Modeling e-hailing and car-pooling services in a coupled morning-evening commute framework

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Jong-Shi Pang, University of Southern California

Michael H. Zhang, University of California, Davis

Maged M. Dessouky, University of Southern California

Wei Gu, University of Southern California

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7. Author(s)

Jong-Shi Pang https://orcid.org/0000-0002-2224-6787
Michael H. Zhang
Maged M. Dessouky https://orcid.org/0000-0002-9630-6201
Wei Gu

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16. Abstract

In this research project, we develop a general equilibrium model to capture the complex interactions between solo-driving, rideshare and e-hailing that allows travelers to switch between different transportation modes in a coupled morning-evening commute. The model is formulated as a mixed complementarity problem. The existence of an equilibrium solution and the properties of the solution are investigated. The proposed model is then validated with a small network and the renowned Sioux-Falls network. The results show that our model captures the mode switches between morning and evening that are missed by decoupled morning and evening commute models. In particular, our numerical examples show that modeling morning and evening commutes separately tends to overestimate the number of drivers and total vehicle miles traveled (VMT) in the network when accounting for travelers' capabilities for mode switching. With a coupled model, transportation planners can better understand appropriate incentives to increase vehicle occupancy and reduce VMT, thereby achieving some social benefits associated with morning and evening commutes.

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Contents

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Disclosure

The Principal Investigator and Co-Investigators, with the assistance of a graduate student, conducted this research titled, Modeling e-hailing and car-pooling services in a coupled morning-evening commute framework, at the Daniel J. Epstein Department of Industrial & System Engineering, Viterbi School of Engineering, University of Southern California and at the Department of Civil and Environmental Engineering, University of California Davis. The research took place from 01/01/2020 to 12/31/2020 and was funded by a grant from Caltrans in the amount of $200,642. The research was conducted as part of the Pacific Southwest Region University Transportation Center research program.

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Abstract

In this research project, we develop a general equilibrium model to capture the complex interactions between solo-driving, rideshare and e-hailing that allows travelers to switch between different transportation modes in a coupled morning-evening commute. The model is formulated as a mixed complementarity problem. The existence of an equilibrium solution and the properties of the solution are investigated. The proposed model is then validated with a small network and the renowned Sioux-Falls network. The results show that our model captures the mode switches between morning and evening that is missed by decoupled morning and evening commute models. In particular, our numerical examples show that modeling morning and evening commutes separately tends to overestimate the number of drivers and total vehicle miles traveled (VMT) in the network when accounting for travelers' capabilities for mode switching. With a coupled model, transportation planners can better understand appropriate incentives to increase vehicle occupancy and reduce VMT, thereby achieving some social benefits associated with morning and evening commutes.

Executive Summary

The emerging shared mobility services, such as e-hailing services provided by Uber, Lyft, Didi, Grab and Ola or rideshare services enabled by SCOOP, WAZE, Zipcar, and Turo provide more travel mode choices for commuters in both morning and evening commutes. Moreover, these new services compete or cooperate in this space to reduce travel demand and hence traffic congestion. There is clearly a need to understand not only how these services are transforming urban transportation network, but also what the impact is on travelers' behavior.

In this research project, we propose a general equilibrium modeling framework, which is capable of capturing the complex interactions between solo-driving, rideshare, and e-hailing, and allows travelers to switch between different transportation modes in a coupled morning-evening commute. Formulated as a mixed complementarity problem, the main constraints of the general equilibrium model include user equilibrium conditions, flow conservation equations, rideshare capacity and a minimum fare threshold.

Then we prove that an equilibrium exists for the proposed model. Also, we show that when the model reaches an equilibrium, (1) the morning (evening) commute also reaches an equilibrium; (2) if travelers' mode choice is fixed, the morning (evening) commute is equivalent to a traditional traffic equilibrium problem; (3) travelers are rational to mode choice, which means that no traveler will choose a more expensive travel mode combination. Furthermore, we provide the conditions under which travelers' mode choice will be unique.

Finally, the proposed model is validated in two networks: a small network and the Sioux-Falls network. The results show that the proposed coupled morning-evening model is effective in capturing the mode switches between morning and evening, which eventually leads to better system performance (e.g., number of drivers, total VMT) compared with a decoupled morning (evening) commute model. For example, in the Sioux-Falls network, the coupled model produces 24.2% fewer drivers and 8.4% less VMT in the system compared with the decoupled model when the inconvenience cost due to ridesharing is higher during the evening commute than in the morning commute.

This is due to the fact that the coupled model can capture the behavior of travelers' capability to switch to e-hailing in the evening commute when ridesharing in the morning commute. A decoupled model cannot capture this effect and most likely will predict that the traveler will drive to work.

1. Introduction

App-based transportation services, such as e-hailing services provided by Uber, Lyft, Didi, Grab and Ola or casual rideshare enabled by SCOOP, WAZE, Zipcar, and Turo are growing rapidly. For example, Uber has hit its milestone in 2018 to serve over 10 billion trips within more than 700 cities of 80 countries (Uber, 2018). There are over 75 million riders and 3.9 million drivers in total, producing more than 14.1 billion of annual net revenue for the Uber company (Iqbal, 2020). These emerging transportation services are transforming the travel behavior of individuals and urban mobility patterns, and provide significant challenges to transportation planners and policy makers on how to assess the impact of these services on transportation systems, and how to facilitate or regulate these services because conventional planning tools are inadequate to model their more complex interactions between drivers, riders, and the private enterprises that link the drivers and riders together.

Due to heavy traffic, commuters suffer from long travel delays in both the morning and evening commutes in many urban areas. The emerging shared mobility transportation services, e-hailing and ridesharing, provide more travel mode choices for commuters in both morning and evening commutes. Furthermore, these new modes of travel compete or cooperate in this space to reduce travel demand and hence traffic congestion. For example, a person can combine a rideshare service in the morning, but use an e-hailing service for the evening return trip to reduce the pairing cost, and provide more flexibility in evening trips.

While these new transportation services significantly increase the options that travelers have for their commute and the travel choice in the morning commute directly impacts the feasible options for the evening commute, the net effect of these new services on the long term efficiency, sustainability, and equity of urban transportation systems remain to be better understood. There is a clear need to not only understand the nature and effect of these new mobility services better, but also to understand, model, and study the interactions between the various modes of transportation, and integrate them in a unified transportation planning model that includes morning and evening commute trips. However, prior research models typically treat these two commute trips separately (Xiao et al., 2016; Ma and Zhang, 2017; Liu and Li, 2017; Su and Wang, 2019; Lin et al., 2020). To the best of our knowledge, there is no research to provide a general equilibrium model to both capture the complex interactions between solo driving, ridesharing, and e-hailing and allow travelers to switch between different transportation modes in a coupled morning-evening commute. In order to address the research gap, we develop a general modeling framework to simultaneously consider the morning and evening commute. The objective of this research is to understand the impact of the new shared mobility modes on the coupling of the morning and evening commute: traffic congestion, travelers' behavior of mode choices, and efficiency of the overall urban transportation systems.

Without a coupled modeling approach, planners could solve a traffic assignment problem using estimated cost and value of time data to predict and understand traveler behavior during the morning commute. The same could be done for the evening commute but there would have to be the added constraint that if an individual chose to drive in the morning that they would also need to drive in the evening. A coupled modelling approach would be able to capture traveler behavior for the entire day and allow for travelers to switch one type of commute mode in the morning to another in the evening. For various reasons, travelers may switch from one type of commute mode in the morning to a different type in the evening: rideshare passengers and e-hailing passengers may switch among these two types. This capturing of mode switches is especially important if the travel cost data is different in the morning and evening times. For example, a traveler with a high inconvenience cost for ridesharing in the afternoon, which may be due to the need to pick up their children from after school activities, will not use this mode in the afternoon, but may consider this option in the morning with the appropriate incentives since they would be able to take e-hailing in the afternoon. With a coupled model, transportation planners can better understand appropriate incentives that captures the entire day to increase vehicle occupancy and reduce vehicles miles traveled. We note that even when the cost structure for the morning and evening commutes are the same, a coupled model could yield a different equilibrium solution (e.g., number of drivers, total VMT, etc) than the separate models if the traffic network is not symmetrical.

In this report, we first develop an equilibrium model that considers multiple transportation modes (solo driving, e-hailing and ridesharing) and integrates the morning and evening commutes. Then we show an equilibrium solution exists for our proposed model. Furthermore, we run some experiments to show the effectiveness of our coupled model in capturing possible mode switch behavior with the appropriate incentives compared with treating the commute trips separately. The remainder of the report is organized as follows. In Section 2, we review the relevant literature. Section 3 provides the coupled morning-evening traffic equilibrium model, while Section 4 analyzes mathematical properties of the proposed model. In Section 5, experimental results are given to validate our model. Section 6 concludes this report and points out some possible directions for future research.

2. Literature Review

There have been extensive efforts to model the emerging shared mobility transportation services, and many of these papers mainly focus on the transportation network companies' (TNCs') daily operations (Furuhata et al., 2013; Mourad et al., 2019; Wang and Yang, 2019; Yan et al., 2019; Tafreshian et al., 2020). In this section, we review the papers that are most relevant to our research, which includes two categories: (1) equilibrium of shared mobility transportation systems, and (2) shared mobility services in the morning commute.

There has been some research that formulates the new shared mobility modes as a Traffic Assignment Problem (Sheffi, 1985; Patriksson, 2015). Xu et al. (2015b) proposed a traffic equilibrium model with rideshare service, while Ban et al. (2019) considered the e-hailing service in their general equilibrium model. Di and Ban (2019) provided a general traffic equilibrium modeling framework, which included travelers' mode choices, rideshare equilibrium and e-hailing operations. Considering an OD-based surge pricing strategy, Ma et al. (2020) proposed a rideshare user equilibrium model with ride-matching constraints. Li et al. (2020) studied a path-based rideshare equilibrium model to simultaneously produce route choices, mode choices, and matching decisions. Instead of using a mixed complementary formulation, Wang et al. (2020) established a convex programming formulation for the rideshare user equilibrium problem. To better understand vacant trips generated by e-hailing service, Xu et al. (2020) put forward a network equilibrium model to capture both cruising and deadheading trips of e-hailing vehicles.

With the traffic equilibrium including shared mobility services as a lower level problem, researchers extended the literature of the Transportation Network Design Problem (Yang and Bell, 1998; Chen et al., 2011; Farahani et al., 2013; Xu et al., 2016). For example, Di et al. (2018) extended the rideshare equilibrium framework of Xu et al. (2015b) to optimize the deployment of high-occupancy toll (HOT) lanes.

Since the shared mobility transportation market may influence traffic congestion, some research explored the equilibrium in shared mobility transportation market, and its impacts on or interaction with the equilibrium in a traffic network. Xu et al. (2015a) combined an elastic demand traffic equilibrium model with an economic pricing model to determine the rideshare price. Under the scenario of mixed e-hailing and taxi market, He and Shen (2015) established a spatial equilibrium model to balance supply and demand in the market, and at the same time evaluated travelers' possible adoption to the emerging e-hailing service; Qian and Ukkusuri (2017) investigated the equilibrium of the competitive market by modeling it as a multiple-leader-follower game: passengers are the leaders who aim to minimize the cost, while drivers are the followers seeking to maximize the profit. Li et al. (2019) studied the impact of regulation on TNCs based on a queuing theoretic market equilibrium model. Ke et al. (2020a) explored the effects of key decision variables of an e-hailing platform (such as price and vehicle fleet size) on its revenue and social welfare. With a macroscopic fundamental diagram to characterize traffic congestion, Ke et al. (2020b) proposed an e-hailing market equilibrium model with congestion effects. Zhang and Nie (2020) put forward a matching-based market equilibrium model to explore the influence of regulation on both e-hailing and rideshare services.

Some papers extended the Morning Commute Problem (Vickrey, 1969; Newell, 1987, 1988; Daganzo and Garcia, 2000; Nie and Zhang, 2009; Shen and Zhang, 2009; Liu and Nie, 2011; Qian et al., 2012; Xiao and Zhang, 2014) by considering the emerging transportation services. Xiao et al. (2016) explored the morning commute problem with rideshare service and parking space limitation. Ma and Zhang (2017) studied the integration of rideshare in the morning commute from home to the central business district under a dynamic rideshare payment. Liu and Li (2017) investigated the dynamic rideshare user equilibrium during morning commute under the fixed-ratio charging-compensation scheme (FCS), while Wang et al. (2019) examined an extended version under the variable-ratio charging-compensation scheme (VCS). Considering parking space constraints, Su and Wang (2019) addressed the problem of regulating the supply of rideshare services in the morning commute. Lin et al. (2020) studied the influence of HOV/HOT lines on dynamic rideshare during morning commute.

In summary, we extend the current literature by providing a general equilibrium model framework to capture both e-hailing and rideshare services in the coupled morning-evening commute.

3. Mathematical Model

3.1. Problem description

We propose an extended path-induced cycle-based traffic equilibrium problem of morning and evening commutes, taking into account the emergent travel trends of ridesharing and e-hailing that offer alternative modes of travel supplementing the traditional mode of commute: solo driving. The goal of the model is to study the morning and evening commute trip flows in the network caused by traffic congestion and the travelers' choices of commute types to minimize their disutilities. Most importantly, our approach is holistic, combining morning travel from an origin to a destination and evening return from the same destination (which therefore is the origin of the evening trip) to the morning's origin; this round trip constitutes the commute cycle (cc). Specifically, each such cycle is composed of a morning trip taken on a path and an evening trip taken on a possibly different (reverse) path with possibly a different mode. The cycle flows encompass travelers' commute behavior; the equilibrium will determine the travelers' cycle selections by equilibrating the cycle flows with the travelers' disutilities associated with the particular round-trip (i.e., cycle) choices based on an extension of Wardrop's user equilibrium principle.

As illustrated in Fig. 1, there are 3 types of commuters: (a) drivers, labelled as 1; (b) rideshare passengers, labelled as 2; and (c) e-hailing passengers, labelled as 3. Trip makers travel from home to work in the morning as one of these 3 types of commuters and from work to home in the evening. For various reasons, travelers may switch from one type of commute mode in the morning to a different type in the evening: rideshare passengers and e-hailing passengers may switch among these two types. As a result, there are 5 types of mode combinations between morning commuters and evening commuters in total (see Fig. 1). Labeled by $j \in \{1, \cdots, 5\}$, each of these 5 mode combination types is incident to a unique pair of commuters, which we label as j_{am} and j_{pm}, both being indices in $\{1, 2, 3\}$, respectively. For instance, the combination $j = 3$ means morning rideshare passenger (2) switched to evening e-hailing passenger (3); thus for this j, we have $j_{\text{am}} = 2$ and $j_{\text{pm}} = 3$. Based on a set of travel costs, the model aims to determine a user equilibrium of trips under a set of reasonable and realistic assumptions and subject to traffic congestion. In the process, the model also determines the switches of the commuter types in the morning and the evening trips. The correspondences between the mode combinations and commuter types are listed in the following table:

mode combination j	morning commuter type j_{am}	evening commuter type j_{pm}
1	1	1
2	2	2
3	2	3
4	3	2
5	3	3

A morning OD pair $k = (o; d)$ becomes the OD pair $\bar{k} = (d; o)$ in the evening. That is to say, the origin and destination of morning OD-pair $k \in \mathcal{K}$ becomes the destination and origin of evening OD-pair $\bar{k}$, respectively. Corresponding to each morning OD-pair k and evening reverse OD-pair $\bar{k}$ is a set $\mathcal{C}_k$ of **commute cycles** c_k each consisting of a pair of morning and evening paths used by this cycle; conversely, associated with each commute cycle $c \in \mathcal{C} \triangleq \bigcup_{k \in \mathcal{K}} \mathcal{C}_k$ is a unique morning OD-pair and its evening reverse OD-pair. Therefore, the association $k \in \mathcal{K} \mapsto c_k \in \mathcal{C}$ is multi-valued while its inverse is single-valued. It is possible that two commute cycles c_k and c'_k associated with the same OD-pair k can use the same morning path and yet differ in the evening paths; similarly, it is also possible for these two commute cycles to differ in the morning paths but use the same evening path. Flows on such commute cycles are the primary decision variables of the model.

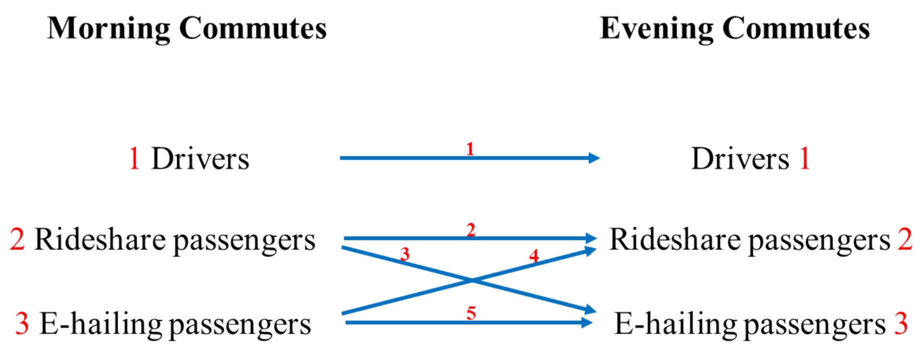

Figure 1. Multiple modes and combinations of morning and evening commutes. The arrow $i \to i'$ means that traveler type $i \in \{1, 2, 3\}$ in morning becomes traveler type $i' \in \{1, 2, 3\}$ in evening.

3.2. Model notations and assumptions

Notations used in this study are summarized as follows, including input sets and parameters in Table 1 and decision variables in Table 2.

Table 1. Input sets and parameters

$\mathcal{N}$	Set of nodes, $n \in \mathcal{N}$
$\mathcal{A}$	Set of (directed) arcs, $a \in \mathcal{A}$
$\mathcal{K}$	Set of morning origin-destination (OD) pairs; subset of $\mathcal{N} \times \mathcal{N}$
$\bar{k}$	Evening return OD-pair corresponding to morning OD-pair $k \in \mathcal{K}$
$\mathcal{C}_k$	Set of commute cycles associated with OD pair k (and its evening reverse pair $\bar{k}$)
p_c^{am}	The morning path used by commute cycle $c \in \mathcal{C}$
p_c^{pm}	The evening path used by commute cycle $c \in \mathcal{C}$
d_k	Total trip demand of OD pair k (morning demand = evening return demand)
$O_k^{1;\text{am}}$	Operation cost of OD pair $k \in \mathcal{K}$ for morning drivers
$O_{\bar{k}}^{1;\text{pm}}$	Operation cost of reverse OD pair $\bar{k}$ for evening drivers
$I_k^{i;\text{am}}$	Unit inconvenience of OD pair $k \in \mathcal{K}$ for morning commuter type $i \in \{1, 2, 3\}$

(Table 1 continued)

$I_{\bar{k}}^{i;\text{pm}}$	Unit inconvenience of reverse OD pair $\bar{k}$ for evening commuter type $i \in \{1,2,3\}$
$E_k^{3;\text{am}}$	Unit payment of OD pair $k \in \mathcal{K}$ for e-hailing passengers in the morning
$E_{\bar{k}}^{3;\text{pm}}$	Unit payment of reverse OD pair $\bar{k}$ for e-hailing passengers in the evening
$\underline{E}_k^{2;\text{am}}$	Minimum unit payment of OD pair $k \in \mathcal{K}$ for morning rideshare commuter
$\underline{E}_{\bar{k}}^{2;\text{pm}}$	Minimum unit payment of reverse OD pair $\bar{k}$ for evening rideshare commuter
$\gamma_k^{2;\text{am}}$	Conversion factor of rideshare under-capacity to surcharge over minimum unit payment of OD pair $k \in \mathcal{K}$ for morning rideshare commuter
$\gamma_{\bar{k}}^{2;\text{pm}}$	Conversion factor of rideshare under-capacity to surcharge over minimum unit payment of reverse OD pair $\bar{k}$ for evening rideshare commuter
$W_k^{i;\text{am}}$	Waiting time of OD pair $k \in \mathcal{K}$ for morning commuter type $i \in \{2,3\}$
$W_{\bar{k}}^{i;\text{pm}}$	Waiting time of reverse OD pair $\bar{k}$ for evening commuter type $i \in \{2,3\}$
M	Capacity in terms of number of rideshare passengers for each private car
$\delta_{a;p}$	Arc-path incidence indicator; $\delta_{a;p} = \begin{cases} 1 \text{ if path } p \text{ uses arc } a \\ 0 \text{ otherwise} \end{cases}$
$t_a(\bullet)$	The Bureau of Public Roads (BPR) travel time function for arc $a \in \mathcal{A}$ as a function of traffic flow
ψ	Conversion factor of time (minutes) to money (dollars)

Table 2. Decision variables

Primary:	
h_c^j	Flow of travelers of commute cycle $c \in \mathcal{C}$ of mode combination type $j \in \{1,...,5\}$
$E_k^{2;\text{am}}$	Unit payment of OD pair $k \in \mathcal{K}$ for morning rideshare commuter type
$E_{\bar{k}}^{2;\text{pm}}$	Unit payment of the reverse OD pair $\bar{k}$ for evening rideshare commuter type
u_k	(Least) disutility of OD pair $k \in \mathcal{K}$

(Table 2 continued)

Derived:	
f_a^{am}	Vehicular flow of arc $a \in \mathcal{A}$ in the morning
f_a^{pm}	Vehicular flow of arc $a \in \mathcal{A}$ in the evening
d_k^j	Travel demand of OD pair $k \in \mathcal{K}$ of mode combination type $j \in \{1, ..., 5\}$
π_c^j	Total cost on commute cycle $c \in \mathcal{C}$ of mode combination type $j \in \{1, ..., 5\}$
α_k^{am}	Average number of morning rideshare passengers of OD pair $k \in \mathcal{K}$
$\alpha_{\bar{k}}^{\text{pm}}$	Average number of evening rideshare passengers of the reverse OD pair $\bar{k}$
$c_a^{\text{am}}(f^{\text{am}})$	Travel cost on arc $a \in \mathcal{A}$ in morning commute as a function of arc flows $f^{\text{am}} \triangleq \{ f_a^{\text{am}} \}_{a \in \mathcal{A}}$
$c_a^{\text{pm}}(f^{\text{pm}})$	Travel cost on arc $a \in \mathcal{A}$ in evening commute as a function of arc flows $f^{\text{pm}} \triangleq \{ f_a^{\text{pm}} \}_{a \in \mathcal{A}}$
$C_c^{i;\text{am}}(h)$	Total cost on morning path p_c^{am} used by commute cycle $c \in \mathcal{C}$ and commuter type $i \in \{1, ..., 3\}$ as a function of commute cycle flow $h \triangleq \{ (h_c^j)_{c \in \mathcal{C}} \}_{j=1}^{5}$
$C_c^{i;\text{pm}}(h)$	Total cost on evening path p_c^{pm} used by commute cycle $c \in \mathcal{C}$ and commuter type $i \in \{1, ..., 3\}$ as a function of commute cycle flow

Assumptions about the model are as follows:

- There is no distinction between rideshare and solo drivers; each is operating a vehicle.
- All drivers are willing to share vehicles and provide rideshare services.
- There is the same passenger capacity in each rideshare vehicle. And the passenger capacity for each e-hailing vehicle is one. That is, each e-hailing vehicle satisfies one unit of demand.
- The rideshare capacity constraints are enforced at the aggregate level, not at the vehicle level. That is, for each OD pair, the total number of rideshare passengers over the total number of drivers has to be less than or equal to the vehicle capacity.

Several other remarks about the model:

• All variables are expressed as real numbers; in particular, the travel demands d_k are considered as traveler (i.e. people) flows, so are d_k^j and the commute cycle flows h_c^j. The am and pm arc flows f_a^{am} and f_a^{pm} are vehicular flows, which are the source of traffic congestion.

• The commute cycle flows h_c^j for $j = 1, 3, 4,$ and 5 contribute to the arc flows (either morning or evening or both), thus to congestion, while the other commute types do not.

• Each e-hailing vehicle is assumed to pick up only one passenger.

• We postulate a minimum per-passenger rideshare fee which becomes the charged payment if the rideshare vehicle is at capacity. As a result of this postulate, the unit passenger rideshare payment is a decision variable to be determined from the model. The payment can be equal to the set minimum, or a higher value if the rideshare is below capacity. This addition to the minimum payment (if positive) is equal to a multiplicative factor of the under-capacity.

3.3. Connections among variables and functions

(1) Relationships between arc and cycle flows:

Arc flows have 3 components as shown in Fig. 2 and are calculated as the summation of the associated path flows as follows. Here $\delta_{a;p_c^{\text{am}}}$ and $\delta_{a;p_c^{\text{pm}}}$ are the arc-path indicators in the morning and in the evening, respectively. Note that the flows of rideshare passengers are not considered here since they have no influence on traffic congestion.

$$\forall\, a \in \mathcal{A} \quad \left\{ \begin{aligned} f_a^{\text{am}} &= \sum_{c \in \mathcal{C}} \left[\sum_{j=1,4,5} h_c^j \times \delta_{a;p_c^{\text{am}}} \right] \\ f_a^{\text{pm}} &= \sum_{c \in \mathcal{C}} \left[\sum_{j=1,3,5} h_c^j \times \delta_{a;p_c^{\text{pm}}} \right] \end{aligned} \right. \tag{1}$$

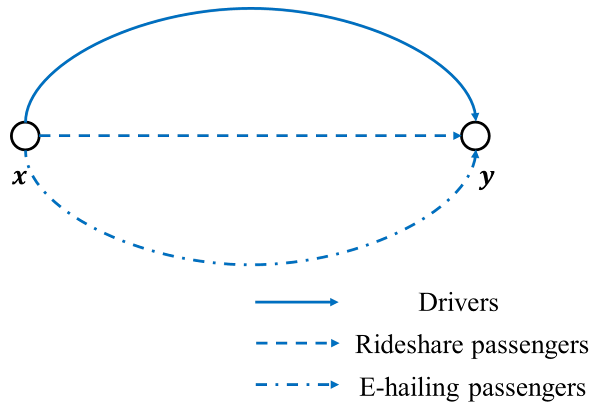

Figure 2. Components of flow between arc (x, y) in network

(the flows of rideshare passengers do not influence traffic congestion)

(2) Arc cost functions:

The notations c_a^{am} and c_a^{pm} offer the possibility that the arc cost on the same arc a may be different in the morning and evening, respectively. Assuming separable arc costs, we have,

$$\begin{aligned} c_a^{\mathrm{am}}(f^{\mathrm{am}}) &= t_a(f_a^{\mathrm{am}}) \quad \forall a \in \mathcal{A} \\ c_a^{\mathrm{pm}}(f^{\mathrm{pm}}) &= t_a(f_a^{\mathrm{pm}}) \quad \forall a \in \mathcal{A} \end{aligned} \tag{2}$$

where $t_a(\bullet)$ represents the Bureau of Public Roads (BPR) travel time function for arc $a \in \mathcal{A}$ as a function of traffic flow. The cost on arc a is a function of the flow on that arc only. Note that the arc costs in the morning are different from those in the evening due to the difference in flows, even on the same arc.

(3) Cost functions for commuter types:

We assume throughout that the commute cycle costs are additive; that is, each such cycle cost is equal to the sum of the costs on the arcs used by the two morning and evening paths, respectively, in the cycle. Besides the travel cost due to congestion, there are many other components that contribute to the total cost, which differ between the commuter types. For each trip, the total travel cost equals to the summation of travel cost due to congestion and specific costs for the commuter type. The cost structures of different commuter types in our model are as follows. Unlike Xu et al.

(2015) which builds the arc costs in a rideshare model, here we directly formulate the commute cycle cost functions.

a) Drivers:

total cost = **travel cost** (BPR function) + **operation cost** (different constants between ODs) + **inconvenience of rideshare** (function of number of passengers) - **income from rideshare** (different constants between ODs, times number of passengers)

$$
\begin{aligned}
C_c^{1;\text{am}}(h) &= \psi \times \sum_{a \in \mathcal{A}} \left(\delta_{a;p_c^{\text{am}}} \times c_a^{\text{am}}(f^{\text{am}}) \right) + \underbrace{O_k^{1;\text{am}}}_{\text{a constant}} + \underbrace{\alpha_k^{2;\text{am}}}_{\text{a variable}} \times \left(I_k^{1;\text{am}} - \underbrace{E_k^{2;\text{am}}}_{\text{a variable}} \right) \\
&\qquad \forall\, c \in \mathcal{C}_k \quad \forall\, k \in \mathcal{K} \\
C_c^{1;\text{pm}}(h) &= \psi \times \sum_{a \in \mathcal{A}} \left(\delta_{a;p_c^{\text{pm}}} \times c_a^{\text{pm}}(f^{\text{pm}}) \right) + \underbrace{O_{\bar{k}}^{1;\text{pm}}}_{\text{a constant}} + \underbrace{\alpha_{\bar{k}}^{2;\text{pm}}}_{\text{a variable}} \times \left(I_{\bar{k}}^{1;\text{pm}} - \underbrace{E_{\bar{k}}^{2;\text{pm}}}_{\text{a variable}} \right) \\
&\qquad \forall\, c \in \mathcal{C}_k \text{ and associated } \bar{k}
\end{aligned}
\tag{3}
$$

Note that the right-hand sides are functions of the arc flows; yet we write the left-hand sides as functions of the commute cycle flows; this is done with the understanding that once the arc flows in the right-hand sides are substituted by their connections to the commute cycle flows through the expressions (1), the substituted left-hand sides are indeed function of the latter flows. The operation cost includes parking, depreciation, insurance, maintenance and so on, which may differ between OD pairs and in the morning and evening; e.g., there is no parking cost in the evening commute.

The inconvenience of drivers with extra passengers includes anxiety for riding with strangers, detours, etc, which is increased with the number of passengers. Income to a driver with extra passengers equals to the total payment by the rideshare passengers in the car. The inconvenience and price of rideshare may differ between OD pairs and could be different between morning and evening. Although the detour is not considered in the model for simplification, it could be viewed as considered here in the inconvenience function: the inconvenience is increased with the number of passengers, which is a relationship between inconvenience, detour distance and number of passengers - the more passengers, the more detour distance, at the same time the more inconvenience.

b) Rideshare passengers:

total cost = **travel cost** (BPR function) + **waiting cost for rideshare** (different constants between ODs) + **inconvenience of rideshare** (function of number of passengers) + **payment for rideshare** (different constants between ODs)

$$
\begin{aligned}
C_c^{2;\text{am}}(h) &= \psi \times \sum_{a \in \mathcal{A}} \left(\delta_{a;p_c^{\text{am}}} \times c_a^{\text{am}}(f^{\text{am}}) \right) + \underbrace{W_k^{2;\text{am}}}_{\text{a constant}} + \underbrace{\alpha_k^{2;\text{am}}}_{\text{a variable}} \times I_k^{2;\text{am}} + \underbrace{E_k^{2;\text{am}}}_{\text{a variable}} \\
&\qquad \forall c \in \mathcal{C}_k \quad \forall k \in \mathcal{K} \\
C_c^{2;\text{pm}}(h) &= \psi \times \sum_{a \in \mathcal{A}} \left(\delta_{a;p_c^{\text{pm}}} \times c_a^{\text{pm}}(f^{\text{pm}}) \right) + \underbrace{W_{\bar{k}}^{2;\text{pm}}}_{\text{a constant}} + \underbrace{\alpha_{\bar{k}}^{2;\text{pm}}}_{\text{a variable}} \times I_{\bar{k}}^{2;\text{pm}} + \underbrace{E_{\bar{k}}^{2;\text{pm}}}_{\text{a variable}} \\
&\qquad \forall c \in \mathcal{C}_k \text{ and associated } \bar{k}
\end{aligned} \tag{4}
$$

Here we treat the waiting cost as different constants between ODs. The inconvenience of rideshare passengers, similar to the inconvenience of drivers, increases with the number of passengers. The total payment of rideshare passengers is the same as the income received by the drivers with extra passengers.

c) E-hailing passengers:

total cost = **travel cost** (BPR function) + **waiting cost for e-hailing** (different constants between ODs) + **inconvenience of e-hailing** (different constants between ODs) + **payment for e-hailing** (different constants between ODs)

$$
\begin{aligned}
C_c^{3;\text{am}}(h) &= \psi \times \sum_{a \in \mathcal{A}} \left(\delta_{a;p_c^{\text{am}}} \times c_a^{\text{am}}(f^{\text{am}}) \right) + \underbrace{W_k^{3;\text{am}} + I_k^{3;\text{am}} + E_k^{3;\text{am}}}_{\text{a constant}} \quad \forall c \in \mathcal{C}_k \quad \forall k \in \mathcal{K} \\
C_c^{3;\text{pm}}(h) &= \psi \times \sum_{a \in \mathcal{A}} \left(\delta_{a;p_c^{\text{pm}}} \times c_a^{\text{pm}}(f^{\text{pm}}) \right) + \underbrace{W_{\bar{k}}^{3;\text{pm}} + I_{\bar{k}}^{3;\text{pm}} + E_{\bar{k}}^{3;\text{pm}}}_{\text{a constant}} \quad \begin{array}{l} \forall c \in \mathcal{C}_k \\ \text{and associated } \bar{k} \end{array}
\end{aligned} \tag{5}
$$

The waiting cost is treated as a kind of constant which differs between ODs. Inconvenience of e-hailing is also a constant that differs between ODs since we assume no pooling for e-hailing. Unit payment for e-hailing passengers should be higher than that of rideshare passengers.

(4) Cost functions for mode combination types:

The total cost of each mode combination and each commute cycle is the summation of the costs in the morning and evening. We have

$$\pi_c^j = C_c^{u;\mathrm{am}}(h) + C_c^{v;\mathrm{pm}}(h) \quad \forall c \in \mathcal{C}, \quad \forall j \in \{1, ..., 5\} \text{ with } u = j_{\mathrm{am}} \text{ and } v = j_{\mathrm{pm}} \tag{6}$$

3.4. Flow conservation equations

Demand distribution equations are used to balance total trip demands with commute cycle flows and ensuring morning trip demands equal evening trip demands

- per mode combination type

$$d_k^j = \sum_{c \in \mathcal{C}_k} h_c^j = d_{\bar{k}}^j \quad \forall j \in \{1, 2, \cdots, 5\}, \quad \forall k \in \mathcal{K} \tag{7}$$

- morning and evening trip demands, aggregated to total trip demands

$$d_k = \sum_{j=1}^{5} d_k^j = \sum_{j=1}^{5} \sum_{c \in \mathcal{C}_k} h_c^j, \quad \forall k \in \mathcal{K} \tag{8}$$

3.5. Rideshare capacity and addition to minimum fare

In this section, we first compute the average number of rideshare passengers in each vehicle for each OD pair and then describe the rideshare payment scheme.

rideshare passenger flow = average number of rideshare passengers per car × number of cars, with the average number slightly adjusted to avoid division by zero

Written as a fraction, the formulas for the (approximate) average number of rideshare passengers per car are as follows: for a small scalar $\varepsilon > 0$,

$$\alpha_k^{\mathrm{am}} = \frac{d_k^2 + d_k^3}{d_k^1 + \varepsilon} \quad \text{and} \quad \alpha_{\bar{k}}^{\mathrm{pm}} = \frac{d_k^2 + d_k^4}{d_k^1 + \varepsilon} \quad k \in \mathcal{K}, \quad \text{associated } \forall \bar{k} \tag{9}$$

For the overall equilibrium model, existence of a solution can be established as long as α_k^{am} and α_k^{pm} are continuous functions of the commute combination demands d_k^j, thus of the commute cycle flows h_c^j. The above are examples of such functions.

Based on the pair of averages $(\alpha_k^{\text{am}}, \alpha_{\bar{k}}^{\text{pm}})$ we propose a corresponding pair additions to the minimum rideshare fares. The averages are decision variables subject to the upper bound M. The under-capacity will translate into added payment to the minimum rideshare fee. We model this preliminary consideration by the following constraints:

$$\begin{aligned} 0 \le \gamma_k^{2;\text{am}} \left(M - \alpha_k^{\text{am}} \right) - \left(E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}} \right) \perp E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}} \ge 0 \ \forall\, k \in \mathcal{K} \\ 0 \le \gamma_{\bar{k}}^{2;\text{pm}} \left(M - \alpha_{\bar{k}}^{\text{pm}} \right) - \left(E_{\bar{k}}^{2;\text{pm}} - \underline{E}_{\bar{k}}^{2;\text{pm}} \right) \perp E_{\bar{k}}^{2;\text{pm}} - \underline{E}_{\bar{k}}^{2;\text{pm}} \ge 0 \ \forall\, \bar{k}, \end{aligned} \tag{10}$$

where $\perp$ is the perpendicularity notation, which in this context has several consequences for the morning and evening rideshares; we describe only the morning ones:

- if $\alpha_k^{\text{am}} = M$ (i.e., if rideshare is at capacity), then $E_k^{2;\text{am}} = \underline{E}_k^{2;\text{am}}$ (i.e., the payment is at its minimum);

- if $E_k^{2;\text{am}} > \underline{E}_k^{2;\text{am}}$ (i.e., if the payment exceeds its minimum), then $\alpha_k^{\text{am}} < M$ (i.e., rideshare must be below capacity) and, more importantly, $E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}} = \gamma_k^{2;\text{am}} \left(M - \alpha_k^{\text{am}} \right)$ (i.e., the addition to the minimum payment is a constant factor of the under-capacity);

- in the other two cases, that is, if $E_k^{2;\text{am}} = \underline{E}_k^{2;\text{am}}$, then rideshare may or may not be at capacity; similarly, if $\alpha_k^{\text{am}} < M$, then rideshare payment may be equal to or exceed the minimum.

An additional consequence of the above complementarity conditions is that the addition to the minimum payment is bounded above by a constant multiplicative factor of the rideshare under-capacity so that the total payment will not be unreasonably high. Unfortunately, given α_k^{am} and α_k^{am}, the complementarity conditions (10) do not determine the additional payments over the minimum payments uniquely. To achieve this uniqueness, we postulate the following models to determine the additional payments, given the average α_k^{am} of rideshare passengers,

$$\begin{aligned} &\underset{E_k^{2;\text{am}}}{\text{minimize}} \;\; \tfrac{1}{2}\left(E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}}\right)^2 - \left(E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}}\right)\left[\gamma_k^{2;\text{am}}\left(M - \alpha_k^{\text{am}}\right)\right] \\ &\text{subject to } 0 \leq E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}} \leq \gamma_k^{2;\text{am}}\left(M - \alpha_k^{\text{am}}\right). \end{aligned} \tag{11}$$

In essence, this yields

$$E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}} = \gamma_k^{2;\text{am}}\left(M - \alpha_k^{\text{am}}\right), \quad \text{if } \alpha_k^{\text{am}} \leq M, \tag{12}$$

which essentially fixes the excess payment to be equal to the multiplicative factor $\gamma_k^{2;\text{am}}$ times the under-capacity when the ratio α_k^{am}, which is a decision variable of the model, satisfies the upper bound M. Since such a bound is guaranteed to be satisfied only through a solution of the model, we need to impose the bound explicitly in defining the model. Thus instead of using (12) directly, we employ the Karush-Kuhn-Tucker optimality conditions of the above simple bounded quadratic program (11) which we write in the form of the following complementarity conditions, where $\lambda_k^{2;\text{am}}$ is a multiplier for the upper bound constraint of $E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}}$ in this program:

$$\begin{aligned} &0 \leq \gamma_k^{2;\text{am}}\left(M - \alpha_k^{\text{am}}\right) - \left(E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}}\right) && \perp \lambda_k^{2;\text{am}} \geq 0 \\ &0 \leq \left(E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}}\right) - \gamma_k^{2;\text{am}}\left(M - \alpha_k^{\text{am}}\right) + \lambda_k^{2;\text{am}} && \perp E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}} \geq 0, \end{aligned} \tag{13}$$

where $\perp$ is the perpendicularity notation, which in this context asserts the complementary slackness condition of the quadratic program (11). From quadratic programming and linear complementarity theory (Cottle et al., 2009), we know that the unique $E_k^{2;\text{am}}$ satisfying the above complementarity conditions is a piecewise affine, thus a Lipschitz continuous function of α_k^{am}, and thus of the commute cycle flows h_c^j. A similar set of pm conditions is as follows:

$$\begin{aligned} &0 \leq \gamma_{\bar{k}}^{2;\text{pm}}\left(M - \alpha_{\bar{k}}^{\text{pm}}\right) - \left(E_{\bar{k}}^{2;\text{pm}} - \underline{E}_{\bar{k}}^{2;\text{pm}}\right) && \perp \lambda_{\bar{k}}^{2;\text{pm}} \geq 0 \\ &0 \leq \left(E_{\bar{k}}^{2;\text{pm}} - \underline{E}_{\bar{k}}^{2;\text{pm}}\right) - \gamma_{\bar{k}}^{2;\text{pm}}\left(M - \alpha_{\bar{k}}^{\text{pm}}\right) + \lambda_{\bar{k}}^{2;\text{pm}} && \perp E_{\bar{k}}^{2;\text{pm}} - \underline{E}_{\bar{k}}^{2;\text{pm}} \geq 0, \end{aligned} \tag{14}$$

3.6. User equilibrium

We apply the user equilibrium principle that describes a complementary relation between the daily commute flows and the travelers' minimum disutiltities; it is based on the combined morning-evening round trips, allowing switches of commute types. This type of equilibrium distinguishes itself from the separate morning equilibrium and evening equilibrium. Yet the disutilities pertain to each OD pair k and the flows h_c^j of all the cycles $c \in \mathcal{C}_k$ joining that OD pair and across the 3 commute types. That is to say, for each OD pair k, the chosen mode combinations $c \in \mathcal{C}_k$ joining this OD pair among the 3 types in Figure 1 will all have travel costs equal to the least disutility of the OD pair in question, and this common cost is the smaller than the travel costs of the unchosen mode combinations joining the same OD pair. This is exactly Wardrop's user equilibrium principle for the commute cycle flows instead of the path flows in a traditional traffic equilibrium problem. This equilibrating process incorporates the switches of commuter types between morning and evening trips.

Thus the user equilibrium conditions for the combined morning and evening commutes among the 5 mode combination types are:

$$0 \leq h_c^j \perp \pi_c^j - u_k \geq 0, \quad \forall\, j \in \{1, \cdots, 5\}; \quad \forall\, k \in \mathcal{K} \text{ and } \forall\, c \in \mathcal{C}_k,$$

where $\perp$ is the perpendicularity notation, which in this context asserts the complementarity between the commute cycle flows and the travelers' deviations from the minimum disutilities. In words, if a traveler chooses the combination $c \in \mathcal{C}_k$, then the cycle cost/disutility of this combination must be the minimum of all costs for this OD pair k.

3.7. The overall equilibrium model

In this section, we summarize the aforementioned sections and develop a general equilibrium model to capture the complicated interactions between drivers, rideshare passengers and e-hailing passengers that allows travelers to switch between different transportation modes in a coupled morning-evening commute. The model is formulated as a mixed complementarity problem as follows.

$$0 \leq h_c^j \perp \pi_c^j - u_k \geq 0, \quad \forall\, j \in \{1, \cdots, 5\}; \quad \forall\, k \in \mathcal{K} \text{ and } \forall\, c \in \mathcal{C}_k, \tag{15}$$

$$d_k = \sum_{j=1}^{5} d_k^j = \sum_{j=1}^{5} \sum_{c \in \mathcal{C}_k} h_c^j, \quad \forall\, k \in \mathcal{K} \tag{16}$$

$$\begin{aligned}
&0 \leq \gamma_k^{2;\text{am}} \left(M - \alpha_k^{\text{am}} \right) - \left(E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}} \right) && \perp \lambda_k^{\text{am}} \geq 0 && \forall k \in \mathcal{K} \\
&0 \leq \left(E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}} \right) - \gamma_k^{2;\text{am}} \left(M - \alpha_k^{\text{am}} \right) + \lambda_k^{\text{am}} && \perp E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}} \geq 0 && \forall k \in \mathcal{K} \\
&0 \leq \gamma_{\bar{k}}^{2;\text{pm}} \left(M - \alpha_{\bar{k}}^{\text{pm}} \right) - \left(E_{\bar{k}}^{2;\text{pm}} - \underline{E}_{\bar{k}}^{2;\text{pm}} \right) && \perp \lambda_{\bar{k}}^{\text{pm}} \geq 0 && \text{associated } \forall \bar{k} \\
&0 \leq \left(E_{\bar{k}}^{2;\text{pm}} - \underline{E}_{\bar{k}}^{2;\text{pm}} \right) - \gamma_{\bar{k}}^{2;\text{pm}} \left(M - \alpha_{\bar{k}}^{\text{pm}} \right) + \lambda_{\bar{k}}^{\text{pm}} && \perp E_{\bar{k}}^{2;\text{pm}} - \underline{E}_{\bar{k}}^{2;\text{pm}} \geq 0 && \text{associated } \forall \bar{k}
\end{aligned} \tag{17}$$

where constraints (15) are the user equilibrium conditions for the combined morning and evening commutes among the 5 mode combination types; constraints (16) are the flow conservation equations to balance total trip demands with commute cycle flows and ensure morning trip demands equal to evening trip demands; constraints (17) ensure solution uniqueness for $E_k^{2;\text{am}}$ and $E_k^{2;\text{pm}}$ if α_k^{am} and $\alpha_{\bar{k}}^{\text{pm}}$ are determined, and satisfy the rideshare capacity, i.e., number of rideshare passengers must be no larger than capacity for each private car times the number of drivers. Here λ_k^{am} and $\lambda_{\bar{k}}^{\text{pm}}$ are multipliers. Actually, constraints (17) are equivalent to constraints (18) as follows:

$$\begin{aligned}
&E_k^{2;\text{am}} - \underline{E}_k^{2;\text{am}} = \gamma_k^{2;\text{am}} \left(M - \alpha_k^{\text{am}} \right) \quad \text{if } \alpha_k^{\text{am}} \leq M \quad \forall\, k \in \mathcal{K} \\
&E_{\bar{k}}^{2;\text{pm}} - \underline{E}_{\bar{k}}^{2;\text{pm}} = \gamma_{\bar{k}}^{2;\text{pm}} \left(M - \alpha_{\bar{k}}^{\text{pm}} \right) \quad \text{if } \alpha_{\bar{k}}^{\text{pm}} \leq M \quad \text{associated } \forall \bar{k}
\end{aligned} \tag{18}$$

4. Model Analysis

In this section, properties of our model are analyzed. First, we show that the proposed (mixed) complementarity formulation is equivalent to a variational inequality, and prove the existence of an equilibrium. Then we prove the relationship between the proposed coupled morning-evening traffic equilibrium model and morning (evening) commute problem. Finally, we present some conditions where in equilibrium the travelers' mode choices are unique.

4.1. Existence of an equilibrium

The primary decision variables of the proposed model are:

- **commute cycle flows**: $\{\, h_c^j \mid j = 1, \cdots, 5 \,\}_{c\in\mathcal{C}}$;
- **average number of rideshare passengers**: $\left\{ \alpha_k^{\mathrm{am}}; \alpha_{\bar{k}}^{\mathrm{pm}} \right._{k\in\mathcal{K}}$;
- **unit rideshare payments**: $\left\{ E_k^{2;\mathrm{am}}; E_{\bar{k}}^{2;\mathrm{pm}} \right._{k\in\mathcal{K}}$;
- **least travel disutilities of OD pairs**: $\{u_k\}_{k\in\mathcal{K}}$.

Among the above variables, the basic ones are the commute cycle flows h_c^j and travel disutilities of the OD pairs. After substituting the flow variables into the expressions of $\left\{ \alpha_k^{\mathrm{am}}; \alpha_{\bar{k}}^{\mathrm{pm}} \right._{k\in\mathcal{K}}$ and also $\left\{ E_k^{2;\mathrm{am}}; E_{\bar{k}}^{2;\mathrm{pm}} \right._{k\in\mathcal{K}}$ we can in turn substitute the latter variables into the rideshare cost functions $C_c^{1;\mathrm{am}}(h)$, $C_c^{1;\mathrm{pm}}(h)$, $C_c^{2;\mathrm{am}}(h)$, and $C_c^{2;\mathrm{pm}}(h)$. The end result is that all the morning and evening commute cycle cost functions $C_c^{i;\mathrm{am}}$ and $C_c^{i;\mathrm{pm}}$ for $i = 1, \cdots, 3$ can be expressed as continuous functions of the flow variables h_c^j. In summarizing the complementarity conditions below, it is understood that all these substitutions are made. This results in the following two sets of conditions for our proposed mathematical model for the combined morning and evening commute user equilibrium problem:

- for all $k \in \mathcal{K}$, all $c \in \mathcal{C}_k$, and all $j = 1, \cdots, 5$ with $u = j_{\mathrm{am}}$ and $v = j_{\mathrm{pm}}$,

$$0 \le h_c^j \perp \underbrace{C_c^{u;\mathrm{am}}(h) + C_c^{v;\mathrm{pm}}(h)}_{\text{denoted } \pi_c^j(h)} - u_k \ge 0, \quad \text{for all } c \in \mathcal{C}_k$$

- for all $k \in \mathcal{K}$,

$$d_k = \sum_{j=1}^{5} \sum_{c \in \mathcal{C}_k} h_c^j.$$

Considering the variable u_k as the multiplier of the OD-demand balancing constraints, this (mixed) complementarity formulation is equivalent to a variational inequality (VI) (Facchinei and Pang, 2003) defined by the pair of mapping Φ and polyhedral set $\mathcal{H}$ as follows:

$$\begin{aligned}
\Phi(h) &\triangleq \left(\pi_c^j(h)\right)_{(c,j)\in\mathcal{C}\times\{1,\cdots,5\}} \\
\mathcal{H} &\triangleq \left\{ \left(h_c^j\right)_{(c,j)\in\mathcal{C}\times\{1,\cdots,5\}} \geq 0 \mid \sum_{j=1}^{5} \sum_{c\in\mathcal{C}_k} h_c^j = d_k, \quad \forall\, k \in \mathcal{K} \right\}.
\end{aligned}$$

Since the maping Φ is continuous and the set $\mathcal{H}$ is compact and convex, it follows VI $(\Phi, \mathcal{H})$ has a solution, thus so does our combined morning-evening commute model with mode switches.

4.2. Relationship with traditional models

Main highlights of the proposed model include: (1) different from the morning commute problem (Xiao et al., 2016; Ma and Zhang, 2017; Liu and Li, 2017; Su and Wang, 2019; Lin et al., 2020), our model handles coupled morning-evening commute; (2) instead of using a discrete choice model (Ben-Akiva and Lerman, 1985; Train, 2009), the number of travelers for each commuter type is derived directly from our model, as the result of the user equilibrium conditions. In this section, we show that our proposed coupled morning-evening traffic equilibrium model could produce morning (evening) commute equilibrium. Besides, the equilibrium of our model also leads to rational traveler behavior, which means that none of the travel mode combinations with higher cost will be selected. What's more, we prove that under a mild condition, the proposed model is equivalent to two separate traditional traffic equilibrium models.

To prove the aforementioned statements, we first derive an extended network for our problem. For each traveler, the total cost for the coupled morning-evening commute consists of three parts: (1) congestion cost on a selected path (consists of arcs) from home to work place in the morning; (2) specified cost for a selected travel mode combination, including the cost of travel mode from home to work place in the morning and the cost of travel mode from work place to home in the evening; (3) congestion cost on a selected path (consists of arcs) from work place to home in the evening. As shown in Fig. 3, we construct five virtual arcs representing the five travel mode combinations (am driver + pm driver, am rideshare passenger + pm rideshare passenger, am rideshare passenger + pm e-hailing passenger, am e-hailing passenger + pm rideshare passenger, am e-hailing passenger + pm e-hailing passenger), with the specified cost (travel cost due to congestion excluded) for relevant mode combination type as the cost function. To reach or leave the work place, each traveler must choose one of the five virtual arcs, which means that each traveler need to choose one mode combination for traveling. With the virtual arcs, now we obtain the extended network for our problem, as shown in Fig. 3.

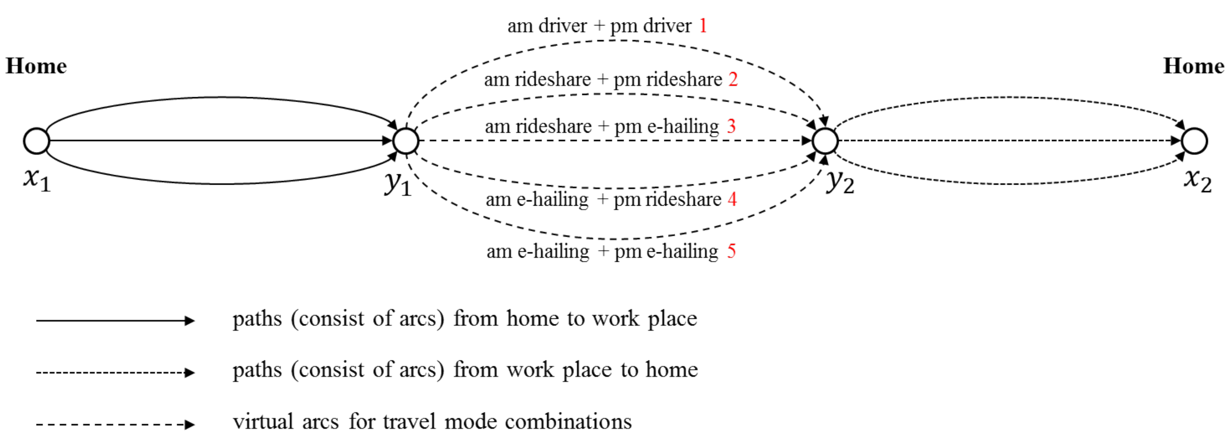

Figure 3. The extended network for our problem

Next, we redefine the cost functions for mode combination types, π_c^j. Here we define the problem of traveling from x_1 to y_1 in Fig. 3 as the **morning commute problem**, and the problem of traveling from y_2 to x_2 in Fig. 3 as the **evening commute problem**. Note that each cycle $c \in \mathcal{C}$ includes two paths: the morning path p_c^{am} with path flow $P_{p_c^{\mathrm{am}}}^{\mathrm{am}}$ and the afternoon path p_c^{pm} with path flow $P_{p_c^{\mathrm{pm}}}^{\mathrm{pm}}$. Denote the total cost for the morning commute problem as $\theta_{p_c^{\mathrm{am}}}^{\mathrm{am}}$, and the total cost for the evening commute problem as $\theta_{p_c^{\mathrm{pm}}}^{\mathrm{pm}}$, then we have,

$$\theta_{p_c^{\mathrm{am}}}^{\mathrm{am}} = \psi \times \sum_{a \in \mathcal{A}} \left(\delta_{a;p_c^{\mathrm{am}}} \times c_a^{\mathrm{am}}(f^{\mathrm{am}}) \right) \quad \forall c \in \mathcal{C} \tag{19}$$

$$\theta^{\mathrm{pm}}_{p^{\mathrm{pm}}_c} = \psi \times \sum_{a\in\mathcal{A}} \left(\delta_{a;p^{\mathrm{pm}}_c} \times c^{\mathrm{pm}}_a(f^{\mathrm{pm}}) \right) \quad \forall c \in \mathcal{C} \tag{20}$$

Denote the specified cost for travel mode combination j and commute cycle c as η^j_c, which is the cost from y_1 to y_2 in Fig. 3. Then the total cost of each mode combination and each commute cycle can be represented as

$$\pi^j_c = \theta^{\mathrm{am}}_{p^{\mathrm{am}}_c} + \eta^j_c + \theta^{\mathrm{pm}}_{p^{\mathrm{pm}}_c} \quad \forall c \in \mathcal{C}, \quad \forall j \in \{1, ..., 5\} \tag{21}$$

Section 4.1 guarantees the existence of an equilibrium for the proposed coupled morning-evening commute model. In Theorem 1 below, we show the properties of the proposed model when it reaches an equilibrium. Notate u^{am}_k and u^{pm}_k as the least disutility of OD pair $k \in \mathcal{K}$ for the morning commute problem and the evening commute problem, respectively; notate $\bar{u}_k$ as the least disutility of OD pair $k \in \mathcal{K}$ for the mode combinations $j \in \{1, \ldots, 5\}$. Here we define that the least cycle disutility u_k is **decomposable** if it can be decomposed into the least disutility for the morning commute problem u^{am}_k, the least disutility for the evening commute problem u^{pm}_k, and the least disutility for mode combinations $\bar{u}_k$, i.e., $u_k = u^{\mathrm{am}}_k + \bar{u}_k + u^{\mathrm{pm}}_k$.

Theorem 1. *Assume that the least cycle disutility is decomposable. When the coupled morning-evening commute problem reaches an equilibrium, we have the following properties:*

(1) The morning commute problem reaches an equilibrium;

(2) Given travelers' mode choice is fixed, the morning commute problem is equivalent to a traditional traffic equilibrium problem;

(3) The evening commute problem also has properties (1) and (2);

(4) Travelers are rational to mode choice, which means that no traveler will choose a more expensive travel mode combination. It coincides with the basic assumption of economic consumer theory (Mas-Colell et al., 1995).

Proof.

(1) As described in Section 3.6, the user equilibrium conditions for the coupled morning-evening commute problem are:

$$0 \leq h_c^j \perp \pi_c^j - u_k \geq 0, \quad \forall j \in \{1, \cdots, 5\}; \quad \forall k \in \mathcal{K} \text{ and } \forall c \in \mathcal{C}_k,$$

which, by definition, is equivalent to

$$\begin{cases} \text{if} \quad \pi_c^j - u_k = 0, \quad \text{then} \quad h_c^j \geq 0 \\ \text{if} \quad \pi_c^j - u_k > 0, \quad \text{then} \quad h_c^j = 0 \end{cases}$$

Since $u_k = u_k^{\text{am}} + \bar{u}_k + u_k^{\text{pm}}$, the user equilibrium conditions for the coupled morning-evening commute problem can be written as:

$$0 \leq h_c^j \perp (\theta_{p_c^{\text{am}}}^{\text{am}} - u_k^{\text{am}}) + (\eta_c^j - \bar{u}_k) + (\theta_{p_c^{\text{pm}}}^{\text{pm}} - u_k^{\text{pm}}) \geq 0, \quad \forall j \in \{1, \cdots, 5\}; \quad \forall k \in \mathcal{K} \text{ and } \forall c \in \mathcal{C}_k,$$

Assume that the morning commute problem doesn't reach an equilibrium, which means that $\exists \theta_{p_c^{\text{am}}}^{\text{am}} - u_k^{\text{am}} > 0$ such that the path flow $P_{p_c^{\text{am}}}^{\text{am}} > 0$, i.e., the relevant cycle flow $h_c^j > 0$. Since we have $\eta_c^j - \bar{u}_k \geq 0$ and $\theta_{p_c^{\text{pm}}}^{\text{pm}} - u_k^{\text{pm}} \geq 0$, there must exist $(\theta_{p_c^{\text{am}}}^{\text{am}} - u_k^{\text{am}}) + (\eta_c^j - \bar{u}_k) + (\theta_{p_c^{\text{pm}}}^{\text{pm}} - u_k^{\text{pm}}) > 0$ such that $h_c^j > 0$, i.e.,

$$\text{if} \quad \pi_c^j - u_k > 0, \quad \text{then} \quad h_c^j > 0$$

which means that the coupled morning-evening commute problem also doesn't reach an equilibrium. Contradiction happens. As a result, the morning commute problem must reach an equilibrium.

(2) From (1) we know that when the coupled morning-evening commute problem reaches an equilibrium, the morning commute problem also reaches an equilibrium. Denote u_k^{am} as the (least) disutility of OD pair $k \in \mathcal{K}$ for the morning commute problem, the equilibrium of morning commute problem can be written as:

$$0 \leq P_{p_c^{\text{am}}}^{\text{am}} \perp \theta_{p_c^{\text{am}}}^{\text{am}} - u_k^{\text{am}} \geq 0, \quad \forall k \in \mathcal{K} \text{ and } \forall c \in \mathcal{C}_k,$$

where p_c^{am} is the morning path of cycle c, with relevant path flow P_c^{am} and path cost $\theta_{p_c^{\text{am}}}^{\text{am}}$.

Since travelers' mode choice is fixed, we have

$$\sum_{c\in\mathcal{C}_k} P^{\text{am}}_{p^{\text{am}}_c} = d^1_k + d^4_k + d^5_k, \quad \forall\, k \in \mathcal{K}$$

Since $\delta_{a;p^{\text{am}}_c}$ is the arc-path incidence matrix in the morning, the morning flow on arc a, notate as f^{am}_a, can be calculated as

$$f^{\text{am}}_a = \sum_{c\in\mathcal{C}} \left(P^{\text{am}}_{p^{\text{am}}_c} \times \delta_{a;p^{\text{am}}_c} \right), \quad \forall\, a \in \mathcal{A}$$

Then the path cost $\theta^{\text{am}}_{p^{\text{am}}_c}$ can be written as

$$\theta^{\text{am}}_{p^{\text{am}}_c} = \psi \times \sum_{a\in\mathcal{A}} \left(\delta_{a;p^{\text{am}}_c} \times c^{\text{am}}_a(f^{\text{am}}_a) \right) \quad \forall\, c \in \mathcal{C}$$

Actually, the formulation above is the same as the standard traffic equilibrium. Based on section 1.4.5 of Facchinei and Pang (2007), we can conclude that the morning arc flows f^{am}_a are unique. The details about the proof are as follows: the path cost $\theta^{\text{am}}_{p^{\text{am}}_c}$ is monotone-plus since it is a VI function with the composition of the following kind: $\Phi(h) = A^T\varphi(Ah)$, for some (rectangular) matrix A and (square) mapping φ that is strictly monotone. In our case, A is the arc-path incidence matrix, φ is the vector function of arc costs, h is the vector of path flows, and Φ is the vector function of path costs. Since the arc costs are separable, then c^{am}_a is strictly monotone, and $\theta^{\text{am}}_{p^{\text{am}}_c}$ becomes monotone. By these properties, $f^{\text{am}}_a = \sum_{c\in\mathcal{C}} \left(P^{\text{am}}_{p^{\text{am}}_c} \times \delta_{a;p^{\text{am}}_c} \right)$, which are the morning arc flows, are unique.

(3) Similar to the proof in (1) and (2), we can prove the same properties for the evening commute problem.

(4) Similar to the proof in (1), we can prove that the sub-problem from y_1 to y_2 in Fig. 3 also reaches an equilibrium. Since d^j_k is the travel demand of OD pair $k \in \mathcal{K}$ of mode combination type $j \in \{1, ..., 5\}$, and $\bar{u}_k$ represents the smallest cost of OD pair $k \in \mathcal{K}$ and travel mode combination $j \in \{1, \ldots, 5\}$, by definition of equilibrium, we have,

$$\begin{cases} \text{if} \quad \eta^j_c - \bar{u}_k = 0, \quad \text{then} \quad d^j_k \geq 0 \\ \text{if} \quad \eta^j_c - \bar{u}_k > 0, \quad \text{then} \quad d^j_k = 0 \quad \text{(also, the relevant} \quad h^j_c = 0) \end{cases}$$

which means that travelers will not choose a travel mode combination that is more expensive. Thus, they are rational to mode choice. □

4.3. The conditions of unique travelers' mode choice

In Section 4.2, we proved that given travelers' mode choice is fixed, the proposed coupled morning-evening traffic equilibrium model is equivalent to two separate traditional traffic equilibrium models. In this section, we provide the conditions under which traveler's mode choice will be unique.

THEOREM 2. *Assume that the least cycle disutility is decomposable. When the coupled morning-evening commute problem reaches an equilibrium, travelers' mode choice (determined by decision variables d_k^j) is unique under any one of the following conditions:*

(1) $O_k^{1;\mathrm{am}}, O_{\bar{k}}^{1;\mathrm{pm}}, I_k^{1;\mathrm{am}}, I_{\bar{k}}^{1;\mathrm{pm}} \ll W_k^{2;\mathrm{am}}, W_{\bar{k}}^{2;\mathrm{pm}}, I_k^{2;\mathrm{am}}, I_{\bar{k}}^{2;\mathrm{pm}}, W_k^{3;\mathrm{am}}, W_{\bar{k}}^{3;\mathrm{pm}}, I_k^{3;\mathrm{am}}, I_{\bar{k}}^{3;\mathrm{pm}}, E_k^{3;\mathrm{am}}, E_{\bar{k}}^{3;\mathrm{pm}}$*;*

(2) $W_k^{3;\mathrm{am}}, W_{\bar{k}}^{3;\mathrm{pm}}, I_k^{3;\mathrm{am}}, I_{\bar{k}}^{3;\mathrm{pm}}, E_k^{3;\mathrm{am}}, E_{\bar{k}}^{3;\mathrm{pm}} \ll O_k^{1;\mathrm{am}}, O_{\bar{k}}^{1;\mathrm{pm}}, I_k^{1;\mathrm{am}}, I_{\bar{k}}^{1;\mathrm{pm}}, W_k^{2;\mathrm{am}}, W_{\bar{k}}^{2;\mathrm{pm}}, I_k^{2;\mathrm{am}}, I_{\bar{k}}^{2;\mathrm{pm}}$*;*

(3) Given α_k^{am} *and* $\alpha_{\bar{k}}^{\mathrm{pm}}$ *are known constants, and* $O_k^{1;\mathrm{am}}, O_{\bar{k}}^{1;\mathrm{pm}}, I_k^{1;\mathrm{am}}, I_{\bar{k}}^{1;\mathrm{pm}}, W_k^{2;\mathrm{am}}, W_{\bar{k}}^{2;\mathrm{pm}}, I_k^{2;\mathrm{am}}, I_{\bar{k}}^{2;\mathrm{pm}} \ll W_k^{3;\mathrm{am}}, W_{\bar{k}}^{3;\mathrm{pm}}, I_k^{3;\mathrm{am}}, I_{\bar{k}}^{3;\mathrm{pm}}, E_k^{3;\mathrm{am}}, E_{\bar{k}}^{3;\mathrm{pm}}$*;*

(4) Given α_k^{am} *and* $\alpha_{\bar{k}}^{\mathrm{pm}}$ *are known constants, and* $O_k^{1;\mathrm{am}}, I_k^{1;\mathrm{am}}, W_k^{2;\mathrm{am}}, I_k^{2;\mathrm{am}} \ll W_k^{3;\mathrm{am}}, I_k^{3;\mathrm{am}}, E_k^{3;\mathrm{am}}$ *and* $O_{\bar{k}}^{1;\mathrm{pm}}, I_{\bar{k}}^{1;\mathrm{pm}}, W_{\bar{k}}^{3;\mathrm{pm}}, I_{\bar{k}}^{3;\mathrm{pm}}, E_{\bar{k}}^{3;\mathrm{pm}} \ll W_{\bar{k}}^{2;\mathrm{pm}}, I_{\bar{k}}^{2;\mathrm{pm}}$*;*

(5) Given α_k^{am} *and* $\alpha_{\bar{k}}^{\mathrm{pm}}$ *are known constants, and* $O_k^{1;\mathrm{am}}, I_k^{1;\mathrm{am}}, W_k^{3;\mathrm{am}}, I_k^{3;\mathrm{am}}, E_k^{3;\mathrm{am}} \ll W_k^{2;\mathrm{am}}, I_k^{2;\mathrm{am}}$ *and* $O_{\bar{k}}^{1;\mathrm{pm}}, I_{\bar{k}}^{1;\mathrm{pm}}, W_{\bar{k}}^{2;\mathrm{pm}}, I_{\bar{k}}^{2;\mathrm{pm}} \ll W_{\bar{k}}^{3;\mathrm{pm}}, I_{\bar{k}}^{3;\mathrm{pm}}, E_{\bar{k}}^{3;\mathrm{pm}}$*;*

(6) Given α_k^{am} *and* $\alpha_{\bar{k}}^{\mathrm{pm}}$ *are known constants, and* $O_k^{1;\mathrm{am}}, I_k^{1;\mathrm{am}}, W_k^{2;\mathrm{am}}, I_k^{2;\mathrm{am}} \ll W_k^{3;\mathrm{am}}, I_k^{3;\mathrm{am}}, E_k^{3;\mathrm{am}}$*;*

(7) Given α_k^{am} *and* $\alpha_{\bar{k}}^{\mathrm{pm}}$ *are known constants, and* $O_{\bar{k}}^{1;\mathrm{pm}}, I_{\bar{k}}^{1;\mathrm{pm}}, W_{\bar{k}}^{2;\mathrm{pm}}, I_{\bar{k}}^{2;\mathrm{pm}} \ll W_{\bar{k}}^{3;\mathrm{pm}}, I_{\bar{k}}^{3;\mathrm{pm}}, E_{\bar{k}}^{3;\mathrm{pm}}$*.*

Proof.

Here we prove condition (6) as an example, and the proofs for the other conditions are similar. From Theorem 1(4) we know that when the coupled morning-evening commute problem reaches an equilibrium, the sub-problem from y_1 to y_2 in Fig. 3 also reaches an equilibrium. By definition of equilibrium, we have,

$$\begin{cases} \text{if} \quad \eta_c^j - \bar{u}_k = 0, \quad \text{then} \quad d_k^j \geq 0 \\ \text{if} \quad \eta_c^j - \bar{u}_k > 0, \quad \text{then} \quad d_k^j = 0 \quad (\text{also, the relevant} \quad h_c^j = 0) \end{cases}$$

Since $O_k^{1;\mathrm{am}}, I_k^{1;\mathrm{am}}, W_k^{2;\mathrm{am}}, I_k^{2;\mathrm{am}} \ll W_k^{3;\mathrm{am}}, I_k^{3;\mathrm{am}}, E_k^{3;\mathrm{am}}$, and η_c^4, η_c^5 consist of $W_k^{3;\mathrm{am}}, I_k^{3;\mathrm{am}}, E_k^{3;\mathrm{am}}$ while $\eta_c^1, \eta_c^2, \eta_c^3$ don't, thus we have η_c^1, η_c^2, $\eta_c^3 \ll \eta_c^4, \eta_c^5$. According to the definition of equilibrium above, we have $d_k^4 = d_k^5 = 0$ and $d_k^1 \geq 0$, $d_k^2 \geq 0$, $d_k^3 \geq 0$.

For each $k \in \mathcal{K}$ and relevant $\bar{k}$, the linear equation system that α_k^{am}, $\alpha_{\bar{k}}^{\mathrm{pm}}$, d_k^j must satisfy in the proposed model is as follows:

$$\begin{cases} d_k^1 + d_k^2 + d_k^3 + d_k^4 + d_k^5 = d_k \\ \alpha_k^{\mathrm{am}} d_k^1 - d_k^2 - d_k^3 = -\alpha_k^{\mathrm{am}} \varepsilon \\ \alpha_{\bar{k}}^{\mathrm{pm}} d_k^1 - d_k^2 - d_k^4 = -\alpha_{\bar{k}}^{\mathrm{pm}} \varepsilon \end{cases}$$

Since $d_k^4 = d_k^5 = 0$, we have

$$\begin{cases} d_k^1 + d_k^2 + d_k^3 = d_k \\ \alpha_k^{\mathrm{am}} d_k^1 - d_k^2 - d_k^3 = -\alpha_k^{\mathrm{am}} \varepsilon \\ \alpha_{\bar{k}}^{\mathrm{pm}} d_k^1 - d_k^2 = -\alpha_{\bar{k}}^{\mathrm{pm}} \varepsilon \end{cases}$$

Under condition (6) we have α_k^{am} and $\alpha_{\bar{k}}^{\mathrm{pm}}$ are known constants, thus the determinant of this linear equation system is

$$\begin{matrix} 1 & 1 & 1 \\ \alpha_k^{\mathrm{am}} & -1 & -1 \\ \alpha_{\bar{k}}^{\mathrm{pm}} & -1 & 0 \end{matrix} = -1 - \alpha_k^{\mathrm{am}} \neq 0$$

Moreover, we have

$$d_k^1 = \frac{d_k - \alpha_k^{\mathrm{am}} \varepsilon}{1 + \alpha_k^{\mathrm{am}}} \geq 0$$

$$d_k^2 = \frac{\alpha_{\bar{k}}^{\mathrm{pm}} (d_k + \varepsilon)}{1 + \alpha_k^{\mathrm{am}}} \geq 0$$

$$d_k^3 = \frac{(\alpha_k^{\mathrm{am}} - \alpha_{\bar{k}}^{\mathrm{pm}})(d_k + \varepsilon)}{1 + \alpha_k^{\mathrm{am}}} \geq 0$$

Thus, the decision variables d_k^j must be unique. □

In the conditions (3), (4), (5), (6), (7) of Theorem 2, we need the uniqueness of α_k^{am} and $\alpha_{\bar{k}}^{\mathrm{pm}}$. Below in Theorem 3, we give the conditions under which α_k^{am} and $\alpha_{\bar{k}}^{\mathrm{pm}}$ will be unique when Theorem 2 (6) is satisfied. Similarly, we can also give the conditions under which α_k^{am} and $\alpha_{\bar{k}}^{\mathrm{pm}}$ will be unique when we have Theorem 2 (3), (4), (5), or (7).

Let's define the specified cost functions for commuter types (driver, rideshare passengers, and e-hailing passengers). Since the congestion cost will not influence travelers' mode choice, here we define the specified cost functions for commuter types as the cost for choosing a travel mode, which means that the congestion cost is excluded. Denote the specified cost function for commuter type i and OD pair k as $\bar{C}_k^{i;\mathrm{am}}$ in the morning and $\bar{C}_{\bar{k}}^{i;\mathrm{pm}}$ in the evening, from Section 3.3(3) and Section 3.5, we have the specified cost functions for the three commuter types as follows,

a) Drivers:

$$\begin{aligned}
\bar{C}_k^{1;\mathrm{am}}(\alpha^{\mathrm{am}}) &= O_k^{1;\mathrm{am}} + \left(I_k^{1;\mathrm{am}} - \underline{E}_k^{2;\mathrm{am}} - M \times \gamma_k^{2;\mathrm{am}}\right) \times \underbrace{\alpha_k^{2;\mathrm{am}}}_{\text{a variable}} + \gamma_k^{2;\mathrm{am}} \times \underbrace{\left(\alpha_k^{2;\mathrm{am}}\right)^2}_{\text{a variable}} \\
&\quad \forall\, c \in \mathcal{C}_k \quad \forall\, k \in \mathcal{K} \\
\bar{C}_{\bar{k}}^{1;\mathrm{pm}}(\alpha^{\mathrm{pm}}) &= O_{\bar{k}}^{1;\mathrm{pm}} + \left(I_{\bar{k}}^{1;\mathrm{pm}} - \underline{E}_{\bar{k}}^{2;\mathrm{pm}} - M \times \gamma_{\bar{k}}^{2;\mathrm{pm}}\right) \times \underbrace{\alpha_{\bar{k}}^{2;\mathrm{pm}}}_{\text{a variable}} + \gamma_{\bar{k}}^{2;\mathrm{pm}} \times \underbrace{\left(\alpha_{\bar{k}}^{2;\mathrm{pm}}\right)^2}_{\text{a variable}} \\
&\quad \forall\, c \in \mathcal{C}_{\bar{k}} \text{ and associated } \bar{k}
\end{aligned} \tag{22}$$

b) Rideshare passengers:

$$\begin{aligned}
\bar{C}_k^{2;\mathrm{am}}(\alpha^{\mathrm{am}}) &= W_k^{2;\mathrm{am}} + \underline{E}_k^{2;\mathrm{am}} + M \times \gamma_k^{2;\mathrm{am}} + (I_k^{2;\mathrm{am}} - \gamma_k^{2;\mathrm{am}}) \times \underbrace{\alpha_k^{2;\mathrm{am}}}_{\text{a variable}} \\
&\quad \forall\, c \in \mathcal{C}_k \quad \forall\, k \in \mathcal{K} \\
\bar{C}_{\bar{k}}^{2;\mathrm{pm}}(\alpha^{\mathrm{pm}}) &= W_{\bar{k}}^{2;\mathrm{pm}} + \underline{E}_{\bar{k}}^{2;\mathrm{pm}} + M \times \gamma_{\bar{k}}^{2;\mathrm{pm}} + (I_{\bar{k}}^{2;\mathrm{pm}} - \gamma_{\bar{k}}^{2;\mathrm{pm}}) \times \underbrace{\alpha_{\bar{k}}^{2;\mathrm{pm}}}_{\text{a variable}} \\
&\quad \forall\, c \in \mathcal{C}_{\bar{k}} \text{ and associated } \bar{k}
\end{aligned} \tag{23}$$

c) E-hailing passengers:

$$\begin{aligned}
\bar{C}_k^{3;\mathrm{am}} &= \underbrace{W_k^{3;\mathrm{am}} + I_k^{3;\mathrm{am}} + E_k^{3;\mathrm{am}}}_{\text{a constant}} \quad \forall\, c \in \mathcal{C}_k \quad \forall\, k \in \mathcal{K} \\
\bar{C}_{\bar{k}}^{3;\mathrm{pm}} &= \underbrace{W_{\bar{k}}^{3;\mathrm{pm}} + I_{\bar{k}}^{3;\mathrm{pm}} + E_{\bar{k}}^{3;\mathrm{pm}}}_{\text{a constant}} \quad \forall\, c \in \mathcal{C}_{\bar{k}} \text{ and associated } \bar{k}
\end{aligned} \tag{24}$$

Then the cost of each mode combination (the cost from y_1 to y_2 in Fig. 3) can be written as,

$$\eta_c^j = \bar{C}_k^{u;\mathrm{am}} + \bar{C}_{\bar{k}}^{v;\mathrm{pm}} \quad \begin{array}{l} \forall c \in \mathcal{C}_k, \quad \forall c \in \mathcal{C}_{\bar{k}}, \quad \forall k \in \mathcal{K} \text{ and associated } \bar{k}, \\ \forall j \in \{1, ..., 5\} \text{ with } u = j_{\mathrm{am}} \text{ and } v = j_{\mathrm{pm}} \end{array} \tag{25}$$

THEOREM 3. *Assume that the least cycle disutility is decomposable. When the coupled morning-evening commute problem reaches an equilibrium and Theorem 2 (6) is satisfied,* α_k^{am} *and* $\alpha_{\bar{k}}^{\mathrm{pm}}$ *are unique under the following conditions:*

(1) $\alpha_{\bar{k}}^{2;\mathrm{pm}} = \frac{W_{\bar{k}}^{3;\mathrm{pm}} + I_{\bar{k}}^{3;\mathrm{pm}} + E_{\bar{k}}^{3;\mathrm{pm}} - W_{\bar{k}}^{2;\mathrm{pm}} - \underline{E}_{\bar{k}}^{2;\mathrm{pm}} - M \times \gamma_{\bar{k}}^{2;\mathrm{pm}}}{I_{\bar{k}}^{2;\mathrm{pm}} - \gamma_{\bar{k}}^{2;\mathrm{pm}}} \in [0, M]$

(2) $M\gamma_k^{2;\mathrm{am}} + \underline{E}_k^{2;\mathrm{am}} + I_k^{2;\mathrm{am}} - I_k^{1;\mathrm{am}} - \gamma_k^{2;\mathrm{am}} > 0$

(3) $O_k^{1;\mathrm{am}} - W_k^{2;\mathrm{am}} - \underline{E}_k^{2;\mathrm{am}} - M\gamma_k^{2;\mathrm{am}} + \bar{C}_{\bar{k}}^{1;\mathrm{pm}}(\alpha^{\mathrm{pm}}) - \bar{C}_{\bar{k}}^{2;\mathrm{pm}}(\alpha^{\mathrm{pm}}) > 0$

(4) $(I_k^{1;\mathrm{am}} - I_k^{2;\mathrm{am}} - \underline{E}_k^{2;\mathrm{am}} + \gamma_k^{2;\mathrm{am}})M + O_k^{1;\mathrm{am}} - W_k^{2;\mathrm{am}} - \underline{E}_k^{2;\mathrm{am}} - M\gamma_k^{2;\mathrm{am}} + \bar{C}_{\bar{k}}^{1;\mathrm{pm}}(\alpha^{\mathrm{pm}}) - \bar{C}_{\bar{k}}^{2;\mathrm{pm}}(\alpha^{\mathrm{pm}}) < 0$

Proof.

From Theorem 1(4), we know that when the coupled morning-evening commute problem reaches an equilibrium, the sub-problem from y_1 to y_2 in Fig. 3 also reaches an equilibrium. According to Theorem 2(6), we have $d_k^4 = d_k^5 = 0$ and $d_k^1 \geq 0$, $d_k^2 \geq 0$, $d_k^3 \geq 0$. With loss of generality, let $\eta_c^1 - \bar{u}_k = \eta_c^2 - \bar{u}_k = \eta_c^3 - \bar{u}_k$ we have,

$$\eta_c^1 = \eta_c^2 = \eta_c^3$$

From $\eta_c^2 = \bar{C}_k^{2;\mathrm{am}}(\alpha^{\mathrm{am}}) + \bar{C}_{\bar{k}}^{2;\mathrm{pm}}(\alpha^{\mathrm{pm}}) = \eta_c^3 = \bar{C}_k^{2;\mathrm{am}}(\alpha^{\mathrm{am}}) + \bar{C}_{\bar{k}}^{3;\mathrm{pm}}(\alpha^{\mathrm{pm}})$, we have $\bar{C}_{\bar{k}}^{2;\mathrm{pm}}(\alpha^{\mathrm{pm}}) = \bar{C}_{\bar{k}}^{3;\mathrm{pm}}(\alpha^{\mathrm{pm}})$,

$$W_{\bar{k}}^{2;\mathrm{pm}} + \underline{E}_{\bar{k}}^{2;\mathrm{pm}} + M \times \gamma_{\bar{k}}^{2;\mathrm{pm}} + (I_{\bar{k}}^{2;\mathrm{pm}} - \gamma_{\bar{k}}^{2;\mathrm{pm}}) \times \alpha_{\bar{k}}^{2;\mathrm{pm}} = W_{\bar{k}}^{3;\mathrm{pm}} + I_{\bar{k}}^{3;\mathrm{pm}} + E_{\bar{k}}^{3;\mathrm{pm}}$$

$$\alpha_{\bar{k}}^{2;\mathrm{pm}} = \frac{W_{\bar{k}}^{3;\mathrm{pm}} + I_{\bar{k}}^{3;\mathrm{pm}} + E_{\bar{k}}^{3;\mathrm{pm}} - W_{\bar{k}}^{2;\mathrm{pm}} - \underline{E}_{\bar{k}}^{2;\mathrm{pm}} - M \times \gamma_{\bar{k}}^{2;\mathrm{pm}}}{I_{\bar{k}}^{2;\mathrm{pm}} - \gamma_{\bar{k}}^{2;\mathrm{pm}}} \in [0, M]$$

which means that

$$0 \leq W_{\bar{k}}^{3;\mathrm{pm}} + I_{\bar{k}}^{3;\mathrm{pm}} + E_{\bar{k}}^{3;\mathrm{pm}} - W_{\bar{k}}^{2;\mathrm{pm}} - \underline{E}_{\bar{k}}^{2;\mathrm{pm}} - M \times \gamma_{\bar{k}}^{2;\mathrm{pm}} \leq M \times (I_{\bar{k}}^{2;\mathrm{pm}} - \gamma_{\bar{k}}^{2;\mathrm{pm}})$$

Known the value of α^{pm}, from $\eta_c^1 = \eta_c^2$ we can derive $\bar{C}_k^{1;\mathrm{am}}(\alpha^{\mathrm{am}}) + \bar{C}_{\bar{k}}^{1;\mathrm{pm}}(\alpha^{\mathrm{pm}}) = \bar{C}_k^{2;\mathrm{am}}(\alpha^{\mathrm{am}}) + \bar{C}_{\bar{k}}^{2;\mathrm{pm}}(\alpha^{\mathrm{pm}})$, which is a quadratic equation for α^{am}. Define a quadratic function $f_k^{\mathrm{am}}(\alpha_k^{2;\mathrm{am}})$ for each OD pair $k \in \mathcal{K}$ as follows:

$$\begin{aligned}
f_k^{\mathrm{am}}(\alpha_k^{2;\mathrm{am}}) = \eta_c^1 - \eta_c^2 &= \bar{C}_k^{1;\mathrm{am}}(\alpha^{\mathrm{am}}) + \bar{C}_{\bar{k}}^{1;\mathrm{pm}}(\alpha^{\mathrm{pm}}) - \bar{C}_k^{2;\mathrm{am}}(\alpha^{\mathrm{am}}) - \bar{C}_{\bar{k}}^{2;\mathrm{pm}}(\alpha^{\mathrm{pm}}) \\
&= \gamma_k^{2;\mathrm{am}} \underbrace{\left(\alpha_k^{2;\mathrm{am}}\right)^2}_{\text{a variable}} + \left(I_k^{1;\mathrm{am}} - I_k^{2;\mathrm{am}} - \underline{E}_k^{2;\mathrm{am}} - M\gamma_k^{2;\mathrm{am}} + \gamma_k^{2;\mathrm{am}}\right) \underbrace{\alpha_k^{2;\mathrm{am}}}_{\text{a variable}} \\
&\quad + O_k^{1;\mathrm{am}} - W_k^{2;\mathrm{am}} - \underline{E}_k^{2;\mathrm{am}} - M\gamma_k^{2;\mathrm{am}} + \bar{C}_{\bar{k}}^{1;\mathrm{pm}}(\alpha^{\mathrm{pm}}) - \bar{C}_{\bar{k}}^{2;\mathrm{pm}}(\alpha^{\mathrm{pm}}) \\
&= \gamma_k^{2;\mathrm{am}} \left(\underbrace{\alpha_k^{2;\mathrm{am}}}_{\text{a variable}} - \frac{M\gamma_k^{2;\mathrm{am}} + \underline{E}_k^{2;\mathrm{am}} + I_k^{2;\mathrm{am}} - I_k^{1;\mathrm{am}} - \gamma_k^{2;\mathrm{am}}}{2\gamma_k^{2;\mathrm{am}}} \right)^2 \\
&\quad + O_k^{1;\mathrm{am}} - W_k^{2;\mathrm{am}} - \underline{E}_k^{2;\mathrm{am}} - M\gamma_k^{2;\mathrm{am}} + \bar{C}_{\bar{k}}^{1;\mathrm{pm}}(\alpha^{\mathrm{pm}}) - \bar{C}_{\bar{k}}^{2;\mathrm{pm}}(\alpha^{\mathrm{pm}}) \\
&\quad - \frac{(M\gamma_k^{2;\mathrm{am}} + \underline{E}_k^{2;\mathrm{am}} + I_k^{2;\mathrm{am}} - I_k^{1;\mathrm{am}} - \gamma_k^{2;\mathrm{am}})^2}{4\gamma_k^{2;\mathrm{am}}} \\
&\quad \forall\, k \in \mathcal{K} \text{ and associated } \bar{k}
\end{aligned}$$

To ensure $\alpha_k^{2;\mathrm{am}}$ to be unique, we need the quadratic function $f_k^{\mathrm{am}}(\alpha_k^{2;\mathrm{am}})$ to have only one solution within $[0, M]$. In order to have only one solution, the quadratic function $f_k^{\mathrm{am}}(\alpha_k^{2;\mathrm{am}})$ needs to satisfy:

$$\frac{M\gamma_k^{2;\mathrm{am}} + \underline{E}_k^{2;\mathrm{am}} + I_k^{2;\mathrm{am}} - I_k^{1;\mathrm{am}} - \gamma_k^{2;\mathrm{am}}}{2\gamma_k^{2;\mathrm{am}}} > 0$$

$$f_k^{\mathrm{am}}(0) > 0$$

$$f_k^{\mathrm{am}}(M) < 0$$

which means that

$$M\gamma_k^{2;\mathrm{am}} + \underline{E}_k^{2;\mathrm{am}} + I_k^{2;\mathrm{am}} - I_k^{1;\mathrm{am}} - \gamma_k^{2;\mathrm{am}} > 0$$

$$O_k^{1;\mathrm{am}} - W_k^{2;\mathrm{am}} - \underline{E}_k^{2;\mathrm{am}} - M\gamma_k^{2;\mathrm{am}} + \bar{C}_{\bar{k}}^{1;\mathrm{pm}}(\alpha^{\mathrm{pm}}) - \bar{C}_{\bar{k}}^{2;\mathrm{pm}}(\alpha^{\mathrm{pm}}) > 0$$

$$(I_k^{1;\mathrm{am}} - I_k^{2;\mathrm{am}} - \underline{E}_k^{2;\mathrm{am}} + \gamma_k^{2;\mathrm{am}})M + O_k^{1;\mathrm{am}} - W_k^{2;\mathrm{am}} - \underline{E}_k^{2;\mathrm{am}} - M\gamma_k^{2;\mathrm{am}} + \bar{C}_{\bar{k}}^{1;\mathrm{pm}}(\alpha^{\mathrm{pm}}) - \bar{C}_{\bar{k}}^{2;\mathrm{pm}}(\alpha^{\mathrm{pm}}) < 0$$

$\square$

5. Computational Results

In this section, we present computational experiments illustrating the importance of having a model that combines the morning and evening commute simultaneously. We study the sensitivity analysis of three parameters: (1) $\left\{ I_k^{2;\text{am}}; I_{\bar{k}}^{2;\text{pm}} \right\}_{k\in\mathcal{K}}$: the inconvenience for each rideshare passenger; (2) $\left\{ \gamma_k^{2;\text{am}}; \gamma_{\bar{k}}^{2;\text{pm}} \right\}_{k\in\mathcal{K}}$: the conversion factors of rideshare under-capacity to surcharge; (3) $\left\{ E_k^{3;\text{am}}; E_{\bar{k}}^{3;\text{pm}} \right\}_{k\in\mathcal{K}}$: unit payments for each e-hailing passenger. In addition, the outputs of the coupled model and the decoupled morning (or evening) commute model are compared. Formulated as a (mixed) complementarity problem, the proposed model is solved using the PATH solver (Ferris and Munson, 1998) coded in AMPL (Fourer et al., 2003). Two networks are used to test the model and the solution approach. One is a small network with hundreds of decision variables, while the other is the well-studied Sioux-Falls network with thousands of decision variables. Network settings in the case study are summarized in Table 3.

Table 3. Summary of the small network and the Sioux-Falls network.

	# ODs	# Variables	# Arcs	# Arcs Used	Arc Capacity	Demands
The Small Network	4	300	9	7	1000-1500	1000-1500
The Sioux-Falls Network	25	1250	76	68	4000-20000	1000-6000

5.1. Results of the small network

The structure of the small network is shown in Fig. 4. There are five nodes in total, in which node 1 is home while the rest of the nodes are work places. The travel demand for each OD pair is listed in Table 4. The BPR function we use is in equation (26) below, and relevant parameters for the arcs are shown in Table 6.

$$t_a(f_a) = t_a^0 \times \left[1 + 0.15 \left(\frac{f_a}{\text{Cap}_a} \right)^4 \right] \quad \forall\, a \in \mathcal{A} \tag{26}$$

where t_a^0 represents the free flow travel time in arc a, and Cap_a is the capacity of arc a.

All other model inputs for the base case are listed in Table 5, which are determined by the following guiding principles to coincide with reality: (1) unit inconvenience for rideshare passengers is not less than unit inconvenience for e-hailing passengers; (2) waiting time for rideshare passenger is not less than that for e-hailing passenger. Also, the parameters are designed based on Theorem 2 and Theorem 3, in order to reach a unique solution. The conversion factor of time to money, ψ, is set to be one dollar per minute.

Table 4. Travel demand for each OD pair.

OD Pair	1-2	1-3	1-4	1-5
Travel Demand	1000	1500	1000	1500

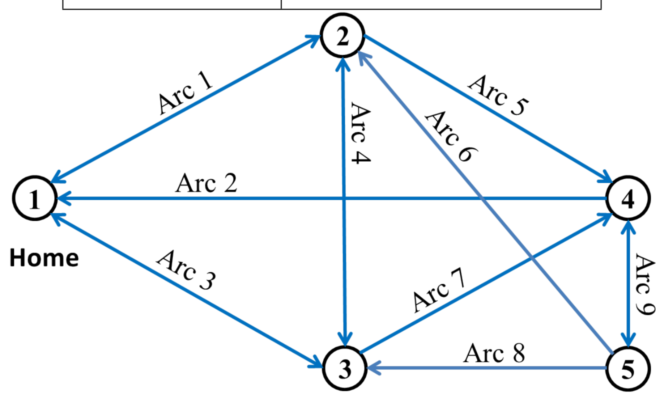

Figure 4. The small network.

Table 5. Other parameters of the base case.

Parameters	Value	Units
$O_k^{1;\text{am}}$, $O_{\bar{k}}^{1;\text{pm}}$	6.95	Dollars
$I_k^{1;\text{am}}$, $I_{\bar{k}}^{1;\text{pm}}$	0.5	Dollars
$W_k^{2;\text{am}}$, $W_{\bar{k}}^{2;\text{pm}}$	3	Dollars
$I_k^{2;\text{am}}$, $I_{\bar{k}}^{2;\text{pm}}$	2.3	Dollars
$W_k^{3;\text{am}}$, $W_{\bar{k}}^{3;\text{pm}}$	2	Dollars
$I_k^{3;\text{am}}$, $I_{\bar{k}}^{3;\text{pm}}$	0.8	Dollars
$E_k^{3;\text{am}}$, $E_{\bar{k}}^{3;\text{pm}}$	4.2	Dollars
$\underline{E}_k^{2;\text{am}}$, $\underline{E}_{\bar{k}}^{2;\text{pm}}$	0	Dollars
$\gamma_k^{2;\text{am}}$, $\gamma_{\bar{k}}^{2;\text{pm}}$	0.2	Dollars/Passengers
M	4	Passengers

Table 6. Parameters of the BPR functions.

Arc a	1	2	3	4	5	6	7	8	9
t_a^0	3	10	5	4	6	8	5	7	5
Cap_a	1000	1200	1500	1000	1200	1500	1000	1200	1500

The results of rideshare prices and travelers' mode choices when changing $I^{2;\text{pm}}$ are shown in Table 7 and Fig. 5, respectively. As demonstrated in Fig. 5(b), when we increase $I^{2;\text{pm}}$, and at the same time keep all the rest of the parameters fixed to the values given in Table 5, rideshare passengers switch to e-hailing passengers during the evening commute. At the same time, the number of drivers almost remains the same. This means that there are fewer rideshare passengers in each vehicle, which leads to higher payment for each passenger, as can be seen in Table 7.

Table 7. Results of rideshare prices when changing $I^{2;\text{pm}}$.

$I^{2;\text{pm}}$	2.3	2.4	2.5	2.6	2.9	3.2	3.5
Rideshare Price	0.500	0.509	0.522	0.533	0.563	0.587	0.606

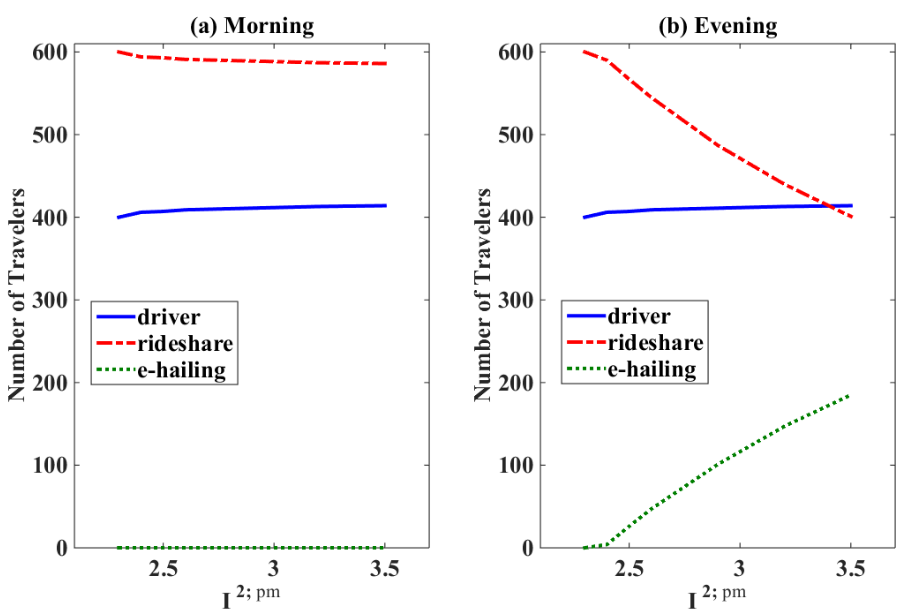

Figure 5. Results of travelers' mode choices when changing $I^{2;\text{pm}}$.

As described in Section 3.5, $\gamma^{2;\text{am}}$ and $\gamma^{2;\text{pm}}$ are actually related to the upper bound of the rideshare prices in the morning and evening, respectively. That is, the larger $\gamma^{2;\text{am}}$ or $\gamma^{2;\text{pm}}$ value gives a higher upper bound on the morning or evening rideshare price. This explains the results illustrated in Table 8, when $\gamma^{2;\text{pm}}$ is increased, the rideshare price becomes larger, due to the higher upper bound of the rideshare price.

The results of the travelers' mode choices when changing $\gamma^{2;\text{pm}}$ can be seen in Fig. 6. Fig. 6(b) shows that when we increase $\gamma^{2;\text{pm}}$ (the evening rideshare price is increased at the same time), the number of rideshare passengers continues to decrease in the evening, switching to first drivers then e-hailing passengers. When $\gamma^{2;\text{pm}}$ is larger than 0.7, the number of rideshare passengers starts to decrease rapidly. Most of them switch to e-hailing passengers, while the number of drivers slightly decreases since the market for rideshare decreases significantly and we do not need so many drivers. In addition, we can see a coupled morning-evening effect in Fig. 6(a). Since the number of drivers should be equal for the morning and the evening, the change of morning drivers coincides with that of evening drivers: it first increases then decreases. As a result, the number of morning rideshare passengers first decreases then increases, due to the conservation of total flows for morning trip demands and evening trip demands.

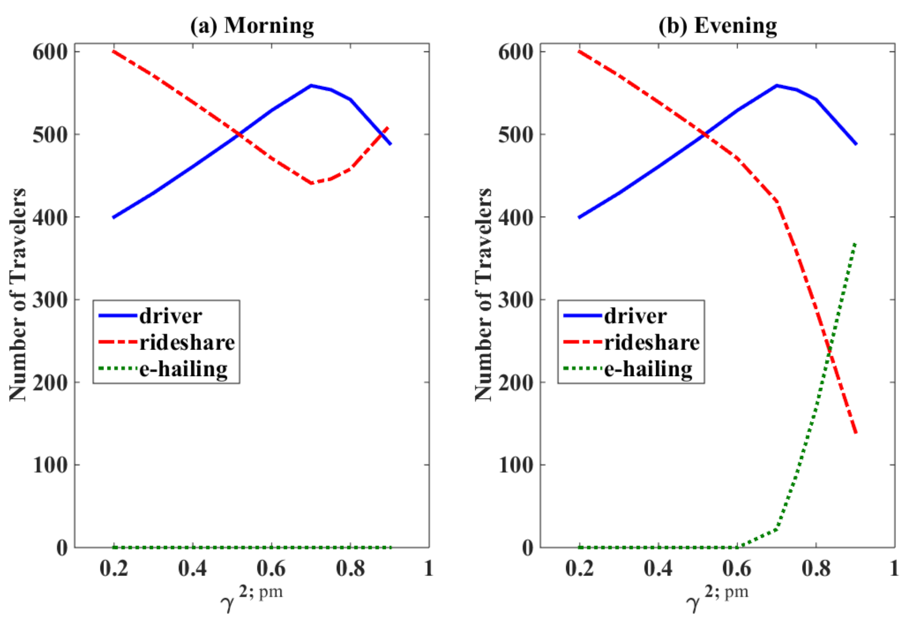

Figure 6. Results of travelers' mode choices when changing $\gamma^{2;\text{pm}}$.

Table 8. Results of rideshare prices when changing $\gamma^{2;\mathrm{pm}}$.

$\gamma^{2;\mathrm{pm}}$	0.2	0.3	0.4	0.5	0.6	0.7	0.75	0.8	0.9
Rideshare Price	0.500	0.801	1.132	1.488	1.867	2.275	2.516	2.773	3.343

The results of how rideshare prices and travelers' mode choices change when changing $E^{3;\mathrm{pm}}$ are shown in Table 9 and Fig. 7, respectively. As illustrated in Fig. 7(b), when $E^{3;\mathrm{pm}}$ becomes higher, e-hailing passengers switch to drivers and even much more rideshare passengers. Consequently, there are more rideshare passengers in each vehicle, which leads to a lower rideshare price for each passenger, as can be seen in Table 9.

Table 9. Results of rideshare prices when changing $E^{3;\mathrm{pm}}$.

$E^{3;\mathrm{pm}}$	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9
Rideshare Price	0.606	0.600	0.594	0.588	0.582	0.576	0.573	0.573

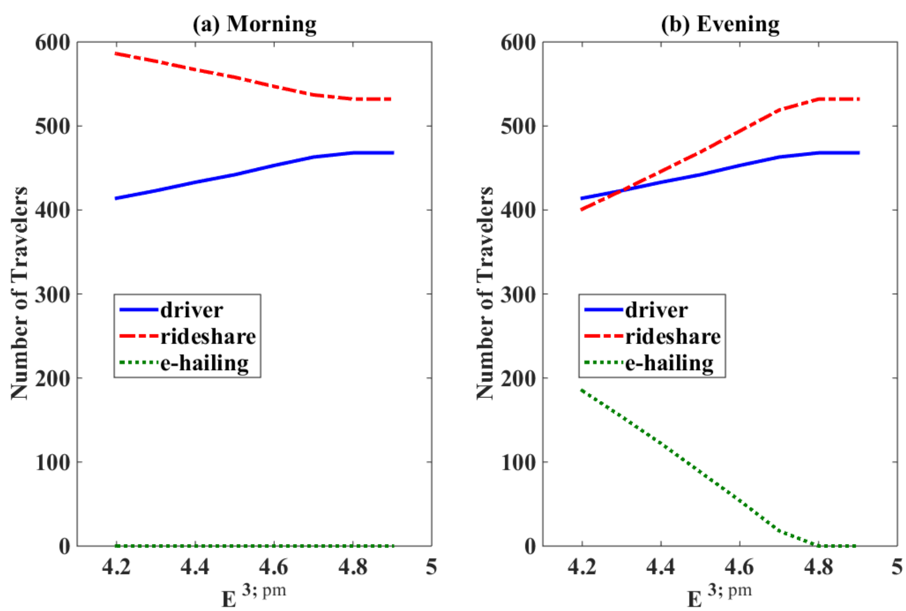

Figure 7. Results of travelers' mode choices when changing $E^{3;\mathrm{pm}}$.

We next compare the equilibrium solution from the proposed coupled morning-evening model against a decoupled morning (evening) commute model when the rideshare inconvenience cost is higher during the evening commute than in the morning commute. We use the same parameters as those in Table 5, except that $I_1^{2;\mathrm{pm}}$ is 3.5 dollars. Recall $I_1^{2;\mathrm{am}}$ is 2.3. We assume that individuals will use the higher cost parameters to determine their mode choice in a decoupled model. Thus, in the decoupled model, since the rideshare inconvenience cost is higher in the evening and all other parameters are the same, an individual will determine whether or not to be a driver using the evening parameter settings. Since once someone decides to be a driver, she/he remains a driver, the results of the decoupled model are obtained as follows: we first optimize the evening commute model to obtain the evening route choice and evening mode choice including being a driver; then we fix the number of drivers, and optimize the morning route choice and morning mode choice between rideshare and e-hailing. The main quantities for comparison include Vehicle Miles Traveled (VMT), and number of each type of travelers.

The comparison between the two models is shown in Table 10. The decoupled model overestimates the number of drivers by 25.6% and the VMT by 2.7% compared with the coupled model because the coupled model is capable of capturing the mode switches between morning and evening, which leads to fewer drivers and less VMT in the system. In this case, 31.6% of the morning rideshare passengers switch to e-hailing service in the evening because of the higher inconvenience cost of ridesharing during the evening commute, may be due to some individuals needing to pick up their children at their after school activities, making the inconvenience of rideshare service higher during the evening. The decoupled model cannot capture this effect and most likely will predict that the traveler will drive to work, thus causing the overestimation of the number of drivers for these parameters setting.

Table 10. Comparisons between coupled model and decoupled model.

	Coupled Model	Decoupled Model
VMT (am)	1242	1561
VMT (pm)	1796	1561
VMT (total)	3038	3121
# Drivers	414	520
# Rideshare Passengers (am)	586	480
# Rideshare Passengers (pm)	401	480
# E-hailing Passengers (am)	0	0
# E-hailing Passengers (pm)	185	0

5.2. Results of the Sioux-Falls network

In this section, we test the proposed model and solution approach on the Sioux-Falls network, which is widely used by researchers to test their models. We follow the settings used in Ben (2020), including the geometry, travel demand for each OD pair, and parameters of the BPR function for each arc. We selected five nodes (1, 2, 4, 7, 9) as homes and another five nodes (13, 19, 20, 23, 24) as work places. To increase the congestion level of the network, we use ten times the travel demands in Ben (2020). Furthermore, we set the travel demand to be a small value (i.e., ten) if it is zero in order to keep the complementarity problem square. The parameters of the travel modes in the base case can be found in Table 11 and are set using the same guiding principles as in Section 5.1. The conversion factor of time to money, ψ, is set to be one dollar per minute.

Similar sensitivity analysis results are revealed as in Section 5.1. As shown in Fig. 8(b), when we increase $I^{2;\mathrm{pm}}$, rideshare passengers switch to e-hailing passengers and drivers, which leads to fewer rideshare passengers in each vehicle and eventually leads to a more expensive rideshare price as in Table 12. As can be seen from Table 13, the rideshare price is higher as $\gamma^{2;\mathrm{pm}}$ becomes larger, since $\gamma^{2;\mathrm{pm}}$ is related to the upper bound of the rideshare price. Consequently, as shown in

Fig. 9(b), there are fewer rideshare passengers and more drivers. When $\gamma^{2;\mathrm{pm}}$ is larger than 1.2, the number of rideshare passengers decreases rapidly, which leads to the significant decrease of the rideshare market. As a result, the number of drivers also decreases and number of e-hailing passengers increases rapidly. Also, the coupled morning-evening effect is shown in Fig. 9(a). The number of morning drivers first increases then decreases, since it is the same as the number of evening drivers; the number of morning rideshare passengers first decreases then increases, due to the total flow conservation for morning trip demands and evening trip demands. As illustrated in Fig. 10(b), when $E^{3;\mathrm{pm}}$ is raised, e-hailing passengers become drivers and even more rideshare passengers. As a result, there are more rideshare passengers in each vehicle and the rideshare price decreases as in Table 14.

Table 11. Parameters of the base case.

Parameters	Value	Units
$O_k^{1;\mathrm{am}}$, $O_{\bar{k}}^{1;\mathrm{pm}}$	9.95	Dollars
$I_k^{1;\mathrm{am}}$, $I_{\bar{k}}^{1;\mathrm{pm}}$	1.5	Dollars
$W_k^{2;\mathrm{am}}$, $W_{\bar{k}}^{2;\mathrm{pm}}$	6	Dollars
$I_k^{2;\mathrm{am}}$, $I_{\bar{k}}^{2;\mathrm{pm}}$	3.3	Dollars
$W_k^{3;\mathrm{am}}$, $W_{\bar{k}}^{3;\mathrm{pm}}$	3.5	Dollars
$I_k^{3;\mathrm{am}}$, $I_{\bar{k}}^{3;\mathrm{pm}}$	2.3	Dollars
$E_k^{3;\mathrm{am}}$, $E_{\bar{k}}^{3;\mathrm{pm}}$	5.7	Dollars
$\underline{E}_k^{2;\mathrm{am}}$, $\underline{E}_{\bar{k}}^{2;\mathrm{pm}}$	0	Dollars
$\gamma_k^{2;\mathrm{am}}$, $\gamma_{\bar{k}}^{2;\mathrm{pm}}$	0.2	Dollars/Passengers
M	4	Passengers

Table 12. Results of rideshare prices when changing $I^{2;\mathrm{pm}}$.

$I^{2;\mathrm{pm}}$	3.3	3.5	3.8	4	4.3	4.5	4.8	5	5.1
Rideshare Price	0.500	0.516	0.539	0.553	0.571	0.581	0.596	0.604	0.608

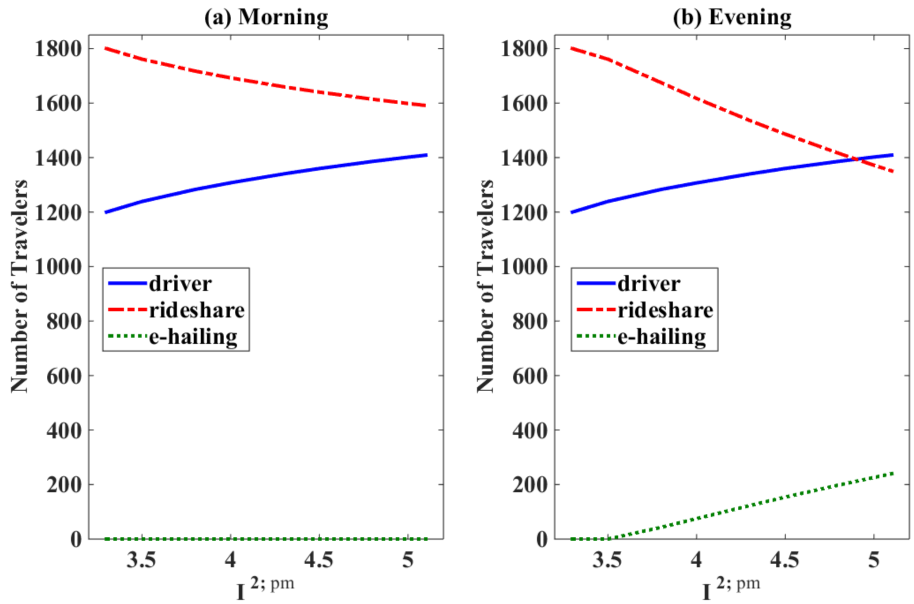

Figure 8. Results of travelers' mode choices when changing $I^{2;\text{pm}}$.

Table 13. Results of rideshare prices when changing $\gamma^{2;\text{pm}}$.

$\gamma^{2;\text{pm}}$	0.2	0.6	1.0	1.1	1.2	1.25	1.3	1.35	1.4
Rideshare Price	0.500	1.867	3.533	3.976	4.425	4.695	5.005	5.331	5.600

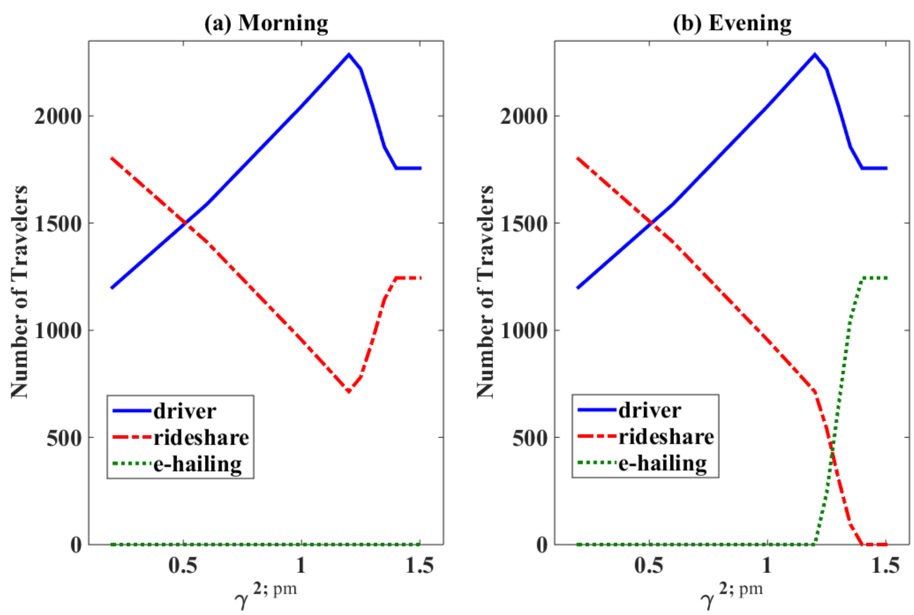

Figure 9. Results of travelers' mode choices when changing $\gamma^{2;\text{pm}}$.

Table 14. Results of rideshare prices when changing $E^{3;\mathrm{pm}}$.

$E^{3;\mathrm{pm}}$	5.5	5.7	5.8	5.9	6.0	6.1	6.2
Rideshare Price	0.616	0.608	0.604	0.600	0.597	0.597	0.597

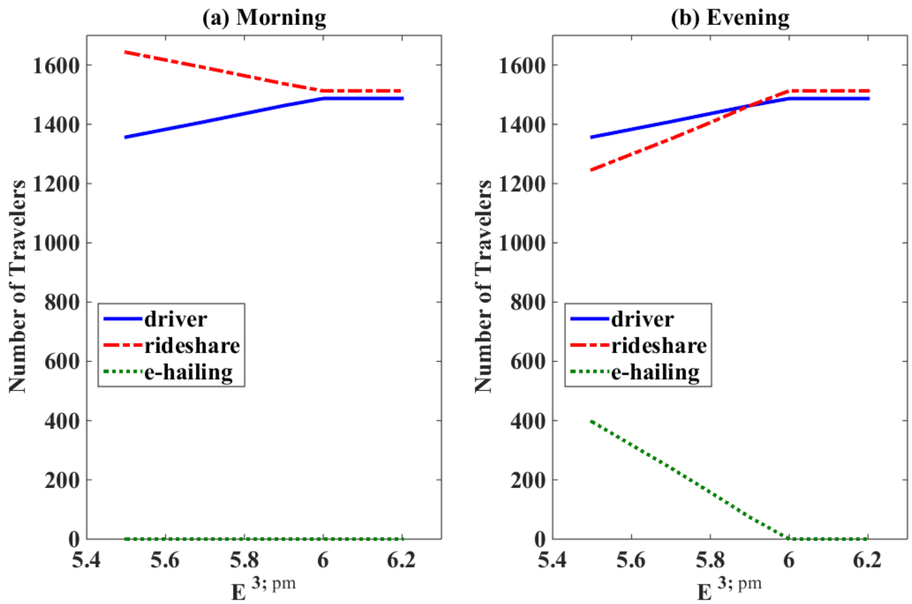

Figure 10. Results of travelers' mode choices when changing $E^{3;\mathrm{pm}}$.

In Table 15, similar results as Section 5.1 can also be found when we compare the coupled morning-evening model with the decoupled morning (evening) commute model. The results of the decoupled model are obtained with the same approach described in Section 5.1. The parameters are the same as those in Table 11, except that $I_3^{2;\mathrm{pm}}$ is 5.1 dollars. Recall $I_3^{2;\mathrm{am}}$ is 3.3. In this example, the decoupled model overestimates the number of drivers by 24.2% and the VMT by 8.4% for the same reasons as outlined in Section 5.1.

Table 15. Comparisons between coupled model and decoupled model.

	Coupled Model	Decoupled Model
VMT (am)	29862	37102
VMT (pm)	38576	37102
VMT (total)	68438	74204
# Drivers	1357	1686
# Rideshare Passengers (am)	1643	1314
# Rideshare Passengers (pm)	1247	1314
# E-hailing Passengers (am)	0	0
# E-hailing Passengers (pm)	396	0

6. Conclusions and Future Research

In this study, we include both e-hailing and ridesharing as travel modes and integrate morning and evening commute trips in a general network equilibrium modeling framework. The model is formulated as a (mixed) complementarity problem. We prove the solution existence for the proposed model, and show its relationship with some traditional traffic models. The proposed model is validated in two networks: a small network and the Sioux-Falls network. The results show that the proposed coupled morning-evening model is effective in capturing the mode switches between morning and evening, compared with a decoupled morning (evening) commute model. In particular, our numerical examples show that modeling morning and evening commutes separately tends to overestimate the number of drivers and total vehicle miles traveled (VMT) in the network when accounting for travelers' capabilities for mode switching. For example, in the Sioux-Falls network, the coupled model produces 24.2% fewer drivers and 8.4% less VMT in the system compared with the decoupled model when the inconvenience cost due to ridesharing is higher during the evening commute than in the morning commute. This is due to the fact that the coupled model can capture the behavior of travelers capability to switch to e-hailing in the evening commute when ridesharing in the morning commute. A decoupled model cannot capture this effect and most likely will predict that the traveler will drive to work. With a coupled model, transportation planners can better understand appropriate incentives to increase vehicle occupancy and reduce VMT.

Further research could focus on including other realistic elements in this modeling framework: deployment of High Occupancy Vehicle (HOV) lanes and rideshare pick-up and drop-off locations, to just name a few. These have the potential of integrating e-hailing and rideshare services seamlessly and more effectively, which could reduce solo driving, and consequently lessen traffic demand, congestion, and VMT.

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8. Data Management Plan

Products of Research

The main research products will be peer-reviewed journal articles, book chapters and/or conference proceedings targeted towards the transportation science research community, plus supplemental materials such as tables, numerical data used for graphs, etc. No personal data will be used in the project, so there is no threat of identity theft.

Data Format and Content

All research products will be available online in digital form. Manuscripts will appear in a common document-viewing format, such as PDF, and supplemental materials such as tables and numerical data will be in a tabular format, such as Microsoft Excel spreadsheet, tab-delimited text, etc.

Data Access and Sharing

All participants in the project will publish the results of their work. Papers will be published in peer-reviewed scientific journals, books published in English, conference proceedings, or as peer-reviewed data reports. Beyond the data posted on USC websites, primary data and other supporting materials created or gathered in the course of work will be shared with other researchers upon reasonable request, at no more than incremental cost and within a reasonable time of the request or, if later, the filing of a patent application covering the results of such research.

All the data used in the research are included in Tables in the final report or are publicly available. For the experiments on the small network, Figure 4 shows the network structure and Tables 4, 5, and 6 list the data parameters used in the experiments.

For the Sioux Falls experiments, the data can be found in the following link:
https://github.com/bstabler/TransportationNetworks/tree/master/SiouxFalls
Ben, S. (2020). Transportation Networks for Research

We follow the settings used in Ben (2020), including the geometry, travel demand for each OD pair, and parameters of the BPR function for each arc. We selected five nodes (1, 2, 4, 7, 9) as homes and another five nodes (13, 19, 20, 23, 24) as work places. To increase the congestion level of the network, we used ten times the travel demands in Ben (2020). Furthermore, we set the travel demand to be a small value (i.e., ten) if it is zero in order to keep the complementarity problem square. The parameters of the travel modes can be found in Table 11.

Reuse and Redistribution

USC's policy is to encourage, wherever appropriate, research data to be shared with the general public through internet access. This public access will be regulated by the university in order to protect privacy and confidentiality concerns, as well to respect any proprietary or intellectual property rights. Administrators will consult with the university's legal office to address any concerns on a case-by-case basis, if necessary. Terms of use will include requirements of attribution along with disclaimers of liability in connection with any use or distribution of the research data, which may be conditioned under some circumstances.