinal joint.

The data and discussion just presented indicate the necessity for short slab units if the stresses produced by restrained warping are to be kept within economical limits. The desirability of slab lengths of approximately 10 feet is indicated and this may, at first thought, seem to be impractical because of the number of transverse joints that would be required. One problem of joint design is to overcome the difficulty of providing satisfactorily for the movements caused by the expansion and contraction of abutting slabs. If very short slabs were used, the very frequency of the joints would largely solve this problem and thus simplify the joint design requirements.

It was shown earlier that the critical temperature-warping stresses in the corner region of a pavement slab occur during the daytime when the sense of the warping stress is opposite to that of the critical stress caused by load. During the night and early morning when the sense of the two stresses is the same, the magnitude of the warping stress is very small. It is evident, therefore, that increasing the thickness of the edge of a pavement slab will be effective in reducing the combined stresses in the corner area and consequently will reduce corner cracking. This conclusion is likewise in agreement with the observations of the survey previously referred to.

SOME EFFECTS OF FREEZING AND THAWING OF SUBGRADE DETERMINED

As mentioned previously, because of the comparatively mild winters in the region where the tests were made there was very little opportunity to study the effects that a frozen subgrade might have on the pavement sections. During the latter part of the winter of 1933–34, however, severe weather caused the subgrade under the test slabs to freeze solidly to a depth of about 2½ inches and frost crystals were found at a slightly greater depth. The earth shoulders were frozen solidly to a depth of about 7 inches and frost crystals were found at depths of 10 or 11 inches.

During this period observations were made with the clinometer to determine the vertical movements that developed in the various parts of the slab, the technic being the same as that used in the measurements of warping. Figure 47 shows the position and shape of the 9-inch constant-thickness section on two different days referred to its position during the preceding September as a base. The first series of observations (Feb. 23) show the position of the slab with the subgrade frozen, while those made on March 6 show its position just after the subgrade had thawed.

These data show that, for the conditions that obtained, the freezing of the subgrade lifted the entire panel almost uniformly to the extent of about one-half inch. It is interesting to note that this lift was produced by a subgrade that was frozen solidly to a depth of only 2½ inches. As soon as the subgrade had thawed completely the measurements showed that the slab settled back about three-fourths of the distance through which it had been lifted, and that during the next several days the slab was slowly settling. Traffic on the pavement would probably accelerate this settling during the period of thawing.

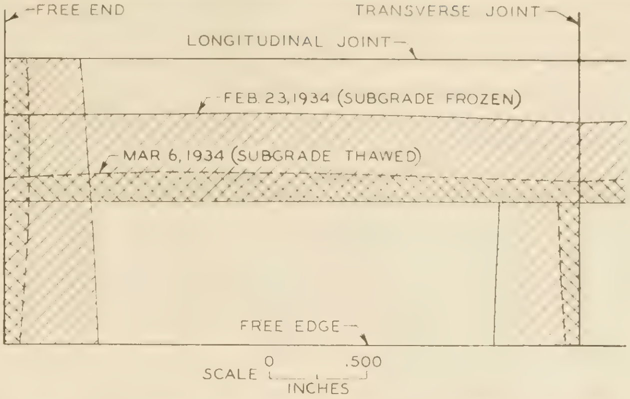

Figure 47.—Effect of Frozen Subgrade on Elevation of 9-inch Uniform-Thickness Slab, Base Elevation as of Sept. 24, 1933.

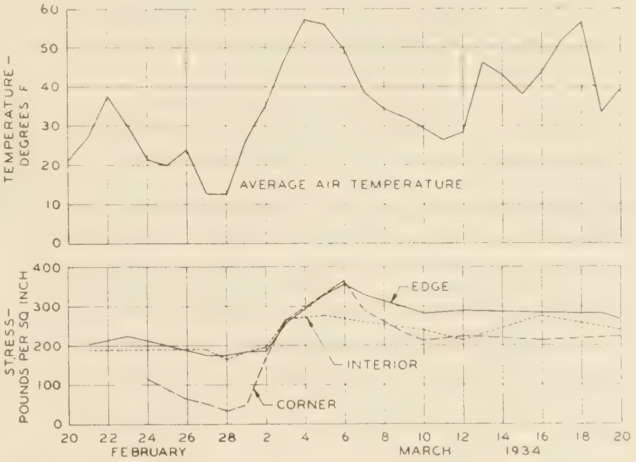

Figure 48.—Daily Variation of Stresses Caused by a 7,000-pound Load on a 9-6-9 Thickened-Edge Section, Over a Period of Freezing and Thawing of the Subgrade.

Tests were made to determine the effect of the freezing and thawing of the subgrade upon the stresses caused by applied loads. The 9–6–9 thickened-edge section was selected for this purpose and loads were applied at a free corner, at the interior, and at a point 6 inches from a free edge. The 8-inch diameter circular bearing plate was used in all cases. The tests were started during the time when the subgrade was frozen solidly and were repeated frequently until after the subgrade had thawed completely. The variations in the maximum stress produced by a 7,000-pound load applied at each of the three positions previously mentioned during the entire period are shown in figure 48, together with the variations in the average daily air temperature.

From these data it appears that the subgrade was in a fairly normal condition after March 10, or about 10 days after it first started to thaw. The subgrade was no doubt still very wet at this time, but it is known that this particular soil remains very wet during the winter even when not subjected to freezing and thawing.

It will be observed that at all three of the points tested the stresses from the applied load were reduced during the period when the soil was frozen. The effect was much greater at the corner than at the other points, probably because a much greater deflection is required to produce a given stress at this point.

As soon as thawing started there was an immediate increase in the stress at all of the points. As the subgrade became completely thawed the stresses were slightly above normal and they remained at this general level during a period of 6 or 8 days.

It is interesting to note that when the subgrade is in what may be termed its "normal" winter condition, but unfrozen, the stresses produced by a given load at the free corner and interior of this section are of approximately the same magnitude while the stress at the edge is but slightly greater.

It appears from these data that the conditions of freezing and thawing that obtained during the tests had no serious effect upon the magnitude of the stresses developed under the applied load. Had the subgrade been frozen to a greater depth or had the subgrade material not been uniform, it is possible that the effects of freezing would have been more serious.

CONCLUSIONS

In this study of the effects of temperature and of moisture on concrete pavement slabs, it has been found that in the locality where the tests were made (Washington, D. C.):

1. The average pavement temperature undergoes an annual change of about 80° F.
2. The maximum temperature differentials observed at the edges of the test sections were:
    - *a.* For a 6-inch uniform-thickness section, 23° F.
    - *b.* For a 9-inch uniform-thickness section, 33° F.
    - *c.* For a 9-6-9 thickened-edge section, 33° F.

   These maxima occur during the hot afternoons of early summer when the upper surface of the pavement is heated by the intense sunlight and the lower surface is kept cool by a subgrade that is still at a relatively low temperature.
3. In the thickened-edge design (sec. no. 5) the temperature differential in the interior of the slab averaged about 4° F. less than that at the thickened edge during the most critical part of the year.
4. There is a cyclic variation in slab length that is entirely dissociated from temperature changes. The annual variation in the length of the test sections from causes other than temperature changes is approximately equivalent to that caused by a temperature change of 30° F., and the maximum length occurs during the late winter when the ground moisture content is greatest. Conversely, the slab is shortest during the late summer when the ground moisture and, so far as could be determined, the concrete moisture are a minimum.
5. The thermal coefficient of expansion of the concrete as determined in the laboratory is 0.0000048 per degree F. This value agrees almost exactly with that determined by measurement of actual temperature expansion in the test sections, indicating: First, that the movement of a pavement slab from thermal expansion can be predicted accurately from laboratory determinations of the thermal coefficient; and second, that in slabs of moderate length the effect of subgrade restraint on slab expansion is so small as to be negligible.
6. The resistance developed in the subgrade to horizontal slab movement is not merely a matter of sliding friction in the commonly accepted sense of the word. It appears to consist of two elements, one an elastic deformation of the soil horizontally that is present for all displacements of the slab, and the other a frictional resistance that develops only after a certain amount of elastic deformation has occurred. The first element appears to be independent of, while the second varies directly with, the slab weight or thickness. Although only one subgrade material was involved in these tests, it seems probable that the relative importance of the two elements may vary considerably with different types of soils.
7. In pavement slabs of moderate length the tensile stresses resulting from contraction will not be large for subgrade soils of the type used in these tests. The thicker the pavement the lower will be the unit stress from this cause, other conditions being the same.
8. The changes in shape of a pavement slab resulting from restrained temperature warping do not cause large changes in the critical stresses from applied loads. In this investigation, the maximum observed condition of upward warping from temperature was found to increase the critical stress resulting from load by about 5 percent for a corner loading and about 20 percent for an edge loading, as compared with the stresses produced by the given load with the slab in the flat or unwarped condition. Maximum downward warping was found to effect a negligible reduction in the load stress at the edge and a reduction of about 20 percent at the corner.
9. For pavement slabs of the size used in this investigation or larger, certain of the stresses arising from restrained temperature warping are equal in importance to those produced by the heaviest of legal wheel loads. The longitudinal tensile stress in the bottom of the pavement, caused by restrained temperature warping, frequently amounts to as much as 350 pounds per square inch at certain periods of the year and the corresponding stress in the transverse direction is approximately 125 pounds per square inch. These stresses are additive to those produced by wheel loads.
10. In long or even moderately long pavement slabs, when conditions are such as to produce large temperature differentials, thickening the edge of the slab may actually decrease the load-carrying capacity of this part of the pavement. In very short pavement slabs, thickening the edge of the slab

may be expected to increase definitely its load-carrying capacity.

11. Since the critical stresses resulting from restrained warping are opposite in sense to those caused by applied loads in the corner region of a pavement, thickening the edge of the slab may be expected to increase the load-carrying capacity of the slab corner.
12. Because of the facts stated in conclusions 10 and 11, it is evident that thickening the edge of a long pavement slab will not tend to reduce transverse cracking but will tend to reduce corner cracking.
13. The annual cyclic variation in moisture conditions within the concrete produces a warping of the slab surface similar to that caused by temperature. The edges of the slab reach their maximum position of upward warping from this cause during the summer and the maximum position of downward warping during the winter, the extent of the upward movement apparently exceeding that of the downward movement considerably.
14. While sufficient information is not available to permit an estimate to be made of the magnitude of the stresses arising from restrained moisture warping, it appears that at the time of year when the stresses from restrained temperature warping are a maximum (the summer months) any stresses caused by restrained moisture warping will be of opposite sense and will thus tend to reduce rather than to increase the state of stress created by restrained temperature warping.

CURRENT STATUS OF UNITED STATES PUBLIC WORKS ROAD CONSTRUCTION

AS PROVIDED BY SECTION 204 OF THE NATIONAL INDUSTRIAL RECOVERY ACT (1934 FUNDS) AND BY THE ACT OF JUNE 18, 1934 (1935 FUNDS)

CLASS 1.—PROJECTS ON THE FEDERAL-AID HIGHWAY SYSTEM OUTSIDE OF MUNICIPALITIES

AS OF OCTOBER 31, 1935

STATE	APPORTIONMENTS		COMPLETED				UNDER CONSTRUCTION				APPROVED FOR CONSTRUCTION			BALANCE OF FUNDS AVAILABLE FOR NEW PROJECTS	
	Sec 204 of the Act of June 16, 1933 (1934 Fund)	Act of June 18, 1934 (1935 Fund)	Total Cost	1934 Public Works Funds	1935 Public Works Funds	Mileage	Estimated Total Cost	1934 Public Works Funds	1935 Public Works Funds	Mileage	1934 Public Works Funds	1935 Public Works Funds	Mileage	1934 Public Works Funds	1935 Public Works Funds
Alabama	$ 3,947,753	$ 2,129,921	$ 7,416,251	$ 3,664,762	$ 649,877	417.1	$ 1,495,617	$ 282,991	$ 1,040,178	62.7		$ 158,785	9.0		$ 281,082
Arizona	3,855,555	1,381,051	5,585,167	3,829,743	1,044,260	385.5	420,756	15,039	313,333	9.4		6,580	.1	$ 10,773	16,879
Arkansas	3,334,167	1,714,000	4,383,860	2,895,474	782,558	197.7	1,324,374	394,978	790,058	69.2		32,191	.1	43,716	109,194
California	7,912,928	3,713,643	12,755,782	7,790,341	1,587,165	415.9	2,683,780	121,227	1,989,200	32.4		56,779		1,360	80,499
Colorado	3,437,265	2,424,504	6,068,320	3,401,939	2,412,804	293.2	28,829	23,817	5,012					11,510	6,689
Connecticut	1,404,213	607,500	2,125,647	1,398,014	415,900	36.2	138,826		138,826	1.8				6,199	52,774
Delaware	877,566	461,697	1,355,369	877,566	456,600	48.8	1,102		1,102						3,996
Florida	2,469,369	1,116,600	4,104,035	2,422,540	830,923	155.4	132,867	29,831	94,809	1.5		173,253	1.7	16,998	17,615
Georgia	5,045,592	2,556,745	5,897,379	4,372,974	1,224,853	384.6	1,115,340	509,281	558,035	43.8	$ 15,717	136,834	8.8	147,619	637,022
Idaho	2,166,858	1,131,910	2,870,866	2,158,429	611,486	232.1	346,134		345,193	16.5		1,670	.1	8,429	173,561
Illinois	4,408,827	2,408,778	3,732,559	3,089,964	581,350	65.3	2,909,847	1,284,028	1,625,004	41.8		202,424		34,836	
Indiana	5,018,921	2,688,632	4,932,428	4,417,417	328,454	136.7	2,800,981	523,593	2,242,173	144.8		3,920		77,910	114,085
Iowa	5,027,830	1,963,361	7,385,725	4,988,630	1,896,876	439.0	108,629	39,200	65,500	.4					985
Kansas	5,044,802	2,354,131	6,861,311	5,004,394	1,679,030	711.7	715,834	31,530	661,756	49.5		13,345	.9	8,878	
Kentucky	3,751,605	1,302,209	4,417,336	3,512,268	641,962	318.8	844,702	182,976	532,648	27.6	8,570	5,000	1.1	47,790	122,599
Louisiana	2,693,135	1,380,419	3,399,544	2,481,602	390,211	86.9	784,764	34,283	750,481	18.7	141,616	227,147	14.2	35,634	12,579
Maine	1,567,012	782,195	2,169,629	1,549,427	589,812	57.5	192,303		192,289	6.8				17,585	94
Maryland	1,782,263	332,836	1,316,429	1,191,392	123,526	25.0	630,329	500,659	129,670	12.8		74,482	3.4	90,213	5,157
Massachusetts	1,101,716	1,582,874	2,127,801	1,101,653	530,306	48.2	1,022,691		1,022,691	11.4				63	29,877
Michigan	6,051,533	3,126,284	7,971,596	5,932,694	1,594,911	327.5	1,543,550	64,600	1,475,250	74.6		56,123	3.0	4,239	
Minnesota	4,561,011	2,533,733	7,177,401	4,515,018	2,371,404	1,082.5	174,714	30,000	106,779	16.2		54,971	1.1	15,993	578
Mississippi	3,489,337	2,832,182	6,146,390	2,854,077	875,755	322.7	2,566,080	570,597	1,609,119	136.6	5,574	294,586	14.2	59,089	52,722
Missouri	5,237,532	2,890,666	5,532,245	4,582,356	336,056	225.0	3,262,066	654,277	2,419,784	100.0		25,508		899	109,318
Montana	4,463,849	2,714,208	7,142,910	4,296,521	2,266,796	635.7	510,273	111,796	398,477	17.5	47,836		7.3	7,695	48,936
Nebraska	3,914,481	1,982,182	5,867,233	3,866,972	881,537	443.8	1,356,503	18,006	1,091,817	34.6		6,264		29,503	2,564
Nevada	2,909,387	1,350,356	4,264,712	2,877,829	1,239,625	464.8	85,927		73,567	39.0		1,650		31,558	35,514
New Hampshire	692,119	465,404	1,184,179	692,119	444,376	23.3	18,628		18,628	.1					2,401
New Jersey	3,173,019	951,379	3,129,342	2,793,057	146,403	46.8	1,029,469	345,862	586,834	8.7		20,970		34,100	197,171
New Mexico	2,846,648	1,676,769	4,227,210	2,691,814	1,375,407	408.9	349,999	62,413	287,586	22.7	37,087			55,334	13,777
New York	10,234,915	3,673,231	14,718,479	9,623,226	1,780,480	275.4	4,269,430	576,975	1,717,202	77.3	12,000	1,000	.2	22,714	174,549
North Carolina	4,761,147	1,930,365	6,298,184	4,175,134	939,851	753.7	915,940	231,240	667,805	51.0	74,297	256,314	4.6	280,476	66,395
North Dakota	2,902,224	1,469,483	3,979,428	2,725,292	648,124	1,345.9	214,649	54,799	95,319	42.6	84,821	218,725	48.4	37,312	507,315
Ohio	7,277,758	3,539,256	9,845,084	7,165,072	2,034,899	252.0	1,392,114	97,161	1,140,239	18.6		325,171	3.1	15,525	38,945
Oklahoma	4,608,399	2,342,590	6,324,410	4,421,227	1,592,969	383.9	831,484	181,499	646,415	25.3		28,063		5,673	75,142
Oregon	3,053,448	1,426,910	4,495,347	2,928,739	1,087,443	239.9	418,959	79,514	299,799	9.3				45,195	39,668
Pennsylvania	6,641,194	4,554,082	11,300,249	6,489,503	4,288,699	215.4	317,421	136,292	172,948	8.7	2,126	87,435	.2	13,272	5,000
Rhode Island	988,230	474,772	1,528,596	988,230	464,572	34.9	3,800		3,800			6,400			
South Carolina	2,729,583	940,954	2,956,054	2,639,674	249,246	254.1	498,984	34,290	464,694	23.8	11,220	52,612	.2	44,399	174,402
South Dakota	3,005,739	1,523,822	4,737,023	2,960,912	1,237,310	797.9	136,631	42,796	93,835	20.4		108,915	16.0	2,031	83,762
Tennessee	4,246,309	2,105,453	6,105,554	4,105,518	1,300,110	231.9	727,966	70,497	561,068	23.8	11,254	143,931	1.5	59,040	100,344
Texas	11,588,643	6,858,253	16,104,360	11,435,999	3,898,336	1,393.9	2,931,747	152,644	2,695,968	165.2		226,012	4.1		37,937
Utah	2,367,205	1,066,345	3,486,754	2,322,418	863,454	300.1	171,622	37,000	65,450	4.1		100,000	.3	7,787	37,441
Vermont	928,184	466,042	1,369,523	912,376	347,618	64.2	132,487	3,922	114,302	3.1		2,713		11,887	1,410
Virginia	3,731,207	1,916,178	5,616,215	3,519,936	1,586,137	248.7	449,588	159,398	248,609	19.5	11,386	7,213	.1	40,487	74,218
Washington	3,057,934	1,553,206	3,457,525	2,849,505	580,199	117.3	1,271,966	198,743	946,546	13.0		6,678		9,686	19,783
West Virginia	2,013,405	1,140,167	2,710,199	1,963,040	690,200	99.0	203,708	24,134	176,070	6.6		20,143	.1	26,231	253,754
Wisconsin	4,697,518	1,818,970	6,268,111	4,659,171	1,343,460	289.6	623,128	32,300	449,646	20.0		23,126		6,047	2,738
Wyoming	2,250,663	1,686,368	3,576,499	2,221,894	1,189,898	656.5	506,384	26,873	453,354	78.0		37,872		1,896	5,244
District of Columbia															
Hawaii	1,693,344	598,778	1,326,739	952,902		27.3	1,140,384	681,972	293,004	13.7	20,973	276,443	1.8	37,497	29,331
TOTALS	184,963,342	93,641,394	257,752,989	174,360,754	54,433,228	16,418.2	45,753,306	8,653,033	31,871,872	1,625.1	484,477	3,481,248	145.6	1,465,078	3,855,046

CURRENT STATUS OF UNITED STATES PUBLIC WORKS ROAD CONSTRUCTION

AS PROVIDED BY SECTION 204 OF THE NATIONAL INDUSTRIAL RECOVERY ACT (1934 FUNDS) AND BY THE ACT OF JUNE 18, 1934 (1935 FUNDS)

CLASS 2.—PROJECTS ON EXTENSIONS OF THE FEDERAL-AID HIGHWAY SYSTEM INTO AND THROUGH MUNICIPALITIES

AS OF OCTOBER 31, 1935

STATE	APPORTIONMENTS		COMPLETED				UNDER CONSTRUCTION				APPROVED FOR CONSTRUCTION			BALANCE OF FUNDS AVAILABLE FOR NEW PROJECTS	
	Sec. 204 of the Act of June 16, 1933 (1934 Fund)	Act of June 18, 1934 (1935 Fund)	Total Cost	1934 Public Works Funds	1935 Public Works Funds	Mileage	Estimated Total Cost	1934 Public Works Funds	1935 Public Works Funds	Mileage	1934 Public Works Funds	1935 Public Works Funds	Mileage	1934 Public Works Funds	1935 Public Works Funds
Alabama	$ 2,389,928	$ 1,064,961	$ 2,455,651	$ 2,091,174	$ 343,359	63.7	$ 462,588	$ 255,706	$ 206,882	12.6	$ 11,885	$ 232,524	4.3	$ 31,161	$ 282,195
Arizona	756,982	289,673	746,297	622,804	86,475	15.1	374,317	133,822	161,998	.7				357	41,199
Arkansas	1,964,534	857,025	2,253,373	1,731,970	418,019	53.4	462,425	100,471	361,402	8.8	116,801	27,136	1.4	15,292	50,468
California	4,213,986	2,219,360	5,851,832	3,856,845	1,219,276	71.7	2,578,887	356,840	822,800	7.0		87,911	.7	301	89,373
Colorado	1,718,633	190,000	1,947,213	1,679,027	171,778	40.8								39,606	18,222
Connecticut	802,407	426,500	1,032,256	802,407	151,883	11.9	218,505		218,505	1.8					56,112
Delaware	460,409	230,849	560,623	460,282	91,834	9.2						42,012		127	97,002
Florida	1,459,649	501,200	1,885,653	1,441,744	146,700	21.8	313,134		283,813	3.3		9,807		17,905	60,880
Georgia	2,724,620	1,278,373	2,709,265	2,296,104	353,436	83.2	403,056	197,779	199,277	9.7	72,102	82,864	2.7	158,636	642,796
Idaho	1,197,829	321,126	1,224,303	1,150,878	27,359	21.3	284,291		283,103	3.0				46,951	10,664
Illinois	7,381,910	2,230,350	6,932,054	6,261,937	328,771	70.7	2,377,330	1,083,605	1,293,725	10.3		85,320	1.5	36,369	522,534
Indiana	4,287,050	2,248,858	4,739,959	4,049,430	511,971	82.5	1,740,546	185,901	1,554,467	23.8	51,719	80,140	2.2		102,280
Iowa	2,614,472	1,280,000	3,364,192	2,443,068	781,335	78.9	455,084	171,365	253,500	3.3				39	245,165
Kansas	2,522,401	1,432,949	3,393,422	2,473,483	751,065	53.2	670,904		670,904	3.8	48,918	10,980	1.3		
Kentucky	1,927,828	958,599	2,082,263	1,579,834	428,512	43.4	651,089	316,558	314,624	5.8	31,436	42,213	1.0		173,248
Louisiana	1,708,577	744,560	1,080,670	791,986	276,341	25.7	1,125,756	896,730	187,071	13.3	10,666	243,793	2.5	9,195	37,355
Maine	960,426	484,379	1,113,059	910,266	197,218	19.5	318,123	45,122	259,099	3.9				5,038	28,062
Maryland	891,132	452,514	434,065	422,396		3.8	1,282,700	468,736		2.2		127,791	.1		324,724
Massachusetts	5,007,199	847,600	3,236,487	3,003,788	157,851	18.5	2,556,680	1,974,333	582,347	1.5				29,078	107,402
Michigan	3,500,637	1,613,142	5,032,635	3,477,717	1,342,100	56.6	226,575	650	225,925	3.8		28,925		22,270	16,192
Minnesota	3,719,143	1,421,494	3,797,274	3,106,574	610,203	123.6	1,033,681	509,834	427,765	10.4	13,805	150,366	.6	88,929	233,161
Mississippi	1,744,669	354,022	1,484,253	1,256,645	175,525	46.3	566,651	445,452	79,162	16.3		9,950	.4	42,572	89,385
Missouri	4,019,501	919,152	3,437,441	3,187,383	159,948	60.2	1,478,440	704,456	664,992	8.0		94,212	.4	127,663	
Montana	1,115,962	113,092	1,191,712	1,083,109	66,149	42.7	5,994		5,994	.8				32,352	40,949
Nebraska	1,957,240	991,091	2,690,910	1,941,205	710,481	48.5	192,629	16,035	176,594	3.2		3,992			100,024
Nevada	500,051	100,000	539,499	473,788	56,234	10.8	67,552	26,150	41,402	.6				112	2,364
New Hampshire	740,334	242,465	853,411	668,379	181,465	18.9	46,123		46,123	.3	22,006		.5	49,949	14,877
New Jersey	3,117,921	1,809,500	3,356,687	2,983,109	207,990	24.3	1,068,330	10,297	838,485	4.5		635,875	1.0	124,515	127,149
New Mexico	1,674,158	529,506	1,844,774	1,568,055	261,203	41.9	151,695		151,695	1.1				106,103	116,609
New York	8,092,500	3,961,690	9,650,901	7,945,645	1,126,300	72.3	2,515,683	80,200	2,384,840	14.1		57,000		66,655	393,550
North Carolina	2,380,573	1,210,236	3,276,664	2,271,729	965,139	107.7	279,947	84,204	195,743	2.7		1,881		24,640	47,473
North Dakota	1,451,112	734,742	1,537,690	1,341,459	187,278	68.8	273,545	95,335	178,155	8.3	14,170	78,504	5.5	148	290,805
Ohio	4,335,686	2,359,503	5,674,168	4,284,639	859,673	74.5	1,610,360	46,452	1,438,470	10.4				4,595	61,361
Oklahoma	2,304,200	1,171,295	2,936,593	2,264,019	571,322	55.4	433,599	37,623	380,294	4.8		96,583	.9	2,558	123,096
Oregon	1,526,724	778,728	2,189,833	1,482,285	646,242	41.3	166,640		130,167	1.0	518			43,921	2,319
Pennsylvania	4,837,988	2,397,703	6,561,695	4,788,944	1,470,175	84.4	325,233	36,605	288,581	1.4		296,345	1.3	12,440	342,602
Rhode Island	512,665	285,760	650,675	508,370	140,964	8.9						144,796	1.1	4,295	
South Carolina	1,364,791	488,000	1,401,653	1,271,150	118,140	44.3	277,851	89,121	164,579	7.2	4,520				205,282
South Dakota	1,502,870	761,910	1,566,166	1,289,105	273,095	52.0	179,282	16,774	162,508	12.5	6,425	290,151	11.0	190,566	36,157
Tennessee	2,123,155	1,121,790	2,562,523	2,054,124	419,205	32.6	567,627	69,031	498,596	3.9		148,190	.8		55,799
Texas	6,642,863	1,795,000	6,650,894	5,889,378	601,556	149.6	1,684,678	488,169	1,040,319	15.4	265,316	132,119	2.6		21,007
Utah	778,826	533,173	1,221,740	647,321	449,873	31.7	240,121	130,060	82,300	2.6		1,000		1,445	
Vermont	500,509	240,611	752,805	486,227	188,069	17.5	52,541		52,542	1.6				14,282	
Virginia	1,948,780	956,021	2,843,763	1,923,878	661,722	44.0	239,173	1,453	195,952	4.3	9,099	52,687	1.4	14,349	45,559
Washington	1,977,260	776,603	2,728,581	1,974,755	732,352	47.5	6,495		6,495			1,415		2,505	36,341
West Virginia	1,342,270	570,085	1,213,156	1,139,949	28,109	19.4	669,664	198,519	441,660	8.0		31,667		3,802	68,650
Wisconsin	2,596,143	1,379,513	3,930,649	2,550,676	1,297,634	73.2	38,915	9,502	24,387	.9		55,985	1.2	35,965	1,508
Wyoming	1,125,332	29,416	1,114,414	1,098,148	13,071	24.2	26,495	22,068	3,845	1.7		12,500		5,116	
District of Columbia	946,445	181,051	1,127,496	946,445	181,051	6.6									
Hawaii															
TOTALS	115,370,280	47,885,170	130,863,292	103,973,633	21,145,451	2,318.0	30,700,934	9,304,958	17,981,087	264.6	679,387	3,396,634	46.4	1,412,302	5,361,998

CURRENT STATUS OF UNITED STATES PUBLIC WORKS ROAD CONSTRUCTION

AS PROVIDED BY SECTION 204 OF THE NATIONAL INDUSTRIAL RECOVERY ACT (1934 FUNDS) AND BY THE ACT OF JUNE 18, 1934 (1935 FUNDS)

CLASS 3.—PROJECTS ON SECONDARY OR FEEDER ROADS

AS OF OCTOBER 31, 1935

STATE	APPORTIONMENTS		COMPLETED				UNDER CONSTRUCTION				APPROVED FOR CONSTRUCTION			BALANCE OF FUNDS AVAILABLE FOR NEW PROJECTS	
	Sec. 204 of the Act of June 16, 1933 (1934 Fund)	Act of June 18, 1934 (1935 Fund)	Total Cost	1934 Public Works Funds	1935 Public Works Funds	Mileage	Estimated Total Cost	1934 Public Works Funds	1935 Public Works Funds	Mileage	1934 Public Works Funds	1935 Public Works Funds	Mileage	1934 Public Works Funds	1935 Public Works Funds
Alabama	$2,032,452	$1,064,960	$2,508,096	$1,932,785	$550,192	176.1	$549,329	$61,735	$487,594	26.7		$22,904	0.3	$37,932	$4,271
Arizona	599,423	971,211	1,216,816	530,962	573,628	102.3	412,022	59,762	344,389	32.0				8,699	53,194
Arkansas	1,449,634	857,024	1,704,635	1,337,753	354,517	205.8	543,110	79,018	462,095	64.0	$32,863	31,680	4.6		8,733
California	3,480,440	1,999,203	5,197,903	3,476,277	924,869	222.2	997,017		943,759	23.8		61,629		4,163	68,946
Colorado	1,718,632	871,502	2,824,503	1,608,632	781,675	295.4	275,492	110,000	89,827	16.1					
Connecticut	659,120	420,868	920,180	659,120	235,769	19.4	140,661		125,141	1.9		58,859	.1		1,099
Delaware	481,113	230,849	527,686	277,564	228,317	68.8	203,783	203,549		2.5					2,532
Florida	1,302,816	1,043,543	1,692,424	1,276,273	394,194	83.3	675,727		624,180	42.1		15,978		26,543	9,191
Georgia	2,320,973	1,278,373	2,146,513	1,935,431	172,808	155.1	613,012	293,138	319,874	44.9		100,176	4.0	92,404	685,515
Idaho	1,121,562	824,450	1,891,835	1,094,530	611,868	217.5	136,457		136,457	6.7		76,125	.3	27,032	
Illinois	5,780,033	4,282,273	7,029,186	5,273,713	1,683,718	373.4	2,691,497	468,420	2,223,077	128.3		90,274	2.6	37,899	285,204
Indiana	731,872	151,473	685,202	604,642	22,167	75.2	174,810	87,850	86,960	24.7				39,380	42,346
Iowa	2,413,358	1,875,000	3,965,234	2,411,257	1,431,019	649.7	456,566		435,400	53.9		1,000	2.2	2,101	7,581
Kansas	2,522,401	1,330,595	3,364,346	2,483,248	877,989	287.6	491,759	39,153	452,606	35.7					
Kentucky	1,837,926	1,557,503	2,898,900	1,813,980	914,379	340.5	692,178		608,088	69.0		29,555	2.5	23,946	5,482
Louisiana	1,426,879	838,953	1,593,860	1,291,973	289,472	69.4	597,350	127,387	469,963	26.7	3,135	74,211	2.5	4,384	5,308
Maine	842,479	445,012	1,397,095	842,403	419,169	104.7	16,332		16,332	.8				75	9,510
Maryland	891,132	1,024,708	1,218,325	876,320	283,690	79.8	269,248	9,909	259,339	11.3		200,111	8.2	4,903	281,568
Massachusetts	488,185	920,000	728,630	474,504	246,398	22.8	590,935		590,935	12.9				13,681	82,667
Michigan	3,184,057	1,713,142	3,809,885	3,025,292	526,975	239.2	1,264,377	117,227	1,128,600	61.4		43,200	2.9	41,538	14,367
Minnesota	2,376,415	1,470,324	3,907,125	2,312,126	1,337,292	403.3	137,996		110,996	13.0				64,289	22,036
Mississippi	1,744,669	354,023	1,383,008	1,366,416	16,500	149.6	559,692	349,342	210,350	46.5		116,646	8.6	28,911	10,527
Missouri	2,923,273	2,363,922	4,305,104	2,828,144	1,390,700	908.2	991,333	93,414	852,163	149.3		121,059	.3	1,716	
Montana	1,859,937	942,434	2,613,003	1,764,962	835,121	313.0	140,462	42,826	97,635	14.0				52,149	9,678
Nebraska	1,957,240	991,091	2,595,465	1,947,268	621,814	449.2	413,643		369,138	54.6		139	.1	9,972	
Nevada	1,136,479	852,000	1,745,185	1,123,828	577,637	215.7	232,198		232,198	20.8		40,000	5.7	12,651	2,165
New Hampshire	477,386	261,593	769,895	475,526	244,016	34.7						11,610	.4	1,860	5,968
New Jersey	55,099	460,000	56,528	55,099		.5	321,492		320,879	2.8					139,121
New Mexico	1,272,129	735,425	1,967,597	1,272,129	693,418	293.3	26,051		26,051	.7					15,956
New York	4,002,686	3,693,000	7,384,831	3,538,046	2,741,939	311.7	1,201,150	327,200	870,400	72.2				137,440	80,661
North Carolina	2,380,573	1,700,340	3,408,235	2,228,195	1,169,844	376.2	674,445	143,949	530,496	54.4				8,429	
North Dakota	1,451,112	734,742	1,529,905	1,330,868	198,200	469.6	434,981	91,607	343,374	104.5	25,629	92,333	11.9	3,008	100,834
Ohio	3,871,148	1,966,253	4,895,840	3,781,486	829,831	375.1	772,760	18,070	722,417	40.0	47,700	323,350	16.8	23,892	90,655
Oklahoma	2,304,199	1,171,295	2,842,275	2,158,656	372,526	296.3	811,029	147,071	623,574	37.4		29,343	.3	6,472	145,852
Oregon	1,526,724	892,176	2,535,214	1,494,881	776,391	173.7	19,526	19,526		.2				12,317	115,785
Pennsylvania	7,411,822	2,639,003	8,936,401	6,538,006	2,069,404	655.2	1,288,961	658,917	553,497	98.0		5,701		214,899	10,401
Rhode Island	497,813	254,040	497,631	447,809	39,789	34.2	211,374		211,374	7.1	50,004		3.0		2,876
South Carolina	1,364,791	1,342,000	1,417,537	1,062,011	327,745	142.4	1,259,359	249,891	990,809	148.0				52,889	23,446
South Dakota	1,502,870	761,911	2,013,290	1,442,863	565,011	592.6	177,456	50,813	126,643	40.3		67,190	7.0	9,194	3,066
Tennessee	2,123,155	1,075,748	2,537,856	1,949,718	479,648	174.2	415,358	103,127	312,231	14.4	3,663	120,014	1.5	66,647	163,855
Texas	6,012,518	3,638,000	8,234,416	5,982,701	1,723,228	956.6	1,893,214		1,893,214	90.4		5,000	14.2	29,817	16,558
Utah	1,048,677	533,173	1,738,377	1,039,141	392,639	236.1	166,380		140,000	16.0				9,536	534
Vermont	438,880	241,354	812,551	435,360	241,354	53.2								3,520	
Virginia	1,736,770	893,188	2,199,326	1,672,495	474,440	254.6	405,183	35,103	357,944	45.8	21,736	12,083		7,437	48,721
Washington	1,080,673	776,603	1,817,676	1,077,997	697,729	124.1	63,750		63,750	.8				2,676	15,124
West Virginia	1,118,559	570,083	1,243,668	1,056,049	142,795	64.2	370,225	57,172	313,053	18.8		29,594	2.4	5,338	84,641
Wisconsin	2,431,220	1,743,354	3,946,579	2,409,749	1,214,719	226.2	558,308		505,107	9.3		18,549		21,471	4,978
Wyoming	1,125,332	571,928	1,513,605	1,122,741	373,375	251.3	198,553		198,553	25.9				2,590	
District of Columbia	972,024	792,791	1,448,986	971,729	477,257	12.2	104,932		104,932	.7		75,395	.6	296	135,207
Hawaii	177,718	351,000	178,209	177,718		4.9	150,713		143,867	4.6					207,133
TOTALS	93,666,378	58,473,436	127,796,572	88,282,376	32,547,175	12,340.3	25,792,186	4,045,176	21,029,261	1,815.9	184,730	1,873,708	103.0	1,154,096	3,023,292

PUBLICATIONS *of the* BUREAU OF PUBLIC ROADS

Any of the following publications may be purchased from the Superintendent of Documents, Government Printing Office, Washington, D. C. As his office is not connected with the Department and as the Department does not sell publications, please send no remittance to the United States Department of Agriculture.

ANNUAL REPORTS

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Report of the Chief of the Bureau of Public Roads, 1927. 5 cents.

Report of the Chief of the Bureau of Public Roads, 1928. 5 cents.

Report of the Chief of the Bureau of Public Roads, 1929. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1931. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1932. 10 cents.

Report of the Chief of the Bureau of Public Roads, 1933.

Report of the Chief of the Bureau of Public Roads, 1934.

DEPARTMENT BULLETINS

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No. 347D . . Methods for the Determination of the Physical Properties of Road-Building Rock. 10 cents.

No. 583D . . Reports on Experimental Convict Road Camp, Fulton County, Ga. 25 cents.

No. 1279D . . Rural Highway Mileage, Income, and Expenditures, 1921 and 1922.

TECHNICAL BULLETINS

No. 55T . . . Highway Bridge Surveys. 20 cents.

No. 265T . . . Electrical Equipment on Movable Bridges. 5 cents.

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No. 62MC . . Standards Governing Plans, Specifications, Contract Forms, and Estimates for Federal-Aid Highway Projects. 5 cents.

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No. 76MP . . The results of Physical Tests of Road-Building Rock. 25 cents.

Federal Legislation and Regulations Relating to Highway Construction. 10 cents.

Supplement No. 1 to Federal Legislation and Regulations Relating to Highway Construction.

No. 191 . . . . Roadside Improvement. 10 cents.

The Taxation of Motor Vehicles in 1932. 35 cents.

REPRINT FROM PUBLIC ROADS

Reports on Subgrade Soil Studies. 40 cents.

Single copies of the following publications may be obtained from the Bureau of Public Roads upon request. They cannot be purchased from the Superintendent of Documents.

SEPARATE REPRINT FROM THE YEARBOOK

No. 1036Y . . Road Work on Farm Outlets Needs Skill and Right Equipment.

TRANSPORTATION SURVEY REPORTS

Report of a Survey of Transportation on the State Highway System of Ohio (1927).

Report of a Survey of Transportation on the State Highways of Vermont (1927).

Report of a Survey of Transportation on the State Highways of New Hampshire (1927).

Report of a Plan of Highway Improvement in the Regional Area of Cleveland, Ohio (1928).

Report of a Survey of Traffic on the Federal-Aid Highway Systems of Eleven Western States (1930).

A complete list of the publications of the Bureau of Public Roads, classified according to subject and including the more important articles in PUBLIC ROADS, may be obtained upon request addressed to the U. S. Bureau of Public Roads, Willard Building, Washington, D. C.

CURRENT STATUS OF UNITED STATES PUBLIC WORKS ROAD CONSTRUCTION

AS PROVIDED BY SECTION 204 OF THE NATIONAL INDUSTRIAL RECOVERY ACT (1934 FUNDS) AND BY THE ACT OF JUNE 18, 1934 (1935 FUNDS)

SUMMARY OF CLASSES 1, 2, AND 3.

AS OF OCTOBER 31, 1935

STATE	APPORTIONMENTS		COMPLETED				UNDER CONSTRUCTION				APPROVED FOR CONSTRUCTION			BALANCE OF FUNDS AVAILABLE FOR NEW PROJECTS	
	Sec. 204 of the Act of June 16, 1933 (1934 Fund)	Act of June 18, 1934 (1935 Fund)	Total Cost	1934 Public Works Funds	1935 Public Works Funds	Mileage	Estimated Total Cost	1934 Public Works Funds	1935 Public Works Funds	Mileage	1934 Public Works Funds	1935 Public Works Funds	Mileage	1934 Public Works Funds	1935 Public Works Funds
Alabama	$ 8,370,133	$ 4,259,842	$ 12,379,999	$ 7,688,721	$ 1,543,428	656.9	$ 2,507,534	$ 600,432	$ 1,734,653	102.0	$ 11,886	$ 414,213	13.6	$ 69,093	$ 567,548
Arizona	5,211,960	2,641,935	7,548,280	4,983,509	1,704,363	502.9	1,207,096	208,622	819,720	42.2		6,580	.1	19,828	111,272
Arkansas	6,748,335	3,428,049	8,341,868	5,965,196	1,555,094	456.9	2,329,909	574,467	1,613,555	142.0	149,665	91,006	6.0	59,007	168,395
California	15,607,354	7,932,206	23,805,517	15,123,463	3,731,310	709.8	6,259,684	478,067	3,755,759	63.2		206,319	.7	5,824	238,818
Colorado	6,874,530	3,486,006	10,840,035	6,689,598	3,366,256	629.3	304,320	133,817	94,839	16.1				51,115	24,911
Connecticut	2,865,740	1,454,868	4,078,082	2,859,541	803,552	67.5	497,992		482,471	5.5		58,859	.1	6,199	109,986
Delaware	1,819,088	923,395	2,443,679	1,615,412	776,751	126.8	204,886	203,549	1,102	2.5		42,012		127	103,530
Florida	5,231,834	2,661,343	7,682,112	5,140,557	1,371,817	260.4	1,121,729	29,832	1,002,802	46.9		199,038	1.7	61,445	87,687
Georgia	10,091,185	5,113,491	10,753,157	8,604,509	1,751,097	622.9	2,131,408	1,000,198	1,077,187	98.3	87,819	319,874	15.5	398,660	1,965,333
Idaho	4,486,249	2,277,486	5,987,005	4,403,836	1,250,713	470.8	766,882		764,753	26.1		77,795	.4	82,413	184,225
Illinois	17,570,770	8,921,401	17,693,800	14,625,614	2,593,839	509.5	7,978,674	2,836,052	5,141,806	180.4		378,018	4.2	109,104	807,738
Indiana	10,037,843	5,088,963	10,357,589	9,071,489	862,591	294.3	4,716,337	797,345	3,883,600	193.3	51,719	84,060	2.2	117,290	258,712
Iowa	10,055,660	5,118,361	14,715,150	9,842,955	4,109,230	1,167.6	1,020,278	210,565	754,400	57.6		1,000	2.2	2,140	253,731
Kansas	10,089,604	5,117,675	13,619,078	9,961,125	3,308,084	1,052.6	1,878,497	70,683	1,785,266	89.0	48,919	24,325	2.2	8,878	
Kentucky	7,517,359	3,818,311	9,398,499	6,906,083	1,984,852	702.7	2,187,969	499,535	1,455,360	102.4	40,006	76,769	4.6	71,735	301,330
Louisiana	5,828,591	2,963,932	6,074,075	4,565,560	956,023	182.0	2,507,869	1,058,401	1,407,515	58.8	155,417	545,151	19.2	49,213	55,242
Maine	3,369,917	1,711,586	4,679,782	3,302,096	1,206,200	181.7	526,759	45,122	467,720	11.5				22,699	37,665
Maryland	3,564,527	1,810,058	2,968,820	2,490,108	407,217	108.7	2,182,277	979,303	389,010	26.4		402,384	11.7	95,116	611,448
Massachusetts	6,597,100	3,350,474	6,092,918	4,579,945	934,555	89.5	4,170,306	1,974,333	2,195,973	25.8				42,822	219,946
Michigan	12,736,227	6,452,568	16,814,115	12,485,702	3,463,986	623.3	3,034,502	182,477	2,829,775	139.7		128,248	5.9	68,048	30,559
Minnesota	10,656,569	5,425,551	14,881,801	9,933,719	4,318,900	1,609.4	1,346,392	539,834	645,540	39.6	13,805	205,337	1.8	169,211	255,774
Mississippi	6,978,675	3,540,227	9,013,651	5,477,139	1,067,781	518.5	3,692,423	1,365,390	1,898,631	199.4	5,574	421,181	23.1	130,572	152,634
Missouri	12,180,306	6,173,740	13,274,789	10,597,882	1,886,704	1,193.4	5,731,840	1,452,147	3,936,939	257.3		240,779	.7	130,277	109,318
Montana	7,439,748	3,769,734	10,947,625	7,144,593	3,168,065	991.4	656,728	154,622	502,106	32.3	47,837		7.3	92,696	99,563
Nebraska	7,828,961	3,964,364	11,153,608	7,755,445	2,213,832	941.4	1,962,774	34,041	1,637,550	92.5		10,395	.1	39,475	102,588
Nevada	4,545,917	2,302,356	6,549,397	4,475,445	1,873,496	691.4	385,677	26,150	347,167	60.4		41,650	5.7	44,322	40,042
New Hampshire	1,909,839	969,462	2,807,486	1,836,024	869,856	76.9	64,751		64,751	.4	22,006	11,610	.9	51,809	23,244
New Jersey	6,346,039	3,220,879	6,542,556	5,831,264	354,394	71.6	2,419,290	356,159	1,746,198	16.0		656,846	1.0	158,615	463,441
New Mexico	5,792,935	2,941,700	8,039,581	5,531,998	2,330,027	744.1	527,745	62,414	465,331	24.6	37,087			161,437	146,341
New York	22,330,101	11,327,921	31,754,211	21,106,918	5,648,719	659.4	7,986,263	984,375	4,972,442	163.6	12,000	58,000	.2	226,808	648,761
North Carolina	9,522,293	4,840,941	12,983,083	8,675,058	3,074,834	1,237.6	1,870,333	459,393	1,394,045	108.1	74,297	258,195	4.6	313,545	113,867
North Dakota	5,804,448	2,938,967	7,047,022	5,397,619	1,033,602	1,884.3	923,175	241,742	616,848	155.4	124,619	389,562	65.8	40,468	898,955
Ohio	15,484,592	7,865,012	20,415,092	15,231,198	3,724,403	701.6	3,775,234	161,683	3,301,126	69.0	47,700	648,521	19.9	44,012	190,961
Oklahoma	9,216,798	4,685,180	12,103,278	8,835,902	2,536,818	735.6	2,076,112	366,193	1,650,283	67.4		153,989	1.2	14,702	344,090
Oregon	6,106,896	3,097,814	9,220,393	5,905,905	2,510,076	455.0	605,124	99,039	429,966	10.5	517			101,434	157,772
Pennsylvania	18,891,004	9,590,788	26,798,346	17,816,453	7,828,278	955.0	1,931,615	831,814	1,015,026	108.1	2,126	389,480	1.5	240,611	358,004
Rhode Island	1,998,708	1,014,572	2,676,902	1,944,409	645,325	78.0	215,174		215,174	7.1	50,004	151,196	4.1	4,295	2,876
South Carolina	5,459,165	2,770,954	5,775,244	4,972,835	695,130	440.8	2,036,194	373,302	1,620,082	179.0	15,740	52,612	.2	97,288	403,130
South Dakota	6,011,479	3,047,643	8,316,479	5,692,880	2,075,416	1,442.5	493,369	110,383	382,986	73.2	6,425	466,256	33.9	201,791	122,986
Tennessee	8,492,619	4,302,991	11,205,934	8,109,360	2,198,963	438.7	1,710,951	242,654	1,371,895	42.1	14,917	412,135	3.8	125,688	319,997
Texas	24,244,024	12,291,253	30,989,671	23,308,078	6,223,120	2,500.1	6,509,639	640,814	5,629,501	271.0	265,315	363,130	20.9	29,817	75,502
Utah	4,194,708	2,132,691	6,446,870	4,008,880	1,705,966	567.9	578,122	167,060	287,750	22.7		101,000	.3	18,768	37,975
Vermont	1,867,573	948,007	2,934,878	1,833,963	777,042	134.9	185,029	3,922	166,843	4.7		2,713		29,688	1,410
Virginia	7,416,757	3,765,387	10,659,304	7,116,309	2,722,299	547.3	1,093,945	195,954	802,505	69.6	42,221	71,985	1.5	62,273	168,599
Washington	6,115,867	3,106,412	8,003,783	5,902,257	2,010,280	289.0	1,342,211	198,743	1,016,791	13.8		8,093		14,867	71,248
West Virginia	4,474,234	2,280,335	5,167,023	4,159,038	861,104	182.6	1,243,596	279,824	930,783	33.3		81,403	2.6	35,372	407,044
Wisconsin	9,724,881	4,941,837	14,145,338	9,619,596	3,855,813	589.0	1,220,351	41,802	979,141	30.2		97,661	1.2	63,483	9,222
Wyoming	4,501,327	2,287,712	6,204,518	4,442,784	1,576,344	932.0	731,432	48,941	655,752	105.6		50,372		9,603	5,244
District of Columbia	1,918,469	973,842	2,576,482	1,918,173	658,309	18.8	104,932		104,932	.7		75,395	.6	296	135,207
Hawaii	1,871,062	949,778	1,504,948	1,130,620		32.2	1,291,097	681,972	436,870	18.3	20,973	276,443	1.8	37,497	236,465
TOTALS	394,000,000	200,000,000	516,412,853	366,616,763	108,125,854	31,076.5	102,246,426	22,003,167	70,882,220	3,705.6	1,348,594	8,751,590	295.0	4,031,476	12,240,336

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