



University Transportation Research Center - Region 2

# Final Report



## Intelligent Wireless Charging for Electric Buses in Smart City

Performing Organization: Columbia University



July 2016



## University Transportation Research Center - Region 2

The Region 2 University Transportation Research Center (UTRC) is one of ten original University Transportation Centers established in 1987 by the U.S. Congress. These Centers were established with the recognition that transportation plays a key role in the nation's economy and the quality of life of its citizens. University faculty members provide a critical link in resolving our national and regional transportation problems while training the professionals who address our transportation systems and their customers on a daily basis.

The UTRC was established in order to support research, education and the transfer of technology in the field of transportation. The theme of the Center is "Planning and Managing Regional Transportation Systems in a Changing World." Presently, under the direction of Dr. Camille Kamga, the UTRC represents USDOT Region II, including New York, New Jersey, Puerto Rico and the U.S. Virgin Islands. Functioning as a consortium of twelve major Universities throughout the region, UTRC is located at the CUNY Institute for Transportation Systems at The City College of New York, the lead institution of the consortium. The Center, through its consortium, an Agency-Industry Council and its Director and Staff, supports research, education, and technology transfer under its theme. UTRC's three main goals are:

### Research

The research program objectives are (1) to develop a theme based transportation research program that is responsive to the needs of regional transportation organizations and stakeholders, and (2) to conduct that program in cooperation with the partners. The program includes both studies that are identified with research partners of projects targeted to the theme, and targeted, short-term projects. The program develops competitive proposals, which are evaluated to insure the most responsive UTRC team conducts the work. The research program is responsive to the UTRC theme: "Planning and Managing Regional Transportation Systems in a Changing World." The complex transportation system of transit and infrastructure, and the rapidly changing environment impacts the nation's largest city and metropolitan area. The New York/New Jersey Metropolitan has over 19 million people, 600,000 businesses and 9 million workers. The Region's intermodal and multimodal systems must serve all customers and stakeholders within the region and globally. Under the current grant, the new research projects and the ongoing research projects concentrate the program efforts on the categories of Transportation Systems Performance and Information Infrastructure to provide needed services to the New Jersey Department of Transportation, New York City Department of Transportation, New York Metropolitan Transportation Council, New York State Department of Transportation, and the New York State Energy and Research Development Authority and others, all while enhancing the center's theme.

### Education and Workforce Development

The modern professional must combine the technical skills of engineering and planning with knowledge of economics, environmental science, management, finance, and law as well as negotiation skills, psychology and sociology. And, she/he must be computer literate, wired to the web, and knowledgeable about advances in information technology. UTRC's education and training efforts provide a multidisciplinary program of course work and experiential learning to train students and provide advanced training or retraining of practitioners to plan and manage regional transportation systems. UTRC must meet the need to educate the undergraduate and graduate student with a foundation of transportation fundamentals that allows for solving complex problems in a world much more dynamic than even a decade ago. Simultaneously, the demand for continuing education is growing – either because of professional license requirements or because the workplace demands it – and provides the opportunity to combine State of Practice education with tailored ways of delivering content.

### Technology Transfer

UTRC's Technology Transfer Program goes beyond what might be considered "traditional" technology transfer activities. Its main objectives are (1) to increase the awareness and level of information concerning transportation issues facing Region 2; (2) to improve the knowledge base and approach to problem solving of the region's transportation workforce, from those operating the systems to those at the most senior level of managing the system; and by doing so, to improve the overall professional capability of the transportation workforce; (3) to stimulate discussion and debate concerning the integration of new technologies into our culture, our work and our transportation systems; (4) to provide the more traditional but extremely important job of disseminating research and project reports, studies, analysis and use of tools to the education, research and practicing community both nationally and internationally; and (5) to provide unbiased information and testimony to decision-makers concerning regional transportation issues consistent with the UTRC theme.

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16. Abstract <b>Abstract</b>  According to the EPA, 28% of all 2011 GHGs for the US are from transportation related sources. <sup>1</sup> These are the second largest sources of GHGs in the US after electricity. The US is also the second highest CO2 emitter after China. <sup>2</sup> These emissions are primarily from burning fossil fuels for transportation usage. While vehicles have become more environmentally friendly with lower emissions, there has still been a steady rise in GHGs from these modes of transportation. The EPA estimates that there has been an increase of 18 percent GHG, which is most likely due to more vehicles on the road.  One method for eliminating air emissions is through environmentally friendly transportation modes. EVs are considered a prime candidate for lowering carbon and other environmentally unfriendly footprints. However, the leading issues with the adoption of EVs in today's market are due to their limited driving range and lack of charging infrastructure along with the long durations of non-operation during recharging. Current technology is addressing both of these concerns through creating better, longer lasting batteries as well as other method from charging both efficiently and quickly.  The adoption of EVs by commercial fleets is an easier implementation strategy since fleets typically have predetermined routes and scheduling. We propose a feasibility study to determine primarily whether wireless charging at specifically designated bus stops throughout New York City can help to increase the feasibility of electric buses for city use, both from an operational and a financial standpoint. We have partnered with the Metropolitan Transit Authority (MTA) in order to obtain data about bus operations. The final deliverable is to provide a statistical model that can be utilized with bus data to determine the efficacy of wireless charging at bus stops. Our model can be adjusted to see to how the placement of varying numbers of charging stations would change the outcome of buses being able to complete their routes.  Using probabilistic modeling, we cluster bus trips to discover patterns of travel for buses, which include time spans that are spent at different stops. Using model selection, we choose the best model parameter, i.e. number of clusters equivalent to number of travel patterns. This probabilistic model allows us to simulate data since it is a generative model and can easily be applied to other bus lines. Also, this model allows us to simplify the optimization problem of finding good spots to install wireless charging pads that cater to as many bus trip types as possible with minimum disruption of the current schedule.			
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# Intelligent Wireless Charging for Electric Buses in Smart City

## 1 Abstract

According to the EPA, 28% of all 2011 GHGs for the US are from transportation related sources.<sup>1</sup> These are the second largest sources of GHGs in the US after electricity. The US is also the second highest CO2 emitter after China.<sup>2</sup> These emissions are primarily from burning fossil fuels for transportation usage. While vehicles have become more environmentally friendly with lower emissions, there has still been a steady rise in GHGs from these modes of transportation. The EPA estimates that there has been an increase of 18% since 1990 for transportation related GHGs, which is most likely due to more vehicles on the road!

One method for eliminating air emissions is through environmentally friendly transportation modes. EVs are considered a prime candidate for lowering carbon and other environmentally unfriendly footprints. However, the leading issues with the adoption of EVs in today's market are due to their limited driving range and lack of charging infrastructure along with the long durations of non-operation during recharging. Current technology is addressing both of these concerns through creating better, longer lasting batteries as well as other method from charging both efficiently and quickly.

The adoption of EVs by commercial fleets is an easier implementation strategy since streets typically have predetermined routes and scheduling. We propose a feasibility study to determine primarily whether wireless charging at specifically designated bus stops throughout New York City can help to increase the feasibility of electric buses for city use both from an operational and a financial standpoint. We have partnered with the Metropolitan Transit Authority (MTA) in order to obtain data about bus operations. The final deliverable is to provide a statistical model that can be utilized with bus data to determine the efficacy of wireless charging at bus stops. Our model can be adjusted to see to how the placement of varying numbers of charging stations would change the outcome of buses being able to continuously and successfully complete their routes.

Using probabilistic modeling we cluster bus trips to discover patterns of travel for buses which include time spans that are spent at different stops. Using model selection, we choose the best model parameter i.e. number of clusters equivalent to number of travel patterns. This probabilistic model allows us to simulate data since it is a generative model and can be easily applied to other bus lines. Also, this model allows us to simplify the optimization problem of finding good spots to install wireless charging pads that cater to as many bus trip types as possible with minimum disruption of the current schedule.

## 2 Background and Problem Statement

Vehicles, both personal and commercial, have become a ubiquitous form of transportation in the developed world. The auto industry is amidst a technological transformation in identifying alternative sources of energy to power vehicles due to two driving forces: environmental pollution prevention and depletion of fuel resources. The EPA states that 28% of all greenhouse gas emissions in 2011 for the United States were from transportation related sources. This drive for developing smarter solutions to create a smarter planet is crucial to advancing the technological science of EVs. As alternative methods for creating energy are being sought, we see an increased interest in electric vehicles as one potential solution for lessening our dependence on fossil fuels.

In 2011, President Obama announced in his State of the Union address that his administration would push to have 1-million electric vehicles on the road by 2015. Similarly, New York and seven other states have joined together in a similar initiative to put 3.3 million zero-emission vehicles on the road by 2025. Currently, the main hurdle with the adoption of EVs is due to their limited driving range and lack of ease in recharging.

Driving behavior varies for a vehicle based on the type of vehicle and its main usage purpose. Commuters use their vehicles very differently from service vehicles. For the purpose of this proposal, we aim to study fleet vehicles.

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<sup>1</sup> "Greenhouse Gas Emissions: Transportation Sector Emissions." EPA. Environmental Protection Agency. Web: <http://www.epa.gov/climatechange/ghgemissions/sources/transportation.html>. 04 May 2014

<sup>2</sup> Houghton, R.A. (2008). Carbon Flux to the Atmosphere from Land-Use Changes: 1850-2005. In TRENDS: A Compendium of Data on Global Change. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tenn., U.S.A.

Fleet vehicles are designed as groups of vehicles that are owned or leased by a business as opposed to an individual or family. Types of fleet vehicles range from cars to vans to trucks, depending on the need of the company. Multiple drivers, multiple paths, or any combination of the two can use any single vehicle in a fleet system.

Structured fleet vehicles are those vehicles that follow a set path and schedule with very little variance in daily activity. These include vehicles such as Federal Express (FedEx), United Parcel Service (UPS), Metropolitan Transit Authority (MTA) buses, etc. In each of the example cases, the buses and trucks have a designated schedule. This allows for very little variability except for times when there is major traffic congestion or other conditions that prohibit from keeping its course. Given this, we know the hours of operation for the vehicles as well as the number of miles traveled for each vehicle. More importantly, we know the location of a vehicle at a given time.

In this study, we examined the Metropolitan Transit Authority (MTA) since it is comprised of over 5,900 buses in a fixed-route service in New York City as well as 2,000 vans and cabs for ADA para-transit service. Buses operate in the city on a continuous cycle with increased coverage during peak transit times. Mass transportation systems are critical in sustaining large metropolitan cities. Currently, bus transit is the second leading carbon emitter/passenger mile after private commuting vehicles.

The MTA has converted much of its fleet to hybrid buses that are designed with electric-drive systems that consist of a battery pack and electric motor. There is regenerative braking that supplies additional power to accelerate and for inclines. This technology combined with the use of a diesel particulate matter and ultra-low-sulfur fuel has reduced emissions of particulate matter by 90%, nitrogen oxides by 40% and greenhouse gases by 30%. Additionally, fuel consumption of hybrid buses is approximately 25% to 35% less than a standard diesel bus.

After a recent meeting with the Chief of Innovation and Technology at the New York City Transit, we have also learned that the MTA has committed to transferring to all electric buses as well as adopting other greener environmentally friendly options for operating the large transit system in NYC. This sentiment is shared by several major U.S. cities.

Still opportunities exist to further reduce the emissions and increased greenness of buses. Fleets of buses for transit (e.g. Greyhound) and school buses have not adopted these technologies. For example, there are school bus emission reduction programs to encourage school buses to convert to better vehicle types. There are several different trials ongoing worldwide for adoption of electric vehicles such as a 5-year trial ongoing in the United Kingdom and a three-month zero-emissions bus trial in Bangalore, India.

### 3 Literature Review

There have been several electric bus trials around the globe. While there is not a lot of academic literature available about these trials, there are several articles published to indicate that this is a key trend for the future of electric buses. A novelty of our study is the quantification of the feasibility of electric buses in more research-based terms for future modeling and simulation as the technologies improve.

Currently ongoing trials in the United States include trials through a Utah-based company, WAVE. WAVE started operating buses on college campuses. While the idea of electric buses is not novel, WAVEs approach to reducing battery size and placing constraints on bus travel speeds to minimize over-usage of the battery is a new method for utilizing wireless charge transfer at designated areas. This downsizing of the battery size helps with cost reduction for the vehicles. WAVE currently has a trial in California and looks to expand its trials to 10 to 20 cities in the upcoming year.

A long 5-year wireless electric bus pilot is ongoing in the United Kingdom since the beginning of 2014. This trial has bus operating recharging the buses to two-thirds of their battery capacities while parked over charging plates when the bus drivers have their scheduled breaks. The novelty is that the trial will try to utilize only two charging plates to support a fleet of eight electric buses.

There was even an electric bus trial in New York City that was run in conjunction with BYD (electric bus manufacturer) and the NYC MTA in 2012. The pilot tested several routes throughout Manhattan and covered a total distance of 1,481 miles. While the results from the trial were favorable, the main issue for the MTA's adoption of the technology is the large initial costs required to convert to the electric buses. Additionally, this pilot test did not address the potential of wireless charging but rather relied on having the bus return to a depot. The main issue with this method is the long amounts of time needed to recharge the vehicle while it remains out of service, adding to the number of buses needed to maintain a working fleet.

Future trials include one that is set to start in 2015 in Berlin. This 4-bus fleet trial will run on a 6.1 kilometers (3.8 mile) pilot project route and will utilize Bombardier's inductive charging system throughout the route to keep recharging. At the proposed transfer rate output of 200kW, the bus will only need a few minutes of charging points at end points of the line in order to recharge and continue on trek.

With several electric bus companies on the current market, as well as several competing wireless charge transfer devices also available, the current technical specifications available for these devices and buses are typically from the manufacturer. The United States Department of Transportation retained a company to produce a report which contains in detail a survey up to 2014 about the technologies that exist for Electric Buses.<sup>3</sup> Current bus chargers due to advanced superconductors and better batteries can now charge as quickly as full charges in 3 minutes. There are some that even boast 10 seconds, although not much literature is available to validate the claim.<sup>4</sup>

Our goal in this study, however, was not to pick a particular technology but to show the feasibility of having wireless charging at bus stops.

## 4 Analysis

The goal of our study is to demonstrate the feasibility of electrical buses that use wireless charging technology throughout their route without disrupting the established pattern of operation for the fleet. We introduce a methodology by which patterns of operation of the transportation route is recognized and using this information we formulate a combinatorial optimization problem whose solution is the location of wire-less charging pads that maintains the operation of the fleet. This methodology provides the necessary framework to investigate any bus route and wireless charging technology and the appropriate numbers and locations to install the charger pads. Subsequently, a cost-benefit analysis can be carried out using our approach in order to decide adoption of a technology.

## 5 Data

The data on which we perform our analyses is provided by New York City MTA and belongs to B63 route in Brooklyn from April 3, 2011 through May 3, 2011. Each record in this data set contains, for a single bus, the time of observation, bus location, bus route, next stop, distance from that stop, and other variables.

- vehicle\_id - the 4-digit ID of the bus
- timestamp - the date and time of the observation
- latitude - the latitude of the bus
- longitude - the longitude of the bus
- phase - the phase of the bus in its duty cycle; current extract includes only observations when the bus is inferred to be IN\_PROGRESS (i.e. driving on the route) or LAYOVER\_DURING (i.e. waiting at a terminal for a trip to begin)
- trip\_id - a GTFS trip\_id representing the stopping pattern inferred for the given bus at the given time
- direction\_id - the GTFS direction id for the direction the bus is traveling
- trip\_headsign - the GTFS destination sign value for the inferred representative trip
- shape\_dist\_traveled - the distance the bus has traveled (in meters) along the precise geographic route of the inferred representative trip
- stop\_id - the GTFS stop\_id of the next stop the bus will serve

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<sup>3</sup> [http://www.calstart.org/Libraries/Publications/Peak\\_Demand\\_Charges\\_and\\_Electric\\_Transit\\_Buses\\_White\\_Paper.s.b.ashx](http://www.calstart.org/Libraries/Publications/Peak_Demand_Charges_and_Electric_Transit_Buses_White_Paper.s.b.ashx) <sup>4</sup>Ibid.



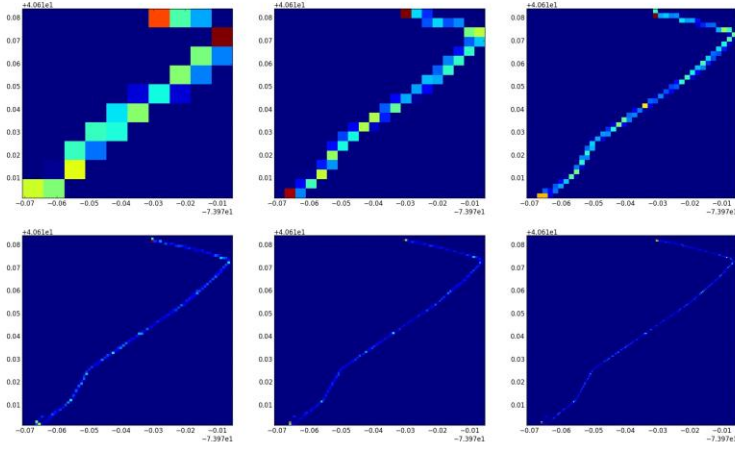


Figure 1: Six matrix histograms (heat maps) showing the same trip imposed upon grids of different granularity parameter  $m$ .

- stop\_sequence - the GTFS stop\_sequence of the next stop the bus will serve distance\_from\_stop - the distance of the bus (in meters) from that next stop

The numbers of individual trips (travels between starting points and final destinations) that are recorded in this data set exceeds tens of thousands. This volume of data can make any optimization problem extremely difficult. In the following sections we discuss how to overcome this challenge while gaining further insight into the dataset by using a generative probabilistic model and preprocessing the data to make it appropriate for our model.

## 6 Preprocessing

In this section we examine the dataset from a perspective that would help with designing our probabilistic model. The reasoning behind choices that are made is explained in the next section, which describes the details of the model. Previously, we saw that the records in the dataset belong to buses from different times of the day and locations. We group these records into individual trips determined by the process in which a particular bus travels from the starting point to the final destination. Note, that the route is the same across all trips. Therefore, we break this route in equidistant segments. Hereby, each record will fall into one of these geographical segments. This allows a single trip to be represented by a histogram of the number of records versus the geographical location. Bus routes generally do not follow a straight line pattern. Thus, we project the trajectory of the route on two dimensional surfaces onto straight lines.

Let  $X_1, X_2, \dots, X_N$  denote the partition of all records into  $N$  individual observed trips. Also, consider the smallest latitude and longitude intervals that contain all the records of the dataset, which are designed by minimum and maximum values of each record. For simplicity, let this geographical area be a square. We divide the latitude and longitude information into  $m$  equal size intervals, which creates an area that can be thought of as a grid. Therefore, the grid has  $g = m \times m$  bins. Following the formation of the grid, each individual record can be assigned to a single bin in the grid. Therefore, all the data from a single partition  $X_i$  can form a two dimensional histogram which is denoted by  $H_i$ . It is worth to note that the superposition of all the two dimensional trip histograms, namely  $H^* = \sum_i^N H_i$ , is a sparse matrix since much of this geographical region is not on the route of the vehicles. Therefore,  $H_i$  for all  $i$  can be encoded into a vector histogram using a map  $R_{H^*}: N^{m \times m} \rightarrow N^M$  where  $M$  is the number of non-zero elements of  $H^*$ . Let  $R_{H^*}(H_i) = h_i$ , where the  $h_{ij}$  is the  $j$ th non-zero component of  $H^*$  ordered first by column numbers followed by row numbers.

### 6.1 Probabilistic Model

We are interested in the way a bus trip unfolds from the starting point until the final destination. The most relevant information is the amount of time that a bus might spend at certain locations. Theoretically, if we know the exact

amount of time information across all observed bus trips and possible distinct location along the route, we can formulate an optimization problem to find the locations for which there should exist a wireless charging pad such that no bus in previously observed trips would have run out of fuel (electricity). There are two major problems with this framing. The exact information is not available, and new trip types might emerge in the future.

In the next step we propose the use of a multinomial mixture model as a probabilistic model in order to capture the nature of the observations. That is we suggest the way that bus trips unfold from their origins to their destinations (which is fixed in our setting) can be categorized into histograms that are generated from a mixture of multinomial distributions. This is incentivized by the observation that during different hours of day and based on various weather patterns, the extent of road traffic, etc., the time-location trajectory of trips can change. However, in similar circumstances it is expected for these trajectories to be similar as well. Therefore, each individual multinomial distribution might represent a circumstance which creates a certain type of bus trip.

Earlier, we discussed obtaining the set  $\mathbf{H} = \{h_1, h_2, \dots, h_N\}$  which consists of all the observed trip histograms. Assuming there are  $K$  mixtures, the model is described below

$$p(h_{1:N} | z_{1:N}, \theta_{1:K}, \pi) = \prod_{i=1}^N p(z_i | \pi) \prod_{k=1}^K p(h_i | \theta_k, z_i) \quad (1)$$

where  $\theta_k$  is a normalized vector of dimension  $M$  and  $\pi$  is the categorical distribution of each cluster. which can also be depicted by the figure 2. The generative model can be described as follows

$$z_i \sim \text{Categorical}(\pi) \quad (2)$$

$$h_i \sim \text{Multinomial}(\theta_{z_i}) \quad (3)$$

For inference, we use the Expectation-Maximization (EM) Algorithm which consists of iteratively updating the hidden parameters  $z_{1:N}$  and  $\theta_{1:K}$  and  $\pi$ . More specifically, the expectation step at each iteration consists of finding cluster assignment of each histogram given  $\pi$  and  $\theta_{1:K}$  that is the conditional probability  $p(z_{1:N} | \pi, \theta_{1:K}, H_{1:N})$ . Then, in the maximization step, the cluster parameters are updated using the membership assignments  $z_{1:N}$ .

- E step: For all  $i = 1, \dots, N$  and  $k^* = 1, \dots, K$

$$z_{ik^*} := \frac{\prod_{j=1}^M \theta_{k^*j}^{h_{ij}}}{\sum_{k=1}^K \left( \prod_{j=1}^M \theta_{kj}^{h_{ij}} \right)} \quad (4)$$

- M step: For all  $m = 1, \dots, M$  and  $k^* = 1, \dots, K$

$$\theta_{k^*m} := \frac{\sum_{i=1}^N z_{ik^*} h_{im}}{\sum_{j=1}^M \sum_{i=1}^N z_{ik^*} h_{ij}} \quad (5)$$

$$\pi := \frac{\sum_{i=1}^N z_i}{\sum_{j=1}^M \sum_{i=1}^N z_{ij}} \quad (6)$$

The EM algorithm requires initialization of parameters which is achieved by randomly selecting  $K$  normalized histograms as the original  $\theta_{1:K}$  parameters and setting  $\pi$  to be a uniform categorical distribution. In order to find the number of clusters  $K$  we do model selection by using the Akaike Information Criterion (AIC).

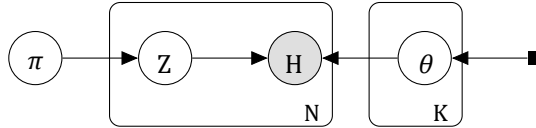


Figure 2: The graphical model which describes the conditional dependencies of the observed variables, namely histograms and hidden parameters which are multinomial distributions.

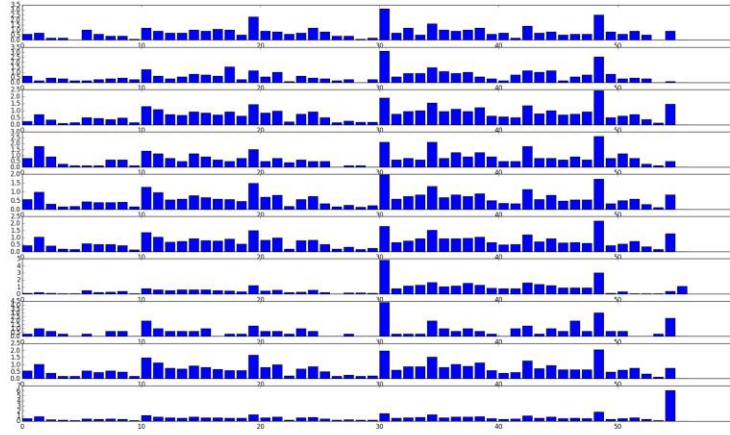


Figure 3: The result of running an instance of multinomial mixture model with 10 clusters on our data. Each histogram shows the distribution of time spent at each bust stop. Note that the stops appear on the histogram with the order that they appear on their physical route. The unit of time in the graphs is 30 sec.

## 7 Optimization

In this section, we formalize the problem of locating suitable places for installing wireless charging pads for a particular bus route. Let  $H = \{h_i\}$  denote the available histogram of distribution of time over stops for each individual trip. These histograms indicate the amount of time that is spent at each individual bus stop across all the observed trips. Without loss of generality allow the stops to be equidistant. This allows us to establish a single unique depletion rate of the battery from any stop to the one right after it. We denote this universal stop-to-stop depletion rate by  $\delta$ . In other words,  $\delta$  is the percentage of the battery's capacity, which is discharged if the vehicle moves from one stop to the one immediately after it. Note that, it is trivial to generalize the problem in order to account for various distances between any two immediate stops by introducing  $\delta_1, \delta_2, \dots, \delta_M$  where  $M$  is the number of stops. Finally, we know that the batteries are charged with a rate  $\rho(t)$  percent for  $t$  unit of time spent at each stop. Here, we assume that  $\rho$  is a linear function in other words  $\rho(t) = r \times t$ .

Our goal is to ensure that the solution to our optimization problem, which consists of a set of charging pad locations, ensures no bus trip type is at the risk of depleting entire electric charge without arriving at the final destination. Subsequently, we would like to decrease the amount of energy that the buses will require by other means (fossil fuels). In our formulation, a solution is an ordered tuple of locations  $\sigma = (\sigma_1, \dots, \sigma_N)$  for a fixed  $N$  where  $1 \leq \sigma_1 < \sigma_2 < \dots < \sigma_N \leq M$ . The following set of equations describes the utility function. Note that,  $f_{ij}(\sigma)$  indicates the remaining battery charge in trip  $i$  where the bus reaches stop  $\sigma_j$  before recharging at that location. For simplicity, let us assume that  $\sigma_0 = 1$  for all possible solutions.

$$U(\sigma) = -\prod_{i=1}^M \prod_{j=1}^N \mathbb{1}_{(f_{ij}(\sigma) < 0)} \times f_{ij}(\sigma) \quad (7)$$

$$f_{ij}(\sigma) = 100 + \sum_{k=1}^{j-1} (-(\sigma_k - \sigma_{k-1})\delta + \rho h_{i\sigma_k}) - (\sigma_j - \sigma_{j-1})\delta \quad (8)$$

Given  $S_N$  the set of all feasible solutions  $\sigma = (\sigma_1, \dots, \sigma_N)$  for a fixed  $N$  where  $1 \leq \sigma_1 < \sigma_2 < \dots < \sigma_N \leq M$  which our problem then can be formalized as

$$\min_{\sigma \in S_N} U(\sigma) \quad (9)$$

Since it is computationally intractable to find the optimal solution to this combinatorial problem, we propose a local search algorithm, which is described as follows. It is crucial to notice this solution would be computationally feasible only if, instead of all the possible trip variations, we consider the categories of trips that we have derived

using the probabilistic model described in the previous section. We initialize the solution to a randomly selected feasible solution. Then one of the stops in the solution is selected according to a uniform distribution. Then a value (direction)  $d$  is drawn from a beta distribution  $\beta(1, \alpha)$  for an alpha parameter. Then the selected stop of the solution is updated to either  $\sigma_i := \sigma_i + \lfloor d \frac{M}{2} \rfloor$  or  $\sigma_i := \sigma_i - \lfloor d \frac{M}{2} \rfloor$  at random. We make sure to choose the feasible one if one of them is not feasible. In the end it, is important to make sure there are not duplicate stops in the solution and that the new solution is an ordered tuple. If the new solution improves the utility, we update our solution. We keep doing the same procedure iteratively for a fixed number of iterations.

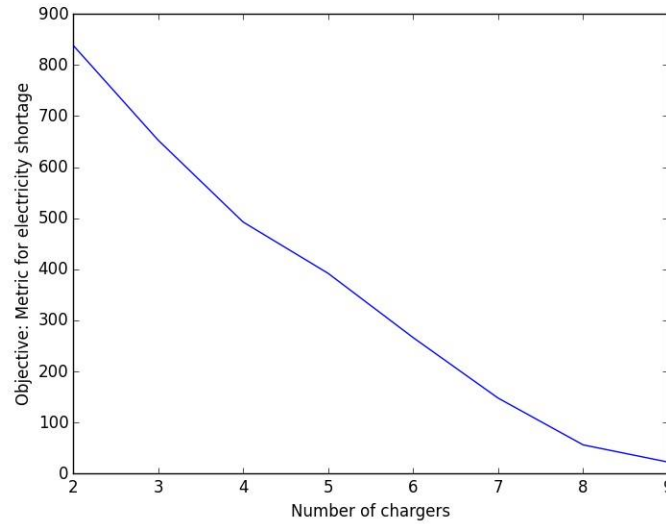


Figure 4: Depicting the utility of the suboptimal solutions to our problem for a fixed  $(\rho, \delta)$  characteristics for different fixed number of chargers.

A long-exposure photograph of a city skyline at night, reflected in a body of water. In the foreground, a bridge or highway has light trails from moving vehicles. The sky is dark, and the city lights are bright and colorful.

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