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TRAFFIC RECORDING STATION—OHIO TRANSPORTATION SURVEY

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DIGEST OF REPORT OF OHIO HIGHWAY TRANSPORTATION SURVEY

Reported by J. GORDON McKAY, Chief of the Division of Highway Economics, Bureau of Public Roads

THE Report of a Survey of Transportation on the State Highway System of Ohio has recently come from the press.[1] This report contains the results of highway-traffic studies of the State, county, and township road systems of Ohio conducted during 1925 under a cooperative agreement between the Bureau of Public Roads, United States Department of Agriculture, and the Ohio State Department of Highways and Public Works.

The investigation was undertaken in order to obtain essential facts concerning traffic on Ohio highways as a basis for planning the development of the Ohio State highway system to serve present and future traffic.

The conclusions are based upon the present density, type, loading, and distribution of traffic units and a traffic classification of State highways, upon present population and population trends, upon predicted future traffic, and upon an economic and physical analysis of other factors affecting the planning of a program of highway improvement.

The first section of the full report contains a summary of principal conclusions, the second contains the detailed data, and the third section the proposed plan of State-highway improvement indorsed by the Ohio State Department of Highways and Public Works and the United States Bureau of Public Roads.

The highway-traffic studies upon which the report is based were conducted under the joint supervision of Thomas H. MacDonald, Chief of the Bureau of Public Roads, and George F. Schlesinger, director, and L. A. Boulay, former director of the Ohio State Department of Highways and Public Works. J. Gordon McKay, Chief of the Division of Highway Economics, Bureau of Public Roads, directed the work of the survey and preparation of the report, assisted by O. M. Elvehjem, E. T. Stein, L. E. Peabody, and B. P. Root, all of the Division of Highway Economics, and Harry J. Kirk, State highway engineer, and Harry E. Neal, traffic engineer of Ohio.

Of the approximately 11,000 miles of State highways on January 1, 1926, there were 5,194 miles that were improved with brick, concrete, asphalt, and bituminous-macadam surfaces, of which more than half had surfaces less than 18 feet in width, and approximately 400 miles were old and worn-out pavement under county maintenance. Surface-treated macadam surfaces, of which there were 1,307 miles in 1925, will require re-treatment or reconstruction when the traffic materially increases. The remainder comprised 3,000 miles of gravel, slag, and stone and 1,282 miles of unimproved highways.

The principal problems now confronting the State are the reconstruction of the old, worn-out surfaces, expensive to maintain, the widening of narrow pavements, the improvement of unimproved sections of the State system, the elimination of the most dangerous railroad grade crossings, and the distribution of present highway revenues satisfactorily to complete the State system of roads.

Anticipating the need for a definite program of reconstruction and development, and realizing the necessity of having as a basis for a sound plan of highway improvement accurate data with respect to the traffic on the various sections of the State highway system, the Director of Highways and Public Works entered into an agreement with the United States Bureau of Public Roads to conduct a cooperative survey of transportation on the highways of the State.

At the City Limits of Zanesville on the National Road in 1913

CONCLUSIONS FROM SURVEY SUMMARIZED

The results of the survey show that during the next five years the State should reconstruct 1,220 miles of the State system, widen 1,594 miles, and build 1,707 miles, the latter comprising 1,007 miles of construction superior to gravel and 700 miles of traffic-bound improvements. The cost is estimated at $100,000,000.

On the basis of the traffic observed during the survey, it is estimated that the State highway system, which comprises 13 per cent of the total rural mileage, provided highway service for a traffic of 2,160,435,000 vehicle-miles, equal to 57.7 per cent of the total motor-vehicle traffic on the rural highways of the State in 1925; that the county highways, which include 27.1 per cent of the rural mileage, provided service for 1,108,870,000 vehicle-miles, 29.6 per cent of the total traffic; and that the township highways, which constitute 59.9 per cent of rural mileage, provided service for 477,055,000 vehicle-miles, or only 12.7 per cent of total rural traffic. The daily traffic on the State system averages over nine times that upon the county and township roads. These facts show the necessity of allotting a sufficient portion of total highway revenues to complete the improvement of the State system of highways.

It is clearly shown by the survey that the principal routes of the State system, comprising what are known as the Federal-aid and main market systems, are with a few exceptions the most important traffic routes of the State. The improvement of these routes, however, has not up to this time been entirely consistent with their traffic importance. There are sections of gravel and

[1] Copies of the full report may be obtained upon request from the Bureau of Public Roads.

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old, worn-out surfaces, narrow 9 to 16 foot pavements, as well as unimproved sections on these heavily traveled routes.

The largest volume of motor-vehicle traffic is found in the areas adjacent to large centers of urban population and on the main, through-traffic routes.

Of the 11,000 miles of the State highway system, 1.2 per cent carried 2,500 or more motor vehicles per day, 7.8 per cent carried 1,500 or more, 29.4 per cent carried 600 or more, and 70.6 per cent carried less than 600 vehicles per day in 1925.

Pan American Highway Engineers and Busses in Which They Inspected Ohio Highways in 1924

The northeastern part of the State is the most important traffic area and is also the region of densest population, motor-vehicle registration, and industrial development. The southwestern area is very close to the northeastern section in traffic importance. The northwestern and southeastern parts of the State are of least present and expected future traffic importance.

Motor-truck traffic is an important part of total traffic on the principal routes. On the State system in 1925, 2.7 per cent of the mileage carried 200 or more trucks per day, 5.7 per cent carried 150 or more, 13.1 per cent 100 or more, and 27.5 per cent carried 60 or more trucks per day. Based on the traffic forecast, it is expected that by 1935 over 1,300 miles of State highways will carry 200 or more trucks per day, while a comparatively small mileage is expected to carry from 500 to 1,000 trucks a day.

Four-fifths of the trucks operating on the State system are small units, 2½ tons capacity or less. The comparatively small number of large-capacity trucks and heavy loads observed may perhaps be attributed in a measure to the gross-load limitation of 20,000 pounds fixed by law in the State.

The average daily density of traffic in 1925 was 538 vehicles on the State system, 132 on county highways, and only 26 on township roads, each mile of road of the State system providing traffic service more than equal to that of 4 miles of county and 20 miles of town roads.

The Federal-aid system, slightly more than half the State system, carried 70.6 per cent of the daily traffic of the State system; the main market roads, approximately one-third of the State-system mileage and included for the most part in the Federal-aid system, carried over half the traffic, and the principal routes of the system, 8.8 per cent of the mileage, carried over one-fourth of the traffic.

Foreign traffic comprises but a small part of the total traffic except on the principal through-traffic routes. More than half of passenger-car traffic consists of cars used for business purposes.

The survey clearly shows that the traffic using the State system is predominantly city passenger-car and motor-truck traffic, farm-owned passenger cars and motor trucks making up only 12.4 per cent and 15.5 per cent, respectively, of the total passenger-car and motor-truck traffic. The improvement and maintenance of State highways is, therefore, primarily the result of the demand for highway service by city motor-vehicle owners.

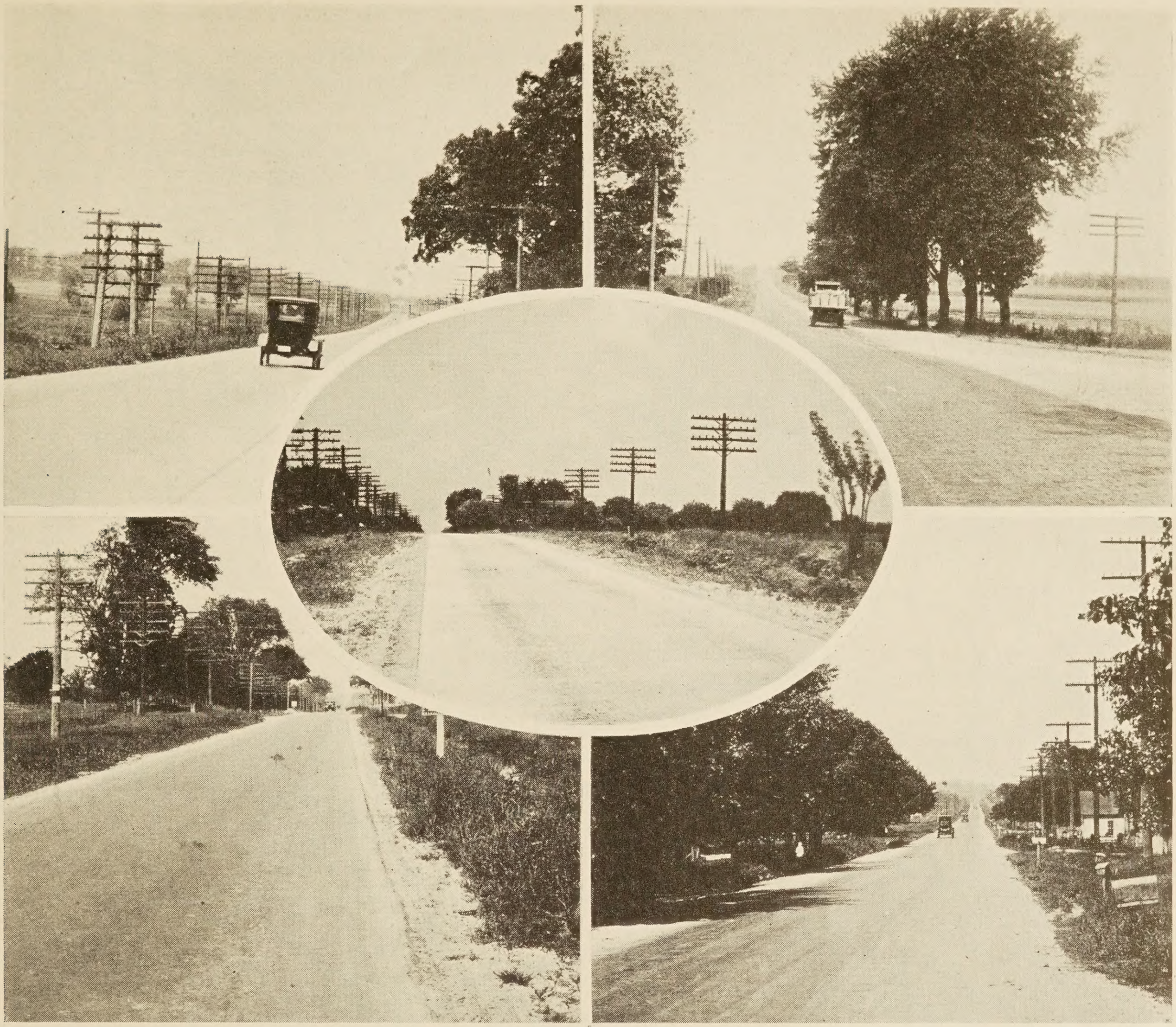
The volume of traffic in a given area is principally produced by the population residing within a radius of 30 miles, since less than 30 per cent of the truck traffic and less than 40 per cent of the passenger-car traffic travels more than 30 miles.

FUTURE TRAFFIC AND HIGHWAY REQUIREMENTS ESTIMATED

The distribution of population is an important factor in planning highway improvements. Of the entire area of the State 80 per cent has a population of less than 80 persons per square mile and is the home of only 22.7 per cent of the total State population, whereas the 9.4 per cent of the area that has a population of 160 or more persons per square mile includes 69.3 per cent of the population of the State.

These variations indicate marked differences in the necessity for highway service in the several parts of the State. In the densely populated areas the highway system should be planned to serve large volumes of traffic between the principal centers of population with tributary feeder routes connecting minor population centers with the primary traffic routes. These routes should be of sufficient width and improved with surfaces adequate to carry the large daily volume of traffic as directly as possible; obstructions to the free movement of traffic, such as railway crossings at grade, sharp curves, heavy grades, and congested traffic sections should be eliminated, and by-pass routes should be constructed to avoid the traffic congestion which occurs when a main traffic route passes through the business center of small villages and cities. In the sparsely populated areas the volume of traffic is smaller and its sources more scattered. A connected system of main routes comprising a smaller mileage, improved with gravel or the lower types of paved surfaces where traffic warrants such improvements, should satisfactorily meet traffic requirements in these areas, except on the main through routes traversing them. The removal of obstacles to the easy movement of traffic is not an important problem in areas of low population and traffic, particularly when the expectancy of future traffic increase is small.

The density of traffic on the various roads of the State system has been used as the basis for an estimate of traffic on the same roads in 1930 and 1935, applying for this purpose the relation between the increase in traffic on the highways and the ratio of population to motor-vehicle registration observed in other States. In 1925 there was one motor vehicle for each 4.7 persons in Ohio. Extending the past trend of this ratio to 1935 it is estimated that there will then be one vehicle for each 2.82 persons. On this basis the registration of 1935 is estimated at 2,607,000 motor vehicles, a registration approximately twice as great as that of 1925. As the yearly increase of motor-vehicle traffic on the highways has been found to be practically in direct proportion to the growth of motor-vehicle registration, it may be expected that traffic on the State highways will increase 51 per cent between 1925 and 1930, and 28 per cent between 1930 and 1935.



TYPES OF ROAD CONSTRUCTED ON THE OHIO STATE SYSTEM IN RECENT YEARS

As a basis for the plan of highway improvement, the State highways are classified in three groups designated as major, medium, and minor traffic highways, according to their average daily traffic. Routes or sections of routes carrying 1,500 or more motor vehicles per day are classed as major routes, those carrying 600 to 1,500 vehicles per day as medium routes, and those carrying less than 600 vehicles daily are classed as minor routes. The routes or sections of routes are classed in this way on the basis of the observed 1925 traffic, and the estimated traffic for 1930 and 1935 is employed in a similar manner to indicate the probable classification in those years.

Experience in many States indicates that ordinary untreated gravel and similar surfaces can not be economically maintained when the traffic exceeds 500 to 600 vehicles per day, and similar experience in Ohio points to approximately 600 vehicles per day as the limit. Above that traffic density the type and design of surface required are largely functions of the frequency of heavy loads, the choice of types including bituminous macadam for the lower densities and the several rigid types for roads of greater density.

If, on the basis of this experience, those sections of the Ohio State system which carry a traffic of 600 or more vehicles per day be considered as requiring a type of surface superior to untreated gravel, it is found that in 1925 over one-third of the 11,000 miles of the State system, or 3,852 miles, required such surfaces, and 10 years later, in 1935, based on the estimated traffic, approximately half the system, or 5,221 miles, should be so improved.

THE METHODS OF THE SURVEY

The traffic survey was begun in December, 1924, and continued for a period of one year. During the survey traffic data were recorded at 1,158 points on Ohio highways. At 358 of these points complete data were recorded one day each month during the year period. At the remaining 800 points counts of passenger cars and motor trucks were obtained on three days during the summer months. Data obtained included counts of passenger cars, motor trucks, motor busses, horse-drawn vehicles, foreign vehicles, and detailed truck and passenger-car data. Motor-truck data included the capacity of the truck, State of registration, place

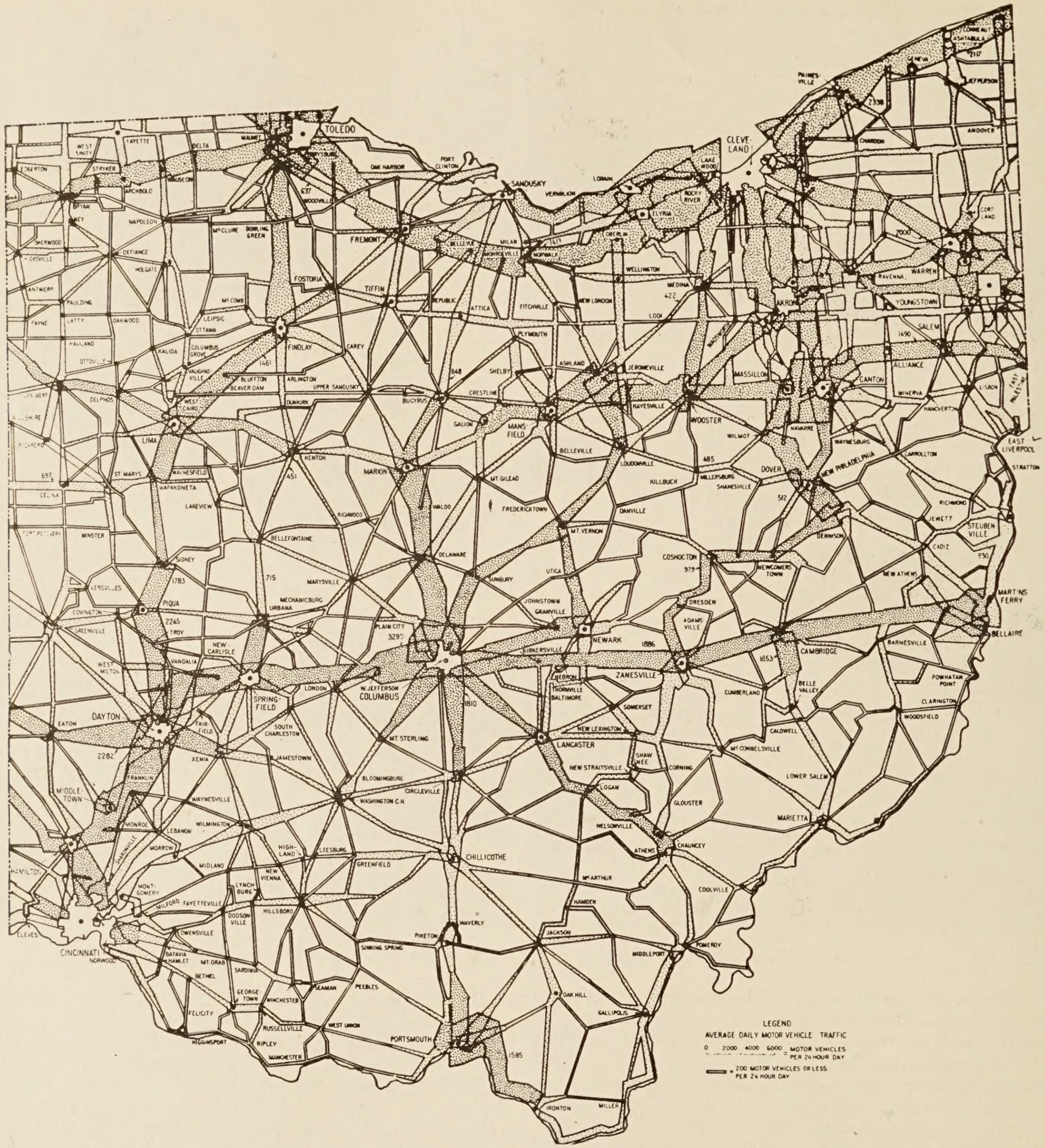


Fig. 1.—Average Daily Density of Motor Vehicle Traffic on the State Highway System

of ownership, origin, destination, type of origin and destination, commodity carried, and tire equipment. At alternate operations at 156 stations total gross and rear-axle weights were measured by means of portable scales. Passenger-car data included State of registration, situs of ownership, purpose of trip, origin, destination, and number of passengers.

Each operation consisted of a 10-hour observation period, alternating between 6 a. m. to 4 p. m. and 10 a. m. to 8 p. m. Special observers tabulated traffic between 8 p. m. and 6 a. m. at selected stations. Complete 24-hour observations were therefore available at these stations, which were made the basis of computation of hourly variations in traffic and of average daily traffic at all stations. Traffic observations for week periods were also made at selected stations to determine variations in traffic by days of the week. Seasonal variations were computed from the monthly operations at all stations. Stations were operated on a carefully planned schedule which permitted operations on the various days of the week and prevented duplicate recording of traffic.

Traffic was observed on practically all sections of the State highway system, and stations were so located as to enable close observation of the variations in traffic on various routes and sections of routes. Stations were also located on representative sections of the county and township highway systems in all sections of the State.

STATE SYSTEM CARRIES LARGE PERCENTAGE OF TOTAL TRAFFIC

There are in Ohio approximately 84,884 miles of rural highway, of which, on January 1, 1926, 11,000 miles were intercounty highways, constituting the State highway system. Of the remaining 73,884 miles, 22,991 were included in the county system, and 50,893 were township roads. Federal-aid highways included in the State highway system aggregated 5,899 miles and the main market road system 3,486 miles.

Upon the 84,884 miles it is estimated that in 1925 there was a motor-vehicle movement of approximately 3,746,360,000 vehicle-miles. The relative importance of the State highway system is indicated by the fact that although it includes but 13 per cent of the entire rural road mileage, it carried 2,160,435,000 vehicle-miles, or 57.7 per cent, of the total motor-vehicle traffic during 1925. The average daily motor-vehicle traffic per mile upon the State highway system is over nine times that upon the county and township roads.

The daily volume of traffic on different parts of the State highway system varies widely. The number of motor vehicles per average 24 hour day varied from 5,583 on route U. S. 30, between Canton and Massillon, to a minimum of less than 20 vehicles on several unimproved sections. The State highway system includes 4,180 miles, on which the average daily motor-vehicle traffic per mile was less than 200 vehicles. County and town roads include a considerable mileage, on which the average daily traffic was less than 5 vehicles per day.

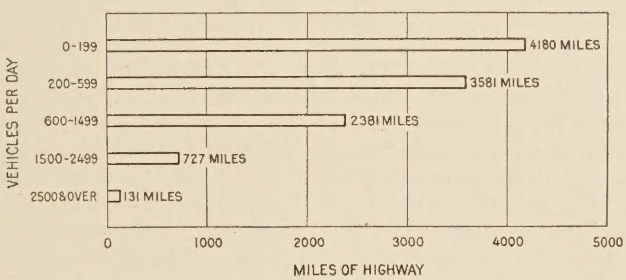


FIG. 2.—MILEAGE OF STATE HIGHWAYS BY TRAFFIC DENSITY CLASSES

During 1925 the density of horse-drawn vehicles was recorded at all survey stations, but it was early apparent that their numbers were so few as to warrant no consideration in highway planning. The average traffic of horse-drawn vehicles on State highways is less than 7 per day.

Motor-bus traffic is important on several of the State routes. It is, however, a specialized movement, and its volume on any highway is the product of several factors which have little effect upon other motor-vehicle traffic. Motor-bus traffic is, therefore, discussed separately in the report, and the term "motor-vehicle traffic" refers only to passenger cars and motor trucks.

The complete report shows the traffic density at each of the 1,158 points where traffic was observed during the survey and shows also, by counties, the routes upon which traffic was observed, the averge density of motor-vehicle traffic for a 24-hour day, the average daily density of motor-truck traffic, the normal maximum traffic, and the estimated traffic for 1930.

The average daily distribution of total traffic on the State highway system is shown in Figure 1. The more important traffic routes of the State are apparent. Figure 2 shows the mileage of State highways by traffic density classes.

NORTH HILL VIADUCT AT AKRON, OHIO. LENGTH 2,800 FEET; ROADWAY, 52 FEET; BUILT 1921

PRINCIPAL TRAFFIC ROUTES DISCUSSED

The largest volume of traffic of both passenger cars and trucks is found in the areas adjacent to large centers of urban population and on the main through routes. The concentration of traffic in the areas immediately adjacent to the larger cities of the State is apparent. The principal through routes are also clearly evident as broad bands serving wide areas in the State and connecting the important cities of Ohio and near-by States. The traffic importance of these principal routes is the result of local traffic of the area augmented by the traffic moving between larger centers of population. Among the more important of the through routes are the following: The Buffalo-Chicago Highway, crossing Ohio near its northern border, connecting the Buffalo and Erie territory with Cleveland and surrounding cities, Toledo and other points in Ohio, and cities in Indiana, Michigan, and Illinois; the National Pike, from Bridgeport through Zanesville, Columbus, Springfield, and Dayton,[2] and connecting Wheeling and eastern points with central Ohio cities and with cities in Indiana, Illinois, and points west; the Lincoln-Harding Highway (route U. S. 30), through Canton, Mansfield, Marion, and Lima, connecting Pittsburgh and Pennsylvania cities with the above cities in Ohio and with Indiana and Illinois cities to the west. Crossing the State in a north and south direction are the Dixie Highway, from Toledo through Dayton to Cincinnati, connecting Detroit and other Michigan cities with areas south of Ohio; the Scioto Trail, from Sandusky, via Marion and Columbus, to Portsmouth; the Cincinnati-Columbus-Cleveland (the "C. C. C.") Highway; and the Cleveland-Marietta Highway, via Akron, Canton, New Philadelphia, and Cambridge.

[2] The National Pike is routed north of Dayton, but due to the condition of sections of this route west of Brandt traffic on the route passes through Dayton.

The principal through routes coincide in general with the routes adopted for uniform marking by the American Association of State Highway Officials in November, 1926, referred to as United States numbered routes, of which the most important east and west routes are U. S. 20, 30, 40, and 50, and the most important north and south, U. S. 21, 23, and 25.

Route U. S. 20 from the Pennsylvania line follows the Buffalo-Chicago Highway west through Ashtabula, Cleveland, Elyria, Norwalk, and Fremont. A few miles west of Fremont it diverges from the present heavily traveled route and continues through Perrysburg and Maumee rather than Toledo, thence north, crossing State route 2 (the present principal traffic route) and west to the Indiana line over a route at present carrying very light traffic. From Conneaut to Fremont traffic on this route averages 2,447 vehicles per day; from Maumee to the Indiana line but 297. When proposed improvements on this portion of route U. S. 20 are completed, through traffic will to a larger extent use this route in preference to route 2.

A Truck and Trailer Combination

Route U. S. 30 follows the Lincoln and Harding Highways across the State, from East Liverpool through Lisbon, Canton, Mansfield, Marion, Lima, and Van Wert. This route is already an important cross-State route, although east of Canton through traffic has followed the route via Salem and East Palestine rather than the route via Lisbon and East Liverpool, as the former provided a more completely improved route to Pittsburgh. Average traffic for the entire length of route U. S. 30 is 1,071 vehicles per day.

Route U. S. 40, the National Pike, from Wheeling, W. Va., passes almost due west through Zanesville, Columbus, and Springfield to the Indiana line. The western end of the route is unimproved at present, with the result that through traffic detours via Dayton and Eaton. Improvement of less than 20 miles in Preble County will open route U. S. 40 as a direct bypass for through traffic north of Dayton. Traffic in 1925 averaged 1,749 vehicles per day on the 159 miles from Brandt east to the Pennsylvania line.

Route U. S. 50 crosses the State from Belpre via Athens, McArthur, Chillicothe, Hillsboro, and Cincinnati to the Indiana line and is one of the less important United States routes, averaging but 452 vehicles per day. It passes through few large centers of population and for much of its length is surfaced with gravel.

Route U. S. 25, the Dixie Highway from the Michigan line north of Toledo to Cincinnati, is the most important north and south through route and is paved throughout practically its whole length. South of Franklin, in Warren and Butler Counties, the new route follows the most direct course between Dayton and Cincinnati. For 136 miles north of Franklin the average traffic is 1,743 vehicles per day.

Route U. S. 23 crosses the State from the Michigan line, north of Toledo, to Portsmouth via Fostoria, Marion, Columbus, and Circleville. South of Marion it follows the Scioto Trail. On account of the relative lack of improvement in its northern portion, as compared with the Dixie Highway to Findlay, the traffic north of Carey is very light. Between Marion and Columbus traffic averaged 2,160, and between Columbus and Portsmouth it averaged 978 vehicles per day.

Route U. S. 21, from Cleveland to Marietta, coincides with the old Cleveland-Marietta highway south of Newcomerstown and is located west of Canton and Akron to avoid the congested urban traffic of this area.

The system of numbered United States highways in Ohio when improved will form a well-balanced network of the more important through-traffic highways of the State. Many of these routes will serve a large volume of local traffic. The total traffic on each route will depend very largely upon the population and development of the immediate areas which it traverses, and traffic upon those routes which pass through the sparsely populated sections of the State will continue to be small as compared with those routes which connect the important sources of local traffic.

Of the 11,000 miles of the State highway system, 131 miles, or 1.2 per cent of the total mileage, carried 2,500 or more motor vehicles per day in 1925; 858 miles, 7.8 per cent of the system, carried 1,500 or more; 3,239 miles, approximately 30 per cent of the total, carried 600 or more; and 7,761 miles, 70.6 per cent, carried less than 600 vehicles per day, of which 4,180 miles carried less than 200, as shown in Figure 2.

The routes carrying the largest daily volume of traffic are with few exceptions in the northeastern, northern, and southwestern parts of the State, and the routes of least traffic importance are in the southeastern and northwestern sections.

On the basis of traffic the State is divided into five separate traffic sections, each of these sections being subdivided in the order of their traffic importance into two or more divisions, somewhat comparable with the distribution of population and industry. Table 1 shows the mileage of State highways, by traffic classes, in the five traffic sections.

Table 1.—*Mileage of State highways, by traffic classes, in the five traffic sections*

Section	All State highways		Daily traffic, over 1,500 vehicles		Daily traffic, 600 to 1,500 vehicles		Daily traffic, less than 600 vehicles	
	Miles	Per cent of total miles	Miles	Per cent of total miles	Miles	Per cent of total miles	Miles	Per cent of total miles
Northeastern	2,821	25.6	454	52.9	836	35.1	1,536	19.8
Southwestern	736	6.7	133	15.5	202	8.5	401	5.2
East-central	1,285	11.7	86	10.0	265	11.1	934	12.0
Northwestern	3,402	30.9	162	18.9	679	28.5	2,556	32.9
Southern	2,756	25.1	23	2.7	399	16.8	2,334	30.1
State total	11,000	100.0	858	100.0	2,381	100.0	7,761	100.0

TRUCK TRAFFIC AN IMPORTANT FACTOR

Although motor-truck traffic on the State highway system is only 9.5 per cent of total motor-vehicle traffic measured in vehicle-miles, motor-truck traffic is an important factor in highway-traffic planning.

The average gross weight of motor trucks using the State system is over twice that of passenger cars, while maximum motor-truck weights are four times the maximum weights of passenger cars. The importance of the motor truck is further emphasized by the fact that many trucks are equipped with cushion and solid tires, which are much less effective in cushioning the impact of the wheels than the pneumatic tires with which the passenger cars are equipped. A study of the rear-wheel tire equipment of motor trucks using the State highway system shows that 15 per cent of all the trucks are equipped with solid tires and a like percentage with cushion tires, and that 95 per cent of the 3 to 7½ ton trucks are equipped with cushion or solid tires on the rear wheels.

Since motor trucks carry heavier gross loads than passenger cars, and the larger capacities are equipped with solid or cushion tires, and because the trucks do not have the refinements in shock-absorbing devices and spring equipment possessed by the passenger cars, the motor truck, where it forms an appreciable part of motor-vehicle traffic, presents a special problem for the highway builder.

The average daily density of motor-truck traffic varies greatly on different routes and in various parts of the State. Of the 11,000 miles of State highways, 299 miles, 2.7 per cent of the total, carried in 1925, 200 or more trucks per day; 629 miles, 5.7 per cent, carried 150 or more; 1,442 miles, 13.1 per cent, carried 100 or more; 3,019 miles, 27.5 per cent, carried 60 or more; and 7,981 miles, 72.5 per cent, carried less than 60 trucks, of which 5,305 miles, 48.2 per cent, carried less than 30, as shown in Figure 3. On routes carrying a small average daily truck traffic, especially those on which there are less than 30 trucks, and many sections on which the daily density ranges from 30 to 59 class the number of trucks is practically negligible in planning highway improvements. An improvement sufficient for passenger-car traffic will, with but few exceptions, prove satisfactory for the small-capacity trucks using these routes. On those routes carrying 60 or more, and particularly the routes carrying 100 or more, the motor truck becomes an important factor in planning the improvement of the highways.

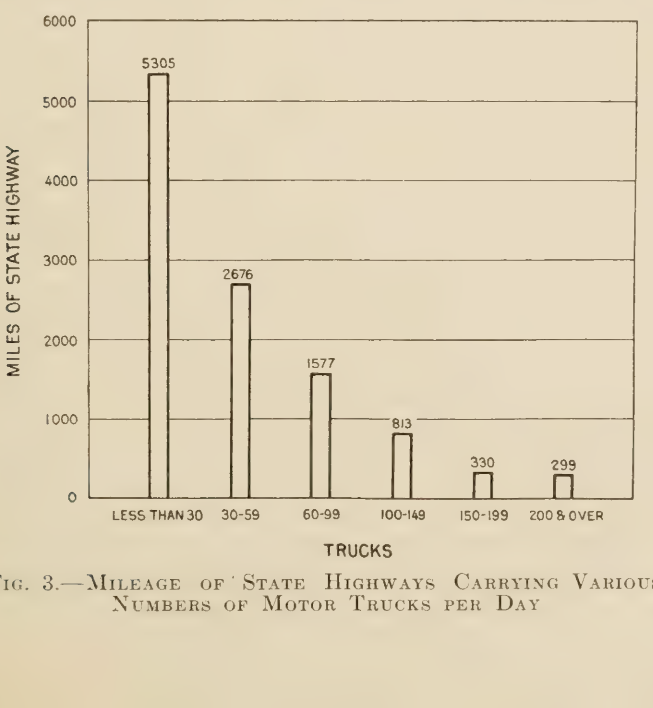


Fig. 3.—Mileage of State Highways Carrying Various Numbers of Motor Trucks per Day

Sections of highway which have a traffic of more than 200 trucks per day include 299 miles, or 2.7 per cent, of the total State highway mileage. On the basis of the traffic forecast there will be 841 miles of State road in 1930 and 1,351 miles in 1935, on which the motor-truck traffic density will exceed 200 trucks per day.

U. S. route 30, Massillon to Canton, carries an average of 485 trucks per day, which is the highest truck density on any section in the State. In the forecast for 1935, truck traffic on this route is estimated at 1,000 per day. Highway sections carrying from 150 to 200 trucks per day comprise 330 miles, or 3 per cent, of the total mileage of State highways.

Milk and Other Dairy Products are Among the Principal Commodities Hauled by Trucks

CITIES INFLUENCE TRUCK TRAFFIC

The more important of the five traffic sections shown in Table 1, from the standpoint of motor-truck density, are the northeastern section, where the average daily density of motor trucks is 77 per mile of State highway, and the southwestern section with a density of 75 trucks. The corresponding density is 53 in the east-central section, and 36 in the northwestern and southern sections.

The comparative importance of the northeastern section is further emphasized by the fact that this section includes 25 per cent of the total mileage of State highways. The southwestern section, although of almost equal truck traffic density, includes only 7 per cent of the mileage of State highways. The northwestern and southern sections, with a density of only 36 trucks per day, include 56 per cent of the State-highway mileage.

The size, location, and industrial development of cities and towns determines very largely the volume of motor trucking on routes in the several sections of the State. Centers of population and industry are the main source and destination of goods transported by motor truck, and the principal trucking routes are those serving the terrritory adjacent to the larger cities and those routes connecting centers of population.

The influence of large cities on motor-truck traffic is clearly shown by the data collected. The nine largest cities in the State, in order of population, are Cleveland, Cincinnati, Toledo, Columbus, Akron, Dayton, Youngstown, Canton, and Springfield. Around these cities with one or two exceptions, is found the greatest volume of motor-truck traffic. The influence of large cities upon truck traffic in the five sections of the State is shown in Table 2.

It is apparent that the high density of truck traffic per mile on the State roads of the northeastern and southwestern sections is caused by the large cities in these areas. The two sections, which together include 30 per cent of the total area of the State, include 58 per cent of the cities of over 10,000 population. The northwestern and southern sections, the least important regions of truck traffic, contain 58 per cent of the area and include only 24 per cent of the cities of over 10,000 population.

A Typical Roadside Market

The more important trucking areas, the northeastern and southwestern sections, have five of the seven cities between 30,000 and 100,000 population, and six of the seven cities of over 100,000 population. Cleveland in the northeast and Cincinnati in the southwest are the predominating traffic influences in these two regions.

The motor-truck registration per square mile, shown in Table 2, indicates the concentration of trucks in the southwestern and northeastern sections. The comparatively low registration per mile in the three remaining sections, especially in the southern section, again reflects the relatively small number of large centers of population and industry and accounts for the lower truck-traffic density in these areas.

Figure 4 shows the distribution of motor trucks by capacity groups, Figure 5 the distribution of loaded motor trucks by gross-weight groups, Figure 6 the mileage of State highways carrying various numbers of 3 to 7½ ton trucks, and Figure 7 the distribution of motor trucks by capacity classes in the five traffic sections of the State.

Table 2.—*Average truck-traffic density on State roads in the five traffic sections of the State compared with the relative area of the sections, their truck registration per square mile, and the number of cities over 10,000 population in each*

Section	Average truck-traffic density	Percentage of total area in the State	Truck registration per square mile (1924)	Cities over 10,000 population, by population classes [1]				
				Total		10,000 to 30,000	30,000 to 100,000	Over 100,000
				Number	Per cent			
Northeastern	77	24	7.9	22	44	15	3	4
Southwestern	75	6	10.9	7	14	3	2	2
East-central	53	12	2.5	9	18	9		
Northwestern	36	30	2.3	7	14	5	1	1
Southern	36	28	1.3	5	10	4	1	
Total	51	100	3.9	50	100	36	7	7

[1] United States census of 1920.

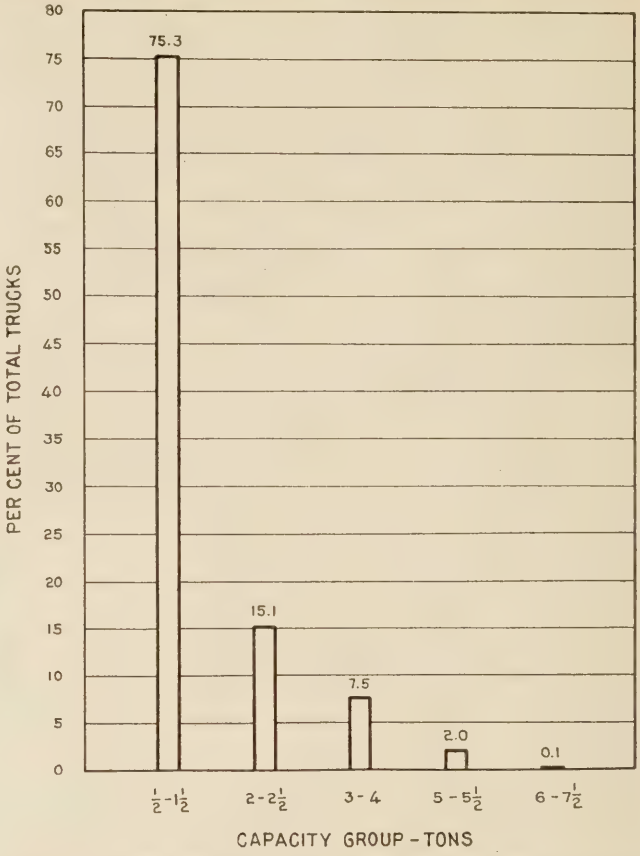


Fig. 4.—Distribution of Motor Trucks by Capacity Groups

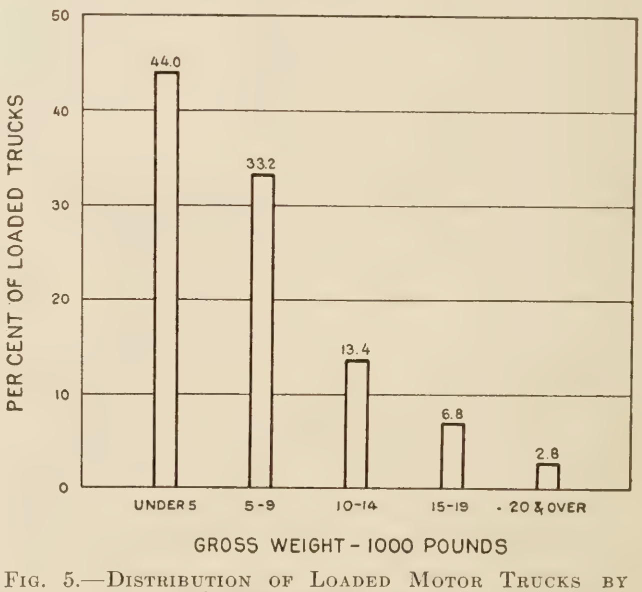


Fig. 5.—Distribution of Loaded Motor Trucks by Gross-Weight Groups

It is possible that the high density of motor-truck traffic in the important traffic areas of the State will eventually present problems difficult of solution. Routes having a large number of motor trucks also carry a large number of passenger cars. On such routes, unless supplementary highway facilities are provided, serious congestion problems will develop. The mileage subject to this danger is not large. In 1925 there were only 15 miles of State highway on

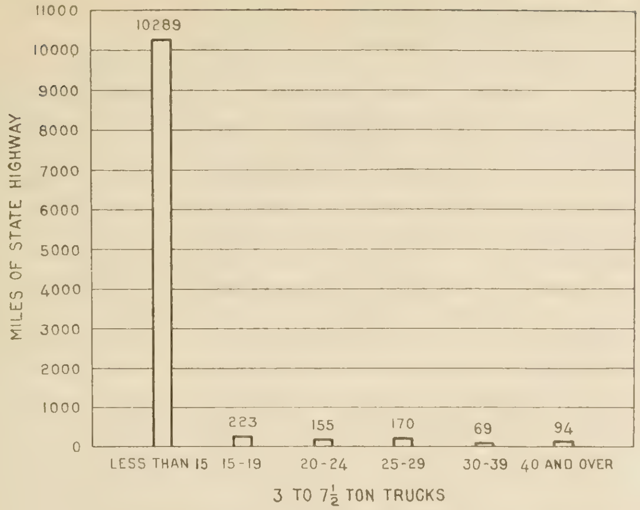


Fig. 6.—Mileage of State Highways Carrying Various Numbers of 3 to 7½ Ton Trucks

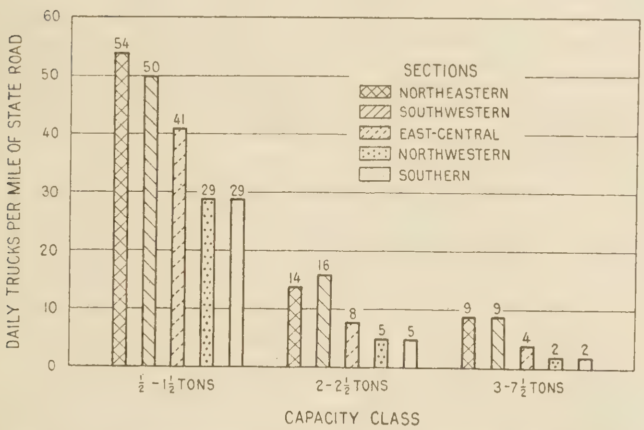


Fig. 7.—Distribution of Motor Trucks by Capacity Classes in the Five Traffic Sections of the State

which the density of motor-truck traffic exceeded 400 trucks per day and only 87 miles where the daily density was greater than 300 trucks.

The relief of congestion which will probably result on a few sections of the State system undoubtedly lies in increasing the width of road and creating separate lanes for truck traffic or in the construction of parallel routes. In either case the segregation of passenger-car and motor-truck traffic would speed up passenger-car traffic and eliminate congestion caused by heavy, slow-moving vehicles.

HIGHWAY UTILIZATION

During 1925 traffic of passenger cars and trucks on the 84,884 miles of rural highways in Ohio was approximately 3,746,360,000 vehicle-miles. The distribution of traffic varies greatly on the several highway systems, on sections of each system, and in the several traffic areas of the State. The three classes into which the rural highways of the State are divided are State highways, of which there are 11,000 miles, county highways, which total 22,991 miles, and township highways, of which there are 50,893 miles. The distribution of vehicle mileage on each of these systems or their traffic use is shown in Table 3 and Figure 8.

The State highway system, 13 per cent of the rural highway mileage, carries 57.7 per cent of the traffic

Table 3.—*Motor-vehicle utilization and mileage of Ohio rural highways, by systems* [1]

Highway system	Mileage	Per cent of total mileage	Daily vehicle-miles	Annual vehicle-miles	Per cent of total vehicle-miles	Average daily traffic density
State highways	11,000	13.0	5,919,000	2,160,435,000	57.7	538
County highways	22,991	27.1	3,038,000	1,108,870,000	29.6	132
Township highways	50,893	59.9	1,307,000	477,055,000	12.7	26
Total	84,884	100.0	10,264,000	3,746,360,000	100.0	

[1] Motor vehicles refer to passenger cars and trucks only, excluding motor busses.

measured in vehicle-miles. Contrasted with this system is the township highway system with 59.9 per cent of the highway mileage and only 12.7 per cent of the vehicle mileage.

The average daily density of traffic on the State system is 538 vehicles, on the county system 132, and on township roads only 26.

The distribution of highway mileage and vehicle mileage by highway systems in each of the five sections of the State is shown in Table 4.

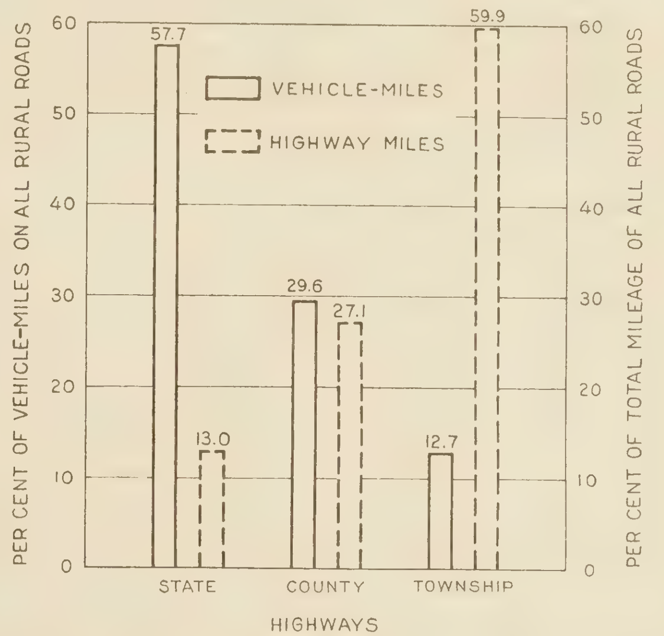


Fig. 8.—Comparison of Traffic on the State, County, and Township Highway Systems, and the Proportionate Mileage of the Three Systems

The State highways in the northeastern section of the State, comprising 25.6 per cent of the State highway mileage, carry 38.3 per cent of the total traffic on the State highway system. In the southwestern section there is 6.7 per cent of the State highway mileage which carries 9.6 per cent of total traffic on the system. In contrast with these areas is the southern section, with 25.1 per cent of the State highway mileage and only 16.1 per cent of the total traffic on the system. Variations in traffic on the county and township system in the five regions are almost equally pronounced.

Traffic on the State highway system also varies greatly on the different routes, ranging from more than 5,500 vehicles per day on the heaviest route observed to less than 50 vehicles per day. The daily use of selected sections of the State highway system is shown in Table 5 and Figure 9.

TABLE 4.—*Motor-vehicle utilization and mileage of Ohio rural highways in the five traffic sections*

Highway systems and sections of State	Mileage of highways	Per cent of highway mileage	Daily vehicle-miles	Per cent of daily vehicle-miles	Average daily density of traffic
State highways:					
Northeastern	2,821	25.6	2,268,000	38.3	804
Southwestern	736	6.7	571,000	9.6	776
East-central	1,285	11.7	650,000	11.0	506
Northwestern	3,402	30.9	1,478,000	25.0	434
Southern	2,756	25.1	952,000	16.1	345
Total	11,000	100.0	5,919,000	100.0	538
County highways:					
Northeastern	5,916	25.7	1,128,000	37.1	191
Southwestern	1,788	7.8	353,000	11.6	197
East-central	2,654	11.6	319,000	10.5	120
Northwestern	7,547	32.8	814,000	26.8	108
Southern	5,086	22.1	424,000	14.0	83
Total	22,991	100.0	3,038,000	100.0	132
Township highways:					
Northeastern	10,625	20.9	428,000	32.8	40
Southwestern	2,579	5.1	102,000	7.8	39
East-central	7,372	14.5	184,000	14.1	25
Northwestern	14,889	29.2	326,000	24.9	22
Southern	15,428	30.3	267,000	20.4	17
Total	50,893	100.0	1,307,000	100.0	26
All highways:					
Northeastern	19,362	22.8	3,824,000	37.3	197
Southwestern	5,103	6.0	1,026,000	10.0	201
East-central	11,311	13.3	1,153,000	11.2	102
Northwestern	25,838	30.5	2,618,000	25.5	101
Southern	23,270	27.4	1,643,000	16.0	71
State total	84,884	100.0	10,264,000	100.0	121

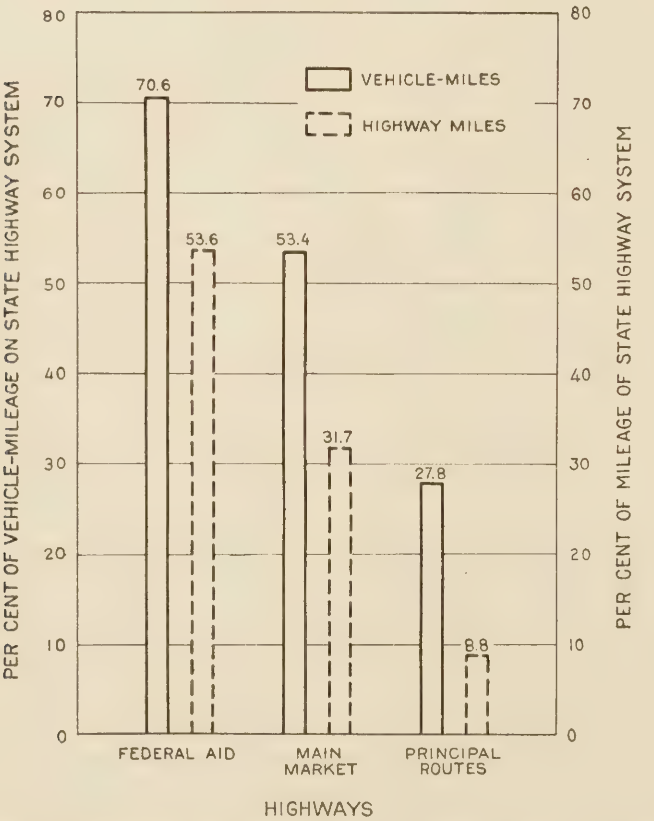


FIG. 9.—PERCENTAGES OF THE TOTAL VEHICLE MILEAGE ON THE INTERCOUNTY SYSTEM THAT IS CARRIED BY THE FEDERAL-AID AND MAIN MARKET HIGHWAYS AND BY THE PRINCIPAL ROUTES OF THE STATB, AND THE MILEAGE OF THE SEVERAL CLASSES OF HIGHWAYS

TABLE 5.—*Average daily traffic on selected sections of the State highway system*

Sections of system	Highway miles	Per cent of State highway system	Per cent of total rural highway system	Average daily vehicle-miles	Per cent of daily vehicle-miles on State highway system	Per cent of daily vehicle-miles on all rural highways	Average daily density of traffic
Federal-aid system [1]	5,899	53.6	6.9	4,177,000	70.6	40.7	708
United States highways	1,878	17.1	2.2	1,848,000	31.2	18.0	984
Main market roads	3,486	31.7	4.1	3,158,000	53.4	30.8	906
Selected principal routes	970	8.8	1.1	1,645,000	27.8	16.0	1,696
State highway system	11,000	100.0	13.0	5,919,000	100.0	57.7	538

[1] With few exceptions the Federal-aid system includes all United States highways and main market highways as well as the roads designated as principal routes.

The Federal-aid system, slightly more than half the State system, carries 70.6 per cent of the daily traffic; main market roads, approximately one-third the mileage, carry over half the traffic; and the principal routes, 8.8 per cent of the mileage, carry over one-fourth of the traffic. These divisions of the State highway system indicate clearly the variation in traffic on this system.

The distribution of passenger-car and motor-truck traffic on the different highway systems is shown in Table 6.

TABLE 6.—*Motor-truck and passenger-car utilization of Ohio rural highways, by systems*

System	Daily truck-miles	Per cent of total truck-miles	Daily passenger-car miles	Per cent of total passenger-car miles	Ratio of truck-miles to total vehicle-miles
State highways	565,000	54.1	5,354,000	58.1	9.5
County highways	332,000	31.8	2,706,000	29.3	10.9
Township highways	147,000	14.1	1,160,000	12.6	11.2
Total	1,044,000	100.0	9,220,000	100.0	10.2

Motor-truck traffic on the township system, and to a lesser degree on the county system, is almost exclusively made up of small-capacity trucks, while on the State system is includes a larger proportion of medium and large capacity trucks.

There is also considerable variation in the relative number of passenger cars and trucks on different routes of the State highway system. Motor trucks vary from less than 6 per cent to more than 20 per cent of total number of vehicles. The extremes of the range are found on routes of minor traffic importance; on nearly all the important traffic routes the percentage falls between 7 and 11. Passenger cars are found to be relatively more important on the principal through routes, and the greatest proportion of trucks is found on important routes in the industrial areas which are not a part of these through routes.

FORECAST OF HIGHWAY TRAFFIC

A knowledge of future traffic, in so far as it can be predicted with reasonable accuracy, is essential in the establishment of a sound plan of highway improvement. A forecast based directly upon past traffic trends is not possible, since there is no historical series of highway-traffic records in Ohio. Such records are available in the States of Maine, Maryland, Massachusetts, Michigan, and Wisconsin. Highway traffic and motor-vehicle registration in each of these States have increased at approximately equal rates, as shown in Figure 10. The great variations in industrial and agricultural development, in population, in motor-vehicles registration, and in the period of the series apparently have had no effect upon the relationship between the rates of traffic increase and motor-vehicle registration growth.

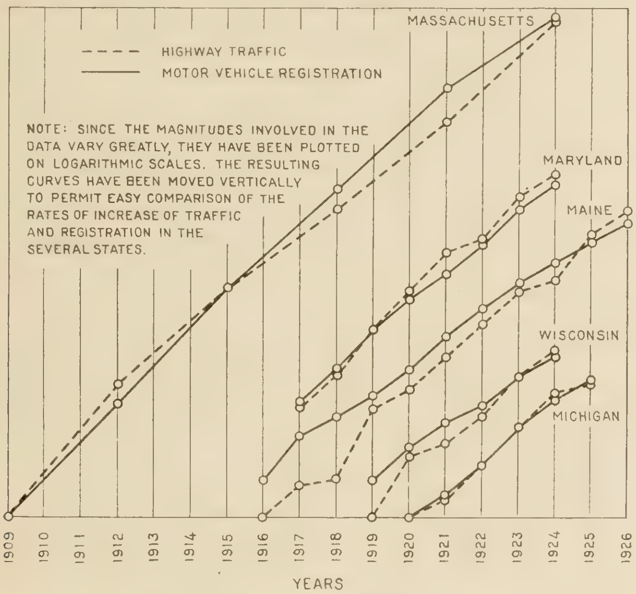


Fig. 10.—Trend of Highway Traffic and Motor-Vehicle Registration in Massachusetts, Maryland, Maine, Wisconsin, and Michigan

In the absence of any comprehensive historical series of traffic records in Ohio it has therefore been assumed that highway traffic in Ohio is increasing directly with the increase in motor-vehicle registration.

The increase of motor-vehicle registration is a function of two variables: (1) The increase in population, and (2) the increase in ownership and use of motor vehicles in proportion to population, measured by the number of persons per motor vehicle. The past trend of both of these factors may be determined from available records.

Population changes are measured accurately by the decennial census and intercensal estimates made by the Bureau of the Census. Population, by years, as estimated by the Bureau of the Census, from 1913 to 1923 and estimates calculated by extension of the census method for 1924 to 1930 and for 1935 is shown in Table 7.

The growth of motor-vehicle registration in proportion to population, i. e., the decrease in persons per car, appears to follow the same general characteristics as the growth of population, an early growth slow in number of vehicles but rapid in rate of increase followed by a gradual decrease in the rate of growth.

The number of persons per car in Ohio during the years 1913 to 1925 and the extension of the trend of persons per car to 1935 is shown in Figure 11. The estimated number of persons per car for each year is shown in Table 7.

Combining estimated population and estimated number of persons per car for each year, the predicted registration for each year is obtained. These estimates are also shown in Table 7.

Table 7.—*Comparison of population and number of motor vehicles in the State of Ohio*

Year	Registration (thousands)		Population [1] (thousands)	Persons per car	
	Actual	Estimated		Actual	Estimated
1913	86	86	5,095	59.24	59.24
1914	123	127	5,197	42.60	40.92
1915	181	179	5,299	29.28	29.60
1916	252	245	5,402	21.44	22.05
1917	347	324	5,504	15.86	16.99
1918	413	416	5,606	13.57	13.48
1919	511	521	5,708	11.17	10.96
1920	621	637	5,810	9.36	9.12
1921	721	763	5,913	8.20	7.75
1922	859	897	6,015	7.00	6.71
1923	1,069	1,038	6,117	5.72	5.89
1924	1,242	1,181	6,219	5.01	5.27
1925	[2] 1,346	1,329	6,321	4.70	4.76
1926	[2] 1,480	1,475	6,424		4.36
1927		1,621	6,526		4.03
1928		1,763	6,628		3.76
1929		1,902	6,730		3.54
1930		2,035	6,833		3.36
1935		2,607	7,344		2.82

[1] Population as of July 1 of each year.
[2] Data not available when forecast was made. Estimate differs by 1.3 per cent rom actual value in 1925, and by 0.3 per cent in 1926.

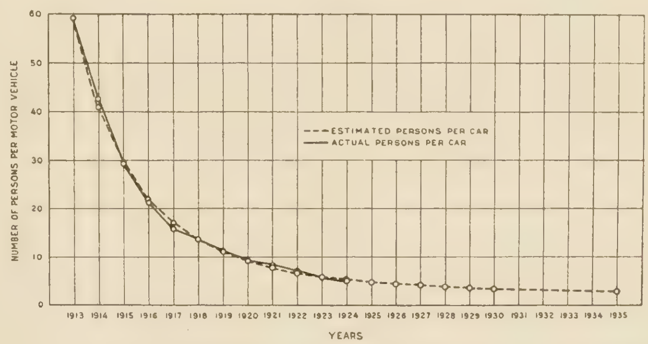


Fig. 11.—The Number of Persons per Car in Ohio for the Years 1913 to 1935 (Based on Estimated Population for Intercensal Years)

On the basis of these predictions it is estimated that motor-vehicle registration in 1930 will be 2,035,000, with 3.36 persons per car, and in 1935, 2,607,000, with 2.82 persons per car. The increase in motor-vehicle registration between 1925 and 1930 is therefore expected to be 51 per cent and the increase between 1930 and 1935, 28 per cent.

The rate of population change varies greatly in different sections of the State, as does also the present number of persons per car.

The rates of decrease in persons per car within these areas, however, follow the same principle and are therefore in close agreement.

To allow for differences in the rate of population change and for differences in the present number of persons per car in the various sections of the State, the number of persons per car, based on estimated population and actual motor-vehicle registration in 1925, was obtained for each county of the State. To this 1925 value for each county was applied the rate of decrease in persons per car for the State between 1925 and 1930. The estimated registration for each county in 1930 was calculated by applying the estimated number of persons per car in 1930 for each county to the estimated 1930 population of that county. From the actual county registration in 1925 and the estimated registration in 1930 the percentage increase in regis-

(Continued on page 88)

ANALYSIS OF CONCRETE ARCHES

PART I.—DERIVATION OF FORMULAS AND ANALYSIS OF SYMMETRICAL ARCHES

By W. P. LINTON, Highway Bridge Engineer, and C. D. GEISLER, Associate Highway Bridge Engineer, United States Bureau of Public Roads[1]

PRACTICALLY all arches are designed by tentatively proportioning the structure using an empirical formula, or basing the design on the results of previous experience, and then computing the stresses in the tentative design and making such changes as seem to be desirable. This is the procedure followed generally in the design of framed structures, but in the case of arches it is a tedious and complicated process. In 1909 the first-named author attempted to systematize this procedure by developing a standard set of forms for tabulating the computations in such a way that the work is as nearly mechanical as possible and both the labor and the probability of error is greatly reduced. It has been found possible to develop a method whereby much of the work is the same for all arches and the results of this portion of the computations can be placed on the tracings which are used for printing the forms. The method has been in use since 1909, and it has been found to be satisfactory in practice.

No attempt will be made to discuss procedure in determining the curve of the arch ring or its thickness at various points, as the method of calculation presented here may be used in combination with any of the empirical or semirational formulas found in various textbooks. The derivation of the formulas on which the method is based is not greatly different from that found in other places. It is not necessary that these derivations be at hand when using the forms for arch design, but they are included for convenience and completeness. The engineer who is familiar with arch design can proceed at once to the explanation of the forms under the heading, Calculations for a Symmetrical Arch.

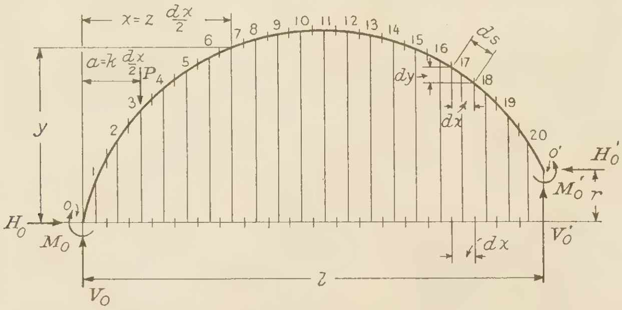


FIG. 1.—SKETCH OF AXIS OF ARCH RING TO SHOW MEANING OF SYMBOLS USED

NOMENCLATURE

The following is the nomenclature used in this discussion. Figures 1, 2, 3, and 4 also illustrate the meaning of many of the symbols, and in each of these figures the origin of coordinates is taken at the left support. All dimensions are in feet unless otherwise stated.

A_c = the area of the concrete in a radial section of the arch ring.

A_s = the area of the steel in a radial section of the arch ring.

$A = A_c + A_s$.

a = the horizontal distance from the origin to the point of application of a concentrated load. $a = k\frac{dx}{2}$ when the span is divided into a number of equal parts, each equal to dx.

$B = \frac{1}{2}\sum_0^l z\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)$, a term in the denominator of the equation for V_o for unsymmetrical arches and is independent of the loading.

$C = \frac{1}{dx}\sum_0^l y\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right) + \sum_0^l \frac{\cos\phi}{A}$, a term in the denominator of certain equations and is independent of the loading.

d' = the distance from the center of the reinforcing steel to the surface of the concrete.

Dl = the change in the span length due to deformation of the arch ring.

Ds = the change in the length of the arch axis due to deformation of the arch ring.

Dr = the change in the relative elevation of the supports due to deformation of the arch ring.

$D(dx)$ = the change in length of dx due to deformation of the arch ring.

$D(ds)$ = the change in length of ds due to deformation of the arch ring.

$D\theta$ = the change in the angle between the end tangents due to deformation of the arch ring.

$D(d\phi)$ = the change in the angle $d\phi$ due to deformation of the arch ring.

$D(dy)$ = the change in length of dy due to deformation of the arch ring.

E = Young's modulus of elasticity.

E_c = Young's modulus of elasticity for concrete.

E_s = Young's modulus of elasticity for steel.

e = the coefficient of expansion due to a change of temperature.

F = a term in the denominator of the equation for V_o.

$F = \frac{1}{2}\sum_0^l z\Delta\left(z - \frac{\Sigma z\Delta}{\Sigma\Delta}\right)$ for unsymmetrical arches and

$F = \frac{1}{2}\sum_0^l z^2\Delta - 200\ \Sigma\Delta$ for symmetrical arches.

f_c = the stress in the concrete.

f_s = the stress in the steel.

$G = \frac{1}{dx}\sum_o^l z\Delta\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)$, a term in the denominator of the equation for V_o in unsymmetrical arches. G is independent of the loading.

H = the horizontal thrust; that is, the horizontal component of the thrust T.

H_o = the horizontal thrust at the left support.

H_x = the horizontal thrust at a point distant x from the left support.

H_t = the horizontal thrust caused by a change of temperature.

h = the thickness of the arch ring at any point.

I = the moment of inertia of a radial section of the arch ring about the axis of the arch ring.

I_c = the moment of inertia of the concrete in a radial section of the arch ring about the axis of the arch ring.

I_s = the moment of inertia of the reinforcing steel about the axis of the arch ring.

I_o = the moment of inertia of the steel about its own axis.

j = the distance from the axis of the arch to the center of the reinforcement.

$k = \frac{2a}{dx}$ when the span is divided into a number of equal parts each equal to dx.

l = the span of the arch axis.

M = the bending moment.

M_o = the bending moment at the left support.

M_x = the bending moment at a point a distance x from the left support.

M_t = the bending moment caused by a change of temperature.

$m_x = V_o x - \Sigma P(x-a)$, a term which would be the bending moment if the arch were a simple beam on two supports. When P is unity, $m_x = [V_o z - (z-k)]\frac{dx}{2}$.

[1] The method of arch analysis described in this article and the one to follow on unsymmetrical arches was developed by W. P. Linton. C. D. Geisler collaborated in preparing the articles for publication.

N = the normal component of the resultant forces acting on a radial section of the arch ring.

$n = \frac{E_s}{E_c} = 15$.

P = a vertical concentrated load.

$p = \frac{N}{A}$, the average stress on a radial section.

r = the difference in elevation of the two supports. $r=0$ for a symmetrical arch.

s = the length of the arch axis.

ds = the length of one division of the arch axis.

T = the resultant thrust on a radial section of the arch.

t = the number of degrees of rise or fall of temperature.

u = the eccentricity of the normal thrust N.

V = the vertical component of the thrust T at any section.

V_o = the vertical reaction at the left support.

V_x = the vertical component of the thrust T at a point distant x from the left support.

x = the abscissa of any point on the arch axis. $x = z\frac{dx}{2}$.

dx = the horizontal projection of one division of the arch axis.

y = the ordinate of any point on the arch axis.

dy = the vertical projection of one division of the arch axis.

$z = \frac{2x}{dx}$ when the span is divided into a number of equal parts each equal to dx.

$\Delta = \frac{ds}{I}$

θ = the angle between the end tangents to the arch axis (fig. 3).

$\sum_o^a$ = the summation of all terms between the left support and the concentrated load distant a from the left support.

$\sum_a^l$ = the summation of all terms from the concentrated load to the right support.

$\sum_o^l$ = the summation of all terms from one support to the other.

$\Sigma = \sum_o^l$. When no limits are specified, it is understood that the summation is to be taken from one support to the other.

ϕ = the angle which any radial section makes with the vertical. (See fig. 3.)

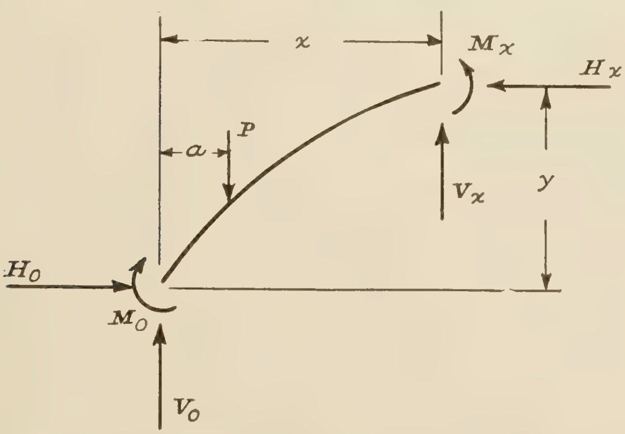


Fig. 2.—Hingeless Arch Acted Upon by Load P

DERIVATION OF FORMULAS

A hingeless arch is statically indeterminate, that is, the stresses in it can not be computed from the principles of statics alone. Let Figure 2 represent a portion of a hingeless arch acted upon by one or more loads P and held in equilibrium by the forces and moments shown. There are six unknown quantities H_o, V_o, M_o, H_x, V_x, and M_x, but only three independent equations can be written from statics. For a single vertical load these equations are as follows:

$$H_x = H_o.$$

$$V_x = V_o - P.$$

$$M_x = M_o + V_o x - H_o y - P(x-a).$$

To determine the six unknown quantities, three additional equations must be supplied from some other source, and they may be derived by considering the elastic properties of the arch ring.

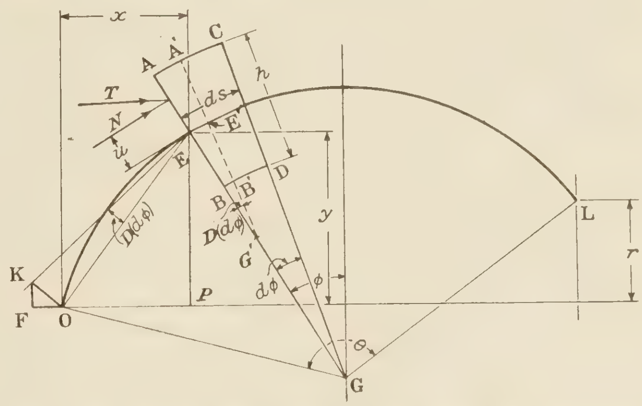


Fig. 3.—Diagram for Use in Derivation of Formulas for Stress in an Elastic Arch

Referring to Figure 3, let ABCD represent a very small portion of the arch ring, cut out by radial planes AB and CD. Let the length of this small segment, measured along its center line, be ds and let the angle between the planes AB and CD be $d\phi$. Both ds and $d\phi$ are very small, and since the depth AB of the arch ring is small compared with the radius BG, the lengths AC and BD may be considered equal to ds without appreciable error in the results.

When the arch is subjected to loads or a change of temperature, there will be stress and deformation in the element ABCD. If we consider ABCD to be separated from the rest of the arch ring, the internal stresses in it must be held in equilibrium by external forces, so let us assume that T is the resultant of all the external forces acting on the plane AB and N the component of T perpendicular to AB. There is also a component of T parallel to AB, but it will cause only shear in the arch ring, which will be small and may be neglected. Neglecting the component parallel to AB is equivalent to assuming that N is the resultant, which we shall do. The forces acting on the plane AB are of course not concentrated in N, but are distributed over the plane, and we shall assume that they vary uniformly from the value at A to the value at B. Then if A is the area of the plane AB, the unit force or the unit internal stress acting at A will be $\left(\frac{N}{A} + \frac{Mh}{2I}\right)$, and at B, $\left(\frac{N}{A} - \frac{Mh}{2I}\right)$.

M, the bending moment is equal to Nu, u being the distance from the axis of the arch to the line of action of N. M is considered positive when u is positive, that is, when N acts above the axis of the arch, and negative when u is negative.

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Consider first the elasticity of only the element ABCD and assume that the arch is fixed at the point L and free to move at point O and that the entire ring, with the exception of the element ABCD, is rigid. The plane CD will then be fixed and the plane AB will move until the internal stresses in the element ABCD are in equilibrium with the bending moment M and the thrust N. The plane AB will move to some position as A′B′. The unit stress in the extreme fiber AC will be $\left(\frac{N}{A}+\frac{Mh}{2I}\right)$, and since we can assume the length of this extreme fiber to be equal to the average length ds of the element, the distance AA′ will be $-\left(\frac{N}{A}+\frac{Mh}{2I}\right)\frac{ds}{E}$. Likewise the unit stress in the fiber BD will be $\left(\frac{N}{A}-\frac{Mh}{2I}\right)$, and the distance BB′ will be $-\left(\frac{N}{A}-\frac{Mh}{2I}\right)\frac{ds}{E}$, in which E is the modulus of elasticity of the material comprising the arch ring. AA′ and BB′ are both negative quantities because they represent decreases in the lengths AC and BD, respectively, and as it is assumed that N is positive, minus signs must be placed in front of the two quantities, as shown.

The movement of the plane AB may be considered as taking place in two separate stages: First, as moving along the axis of the arch ring from E to E′, during which it remains parallel to its original position; and second, as rotating around the point E′, through an angle $D(d\phi)$ and taking the position A′B′. The distance EE′ which the plane AB moves along the axis of the arch ring is called $D(ds)$ because that is the change in the length of ds. Likewise, the angle through which it turns we call $D(d\phi)$ because that is the change in the angle $d\phi$.

When ds and $d\phi$ increase in magnitude, $D(ds)$ and $D(d\phi)$ will be considered as positive, and when ds and $d\phi$ decrease in magnitude $D(ds)$ and $D(d\phi)$ will be considered as negative. In the figure, the angle $d\phi$ is decreased by the deformation; therefore the angle $D(d\phi)$ is negative.

The angle $D(d\phi)$ is equal to $\frac{AA'-BB'}{h}$; and since AA′ is equal to $-\left(\frac{N}{A}+\frac{Mh}{2I}\right)\frac{ds}{E}$, and BB′ is equal to $-\left(\frac{N}{A}-\frac{Mh}{2I}\right)\frac{ds}{E}$, the angle $D(d\phi)$ is equal to

$$\left[-\left(\frac{N}{A}+\frac{Mh}{2I}\right)\frac{ds}{E}+\left(\frac{N}{A}-\frac{Mh}{2I}\right)\frac{ds}{E}\right]\frac{1}{h},$$

which reduces to $-\frac{Mds}{EI}$.

$$\text{Therefore } D(d\phi)=-M\frac{ds}{EI} \text{------------------} (1)$$

Now consider the effect on the point O of the rotating of the plane AB through the angle $D(d\phi)$. The arch ring is free to move at O. Since that part of the ring between O and E is rigid, when the plane AB turns through the angle $D(d\phi)$ the line EO will likewise describe the same angle. During this rotation the point O will move through an arc of a circle to K. OK will then be equal to OE times the angle $D(d\phi)$; and since the angle $D(d\phi)$ is negative, OK $= -$ OE $D(d\phi)$. Since the angle $D(d\phi)$ is very small, we may consider that OK is a straight line and that the angle EOK is a right angle. From geometry, the angle FOK is equal to the angle PEO.

Then, $\frac{OF}{OK}=\frac{EP}{OE}=\frac{y}{OE}$. (OF represents an increase in span of arch and is therefore positive.)

$$OF=OK\frac{y}{OE}=-OE\,D(d\phi)\frac{y}{OE}$$

$$\text{Or, } OF=-D(d\phi)y$$

$$\text{But, } D(d\phi)=-M\frac{ds}{EI}$$

$$\text{Therefore, } OF=+My\frac{ds}{EI} \text{---------} (2)$$

Also, $\frac{FK}{OK}=\frac{OP}{OE}=\frac{x}{OE}$. (FK represents an upward movement of the arch at point O and is therefore positive.)

$$FK=OK\frac{x}{OE}=-OE\,D(d\phi)\frac{x}{OE}$$

$$\text{Or, } FK=-D(d\phi)\,x$$

$$\text{By substitution, } FK=+Mx\frac{ds}{EI} \text{------------------} (3)$$

Now let us return and consider the effect on the point O of the other stage of the movement of the plane AB, that is, its movement along the axis of the arch during which it remains parallel to its original position. It is apparent that during this movement of the plane AB the change in position of the point O will correspond exactly with that of the point E; that is, point O will move a distance equal to $D(ds)$. Now $D(ds)$ is equal to $\frac{1}{2}$(AA′ + BB′), or to

$$\frac{1}{2}\left[-\left(\frac{N}{A}+\frac{Mh}{2I}\right)\frac{ds}{E}-\left(\frac{N}{A}-\frac{Mh}{2I}\right)\frac{ds}{E}\right],$$

which reduces to $-\frac{Nds}{AE}$.

$$\text{Therefore, } D(ds)=-\frac{Nds}{AE} \text{----------------------} (4)$$

dx and dy are the horizontal and vertical projections of ds, and since ϕ is equal to the angle which ds makes with the horizontal, $dx=ds\cos\phi$, and $dy=ds\sin\phi$; also $D(dx)=D(ds)\cos\phi$ and $D(dy)=D(ds)\sin\phi$.

$$\text{By substitution, } D(dx)=-\frac{Nds}{AE}\cos\phi$$

$$\text{and } D(dy)=-\frac{Nds}{AE}\sin\phi$$

Substituting the values of $\cos\phi=\frac{dx}{ds}$, and $\sin\phi=\frac{dy}{ds}$,

$$D(dx)=-\frac{Ndx}{AE} \text{----------------------------} (5)$$

$$\text{and } D(dy)=-\frac{Ndy}{AE} \text{----------------------------} (6)$$

Equations 5 and 6 give the horizontal and vertical changes in position of the point O caused by the thrust N.

We now have the changes in position of the point O caused by both the bending moment M and the thrust N acting on the element ABCD when the point O is free to move and all of the arch is rigid except the element ABCD. So far we have not considered what caused the bending moment, M, and the thrust, N, but in the preceding discussion it does not matter. Let us consider, however, that the moment and thrust were produced by vertical loads. Then if the element ABCD undergoes a change of temperature there will be an additional movement of the point O. Since O and the plane, AB, are free to move, this movement will be simply a change of length of ds, which we may call $D_t(ds)$, and is equal to $et(ds)$, in which t is the number of degrees of change of temperature and e is the coefficient of expansion of the material comprising the arch ring. Thus $D_t(ds)=et(ds)$. In the same manner we find that:

$$D_t(dx)=et(dx) \qquad (7)$$

$$\text{and } D_t(dy)=et(dy) \qquad (8)$$

We now have in equations 1 to 8, inclusive, expressions for the change in the angle $d\phi$ and the horizontal and vertical displacements of point O with respect to point L as produced by the action of both vertical loads and temperature and when only the element ABCD is affected.

Let $D(dl)$ equal the horizontal movement of the point O with respect to L, and let an increase in span length be considered as positive.

Let $D(dr)$ equal the change in elevation of the point O with respect to the point L, and let an increase in elevation of point O be considered as positive.

$$\text{Then, } D(dl)=\text{OF}+D(dx)+D_t(dx)$$

$$D(dr)=\text{FK}-D(dy)-D_t(dy)$$

$$\text{Or, } D(dl)=+My\frac{ds}{EI}-\frac{Ndx}{AE}+et(dx) \qquad (9)$$

$$D(dr)=+Mx\frac{ds}{EI}+\frac{Ndy}{AE}-et(dy) \qquad (10)$$

In equations 1, 9, and 10 we have expressions for the change in the angle $d\phi$ and the horizontal and vertical displacement of point O relative to point L as produced by the deformation of the element ABCD. If the entire arch ring were elastic, we would find the total displacement of point O relative to point L, as caused by the deformation of the entire arch ring, by taking the sum of the movements of point O as caused by the deformation of all of the elements. Therefore, let

$D\theta$ = the change in the angle θ caused by the deformation of the entire arch ring,

Dl = the change in span length caused by the deformation of the entire arch ring, and

Dr = the change in the elevation of point O relative to point L as caused by the deformation of the entire arch ring.

$$\text{Then, } D\theta=-\frac{1}{E}\sum_0^l M\frac{ds}{I} \qquad ^3\ (11)$$

$$Dl=\frac{1}{E}\sum_0^l My\frac{ds}{I}-\frac{1}{E}\sum_0^l\frac{N}{A}dx+etl \qquad (12)$$

$$Dr=\frac{1}{E}\sum_0^l Mx\frac{ds}{I}+\frac{1}{E}\sum_0^l\frac{N}{A}dy-etr \qquad (13)$$

in which E is a constant and may be taken outside of the summation sign. $\sum_0^l dx$ is equal to l, and $\sum_0^l dy$ is equal to r.

In the previous discussion it has been assumed that the arch ring is fixed at the right end and free to move at the left end, and we have derived equations 11, 12, and 13, which give the amount of the change in the inclination of the tangent to the arch axis at the free end and also the horizontal and vertical movements at that end.

Let us now imagine a horizontal thrust, H_o, a vertical reaction, V_o, and a bending moment, M_o, all applied at the free end and of exactly sufficient intensity to bring the free end of the arch ring back to its original position. Under these conditions $D\theta$, Dl, and Dr will each be equal to zero, so we may write:

$$\sum_0^l M\Delta=0 \qquad (14)$$

$$\sum_0^l My\Delta-\sum_0^l\frac{N}{A}dx+etlE=0 \qquad (15)$$

$$\sum_0^l Mx\Delta+\sum_0^l\frac{N}{A}dy-etrE=0 \qquad (16)$$

in which Δ is equal to $\frac{ds}{I}$.

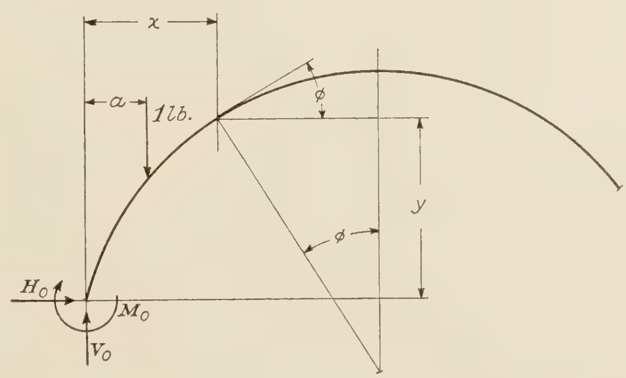

FIG. 4.—ARCH RING WITH UNIT LOAD AT POINT DISTANT a FROM SUPPORT

In these equations, M and N are the bending moment and thrust at any point in the arch ring, of which point x and y are the coordinates. These equations are true for any condition of loading, but for our present purpose we shall consider only a single load of unity placed a distance, a, from the left support (fig. 4).

[3] Throughout this article where limits are given with the summation sign such as $\sum_0^l$ it is intended to indicate that these limits are values of x or in this instance $\sum_{x=o}^{x=l}$. Where no limits are given, they should be assumed as being between $x=l$ and $x=o$.

Then:

$$\left.\begin{array}{ll} M=M_o+V_ox-H_oy, & \text{when } x<a \\ M=M_o+V_ox-H_oy-(x-a), & \text{when } x>a \end{array}\right\} \text{-- (17)}$$

$$N=H\cos\phi+V\sin\phi \text{ ------------------- (18)}$$

in which, M_o, V_o, and H_o are, respectively, the bending moment, the vertical reaction, and the horizontal thrust at the left support.

In the following mathematical transformations, x and y are the variables, while a remains constant. Therefore, M_o, V_o, and H_o are constants because they depend on a and not on x or y. For that reason they may be taken outside of the summation signs.

Substituting equations 17 and 18 in equations 14, 15, and 16 we have:

$$\sum_o^a[M_o+V_ox-H_oy]\Delta$$

$$+\sum_a^l[M_o+V_ox-H_oy-(x-a)]\Delta=0$$

$$\sum_o^a[M_o+V_ox-H_oy]y\Delta$$

$$+\sum_a^l[M_o+V_ox-H_oy-(x-a)]y\Delta$$

$$-\sum_o^l\left(\frac{H\cos\phi+V\sin\phi}{A}\right)dx+etlE=0$$

$$\sum_o^a[M_o+V_ox-H_oy]x\Delta$$

$$+\sum_a^l[M_o+V_ox-H_oy-(x-a)]x\Delta$$

$$+\sum_o^l\left(\frac{H\cos\phi+V\sin\phi}{A}\right)dy-etrE=0$$

These three equations may be written:

$$M_o\sum_o^l\Delta+V_o\sum_o^l x\Delta-H_o\sum_o^l y\Delta-\sum_a^l(x-a)\Delta=0$$

$$M_o\sum_o^l y\Delta+V_o\sum_o^l xy\Delta-H_o\sum_o^l y^2\Delta-\sum_a^l(x-a)y\Delta$$

$$-\sum_o^l H\frac{\cos\phi}{A}dx-\sum_o^l V\frac{\sin\phi}{A}dx+etlE=0$$

$$M_o\sum_o^l x\Delta+V_o\sum_o^l x^2\Delta-H_o\sum_o^l xy\Delta-\sum_a^l(x-a)x\Delta$$

$$+\sum_o^l H\frac{\cos\phi}{A}dy+\sum_o^l V\frac{\sin\phi}{A}dy-etrE=0$$

In the last two equations H and V are still inside the summation sign, but $H=H_o$ and is therefore constant.

Also, $V=V_o$, for $x<a$
$V=V_o-1$, for $x>a$

Making this substitution in the three equations, we have:

$$M_o\sum_o^l\Delta+V_o\sum_o^l x\Delta-H_o\sum_o^l y\Delta-\sum_a^l(x-a)\Delta=0$$

$$M_o\sum_o^l y\Delta+V_o\sum_o^l xy\Delta-H_o\sum_o^l y^2\Delta-\sum_a^l(x-a)y\Delta$$

$$-H_o\sum_o^l\frac{\cos\phi}{A}dx-V_o\sum_o^a\frac{\sin\phi}{A}dx$$

$$-(V_o-1)\sum_a^l\frac{\sin\phi}{A}dx+etlE=0$$

$$M_o\sum_o^l x\Delta+V_o\sum_o^l x^2\Delta-H_o\sum_o^l xy\Delta-\sum_a^l(x-a)x\Delta$$

$$+H_o\sum_o^l\frac{\cos\phi}{A}dy+V_o\sum_o^a\frac{\sin\phi}{A}dy$$

$$+(V_o-1)\sum_a^l\frac{\sin\phi}{A}dy-etrE=0$$

These three equations may be written:

$$M_o\sum_o^l\Delta+V_o\sum_o^l x\Delta-H_o\sum_o^l y\Delta-\sum_a^l(x-a)\Delta=0 \text{ ------ (19)}$$

$$M_o\sum_o^l y\Delta+V_o\sum_o^l xy\Delta-H_o\sum_o^l y^2\Delta-\sum_a^l(x-a)y\Delta$$

$$-H_o\sum_o^l\frac{\cos\phi}{A}dx-V_o\sum_o^l\frac{\sin\phi}{A}dx$$

$$+\sum_a^l\frac{\sin\phi}{A}dx+etlE=0 \text{ ---------------------- (20)}$$

$$M_o\sum_o^l x\Delta+V_o\sum_o^l x^2\Delta-H_o\sum_o^l xy\Delta-\sum_a^l(x-a)x\Delta$$

$$+H_o\sum_o^l\frac{\cos\phi}{A}dy+V_o\sum_o^l\frac{\sin\phi}{A}dy$$

$$-\sum_a^l\frac{\sin\phi}{A}dy-etrE=0 \text{ ---------------------- (21)}$$

From equation 19

$$M_o=H_o\frac{\Sigma y\Delta}{\Sigma\Delta}-V_o\frac{\Sigma x\Delta}{\Sigma\Delta}+\frac{1}{\Sigma\Delta}\sum_a^l(x-a)\Delta$$

Substituting this value of M_o in equations 20 and 21 and collecting the terms containing H_o and V_o, we have:

$$V_o\left[-\Sigma y\Delta\frac{\Sigma x\Delta}{\Sigma\Delta}+\Sigma xy\Delta-\Sigma\frac{\sin\phi}{A}dx\right]$$

$$+H_o\left[\Sigma y\Delta\frac{\Sigma y\Delta}{\Sigma\Delta}-\Sigma y^2\Delta-\Sigma\frac{\cos\phi}{A}dx\right]$$

$$=-\frac{\Sigma y\Delta}{\Sigma\Delta}\sum_a^l(x-a)\Delta+\sum_a^l(x-a)y\Delta$$

$$-\sum_a^l\frac{\sin\phi}{A}dx-etlE, \text{ and}$$

$$V_o\left[-\frac{(\Sigma x\Delta)^2}{\Sigma\Delta}+\Sigma x^2\Delta+\Sigma\frac{\sin\phi}{A}dy\right]$$

$$+H_o\left[\Sigma x\Delta\frac{\Sigma y\Delta}{\Sigma\Delta}-\Sigma xy\Delta+\Sigma\frac{\cos\phi}{A}dy\right]$$

$$=-\frac{\Sigma x\Delta}{\Sigma\Delta}\sum_a^l(x-a)\Delta+\sum_a^l(x-a)x\Delta$$

$$+\sum_a^l\frac{\sin\phi}{A}dy+etrE=0$$

These two equations may be written:

$$V_o\left[\Sigma x\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-\Sigma\frac{\sin\phi}{A}dx\right]$$

$$-H_o\left[\Sigma y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\cos\phi}{A}dx\right]$$

$$=\sum_a^l(x-a)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-\sum_a^l\frac{\sin\phi}{A}dx-etlE \quad (22)$$

$$V_o\left[\Sigma x\Delta\left(x-\frac{\Sigma x\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\sin\phi}{A}dy\right]$$

$$-H_o\left[\Sigma x\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-\Sigma\frac{\cos\phi}{A}dy\right]$$

$$=\sum_a^l(x-a)\Delta\left(x-\frac{\Sigma x\Delta}{\Sigma\Delta}\right)+\sum_a^l\frac{\sin\phi}{A}dy+etrE \quad (23)$$

Let $x=z\frac{dx}{2}$ and $a=k\frac{dx}{2}$.

When the span is divided into 20 equal parts each of length dx, $l=40\frac{dx}{2}$. We can consider dx as a constant and z as a variable.

Making these substitutions, and dividing equation 22 by dx and equation 23 by $\frac{(dx)^2}{2}$ we have:

$$V_o\left[\frac{1}{2}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-\Sigma\frac{\sin\phi}{A}\right]$$

$$-H_o\left[\frac{1}{dx}\Sigma y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\cos\phi}{A}\right]$$

$$=\frac{1}{2}\sum_a^l(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-\sum_a^l\frac{\sin\phi}{A}-20\,etE \quad (24)$$

$$V_o\left[\frac{1}{2}\Sigma z\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)+\frac{2}{(dx)^2}\Sigma\frac{\sin\phi}{A}dy\right]$$

$$-H_o\left[\frac{1}{dx}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-\frac{2}{(dx)^2}\Sigma\frac{\cos\phi}{A}dy\right]$$

$$=\frac{1}{2}\sum_a^l(z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)+\frac{2}{(dx)^2}\sum_a^l\frac{\sin\phi}{A}dy$$

$$+\frac{2}{(dx)^2}etrE \quad (25)$$

In the above equations the terms containing $\sin\phi$ and $\cos\phi$ are those which include the effect of the axial or direct stress. To neglect them would be to neglect the axial stress which is sometimes done because it is usually a small part of the total stress. It may not be desirable to neglect this stress entirely, but we can shorten the work considerably by neglecting some of the terms.

The terms $\Sigma\frac{\sin\phi}{A}$, $\frac{2}{(dx)^2}\Sigma\frac{\sin\phi}{A}dy$, $\frac{2}{(dx)^2}\Sigma\frac{\cos\phi}{A}dy$, $\sum_a^l\frac{\sin\phi}{A}$, and $\frac{2}{(dx)^2}\sum_a^l\frac{\sin\phi}{A}dy$ are all very small compared with the terms to which they are added, and probably no accuracy will be sacrificed by neglecting them. The data upon which all of the terms depend are obtained by scaling a large-scale drawing, and this drawing, while probably more accurate in dimensions than the dimensions of the arch as built is not sufficiently accurate to make the above-mentioned terms significant.

Omitting the above-mentioned terms, we have:

$$V_o\frac{1}{2}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-H_o\left[\frac{1}{dx}\Sigma y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\cos\phi}{A}\right]$$

$$=\frac{1}{2}\sum_a^l(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-20\,etE \quad (26)$$

$$V_o\frac{1}{2}\Sigma z\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)-H_o\frac{1}{dx}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$$

$$=\frac{1}{2}\sum_a^l(z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)+\frac{2}{(dx)^2}etr\,E \quad (27)$$

In these two equations the coefficients of H_o and V_o contain no summations except those for the entire arch ring. For that reason these coefficients are independent of the loading and may be considered as constants.

Therefore we may let:

$$\boldsymbol{B}=\frac{1}{2}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right) \quad (28)$$

$$\boldsymbol{C}=\frac{1}{dx}\Sigma y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\cos\phi}{A} \quad (29)$$

$$\boldsymbol{F}=\frac{1}{2}\Sigma z\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right) \quad (30)$$

$$\boldsymbol{G}=\frac{1}{dx}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right) \quad (31)$$

Then the two equations may be written:

$$V_o\boldsymbol{B}-H_o\boldsymbol{C}=\frac{1}{2}\sum_a^l(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-20\,etE$$

$$V_o\boldsymbol{F}-H_o\boldsymbol{G}=\frac{1}{2}\sum_a^l(z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)+\frac{2}{(dx)^2}etrE$$

From the first of these two equations:

$$H_o=V_o\frac{\boldsymbol{B}}{\boldsymbol{C}}-\frac{1}{\boldsymbol{C}}\left[\frac{1}{2}\sum_a^l(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-20\,etE\right] \quad (32)$$

and by substitution:

$$V_o=\frac{\frac{1}{2}\sum_a^l(z-k)\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)+\frac{2}{(dx)^2}etrE}{F-\frac{\boldsymbol{BG}}{\boldsymbol{C}}}$$

$$\frac{\frac{\boldsymbol{G}}{\boldsymbol{C}}\left[\frac{1}{2}\sum_a^l(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)-20\ etE\right]}{F-\frac{\boldsymbol{BG}}{\boldsymbol{C}}} \quad \text{(33)}$$

It is more convenient to consider the effect of the loads and temperature separately. When the effect of the loads is considered without a change of temperature, t in the formulas becomes zero. Then the effect of a change of temperature may be computed by making the terms which contain the effect of the loads equal to zero.

Making this separation and collecting the formulas which are used in computing the stresses in an arch ring, we have:

$$V_o=\frac{\frac{1}{2}\sum_a^l(z-k)\ \Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)}{\boldsymbol{F}-\frac{\boldsymbol{BG}}{\boldsymbol{C}}}$$

$$\frac{\frac{\boldsymbol{G}}{\boldsymbol{C}}\frac{1}{2}\sum_a^l(z-k)\ \Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{\boldsymbol{F}-\frac{\boldsymbol{BG}}{\boldsymbol{C}}} \quad \text{(34)}$$

$$H_o=\frac{V_o\boldsymbol{B}-\frac{1}{2}\sum_a^l(z-k)\ \Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{\boldsymbol{C}} \quad \text{(35)}$$

$$M_o=\frac{dx}{\Sigma\Delta}\frac{1}{2}\sum_a^l(z-k)\ \Delta+H_o\frac{\Sigma y\Delta}{\Sigma\Delta}-V_o\frac{dx}{2}\frac{\Sigma z\Delta}{\Sigma\Delta} \quad \text{(36)}$$

$$V_x=\begin{cases}V_o, \text{ for } x<a\\ V_o-1, \text{ for } x>a\end{cases} \quad \text{(37)}$$

$$H_x=H_o \quad \text{(38)}$$

$$M_x=\begin{cases}M_o+V_oz\frac{dx}{2}-H_oy, \text{ for } x<a\\ M_o+[V_oz-(z-k)]\frac{dx}{2}-H_oy, \text{ for } x>a\end{cases} \quad \text{(39)}$$

$$N_x=H_o\cos\phi+V_x\sin\phi \quad \text{(40)}$$

$$V_t=\frac{\frac{2r}{(dx)^2}+20\frac{\boldsymbol{G}}{\boldsymbol{C}}}{\boldsymbol{F}-\frac{\boldsymbol{BG}}{\boldsymbol{C}}}etE \quad \text{(41)}$$

$$H_t=\frac{V_t\boldsymbol{B}+20\ etE}{\boldsymbol{C}} \quad \text{(42)}$$

$$M_t=-H_t\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+V_t\frac{dx}{2}\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right) \quad \text{(43)}$$

$$f_c=\frac{N}{A}\pm M\frac{h}{2I} \quad \text{(44)}$$

Equations 34 to 44, inclusive, are applicable to all concrete arches with fixed ends, whether symmetrical or unsymmetrical. However, if the arch is symmetrical, some of the terms in the above equations will become zero and the equations will be simplified.

If the arch is symmetrical, the values of Δ will be symmetrical—that is, the value of Δ for point 1′ will be the same as for point 1 and for point 2′ the same as for point 2, etc. The value of z for points 1 and 1′ combined is $(1+39)=40$ and for points 2 and 2′ combined it is $(3+37)=40$ and in the same way for all pairs of symmetrical points combined the value of z is always 40. Therefore, for symmetrical arches $\Sigma z\Delta=20\Sigma\Delta$. The values of y are also symmetrical and we have $\Sigma zy\Delta=20\ \Sigma y\Delta$.

These values transform equation 28 as follows:

$$\begin{aligned}\boldsymbol{B}&=\frac{1}{2}\Sigma z\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)=\frac{1}{2}\Sigma zy\Delta-\frac{1}{2}\Sigma z\Delta\frac{\Sigma y\Delta}{\Sigma\Delta}\\&=\frac{1}{2}\times20\Sigma y\Delta-\frac{1}{2}\times20\Sigma\Delta\frac{\Sigma y\Delta}{\Sigma\Delta}\\&=10\Sigma y\Delta-10\Sigma y\Delta=0\end{aligned}$$

Therefore, for symmetrical arches

$$\boldsymbol{B}=0$$

$$\begin{aligned}\boldsymbol{F}&=\frac{1}{2}\Sigma z\Delta\left(z-\frac{\Sigma z\Delta}{\Sigma\Delta}\right)=\frac{1}{2}\Sigma z^2\Delta-\frac{1}{2}\Sigma z\Delta\frac{\Sigma z\Delta}{\Sigma\Delta}\\&=\frac{1}{2}\Sigma z^2\Delta-\frac{1}{2}\times20\Sigma\Delta\frac{20\Sigma\Delta}{\Sigma\Delta}\\&=\frac{1}{2}\Sigma z^2\Delta-200\Sigma\Delta\end{aligned}$$

$\boldsymbol{G}=0$ for the same reason as given for $\boldsymbol{B}$

Therefore, for symmetrical arches we have the following formulas:

FORMULAS FOR SYMMETRICAL ARCHES

$$\boldsymbol{C}=\frac{1}{dx}\Sigma y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\cos\phi}{A}$$

$$\boldsymbol{F}=\frac{1}{2}\Sigma z^2\Delta-200\Sigma\Delta$$

$$V_o=\frac{\frac{1}{2}\sum_a^l(z-k)\ (z-20)\ \Delta}{\boldsymbol{F}} \quad \text{(45)}$$

$$H_o=\frac{-\frac{1}{2}\sum_a^l(z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{\boldsymbol{C}} \quad \text{(46)}$$

$$M_o=\frac{dx}{\Sigma\Delta}\frac{1}{2}\sum_a^l(z-k)\Delta+H_o\frac{\Sigma y\Delta}{\Sigma\Delta}-20\frac{dx}{2}V_o \quad \text{(47)}$$

$$V_x=\begin{cases}V_o, \text{ when } x<a\\ V_o-1, \text{for a single unit load when } x>a\end{cases} \quad \text{(48)}$$

$$H_x = H_o$$

$$M_x = M_o + \left[V_o z - \overset{z>k}{(z-k)} \right] \frac{dx}{2} - H_o y \quad \text{(49)}$$

$$N_x = H \cos \phi + V_x \sin \phi \quad \text{(50)}$$

$$V_t = 0 \quad \text{(51)}$$

$$H_t = 20 \frac{etE}{C} \quad \text{(52)}$$

$$M_t = -H_t \left(y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right) \quad \text{(53)}$$

$$f_c = \frac{N}{A} \pm M \frac{h}{2I} \quad \text{(54)}$$

The term $\frac{\cos \phi}{A}$ in some of the equations includes the effect of the so-called axial or rib shortening stress. This stress is sometimes considered separately from the other stresses and sometimes is neglected entirely. If the rise of the arch is relatively large, the axial stress is small and little accuracy is sacrificed by neglecting it, but if the arch is flat its effect is considerable and it should not be neglected.

Since it is easily taken care of on the forms, there appears to be no good reason for neglecting it or separating it from the rest of the stress, so it will be left in the formula and on the forms in its correct place, and the axial stress will be included in all cases.

CALCULATIONS FOR A SYMMETRICAL ARCH

The method of making the calculations for a symmetrical arch can best be explained by working out an actual example, and each step will be taken up in order and explained in detail. All calculations are entered in Tables 1 to 9, forms for which can be provided on five sheets of letter-size paper, and these forms are grouped in the text as used by the authors. Numerical values and plus and minus signs, which are the same for all arches, are shown in boldface type. In actual practice, blank forms made by the white-print process and showing column headings and other information common to all symmetrical arches are used. The formulas to be used are those developed for symmetrical arches. The formulas and forms for unsymmetrical arches could be used, but most arches are symmetrical, and the work is much reduced by using the simplified rather than the general formulas.

Assume that a 60-foot arch is to be designed for a uniform live load of 125 pounds per square foot (concentrated loads may be used without difficulty) and that the arch must withstand temperature stresses caused by a rise in temperature of 30° F. or a drop of 40° F. The tentative proportions of the arch and its reinforcement are determined and half of the arch ring plotted on detail paper on a scale of 1 inch to 2 feet. The rather large scale is used because the stresses to be determined will depend on scaled dimensions from this drawing. A similar drawing on a smaller scale is made on letter-size paper to accompany the tables of calculations as illustrated in Figure 5 (sheet 1 of forms).

After plotting one-half of the arch ring, draw the axis of the arch ring, which is a curve lying halfway between the intrados and extrados. Draw a vertical line through the springing line of the intrados until it intersects the arch axis as shown in Figure 5. This point is the origin of coordinates and is referred to as the origin, or point O. Through the origin draw a horizontal line, and on this line divide the half span into 10 equal parts. The length of each part will be equal to dx. On sheet 2 (Tables 1, 2, and 3) of the forms record in the places indicated the values of l, dx and the rise, which is the vertical distance from the arch axis at the crown to the horizontal line drawn through the origin. At the center of each dx erect a perpendicular to intersect with the arch axis, and mark these intersections 1, 2, 3, etc. In working out this example maximum stresses will be determined at points 0, 3, 8, and 10½ and the abutment pressure on the foundation determined.

The general method of procedure is to place a load of unity successively at each of the points on the arch ring and compute coefficients which can be applied to the actual dead and live loads for determining moments, shears, and thrusts.

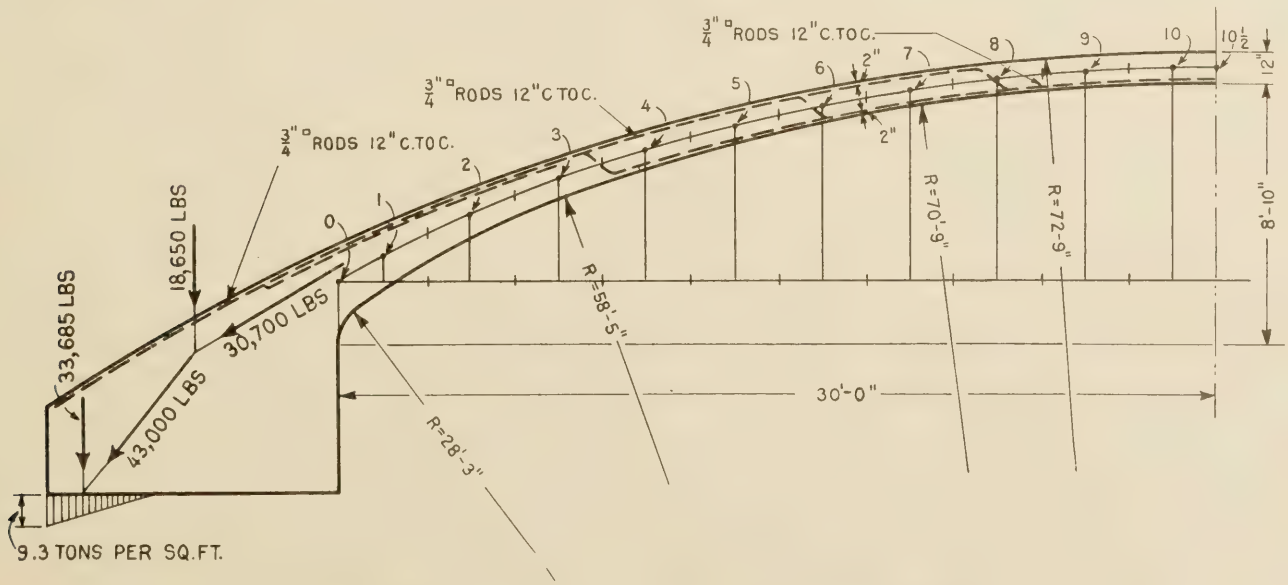

FIG. 5.—SKETCH OF ARCH TO ACCOMPANY CALCULATION SHEETS

TABLE 1.—*Computations for* Δ

1	2	3	4	5	6	7	8	9	10	11
Points	h	h^3	$I_c=\frac{h^3}{12}$	$\frac{h}{2}$	$\frac{h}{2}-d'$	$\left(\frac{h}{2}-d'\right)^2$	$I_s=\left(\frac{h}{2}-d'\right)^2 nA_s$	$I=I_c+I_s$	ds	$\Delta=\frac{ds}{I}$
0	2.30	12.167	1.014	1.15	0.98	0.960	0.112	1.126	——	——
1	1.98	7.762	.647	.99	.82	.672	.079	.726	3.43	4.7
2	1.51	3.443	.287	.75	.58	.336	.039	.326	3.29	10.1
3	1.30	2.197	.183	.65	.48	.230	.013	.196	3.17	16.2
4	1.18	1.643	.137	.59	.42	.176	.010	.147	3.13	21.3
5	1.10	1.331	.111	.55	.38	.144	.008	.119	3.09	26.0
6	1.05	1.158	.096	.52	.35	.122	.007	.103	3.05	29.6
7	1.02	1.061	.088	.51	.34	.116	.007	.095	3.02	31.8
8	1.02	1.061	.088	.51	.34	.116	.007	.095	3.01	31.7
9	1.00	1.000	.083	.50	.33	.109	.006	.089	3.00	33.7
10	1.00	1.000	.083	.50	.33	.109	.006	.089	3.00	33.7
									$\frac{1}{2}\Sigma\Delta=$	238.8

TABLE 2.—*Computations for* V_o

Points	12	13	14	15	16	17	18
	$z-20$	$(z-20)\Delta$	$\Sigma Q=\sum_a^l (z-20)\Delta$	$\frac{1}{2}\sum_a^l (z-k)Q$	z^2	$z^2\Delta$	V_o
0	—	——	——	——	——	——	——
1	− 19	89.3	+ 89.3	+ 20,091.6	1,522	7,153.4	1.00000
2	− 17	− 171.7	+ 261.0	+ 20,002.3	1,378	13,917.8	.99556
3	− 15	− 243.0	+ 504.0	+ 19,741.3	1,250	20,250.0	.98256
4	− 13	− 276.9	+ 780.9	+ 19,237.3	1,138	24,239.4	.95748
5	− 11	− 286.0	+ 1,066.9	+ 18,456.4	1,042	27,092.0	.91861
6	− 9	− 266.4	+ 1,333.3	+ 17,389.5	962	28,475.2	.86551
7	− 7	− 222.6	+ 1,555.9	+ 16,056.2	898	28,556.4	.79915
8	− 5	− 158.5	+ 1,714.4	+ 14,500.3	850	26,945.0	.72171
9	− 3	− 101.1	+ 1,815.5	+ 12,785.9	818	27,566.4	.63638
10	− 1	− 33.7	+ 1,849.2	+ 10,970.4	802	27,027.4	.54662
		− 1,849.2	+ 10,970.4		$\Sigma z^2\Delta=$	231,223.2	

TABLE 3.—*Computations for* H_o

19	20	21	22	23	24	25	26	27	28	29	30
Points	y	Δ	$y\Delta$	$y-\frac{\Sigma y\Delta}{\Sigma\Delta}$	$\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$	$\Sigma Q=\sum_a^l \Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$	$\frac{1}{2}\sum_a^l (z-k)Q$	$y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$	$\cos\phi$	$\frac{\cos\phi}{A}$	H_o
0	0	——	——	——	——	——	——	——	0.838	——	——
1	0.88	4.7	4.136	− 4.976	− 23.387	+ 23.352	0.000	− 20.581	.874	0.441	0.000
2	2.32	10.1	23.432	− 3.536	− 35.714	+ 59.066	− 23.352	− 82.856	.911	.603	+ .062
3	3.51	16.2	56.862	− 2.346	− 38.005	+ 97.071	− 82.418	− 133.398	.946	.728	+ .219
4	4.48	21.3	95.426	− 1.376	− 29.309	+ 126.380	− 179.489	− 131.304	.948	.804	+ .478
5	5.29	26.0	137.540	− 0.566	− 14.716	+ 141.096	− 305.869	− 77.848	.971	.883	+ .815
6	5.93	29.6	175.528	+ 0.074	+ 2.190	+ 138.906	− 446.965	+ 12.987	.984	.938	+ 1.191
7	6.48	31.8	206.064	+ 0.624	+ 19.843	+ 119.063	− 585.871	+ 128.583	.993	.974	+ 1.561
8	6.85	31.7	217.145	+ 0.994	+ 31.510	+ 87.553	− 704.934	+ 215.844	.996	.976	+ 1.878
9	7.10	33.7	239.270	+ 1.244	+ 41.923	+ 45.630	− 792.487	+ 297.653	1.000	1.000	+ 2.111
10	7.21	33.7	242.977	+ 1.354	+ 45.630	0.000	− 838.117	+ 328.992	1.000	1.000	+ 2.232
		$\frac{1}{2}\Sigma y\Delta=$	1,398.378		0.000	838.117	$\frac{1}{2}\Sigma=$	+ 538.072	$\frac{1}{2}\Sigma=$	8.347	

Span $=l=$ 60.00 ft.

Rise $=$ 7.22 ft.

$$dx=\frac{l}{20}=3.00$$

$$\frac{1}{2}dx=1.50$$

$$\frac{\Sigma y\Delta}{\Sigma\Delta}=5.85585$$

$$F=\frac{1}{2}\Sigma z^2\Delta-200\,\Sigma\Delta=20{,}091.6$$

$$C=\frac{1}{dx}\sum_o^l y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\frac{\cos.\ \phi}{A}=375.409$$

$$V_o=\frac{\frac{1}{2}\sum_a^l (z-k)(z-20)\Delta}{F}\qquad H_o=\frac{-\frac{1}{2}\sum_a^l (z-k)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{C}\qquad H_t=\frac{20\,etE}{C}=\frac{34560}{C}t=\frac{+2{,}762}{-3{,}682}\text{ for }t=\frac{+30}{-40}$$

$$\text{Col. }15=\frac{1}{2}\sum_a^l (z-k)\ (z-20)\Delta\qquad \text{Col. }26=\frac{1}{2}\sum_a^l (z-k)\ \Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)$$

Table 1.—Computations for Δ.—Scale accurately the thickness of the arch ring h at each point and set down the result in column 2. Columns 3 to 9, inclusive, are used in finding the moments of inertia of the sections of the arch ring at the various points. $I=I_c+I_s$, in which I is the total moment of inertia of the section about the axis of the arch, I_c the moment of inertia of the concrete, and I_s the moment of inertia of the steel. It is convenient to consider a strip of the arch ring 1 foot in width and scale all dimensions in feet, then $I_c=\frac{bh^3}{12}$, or, since $b=1$, $I_c=\frac{h^3}{12}$, which is tabulated in column 4.

$I_s=I_o+A_sj^2$ in which I_s is the moment of inertia of the steel about the axis of the arch, I_o is the moment of inertia of the steel about its own axis, A_s is the area of the steel, and j is the distance from the axis of the arch to the axis of the steel. Since the strength of steel is equivalent to n times that of the same area of concrete, and all of the dimensions are in feet, A_s in the above formula must be taken as equal to n times the total area of steel in square inches divided by 144.

$$n=\frac{E_s}{E_c}=15\qquad j=\frac{h}{2}-d'$$

in which d' is the distance from the surface of the concrete to the center of the steel. I_o is so small that it may be neglected.

Having computed I for each section and having the results tabulated in column 9, next scale ds for each section along the axis of the arch ring. The values of Δ are then computed. This should be carefully done because there is no check on the work and the accuracy of all of the rest of the columns will depend on the accuracy of column 11.

Special method of computation for terms involving $\frac{1}{2}\sum_a^l(z-k)$.—Column 12 (also 16 and 32) is the same for all arches and is already filled out on the printed form. In preparing the forms the values of z for the different points have been determined from the formula $x=z\frac{dx}{2}$. By inspection of Figure 1 it is evident that the value of z for point 1 is 1, for point 2 it is 3, for point 10 it is 19, etc.

The numerators of the formulas for V_o and H_o and one term of the formula for M_o contain a term which may be written in the form,

$$\frac{1}{2}\sum_a^l(z-k)\ Q$$

In each of these formulas $\boldsymbol{Q}$ has a different value, but the method of procedure in arriving at the value of the above expresssion is the same in each case and the term $\boldsymbol{Q}$ is used in order that a single explanation may serve for three cases.

In the formula for V_o, $\boldsymbol{Q}=(z-20)\ \Delta$

In the formula for H_o, $\boldsymbol{Q}=\Delta\ \left(y-\frac{\Sigma\ y\Delta}{\Sigma\Delta}\right)$

In the formula for M_o, $\boldsymbol{Q}=\Delta$.

In each case $\boldsymbol{Q}$ depends solely on the properties of the arch and has the values $\boldsymbol{Q}_1$, $\boldsymbol{Q}_2$, etc., for the points along the arch ring.

We will consider first the formula for M_o because we have there the values of $\boldsymbol{Q}=\Delta$ for the entire arch ring while in the other two cases we have the values of $\boldsymbol{Q}$ for only one-half of the arch ring. Column 35 of Table 4 is copied from column 11 of Table 1. Table 4, A, has been inserted to show the manner in which the values in columns 36 and 37 are secured. This table and Table 4, B, are not part of the forms and are inserted for purposes of explanation only. Column 36 of Table 4, A, shows that $\Sigma\boldsymbol{Q}$ for any point is the sum of values of $\boldsymbol{Q}$ for all points to the right of the point considered and is a matter of successive addition starting at the bottom of the column.

To understand the derivation of column 37 in Tables 4 and 4, A, it is necessary to study Table 4, B. In this table it is considered that a unit load is successively placed at each point and in each case the value of $\frac{1}{2}\sum_a^l (z-k)\boldsymbol{Q}$ determined for each point to the right of the load. Values are not determined to the left of the load since the formula requires the values only where z is greater than k. At the point of load application z is equal to k and the expression becomes zero. The determinations of z and k for Table 4, B, are obvious from an examination of Figure 1. The value of $\frac{1}{2}\sum_a^l (z-k)\ \boldsymbol{Q}$ for a unit load at any point is the sum of the column headed $\frac{1}{2}\ (z-k)\ \boldsymbol{Q}$ and bearing the proper point designation. An inspection will show that the totals of these columns are the same as the quantities obtained in column 37 of Table 4, A, by the simple process of successive addition.

Table 2.—Computations for V_o.—We can now return to Table 2 of the forms and compute the values of V_o from the formula

$$V_o=\frac{\frac{1}{2}\sum_a^l(z-k)(z-20)\Delta}{\frac{1}{2}\Sigma z^2\Delta-200\Sigma\Delta}$$

In this formula $\boldsymbol{Q}$, as used in the preceding explanation is equal to $(z-20)\Delta$. The values of $(z-20)$ are the same for all arches when the arch ring is divided into 20 parts so they are permanently printed in column 12 of the forms. The values of $(z-20)\Delta$ are computed by multiplying each term in column 11 by the corresponding term in column 12 and the results written in column 13. If Table 2 were made out for the entire arch ring the computation of columns 14 and 15 would be exactly as explained for columns 36 and 37 in Table 4, A, but due to the symmetry of the arch ring it is necessary to use only the points on one side of the crown.

The values of Δ are symmetrical. That is the value of Δ_{20} is the same as Δ_1, etc. The values of $(z-20)$ on the right-hand side of the arch are numerically the same as those on the left but with the opposite algebraic sign. Therefore the values of $\boldsymbol{Q}=(z-20)\Delta$ on the right-hand half of the arch are the same numerically as those on the left but of opposite algebraic sign. Referring to Table 4, A, we see that the quantity in column 36 for point 10 is $(\boldsymbol{Q}_{20}+\boldsymbol{Q}_{19}\ \cdot\ \cdot\ \cdot\ \boldsymbol{Q}_{11})$ which is equal to $-(\boldsymbol{Q}_1+\boldsymbol{Q}_2\ \cdot\ \cdot\ \cdot\ \boldsymbol{Q}_{10})$. Therefore we find the sum of column 13 and write it opposite point 10 in column 14 but with opposite algebraic sign. We then continue the process of successive addition to the top of the column as previously explained.

A Filled-Spandrel Arch of the Type Illustrated by the Example

Referring to Table 4, A, we see that the quantity in column 37 opposite point 10 which corresponds to the bottom figure in column 15 of Table 2 is $(10\boldsymbol{Q}_{20}+9\boldsymbol{Q}_{19}\ \cdot\ \cdot\ \cdot\ \boldsymbol{Q}_{11})$ and this is equal to the sum of column 14. Therefore we find the sum of column 14 and write it in the bottom space of column 15. Next we add the figure in column 14 opposite point 9 and set down the result in column 15 opposite point 9 and continue the process of successive addition to the top of the column. Examination of column 37 of Table 4, A, and column 14 of Table 2 will demonstrate the correctness of this procedure. As a partial check on the numerical work it should be noted that the top figure of column 14 is numerically the same as the top figure of column 13. Column 15 now contains the values of $\frac{1}{2}\Sigma(z-k)(z-20)\Delta$ for a unit load placed successively at each point on the left half of the arch ring.

The denominator of the equation for V_o; that is, $\boldsymbol{F}=\frac{1}{2}\Sigma z^2\Delta-200\Sigma\Delta$, is independent of the loading and is easily computed. The values of z^2 are permanently printed in column 16 of the forms and for convenience the symmetrical values are combined; that is, for points 1 and 20, $z^2=(1^2+39^2)=1522$. For points 2 and 19 combined, $z^2=(3^2+37^2)=1378$, etc.

Column 17 is computed by multiplying each term in column 16 by the corresponding values of Δ in column 11. The sum of column 17 is equal to $\Sigma z^2\Delta$ and the sum of column 11 is equal to $\frac{1}{2}\Sigma\Delta$. The value of $\boldsymbol{F}$ is then computed and entered on the form. As a check on the numerical work it should be noted that the top figure of column 15 is equal to the value of $\boldsymbol{F}$.

V_o can now be computed and tabulated in column 18 by dividing each term in column 15 by $\boldsymbol{F}$ or, since it is easier to multiply than to divide on most calculating machines, it will be quicker to find the reciprocal of $\boldsymbol{F}$ and multiply.

Table 4, A.—*Supplementary table to explain method of computing* $\frac{1}{2}\sum_a^l (z-k)\,Q$

Points	35 Q	36 $\sum_a^l Q$	37 $\frac{1}{2}\sum_a^l (z-k)\,Q$
1	Q_{1}	$Q_{20}+Q_{19}$ . . . . $+Q_{2}$	$19\,Q_{20}+18\,Q_{19}$ . . . . $+2\,Q_{3}+Q_{2}$
2	Q_{2}	$Q_{20}+Q_{19}$ . . . . $+Q_{3}$	$18\,Q_{20}+17\,Q_{19}$ . . . . $+2\,Q_{4}+Q_{3}$
3	Q_{3}	$Q_{20}+Q_{19}$ . . . . $+Q_{4}$	$17\,Q_{20}+16\,Q_{19}$ . . . . $+2\,Q_{5}+Q_{4}$
4	Q_{4}	$Q_{20}+Q_{19}$ . . . . $+Q_{5}$	$16\,Q_{20}+15\,Q_{19}$ . . . . $+2\,Q_{6}+Q_{5}$
5	Q_{5}	$Q_{20}+Q_{19}$ . . . . $+Q_{6}$	$15\,Q_{20}+14\,Q_{19}$ . . . . $+2\,Q_{7}+Q_{6}$
6	Q_{6}	$Q_{20}+Q_{19}$ . . . . $+Q_{7}$	$14\,Q_{20}+13\,Q_{19}$ . . . . $+2\,Q_{8}+Q_{7}$
7	Q_{7}	$Q_{20}+Q_{19}$ . . . . $+Q_{8}$	$13\,Q_{20}+12\,Q_{19}$ . . . . $+2\,Q_{9}+Q_{8}$
8	Q_{8}	$Q_{20}+Q_{19}$ . . . . $+Q_{9}$	$12\,Q_{20}+11\,Q_{19}$ . . . . $+2\,Q_{10}+Q_{9}$
9	Q_{9}	$Q_{20}+Q_{19}$ . . . . $+Q_{10}$	$11\,Q_{20}+10\,Q_{19}$ . . . . $+2\,Q_{11}+Q_{10}$
10	Q_{10}	$Q_{20}+Q_{19}$ . . . . $+Q_{11}$	$10\,Q_{20}+9\,Q_{19}$ . . . . $+2\,Q_{12}+Q_{11}$
11	Q_{11}	$Q_{20}+Q_{19}$ . . . . $+Q_{12}$	$9\,Q_{20}+8\,Q_{19}$ . . . . $+2\,Q_{13}+Q_{12}$
12	Q_{12}	$Q_{20}+Q_{19}$ . . . . $+Q_{13}$	$8\,Q_{20}+7\,Q_{19}$ . . . . $+2\,Q_{14}+Q_{13}$
13	Q_{13}	$Q_{20}+Q_{19}$ . . . . $+Q_{14}$	$7\,Q_{20}+6\,Q_{19}$ . . . . $+2\,Q_{15}+Q_{14}$
14	Q_{14}	$Q_{20}+Q_{19}$ . . . . . . $+Q_{15}$	$6\,Q_{20}+5\,Q_{19}$ . . . . . . $+2\,Q_{16}+Q_{15}$
15	Q_{15}	$Q_{20}+Q_{19}$ . . . . $+Q_{16}$	$5\,Q_{20}+4\,Q_{19}$ . . . . . $+2\,Q_{17}+Q_{16}$
16	Q_{16}	$Q_{20}+Q_{19}$ . . $+Q_{17}$	$4\,Q_{20}+3\,Q_{19}$ . . . $+2\,Q_{18}+Q_{17}$
17	Q_{17}	$Q_{20}+Q_{19}+Q_{18}$	$3\,Q_{20}+2\,Q_{19}+Q_{18}$
18	Q_{18}	$Q_{20}+Q_{19}$	$2\,Q_{20}+Q_{19}$
19	Q_{19}	Q_{20}	Q_{20}
20	Q_{20}	0	0

Table 4, B.—*Supplementary table to explain the method of computing* $\frac{1}{2}\sum_a^l (z-k)\,Q$

Points	z	Q	Load at point 20 $k=39$			Load at point 19 $k=37$			Load at point 18 $k=35$			Load at point 17 $k=33$			Load at point 16 $k=31$			Load at point 15 $k=29$			Load at point 14 $k=27$			Load at point 13 $k=25$			Load at point 12 $k=23$			Load at point 11 $k=21$		
			$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$
1	1	Q_{1}																														
2	3	Q_{2}																														
3	5	Q_{3}																														
4	7	Q_{4}																														
5	9	Q_{5}																														
6	11	Q_{6}																														
7	13	Q_{7}																														
8	15	Q_{8}																														
9	17	Q_{9}																														
10	19	Q_{10}																														
11	21	Q_{11}																											0	0	0
12	23	Q_{12}																								0	0	0	2	1	$1Q_{12}$
13	25	Q_{13}																						0	0	0	2	1	Q_{13}	4	2	$2Q_{13}$
14	27	Q_{14}																				0	0	0	2	1	Q_{14}	4	2	$2Q_{14}$	6	3	$3Q_{14}$
15	29	Q_{15}																		0	0	0	2	1	Q_{15}	4	2	$2Q_{15}$	6	3	$3Q_{15}$	8	4	$4Q_{15}$
16	31	Q_{16}															0	0	0	2	1	Q_{16}	4	2	$2Q_{16}$	6	3	$3Q_{16}$	8	4	$4Q_{16}$	10	5	$5Q_{16}$
17	33	Q_{17}												0	0	0	2	1	Q_{17}	4	2	$2Q_{17}$	6	3	$3Q_{17}$	8	4	$4Q_{17}$	10	5	$5Q_{17}$	12	6	$6Q_{17}$
18	35	Q_{18}									0	0	0	2	1	Q_{18}	4	2	$2Q_{18}$	6	3	$3Q_{18}$	8	4	$4Q_{18}$	10	5	$5Q_{18}$	12	6	$6Q_{18}$	14	7	$7Q_{18}$
19	37	Q_{19}				0	0	0	2	1	Q_{19}	4	2	$2Q_{19}$	6	3	$3Q_{19}$	8	4	$4Q_{19}$	10	5	$5Q_{19}$	12	6	$6Q_{19}$	14	7	$7Q_{19}$	16	8	$8Q_{19}$
20	39	Q_{20}	0	0	0	2	1	Q_{20}	4	2	$2Q_{20}$	6	3	$3Q_{20}$	8	4	$4Q_{20}$	10	5	$5Q_{20}$	12	6	$6Q_{20}$	14	7	$7Q_{20}$	16	8	$8Q_{20}$	18	9	$9Q_{20}$

Points	z	Q	Load at point 10 $k=19$			Load at point 9 $k=17$			Load at point 8 $k=15$			Load at point 7 $k=13$			Load at point 6 $k=11$			Load at point 5 $k=9$			Load at point 4 $k=7$			Load at point 3 $k=5$			Load at point 2 $k=3$			Load at point 1 $k=1$		
			$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$	$(z-k)$	$\frac{1}{2}(z-k)$	$\frac{1}{2}(z-k)Q$
1	1	Q_{1}																											0	0	0
2	3	Q_{2}																								0	0	0	2	1	Q_{2}
3	5	Q_{3}																						0	0	0	2	1	Q_{3}	4	2	$2Q_{3}$
4	7	Q_{4}																				0	0	0	2	1	Q_{4}	4	2	$2Q_{4}$	6	3	$3Q_{4}$
5	9	Q_{5}																		0	0	0	2	1	Q_{5}	4	2	$2Q_{5}$	6	3	$3Q_{5}$	8	4	$4Q_{5}$
6	11	Q_{6}															0	0	0	2	1	Q_{6}	4	2	$2Q_{6}$	6	3	$3Q_{6}$	8	4	$4Q_{6}$	10	5	$5Q_{6}$
7	13	Q_{7}												0	0	0	2	1	Q_{7}	4	2	$2Q_{7}$	6	3	$3Q_{7}$	8	4	$4Q_{7}$	10	5	$5Q_{7}$	12	6	$6Q_{7}$
8	15	Q_{8}									0	0	0	2	1	Q_{8}	4	2	$2Q_{8}$	6	3	$3Q_{8}$	8	4	$4Q_{8}$	10	5	$5Q_{8}$	12	6	$6Q_{8}$	14	7	$7Q_{8}$
9	17	Q_{9}				0	0	0	2	1	Q_{9}	4	2	$2Q_{9}$	6	3	$3Q_{9}$	8	4	$4Q_{9}$	10	5	$5Q_{9}$	12	6	$6Q_{9}$	14	7	$7Q_{9}$	16	8	$8Q_{9}$
10	19	Q_{10}	0	0	0	2	1	Q_{10}	4	2	$2Q_{10}$	6	3	$3Q_{10}$	8	4	$4Q_{10}$	10	5	$5Q_{10}$	12	6	$6Q_{10}$	14	7	$7Q_{10}$	16	8	$8Q_{10}$	18	9	$9Q_{10}$
11	21	Q_{11}	2	1	Q_{11}	4	2	$2Q_{11}$	6	3	$3Q_{11}$	8	4	$4Q_{11}$	10	5	$5Q_{11}$	12	6	$6Q_{11}$	14	7	$7Q_{11}$	16	8	$8Q_{11}$	18	9	$9Q_{11}$	20	10	$10Q_{11}$
12	23	Q_{12}	4	2	$2Q_{12}$	6	3	$3Q_{12}$	8	4	$4Q_{12}$	10	5	$5Q_{12}$	12	6	$6Q_{12}$	14	7	$7Q_{12}$	16	8	$8Q_{12}$	18	9	$9Q_{12}$	20	10	$10Q_{12}$	22	11	$11Q_{12}$
13	25	Q_{13}	6	3	$3Q_{13}$	8	4	$4Q_{13}$	10	5	$5Q_{13}$	12	6	$6Q_{13}$	14	7	$7Q_{13}$	16	8	$8Q_{13}$	18	9	$9Q_{13}$	20	10	$10Q_{13}$	22	11	$11Q_{13}$	24	12	$12Q_{13}$
14	27	Q_{14}	8	4	$4Q_{14}$	10	5	$5Q_{14}$	12	6	$6Q_{14}$	14	7	$7Q_{14}$	16	8	$8Q_{14}$	18	9	$9Q_{14}$	20	10	$10Q_{14}$	22	11	$11Q_{14}$	24	12	$12Q_{14}$	26	13	$13Q_{14}$
15	29	Q_{15}	10	5	$5Q_{15}$	12	6	$6Q_{15}$	14	7	$7Q_{15}$	16	8	$8Q_{15}$	18	9	$9Q_{15}$	20	10	$10Q_{15}$	22	11	$11Q_{15}$	24	12	$12Q_{15}$	26	13	$13Q_{15}$	28	14	$14Q_{15}$
16	31	Q_{16}	12	6	$6Q_{16}$	14	7	$7Q_{16}$	16	8	$8Q_{16}$	18	9	$9Q_{16}$	20	10	$10Q_{16}$	22	11	$11Q_{16}$	24	12	$12Q_{16}$	26	13	$13Q_{16}$	28	14	$14Q_{16}$	30	15	$15Q_{16}$
17	33	Q_{17}	14	7	$7Q_{17}$	16	8	$8Q_{17}$	18	9	$9Q_{17}$	20	10	$10Q_{17}$	22	11	$11Q_{17}$	24	12	$12Q_{17}$	26	13	$13Q_{17}$	28	14	$14Q_{17}$	30	15	$15Q_{17}$	32	16	$16Q_{17}$
18	35	Q_{18}	16	8	$8Q_{18}$	18	9	$9Q_{18}$	20	10	$10Q_{18}$	22	11	$11Q_{18}$	24	12	$12Q_{18}$	26	13	$13Q_{18}$	28	14	$14Q_{18}$	30	15	$15Q_{18}$	32	16	$16Q_{18}$	34	17	$17Q_{18}$
19	37	Q_{19}	18	9	$9Q_{19}$	20	10	$10Q_{19}$	22	11	$11Q_{19}$	24	12	$12Q_{19}$	26	13	$13Q_{19}$	28	14	$14Q_{19}$	30	15	$15Q_{19}$	32	16	$16Q_{19}$	34	17	$17Q_{19}$	36	18	$18Q_{19}$
20	39	Q_{20}	20	10	$10Q_{20}$	22	11	$11Q_{20}$	24	12	$12Q_{20}$	26	13	$13Q_{20}$	28	14	$14Q_{20}$	30	15	$15Q_{20}$	32	16	$16Q_{20}$	34	17	$17Q_{20}$	36	18	$18Q_{20}$	38	19	$19Q_{20}$

Table 3.—Computations for H_o.—Table 3 is used in computing H_o from the formula

$$H_o=\frac{-\frac{1}{2}\sum_a^l\left(z-k\right)\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)}{\boldsymbol{C}}$$

In this formula the denominator $\boldsymbol{C}$ is equal to

$$\frac{1}{dx}\sum_0^l y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\sum_0^l\frac{\cos\phi}{A}$$

and is independent of the loading. The numerator depends on the position of the unit load and is computed in a way similar to that used for the formula for V_o. Values of y are scaled from the drawing and tabulated in column 20. For convenience the values of Δ are copied in column 21. The values of $y\Delta$ are found by multiplying, and then the value of $\frac{\Sigma y\Delta}{\Sigma\Delta}$ may be found by dividing the sum of column 22 by the sum of column 11. Column 23 is computed by subtracting the constant value of $\frac{\Sigma y\Delta}{\Sigma\Delta}$ from each value of y. Then column 24 may be computed by multiplying each term in column 23 by the corresponding value of Δ. The sum of column 24 should be zero. If it is not it is due to some inaccuracy which should be found before going further. The error will be in column 21, 22, 23, or 24. Of course allowance should be made for the fact that decimal places were dropped in the value of $\frac{\Sigma y\Delta}{\Sigma\Delta}$ and that inaccuracy is carried into columns 24 and 25. For that reason the sum of column 24 will usually not check exactly, but it is not difficult to determine if the inaccuracy is from that cause or a mistake.

Columns 25 and 26 are computed from column 24 in exactly the same manner as columns 14 and 15 were computed from column 13.

The bottom figure in column 25 is equal to the sum of the figures of column 24 for the right-hand half of the arch, which are not shown, but they are the same as the figures which are shown and their sum is also zero. To this zero we add the last figure shown in column 24 and write the result in the next space above in column 25 and continue the process of successive addition to the top of the column. The top figure of column 25 should be approximately equal numerically to the top figure of column 24.

The bottom figure in column 26 is the sum of the figures in the half of column 25 which is not shown plus the figure opposite point 10, but since the latter figure is always zero it makes no difference in the sum. This sum is equal to minus the sum of the figures shown. This may be understood from consideration of Table 4, A. Thus we find the sum of the figures shown in column 25 and write it in the bottom space of column 26. Then we add to it the figure opposite point 9 in column 25 and continue the process of successive addition to the top of the column, observing the correct algebraic sign. Then all of the figures in column 26 will be negative and the top figure will be zero.

The denominator of the equation for H_o that is

$$\boldsymbol{C}=\frac{1}{dx}\,\Sigma y\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma\Delta}\right)+\Sigma\,\frac{\cos\phi}{A}$$

is independent of the loading and is computed in columns 27, 28, and 29 as indicated by the headings of the columns. Cos ϕ is tabulated in column 28. It might be computed from the formula $\cos\phi=\frac{dx}{ds}$, but since $\cos\phi$ varies so little for small angles it will be found more accurate to compute $\sin\phi$ from dimensions scaled on the drawing and then find the corresponding values of $\cos\phi$ in a table of trigonometric functions.

Column 29 is computed by dividing each term in column 28 by the area A. Since a strip of the arch ring 1 foot wide is being considered, A is equal to h plus n times the area of steel in square feet. Since the area of steel in square feet is small and the effect of $\cos\phi$ is relatively small, it is sufficiently accurate to assume, for this purpose, that A is equal to h. $\boldsymbol{C}$, the denominator of the equation for H_o, is found by dividing the sum of column 27 by $\frac{1}{2}\,dx$ and adding twice the sum of column 29. The value of $\boldsymbol{C}$ should be set down in the space provided on the same sheet with Table 3. H_o may now be computed for a load of unity at each point by dividing each term of column 26 by $\boldsymbol{C}$ and the results are tabulated in column 30. Note that the algebraic sign is changed because the formula for H_o is preceded by a minus sign.

Table 4.—Computation for M_o.—M_o is computed in Table 4 from the formula

$$M_o=H_o\,\frac{\Sigma y\Delta}{\Sigma\Delta}-20\,\frac{dx}{2}\,V_o+\frac{dx}{\Sigma\Delta}\times\frac{1}{2}\sum_a^l(z-k)\,\Delta$$

The values of z are permanently printed in column 32. The values of H_o, V_o and Δ have been previously found and for convenience are copied in columns 33, 34, and 35. The values of H_o and Δ are symmetrical so the values found for the left-hand half of the arch ring may be copied for the right-hand half. The values of V_o for the right half of the arch may be found by subtracting each value for the left half from unity.

Column 36 is computed by summing up column 35, starting with zero in the bottom space for point 1′. Column 37 is computed by summing up column 36 in the same way starting with zero at the bottom opposite point 1′ as previously explained. Point 1′ for symmetrical arches is the same as point 20 used in the explanation. Column 38 is computed by multiplying each term in column 37 by $\frac{dx}{\Sigma\Delta}$. Column 39 is computed by multiplying each value of H_o by $\frac{\Sigma y\Delta}{\Sigma\Delta}$. Column 40 is computed by multiplying each value of V_o by $-20\,\frac{dx}{2}$. The sum of column 39 should equal the sum of column 33 multiplied by $\frac{\Sigma y\Delta}{\Sigma\Delta}$ and the sum of column 40 should equal $-100\,dx$. M_o in column 41 is computed by taking the algebraic sum of the corresponding terms in columns 38, 39, and 40. The algebraic sum of column 41 should equal the algebraic sum of columns 38, 39, and 40.

TABLE 4.—*Computations for M_o*

31	32	33	34	35	36	37	38	39	40	41
Points	z	H_o	V_o	Δ	$\sum_a^l \Delta$	$\frac{1}{2}\sum_a^l (z-k)\Delta$	$\frac{dx}{\Sigma\Delta}$ Col. 37	$\frac{\Sigma y\Delta}{\Sigma\Delta}$	$-20\frac{dx}{2}V_o$	M_o
0	0									
1	1	0.000	1.0000	4.7	472.9	4,537.2	28.500	0.000	− 30.000	− 1.500
2	3	.062	.99556	10.1	462.8	4,064.3	25.529	.363	− 29.867	− 3.975
3	5	.219	.98256	16.2	446.6	3,601.5	22.622	1.282	− 29.477	− 5.573
4	7	.478	.95748	21.3	425.3	3,154.9	19.817	2.799	− 28.724	− 6.108
5	9	.815	.91861	26.0	399.3	2,729.6	17.146	4.772	− 27.558	− 5.640
6	11	1.191	.86551	29.6	369.7	2,330.3	14.637	6.974	− 25.965	− 4.354
7	13	1.561	.79915	31.8	337.9	1,960.6	12.315	9.141	− 23.974	− 2.518
8	15	1.878	.72171	31.7	306.2	1,622.7	10.193	10.997	− 21.651	− 0.461
9	17	2.111	.63638	33.7	272.5	1,316.5	8.269	12.362	− 19.091	+ 1.540
10	19	2.232	.54602	33.7	238.8	1,044.0	6.558	13.070	− 16.381	+ 3.247
10′	21	2.232	.45398	33.7	205.1	805.2	5.058	13.070	− 13.619	+ 4.509
9′	23	2.111	.36362	33.7	171.4	600.1	3.769	12.362	− 10.909	+ 5.222
8′	25	1.878	.27829	31.7	139.7	428.7	2.693	10.997	− 8.349	+ 5.341
7′	27	1.561	.20085	31.8	107.9	289.0	1.815	9.141	− 6.026	+ 4.930
6′	29	1.191	.13449	29.6	78.3	181.1	1.137	6.974	− 4.035	+ 4.076
5′	31	.815	.08139	26.0	52.3	102.8	.646	4.772	− 2.442	+ 2.976
4′	33	.478	.04252	21.3	31.0	50.5	.317	2.799	− 1.276	+ 1.840
3′	35	.219	.01744	16.2	14.8	19.5	.122	1.282	− .523	+ .881
2′	37	.062	.00444	10.1	4.7	4.7	.030	.363	− .133	+ .260
1′	39	0.000	0.0000	4.7	0.0	0.0	0.000	0.000	0.000	0.000
0′	40									
		21.094	10.0000				181.173	123.520	− 300.000	+ 4.693

$$M_o=\frac{dx}{\Sigma\Delta}\frac{1}{2}\sum_a^l (z-k)\Delta+H_o\frac{\Sigma y\Delta}{\Sigma\Delta}-20\frac{dx}{2}V_o$$

$$\frac{dx}{\Sigma\Delta}=.0062814 \qquad -20\frac{dx}{2}=-30.0$$

For check:

$\frac{\Sigma y\Delta}{\Sigma\Delta}\Sigma\text{Col. } 33=123.523=\Sigma\text{Col. } 39.$

$-20\frac{dx}{2}\Sigma\text{Col. } 34=-300.0=\Sigma\text{Col. } 40.$

$\Sigma\text{Col. } 38+\Sigma\text{Col. } 39+\Sigma\text{Col. } 40=+4.693 =\Sigma\text{Col. } 41.$

Tables 5, 6, and 7.—Computation of bending moments at points 3, 8, and 10½.—We now have the values of H_o, V_o, and M_o for a load of unity placed at any point on the arch. It must now be decided at which other points the stresses are desired. Usually three other points are sufficient, and they should be selected with care in order to get the critical points. One should be at or near the crown, one near the haunch, and probably the best place for the third is between the haunch and the left abutment. Points 3 and 8 are chosen because they are near changes in the reinforcement; and point 10½, being the crown, is also chosen.

Columns 46 to 50, inclusive, are used in finding the moment at point 3, from the formula $M_3=M_o-H_oy+m_3$. In computing the moment at any particular point, z and y are constant, while k varies with the position of the unit load. We now desire the moment at a particular point caused by a load at each point. For point 3 the value of z is 5 and the value of y can be found in column 20. These figures should be set down in the spaces indicated in the heading for columns 46 to 50. For convenience H_o, V_o, and M_o are copied in columns 43, 44, and 45. The values of m_x are computed from the formula $m_x=\left[V_oz-(z-k)\right]\frac{dx}{2}$ where k is less than z, and from the formula $m_x=V_oz\frac{dx}{2}$ where k is greater than z. Column 46 is computed by multiplying the values of V_o by z ($z=5$ for point 3). It is only necessary to fill out column 46 for a load at those points where

TABLE 5.—*Computations for M_3* TABLE 6.—*Computations for M_8* TABLE 7.—*Computations for $M_{10\frac{1}{2}}$*

42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
				Point 3, $z_3=5$; $y_3=3.51$					Point 8, $z_8=15$; $y_8=6.85$					Point 10½, $z_{10\,1/2}=20$; $y_{10\,1/2}=7.21$				
Points	H_o	V_o	M_o	V_oz_3	$V_oz_3-(z_3-k)$	m_3	$-H_oy_3$	M_3	V_oz_8	$V_oz_8-(z_8-k)$	m_8	$-H_oy_8$	M_8	$V_oz_{10\,1/2}$	$V_oz_{10\,1/2}-(z_{10\,1/2}-k)$	$m_{10\,1/2}$	$-H_oy_{10\,1/2}$	$M_{10\,1/2}$
0																		
1	0.000	1.00000	−1.500	5.000	1.0000	1.500	0.000	0.000	15.000	1.0000	1.500	0.000	0.000	20.000	1.000	1.500	0.000	0.000
2	.062	.99556	−3.975	4.978	2.978	4.467	− .218	+ .274	14.933	2.933	4.400	− .425	.000	19.911	2.911	4.367	− .447	− .055
3	.219	.98256	−5.573			7.369	− .769	+1.027	14.738	4.738	7.107	− 1.500	+ .034	19.651	4.651	6.976	− 1.579	− .176
4	.478	.95748	−6.108			7.181	− 1.678	− .605	14.362	6.362	9.543	− 3.274	+ .161	19.150	6.150	9.225	− 3.446	− .329
5	.815	.91861	−5.640			6.890	− 2.861	−1.611	13.779	7.779	11.669	− 5.583	+ .446	18.372	7.372	11.058	− 5.876	− .458
6	1.191	.86551	−4.354			6.491	− 4.180	−2.043	12.983	8.983	13.475	− 8.158	+ .963	17.310	8.310	12.465	− 8.587	− .476
7	1.561	.79915	−2.518			5.994	− 5.479	−2.003	11.987	9.987	14.981	−10.693	+1.770	15.983	8.983	13.475	− 11.255	− .298
8	1.878	.72171	−0.461			5.413	− 6.592	−1.640			16.238	−12.864	+2.913	14.434	9.434	14.151	− 13.540	+ .150
9	2.111	.63638	+1.540			4.773	− 7.409	−1.096			14.319	−14.460	+1.399	12.728	9.728	14.592	− 15.220	+ .912
10	2.232	.54602	+3.247			4.095	− 7.834	− .492			12.285	−15.289	+ .243	10.920	9.920	14.880	− 16.093	+ 2.034
10′	2.232	.45398	+4.509			3.405	− 7.834	+ .080			10.215	−15.289	− .565			13.619	− 16.093	+ 2.035
9′	2.111	.36362	+5.222			2.727	− 7.409	+ .540			8.181	−14.460	−1.057			10.909	− 15.220	+ .911
8′	1.878	.27829	+5.341			2.087	− 6.592	+ .836			6.262	−12.864	−1.261			8.349	− 13.540	+ .150
7′	1.561	.20085	+4.930			1.506	− 5.479	+ .957			4.519	−10.693	−1.244			6.025	− 11.255	− .300
6′	1.191	.13449	+4.076			1.009	− 4.180	+ .905			3.026	− 8.158	−1.056			4.035	− 8.587	− .476
5′	.815	.08139	+2.976			.610	− 2.861	+ .725			1.831	− 5.583	− .776			2.442	− 5.876	− .458
4′	.478	.04252	+1.840			.319	− 1.678	+ .481			.957	− 3.274	− .477			1.276	− 3.446	− .330
3′	.219	.01744	+ .881			.131	− .769	+ .243			.392	− 1.500	− .227			.523	− 1.579	− .175
2′	.062	.00444	+ .260			.033	− .218	+ .075			.100	− .425	− .065			.133	− .447	− .054
1′	0.000	0.00000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0′																		
	21.094	10.00000	+4.693			66,000	−74.040	−3.347			141.000	−144.492	+1.201			150.000	−152.086	+ 2.607

$$M_x=M_o+m_x-H_oy$$

$$m_x=\left[V_oz_x-(z_x-k)\right]\frac{dx}{2} \quad z>k$$

For check:

Σ Col. 45 + Σ Col. 48 + Σ Col .49 = Σ Col. 50.

Σ Col. 45 + Σ Col. 53 + Σ Col. 54 = Σ Col. 55.

Σ Col. 45 + Σ Col. 58 + Σ Col. 59 = Σ Col. 60.

k is less than z. Column 47 is computed by subtracting $(z-k)$ from the figures in column 46. The values of m_x may now be computed for a load at those points where k is less than z by multiplying the figures in column 47 by $\frac{dx}{2}$ and for the remaining points by multiplying the values of V_o by $z\frac{dx}{2}$.

The value of $-H_oy$ in column 49 is computed by multiplying each value of H_o by $-y$ as given in the heading. The bending moment $M_3 = M_o + m_3 - H_oy$ is now computed by adding algebraically the corresponding values of columns 45, 48, and 49 and the results tabulated in column 50. The bending moments at the two other points are computed in the same way, using columns 51 to 60, inclusive. We now have in the tabulation which includes Tables 5, 6, and 7 (sheet 4 of assembly of data) the vertical shear, the horizontal thrust, and the bending moment at point 0 and the bending moment at three other points caused by a load of unity at each of the points of the arch ring.

Table 8.—Computations for the dead-load moments, thrust, and shears.—The values of H_o, V_o, M_o and the unit-load bending moments at the points 3, 8, and 10½ are copied in columns 62 to 67 of Table 8. The dead load which is applied at each point is then computed and tabulated in column 68. This may be conveniently tabulated in a supplementary table as follows: The dead load at any point is equal to $150hds + 110dx\ h_f$, in which the weight of the concrete is assumed to be 150 pounds per cubic foot and the weight of the earth fill 110 pounds per cubic foot. Values of h and ds are found in columns 2 and 10, respectively, of Table 1, and h_f is the depth of the fill above the arch ring and dx for this arch is 3 feet.

Computation of dead load

Point	$h\,ds$	$150\,h\,ds$	h_f	$330\,h_f$	Dead load
					Pounds
1	6.79	1,020	6.00	1,980	3,000
2	4.96	744	4.80	1,585	2,329
3	4.11	616	3.82	1,260	1,876
4	3.69	553	3.00	990	1,543
5	3.40	510	2.25	742	1,252
6	3.20	480	1.70	560	1,040
7	3.08	462	1.20	396	858
8	3.04	456	.90	297	753
9	3.00	450	.70	232	682
10	3.00	450	.67	220	670

Column 69 to 74, inclusive, are computed by multiplying the dead load in column 68 by the proper figure from columns 62 to 67, inclusive. The algebraic sums of columns 69, 70, and 71 are the horizontal thrust, vertical shear, and bending moment, respectively, at point 0 resulting from the dead load. The algebraic sums of columns 72, 73, and 74 are the dead-load bending moments at points 3, 8, and 10½.

Table 9—Computation of stresses.—Column 75 in Table 9 is now to be filled out. The values of cos ϕ for each point desired are taken from column 28. The values of sin ϕ may be found by scaling the drawing. Since the stresses are desired in pounds per square inch, and all previous dimensions have been in feet, the area in column 75 should be 144 times h (taken from column 2) plus n times the area of steel in square inches. The values of $\frac{h}{2I}$ are computed from columns 2 and 9 and this result should be divided by 144 to get results in pounds per square inch. The values of M, H, and V for the dead load are taken from the sums of columns 69 to 74, inclusive, and set down in the proper places in

TABLE 8.—*Computations of H, V, and M for dead load*

61	62	63	64	65	66	67	68	69	70	71	72	73	74
Point	Unit loads						D. L.	Dead load					
	H_o	V_o	M_o	M_3	M_8	$M_{10\frac{1}{2}}$		H_o	V_o	M_o	M_3	M_8	$M_{10\frac{1}{2}}$
1	0	1.00000	− 1.500	0	0	0	3,000	0	3,000	− 4,500	0	0	0
2	.062	.99556	− 3.975	+ .274	+ .000	− .055	2,330	144	2,320	− 9,262	+ 638	0	− 128
3	.219	.98256	− 5.573	+ 1.027	+ .034	− .176	1,880	412	1,847	− 10,477	+ 1,931	+ 64	− 331
4	.478	.95748	− 6.108	− .605	+ .161	− .329	1,540	736	1,475	− 9,406	− 932	+ 248	− 507
5	.815	.91861	− 5.640	− 1.611	+ .446	− .458	1,250	1,019	1,148	− 7,050	− 2,014	+ 558	− 573
6	1.191	.86551	− 4.354	− 2.043	+ .963	− .476	1,040	1,239	900	− 4,528	− 2,125	+ 1,002	− 495
7	1.561	.79915	− 2.518	− 2.003	+ 1.770	− .298	860	1,342	687	− 2,165	− 1,723	+ 1,522	− 256
8	1.878	.72171	− 0.461	− 1.640	+ 2.913	+ .150	750	1,409	541	− 346	− 1,230	+ 2,185	+ 113
9	2.111	.63638	+ 1.540	− 1.096	+ 1.399	+ .912	680	1,435	433	+ 1,047	− 745	+ 951	+ 620
10	2.232	.54602	+ 3.247	− .492	+ .243	+ 2.034	670	1,495	366	+ 2,175	− 330	+ 163	+ 1,363
10′	2.232	.45398	+ 4.509	+ .080	− .565	+ 2.035	670	1,495	304	+ 3,021	+ 54	− 379	+ 1,363
9′	2.111	.36362	+ 5.222	+ .540	− 1.057	+ .911	680	1,435	247	+ 3,551	+ 367	− 719	+ 619
8′	1.878	.27829	+ 5.341	+ .836	− 1.261	+ .150	750	1,409	209	+ 4,006	+ 627	− 946	+ 113
7′	1.561	.20085	+ 4.930	+ .957	− 1.244	− .300	860	1,342	173	+ 4,240	+ 823	− 1,070	− 258
6′	1.191	.13449	+ 4.076	+ .905	− 1.056	− .476	1,040	1,239	140	+ 4,239	+ 941	− 1,098	− 495
5′	.815	.08139	+ 2.976	+ .725	− .776	− .458	1,250	1,019	102	+ 3,720	+ 906	− 970	− 573
4′	.478	.04252	+ 1.840	+ .481	− .477	− .330	1,540	736	65	+ 2,834	+ 741	− 735	− 508
3′	.219	.01744	+ .881	+ .243	− .227	− .175	1,880	412	33	+ 1,656	+ 457	− 427	− 329
2′	.062	.00444	+ .260	+ .075	− .065	− .054	2,330	144	10	+ 606	+ 175	− 151	− 126
1′	0	0	0	0	0	0	3,000	0	0	0	0	0	0
	21.094	10.0000	+ 34.822	+ 6.143	+ 7.929	+ 6.192	28,000	18,462	14,000	+ 31,095	+ 7,660	+ 6,693	+ 4,191
			− 30.129	− 9.490	− 6.728	− 3.585				− 47,734	− 9,099	− 6,495	− 4,579
			+ 4.693	− 3.347	+ 1.201	+ 2.607				− 16,639	− 1,439	+ 198	− 388

$$M_t = -H_t\left(y - \frac{\Sigma y\Delta}{\Sigma\Delta}\right)$$

At point 0, $M_t = -\begin{Bmatrix}+2{,}762\\-3{,}682\end{Bmatrix}\times(-5.856) = \begin{Bmatrix}+16{,}174\\-21{,}562\end{Bmatrix}$

At point 3, $M_t = -\begin{Bmatrix}+2{,}762\\-3{,}682\end{Bmatrix}\times(-2.346) = \begin{Bmatrix}+\ 6{,}480\\-\ 8{,}638\end{Bmatrix}$

At point 8, $M_t = -\begin{Bmatrix}+2{,}762\\-3{,}682\end{Bmatrix}\times(+0.994) = \begin{Bmatrix}-\ 2{,}745\\+\ 3{,}660\end{Bmatrix}$

At point 10½, $M_t = -\begin{Bmatrix}+2{,}762\\-3{,}682\end{Bmatrix}\times(+1.364) = \begin{Bmatrix}-\ 3{,}470\\+\ 4{,}985\end{Bmatrix}$

TABLE 9.—*Computation of maximum stresses*

75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
										Extrados	Intrados	Maximum stresses			
		H	M	V	$H \cos \phi$	$V \sin \phi$	N	$\frac{N}{A}$	$M\frac{h}{2I}$	$\frac{N}{A}+M\frac{h}{2I}$	$\frac{N}{A}-M\frac{h}{2I}$	Extrados		Intrados	
												+	−	+	−
Point 0	D. L.	+ 18,462	− 16,639	14,000	+ 15,471	7,672	+ 23,142	+ 66	− 118	− 52	+ 184	− 52	− 52	+ 184	+ 184
$\sin \phi=.548.$	+ C. L. L.	+	+		+		+	+	+						
$\cos \phi=.838.$	+ U. L. L.	+ 5,584	+ 13,058	1,035	+ 4,679	567	+ 5,246	+ 15	+ 93	+ 108	− 78	+ 108			− 78
Area=348.	− C. L. L.	+	−		+		+	+	−						
	− U. L. L.	+ 2,326	− 11,298	2,715	+ 1,949	1,488	+ 3,437	+ 10	− 80	− 70	+ 90		− 70	+ 90	
$\frac{h}{2I}=.00709.$	+ T.	+ 2,762	+ 16,174	0	+ 2,315	0	+ 2,315	+ 7	+ 115	+ 122	− 108	+ 122			− 108
	− T.	− 3,682	− 21,562	0	− 3,086	0	− 3,086	− 9	− 153	− 162	+ 144		− 162	+ 144	
									Total excluding temperature				− 122	+ 274	
											Total	+ 178	− 284	+ 418	− 2
Point 3	D. L.	+ 18,462	− 1,439	8,670	+ 17,465	2,982	+ 20,447	+ 104	− 33	+ 71	+ 137	+ 71	+ 71	+ 137	+ 137
$\sin \phi=.344.$	+ C. L. L.	+	+		+		+	+	+						
$\cos \phi=.946.$	+ U. L. L.	+ 4,060	+ 2,304	958	+ 3,841	330	+ 4,171	+ 21	+ 53	+ 74	− 32	+ 74			− 32
Area=196.	− C. L. L.	+	−		+		+	+	−						
	− U. L. L.	+ 3,850	− 3,559	2,042	+ 3,642	702	+ 4,344	+ 22	− 82	− 60	+ 104		− 60	+ 104	
$\frac{h}{2I}=.02303.$	+ T.	+ 2,762	+ 6,480	0	+ 2,613	0	+ 2,613	+ 13	+ 149	+ 162	− 136	+ 162			− 136
	− T.	− 3,682	− 8,638	0	− 3,483	0	− 3,483	− 18	− 199	− 217	+ 181		− 217	+ 181	
									Total excluding temperature			+ 145		+ 241	
											Total	+ 307	− 206	+ 422	− 31
Point 8	D. L.	+ 18,462	+ 198	2,100	+ 18,388	216	+ 18,604	+ 120	+ 7	+ 127	+ 113	+ 127	+ 127	+ 113	+ 113
$\sin \phi=.103.$	+ C. L. L.	+	+		+		+	+	+						
$\cos \phi=.996.$	+ U. L. L.	+ 3,955	+ 2,973	534	+ 3,939	55	+ 3,994	+ 26	+ 111	+ 137	− 85	+ 137			− 85
Area=155.	− C. L. L.	+	−		+		+	+	−						
	− U. L. L.	+ 3,955	− 2,523	591	+ 3,939	61	+ 4,000	+ 26	− 94	− 68	+ 120		− 68	+ 120	
$\frac{h}{2I}=.03728.$	+ T.	+ 2,762	− 2,745	0	+ 2,751	0	+ 2,751	+ 18	− 102	− 84	+ 120		− 84	+ 120	
	− T.	− 3,682	+ 3,660	0	− 3,667	0	− 3,667	− 24	+ 136	+ 112	− 160	+ 112			− 160
									Total excluding temperature			+ 264		+ 233	
											Total	+ 376	− 25	+ 353	− 132
Point 10½	D. L.	+ 18,462	− 388	0	+ 18,462	0	+ 18,462	+ 121	− 15	+ 106	+ 136	+ 106	+ 106	+ 136	+ 136
$\sin \phi=0.$	+ C. L. L.		+		+		+	+	+						
$\cos \phi=1.00.$	+ U. L. L.	+ 4,666	+ 2,322	0	+ 4,666	0	+ 4,666	+ 31	+ 91	+ 122	− 60	+ 122			− 60
Area=152.	− C. L. L.	+	−		+		+	+	−						
	− U. L. L.	+ 3,245	− 1,344	0	+ 3,245	0	+ 3,245	+ 21	− 52	− 31	+ 73		− 31	+ 73	
$\frac{h}{2I}=.03901.$	+ T.	+ 2,762	− 3,470	0	+ 2,762	0	+ 2,762	+ 18	− 135	− 117	+ 153		− 117	+ 153	
	− T.	− 3,682	+ 4,985	0	− 3,682	0	− 3,682	− 24	+ 194	+ 170	− 218	+ 170			− 218
									Total excluding temperature			+ 228		+ 269	
											Total	+ 398	− 42	+ 362	− 142

$N=H \cos \phi+V \sin \phi$ $\qquad f_c=\frac{N}{A} \pm M\frac{h}{2I}$

columns 77, 78, and 79. It should be remembered that the V_o given in the sum of column 70 is the dead-load vertical shear at point 0, and to get the vertical shear at any other point the loads between point 0 and the point under consideration must be subtracted from V_o.

We will consider next the live-load thrusts, moments, and shears. A uniform live load of 125 pounds per square foot has been assumed, and since we are considering a strip of the arch ring 1 foot wide, the live load at one point is 125*dx*, or 375 pounds. We wish to find the maximum stress at each of the points, 0, 3, 8, and 10½ and the stress is a function of the moment, thrust, and shear. It is not necessarily maximum at the same time that the moment is, but it is so nearly so that this is assumed to be true. Inspecting column 64 it is seen that the maximum positive moment at point 0 is produced by placing a live load at all of the points which give a positive moment, and no load should be placed at the points which give a negative moment. Thus the maximum positive live-load moment at point 0 is found by multiplying the sum of all of the positive quantities in column 64 by 375 pounds as is done in the table following.

The horizontal thrust which occurs at the same time as this maximum moment is found by adding the quantities in column 62 for the points which give a positive moment and multiplying the sum by 375. The same procedure is followed for the vertical shear, using the quantities in column 63. The maximum negative moment and the thrust and shear which occur at the same time are found by placing the loads at all of the points which give negative moments. The live-load moments, thrusts, and shears at the other points are found in the same way, but it should be remembered that the values of V_o in column 63 are the values of the reaction at the left support, and if there are any loads to the left of the point under consideration the shear is found by adding the proper quantities in the V_o-column and then subtracting 1.00 for each load to the left. Then the quantity thus obtained is multiplied by the live load per load point as shown in the following computations:

MOMENTS, THRUSTS, AND SHEARS DUE TO LIVE LOAD

Live load = 125 pounds per square foot
= 125×3 = 375 pounds per load point

Point 0:

+M = 375×34.822 =	+13,058
H = 375×14.890 =	5,584
V = 375× 2.759 =	1,035
−M = 375×30.129 =	−11,298
H = 375× 6.204 =	2,326
V = 375× 7.241 =	2,715

Point 3:

+M = 375× 6.143 =	+2,304
H = 375×10.828 =	4,060
V = 375×(4.555−2) =	958
−M = 375× 9.490 =	−3,559
H = 375×10.266 =	3,850
V = 375× 5.445 =	2,042

Point 8:

+M = 375× 7.929 =	+2,973
H = 375×10.547 =	3,955
V = 375×(8.423−7) =	534
−M = 375× 6.728 =	−2,523
H = 375×10.547 =	3,955
V = 375× 1.577 =	591

Point 10½:

$+M=375\times 6.192=$	$+2{,}322$
$H=375\times 12.442=$	$4{,}666$
$V=375\times(3-3)=$	0
$-M=375\times 3.585=$	$-1{,}344$
$H=375\times 8.652=$	$3{,}245$
$V=375\times(7-7)=$	0

The horizontal thrust due to a change in temperature is computed from the formula $H_t=\frac{20\ etE_c}{C}$. If E_c be taken as 2,000,000 pounds per square inch or 144 x 2,000,000 pounds per square foot and e be taken as 0.000006, the formula becomes $H_t=\frac{34560}{C}t$. H_t is constant for all points of the arch ring, and its value is calculated and recorded in column 77. M_t is computed from the formula $M_t=-H_t\left(y-\frac{\Sigma\, y\,\Delta}{\Sigma\Delta}\right)$. Values of $\left(y-\frac{\Sigma\, y\,\Delta}{\Sigma\Delta}\right)$ are found in column 23 except for point 0 and the value for this point can be quickly computed. The value of V_t is zero in every case.

$H\cos\phi$ and $V\sin\phi$ are computed and tabulated in columns 80 and 81, respectively. N in column 82 is computed by adding the corresponding values in columns 80 and 81. Column 83 is computed by dividing N in column 82 by A in column 75. $M\frac{h}{2I}$ in column 84 is computed by multiplying M in column 78 by $\frac{h}{2I}$. The stress in the concrete at the extrados $\left(\frac{N}{A}+M\frac{h}{2I}\right)$ is computed for column 85 by adding the corresponding values in columns 83 and 84. The stress in the concrete at the intrados is computed in column 86 by subtracting column 84 from column 83, observing the correct algebraic sign.

Columns 85 and 86 give the stresses due to various conditions of loading and temperature, but those conditions do not all obtain at one time. To find the maximum stress which may obtain, columns 87 to 90 are used, as indicated in the headings. For each case the maximum stress is caused by a combination of the dead load, one position of the live load, and one condition of the temperature. These stresses are tabulated in columns 87 to 90, inclusive, and the algebraic sum taken. The correct algebraic sign must be observed in all cases.

These maximum stresses have been computed upon the assumption that the arch ring is uniformly elastic and acts in accordance with Hook's law under all conditions—that is, we have assumed that the concrete will withstand the tensile stresses as well as the compressive. If the computed tensile stresses do not exceed the allowable tensile stress for concrete, the above assumption is practically true, but if we find that the tension in the concrete exceeds the allowable stress at any point the assumption is not true and we must assume that the concrete cracks at that point and all of the tension is taken by the steel reinforcement. The stresses in the steel and concrete at that point should then be computed by another method.

This may be done by dividing the maximum moment by the maximum thrust (columns 78 and 82) to find the eccentricity of the thrust and then computing the stresses by the theory of bending and direct stress in reinforced concrete for which formulas and diagrams may be found in any textbook on reinforced concrete.

Stresses in steel and concrete.—Columns 87 to 90, inclusive, give the stresses in the concrete computed upon the assumption that the concrete will withstand the stress to which it is subjected. It is found that the tension in the concrete at the abutment under certain conditions is 284 pounds per square inch. Under these circumstances the concrete will crack and all of the tension must be taken by the steel.

A Two-Ribbed Open-Spandrel Arch Designed by the Method Described Here but with Some Variation on Account of the Position of Load Points as Fixed by the Location of Columns

When the tensile strength of the concrete is exceeded, the stresses in the steel and concrete may be found from the diagram on page 405 of Hool and Johnson's "Concrete Engineers' Handbook." It will be noted that in this case there is reinforcement on the tension side only. The nomenclature used in the following computations, if not previously explained, is found on page 403 of Hool and Johnson.

At point 0:

	M	N
D. L.=	−16, 639	+23, 142
L. L.=	−11, 298	+3, 437
T.=	−21, 562	−3, 086
	−49, 499	+23, 493

$u=\frac{-49{,}499}{+23{,}493}=-2.11$ ft.

$e'=u+\left(\frac{h}{2}-d'\right)=2.11+0.98=3.09$ ft. $=37.1$ in.

$d=h-d'=2.30-0.17=2.13$ ft. $=25.6$ in.

$\frac{e'}{d}=\frac{37.1}{25.6}=1.45.$

Area of steel $=1.125$ sq. in. (see fig. 5):

$p=\frac{1.125}{144\times 2.13}=0.0037$ per cent steel.

$K=\frac{Ne'}{bd^2}=\frac{23{,}493\times 37.1}{12\times(25.6)^2}=111.$

From diagram (Hool and Johnson) $f_c=630$ and $f_s=14{,}000$.

Therefore the arch is satisfactory at this point.

At point 3 the worst conditions of loading and temperature give 206 pounds tension in the concrete, which will cause the concrete to crack.

At point 3:

	M	N
D. L.=	−1, 439	+20, 446
L. L.=	−3, 559	+4, 344
T.=	−8, 638	−3, 483
	−13, 636	+21, 308

$u=\frac{-13{,}636}{+21{,}308}=0.64$ ft.

$e'=0.64+0.48=1.12$ ft. $=13.4$ in.

$d=13.6$ in.

$\frac{e'}{d}=\frac{13.4}{13.6}=0.985.$

$p=\frac{0.5625}{144\times 1.13}=0.00346$ per cent steel.

$K=\frac{Ne'}{bd^2}=\frac{21{,}305\times 13.4}{12\times(13.6)^2}=128.$

From diagram, $f_c=575$ and $f_s=7{,}500$.

At point 8:

	M	N
D. L.=	+198	+18, 604
L. L.=	+2, 973	+3, 994
T.=	+3, 660	−3, 667
	+6, 831	+18, 931

$u=\frac{+6{,}831}{+18{,}931}=0.361$

$e'=0.361+0.34=0.701$ ft. $=8.4$ in.

$d=10.2$ in.

$\frac{e'}{d}=\frac{8.4}{10.2}=0.824$

$p=\frac{0.5625}{12\times 10.2}=0.0046.$

$K=\frac{Ne'}{bd^2}=\frac{18{,}931\times 8.4}{12\times(10.2)^2}=127.$

From diagram, $f_c=460$ and $f_s=2{,}600$.

At point 10½:

	M	N
D. L.=	−388	+18, 462
L. L.=	+2, 322	+4, 666
T.=	+4, 985	−3, 682
	+6, 919	+19, 446

$u=\frac{+6{,}919}{+19{,}446}=+0.356$ ft.

$e'=0.356+0.333=0.689$ ft. $=8.3$ in.

$d=10.0$ in.

$\frac{e'}{d}=\frac{8.3}{10}=0.83.$

$p=\frac{0.5625}{12\times 10}=0.0047$ per cent steel.

$K=\frac{Ne'}{bd^2}=\frac{19{,}446\times 8.3}{12\times(10)^2}=134.$

From diagram, $f_c=490$ and $f_s=2{,}900$.

Foundation pressure.—The maximum pressure on the foundation at the back of the abutment will be caused by the dead load, the live load which causes positive moment at the abutment, and a rise of temperature. The live load which causes a negative moment at the abutment may in some cases increase the foundation pressure, but usually it does not. It will increase the thrust, but on account of the moment being of opposite sign, it decreases the eccentricity which decreases the maximum pressure at the back of the abutment.

	M	H	V
D. L.=	−16, 639	+18, 462	14, 000
L. L.=	+13, 058	+5, 584	1, 035
T.=	+16, 174	+2, 762	0
	+12, 593	+26, 808	15, 035

$u=\frac{+12{,}593}{+26{,}808}=+0.47$ ft.

The thrust at the abutment (point 0) may be obtained by plotting H and V on the drawing and scaling the thrust, which is found to be 30,700 pounds and is applied 0.47 feet above point 0.

The weight of the abutment and the fill above the abutment is 18,650 pounds, and the center of gravity of these loads is 4.9 feet back of the face of the abutment. This weight is combined graphically with the thrust from the arch, and we obtain a total pressure of 43,000 pounds applied 1.21 feet from the back of the abutment. The vertical component of this pressure is 33,685 pounds and the horizontal component is 26,808 pounds.

The maximum vertical pressure on the foundation is therefore $\frac{33{,}685\times 2}{3\times 1.21}=18{,}560$ per square foot, or 9.3 tons per square foot. The horizontal pressure of 26,808 pounds must be taken by friction on the foundation and pressure against the vertical rock back of the abutment.

(Continued from page 71)

tration for the county was computed. This method results in the same total for the State and reflects differences in motor-vehicle registration in 1925 as well as differences in rates of population growth in the 88 counties. The expected county registration growth, 1925 to 1930, varies from 41.3 to 74.9 per cent, the latter rate resulting from a rapid rate of population increase.

Since traffic increases at the same rate as motor-vehicle registration, the expected traffic at each survey station was determined by applying the county rate of registration increase to each traffic station in the county. The resulting forecast of traffic at each station is shown in the appendix of the full report.

Industrial and suburban development, as well as changes affecting the present highway system as to location of routes, routing of traffic, and detours and conditions of improvement influence traffic on short sections of highway, and it is not expected that these estimates will in all cases reflect exactly the actual traffic in 1930, but it is believed that they will reflect with reasonable accuracy highway traffic on the State highway system as a whole.

In certain areas, particularly areas of sparse population adjacent to centers of population, a very important part of the traffic originates in the centers of population. In such cases a traffic forecast based on the population and motor-vehicle registration in the sparesely populated area will not reflect the influence of traffic originating outside the county boundaries.

To allow for such variations, and also because population estimates based on arithmetical progressions[3] are less accurate when applied to smaller areas and when forecasted over a greater number of years, the traffic forecast for 1935 was computed on the basis of the State increase in registration between 1930 and 1935 rather than for increases in each county. The 1935 forecast therefore represents an increase of 28 per cent over the forecast in 1930 for all sections of the State. Because of the longer period of time and the greater probability of changes in the rate of population growth in motor-vehicle use and in changes in the State highway system, the traffic forecast for 1935 is expected to be less accurate than the forecast for 1930. It has therefore been applied to highway sections rather than to individual points and is expected to reflect traffic conditions in 1935 within the limits ot accuracy required in the establishment of a sound plan of highway improvement.

[3] The method used by the Bureau of the Census.

ROAD PUBLICATIONS OF BUREAU OF PUBLIC ROADS

Applicants are urgently requested to ask only for those publications in which they are particularly interested. The Department can not undertake to supply complete sets nor to send free more than one copy of any pyblication to any one person. The editions of some of the publications are necessarily limited, and when the Department's free supply is exhausted and no funds are available for procuring additional copies, applicants are referred to the Superintendent of Documents, Government Printing Office, this city, who has them for sale at a nominal price, under the law of January 12, 1895. Those publications in this list, the Department supply of which is exhausted, can only be secured by purchase from the Superintendent of Documents, who is not authorized to furnish publications free.

ANNUAL REPORTS

Report of the Chief of the Bureau of Public Roads, 1924.
Report of the Chief of the Bureau of Public Roads, 1925.

DEPARTMENT BULLETINS

No. 105D. Progress Report of Experiments in Dust Prevention and Road Preservation, 1913.
*136D. Highway Bonds. 20c.
220D. Road Models.
257D. Progress Report of Experiments in Dust Prevention and Road Preservation, 1914.
*314D. Methods for the Examination of Bituminous Road Materials. 10c.
*347D. Methods for the Determination of the Physical Properties of Road-Building Rock. 10c.
*370D. The Results of Physical Tests of Road-Building Rock. 15c.
386D. Public Road Mileage and Revenues in the Middle Atlantic States, 1914.
387D. Public Road Mileage and Revenues in the Southern States, 1914.
388D. Public Road Mileage and Revenues in the New England States, 1914.
390D. Public Road Mileage and Revenues in the United States, 1914. A Summary.
407D. Progress Reports of Experiments in Dust Prevention and Road Preservation, 1915.
*463D. Earth, Sand-Clay, and Gravel Roads. 15c.
*532D. The Expansion and Contraction of Concrete and Concrete Roads, 10c.
*537D. The Results of Physical Tests of Road-Building Rock in 1916, Including all Compression Tests, 5c.
*583D. Reports on Experimental Convict Road Camp, Fulton County, Ga. 25c.
*660D. Highway Cost Keeping. 10c.
*670D. The Results of Physical Tests of Road-Building Rock in 1916 and 1917. 5c.
*691D. Typical Specifications for Bituminous Road Materials. 10c.
*724D. Drainage Methods and Foundations for County Roads. 20c.
*1077D. Portland Cement Concrete Roads. 15c.
*1132D. The Results of Physical Tests of Road-Building Rock from 1916 to 1921, Inclusive. 10c.
*1216D. Tentative Standard Methods of Sampling and Testing Highway Materials, Adopted by the American Association of State Highway Officials and Approved by the Secretary of Agriculture for Use in Connection with Federal aid Road Construction. 15c.

DEPARTMENT BULLETINS—Continued

No. 1259D. Standard Specifications for Steel Highway Bridges, adopted by the American Association of State Highway Officials and approved by the Secretary of Agriculture for use in connection with Federal-aid road work.
1279D. Rural Highway Mileage, Income, and Expenditures, 1921 and 1922.

DEPARTMENT CIRCULARS

No. 94C. TNT as a Blasting Explosive.
331C. Standard Specifications for Corrugated Metal Pipe Culverts.

MISCELLANEOUS CIRCULARS

No. 60M. Federal Legislation Providing for Federal Aid in Highway Construction.
62M. Standards Governing Plans, Specifications, Contract Forms, and Estimates for Federal Aid Highway Projects.

FARMERS' BULLETINS

No. *338F. Macadam Roads. 5c.
*505F. Benefits of Improved Roads. 5c.

SEPARATE REPRINTS FROM THE YEARBOOK

No *739Y. Federal Aid to Highways, 1917. 5c,
*849Y. Roads. 5c.
914Y. Highways and Highway Transportation.

REPRINTS FROM THE JOURNAL OF AGRICULTURAL RESEARCH

Vol. 5, No. 17, D- 2. Effect of Controllable Variables upon the Penetration Test for Asphalts and Asphalt Cements.
Vol. 5, No. 19, D- 3. Relation Between Properties of Hardness and Toughness of Road-Building Rock.
Vol. 5, No. 24, D- 6. A New Penetration Needle for Use in Testing Bituminous Materials.
Vol. 6, No. 6, D- 8. Tests of Three Large-Sized Reinforced-Concrete Slabs Under Concentrated Loading.
Vol. 10, No. 5, D-12. Influence of Grading on the Value of Fine Aggregate Used in Portland Cement Concrete Road Construction.
Vol. 11, No. 10, D-15. Tests of a Large-Sized Reinforced-Concrete Slab Subjected to Eccentric Concentrated Loads.

* Department supply exhausted.

UNITED STATES DEPARTMENT OF AGRICULTURE

BUREAU OF PUBLIC ROADS

STATUS OF FEDERAL AID HIGHWAY CONSTRUCTION

AS OF

MAY 31, 1927

STATES	FISCAL YEARS 1917-1926			FISCAL YEAR 1927									BALANCE OF FEDERAL AID FUND AVAILABLE FOR NEW PROJECTS	STATES
	PROJECTS COMPLETED PRIOR TO JULY 1, 1926			PROJECTS COMPLETED SINCE JUNE 30, 1926			*PROJECTS UNDER CONSTRUCTION			PROJECTS APPROVED FOR CONSTRUCTION				
	TOTAL COST	FEDERAL AID	MILES	TOTAL COST	FEDERAL AID	MILES	ESTIMATED COST	FEDERAL AID ALLOTTED	MILES	ESTIMATED COST	FEDERAL AID ALLOTTED	MILES		
Alabama	$ 18,226,411.34	$ 8,725,985.09	1298.3	$ 1,834,960.34	$ 889,114.85	101.9	$ 6,566,145.16	$ 3,127,211.59	381.3	$ 273,351.16	$ 136,675.57	14.3	$ 3,017,687.90	Alabama
Arizona	10,949,878.25	5,863,772.35	729.8	766,036.23	525,632.71	48.9	1,135,441.23	768,754.15	72.7	735,909.00	608,802.02	31.2	2,907,281.77	Arizona
Arkansas	18,364,544.50	7,666,698.35	1323.0	3,972,470.13	1,858,494.40	237.6	2,854,174.10	1,386,642.91	211.5	925,378.85	262,746.03	18.0	1,709,118.31	Arkansas
California	27,142,596.90	13,003,592.30	1058.0	7,797,030.13	3,861,460.35	243.2	7,442,439.26	3,457,220.34	154.0	192,532.27	99,161.19	6.6	4,134,817.82	California
Colorado	13,905,904.64	7,127,288.18	745.0	1,533,319.75	774,541.81	81.1	6,426,488.21	3,117,393.14	278.9	35,740.10	20,057.34	2.9	2,663,051.53	Colorado
Connecticut	5,414,567.19	2,100,585.80	117.1	982,825.10	343,414.74	20.1	5,313,766.77	1,477,884.88	64.0	1,246,486.94	290,040.13	13.0	594,440.45	Connecticut
Delaware	4,918,052.29	1,781,665.60	124.3	1,318,974.26	563,906.92	35.2	525,119.13	249,574.90	24.5	271,652.15	63,316.77	8.4	181,219.01	Delaware
Florida	3,832,680.26	1,824,362.32	132.9	3,644,176.05	1,803,550.28	112.2	8,615,396.14	3,777,276.07	186.3	1,190,401.52	398,091.78	24.2	1,181,124.55	Florida
Georgia	24,791,206.97	11,664,237.86	1794.0	5,915,181.39	2,869,016.13	338.3	9,361,968.86	4,587,746.91	375.5				1,290,161.10	Georgia
Idaho	11,061,193.14	5,882,112.70	724.7	2,164,317.31	1,193,414.46	110.8	2,124,109.00	1,333,304.65	164.1	665,217.07	400,661.56	42.1	685,326.62	Idaho
Illinois	44,116,611.86	20,619,995.74	1377.7	4,422,370.30	2,161,520.86	153.1	9,190,129.95	4,396,670.88	330.9	5,440,226.91	2,696,826.04	184.3	3,111,613.48	Illinois
Indiana	16,949,425.87	8,172,125.19	534.3	5,659,150.59	2,694,046.53	177.7	16,131,498.35	7,543,094.26	468.0	2,027,795.86	1,007,570.88	73.2	714,290.14	Indiana
Iowa	29,062,375.40	11,926,302.10	2114.8	5,015,643.50	2,356,134.60	353.4	12,925,811.69	5,849,382.90	538.8	3,170,180.51	1,270,144.71	86.1	128,597.69	Iowa
Kansas	32,826,601.64	12,590,489.25	1160.6	4,182,914.62	1,933,261.65	302.8	12,979,798.12	5,141,155.09	648.9	2,609,194.84	1,016,180.59	194.1	851,856.42	Kansas
Kentucky	20,737,706.10	8,492,082.25	758.3	2,245,656.73	915,591.55	100.1	9,526,516.34	4,501,792.53	427.3	796,169.20	387,426.88	47.3	333,862.79	Kentucky
Louisiana	13,830,592.68	6,144,739.99	1055.9	2,046,959.52	949,152.22	122.8	4,092,337.34	1,922,166.10	137.9	26,200.66	13,100.33	3.0	1,256,557.36	Louisiana
Maine	8,747,552.76	4,192,507.39	303.6	1,817,247.30	665,945.28	54.0	2,231,038.93	821,344.43	63.4	744,122.14	215,406.00	17.0	1,250,418.90	Maine
Maryland	10,924,943.10	5,112,991.22	423.3	865,260.83	411,947.05	54.5	814,475.47	373,185.64	33.9	1,238,679.25	589,300.00	59.0	72,752.09	Maryland
Massachusetts	18,353,757.71	6,657,260.62	374.5	2,316,488.31	768,667.53	35.9	4,881,500.94	1,287,714.57	77.8	774,042.27	190,341.12	10.9	2,293,842.16	Massachusetts
Michigan	25,997,240.78	11,827,052.30	963.0	5,443,350.45	2,311,166.69	121.0	11,547,646.33	5,239,078.35	347.4	1,613,486.00	755,705.00	40.6	2,424,053.66	Michigan
Minnesota	37,170,985.95	15,586,116.56	3181.9	7,928,662.52	3,460,029.11	461.6	6,391,949.14	1,889,998.90	340.9	1,204,533.98	225,900.00	48.5	550,476.43	Minnesota
Mississippi	15,146,088.52	7,414,534.10	1129.0	2,522,210.42	1,258,294.62	165.2	6,830,314.93	3,319,747.70	352.9	1,088,722.76	530,445.75	72.0	912,874.83	Mississippi
Missouri	28,989,166.92	13,736,014.85	1543.2	12,095,884.16	5,322,158.35	359.8	10,295,928.65	4,204,049.48	302.5	1,286,312.79	599,394.16	52.5	1,329,994.16	Missouri
Montana	11,400,983.81	6,333,465.89	1054.9	1,454,011.91	953,822.80	96.7	1,615,288.16	1,370,643.96	126.1	2,618,436.39	1,507,305.62	263.1	4,811,145.73	Montana
Nebraska	11,533,401.62	5,474,202.52	1768.3	4,356,776.62	2,142,485.17	440.9	11,489,881.82	5,655,665.07	1201.3	2,979,536.94	1,485,232.10	231.1	1,462,788.14	Nebraska
Nevada	7,558,195.51	5,130,934.59	538.8	2,776,355.76	2,382,094.86	301.0	1,612,989.97	1,395,831.20	196.5				834,864.35	Nevada
New Hampshire	4,992,558.60	2,377,450.07	237.6	876,339.16	401,477.98	27.2	753,035.97	351,317.49	22.8	335,675.36	147,698.10	8.9	257,173.36	New Hampshire
New Jersey	16,346,301.01	5,098,342.21	290.3	5,881,939.07	2,397,022.27	26.0	5,792,261.31	1,273,543.56	82.8	1,699,048.74	433,065.00	28.9	200,057.96	New Jersey
New Mexico	12,404,337.77	7,339,657.38	1427.0	912,428.18	587,767.43	78.2	3,354,942.73	2,596,588.47	233.6				1,635,135.72	New Mexico
New York	43,224,279.79	17,911,957.19	1197.0	10,758,701.76	3,701,463.46	236.9	34,533,153.00	8,800,778.95	549.5	9,792,833.00	2,227,927.50	130.0	5,038,284.90	New York
North Carolina	27,009,419.47	11,177,337.94	1257.9	8,286,429.74	3,341,565.22	202.2	2,472,083.16	1,191,705.92	78.0	1,006,095.75	432,189.75	26.5	1,287,763.17	North Carolina
North Dakota	12,313,311.40	6,031,859.78	2193.1	3,559,649.54	1,710,135.10	522.4	4,908,888.68	2,687,652.06	704.3	1,871,839.41	930,501.95	271.4	583,461.11	North Dakota
Ohio	47,699,532.90	17,371,787.03	1364.1	4,921,858.59	1,959,589.73	150.8	11,343,262.56	4,389,902.25	329.8	714,896.16	244,093.22	13.0	4,528,632.77	Ohio
Oklahoma	28,247,950.33	13,159,999.15	1178.9	2,134,006.75	957,590.06	89.2	3,348,529.42	1,433,037.86	196.3	1,713,513.16	766,641.99	126.3	1,494,408.94	Oklahoma
Oregon	17,027,878.42	8,593,214.79	939.2	2,288,382.35	1,314,004.60	105.6	2,357,170.76	1,239,049.68	57.6	360,519.22	196,950.85	25.0	718,329.08	Oregon
Pennsylvania	61,366,150.80	21,560,732.04	1188.8	14,599,594.29	4,240,910.49	311.3	16,613,655.87	5,044,782.32	333.7	2,808,736.18	927,083.64	57.3	2,901,007.51	Pennsylvania
Rhode Island	3,988,616.09	1,558,829.06	86.7	1,244,797.29	439,650.00	29.3	778,986.12	205,665.00	13.7	957,110.35	237,810.00	15.8	591,239.94	Rhode Island
South Carolina	15,020,639.90	6,766,322.93	1481.9	1,981,400.03	760,665.87	86.5	6,168,533.01	2,607,525.08	209.3	512,631.79	231,100.29	29.4	490,897.83	South Carolina
South Dakota	17,468,373.19	8,603,826.97	2181.2	1,793,690.05	903,698.57	321.7	3,312,578.00	1,753,802.29	574.7	649,120.64	357,016.32	128.5	768,509.85	South Dakota
Tennessee	21,624,631.57	10,276,584.02	780.0	2,562,769.67	1,229,742.90	78.5	8,428,199.54	3,503,402.56	235.9				1,885,627.52	Tennessee
Texas	69,183,673.48	27,440,254.72	4920.2	8,808,957.46	4,036,348.78	537.3	15,263,952.98	6,773,705.87	534.8	1,277,063.78	608,645.18	53.4	6,244,748.45	Texas
Utah	8,253,178.03	5,098,440.68	546.4	901,199.30	668,639.27	82.5	2,198,402.66	1,672,855.84	161.9	366,014.46	263,912.69	30.1	961,836.52	Utah
Vermont	4,242,042.64	2,017,699.51	134.5	795,075.59	331,156.50	18.2	2,136,881.86	819,113.52	41.7	873,280.34	329,018.91	21.5	137,143.56	Vermont
Virginia	21,990,249.44	10,385,728.11	1005.5	4,720,759.73	2,084,907.11	157.6	4,608,575.46	1,897,303.04	105.3	1,071,311.26	393,530.29	21.9	182,759.45	Virginia
Washington	17,078,511.63	7,782,909.46	668.6	1,105,994.34	463,642.49	42.5	3,690,460.66	1,750,600.00	68.2	237,489.59	65,000.00	10.2	1,215,156.05	Washington
West Virginia	9,473,716.44	4,141,062.65	392.9	951,130.88	432,685.36	26.5	7,132,862.00	2,934,191.85	238.2	109,976.65	54,988.32	5.7	583,218.82	West Virginia
Wisconsin	24,856,508.19	10,392,705.73	1592.1	2,824,555.10	1,352,063.54	130.4	8,043,233.89	3,786,749.30	328.1	2,138,514.26	949,398.50	56.8	2,828,352.93	Wisconsin
Wyoming	10,928,302.56	6,040,887.05	1133.5	1,722,409.59	1,098,380.00	182.3	1,870,476.16	1,222,454.22	122.1	65,821.31	42,256.00	16.7	1,096,665.73	Wyoming
Hawaii				343,664.15	97,440.00	6.5	1,786,323.89	562,362.64	29.7				805,975.36	Hawaii
TOTALS	966,692,834.36	426,178,703.58	52526.6	179,053,966.84	79,879,412.15	8079.4	323,818,603.88	136,692,620.28	12759.3	61,715,800.97	24,608,660.07	2620.6	77,140,603.92	TOTALS

* Includes projects reported completed (final vouchers not yet paid) totaling: Estimated cost $ 70,576,344.58 Federal aid $ 30,543,223.16 Miles 2758.8