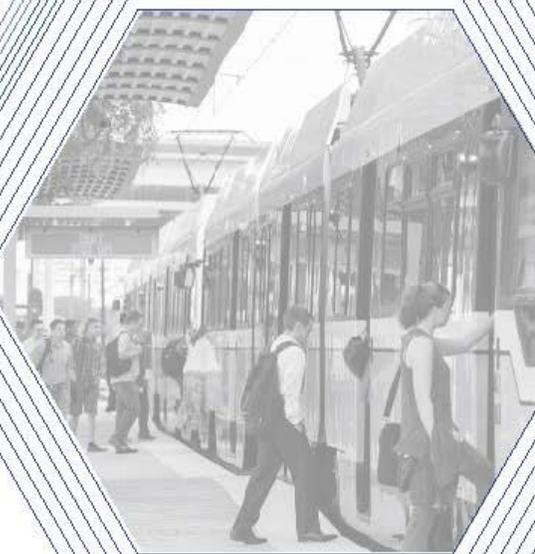
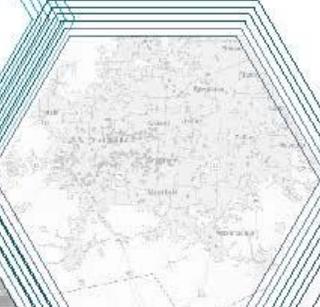


Safety Effects of Transit Signal Priority: Magnitude and Mechanism

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FINAL REPORT

SAFETY EFFECTS OF TRANSIT SIGNAL PRIORITY: MAGNITUDE AND MECHANISM

FINAL PROJECT REPORT

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Abstract

Previous studies have found a correlation between the implementation of transit signal priority (TSP) and a reduction in vehicle crash rates through aggregated analysis. To understand how TSP affects safety, this study engages a more in-depth evaluation. In this study, the effects of TSP are evaluated on a surrogate safety measure - bus movement roughness - which the authors expect to be closely linked with the risks of bus-related crashes and in-vehicle passenger incidents. Metro Transit of Minneapolis-St. Paul implemented TSP at 30 signalized intersections on bus route #5 in early 2019. Bus crashes and incidents, automatic vehicle location (AVL) records, TSP request log, weather and roadway geometric data were obtained for April-June 2018 and 2019, covering the time periods before and after TSP implementation. Linear regression and tree-based model analysis was carried out on 23,123 event-level observations, with an event defined as a bus approaching and clearing a signalized intersection. The analysis results indicate that the overall bus movement roughness did not change significantly after TSP implementation. However, with a TSP request, the bus movement roughness is significantly smoother than without a TSP request. Intersections with more incidents and more crashes are significantly associated with rougher bus movements. Although the effect of TSP on bus movement roughness is significant, its impact is slight, comparing with the impact of bus approaching speed and road geometric features, which also interact with each other. Implications for transit agencies are: 1) TSP helps improve bus safety and can be implemented without much concerns in terms of safety. 2) Bus operational features and road geometric design factors should be integrated in an improvement plan to achieve an optimal safety outcome.

Chapter I: Background

The effects of transit signal priority (TSP) on bus safety were evaluated for the bus Route 5 of Minneapolis-St. Paul Metro Transit. Route 5 is a local bus route that runs north-south through the Cities of Brooklyn Center, Minneapolis, Richfield, and Bloomington, all within Hennepin County, Minnesota. The bus route map is shown in Figure 1. Route 5 runs a high-frequency service (with a bus every 15 minutes) in Minneapolis between North 26th Avenue and East 56th Street.

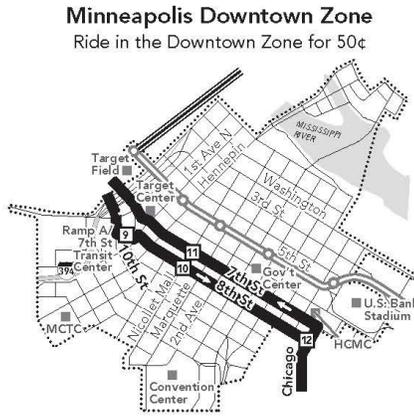
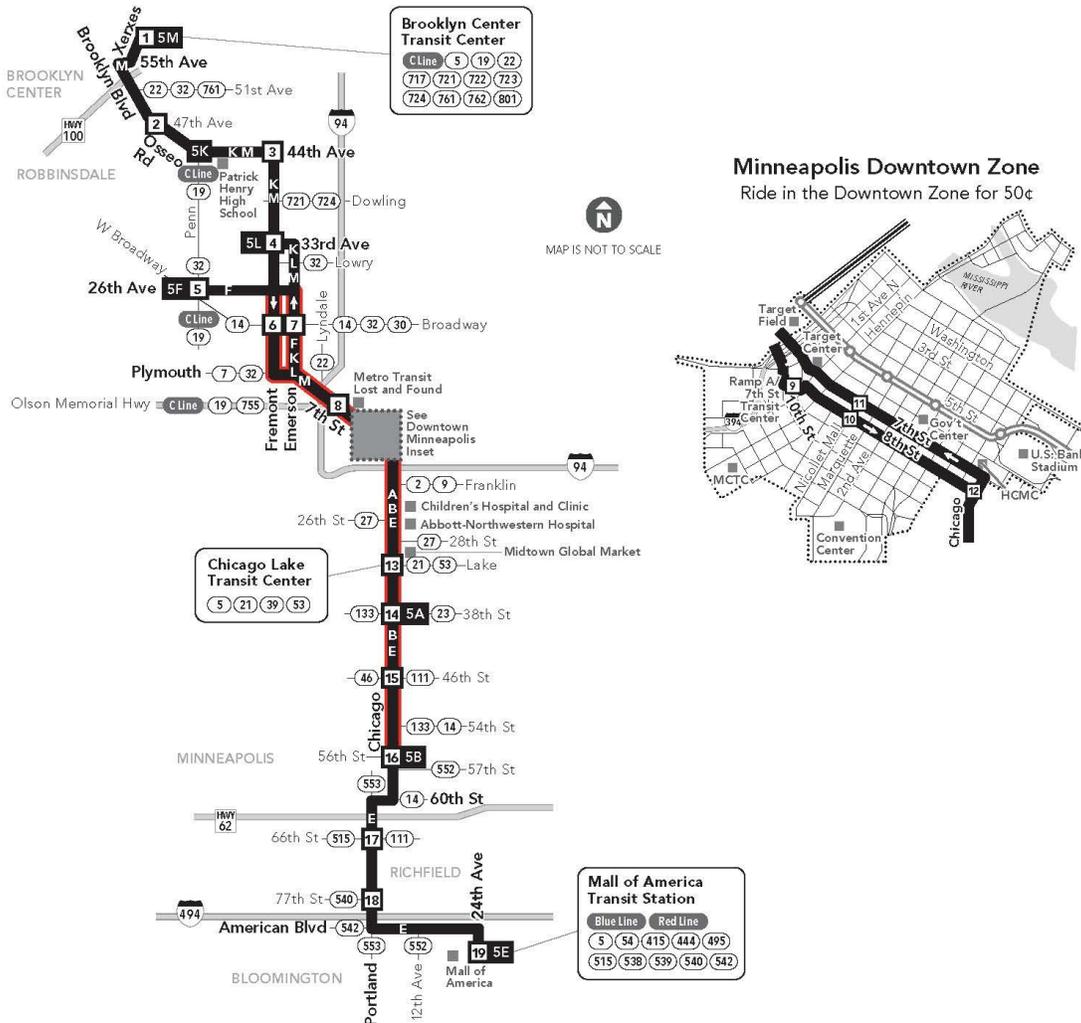
In order to improve the on-time performance and reduce travel time of Route 5 buses, as well as to prepare for future conversion of local bus Route 5 into a D-Line Bus Rapid Transit (BRT) service, Metro Transit started installing TSP at 30 signalized intersections along Route 5 in late 2018 and completed the installation and testing processes in March 2019. By late March 2019, all 30 intersections had the TSP activated, with buses sending out requests and receiving signal priority. The 30 signalized intersections are located along two sections of Route 5 within the central-outer city transition area. The North Section is from North 7th Street and Olsen Memorial Highway to Fremont Avenue North and 42nd Avenue North, with 18 TSP intersections. The South Section is from Chicago Avenue and East Franklin Avenue to Chicago Avenue and East 54th Street, with 12 TSP intersections.

TSP equipment and system for Route 5 were provided and installed by EMTRAC. The TSP system uses buses' GPS location and schedule adherence status to determine the activation of signal priority, as illustrated in Figure 2. If TSP was to be activated, the system determines whether to extend the green interval or shorten the red interval, based on the real-time signal controller status. A more detailed procedure of TSP system detecting a bus approaching a signalized intersection is illustrated in Figure 3. A bus approaching an intersection will be pre-checked two blocks away from the intersection. At this location, bus velocity and schedule adherence information are obtained but TSP is not activated. When the bus arrives into the adjacent block upstream of the intersection, TSP request will be sent out via frequency-hopping spread spectrum (FHSS), from an in-vehicle device, to the signal controller equipped with EMTRAC priority detector. The signal controller adjusts signal timing when receives priority request from the EMTRAC detector. When the bus has gone through the signal and arrives into the check-out zone, the TSP request will stop, and the signal controller will start recovering from the timing adjustment.

5 Local Bus Route



Effective 12/7/19



Timepoint on schedule Find the timepoint nearest your stop, and use that column of the schedule. Your stop may be between timepoints.	High Frequency Service Service every 15 minutes on weekdays 6 am – 7 pm and on Saturdays 9 am – 6 pm.
Regular Route Bus will pick up or drop off customers at any bus stop along this route.	Route Ending Point Trips with the indicated number/letter end at this point. Number/letter is found in schedules and on bus destination signs.
METRO Line and Stations METRO trains or buses will pick up or drop off customers at any station along this route.	Route Letter Indicates which trips travel on this section of the route. Letter is found in schedules and on bus destination signs.
Northstar Commuter Line Transfers from Northstar to buses or light rail are free. Transfers from buses or light rail to Northstar require an additional fare.	Connecting Routes See those route schedules for details.

Figure 1 Metro Transit Route 5 map (1)

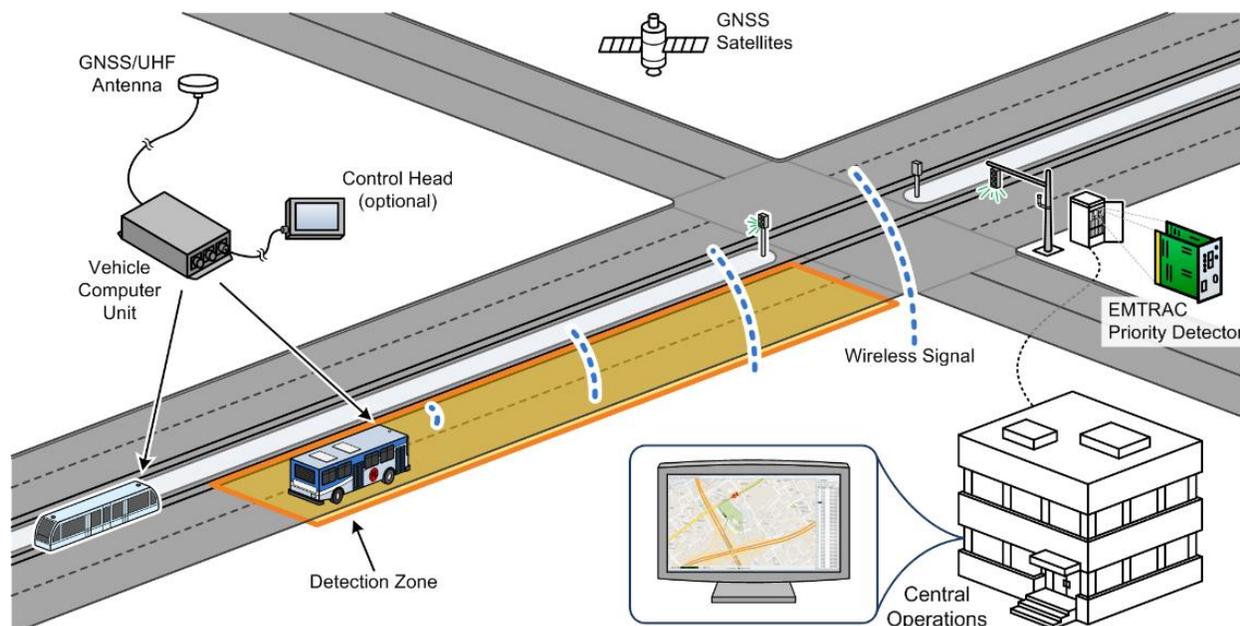
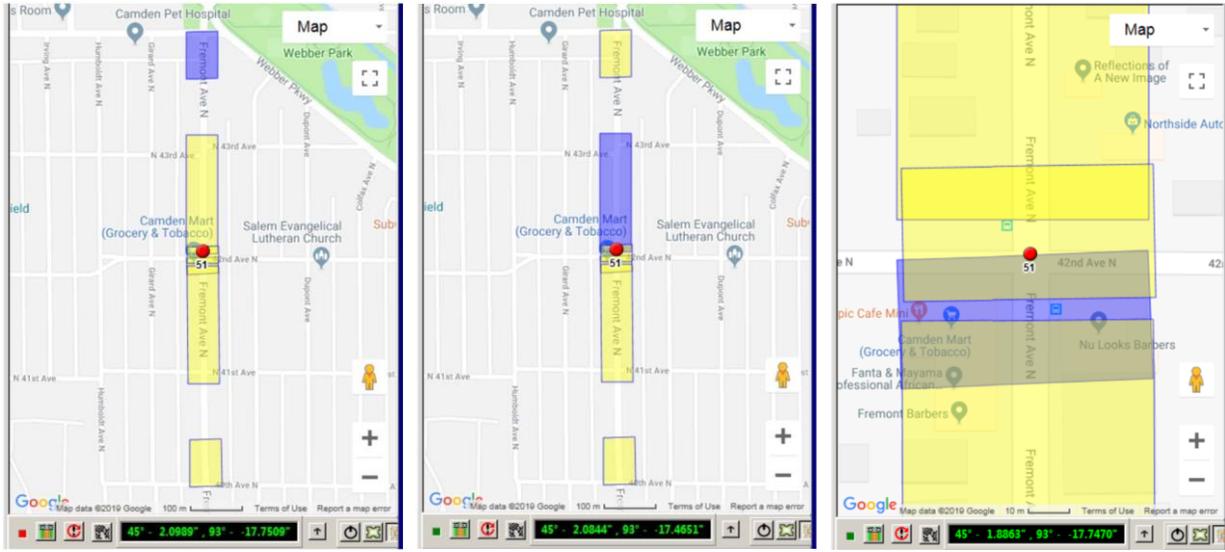


Figure 2 EMTRAC GPS-based TSP (2)

The TSP implementation on Route 5 did not involve stop-relocation or other changes in the bus routes' facilities. The only geometric changes took place on the one-way thoroughfare of Fremont Avenue North and Emerson Avenue North between Plymouth Avenue North and North 33rd Avenue. The changes include bike lane conversion and sidewalk bulb-out (i.e., curb extension), which were implemented around the same time with the TSP installation. All geometric changes are considered in the evaluation.

In terms of operational effects, Metro Transit is obtaining data and evaluating bus travel time changes, schedule adherence, and other measures. Generally, TSP implemented in the United States reduce the average bus travel time between 2% and 20%, with 8% to 12% as the most typical range (3). Slight increase in side-street traffic delay may be seen with TSP implementations, but GPS-based TSP using conditional priority logics is able to keep this increase in side-street delay to a minimum (4).

In parallel with the operational evaluation of the Route 5 TSP, this study is carried out to evaluate the system's potential effects on safety. The safety aspect of TSP's effects did not receive much attention until recent years. There have been several studies published on this topic, covering implementations in multiple cities of multiple countries. All previous studies were aggregated analyses of a before-after effectiveness of TSP on traffic crashes. These previous studies are reviewed, and their findings are summarized in the following Literature Review section.



(a) Pre-check

(b) Check in

(c) Check out

Note: active zone highlighted in blue

Figure 3 TSP detection procedure: example of Route 5 southbound at Fremont Ave and 42nd Ave

Chapter II: Literature Review

Mixed results were reported in previous studies on TSP's effects on traffic crashes. Most of the studies found that the implementation of TSP is related to the reduction in number of crashes at the intersection level and the corridor level (7-11). However, there are also studies indicating that TSP may lead to increase in number of crashes (5,6).

Two evaluations of TSP implementations in Toronto, Canada, reported correlations between TSP and increase in traffic crashes. Shahla et al. studies Toronto's bus and streetcar TSP system using a cross-sectional study method with negative binomial regression models (5). Four years (1999–2003) of crash data were analyzed. Modeling results showed that the implementation of TSP was correlated with the increase of crashes at signalized intersections. In terms of all crashes, the implementation of streetcar TSP brought a 31% increase in crashes, while bus TSP brought a 28% crash increase. From two different models, TSP was found related to a 25% or 51% increase in the combined streetcar/bus-involved crashes. Li et al. studied the same TSP system using Vissim micro-simulation and Surrogate Safety Assessment Model (SSAM) (6). Five years (2006-2010) of data were used in the analysis. Intersection-level vehicular conflicts were reported from the simulation models and were converted to crashes using a negative binomial regression models reflecting the relationship between the numbers of different types of crashes and the numbers of different types of conflicts. The analysis results showed that removing TSP from the existing, TSP-enabled, intersections could lead to a reduction in all crashes by as much as 1.6%, a 2.9% reduction in angle crashes, a 1.9% reduction in rear-end crashes, and a 2.1% reduction in side-swipe crashes. However, with the removal of TSP, changes in transit-involved crashes could range from a reduction of 0.1% to an increase of 1.6%. These results indicated that the enabling of TSP would increase traffic crash numbers.

Three studies of TSP implementations in Melbourne, Australia, showed that TSP helped reduce number of crashes. Goh et al. explored the traffic safety impacts of bus priority treatments including bus lanes and TSP using Empirical Bayes (EB) before-after analysis method (7). Particularly, in terms of TSP, the results showed an 11.1% reduction in expected fatal and injury (FI) crashes in the “after” period with TSP implemented, compared with the “after” period crash frequency estimates without TSP. Another cross-sectional evaluation of TSP's effects on bus-involved crashes by Goh et al., using negative binomial regression and neural network modeling methods, showed that Melbourne TSP, in collaboration with non-TSP transit preferential treatments (e.g., bus lane, stop relocation), helped reduce 53.5% of bus-involved crashes (8). Naznin et al. assessed streetcar TSP at 29 intersections using EB before-after study method (9). A statistically significant reduction of 13.9% in streetcar-involved crashes was found after the implementation of TSP. A disaggregate-level simple before–after analysis also indicated reductions in total and FI crashes as well as vehicle, pedestrian, and motorcycle-involved crashes.

Table 1 Effects of TSP on the number of traffic crashes based on previous studies

Study	Method	Crash Type	Effectiveness	95% CI ^a
<i>Canada</i>				
Shahla et al. 2009 (5)	NB ^b Regression	Streetcar/Bus-Involved	1.52	(1.17, 1.96)
			1.26	(1.00, 1.58)
		All, Streetcar TSP	1.32	(1.23, 1.42)
			All, Bus TSP	1.28
Li et al. 2017 (6)	Micro-simulation	All	0.99 ^d	N/A
		Transit-Involved	1.02 ^d	N/A
<i>Australia</i>				
Goh et al. 2013 (7)	EB ^c Before-After Study	FI	0.89	(0.67, 1.10)
Goh et al. 2014 (8)	NB Regression; Neural Network	Bus-Involved	0.47	N/A
Naznin et al. 2016 (9)	EB Before-After Study	Streetcar-Involved	0.86	(0.70, 1.02)
<i>United States</i>				
Song & Noyce 2018 (<i>Error! Reference source not found.</i>)	EB Before-After Study	All	0.87	(0.84, 0.91)
		PDO	0.84	(0.80, 0.88)
		FI	0.95	(0.89, 1.01)
Song & Noyce 2019 (11)	Time Series Analysis	All	0.95	(0.91, 0.99)
		PDO	0.9	(0.82, 0.98)
		FI	1.03	(1.02, 1.04)
		Pedestrian-Involved	1.06	(0.83, 1.29)
		Bike-Involved	2.95	(2.39, 3.51)

Note: a. 95% confidence interval; b. Negative binomial; c. Empirical Bayes; d. These effects are from removing TSP comparing with keeping TSP in the simulation model

One of the two studies of TSP safety investigated the implementations in the Seattle metropolitan area, and reported a reduction in number of traffic crashes after TSP deployment. Song and Noyce assessed the effects of TSP implemented with King County Metro Transit's RapidRide BRT system in 2010-2014, using EB before-after study. The study results showed that the implementation of TSP was significantly correlated with a 13% reduction in all crashes, a 16% reduction in property-damage-only (PDO) crashes, and a 5% reduction in FI crashes. Another study evaluated the TSP implementations in Portland, Oregon. Song and Noyce carried out time series analysis with Portland data from 1995 to 2010 (11). It was found that with the implementation of TSP in late 2002, the number of all crashes in the post-intervention period (2003-2010) was 4.5% lower comparing with the no-TSP scenario, and the number of PDO crashes was 10% lower. No significant change was found in the number of FI crashes.

The findings from the cited studies, regarding the safety effects of TSP, are summarized in Table 1. Through reviewing prior studies on TSP's safety effects, the following three potential improvements are identified for this and future studies to make.

- Prior studies yielded mixed results about TSP's effects on safety, thus more studies are still needed to better understand what made TSP affect traffic safety differently.
- Prior studies only used crash numbers as the measure of safety performance. The number of in-vehicle passenger incidents is also an important safety measure that transit agencies should pay attention to.
- Prior studies evaluated the safety effects of TSP at an aggregated level and generally evaluated before-after changes without potential confounding factors controlled. As TSP is usually implemented along with other transit improvements (e.g., relocation of bus stop, addition of bus lanes). Further studies are needed to evaluate TSP's effects with as many potential confounding factors controlled as possible.

For the Route 5 evaluation, as the TSP system is newly implemented, aggregated analysis solely may not provide enough insights about the safety effects. As detailed vehicle location and TSP request data is available for Route 5 from Metro Transit, there is an opportunity to carry out a more in-depth, event-level analysis of TSP's effect on safety.

Chapter III: Data

The data used in this study was provided by Metro Transit. Except for geometric change information, all data was collected for Route 5 from the months of April – June in 2018 and 2019, to ensure similar climate and travel patterns for the before and after periods.

Bus Route, Schedule, TSP Activation Status, and Geometric Features

Bus route maps and schedules for all Route 5 weekday, Saturday, and Sunday services were provided by Metro Transit. The bus schedules were adjusted several times during both the before and after periods. The schedules were used to validate the bus trip and timestamp information in the automatic vehicle location (AVL) dataset. TSP activation status was provided. All 30 intersections analyzed in this study had TSP installed and activated by March 2019.

Geometric features including one-way, nearside bus stop, number of lanes, roadside parking, bike lanes, bulb-outs, and curb cuts were collected through historical Google Maps Street Views and related Metropolitan Council street improvement project documents. Geometric features of all 30 TSP intersections, during the before and after periods, were all coded and recorded by bus operating direction (i.e., northbound and southbound) in an intersection summary data set. All geometric changes along Emerson Avenue North and Fremont Avenue North were coded and recorded based on the Emerson & Fremont Pedestrian & Bicycle Enhancements Project CAD drawings from Alliant Engineering and Minneapolis Public Works, who designed and implemented the geometric changes.

Bus Crashes and In-vehicle Passenger Incidents

Bus crashes and in-vehicle passenger incidents are archived by Metro Transit regularly. Metro Transit removed all sensitive information from the dataset before sharing. The TSP system was implemented quite recently, and only three months' worth of crash and incident data for the before and after periods was obtained. Crashes and incidents were recorded by Metro Transit with a set of internal codes. By examining the data, the authors determined that the following types of crashes and incidents are directly or indirectly related to traffic signal and TSP:

- #24: Vehicle hit bus (except #28) (includes vehicle backing or rolling back)
- #62: Falls, bumps, etc. - bus starting
- #63: Falls, bumps, etc. - bus stopping
- #64: Falls, bumps, etc. - bus turning at curves or corners
- #65: Falls, bumps, etc. - bus running straight

The number of bus-related crashes and in-vehicle incidents on Route 5 are illustrated in Figure 4. In April-June 2019 (after TSP implementation), no Type #24 and #62 crashes/incidents happened on Route 5. There is also a reduction in Type #63, #64, and #65 crashes/incidents, comparing with April-June 2018.

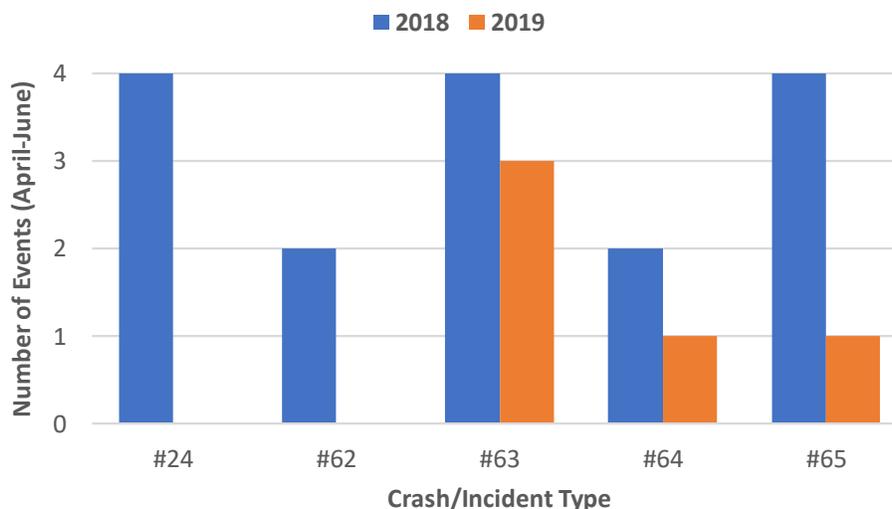


Figure 4 Route 5 bus crashes and in-vehicle incidents in April-June 2018 and 2019

Automated Vehicle Location (AVL)

The AVL data set was provided for April-June 2018 and 2019, with records of all Route 5 buses in operation during the periods. The AVL data set includes date, timestamp, bus ID, block ID, trip ID, vehicle direction, latitude and longitude of coordinates, and schedule adherence (in minutes). When a Route 5 bus is in operation, values for all the data items in the AVL data set are collected every 8 seconds. The AVL data provides information about the movements of buses and bus activities at intersections and its upstream and downstream road segments.

TSP Requests

A total of 81,392 TSP requests were sent out by Route 5 buses during April-June 2019. Date, timestamp, intersection ID, bus ID, and direction were included in the TSP request data set. Figure 5 shows the daily TSP requests throughout the period from April 1st to June 30th, 2019. Figure 6 illustrates the daily average TSP requests by intersection.

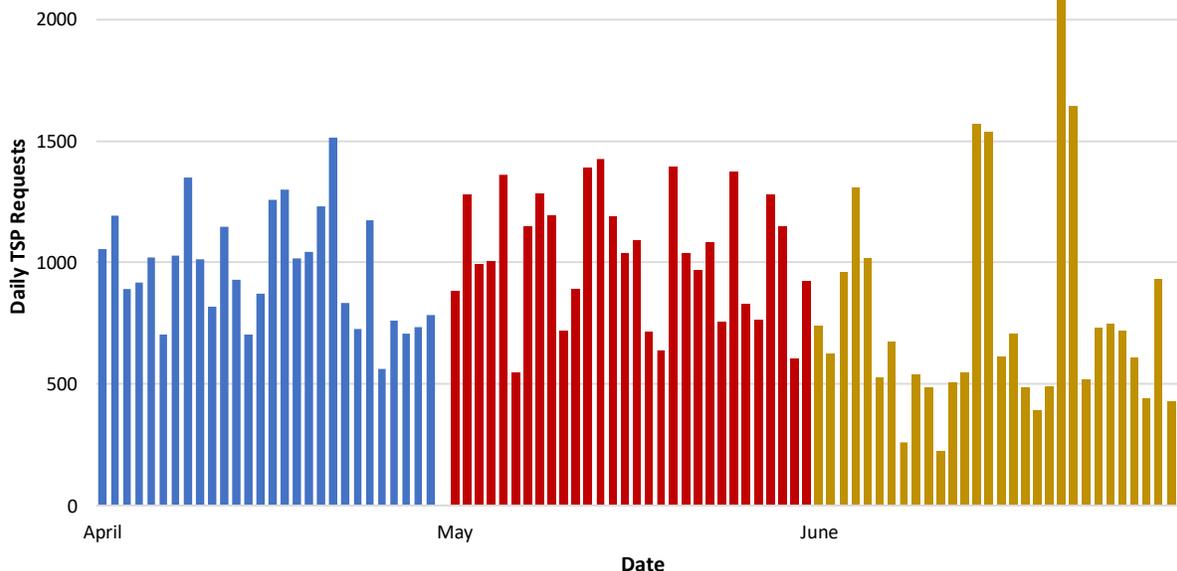
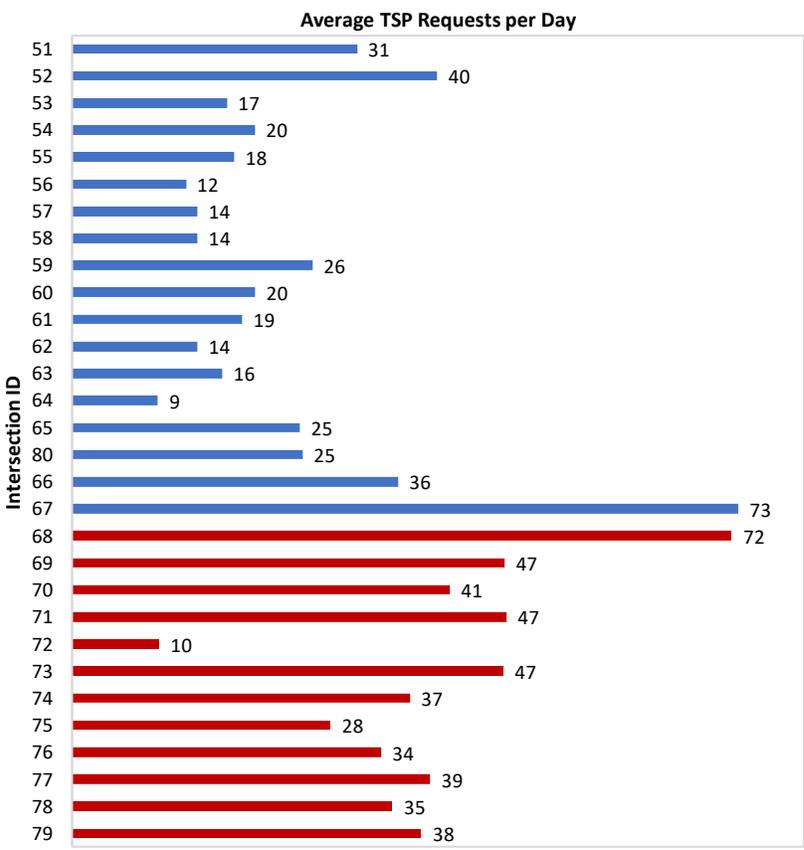


Figure 5 Distribution of TSP requests over time

Weather

Hourly weather data was obtained from the Local Climatological Database of National Ocean and Atmospheric Administration (NOAA) (12). Data from weather observation stations at two Minneapolis area airports were collected. As Route 5 has two sections with TSP implemented, one north of Downtown Minneapolis and the other south of Downtown Minneapolis, the weather data from Crystal Airport (MIC) station was assigned to the North Section and the data from Minneapolis-St. Paul International Airport (MSP) was assigned to the South Section. Weather variables include air temperature (°F), precipitation (in), relative humidity (%), visibility (mi), and wind speed (mph). The weather data covered both the before and after periods.



Section	ID	Route Street	Cross Street
North	51	FREMONT	42ND
	52	FREMONT	DOWLING
	53	FREMONT	LOWRY
	54	FREMONT	26TH
	55	FREMONT	24TH
	56	FREMONT	BROADWAY
	57	FREMONT	17TH
	58	FREMONT	PLYMOUTH
	59	EMERSON	LOWRY
	60	EMERSON	26TH
	61	EMERSON	24TH
	62	EMERSON	BROADWAY
	63	EMERSON	16TH
	64	EMERSON	PLYMOUTH
	65	EMERSON	VAN WHITE
	80	7TH	LYNDALE SB
	66	7TH	LYNDALE NB
67	7TH	OLSON	
South	68	CHICAGO	FRANKLIN
	69	CHICAGO	24TH
	70	CHICAGO	26TH
	71	CHICAGO	28TH
	72	CHICAGO	29TH
	73	CHICAGO	LAKE
	74	CHICAGO	35TH
	75	CHICAGO	36TH
	76	CHICAGO	38TH
	77	CHICAGO	46TH
	78	CHICAGO	MINNEHAHA
	79	CHICAGO	54TH

Figure 6 Average daily TSP requests by intersection

Chapter IV: Methodology

Previous studies have helped improve the understanding of TSP's effects on crashes at an aggregated level (i.e., intersection level or corridor level). However, those studies only provided an idea about the magnitude of TSP's safety effects. There is still not a clear idea about how TSP affects safety. In this study, two aspects of TSP's safety effects, the magnitudes and the mechanism, are both evaluated. The evaluation is done at both corridor level and event level.

The corridor-level evaluation is done using simple before-after comparison of bus-related crash and in-vehicle passenger incident numbers. Due to the short period of time since the Route 5 TSP implementation, complicated before-after analysis cannot be carried out. Since the previous studies have provided a good understanding of how much TSP can affect safety at the corridor level or intersection level, this part of the evaluation is not the key focus point of this study.

The event-level evaluation is the key focus point of this study. In this study, an event is defined as a bus approaching and clearing a signalized intersection. For one intersection on Route 5, each day, there are over 70 of these events happening on each bus operating direction. For both operating directions of the 30 TSP intersections, there are over 4,200 events happening each day. A cross-sectional analysis is carried out to evaluate the safety of those events, before and after the implementation of TSP, and with and without TSP request. Since bus-related crashes and in-vehicle passenger incidents are rare during the short periods covered by the available data sets, directly modeling the relationship between TSP and crashes/incidents would not provide informative results. With the available data, modeling the relationship between TSP and a surrogate safety measure could provide more insights.

Surrogate Safety Measure

From a probabilistic viewpoint, Davis et al. proposed a conceptual framework as shown in Figure 7 (13). The framework shows the relationship between surrogate measures and safety outcomes, implying that some measurable evasive actions, together with initial conditions (including environmental and human factors), determines the probability of the occurring of a safety outcome such as a traffic crash or a passenger incident. Therefore, by using measurable variables describing initial conditions, and measurable variables describing road user actions, risks that leading to safety outcomes can be estimated.

Traditionally, measures such as time to collision (TTC), post-encroachment time (PET), and deceleration rate are used as safety surrogates for safety risk estimation (14). Depending on the interested road user groups, safety surrogates have been developed for specific groups such as pedestrians. Combinations of surrogate indicators have also been developed to not only capture the probability of a crash, but also the severity of a crash.

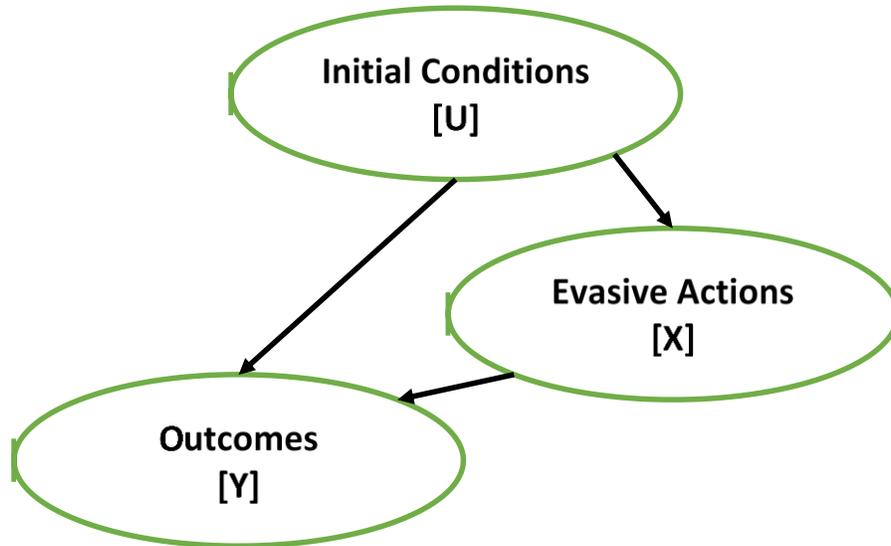


Figure 7 Causal relationships between initial conditions, evasive actions, and safety outcomes (13)

Several previous studies have used speed profile or vehicle trajectory as a surrogate to measure safety risks (15-18). Change in speed and trajectory can reflect the operating status of a vehicle. When an emergency situation occurs or a crash happens, vehicle speed and trajectory are expected to change dramatically. A time-space diagram combines information from speed profile and a part of vehicle trajectory (only longitudinal, no lateral), thus is expected to also be a feasible surrogate safety measure. Any change in vehicle speed is reflected directly by the change in the slope of a time-space curve, with an example in Figure 8 showing a set of buses' time-space curve when they went through the intersection of Fremont Avenue North and 42nd Avenue North. The time interval between each pair of consecutive timestamps is 8 seconds, and the distance is calculated from the starting point of the intersection's upstream TSP check-in zone. The change in time-space diagrams are what is interested in as a surrogate safety measure. A dramatic change in the curve slope suggests sudden braking or stopping, which are closely linked to rear-end crashes and in-vehicle passenger incidents such as fall-overs.

To quantify the dramatic change in time-space diagrams, a curve “roughness” measure is introduced. A rougher curve has more dramatic changes, meaning more sudden braking or stopping. On the contrary, a smoother curve reflects fewer sudden braking or stopping. To calculate the roughness of a curve, the bus distance data points, from an event of it approaching and passing the intersection, provide an array of values, X . The roughness of a time-space curve is then calculated as:

$$Roughness = \frac{s.d.(diff(X))}{abs(mean(diff(X)))} \quad (1)$$

Where: $s.d.()$ = standard deviation;
 $diff()$ = lagged differences, lag = 1 used here, i.e., successive differences;
 $abs()$ = absolute value; and
 $mean()$ = mean value.

A roughness value closer to 0 means a smoother curve.

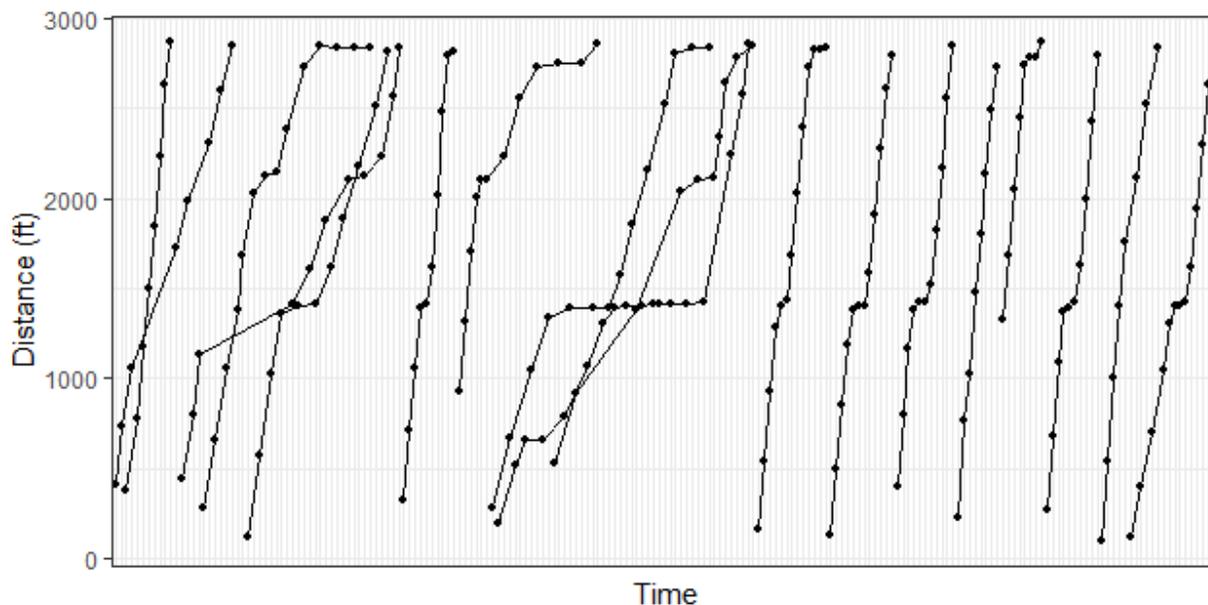


Figure 8 An example set of bus time-space diagrams

Data Processing

To formulate a data set suitable for statistical analysis, the raw data sets are cleaned and merged following the procedure shown in Figure 9. The AVL data is filtered using intersection upstream and downstream limits, and merged with weather data, to get event-level data including calculated time-space diagram roughness, upstream approaching speed, schedule adherence, and weather conditions. The upstream and downstream limits of an intersection are determined by the limits of TSP activation zones. About one block upstream and one block downstream from the intersection is bordered by the limits. Since the signalized intersections are mostly spaced more than one block away from each other, there is no overlap between the buffers bordered by the limits. The upstream approaching speed is calculated using the first three AVL coordinates on each time-space curve. Merging with intersection attributes, 30 intersections' event-level data sets were combined. The combined event-level data set was then merged with TSP request data, using intersection ID, operating direction, date, and bus ID as matching criteria. The merged final event-level data set was ready to use for statistical analysis.

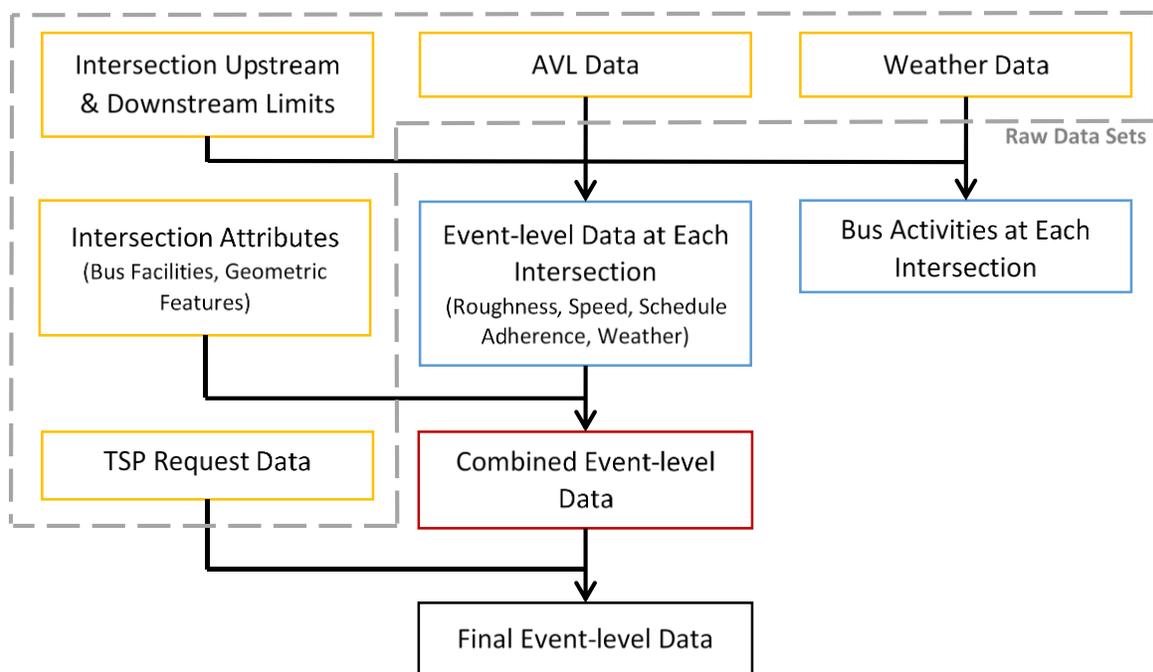


Figure 9 Data processing procedure

To reduce potential bias in modeling due to outliers in the calculated measures such as roughness and speed, the extremely large outliers were excluded from these two variables. The density distributions of these two variables after data cleaning are shown in Figure 10 (a) and (b). The cleaned final data set consists of 23,123 rows of events where a bus approaching and clearing a signalized intersection, and 23 variables for statistical analysis. The records cover all 30 selected intersections, as well as before and after periods of April-June 2018 and 2019.

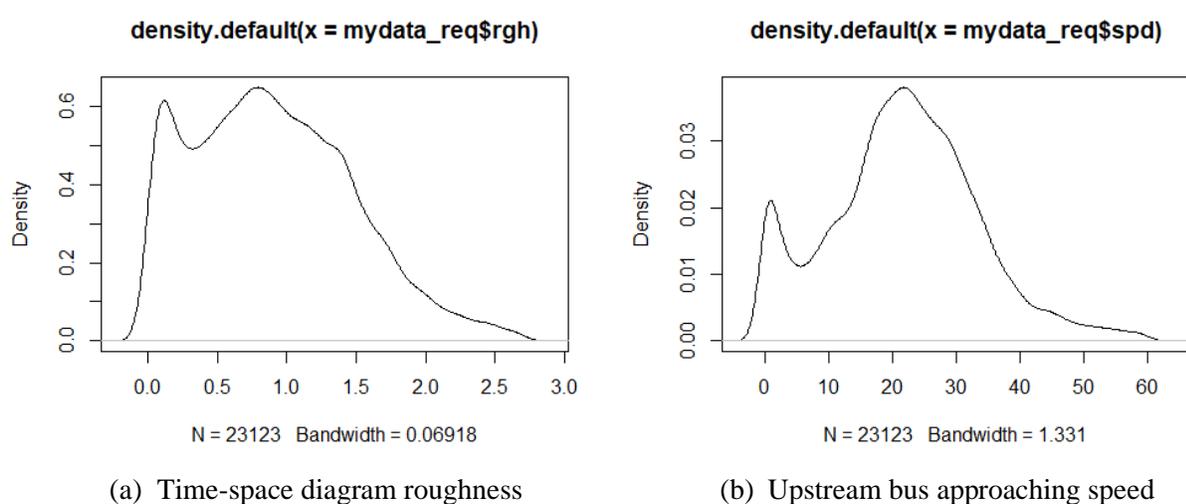


Figure 10 Density distributions of variables roughness and speed

The descriptive statistics of all variables in the final data set are summarized in Table 2. The surrogate safety measure, roughness, is the dependent variable of our interest. After excluding extreme outliers, the minimum value of roughness is 0.00, the maximum value is 2.69, the mean value is 0.90, and the standard deviation is 0.57. The dependent variables are grouped into four categories, bus operations-related variables, weather-related variables, roadway and intersection-related variables, and high-incident intersection indicators. The variable of special interest here is the TSP request indicator. About 8% of the 23,123 events have a bus requesting TSP. The high-incident intersections are those intersections where most of the bus crashes and in-vehicle passenger incidents happened during the before period (April-June 2018). Those intersections are # 62 Emerson Avenue North and Broadway, #67 7th Avenue North and Olson Memorial Highway, and # 73 Chicago Avenue and East Lake Street.

Analytical Methods

A linear regression analysis is carried out to estimate the effect of TSP implementation and TSP request on the movement of buses which is measured by the roughness of buses' time-space diagram. A regression tree and a random forest are modeled to explore the potential relationships between TSP request and other factors that affects the surrogate safety measure at the same time. Importance of variables are compared to understand TSP's influential level comparing with weather and geometric features in affecting bus and passenger safety.

Since the dependent variable, roughness, is continuous, linear regression is selected as an analytical method. Linear regression models the dependent variable as a linear combination of the independent variables:

$$v = X\beta + \varepsilon \quad (2)$$

Where y = a vector of observed values for the dependent variable;

X = an n-dimensional column vector of independent variables, and n = number of observations;

β = a vector of estimated parameters, with β_0 being an intercept term and β_1, β_2, \dots being estimated parameters for independent variables; and

ε = a vector of error terms.

Table 2 Descriptive statistics

Variable	Description	Min	Max	Mean	S.D.
rg	Time-space diagram roughness	0.00	2.69	0.90	0.57
Bus operations					
after	Record from after (2019) period (yes=1, no=0)	0.00	1.00	0.46	0.50
tsp_req	Sent TSP request (yes=1, no=0)	0.00	1.00	0.08	0.27
dir	Direction (southbound=1, northbound=0)	0.00	1.00	0.45	0.50
spd	Upstream bus approaching speed (mph)	0.00	59.89	21.70	11.78
adh	Bus schedule adherence (min)	-60.00	22.00	-3.98	5.01
late	Bus late arrival (if adh \leq -1; yes=1, no=0)	0.00	1.00	0.78	0.41
Weather					
temp	Air temperature ($^{\circ}$ F)	24.00	98.00	66.66	12.22
precip	Precipitation (in)	0.00	1.71	0.01	0.04
humid	Relative humidity (%)	17.00	100.00	55.80	19.87
visib	Visibility (mi)	0.12	10.00	9.75	1.02
wind	Wind speed (mph)	0.00	28.00	9.57	4.85
Roadway and Intersection					
oneway	One-way street (yes=1, no=0)	0.00	1.00	0.14	0.34
nearside	Having a nearside bus stop (yes=1, no=0)	0.00	1.00	0.65	0.48
numlanes	Number of lanes on road segment	1.00	2.00	1.06	0.23
totlane	Number of lanes at intersection approach	1.00	3.00	1.85	0.47
ltlane	Number of left-turn lanes	0.00	1.00	0.50	0.50
rtlane	Number of right-turn lanes	0.00	1.00	0.29	0.45
bulbout	Bulb-out at intersection approach (yes=1, no=0)	0.00	1.00	0.03	0.18
sideparking	Road-side parking within a block (yes=1, no=0)	0.00	1.00	0.85	0.36
rightbike	Right-side bike lane (yes=1, no=0)	0.00	1.00	0.44	0.50
leftbike	Left-side bike lane (yes=1, no=0)	0.00	1.00	0.10	0.30
curbcut	Curb cut (driveway) within a block (yes=1, no=0)	0.00	1.00	0.42	0.49
High-incident Intersection Indicators					
int62	Emerson & Broadway (yes=1, no=0)	0.00	1.00	0.01	0.10
int67	7th & Olson (yes=1, no=0)	0.00	1.00	0.00	0.05
int73	Chicago & Lake (yes=1, no=0)	0.00	1.00	0.08	0.27

Note: Number of observations: 23,123. Min = minimum; Max = maximum; S.D. = standard deviation.

A linear regression model has five assumptions about the generation of observations (19).

- The linear relationship between dependent variable and the set of independent variables;
- The expected value of the error term is zero;
- The error terms all have the same variance and are not correlated;
- The observations on the independent variables can be considered fixed in repeated samples;
- The number of observations is greater than the number of independent variables and there are no exact linear relationships between any pair of the independent variables.

These assumptions are checked through the model diagnostic process. Before conducting linear regression, the correlations between each pair of the 26 variables (1 independent variable + 25 dependent variables) are checked, with no strong correlations (> 0.70) found except for the

correlation between one-way street and left-side bike lane (0.83). That is because all left-side bike lanes are on the one-way road segments in the sample. Therefore, the left-side bike lane variable was dropped from the model since the one-way street variable is of more interest.

Tree-based models are generally used as tools for prediction. Although prediction is not the focus of this study, tree-based models help estimate the relationships between dependent and independent variables, as well as intuitively illustrate the relationship between independent variables and compare their importance in affecting the dependent variable. With these features, tree-based models are selected to provide insights about the mechanism of TSP in affecting bus and passenger safety. Regression tree is a predictive modeling approach using decision tree that splits the observations (i.e., root) about an item with independent variables and reach conclusions (i.e., leaves) about the item's target values. The Classification and Regression Tree (CART) method used in this study was developed by Breiman et al. (20). When growing a regression tree, the observations are split recursively, following a rule that measures the “goodness of split” for every split of the tree. To split the feature space (i.e., to select the proper variable and the appropriate threshold to partition the space), a search over every independent variable and every possible threshold takes place, aiming at minimizing a specified cost. For regression tree, such a cost is defined as the sum of squared errors, S :

$$S = \sum_{c \in \text{leaves}(T)} \sum_{i \in c} (y_i - m_c)^2 \quad (3)$$

Where y_i is an observation on leaf c ; and

$$m_c = \frac{1}{n} \sum_{i \in c} y_i \text{ is the prediction on leaf } c.$$

A random forest is an ensemble of trees that are created with bootstrapped (randomly selected with replacements) samples. A subset of independent variables is randomly selected for splitting, allowing for assessments of the importance of variables. Each tree is grown to the largest extent possible. The algorithm creates many trees to form a “forest.” The forest error rate depends on the correlation between any two trees in the forest (as correlation increases, error increases), and the strength of each individual tree in the forest (as strength increases, error decreases). The final prediction is made by averaging across all trees. (21)

Chapter V: Results

In this section, the linear regression model and tree-based model results are summarized. Insights of the magnitude and mechanism of TSP's effects on the surrogate safety measure are also discussed.

Linear Regression Models

Two linear regression models are developed, and the results are summarized in Table 3. The linear regression models are developed to estimate the relationships between the surrogate safety measure, roughness of bus time-space diagram, to the variables of interest covering aspects including bus operations, weather, roadway and intersection geometries, and high-incident intersections. The relationship between TSP request and bus time-space diagram roughness is of the most interest.

Model 1 includes all independent 22 variables of interest. Model 2 includes 17 variables of interest, excluding the 5 weather-related variables. All 23,123 observations in the final data set are used for the modeling. Both models were significant (Model 1: $F(22, 23,100) = 508.5$, $p < 0.000$, $R^2 = 0.326$; Model 2: $F(17, 23,105) = 652.7$, $p < 0.000$, $R^2 = 0.324$).

From both models, it is found that roughness is significantly associated with TSP request ($p < 0.01$). Roughness is a measure calculated from the time-space diagram of a bus, with no unit. TSP request is coded as 1 = yes and 0 = no. With TSP requested, the bus events' roughness reduces about 0.04, comparing with the bus events where TSP is not requested. It means that with TSP requested, a bus approaches and clears a signalized intersection more smoothly than without TSP requested.

Coefficient estimates of other variables also provide insights about other factors affecting the surrogate safety measure. The overall bus movement roughness did not change significantly after TSP was implemented, comparing with before TSP was implemented ($\beta_{\text{after}} = 0.008$, $p < 0.001$). Southbound buses had rougher trips than northbound buses ($\beta_{\text{dir}} = 0.119$, $p < 0.001$). A higher approaching speed is associated with a smoother bus movement ($\beta_{\text{spd}} = -0.014$, $p < 0.001$). If a bus is late, it experiences a rougher movement ($\beta_{\text{late}} = 0.132$, $p < 0.001$). The relative humidity is associated with the roughness significantly, but only have a very slight impact ($\beta_{\text{humid}} = -0.001$, $p < 0.001$). Bus movement roughness is also significantly related to roadway and intersection geometric features ($p < 0.001$). One-way street, more lanes, the existence of a turn lane, and road-side parking are associated with a rougher bus movement through a signalized intersection. A near-side bus stop, sidewalk bulb-out, right-side bike lane, and curb cuts are associated with a smoother bus movement. With these geometric features, buses tend to not make dramatic changes in their movements. The intersections with higher numbers of bus-involved crashes and passenger incidents are all significantly associated with rougher bus movements ($\beta_{\text{int62}} = 0.094$, $p < 0.01$; $\beta_{\text{int67}} = 0.646$, $p < 0.001$; $\beta_{\text{int73}} = 0.550$, $p < 0.001$), comparing with other intersections.

Table 3 Linear regression models

Variable	Model 1				Model 2			
	Estimate	S.E.	t-value	Significance	Estimate	S.E.	t-value	Significance
(Intercept)	1.016	0.053	19.33	***	1.058	0.026	40.75	***
after	0.008	0.007	1.13		0.003	0.007	0.41	
tsp_req	-0.039	0.012	-3.19	**	-0.038	0.012	-3.11	**
dir	0.119	0.008	15.22	***	0.119	0.008	15.45	***
spd	-0.014	0.000	-48.90	***	-0.014	0.000	-49.13	***
late	0.132	0.008	16.84	***	0.131	0.008	16.79	***
temp	0.001	0.000	1.75	.				
precip	0.020	0.082	0.24					
humid	-0.001	0.000	-5.16	***				
visib	0.005	0.004	1.29					
wind	0.001	0.001	2.13	*				
oneway	0.058	0.013	4.41	***	0.056	0.013	4.26	***
nearside	-0.289	0.010	-29.32	***	-0.288	0.010	-29.22	***
numlanes	0.143	0.016	8.89	***	0.145	0.016	9.01	***
ltlane	0.050	0.010	4.85	***	0.048	0.010	4.68	***
rtlane	0.069	0.014	5.02	***	0.068	0.014	4.90	***
bulbout	-0.442	0.021	-21.11	***	-0.442	0.021	-21.07	***
sideparking	0.104	0.014	7.49	***	0.104	0.014	7.54	***
rightbike	-0.201	0.010	-19.48	***	-0.200	0.010	-19.37	***
curbcut	-0.119	0.007	-16.42	***	-0.119	0.007	-16.29	***
int62	0.094	0.035	2.69	**	0.099	0.035	2.83	**
int67	0.646	0.070	9.29	***	0.648	0.070	9.31	***
int73	0.550	0.016	33.44	***	0.550	0.016	33.41	***
R-squared				0.326				0.324
Adjusted R-squared				0.326				0.324
F-statistic				F(22, 23,100) = 508.5***				F(17, 23,105) = 652.7***

Note: S.E. = standard error.

Significance codes: “***” 0.001; “**” 0.01; “*” 0.05; “.” 0.1; “ ” 1.

The diagnostics of Model 1 and Model 2 are supported by the plots shown in Figure 11 and Figure 12, respectively. There are four plots in each diagnostic set, the residuals v.s. fitted plot, the normal Q-Q plot, the scale-location plot, and the residual v.s. leverage plot. The four plots are used to check the assumptions about linearity, residual normality, residual homoscedasticity, and extreme and influential cases (22).

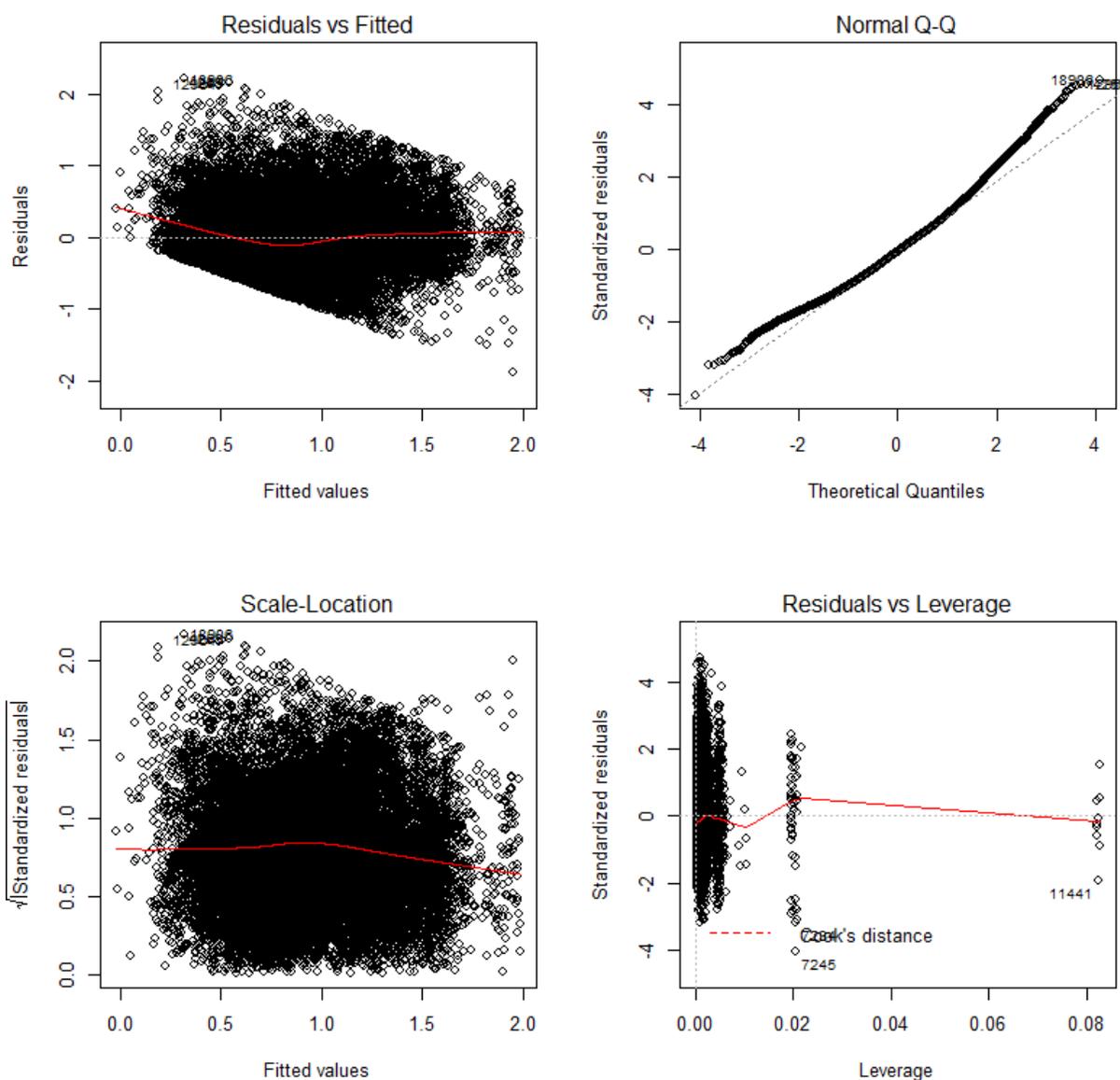


Figure 11 Linear regression Model 1 diagnostic

The residual plot in Figure 11 shows no obvious patterns between residuals and fitted values from Model 1. Thus, the assumption about linear relationship between the dependent variable and the independent variables is supported. The Q-Q plot shows that the normal probability plot of residuals approximately follows the reference line, confirming the residual normality assumption. The scale-location plot shows that the variances of the residual points did not change much when the value of the fitted value changes. Therefore, the plot supports the residual homoscedasticity assumption, that the error terms all have the same variance and are not correlated. The residual v.s. leverage plot shows that some points have larger standardized residuals and there are some points with higher leverage values, but with a large sample size these are expected. No influential points are shown on the plot, with no Cook's distance lines

shown. Therefore, the diagnostic for possible outliers and high-leverage points is acceptable. Similar diagnostic conclusions are drawn for Model 2, according to the four plots in Figure 12, which present similar patterns with Model 1 diagnostic plots.

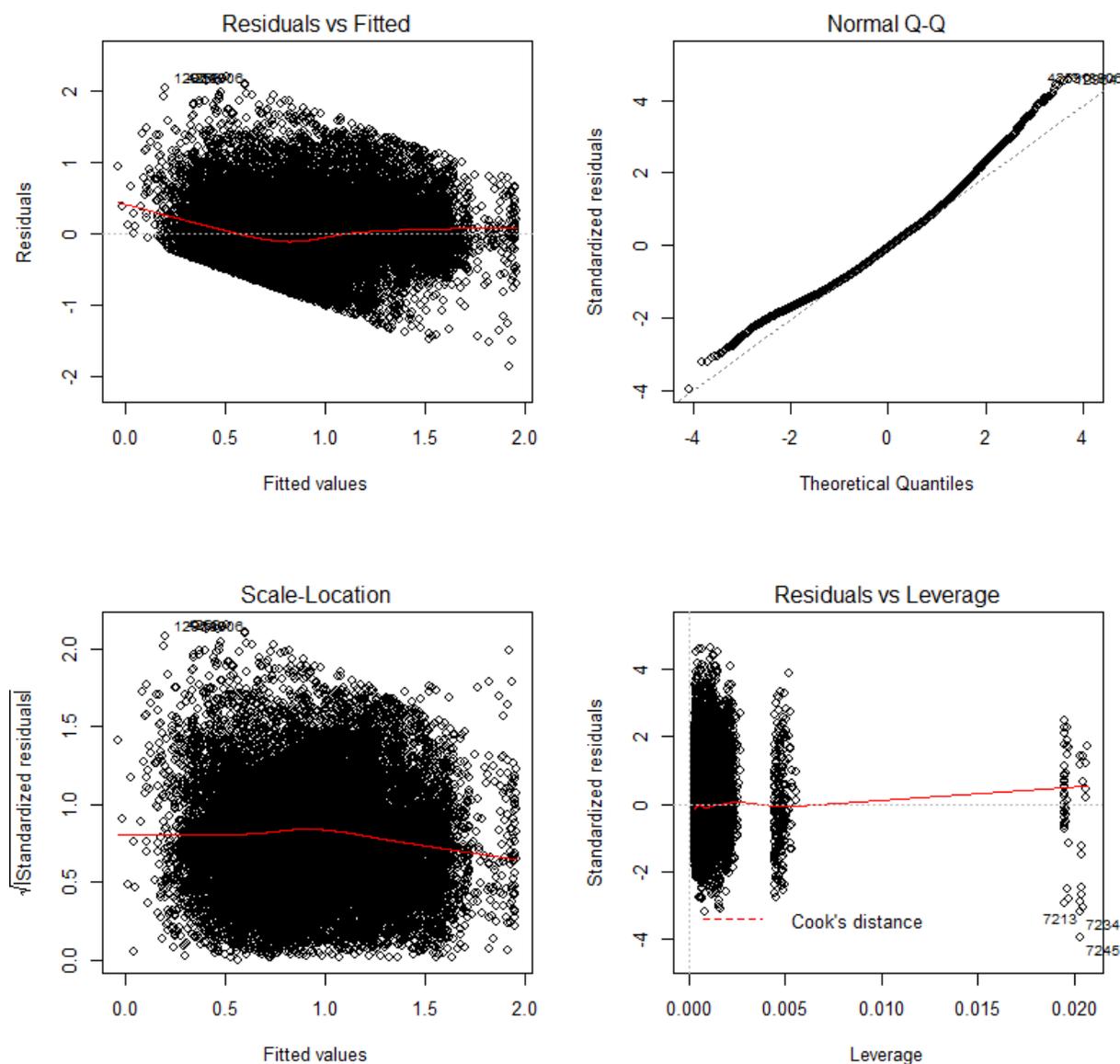


Figure 12 Linear regression Model 2 diagnostic

Tree-based Models

Tree-based models are estimated with hope to obtain insights about the interactive relationship between independent variables and how big of a role each of the independent variables plays in affecting bus movement roughness. A regression tree model is developed to intuitively show the structure of interactive relationships between variables. A full tree is grown

without any pruning to explore how many variables can be used to split the 23,123 observations until there is no significant improvement in minimizing the impurity cost. As seen in Figure 13, a six-level tree is grown. This is the full tree that selected the most suitable independent variables to reach an optimal overall cost. The most interested variable, TSP request, is not included in the tree, which implies that TSP request does not have an influence on the bus movement roughness as big as the variables that are included in the tree. Therefore, finding the interactions between TSP request and other independent variables was not possible from the regression tree model. However, the tree model still provides some insights about the mechanism. The tree shows that some roadway and intersection geometric attributes (near-side bus stop, existence of turn lane, right-side bike lane, and one-way street) and bus approaching speed are important factors affecting the roughness of bus movements. Also, variables such as near-side bus stop and bus approaching speed are not simply affecting bus movement roughness directly, as what might be interpreted from results of the linear regression models. The confounding relationships shown in the tree model reflect that there exists complexity in the process of these factors affecting the outcome variable, bus movement roughness.

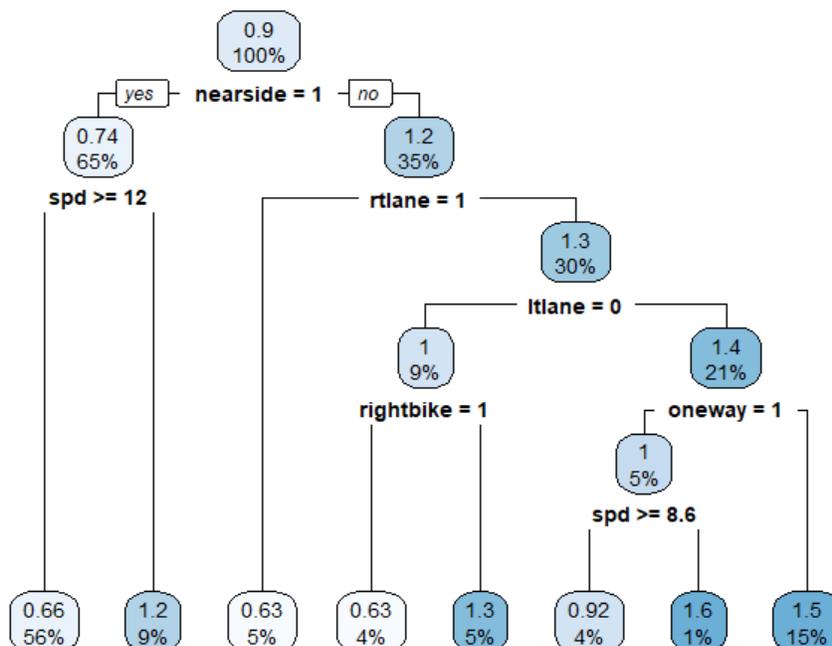


Figure 13 Regression tree for roughness

The random forest model creates hundreds of trees with randomly sampled observations and a randomly selected set of variables. A visualized structure of tree is not provided by the random forest model, but a variable importance comparison is provided to show the power of the variables considered in affecting the outcome variable. Figure 14 (a) shows the number of trees estimated v.s. the model prediction error. With around 490 trees, the model reached minimum error. Figure 14 (b) and (c) present the variable importance ranked by the increase in two types of error (the mean squared error and the node purity/impurity), respectively. Comparing with speed, roadway geometry, and weather factors, TSP request has the lowest power in affecting the roughness of bus movements through the studied intersections.

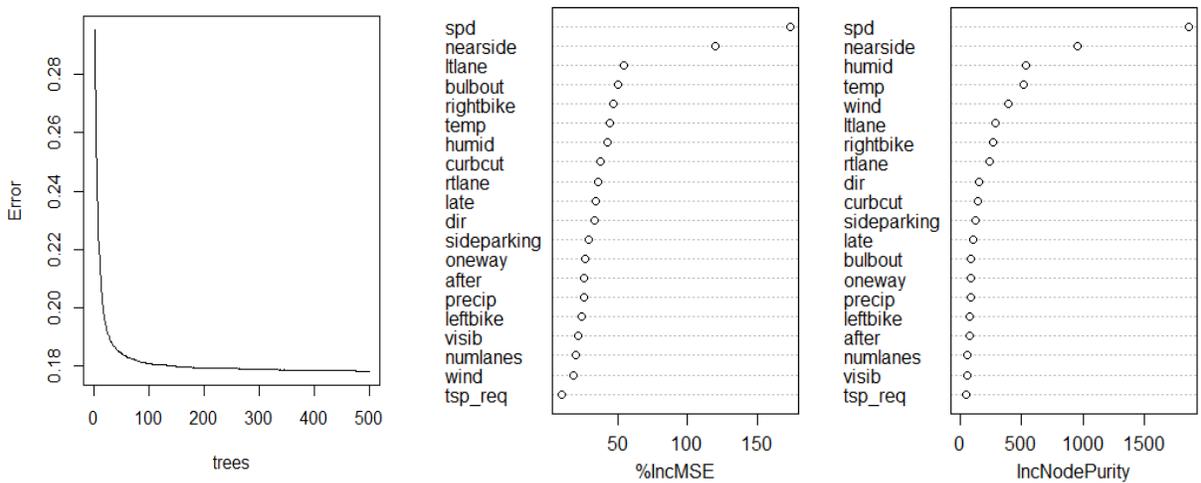


Figure 14 Random forest results

Chapter VI: Conclusions

With the objective of evaluating the effects of TSP, implemented on bus Route 5 of Minneapolis-St. Paul Metro Transit, this study was carried out. An empirical before-after comparison of bus-related crashes and in-vehicle passenger incidents showed a reduction in types of crashes/incidents that are considered potentially associated to traffic signal timing and TSP. With this knowledge in mind, and a concern about limited insights from modeling with a very small sample of crashes/incidents, statistical analysis of a surrogate safety measure was conducted. The roughness of bus movement in an event of it approaching and clearing a signalized intersection is the surrogate measure used in this study to reflect safety risks.

Linear regression and tree-based models including regression trees and random forests were used to evaluate the magnitude and mechanism of TSP request in affecting bus safety. From the modeling results it was found that TSP request is significantly associated with smoother bus movements through signalized intersections. With TSP requested, Route 5 buses go through the intersections more smoothly without many sudden accelerations, deceleration, and stops. As a smoother bus movement is closely linked to a safer and more comfortable bus trip. The cumulative benefit from safer bus trips is a safer bus corridor. Therefore, this finding is consistent with what was found by most of the previous studies on TSP's safety effects, that the implementation of TSP helps to improve bus corridors' safety performance.

Although TSP request is shown to significantly smoothen bus movements, there are other factors that were considered in the analysis showing more power in affecting the roughness of bus movements. The random forest model ranked bus approaching speed, roadway and intersection geometric features, and weather-related factors higher than TSP request, in terms of variable importance. Also, the regression tree did not pick TSP request as a variable to split the observation data set. Therefore, from the tree-based models, we can only get the insight that TSP requests' effect on bus safety is slight, but still not clear about its interaction with other confounding variables. It is possible that the closer confounding factors of TSP request were not included in this analysis, due to data availability. As TSP request records are used to distinguish the buses that may benefit from TSP, we were not sure about which buses were granted TSP. Signal activity logs are not stored for long periods as we needed for this analysis. Therefore, there exists a probability of buses requested TSP but were not granted TSP by the signal controllers, so TSP's effect may be slightly under-estimated in this analysis. The slightness of TSP's effect on bus safety is expected. As in previous studies, the effect of TSP on crash numbers was also estimated to be small, with some results not significant. Also, the effects of TSP estimated in this study were on a surrogate safety measure. There are other possible ways that TSP may affect some aspects of bus safety.

Two implications from this study, for transit agencies, are: 1) TSP does not harm bus safety but improves bus safety, thus, this technology should be implemented when needed without too much concerns in terms of safety. 2) Bus operational and roadway/intersection geometric factors also affect bus safety significantly, thus, their effects and intertwined relationships need to be evaluated to come up with an integrated operational and geometric design improvements to obtain the optimal safety outcome.

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