Transportation Consortium of South-Central States

Tran-SET

Solving Emerging Transportation Resiliency, Sustainability, and Economic Challenges through the Use of Innovative Materials and Construction Methods: From Research to Implementation

Selecting the Most Feasible Construction Phasing Plans for Urban Highway Rehabilitation Projects

Project No. [19ITSLSU07]

Lead University: [Texas A&M University]

Collaborative Universities: [Louisiana State University]

Final Report

August 2020

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16. Abstract

Despite the abundance of research that has aimed to understand the effects of highway work zones, very little definitive information is available concerning the determination of work zone length (WZL). Quantitative studies that holistically model WZL are very rare. To fill this gap, this study identifies critical factors affecting WZL and develops decision support models that determine the optimal WZL in a balanced tradeoff between motorists' inconvenience due to traffic disruption and their opportunity cost. A high-confidence dataset was created by conducting a series of scheduling and traffic simulations and analyses. The results revealed that traffic loading and work zone duration are critical factors, with traffic loading at approximately 41,000 vehicles-per-day being an important benchmarking point. Based on these findings, a decision support model was developed to determine the most feasible WZL. As the first of its kind, this study will help state transportation agencies devise sounder construction phasing plans by providing a point of reference when establishing WZL in a viable way to minimize traffic disruption during construction.

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SI* (MODERN METRIC) CONVERSION FACTORS

APPROXIMATE CONVERSIONS TO SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
		LENGTH		
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
		AREA		
in^2	square inches	645.2	square millimeters	mm^2
ft^2	square feet	0.093	square meters	m^2
yd^2	square yard	0.836	square meters	m^2
ac	acres	0.405	hectares	ha
mi^2	square miles	2.59	square kilometers	km^2
		VOLUME		
fl oz	fluid ounces	29.57	milliliters	mL
gal	gallons	3.785	liters	L
ft^3	cubic feet	0.028	cubic meters	m^3
yd^3	cubic yards	0.765	cubic meters	m^3
		NOTE: volumes greater than 1000 L shall be shown in m^3		
		MASS		
oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")
		TEMPERATURE (exact degrees)		
^oF	Fahrenheit	5 (F-32)/9 or (F-32)/1.8	Celsius	^oC
		ILLUMINATION		
fc	foot-candles	10.76	lux	lx
fl	foot-Lamberts	3.426	candela/m^2	cd/m^2
		FORCE and PRESSURE or STRESS		
lbf	poundforce	4.45	newtons	N
lbf/in^2	poundforce per square inch	6.89	kilopascals	kPa

APPROXIMATE CONVERSIONS FROM SI UNITS

Symbol	When You Know	Multiply By	To Find	Symbol
		LENGTH		
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
		AREA		
mm^2	square millimeters	0.0016	square inches	in^2
m^2	square meters	10.764	square feet	ft^2
m^2	square meters	1.195	square yards	yd^2
ha	hectares	2.47	acres	ac
km^2	square kilometers	0.386	square miles	mi^2
		VOLUME		
mL	milliliters	0.034	fluid ounces	fl oz
L	liters	0.264	gallons	gal
m^3	cubic meters	35.314	cubic feet	ft^3
m^3	cubic meters	1.307	cubic yards	yd^3
		MASS		
g	grams	0.035	ounces	oz
kg	kilograms	2.202	pounds	lb
Mg (or "t")	megagrams (or "metric ton")	1.103	short tons (2000 lb)	T
		TEMPERATURE (exact degrees)		
^oC	Celsius	1.8C+32	Fahrenheit	^oF
		ILLUMINATION		
lx	lux	0.0929	foot-candles	fc
cd/m^2	candela/m^2	0.2919	foot-Lamberts	fl
		FORCE and PRESSURE or STRESS		
N	newtons	0.225	poundforce	lbf
kPa	kilopascals	0.145	poundforce per square inch	lbf/in^2

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ACRONYMS, ABBREVIATIONS, AND SYMBOLS

AADT	Annual Average Daily Traffic
CA	Cellular Automata
CA4PRS	Constructability Analysis for Pavement Rehabilitation Strategies
CORSIM	CORridor SIMulation
CWZ	Construction Work Zone
DRUC	Daily Road User Cost
DTD	Daily Traffic Delay
EAADT	Equivalent Annual Average Daily Traffic
FHWA	Federal Highway Administration
FCD	Floating Car Data
GRG	Generalized Reduced Gradient
GA	Genetic Algorithm
PCE	Passenger Car Equivalent
PVPH	Per Vehicle Per Hour
3R	Resurfacing, Restoration, Rehabilitation
RUC	Road User Cost
STA	State Transportation Agency
SGD	Stochastic Gradient Descent
VOT	Value of Time
VPD	Vehicles Per Day
VPHPL	Vehicle Per Hour Per Lane
WZL	Work Zone Length

EXECUTIVE SUMMARY

Maintenance and rehabilitation work on the existing highway system are becoming increasingly critical to maintain the long-term mobility and safety performance of the aging highway infrastructure. However, the closed construction work zones inevitably cause significant adversarial mobility and safety impact to the traveling public. A sound construction phasing plan that strikes a balance between traffic impact and constructability is crucial to determine the most feasible work zone length (WZL) in order to minimize traffic inconvenience to motorists while ensuring on-time completion of the project.

Despite the wealth of research that has aimed to understand the effects of highway work zones, very little definitive information is available concerning the determination of WZL. Quantitative studies that holistically model WZL are very rare. To fill this gap, this study identifies critical factors affecting WZL and develops decision support models that determine the optimal WZL in a balanced tradeoff between motorists' inconvenience due to traffic disruption and their opportunity cost. A high-confidence dataset was created by conducting a series of scheduling and traffic simulations and analyses. The results revealed that traffic loading and work zone duration are critical factors, with traffic loading at approximately 41,000 vehicles-per-day being an important benchmarking point. Based on these findings, a decision support model was developed to determine the most feasible WZL. As the first of its kind, this study will help state transportation agencies devise sounder construction phasing plans by providing a point of reference when establishing WZL in a viable way to minimize traffic disruption during construction.

1. INTRODUCTION

The aging highway infrastructure directly affects the health of the nation's economy unfavorably in many ways, due to the decreased mobility and safety performance provided to the traveling public. Thus, maintenance and rehabilitation work on the existing highway system are becoming increasingly critical to improve the long-term mobility and safety performance. When a certain section of highway is under construction, it is called a construction work zone (CWZ). During construction, traffic and construction work exist in close proximity to one another *(1, 2).* A CWZ typically includes closed or narrowed lanes, and highway traffic is influenced by traffic on nearby roadways, as well as earlier flow through the same location. Due to sudden speed drops and mandatory lane changes caused by CWZs, delays and rerouting account for the average driver wasting 67 hours on the road and 32 gallons of fuel annually, and 97,000 crashes each year. This implies that CWZs create both spatial and temporal restrictions on highways with reduced capacity and have an adverse traffic impact on motorists *(3)*. An earlier study showed that around 20% of highways in the national highway system have scheduled construction work during the peak construction season, and approximately 24% of non-recurring delays on highways are caused by CWZs *(4)*.

To minimize the impact that a CWZ has on the traveling public, it is very common to use a construction phasing plan that defines the length of the CWZ closure and describes how the entire project should be split. In practice, such construction phasing plans are typically developed by estimating and comparing road user cost (RUC) for project alternatives under consideration. However, when a phasing plan is poorly conceived, it can cause significant traffic inconvenience, resulting in excessive RUC for the traveling public *(2, 5)*. State transportation agencies (STAs) commonly implement mobility assessments of rehabilitation project's phasing plans based on daily commuters and business interests. These assessments are critical, but they are also difficult to perform and expensive to conduct, with the primary problem being a lack of effective analytical tools and methods. There is an urgent need to develop a means of correcting and improving the results of CWZ phasing plan modeling to minimize the mobility impact of CWZs to the traveling public.

2. OBJECTIVES

To meet the pressing issue of a lack of practical methodology to improve CWZ phasing plan, this project aims to develop a practical and easy-to-use toolkit to assist STAs improve construction phasing plans of CWZs. The specific objectives of this study were twofold:

a) identify critical factors affecting decisions related to lane closure length in rural highway rehabilitation projects and
b) develop a novel decision support model for determining the optimal work zone length (WZL), accounting for all critical factors.

This research will greatly benefit STAs and the traveling public by significantly improving mobility around the CWZs and positively affecting regional development.

3. LITERATURE REVIEW

Research studies have shown that the level of traffic inconvenience during construction is closely tied to the length of the CWZ, duration of lane closure, and level of traffic loading. There exists a tradeoff between motorists' inconvenience and project constructability: as the WZL increases, so does the public's inconvenience, since the approaching traffic flow needs pass through the CWZ with reduce the speed *(6)*; but setting the WZL too short will increase the closures needed to complete the project, reducing the efficiency of the construction activities due to the fact that there are more repeated work zone setup steps *(7)*. Therefore, it is imperative to develop construction phasing plans that strike a balance and viable tradeoff between reducing disruption to the traveling public and minimizing construction time.

Studies on how best to determine CWZ length are rare, and those that do exist lack definitive practicality that accounts for critically influential factors *(8, 9)*. Specifically, the core problem is a lack of a standardized methodology and set of analytical tools for proactively estimating WZL that consider potentially critical latent variables, such as traffic state, schedule, capacity, and user cost. There is a lack of practical methods and analytical models that proactively assess the trade-off between traffic impact and WZL with an up-to-date quantitative model. As a result, no comprehensive practical methods or tools for phasing plan are currently available that mirror the unique dynamics of changes resulting from highway rehabilitation work.

Haseman et al. *(10)* collected 1.4 million travel time records over 12-week period to evaluate quantifiable travel mobility metrics for a rural highway work zone. This study conveyed a conclusion that travel time could be significantly delayed by construction activities as well as safety incidents in and between the construction work zones. This study concluded that both traffic delay and accident costs would have an adversarial effect on the total project cost, which need to be considered in the project scoping phase through a robust transportation management planning process *(6)*.

McCoy et al. *(11)* developed a method for optimizing WZL by minimizing RUC and traffic control cost for rural four-lane highways. That study provided researchers with a framework for optimizing WZL by minimizing the total cost, including costs related to user delay, vehicle operation, construction operation, and accidents. However, the study was conducted in the 1970s. Many variables (such as unit cost factors) change over time. The method is now outdated and likely not applicable in today's environment. Based on McCoy's method, Martinelli and Xu *(12)* updated the variables and added a vehicle queue delay cost that took traffic control and operating costs into consideration, increasing the accuracy of this method under various conditions. However, these studies were based solely on cost factors, and other critical factors like construction duration were absent.

Chien and Schonfeld *(6)* developed an optimization method by formulating a total cost function and leveraging it to optimize WZL. The function they proposed was minimized by using a classical optimization approach (i.e., setting derivatives to equal zero and then solving) involving components that significantly influence WZL, including but not limited to work zone speed, approaching traffic flow, work zone setup cost, maximum discharge rate, total agency cost, and construction duration. The proposed modeling method demonstrated how WZL could be optimized based on significant factors. Their model was more comprehensive and complete than those proposed in earlier studies. However, the authors assumed that if the work zone capacity was more than the annual average daily traffic (AADT) volume, no queue would form. Since traffic flow varies by day, this assumption does not hold, at least for part of the day *(2)*.

Chien and Tang *(13)* developed a genetic algorithm model to optimize work zone length and staging plan. This study attempted to incorporate the total project cost concept that includes maintenance cost, road user cost, accident cost, and vehicle operating cost with a set of project peculiarities. The total cost model developed in this study is a nonlinear, discontinuous function that models an interdependent relationship between key parameters such as work zone length, number of work zones, and number of detoured vehicles. It appears that this model is applicable to estimate the total project cost within a limited set of input data.

Watts et al. *(14)* developed a method for estimating RUC derived from vehicle volume and lane closure length in rural freeway work zones. Based on that research, Choi *(15)* presented case studies reporting RUC calculations for projects with lane closures in construction work zones. That research specified several factors contributing to RUC, including AADT, work zone speed, and WZL. AADT was highlighted in that research because the results indicated that RUC values were consistently high in areas of high traffic volume (such as those with high AADT).

Zhao et al. *(16)* developed a comprehensive model to optimize CWZ construction schedule with x. The FCD in this study were mainly collected by mobile devices (e.g., smartphone) on board of traveling vehicles by INRIX via VPP Suite, which became more accurate and abundant in recent years. Their model considered the effects of various factors, such as traffic assignment and safety cost. However, their proposed model requires a large amount of input information that might not always be available for a project, reducing the practicality of its implementation.

In recent years, there has been a trend of researchers using simulation models to study how the CWZ could affect the traffic *(8, 9, 17)*. Marzouk and Fouad *(9)* developed a simulation model that tries to capture both resurfacing construction activities and traffic flows. This study identified 6 critical factors, including traffic volume, WZL, average production rate, and average headway. However, the optimization method in this study was implemented with a Genetic Algorithm (GA), which became less popular recently due to its demanding computational process.

Microscopic simulation tools can represent nearly actual traffic flows (e.g., CORSIM), but these tools are often time-consuming, costly, and sometimes computationally expensive. To combat this drawback, Cellular Automata (CA) models based on Nagel and Schreckenberg *(18)* were used by researchers for the higher computational efficiency while still being able to reproduce realistic traffic flows. For example, Meng and Weng *(17)* developed a CA model with improved computational efficiency with driving behavior rules calibrated with field data. The results of their model were statistically validated with state-of-the-art simulation software to prove its accuracy. Though, it is worth mentioning that the scope of this study focused on the efficiency and validity of the simulation method itself rather than optimizing the WZL or phasing plan.

Fei et al. *(8)* proposed a meticulous two-lane CA model that considered differences in driving behavior and vehicle acceleration rates. This study found that the flow rate of the CWZ traffic no longer increases after the speed limit reaches 40 km/h (25 mph) when the traffic volume is relatively low, as the simulated flow rate for 40 km/h, 50 km/h (31 mph), and 60 km/h (37 mph) were identical. Even though this study had several key findings of the merging behavior of CWZ traffic, it did not focus on the optimization of the phasing plan.

Construction Analysis For Pavement Rehabilitation Strategies (CA4PRS), a construction management software has been applied to analyze cost and benefit for different work zone construction alternatives *(19)*. This tool takes constructability, road user cost, resource constraints, and lead-lag relation into consideration, which allows researchers to create traffic simulation for different work zone alternatives. However, optimizing construction scheduling plan with CA4PRS requires numerous trials, while the delay and RUC calculation must rely on external traffic analysis tools, such traffic simulation model or capacity analysis model. Table 1 summarizes the existing body of knowledge pertaining to work zone length estimation methods. As Table 1 shows, specific to the determination of optimal work zone length, very few studies explored the ways to determine WZL as a function of critical influential factors. Those that do exist lack definitive information regarding the trade-off between traffic impact and WZL with an up-to-date quantitative model.

Table 1. Summary of the Literature Review

Author/Date	Method	Topic/Focus	Findings
Nagel & Schreckenberg (1992)	• Boolean Simulation Model • Quantitative Comparison with Realistic Traffic	• Traffic Flow	• Discrete modelling is more computational efficient and can capture driver's behavior.
Martinelli & Xu (1996)	• Statistical Analysis	• Traffic Delay • Optimal WZL	• Highway capacity & LOS analyses to estimate traffic delay • Traffic control-RUC tradeoff model
Chien & Schonfeld (2001)	• Cost Objective Model • Traffic Simulation • Statistical Analysis	• Optimal WZL	• Shorten the WZL or increase CWZ speed can reduce RUC
Jiang & Adeli (2003)	• Boltzmann-simulated Annealing Neural Network • Statistical Analysis	• Traffic Delay • Optimal WZL • Traffic Flow	• WZL and ADT can be used to predict traffic delay and RUC
Haseman et al. (2010)	• Bluetooth Probe Data from Field Collection • Empirical Analysis	• CWZ Traffic Delay	• Prove that WZL affects RUC with field data • Prove that CWZ can cause road capacity drop with field data
Meng & Weng (2010)	• Cellular Automata (CA) Model • Case Study • Statistical Comparison	• Work Zone Configuration	• CA model can estimate traffic delay with high efficiency and accuracy
Watts et al. (2012)	• Microsimulation model • Case Study • Statistical Comparison	• Traffic Flow • Optimal WZL • Road User Cost	• WZL and RUC have a positive linear relationship
Chien & Tang (2014)	• Objective Total Cost Function • Genetic Algorithm Optimization • Statistical Analysis	• Total Cost • Work Zone Schedule	• A practical and cost-effective CWZ schedule to minimize RUC

Marzouk & Fouad (2014)	• Traffic Simulation • Genetic Algorithm Optimization • Numerical Example Demonstration	• Optimal WZL	• An efficient framework to estimate project duration and total RUC • Single objective optimization algorithm to minimize RUC
Fei et al. (2016)	• Meticulous Two-Lane CA Model Traffic Simulation Empirical Analysis	• Traffic Optimization	• 40 km/h is an optimal speed limit for CWZ
Zhao et al. (2019)	• Traffic Simulation Case Study Statistical Comparison	• Optimal WZL • Optimal CWZ Characteristics	• A practical method for CWZ schedule that minimize RUC
Choi J (2020)	• Case Study Statistical Comparison	• Road User Cost • Lane Closure	• AADT is a determinant for RUC

4. METHODOLOGY

This study blends existing traffic simulation techniques with a stochastic analysis to model WZL as a function of RUC and traffic load, simultaneously capturing the mobility impact, production rate, and project schedule. The objectives of the study were achieved by enacting a four-stage methodology that articulated a new data creation technique and modeling framework, where WZL was assessed by implementing a balanced tradeoff between travelers' inconvenience measured by RUC and constructability evaluated by project duration. These steps were as follows (see Figure 1):

1. **Data creation**. A rich set of 285 traffic and 84 schedule datapoints was created by a series of macroscopic traffic simulations established using the Constructability Analysis for Pavement Rehabilitation Strategies (CA4PRS) software.
2. **Critical factor identification.** Critical factors affecting WZL determination were investigated by calculating descriptive statistics and conducting factor and schedule-traffic interdependency analyses.
3. **Preliminary modeling.** Two preliminary predictive models assessing the effects of traffic delay and project schedules were developed to quantify the impact of WZL on road users and project schedule.
4. **Decision support model.** Leveraging the two predictive models developed in the previous stage, a decision-support model quantifying the total RUC throughout the entire project duration was developed, and a subsequent WZL sensitivity table (see Table 4 and Table 5) was developed.

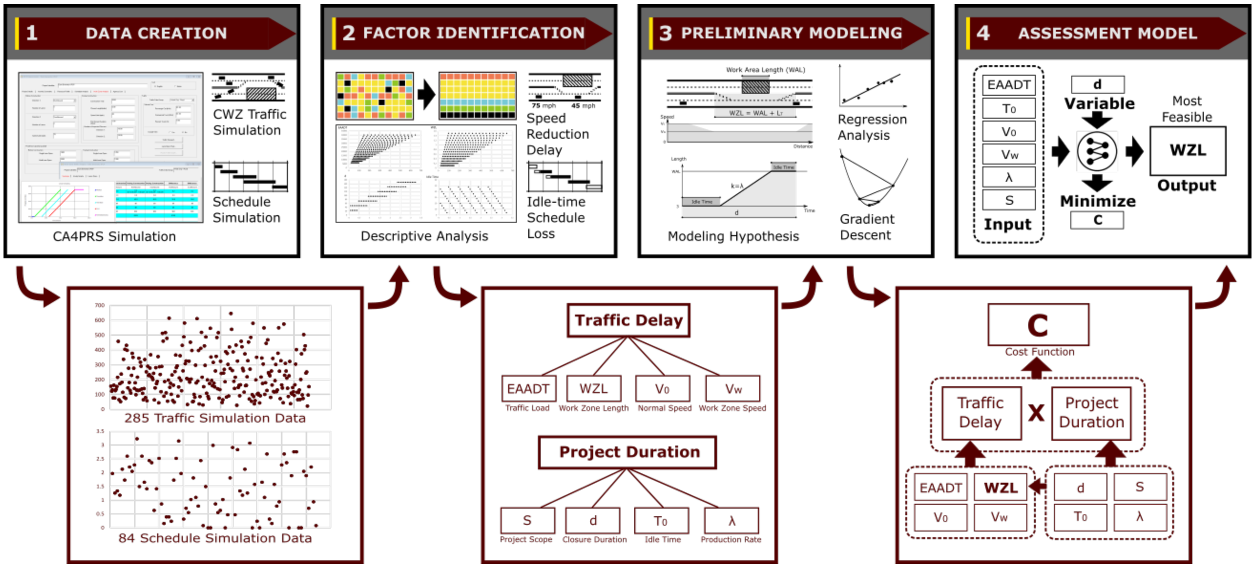

Figure 1. The four-step research objective and methods implemented in this study.

This study relied on the following assumptions and had the following limitations:

- This study focused on a typical work zone configuration for four-lane rural highways, where one lane is closed in each direction during construction.
- This study was limited to resurfacing, restoration, and/or rehabilitation (i.e., 3R) types of concrete pavement projects.
- For the project schedule analysis, it was assumed that contractors' productivity levels would not be significantly different from one another.
- The schedule estimate model was based on the assumption that generic resources, production rates, and sequences of construction would be implemented.
- For the traffic simulations, a conventional rural traffic pattern was referenced and adopted.
- Changes in traffic demand by no-shows and trip mode adjustments were assumed to be minimal because the scope of this study was confined to rural highway networks.
- The effects of microscopic factors (e.g., lane merging, detours, demand reduction, etc.) were considered minimal and thus not incorporated into the analysis.

4.1 Data Generation

According to the Federal Highway Administration, the CA4PRS software is reasonably accurate in predicting optimum pavement construction production rates and reliable mobility impacts *(19, 20)*. Thus, it can be used to back-analyze historical 3R highway projects. CA4PRS is a software package endorsed by the Federal Highway Administration (FHWA) that is capable of assisting transportation agencies with integrated scheduling, traffic, and cost analyses for highway rehabilitation projects. Since 1999, the accuracy and reliability of CA4PRS has been proven in relation to numerous highway 3R projects in California, Washington, Minnesota, and Texas. Therefore, the research team believed it would be an effective tradeoff simulation tool for creating the study data of competing phasing plans. Specifically, it was used in this research as a primary analytical tool for creating traffic impact data and simulating traffic delay effects across various levels of traffic loading (measured according to AADT volume).

The CA4PRS software has three interactive analytical modules: a scheduling module to estimate the duration of the project, a traffic module to quantify the impact of the time delay caused by work zone lane closure, and a cost module to compare the differences among various design and construction alternatives. Specific to this study, the traffic module for CA4PRS was utilized to simulate the traffic impact with conventional rural highway traffic conditions. On the other hand, the schedule module was used to simulate the project schedule, using generic rural highway construction peculiarities and constraints as input. Based on the latest AADT information for rural 2 x 2 lane highway networks across the state of Texas *(21, 22)*, a series of traffic simulations were performed for various AADT volumes ranging from 6,000 vehicles-per-day (vpd) to 40,000 vpd at an interval of 2,000 vpd. For the respective AADT simulation datapoints, WZL values at increments of 0.1 mile and ranging from 0.1 to 1.5 miles were simulated to examine how traffic delay might be affected by the choice of WZL and level of traffic loading (see Figure 2). For schedule simulations, a typical 12-inch concrete rehabilitation sectional profile was selected with a continuous closure, continuous operation, and concurrent working methods.

Performing large numbers of macroscopic traffic simulations can be tedious and time consuming, but they are effective for generating important benchmarking datasets such as daily traffic delay (DTD), maximum delay, and maximum queue length. Table 2 presents key parameters incorporated into the traffic simulations in the present research. Since the primary scope of this study was confined to rural highways, detours were not considered. The roadway capacity information specified in the FHWA guidelines *(23)* was adopted. The truck percentage was converted into passenger car count by applying a passenger car equivalent (PCE) factor, as suggested in the guidelines. In this study, the concept of equivalent AADT (i.e., EAADT), which reflects the weighted value of truck percentage (τ), was used and computed by the following equation:

$$EAADT = AADT \times [1 + \tau \times (PCE - 1)] \quad (1)$$

Table 2. CA4PRS Traffic Simulation Parameters

Roadway Capacity Parameters			
Basic Capacity (vphpl)	2,200	Work Zone Capacity Adjustment Factor	0.70
Lane Width Factor	0.95	Shoulder/Lateral Factor	0.95
Traffic Demand Parameters			
EAADT (vpd)	6,000-42,000	Detour (%)	0
Normal Speed Limit (mph)	75	CWZ Speed Limit (mph)	45
Lane Closure Parameters			
Lanes Closed (lane)	2	Lanes Remaining Open (lane)	2
Lane Closure Length (mile)	0.1-1.5	Lane Closure Per Day (hour)	24

When devising competing construction phasing plans, it is imperative to note that the duration of the lane closure (i.e., the number of days lanes must be closed during construction) is one of the major contributing factors to the level of traffic inconvenience. With this practical prior knowledge, the research team took into account project duration when modeling the optimal WZL. CA4PRS's deterministic schedule simulations were implemented by experimenting the effects of various levels of activity constraints on WZL; namely, for the idle times of non-value-adding construction activities such as mobilization/demobilization, lag time, and concrete curing time, a series of simulations were performed, ranging from 4 hours to 48 hours at an interval of 4 hours. Table 3 presents key parameters considered in the schedule simulations. In this study, the idle time (T_0) was defined as follows:

$$T_0 = T_m + T_a + Max(T_d, T_c) \quad (2)$$

where T_m is the mobilization time, T_a is the activity lag time, T_d is the demobilization time, and T_c is the concrete curing time.

Table 3. CA4PRS Schedule Simulation Parameters

Construction Scenarios			
Closure Method	Continuous	Operation Method	Continuous
Construction Window	24 hours/day	Working Method	Concurrent
Activity Constraints			
Mobilization Hours	1.0-6.0	Demobilization Hours	1.0-8.0
Demolition to JPCP Installation Lag Hours	1.0-18.0	Fast-Track Concrete Curing Time (Hours)	2.0-24.0
Resource Profile			
Demolition Hauling Truck Rated Capacity	26.5 ton	Demolition Hauling Truck Packing Efficiency	0.55
Demolition Hauling Trucks per Hour per Team	10	Demolition Hauling Truck Teams	2
Base Delivery Truck Rated Capacity	13.1 cu. yd	Base Delivery Truck Packing Efficiency	0.9
Base Delivery Trucks per Hour	10		
Batch Plant Capacity	117.7 cu. yd/hour	Number of Batch Plants	1
Concrete Delivery Truck Capacity	7.8 cu. yd	Concrete Delivery Trucks Per Hour	15
Concrete Delivery Truck Packing Efficiency	1		
Paver Speed	6.6 ft/min	Number of Pavers	1
Section Profile			
Existing Surface Type	Concrete	Existing Surface Depth	8 inches
Pavement Surface Type	JPCP	Surface Depth	12 inches
Base Type	CTB	Base Depth	6 inches

4.2 Descriptive Factor Analysis

A descriptive factor analysis was performed on the 285 traffic datapoints, yielding two major findings. Firstly, an inflection point of EAADT ($\bar{A}$) on the level of traffic delay was discovered at around 41,000 vpd (see Figure 2), which provided an important benchmarking point. Secondly, as shown in Figure 2, the level of traffic inconvenience was strongly tied to WZL and EAADT for highway networks where the EAADT was less than $\bar{A}$. For urban highway networks where the EAADT was higher than $\bar{A}$, simulations did not prove a positive linear relationship because other externalities such as detours, trip mode adjustments, socioeconomic characteristics, travelers' behavior, etc., also played pivotal roles in the effect that traffic delay had. This finding led this study to focus fully on rural highway networks. This research hypothesized that the DTD was the result of speed-reduction delay (D_S) and congestion-induced delay (D_C). Since the scope of this study included only traditional rural four-lane highway configurations, the effect of D_C was assumed to be minimal and thus was negligible. This study further hypothesized that D_S was a combination of two latent attributes: acceleration/deceleration and low-speed pass-through. More specifically, the effects of normal travel speed (V_0) and travel speed limit around the work zone (V_w) on speed reduction delay (D_S) were analyzed in miles-per-hour (mph).

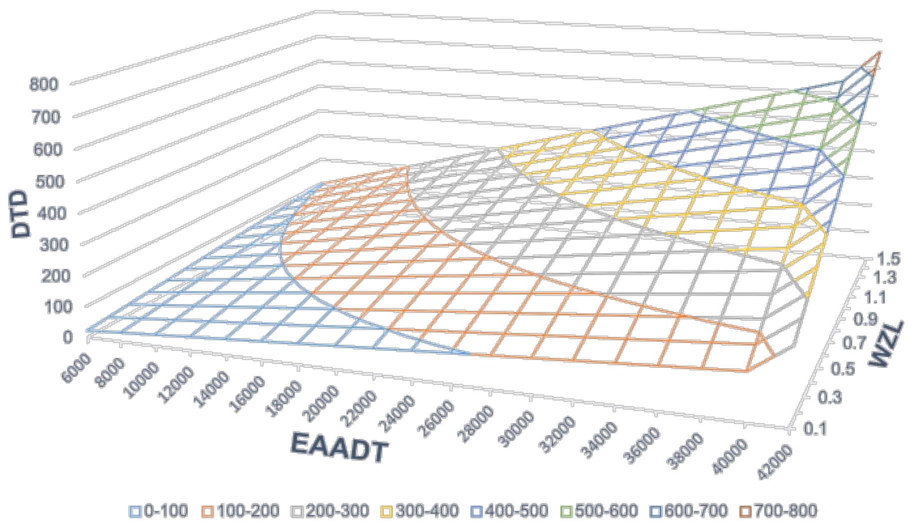

Figure 2. Daily traffic delay (DTD) versus traffic loading (AADT) for various WZL scenarios.

$$D_S = AADT \times \left[\frac{(V_0 - V_w)}{a} + WZL \times \left(\frac{1}{V_w} - \frac{1}{V_0}\right)\right] \quad (3)$$

where a is a coefficient related to the vehicle's acceleration and deceleration ability, and WZL is the work zone length measured in miles. The acceleration/deceleration delay was only associated with the speed difference between V_0 and V_w, while the low-speed pass-through delay was determined by V_0, V_w, and WZL.

As stated previously, the duration of project would have a significant impact on the level of traffic disruption. As shown in Figure 3, two descriptive factor analyses were performed on the 84 schedule datasets to investigate the trends between idle time, closure duration, production rate, and project duration, conveying that idle time, production rate and project scope (i.e., centerline miles to be rebuilt) were the most critical factors in relation to lane closure duration estimates and their respective effects on traffic delay.

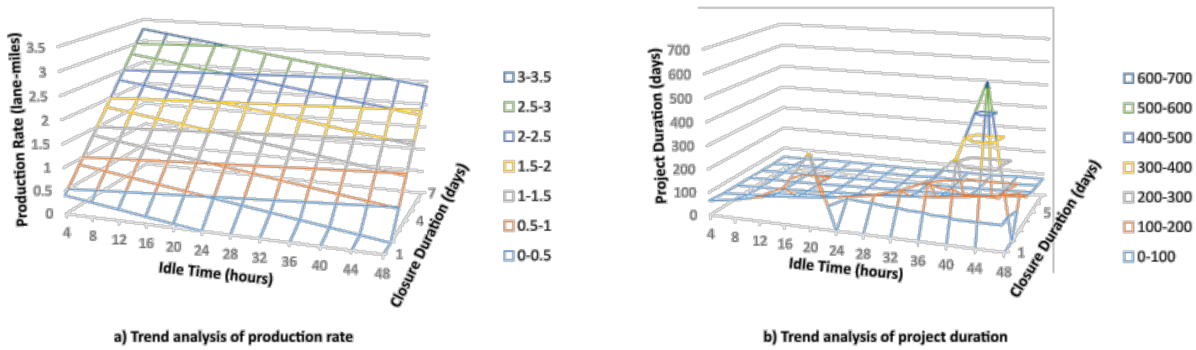

Figure 3. Descriptive analysis of the schedule data.

The schedule factor analysis revealed that the number of lane closure days should be linked to the holistic modeling of WZL and is significantly affected by production rate. For any typical 3R-like concrete pavement rehabilitation project, using certain idle times for non-value-adding activities such as site preparation, resource mobilization/demobilization, concrete curing time, and lag time between activities is unavoidable. In this regard, assuming that the production rate (μ) is measured in lane-miles to be constructed per day and the duration of each closure is *d* and measured in days, then the actual production rate per closure (*U*) can be calculated with:

$$U = [d - T_0] \times \mu \quad (4)$$

where T_0 is the idle time defined by Eq. 3. It is important to note that if the calculated productive time ($d - T_0$) value is zero or negative, the project cannot be finished under the given *d* value since there is not enough time to produce any work within the duration of each closure, making the entire phasing plan invalid.

4.3 Preliminary Modeling

The findings from the data simulations and follow-up descriptive analysis provided the research team with a valuable foundation for this study. The latent attributes of WZL were identified in association with the level of traffic inconvenience, enabling the research team to strategically determine their focus and how to streamline the subsequent modeling procedures *(24)*. In order to model WZL in a way that balanced ideal tradeoffs between travelers' inconvenience and constructability, it was essential to develop the following two preliminary models.

A Stochastic Gradient Descent (SGD) non-linear regression analysis using a Generalized Reduced Gradient (GRG) tool was performed to find the best fit values of the variables for Eq. 3 and Eq. 4 for all valid data points. This tool was developed as a non-linear solving plug-in for Microsoft's Excel by Frontline System based on the work of Lasdon et al. (1974) *(25)*, and Lasdon et al. (1975) *(26)*. SGD can find local optimal solutions for non-linear problems by modifying the target parameters in a way that can minimize the value of a cost function (in this case the sum squared error) using a one or a few data points at each iteration. By doing this, it significantly reduces the computational steps required to converge while still maintain an acceptable accuracy of non-linear regression analysis, with the risk of reaching a local (instead of global) optimum. For more details on the SGD solver of Microsoft Excel, the reader is referred to Frontline Systems *(27)*.

In general, the SGD analysis plug-in for Excel can automatically calibrate non-linear regression models by tweaking a set of target variables in the model to maximizing or minimizing the value of a cost function in a target cell, usually representing the sum square error (SSE) of the model *(28)*. In the traffic analysis, the generated traffic data were used as observations, and the plug-in tool was conducting an SGD on the value of the parameter a in Eq. (3) to minimize the SSE, which is the sum square of the difference between the value of the generated traffic data and the value calculated with Eq. (3) with the current a value. Similarly, for schedule analysis, the tool was used to estimate the value of μ in Eq. (4) with the schedule simulation data.

5. ANALYSIS AND FINDINGS

5.1 Stochastic Gradient Descent Regression Analysis

Speed reduction delay (D_S) was defined in Eq. 2. This study leveraged the 270 traffic simulation datasets that were under the EAADT threshold $\bar{A}$ to estimate the value of the coefficient *a* through a regression analysis with Excel GRG solving algorithm. This resulted in the following quantitative model:

$$D_S = AADT \times \left[\frac{(V_0 - V_w)}{10{,}674\ mph/h} + WZL \times \left(\frac{1}{V_w} - \frac{1}{V_0}\right)\right] \tag{5}$$

In this study, the latest value of time (VOT) published by the Texas Department of Transportation *(29)* was used to calculate the daily road user cost (DRUC). The VOT for passenger cars (VOT_C) was \$30.12 per-vehicle-per-hour (pvph), and for commercial trucks (VOT_T) the value was \$41.33 pvph. Using those values, DRUC can be calculated based on DTD, PCE, and τ, using the following equation:

$$DRUC = DTD \times \frac{VOT_C \times (1 - \tau) + VOT_T \times \tau}{1 + \tau \times (PCE - 1)} \tag{6}$$

Similar to the traffic impact analysis, the GRG solving algorithm was adopted to conduct a regression analysis upon the 76 valid datasets (after removing eight invalid datapoints) to estimate the optimal value for the coefficients of Eq. 4. The resulted quantitative model is:

$$U = [d - T_0] \times 0.4724\ lane\ miles/day \tag{7}$$

Based on the preliminary traffic model, the level of traffic disruption is affected significantly by WZL. In practice, according to the guidelines specified in transportation management plans, WZL should be longer than the section to be rebuilt during a lane closure, with buffer space in and between the work zone. Therefore, WZL should be equivalent to the value of the production rate U calculated with Eq. 4, plus the total length of the established buffer space in and between the work zone, which is defined as L_T. The length of L_T was set to be 0.30 mile (1,570 ft) according to the Manual on Uniform Traffic Control Devices guidelines for traffic control plans *(13, 30)*. The following preliminary quantitative schedule model was developed:

$$WZL\ (miles) = U + L_T = [d - T_0] \times 0.4724 + 0.30 \tag{8}$$

The project scope S (i.e., lane-miles to be rebuilt for the project) is known from the very early stages of project scoping. With the estimated production rate per closure *U*, the number of lane closure working days *D* can be calculated with the following equation:

$$D = \frac{S}{U} \times d \tag{9}$$

5.2 Decision Support Model to Determine the Most Realistic Work Zone Length

To find the most feasible WZL, the proposed decision-support model computes the total RUC by accounting for the closure duration (*d*), with the identified critical factors such as project duration (*S*), traffic loading (*AADT*),truck percentage (τ), normal vehicle speed (V_0), work zone speed limit (V_w), idle time (T_0), *and* production rate (μ). For a value of *d* to be valid, it needs to be greater than the idle time T_0 to produce any actual work. Also, in most practices, a single lane closure with a continuous work shift rarely goes beyond seven continuous days. Thus, in this study, the valid range of *d* was reasonably set to be:

$$T_0 < d \leq 7 \tag{10}$$

Specifically, for each tested value of *d*, the model evaluates the total traffic impact in the total RUC (*C*) of a work zone over the entire duration of the construction, which integrated the result from both the preliminary predictive models of traffic and schedule as follows:

$$C = DRUC \times D \tag{11}$$

The value of *C* can be computed by conducting the following five steps:

1. WZL is estimated by Eq. 8.
2. DRUC is calculated by Eq. 5 based on the value of WZL.
3. A respective production rate per closure, *U*, is estimated with Eq. 7.
4. Project duration, *D*, is computed using Eq. 9.
5. C is calculated using Eq. 11.

The five steps above will be repeatedly performed on all the probable values of *d* (i.e., closure duration). And the model will find the *d* value and its corresponding *WZL* that cause a minimal amount of traffic impact throughout the entire construction. For example, assuming a generic production rate and sequence of construction with the 36-hour idle time, the effects of traffic loading (*EAADT*) on RUC and working days per closure (*d*) were examined at an interval of 10,000 ranging from 10,000 to 40,000. The results (see Figure 4) indicated that an optimal value of *d* that minimizes the total RUC exists.

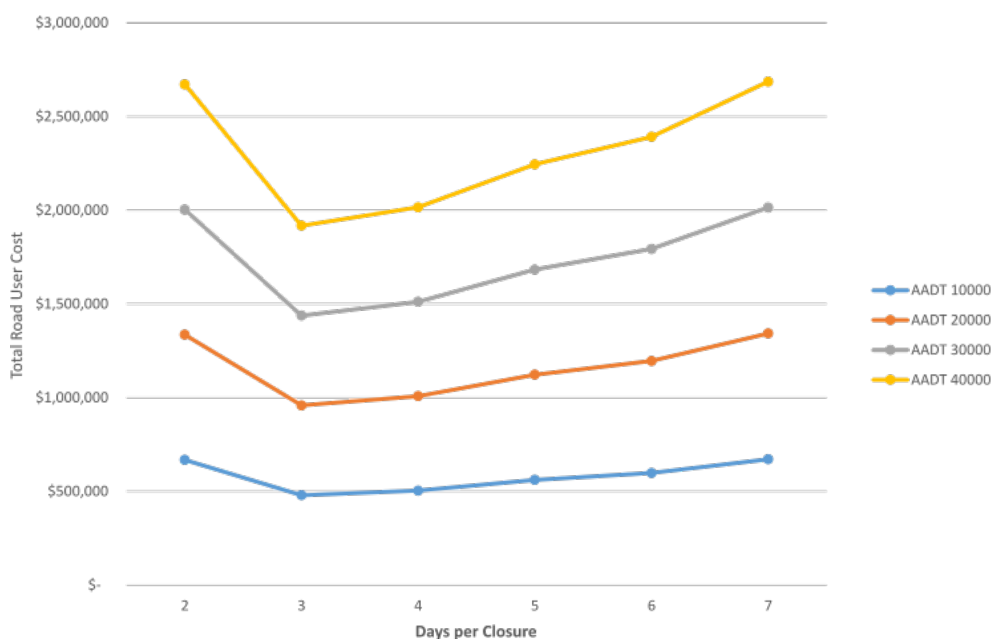

Figure 4. Effect of traffic loading on RUC and project duration

The research team then performed this process on all pairs of *EAADT* and T_0 values to determine the most feasible *WZL* and *d* value. Based on these results, two lookup tables (Table 4 and Table 5) were developed that can intuitively guide agency engineers to perform a tradeoff analysis, thus determining the most feasible WZL by balancing the tradeoff point between the given traffic loading, constructability, and project schedule considerations.

Table 4. WZL Lookup Table for the Various Levels of Traffic Loading and Construction Idle Time

Optimal Work Zone Length (WZL)

	Idle Time																		
EAADT	**4**	**8**	**12**	**16**	**20**	**24**	**28**	**32**	**36**	**40**	**44**	**48**	**52**	**56**	**60**	**64**	**68**	**72**	**76**
6000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
8000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
10000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
12000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
14000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
16000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
18000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
20000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
22000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
24000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
26000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
28000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
30000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
32000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
34000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
36000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
38000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17
40000	0.69	0.61	0.54	0.93	0.85	0.77	0.69	1.09	1.01	0.93	0.85	1.24	1.17	1.09	1.01	1.40	1.32	1.24	1.17

Table 5. Project Duration Estimate Lookup Table for the Various Levels of Traffic Loading and Construction Idle Time

Working Days per Closure

	Idle Time																		
EAADT	**4**	**8**	**12**	**16**	**20**	**24**	**28**	**32**	**36**	**40**	**44**	**48**	**52**	**56**	**60**	**64**	**68**	**72**	**76**
6000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
8000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
10000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
12000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
14000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
16000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
18000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
20000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
22000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
24000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
26000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
28000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
30000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
32000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
34000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
36000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
38000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
40000	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5

5.3 Implementation Use Case

To best use these two lookup tables, engineers first need to compute the project specific EAADT using Eq. (1) based on AADT and truck percentage τ of the construction work zone. The expected idle time can be estimated based on construction methods, manuals, and/or historical data. On the other hand, Eq. (2) could be used as a shortcut to quickly estimate the idle time by adding up construction mobilization time, activity lag time, and concrete curing time or asphalt cooling time. With this information of EAADT and idle time estimated, engineers can then leverage the lookup tables to determine the optimal WZL along with the project duration estimate per closure in working days.

6. CONCLUSIONS

This study presents a new decision support model that can be used to determine the optimal WZL in a balanced tradeoff between motorists' traffic disruption and project constructability. An extensive literature review revealed that despite the wealth of research studies examining the effects of highway work zones, there is a definite lack of hands-on methods for modeling WZL through its latent attributes. This lack of a standardized methodology and set of analytical tools for proactively estimating the optimal WZL was the point of departure for this research, and this practical need motivated the research team to initiate this study. A rich set of high-confidence data was assembled by conducting a series of schedule and traffic simulations.

The results of the final decision-support model (see Table 4 and Table 5) supports the existence of the tradeoff relationship between the traffic impact and project constructability and found the optimal point for all EAADT and idle time range within the scope of a rural highway CWZ. With the assistance of the two provided tables, agency engineers can easily find the most feasible WZL and phasing plan that suits the challenge the project team is facing by finding the matching EAADT (calculated with AADT and truck percentage) and estimated construction idle time. According to the two tables, for any conditions with EAADT lower than 41,000 vpd, the longer the estimated idle times is, the longer the WZL should be, as well as a longer closure duration for each construction windows to reduce the costly non-productive work hours. On the other hand, if the idle time is anticipated to be short, the WZL should be reduced to minimize the direct traffic disruptions to the traveling public, with more frequent phasing for construction and shorter closure duration.

Based on the descriptive factor analysis, a critical benchmarking point of EAADT at 41,000 vpd was discovered. Additionally, a series of statistical analyses further validated that the level of traffic disruption was affected by WZL, construction idle time, closure duration, and traffic loading in rural corridors where the EAADT was lower than 40,000 vpd. The results also reveal that traffic loading at 41,000 vpd suggests an important benchmarking point.

This study is the first of its kind and will help STAs make better-informed decisions by providing a point of reference when establishing WZL in construction phasing plans. Use of the models will facilitate a more realistic determination of WZL.

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