
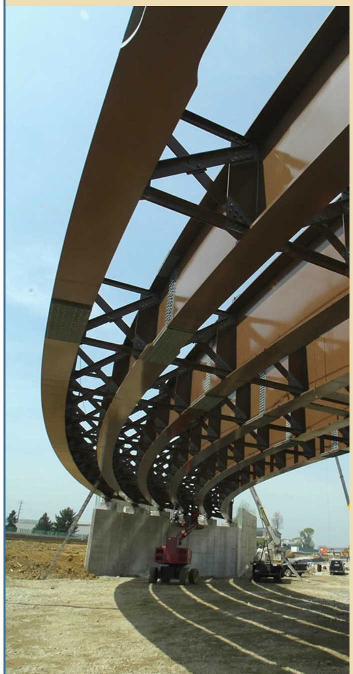

U.S. Department of Transportation
Federal Highway Administration

Steel Bridge Design Handbook

Design Example 2A: Two-Span Continuous Straight Composite Steel I-Girder Bridge

Publication No. FHWA-IF-12-052 - Vol. 21

November 2012

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16. Abstract

The purpose of this example is to illustrate the use of the AASHTO LRFD Bridge Design for the design of a continuous two span steel I-girder bridge. The design process and corresponding calculations for steel I-girders are the focus of this example, with particular emphasis placed on illustration of the optional moment redistribution procedures. All aspects of the girder design are presented, including evaluation of the following: cross-section proportion limits, constructibility, serviceability, fatigue, and strength requirements. Additionally, the weld design for the web-to-flange joint of the plate girders is demonstrated along with all applicable components of the stiffener design and cross frame member design.

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FOREWORD

It took an act of Congress to provide funding for the development of this comprehensive handbook in steel bridge design. This handbook covers a full range of topics and design examples to provide bridge engineers with the information needed to make knowledgeable decisions regarding the selection, design, fabrication, and construction of steel bridges. The handbook is based on the Fifth Edition, including the 2010 Interims, of the AASHTO LRFD Bridge Design Specifications. The hard work of the National Steel Bridge Alliance (NSBA) and prime consultant, HDR Engineering and their sub-consultants in producing this handbook is gratefully acknowledged. This is the culmination of seven years of effort beginning in 2005.

The new *Steel Bridge Design Handbook* is divided into several topics and design examples as follows:

- Bridge Steels and Their Properties
- Bridge Fabrication
- Steel Bridge Shop Drawings
- Structural Behavior
- Selecting the Right Bridge Type
- Stringer Bridges
- Loads and Combinations
- Structural Analysis
- Redundancy
- Limit States
- Design for Constructibility
- Design for Fatigue
- Bracing System Design
- Splice Design
- Bearings
- Substructure Design
- Deck Design
- Load Rating
- Corrosion Protection of Bridges
- Design Example: Three-span Continuous Straight I-Girder Bridge
- Design Example: Two-span Continuous Straight I-Girder Bridge
- Design Example: Two-span Continuous Straight Wide-Flange Beam Bridge
- Design Example: Three-span Continuous Straight Tub-Girder Bridge
- Design Example: Three-span Continuous Curved I-Girder Beam Bridge
- Design Example: Three-span Continuous Curved Tub-Girder Bridge

These topics and design examples are published separately for ease of use, and available for free download at the NSBA and FHWA websites: http://www.steelbridges.org, and http://www.fhwa.dot.gov/bridge, respectively.

The contributions and constructive review comments during the preparation of the handbook from many engineering processionals are very much appreciated. The readers are encouraged to submit ideas and suggestions for enhancements of future edition of the handbook to Myint Lwin at the following address: Federal Highway Administration, 1200 New Jersey Avenue, S.E., Washington, DC 20590.

M. Myint Lwin, Director
Office of Bridge Technology

1.0 INTRODUCTION

The purpose of this example is to illustrate the use of the Fifth Edition of the *AASHTO LRFD Bridge Design Specifications* [1], referred to herein as *AASHTO LRFD (5th Edition, 2010)* for the design of a continuous steel I-girder bridge. The design process and corresponding calculations for steel I-girders are the focus of this example, with particular emphasis placed on illustration of the optional moment redistribution procedures. All aspects of the girder design are presented, including evaluation of the following: cross-section proportion limits, constructibility, serviceability, fatigue, and strength requirements. Additionally, the weld design for the web-to-flange joint of the plate girders is demonstrated along with all applicable components of the stiffener design and lateral bracing design.

The moment redistribution procedures allow for a limited degree of yielding at the interior supports of continuous-span girders. The subsequent redistribution of moment results in a decrease in the negative bending moments and a corresponding increase in positive bending moments. The current moment redistribution procedures utilize the same moment envelopes as used in a conventional elastic analysis and do not require the use of iterative procedures or simultaneous equations. The method is similar to the optional provisions in previous AASHTO specifications that permitted the peak negative bending moments to be decreased by 10% before performing strength checks of the girder. However, in the present method this empirical percentage is replaced by a calculated quantity, which is a function of geometric and material properties of the girder. Furthermore, the range of girders for which moment redistribution is applicable is expanded compared to previous editions of the specifications, in that girders with slender webs may now be considered. The result of the use of these procedures is considerable economical savings. Specifically, inelastic design procedures may offer cost savings by (1) requiring smaller girder sizes, (2) eliminating the need for cover plates (which have unfavorable fatigue characteristics) in rolled beams, and (3) reducing the number of flange transitions without increasing the amount of material required in plate girder designs, leading to both material and, more significantly, fabrication cost savings.

2.0 DESIGN PARAMETERS

The bridge cross-section for the tangent, two-span (90 ft - 90 ft) continuous bridge under consideration is given below in Figure 1. The example bridge has four plate girders spaced at 10.0 ft and 3.5 ft overhangs. The roadway width is 34.0 ft and is centered over the girders. The reinforced concrete deck is 8.5 inch thick, including a 0.5 inch integral wearing surface, and has a 2.0 inch haunch thickness.

The framing plan for this design example is shown in Figure 2. As will be demonstrated subsequently, the cross frame spacing is governed by constuctibility requirements in positive bending and by moment redistribution requirements in negative bending.

The structural steel is ASTM A709, Grade 50W, and the concrete is normal weight with a compressive strength of 4.0 ksi. The concrete slab is reinforced with nominal Grade 60 reinforcing steel.

The design specifications are the *AASHTO LRFD (5th Edition, 2010) Bridge Design Specifications*. Unless stated otherwise, the specific articles, sections, and equations referenced throughout this example are contained in these specifications.

The girder design presented herein is based on the premise of providing the same girder design for both the interior and exterior girders. Thus, the design satisfies the requirements for both interior and exterior girders. Additionally, the girders are designed assuming composite action with the concrete slab.

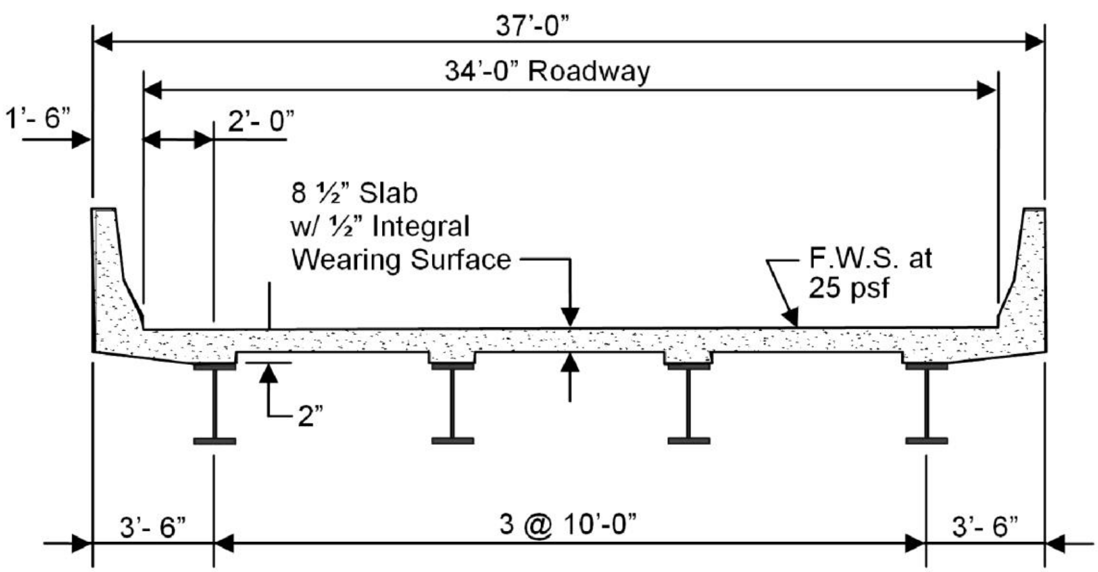

Figure 1 Sketch of the Typical Bridge Cross Section

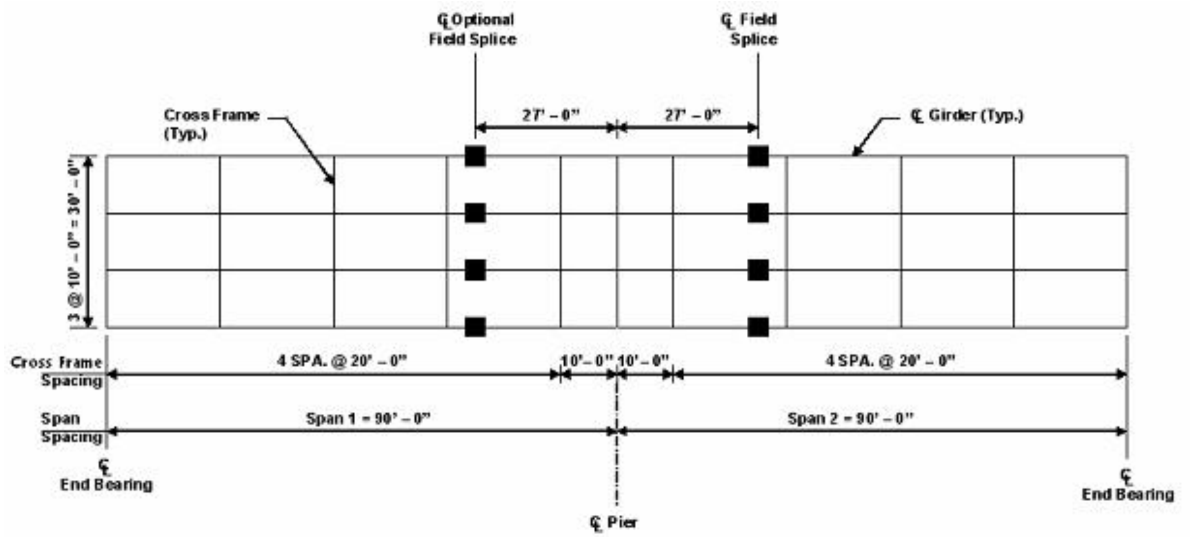

Figure 2 Sketch of the Superstructure Framing Plan

3.0 GIRDER GEOMETRY

The girder elevation is shown in Figure 3. As shown in Figure 3, section transitions are provided at 30% of the span length (27 feet) from the interior pier. The design of the girder from the abutment to 63 feet from the abutment is primarily based on positive bending moments; thus, this section of the girder is referred to as either the "positive bending region" or "Section 1" throughout this example. Alternatively, the girder geometry at the pier is controlled by negative bending moments; consequently the region of the girder extending from 0 to 27 feet on each side of the pier will be referred to as the "negative bending region" or "Section 2". The rationale used to develop the cross-sectional geometry of these sections and a demonstration that this geometry satisfies the cross-section proportion limits specified in Article 6.10.2 is presented herein.

3.1 Web Depth

Selection of appropriate web depth has a significant influence on girder geometry. Thus, initial consideration should be given to the most appropriate web depth. In the absence of other criteria the span-to-depth ratios given in Article 2.5.2.6.3 may be used as a starting point for selecting a web depth. As provided in Table 2.5.2.6.3-1, the minimum depth of the steel I-beam portion of a continuous-span composite section is 0.027L, where L is the span length. Thus, the minimum steel depth is computed as follows.

$$0.027(90\text{ ft})(12\text{ in./ft}) = 29.2\text{ inches}$$

Preliminary designs were evaluated for five different web depths satisfying the above requirement. These web depths varied between 36 inches and 46 inches and in all cases girder weight decreased as web depth increased. However, the decrease in girder weight became much less significant for web depths greater than 42 inches.

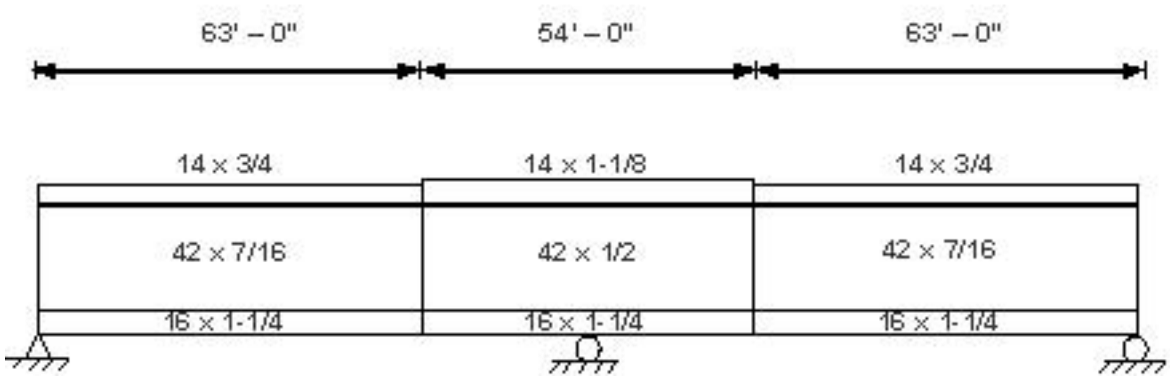

Figure 3 Sketch of the Girder Elevation

3.2 Web Thickness

The thickness of the web was selected to satisfy shear requirements at the strength limit state without the need for transverse stiffeners. This resulted in a required web thickness of 0.5 inch at the pier and 0.4375 inch at the abutments. The designer may also want to examine the economy of using a constant 0.5 inch web throughout.

In developing the preliminary cross-section it should also be verified that the selected dimensions satisfy the cross-section proportion limits required in Article 6.10.2. The required web proportions are given in Article 6.10.2.1 where, for webs without longitudinal stiffeners, the web slenderness is limited to a maximum value of 150.

$$\frac{D}{t_w} \leq 150 \qquad \text{Eq. (6.10.2.1.1-1)}$$

Thus, the following calculations demonstrate that Eq. 6.10.2.1.1-1 is satisfied for both the positive and negative moment regions of the girder, respectively.

$$\frac{D}{t_w} = \frac{42}{0.4375} = 96 \leq 150 \qquad \text{(satisfied)}$$

$$\frac{D}{t_w} = \frac{42}{0.5} = 84 \leq 150 \qquad \text{(satisfied)}$$

3.3 Flange Geometries

The width of the compression flange in the positive bending region was controlled by constructability requirements as the flange lateral bending stresses are directly related to the section modulus of the flange about the y-axis of the girder as well as the lateral bracing distance. Various lateral bracing distances were investigated and the corresponding flange width required to satisfy constructability requirements for each case was determined. Based on these efforts it was determined that a minimum flange width of 14 in. was needed to avoid the use of additional cross-frames. Thus, this minimum width was used for the top flanges.

All other plate sizes were iteratively selected to satisfy all applicable requirements while producing the most economical girder design possible. The resulting girder dimensions are illustrated in Figure 3.

Article 6.10.2.2 specifies four flange proportions limits that must be satisfied. The first of these is intended to prevent the flange from excessively distorting when welded to the web of the girder during fabrication.

$$\frac{b_f}{2t_f} \leq 12.0 \qquad \text{Eq. (6.10.2.2-1)}$$

Evaluation of Eq. 6.10.2.2-1 for each of the three flange sizes used in the example girder is demonstrated below.

$$\frac{b_f}{2t_f} = \frac{14}{2(0.75)} = 9.33 \le 12.0 \qquad \text{(satisfied)}$$

$$\frac{b_f}{2t_f} = \frac{14}{2(1.125)} = 6.22 \le 12.0 \qquad \text{(satisfied)}$$

$$\frac{b_f}{2t_f} = \frac{14}{2(1.25)} = 6.4 \le 12.0 \qquad \text{(satisfied)}$$

The second flange proportion limit that must be satisfied corresponds to the relationship between the flange width and the web depth. The ratio of the web depth to the flange width significantly influences the flexural capacity of the member and is limited to a maximum of 6, which is the maximum value for which the moment capacity prediction equations for steel I-girders are proven to be valid.

$$b_f \ge \frac{D}{6} = \frac{42}{6} = 7.0 \qquad \text{Eq. (6.10.2.2-2)}$$

It is shown below that Eq. 6.10.2.2-2 is satisfied for both flange widths utilized in this design example.

$b_f = 14.0$ inch (satisfied)
$b_f = 16.0$ inch (satisfied)

Equation 3 of Article 6.10.2.2 limits the thickness of the flange to a minimum of 1.1 times the web thickness. This requirement is necessary to ensure that some web shear buckling restraint is provided by the flanges, and that the boundary conditions at the web-flange junction assumed in the development of the web-bend buckling and flange local buckling are sufficiently accurate.

$$t_f \ge 1.1\, t_w \qquad \text{Eq. (6.10.2.2-3)}$$

Evaluation of Eq. 6.10.2.2-3 for the minimum flange thickness used in combination with each of the web thicknesses utilized in the example girder is demonstrated below.

$$t_f = t_{f\text{-min}} = 0.75 \ge 1.1(0.4375) = 0.48 \qquad \text{(satisfied)}$$

$$t_f = t_{f\text{-min}} = 1.125 \ge 1.1(0.5) = 0.55 \qquad \text{(satisfied)}$$

Equation 6.10.2.2-4 sets limits for designed sections similar to the previsions of previous specifications. This provision prevents the use of extremely mono symmetric sections ensuring more efficient flange proportions and results in a girder section suitable for handling during erection.

$$0.1 \le \frac{I_{yc}}{I_{yt}} \le 10 \qquad \text{Eq. (6.10.2.2-4)}$$

where: I_{yc} = moment of inertia of the compression flange of the steel section about the vertical axis in the plane of the web (in.^{4})

I_{yt} = moment of inertia of the tension flange of the steel section about the vertical axis in the plane of the web (in.^{4})

Computing the ratio between the top and bottom flanges for the positive and negative bending regions, respectively, shows that this requirement is satisfied for the design girder.

$$0.1 \le \frac{(0.75)(14)^3/12}{(1.25)(16)^3/12} = \frac{171.5}{426.7} = 0.40 \le 10 \qquad \text{(satisfied)}$$

$$0.1 \le \frac{(1.125)(14)^3/12}{(1.25)(16)^3/12} = \frac{257.25}{426.7} = 0.60 \le 10 \qquad \text{(satisfied)}$$

4.0 LOADS

This example considers all applicable loads acting on the super-structure including dead loads, live loads, and wind loads as discussed below. In determining the effects of each of these loads, the approximate methods of analysis specified in Article 4.6.2 are implemented.

4.1 Dead Loads

The dead load, according to Article 3.5.1, is to include the weight of all components of the structure, appurtenances and utilities, earth cover, wearing surface, future overlays, and planned widening. Dead loads are divided into three categories: dead load of structural components and non-structural attachments (DC) and the dead load of wearing surface and utilities (DW). Alternative load factors are specified for each of these three categories of dead load depending on the load combination under consideration.

4.1.1 Component And Attachment Dead Load (DC)

For composite girders consideration is given to the fact that not all loads are applied to the composite section and the DC dead load is separated into two parts: the dead load acting on the section before the concrete deck is hardened or made composite (DC1), and the dead load acting on the composite section (DC2). DC1 is assumed to be carried by the steel section alone. DC2 is assumed to be carried by the long-term composite section. In the positive bending region the long-term composite section is comprised of the steel girder and an effective width of the concrete slab. Formulas are given in the specifications to determine the effective slab width over which a uniform stress distribution may be assumed. The effective width of the concrete slab is transformed into an equivalent area of steel by multiplying the width by the ratio between the steel modulus and one-third the concrete modulus, which is typically referred to as a modular ratio of 3n. The reduced concrete modulus is intended to account for the effects of concrete creep. In the negative bending region the composite section is comprised of the steel section and the longitudinal steel reinforcing within the effective width of the slab.

DC1 includes the girder self weight, weight of concrete slab (including the haunch and overhang taper), deck forms, cross frames, and stiffeners. The unit weight for steel (0.490 k/ft^3) used in this example is taken from Table 3.5.1-1, which provides approximate unit weights of various materials. Table 3.5.1-1 also lists the unit weight of normal weight concrete as 0.145 k/ft^3; the concrete unit weight is increased to 0.150 k/ft^3 in this example to account for the additional weight of the steel reinforcement within the concrete. The dead load of the stay-in-place forms is assumed to be 15 psf. To account for the dead load of the cross-frames, stiffeners and other miscellaneous steel details a dead load of 0.015 k/ft. is assumed. It is also assumed that these dead loads are equally distributed to all girders as permitted by Article 4.6.2.2.1 for the line-girder type of analysis implemented herein. Thus, the total DC1 loads used in this design are as computed below.

Slab = (8.5/12) x (37) x (0.150)/4	= 0.983 k/ft
Haunch (average wt/length)	= 0.017 k/ft
Overhang taper = 2 x (1/2) x [3.5-(7/12)] x (2/12) x 0.150/4	= 0.018 k/ft
Girder (average wt/length)	= 0.174 k/ft
Cross-frames and misc. steel	= 0.015 k/ft
Stay-in-place forms = 0.015 x (30-3 x (12/12))/4	= 0.101 k/ft
Total DC1	=1.308 k/ft

DC2 is composed of the weight from the barriers, medians, and sidewalks. No sidewalks or medians are present in this example and thus the DC2 weight is equal to the barrier weight alone. The parapet weight is assumed to be equal to 520 lb/ft. Article 4.6.2.2.1 specifies that when approximate methods of analysis are applied DC2 may be equally distributed to all girders or a larger proportion of the concrete barriers may be applied to the exterior girder. In this example, the barrier weight is equally distributed to all girders, resulting in the DC2 loads computed below.

Barriers = (0.520 x 2)/4 = 0.260 k/ft

DC2 = 0.260 k/ft

4.1.2 Wearing Surface Dead Load (DW)

Similar to the DC2 loads, the dead load of the future wearing surface is applied to the long-term composite section and is assumed to be equally distributed to each girder. A future wearing surface with a dead load of 25 psf is assumed. Multiplying this unit weight by the roadway width and dividing by the number of girders gives the following.

Wearing surface = (0.025) x (34)/4 = 0.213 k/ft

DW = 0.213 k/ft

4.2 Vehicular Live Loads

The *AASHTO LRFD (5th Edition, 2010)* Specifications consider live loads to consist of gravity loads, wheel load impact (dynamic load allowance), braking forces, centrifugal forces, and vehicular collision forces. Live loads are applied to the short-term composite section. In positive bending regions, the short-term composite section is comprised of the steel girder and the effective width of the concrete slab, which is converted into an equivalent area of steel by multiplying the width by the modular ratio between steel and concrete. In other words, a modular ratio of n is used for short-term loads where creep effects are not relevant. In negative bending regions the short-term composite section consists of the steel girder and the longitudinal reinforcing steel, except for live-load deflection and fatigue requirements in which the concrete deck may be considered in both negative and positive bending.

4.2.1 General Vehicular Live Load (Article 3.6.1.2)

The *AASHTO LRFD (5th Edition, 2010)* vehicular live loading is designated as the HL-93 loading and is a combination of the design truck or tandem plus the design lane load. The design truck, specified in Article 3.6.1.2.2, is composed of an 8-kip lead axle spaced 14 feet from the closer of two 32-kip rear axles, which have a variable axle spacing of 14 feet to 30 feet. The transverse spacing of the wheels is 6 feet. The design truck occupies a 10 feet lane width and is positioned within the design lane to produce the maximum force effects, but may be no closer than 2 feet from the edge of the design lane, except for in the design of the deck overhang.

The design tandem, specified in Article 3.6.1.2.3, is composed of a pair of 25-kip axles spaced 4 feet apart. The transverse spacing of the wheels is 6 feet.

The design lane load is discussed in Article 3.6.1.2.4 and has a magnitude of 0.64 klf uniformly distributed in the longitudinal direction. In the transverse direction, the load occupies a 10 foot width. The lane load is positioned to produce extreme force effects, and therefore, need not be applied continuously.

For both negative moments between points of contraflexure and interior pier reactions a special loading is used. The loading consists of two design trucks (as described above but with the magnitude of 90% the axle weights) in addition to the lane loading. The trucks must have a minimum headway of 50 feet between the two loads. The live load moments between the points of dead load contraflexure are to be taken as the larger of the HL-93 loading or the special negative loading.

Live load shears are to be calculated only from the HL-93 loading, except for interior pier reactions, which are the larger of the HL-93 loading or the special negative loading.

The dynamic load allowance, which accounts for the dynamic effects of force amplification, is only applied to the truck portion of the live loading, and not the lane load. For the strength and service limit states, the dynamic load allowance is taken as 33 percent, and for the fatigue limit state, the dynamic load allowance is taken as 15 percent.

4.2.2 Optional Live Load Deflection Load (Article 3.6.1.3.2)

The loading for the optional live load deflection criterion consists of the greater of the design truck, or 25 percent of the design truck plus the lane load. A dynamic load allowance of 33 percent applies to the truck portions (axle weights) of these load cases. During this check, all design lanes are to be loaded, and the assumption is made that all components deflect equally.

4.2.3 Fatigue Load (Article 3.6.1.4)

For checking the fatigue limit state, a single design truck with a constant rear axle spacing of 30 feet is applied.

4.3 Wind Loads

Article 3.8.1.2 discusses the design horizontal wind pressure, P_D, which is used to determine the wind load on the structure. The wind pressure is computed as follows:

$$P_D = P_B \frac{V_{DZ}^2}{10,000} \qquad \text{Eq. (3.8.1.2.1-1)}$$

where: P_B = base wind pressure of 0.050 ksf for beams (Table 3.8.1.2.1-1)
V_{DZ} = design wind velocity at design elevation, Z (mph)

In this example it is assumed the superstructure is less than 30 feet above the ground, at which the wind velocity is prescribed to equal 100 mph, which is designated as the base wind velocity, V_B. With V_{DZ} equal to the base wind velocity of 100 in Eq. 3.8.1.2.1-1 the horizontal wind pressure, P_D, is determined as follows.

$$P_D = 0.050 \frac{100^2}{10,000} = 0.050 \text{ ksf}$$

4.4 Load Combinations

The specifications define four limit states: the service limit state, the fatigue and fracture limit state, the strength limit state, and the extreme event limit state. The subsequent sections discuss each limit state in more detail; however for all limit states the following general equation from Article 1.3.2.1 must be satisfied, where different combinations of loads (i.e., dead load, wind load) are specified for each limit state.

$$\eta_D \eta_R \eta_I \Sigma \gamma_i Q_i \leq \varphi R_n R_r$$

where:

η_D = Ductility factor (Article 1.3.3)
η_R = Redundancy factor (Article 1.3.4)
η_I = Operational importance factor (Article 1.3.5)
γ_i = Load factor
Q_i = Force effect
φ = Resistance factor
R_n = Nominal resistance
R_r = Factored resistance

The factors relating to ductility and redundancy are related to the configuration of the structure, while the operational importance factor is related to the consequence of the bridge being out of service. The product of the three factors results in the load modifier, η, and is limited to the range between 0.95 and 1.00. In this example, the ductility, redundancy, and operational importance factors are each assigned a value equal to one. The load factors are given in Tables 3.4.1-1 and 3.4.1-2 of the specifications and the resistance factors for the design of steel members are given in Article 6.5.4.2.

5.0 STRUCTURAL ANALYSIS

The *AASHTO LRFD (5^{th} Edition, 2010)* specifications allow the designer to use either approximate (e.g., line girder) or refined (e.g., grid or finite element) analysis methods to determine force effects; the acceptable methods of analysis are detailed in Section 4 of the specifications. In this design example, the line girder approach is employed to determine the girder moment and shear envelopes. Using the line girder approach, vehicular live load force effects are determined by first computing the force effects due to a single truck or loaded lane and then by multiplying these forces by multiple presence factors, live-load distribution factors, and dynamic load allowance factors as detailed below.

5.1 Multiple Presence Factors (Article 3.6.1.1.2)

Multiple presence factors account for the probability of multiple lanes on the bridge being loaded simultaneously. These factors are specified for various numbers of loaded lanes in Table 3.6.1.1.2-1 of the specifications. There are two exceptions when multiple presence factors are not to be applied. These are when (1) distribution factors are calculated using Article 4.6.2.2.1 as these equations are already adjusted to account for multiple presence effects and (2) when determining fatigue truck moments, since the fatigue analysis is only specified for a single truck. Thus, for the present example, the multiple presence factors are only applicable when distribution factors are computed using the lever rule at the strength and service limit states as demonstrated below.

5.2 Live-Load Distribution Factors (Article 4.6.2.2)

The distribution factors approximate the amount of live load (i.e., percentage of a truck or lane load) distributed to a given girder. These factors are computed based on a combination of empirical equations and simplified analysis procedures. Empirical equations are provided in Article 4.6.2.2.1 of the specifications and are specifically developed based on the location of the girder (i.e. interior or exterior), the force effect considered (i.e., moment or shear), and the bridge type. These equations are valid only if specific parameters of the bridge are within the ranges specified in the tables given in Article 4.6.2.2.1. If the limits are not satisfied, a more refined analysis must be performed. This design example satisfies all limits for use of the empirical distribution factors, and therefore, the analysis using the approximate equations follows.

Distribution factors are a function of the girder spacing, slab thickness, span length, and the stiffness of the girder, which depends on the proportions of the section. Since the factor depends on the girder proportion that is not initially known, the stiffness term may be assumed to be equal to one for preliminary design. In this section, calculation of the distribution factors is presented based on the girder proportions previously shown in Figure 3.

5.2.1 Live-Load Lateral Distribution Factors – Positive Flexure

In positive bending regions, the stiffness parameter required for the distribution factor equations, K_g, is determined based on the cross section in Figure 4.

$$K_g = n(I + Ae_g^2) \qquad \text{Eq. (4.6.2.2.1-1)}$$

where:

n = modular ratio
I = moment of inertia of the steel girder
A = area of the steel girder
e_g = distance between the centroid of the girder and centroid of the slab

The required section properties of the girder (in addition to other section properties that will be relevant for subsequent calculations) are determined as follows.

$e_g = 8.0 / 2 + 2.0 + 26.01 - 0.75 = 31.26$ in.

$n = 8$

$K_g = n(I + Ae_g^2) = 8(15{,}969 + 48.88(31.26)^2) = 509{,}871$ in.^{4}

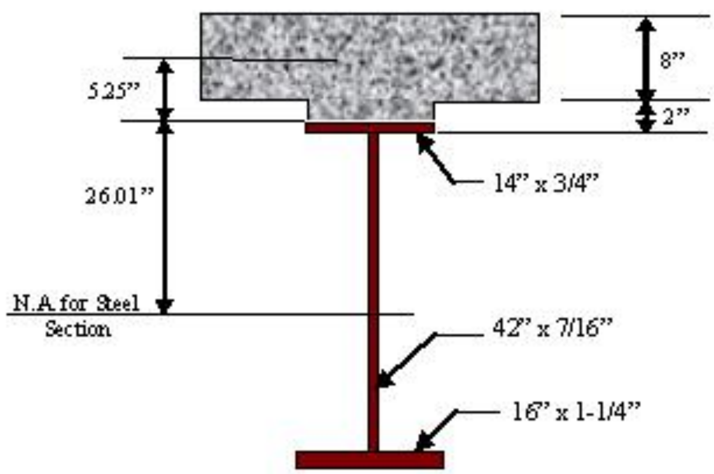

Figure 4 Sketch of Section 1, Positive Bending Region

Table 1 Section 1 Steel Only Section Properties

Component	A	D	Ad	Ad^2	I_o	I
Top Flange 14" x ¾"	10.50	21.38	224.5	4797	0.49	4798
Web 42" x $^{7}/_{16}$"	18.38				2701	2701
Bottom Flange 16" x 1-¼"	20.00	-21.63	-432.5	9353	2.60	9356
	48.88		-208.0			16,855
					-4.26(208)=	-886
					I_{NA} =	15,969 in^4

$$d_s = \frac{-208.0}{48.88} = -4.26 \text{ in.}$$

$$d_{TOP\ OF\ STEEL} = 21.75 + 4.26 = 26.01 \text{ in.} \qquad S_{TOP\ OF\ STEEL} = \frac{15{,}969}{26.01} = 614.0 \text{ in.}^3$$

$$d_{BOT\ OF\ STEEL} = 22.25 - 4.26 = 17.99 \text{ in} \qquad S_{BOT\ OF\ STEEL} = \frac{15{,}969}{19.99} = 887.7 \text{ in.}^3$$

5.2.1.1 Interior Girder – Strength and Service Limit States

For interior girders, computation of the distribution factors for the strength and service limit states is based on the empirical equations given in Article 4.6.2.2.2 as described below.

5.2.1.1.1 Bending Moment

The empirical equations for distribution of live load moment at the strength and service limit states are given in Table 4.6.2.2.2b-1. Alternative expressions are given for one loaded lane and multiple loaded lanes, where the maximum of the two equations governs as shown below. It is noted that the maximum number of lanes possible for the 34 feet roadway width considered in this example is two lanes.

$$DF = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1} \quad \text{for one-lane loaded}$$

where:
S = girder spacing
L = span length
t_s = slab thickness
K_g = stiffness term

$$DF = 0.06 + \left(\frac{10.0}{14}\right)^{0.4}\left(\frac{10.0}{90}\right)^{0.3}\left(\frac{509871}{12.0(90)(8.0)^3}\right)^{0.1} = 0.508 \text{ lanes}$$

$$DF = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0Lt^3}\right)^{0.1} \quad \text{for two or more lanes loaded}$$

$$DF = 0.075 + \left(\frac{10.0}{9.5}\right)^{0.6}\left(\frac{10.0}{90}\right)^{0.2}\left(\frac{509871}{12.0(90)(8.0)^3}\right)^{0.1} = 0.734\,\text{lanes} \qquad \text{(governs)}$$

Thus, the controlling distribution factor for moment of an interior girder in the positive moment region at the strength or service limit state is 0.734 lanes.

5.2.1.1.2 Shear

The empirical equations for distribution of live load shear in an interior girder at the strength and service limit states are given in Table 4.6.2.2.3a-1. Similar to the equations for moment given above, alternative expressions are given based on the number of loaded lanes.

$$DF = 36.0 + \frac{S}{25.0} \quad \text{for one lane loaded}$$

$$DF = 36.0 + \frac{10.0}{25.0} = 0.760 \text{ lanes}$$

$$DF = 0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^2 \quad \text{for two lanes loaded}$$

$$DF = 0.2 + \frac{10.0}{12} - \left(\frac{10.0}{35}\right)^2 = 0.952 \text{ lanes} \qquad \text{(governs)}$$

5.2.1.2 Exterior Girder – Strength and Service Limit States

The live load distribution factors for an exterior girder for checking the strength limit state are determined as the governing factors calculated using a combination of the lever rule, approximate formulas, and a special analysis assuming that the entire cross section deflects and rotates as a rigid body. Each method is illustrated below.

5.2.1.2.1 Bending Moment

Lever Rule:

As specified in Table 4.6.2.2.2d-1, the lever rule is one method used to determine the distribution factor for the exterior girder. The lever rule assumes the deck is hinged at the interior girder, and statics is employed to determine the percentage of the truck weight resisted by the exterior girder, i.e., the distribution factor. It is specified that the truck is to be placed such that the closest wheel is two feet from the barrier or curb, which results in the truck position shown in Figure 5 for the present example. The calculated reaction of the exterior girder is multiplied by the multiple presence factor for one lane loaded, m_1, to determine the distribution factor.

$$DF = \left(0.5 + 0.5\left(\frac{10-6}{10}\right)\right)m_1$$

m_1 = 1.20 (from Table 3.6.1.1.2-1)

DF = 0.7 x 1.2 = 0.840 lanes

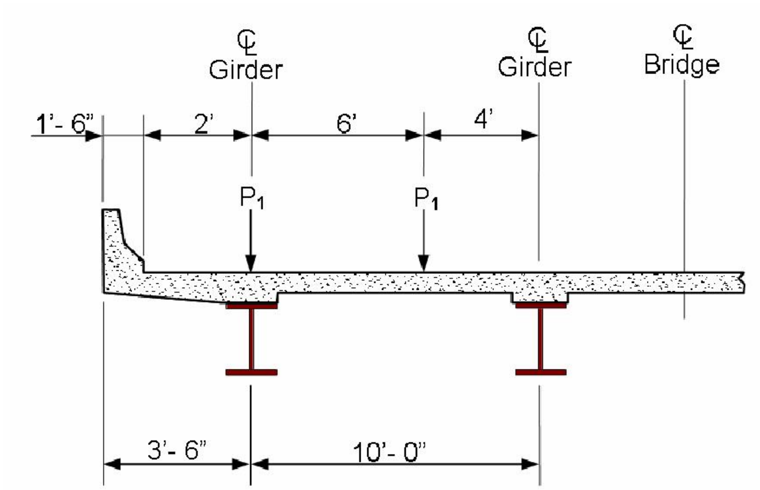

Figure 5 Sketch of the Truck Location for the Lever Rule

Modified Interior Girder Distribution Factor:
The modification factor, e, is found in Table 4.6.2.2.2d-1 and is given below.

$$e = 0.77 + \frac{d_e}{9.1}$$

In the above equation d_e is the distance between the center of the exterior girder and the interior face of the barrier or curb in feet. Thus, for the present example d_e is equal to 2.

$$e = 0.77 + \frac{2.0}{9.1} = 0.990$$

Multiplying the one-lane loaded distribution factor for moment in the positive moment region of an interior girder (which was previously determined to be 0.508 lanes) by the correction factor of 0.990 gives the following.

DF = 0.990(0.508) = 0.503 lanes

Similarly, modifying the interior girder distribution factor for two or more lanes loaded gives the following.

DF = 0.990(0.734) = 0.727

Special Analysis:

The special analysis assumes the entire bridge cross-section behaves as a rigid body rotating about the transverse centerline of the structure and is discussed in the commentary of Article 4.6.2.2.2d. The reaction on the exterior beam is calculated from the following equation:

$$R = \frac{N_L}{N_b} + \frac{X_{ext} \sum^{N_L} e}{\sum^{N_b} x^2} \qquad \text{Eq. (C4.6.2.2.2d-1)}$$

where:

N_L = number of lanes loaded

N_b = number of beams or girders

X_{ext} = horizontal distance from center of gravity of the pattern of girders to the exterior girder (ft.)

e = eccentricity of a design truck or a design lane load from the center of gravity of the pattern of girders (ft.)

x = horizontal distance from the center of gravity of the pattern of girders to each girder (ft.)

Figure 6 shows the truck locations for the special analysis. Here it is shown that the maximum number of trucks that may be placed on half of the cross-section is two. Thus, we proceed with calculation of the distribution factors using the special analysis procedure for one loaded lane and two loaded lanes.

$$\text{DF} = 1.2\left(\frac{1}{4} + \frac{(15)(12)}{2\left((15)^2 + (5)^2\right)}\right) = 0.732 \text{ for one lane loaded}$$

$$\text{DF} = 1.0\left(\frac{2}{4} + \frac{(15)(12+0)}{2\left((15)^2 + (5)^2\right)}\right) = 0.860 \text{ for two lanes loaded} \quad \text{(governs)}$$

Based on the computations for exterior girder distribution factors for moment in the positive bending region shown above, it is determined that the controlling factor for this case is equal to 0.860, which is based on the special analysis with two lanes loaded. Compared to the interior girder distribution factor for moment in the positive bending region, which was computed to be 0.734, it is shown that the exterior girder distribution factor is larger, and therefore controls the bending strength design at the strength and service limit states in the positive bending region.

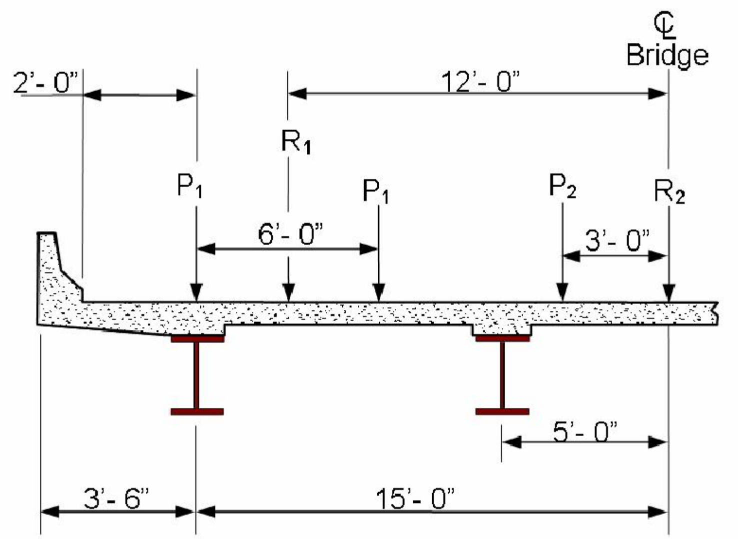

Figure 6 Sketch of the Truck Locations for Special Analysis

5.2.1.2.2 Shear

The distribution factors computed above using the lever rule, approximate formulas, and special analysis methods are also applicable to the distribution of shear force.

Lever Rule:
The above computations demonstrate that the distribution factor is equal to 0.840 lanes based on the lever rule.

DF = 0.840 lanes

Modified Interior Girder Distribution Factor:
The shear modification factor is computed using the following formula.

$$e = 0.60 + \frac{d_e}{9.1}$$

$$e = 0.60 + \frac{2}{9.1} = 0.820$$

Applying this modification factor to the previously computed interior girder distribution factors for shear for one lane loaded and two or more lanes loaded, respectively, gives the following.

DF = 0.820(0.760) = 0.623 lanes

DF = 0.820(0.952) = 0.781 lanes

Special Analysis:
It was demonstrated above that the special analysis yields the following distribution factors for one lane and two or more lanes loaded, respectively.

DF = 0.732 lanes

DF = 0.860 lanes (governs)

Thus, the controlling distribution factor for shear in the positive bending region of the exterior girder is 0.860, which is less than that of the interior girder. Thus, the interior girder distribution factor of 0.952 controls the shear design in the positive bending region.

5.2.1.3 Fatigue Limit State

As stated in Article 3.6.1.1.2, the fatigue distribution factor is based on one lane loaded, and does not include the multiple presence factor, since the fatigue loading is specified as a single truck load. Because the distribution factors calculated from empirical equations incorporate the multiple presence factors, the fatigue distribution factors are equal to the strength distribution factors divided by the multiple presence factor for one lane, as described subsequently.

5.2.1.3.1 Bending Moment

Upon reviewing the moment distribution factors for one lane loaded computed above, it is determined that the maximum distribution factor results from the lever rule calculations. Dividing this distribution factor of 0.840 by the multiple presence factor for one lane loaded results in the following distribution factor for fatigue moment in the positive bending region.

$$DF = \frac{0.840}{1.20} = 0.700 \text{ lanes}$$

5.2.1.3.2 Shear

Similarly, based on the above distribution factors for shear due to one lane loaded, the controlling distribution factor is calculated by again dividing the lever rule distribution factor by the multiple presence factor.

$$DF = \frac{0.840}{1.20} = 0.700 \text{ lanes}$$

5.2.1.4 Distribution Factor for Live-Load Deflection

Article 2.5.2.6.2 states that all design lanes must be loaded when determining the live load deflection of the structure. In the absence of a refined analysis, an approximation of the live load deflection can be obtained by assuming that all girders deflect equally and applying the appropriate multiple presence factor. The controlling case occurs when two lanes are loaded, and the calculation of the corresponding distribution factor is shown below.

$$\text{DF} = m\left(\frac{N_L}{N_b}\right) = 1.0\left(\frac{2}{4}\right) = 0.500\,\text{lanes}$$

Table 2 summarizes the governing distribution factors for the positive bending region.

Table 2 Positive Bending Region Distribution Factors

	Interior Girder	Exterior Girder
Bending Moment	0.734	0.860
Shear	0.952	0.860
Fatigue (Bending Moment)	0.423	0.700
Fatigue (Shear)	0.633	0.700
Deflection	0.500	0.500

5.2.2 Live-Load Lateral Distribution Factors – Negative Flexure

Many of the distribution factors are the same in both the positive and negative bending regions. This section demonstrates the computation of the distribution factors that are unique to the negative bending region. Specifically, the distribution factor for the interior girder at the strength and service limit states is directly influenced by to the girder proportions. As in the above calculations for the positive moment region, this process begins with determining the stiffness parameter, K_g of the section. The cross section for the negative bending region is shown in Figure 7. The section properties of the girder are determined as follows.

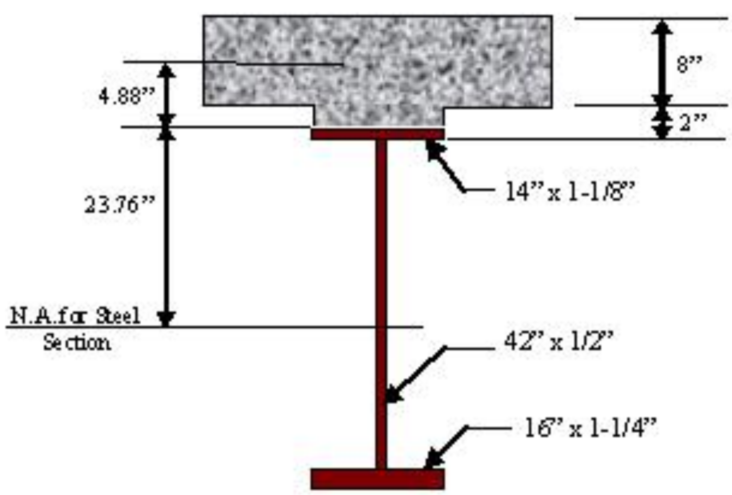

Figure 7 Sketch of Section 2, Negative Bending Region

Table 3 Section 2 Steel Only Section Properties

Component	A	D	Ad	Ad^2	I_o	I
Top Flange 14" x 1-1/8"	15.75	21.56	340	7323	1.66	7325
Web 42" x 1/2"	21.00				3087	3087
Bottom Flange 16" x 1-1/4"	20.00	-21.63	-432.5	9352	2.60	9355
	56.75		-92.5			19,767
					-1.63(92.5) =	-151
					I_{NA} =	19,616 in^4

$$d_s = \frac{-92.5}{56.75} = -1.63 \text{ in.}$$

$$d_{\text{TOP OF STEEL}} = 22.125 + 1.63 = 23.76 \text{ in.} \qquad S_{\text{TOP OF STEEL}} = \frac{19,616}{23.76} = 825.6 \text{ in.}^3$$

$$d_{\text{BOT OF STEEL}} = 22.25 - 1.63 = 20.62 \text{ in.} \qquad S_{\text{BOT OF STEEL}} = \frac{19,616}{20.62} = 951.3 \text{ in.}^3$$

$e_g = 8.0 / 2 + 2.0 + 23.76 - 1.125 = 28.64$ in.

n = 8

$K_g = n(I + Ae_g^2) = 8(19,616 + 56.75(28.64)^2) = 529,321$ in.^{4}

As discussed above, the distribution factors for interior girders at the strength and service limit states are computed based on the empirical equations given in Article 4.6.2.2.2.

The applicable equations for moment distribution factors from Table 4.6.2.2.2b-1 are as shown below.

$$DF = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1} \text{ for one lane loaded}$$

$$DF = 0.06 + \left(\frac{10.0}{14}\right)^{0.4}\left(\frac{10.0}{90}\right)^{0.3}\left(\frac{529,321}{12.0(90)(8.0)^3}\right) = 0.510 \text{ lanes}$$

$$DF = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0Lt^3}\right)^{0.1} \text{ for two or more lanes loaded}$$

$$DF = 0.075 + \left(\frac{10.0}{9.5}\right)^{0.6}\left(\frac{10.0}{90}\right)^{0.2}\left(\frac{529,321}{12.0(90)(8.0)^3}\right) = 0.737 \text{ lanes}$$

Table 4 summarizes the distribution factors for the negative bending region, where it is shown that the exterior girder controls all aspects of the design expect for shear at the strength and service limit states.

Table 4 Negative Bending Region Distribution Factors

	Interior Girder	Exterior Girder
Bending Moment	0.737	0.860
Shear	0.952	0.860
Fatigue (Bending Moment)	0.425	0.700
Fatigue (Shear)	0.633	0.700
Deflection	0.500	0.500

5.2.3 Dynamic Load Allowance

The dynamic effects of the truck loading are taken into consideration by the dynamic load allowance, IM. The dynamic load allowance, which is discussed in Article 3.6.2 of the specifications, accounts for the hammering effect of the wheel assembly and the dynamic response of the bridge. IM is only applied to the design truck or tandem, not the lane loading. Table 3.6.2.1-1 specifies IM equal to 1.33 for the strength, service, and live load deflection evaluations, while IM of 1.15 is specified for the fatigue limit state.

6.0 ANALYSIS RESULTS

6.1 Moment and Shear Envelopes

Figures 8 through 11 show the moment and shear envelopes for this design example, which are based on the data presented in Tables 5 through 11. These figures show distributed moments for the exterior girder and distributed shears for an interior girder, which are the controlling girders for each force effect, based on the distribution factors computed above. The envelopes shown are determined based on the section properties of the short-term composite section.

As previously mentioned, the live load in the positive bending region between the points of dead load contraflexure is the result of the HL-93 loading. In the negative bending region between the points of dead load contraflexure, the moments are the larger of the HL-93 loading and the special negative-moment loading, which is composed of 90 percent of both the truck-train moment and lane loading moment.

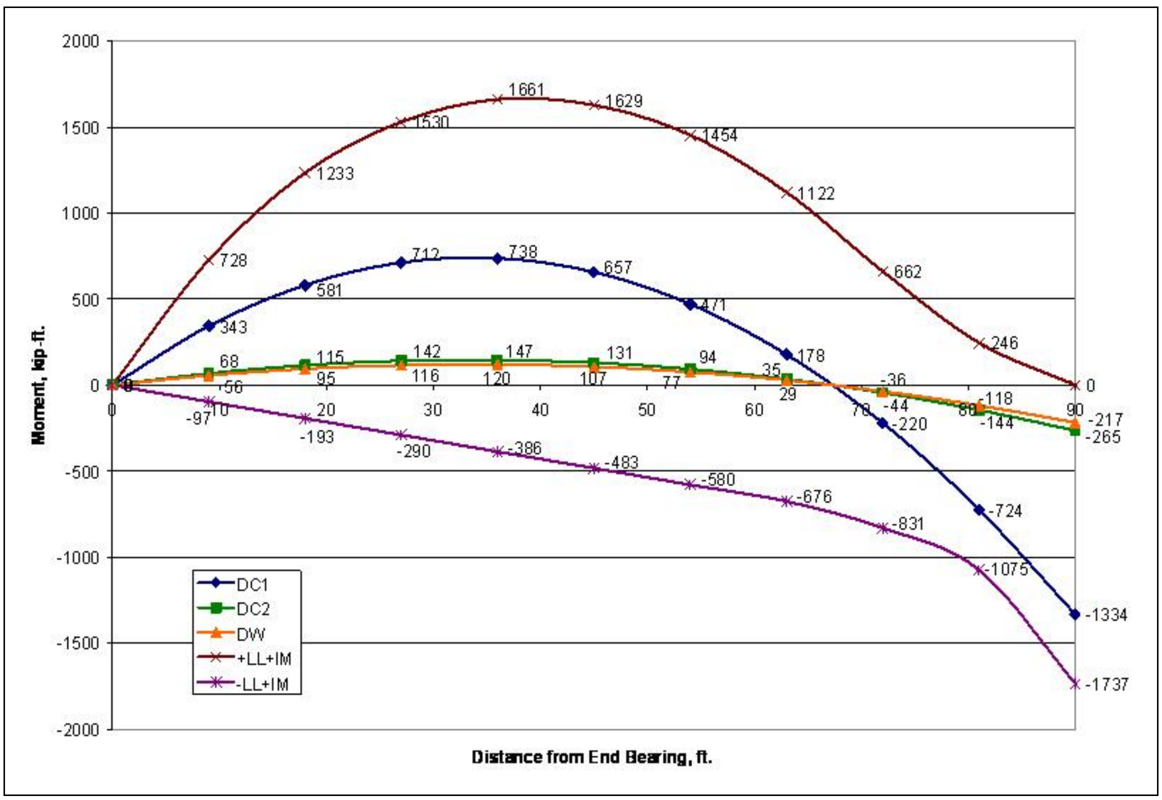

Figure 8 Dead and Live Load Moment Envelopes

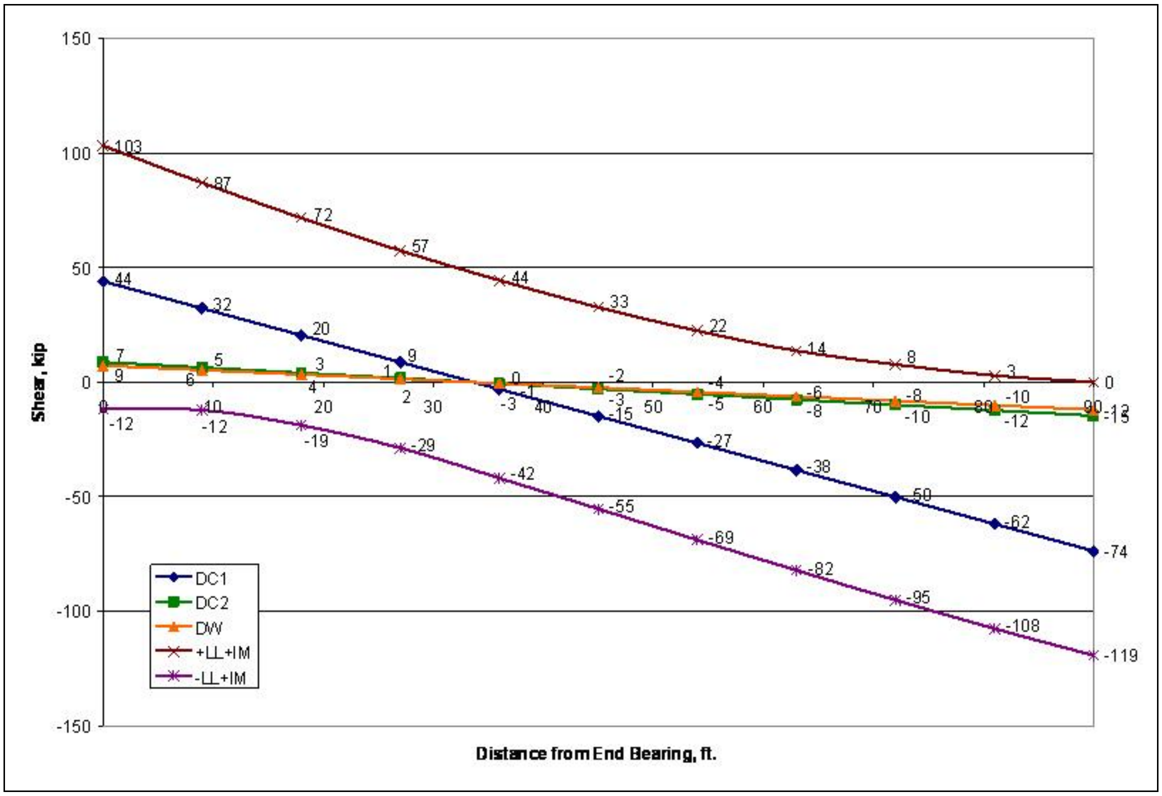

Figure 9 Dead and Live Load Shear Envelopes

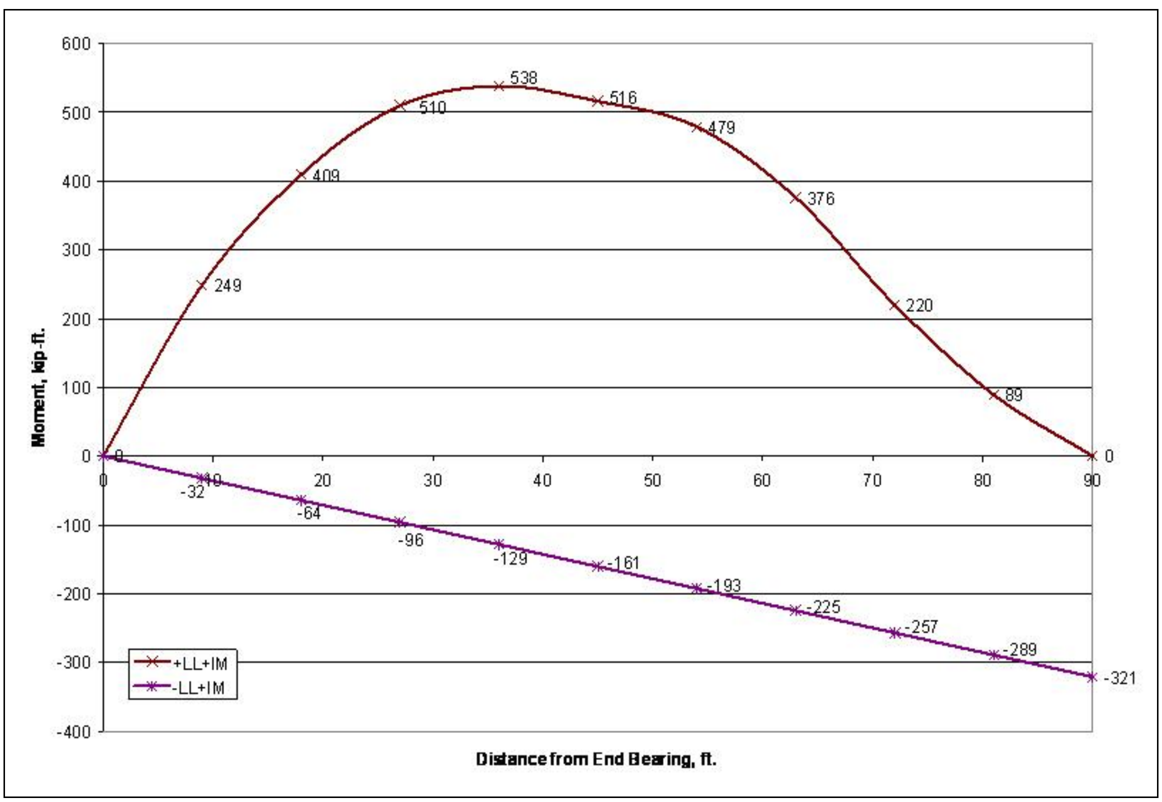

Figure 10 Fatigue Live Load Moments

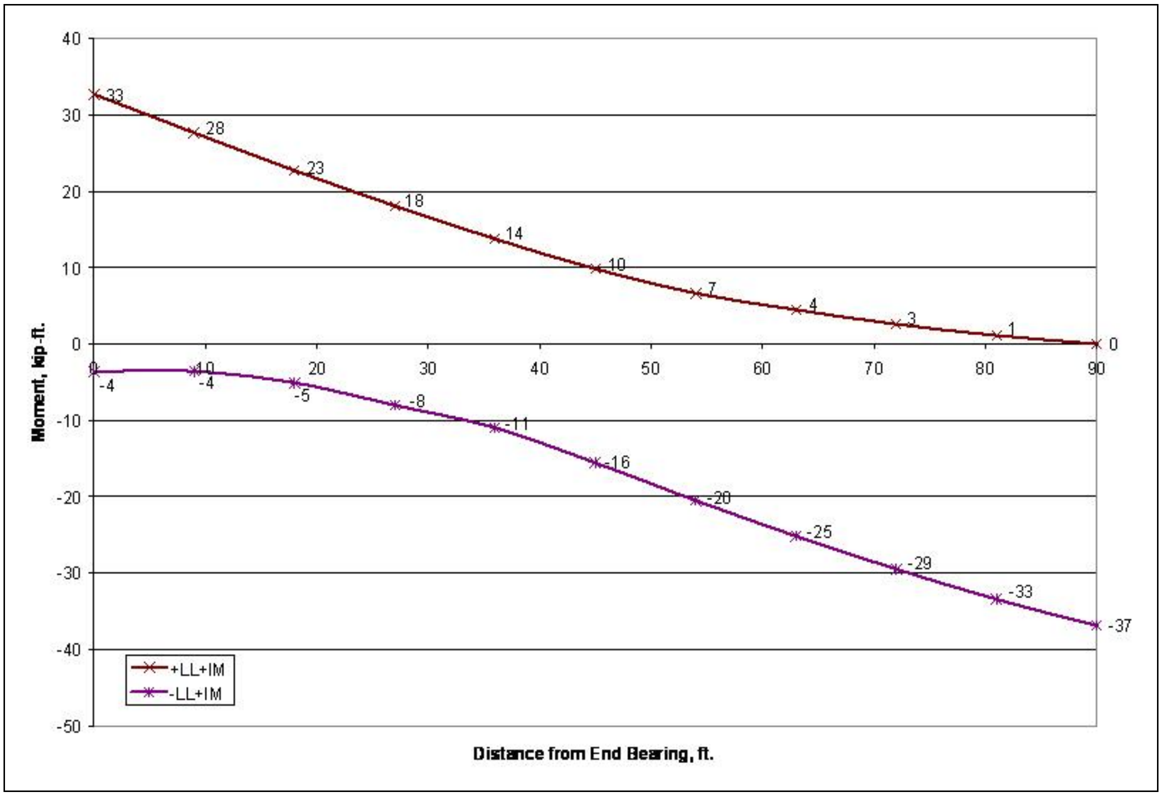

Figure 11 Fatigue Live Load Shears

Table 5 Unfactored and Undistributed Moments (kip-ft)

	Non-Com. Dead	Com. Dead	Wearing Surf.	Truck Load		Lane Load		Tandem		Double Truck		Double Tandem	
Span 1	DC1	DC2	DW	pos.	neg.	pos.	neg.	pos.	neg.	pos.	neg.	pos.	neg.
0.00	0	0	0	0	0	0	0	0	0	0	0	0	0
0.10	343	68	56	485	-60	201	-33	381	-44	0	0	0	0
0.20	581	115	95	816	-120	349	-65	652	-87	0	0	0	0
0.30	712	142	116	1002	-180	446	-98	817	-131	0	0	0	0
0.40	738	147	120	1083	-240	491	-131	883	-174	0	0	0	0
0.50	657	131	107	1059	-300	485	-163	866	-218	0	0	0	0
0.60	471	94	77	951	-359	426	-196	779	-261	0	0	0	0
0.70	178	35	29	743	-419	316	-229	625	-305	0	0	0	0
0.80	-220	-44	-36	463	-479	154	-261	423	-348	0	-479	0	-611
0.90	-724	-144	-118	148	-539	30	-385	192	-392	0	-755	0	-687
1.00	-1334	-265	-217	0	-599	0	-653	0	-436	0	-1196	0	-764

Table 6 Unfactored and Undistributed Live Load Moments (kip-ft)

Span 1	vehicle positive	vehicle negative	special negative	standard negative	1.33 Vehicle + Lane positive	Distribution Factors	Positive	Negative
0.00	0	0	0	0	0	0.86	0	0
0.10	485	-60	-29	-112	846	0.86	728	-97
0.20	816	-120	-59	-225	1434	0.86	1233	-193
0.30	1002	-180	-88	-337	1779	0.86	1530	-290
0.40	1083	-240	-118	-449	1932	0.86	1661	-386
0.50	1059	-300	-147	-562	1894	0.86	1629	-483
0.60	951	-359	-176	-674	1691	0.86	1454	-580
0.70	743	-419	-206	-786	1304	0.86	1122	-676
0.80	463	-479	-967	-899	770	0.86	662	-831
0.90	192	-539	-1250	-1102	285	0.86	246	-1075
1.00	0	-599	-2020	-1450	0	0.86	0	-1737

Table 7 Strength I Load Combination Moments (kip-ft)

Span 1	1.25 DC1	1.25 DC2	1.5 DW	1.75 (LL+IM) positive	1.75 (LL+IM) negative	Strength I positive	Strength I negative
0.00	0	0	0	0	0	0	0
0.10	429	85	84	1273	-169	1871	429
0.20	726	144	142	2158	-338	3170	674
0.30	890	177	174	2677	-507	3918	734
0.40	922	183	180	2907	-676	4192	609
0.50	821	163	161	2850	-845	3995	300
0.60	588	117	115	2545	-1014	3365	-194
0.70	223	44	44	1963	-1183	2274	-873
0.80	-275	-55	-54	1159	-1455	775	-1838
0.90	-905	-180	-177	430	-1882	-833	-3144
1.00	-1668	-332	-326	0	-3039	-2326	-5365

Table 8 Service II Load Combination Moments (kip-ft)

Span 1	1.0 DC1	1.0 DC2	1.0 DW	1.3 (LL+IM) positive	1.3 (LL+IM) negative	Service II positive	Service II negative
0.00	0	0	0	0	0	0	0
0.10	343	68	56	946	-126	1413	342
0.20	581	115	95	1603	-251	2394	540
0.30	712	142	116	1989	-377	2959	593
0.40	738	147	120	2160	-502	3164	502
0.50	657	131	107	2117	-628	3012	267
0.60	471	94	77	1890	-753	2531	-113
0.60	471	94	77	1890	-753	2531	-113
0.70	178	35	29	1458	-879	1701	-636
0.80	-220	-44	-36	861	-1081	561	-1380
0.90	-724	-144	-118	319	-1398	-667	-2384

Table 9 Unfactored and Undistributed Shears (kip)

Span 1	Non-Com. Dead	Com. Dead	Wearing Surf.	Truck Load		Lane Load		Tandem	
	DC1	DC2	DW	positive	negative	positive	negative	positive	negative
0.00	44	9	7	63	-7	25	-4	49	-5
0.10	32	6	5	54	-7	20	-4	42	-5
0.20	20	4	3	45	-10	15	-5	36	-11
0.30	9	2	1	37	-18	11	-7	30	-17
0.40	-3	-1	0	29	-26	8	-9	25	-23
0.50	-15	-3	-2	22	-34	5	-12	19	-29
0.60	-27	-5	-4	15	-42	3	-16	14	-34
0.70	-38	-8	-6	10	-50	2	-20	10	-38
0.80	-50	-10	-8	5	-56	1	-25	6	-43
0.90	-62	-12	-10	2	-62	0	-30	2	-46
1.00	-74	-15	-12	0	-67	0	-36	0	-49

Table 10 Unfactored and Undistributed Live Load Shears (kip)

Span 1	vehicle		(1.33 V Vehicle + V Lane)			Live Load	
	positive	negative	positive	negative	Distribution	Positive	Negative
0.00	63	-7	108	-12	0.952	103	-12
0.10	54	-7	91	-13	0.952	87	-12
0.20	45	-11	75	-20	0.952	72	-19
0.30	37	-18	60	-30	0.952	57	-29
0.40	29	-26	47	-44	0.952	44	-42
0.50	22	-34	34	-58	0.952	33	-55
0.60	15	-42	24	-72	0.952	22	-69
0.70	10	-50	14	-86	0.952	14	-82
0.80	6	-56	8	-100	0.952	8	-95
0.90	2	-62	3	-113	0.952	3	-108
1.00	0	-67	12	-108	0.952	12	-103

Table 11 Strength I Load Combination Shear (kip)

Span 1	1.25 DC1	1.25 DC2	1.5 DW	1.75 (LL+IM)		Strength I	
				positive	negative	positive	negative
0.00	55	11	11	181	-21	257	56
0.10	40	8	8	152	-21	209	35
0.20	26	5	5	126	-33	161	3
0.30	11	2	2	101	-50	116	-35
0.40	-4	-1	-1	78	-73	72	-79
0.50	-19	-4	-4	57	-97	31	-123
0.60	-33	-7	-6	39	-120	-7	-167
0.70	-48	-10	-9	24	-144	-43	-211
0.80	-63	-12	-12	13	-167	-74	-254
0.90	-77	-15	-15	5	-188	-103	-296
1.00	-92	-18	-18	0	-209	-128	-337

6.2 Live Load Deflection

As provided in Article 3.6.1.3.2, control of live-load deflection is optional. Evaluation of this criterion is based on the flexural rigidity of the short-term composite section and consists of two load cases: deflection due to the design truck, and deflection due to the design lane plus 25 percent of the design truck. The dynamic load allowance of 33 percent is applied to the design truck load only for both loading conditions. For this example, the live load is distributed using a distribution factor of 0.500 calculated earlier.

The maximum deflection due to the design truck is 0.917 inches. Applying the impact and distribution factors gives the following.

$$\Delta_{LL+IM} = 0.500 \text{ x } 1.33 \text{ x } 0.917 = 0.610 \text{ in.} \qquad \text{(governs)}$$

The deflection due to 25% of the design truck plus the lane loading is equal to the following.

$$\Delta_{LL+IM} = 0.500 \text{ (} 1.33 \text{ x } 0.25 \text{ x } 0.917 + 0.475 \text{) } = 0.390 \text{ in.}$$

Thus the governing deflection equal to 0.610 inch will be used to assess the girder design based on the live-load deflection criterion.

7.0 LIMIT STATES

As discussed previously, there are four limit states applicable to the design of steel I-girders. Each of these limit states is described below.

7.1 Service Limit State (Articles 1.3.2.2 and 6.5.2)

The intent of the Service Limit State is to ensure the satisfactory performance and rideability of the bridge structure by preventing localized yielding. For steel members, these objectives are intended to be satisfied by limiting the maximum levels of stress that are permissible. The optional live-load deflection criterion is also included in the service limit state and is intended to ensure user comfort.

7.2 Fatigue and Fracture Limit State (Article 1.3.2.3 and 6.5.3)

The intent of the Fatigue and Fracture Limit State is to control crack growth under cyclic loading. This is accomplished by limiting the stress range to which steel members are subjected. The allowable stress range varies for various design details and member types. The fatigue limit state also restricts the out-of-plane flexing of the web. Additionally, fracture toughness requirements are stated in Article 6.6.2 of the specifications and are dependent on the temperature zone.

7.3 Strength Limit State (Articles 1.3.2.4 and 6.5.4)

The strength limit state ensures the design is stable and has adequate strength when subjected to the highest load combinations considered. The bridge structure may experience structural damage (e.g., permanent deformations) at the strength limit state, but the integrity of the structure is preserved.

The suitability of the design must also be investigated to ensure adequate strength and stability during each construction phase. The deck casting sequence has a significant influence on the distribution of stresses within the structure. Therefore, the deck casting sequence should be considered in the design and specified on the plans to ensure uniformity between predicted and actual stresses. In addition, lateral flange bending stresses resulting from forces applied to the overhang brackets during construction should also be considered during the constructability evaluation.

7.4 Extreme Event Limit State (Articles 1.3.2.5 and 6.5.5)

The extreme event limit state is to ensure the structure can survive a collision, earthquake, or flood. The collisions investigated under this limit state include the bridge being struck by a vehicle, vessel, or ice flow. This limit state is not addressed by this design example.

8.0 SAMPLE CALCULATIONS

This example presents sample calculations for the design of positive and negative bending sections of the girders for the strength, fatigue and fracture, and service limit states. In addition, calculations evaluating the constructibility of the bridge system are included and the optional provisions for moment redistribution are presented. Also presented are the cross-frame design, stiffener design, and weld design. The moment and shear envelopes provided in Figs. 8 through 11 are referenced in the following calculations.

8.1 Section Properties

The section properties for Section 1 and Section 2 are first calculated and will be routinely used in the subsequent evaluations of the various code checks. The structural slab thickness is taken as the slab thickness minus the integral wearing surface (8 inches) and the modular ratio (n) is taken to be 8 inches these calculations.

8.1.1 Section 1 – Positive Bending Region

Section 1 represents the positive bending region and was previously shown in Figure 4. The longitudinal reinforcement is neglected in the computation of these section properties.

8.1.1.1 Effective Flange Width (Article 4.6.2.6)

Article 4.6.2.6 of the *AASHTO LRFD (5th Edition, 2010)* Specifications governs the determination of the effective flange width of the concrete slab when designing composite sections.

For the interior girders of this example, b_{eff} in the positive bending region is determined as one-half the distance to the adjacent girder one each side of the girder being analyzed.

$$b_{eff} = \frac{120}{2} + \frac{120}{2} = 120.0\text{in.}$$

For the exterior girders of this example, b_{eff} in the positive bending region is determined as one-half the distance to the adjacent girder plus the full overhang width.

$$b_{eff} = \frac{120}{2} + 42 = 102.0\text{in.}$$

The exterior girder has both a smaller effective width and a larger live load distribution factor than the interior girder therefore moment design of the positive bending region is controlled by the exterior girder.

8.1.1.2 Elastic Section Properties: Section 1

As discussed above, the section properties considered in the analysis of the girder vary based on the loading conditions. Specifically, live loads are applied to the termed the short-term composite section, where the modular ratio of 8 is used in the computations. Alternatively, dead loads are applied to the long-term composite section. The long-term composite section is considered to be comprised of the full steel girder and one-third of the concrete deck to account for the reduction in strength that may occur in the deck over time due to creep effects. This is reflected in the section property calculations through use of a modular ratio equal to 3 times the typical modular ratio (3n), or in this example, 24. The section properties for the short-term and long-term composite sections are computed below, in Tables 12 and 13. Recall that the section properties for the steel section (girder alone) were previously computed in for the purpose of determining live load distribution factors.

Table 12 Section 1 Short Term Composite (n) Section Properties (Exterior Girder)

Component	A	d	Ad	Ad^2	I_o	I
Steel Section	48.88		-208.0			16,855
Concrete Slab (8"x 102"/8)	102.0	27.00	2,754	74,358	544	74,902
	150.88		2,546			91,757
				-16.87(2,546) =		-42,951
						48,806 in⁴

$$d_s = \frac{2{,}546}{150.88} = 16.87 \text{ in.}$$

$$d_{TOP\ OF\ STEEL} = 21.75 - 16.87 = 4.88 \text{ in.} \qquad S_{TOP\ OF\ STEEL} = \frac{48{,}806}{4.88} = 10{,}001 \text{ in.}^3$$

$$d_{BOT\ OF\ STEEL} = 22.25 + 16.87 = 39.12 \text{ in.} \qquad S_{BOT\ OF\ STEEL} = \frac{48{,}806}{39.12} = 1{,}248 \text{ in.}^3$$

Table 13 Section 1 Long Term Composite (3n) Section Properties (Exterior Girder)

Component	A	d	Ad	Ad^2	I_o	I
Steel Section	48.88		-208.0			16,855
Concrete Slab (8"x 102"/24)	34.00	27.00	918.0	24,786	181.3	24,967
	82.88		710.0			41,822
				-8.57(710.0) =		-6,085
						35,737 in⁴

$$d_s = \frac{710}{82.88} = 8.57 \text{ in.}$$

$$d_{TOP\ OF\ STEEL} = 21.75 - 8.57 = 13.18 \text{ in.} \qquad S_{TOP\ OF\ STEEL} = \frac{35{,}737}{13.18} = 2{,}711 \text{ in.}^3$$

$$d_{BOT\ OF\ STEEL} = 22.25 + 8.57 = 30.82 \text{ in.} \qquad S_{BOT\ OF\ STEEL} = \frac{35{,}737}{30.82} = 1{,}159 \text{ in.}^3$$

8.1.1.3 Plastic Moment: Section 1

The plastic moment M_p may be determined for sections in positive flexure using the procedure outlined in Table D6.1-1 as demonstrated below. The longitudinal deck reinforcement is conservatively neglected in these computations. The plastic forces acting in the slab (P_s), compression flange (P_c), web (P_w), and tension flange (P_t) are first computed.

$P_s = 0.85f'_c b_s t_s \quad = 0.85(4.0)(102)(8) \quad = 2{,}611$ kips

$P_c = F_{yc} b_c t_c \quad = (50)(14)(0.75) \quad = 525$ kips

$P_w = F_{yw} D t_w \quad = (50)(42)(0.4375) \quad = 919$ kips

$P_t = F_{yt} b_t t_t \quad = (50)(16)(1.25) \quad = 1{,}000$ kips

The plastic forces for each element of the girder are then compared to determine the location of the plastic neutral axis (PNA). The position of the PNA is determined by equilibrium, no net axial force when considering the summation of plastic forces. Table D.6.1-1 provides seven cases, with accompanying conditions for use, to determine the location of the PNA and subsequently calculate the plastic moment.

Following the conditions set forth in Table D6.1-1, the PNA is generally located as follows:

CASE I

$P_t + P_w \geq P_c + P_s$

$1{,}000 + 919 \geq 525 + 2{,}611$

$1{,}919 < 3{,}136$ Therefore, PNA is not in the web

CASE II

$P_t + P_w + P_c \geq P_s$

$1{,}000 + 919 + 525 \geq 2{,}611$

$2{,}444$ kips $< 2{,}611$ kips Therefore, PNA is not in the top flange

Therefore, the plastic neutral axis is in the concrete deck and $\bar{y}$ is computed using the following equation derived from that provided in Table D6.1-1 when deck reinforcement is ignored:

$$\bar{y} = (t_s)\left[\frac{P_c + P_w + P_t}{P_s}\right]$$

$$\bar{y} = (8.0)\left[\frac{525 + 919 + 1000}{2611}\right] = 7.49 \text{ inches from the top of the concrete slab}$$

The plastic moment M_p is then calculated using the following equation derived from that provided in Table D6.1-1 when deck reinforcement is ignored.

$$M_p = \left(\frac{\bar{y}^2 P_s}{2t_s} \right) + \left[P_c d_c + P_w d_w + P_t d_t \right]$$

The distance from the PNA to the centroid of the compression flange, web, and tension flange (respectively) is as follows:

d_c = 8.0 + 2.0 – 0.5(0.75) - 7.49 = 2.135 in.

d_w = 8.0 + 2.0 + 0.5(42.0) – 7.49 = 23.51 in.

d_t = 8.0 + 2.0 + 42.0 + 0.5(1.25) – 7.49 = 45.135 in.

Substitution of these distances and the above computed plastic forces, into the preceding equation, gives the following:

$$M_p = \left(\frac{(7.49)^2 (2611)}{2(8.0)} \right) + \left[(525)(2.135) + (919)(23.51) + (1000)(45.135) \right]$$

M_p = 77,016 k-in = 6,418 k-ft.

8.1.1.4 Yield Moment: Section 1

The yield moment, which is the moment which causes first yield in either flange (neglecting flange lateral bending) is detailed in Article D6.2.2 of the specifications. This computation method for the yield moment recognizes that different stages of loading (e.g. composite dead load, non-composite dead load, and live load) act on the girder when different cross-sectional properties are applicable. The yield moment is determined by solving for M_{AD} using Equation D6.2.2-1 (given below) and then summing M_{D1}, M_{D2}, and M_{AD}, where, M_{D1}, M_{D2}, and M_{AD} are the factored moments applied to the noncomposite, long-term composite, and short-term composite section, respectively.

$$F_{yt} = \frac{M_{D1}}{S_{NC}} + \frac{M_{D2}}{S_{LT}} + \frac{M_{AD}}{S_{ST}} \qquad \text{Eq. (D6.2.2-1)}$$

Due to the significantly higher section modulus of the short-term composite section about the top flange, compared to the short-term composite section modulus taken about the bottom flange, the minimum yield moment results when using the bottom flange section modulus values.

Computation of the yield moment for the bottom flange is thus demonstrated below. First the known quantities are substituted into Equation D6.2.2-1 to solve for M_{AD}.

$$50 = 1.0\left[\frac{1.25(1738)(12)}{887.7} + \frac{1.25(147)(12) + 1.50(120)(12)}{1,159} + \frac{M_{AD}}{1,248}\right]$$

M_{AD} = 42,136 k-in. = 3,511 k-ft.

M_y is then determined by applying the applicable load factors and summing the dead loads and M_{AD}.

M_y = 1.25(738) + 1.25(147) + 1.50(120) + 3,489 = 4,776 k-ft   Eq. (D6.2.2-2)

8.1.2 Section 2 – Negative Bending Region

This section details the calculations to determine the section properties of the composite girder in the negative bending region, which was previously illustrated in Figure 7.

8.1.2.1 Effective Flange Width (Article 4.6.2.6)

As discussed previously, the effective flange width for interior girders is computed as one-half the distance to the adjacent girder one each side of the girder being analyzed.

$$b_{eff} = \frac{120}{2} + \frac{120}{2} = 120.0\text{in.}$$

For an exterior girder, b_{eff} is determined as one-half the distance to the adjacent girder plus the full overhang width.

$$b_{eff} = \frac{120}{2} + 42 = 102.0\text{in.}$$

8.1.2.2 Minimum Negative Flexure Concrete Deck Reinforcement (Article 6.10.1.7)

The total area of the longitudinal reinforcement, provided in negative bending regions, shall not be less than one percent of the total cross-sectional area of the concrete deck. This provision is intended to prevent cracking of the concrete deck in regions where the tensile stress due to the factored construction load or the service II load exceeds ϕf_r, which is typically the case in negative bending regions. (f_r is the modulus of rupture of the concrete and is equal to $0.24(f_r)^{0.5}$ for normal weight concrete, and ϕ is equal to 0.90)

The total area of the concrete deck in this example is computed as follows.

$$A_{deck} = \frac{8}{12}(37.0) + 2\left[\left(\frac{1}{2}\right)\left(\frac{1}{12}\right)(2.0)\left(3.5 - \frac{14/2}{12}\right)\right] = 25.15\text{ft.}^2 = 3,622\text{in.}^2$$

The minimum area of reinforcing steel required is taken as:

$$0.01(3{,}622) = 36.22 \text{ in.}^2$$

Reinforcement is to be distributed uniformly across the deck width. The area of reinforcement required within the effective width (102 inches) of an exterior girder is determined as shown below.

$$\frac{36.22\text{in}^2}{37.0\text{ft}} = 0.98\text{in}^2/\text{ft.} = 0.816\text{in}^2/\text{ft.}$$

$$0.0816(102) = 8.32 \text{ in.}^2$$

This reinforcement is to be placed in two layers with two-thirds of the reinforcement in the top layer and the remaining one-third placed in the bottom layer. Therefore, the area of the top reinforcement is 5.55 in^2 and the area of the bottom reinforcement is 2.77 in^2. Additionally, the reinforcement should not use bar sizes exceeding No. 6 bars, have a yield strength greater than 60 ksi, or use bar spacing exceeding 12.0 inches.

8.1.2.3 Elastic Section Properties: Section 2

Similar to the computation of section properties presented above for Section 1, section properties for the short-term and long-term composite sections in Section 2 are presented below. The section consisting of the girder and reinforcing steel only is included in the composite section, in regions of negative bending, as it is assumed that the concrete is not effective in tension.

Table 14 Section 2 Short Term Composite (n) Section Properties

Component	A	d	Ad	Ad^2	I_o	I
Steel Section	56.75		-92.5			19,767
Concrete Slab (8"x 102"/8)	102.0	27.00	2,754	74,358	544	74,902
	158.75		2,662			94,669
				-16.77(2,662) =		-44,642
						50,027 in^4

$$d_s = \frac{2{,}662}{158.75} = 16.77 \text{ in.}$$

$$d_{TOP\ OF\ STEEL} = 22.125 - 16.77 = 5.36 \text{ in.} \qquad S_{TOP\ OF\ STEEL} = \frac{50{,}027}{5.36} = 9{,}333 \text{ in.}^3$$

$$d_{BOT\ OF\ STEEL} = 22.25 + 16.77 = 39.02 \text{ in.} \qquad S_{BOT\ OF\ STEEL} = \frac{50{,}027}{39.02} = 1{,}282 \text{ in.}^3$$

Table 15 Section 2 Long Term Composite (3n) Section Properties

Component	A	d	Ad	Ad^2	I_o	I
Steel Section	56.75		-92.5			19,767
Concrete Slab (8" x 102"/24)	34.0	27.00	918	24,786	181.3	24,967
	90.75		825.5			44,734
				-9.10(825.5) =		-7,512
						37,222 in^4

$$d_s = \frac{825.5}{90.75} = 9.10 \text{ in.}$$

$$d_{TOP\ OF\ STEEL} = 22.125 - 9.10 = 13.03 \text{ in.} \qquad S_{TOP\ OF\ STEEL} = \frac{37{,}222}{13.03} = 2{,}857 \text{ in.}^3$$

$$d_{BOT\ OF\ STEEL} = 22.25 + 9.10 = 31.35 \text{ in.} \qquad S_{BOT\ OF\ STEEL} = \frac{37{,}222}{31.35} = 1{,}187 \text{ in.}^3$$

Table 16 Section 2 Steel Section and Longitudinal Reinforcement Section Properties

Component	A	d	Ad	Ad^2	I_o	I
Steel Section	56.75		-92.5			19,767
Top Long. Reinforcement	5.09	28.75	146.3	4,207		4,207
Bot. Long. Reinforcement	2.54	24.25	61.6	1,494		1,494
	64.38		115.4			25,468
				-1.79(115.4) =		-207
						25,261 in^4

$$d_s = \frac{115.4}{64.83} = 1.79 \text{ in.}$$

$$d_{TOP\ OF\ STEEL} = 22.125 - 1.79 = 20.34 \text{ in.} \qquad d_{TOP\ OF\ REIN.} = 29.00 - 1.79 = 27.21 \text{ in}$$

$$d_{BOT\ OF\ STEEL} = 22.25 + 1.79 = 24.04 \text{ in}$$

The design section modulus at the top of the composite section shall be calculated relative the first element to yield, either the top flange or the reinforcing steel. Using the computed distances from the neutral axis to each element it is determined that the reinforcing steel is the first to yield, as demonstrated below:

$$x / 26.94 = 50 / 20.07$$

$$x = 67.2 \text{ ksi} > F_{yr} = 60 \text{ ksi}$$

Therefore, the reinforcing steel yields first.

$S_{REIN.} = 25{,}706 / 26.94 = 954.2 \text{ in.}^3$ (Controls)

$S_{BOT\ OF\ STEEL} = 25{,}706 / 24.31 = 1{,}057 \text{ in.}^3$

8.1.2.4 Plastic Moment: Section 2

Similar to the calculation of the plastic moment for Section 1, Table D6.1-2 is used to determine the plastic moment (M_p) for the negative bending section as demonstrated below. The concrete slab is assumed to crack and is neglected in the computation of M_p. The plastic force acting in each element of the girder is first computed.

$P_c = F_{yc}b_ct_c$ = (50)(16)(1.25) = 1,000 kips

$P_w = F_{yw}Dt_w$ = (50)(42)(0.50) = 1,050 kips

$P_t = F_{yt}b_tt_t$ = (50)(14)(1.125) = 788 kips

$P_{rb} = F_{yrb}A_{rb}$ = (60)(2.77) = 166 kips

$P_{rt} = F_{yrt}A_{rt}$ = (60)(5.55) = 333 kips

The plastic forces in each element are used to determine the general location of the plastic neutral axis as follows:

CASE I

$P_c + P_w \geq P_t + P_{rb} + P_{rt}$

$1{,}000 + 1{,}050 \geq 788 + 166 + 333$

$2{,}050 \geq 1{,}287$ Therefore, the plastic neutral axis is in the web.

The location of plastic neutral axis ($\bar{y}$) is determined by the following equation:

$$\bar{y} = \left(\frac{D}{2}\right)\left[\frac{P_c - P_t - P_{rt} - P_{rb}}{P_w} + 1\right]$$

$$\bar{y} = \left(\frac{42}{2}\right)\left[\frac{1000 - 788 - 333 - 166}{1050} + 1\right] = 15.26\text{in.}$$

The plastic moment (M_p) is then computed as follows:

$$M_p = \frac{P}{2D}\left[\bar{y}^2 + \left(D - \bar{y}\right)^2\right] + \left[P_{rt}d_{rt} + P_{rb}d_{rb} + P_td_t + P_cd_c\right]$$

where: d_{rt} = 15.26 + 2 + 8.0 – 2.25 = 23.01 in.

d_{rb} = 15.26 + 2 + 1.25 = 18.51 in.

d_t = 15.26 + 1.125/2 = 15.82 in.

d_c = 42.0 – 15.26 + 1.25/2 = 27.37 in.

$$M_p = \left[\frac{1{,}050}{2(42.0)}\right]\left[(15.26)^2 + (42.0-15.26)^2\right] + \left[(305)(23.01) + (152)(18.51) + (788)(15.82) + (1000)(27.37)\right]$$

M_p = 61,515 k-in. = 5,126 k-ft.

8.1.2.5 Yield Moment: Section 2

The process for determining the yield moment of the negative bending section is similar to the process for the positive bending section. The one difference, though, is that since the composite short-term and the composite long-term bending sections are both composed of the steel section and the reinforcing steel only, the section modulus is the same for both the short-term and long-term composite sections.

The yield moment is the lesser of the moment which causes first yielding of the section, either yielding in the bottom flange or yielding in the steel reinforcing. Because, for the negative bending region it is not clear which yield moment value will control, the moments causing first yield in both compression and tension are computed.

The moment causing yielding in compression flange is first computed based on Equation D6.2.2-1.

$$F_{yf} = \frac{M_{D1}}{S_{NC}} + \frac{M_{D2}}{S_{LT}} + \frac{M_{AD}}{S_{ST}} \qquad \text{Eq. (D6.2.2-1)}$$

$$50 = 1.0\left[\frac{1.25(1{,}334)(12)}{958.6} + \frac{1.25(265)(12) + 1.50(217)(12)}{1{,}057} + \frac{M_{AD}}{1{,}057}\right]$$

M_{AD} = 22,905 k-in. = 1,909 k-ft

M_{yc} = (1.25)(1,334) + (1.25)(265) + (1.50)(217) + 1,909 = 4,233 k-ft

Similarly, the moment which causes yielding in tension (in the steel reinforcing) is computed as follows:

$$50 = 1.0\left[\frac{1.25(1{,}334)(12)}{825.6} + \frac{1.25(265)(12) + 1.50(217)(12)}{954} + 954\right]$$

M_{AD} = 16,697 k-in. = 1,391 k-ft

M_{yt} = (1.25)(1,334) + (1.25)(265) + (1.50)(217) + 1,391 = 3,715 k-ft

M_y = 3,715 k-ft. (governs)

8.2 Exterior Girder Check: Section 2

This design example illustrates the use of the optional moment redistribution procedures, where moment is redistributed from the negative bending region to the positive bending region; therefore the negative bending region will be checked first in order to determine the amount of moment that must be redistributed to the positive bending region.

8.2.1 Strength Limit State (Article 6.10.6)

8.2.1.1 Flexure (Appendix A)

For sections in negative flexure, the flexural capacity of the member can be determined for general steel I-girders using Article 6.10.8, which limits the maximum capacity to the yield moment of the section. Alternatively, Appendix A permits girder capacities up to M_p and may be used for girders: having a yield strength less than or equal to 70 ksi, with a compact or non-compact web (which is defined by Eq. A6.1-1), and satisfying Eq. A6.1-2 (given below). The applicability of Appendix A for this design example is evaluated below.

The first requirement that the nominal yield strength must be less than 70 ksi is easily evaluated.

$$F_{yf} = 50 \text{ ksi} \leq 70 \text{ ksi} \qquad \text{(satisfied)}$$

The web slenderness requirement is evaluated using Eq. A6.1-1.

$$\frac{2D_c}{t_W} < 5.7\sqrt{\frac{E}{F_{YC}}} \qquad \text{Eq. (A6.1-1)}$$

As computed above the elastic neutral axis is located 24.31 inches from the bottom of the composite negative bending section. Subtracting the bottom flange thickness gives the web depth in compression in the elastic range (D_c) as computed below.

$$D_c = 24.31 - 1.25 = 23.06 \text{ in.}$$

Substituting the applicable values into Eq. A6.1-1 shows that the equation is satisfied.

$$\frac{2(23.06)}{0.5} \leq 5.7\sqrt{\frac{29{,}000}{50}}$$

$$92.24 < 137.27 \qquad \text{(satisfied)}$$

Equation A6.1-2 prevents the use of extremely mono-symmetric girders, which analytical studies indicate have significantly reduced torsional rigidity.

$$\frac{I_{yc}}{I_{yt}} \geq 0.3 \qquad \text{Eq. (A6.1-2)}$$

$$\frac{(1/12)(1.25)(16)^3}{(1/12)(1.125)(14)^3} = 1.7 > 0.3 \qquad \text{(satisfied)}$$

Thus, Appendix A is applicable.

The strength requirements specified by Appendix A are given in Section A6.1.1. Since the compression flange is discretely braced, the flexural capacity of the compression flange must exceed the maximum negative moment due to the factored Strength I loading plus one-third of the lateral bending stress multiplied by the section modulus for the compression flange, see Eq. A6.1.1-1.

$$M_u + \frac{1}{3} f_l S_{xc} \leq \phi_f M_{nc} \qquad \text{Eq. (A6.1.1-1)}$$

However, because the lateral bending forces are zero at the Strength I limit state for the straight girders considered in this example, the left side of the equation reduces to only the maximum moment. Similarly, the tensile moment capacity must also be greater than the maximum factored loading.

$$M_u \leq \phi_f M_{nt}$$

Use of Appendix A begins with the computation of the web plastification factors, as detailed in Article A6.2 and calculated below. If the section has a web which satisfies the compact web slenderness limit of Eq. A6.2.1-1, the section can reach M_p provided the flange slenderness and lateral torsional bracing requirements are satisfied.

$$\frac{2D_{cp}}{t_W} < \lambda_{PW(D_{CP})}, \qquad \text{Eq. (A6.2.1-1)}$$

$$\text{where: } \lambda_{pw(D_{CP})} = \frac{\sqrt{\dfrac{E}{F_{yc}}}}{\left(0.54\dfrac{M_p}{R_h M_y} - 0.1\right)^2} \leq \lambda_{rw}\left(\frac{D_{cp}}{D_c}\right) \qquad \text{Eq. (A6.2.1-2)}$$

The web depth in compression at M_p is computed by subtracting the previously determined distance between the top of the web and the plastic neutral axis from the total web depth.

D_{cp} = 42.0 – 15.26 = 26.74 in.

The hybrid factor, R_h, is determined from Article 6.10.1.10.1, and is 1.0 for this example since the design has a homogeneous material configuration. Therefore, λ_{pw} is computed as follows.

$$\lambda_{pw(D_{cp})} = \frac{\sqrt{\frac{29000}{50}}}{\left(0.54\frac{61516}{(1.0)(3715)(12)} - 0.09\right)^2} \leq 157.87\left(\frac{26.74}{23.06}\right) = 183.1$$

$\lambda_{pw(D_{cp})} = 56.11 < 183.1$ (satisfied)

The web slenderness classification is then determined as follows.

$$\frac{2(D_{cp})}{t_w} = \frac{2(26.74)}{0.5} = 107.0 > \lambda_{pw(D_c)} = 56.11 \quad \text{(not compact)}$$

As shown, the section does not qualify as compact. However, it was previously demonstrated, when evaluating the Appendix A applicability, that the web does qualify as non-compact. Therefore, the applicable web plastification factors are specified by Eqs. A6.2.2-4 and A6.2.2-5 and are calculated as follows.

$$R_{pc} = \left[1 - \left(1 - \frac{R_h M_{yc}}{M_p}\right)\left(\frac{\lambda_w - \lambda_{pw(D_C)}}{\lambda_{rw} - \lambda_{pw(D_C)}}\right)\right]\frac{M_p}{M_{yc}} \leq \frac{M_p}{M_{yc}} \quad \text{Eq. (A6.2.2-4)}$$

where: $\lambda_{pw(D_c)}$ = limiting slenderness ratio for a compact web corresponding to $2D_c/t_w$

$$\lambda_{pw(D_c)} = \lambda_{pw(D_{cp})}\left(\frac{D_c}{D_{cp}}\right) = (56.11)\left(\frac{23.06}{26.74}\right) = 48.39 < \lambda_{rw} = 157.87 \quad \text{Eq. (A6.2.2-6)}$$

$$R_{pc} = \left[1 - \left(1 - \frac{(1.0)(4233)(12)}{61515}\right)\left(\frac{92.24 - 48.39}{137.27 - 48.39}\right)\right]\frac{61{,}515}{(4233)(12)} \leq \frac{61515}{(4233)(12)}$$

$R_{pc} = 1.107 \leq 1.211 = 1.107$

$$R_{pc} = \left[1 - \left(1 - \frac{R_h M_{yt}}{M_p}\right)\left(\frac{\lambda_w - \lambda_{pw(D_C)}}{\lambda_{rw} - \lambda_{pw(D_C)}}\right)\right]\frac{M_p}{M_{yt}} \leq \frac{M_p}{M_{yt}} \quad \text{Eq. (A6.2.2-5)}$$

$$R_{pt} = \left[1 - \left(1 - \frac{(1.0)(3715)(12)}{61515}\right)\left(\frac{92.24 - 48.39}{137.27 - 48.39}\right)\right]\frac{61{,}515}{(3715)(12)} \leq \frac{61515}{(3715)(12)}$$

$R_{pt} = 1.192 \leq 1.380 = 1.192$

The flexural resistance based on the compression flange is determined from Article A6.3 and is taken as the minimum of the local buckling resistance from Article A6.3.2 and the lateral torsional buckling resistance from Article A6.3.3.

To evaluate the local buckling resistance, the flange slenderness classification is first determined, where the flange is considered compact if the following equation is satisfied.

$$\lambda_f \leq \lambda_{pf}$$

where: $\lambda_f = \dfrac{b_{fc}}{2t_{fc}}$ Eq. (A6.3.2-3)

$$\lambda_{pf} = 0.38\sqrt{\frac{E}{f_{yc}}}$$ Eq. (A6.3.2-4)

$$\lambda_f = \frac{b_{fc}}{2t_{fc}} \leq \lambda_{pf} = 0.38\sqrt{\frac{E}{F_{yc}}}$$

$$\lambda_f = \frac{16.0}{2(1.25)} \leq \lambda_{pf} = 0.38\sqrt{\frac{29,000}{50}}$$

$\lambda_f = 6.40 \leq \lambda_{pf} = 9.15$ (satisfied)

Therefore, the compression flange is considered compact, and the flexural capacity based on local buckling of the compression flange is governed by Eq. A6.3.2-1.

$M_{nc} = R_{pc}\, M_{yc} = (1.107)(4,233) = 4,686$ k-ft Eq. (A6.3.2-1)

Similarly, to evaluate the compressive flexural resistance based on lateral-torsional buckling, the lateral bracing distance must be first classified. Lateral bracing distances satisfying the following equation are classified as compact.

$$L_b \leq L_p$$

where: $L_b = (10.0)(12) = 120$ ft

$$120' \leq L_p = r_t\sqrt{\frac{E}{F_{yc}}}$$ Eq. (A6.3.3-4)

where: r_t = effective radius of gyration for lateral torsional buckling (in.)

$$r_t = \frac{b_{fc}}{\sqrt{12\left(1 + \frac{1}{3}\frac{D_c\, t_w}{b_{fc}\, t_{fc}}\right)}} = \frac{16.0}{\sqrt{12\left(1 + \frac{1}{3}\frac{(22.79)(0.5)}{(16.0)(1.25)}\right)}}$$ Eq. (A6.3.3-10)

$r_t = 4.234$ in.

$$\therefore L_b > L_p = 4.234\sqrt{\frac{29{,}000}{50}} = 102.0 \qquad \text{(not compact)}$$

Because the lateral bracing distance does not satisfy the compact limit, the non-compact limit is next evaluated.

$$L_p < L_b \le L_r$$

where: L_r = limiting unbraced length to achieve the nominal onset of yielding in either flange under uniform bending with consideration of compression flange residual stress effects (in.)

$$L_r = 1.95 r_t \frac{E}{F_{yr}}\sqrt{\frac{J}{S_{xc}h}}\sqrt{1+\sqrt{1+6.76\left(\frac{F_{yr}S_{xc}h}{EJ}\right)^2}} \qquad \text{Eq. (A6.3.3-5)}$$

F_{yr} = smaller of the compression flange stress at the nominal onset of yielding of either flange, with consideration of compression-flange residual stress effects but without consideration of flange lateral bending, or the specified minimum yield strength of the web

$$F_{yr} = \min\left(0.7F_{yc}, R_h F_{yt}\frac{S_{xt}}{S_{xc}}, F_{yw}\right) \qquad \text{Eq. (A6.3.3-9)}$$

S_{xt} = (3715) (12) / 50 = 891.6 in.^{3}

S_{xx} = (4233) (12) / 50 = 10166 in.^{3}

$$F_{yr} = \min\left(0.7(50),(1.0)(50)\frac{891.6}{1016},50\right)$$

F_{yr} = min(35,43.9, 50)

F_{yr} = 35.0 ksi > 0.5 F_{yc} = 25ksi (satisfied)

J = St. Venant torsional constant

$$J = \frac{1}{3}\left(Dt_w^{\,3} + b_{fc}t_{fc}^{\,3}\left(1-0.63\frac{t_{fc}}{b_{fc}}\right)+b_{ft}t_{ft}\left(1-0.63\frac{t_{ft}}{b_{ft}}\right)\right) \qquad \text{Eq. (A6.3.3-9)}$$

$$J = \frac{1}{3}\left((42)(0.5)^3+(16)(1.25)^3(.95)+(14)(1.125)^3(.95)\right)$$

J = 17.96 in.^{3}

h = depth between the centerline of the flanges

h = 1.125/2 + 42 + 1.25/2 = 43.19 in.

$$L_r = 1.95(4.234)\frac{29000}{35}\sqrt{\frac{17.96}{(1016)(43.19)}}\sqrt{1+\sqrt{1+6.76\left(\frac{35(1016)(43.19)}{(29000)(18.81)}\right)^2}}$$

L_r = 400.8 in.

L_B = $120 \le L_r = 400.8$ (satisfied)

Therefore, the lateral bracing distance is classified as non-compact and the lateral torsional buckling resistance is controlled by Eq. A6.3.3-2 of the Specifications.

$$M_{nc} = C_b\left[1-\left(1-\frac{F_{yr}S_{xc}}{R_{pc}M_{yc}}\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right]R_{pc}M_{yc} \le R_{pc}M_{yc} \qquad \text{Eq. (A6.3.3-2)}$$

where: C_b = moment gradient modifier

The moment gradient modifier is discussed in Article A6.3.3 and is calculated in the following manner.

$$C_b = 1.75 - 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \le 2.3 \qquad \text{Eq. (A6.3.3-7)}$$

where: M_1 = M_o when the variation in moment between brace points in concave and otherwise

M_1 = $2M_{mid} - M_2 \ge M_0$

M_{mid} = major-axis bending moment at the middle of the unbraced length

M_0 = moment at the brace point opposite to the one corresponding to M_2

M_2 = largest major-axis bending moment at either end of the unbraced length causing compression in the flange under consideration

For the critical moment location at the interior pier, the variation in moment is concave throughout the unbraced length and the applicable moment values are as follows.

M_2 = 5,365 k-ft.

M_0 = 2,999 k-ft.

$M_1 = M_0$ = 2,999 k-ft Eq. (A6.3.3-11)

$$C_b = 1.75 - 1.05\left(\frac{2,999}{5,365}\right) + 0.3\left(\frac{2,999}{5,365}\right)^2 = 1.26 \le 2.3$$

$C_b = 1.26$

Therefore, M_{nc} is equal to the following.

$$M_{nc} = (1.26)\left[1-\left(1-\frac{(35.0)(1016)}{(1.107)(4233)(12)}\right)\left(\frac{120-1{,}016}{400.8-120}\right)\right](1.107)(4233) \leq 1.107(4233)$$

$M_{nc} = 5{,}773 \leq 4{,}686$

$M_{nc} = 4{,}686$ k-ft

As previously stated, the flexural capacity based on the compression flange is the minimum of the local buckling resistance and the lateral torsional buckling resistance, which in this design example are equal.

$M_{nc} = 4{,}686$ k-ft

Multiplying the nominal moment capacity by the applicable resistance factor gives the following.

$\phi_f M_{nc} = (1.0)(4{,}686)$

$\phi_f M_{nc} = 4{,}686$ k-ft.

Comparing this moment resistance to the Strength I factored moment at the pier shows that the factored moment is greater than the moment resistance. Thus, moment redistribution may be considered.

$M_u = 5{,}365 > \phi_f M_{nc} = 4{,}695$ k-ft

The moment capacity is also evaluated in terms of the tensile moment capacity. For a continuously braced tension flange at the strength limit state, the section must satisfy the requirements of Article A6.1.4.

$M_u \leq \phi_f R_{pt} M_{yt}$ Eq. (A6.1.4-1)

$\phi_f M_{nt} = \phi_f R_{pt} M_{yt}$

$\phi_f M_{nt} = (1.0)(1.192)(37.15)$

$\phi_f M_{nt} = 4{,}428$ k-ft

Not only is this moment capacity less than the applied Strength I factored moment of 5,365 k-ft, it is also less than the moment capacity determined based on the resistance of the section in compression. Thus, the tensile moment capacity will govern the moment resistance for the negative bending region of the girder.

$\phi_f M_{nt} = 4{,}428$ k-ft. $< M_u = 5{,}365$ k-ft.

$\phi_f M_{nt}$ = 4,428 k-ft. < M_{nc} = 4,686 k-ft.

$\phi_f M_n$ = 4,428 k-ft.

8.2.1.2 Moment Redistribution (Appendix B, Sections B6.1 – B6.5)

Article B6.2 defines the applicability of the Appendix B provisions. Specifically the sections must be straight continuous span I-sections that are not skewed more than 10 degrees and do not have staggered cross-frames. The specified minimum yield strength of the section must not exceed 70 ksi. In addition, the section must satisfy web proportions (Article B6.2.1), compression flange proportions (Article B6.2.2), section transition (Article B6.2.3), compression flange bracing (Article B6.2.4), and shear (Article B6.2.5) requirements, which are discussed below.

8.2.1.2.1 Web Proportions

Equations B6.2.1-1, B6.2.1-2, and B6.2.1-3 specify the web proportion limits that must be satisfied.

$$\frac{D}{t_w} \leq 150 \qquad \text{Eq. (B6.2.1-1)}$$

$$\frac{D}{t_w} = \frac{42.0}{0.5} = 84.0 \leq 150 \qquad \text{(satisfied)}$$

$$\frac{2D_c}{t_w} \leq 6.8\sqrt{\frac{E}{F_{yc}}} \qquad \text{Eq. (B6.2.1-2)}$$

$$\frac{2(23.06)}{0.50} = 92.24 \leq 6.8\sqrt{\frac{29{,}000}{50}} = 163.8 \qquad \text{(satisfied)}$$

$$D_{cp} \leq 0.75D \qquad \text{Eq. (B6.2.1-3)}$$

$$D_{cp} = 26.74 \leq 0.75(42.0) = 31.50 \qquad \text{(satisfied)}$$

8.2.1.2.2 Compression Flange Proportions

Section B6.2.2 requires that the following two compression flange proportion limits must be satisfied.

$$\frac{b_{fc}}{2t_{fc}} \leq 0.38\sqrt{\frac{E}{F_{yc}}} \qquad \text{Eq. (B6.2.2-1)}$$

$$\frac{16}{2(1.25)} = 6.40 \leq 0.38\sqrt{\frac{29{,}000}{50}} = 9.15 \qquad \text{(satisfied)}$$

$$b_{fc} \geq \frac{D}{4.25}$$ Eq. (B6.2.2-2)

$$b_{fc}=16.0 \geq \frac{42}{4.25}=9.88$$ (satisfied)

8.2.1.2.3 Compression Flange Bracing Distance

The compression flange bracing distance must satisfy the following:

$$L_b \leq \left[0.1 - 0.06\left(\frac{M_1}{M_2}\right)\right]\frac{r_t E}{F_{yc}}$$ Eq. (B6.2.4-1)

$$L_b=120.0 \leq \left[0.1\text{-}0.06\left(\frac{2{,}999}{5{,}365}\right)\right]\frac{(4.234)(29{,}000)}{50}=163.2$$ (satisfied)

8.2.1.2.4 Shear

Additionally, the applied shear under the Strength I loading must be less than the shear buckling resistance of the girder as specified by the following:

$$V \leq \phi_v V_{cr}$$ Eq. (B6.2.5-1)

where: V_{cr} = shear buckling resistance (kip)

V_{cr} = CV_p (for unstiffened webs) Eq.(6.10.9.2-1)

V_p = plastic shear force (kip)

V_p = $0.58\ F_{yw}Dt_w$ Eq. (6.10.9.2-2)

C = ratio of the shear buckling resistance to the shear yield strength determined as specified in Article 6.10.9.3.2, with the shear buckling coefficient, k, taken equal to 5.0

Alternative equations are provided for computing the value of C based on the web slenderness of the girder. First the web slenderness is evaluated using the following equation.

$$\frac{D}{t_w} \leq 1.12\sqrt{\frac{Ek}{F_{yw}}}$$

$$\frac{42.0}{0.50}=84.0>1.12\sqrt{\frac{(29{,}000)(5)}{50}}=60.31$$ (not satisfied)

The web slenderness is next evaluated using the following equation.

$$1.12\sqrt{\frac{Ek}{F_{yw}}} \le \frac{D}{t_w} \le 1.40\sqrt{\frac{Ek}{F_{yw}}}$$

$$1.12\sqrt{\frac{Ek}{F_{yw}}} = 60.31 < \frac{D}{t_w} = 84.0 > 1.40\sqrt{\frac{Ek}{F_{yw}}} = 75.4 \qquad \text{(not satisfied)}$$

Thus, the governing equation for computing the ratio C is given by Eq. 6.10.9.3.2-6, which is applicable when:

$$\frac{D}{t_w} = 84.0 > 1.40\sqrt{\frac{Ek}{F_{yw}}} = 75.4 \qquad \text{(satisfied)}$$

$$C = \frac{1.57}{\left(\frac{D}{t_w}\right)^2}\left(\frac{Ek}{F_{yc}}\right) \qquad \text{Eq. (6.10.9.3.2-6)}$$

$$C = \frac{1.57}{(84.0)^2}(2{,}900) = 0.645$$

The shear buckling resistance is then computed as follows.

$$V_{cr} = CV_p = (0.645)(0.58)(50)(42)(0.5) = 392.8 \text{ kips}$$

The shear requirement for Appendix B can then be evaluated.

$$V = 337 \text{ kips} \le \varphi V_{cr} = (1.0)(392.8) = 392.8 \text{ kips} \qquad \text{(satisfied)}$$

The provisions of Article B6.2.1 through B6.2.6 are satisfied for this section. Therefore, moments may be redistributed in accordance with Appendix B

The effective plastic moment, determined from Article B6.5, is a function of the geometry and material properties of the section. Furthermore, alternative equations are provided for girders that satisfy the requirements for enhanced moment rotation characteristics, i.e., classification as ultracompact sections. To be classified as ultracompact, the girder must either (1) contain transverse stiffeners at a location less than or equal to one-half the web depth from the pier or (2) satisfy the web compactness limit given by Eq. B6.5.1-1.

$$\frac{2D_{cp}}{t_w} \le 2.3\sqrt{\frac{E}{F_{yc}}} \qquad \text{Eq. (B6.5.1-1)}$$

$$\frac{2(26.74)}{0.50} = 107.0 > 2.3\sqrt{\frac{29{,}000}{50}} = 55.4 \qquad \text{(not satisfied)}$$

Therefore, the section does not satisfy the web compactness limit and because the section uses an unstiffened web, the girder does not satisfy the transverse stiffener requirement. Thus, the girder is not considered to be ultracompact and the applicable M_{pe} equation at the strength limit state is thus Eq. B6.5.2-2.

$$M_{pe} = \left[2.63 - 2.3 \frac{b_{fc}}{t_{fc}} \sqrt{\frac{F_{yc}}{E}} - 0.35 \frac{D}{b_{fc}} + 0.39 \frac{b_{fc}}{t_{fc}} \sqrt{\frac{F_{yc}}{E}} \frac{D}{b_{fc}} \right] M_n \leq M_n \qquad \text{Eq. (B6.5.2-2)}$$

$$M_{pe} = \left[2.63 - 2.3 \frac{16}{1.25} \sqrt{\frac{50}{29000}} - 0.35 \frac{42}{16} + 0.39 \frac{16}{1.25} \sqrt{\frac{50}{29000}} \frac{42}{16} \right] 4428 \leq 4428$$

M_{pe} = 4,574 ≤ 4,428 = 4,428 k-ft

The redistribution moment, M_{rd}, for the strength limit state is taken as the larger of the values calculated from Eqs. B6.4.2.1-1 and B6.4.2.1-2.

$$M_{rd} = |M_e| + \frac{1}{3} f_l S_{xc} - \phi_f M_{pe} \qquad \text{Eq. (B6.4.2.1-1)}$$

$$M_{rd} = |M_e| + \frac{1}{3} f_l S_{xt} - \phi_f M_{pe} \qquad \text{Eq. (B6.4.2.1-2)}$$

where: M_e = critical elastic moment envelope value at the interior-pier section due to the factored loads

Since the lateral bending stresses are negligible for this example, the previous equations reduce to the following equation.

$$M_{rd} = |M_e| - \phi_f M_{pe}$$

In addition, the redistribution moment is limited to 20 percent of the elastic moment by Eq. B6.4.2.1-3.

$$0 \leq M_{rd} \leq 0.2 |M_e| \qquad \text{Eq. (B6.4.2.1-3)}$$

Therefore, the redistribution moment is computed as follows, which is shown to satisfy the 20% limit.

$M_{rd} = |M_e| - \phi_f M_{pe}$ = 5,365 - (1.0)(4,429)

M_{rd} = 937 k-ft = 17.5% M_e ≤ 20% M_e

Therefore, the negative bending region of the girder satisfies strength requirements when the effective plastic moment equations given in Appendix B are used to evaluate girder capacity.

8.2.1.3 Moment Redistribution - Refined Method (Appendix B, Section B6.6)

Article B6.6 of Appendix B contains specifications for computing redistribution moments using a direct method of analysis. Using this analysis procedure, the effective plastic moments are computed based on the rotation at which the continuity curve intersects the moment-rotation curve, as opposed to assuming that this intersection occurs at a plastic rotation of 30 mrads, as assumed in the effective plastic moment equations utilized above.

In cases such as this example, where the effective plastic moment is equal to the nominal moment capacity of the negative bending section, there is no advantage to be gained by using the refined method. This is because the peak value of the moment-rotation curve is equal to M_n , the maximum value of M_{pe} possible, irrespective of using the effective plastic moment equations from Article B6.5 or the refined method of Article B6.6. However, in other cases the use of the refined method may lead to higher values of M_{pe}, further increasing the economic benefits of using the moment redistribution procedures. For this reason, use of the refined method for the present design is demonstrated below.

The first step in using the refined method for moment redistribution is to determine the moment-rotation curve for the negative bending section. This is done using Figure B6.6.2-1 from the *AASHTO LRFD (5th Edition, 2010)* Specifications, which is reproduced in Figure 12. From Figure 12 it is observed that the moment-rotation relationship is a function of the single parameter, θ_{RL}, which is the rotation at which the moment begins to decrease below the nominal moment capacity. Similar to the equations for M_{pe} given for the simplified method introduced above, alternative equations for θ_{RL} are given based on whether the negative bending section satisfies the criteria for enhanced moment rotation characteristics given by Section B6.5. It has been shown above that the negative bending section does not satisfy either of the requirements for sections with enhanced moment-rotation performance. Thus, θ_{RL} is given in radians by Eq. B6.6.2-2.

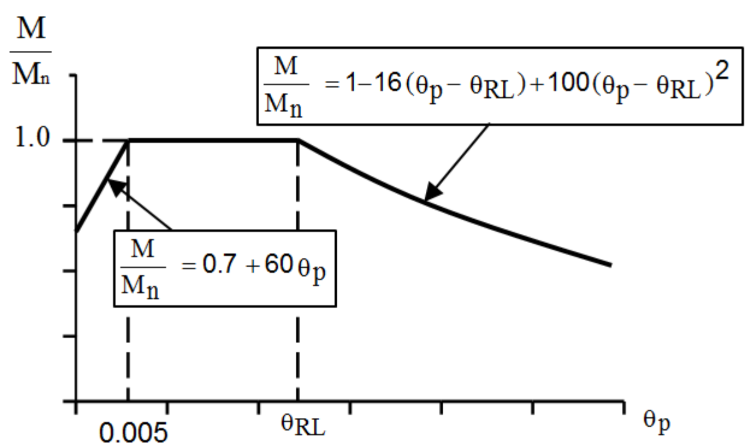

Figure 12 AASHTO LRFD Moment-Rotation Model

$$\theta_{RL} = 0.128 - 0.143\frac{b_{fc}}{t_{fc}}\sqrt{\frac{F_{yc}}{E}} - 0.0216\frac{D}{b_{fc}} + 0.0241\frac{b_{fc}}{t_{fc}}\frac{D}{b_{fc}}\sqrt{\frac{F_{yc}}{E}} \qquad \text{Eq. (B6.6.2-2)}$$

Substituting the applicable values into Eq. B6.6.2-2 gives the following.

$$\theta_{RL} = 0.128 - 0.143\frac{16}{1.25}\sqrt{\frac{50}{29000}} - 0.0216\frac{42}{16} + 0.0241\frac{16(42)}{0.5(16)}\sqrt{\frac{50}{29000}} = 0.079$$

Thus, θ_{RL} is equal to 0.079 radians or 79 mrads. Recalling that the nominal moment capacity of the negative moment section of this girder is 4428 ft-kips, the predicted moment-rotation relationship of the example girder is as illustrated in Figure 13.

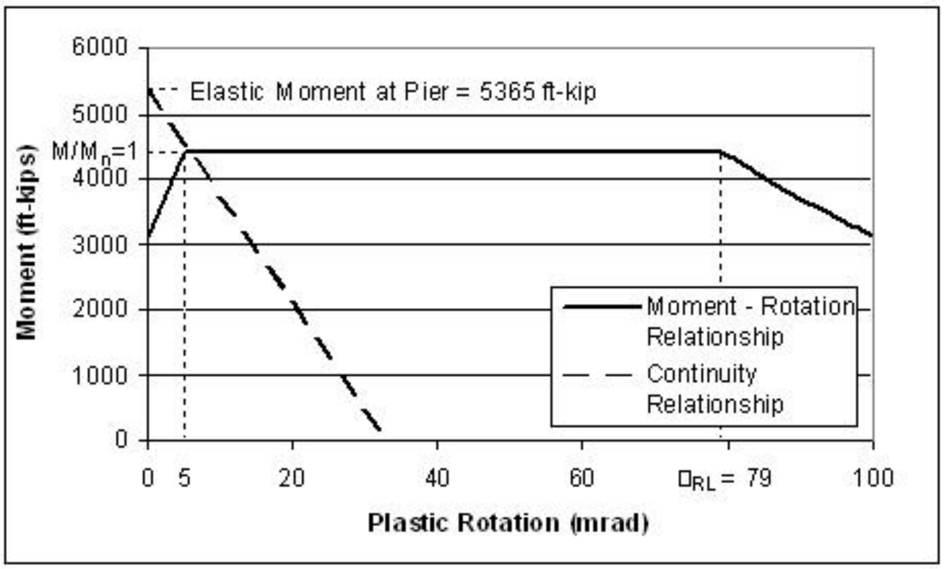

Figure 13 Determination of M_{pe} Using Refined Method

In addition to the moment-rotation relationship, the continuity relationship must also be determined. The continuity relationship is a linear relationship between the elastic moment at the pier (where no plastic rotation occurs) and the rotation assuming no continuity at the pier. The elastic moment at the pier has previously been determined to equal 5365 ft-kips, which is the y-intercept for the continuity relationship. To determine the x-intercept of the continuity relationship, the beam is analyzed assuming that a hinge exists at each pier, and rotations due to applied moments equal to the elastic moment are computed as shown in Figure 14. In this analysis, the *AASHTO LRFD (5th Edition, 2010)* Specifications stipulate that the section properties of the short-term composite section shall be used. Thus, the applicable moment of inertia of the positive bending section is 48,806 in^4 and the moment of inertia value used for the negative bending section is 50,027 in^4. From basic structural analysis, or the use of structural analysis software, the rotation at the pier for the situation depicted in Figure 14 is then 32.88 mrads, which is the x-intercept for the continuity relationship. Based on the x- and y- intercepts of the continuity relationship, the continuity equation is thus expressed as

$$M = 5365 \text{ ft-kips} - 163.17 \text{ ft-kips} * \theta_p$$

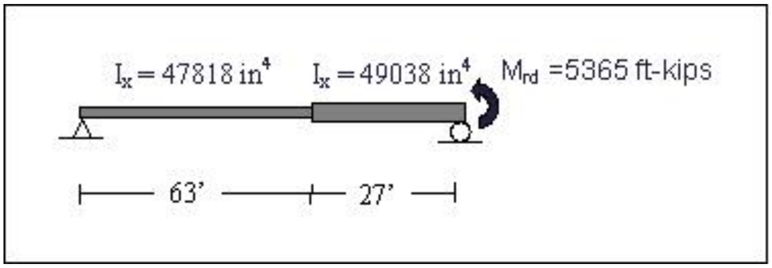

Figure 14 Determination of Rotation at Pier Assuming No Continuity

The moment at the intersection of the continuity relationship and the moment-rotation relationship is the effective plastic moment. From Figure 13 it is illustrated that this moment is equal to the nominal moment capacity of 4428 ft-kips. The effective plastic moment can also be determined mathematically by iteratively selecting θ_p values to be substituted into both the moment-rotation and continuity curves until the moment converges. Alternatively, for the present girder it is known that the moment is equal to M_n for θ_p values between 5 and 79 mrads. Solving the continuity equation for the value of θ_p at M_n gives a rotation of:

$$\theta_p = (5365 - 4428) / 163.17 = 5.7 \text{ mrads.}$$

Since this value is between 5 and 79 mrads, it is mathematically determined that the effective plastic moment is equal to M_n. Once M_{pe} is determined, the moment redistribution analysis proceeds in the same manner used in the simplified method outlined above, where the redistribution moments are computed as the difference between the elastic and the effective plastic moments as specified in Sections B6.3 and B6.4 and the girder is determined to satisfy strength requirements if the redistribution moment is less than 20% of the elastic moment.

8.2.1.4 Shear (6.10.6.3)

As computed above the shear resistance of the negative bending region is governed by Article 6.10.9.2 because the girder is comprised of an unstiffened web, i.e., no transverse stiffeners are provided. The shear resistance of the section was previously calculated to be:

$$V_n = V = CV_p = 392.8 \text{kips} \qquad \text{Eq. (6.10.9.2-1)}$$

The applied shear at the pier at the strength limit state is 337 kips, thus the shear requirements are satisfied.

$$V = 337 \text{kips} \le \varphi_V V_{cr} = (1.0)(392.8) = 392.8 \text{kips} \qquad \text{(satisfied)}$$

8.2.2 Constructibility (Article 6.10.3)

Article 2.5. requires the engineer to design bridge systems such that the construction is not difficult or results in unacceptable locked-in forces. In addition, Article 6.10.3 states the main load-carrying members are not permitted to experience nominal yielding, or reliance on post-buckling resistance during the construction phases. The sections must satisfy the requirements of Article 6.10.3 at each construction stage. The applied loads to be considered are specified in Table 3.4.1-1 and the applicable load factors are provided in Article 3.4.2.

The girders are considered to be non-composite during the initial construction phase. The influence of various segments of the girder becoming composite at various stages of the deck casting sequence is then considered. The effects of forces from deck overhang brackets acting on the fascia girders are to be included in the constructibility check.

8.2.2.1 Deck Placement Analysis

Temporary moments the noncomposite girders experience during the casting of the deck can be significantly higher than those which may be calculated based on the final conditions of the system. An analysis of the moments during each casting sequence must be conducted to determine the maximum moments in the structure. The potential for uplift should also be investigated if the casting of the two end pours does not occur simultaneously.

Figure 15 depicts the casting sequence assumed in this design example. As required in Article 6.10.3.4, the loads are applied to the appropriate composite sections during each casting sequence. For example, it is assumed during Cast One that all sections of the girder are non-composite. Similarly, the dead load moments due to the steel components are also based on the non-composite section properties. However, to determine the distribution of moments due to Cast Two, the short-term composite section properties are used in the regions of the girders that were previously cast in Cast One, while the non-composite section properties are used in the region of the girder where concrete is cast in Cast Two. The moments used in the evaluation of the constructability requirements are then taken as the maximum moments that occur during any stage of construction, i.e., the sum of the moments due to the steel dead load and the first casting phase or the sum of the moments due to the steel dead load and both casting phases. Additionally, while not required, the dead load moment resulting from applying all dead load to the short-term composite section (DC1) is also considered.

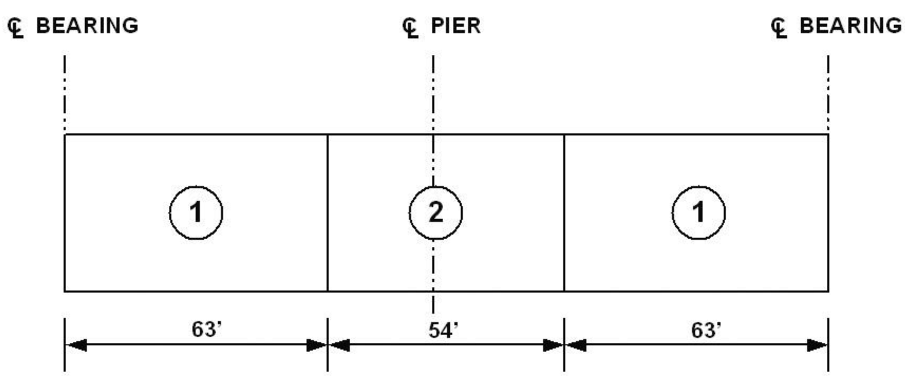

Figure 15 Deck Placement Sequence

The results of the deck placement analysis are shown in Table 17 where the maximum dead load moments in the positive and negative bending regions are indicated by bold text. Note that the maximum positive bending moment during construction occurs during Cast 2, and that the maximum negative bending moment occurs when it is assumed that the loads are simultaneously applied to the composite section.

Table 17 Moments from Deck Placement Analysis (kip-ft)

x/L	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Dist. (ft.)	0	9	18	27	36	45	54	63	72	81	90
Steel Wt.	0	49	82	101	106	96	71	31	-22	-90	-173
SIP Forms	0	27	46	56	58	53	39	16	-13	-50	-94
Cast 1	0	260	437	532	544	474	321	86	-181	-447	-714
Cast 2	0	301	518	654	707	677	565	370	105	-238	-656
Σ Cast 1	0	335	565	689	708	622	430	133	-216	-587	-981
Σ Cast 2	0	376	646	811	**871**	825	674	417	70	-377	-923
DC1	0	343	581	712	738	657	471	178	-220	-724	**-1334**

Article 6.10.1.6 states that when checking the flexural resistance based on lateral torsional buckling f_{bu} is the largest compressive stress in the flange under consideration, without consideration of flange lateral bending, throughout the unbraced length. When checking the flexural resistance based on yielding, flange local buckling or web bend buckling, f_{bu} is the stress at the section under consideration. The maximum factored flexural stresses due to the deck casting sequence are calculated below. The controlling section during the constructibility check for Section 2 is at the pier.

8.2.2.1.1 Strength I

Top Flange

$$f_{bu}=\frac{1.0(1.25)(1{,}334)(12)}{825.6}=24.24\text{ ksi}$$

Bottom Flange

$$f_{bu}=\frac{1.0(1.25)(-1{,}334)(12)}{958.6}=-20.87\text{ ksi}$$

8.2.2.1.2 Strength IV

Top Flange

$$f_{bu}=\frac{1.0(1.50)(1{,}334)(12)}{825.6}=29.08\text{ ksi}$$

Bottom Flange

$$f_{bu}=\frac{1.0(1.50)(-1{,}334)(12)}{958.6}=-25.05\text{ ksi}$$

8.2.2.2 Deck Overhang Loads

The deck overhang bracket configuration assumed in this example is shown in Figure 16. Typically the brackets are spaced between 3 and 4 feet, but the assumption is made here that the loads are uniformly distributed, except for the finishing machine. Half of the overhang weight is assume to be carried by the exterior girder, and the remaining half is carried by the overhang brackets.

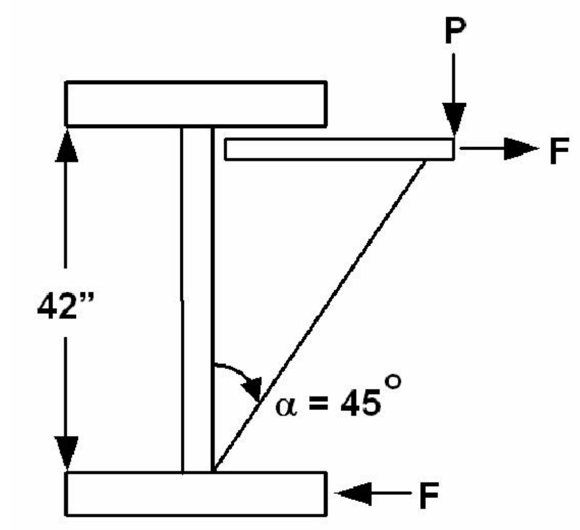

Figure 16 Deck Overhang Bracket Loads

The following calculation determines the weight of deck overhang acting on the overhang bracket.

$$P = 0.5(150)\left[\frac{8.5}{12}(3.5) + \left[\frac{1}{12}\left(\frac{2.0}{2}\right)\left(3.5 - \frac{14/2}{12}\right)\right] + \frac{1.25}{12}\left(\frac{14/2}{12}\right)\right] = 208.7 \text{ lbs/ft}$$

The following is a list of typical construction loads assumed to act on the system before the concrete slab gains strength. The magnitudes of load listed are those that are applied to only the overhang brackets.

Overhang Deck Forms:	P = 40 lb/ft
Screed Rail:	P = 85 lb/ft
Railing:	P = 25 lb/ft
Walkway:	P = 125 lb/ft
Finishing Machine:	P = 3,000 lb

The weight of the finishing machine is estimated as one-half of the total finishing machine truss weight. The lateral force acting on the girder section due to the vertical loading is computed as follows.

$F = P\tan\alpha$

where: $\alpha = \tan^{-1}\left(\frac{42\text{in.}}{42\text{in.}}\right) = 45^{\circ}$

The equations provided in Article C6.10.3.4 to determine the lateral bending moment can be employed in the absence of a more refined method. From the article, the following equation determines the lateral bending moment for a uniformly distributed lateral bracket force:

$$M_l = \frac{F_l L_b^2}{12}$$

where: M_ℓ = lateral bending moment in the top flange due to the eccentric loadings from the form brackets

F_ℓ = statically equivalent uniformly distributed lateral force due to the factored loads

The equation which estimates the lateral bending moment due to a concentrated lateral force at the middle of the unbraced length is as follows.

$$M_l = \frac{P_l L_b}{8}$$

where: P_ℓ = statically equivalent concentrated force placed at the middle of the unbraced length

For simplicity, the largest value of f_ℓ within the unbraced length is conservatively used in the design checks, i.e., the maximum value of f_ℓ within the unbraced length is the assumed stress level throughout the unbraced length. The unbraced length for the section under consideration is 10 feet.

Article 6.10.1.6 specifies the process for determining the lateral bending stress. The first-order lateral bending stress may be used if the following limit is satisfied.

$$L_b \leq 1.2 L_p \sqrt{\frac{C_b R_b}{f_{bm}/F_{yc}}} \qquad \text{Eq. (6.10.1.6-2)}$$

where: L_p = limiting unbraced length from Article 6.10.8.2.3 of the Specifications

C_b = moment gradient modifier

R_b = web load-shedding factor

F_{yc} = yield strength of the compression flange

The moment gradient modifier is discussed in Article A6.3.3 and is calculated in the following manner.

$$C_b = 1.75 - 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \le 2.3$$ Eq. (A6.3.3-7)

where: M_1 = $2M_{mid} - M_2 \ge M_0$ Eq. (A6.3.3-12)

M_{mid} = major-axis bending moment at the middle of the unbraced length

M_0 = moment at the brace point opposite to the one corresponding to M_2

M_2 = largest major-axis bending moment at either end of the unbraced length causing compression in the flange under consideration

The following values correspond to the results of the deck placement analysis.

$M_2 = 1{,}334$ k-ft.	$M_1 = 2M_{mid} - M_2 \ge M_0$
$M_0 = 668$ k-ft.	$M_1 = 2(995) - (1{,}334) = 656 \le 668$
$M_{mid} = 995$ k-ft.	$M_1 = 668$ k-ft.

Thus, C_b is calculated as follows.

$$C_b = 1.75 - 1.05\left(\frac{668}{1{,}334}\right) + 0.3\left(\frac{668}{1{,}334}\right)^2 = 1.30 \le 2.3$$

According to Article 6.10.1.10.2, the web load-shedding factor, R_b, is 1.0 when checking constructibility. Thus, Eq. 6.10.1.6-2 is evaluated as follows.

$$L_b = 120\text{ in.} \le 1.2(103.2)\sqrt{\frac{(1.30)(1.0)}{29.08/50}} = 185.1\text{ in.}$$

Hence, it is shown that the first-order elastic analysis is applicable.

According to Article 3.4.2, a load factor of 1.5 is applied to construction loads for all strength limit states. For other dead loads, a load factor of 1.25 is used for the Strength I load combination, while a load factor of 1.5 is used for dead load under the Strength IV load combination. Additionally, live load is not considered under the Strength IV load combinations.

8.2.2.2.1 Strength I

The lateral bending forces at the Strength I limit state are computed as follows.

Dead loads:

P = [1.25(209) + 1.5(40 + 85 + 25 + 125)] = 673.8 lbs/ft.

F = F_ℓ = P tan α = 673.8 tan (45°) = 673.8 lbs/ft.

$$M_l = \frac{F_l L_b^{\,2}}{12} = \frac{(0.6738)(10)^2}{12} = 5.62\,\text{k-ft}$$

Top Flange: $f_l = \dfrac{M_l}{S_l} = \dfrac{5.62(12)}{1.125(14)^2/6} = 1.84\,\text{ksi}$

Bottom Flange: $f_l = \dfrac{M_l}{S_l} = \dfrac{5.62(12)}{1.25(16)^2/6} = 1.26\,\text{ksi}$

Live loads:

P = [1.5(3,000)] = 4,500 lbs.

F = P_ℓ = P tan a = 4,500 tan (45°) = 4,500 lbs.

$$M_l = \frac{P_l L_b}{8} = \frac{(4.5)(10)}{8} = 5.63\,\text{k-ft}$$

Top Flange: $f_L = \dfrac{M_L}{S_L} = \dfrac{5.63(12)}{1.125(14)^2/6} = 1.84\,\text{ksi}$

Bottom Flange: $f_L = \dfrac{M_L}{S_L} = \dfrac{5.63(12)}{1.25(16)^2/6} = 1.26\,\text{ksi}$

Total:

Top flange: f_ℓ = 1.84 + 1.84 = 3.68 ksi

Bot. flange: f_ℓ = 1.26 + 1.26 = 2.52 ksi

8.2.2.2.2 Strength IV

The computation of the lateral bending forces at the Strength IV limit state is demonstrated below.

Dead loads:

P = [1.5(209 + 40 + 85 + 25 + 125)] = 726.0 lbs/ft.

F = F_ℓ = P tan α = 726.0 tan (45°) = 726.0 lbs/ft.

$$M_l = \frac{F_l L_b^2}{12} = \frac{(0.7260)(10)^2}{12} = 6.05 \text{ k-ft}$$

Top Flange: $f_l = \dfrac{M_l}{S_l} = \dfrac{6.05(12)}{1.125(14)^2/6} = 1.98 \text{ ksi}$

Bottom Flange: $f_l = \dfrac{M_l}{S_l} = \dfrac{6.05(12)}{1.25(14)^2/6} = 1.36 \text{ ksi}$

Live loads:

Not applicable

Total:

Top flange: f_ℓ = 1.98 ksi

Bot. flange: f_ℓ = 1.36 ksi

According to Article 6.10.1.6, the lateral bending stresses must be less than 60 percent of the yield stress of the flange under consideration. It is shown above that the lateral bending stresses are highest at the pier under the Strength I load combination. Thus, evaluation of Eq. 6.10.1.6-1 at the Strength I limit state is shown below.

$$f_l \leq 0.6F_y \qquad (6.10.1.6\text{-}1)$$

Top flange: f_ℓ = 3.68 ksi < $0.6F_{yf}$ = 30 ksi (satisfied)

Bot. flange: f_ℓ = 2.52 ksi < $0.6F_{yf}$ = 30 ksi (satisfied)

8.2.2.3 Flexure (Article 6.10.3.2)

During construction, both the compression and tension flanges are discretely braced. Therefore, Article 6.10.3.2 requires the noncomposite section to satisfy Eqs. 6.10.3.2.1-1, 6.10.3.2.1-2, and 6.10.3.2.1-3, which ensure the flange stress is limited to the yield stress, the section has sufficient strength under the lateral torsional and flange local buckling limit states, and web bend buckling does not occur during construction, respectively.

8.2.2.3.1 Compression Flange:

Flange nominal yielding: The allowable stress in the compression flange is limited to the nominal yield strength of the flange multiplied by the hybrid factor.

$$F_{bu} + f_l \leq \varphi_f R_h F_{yc} \qquad (6.10.3.2.1\text{-}1)$$

Since the section under considerations has a homogeneous material configuration, the hybrid factor is 1.0, as stated in Article 6.10.1.10.1. Thus, Eq. 6.10.3.2.1-1 is evaluated as follows.

$$25.05 + 1.36 \leq (1.0)(1.0)(50)$$

$$26.41\,\text{ksi} \leq 50\,\text{ksi} \qquad \text{(satisfied)}$$

Flexural Resistance: The flexural resistance of the noncomposite section is required to be greater than the maximum bending moment as a result of the deck casting sequence plus one third of the lateral bending stresses, as expressed by:

$$f_{bu} + \frac{1}{3} f_l \leq \phi_f F_{nc} \qquad \text{Eq. } (6.10.3.2.1\text{-}2)$$

According to Article 6.10.3.2.1, the flexural resistance, F_{nc}, is determined as specified in Article 6.10.8.2 or Article A6.3.3, if applicable. Two requirements provided in Article A6.1 must be satisfied for Article A6.3.3 to be applicable.

$$F_{yf} = 50\text{ksi} \leq 70\text{ksi} \qquad \text{(satisfied)}$$

$$\frac{2D_c}{t_w} < 5.7\sqrt{\frac{E}{F_{yc}}} \qquad \text{Eq. } (A6.1\text{-}1)$$

D_c = 20.62 – 1.25 = 19.37 in.

$$\frac{2(19.37)}{(0.5)} < 5.7\sqrt{\frac{(29{,}000)}{(50)}}$$

$$77.48 < 137.27 \qquad \text{(satisfied)}$$

Therefore, Appendix A is applicable.

The sections for which Appendix A is applicable have either compact or noncompact web sections where the web classification dictates the equations used to determine the moment capacity. The section qualifies as a compact web section if Eq. A6.2.1-1 is satisfied.

$$\frac{2D_{cp}}{t_w} < \lambda_{pw(D_{CP})}$$ Eq. (A6.2.1-1)

where: D_{cp} = depth of web in compression at the plastic moment

R_h = hybrid factor

$\lambda_{pw(Dcp)}$ = limiting slenderness ratio for a compact web corresponding to $2D_{cp}/t_w$

$$= \frac{\sqrt{\dfrac{E}{F_{yc}}}}{\left(0.54\dfrac{M_p}{R_h M_y} - 0.1\right)^2}$$ Eq. (A6.2.1-2)

The location of the plastic neutral axis of the steel section must be determined to calculate the depth of web in compression. The equations from Appendix D are employed for this purpose.

P_c	=	(16)(1.25)(50)	= 1,000 kips
P_t	=	(14)(1.125)(50)	= 788 kips
P_w	=	(42)(0.5)(50)	= 1,050 kips
Pc + Pw	=	1,000 + 1,050 kips	= 2,050 ≥ Pt =788 kips

Therefore, the location of the plastic neutral axis is in the web (Table D6.1-2, Case I) and the precise location is computed as follows.

$$\bar{y} = \left(\frac{D}{2}\right)\left[\frac{P_c - P_t}{P_w} + 1\right] = \left(\frac{42}{2}\right)\left[\frac{1{,}000 - 788}{1{,}050} + 1\right] = 25.24\,\text{in.}$$

The plastic neutral axis is located 25.24 inches below the bottom of the top flange. The plastic moment can be determined from the following equation:

$$M_p = \frac{P_w}{2D}\left[\bar{y}^2 + \left(D - \bar{y}\right)^2\right] + P_t d_t + P_c d_c$$

$$M_p = \frac{1{,}050}{2(42)}\left[(25.24)^2 + \left(42 - 25.24\right)^2\right] + (788)(25.24 + 1.125/2) + (1{,}000)(42 - 25.24 + 1.25/2)$$

M_p = 49,192 kip-in = 4,099 kip-ft

From the above calculations, the depth of web in compression can be calculated.

D_{cp} = 42.0 – 25.24 = 16.76 in.

Furthermore, the web slenderness is now evaluated.

$$\lambda_{pw(D_{cp})} = \frac{\sqrt{\dfrac{29,000}{50}}}{\left(0.54\dfrac{49,192}{(1.0)(50)(825.6)} - 0.1\right)^2} = 81.53$$

$$\frac{2D_{cp}}{t_w} = \frac{2(16.76)}{0.5} = 67.04 < \lambda_{pw(D_{cp})} = 81.53 \qquad \text{(satisfied)}$$

Therefore, the section qualifies as a compact web section, and the web plastification factors are determined from Eqs. A6.2.1-4 and A6.2.1-5, where M_{yc} and M_{yt} are the yield moments with respect to the compression and tension flanges, respectively.

$$R_{pc} = \frac{M_p}{M_{yc}} = \frac{49,192}{(50)(958.6)} = 1.026 \qquad \text{Eq. (A6.2.1-4)}$$

$$R_{pt} = \frac{M_p}{M_{yt}} = \frac{49,192}{(50)(825.6)} = 1.192 \qquad \text{Eq. (A6.2.1-5)}$$

As previously discussed, the lateral torsional buckling resistance is provided in Article A6.3.3. If the following equation is satisfied the lateral brace spacing is classified as compact.

$L_b \leq L_p$

where: L_b = unbraced length (in.)

L_p = limiting unbraced length to achieve the nominal flexural resistance $R_{pc}M_{yc}$ under uniform bending (in.)

$$r_t\sqrt{\frac{E}{F_{yc}}} \qquad \text{Eq. (A6.3.3-4)}$$

r_t = effective radius of gyration for lateral torsional buckling (in.)

$$r_t = \frac{b_{fc}}{\sqrt{12\left(1 + \dfrac{1}{3}\dfrac{D_c t_w}{b_{fc} t_{fc}}\right)}} = \frac{16}{\sqrt{12\left(1 + \dfrac{1}{3}\dfrac{(19.37)(0.5)}{(16)(1.25)}\right)}} = 4.286\,\text{in.} \qquad \text{Eq. (A6.3.3-10)}$$

$$L_b = (10.0)(12) = 120\,\text{in.} > L_p = r_t\sqrt{\frac{E}{F_{yc}}} = 103.2\,\text{in.}$$

Therefore, the unbraced spacing is not compact, and the following inequality is evaluated to determine if the unbraced distance is classified as non-compact.

$L_b \leq L_r$

where: L_r = limiting unbraced length to achieve the nominal onset of yielding in either flange under uniform bending (in.) with consideration of compression flange residual stresses

$$= 1.95 r_t \frac{E}{F_{yr}} \sqrt{\frac{J}{S_{xc} h}} \sqrt{1 + \sqrt{1 + 6.76 \left(\frac{F_{yr} S_{xc} h}{EJ} \right)^2}} \qquad \text{Eq. (A6.3.3-5)}$$

$$1.95(4.286) \frac{29{,}000}{35} \sqrt{\frac{17.96}{(958.6)(43.19)}} \sqrt{1 + \sqrt{1 + 6.76 \left(\frac{35}{29{,}000} \frac{(958.6)(43.19)}{18.81} \right)^2}} = 407.4 \text{ in.}$$

F_{yr} = smaller of the compression flange stress at the nominal onset of yielding of either flange, with consideration of compression flange residual stress effects but without consideration of flange lateral bending, or the specified minimum yield strength of the web.

$$= \min \left(0.7 F_{yc}, R_h F_{yt} \frac{S_{xt}}{S_{xc}}, F_{yw} \right)$$

$$= \min \left(0.7(50), (1.0)(50) \frac{825.6}{958.6}, 50 \right) = 35.0 \text{ ksi}$$

J = St. Venant torsional constant

$$= \frac{1}{3} \left(D t_w^{\,3} + b_{fc} t_{fc}^{\,3} \left(1 - 0.63 \frac{t_{fc}}{b_{fc}} \right) + b_{ft} t_{ft} \left(1 - 0.63 \frac{t_{ft}}{b_{ft}} \right) \right) \text{Eq.(A6.3.3-9)}$$

h = depth between the centerline of the flanges = 43.19 in.

$L_b = 120 \text{ in.} \leq L_r = 407.4 \text{ in.}$

Therefore, the section has a noncompact unbraced length, and the lateral torsional buckling resistance is controlled by equation A6.3.3-2 of the specifications.

$$M_{nc} = C_b \left[1 - \left(1 - \frac{F_{yr} S_{xc}}{R_{pc} M_{yc}} \right) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] R_{pc} M_{yc} \leq R_{pc} M_{yc} \qquad \text{Eq.(A6.3.3-2)}$$

$$M_{nc} = (1.30) \left[1 - \left(1 - \frac{(35)(958.6)}{(1.026)(50)(958.6)} \right) \left(\frac{120 - 103.2}{416.9 - 103.2} \right) \right] (1.026)(50)(958.6) \leq R_{pc} M_{yc}$$

M_{nc} = 62,841 ≤ 49,176 = 49,176 k-in = 4,098 k-ft

Article 6.10.3.2.1 prescribes that the nominal flexural resistance, F_{nc}, can be taken as the M_{nc} determined from Article A6.3.3 divided by S_{xc}.

$$F_{nc} = \frac{4,098(12)}{958.6} = 51.3\,\text{ksi}$$

Equation 6.10.3.2.1-2 may now be evaluated as follows.

$$f_{bu} + \frac{1}{3} f_l \le \phi_f F_{nc} = 25.05 + \frac{1}{3}(1.36) \le (1.0)(51.3) \qquad \text{Eq. (6.10.3.2.1-2)}$$

$$25.50\,\text{ksi} \le 51.3\,\text{ksi} \qquad \text{(satisfied)}$$

Thus, the moment capacity of the non-composite section is sufficient to resist the applicable construction loading.

Web Bend Buckling: The flange stresses due to the construction loads are limited to a maximum of the web bend buckling stress by:

$$f_{bu} \le \phi_f F_{crw} \qquad \text{Eq. (6.10.3.2.1-3)}$$

The nominal elastic bend-buckling resistance for web, F_{crw}, is determined according to Article 6.10.1.9 of the Specifications.

$$F_{crw} = \frac{0.9Ek}{\left(D/t_w\right)^2} \qquad \text{Eq. (6.10.1.9.1-1)}$$

where: k = bend buckling coefficient

$$= \frac{9}{\left(D_c/D\right)^2} \qquad \text{Eq. (6.10.1.9.1-2)}$$

From previous calculations: D_c = 19.37 in.

Therefore, $k = \dfrac{9}{\left(19.37/42.0\right)^2} = 42.31$

$$F_{crw} = \frac{0.9(29,000)(42.31)}{\left(\frac{42}{0.50}\right)^2} = 156.5\,\text{ksi} \le R_n F_{yc} = 50\,\text{ksi}$$

$$f_{bu} = 25.05\,\text{ksi} \le \phi_f F_{crw} = (1.0)(50) = 50\,\text{ksi} \qquad \text{(satisfied)}$$

8.2.2.3.2 Tension Flange:

Flange Nominal Yielding: For a discretely braced tension flange, the allowable stress in the tension flange due to the factored loading must be less than the nominal yield strength multiplied by the hybrid factor.

$$f_{bu} + f_l \leq \phi_f R_h F_{yt} \qquad \text{Eq. (6.10.3.2.2-1)}$$

$$29.08 + 1.98 \leq (1.0)(1.0)(50)$$

$$31.06 \leq 50 \qquad \text{(satisfied)}$$

8.2.2.4 Shear (Article 6.10.3.3)

The required shear capacity during construction is specified by Eq. 6.10.3.3-1. Later in this design example, the unstiffened shear strength of the girder is demonstrated to be sufficient to resist the applied shear under the strength load combination. Therefore, the section will have sufficient strength for the constuctibility check.

$$V_u \leq \phi_v V_{cr} \qquad \text{Eq. (6.10.3.3-1)}$$

8.2.3 Service Limit State (Article 6.10.4)

Plastic deformations are controlled under the service limit state, which is specified in Article 6.10.4.

8.2.3.1 Permanent Deformations (Article 6.10.4.2)

The Service II limit state is intended to prevent permanent deformations that may negatively impact the rideability of the structure by limiting the stresses in the section under expected severe traffic loadings. Specifically, under the Service II load combination, the top flange of composite sections must satisfy:

$$f_f \leq 0.95 R_h F_{yf} \qquad \text{Eq. (6.10.4.2.2-1)}$$

Because the bottom flange is discretely braced (as opposed to the top flange), Eq. 6.10.4.2.2-2 must be satisfied for the bottom flange of composite sections.

$$f_f + \frac{f_l}{2} \leq 0.95 R_h F_{yf} \qquad \text{Eq. (6.10.4.2.2-2)}$$

Under the service limit state, the lateral force effects due to wind-load and deck overhang are not considered. Therefore, for bridges with straight, non-skewed girders such as the present design example the lateral bending forces are zero and Eq. 6.10.4.2.2-2 reduces to Eq. 6.10.4.2.2-1.

Appendix B permits the redistribution of moment at the service load level before evaluating the above equations. Article B6.5.2 specifies the effective plastic moment for the service limit state is as follows.

$$M_{pe} = \left[2.90 - 2.3 \frac{b_{fc}}{t_{fc}} \sqrt{\frac{F_{yc}}{E}} - 0.35 \frac{D}{b_{fc}} + 0.39 \frac{b_{fc}}{t_{fc}} \sqrt{\frac{F_{yc}}{E}} \frac{D}{b_{fc}} \right] M_n \leq M_n \qquad \text{Eq. (B6.5.2-1)}$$

$$M_{pe} = \left[2.90 - 2.3 \frac{16}{1.25} \sqrt{\frac{50}{29000}} - 0.35 \frac{42}{16} + 0.39 \frac{16}{1.25} \sqrt{\frac{50}{29000}} \frac{42}{16} \right] 4428 \leq 4428$$

M_{pe} = 5,770 k-ft ≤ 4,428 k-ft

M_{pe} = 4,428 k-ft > M_u = 4,075 k-ft   Therefore, No redistribution at Service II.

Because the effective plastic moment is greater than the maximum factored load for the Service II load combination, it is assumed that there is no moment redistribution at this limit state. The stresses under Service II are computed using the following equation.

$$f_f = \frac{M_{DC1}}{S_{nc}} + \frac{M_{DC2} + M_{DW}}{S_{lt}} + \frac{1.3 M_{LL+IM}}{S_{st}}$$

As permitted by Article 6.10.4.2.1, since shear connectors are provided throughout the span length, the stresses in the member as a result of the Service II load combination are computed assuming the concrete slab is fully effective in both the positive and negative bending region. The stress in the compression flange is thus computed as follows.

$$f_f = \frac{(-1334)(12)}{958.6} + \frac{(-265-217)(12)}{1187} + \frac{1.3(-1737)(12)}{1282} = -42.71\text{ksi}$$

Comparing this stress to the allowable stress shows that Eq. 6.10.4.2.2-1 is satisfied.

$$f_f = |-42.71\text{ksi}| \leq 0.95 R_h F_{yf} = 0.95(1.0)(50) = 47.50\text{ksi} \qquad \text{(satisfied)}$$

Similarly, the computation of the stress in the tension flange is computed as follows.

$$f_f = \frac{(-1334)(12)}{825.6} + \frac{(-265-217)(12)}{2857} + \frac{1.3(-1737)(12)}{9333} = 24.31\text{ksi}$$

Thus, it is also demonstrated that Eq. 6.10.4.2.2-2 is satisfied for the tension flange.

$$\mathrm{f_f} = 24.84\text{ksi} \leq 0.95 \mathrm{R_h F_{yf}} = 0.95(1.0)(50) = 47.50\text{ksi} \qquad \text{(satisfied)}$$

The compression flange stress at service loads is also limited to the elastic bend-buckling resistance of the section by Eq. 6.10.4.2.2 -4.

$$f_c \leq F_{crw} \qquad \text{Eq. (6.10.4.2.2-4)}$$

where: f_c = compression flange stress at the section under consideration due to the Service II loads calculated without consideration of flange lateral bending

F_{crw} = nominal elastic bend buckling resistance for webs with or without longitudinal stiffeners, as applicable, determined as specified in Article 6.10.1.9

From Article 6.10.1.9, the bend-buckling resistance for the web is determined using the following equation.

$$F_{crw} = \frac{0.9Ek}{\left(\frac{D}{t_w}\right)^2} \le \min\left(R_h F_{yc}, \frac{F_{yw}}{0.7}\right) \qquad \text{Eq. (6.10.1.9.1-1)}$$

$$\text{where: } k = \text{bend-buckling coefficient} = \frac{9}{(D_c / D)^2} \qquad \text{Eq. (6.10.1.9.1-2)}$$

The depth of web in compression is calculated using the method described in Article D6.3.1, which states Eq. D6.3.1-1 is to be used when checking composite sections in negative flexure at the service limit state.

$$D_c = \left[\frac{-f_c}{|f_c| + f_t}\right] d - t_{fc} \ge 0 \qquad \text{Eq. (D6.3.1-1)}$$

where: f_t = the sum of the various tension-flange stresses caused by the different loads (ksi)

d = depth of steel section (in.)

$$D_c = \left[\frac{42.84}{42.84 + 24.84}\right] 44.375 - 1.25 \ge 0$$

$$D_c = 26.84 \ge 0 = 26.84 \text{in.}$$

Therefore, k and F_{crw} are computed as follows.

$$k = \frac{9}{(26.84/42.0)^2} = 22.04$$

$$F_{crw} = \frac{0.9(29,000)(22.04)}{\left(\frac{42}{0.50}\right)^2} = 81.53 \text{ksi} < R_h\ F_{yc} = 50 \text{ ksi}$$

It can then be demonstrated that Eq. 6.10.4.2.2-4 is satisfied as shown below.

$$f_c = |\text{-}\ 42.71 \text{ksi}| \le F_{crw} = 50 \text{ksi} \qquad \text{(satisfied)}$$

8.2.4 Fatigue and Fracture Limit State (Article 6.10.5)

The fatigue and fracture limit state incorporates three distinctive checks: fatigue resistance of details (Article 6.10.5.1), fracture toughness (Article 6.10.5.2), and a special fatigue requirement for webs (Article 6.10.5.3). The first requirement involves the assessment of the fatigue resistance of details as specified in Article 6.6.1 using the Fatigue load combination specified in Table 3.4.1-1 and the fatigue live load specified in Article 3.6.1.4. The fracture toughness requirements in Article 6.10.5.2 specify that the fracture toughness must satisfy the requirements of Article 6.6.2. The special fatigue requirement for the web controls the elastic flexing of the web to prevent fatigue cracking. The factored fatigue load for this check is taken as twice the result of the Fatigue load combination.

8.2.4.1 Load Induced Fatigue (Article 6.6.1.2)

Article 6.10.5.1 requires that fatigue be investigated in accordance with Article 6.6.1. Article 6.6.1 requires that the live load stress range be less than the fatigue resistance. The fatigue resistance $(\Delta F)_n$ varies based on the fatigue category to which a particular member or detail belongs and is computed using Eq. 6.6.1.2.5-1 for the Fatigue I load combination and infinite fatigue life; or Eq. 6.6.1.2.5-2 for Fatigue II load combination and finite fatigue life.

$$(\Delta F)_n = (\Delta F)_{TH} \qquad \text{Eq. (6.6.1.2.5-1)}$$

$$(\Delta F)_n = \left(\frac{A}{N}\right)^{\frac{1}{3}} \qquad \text{Eq. (6.6.1.2.5-2)}$$

where: N = $(365)(75)n(ADTT)_{SL}$ Eq. (6.6.1.2.5-3)

A = constant from Table 6.6.1.2.5-1

n = number of stress range cycles per truck passage taken from Table 6.6.1.2.5-2

$(ADTT)_{SL}$ = single-lane ADTT as specified in Article 3.6.1.4

$(\Delta F)_{TH}$ = constant-amplitude fatigue threshold taken from Table 6.6.1.2.5-3

For this example infinite fatigue life is desired, and thus the Fatigue I Load combination and Eq. (6.6.1.2.5-1) are considered.

The fatigue resistance of the base metal at the weld joining the bracing connection plate located 10 feet from the pier to the flanges is evaluated below. From Table 6.6.1.2.3-1, it is determined that this detail is classified as fatigue category C'. The constant-amplitude fatigue threshold, $(\Delta F)_{TH}$, for a category C' detail is 12 ksi (see Table 6.6.1.2.5-3).

Substituting the values into Eq. 6.6.1.2.5-1, the allowable stress range $(\Delta F)_n$ is determined to be 12.00 ksi. It is noted that this is the minimum allowable stress range throughout the structure.

$$(\Delta F)_n = (\Delta F)_{TH} = 12.00\text{ksi}$$

The applied stress range is taken as the result of the fatigue loading with a dynamic load allowance of 15 percent applied and distributed laterally by the previously calculated distribution factor for fatigue. It is demonstrated below that the applied stress range in the top and bottom flange is acceptable.

Bottom of Top Flange:

$$\gamma(\Delta f) = (1.50)\left[\frac{(104)(12)(4.24)}{50{,}027} + \frac{|-285|(12)(4.24)}{50{,}027}\right]$$

$\gamma(\Delta f) = 0.59 \text{ ksi} \leq (\Delta F)_n = 12.00 \text{ ksi}$ (satisfied)

Top of Bottom Flange:

$$\gamma(\Delta f) = (1.50)\left[\frac{(104)(12)(37.77)}{50{,}027} + \frac{|-285|(12)(37.77)}{50{,}027}\right]$$

$\gamma(\Delta f) = 5.29 \text{ ksi} \leq (\Delta F)_n = 12.00 \text{ ksi}$ (satisfied)

8.2.4.2 Distortion Induced Fatigue (Article 6.6.1.3)

A positive connection is to be provided for all transverse connection-plate details to both the top and bottom flanges to prevent distortion induced fatigue.

8.2.4.3 Fracture (Article 6.6.2)

The appropriate Charpy V-notch fracture toughness, found in Table 6.6.2-2, must be specified for main load-carrying components subjected to tensile stress under Strength I load combination.

8.2.4.4 Special Fatigue Requirement for Webs (Article 6.10.5.3)

Article 6.10.5.3 requires that the shear force applied due to the fatigue loading must be less than the shear-buckling resistance of interior panels of stiffened webs.

$V_u \leq V_{cr}$ Eq. (6.10.5.3-1)

However designs utilizing unstiffened webs at the strength limit state, as is the case here, automatically satisfy this criterion. Thus, Eq. 6.10.5.3-1 is not explicitly evaluated herein.

8.3 Exterior Girder Check: Section 1-1

8.3.1 Constructibility (Article 6.10.3)

The constructibility of the system in the positive bending region will be evaluated in a similar manner to the constructibility check in the negative bending region.

8.3.1.1 Deck Placement Analysis

The results from the deck casting sequence analysis, previously presented in Table 17, are referenced for the following calculations of f_{bu}.

8.3.1.1.1 Strength I:

Top Flange: $$f_{bu} = \frac{1.0(1.25)(871)(12)}{614.0} = 21.28\,\text{ksi}$$

Bottom Flange: $$f_{bu} = \frac{1.0(1.25)(871)(12)}{887.7} = 14.72\,\text{ksi}$$

8.3.1.1.2 Strength IV:

Top Flange: $$f_{bu} = \frac{1.0(1.5)(871)(12)}{614.0} = 25.53\,\text{ksi}$$

Bottom Flange: $$f_{bu} = \frac{1.0(1.5)(871)(12)}{887.7} = 17.66\,\text{ksi}$$

8.3.1.2 Deck Overhang Loads

The deck overhang loads are the same for the positive bending region as the negative bending region; however the lateral bending stresses may differ due to potentially varying amplification factors, which are a function of the vertical bending stresses (f_{bm}) and the unbraced length. To compute the amplification factor in the positive bending region, the following equation is first evaluated to determine if first-order elastic analysis is applicable.

$$L_b \le 1.2L_p\sqrt{\frac{C_bR_b}{f_{bm}/F_{yc}}}$$ Eq. (6.10.1.6-2)

L_p is determined using Eq. 6.10.8.2.3-4.

$$L_p = r_t\sqrt{\frac{E}{F_{yc}}}$$ Eq. (6.10.8.2.3-4)

where: $r_t = \dfrac{b_{fc}}{\sqrt{12\left(1+\dfrac{1}{3}\dfrac{D_c\,t_w}{b_{fc}\,t_{fc}}\right)}}$ Eq. (6.10.8.2.3-10)

$$r_t = \frac{14}{\sqrt{12\left(1+\dfrac{1}{3}\dfrac{(25.26)(0.4375)}{(14)(0.75)}\right)}} = 3.477\,\text{in.}$$

Therefore, L_p is computed as follows.

$$L_p = 3.477\sqrt{\frac{29{,}000}{50}} = 83.74\,\text{in.}$$

As previously described, the moment gradient modifier is determined from the following equation.

$$C_b = 1.75 - 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \le 2.3 \qquad \text{Eq. (A6.3.3-7)}$$

The maximum positive bending stresses due to the deck casting occur at 36 feet from the pier. Thus, the critical lateral bracing segment is the lateral bracing panel that begins at 20 feet from the pier and ends at 40 feet from the pier. The applicable moment values for this lateral bracing segment are given below.

M_2	= 850 k-ft.	M_1	= $2M_{mid} - M_2 \ge M_0$
M_{mid}	= 831 k-ft.	M_1	= 2(831) – (850) = 812 ≥ 683
M_0	= 683 k-ft.	M_1	= 812 k-ft.

C_b is then computed as follows.

$$C_b = 1.75 - 1.05\left(\frac{812}{850}\right) + 0.3\left(\frac{812}{850}\right)^2 = 1.02 \le 2.3 = 1.02$$

According to Article 6.10.1.10.2, the web load-shedding factor, R_b, is 1.0 for constructability evaluations. The maximum vertical bending stress occurs in the top flange under the Strength IV load combination and was computed above to equal 25.53 ksi. Lastly, F_{yc} is equal to 50 ksi. The information required for evaluation of Eq. 6.10.1.6-2 is now known.

$$L_b = 240\,\text{in.} > 1.2(83.74)\sqrt{\frac{(1.02)(1.0)}{25.53/50}} = 142.0 \qquad \text{(not satisfied)}$$

Therefore, the first-order elastic analysis is not applicable, and the second-order compression flange lateral bending stresses are calculated below.

$$f_l = \left(\frac{0.85}{1 - \frac{f_{bm}}{F_{cr}}} \right) f_{l1} \geq f_{l1} \qquad \text{Eq. (6.10.1.6-4)}$$

To calculate the amplification factor (the term in bracket in Eq. 6.10.1.6-4), the elastic lateral torsional buckling stress, F_{cr}, must be determined, which can be calculated from Appendix A or Section 6.10.8. As discussed above, Appendix A is applicable if the flange nominal yield strength is less than or equal to 70 ksi and the web is classified as either compact or non-compact. The following calculations demonstate that Appendix A is applicable.

$$F_{yf} = 50\,\text{ksi} \leq 70\,\text{ksi} \qquad \text{(satisfied)}$$

$$\frac{2D_c}{t_w} < 5.7\sqrt{\frac{E}{F_{yc}}} \qquad \text{Eq. (A6.1-1)}$$

$D_c = 26.01 - 0.75 = 25.26$ in.

$$\frac{2(25.26)}{(0.4375)} < 5.7\sqrt{\frac{29{,}000}{50}} = 115.47 < 137.27 \qquad \text{(satisfied)}$$

Since Appendix A is applicable, the elastic lateral torsional buckling stress is determined from Article A6.3.3.

$$F_{cr} = \frac{C_b \pi^2 E}{\left(L_b / r_t\right)^2} \sqrt{1 + 0.0779 \frac{J}{S_{xc} h} \left(L_b / r_t\right)^2} \qquad \text{Eq. (A6.3.3-8)}$$

With the exception of the variable J, the variables in Eq. A6.3.3-8 have been previously computed above.

$$J = \frac{1}{3}\left(D t_w^{\,3} + b_{fc} t_{fc} \left(1\text{-}0.63 \frac{t_{fc}}{b_{fc}} \right) + b_{ft} t_{ft}^{\,3} \left(1\text{-}0.63 \frac{t_{ft}}{b_{bt}} \right) \right) \qquad \text{Eq. (A6.3.3-9)}$$

$$J = \frac{1}{3}\left((42)(0.4375)^3 + (14)(0.75)^3(0.966) + (16)(1.25)^3(0.95) \right) = 12.97\,\text{in.}$$

Therefore,

$$\text{F}_{\text{cr}} = \frac{(1.02)\pi^2(29{,}000)}{(240/3.477)^2} \sqrt{1 + 0.0779 \frac{13.56}{(610.0)(44.0)} (240/3.477)^2} = 66.77\,\text{ksi}$$

The amplification factor for Strength I is then equal to 1.248.

$$\mathrm{AF} = \frac{0.85}{\left(1 - \dfrac{21.28}{66.74}\right)} = 1.248 > 1.0 = 1.248$$

Similarly, the amplification factor for Strength IV is equal to 1.377.

$$\mathrm{AF} = \frac{0.85}{\left(1 - \dfrac{25.53}{66.74}\right)} = 1.377 > 1.0 = 1.377$$

Since the construction load magnitudes are the same for both the positive and negative bending regions, the previously computed lateral forces on the flanges due to the vertical load on the overhang brackets is applicable; however, the lateral moment is not the same due to the different lateral bracing distance.

8.3.1.2.1 Strength I:

Dead loads:

$F = F_{\perp} = 673.8$ lbs/ft.

$$M_l = \frac{F_l L_b^2}{12} = \frac{(0.6738)(20)^2}{12} = 22.46\,\text{k-ft}$$

Top Flange: $f_l = \dfrac{M_l}{S_l} = \dfrac{22.46(12)}{0.75(14)^2/6} = 11.00\,\text{ksi}$

Bottom Flange: $f_l = \dfrac{M_l}{S_l} = \dfrac{22.46(12)}{1.25(16)^2/6} = 5.05\,\text{ksi}$

Live loads:

$F = P_{\perp} = 4{,}500$ lbs.

$$M_l = \frac{P_l L_b}{8} = \frac{(4.5)(20)}{8} = 11.25\,\text{k-ft}$$

Top Flange: $f_l = \dfrac{M_l}{S_l} = \dfrac{11.25(12)}{0.75(14)^2/6} = 5.51\,\text{ksi}$

Bottom Flange: $f_l = \dfrac{M_l}{S_l} = \dfrac{11.25(12)}{1.25(14)^2/6} = 2.53\,\text{ksi}$

Total (w/ Amplification):

Top: $f_\ell = 11.00 + 5.51 = 16.51(1.248) = 20.60$ ksi

Bot.: $f_\ell = 5.05 + 5.51 = 10.56(1.00) = 10.56$ ksi

8.3.1.2.2 Strength IV:

Dead loads:

$F = F_1 = 726.0$ lbs/ft.

$$M_l = \frac{F_l L_b^2}{12} = \frac{(0.7260)(20)^2}{12} = 24.20 \text{ k-ft}$$

Top Flange: $f_l = \frac{M_l}{S_l} = \frac{24.20(12)}{0.75(14)^2/6} = 11.85 \text{ ksi}$

Bottom Flange: $f_l = \frac{M_l}{S_l} = \frac{24.20(12)}{1.25(16)^2/6} = 5.45 \text{ ksi}$

Finishing Machine:

Not applicable

Total (w/ Amplification):

Top: $f_\ell = 11.85(1.377) = 16.32$ ksi

Bot.: $f_\ell = 5.45(1.00) = 5.45$ ksi

Article 6.10.1.6 requires that the lateral bending stresses not exceed 60% of the nominal yield stress of the flange under consideration. Comparing the lateral stresses at the Strength I and Strength IV, computed above, it is shown that the lateral stresses at both limit states satisfy this requirement although the stresses are highest for the Strength I load combination.

$f_{\ell\,top} = 20.60 \text{ ksi} < 0.6F_{yf} = 30 \text{ ksi}$

$f_{\ell\,bottom} = 10.56 \text{ ksi} < 0.6F_{yf} = 30 \text{ ksi}$

8.3.1.3 Flexure (Article 6.10.3.2)

8.3.1.3.1 Compression Flange

For discretely braced compression flanges, three requirements must be satisfied during construction. These are related to prevention of yielding, provision for adequate the flexural resistance, and controlling web bend buckling.

Flange nominal yielding:
The total (vertical and lateral) stress in the compression flange is limited to the product of the nominal yield strength of the flange, the hybrid girder factor, and the flexural resistance factor by:

$$f_{bu} + f_l \leq \phi_f R_h F_{yc} \qquad \text{Eq. (6.10.3.2.1-1)}$$

$$25.63 + 1.36 \leq (1.0)(1.0)(50) = 26.99 \text{ ksi} \leq 50 \text{ ksi} \qquad \text{(satisfied)}$$

Flexural Resistance:
The flexural resistance of the section must be evaluated using Eq. 6.10.3.2.1-2, which requires the calculation of the nominal flexural resistance of the flange while in the noncomposite state.

$$f_{bu} + \frac{1}{3} f_l \leq \phi_f F_{nc} \qquad \text{Eq. (6.10.3.2.1-2)}$$

Article 6.10.3.2 states the nominal flexural resistance of the flange can be determined from computing F_{nc} in Article 6.10.8.2 or computing the lateral torsional buckling resistance, M_{nc}, from Article A6.3.3 divided by the compression flange section modulus. Since it was demonstrated in the previous section that the section satisfies the requirements for Appendix A applicability, the lateral torsional buckling resistance from Article A6.3.3 is now calculated.

Computation of the lateral torsional buckling resistance begins with determining the web plastification factors. As previously stated, if the section satisfies Eq. A6.2.1-1 the web is considered compact, and the web plastification factors are determined by dividing the plastic moment by the yield moment.

$$\frac{2D_{cp}}{t_w} < \lambda_{pw(D_{CP})} \qquad \text{Eq. (A6.2.1-1)}$$

The depth of web in compression of the non-composite section at the plastic moment is determined from the equations in Appendix D. First the girder component forces are determined.

P_c = (14)(0.75)(50) = 525 kips

P_t = (16)(1.25)(50) = 1,000 kips

P_w = (42)(0.4375)(50) = 918.8 kips

Pc + Pw = 525 +918.8 kips = 1,443.8 $\geq$ Pt = 1,000 kips

Therefore, the plastic neutral axis is in the web.

The neutral axis location is then determined as follows.

$$\bar{y} = \left(\frac{D}{2}\right)\left[\frac{P_c - P_t}{P_w} + 1\right]$$

$$\bar{y} = \left(\frac{42}{2}\right)\left[\frac{525-1{,}000}{918.8}+1\right] = 10.14 \text{ in.}$$

The plastic neutral axis is located 10.14 inches above the top of the bottom flange. It is also convenient to compute the plastic moment capacity of the non-composite girder at this point.

$$M_p = \frac{P_w}{2D}\left[\bar{y}^2 + \left(D-\bar{y}\right)^2\right] + P_c d_c + P_t d_t$$

$$M_p = \frac{918.8}{2(42)}\left[(10.14)^2 + (42-10.14)^2\right] + (1{,}000)(10.14+1.25/2) + (525)(42-10.14+0.75/2)$$

M_p = 39,916 k-in. = 3,326 k-ft.

Subtracting $\bar{y}$ from the total web depth gives $D_{cp.}$

D_{cp} = 42.0 – 10.14 = 31.86 in.

$\lambda_{pw(D_{cp})}$ is then computed.

$$\lambda_{pw(D_{CP})} = \frac{\sqrt{E/F_{yc}}}{\left(0.54\left(\frac{M_p}{R_h M_y}\right)-0.1\right)^2} \qquad \text{Eq. (A6.2.1-2)}$$

$$\lambda_{pw(D_{CP})} = \frac{\sqrt{29{,}000/50}}{\left(0.54\left(\frac{39{,}916}{(1.0)(50)(614.0)}\right)-0.1\right)^2} = 66.43$$

Eq. A6.2.1-1 is then evaluated where it is determined that the requirements for a compact flange are not satisfied.

$$\frac{2D_{cp}}{t_w} = \frac{2(31.86)}{0.4375} = 145.65 > \lambda_{pw(D_{CP})} \qquad \text{(not satisfied)}$$

The non-compact flange requirements, which are expressed by Eq. A6.2.2-1, are next evaluated.

$$\lambda_w = \frac{2D_c}{t_w} < \lambda_{rw} = 5.7\sqrt{\frac{E}{F_{yc}}} \qquad \text{Eq. (A6.2.2-1)}$$

$$\lambda_w = 115.47 < \lambda_{rw} = 137.27 \qquad \text{(satisfied)}$$

Therefore, the web plastification factors are governed by equations A6.2.2-4 and A6.2.2-5:

$$R_{pc} = \left[1 - \left(1 - \frac{R_h M_{yc}}{M_p}\right)\left(\frac{\lambda_w - \lambda_{pw(D_C)}}{\lambda_{rw} - \lambda_{pw(D_C)}}\right)\right]\frac{M_p}{M_{yc}} \le \frac{M_p}{M_{yc}} \qquad \text{Eq. (A6.2.2-4)}$$

where: $\lambda_{pw(D_C)} = \lambda_{pw(D_{CP})}\left(\frac{D_c}{D_{cp}}\right)$ Eq. (A6.2.2-6)

$$\lambda_{pw(D_C)} = 66.43\left(\frac{25.26}{31.86}\right) = 52.67$$

Thus,

$$R_{pc} = \left[1 - \left(1 - \frac{(1.0)(614)(50)}{39{,}916}\right)\left(\frac{115.47 - 52.67}{137.27 - 52.67}\right)\right]\frac{39{,}916}{(614)(50)} \le \frac{39{,}916}{(614)(50)}$$

R_{pc} = 1.077 ≤ 1.3 =1.077

$$R_{pc} = \left[1 - \left(1 - \frac{R_h M_{yt}}{M_p}\right)\left(\frac{\lambda_w - \lambda_{pw(D_C)}}{\lambda_{rw} - \lambda_{pw(D_C)}}\right)\right]\frac{M_p}{M_{yt}} \le \frac{M_p}{M_{yt}} \qquad \text{Eq. (A6.2.2-5)}$$

$$R_{pt} = \left[1 - \left(1 - \frac{(1.0)(888)(50)}{39{,}916}\right)\left(\frac{115.47 - 52.67}{137.27 - 52.67}\right)\right]\frac{39{,}916}{(888)(50)} \le \frac{39{,}916}{(888)(50)}$$

R_{pt} = 0.974 ≤ 0.899 =0.899

The series of equations that govern the lateral torsional buckling resistance is based on the classification of the lateral brace spacing, where compact lateral bracing distances are classified by the following equation.

$L_b \le L_p$

$$L_b = (20.0)(12) = 240\,\text{in.} > L_p = r_t\sqrt{E/F_{yc}} = 83.74\,\text{in.} \qquad \text{(not satisfied)}$$

Therefore, the lateral brace spacing is not classified as compact and the non-compact lateral bracing classification is evaluated as follows.

$L_b \le Lr$

$$L_b = 240 \le L_r = 1.95 r_t \frac{E}{F_{yr}}\sqrt{\frac{J}{S_{xc}h}}\sqrt{1 + \sqrt{1 + 6.76\left(\frac{F_{yr}S_{xc}h}{EJ}\right)^2}} \qquad \text{Eq. (A6.3.3-5)}$$

where: $F_{yr} = \min\left(0.7F_{yc}, R_h F_{yt}\frac{S_{xt}}{S_{xc}}, F_{yw}\right)$

$$F_{yr} = \min\left(0.7(50),(1.0)(50)\frac{887.7}{614.0},50\right) = \min(35,72.3,50) = 35.0\,\text{ksi}$$

J = 13.56 in.^{3}

H = 43.0 in.

$$L_r = 1.95(3.477)\frac{29000}{35}\sqrt{\frac{13.56}{(614)(43.0)}}\sqrt{1+\sqrt{1+6.76\left(\frac{(35)(614)(43.0)}{(29000)(13.56)}\right)^2}} = 341.4\text{in.}$$

$L_b = 240 \le L_r = 341.4$ (satisfied)

Because the lateral bracing distance is non-compact, the lateral torsional buckling resistance is controlled by Eq. A6.3.3-2 of the specifications.

$$M_{nc} = C_b\left[1-\left(1-\frac{F_{yr}S_{xc}}{R_{pc}M_{yc}}\right)\left(\frac{L_b-L_p}{L_r-L_p}\right)\right]R_{pc}M_{yc} \le R_{pc}M_{yc} \qquad \text{Eq. (A6.3.3-2)}$$

$$M_{nc} = (1.02)\left[1-\left(1-\frac{(35)(614)}{(1.077)(50)(614)}\right)\left(\frac{240-83.74}{341.4-83.74}\right)\right](1.077)(50)(614) \le (1.077)(50)(614)$$

M_{nc} = 26,566 ≤ 33,064 = 26,566 k-in. = 2,214 k-ft.

The lateral torsional buckling capacity is then expressed in terms of allowable stress by dividing the above moment capacity by the section modulus.

$$F_{nc} = \frac{2,214(12)}{614} = 43.27\,\text{ksi}$$

Equation 6.10.3.2.1-2 can now be evaluated.

$$f_{bu} + \frac{1}{3}f_l \le \phi_f F_{nc} \qquad \text{Eq. (6.10.3.2.1-2)}$$

$$25.53 + \frac{1}{3}(16.32) \le (1.0)(43.27) = 30.97\,\text{ksi} \le 43.27\,\text{ksi} \qquad \text{(satisfied)}$$

<u>Web Bend-Buckling Resistance:</u>
The section must satisfy Eq. 6.10.3.2.1-3 of the specifications to ensure the section has adequate web bend buckling resistance during construction.

$$f_{bu} =\leq \phi_f F_{crw}$$ Eq. (6.10.3.2.1-3)

where: $$F_{crw} = \frac{0.9Ek}{\left(\frac{D}{t_w}\right)^2} \leq \min\left(R_h F_{yc}, \frac{F_{yw}}{0.7}\right)$$ Eq. (6.10.1.9.1-1)

$$k = \frac{9}{\left(D_c/D\right)^2}$$ Eq. (6.10.1.9.1-2)

The depth of web in compression was previously calculated to be 25.26 in.

$$k = \frac{9}{\left(25.26/42.0\right)^2} = 24.88$$

$$F_{crw} = \frac{0.9(29,000)(24.88)}{\left(42/0.4375\right)^2} = 70.46\,\text{ksi} > R_n F_{yc}$$

Therefore, F_{crw} = 50 ksi

$$f_{bu} = 25.53 \text{ ksi} \leq \phi_f F_{crw} = (1.0)(50.0) = 50.0 \text{ ksi}$$ (satisfied)

8.3.1.3.2 Tension Flange

The section must satisfy the tension flange nominal yielding check under the construction loading.

Tension Flange Nominal Yielding:

$f_{bu} + f_l \leq \phi R_h F_{yt}$ Eq. (6.10.3.2.2-1)

$21.28 + 20.60 \leq (1.0)(1.0)(50)$

$41.88 \leq 50.0$ ksi (satisfied)

8.3.1.4 Shear (Article 6.10.3.3)

As previously stated, since the design does not require any transverse stiffeners, the shear check under the construction loading is automatically satisfied.

8.3.2 Service Limit State (Article 6.10.4)

Serviceability requirements of steel I-girder provisions are specified in Article 6.10.4. The evaluation of the positive bending region based on these requirements follows.

8.3.2.1 Elastic Deformations (Article 6.10.4.1)

Since the design bridge is not designed to permit pedestrian traffic, the live load deflection will be limited to L/800. It is shown below that the maximum deflection along the span length using the service loads and a line girder approach is less than the L/800 limit. It is noted, however, that satisfying this requirement is optional.

δ = 0.610 in. < L/800 = (90 x 12) / 800 = 1.35 in.

8.3.2.2 Permanent Deformations (Article 6.10.4.2)

The positive bending section must be evaluated for permanent deformations, which are governed by Eq. 6.10.4.2.2-1.

$$f_f \leq 0.95 R_h F_{yf} \qquad \text{Eq. (6.10.4.2.2-1)}$$

where: $f_f = \dfrac{M_{DC1}}{S_{nc}} + \dfrac{M_{DC2} + M_{DW}}{S_{lt}} + \dfrac{1.3 M_{LL+IM}}{S_{st}}$

It is noted that the moment values in the above equation represent the moments resulting from elastic analysis since it has previously been determined that moment redistribution is not applicable at the service limit state.

The stress in the compression flange is shown below to equal 18.97 ksi, which satisfies the requirements of Eq. 6.10.4.2.2-1.

$$f_f = \frac{(738)(12)}{614} + \frac{(147+120)(12)}{2{,}711} + \frac{1.3(2160)(12)}{10{,}001} = 18.97\,\text{ksi}$$

f_f = 18.97 ksi $\leq 0.95 R_h F_{yf}$ = 0.95(1.0)(50) = 47.5 ksi (satisfied)

Similarly, the stress in the tension flange is computed to equal 39.74 ksi, also satisfying Eq. 6.10.4.2.2-1.

$$f_f = \frac{(738)(12)}{614} + \frac{(147+120)(12)}{1{,}159} + \frac{1.3(2160)(12)}{1{,}248} = 39.74\,\text{ksi}$$

f_f = 39.74 ksi $\leq 0.95 R_h F_{yf}$ = 0.95(1.0)(50) = 47.5 ksi (satisfied)

Thus, all service requirements are satisfied.

8.3.3 Fatigue and Fracture Limit State (Article 6.10.5)

8.3.3.1 Load Induced Fatigue (Article 6.6.1.2)

The fatigue calculation procedures in the positive bending region are similar to those previously presented for the negative bending region. In this section the fatigue requirements are evaluated for a bolted connection plate, connecting to the girder flanges for the cross frame located 40 feet from the abutment. This connection type was selected over a typical welded detail due to the higher fatigue category classification of the bolted connection, and the difficulty of satisfying fatigue requirements with a welded connection detail. Specifically, a considerably larger section would be required to satisfy the fatigue requirements if a fatigue category C' detail was to be used. Specifically, the bolted connection is classified as fatigue category B (see Table 6.6.1.2.3-1), which corresponds to a constant-amplitude fatigue threshold of 16 ksi, compared to a constant-amplitude fatigue threshold of 12 ksi for a C' detail.

The permissible stress range is computed using Eq. 6.6.1.2.5-1 for the Fatigue I load combination and infinite fatigue life.

$$(\Delta F)_n = (\Delta F)_{TH} \qquad \text{Eq. (6.6.1.2.5-1)}$$

$$(\Delta F)_n = (\Delta F)_{TH} = 16.00\text{ksi}$$

Bottom of Top Flange:

$$\gamma(\Delta f) = (1.50)\left[\frac{(528)(12)(4.13)}{48{,}806} + \frac{|-143|(12)(4.13)}{48{,}806}\right]$$

$$\gamma(\Delta f) = 1.02 \text{ ksi} \le (\Delta F)_n = 16.00 \text{ ksi} \qquad \text{(satisfied)}$$

Top of Bottom Flange:

$$\gamma(\Delta f) = (1.50)\left[\frac{(528)(12)(37.94)}{48{,}806} + \frac{|-143|(12)(37.94)}{48{,}806}\right]$$

$$\gamma(\Delta f) = 9.39 \text{ ksi} \le (\Delta F)_n = 12.00 \text{ ksi} \qquad \text{(satisfied)}$$

8.3.3.2 Special Fatigue Requirement for Webs (Article 6.10.5.3)

The following shear requirement must be satisfied at the fatigue limit state.

$$V \le \phi_v V_{cr} \qquad \text{Eq. (6.10.5.3-1)}$$

However this is an unstiffened web. Therefore, this limit is not explicitly evaluated.

8.3.4 Strength Limit State (Article 6.10.6)

8.3.4.1 Flexure (Article 6.10.6.2)

For compact sections in positive bending Equation 6.10.7.1.1-1 must be satisfied sections at the strength limit state.

$$M_u + \frac{1}{3} f_l S_{xt} \leq \phi_f M_n \qquad \text{Eq. (6.10.7.1.1-1)}$$

However, the lateral bending stresses are neglible for the straight, composite girder considered herein. The following requirements must be satisfied for a section to qualify as compact:

$F_y = 50$ ksi ≤ 70 ksi (satisfied)

$$\frac{D}{t_w} = \frac{42.0}{0.4375} = 96.0 \leq 150 \qquad \text{(satisfied)}$$

$$\frac{2D_{cp}}{t_w} = \frac{2(0)}{0.4375} = 0 \leq 3.76\sqrt{E/F_{yc}} \qquad \text{(satisfied)}$$

Therefore, the section is compact, and the nominal flexural resistance is based on Article 6.10.7.1.2. Additionally, the following requirement must be evaluated.

$D_p \leq 0.1D_t$

The depth of web in compression at the plastic moment was determined.

$D_p = 7.49$ in.

D_t = total depth of the composite section

$D_t = 8 + 2 + 42 + 1.25 = 53.25$ in.

$Dp = 7.49 > 0.1Dt = 0.1(53.25) = 5.33$ (not satisfied)

Therefore, the nominal flexural capacity is determined from.

$$M_n = M_p\left(1.07 - 0.7\frac{D_p}{D_t}\right) \qquad \text{Eq. (6.10.7.1.2-2)}$$

$$M_n = 6{,}418\left(1.07 - 0.7\frac{7.49}{53.25}\right) = 6{,}235\text{k - ft}$$

From elastic analysis procedures, the maximum positive moment under the Strength I load combination is 4,192 k-ft., which is at a distance of 36 feet from the left support. The redistribution moment must then be added to this moment to determine the total applied moment. The redistribution moment varies linearly from zero at the end-supports to a maximum at the interior pier of 936 k-ft. Thus, the redistribution moment at 36' from the pier is simply computed as follows.

M_{rd}= 36/90*(936) = 0.4(936) = 374 k-ft

The total design moment is then the sum of the redistribution moment and the elastic moment.

$M_u = 4{,}192 + 374 = 4{,}566$ k-ft

The bending strength of the positive bending region is then shown to be sufficient.

$$M_u \le \phi_f M_n$$

$4{,}566 \text{ k-ft.} \le (1.0)(6{,}235) = 6{,}235 \text{ k-ft}$ (satisfied)

8.3.4.2 Ductility Requirements (6.10.7.3)

Sections in positive bending are also required to satisfy Eq. 6.10.7.3-1 , which is a ductility requirement intended to prevent crushing of the concrete slab.

$$D_p \le 0.42 D_t \qquad \text{Eq. (6.10.7.3-1)}$$

$D_p = 7.49 \text{ in.} \le 0.42(53.25) = 22.37 \text{ in.}$ (satisfied)

8.3.4.3 Shear (6.10.6.3)

The shear requirements at the strength limit state are expressed by:

$$V_u \le \phi_v V_n \qquad \text{Eq. (6.10.9.1-1)}$$

where: V_n = $V_{cr} = CV_p$ Eq. (6.10.9.2-1)

V_p = plastic shear force (kip)

V_p = $0.58F_{yw}Dt_w$ Eq. (6.10.9.2-2)

C = ratio of the shear buckling resistance to the shear yield strength determined from Article 6.10.9.3.2

The computation of C is based on the web slenderness classification. Thus, the web slenderness is first evaluated in terms of the following equation.

$$\frac{D}{t_w} \le 1.12\sqrt{\frac{Ek}{F_{yw}}}$$

$$\frac{D}{t_w} = \frac{42.0}{0.4375} = 96.0 > 1.12\sqrt{Ek/F_{yw}} = 1.12\sqrt{(29{,}000)(5)/50} = 60.31$$ (not satisfied)

The web slenderness is next evaluated in terms of the following equation.

$$1.12\sqrt{\frac{Ek}{F_{yw}}} < \frac{D}{t_w} = 96.0 \le 1.40\sqrt{\frac{Ek}{F_{yw}}}$$

$$1.12\sqrt{\frac{Ek}{F_{yw}}} = 60.31 < \frac{D}{t_w} = 96.0 > 1.40\sqrt{\frac{Ek}{F_{yw}}} = 75.4 \qquad \text{(not satisfied)}$$

Lastly, the web slenderness is evaluated as follows.

$$\frac{D}{t_w} = 96.0 > 1.40\sqrt{\frac{Ek}{F_{yw}}} = 75.4 \qquad \text{(satisfied)}$$

Thus, C is calculated according to Eq. 6.10.9.3.2-6.

$$C = \frac{1.57}{\left(D/t_w\right)^2}\left(\frac{Ek}{F_{yw}}\right) = \frac{1.57}{(96.0)^2}(2{,}900) = 0.494 \qquad \text{Eq. (6.10.9.3.2-6)}$$

Therefore, the shear capacity is equal to:

$V_{cr} = CV_p$ =(0.494)(0.58)(50)(42)((0.4375) = 263.2 kips

Thus, the shear requirements at the strength limit state (and consequently all other limit states as previously discussed) are satisfied.

V = 257 kips ≤ $\phi_v V_{cr}$ = (1.0)(263.2) = 263.2 kips (satisfied)

8.4 Cross-frame Design

The cross-frames alone provide restoring forces during construction to enable the girders to deflect equally. Once the system acts compositely, the concrete slab also contributes to providing restoring forces and continuously braces the top flanges at the girder. Therefore, the engineer may opt to provide temporary cross-frames that are only required during the construction phase. However, it is assumed in this design example that all cross frames are permanent. Although several styles of cross-frames may be used (refer to Chapter 8 for a more complete discussion), a typical K-shaped cross-frame (as shown in Figure. 17) is used for this example. The design of the intermediate and end cross-frames is demonstrated in the sections that follow.

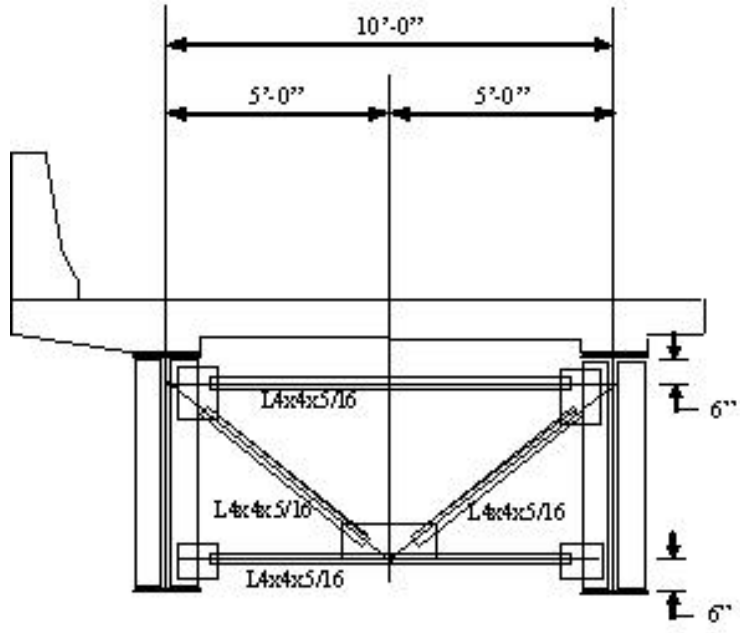

Figure 17 Intermediate Cross Frame

8.4.1 Intermediate Cross-frame Design

This section describes the design process for an intermediate cross-frame. The cross-frames are most critical while the system is in the noncomposite stage under wind loading. The wind load per unit length on the bottom flange is given by Article 4.6.2.7.

$$w = \frac{P_D d}{2} = \frac{(0.050)(44.25/12)}{2} = 0.092\,\text{k/ft.}$$

8.4.1.1 Bottom Strut

The bottom strut is in compression under the wind loading; therefore, the limiting slenderness ratio for bracing members in compression must be satisfied as specified in Article 6.9.3. For bracing members, the slenderness ratio is limited to 140, which is applicable for the major as well as the minor axes.

The unbraced length of the bottom strut is assumed to be 4'-9" about the minor principle axis, and 9'-6" about the major principle axis. Article 4.6.2.5 states that the effective-length factor K for trusses and frames, with bolted or welded connections at both ends, may be taken as 0.750. Therefore:

$$\left(r_z\right)_{\min} = \frac{0.75(4.75)(12)}{140} = 0.305\,\text{in.} \qquad \left(r_y\right)_{\min} = \frac{0.75(9.5)(12)}{140} = 0.611\,\text{in.}$$

The cross-frames will be composed of single-angle members, and the angle capacity will be determined from the AISC LRFD Specifications for Design of Single-Angle Members from the Third Edition of the AISC LRFD Manual of Steel Construction. Based on the required slenderness values and the minimum structural steel thickness of $^5/_{16}$" specified in Article 6.7.3, an L4 x 4 x $^5/_{16}$ member will be selected. The required section properties are calculated below and depicted in Figure 18. In these computations it is assumed that the connection plate is ½-inch thick.

$$A = 2.40 \text{ in.}^2$$

$$r_z = 0.781 \text{ in.}$$

$$I_z = Ar_z^2 = (2.40)(0.781)^2 = 1.46 \text{ in.}^4$$

$$I_w = I_x + I_y - I_z = 3.67 + 3.67 - 1.46 = 5.88 \text{ in.}^4$$

$$r_w = \sqrt{\frac{I_w}{A}} = \sqrt{\frac{5.88}{2.40}} = 1.57 \text{ in.}$$

$$r_x = r_y = 1.24 \text{ in.}$$

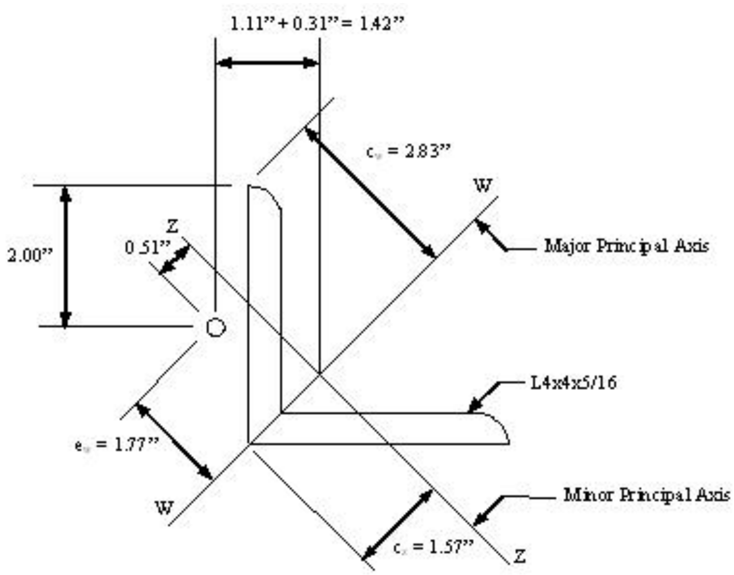

Figure 18 Single Angle for Intermediate Cross Frame

The horizontal wind force applied to the brace point can be calculated in the following manner, where L_b is taken as the maximum cross frame spacing and the wind load per unit length (w) is 0.092 k/ft. as previously determined:

$$P_w = wL_b = (0.092)(20.0) = 1.84 \text{ kips}$$

The bottom struts in the exterior bays of the system must carry the entire wind force P_w; therefore, all of the bottom struts will be conservatively designed to satisfy the requirements of the exterior bay struts.

The Strength III load combination controls the lateral bracing design due to having the largest load factor for wind, which is specified to be 1.40. The following calculation determines the factored axial wind force in the bottom strut, including the η factor.

$$P_u = 1.00(1.40)(1.84) = 2.58 \text{ kips}$$

Connected through one leg only, the strut is eccentrically loaded. The member then experiences both flexure and axial compression; therefore the design must satisfy the interaction equation given in Section 16.3 of the LRFD Manual of Steel Construction.

8.4.1.1.1 Axial Compression

The axial compressive resistance of the selected angle is calculated from the following equation from the AISC SAM Section 4:

$$P_r = \phi_c P_n$$

where: ϕ_c = resistance factor for axial compression

= 0.90

P_n = $A_g F_{cr}$

A_g = gross area of the member

F_{cr} = critical buckling stress

The critical buckling stress based on buckling about the minor principle axis (Z-Z) is determined as follows.

$$F_{cr} = Q\left(0.658^{Q\lambda_c^2}\right)F_y \quad \text{if } \lambda_c\sqrt{Q} \le 1.5$$

$$\lambda_c = \left(\frac{Kl}{r\pi}\right)\sqrt{\frac{F_y}{E}} = \left(\frac{(0.75)(4.75)(12)}{(0.781)\pi}\right)\sqrt{\frac{50}{29{,}000}} = 0.723$$

The appropriate equation to be used for the calculation of Q is selected based on the b/t ratio of the angle. The aspect ratio is first evaluated in terms of the following equation.

$$\frac{b}{t} \le 0.446\sqrt{\frac{E}{F_y}} = 10.7$$

$$\frac{b}{t} = \frac{4.0}{0.3125} = 12.80 > 10.70 \qquad \text{(not satisfied)}$$

Since the above equation is not satisfied, the following equation is evaluated.

$$\frac{b}{t} \leq 0.910\sqrt{\frac{E}{F_y}} = 21.9$$

$$\frac{b}{t} = 12.80 < 21.9 \qquad \text{(satisfied)}$$

Therefore, the reduction factor for local buckling Q is calculated from the equation below.

$$Q = 1.34 - 0.761\frac{b}{t}\sqrt{F_y/E} = 1.34 - 0.761\frac{4.0}{0.3125}\sqrt{50/29,000} = 0.936 \qquad \text{Eq. (4-3b)}$$

$$\lambda_c\sqrt{Q} = 0.723\sqrt{0.936} = 0.699 \leq 1.5 \qquad \text{(satisfied)}$$

Therefore, F_{cr} is equal to the following.

$$F_{cr} = Q\left(0.658^{Q\lambda_c^2}\right)F_y = (0.936)\left(0.658^{(0.936)(0.723)^2}\right)(50) = 38.13\text{ksi} \qquad \text{Eq. (4-1)}$$

The critical buckling stress based on buckling about the geometric axis(Y-Y) is computed as:

$$\lambda_c = \left(\frac{Kl}{r\pi}\right)\sqrt{\frac{F_y}{E}} = \left(\frac{(0.75)(9.5)(12)}{(1.240)\pi}\right)\sqrt{\frac{50}{29,000}} = 0.911$$

Q = 0.936 (same as above)

$$\lambda_c\sqrt{Q} = 0.911\sqrt{0.936} = 0.881 \leq 1.5 \qquad \text{(satisfied)}$$

Therefore, F_{cr} is again computed using Eq. 4-1.

$$F_{cr} = (0.936)\left(0.658^{(0.936)(0.911)^2}\right)(50) = 33.81\text{ksi} \qquad \text{(governs)}$$

Thus, the lower critical stress occurs for buckling about the geometric axis. Consequently, P_n and P_r are computed as follows.

P_n = (2.40)(33.81) = 81.14 ksi

P_r = 0.90(81.14) = 73.03 ksi

Therefore, the compressive capacity of the bottom strut is shown to well exceed the required capacity.

P_u = 2.58 kips < 73.03 ksi (satisfied)

8.4.1.1.2 Flexure: Major-Axis Bending (W-W)

The major-axis (W-W) bending capacity of the angle is based on either the local buckling (Section 5.1.1) or the lateral-torsional buckling (Section 5.1.3) resistance.

Local Buckling:
The local buckling limit state must be checked when the tip of the angle is in compression. The applicable equation for calculation of the local buckling capacity is determined based on the b/t ratio of the bracing member. The b/t ratio is first evaluated in terms of the following equation.

$$\frac{b}{t} \le 0.54\sqrt{\frac{E}{F_y}} = 13.0$$

$$\frac{b}{t} = \frac{4.0}{0.3125} = 12.8 < 13.0 \qquad \text{(satisfied)}$$

Therefore, the moment resistance is computed according to Eq. 5.1a.

$$M_{nw} = 1.5F_yS_c \qquad \text{Eq. (5.1a)}$$

$$M_{nw} = 1.5(50)(5.88/2.83) = 155.8 \text{ k-in.}$$

The applied moment about the major axis is computed using the following procedure.

$$M_{uw} = B_{1w}M_w = B_{1w}P_ue_w$$

where: $$B_{1w} = \frac{C_m}{1 - P_u/P_{e1w}} \ge 1.0 \qquad \text{Eq. (6-2)}$$

$$C_m = 0.6 - 0.4(M_1/M_2) \qquad \text{Eq. (C1-3)}$$

The C_m factor accounts for the moment gradient, and is to be taken as 1.0 for equal end moments.

$$C_m = 1.0$$

$$\lambda_{cw} = \left(\frac{(0.75)(4.75)(12)}{(1.57)\pi}\right)\sqrt{\frac{50}{29{,}000}} = 0.360$$

$$P_{e_{1w}} = \frac{A_gF_y}{\lambda_{cw}^2} = \frac{(2.4)(50)}{(0.36)^2} = 925.9 \text{kips}$$

$$B_{1w} = \frac{1.0}{1 - 2.58/925.9} \ge 1.0$$

P_u has previously been calculated as 2.58 kips and e_w was shown in Figure 17 to be 1.77 inches Thus, M_{uw} is equal to 4.57 as shown below.

$$M_{uw} = (1.00)(2.58)(1.77) = 4.57 \text{ k-in}$$

Thus, compared to the local buckling resistance of 155.8 k-in., the applied moment is satisfactory.

$$M_{uw} = 4.57 \text{ k-in} < M_{nw} = 155.8 \text{ k-in.} \qquad \text{(satisfied)}$$

Lateral-Torsional Buckling:
The elastic lateral-torsional buckling capacity M_{ob} is determined from Eq. 5-5 of Section 5.3.1 of the AISC SAM.

$$M_{ob} = C_b \frac{0.46Eb^2t^2}{l} \qquad \text{Eq. (5-5)}$$

where: C_b = 1.0 for members (as considered here) with uniform moment throughout their unbraced length according to the commentary of AISC SAM Section 5.1.3.

$$M_{ob} = 1.0\frac{0.46(29{,}000)(4.0)^2(0.3125)^2}{(9.5)(12)} = 182.8 \text{ k-ft.}$$

The yield moment for the major principal axis bending is determined below:

$$M_y = F_yS_w = F_y\left(\frac{I_w}{c_w}\right) = 50\left(\frac{5.88}{2.83}\right) = 103.9 \text{ k-ft.}$$

Therefore, $M_{ob} > M_y$ and the lateral torsional buckling capacity is thus computed according to Eq. 5-3b.

$$M_{nw} = \left\lfloor 1.92 - 1.17\sqrt{M_y/M_{ob}} \right\rfloor M_y \le 1.5M_y \qquad \text{Eq. (5-3b)}$$

$$M_{nw} = \left\lfloor 1.92 - 1.17\sqrt{(103.9)/(182.8)} \right\rfloor (103.9) \le 1.5(103.9)$$

$$M_{nw} = 107.8 \le 155.9 = 107.8 \text{ k-in.}$$

Thus, the lateral torsional buckling capacity about the major axis is sufficient to resist the applied moment of 4.57 k-in.

$$M_{nw} = 107.8 \text{ k-in.} > M_{uw} = 4.57 \text{ k-in.} \qquad \text{(satisfied)}$$

8.4.1.1.3 Flexure: Minor-Axis Bending(Z-Z)

The flexural capacity about the minor axis of a member where the corner of the angle is in compression (which causes the tip of the angle leg to be in tension) is calculated using Eq. 5-2.

$$M_{nz} = 1.50M_y = 1.50F_y\left(\frac{I_z}{c_z}\right) = 1.50(50)\left(\frac{1.46}{1.57}\right) = 69.7\,\text{k-ft.} \qquad \text{Eq. (5-2)}$$

The applied moment about the minor axis is computed using the following equation.

$$M_{uz} = B_{1z}M_z = B_{1z}P_ue_z$$

where: $B_{1z} = \dfrac{C_m}{1-P_u/P_{e1z}} \geq 1.0$

$$C_m = 1.0$$

$$\lambda_{cz} = \left(\frac{(0.75)(4.75)(12)}{(0.781)\pi}\right)\sqrt{\frac{50}{29{,}000}} = 0.723$$

$$P_{e_{1w}} = \frac{A_gF_y}{\lambda_{cw}{}^2} = \frac{(2.4)(50)}{(0.723)^2} = 229.6\,\text{kips}$$

$$B_{1w} = \frac{1.0}{1-2.58/229.6} \geq 1.0$$

Therefore: $M_{uz} = B_{1z}M_z = B_{1z}P_ue_z = (1.01)(2.58)(0.51) = 1.33$ k-in.

Thus, the flexural capacity about the minor axis is sufficient.

$M_{uz} = 1.33$ k-in. $< M_{nz} = 69.7$ k-in. (satisfied)

8.4.1.1.4 Flexure and Axial Compression:

The interaction between flexure and axial compression must also be checked according to Section 6.1.1 of AISC SAM. This evaluation begins with determining the ratio between the applied axial load and the axial capacity.

$$\frac{P_u}{\phi P_n} = \frac{2.58}{73.03} = 0.04 < 0.2$$

Because the ratio between the applied axial force and the axial capacity is less than 0.2, Eq. 6-1b must be satisfied.

$$\left|\frac{P_u}{2\phi P_n}+\left(\frac{M_{uw}}{\phi_b M_{nw}}+\frac{M_{uz}}{\phi_b M_{nZ}}\right)\right|\leq 1.0 \qquad \text{Eq. (6-1b)}$$

According to Article 6.5.4.2, ϕ_b and ϕ_c are to be taken equal to 1.0 and 0.9 respectively. Thus, it is demonstrated that Eq. 6-1b is satisified.

$$\left|\frac{2.58}{2(0.9)(73.03)}+\left(\frac{4.57}{1.00(107.8)}+\frac{1.33}{1.00(69.7)}\right)\right|=0.081\leq 1.0 \qquad \text{(satisfied)}$$

8.4.1.2 Diagonals

The diagonals carry a compressive force that is the result of wind loads and reactions from the loads carried in the top strut. It is assumed that each bay carries a portion of P_w, and the two diagonals carry equal loads. From statics the following equation can be derived to determine the axial force in the diagonals.

$$\left(P_w\right)_{diag.}=\sqrt{a^2+b^2}\left(\frac{P_w}{2na}\right)$$

where:

a = one-half the transverse girder spacing

b = vertical distance between working points for the diagonals

P_w = total applied wind-load force

n = number of bays

$$\left(P_w\right)_{diag.}=\sqrt{\left(\frac{(10.0)(12)}{2}\right)^2+(30)^2}\left(\frac{1.84}{2(3)\left(10.0(12)/2\right)}\right)=0.34\,\text{kips}$$

The axial force in each diagonal due to the wind loading under the Strength III load combination is as follows:

P_u = 1.00(1.40)(0.34) = 0.48 kips

The unbraced length of the diagonal in compression, taken as the distance between the working points, is calculated below:

$$l=\sqrt{\left(\frac{10.0(12)}{2}\right)^2+(30.0)^2}=67.08\,\text{in.}$$

A similar analysis was conducted for the diagonals as was conducted for the bottom strut, and the L 4 x 4 x $^5/_{16}$" member was determined to be adequate for the design wind loading.

8.4.2 End Cross-frame Design

The lateral wind forces are transmitted from the deck to the substructure by the end cross-frames. The following section describes the design of end cross-frames (see Figure 19).

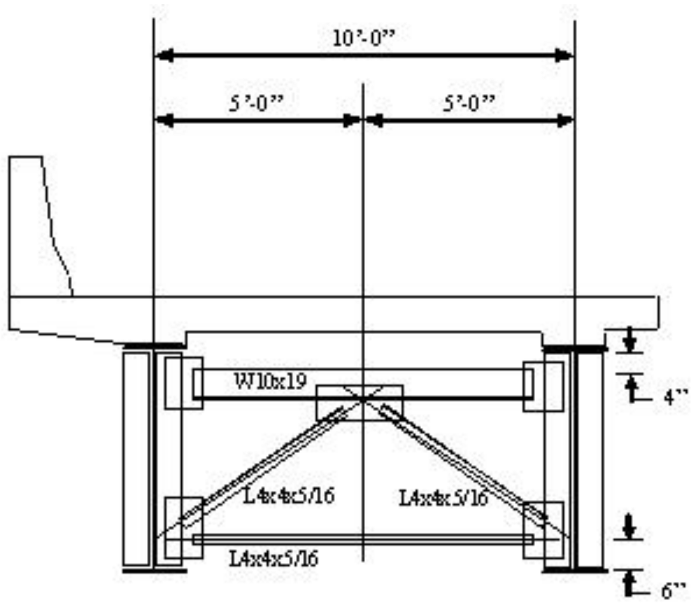

Figure 19 End Cross Frame

8.4.2.1 Top Strut

The top strut of the end cross-frames carries the compressive forces that are a result of the wind load on the structure and vehicles, dead load of the slab, including the haunch, and the wheel loads, including the dynamic load allowance. The total wind pressure P_D, calculated previously, is 0.050 ksf. The total height of the structure is as follows:

Barrier	=	42.00 in.
Deck	=	8.50 in.
Haunch	=	2.00 in.
Girder - top flange	=	43.25 in.
	=	93.75 in. = 7.98 ft

The wind load per unit length on the structure is computed as follows:

$$w_s = (7.98)(0.050) = 0.40 \text{ kips/ft}$$

From Article 3.8.1.3, the wind load per unit length acting normal to the vehicles at a distance of 6.0 feet above the roadway is:

$w_L = 0.10$ kips/ft

The wind loads on the end cross-frames is assumed to be half of the total wind load and is computed below.

$$P_{WL} = 0.40\left(\frac{90.0}{2}\right) = 18.0\,\text{kips}$$

$$P_{WL} = 0.10\left(\frac{90.0}{2}\right) = 4.5\,\text{kips}$$

Each bay is assumed to carry an equal portion of the wind load; therefore, the axial force in the top strut is calculated as follows:

$(P_{WS})_{top\ strut} = 18.00/3 = 6.00$ kips

$(P_{WL})_{top\ strut} = 4.50/3 = 1.50$ kips

The dead load from the slab, concrete haunch, and steel girder acting on the top strut is computed below:

Slab	=	8.50 (14.00 + 12.00 + 7.50)(1/144)(0.150) = 0.30
Concrete Haunch	=	7.50 (14.00 + 12.00 + 7.50/2)(1/144)(0.150) = 0.23
Steel Girder	=	0.03
	=	0.56 kip/ft.

As specified in Article 3.6.1.2.4, the design lane is a 0.64 kips/ft. load distributed over a 10.0 foot width.

$$w_{LL} = \frac{0.64}{10.0(12)}\left(14.0 + 12.0 + \frac{7.50}{2}\right) = 0.16\,\text{kips/ft}$$

The design truck wheel load plus the dynamic load allowance is discussed in Article 3.6.1.2.2 and is as follows.

$$P_{LL} = \frac{32.0}{2}(1.33) = 21.28\,\text{kips}$$

Figure 20 illustrates the position of the above computed live loads that produce the maximum moment and shear in the strut. The maximum moments and reactions in the top strut are then as follows.

M_{DC} = 1.75 k-ft

M_{LL+IM} = 18.30 k-ft

R_{DC} = 3.50 kips

R_{LL+IM} = 25.1 kips

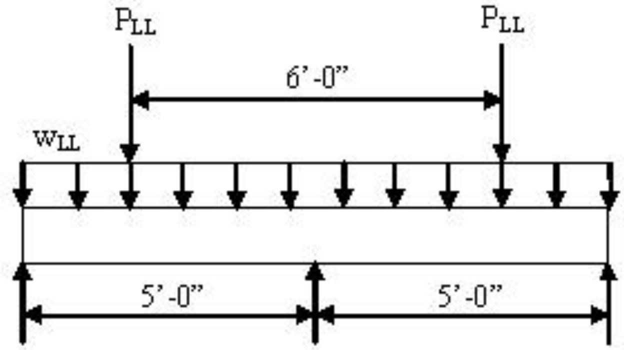

Figure 20 Live load on Top Strut

The Strength I load combination governs the design of the top strut of the end cross-frame design. Thus, the controlling moments and shears are computed as follows.

8.4.2.1.1 Strength I:

M_u = 1.00[1.25(1.75) + 1.75(18.30)] =34.21 k-ft

$$V_u = 1.0\left[1.25\left(\frac{3.5}{2}\right) + 1.75\left(\frac{25.1}{2}\right)\right] = 24.15 \text{kips}$$

To choose a preliminary member for the top strut, the required section modulus assuming the moment capacity of the member is M_p is computed.

$$M_r = \phi_f M_n = \phi_f M_p = \phi_f F_y Z$$

$$Z = \frac{34.2(12)}{1.0(50)} = 8.21 \text{in.}$$

In addition to meeting the flexural requirements, the minimum material thickness requirements must also be considered when selecting the member. Therefore, a W10 x 19 is selected as a trial member.

To determine the flexural capacity of the W10x19 section, the applicability of Appendix A is first evaluated.

F_y = 50 ksi ≤ 70 ksi

$$\frac{2D_c}{t_w} = \frac{2(9.41/2)}{0.25} = 37.64 \le 5.7\sqrt{E/F_{yc}} = 137.3 \qquad \text{Eq. (A6.1-1)}$$

Therefore, Appendix A is applicable. The web slenderness is then evaluated based on Eq. A6.2.1-1.

$$\frac{2D_{cp}}{t_w} \le \lambda_{pw(D_{CP})} \qquad \text{Eq. (A6.2.1-1)}$$

$$\lambda_{pw(D_{CP})} = \frac{\sqrt{E/F_{yc}}}{\left(0.54\left(\frac{M_p}{R_h M_y}\right) - 0.1\right)^2} \qquad \text{Eq. (A6.2.1-2)}$$

$$\lambda_{pw(D_{CP})} = \frac{\sqrt{29{,}000/50}}{\left(0.54\left(\frac{(21.6)(50)}{(1.0)(18.8)(50)}\right) - 0.1\right)^2} = 88.92$$

$$\frac{2D_c}{t_w} = \frac{2(9.41/2)}{0.25} = 37.64 \le 88.92 \qquad \text{(satisfied)}$$

Therefore, the web is compact and the web plastification factors are thus computed as follows.

$$R_{pc} = \frac{M_p}{M_{yc}} = \frac{(21.6)(50)}{(18.8)(50)} = 1.149 \qquad \text{Eq. (A6.2.1-3)}$$

$$R_{pt} = \frac{M_p}{M_{yt}} = \frac{(21.6)(50)}{(18.8)(50)} = 1.149 \qquad \text{Eq. (A6.2.1-4)}$$

The flange slenderness must also be evaluated. The following calculations show that the compression flange is compact.

$$\lambda_{pf} = 0.38\sqrt{E/F_{yc}} = 9.15$$

$$\lambda_f = \frac{b_{fc}}{2t_{fc}} = \frac{4.02}{2(0.395)} = 5.09 \le 9.15 \qquad \text{(satisfied)}$$

Therefore, the flexural capacity of the section based on local buckling is equal to the product of the web plastification factor and the yield moment, as specified in Eq. A6.3.2-1.

$$M_{nc(FLB)} = R_{pc}M_{yc} = 1.149(50)(18.8)/12 = 90.0\text{k-ft.}$$

The flexural capacity based on lateral torsional buckling must also be investigated. Alternative equations are used to compute the lateral torsional buckling capacity based on the lateral bracing distance classification. The lateral bracing distance classifications are based on the value of r_t.

$$r_t = \frac{b_{fc}}{\sqrt{12\left(1+\frac{1}{3}\left(\frac{D_c t_w}{b_{fc} t_{fc}}\right)\right)}} = \frac{4.02}{\sqrt{12\left(\frac{1}{3}\left(\frac{(9.41/2)(0.25)}{(4.02)(0.395)}\right)\right)}} = 1.039\,\text{in.} \qquad \text{Eq. (A6.3.3-10)}$$

The lateral bracing distance is classified as compact if Eq. A6.3.3-4 is satisfied.

$$L_p = r_t \sqrt{E/F_{yc}} = 25.03\,\text{in.} \qquad \text{Eq. (A6.3.3-4)}$$

$$L_b = (5.0)(12) = 60\text{ in.} > 25.03\text{ in.} \qquad \text{(not satisified)}$$

Therefore, the lateral bracing distance is next evaluated compared to the non-compact lateral bracing limit.

$$L_b = 60 \le L_r = 1.95 r_t \frac{E}{F_{yr}} \sqrt{\frac{J}{S_{xc} h}} \sqrt{1+\sqrt{1+6.76\left(\frac{F_{yr} S_{xc} h}{EJ}\right)^2}} \qquad \text{Eq. (A6.3.3-5)}$$

where: $F_{yr} = \min\left(0.7F_{yc}, R_h F_{yt}\frac{S_{xt}}{S_{xc}}, F_{yw}\right)$

$$F_{yr} = \min\left(0.7(50), (1.0)(50)\frac{18.8}{18.8}, 50\right) = \min(35, 50, 50) = 35.0\,\text{ksi}$$

$$J = \frac{1}{3}\left(Dt_w^{\,3} + b_{fc}t_{fc}\left(1 - 0.63\frac{t_{fc}}{b_{fc}}\right) + b_{ft}t_{ft}^{\,3}\left(1 - 0.63\frac{t_{ft}}{b_{bt}}\right)\right) \qquad \text{Eq. (A6.3.3-9)}$$

$$J = \frac{1}{3}\left((9.41)(0.25)^3 + 2(4.02)(.395)^3(.938)\right) = 0.204\,\text{in.}^3$$

$$h = 9.81\,\text{in.}$$

$$L_r = 1.95(1.039)\frac{29,000}{35}\sqrt{\frac{0.204}{(18.8)(9.81)}}\sqrt{1+\sqrt{1+6.76\left(\frac{35}{29,000}\frac{(18.8)(9.81)}{0.204}\right)^2}} = 110\,\text{in.}$$

$$L_b = 60\,\text{in.} < L_r = 110\,\text{in.}$$

Therefore, the lateral torsional buckling resistance is controlled by equation A6.3.3-2 of the specifications.

$$M_{nc} = C_b\left[1-\left(1-\frac{F_{yr}S_{xc}}{R_{pc}M_{yc}}\right)\left(\frac{L_b-L_p}{L_r-L_p}\right)\right]R_{pc}M_{yc} \le R_{pc}M_{yc}$$

$$R_{pc}M_{yc} = (1.149)(50)(18.8) = 1{,}080\,\text{k-ft.}$$

$$M_{nc} = \left[1-\left(\frac{(35)(18.8)}{(1.149)(50)(18.8)}\right)\left(\frac{60-25.03}{112.7-25.03}\right)\right] = 911.7\,\text{k-ft.}$$

$$\therefore M_{nc(LTB)} = 911.7\,\text{k-in.} = 75.9\,\text{k-ft.}$$

Comparing the flange local buckling and lateral torsional buckling capacities, it is determined that the lateral torsional buckling capacity controls the design of the top strut.

$$M_{nc} = \min(90.0, 75.9)$$

$$\phi_f M_{nc} = (1.0)(75.9) = 75.9 \text{ k-ft.}$$

Thus, the moment capacity is sufficient.

$$\phi_f M_{nc} = 75.9 \text{ k-ft.} > M_u = 34.21 \text{ k-ft.} \qquad \text{(satisfied)}$$

In addition to the flexural capacity, the shear capacity must be evaluated to ensure the member is acceptable. The shear capacity of the member is computed below:

$$V_n = V_{cr} = CV_p \qquad \text{Eq. (6.10.9.2-1)}$$

where: $V_p = 0.58F_{yw}Dt_w = 0.58(50)(9.41)(0.25) = 68.22\,\text{kips}$ Eq. (6.10.9.2-2)

The formula used to compute C varies depending on the web slenderness as shown below.

$$1.12\sqrt{\frac{Ek}{F_{yw}}} = 1.12\sqrt{\frac{(29{,}000)(5.0)}{50}} = 60.31$$

$$\frac{D}{t_w} = \frac{9.41}{0.25} = 37.64 < 60.31$$

Therefore, C = 1.0

$$\phi_v V_n = (1.0)(68.22) = 68.22\,\text{kips} \ge V_u = 24.15\,\text{kips} \qquad \text{(satisfied)}$$

Thus, the shear requirements are satisfied.

The member must also be evaluated for combined axial compression and flexure, for which the Strength III and Strength V load combinations are most likely to govern.

8.4.2.1.2 Strength III:

$P_u = 1.00[1.25(0.00) + 1.40(6.00)] = 8.40$ kips

$M_{ux} = 1.00[1.25(1.75) + 1.40(0.00)] = 2.19$ k-ft.

Article 6.9.2.1 specifies the axial capacity as follows.

$$P_r = \phi_c P_n \qquad \text{Eq. (6.9.2.1-1)}$$

where: $\phi_c = 0.90$

The equation used for determining P_n is selected based on the value of the following slenderness parameter.

$$\lambda = \left(\frac{Kl}{r_s\pi}\right)^2 \frac{F_y}{E} = \left(\frac{(0.75)(9.5)(12)}{(0.874)\pi}\right)^2 \frac{50}{29{,}000} = 1.67 \qquad \text{Eq. (6.9.4.1-2)}$$

Because λ is less than 2.25, P_n is computed as follows.

$$P_n = 0.66^{\lambda} F_y A_s = (0.66)^{1.67}(50)(5.62) = 140.4\,\text{kips} \qquad \text{Eq. (6.9.4.1-1)}$$

Thus, the factored resistance is equal to 126.4 kips.

$P_r = 0.90(140.4) = 126.4$ kips

The width-to-thickness ratios of the flange and web are then evaluated to determine the governing equation for moment resistance.

Flange:

$$\frac{(b/2)}{t_f} = \frac{(4.02/2)}{0.395} = 5.09 \le 0.56\sqrt{\frac{E}{F_y}} = 13.49$$

Web:

$$\left(\frac{d - 2t_f - 2k}{t_w}\right) = \left(\frac{10.2 - 2(0.395) - 2(0.695)}{0.25}\right) = 32.08 \le 1.49\sqrt{\frac{E}{F_y}} = 1.49 \qquad \text{Eq. (6.9.4.2-1)}$$

Thus, the moment resistance is equal to M_p.

$M_{rx} = \phi_f M_p = (1.00)(50)(21.6) = 1{,}080$ k-in. $= 90.0$ k-ft.

The combined influence of axial force and moment must then satisfy the following equation.

$$\frac{P_u}{P_r} = \frac{8.4}{126.4} < 0.2 \text{ then,}$$

$$\frac{P_u}{2P_r} + \frac{M_{ux}}{M_{rx}} \le 1.0 \qquad \text{Eq. (6.9.2.2-1)}$$

$$\frac{8.4}{2(126.4)} + \frac{2.19}{90.0} = 0.06 \le 1.0 \qquad \text{(satisfied)}$$

8.4.2.1.3 Strength V:

Similarly, the applied axial force and moment due to the Strength V load combination are computed below.

P_u = 1.00[1.25(0.00) + 1.35(0.00) + 1.40(6.00) + 0.40(1.50)] = 9.00 kips

M_{ux} = 1.00[1.25(1.75) + 1.35(18.30) + 0.40(0.00) + 1.40(0.00)] = 26.89 k-ft.

The axial load and moment interaction equation 6.9.2.2-1 is also shown to be satisfied for this load combination below.

$$\frac{9.0}{2(126.4)} + \frac{26.89}{90.0} = 0.333 \le 1.0 \qquad \text{(satisfied)}$$

8.4.2.2 Diagonals

The diagonals carry a compressive force that is the result of wind loads and reactions from the loads carried in the top strut. The geometry of the end cross-frames was previously illustrated in Figure 19. As previously discussed, the design of the cross-frame is based on the assumption that each bay carries an equal portion of the total wind forces. The axial force is computed below using the same process used earlier in this cross-frame design example.

$$P_{WS} = 18.0 \text{kips}$$

$$P_{WL} = 4.5 \text{kips}$$

$$\left(P_W\right)_{\text{diag.}} = \sqrt{a^2 + b^2}\left(\frac{P_W}{2na}\right)$$

$$\left(P_{WS}\right)_{\text{diag.}} = \sqrt{\left(\frac{10.0(12)}{2}\right)^2 + 30^2}\left(\frac{18.0}{2(3)\left(10.0(12)/2\right)}\right) = 3.35 \text{kips}$$

$$\left(P_{WL}\right)_{\text{diag.}} = \sqrt{\left(\frac{10.0(12)}{2}\right)^2 + 30^2}\left(\frac{4.5}{2(3)\left(10.0(12)/2\right)}\right) = 0.84 \text{kips}$$

The axial force in the diagonal as a result of the dead-load reaction R_{DC} on the top strut is computed below.

$$(P_{DC})_{\text{diag.}} = \sqrt{\left(\frac{10.0(12)}{2}\right)^2 + 30^2}\left(\frac{3.5}{2(30)}\right) = 3.91\,\text{kips}$$

The axial force in the diagonal as a result of the live-load reaction R_{LL+IM} on the top strut is computed as follows.

$$(P_{LL+IM})_{\text{diag.}} = \sqrt{\left(\frac{10.0(12)}{2}\right)^2 + 30^2}\left(\frac{25.1}{2(30)}\right) = 28.06\,\text{kips}$$

The following calculations determine the controlling load combination.

8.4.2.2.1 Strength I:

P_u = 1.00[1.25(3.91) + 1.75(28.06)] = 54.0 kips (governs)

8.4.2.2.2 Strength III:

P_u = 1.00[1.25(3.91) + 1.40(3.35)] = 9.58 kips

8.4.2.2.3 Strength V:

P_u = 1.00[1.25(3.91) + 1.35(28.06) + 0.40(3.35) + 0.40(0.84)] = 44.4 kips

The initial member selection will be based on the compressive strength slenderness requirements of the member and minimum material thickness requirements. The distance between the working points will be taken as the unbraced length ℓ.

$$\frac{Kl}{r} \leq 140$$

$$l = \sqrt{\left(\frac{10.0(12)}{2}\right)^2 + 30^2} = 67.08\,\text{in.}$$

$$r = \frac{0.75(67.08)}{140} = 0.359\,\text{in.}$$

Thus an L4 x 4 x 5/8 is selected as the trial member, assuming a ½-inch connection plate.The member must be evaluated for individual and combined influences of flexure and axial compression as detailed below.

<u>Axial Compression:</u>
The axial compressive resistance of the member is computed from the following equation:

$P_r = \phi_c P_n$

where: ϕ_c = the resistance factor for axial compression

$\phi_c = 0.90$

$P_n = A_g F_{cr}$

Alternative equations are given for F_{cr} based on the value of the slenderness parameter, λ_c.

$$\lambda_c = \left(\frac{Kl}{r\pi}\right)\sqrt{\frac{F_y}{E}} = \left(\frac{(0.75)(67.08)}{0.774\pi}\right)\sqrt{\frac{50}{29{,}000}} = 0.859$$

The value of Q must also be determined, which is based on the b/t ratio of the angle.

$$\frac{b}{t} = \frac{4.0}{0.625} = 6.40 \le 0.446\sqrt{\frac{E}{F_y}} = 10.7$$

$$\therefore Q = 1.0$$

The product of λ_c and Q is then used to determine the controlling equation for F_{cr}.

$$\lambda_c\sqrt{Q} = 0.859(1.0) = 0.859 \le 1.50$$

Because the product of of λ_c and Q is less than 1.5, F_{cr} is computed according to the following equation.

$$F_{cr} = Q\left(0.658^{Q\lambda_c^2}\right)F_y = (1.0)\left(0.658^{(1.0)(0.859)^2}\right)(50) = 36.71 \qquad \text{Eq. (4-1)}$$

The nominal and factored axial capacities are then as follows.

$P_n = (4.61)(36.77) = 169.5$ ksi

$P_r = 0.90(169.5) = 152.6$ ksi

Thus, the member is sufficient to resist the applied axial force of 54.0 kips.

$P_u = 54.0$ kips $< P_r = 152.6$ kips (satisfied)

8.4.2.2.4 Flexure: Major-Axis Bending (W-W)

The flexural capacity of the major–axis is based on the minimum of the resistance determined from the local buckling and lateral torsional buckling equations, which is governed by the AISC SAM Section 5.3.1a. The applied moment about the major axis is computed as follows.

$M_{uw} = B_{1w}M_w = B_{1w}P_u e_w$

$$B_{1w} = \frac{C_m}{1 - P_u / P_{e1w}} \geq 1.0 \qquad \text{Eq. (6-2)}$$

$$C_m = 1.0$$

$$\lambda_{cw} = \left(\frac{(0.75)(67.08)}{1.51\pi}\right)\sqrt{\frac{50}{29,000}} = 0.440$$

$$P_{e1w} = \frac{A_g F_y}{\lambda_{cw}^{\ 2}} = \frac{(4.61)(50)}{(0.441)^2} = 1,190.6\,\text{kips}$$

$$B_{1w} = \frac{1.0}{1 - \dfrac{54.0}{1,190.6}} = 1.05$$

$M_{uw} = B_{1w}M_w = B_{1w}P_ue_w = (1.05)(54.0)(1.77) = 100.4$ k-in

<u>Local Buckling:</u>
The following b/t ratio of the angle is used to determine the governing equation for local buckling capacity.

$$\frac{b}{t} = \frac{4.0}{0.625} = 6.40 \leq 0.54\sqrt{\frac{E}{F_y}} = 13.00$$

Therefore, the nominal moment local buckling capacity is 277.74 k-in, which is sufficient.

$M_{nw} = 1.5F_yS_c$ Eq. (5.1a)

$M_{nw} = 1.5(50)(10.48/2.83) = 277.74$ k-in.

$M_{nw} = 277.7$ k-in. $> M_{uw} = 100.4$ k-in. (satisfied)

<u>Lateral-Torsional Buckling:</u>
The following calculations determine the elastic lateral-torsional buckling capacity M_{ob}, in accordance with Section 5.3.1 of the SAM.

$$M_{ob} = C_b\left(\frac{0.46Eb^2t^2}{l}\right) = 1.0\left(\frac{0.46(29,000)(4.0)^2(0.625)^2}{67.08}\right) = 1,243\,\text{k-in.} \qquad \text{Eq. (5-5)}$$

M_y is then computed as follows.

$$M_y = F_yS_w = F_y\left(\frac{I_w}{c_w}\right) = 50\left(\frac{10.48}{2.83}\right) = 185.2\text{ k-in.}$$

Since $M_{ob} > M_y$, the nominal lateral torsional buckling resistance of the member about the major principal axis is computed from the following equation:

$$1.5M_y = 1.5(185.2) = 277.8\,\text{k-in.}$$

$$M_{nw} = \left\lfloor 1.92 - 1.17\sqrt{M_y / M_{ob}} \right\rfloor M_y \le 1.5M_y \qquad \text{Eq. (5-3b)}$$

$$\mathrm{M_{nw}} = \left\lfloor 1.92 - 1.17\sqrt{(185.2)/(1{,}243)} \right\rfloor (185.2) = 271.9$$

$$\therefore \mathrm{M_{nw}} = 271.9\,\text{k-in.}$$

Thus, the lateral torsional buckling capacity is also sufficient to resist the applied loads.

M_{nw} = 271.9 k-in. > M_{uw} = 100.4 k-in. (satisfied)

8.4.2.2.5 Flexure: Minor-Axis Bending (Z-Z):

The nominal flexural resistance of the section about the minor axis is calculated below:

$$M_{nz} = 1.5M_y = 1.5F_y\left(\frac{I_z}{c_z}\right) = 1.5(50)\left(\frac{2.76}{1.67}\right) = 124.0\,\text{k-in.} \qquad \text{Eq. (5-2)}$$

The applied moment about the minor axis is computed using the following equation.

$M_{uz} = B_{1z}M_z = B_{1z}P_u e_z$

$$B_{1z} = \frac{C_m}{1 - P_u / P_{e1z}} \ge 1.0$$

$$C_m = 1.0$$

$$\lambda_{cw} = \left(\frac{(0.75)(67.08)}{1.51\pi}\right)\sqrt{\frac{50}{29{,}000}} = 0.440$$

$$P_{e1z} = \frac{A_g F_y}{\lambda_{cz}^{\;2}} = \frac{(4.61)(50)}{(0.859)^2} = 312.4\,\text{kips}$$

$$B_{1w} = \frac{1.0}{1 - \dfrac{54.0}{312.4}} = 1.21$$

M_{uz} = (1.21)(54.0)(0.61) = 39.9 k-in.

Thus, the moment capacity about the minor axis is sufficient.

M_{uz} = 39.9 k-in. < M_{nz} = 124.0 k-in. (satisfied)

8.4.2.2.6 Flexure and Axial Compression:

The member is checked for the combined flexural and axial compressive forces according to Section 6.1.1 in the AISC SAM, which specifies that the ratio between the ultimate axial force and the axial capacity be used to determine the governing equation.

$$\frac{P_u}{\phi P_n} = \frac{54.0}{152.3} = 0.35$$

Because $P_u/\phi P_n$ is greater than 0.2, the following equation must be satisfied.

$$\left| \frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{uw}}{\phi_b M_{nw}} + \frac{M_{uz}}{\phi_b M_{nz}} \right) \right| \le 1.0 \qquad \text{Eq. (6-1a)}$$

Using ϕ_b equal to 1.00 and the other values computed above gives the following.

$$\left| \frac{54.0}{152.3} + \frac{8}{9}\left(\frac{99.4}{(1.0)(271.9)} + \frac{41.8}{(1.0)(124.0)} \right) \right| = 0.98 \qquad \text{(satisfied)}$$

Thus, the diagonal member is acceptable.

8.5 Stiffener Design

8.5.1 Bearing Stiffener Design

Bearing stiffeners must be provided at locations of concentrated loads for the webs of sections that do not satisfy the provisions of Article D6.5. Specifically, Article D6.5 specifies the web strength of steel I-girders with respect to the limit states of web local yielding and web crippling. Both of these limit states are evaluated below for the abutment and pier locations, assuming a 10 in. bearing length at each location.

The requirement to prevent web local yielding is expressed by Eq. D6.5.2-1.

$$R_u \le \phi_b R_n$$

The web local yielding capacity, R_n, is given by Eq. D6.5.2-2 for interior pier reactions and by Eq. D6.5.2-3 for abutment reactions.

$$R_n = (5k + N)F_{yw}t_w$$

where:

k = distance from the outer face of the flange resisting the bearing force to the web toe of the fillet

= 1.25 + 0.3125 = 1.5625 in

N = bearing length = 10 in.

F_{yw} = 50 ksi

t_w = 0.5 in.

Substituting the above values into Eq. D6.5.2-2 gives the following.

$R_n = [(5)(1.5625) + 10](50)(0.5)$

$R_n = 445$ kips

Then evaluating Eq. D6.5.2-1, where ϕ_b is equal to 1.00 and R_u at the pier is equal to 337 kips, shows that the web yielding requirements are satisfied at the pier.

$337 \leq (1.00)(445) = 445$ (satisfied)

Equation D6.5.2-3 is now used to evaluate the web yielding capacity at the abutments.

$$R_n = (2.5k + N)F_{yw}t_w \qquad \text{(D6.5.2-2)}$$

$R_n = [(2.5)(1.5625) + 10](50)(0.4375)$

$Rn = 304$ kips

Again evaluating Eq. D6.5.2-1, where R_u at the pier is equal to 258 kips, shows that the web yielding requirements are also satisfied at the abutments.

$258 \leq (1.00)(304) = 304$ (satisfied)

The requirements to prevent web crippling are expressed by Eq. D6.5.3-1.

$R_u \leq \phi_b R_n$

For interior pier reactions, the web crippling capacity, R_n, is given by Eq. D6.5.3-2.

$$R_n = 0.8t_w^2\left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}}$$

where: d = depth of the steel section

d = 44.375 in.

t_f = thickness of the flange resisting the concentrated load

t_f = 1.25 in.

$$R_n = 0.8(0.5)^2 \left[1 + 3\left(\frac{10}{44.375}\right)\left(\frac{0.5}{1.25}\right)^{1.5}\right]\sqrt{\frac{(29,000)(50)(1.25)}{0.5}} = 446\,\text{kips}$$

Evaluation of Eq. D6.5.3-1 where ϕ_w is equal to 0.80 then shows that the pier section has sufficient web crippling resistance.

$337 \leq (0.80)(446) = 356$ (satisfied)

For abutment reactions, R_n is expressed by either Eq. D6.5.3-3 or D6.5.3-4 depending on the ratio between the bearing length and the steel section depth. For the present example with N/d = 10/44 = 0.23 at the abutments, Eq. D6.5.3-4 applies.

$$R_n = 0.4t_w^2 \left[1 + 3\left(\frac{4N}{d} - 0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_{yw}t_f}{t_w}} \qquad \text{Eq. (D6.5.3-4)}$$

$$R_n = 0.4(0.4375)^2 \left[1 + 3\left(\frac{(4)(10)}{44} - 2\right)\left(\frac{0.4375}{1.25}\right)^{1.5}\right]\sqrt{\frac{(29,000)(50)(1.25)}{0.4375}} = 179\,\text{kips}$$

Evaluating Eq. D6.5.3-1 at the abutments thus shows that bearing stiffeners must be provided to prevent web crippling.

$258 > (0.80)(179) = 143$ (not satisfied)

The bearing stiffeners are typically plates welded to both sides of the web that extend the full depth of the web, and are as close to the outer edges of the flanges as practical. The plates are to bear against or to be welded to the flange that the load is transmitted through. This example illustrates the design of bearing stiffeners at Abutment 1.

8.5.1.1 Projecting Width (Article 6.10.11.2.2)

The width, b_t, of projecting stiffener elements must satisfy the following:

$$b_t \leq 0.48t_p\sqrt{\frac{E}{F_{ys}}} \qquad \text{Eq. (6.10.11.2.2-1)}$$

It will be assumed that 6 inches wide plates are welded to each side of the web. Eq. 6.10.11.2.2-1 is then rearranged to determine the minimum allowable thickness of the stiffener.

$$\left(t_p\right)_{\min.} = \frac{b_t}{0.48\sqrt{E/F_{ys}}} = \frac{6.0}{0.48\sqrt{29{,}000/50}} = 0.52\,\text{in.}$$

Thus, a 6 inch by 5/8 inch plate will be used to evaluate the bearing stiffener requirements.

8.5.1.2 Bearing Resistance (Article 6.10.11.2.3)

The factored resistance for the bearing stiffeners shall be taken as:

$$\left(R_{sb}\right)_r = \phi_b \left(R_{sb}\right)_n \qquad \text{Eq. (6.10.11.2.3-1)}$$

where: ϕ_b = resistance factor for bearing =1.0 (Article 6.5.4.2)

$(R_{sb})_n$ = nominal bearing resistance for bearing stiffeners

= $1.4A_{pn}F_{ys}$ Eq. (6.10.11.2.3-2)

A_{pn} = area of the projecting elements for the stiffener outside of the web-to-flange fillet welds but not beyond the edge of the flange

In this design example, it is assumed the clip provided at the base of the stiffener to clear the web-to-flange weld is 1.5 inches in length.

A_{pn} = 2(6.0 - 1.5)(0.625) = 5.63 in.^{2}

$(R_{sb})_n$ = 1.4(5.63)(50) = 394 kips

$(R_{sb})_r$ = (1.00)(394) = 394 kips > R_u = 257.5 kips (satisfied)

The 6 inch by 5/8 inch bearing stiffeners have adequate bearing resistance.

8.5.1.3 Axial Resistance of Bearing Stiffeners (Article 6.10.11.2.4)

The factored axial resistance is calculated from Article 6.9.2.1 of the specifications, where the radius of gyration is computed about the mid-thickness of the web, and the effective length is taken as 0.75D. For stiffeners welded to the web, part of the web is considered in the effective column section. The strip of web included in the effective column is not more than $9t_w$ on each side of the stiffeners. Therefore, the area of the effective column section is computed below:

A_s = 2[(6.0)(0.625) + 9(0.4375)(0.4375)] = 10.95 in.^{2}

The moment of inertia of the effective column section is computed as follows:

$$I_s = \frac{0.625(6.0 + 0.4375 + 6.0)^3}{12} = 100.2\,\text{in.}^4$$

The radius of gyration computed about the mid-thickness of the web is computed as:

$$r_s = \sqrt{\frac{I_s}{A_s}} = \sqrt{\frac{100.2}{10.95}} = 3.03\,\text{in.}$$

The effective length is computed as follows:

Kl = 0.75D = 0.75(42.0) = 31.50 in.

The bearing stiffeners must satisfy the limiting slenderness ratio, stated in Article 6.9.3, which is 120 for main members in compression.

$$\frac{Kl}{r_s} = \frac{31.5}{3.03} = 10.40 \le 120 \qquad \text{(satisfied)}$$

As previously mentioned, the factored axial resistance of the effective column section is calculated from Article 6.9.2.1 using the specified minimum yield strength of the stiffener.

$$P_r = \phi_c P_n \qquad \text{Eq. (6.9.2.1-1)}$$

where:

ϕ_c = resistance factor for axial compression = 0.90 (Article 6.5.4.2)

P_n = nominal compressive resistance from Article 6.9.4.1, which is determined based on the value of λ

Determine P_n using Article 6.9.4.1. First, determine the elastic critical buckling load, P_e, per Article 6.9.4.1.2

$$P_e = \frac{\pi^2 E}{\left(\frac{K\ell}{r_s}\right)^2} A_g \qquad \text{Eq. (6.9.4.1.2-1)}$$

$$P_e = \frac{\pi^2 (29000)}{(10.40)^2}(10.95) = 2{,}646\text{kip}$$

$$P_o = QF_yA_g \qquad \text{Article 6.9.4.1.1}$$

where,

P_o = Equivalent nominal yield resistance

Q = Slender Element Reduction factor, taken as 1.0 for bearing stiffeners

$P_o = QF_yA_g = (1.0)(50)(10.95) = 547.5$ kip

$P_e / P_o = 2646 / 547.5 = 4.83 > 0.44$

Therefore, Eq. 6.9.4.1.1-1 applies.

$$P_n = \left[0.658^{\left(\frac{P_o}{P_e}\right)}\right]P_o \qquad \text{Eq. (6.9.4.1.1-1)}$$

$$P_e = \left[0.658^{547.5/5646}\right](547.5) = 502.1\,\text{kip}$$

$P_r = 0.90(502.1) = 451.9$ kips $> R_u = 257.5$ kips (satisfied)

8.5.1.4 Bearing Stiffener-to-Web Welds

Adequate shear strength of the welds joining the bearing stiffener to the web must also be verified. First the weld shear strength, which is the area of the weld multiplied by 60 percent of the yield strength of the weld metal, is determined.

$$R_r = 0.6\phi_{e2}F_{exx} \qquad \text{Eq. (6.13.3.2.4b-1)}$$

where:

ϕ_{e2} = resistance factor for shear in the throat of the weld metal = 0.80

F_{exx}= classification strength of the weld metal = 70 ksi for this example

$R_r = 0.6(0.80)(70) = 33.6$ ksi

The minimum size fillet weld permissible in this situation is 0.25 inches, according to Table 6.13.3.4-1. Using this weld size the shear strength per unit length of weld is as follows.

$V = 33.6(0.707)(0.25) = 5.94$ k/in.

The length of the weld, allowing 2.5 inches for clips at both the top and bottom of the stiffener, is:

$L = 42.0 – 2(2.5) = 37.0$ in.

The total factored resistance of the weld connecting the stiffener to the web of the section is then 879 kips which is greater than the required shear strength of 257.5 kips.

$4(37.0)(5.94) = 879.1$ kips > 257.5 kips (satisfied)

8.6 Weld Design

This section outlines the weld design for the web-to-flange junction. The weld design strength is checked against the shear flow associated with the design loads. The horizontal shear flow at the end bearing is computed from the following equation:

$$s = \frac{VQ}{I}$$

where: V = shear force

Q = statical moment of the area about the neutral axis

I = moment of inertia

Similar to previous calculations, the shear flow will be computed by considering the cross sectional properties applicable to various applied forces. Thus, the statical moment of the area about the neutral axis will be computed for each applicable section.

8.6.1 Steel Section:

Top flange: Q =(10.50)(25.63) = 269.1 $in.^3$

Bottom flange: Q = (20.0)(17.37) = 347.4 $in.^3$

8.6.2 Long-term Section:

Top flange: Q = (10.50)(12.81) = 134.5 $in.^3$

Slab: Q = (34.00)(18.43) = $\underline{626.6}$ $in.^3$

= 761.1 $in.^3$

Bottom flange: Q = (20.0)(30.20) = 604.0 $in.^3$

8.6.3 Short-term Section:

Top flange: Q = (10.5)(4.51) = 47.4 $in.^3$

Slab: Q = (102.0)(10.13) = $\underline{1033.3}$ $in.^3$

= 1,081 $in.^3$

Bottom flange: Q = (20.0)(38.50) = 770.0 $in.^3$

The shear flow under each loading is thus computed as follows, where it is determined that the bottom flange experiences the highest level of shear flow.

Top Flange:

DC1:	$s = (1.25)(44)(269.1)/15{,}969$	=	0.93
DC2:	$s = (1.25)(7)(761.1)/35{,}737$	=	0.19
DW:	$s = (1.55)(9)(761.1)/35{,}737$	=	0.29
LL+IM	$s = (1.75)(103)(1080.7)/48{,}806$	=	<u>4.00</u>
		=	5.41 kip/in

Bottom Flange:

DC1:	$s = (1.25)(44)(347.4)/15{,}969$	=	1.20
DC2:	$s = (1.25)(7)(604.0)/35{,}737$	=	0.15
DW:	$s = (1.55)(9)(604.0)/35{,}737$	=	0.23
LL+IM	$s = (1.75)(103)(770.0)/48{,}806$	=	<u>2.84</u>
		=	4.42 kip/in

Thus, the applied shear flow of 5.41 k/in., must be evaluated in comparison to the shear flow capacity of both the fillet welds and the base metal. The specifications limit the minimum size of a fillet weld in which the base metal is thicker than 0.75" to 5/16." Therefore, a 5/16" fillet weld is assumed on each side of the plate. The factored resistance of the weld metal is determined as follows:

$$R_r = 0.6\phi_{e2}F_{exx} \qquad \text{Eq. (6.13.3.2.4b-1)}$$

where: ϕ_{e2} = resistance factor for shear on the throat of the weld metal

= 0.80 (Article 6.5.4.2)

F_{exx} = classification strength of the weld metal= 70 ksi

R_r = 0.6(0.80)(70) = 33.6 ksi

The allowable shear flow for the 5/16 inch welds is:

$$v = 33.6(0.707)(0.3125)(2) = 14.85 \text{ k/in.}$$

From Article 6.13.5.3, the factored shear resistance of the connected material is computed as follows:

$R_r = \phi_v R_n$ Eq. (6.13.5.3-1)

$R_n = 0.58A_gF_y$ Eq. (6.13.5.3-2)

where:

R_n = nominal resistance in shear

R_n = 0.58(1.00)(50) = 29.0 ksi

A_g = gross area of the connection element

Fy = minimum yield strength of connection element

Φv = resistance factor for shear = 1.00

The allowable shear flow on the connected material is:

v = 29.0(0.4375) = 12.69 k/in. (governs)

Since, v = 12.69 k/in. > v_u = 5.56 k/in., the 5/16" fillet weld is adequate for the web-to-flange weld.

9.0 REFERENCES

1. AASHTO (2010). *AASHTO LRFD Bridge Design Specifications*, 5th Edition with 2010 Interims, American Association of State Highway and Transportation Officials, Washington, DC.