Adaptive Signal Control Algorithms for Connected Vehicles

Final Report

by

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DISCLAIMER

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16. Abstract

Traffic congestion at a signalized intersection greatly reduces the travel time reliability in urban areas. Adaptive signal control system (ASCS) is an advanced traffic signal control technology that regulates the signal phasing and timings considering the traffic patterns in real-time in order to reduce traffic congestion. Real-time prediction of traffic queue length can be used to adjust the signal phasing and timing for different traffic movements at a signalized intersection with ASCS. The accuracy of the queue length prediction model varies based on the many factors, such as the stochastic nature of the vehicle arrival rates at an intersection, time of the day, weather and driver characteristics. In addition, accurate queue length prediction for multilane undersaturated and saturated traffic scenarios at signalized intersections is challenging. The objective of this study is to develop short-term queue length prediction models for signalized intersections that can be leveraged by adaptive traffic signal control systems using four variations of Grey systems: (i) the first-order single variable Grey model (GM(1,1)); (ii) GM(1,1) with Fourier error corrections (EGM); (iii) the Grey Verhulst model (GVM); and (iv) GVM with Fourier error corrections (EGVM). The efficacy of the Grey models is that they facilitate fast processing as these models do not require the large amount of data that would be needed in artificial intelligence models and these models are able to adapt to stochastic changes, unlike statistical models. We have conducted a case study using queue length data from five intersections with adaptive traffic signal control on a calibrated roadway network in Lexington, South Carolina, utilizing a microscopic traffic simulator. Grey models were compared with five other baseline time-series forecasting models: Autoregressive (AR) model, Logistic Smooth Transition Autoregressive (LSTAR) model, Neural Networks (NNETS) model, Additive non-linear Autoregressive model (AAR) and long short-term memory (LSTM) neural network model. Based on our analyses, we found that EGVM could reduce the prediction error over the closest competing models, i.e., LSTM and AAR, in predicting average and maximum queue lengths by 40% and 42% in terms of Root Mean Squared Error (RMSE), and 51% and 50% in terms of Mean Absolute Error (MAE), respectively.

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EXECUTIVE SUMMARY

Real-time prediction of traffic queue length can be used to adjust the green timing for different traffic movements. Existing systems mainly use inductive loop detectors to detect queue lengths. Inductive loop detectors are embedded within the roadway pavement. There are several disadvantages of the loop detector based sensors: (i) low coverage area (certain section of a traffic lane); (ii) detection susceptibility to environmental conditions; and (iii) high cost for deployment and maintenance. Emerging connected vehicle technology can overcome the challenges of existing queue length estimation methods by providing real-time information to the traffic signal control using Vehicle-to-Infrastructure (V2I) wireless communication.

In a connected vehicle environment, the information of the arrival rate of vehicles for all movements at a signalized intersection is available via V2I communication. However, these arrival rates are stochastic in nature depending on different factors, such as the time of the day, weather and driving characteristics. These factors adversely affect the performance of queue length prediction models and reduce prediction accuracy significantly. Moreover, accurate queue length predictions for multilane scenarios and robustness for both under saturated and saturated traffic scenarios at a signalized intersection are challenging.

The objective of this study is to develop a robust short-term queue length prediction model for adaptive traffic signal control systems using four variations of Grey Systems: (i) the basic Grey model (GM(1,1)); (ii) GM(1,1) with Fourier error corrections (EGM); (iii) the Grey Verhulst model (GVM), and (iv) GVM with Fourier error corrections (EGVM). GM requires a low sample size to update its parameter (as low as only four data points). The efficacies of the Grey models are that they are fast, unlike artificial intelligence models as it does not require a large amount of data for training, and can adapt to stochastic changes of the arrival rate of vehicles at a signalized intersection. Grey models are evaluated using queue length data from five signalized intersections with adaptive traffic signal controls in Lexington, South Carolina. Grey models were compared with five other baseline time-series forecasting models: Autoregressive (AR) model, Logistic Smooth Transition Autoregressive (LSTAR) model, Neural Networks (NNETS) model, Additive non-linear Autoregressive model (AAR) and long short-term memory (LSTM) neural network model, to evaluate the performance of different variations of Grey models.

The study shows the effectiveness of Grey Systems in queue length prediction. As the experiments demonstrated, the EGVM model provides better overall performance and more accurate forecasts for different traffic conditions in both single lane and multilane scenarios. The EGVM model is able to predict accurately for both under congested and congested scenarios, which establishes the efficacy of the model for predicting queue length and using it as an input to the adaptive signal control systems. The EGVM model is identified as the best model because it outperforms other grey models (i.e., GM, EGM, and GVM) for average and maximum queue length prediction. The analysis showed that GVM models could provide approximately one-meter precision in queue length prediction. GVM models provide more accurate results than LSTM requiring only a fraction of the input data (4 vs 2400 observations) and has a very low computational time due to the absence of the training phase. Based on our analyses, we found that EGVM could reduce the prediction error over the closest competing models, LSTM and AAR, in predicting average and maximum queue lengths by 40% and 42% in terms of Root Mean Squared Error (RMSE), and 51% and 50% in terms of Mean Absolute Error (MAE), respectively.

One limitation of the work in this study is that the models are dependent on the accuracy of the historical queue length estimations. However, an accurate queue length estimation is not a trivial task, so this work needs to be combined with a reliable queue length estimation framework for

the effective utilization of the EGVM. Future work should also include the following: (1) inclusion of mid-term and long-term forecasts (2) modifications to the basic grey systems equations and a study on the applicability of multivariable grey models, and (3) consideration of seasonal behavior inducing model structure.

CHAPTER 1

Introduction and Background

Traffic congestion at a signalized intersection negatively impacts the travel time reliability in urban areas (Qi et al. (2016), Ma et al. (2018)). Adaptive signal control systems (ASCS) are an advanced traffic signal control technology that regulates the phasing as well as red, yellow and green timings considering the traffic patterns (i.e., the arrival rate of vehicles at a signalized intersection from different approaches) in real-time to reduce traffic congestion (Radin et al. (2018)). Major benefits of ASCS include: (i) real-time distribution of green timings based on the arrival rate of vehicles for all traffic movements; and (ii) reduction of travel times through intersections by ensuring progression through green signal timing window (Radin et al. (2018)).

Real-time prediction of traffic queue length can be used to adjust the green timing for different traffic movements. Existing systems mainly use inductive loop detectors to detect queue lengths. Inductive loop detectors are installed on the roadway pavement (Tiaprasert et al. (2015)). There are several disadvantages of the loop detector-based sensors, such as (i) low coverage area (only cover a small length of a traffic lane); (ii) detection susceptibility to environmental conditions; and (iii) high cost for deployment and maintenance. Emerging connected vehicle technology can overcome the challenges of existing queue length estimation methods by providing real-time information to the traffic signal control using Vehicle-to-Infrastructure (V2I) wireless communication (Tiaprasert et al. (2015)).

In a connected vehicle environment, the information of the arrival rate of vehicles for all movements at a signalized intersection is available via V2I communication. However, these arrival rates are stochastic in nature depending on different factors, such as the time of the day, weather and driving characteristics (Yang et al. (2018)). These factors adversely affect the performance of queue length prediction models and reduce prediction accuracy significantly. Moreover, accurate queue length predictions for multi-lane scenarios and robustness of the predictions for both under saturated and saturated roadway traffic scenarios at a signalized intersection are challenging (Zhan et al. (2015)).

Recent studies use statistical and machine learning models for predicting queue length at signalized intersections (Tiaprasert et al. (2015), Comert (2016)). Machine learning models, such as a recurrent neural network (RNN) based time series models, require a large amount of data for training a queue prediction model for different scenarios (such as single-lane or multilane roadways and level of congestion) to achieve a higher level of accuracy. However, it increases the computational resource need for real-time applications. It also increases the need for large amounts of data for extensive training considering different roadway traffic scenarios. The advantage of the RNN models is that after training, it can capture the stochastic roadway traffic pattern. On the other hand, although statistical models do not require a large amount of data for training, they need to re-estimate model parameters based on the traffic patterns, which reduces the applicability of the statistical model for real-world applications (Comert (2016)).

Recently, Grey models (GM) have become popular for traffic data prediction, as these models do not assume any underlying distribution for the data generation process; they are able to handle autocorrelated observations and require low computational cost (Bezuglov and Comert (2016)). Furthermore, GM requires a low sample size to update its parameters (as low as only four data points) (Liu et al. (2010)). A study by An et al. showed that the accuracy of the first-order single variable Grey Model (GM(1,1)) is higher than the backpropagation neural network (NN) and radial basis function NN model to predict monthly average daily traffic volumes (An et al. (2012)).

Similarly, Gao et al. found that GM(1,1) prediction accuracy of average hourly traffic volumes surpasses the performance of support vector machine (SVM) and artificial neural network networks (Gao et al. (2010)). However, no previous study has used Grey models for predicting traffic queue length using connected vehicle data for ASCS. In addition, the efficacies of the Grey models are that it does not require a large amount of data, and is able to adapt to stochastic changes of the arrival rate of vehicles at a signalized intersection.

The objective of this study is to develop a robust short-term queue length prediction model for adaptive traffic signal control systems using four variations of Grey Systems: (i) the basic Grey model (GM(1,1)); (ii) GM(1,1) with Fourier error corrections (EGM); (iii) the Grey Verhulst model (GVM), and (iv) GVM with Fourier error corrections (EGVM). Grey models are evaluated using queue length data from five signalized intersections with adaptive traffic signal controls in Lexington, South Carolina. Grey models were compared with five other baseline time-series forecasting models: Autoregressive (AR) model, Logistic Smooth Transition Autoregressive (LSTAR) model, Neural Networks (NNETS) model, Additive non-linear Autoregressive model (AAR) and long short-term memory (LSTM) neural network model, to evaluate the performance of different variations of Grey models.

The rest of the report is organized as follows. Chapter 2 presents related work focusing on queue length estimations and predictions at signalized intersections. Chapter 3 focuses on the Grey models and covers GM(1,1), the Grey Verhulst model, and two variations of these models to improve their prediction accuracy. Chapter 4 presents the compared time series models and detailed numerical experiments to evaluate the prediction performance of the Grey models. Finally, Chapter 5 summarizes the findings and addresses possible future research directions.

CHAPTER 2

Related Work

There have been many studies focusing on queue length estimations and predictions at signalized intersections. Different studies have used different types of models and inputs for estimating or predicting queue lengths at intersections. Below we segment the literature into prediction and estimation studies.

2.1 Queue Length Prediction

Li et al. developed a queue length prediction model for multi-lane signalized intersections (2018). The authors used the Lighthill-Whitham-Richards shockwave theory and Robertson's platoon dispersion model to predict the arrival of vehicles five seconds in advance for each lane and integrated the predictions of different lanes using a Kalman filter. The authors achieved an average RMSE of 2.33 vehicles, MAE of 1.82 vehicles, and MAPE of 16.12% for maximum queue length prediction. However, this model does not consider several aspects of real-world traffic flow that affect queue lengths, such as lane changing, heterogeneous traffic and dynamic correction of travel times. Zeng et al. developed a queue length prediction model using stochastic fluid theory (2017). The authors used the two-fluid theory for considering road traffic and congested traffic for predicting queue lengths. The average relative prediction error of the model is 24.7% for a single-lane scenario and 38.2% for a multilane scenario. The authors attributed the higher error in a multilane scenario to overtaking behavior among different lanes (Zeng et al. (2017)).

2.2 Queue Length Estimation

The estimation models can be divided into three categories, statistical, analytical, and machine learning models.

2.2.1 Statistical Models

Comert developed stochastic models and formulated the analytical expressions of estimators, which were used for estimating queue length from probe vehicle data (e.g., location, time, and count). The developed models estimate cycle-to-cycle queue lengths within ±5% error. However, this study does not deal with predicting future queue lengths (Comert (2016)). Hao et al. developed seven Bayesian network models for estimating cycle-by-cycle queue lengths for seven different traffic scenarios. The input to the models is mobile traffic sensor data collected between the upstream and downstream of an intersection. Hao et al. proved that the stochastic approach at low penetration rates is more robust compared to deterministic approaches. However, this model suffers from the lack of availability of actual ground truth data, since the model predicts queue length distribution by cycle, but in the real world, only a queue is observed at a certain instant (Hao et al. (2014)). Zhan et al. developed a lane-based real-time queue length estimation method using license plate data. The developed model includes a Gaussian Process-based interpolation method and a car-following model for reconstructing the equivalent cumulative arrival-departure curve of each lane and estimating queue lengths. The RMSE and MAE of queue length estimation are below 3.2 vehicles and 2.4 vehicles (approximately 12 m and 16 m based on average vehicle length), respectively. However, this model also has some limitations, such as lane changing effects not considered and the model may infer incorrect arrival times (Zhan et al. (2015)).

2.2.2 Analytical Models

Hao and Ban developed a queue length estimation method to solve the long queue problem using short vehicle trajectories from mobile sensors (Hao and Ban (2015)). The method is based on vehicle trajectory reconstruction models to estimate the missing acceleration/deceleration process. Their method was able to reduce the mean absolute error for long queue length estimation from 3.79 vehicles to 1.61 vehicles (approximately 18.95 m to 8.05 m based on average vehicle length). However, Hao and Ban do not deal with predicting future queue lengths. Moreover, this model is inapplicable for multi-lane intersections and heavily congested scenarios. It also requires the input data to be high precision and a low sampling rate (Hao and Ban (2015)). Wang et al. developed a queue estimation method for signalized intersections using data from both probe vehicles and point detectors. The authors used shockwave theory to model the queue dynamics. The models showed a mean absolute percent error (MAPE) of 17.09% and 12.28% for 2 different scenarios. However, this model has some limitations in estimating queue length when there is a residual queue at the intersection (Wang et al. (2017)). Tiaprasert et al. proposed a queue length estimation model using connected vehicle technology for adaptive signal control (Tiaprasert et al. (2015)). Tiaprasert et al. applied a discrete wavelet transform (DWT) to queue estimation in order to make it robust against randomness in penetration ratio. The authors showed that the queue length estimation algorithm works in both undersaturated and saturated traffic conditions, which is essential for applying it in adaptive signal control (Tiaprasert et al. (2015)).

2.2.3 Machine Learning Models

An et al. developed a real-time queue length estimation model including a breakpoint misidentification checking process and two input-output models (upstream-based and local-based), and used event-based data as input. The model was able to improve on the generic breakpoint model as the maximum queue length estimation MAE was found to be 10.88 m compared to 32.2 m. However, as the model needed to be trained with ground truth data for parameter estimation; two limitations of the model related to parameter estimation are the validity of the parameters for different time periods and transferability of the parameters among intersections (An et al. (2018)). Gao et al. proposed a cycle-by-cycle queue length estimation model, which is a weighted combination of two submodels: shockwave sensing and backpropagation neural network sensing. The input to the model is connected vehicle data. The authors showed that their model has higher accuracy than probability distribution models for low penetrations of connected vehicles, with 85% accuracy for low penetration rates and 95% accuracy for high penetration rates. This model also performs well for both undersaturated and saturated conditions, which is crucial for adaptive signal control. However, it suffers from the data requirements for training the backpropagation neural network model (Gao et al. (2019)).

From the review of literature, it is evident that queue length prediction has some research gaps, which include accuracy for multilane scenarios and robustness for both under saturated and saturated scenarios (to be effective for adaptive signal control). Through our development and evaluation of the Grey model, we will investigate these gaps in the literature.

CHAPTER 3

Grey System Models for Queue Length Prediction

The Grey Systems theory was developed by Ju-Long (1982) and since then it has become the preferred method to study and model systems in which the structure or operation mechanism is not completely known (Ju-Long (1982)). Grey System theory applications have been applied mainly in the area of finance (Kayacan et al. (2010)). Its application in transportation is limited; examples include prediction of average speed, travel time (Bezuglov and Comert (2016)), traffic volume (Gao et al. (2010), Liu et al. (2014)), accident analysis (Na et al. (2010)), and pavement degradation (Du and Shen, 2005)).

According to the Grey Systems theory, the unknown parameters of the system are represented by discrete or continuous Grey numbers. The theory introduces a number of properties and operations on the Grey numbers, such as the core of the number, its degree of Greyness, and whitenization (calculation to represent white/observed data value) of the Grey number. The latter operation generally describes the preference of the number towards the range of its possible values (Liu et al. (2014)).

In order to model time series, the theory suggests a family of Grey models, where the basic one is the first order Grey model with one variable which will be referred to as GM(1,1). The principles and estimation of GM(1,1) are briefly discussed here (Ju-Long (1989)).

Suppose that $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n))$ denotes a sequence of nonnegative observations of a stochastic process (i.e., average and maximum queue lengths) and $X^{(1)} = (x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n))$ is an accumulation sequence of queue lengths, $X^{(1)}$ computed as in Eq. (1).

$$x^{(1)}(k) = \sum_{j=1}^{k} x^{(0)}(j) \tag{1}$$

Then, Eq. (2) defines the original form of the GM(1,1).

$$x^{(0)}(k) + ax^{(1)}(k) = b \tag{2}$$

Let $Z^{(1)} = (z^{(1)}(2), z^{(1)}(3), ..., z^{(1)}(n))$ be a mean sequence of $X^{(1)}$ calculated by formula Eq. (3).

$$z^{(1)}(k) = [x^{(1)}(k-1) + x^{(1)}(k)]/2 \tag{3}$$

where, $k = 2, 3, \cdots, n$.

Eq. (4) gives the basic form of GM(1,1).

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{4}$$

If $\hat{a} = (a, b)^T$ and

$$\boldsymbol{Y} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} z^{(1)}(2) & 1 \\ z^{(1)}(3) & 1 \\ \vdots & \vdots \\ z^{(1)}(n) & 1 \end{bmatrix}$$

Then, as in Liu and Lin (2006), the least-squares estimate of the GM(1,1) model is $\hat{a}$ = $(\boldsymbol{B}^T\boldsymbol{B})^{-1}\boldsymbol{B}^T\boldsymbol{Y}$ and Eq. (5) is the whitenization equation of the GM(1,1) model (GM).

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \quad (5)$$

Suppose that $\hat{x}^{(0)}(k)$ and $\hat{x}^{(1)}(k)$ represent the time response sequence (one-step prediction) and the accumulated time response sequence of GM at time k respectively. Then, the latter can be obtained by solving Eq. (6):

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}, k = 1,2,\dots,n \quad (6)$$

According to the definition in Eq. (1), the restored values of $\hat{x}^{(0)}(k+1)$ are calculated as $\hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$:

$$\hat{x}^{(0)}(k+1) = (1-e^{a})\left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak}, k = 1,2,\dots,n \quad (7)$$

Eq. (7) gives the method to produce time-series forecasts. However, for longer time series, a rolling GM(1,1) is preferred. The rolling model observes a window of a few sequential data points in the series: $x^{(0)}(k+1), x^{(0)}(k+2), \dots, x^{(0)}(k+w)$, where $w \geq 4$ is the window size. Then, the model forecasts one or more future data points: $\hat{x}^{(0)}(k+w+1)$, $\hat{x}^{(0)}(k+w+2)$, i.e., one and two-step forecasts, respectively.

3.1 Grey Verhulst Model (GVM)

The response sequence Eq. (7) implies that the basic GM(1,1) works the best when the time series demonstrate a steady growth or decline and may not perform well when the data has oscillations or saturated sigmoid sequences. For the latter case, the Grey Verhulst model (GVM) is generally used (Liu et al. (2010)). The basic form of the GVM is present by Eq. (8).

$$x^{(0)}(k) + az^{(1)}(k) = b\left(z^{(1)}(k)\right)^2 \quad (8)$$

Eq. (9) provides the whitenization formula of GVM. It practically represents the assumed structure of the data generation process.

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b\left(x^{(1)}\right)^2 \quad (9)$$

Similar to the GM(1,1), the least-squares estimate is applied to find $\hat{a} = (\boldsymbol{B}^T\boldsymbol{B})^{-1}\boldsymbol{B}^T\boldsymbol{Y}$, where

$$\boldsymbol{Y} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} -z^{(1)}(2) & z^{(1)}(2)^2 \\ -z^{(1)}(3) & z^{(1)}(3)^2 \\ \vdots & \vdots \\ -z^{(1)}(n) & z^{(1)}(n)^2 \end{bmatrix}$$

The forecasts $\hat{x}^{(0)}(k+1)$ are calculated using Eq. (10).

$$\hat{x}^{(0)}(k+1) = \frac{ax^{(0)}(1)(a-bx^{(0)}(1))}{bx^{(0)}(1)+(a-bx^{(0)}(1))e^{a(k-1)}} * \frac{e^{a(k-2)}(1-e^{a})}{bx^{(0)}(1)+(a-bx^{(0)}(1))e^{a(k-2)}} \quad (10)$$

3.2 Error Corrections to Grey Models

The accuracy of the Grey models can be improved by a few methods. Suppose that $\epsilon^{(0)} = \epsilon^{(0)}(1)$, $\epsilon^{(0)}(2)$, $\epsilon^{(0)}(3)$..., $\epsilon^{(0)}(n)$ is the error sequence of $X^{(0)}$, where $\epsilon^{(0)}(k) = x^{(0)}(k) - \hat{x}^{(0)}(k)$. These errors, $\epsilon^{(0)}$ can be expressed using the Fourier series (Tan and Chang (1996)) as shown in Eq. (11).

$$\epsilon^{(0)}(k) \cong \frac{1}{2}a_0 + \sum_{i=1}^{z}\left(a_i \cos(\frac{2\pi i}{T}k) + b_i \sin(\frac{2\pi i}{T}k)\right) \tag{11}$$

where, $k = 2,3,\ldots,n$, $T = n - 1$, and $z = \left(\frac{n-1}{2}\right) - 1$.

The solution is found via the least squares estimate, presuming that $\epsilon^{(0)} \approx \boldsymbol{PC}$ where $\mathbf{C}$ is a vector of coefficients: $\boldsymbol{C} = [a_0 a_1 b_1 a_2 \ldots a_n b_n]^T$ and matrix $\boldsymbol{P}$ is:

$$P = \begin{bmatrix} \frac{1}{2} & cos(2\frac{2\pi}{T}) & sin(2\frac{2\pi}{T}) & \ldots & cos(2\frac{2\pi z}{T}) & sin(2\frac{2\pi z}{T}) \\ \frac{1}{2} & cos(3\frac{2\pi i}{T}) & sin(3\frac{2\pi}{T}) & \ldots & cos(3\frac{2\pi z}{T}) & sin(3\frac{2\pi z}{T}) \\ \vdots & & & & & \\ \frac{1}{2} & cos(n\frac{2\pi i}{T}) & sin(n\frac{2\pi}{T}) & \ldots & cos(n\frac{2\pi z}{T}) & sin(n\frac{2\pi z}{T}) \end{bmatrix}$$

Then $C \cong (P^T P)^{-1} P^T \epsilon^{(0)}$.As a result, the predicted value of the time series must be corrected according to Eq. (12):

$$\hat{x}_f^{(0)}(k) = \hat{x}^{(0)}(k) + \epsilon^{(0)}(k), k = 2, 3, \ldots, n \tag{12}$$

To summarize, the Grey system models presented in this chapter provide short-term queue length predictions regardless of aggregation levels on previous queue length observations. Note that the accuracy of this framework is bounded by the accuracy of the observed queue length data, similar to other supervised learning methods.

CHAPTER 4

Analysis and Results

In this chapter, results are presented regarding the performance of the Grey models (GMs). This chapter also presents five baseline time-series forecasting models and their prediction results compared to Grey models.

4.1 Data Description

In order to evaluate the performance of the Grey System models, a case study has been performed. A calibrated microsimulation model has been developed in VISSIM for the US 378 (Sunset Drive) corridor in Lexington, South Carolina. A portion of the corridor has been chosen for analysis that includes five signalized intersections. All the signalized intersections operate under an adaptive signal control system. Traffic data and travel times have been collected for the afternoon peak period and the VISSIM model has been calibrated to this data. As we are interested in queue lengths, a congested scenario is required in order to study the patterns of queue buildups and progressions. The first intersection is a T-intersection, while the other four are 4-way intersections. Along with the five intersections, there are 33 driveways on this corridor, which creates disruptions and stop-go conditions. These can contribute to the queue length patterns at the intersections. A screenshot of the VISSIM simulation environment is shown in Figure 1, including the detectors and queue counters placed at intersections.

In order to get the queue length data, queue counters are placed at each intersection. Each queue counter corresponds to a lane group. A lane group is a group of lanes that allow traffic to move simultaneously in an intersection approach. A lane group is defined based on the definitions from Highway Capacity Manual (HCM, 2010): (i) an exclusive left-turn lane or an exclusive right-turn lane; (ii) any shared lane; and (iii) if any lanes are not exclusive turn lanes or shared lanes, those lanes should be combined into one lane group. There are 31 queue counters in total for five intersections. By running the simulation, we have collected the average and maximum queue length data for each queue counter at each intersection. Please note that some queue counters correspond to multiple lanes. That is why we have divided the dataset into two segments: average queue length and maximum queue length. The intersections are numbered from west to east. Intersection 1 is a T-intersection, so it requires the least number of queue counters (i.e. 4). The queue length data has been collected per second. The information about queue counters is given in Table 1.

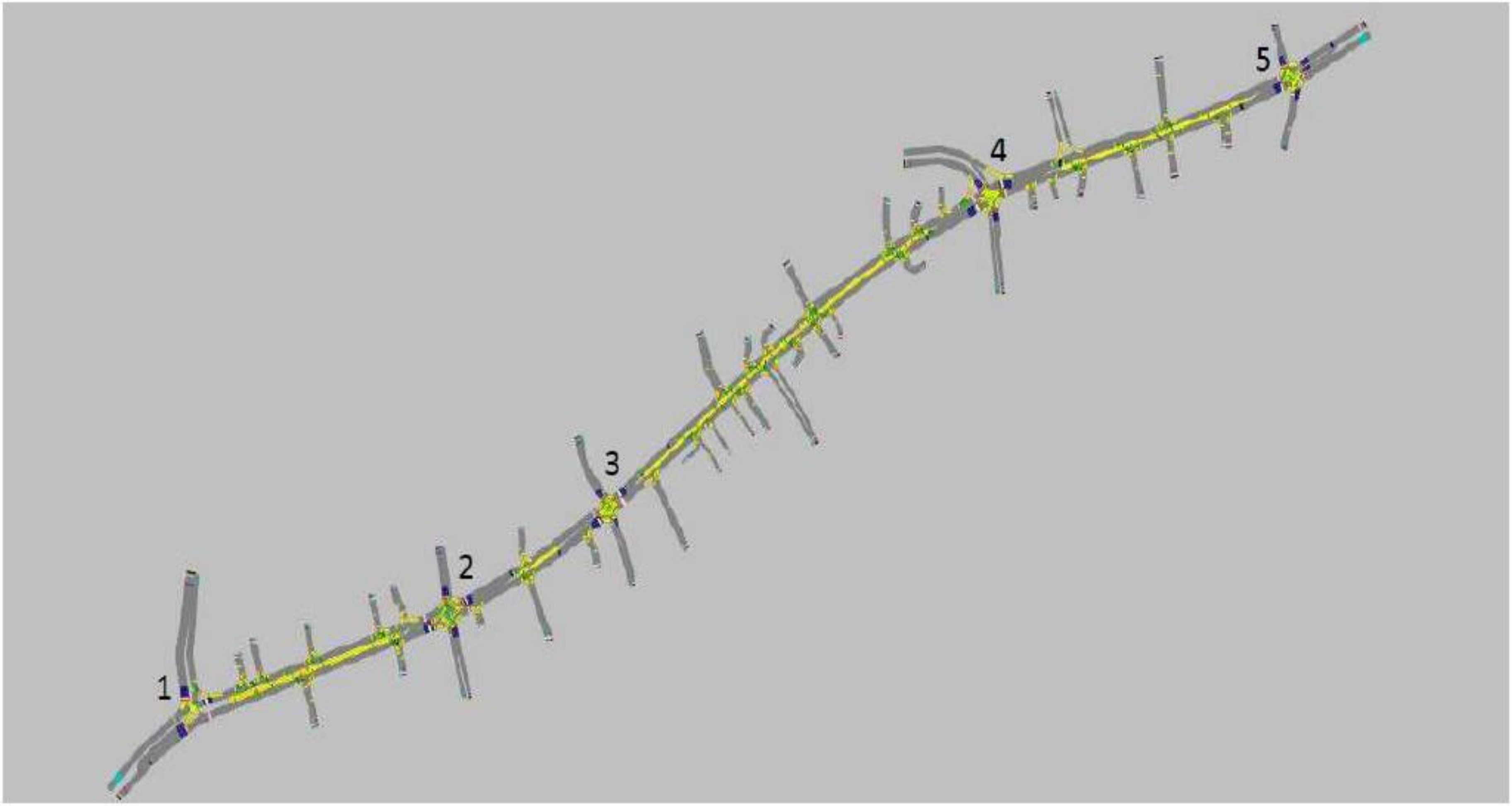


Figure 1 US 378 Lexington, South Carolina corridor with intersections (1 to 5) simulated in VISSIM

Table 1 Queue counter information

Queue Counter	Number of Lanes	Intersection
1	2	1
2	2	1
3	2	1
4	3	1
5	2	2
6	1	2
7	1	2
8	2	2
9	1	2
10	2	2
11	2	3
12	1	3
13	1	3
14	2	3
15	1	3
16	2	3
17	2	4
18	1	4
19	1	4
20	1	4
21	2	4
22	1	4
23	2	4
24	2	5
25	1	5
26	1	5
27	1	5
28	2	5
29	1	5
30	1	5
31	1	5

Average and maximum queue length data of all 31 counters are collected for one hour from four different simulation runs. From Table 1, it can be observed that different queue counters yield different types of queue length patterns based on the number of lanes, intersection, signal phasing, and timings, etc. For example, in the case of queue counter number 6 (denoted as QC6), the number of lanes is one, so the average and maximum queue length is the same. However, for QC4, the number of lanes is three, so there will be variations between the maximum and average queue lengths. The variation of average and maximum queue lengths among three different queue counters, QC12 (one lane), QC1 (two lanes) and QC3 (three lanes), is shown in Figure 2. From Figure 2, it can be observed that the variation in average queue length is higher than the maximum queue length, which indicates the existence of one more congested lane compared to the other lanes in the lane(s) group. The queue buildup for QC12 is more severe at certain times, which takes time to dissipate. On the other hand, QC1 and QC4 have a more distributed queue accumulation and dissipation due to the higher number of lanes.

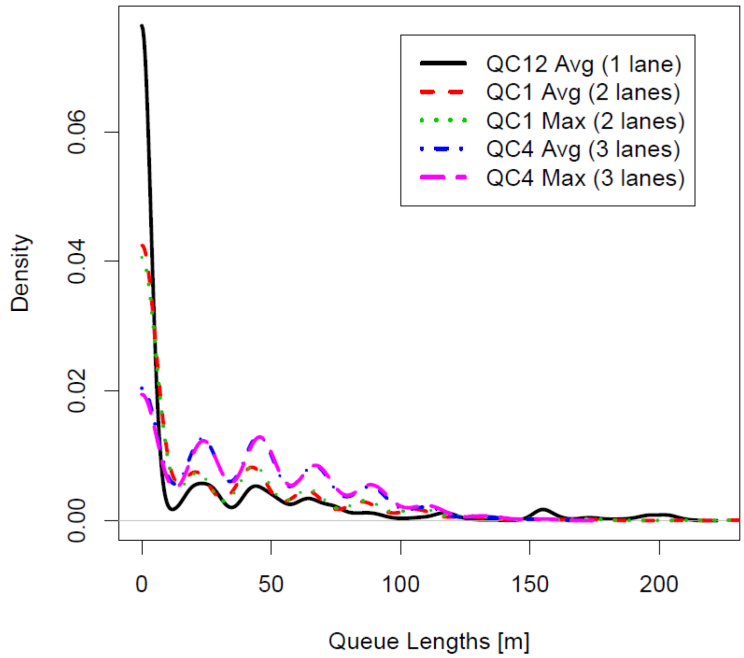
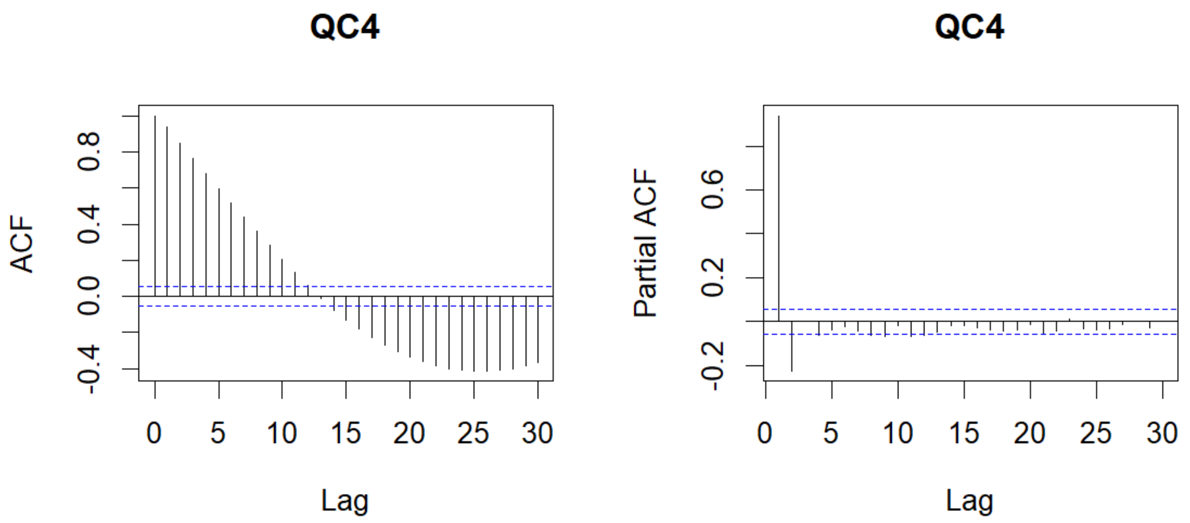


Figure 2 Comparison of average (Avg) and maximum (Max) queue length densities for different Queue Counters (QC1, QC4, and QC12)

Autocorrelation presence within the time series data assists in prediction if the models can capture them. Although several other covariates would influence (hidden or unobserved) the response variable of interest, we can simply use historical data to be able to predict future values. These conditions constitute the main motivation behind Grey system models. The autocorrelations can be shown simply using autocorrelation functions (ACF) and partial ACF or formal statistical tests. As an example, for QC4 average queue lengths, Figure 3 shows the presence of negative autocorrelation in the data (Figure 3). The partial ACF plot also reveals that ACF values become insignificant after two significant lags which suggests that the autoregressive (AR) component in the time series to be fit is low (e.g., AR(1) to AR(3)). A formal Durbin-Watson test also results in a p-value of 0.056, which barely rejects the null hypothesis of no autocorrelation. Although other parts of data used may fail to reject, the queue length data from our experiments show autocorrelations. Note that if data contains a sequence of identical observations (e.g., last observation-imputation due to missing data and series of identical values), low Gaussian noise (N(0,0.0001)) is introduced in Grey system for prediction.



(a) **(b)**

Figure 3 Example of Autocorrelation functions (ACF) of queue length time series data on Queue Counter 4 (QC4).

4.2 Baseline Time-series Forecasting Models

Based on the above discussions, time-series models can be used for forecasting queue lengths. We considered five time-series forecasting models for comparison with the Grey models: 1) Autoregressive (AR) model, 2) Logistic Smooth Transition Autoregressive (LSTAR) model, 3) Neural Networks (NNETS) model, 4) Additive non-linear Autoregressive model (AAR), and 5) long short-term memory (LSTM) neural network model. All these models are fit to the 67% of all queue counter datasets and tested on 33% of the data that contains 1-hour time series data. The description of each model is given below.

4.2.1 Autoregressive (AR) model

The AR model is a linear regression model where the future value of a time-series is predicted based on current and past values from that same time series. Eq.(13) presents a linear model that looks back 3-time steps (AR(3)).

$$Z_{t+1} = \mu + \varphi_1 Z_t + \varphi_2 Z_{t-1} + \varphi_3 Z_{t-2} + a_{t+1} \quad (13)$$

where, Z_t denotes average or maximum queue length observations at time *t;* μ represents an intercept; φ_1, φ_2, and φ_3 are weights of previous observations; and a_{t+1} denotes white noise.

4.2.2 Logistic Smooth Transition Autoregressive (LSTAR) model

LSTAR refers to Logistic Smooth Transition Autoregressive (STAR) model. These models are characterized by higher flexibility in model parameters. It is given in equation (14).

$$Z_{t+1} = \begin{cases} (\varphi 1 \ + \ \varphi 10Zt \ + \ \varphi 11Zt - \delta + \ldots + \varphi 1LZt - (L-1)\delta)(1 \ - \ G(Zt, \gamma, th)) + \\ (\varphi 2 \ + \ \varphi 20Zt \ + \ \varphi 21Zt - \delta + \ldots + \varphi 2HZt - (H-1)\delta)G(Zt, \gamma, th) \ + \ Q_{t+1} \end{cases} \quad (14)$$

where, Z_t denotes average or maximum queue length at time t and $G(Z_t, \gamma, th) = [1 + e^{-\gamma(Z_{t-1}-th)}]^{-1}$ is logistics transition function. L = 1 to 5 and H = 1 to 5 are low and high regimes, δ is the delay of the transition variable, and *th* is the threshold value.

4.2.3 Neural Networks (NNETS) model

Eq.(15) gives the mathematical formulation of the neural networks nonlinear autoregressive model (NNETS).

$$Z_{t+1} = \beta_0 + \sum_{j=1}^{D} \beta_i g\left(\gamma_{0j} + \sum_{i=1}^{m} \gamma_{ij} (Z_{t-(i-1)\delta})\right) \tag{15}$$

where *m* denotes embedding dimension, *D* is the number of hidden layers of the neural network, and β_i, γ_{0j}, and γ_{ij} represent the weights.

4.2.4 Additive non-linear Autoregressive (AAR) model

The additive nonlinear AR model refers to a model, which forecasts a time-series based on an additive combination of multiple input variables. Eq.(16) presents an AAR model.

$$Z_{t+1} = \mu + \sum_{j=0}^{m-1} s_j (Z_{t-(j)\delta}) \tag{16}$$

where s represents nonparametric univariate smoothing functions that depend on Z_ts and δ is the delay parameter. Splines from Gaussian family are fitted in the form of $Z_{t+1} \sim \sum_{i=0}^{m-1} s(Z_t, \ldots, Z_{t-j})$.

4.2.5 Long short-term memory (LSTM) neural network model

Long-Short Term Memory (LSTM) neural network model consists of one or multiple layers of LSTM neurons, which are different from traditional neurons in a neural network model. LSTM neurons contain forget gates and cell state (memory) along with input and output gates. The cell state retains the long-term dependencies and the forget gate removes all other connections to previous inputs. LSTM neurons are specially designed for capturing the long-term dependencies on the past time steps in terms of forecasting future time steps of a time-series.

The parameters and hyperparameters used for each of these models in this experiment are given in Table 2. Based on the Akaike information criteria (AIC) values, the best models are selected at each run for LSTAR and AR model. Similar analysis and justification can be found in Bezuglov and Comert (Bezuglov and Comert (2016)). Parameters of these models are estimated using maximum queue length data from queue counter 31, QC31, and are presented in Table 2.

Table 2 Estimated model parameters using maximum queue length data from queue counter 31 (i.e., QC31)

Model		Parameters
LSTAR(2,2,2)	Low regime, LLow regime, L	$\varphi_1 = 0.379$, $\varphi_{10} = 0.948$, $\varphi_{11} = 0.198$, $\varphi_{12} = -0.015$
	High regime, H	$\varphi_2 = 2.019$ $\varphi_{20} = -0.001$ $\varphi_{21} = -0.311$ $\varphi_{22} = -0.010$
	Threshhold, *Th*	*g=100, X_t=Z_t, Th= 17.74*
Autoregressive Model AR(3)		$\mu = 0.228$ $\varphi_1 = 0.979$ $\varphi_2 = 0.016$, $\varphi_3 = -0.039$$\mu = 0.228$ $\varphi_1 = 0.979$ $\varphi_2 = 0.016$, $\varphi_3 = -0.039$
Neural Networks Model (NNETS)		*D=4, m=4, Batch Size = 25*
Additive Autoregressive Model (AAR)		*m=3*
LSTM Neural Network Model (LSTM)		*Epochs=200, LSTM neurons=5, Batch Size=128, Activation=ReLU, Lag=2*

4.3 Results and Discussions

This section describes the findings related to queue length predictions using Grey models and their comparison with five baseline models, which are Autoregressive (AR) model, Logistic Smooth Transition Autoregressive (LSTAR) model, Neural Networks (NNETS) model, Additive non-linear Autoregressive model (AAR) models, and long short-term memory (LSTM) neural network model. The results of our analyses are presented in the following subsections.

4.3.1 Overall comparison

Figure 4 demonstrates the average and maximum queue length prediction errors in terms of $RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\widehat{Z}_i - Z_i)^2}{n}}$ and $MAE = \frac{\sum_{i=1}^{n}|\widehat{Z}_i - Z_i|}{n}$. Figure 4 contains box plots of RMSE and MAE for all models. Simple GM model and GM model with error corrections do not perform well in predicting queue length data. This is mostly due to the traffic signal generating periodic data. GM model performance can improve if queue lengths are predicted cycle-by-cycle without considering zero queue length. GVM and EGVM are able to capture periodicity with quadratic structures, thus, predicting with higher accuracy compared to GM and EGM models. As shown in Figure 4, LSTM, AR, LSTAR, NNETS, and AAR show similar performance. Their prediction accuracy is much higher than the GM and EGM models, but lower than GVM and EGVM models. All models show slightly worse accuracy for maximum queue length prediction compared to average queue length prediction because the variability of the maximum queue lengths has more randomness. Figure 4 exhibits the following RMSE and MAE errors related to GM, EGM, GVM, EGVM, AR, LSTAR, NNETS, AAR and LSTM models:

$RMSE_{\text{Average queue length}}$ = [GM, EGM, GVM, EGVM, AR, LSTAR, NNETS, AAR, LSTM]
= [22.83, 21.25, 4.10, 2.94, 5.01, 6.78, 5.08, 4.96, 5.71]

$RMSE_{\text{Maximum queue length}}$ = [GM, EGM, GVM, EGVM, AR, LSTAR, NNETS, AAR, LSTM]
= [21.09, 19.98, 4.86, 3.69, 6.40, 11.73, 7.77, 6.47, 13.10]

$MAE_{\text{Average queue length}}$ = [GM, EGM, GVM, EGVM, AR, LSTAR, NNETS, AAR, LSTM]
= [4.43, 4.30, 1.42, 0.91, 1.95, 2.64, 2.06, 1.85, 2.44], and

$MAE_{\text{Maximum queue length}}$ = [GM, EGM, GVM, EGVM, AR, LSTAR, NNETS, AAR, LSTM]
= [3.96, 3.90, 1.71, 1.10, 2.22, 4.56, 3.23, 2.22, 7.49]

Results show that GVM and EGVM models are able to achieve an error-bound of ±5m in terms of RMSE and ±1 m in terms of MAE for both average and maximum queue lengths. Compared models are able to achieve ±2 m error bound in terms of MAE for average queue length. However, for maximum queue lengths, the error bound increases to ±8 m in terms of MAE. Therefore, according to our experiments, GVM and EGVM models are more accurate and robust across all roadway traffic scenarios and error types (i.e., RMSE and MAE).

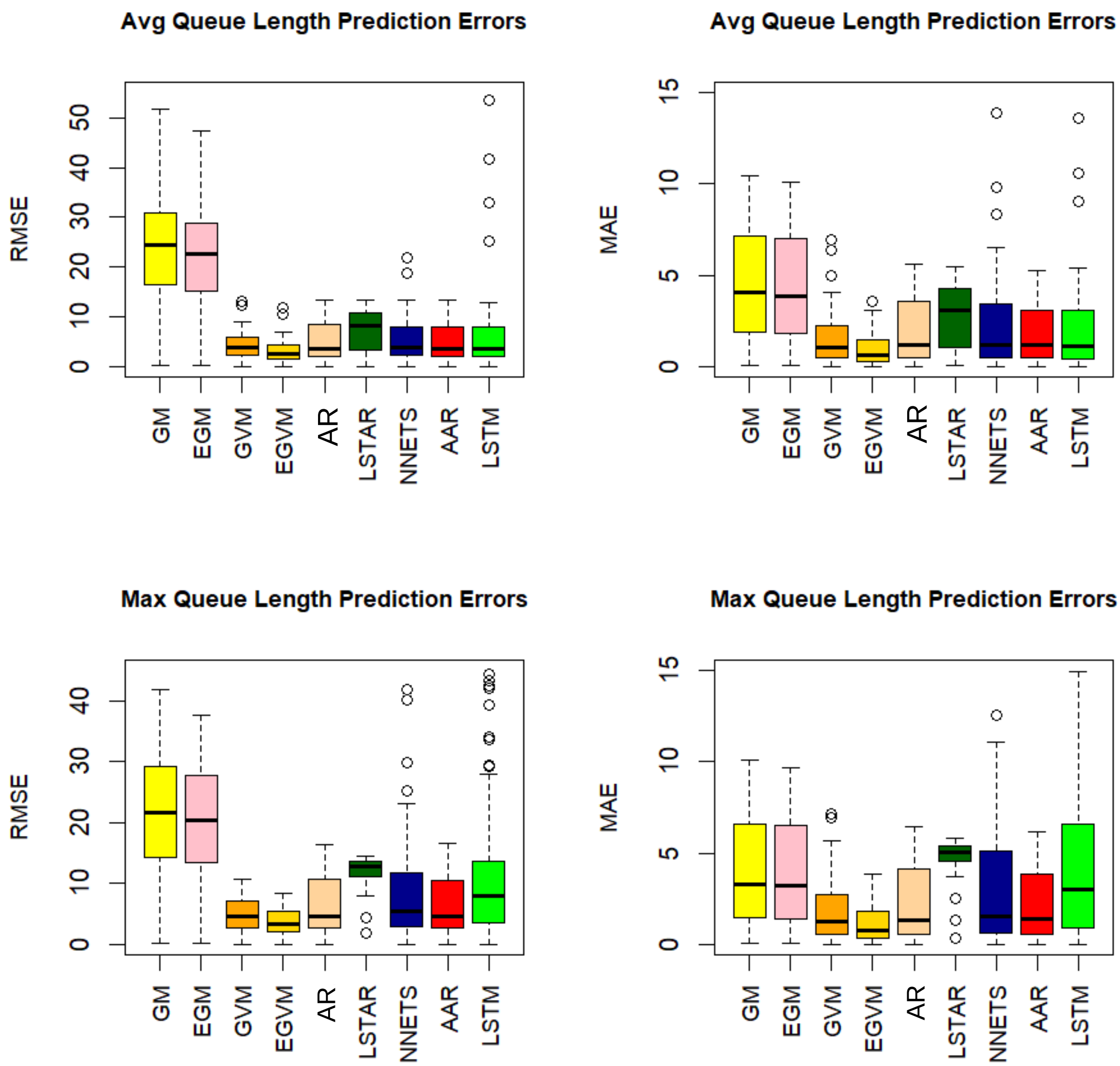


Figure 4 Comparison of the performance of different models in predicting queue length

Figure 5 presents the comparison between the performance of EGVM, LSTM, and AR models in predicting average queue lengths of QC4 as an example. We observe that LSTM overestimates when there are any abrupt changes in queue lengths. On the other hand, EGVM is able to capture sudden changes in queue length. AR model shows almost similar behavior to LSTM. The reason is that the LSTM model that we have used in this study is a basic model with minimum features (univariate single-step prediction). Moreover, LSTM is a data-intensive model but limited (i.e., one hour) data has been included in our study. Lastly, computational times are provided in Table 3 per 3600 observations across all data with training time that include 2400 and testing time that include 1200 observations. GM models do not require any training time and they are updated with low window size (of 4 past observations). LSTM requires more time to learn from the data. EGVM is the best option considering both accuracy and computational time. For robust, adaptive, and accurate predictions with low computational times and low sample size, GVM and EGVM models provide an accurate prediction of queue length.

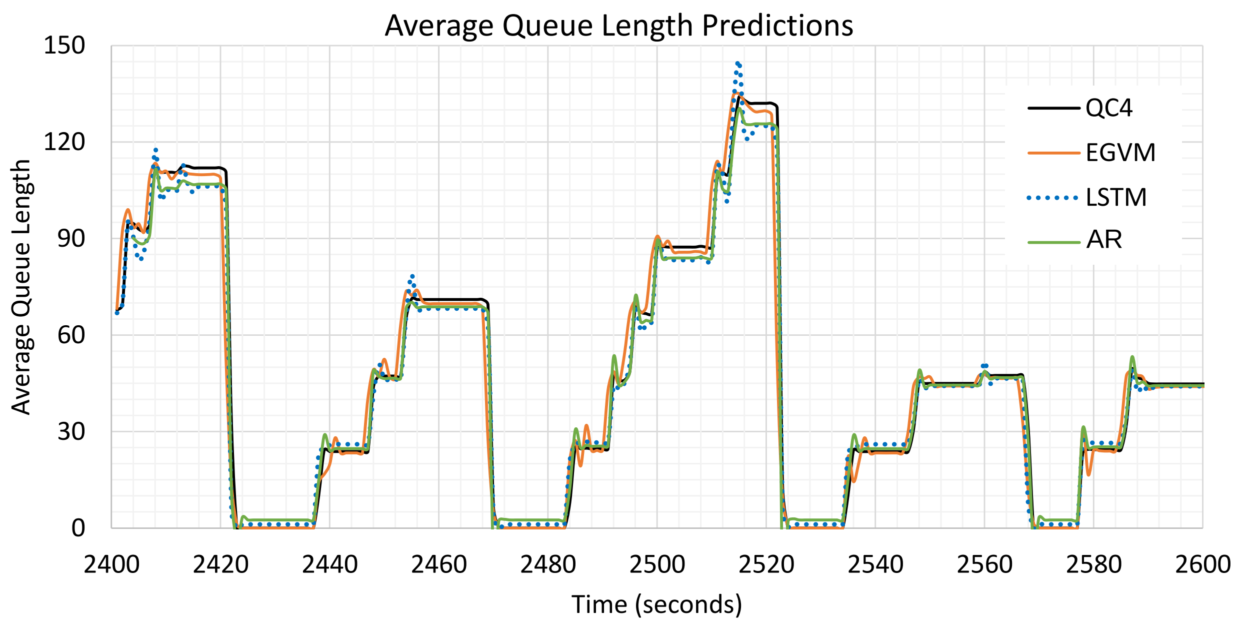


Figure 5 Comparison between the performance of EGVM, LSTM, and AR models in predicting average queue lengths

Table 3 Average computational times (in seconds) of different models

		GM	EGM	GVM	EGVM	AR	LSTAR	NNETS	AAR	LSTM
Train	Avg	-	-	-	-	0.016	7.160	0.513	0.042	50.066
	Max	-	-	-	-	0.016	6.580	0.421	0.044	49.883
Test	Avg	0.115	0.863	0.128	0.883	0.391	0.499	0.480	4.696	0.958
	Max	0.109	0.812	0.115	0.848	0.393	0.482	0.473	4.583	0.958

4.3.2 Model performance comparison (single lane vs multilane)

As stated in the literature review, we found that two major challenges of the queue length prediction models are their prediction capability for multilane scenarios compared to single lane scenarios and their performance in undersaturated and saturated scenarios. Figure 6 shows all queue length prediction results for single lane scenarios and Figure 7 shows all queue length prediction results for multilane scenarios.

Overall, all model performances degrade in multilane scenarios due to many factors; e.g., lane changing behavior of arriving vehicles. However, the EGVM model is still able to maintain a reasonable accuracy for multilane scenarios compared to single lane scenarios. The average RMSE of the EGVM model for a multilane scenario is 3.55 m compared to 1.88 m for single lane scenarios. The average MAE of the EGVM model for multilane scenarios is 1.10 m, compared to 0.44 m for single lane scenarios. These errors indicate that the EGVM model can be used for both single lane and multilane scenarios.

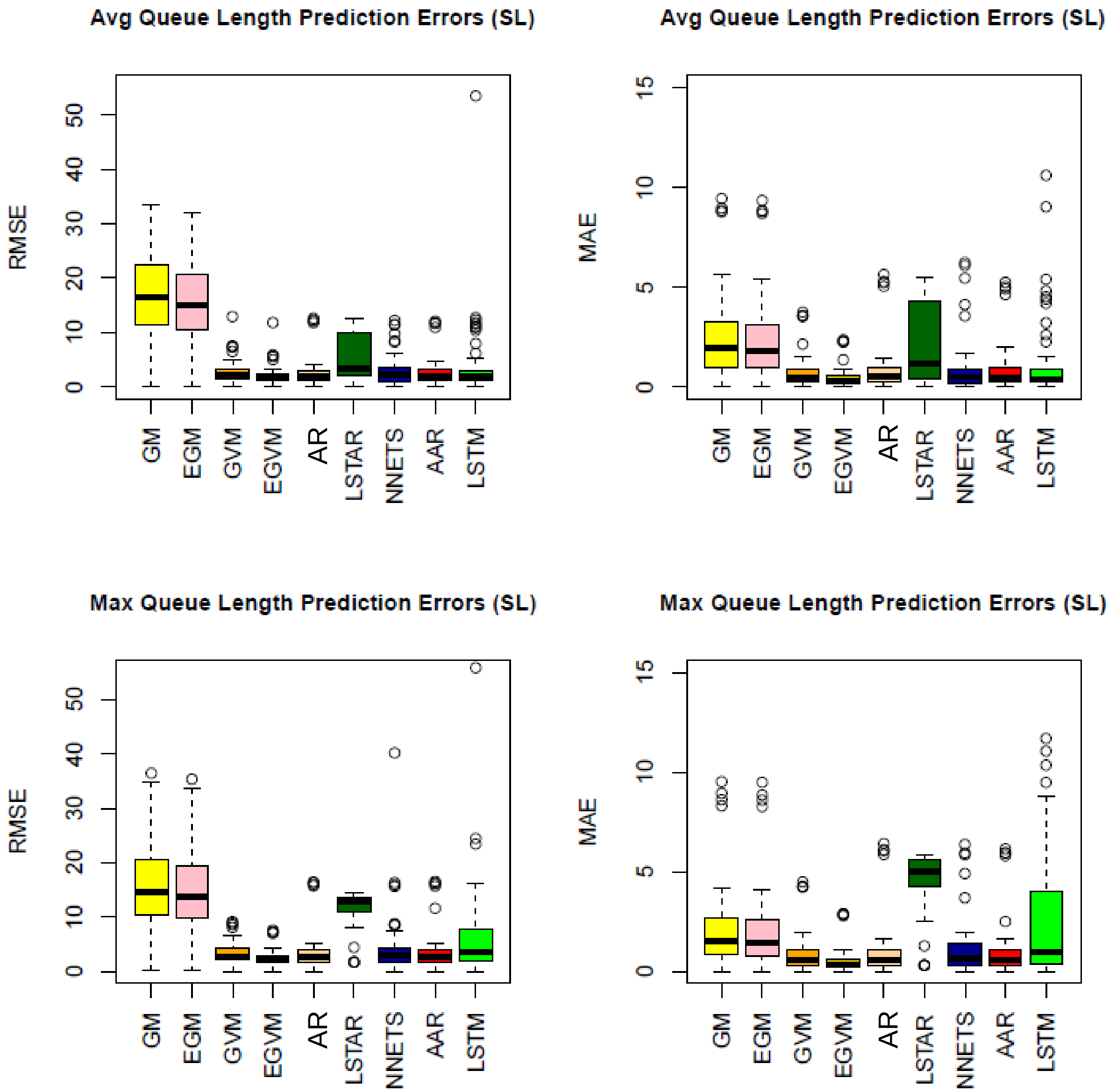


Figure 6 Prediction performances on single-lane scenarios

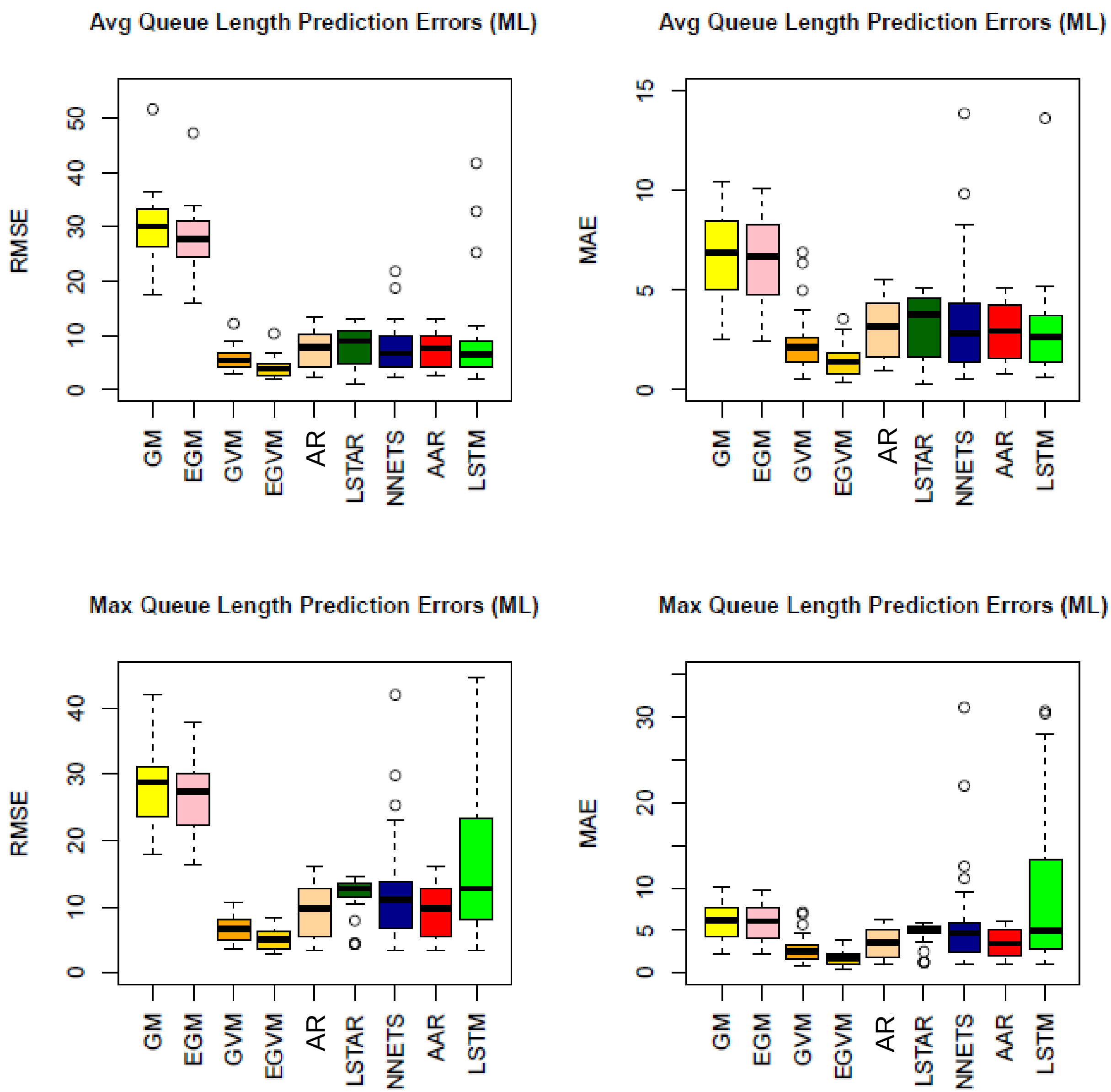


Figure 7 Prediction performances on multi-lane scenarios

4.3.3 Model performance comparison (undersaturated vs saturated)

Within multilane scenarios, we also investigated one queue counter that is operating in saturated conditions (volume/capacity>1), QC2, and another queue counter that is operating in undersaturated conditions (volume/capacity<1), QC8. The comparison of RMSE and MAE values is shown in Figure 8. From Figure 8, we observed that the EVGM model has shown similar performance compared to other models for QC2. However, it provides better performance compared to other models for QC8. The RMSE and MAE for QC8 are lower than QC2, which is expected as the congested scenario will create operational issues, such as residual queue and spillback, which could decrease the accuracy of the model. Therefore, the EGVM model can predict queue length with high accuracy in undersaturated conditions while maintaining accuracy comparable to other models for saturated (or congested) conditions.

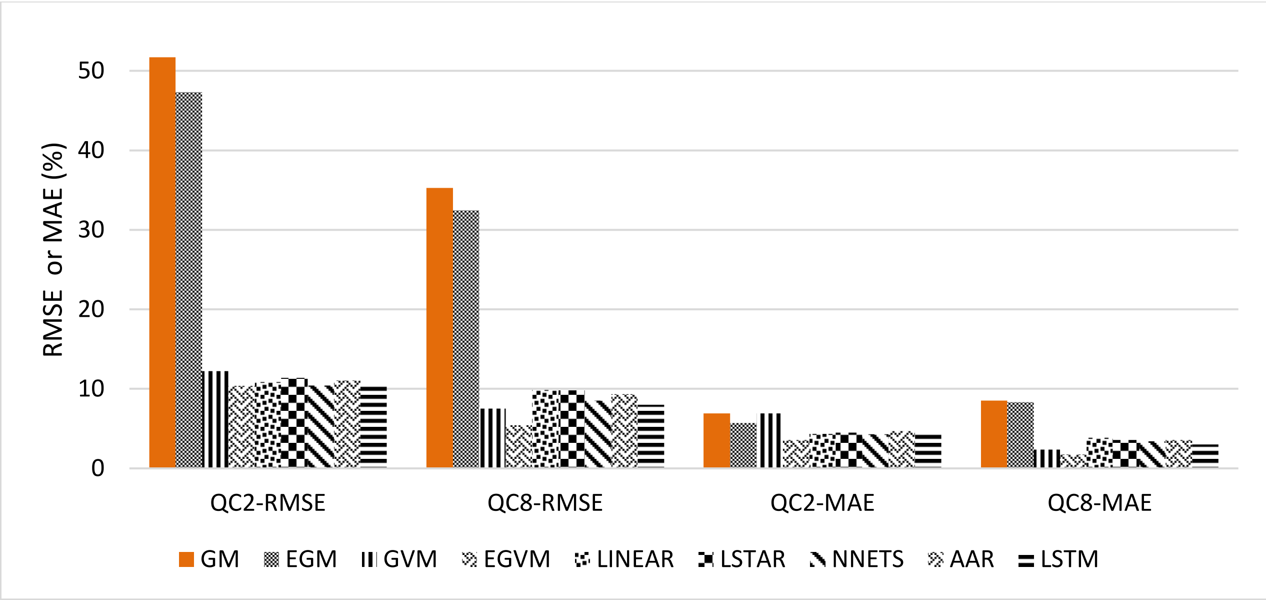


Figure 8 Performance of Grey models and other baseline models for undersaturated and saturated scenarios

CHAPTER 5

Conclusions

The study developed a robust short-term queue length prediction model for adaptive traffic signal control systems using four variations of Grey Systems: (i) the basic Grey model (GM(1,1)); (ii) GM(1,1) with Fourier error corrections (EGM); (iii) the Grey Verhulst model (GVM); and (iv) GVM with Fourier error corrections (EGVM). This study shows the effectiveness of Grey Systems in queue length prediction. The EGVM model provides the most accurate queue length predictions in both single lane and multilane scenarios. In addition, the EGVM model can predict accurately for both undersaturated and saturated traffic conditions, which establish the efficacy of the model for predicting queue length and using it as an input to the adaptive signal control systems.

The EGVM model is identified as the best model because it outperforms the models, considered in this study, for average and maximum queue length prediction. The analysis showed that GVM models (i.e., GVM and EGVM) could provide approximately one-meter precision in queue length prediction. Both GVM models (GVM and EGVM) provide a more accurate prediction than LSTM using only a fraction of the input data (4 vs 2400 observations) and require a very low computational time. This study also showed that the basic GM(1,1), even with error correction, was inferior to other baseline time series prediction models (i.e., Autoregressive (AR) model, Logistic Smooth Transition Autoregressive (LSTAR) model, Neural Networks (NNETS) model, Additive non-linear Autoregressive model (AAR) models, and long short-term memory (LSTM) neural network model) in terms of (RMSE and MAE).

One limitation of this study is that the models are dependent on the accuracy of the historical queue length estimations (i.e., the ground truth). However, an accurate queue length estimation is not a trivial task, so this work needs to be combined with a queue length estimation framework for the effective utilization of the EGVM and GVM models presented in this study. Future work should also include the following: (1) inclusion of mid-term and long-term forecasts of queue length (2) modifications to the basic Grey system equations and a study on the applicability of multivariable Grey models, and (3) consideration of seasonal behavior inducing model structures.

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