

# **Analysis of Variability in Heavy Truck Braking Systems**



U.S. Department of Transportation  
**Federal Motor Carrier Safety Administration**

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## **FOREWORD**

This report provides a summary of analyses conducted to measure variability in stopping distance tests conducted on commercial truck tractors. The data used were retrieved from tests performed under the controlled conditions specified for FMVSS 121 air brake system compliance testing. The report also explores factors affecting FMVSS-121 stopping distance and stopping distance variability, such as brake type, weight, wheelbase, and anti-lock brake system (ABS) system configuration. Results may be of interest to truck manufacturers, carriers, platooning technology developers, and others interested in truck tractor-related braking factors. This publication is the final report for this effort.

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16. Abstract <b>In support of the Federal Motor Carrier Safety Administration's (FMCSA) ongoing interest in truck platooning, this report summarizes analyses conducted to measure variability in stopping distance tests conducted on commercial truck tractors. The data used were retrieved from tests performed under the controlled conditions specified for FMVSS 121 air brake system compliance testing. The report also explores factors affecting FMVSS-121 stopping distance and stopping distance variability, such as brake type, weight, wheelbase, and tractor antilock braking system (ABS).</b>  <b>This analysis uses existing test data collected between 2010 and 2019. The FMVSS-121 data may not exactly reflect many common braking situations experienced by platooning vehicles (the typical braking event does not employ a truck's full braking capacity), but the data do provide insight into the variability of full-system stopping distance performance. This analysis seeks to identify the variability of the service brake stopping distance as defined by 49 CFR 571.121, S5.3.1 Stopping Distance—trucks and buses. Knowledge of this variability may provide a basis for more focused and platoon-relevant testing.</b>			
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# SI\* (MODERN METRIC) CONVERSION FACTORS

Approximate Conversions to SI Units				
Symbol	When You Know	Multiply By	To Find	Symbol
<b>Length</b>				
in	inches	25.4	millimeters	mm
ft	feet	0.305	meters	m
yd	yards	0.914	meters	m
mi	miles	1.61	kilometers	km
<b>Area</b>				
in <sup>2</sup>	square inches	645.2	square millimeters	mm <sup>2</sup>
ft <sup>2</sup>	square feet	0.093	square meters	m <sup>2</sup>
yd <sup>2</sup>	square yards	0.836	square meters	m <sup>2</sup>
ac	Acres	0.405	hectares	ha
mi <sup>2</sup>	square miles	2.59	square kilometers	km <sup>2</sup>
<b>Volume (volumes greater than 1,000L shall be shown in m<sup>3</sup>)</b>				
fl oz	fluid ounces	29.57	milliliters	mL
gal	gallons	3.785	liters	L
ft <sup>3</sup>	cubic feet	0.028	cubic meters	m <sup>3</sup>
yd <sup>3</sup>	cubic yards	0.765	cubic meters	m <sup>3</sup>
<b>Mass</b>				
oz	ounces	28.35	grams	g
lb	pounds	0.454	kilograms	kg
T	short tons (2,000 lb)	0.907	megagrams (or "metric ton")	Mg (or "t")
<b>Temperature (exact degrees)</b>				
°F	Fahrenheit	5(F-32)/9 or (F-32)/1.8	Celsius	°C
<b>Illumination</b>				
fc	foot-candles	10.76	lux	lx
fl	foot-Lamberts	3.426	candela/m <sup>2</sup>	cd/m <sup>2</sup>
<b>Force and Pressure or Stress</b>				
lbf	poundforce	4.45	newtons	N
lbf/in <sup>2</sup>	poundforce per square inch	6.89	kilopascals	kPa
Approximate Conversions from SI Units				
Symbol	When You Know	Multiply By	To Find	Symbol
<b>Length</b>				
mm	millimeters	0.039	inches	in
m	meters	3.28	feet	ft
m	meters	1.09	yards	yd
km	kilometers	0.621	miles	mi
<b>Area</b>				
mm <sup>2</sup>	square millimeters	0.0016	square inches	in <sup>2</sup>
m <sup>2</sup>	square meters	10.764	square feet	ft <sup>2</sup>
m <sup>2</sup>	square meters	1.195	square yards	yd <sup>2</sup>
Ha	hectares	2.47	acres	ac
km <sup>2</sup>	square kilometers	0.386	square miles	mi <sup>2</sup>
<b>Volume</b>				
mL	milliliters	0.034	fluid ounces	fl oz
L	liters	0.264	gallons	gal
m <sup>3</sup>	cubic meters	35.314	cubic feet	ft <sup>3</sup>
m <sup>3</sup>	cubic meters	1.307	cubic yards	yd <sup>3</sup>
<b>Mass</b>				
g	grams	0.035	ounces	oz
kg	kilograms	2.202	pounds	lb
Mg (or "t")	megagrams (or "metric ton")	1.103	short tons (2,000 lb)	T
<b>Temperature (exact degrees)</b>				
°C	Celsius	1.8c+32	Fahrenheit	°F
<b>Illumination</b>				
lx	lux	0.0929	foot-candles	fc
cd/m <sup>2</sup>	candela/m <sup>2</sup>	0.2919	foot-Lamberts	fl
<b>Force and Pressure or Stress</b>				
N	newtons	0.225	poundforce	lbf
kPa	kilopascals	0.145	poundforce per square inch	lbf/in <sup>2</sup>

\* SI is the symbol for the International System of Units. Appropriate rounding should be made to comply with Section 4 of ASTM E380. (Revised March 2003, Section 508-accessible version September 2009.)

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## LIST OF ACRONYMS, ABBREVIATIONS, AND SYMBOLS

<b>Acronym</b>	<b>Definition</b>
4S4M ABS	antilock braking system with four wheel-sensors and four modulators
6S4M ABS	antilock braking system with six wheel-sensors and four modulators
6S6M ABS	antilock braking system with six wheel-sensors and six modulators
ABS	antilock braking system
CMV	commercial motor vehicle
disc/disc	tractor with disc-brake-equipped steer axle and disc-brake-equipped drive axles
disc/drum	tractor with disc-brake-equipped steer axle and drum-brake-equipped drive axles
drum/drum	tractor drum-brake-equipped steer axle and drum-brake-equipped drive axles
FMVSS	Federal Motor Vehicle Safety Standards
GAWR	gross axle weight rating
GVW	gross vehicle weight
GVWR	gross vehicle weight rating
FMCSA	Federal Motor Carrier Safety Administration
USDOT	U.S. Department of Transportation

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# EXECUTIVE SUMMARY

## PURPOSE, RATIONALE, AND BACKGROUND

This study analyzes variations in stopping distance, a factor critical to determining the best order for trucks operating in a platoon. To minimize the chance of collision within the platoon during a braking event, the vehicle with the shortest stopping distance should be placed at the rear of the platoon, while the vehicle with the longest stopping distance should be placed at the front of the platoon. However, stopping distance is somewhat variable even under ideal conditions. This brake performance variability is of interest to the Federal Motor Carrier Safety Administration (FMCSA) as a part of wider efforts to explore platooning technologies, and the truck tractor braking system is a key element of a tractor-trailer's overall braking performance.

## PROCESS

To explore the parameters affecting variability in stopping distance, Oak Ridge National Laboratory (ORNL) has subcontracted with Link Engineering (LINK) to obtain anonymized Federal Motor Vehicle Safety Standards-121 (FMVSS-121) air brake system stopping distance data from a variety of truck tractors. Performing new tests would have been prohibitively expensive, so this analysis builds on previously collected data. The FMVSS-121 data selected for this analysis may not exactly reflect many common braking situations experienced by platooning vehicles (the typical braking event does not employ a truck's full braking capacity), but the data do provide insight into the variability of full-system stopping distance, and by extension, braking performance capability. This analysis seeks to identify the variability of the service brake stopping distance as defined by 49 CFR 571.121, S5.3.1 Stopping Distance—trucks and buses.

LINK was contracted to provide stopping distance data on three-axle tractors equipped with brake systems capable of meeting current FMVSS-121 air brake system requirements. LINK reviewed and summarized a database of over 800 vehicle tests performed between 2010 and 2019. LINK identified a total of 105 tests which all have the following characteristics:

- Three-axle tractors.
- Front gross axle weight rating (GAWR) of 12,000–14,000 lbs.
- Rear drive tandem axle GAWR of 38,000–46,000 lbs.
- Brake system designed to meet the reduced stopping distance requirements of the FMVSS-121 air brake system requirements.

The test data received from LINK were anonymized. Only supporting information about each test vehicle was collected, such as weight, brake type, wheelbase, and antilock braking system (ABS) configuration type, which allowed variability analyses for those parameters. Hypothesis testing was conducted to determine the relative effect of these characteristics on both overall stopping distance and stopping distance variability.

## **STUDY FINDINGS**

Key findings from the investigation address overall stopping distance and stopping distance variability.

### **Stopping Distance**

Several of the examined parameters affected both tractor stopping distance and individual-vehicle stopping distance variability. First, the average stopping distance for disc/disc brakes was shorter than either drum/drum or disc/drum brakes. Second, tractors with a GVWR of 45,000–50,000 lbs had shorter stopping distances than any other examined weight category, but these data did not support any further statements regarding links between GVWR and stopping distance. Third, the 151–200-in. wheelbase category of vehicles had the longest average stopping distance. Finally, tractors with the 6S6M ABS had stopping distances shorter than either the 4S4M or the 6S4M.

### **Stopping Distance Variability**

Brake type did not have a statistically significant effect on stopping distance variability. Weight did have a significant effect; the 50,000–55,000-lb GVWR range had more variability than both the next lower (45,000–50,000 lb) and next higher (55,000–60,000 lb) ranges. Vehicles with a 251–300-in. wheelbase had a lower stopping distance variability than those with a 151–200-in. wheelbase. The 6S6M ABS had a lower stopping distance variability than the 4S4M ABS.

The stopping distance variability was used to calculate a stopping distance range for an individual vehicle's 60-mi/hr full-system stopping distances. These ranges were calculated for both two standard deviations (95.4 percent of observations) and three standard deviations (99.7 percent of observations). These ranges are centered on an individual vehicle's average full-system 60-mi/hr stop under the conditions specified in FMVSS 121 (one tractor and unbraked control trailer loaded to the tractor gross vehicle weight). As such, these results cannot be applied to variability for a standard over-the-road tractor-trailer combination.

## **CONCLUSIONS**

The stopping distance data used in this analysis (which reflects application of full braking capacity) can inform platooning research and technology development because edge conditions may require maximum brake performance from trucks operating in a platoon. It is also important to note that the FMVSS 121 ABS stopping distance performance test is conducted using truck tractors under ideal braking system conditions, whereas a typical platooning tractor may not operate under similar circumstances. To better reflect the tested condition, tractors used for platooning should be well-maintained (without brake defects).

These tests were performed under ideal environmental conditions and involved full-system stops, while platooning situations would be more likely to involve lower-pressure stops under a variety of weather and road conditions. Platooning vehicles include a braked trailer and are often loaded to near gross combination vehicle weight rating; in contrast, the tests used here involved an unbraked control trailer to load the tractor to its GVWR.

Despite these differences, several observations are relevant to platooning research, particularly the effect of brake type on stopping distance variability. In this study, the stopping distance variability shows that for 60-mi/hr full effectiveness stops for a tractor and unbraked control trailer loaded to the tractor GVWR, drum/drum brakes have a 95 percent probability that the vehicle will have a stopping distance between 208.8 feet and 255.5 feet. Given the same test conditions, disc/drum brakes have a 95 percent probability that the vehicle will have a stopping distance between 196.3 feet and 250.7 feet. Disc/disc brakes have a 95 percent probability that the vehicle will have a stopping distance between 192.8 feet and 249.1 feet. Further testing may produce results more directly relevant to platooning and other applications. Specifically, FMVSS-121-type air brake system testing of a platoon with two or more vehicles could help bridge the gap between this analysis and a real-world platooning environment.

# 1. INTRODUCTION

## 1.1 BACKGROUND

A key parameter in determining the position of each truck in a platoon is its stopping distance capability under its current load. To minimize the chance of collisions within the platoon during a braking event, the vehicle with the shortest stopping distance should be placed at the rear of the platoon, and the vehicle with the longest stopping distance should be placed at the front of the platoon. But stopping distance is subject to some variability even under ideal conditions. This brake performance variability is of interest to the Federal Motor Carrier Safety Administration (FMCSA) as a part of wider efforts to explore platooning technologies, and the tractor's braking system performance is a key element of a tractor-trailer's overall braking performance.

Several factors influence stopping distance variation, such as brake performance, tire performance, dynamic weight transfer, and air brake system performance. Even if two vehicles are otherwise identical in braking system and tractor design, other factors will still likely result in differences in stopping distances, such as tire traction, system response time, and variability in each individual wheel end's brake performance. In addition to these natural variations, a manufacturer may not maximize braking capability even while remaining compliant with Federal Motor Vehicle Safety Standards (FMVSS) 121 requirements. Manufacturing decisions involve a balance of various engineering criteria that influence the design of a brake system, such as wear performance, noise, and cost of components. While the original equipment manufacturer is responsible for ensuring a system meets the relevant standards, fleets also make purchasing choices regarding components including brake type and antilock braking system (ABS) configuration.

## 1.2 TESTING OVERVIEW

To explore the parameters affecting variability in stopping distance, Oak Ridge National Laboratory (ORNL) has subcontracted with Link Engineering (LINK) to obtain anonymized FMVSS-121 ABS stopping distance data from a variety of truck tractors. Performing new tests would have been prohibitively expensive, so this analysis builds on previously collected data. The FMVSS-121 data selected for this analysis may not exactly reflect many common braking situations experienced by platooning vehicles (the typical braking event does not employ the full braking capacity), but the data do provide insight into the variability of full-system stopping distance, and by extension, braking performance capability. This data analysis seeks to identify the variability of the service brake stopping distance as defined by 49 CFR 571.121, S5.3.1 Stopping Distance—trucks and buses.

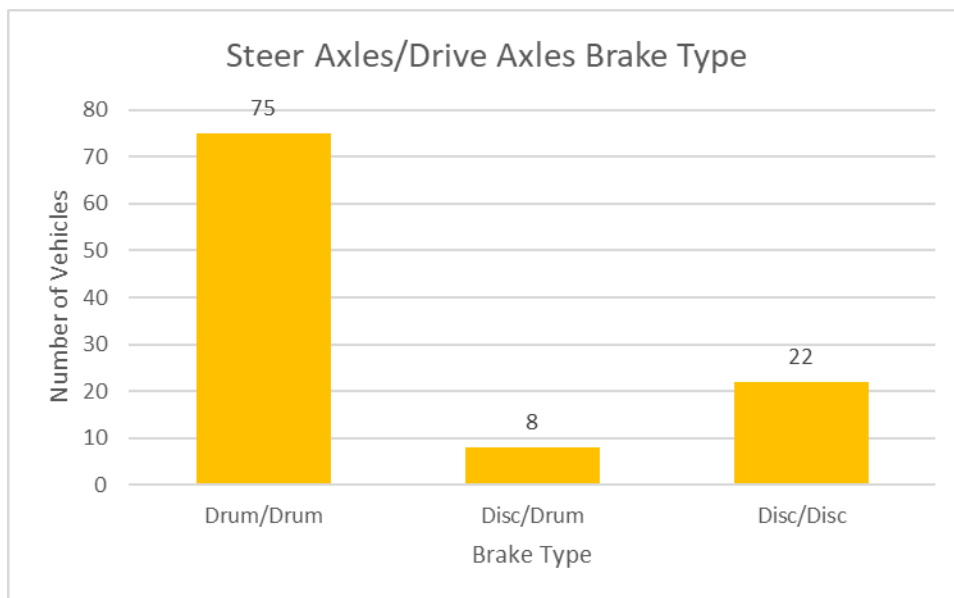
To conduct the FMVSS-121 test, new brake components on a truck tractor are first burnished under controlled conditions per FMVSS 121 to ensure optimal brake performance. Service brake stopping distance requirements of 49 CFR 571.121, S5.3.1 Stopping Distance—trucks and buses specify that truck tractors be loaded to GVWR with an unbraked control trailer. The standard test

specifies six stops from 60 mi/hr.<sup>(1)</sup> To meet the standard, at least one stop must be at 250 feet or less per the requirements of 49 CFR 571.121 Table 1—Stopping Sequence. The three-axle tractor test data received from LINK were anonymized to remove the manufacturers’ names from the tractor, tires, and brake system components. Only supporting information about each test vehicle was provided for this analysis, such as weight, brake type, wheelbase, and ABS type, which allowed variability analysis for those parameters.

### 1.3 GENERAL STATISTICS

General statistics regarding the test data are shown in the following tables and figures. The full set of test data is available in Appendix A. The 105 vehicle brake performance datasets of six runs each included various subcategories, such as brake type, GVWR, wheelbase, and ABS type. Distributions of these subcategories are shown in the following figures. A minimum of five observations are required for each subcategory to support comparisons between them.

The brake type distribution shown in Figure 1 below shows general stopping distance information in a visual format. While more observations for the disc/drum and disc/disc brake configurations would have been ideal, there are enough observations in each category to perform some statistical analyses comparing different brake types.



**Figure 1. Bar graph. Number of vehicles tested by brake type.**

The majority of this report is focused on stopping distance characteristics (such as variability) for specific vehicles rather than broader populations of vehicles. However, an analysis of the data collected does support these generalizations, and in order to support such analysis a single

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<sup>1</sup> 49 CFR § 571.121 - Standard No. 121; Air brake systems, available at <https://www.govinfo.gov/app/details/CFR-2017-title49-vol6/CFR-2017-title49-vol6-sec571-121/summary>

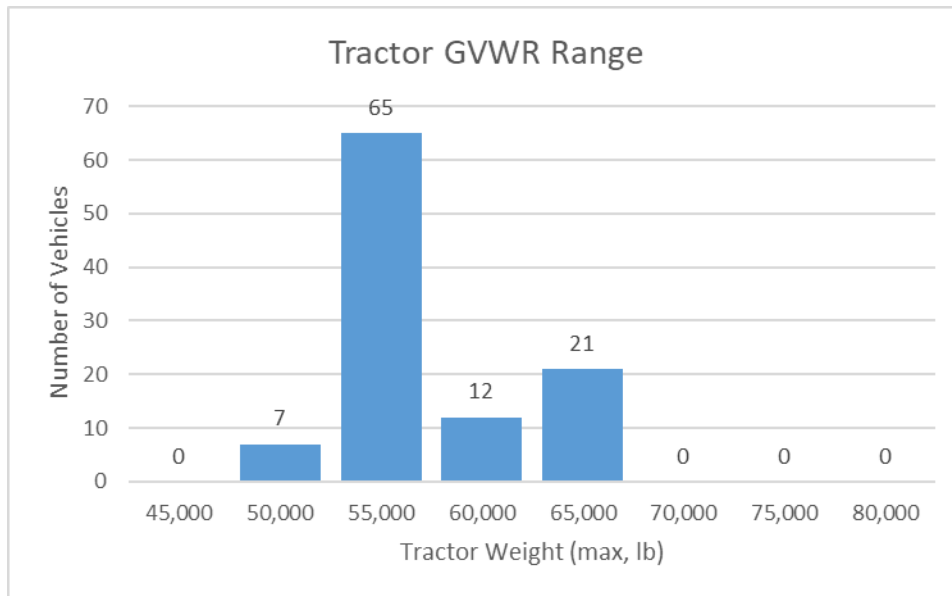
stopping distance observation (of the available six for each vehicle) was selected at random and used to calculate general statistics for each subcategory of data. The relevant statistical information by brake type appears below in 1.

**Table 1. Overall statistical parameters by brake type.**

<b>Brake Type Steer Axle/Drive Axles</b>	<b>Number of Vehicles</b>	<b>Sample Mean Stopping Distance (ft)</b>	<b>Sample Standard Deviation (ft)</b>	<b>Lower Limit for 95% Confidence Interval (ft)</b>	<b>Upper Limit for 95% Confidence Interval (ft)</b>
Drum/Drum	75	232.1	11.90	208.8	255.5
Disc/Drum	8	223.5	13.87	196.3	250.7
Disc/Disc	22	220.9	14.36	192.8	249.1
Overall	105	229.2	13.33	203.1	255.3

The confidence intervals shown in Table 1 indicate that a drum/drum tractor in good condition is expected to stop over an interval of 208.8 ft to 255.5 ft in an FMVSS-121 stopping distance test. This statement can be known with 95 percent confidence.

Similar statistics were generated for tractor GVWR range. Whenever a subcategory encompasses a range for a parameter (such as weight, shown below in Figure 2), the label denotes the high end of the range. For example, Figure 2 indicates that 65 test tractors were in the 50,001–55,000-lb weight range.



**Figure 2. Bar graph. Number of vehicles tested by tractor GVWR range.**

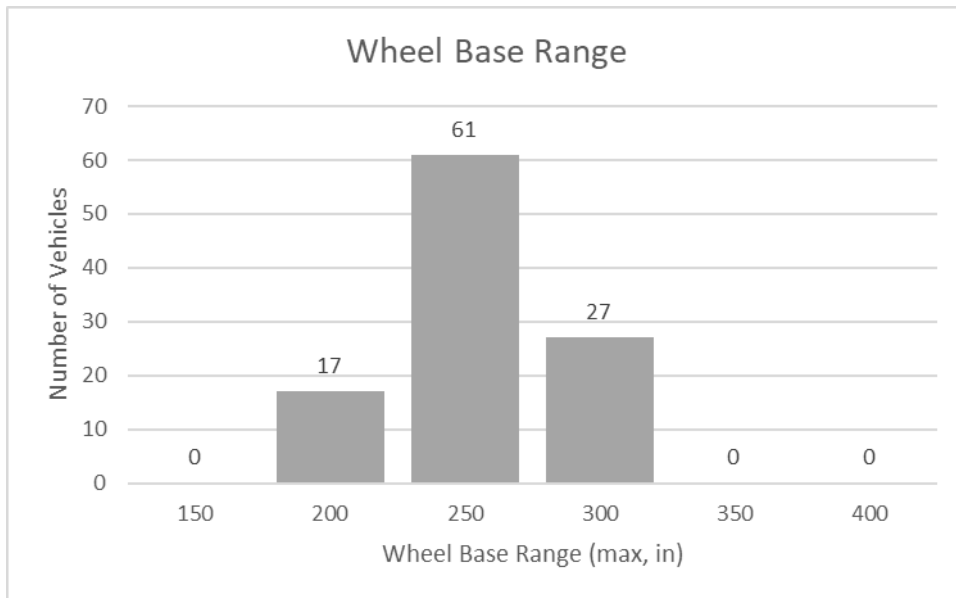
The relevant statistical information by GVWR appears below in Table 2.



**Table 2. Overall statistical parameters by GVWR.**

Tractor GVWR (lb)	Number of Vehicles	Sample Mean Stopping Distance (ft)	Sample Standard Deviation (ft)	Lower Limit for 95% Confidence Interval (ft)	Upper Limit for 95% Confidence Interval (ft)
45,000–50,000	7	217.4	9.62	198.6	236.3
50,000–55,000	65	231.0	13.81	203.9	258.0
55,000–60,000	12	227.5	13.71	200.6	254.4
60,000–65,000	21	227.8	11.31	205.6	249.9

Wheelbase information was also collected (Figure 3) and roughly indicates tractor size. All else being equal, tractors with higher vehicle weight ratings have larger wheelbases.



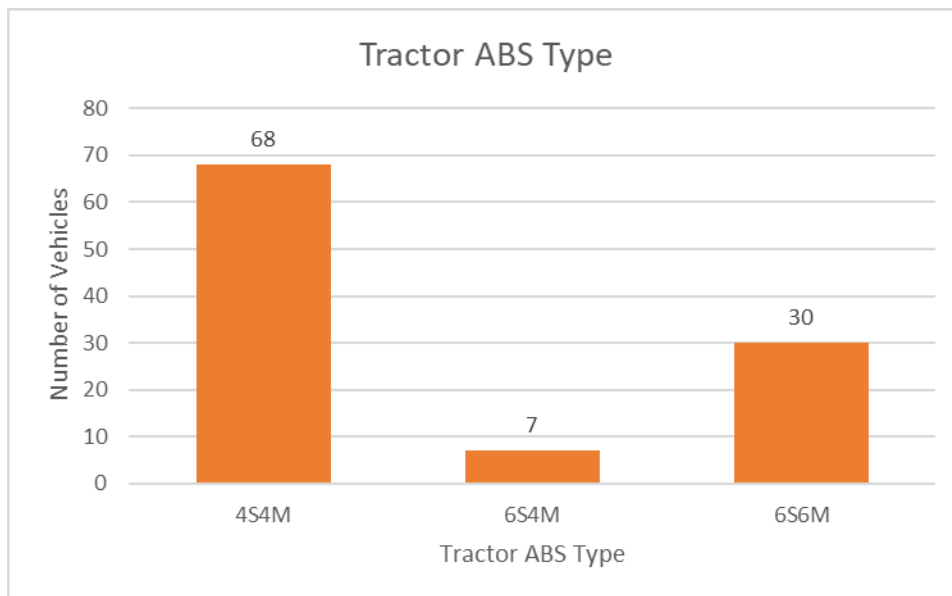
**Figure 3. Bar graph. Number of vehicles tested by wheelbase.**

The relevant statistical information by wheelbase range appears below in Table 3.

**Table 3. Overall statistical parameters by wheelbase range.**

Wheelbase (in.)	Number of Vehicles	Sample Mean Stopping Distance (ft)	Sample Standard Deviation (ft)	Lower Limit for 95% Confidence Interval (ft)	Upper Limit for 95% Confidence Interval (ft)
151–200	17	234.5	12.17	210.6	258.3
201–250	61	228.9	14.09	201.3	256.5
251–300	27	225.8	11.71	202.8	248.7

Tractor ABS type was also collected.<sup>(2)</sup> Approximately two-thirds of the test vehicles had a 4S4M system, and most of the remaining vehicles had a 6S6M system, as shown in Figure 4.



**Figure 4. Bar graph. Number of vehicles tested by tractor ABS type.**

The relevant statistical information by tractor ABS type appear below in Table 4.

**Table 4. Overall statistical parameters by tractor ABS type.**

Tractor ABS Type	Number of Vehicles	Sample Mean Stopping Distance (ft)	Sample Standard Deviation (ft)	Lower Limit for 95% Confidence Interval (ft)	Upper Limit for 95% Confidence Interval (ft)
4S4M	68	229.7	12.56	205.1	254.3
6S4M	7	234.6	16.82	201.6	267.5
6S6M	30	226.2	14.23	198.3	254.1

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<sup>2</sup> The tractor ABS designation indicates the number of wheel sensors and modulators. For example, a 6S4M antilock braking system has six wheel sensors and four modulators.

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## 2. STOPPING DISTANCE COMPARISON

This section examines the average service brake stopping distances of all tested vehicles (i.e., the average of each specific vehicle's six full-system stops). Data are compared within subcategories of the following parameters: brake type, tractor GVWR, wheelbase measurement, and ABS type.

In each case, the actual distribution of average full-system stopping distance is shown. Null-hypothesis testing is employed to determine whether the apparent differences in stopping distance between the various subcategories are statistically significant. Finally, the mean and standard deviation of these average stopping distances is used to show the probability distribution for the average stopping distance for each subcategory of data.

### 2.1 BRAKE TYPE

A histogram of average stopping distance by brake type is shown in Figure 5. The data appears to be approximately normally distributed for each type of brake, indicating that hypothesis testing based on a normal distribution is appropriate.

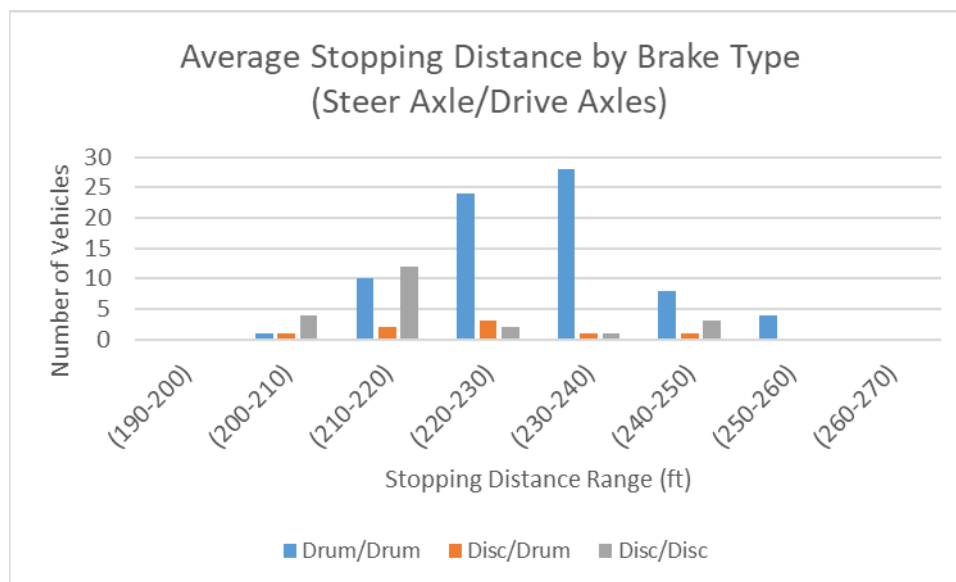


Figure 5. Histogram. Test vehicle distribution of average stopping distance by brake type.

From Figure 5, the drum/drum brakes (blue) appear to be associated with the longest stopping distance, followed by the disc/drum brakes (orange). The disc/disc brakes (gray) appear to be associated with the shortest full-system stopping distance. To confirm that these observations are statistically defensible, the means and standard deviations were used to perform a test of hypothesis. A test of hypothesis seeks to reject with sufficient confidence the hypothesis that two means are equal. If this can be done, the alternative hypothesis (that one mean is greater than the other) is accepted.

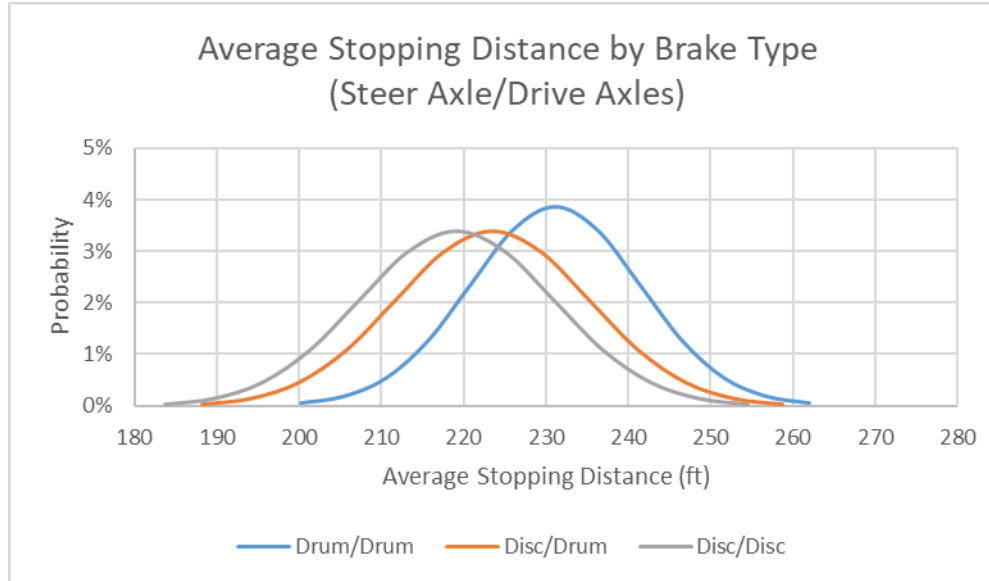
The results of this test are shown in Table 5. Here, each combination of brake types is compared to the other combinations, and the associated confidence level is shown in the relevant cell.

Where the null hypothesis can be rejected with at least a 95 percent confidence level, the cell text is bold. This test requires a minimum of five observations (i.e., n=5), so the small number of disc/drum brakes is adequate. For example, in comparing the drum/drum average stopping distance to that of the disc/drum, the null hypothesis is that the mean drum/drum stopping distance is equal to the mean disc/drum stopping distance. The alternative hypothesis is that the mean drum/drum stopping distance is greater than the mean disc/drum stopping distance. The null hypothesis could only be rejected with 94.39 percent confidence (under the set 95 percent threshold), so the null hypothesis cannot be rejected in favor of the alternative hypothesis. A more detailed walkthrough of the null hypothesis test is available in Appendix B.

**Table 5. Tests of hypothesis for differences in stopping distance by brake type (steer axle/drive axles).**

<b>Brake Type Steer Axle/ Drive Axles</b>	<b>Mean (ft)</b>	<b>Drum/Drum n = 75</b>	<b>Disc/Drum n = 8</b>	<b>Disc/Disc n = 22</b>
Drum/Drum	231.13	--	94.39%	<b>99.99%</b>
Disc/Drum	223.42	--	--	<b>99.50%</b>
Disc/Disc	219.10	--	--	--

Given this information and method, the results show that the average stopping distance for disc/disc brakes is shorter than either drum/drum or disc/drum brakes. For a sample of vehicles, the stopping distance probability distribution can be summarized in Figure 6, which was constructed from the mean and standard deviations for each subset of data.



**Figure 6. Chart. Probability distribution of stopping distances for various brake types derived from test data parameters.**

Here, there is substantial overlap in stopping distance between the various brake types, but drum/drum brakes are clearly associated with the longest full-system stopping distance. The narrower and sharper the curve, the more consistent and predictable the average stopping

distance within the category of brakes. A later section of this report addresses stopping distance variability for a given vehicle (Section 3: Stopping Distance Variability).

Finally, because higher weights are associated with longer stopping distances (all else being equal), the distribution of tractor gross vehicle weight (GVW) was examined for each brake type to confirm that the difference in stopping distance is attributable to brake type rather than unequal distribution of test weights between the brake types. As shown in Figure 7, the GVWs are nearly identical, and the disc/disc brake category may be at a slight disadvantage due to the slightly higher average GVW within that category. Given that disc/disc brakes had the shortest average stopping distance, the small weight discrepancy did not alter the efficacy rankings of the three tested brake types.

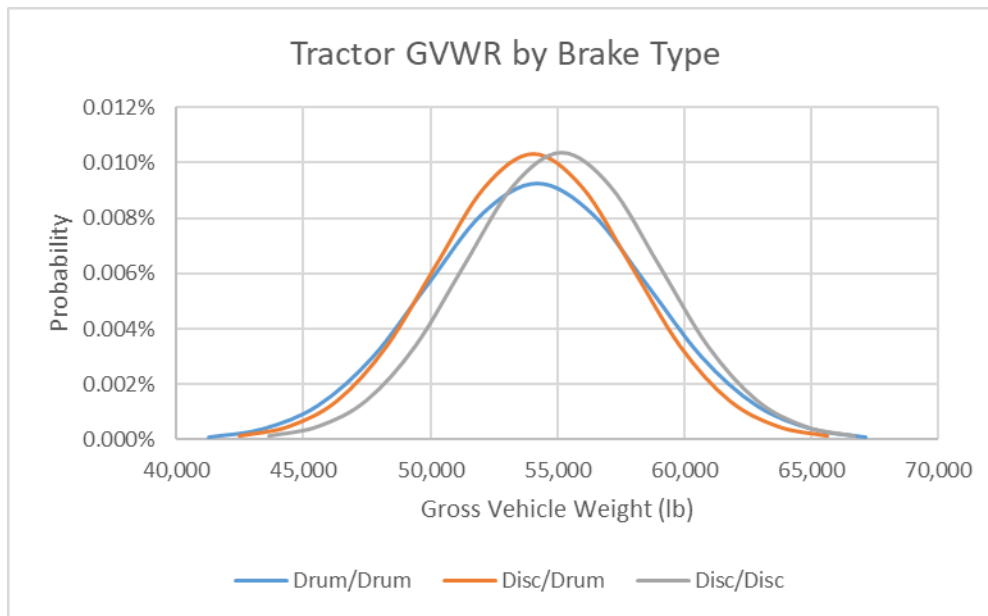
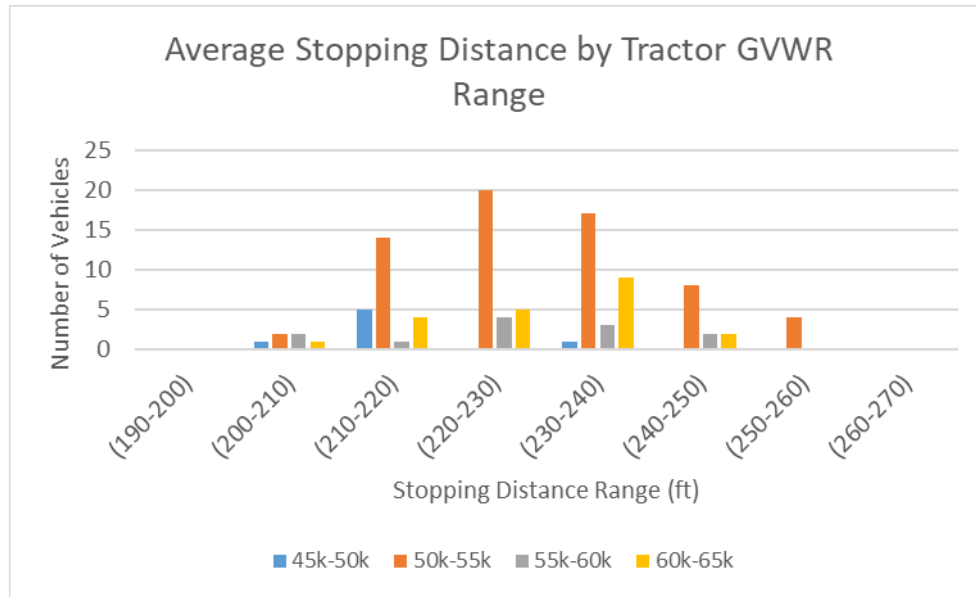


Figure 7. Chart. Probability distribution of tractor gross vehicle weight by brake type derived from test data parameters.

## 2.2 TRACTOR WEIGHT

A histogram of average stopping distance for various GVWR intervals is shown in Figure 8.



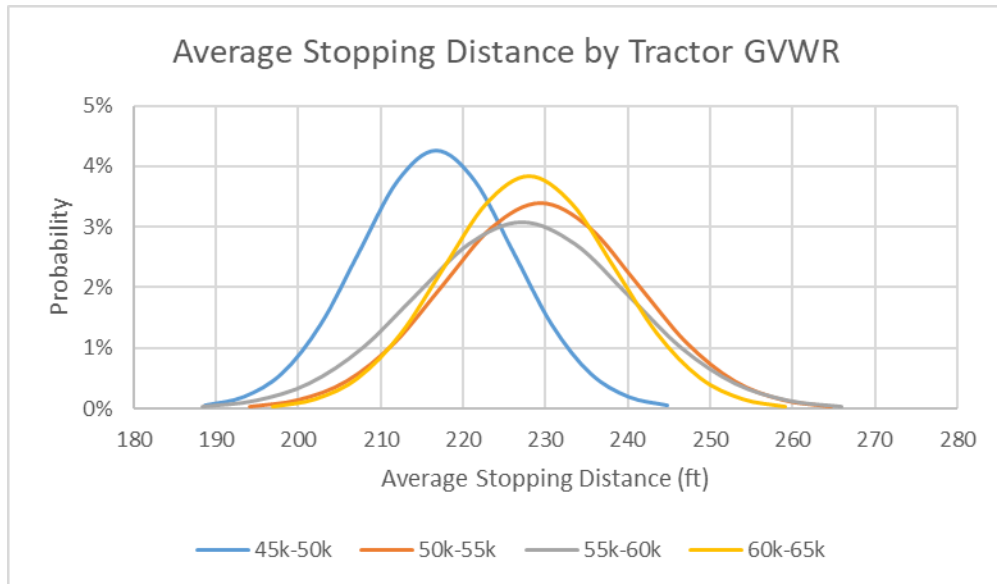
**Figure 8. Histogram. Test vehicle distribution of average stopping distance by weight range.**

As expected, the higher weights in general correspond to longer stopping distances. To determine the extent to which this observation is statistically significant, hypothesis testing was performed as described previously, with the null hypothesis ( $H_0$ ) that the mean stopping distance from two subcategories are equal, and the alternative hypothesis ( $H_a$ ) that the one is greater than the other. For example, in comparing the 50,000–55,000-lb average stopping distance to that of the 45,000–50,000 lb, the null hypothesis is that both means are equal. The alternative hypothesis is that the mean 50,000–55,000 lb stopping distance is greater than the mean 45,000–50,000 lb stopping distance. The null hypothesis could be rejected with 99.47 percent confidence (above the set 95 percent threshold), so the null hypothesis can be rejected in favor of the alternative hypothesis. A more detailed walk-through of the null hypothesis test is available in Appendix B. The results are shown in Table 6.

**Table 6. Tests of hypothesis for differences in stopping distance by tractor GVWR.**

Tractor GVWR (lb)	Mean (ft)	45k-50k n = 7	50k-55k n = 65	55k-60k n = 12	60k-65k n = 21
45,000–50,000	216.71	--	99.47%	97.06%	98.98%
50,000–55,000	229.38	--	--	70.56%	68.68%
55,000–60,000	227.17	--	--	--	58.03%
60,000–65,000	228.06	--	--	--	--

Based on these results, tractors with a GVWR of 45,000–50,000 lbs had shorter stopping distances than any other examined weight category, but it is not possible to make any further statements regarding the link between weight and stopping distance based on these data. The probability distribution is shown in Figure 9.

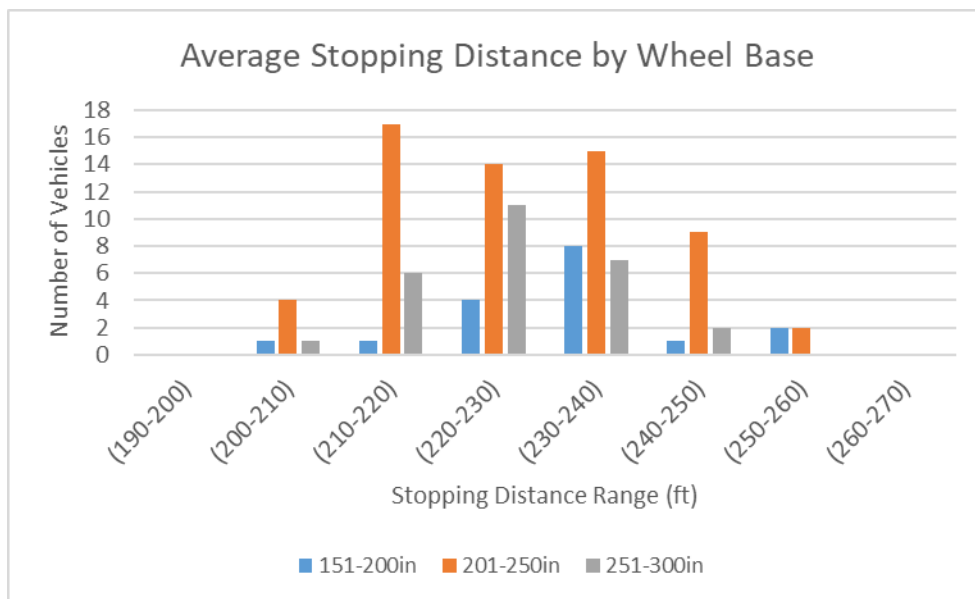


**Figure 9. Chart. Probability distribution of stopping distance for various tractor GVWRs derived from test data parameters.**

As expected from Table 6, the trace for 45,000–50,000 lb trucks shows a markedly lower average stopping distance.

### 2.3 WHEELBASE

The next parameter of interest in wheelbase, which serves as an indicator of tractor size. The relevant histogram appears as Figure 10. As with other categories, a normal distribution is evident.



**Figure 10. Histogram. Test vehicle distribution of average stopping distance by wheelbase.**

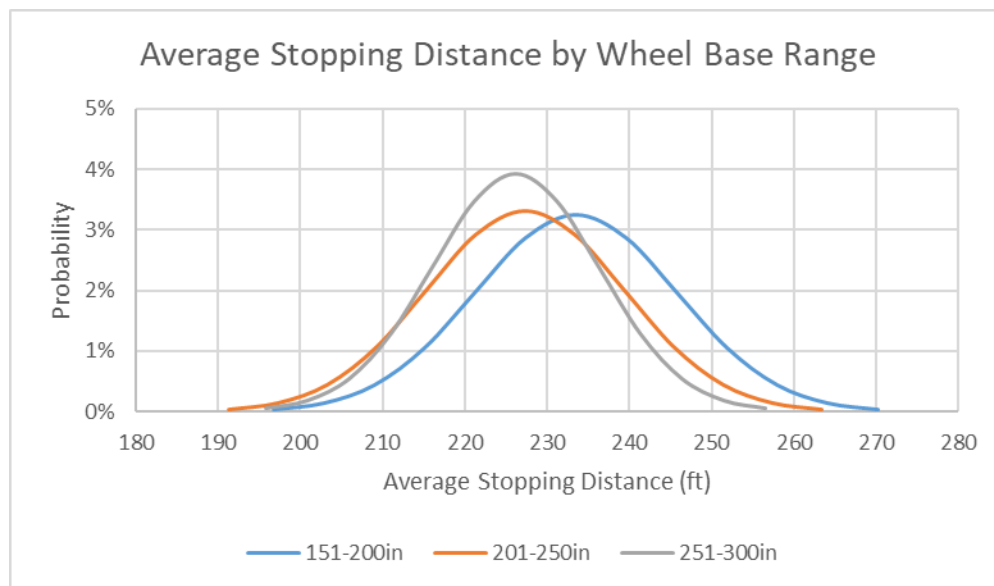


As before, hypothesis testing was used to determine whether any category could be said to represent a higher or lower stopping distance. The null hypothesis ( $H_0$ ) was that the mean stopping distance from two subcategories are equal, and the alternative hypothesis ( $H_a$ ) that the one is greater than the other. For example, in comparing the 151–200-in. wheelbase average stopping distance to that of the 201–250-in. wheelbase, the null hypothesis is that the two mean stopping distances are equal. The alternative hypothesis is that the mean 151–200-in. wheelbase stopping distance is greater than the mean 201–250-in. wheelbase stopping distance. The null hypothesis could be rejected with 96.32 percent confidence (above the set 95 percent threshold), so the null hypothesis can be rejected in favor of the alternative hypothesis. A more detailed walk-through of the null hypothesis test is available in Appendix B. The results are shown in Table 7.

**Table 7. Tests of hypothesis for differences in stopping distance by wheelbase range.**

Wheelbase (in.)	Mean (ft)	151-200in n = 17	201-250in n = 61	251-300in n = 27
151-200	233.5	--	<b>96.32%</b>	<b>99.99%</b>
201-250	227.3	--	--	93.05%
251-300	226.2	--	--	--

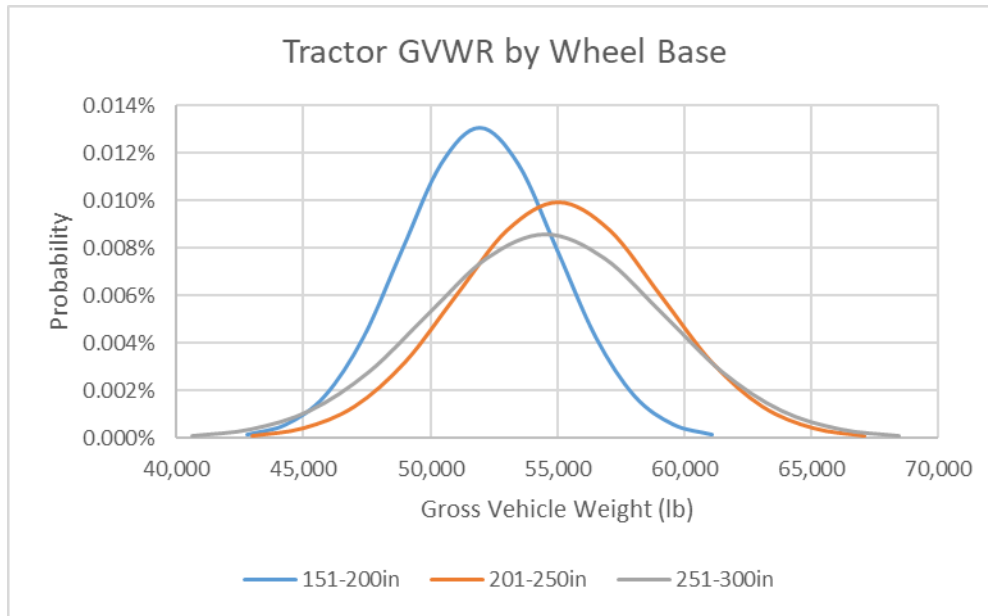
Based on these results, the 151–200-in. wheelbase vehicles have the longest average stopping distance. The probability distribution of the average tractor full-system stopping distance, derived from test data parameters, is shown in Figure 11.



**Figure 11. Chart. Probability distribution of stopping distance for various wheelbases derived from test data parameters.**

To confirm that this observation is not a function of higher GVWRs for the vehicles in this category, a probability distribution of the tractor GVWR by wheelbase was generated from test

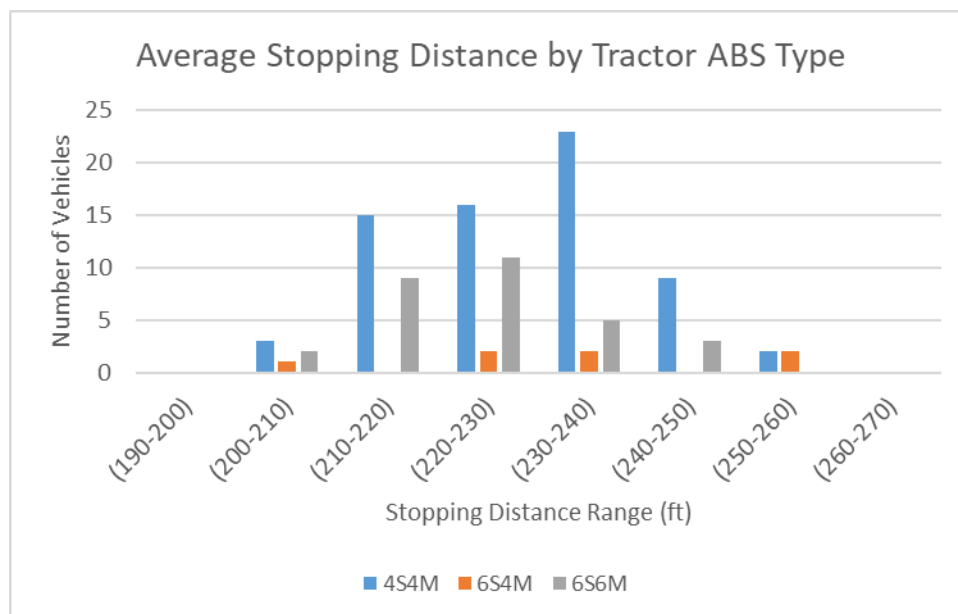
data parameters, as shown in Figure 12. This figure shows that the vehicles associated with the lowest wheelbase range had lower GVWRs.



**Figure 12. Chart. Probability distribution of tractor gross vehicle weight by wheelbase range derived from test data parameters.**

## 2.4 TRACTOR ABS SYSTEM CONFIGURATION TYPE

The histogram for ABS type is shown in Figure 13.



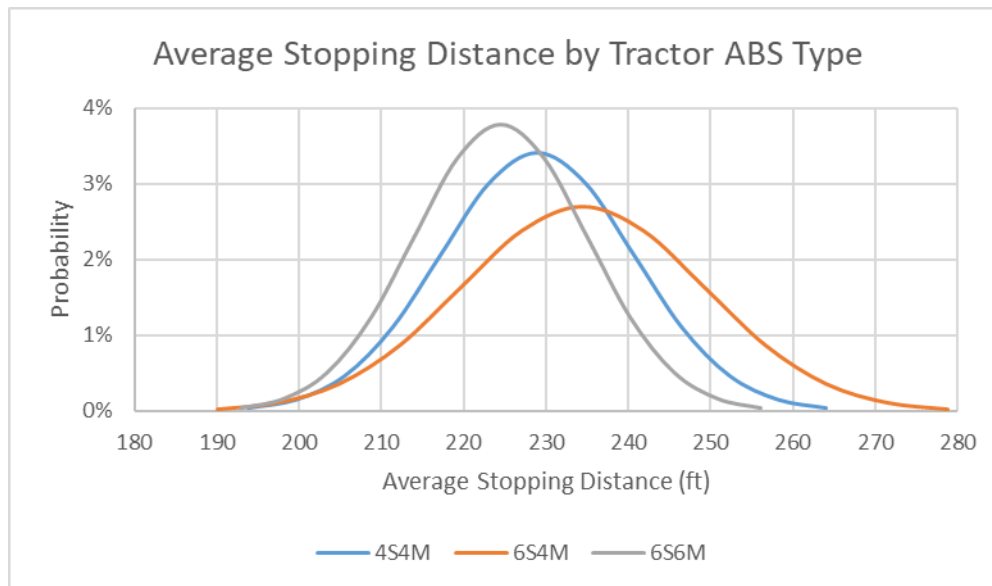
**Figure 13. Histogram. Test vehicle distribution of average stopping distance by tractor ABS type.**

Null hypothesis test results are shown in Table 8. For example, in comparing the 4S4M average stopping distance to that of the 6S4M, the null hypothesis is that the mean 4S4M stopping distance is equal to the mean 6S4M stopping distance. The alternative hypothesis is that the mean 4S4M stopping distance is greater than the mean 6S4M stopping distance. The null hypothesis could only be rejected with 87.66 percent confidence (under the set 95 percent threshold), so the null hypothesis cannot be rejected in favor of the alternative hypothesis. A more detailed walkthrough of the null hypothesis test is available in Appendix B.

**Table 8. Tests of hypothesis for differences in stopping distance by tractor ABS type.**

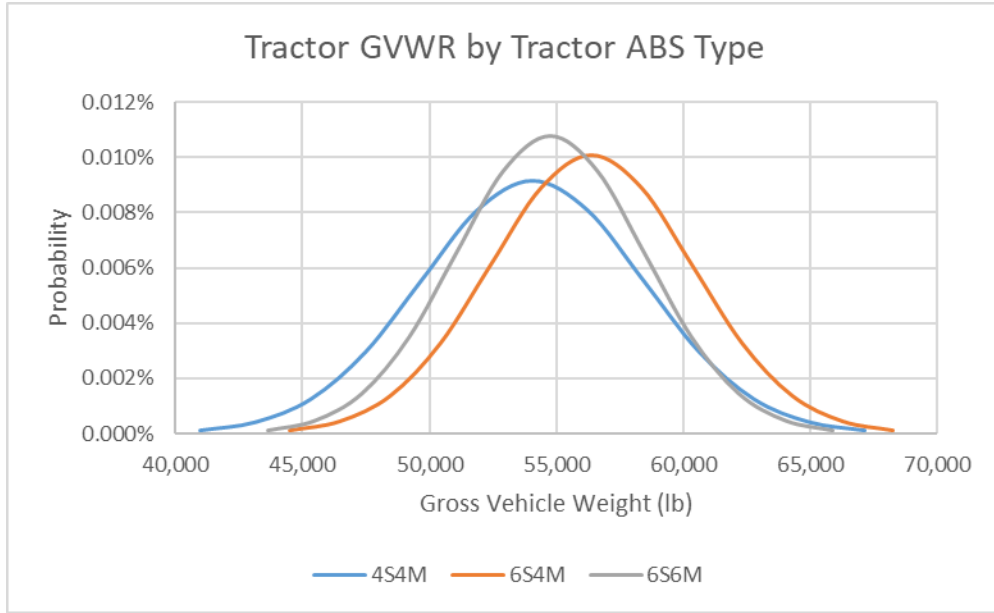
Tractor ABS Type	Mean (ft)	4S4M n = 68	6S4M n = 7	6S6M n = 30
4S4M	228.90	--	81.32%	<b>99.99%</b>
6S4M	234.45	--	--	<b>99.99%</b>
6S6M	224.52	--	--	--

Given these results, the 6S6M ABS is associated with a shorter stopping distance than either the 4S4M or 6S4M. The difference between the 4S4M and 6S4M is not statistically significant. The anticipated probability distribution is shown in Figure 14; as expected from the null hypothesis test results, the 6S6M stopping distance is lower than both the 4S4M and 6S4M.



**Figure 14. Chart. Probability distribution of stopping distance for various ABS types derived from test data parameters.**

To confirm that the differences in stopping distance related to ABS type were not strongly associated with different GVWRs (and thus test weights), a probability distribution of the tractor GVWR by ABS type was generated from test data parameters, as shown in Figure 15. As shown here, the GVWR was fairly consistent across the ABS types, confirming that the conclusions made above concerning stopping distance and ABS type are not simply side-effects of weight variability.



**Figure 15. Chart. Probability distribution of tractor gross vehicle weight by ABS type derived from test data parameters.**

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### 3. STOPPING DISTANCE VARIABILITY

While platooning order may be influenced by average stopping distance, the stopping distance variability must also be considered for spacing considerations. This is evident in this dataset, in which each vehicle performed six 60-mi/hr full-system stopping distance tests with some variation in result. The standard deviation for each set of stopping distances serves as the “stopping distance variability” for the following analysis. This parameter is examined much like stopping distance in the previous section; various subcategories of data are compared, and tests of significance are conducted to determine whether the observed difference in stopping distance variability is statistically significant.

#### 3.1 BRAKE TYPE

A histogram of stopping distance variability for the various brake types is shown below in Figure 16. Stopping distance variability, unlike stopping distance itself, has a skewed distribution.

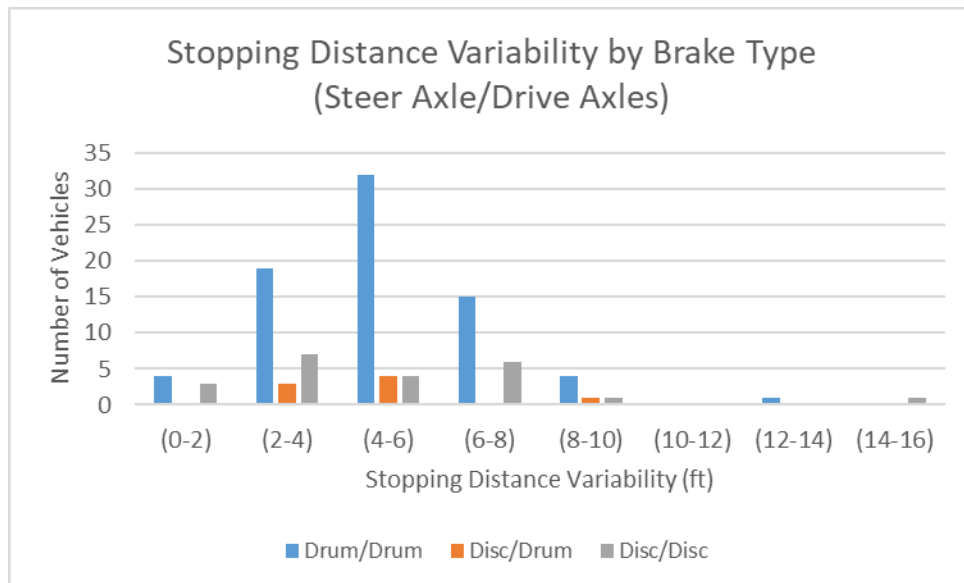


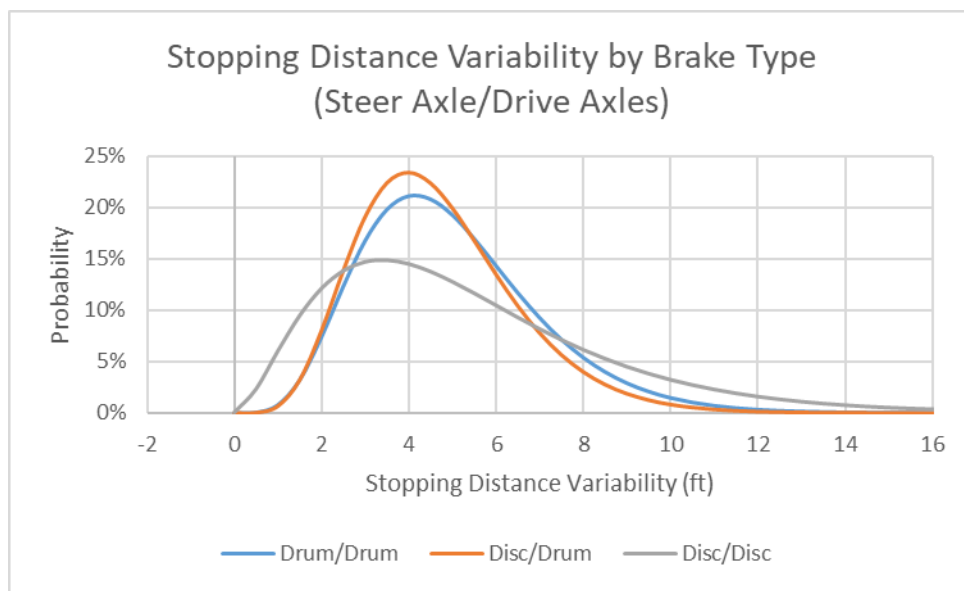
Figure 16. Histogram. Test vehicle distribution of stopping distance variability by brake type.

Because the stopping distance variability cannot be zero, the standard test of hypothesis used for comparing stopping distances cannot be used here. Instead, the non-parametric Mann-Whitney U test was used to conduct hypothesis testing, comparing medians of the subcategories of data (here, brake type) at a 95 percent confidence level. For example, in comparing the drum/drum median stopping distance variability to that of the disc/drum, the null hypothesis is that the median drum/drum stopping distance variability is equal to the median disc/drum stopping distance variability. The alternative hypothesis is that the median drum/drum stopping distance variability is greater than the median disc/drum stopping distance variability. The null hypothesis could not be rejected in favor of the alternative hypothesis, suggesting that the stopping distance variability is similar for all brake types. The results of the null hypothesis tests for various brake types are summarized in Table 9, and more detailed information regarding the Mann-Whitney U test is available in Appendix C.

**Table 9. Tests of hypothesis for stopping distance variability based on brake type (steer axle/drive axles).**

<b>Brake Type Steer Axle/ Drive Axles</b>	<b>Median Variability (ft)</b>	<b>Drum/Drum n = 75</b>	<b>Disc/Drum n = 8</b>	<b>Disc/Disc n = 22</b>
Drum/Drum	4.86	--	Cannot reject H <sub>0</sub>	Cannot reject H <sub>0</sub>
Disc/Drum	4.29	--	--	Cannot reject H <sub>0</sub>
Disc/Disc	5.18	--	--	--

In graphing the probability distribution for stopping distance variability (Figure 17), a folded normal distribution is used because the variability cannot be less than zero.



**Figure 17. Chart. Probability distribution of stopping distance variability for various brake types derived from test data parameters.**

This graph shows that the typical stopping distance variability (standard deviation of the stopping distance) is around 5 ft. For each trace, an “effective maximum variability” can be calculated—the point where there is only a 1 percent chance of encountering a higher variability. Put another way, the stopping distance variability will be less than the “effective maximum variability” 99 percent of the time. This value, which is different for each brake type, is shown in Table 10.

**Table 10. Effective maximum variability for various brake types.**

<b>Brake Type Steer Axle/ Drive Axles</b>	<b>Effective Max Variability (ft)</b>
Drum/Drum	10.9
Disc/Drum	9.9
Disc/Disc	15.4

Because the stopping distance variability corresponds to a standard deviation of the stopping distance, this standard deviation can be used (in conjunction with the assumption that the stopping distances for a given vehicle are normally distributed) to calculate the range for a particular tractor’s full-system stopping distance, as shown in Table 11.

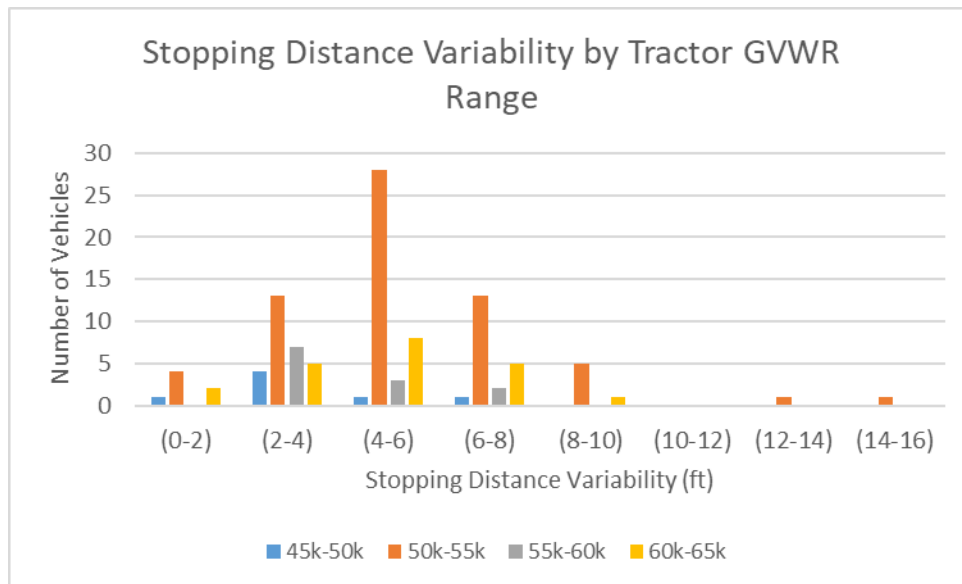
**Table 11. Estimated stopping distance range for various brake types.**

Brake Type Steer Axle/Drive Axles	95.4% Stopping Distance Range (ft)	99.7% Stopping Distance Range (ft)
Drum/Drum	±21.7	±32.6
Disc/Drum	±19.8	±29.7
Disc/Disc	±30.7	±46.1

For two standard deviations (95.4 percent of observations), the tractor full-system stopping distances fall within the range shown in the second column of the table. For three standard deviations (99.7 percent of observations), the stopping distance ranges are as shown in the rightmost column. These ranges are for a full-system 60-mi/hr stop for one tractor and unbraked control trailer loaded to the tractor GVWR. Thus, if a tractor with drum/drum brakes performs 100 60-mi/hr full-system FMVSS-121 stops, it is expected that approximately 95 of them will be within ±21.7 ft of that vehicle’s average stopping distance, and all of that vehicle’s stops will be within ±32.6 ft of the vehicle’s average stopping distance.

### 3.2 TRACTOR WEIGHT

A similar stopping distance variability comparison was made for weight ranges. The histogram appears in Figure 18.



**Figure 18. Histogram. Test vehicle distribution of stopping distance variability by tractor GVWR range.**

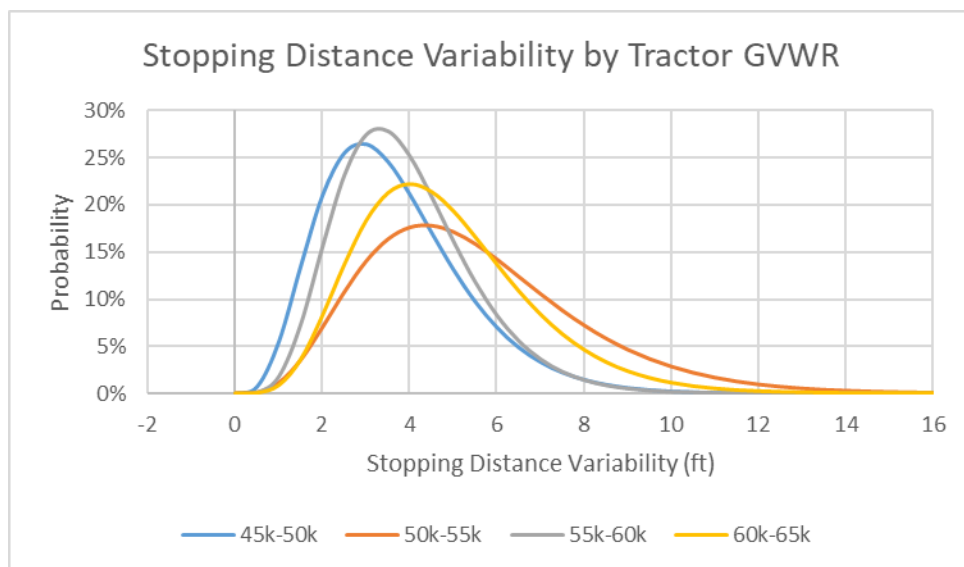


The corresponding Mann-Whitney tests of hypothesis are summarized in Table 12. For example, in comparing the 50,000–55,000-lb median stopping distance variability to that of the 45,000–50,000-lb range, the null hypothesis is that the median stopping distance variabilities are equal. The alternative hypothesis is that the median 50,000–55,000-lb stopping distance variability is greater than the median 45,000–50,000-lb stopping distance variability. The null hypothesis could be rejected in favor of the alternative hypothesis with at least 95 percent confidence, indicating that the 50,000–55,000-lb stopping distance variability is greater than the 45,000–50,000-lb stopping distance variability. More detailed information regarding the Mann-Whitney U test is available in Appendix C.

**Table 12. Tests of Hypothesis for Stopping Distance Variability Based on Tractor GVWR.**

Tractor GVWR (lb)	Median Variability (ft)	45k-50k n = 7	50k-55k n = 65	55k-60k n = 12	60k-65k n = 21
45,000-50,000	2.88	--	<b>Reject H<sub>0</sub></b>	Cannot reject H <sub>0</sub>	Cannot reject H <sub>0</sub>
50,000-55,000	5.19	--	--	<b>Reject H<sub>0</sub></b>	Cannot reject H <sub>0</sub>
55,000-60,000	3.42	--	--	--	Cannot reject H <sub>0</sub>
60,000-65,000	4.54	--	--	--	--

Thus, the 50,000–55,000-lb weight range was found to have more variability than both the next lower (45,000–50,000 lb) and next higher (55,000–60,000 lb) ranges. The corresponding probability distribution for the stopping distance variability by tractor weight is shown in Figure 19.



**Figure 19. Chart. Probability Distribution of Stopping Distance Variability for Various Tractor Weights Derived from Test Data Parameters**

Based on this probability distribution, an effective maximum variability for each GVWR can be calculated as shown in Table 13.

**Table 13. Effective maximum variability various tractor weights.**

<b>Tractor Weight (GVWR) (lbs)</b>	<b>Effective Max Variability (ft)</b>
45,000–50,000	8.5
50,000–55,000	12.7
55,000–60,000	8.3
60,000–65,000	10.4

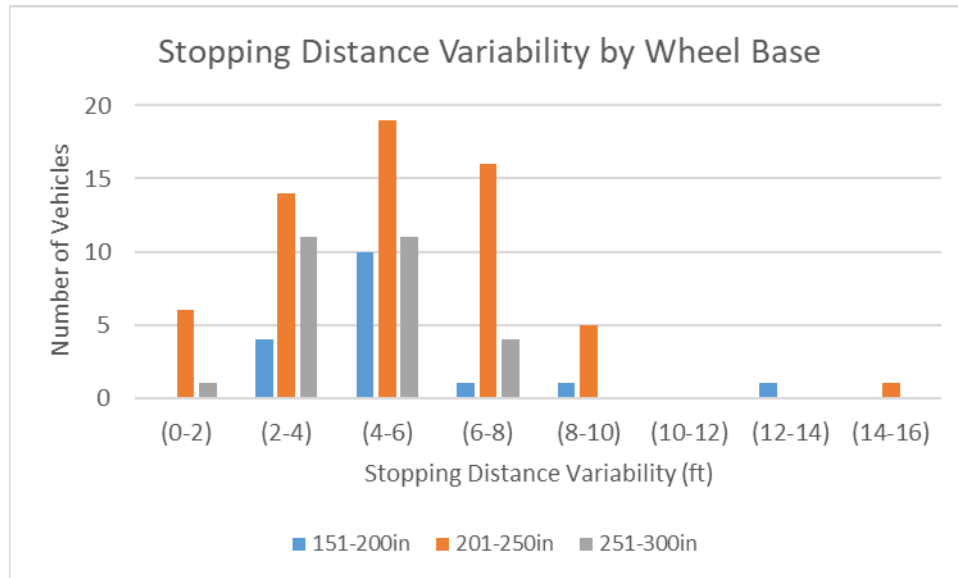
From these effective maximum variabilities, the 60-mi/hr service brake stopping distance range for a given tractor with a particular tractor gross vehicle weight rating can be estimated as shown in Table 14.

**Table 14. Estimated stopping distance range for various tractor weights.**

<b>Tractor Weight (GVWR) (lbs)</b>	<b>95.4% Stopping Distance Range (ft)</b>	<b>99.7% Stopping Distance Range (ft)</b>
45,000–50,000	±17.1	±25.6
50,000–55,000	±25.4	±38.2
55,000–60,000	±16.6	±24.8
60,000–65,000	±20.7	±31.1

### **3.3 WHEELBASE**

The wheelbase is representative of the truck tractor overall size. A histogram of stopping distance variability as compared to wheelbase is shown in Figure 20.



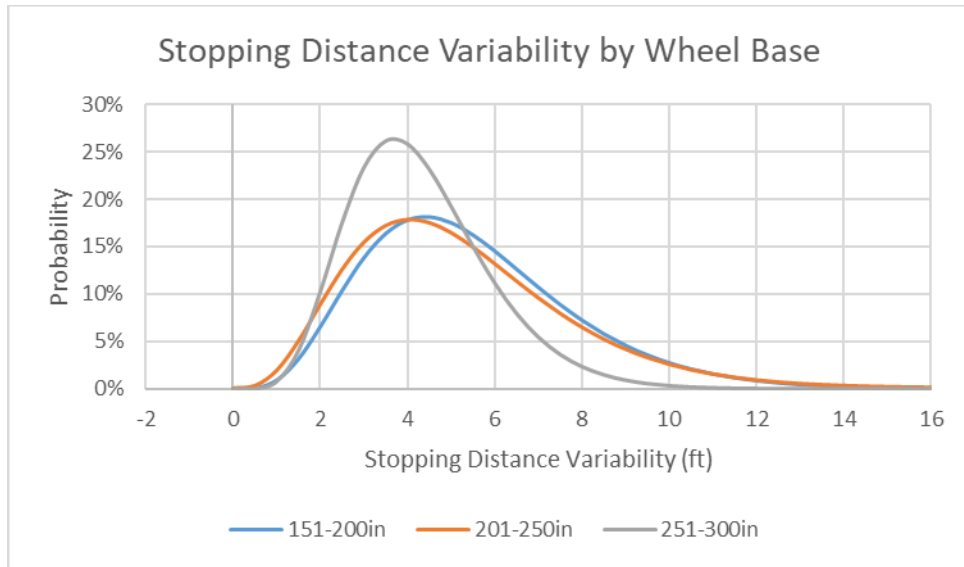
**Figure 20. Histogram. Test vehicle distribution of stopping distance variability by wheelbase.**

The corresponding Mann-Whitney tests of hypothesis are summarized in Table 15. For example, in comparing the 151–200-in. wheelbase median stopping distance variability to that of the 201–250-in. wheelbase, the null hypothesis is that the median stopping distances are equal. The alternative hypothesis is that the median 151–200-in. wheelbase stopping distance variability is greater than the median 201–250-in. wheelbase stopping distance variability. The null hypothesis could not be rejected in favor of the alternative hypothesis with at least 95 percent confidence, suggesting that the stopping distance variability for these two-wheel base ranges are similar. More detailed information regarding the Mann-Whitney U test is available in Appendix C.

**Table 15. Tests of hypothesis for stopping distance variability based on wheelbase range.**

Wheelbase (in.)	Median Variability (ft)	151-200 in n = 17	201-250 in n = 61	251-300 in n = 27
151–200	5.457	--	Cannot reject $H_0$	<b>Reject <math>H_0</math></b>
201–250	5.221	--	--	Cannot reject $H_0$
251–300	4.287	--	--	--

While not readily apparent from Figure 20, the 251–300-in. wheelbase has a statistically significant lower stopping distance variability than the other wheelbase ranges. The probability distribution of stopping distance variability is shown in Figure 21.



**Figure 21. Chart. Probability distribution of stopping distance variability for various wheelbases derived from test data parameters.**

Based on this probability distribution, the effective maximum variability can be calculated for each wheelbase range as shown in Table 16.

**Table 16. Effective maximum variability for various wheelbases.**

Wheelbase (in.)	Effective Max Variability (ft)
151–200	12.5
201–250	12.6
251–300	8.9

These effective maximum variability values can be used to estimate full-system stopping distance ranges for a tractor with a particular wheelbase, as shown in Table 17.

**Table 17. Estimated Stopping Distance Range for Various Wheelbases.**

Wheelbase (in.)	95.4% Stopping Distance Range (ft)	99.7% Stopping Distance Range (ft)
151-200	±25.1	±37.6
201-250	±25.2	±37.8
251-300	±17.8	±26.6

### 3.4 TRACTOR ABS CONFIGURATION TYPE

The final parameter under consideration is tractor ABS type. The stopping distance variability is shown in Figure 22.

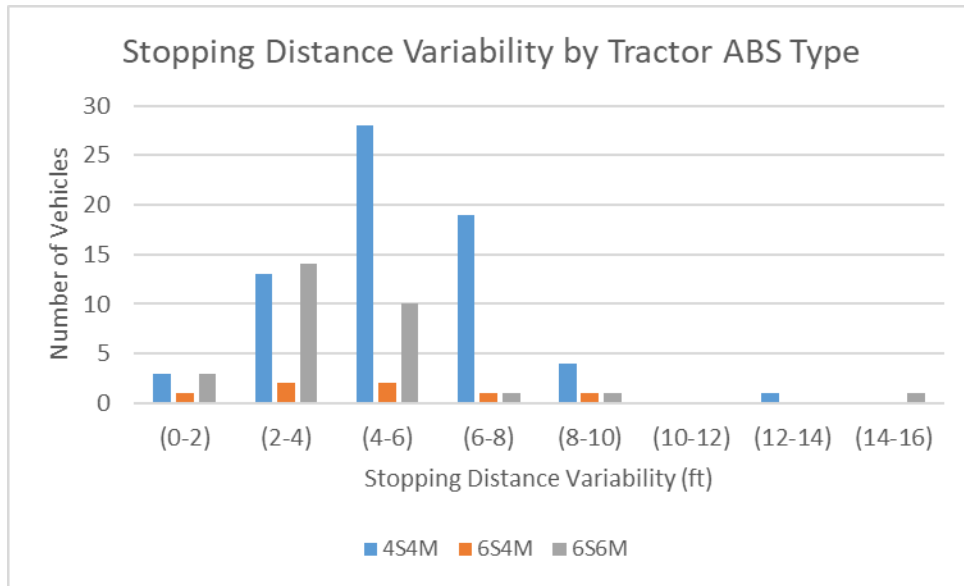


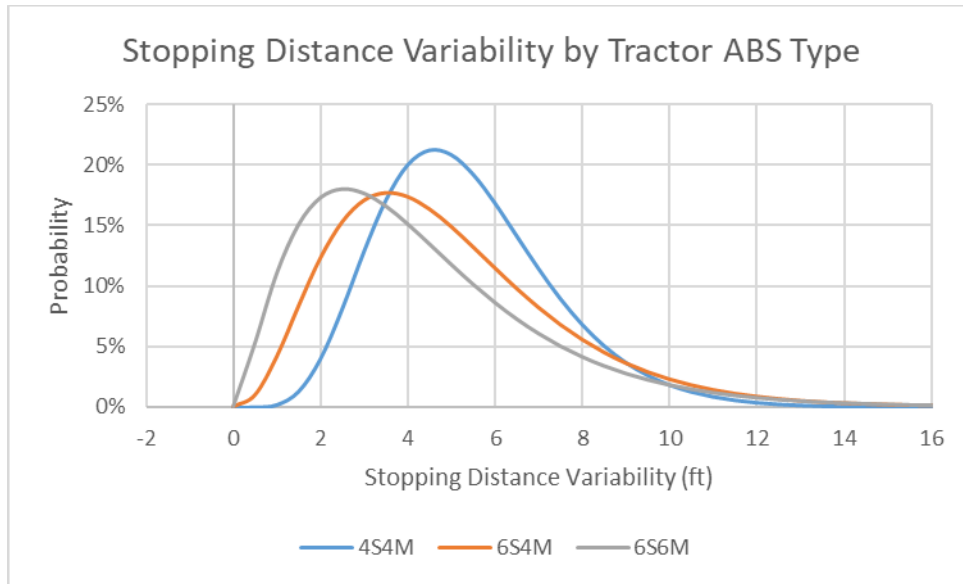
Figure 22. Chart. Test vehicle distribution of stopping distance variability by ABS type.

The corresponding Mann-Whitney tests of hypothesis are summarized in Table 18. For example, in comparing the 4S4M median stopping distance variability to that of the 6S4M, the null hypothesis is that the two median stopping distance variabilities are equal. The alternative hypothesis is that the median 4S4M stopping distance variability is greater than the median 6S4M stopping distance variability. The null hypothesis could not be rejected in favor of the alternative hypothesis with at least 95 percent confidence, suggesting that the stopping distance variability for these ABS types is similar. More detailed information regarding the Mann-Whitney U test is available in Appendix C.

Table 18. Tests of hypothesis for stopping distance variability based on ABS type.

Tractor ABS Type	Median Variability (ft)	4S4M n = 68	6S4M n = 7	6S6M n = 30
4S4M	5.24	--	Cannot reject $H_0$	<b>Reject <math>H_0</math></b>
6S4M	4.54	--	--	Cannot reject $H_0$
6S6M	3.74	--	--	--

The only clear observation is that the 4S4M ABS has a greater stopping distance variability than the 6S6M. The probability distribution in Figure 23 was generated from the standard deviation and mean of the stopping distance variability for the ABS type subcategories.



**Figure 23. Chart. Probability distribution of stopping distance variability for various ABS types derived from test data parameters.**

Based on this theoretical distribution, the effective maximum variability for a given tractor with a particular ABS type can be estimated as shown in Table 19. As noted previously, this effective maximum variability is the point on the associated curve in Figure 23 where there is only a 1 percent chance that the stopping distance variability will be greater. Figure 23 shows that the effective maximum variability is around 12 ft. for all ABS types. The exact effective maximum variabilities are shown in Table 19.

**Table 19. Effective maximum variability and stopping distance range for various brake types.**

Tractor ABS Type	Effective Max Variability (ft)
4S4M	11.1
6S4M	12.7
6S6M	12.9

These effective maximum variabilities can be used to estimate full-system stopping distance range for a given tractor with a particular ABS type as shown in Table 20.

**Table 20. Estimated stopping distance range for various brake types.**

Tractor ABS Type	95.4% Stopping Distance Range (ft)	99.7% Stopping Distance Range (ft)
4S4M	±22.1	±33.2
6S4M	±25.5	±38.2
6S6M	±25.8	±38.7

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## 4. CONCLUSIONS

### 4.1 SUMMARY

#### 4.1.1 Stopping Distance

Several of the examined parameters affected both tractor stopping distance and stopping distance variability. First, the average stopping distance for disc/disc brakes was shorter than either drum/drum or disc/drum brakes. Second, tractors with a GVWR of 45,000–50,000 lbs had shorter stopping distances than any other examined weight category, but from these data it is not possible to make any further statements regarding the link between GVWR and stopping distance. Third, the 151–200-in. wheelbase category of vehicles had the longest average stopping distance. Finally, tractors with the 6S6M ABS system had stopping distances shorter than those with either the 4S4M or the 6S4M ABS systems.

#### 4.1.2 Stopping Distance Variability

Brake type did not have a statistically significant effect on stopping distance variability. Weight, however, had a significant effect; the 50,000–55,000-lb GVWR range had more variability than both the next lower (45,000–50,000 lb) and next higher (55,000–60,000 lb) ranges. The vehicles with a 251–300-in. wheelbase had a lower stopping distance variability than those with a 151–200-in. wheelbase. The 6S6M ABS system had a lower stopping distance variability than the 4S4M ABS system.

The stopping distance variability was used to calculate a stopping distance range for 60-mi/hr full-system stopping distances. These ranges were calculated for both two standard deviations (95.4 percent of observations) and three standard deviations (99.7 percent of observations). Because these ranges are for a full-system 60-mi/hr stop under the conditions specified in FMVSS 121 (one tractor and unbraked control trailer loaded to the tractor GVWR), these results cannot be applied to variability for a standard over-the-road tractor-trailer combination vehicle.

### 4.2 IMPLICATIONS FOR PLATOONING

The full system brake performance data examined in this study may differ from “normal” platoon operations braking performance, but these data can still inform platooning research, because rare edge-case emergency situations may require full system brake performance from trucks operating in a platoon. While platooning capitalizes on the reduced drag associated with close following distances, these following distances must account for variability in stopping distance performance, particularly the effect of brake type. In this study, the stopping distance variability shows that for 60-mi/hr full effectiveness stops for a tractor and unbraked control trailer loaded to the tractor GVWR, drum/drum brakes have a 95 percent probability that the vehicle will have a stopping distance between 208.8 feet and 255.5 feet. Given the same test conditions, disc/drum brakes have a 95 percent probability that the vehicle will have a stopping distance between 196.3 feet and 250.7 feet. Disc/disc brakes have a 95 percent probability that the vehicle will have a stopping distance between 192.8 feet and 249.1 feet.



It is important to note that the FMVSS 121 ABS stopping distance performance test is conducted using truck tractors under ideal braking system conditions, whereas a typical platooning tractor may not be this condition due to several factors (in addition to further variability introduced by the braking performance of the trailer). To better reflect conditions used to select following distances, tractors used for platooning should have well-maintained braking systems.

These tests were performed under ideal environmental conditions and involved full-system stops, whereas normal platooning situations would be more likely to involve lower air pressure braking events under a variety of tire, weather, and road conditions. Platooning vehicles are often loaded to near the gross combination vehicle weight rating and consist of tractors and trailers with brake systems providing braking force at each wheel end; by contrast, the tests used here involved an unbraked control trailer to load the tractor to its GVWR.

FMVSS-121-style brake performance testing of a platoon with two or more vehicles would help bridge the gap between this analysis (based on FMVSS 121 conformance testing of tractors) and a real-world platooning environment (in which the tractor-trailer configuration is standard). Such testing may also allow correlations to be drawn between the FMVSS-121 ABS stopping distance performance test for a tractor and the full-system stopping distance performance of a platoon-ready tractor-trailer combination vehicle.

## APPENDIX A: ANONYMIZED FMVSS-121 TEST DATA

The anonymized data provided by LINK appears below in Table 21.

**Table 21. Summary test data.**

Test Number	Front GA WR (lbs)	Rear GA WR (lbs)	GV WR (lbs)	Wheel base (In)	Brake Type	ABS System	Tire Spec	Full System Stop 1	Full System Stop 2	Full System Stop 3	Full System Stop 4	Full System Stop 5	Full System Stop 6
1	14,600	46,000	60,600	269	Disc/Disc	4S4 M	295/75 R22.5	228	209	212	211	208	211
2	12,000	40,000	52,000	212	Disc/Disc	4S4 M	11R24.5	227	225	221	213	207	213
3	14,600	46,000	60,600	224	Disc/Disc	4S4 M	295/75 R22.5	248	245	244	238	240	243
4	14,600	46,000	60,600	232	Disc/Disc	4S4 M	12R22.5	212	220	226	220	222	214
5	13,200	38,000	51,200	214	Disc/Disc	4S4 M	295/75 R22.5	197	218	221	217	219	218
6	12,000	40,000	52,000	233	Disc/Disc	4S4 M	275/80 R22.5	247	250	250	241	238	230
7	12,000	40,000	52,000	233	Disc/Disc	4S4 M	275/80 R22.5	202	197	204	210	206	193
8	12,350	40,000	52,350	226	Disc/Disc	4S4 M	11R22.5	217	205	212	210	211	209
9	13,200	40,000	53,200	226	Disc/Disc	4S4 M	11R22.5	220	204	208	208	206	211
10	12,000	40,000	52,000	224	Disc/Disc	4S4 M	295/75 R22.5	226	227	213	218	216	214
11	14,000	46,000	60,000	182	Disc/Disc	6S6 M	315/80 R22.5	208	202	199	199	205	202
12	12,350	40,000	52,350	212	Disc/Disc	6S6 M	275/80 R22.5	220	220	216	213	207	208
13	14,600	46,000	60,600	225	Disc/Disc	6S6 M	385/65 R22.5	230	223	222	239	234	235
14	14,600	46,000	60,600	226	Disc/Disc	6S6 M	11R24.5	228	221	220	222	218	225
15	12,500	38,000	50,500	220	Disc/Disc	6S6 M	295/75 R22.5	219	252	263	245	253	233
16	13,200	40,000	53,200	225	Disc/Disc	6S6 M	275/80 R22.5	235	231	225	226	222	222
17	12,500	46,000	58,500	233	Disc/Disc	6S6 M	295/75 R22.5	209	208	214	211	207	211
18	12,500	46,000	58,500	233	Disc/Disc	6S6 M	295/75 R22.5	220	219	215	216	220	213
19	13,200	40,000	53,200	220	Disc/Disc	6S6 M	295/75 R22.5	217	214	217	219	215	215
20	13,200	40,000	53,200	220	Disc/Disc	6S6 M	295/75 R22.5	217	210	213	212	213	214
21	13,200	40,000	53,200	220	Disc/Disc	6S6 M	295/75 R22.5	215	215	216	215	213	219

Test Number	Front GA WR (lbs)	Rear GA WR (lbs)	GV WR (lbs)	Wheel base (In)	Brake Type	ABS System	Tire Spec	Full System Stop 1	Full System Stop 2	Full System Stop 3	Full System Stop 4	Full System Stop 5	Full System Stop 6
22	13,200	40,000	53,200	220	Disc/Disc	6S6M	295/75R22.5	217	214	217	219	215	215
23	13,200	34,000	47,200	256	Disc/Drum	4S4M	295/75R22.5	204	210	204	202	206	203
24	13,200	40,000	53,200	256	Disc/Drum	4S4M	295/75R22.5	216	208	216	215	211	216
25	14,000	40,000	54,000	270	Disc/Drum	4S4M	12R22.5	242	235	234	237	231	241
26	14,600	40,000	54,600	270	Disc/Drum	4S4M	12R22.5	215	213	215	214	220	220
27	12,000	40,000	52,000	212	Disc/Drum	4S4M	11R24.5	237	231	221	222	218	215
28	14,600	44,000	58,600	226	Disc/Drum	4S4M	285/75R22.5	237	247	246	241	239	232
29	13,200	40,000	53,200	230.5	Disc/Drum	4S4M	11R24.5	234	230	228	226	218	223
30	14,600	45,000	59,600	270	Disc/Drum	6S6M	12R22.5	229	232	224	222	223	221
31	13,200	40,000	53,200	256	Drum/Drum	4S4M	295/75R22.5	229	229	236	232	230	221
32	12,000	40,000	52,000	189	Drum/Drum	4S4M	295/75R22.5	246	246	236	233	239	240
33	12,350	38,000	50,350	174	Drum/Drum	4S4M	11R22.5	259	247	260	251	250	246
34	12,350	38,000	50,350	174	Drum/Drum	4S4M	11R22.5	235	246	233	241	242	237
35	12,500	38,000	50,500	162	Drum/Drum	4S4M	11R22.5	219	225	230	226	233	232
36	12,500	38,000	50,500	162	Drum/Drum	4S4M	11R22.5	217	235	228	232	231	223
37	14,600	46,000	60,600	212	Drum/Drum	4S4M	11R22.5	233	221	218	214	217	222
38	12,000	40,000	52,000	212	Drum/Drum	4S4M	295/75R22.5	238	230	231	236	220	229
39	12,000	40,000	52,000	212	Drum/Drum	4S4M	295/75R22.5	238	230	231	236	220	229
40	14,600	46,000	60,600	212	Drum/Drum	4S4M	11R24.5	212	213	208	212	217	205
41	12,000	40,000	52,000	212	Drum/Drum	4S4M	11R24.5	236	226	225	219	218	218
42	14,600	46,000	60,600	212	Drum/Drum	4S4M	11R24.5	240	234	230	237	230	232
43	14,600	46,000	60,600	212	Drum/Drum	4S4M	11R24.5	219	217	219	215	219	220
44	12,000	40,000	52,000	231	Drum/Drum	4S4M	295/75R22.5	244	241	228	229	225	230
45	13,200	34,000	47,200	256	Drum/Drum	4S4M	295/75R22.5	220	217	217	215	219	218
46	13,200	40,000	53,200	256	Drum/Drum	4S4M	295/75R22.5	241	235	242	232	237	232

Test Number	Front GA WR (lbs)	Rear GA WR (lbs)	GV WR (lbs)	Wheel base (In)	Brake Type	ABS System	Tire Spec	Full System Stop 1	Full System Stop 2	Full System Stop 3	Full System Stop 4	Full System Stop 5	Full System Stop 6
47	13,200	34,000	47,200	256	Drum/Drum	4S4 M	295/75 R22.5	208	223	221	210	218	209
48	13,200	46,000	59,200	270	Drum/Drum	4S4 M	12R22.5	230	230	234	222	224	229
49	14,600	40,000	54,600	270	Drum/Drum	4S4 M	12R22.5	247	239	225	231	231	229
50	13,200	40,000	53,200	256	Drum/Drum	4S4 M	295/75 R22.5	225	227	220	216	214	221
51	13,200	34,000	47,200	256	Drum/Drum	4S4 M	295/75 R22.5	214	214	219	210	214	215
52	12,500	38,000	50,500	162	Drum/Drum	4S4 M	11R22.5	265	230	234	237	233	238
53	13,200	46,000	59,200	270	Drum/Drum	4S4 M	12R22.5	237	248	252	252	252	243
54	13,200	38,000	51,200	256	Drum/Drum	4S4 M	295/75 R22.5	221	222	225	228	229	219
55	12,000	40,000	52,000	212	Drum/Drum	4S4 M	11R24.5	257	246	246	244	245	238
56	12,500	38,000	50,500	157	Drum/Drum	4S4 M	295/75 R22.5	245	232	237	242	234	241
57	14,600	44,000	58,600	233	Drum/Drum	4S4 M	11R22.5	248	236	236	235	228	235
58	13,200	38,000	51,200	256	Drum/Drum	4S4 M	295/75 R22.5	248	243	247	241	238	246
59	12,000	38,000	50,000	217	Drum/Drum	4S4 M	295/75 R22.5	214	217	215	212	211	210
60	14600	40000	54,600	172	Drum/Drum	4S4 M	12R22.5	252	229	233	231	233	231
61	14,600	46,000	60,600	212	Drum/Drum	4S4 M	11R24.5	253	240	244	243	249	248
62	14,600	46,000	60,600	212	Drum/Drum	4S4 M	11R24.5	218	220	224	229	231	236
63	14,600	40,000	54,600	172	Drum/Drum	4S4 M	12R22.5	236	242	238	243	243	252
64	14,600	40,000	54,600	172	Drum/Drum	4S4 M	12R22.5	235	224	226	227	222	225
65	12,500	38,000	50,500	157	Drum/Drum	4S4 M	295/75 R22.5	239	238	239	240	236	233
66	12,350	40,000	52,350	172	Drum/Drum	4S4 M	295/75 R22.5	239	244	238	237	239	227
67	14,600	40,000	54,600	172	Drum/Drum	4S4 M	12R22.5	230	228	224	223	217	223
68	12,000	34,000	46,000	240	Drum/Drum	4S4 M	295/75 R22.5	231	243	239	234	237	230
69	14,600	46,000	60,600	231	Drum/Drum	4S4 M	11R24.5	243	227	228	223	224	217
70	12,350	38,000	50,350	174	Drum/Drum	4S4 M	11R22.5	259	247	260	251	250	246
71	12,500	40,000	52,500	229	Drum/Drum	4S4 M	11R24.5	239	231	235	234	232	227

Test Number	Front GA WR (lbs)	Rear GA WR (lbs)	GV WR (lbs)	Wheel base (In)	Brake Type	ABS System	Tire Spec	Full System Stop 1	Full System Stop 2	Full System Stop 3	Full System Stop 4	Full System Stop 5	Full System Stop 6
72	14,600	46,000	60,600	254	Drum/Drum	4S4M	11R24.5	235	238	246	243	236	232
73	12,000	40,000	52,000	231	Drum/Drum	4S4M	295/75R22.5	244	241	228	229	225	230
74	12,000	40,000	52,000	237	Drum/Drum	4S4M	295/75R22.5	247	244	243	242	239	235
75	14,600	46,000	60,600	237	Drum/Drum	4S4M	295/75R22.5	233	234	231	234	233	234
76	14,600	46,000	60,600	237	Drum/Drum	4S4M	295/75R22.5	234	234	243	237	243	237
77	12,000	46,000	58,000	237	Drum/Drum	4S4M	295/75R22.5	226	229	221	225	225	220
78	14,600	46,000	60,600	270	Drum/Drum	4S4M	12R22.5	227	233	238	234	225	224
79	12,000	40,000	52,000	212	Drum/Drum	4S4M	11R24.5	236	226	225	219	218	218
80	12,000	34,000	46,000	180	Drum/Drum	4S4M	295/75R22.5	211	219	215	221	214	219
81	13,200	46,000	59,200	270	Drum/Drum	4S4M	12R22.5	242	237	240	237	233	239
82	14,600	46,000	60,600	225	Drum/Drum	6S4M	295/75R22.5	242	235	236	245	235	234
83	14,600	46,000	60,600	225	Drum/Drum	6S4M	295/75R22.5	221	205	204	206	205	212
84	14,600	46,000	60,600	225	Drum/Drum	6S4M	295/75R22.5	236	237	231	238	232	229
85	13,200	40,000	53,200	225	Drum/Drum	6S4M	295/75R22.5	253	256	257	255	252	242
86	13,200	40,000	53,200	225	Drum/Drum	6S4M	295/75R22.5	228	228	231	227	229	231
87	13,200	40,000	53,200	225	Drum/Drum	6S4M	11R24.5	227	225	227	229	233	231
88	13,200	40,000	53,200	225	Drum/Drum	6S4M	295/75R22.5	267	256	248	246	244	242
89	14,600	45,500	60,100	270	Drum/Drum	6S6M	12R22.5	228	222	224	223	223	223
90	13,200	40,000	53,200	230	Drum/Drum	6S6M	295/75R22.5	244	231	224	229	220	226
91	13,200	46,000	59,200	270	Drum/Drum	6S6M	12R22.5	230	233	234	236	233	231
92	13,200	40,000	53,200	256	Drum/Drum	6S6M	275/80R22.5	230	226	234	218	224	221
93	12,500	38,000	50,500	256	Drum/Drum	6S6M	295/75R22.5	232	221	225	224	226	220
94	12,500	38,000	50,500	198	Drum/Drum	6S6M	285/70R22.5	230	230	232	236	231	234
95	12,000	40,000	52,000	225	Drum/Drum	6S6M	295/75R22.5	259	248	249	244	247	248
96	12,000	40,000	52,000	225	Drum/Drum	6S6M	295/75R22.5	220	210	218	220	215	217

Test Number	Front GA WR (lbs)	Rear GA WR (lbs)	GV WR (lbs)	Wheel base (In)	Brake Type	ABS System	Tire Spec	Full System Stop 1	Full System Stop 2	Full System Stop 3	Full System Stop 4	Full System Stop 5	Full System Stop 6
97	12,000	40,000	52,000	225	Drum/Drum	6S6 M	295/75 R22.5	234	226	234	230	224	224
98	12,000	40,000	52,000	225	Drum/Drum	6S6 M	295/75 R22.5	251	244	239	253	242	241
99	12,000	40,000	52,000	225	Drum/Drum	6S6 M	295/75 R22.5	219	216	212	223	215	218
100	12,000	40,000	52,000	225	Drum/Drum	6S6 M	295/75 R22.5	221	215	214	215	216	213
101	13,200	40,000	53,200	225	Drum/Drum	6S6 M	275/80 R22.5	240	237	240	231	230	230
102	14,600	46,000	60,600	225	Drum/Drum	6S6 M	295/75 R22.5	236	237	231	238	232	229
103	13,200	40,000	53,200	251	Drum/Drum	6S6 M	275/80 R22.5	225	229	229	223	220	222
104	14,600	45,000	59,600	270	Drum/Drum	6S6 M	12R22.5	228	222	224	223	223	223
105	12,500	38,000	50,500	256	Drum/Drum	6S6 M	295/75 R22.5	232	221	225	224	226	220

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## APPENDIX B: NULL HYPOTHESIS TESTS FOR STOPPING DISTANCE COMPARISON

This section outlines the procedure for statistical tests applied to the stopping distances for several vehicles. In this example, the average stopping distance for Subcategory 1 is greater than that for Subcategory 2 in the test data. The example uses the symbols shown in Table 22.

Table 22. Sample data values for stopping distance comparison.

Value	Subcategory 1	Subcategory 2
Number of Observations	$n_1$	$n_2$
Mean Stopping Distance	$\bar{x}_1$	$\bar{x}_2$
Standard Deviation of Stopping Distance	$s_1$	$s_2$
Variance of Stopping Distance	$s_1^2$	$s_2^2$

### HYPOTHESES

#### Null Hypothesis

Vehicles in Subcategory 1 and Subcategory 2 have similar stopping distance.

$$H_0: \mu_1 = \mu_2$$

Equation 1. Null hypothesis.

#### Alternate Hypothesis

Vehicles in Subcategory 1 have greater stopping distance than those in Subcategory 2.

$$H_a: \mu_1 > \mu_2$$



**Equation 2. Alternate hypothesis.**

**STATISTICS FOR T-TEST**

The two-sample  $t$  test statistic is calculated as follows:<sup>(3)</sup>

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

**Equation 3. T-test.**

Calculation of degrees of freedom:<sup>(4)</sup>

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$

**Equation 4. Degrees of freedom.**

This degrees-of-freedom equation requires that both sample sizes  $n_1$  and  $n_2$  be 5 or larger.

**PROBABILITY CALCULATION**

To reject the null hypothesis in favor of the alternate hypothesis with a given level of confidence,  $P(T \geq t) \geq (1 - \alpha)$ . This for the one-sided test is shown below:

- In Excel, this P-value is calculated by T.DIST.RT(x, deg\_freedom)
  - $x = t$
  - $\text{deg\_freedom} = v$

To reject the null hypothesis with 95 percent confidence,  $\alpha = 0.05$  (i.e.,  $P$  must be least 0.95).

**CONFIDENCE INTERVAL**

The two-sample  $t$  confidence interval quantifies how much greater the variability for stopping distance is for the drum/drum configuration than for the disc/drum configuration (i.e.,  $\mu_1 - \mu_2$ ).<sup>(5)</sup>

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<sup>3</sup> David S. Moore and George P. McCabe, *Introduction to the Practice of Statistics*, 5 ed. (New York: W.H. Freeman and Company, 2006), 489.

<sup>4</sup> *Ibid.*, 495.

<sup>5</sup> *Ibid.*, 492.

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

**Equation 5. Confidence interval.**

In this equation,  $t^*$  is the value for the  $t(k)$  density curve with area  $C$  between  $-t^*$  and  $t^*$  (for the number of degrees of freedom previously calculated).

In Excel,  $t^*$  can be calculated by `T.INV.2T(probability,deg_freedom)`

- probability is  $\alpha$  (e.g.,  $\alpha=0.05$  for 95 percent confidence interval)
  - `deg_freedom = v`

**VALUES OF  $T$  FOR VARIOUS SIGNIFICANCE LEVELS ( $T^*$ )**

In Excel, this may be calculated by either of the following for a right-hand-sided  $T$ -test:

- `T.INV.2T(probability,deg_freedom)`
  - probability =  $2\alpha$  (Note that  $\alpha$  must be doubled for a one-side probability because `T.INV.2T` is a two-sided probability.)
  - `deg_freedom = v`
- `-T.INV(probability,deg_freedom)` (Note that the negative sign is necessary because `T.INV` is a left-handed probability rather than a right-handed probability.)
  - probability =  $\alpha$
  - `deg_freedom = v`

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## APPENDIX C: MANN-WHITNEY NULL HYPOTHESIS TESTS FOR STOPPING DISTANCE VARIABILITY COMPARISON

This section outlines the procedure for the Mann-Whitney U test of hypothesis for the standard derivations of stopping distances for several vehicles.<sup>(6)</sup> Thus, beyond this first chart, references to mean refer to mean of the stopping distance standard deviation. References to standard deviation refer to standard deviation of the stopping distance variability (which is quantified by the standard deviation of each set of stopping distance tests). Thus, what is being compared is the variability in stopping distance.

**Table 23. Sample data values for stopping distance variability comparison.**

Value	Subcategory 1	Subcategory 2
Number of observations	$n_1$	$n_2$
Sample Median Variability	$M_1$	$M_2$
Population Median Variability	$m_1$	$m_2$

### HYPOTHESES

#### Null Hypothesis

Vehicles in Subcategory 1 and Subcategory 2 have similar variability in stopping distance (i.e., the medians are equivalent).

$$H_0: m_1 = m_2$$

**Equation 6. Null hypothesis.**

#### Alternate Hypothesis

Vehicles in Subcategory 1 have greater variability in stopping distance than those in Subcategory 2 (i.e., the medians are different).

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<sup>6</sup> Based on the procedure outlined in <https://psych.unl.edu/psycrs/handcomp/hcman.PDF>.

$$H_a: m_1 > m_2$$

**Equation 7. Alternate hypothesis.**

## CALCULATION OF THE U-VALUE

Each sample is given an overall rank  $r$  from smallest to largest across both subcategories. Thus, each sample will have a rank from 1 to  $(n_1 + n_2)$ . The sum of these overall ranks is calculated for each subcategory.

$$R_1 = \sum_{i=1}^{n_1} r_i$$

**Equation 8. Rank for  $n_1$ .**

$$R_2 = \sum_{i=1}^{n_2} r_i$$

**Equation 9. Rank for  $n_2$ .**

These rank sums are used to calculate a U-value for each subcategory.

$$U_1 = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1$$

**Equation 10. U-value for subcategory 1.**

$$U_2 = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - R_2$$

**Equation 11. U-value for subcategory 2.**

The lower of these two values is used in further calculations.

$$U = \min(U_1, U_2)$$

**Equation 12. Selection of U-value for subsequent calculations.**

One of two methods is used to determine whether the null hypothesis can be rejected in favor of the alternative hypothesis, depending on the sample size.

**FOR  $\max(n_1, n_2) \leq 20$**

For small sample sizes, the calculated U-value is compared to the appropriate critical U-value ( $U_{crit}$ ) from a lookup table based on the sample sizes  $n_1$  and  $n_2$ . Such a table is shown below for a 95 percent confidence level (Figure 20).

**Table 24. Lookup table for one-sided 95 percent confidence Mann-Whitney U.**

n1/n2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3	0	0	1	2	2	3	4	4	5	5	6	7	7	8	9	9	10	11
4	0	1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17	18
5	1	2	4	5	6	8	9	11	12	13	15	16	18	19	20	22	23	25
6	2	3	5	7	8	10	12	14	16	17	19	21	23	25	26	28	30	32
7	2	4	6	8	11	13	15	17	19	21	24	26	28	30	33	35	37	39
8	3	5	8	10	13	15	18	20	23	26	28	31	33	36	39	41	44	47
9	4	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54
10	4	7	11	14	17	20	24	27	31	34	37	41	44	48	51	55	58	62
11	5	8	12	16	19	23	27	31	34	38	42	46	50	54	57	61	65	69
12	5	9	13	17	21	26	30	34	38	42	47	51	55	60	64	68	72	77
13	6	10	15	19	24	28	33	37	42	47	51	56	61	65	70	75	80	84
14	7	11	16	21	26	31	36	41	46	51	56	61	66	71	77	82	87	92
15	7	12	18	23	28	33	39	44	50	55	61	66	72	77	83	88	94	100
16	8	14	19	25	30	36	42	48	54	60	65	71	77	83	89	95	101	107
17	9	15	20	26	33	39	45	51	57	64	70	77	83	89	96	102	109	115
18	9	16	22	28	35	41	48	55	61	68	75	82	88	95	102	109	116	123
19	10	17	23	30	37	44	51	58	65	72	80	87	94	101	109	116	123	130
20	11	18	25	32	39	47	54	62	69	77	84	92	100	107	115	123	130	138

The null hypothesis ( $H_0$ ) can be rejected in favor of the alternative hypothesis ( $H_a$ ) if  $U \leq U_{crit}$ .

**FOR  $\max(n_1, n_2) > 20$**

For larger sample sizes, the U-value can be used directly to calculate a Z-value associated with a normal distribution.

$$Z = \frac{\left|U - \frac{n_1 n_2}{2}\right|}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}$$

**Equation 13. Calculation of Z-value.**

The null hypothesis ( $H_0$ ) can be rejected in favor of the alternative hypothesis ( $H_a$ ) if  $Z \leq Z_{crit}$ . For a 95 percent confidence level, this critical Z-value is shown below.

$$Z_{crit} = 1.96$$

**Equation 14. Critical Z-value.**