# BEHAVIORAL MODELS AND CHARACTERISTICS OF BICYCLE-AUTOMOBILE MIXED-TRAFFIC: PLANNING AND ENGINEERING IMPLICATIONS

by

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16. Abstract

This report addresses an important need for fundamental understanding of bicycle-automobile mixed-traffic. It presents models of (1) gap acceptance behavior and (2) bicyclist behavior at the onset of a yellow traffic signal indication, in addition to analysis of (3) coordinating traffic signals to provide (simultaneous) progression for both bicycles and automobiles. Fundamental insights into mixed-traffic behavior are derived and applied to selected problems in mixed-traffic engineering and operations.

Discrete choice (probit) models are developed for both motorist and cyclist gap acceptance behavior. An important fundamental insight from these models is that both cyclists and motorists (on average) require a longer gap when the gap is closed by a large vehicle (e.g. bus), and both will accept a shorter gap when the gap is closed by a bicycle, relative to a gap closed by a passenger car.

A methodology for determining and adequate clearance interval (normally consisting of part yellow change and part all-red clearance intervals) for bicycles is developed from a deterministic model based on kenematic relations. The bicyclists behavior at the onset of a yellow signal indication are obtained from both models (e.g. the reasons that bicycles are required longer clearance intervals at sufficiently wide intersections).

Finally, a conceptual foundation, consisting of three primary contributions, is developed for analyzing bicycle-automobile mixed-traffic progression along signalized streets. The principal considerations for bicycle progression are articulated. Several concepts and techniques that provide improved (or alternative) multiobjective solutions are presented and analyzed. A multiobjective formulation framework for solving the mixed-traffic design is proposed. This framework formally incorporates the elements introduced as part of the first two contributions and provides a way to handle the inherent competing objectives.

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Bicycle-Automobile Mixed Traffic, Gap Acceptance Behavior Yellow Change, All-Red Clearance Interval, Bicyclist Behavior, Mixed-Traffic Progression

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# ABSTRACT

This report addresses an important need for fundamental understanding of bicycle-automobile mixed-traffic. It presents models of (1) gap acceptance behavior and (2) bicyclist behavior at the onset of a yellow traffic signal indication, in addition to analysis of (3) coordinating traffic signals to provide (simultaneous) progression for both bicycles and automobiles. Fundamental insights into mixed-traffic behavior are derived and applied to selected problems in mixed-traffic engineering and operations.

Discrete choice (probit) models are developed for both motorist and cyclist gap acceptance behavior. An important fundamental insight from these models is that both cyclists and motorists (on average) require a longer gap when the gap is closed by a large vehicle (e.g. bus), and both will accept a shorter gap when the gap is closed by a bicycle, relative to a gap closed by a passenger car.

A methodology for determining an adequate clearance interval (normally consisting of part yellow change and part all-red clearance intervals) for bicycles is developed from a deterministic model based on kinematic relations. A probability of stopping model is calibrated from observations of actual bicyclist behavior. It was show to be a useful tool to evaluate clearance intervals, because it reflects actual bicyclist behavior. Fundamental insights into bicyclist behavior at the onset of a yellow signal indication are obtained from both models (e.g. the reasons that bicycles may require longer clearance intervals, relative to automobiles, at sufficiently wide intersections).

Finally, a conceptual foundation, consisting of three primary contributions, is developed for analyzing bicycle-automobile mixed-traffic progression along signalized streets. The principal considerations for bicycle progression are articulated. Several concepts and techniques that provide improved (or alternative) multiobjective solutions are presented and analyzed. A multiobjective formulation framework for solving the mixed-traffic design problem is proposed. This framework formally incorporates the elements introduced as part of the first two contributions and provides a way to handle the inherent competing objectives.

# Executive Summary

Recent emphasis on alternatives to automobile transportation has brought to light deficiencies in basic research performed in bicycle traffic science. This report addresses an important need for fundamental understanding of bicycle-automobile mixed-traffic. Such understanding allows engineers to mitigate traffic hazards to bicyclists through improved facility design and operation. This report presents models of (1) gap acceptance behavior and (2) bicyclist behavior at the onset of a yellow traffic signal indication, in addition to analysis of (3) coordinating traffic signals to provide (simultaneous) progression for both bicycles and automobiles. Fundamental insights into mixed-traffic behavior are derived and applied to selected problems in mixed-traffic engineering and operations.

The study of gap acceptance behavior is of importance in reducing traffic hazards because a large percentage of bicycle-automobile crashes occur at intersections. Such study is also important to the analysis of intersection capacity and delay. From roadside observations, discrete choice (probit) models are developed for both motorist and cyclist gap acceptance behavior. An important fundamental insight from these models is that both cyclists and motorists (on average) require a longer gap when the gap is closed by a large vehicle (e.g. bus), and both will accept a shorter gap when the gap is closed by a bicycle, relative to a gap closed by a passenger car. In addition, the models show that cyclists making a right-turn from a minor (stop-controlled) street accept relatively short gaps in automobile traffic.

Several prior studies indicate that a disproportionate number of bicycle-automobile crashes occur because the standard automobile clearance interval (consisting of part yellow change and part all-red clearance intervals) may be too short for bicycles at some intersections. A methodology for determining an adequate clearance interval for bicycles is developed from a deterministic model based on kinematic relations. A probability of stopping model is calibrated from observations of actual bicyclist behavior at the onset of yellow. This model is shown to be a useful tool to evaluate clearance intervals, because it reflects actual bicyclist behavior. Fundamental insights into bicyclist behavior at the onset of a yellow signal indication are obtained from both models (e.g. the reasons that bicycles may require longer clearance intervals, relative to automobiles, at sufficiently wide intersections).

Finally, a conceptual foundation, consisting of three primary contributions, is developed for analyzing bicycle-automobile mixed-traffic progression along signalized streets. The principal considerations for bicycle progression are articulated. Several concepts and techniques that provide improved (or alternative) multiobjective solutions are presented and analyzed. A multiobjective formulation framework for solving the mixed-traffic design problem is proposed. This framework formally incorporates the elements introduced as part of the first two contributions and provides a way to handle the inherent competing objectives. It is shown that progression designs can be used to mitigate hazards to bicyclists in several ways, including reducing the probability that cyclists arrive at an intersection near the onset of yellow.

## TABLE OF CONTENTS

## LIST OF FIGURES

# LIST OF TABLES

# DISCLAIMER

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# 1 INTRODUCTION

## 1.1 MOTIVATION AND PROBLEM DEFINITION

Recent emphasis on environmental degradation and fuel consumed by personal automobile transport are two reasons to consider greater use of bicycles for transportation purposes. Bicycles consume no fossil fuels in use and appear to be far less damaging to the earth's environment than automobiles. Traffic congestion in urban areas is another problem that many believe can be partially alleviated by diverting some auto trips to bicycles. Bicycles have been estimated to require from only one-twelfth to one-half (depending on the automobile operating speed) of the roadway width to carry equal numbers of people per unit time as automobiles (assuming average automobile occupancy in developed countries) (Lowe, 1989).

Some claim that bicycles make communities more "livable", by reducing noise pollution and providing more opportunities for people to meet, greet, and talk to each other, than if insulated from each other in cars. Bicycles also cost much less to purchase and operate than automobiles, making them accessible to individuals across a wider income spectrum. As such, they provide an affordable complementary mode to transit users in areas or time periods not served well by transit. Current awareness and interest in physical fitness also provides motivation for using bicycles for transportation, hence efficiently combining exercise with otherwise unproductive work commute or other travel time.

These and other reasons have prompted residents in the U.S. to organize and lobby governmental agencies and officials for improvements in bicycle transportation systems. These groups include national organizations such as the League of American Bicyclists and the Bicycle Federation of America, state organizations such as the Texas Bicycle Coalition and local (city) organizations such as the Cascade Bicycle Club in Seattle, Washington and the Houston Area Bicycle Alliance in Texas. Due to pressure from organized citizens and recognition by public agencies and officials, almost all states now have bicycle coordinators in their Departments of Transportation and many cities have bicycle program managers on their staffs. The primary national legislation affecting transportation, the Intermodal Surface Transportation Efficiency Act (ISTEA), reflects a very positive promotional aspect to bicycle transportation, by providing funding and planning mandates for it. In addition, the National Bicycling and Walking Study calls for a doubling of bicycle trips and a concurrent 10 percent reduction in total bicyclist injuries and deaths in traffic crashes (FHWA, 1994).

Finally, one must consider the success of bicycles for transportation in various countries. In addition to countries where bicycle use is predicated mainly on economic reasons, such as Cuba and China, several economically prosperous countries (e.g. Japan, Germany, and The

Netherlands) successfully incorporate the bicycle as an important mode in their transportation systems (Hook, 1994; Dutch Ministry of Transport, 1992).

There are also many drawbacks and obstacles (both perceived and real) to using bicycles for transportation, especially in the United States. Though many U.S. citizens state a desire (or willingness) to use bicycles for transportation (FHWA, 1994; Taylor and Mahmassani, 1996), the percentage of those who actually do so is very low compared with the automobile. In 1992 there were an estimated 4.3 million people in the U.S. who bicycled to work occasionally (BIA, 1994). However, only 0.4 percent of work trips in the 1990 Census were reported to be by bicycle (FHWA, 1994), and only 0.7 percent of all trips were made by bicycle in 1990, according to the Nationwide Personal Transportation Study (FHWA, 1991). Some of the obstacles and drawbacks to bicycle use include being slower than automobiles, not protected enough from accidents or bad weather, physically demanding, not socially acceptable, not secure enough from muggings and other attacks, not comfortable enough, and unable to carry enough cargo. These and other reasons must be dealt with to increase the safety and modal share of bicycling and attain the benefits described above.

Two main reasons seem to stand out as to why bicycle transportation is not more popular in the U.S.: (1) long trip distances in most U.S. cities and (2) safety concerns or a lack of safe places to ride a bicycle. The former reason will not be addressed in this work; suggested solutions involve mixed land-use planning and development, and linking bicycle trips with transit trips for longer distances (Lowe, 1989). The identification and provision of safe places for people to ride bicycles to meet their mobility needs is the main motivation behind this study.

Safety concerns are real, not just perceived. In the late 1980's and early 1990's, about 875 cyclists were killed annually in the U.S. in crashes between bicycles and automobiles, compared to about 34,000 automobile drivers and passengers (IIHS, 1991). Though vehicle-miles-traveled statistics are much less reliable for bicycles than for autos, it is certain that the number of deaths per mile traveled is significantly higher for bicyclists than for auto drivers/passengers. In addition, many non-fatal bicycle accidents occur, but go unreported in State motor vehicle crash files (Stutts et al., 1988; Rodgers, 1995).

Though some cycling accidents are caused by dogs, inanimate objects, and collisions with other bicycles or pedestrians, reducing accidents involving autos and bicycles seems to warrant more study at this time. There are two broad categories of solutions to this problem: (1) separate bicycle and automobile traffic and (2) improve the traffic environment for bicycle-automobile mixed-traffic. Though separation, using trails and paths, should probably be pursued whenever possible, it has two major drawbacks as a comprehensive solution. First, separate bicycle paths and trails almost always attract significant pedestrian use, thereby limiting bicycle speeds and correspondingly, the transportation potential of the facility. Second, and most important, both the spatial and economic requirements of separate full-fledged networks of bicycle facilities would be prohibitive in existing urban areas. In addition, cycling to and from such separate facilities would by necessity involve travel on shared right-of-way. Therefore, the environment for bicycle-automobile mixed-traffic must be considered if bicycle transportation is to achieve its full potential and maximum safety.

One can divide possible traffic environment improvements into two main categories: (1) education and training for both cyclists and motorists on how to "share the road" and (2) engineering, design, and operational changes to the physical roadway environment. The former will not be addressed in this study. However, previous studies indicate that cyclist and motorist errors are the apparent cause of a majority of bicycle/automobile accidents (Forester, 1983; Hunter, 1994; Ardekani et al., 1995). Therefore, educational and training programs should be looked upon to solve a portion of the mixed-traffic safety problem. Though the potential of such programs is far from being exhausted, there has been and continues to be much more work in that area than in the latter category of engineering, design, and operational changes (Stutts et al., 1992).

Subdividing further, one can make engineering, design, and operational decisions based on (1) engineering judgment, (2) results of scientific studies, or (3) some combination of both. Currently, the majority of these decisions are made mainly using engineering judgment in combination with a few scientific studies; most of which are based on user surveys or analysis of police report accident data. In fact, many published design guides (North Carolina DOT, 1994; Oregon DOT, 1988; Florida DOT, 1982) rely heavily on engineering judgment. In addition, many popular publications (Williams) and bicycle engineering courses (Sorton, 1993) base their "good practice" recommendations to a large degree on engineering judgment. Relatively few scientific studies have been performed to address these problems.

A need exists for systematic and reliable scientific study, using theoretical, experimental, and observational analyses to characterize bicycle-automobile mixed-traffic. From this characterization comes fundamental knowledge, techniques, and procedures to solve bicycle-automobile mixed-traffic engineering and operation problems. In addition, certain "laws" of mixed-traffic behavior can be identified and established to provide the foundation for a bicycle-automobile traffic science, which should help solve further problems, and provide basic insight into bicycle-automobile mixed-traffic. The contribution of this report lies in this area of theoretical, experimental, and observational scientific studies of bicycle-automobile mixed-traffic.

## 1.2 RESEARCH OBJECTIVES

To more clearly frame the objectives and contributions of this research, one can distinguish between traffic science and traffic engineering. Traffic science can be thought of as the set of fundamental "laws" that characterize traffic. These laws can then be used to analyze situations and solve problems in traffic engineering.

Naturally, the development of traffic science laws requires observing (large and diverse samples of) real traffic. Many fundamental automobile traffic science insights or laws were discovered through attempts to analyze a particular traffic situation or solve a selected traffic problem. This will be the primary approach taken herein: to study various situations or problems in traffic engineering involving bicycle-automobile mixed-traffic, attempt to characterize the situation, and derive basic insights and underlying laws to serve as a basis for solving the problem. These general insights might subsequently be applied to a variety of bicycle-automobile mixed-traffic situations.

As part of this approach, previous studies of bicycle-automobile mixed-traffic will be used to the extent possible. In addition, existing automobile-only theories will be relied upon, when appropriate, to provide a starting point for theories on mixed-traffic processes. Finally, previous automobile-only studies might provide suggestions regarding observational and experimental study procedures that can be used to collect and analyze data for mixed-traffic situations as well.

The main objectives of this research are to:

(1) Study and characterize individual choice behavior in bicycle-automobile mixed-traffic situations, with particular attention to (a) bicycle/rider unit behavior at the onset of a yellow traffic signal indication and (b) gap acceptance behavior of cyclists (and motorists) for a variety of mixed-traffic gap types.

(2) Investigate the implications of traffic control design for bicycle-automobile mixed-traffic, with particular attention to arterial progression.

(3) Develop techniques and procedures to help solve basic traffic engineering problems associated with mixed-traffic situations, such as determining the duration of the yellow change interval at an isolated intersection and determining cycle times, offsets, and splits for arterial progression.

(4) Derive, from the above studies, applicable traffic science laws and fundamental insights that contribute to the understanding of bicycle-automobile mixed-traffic.

(5) Develop an organizing framework for bicycle-automobile mixed-traffic science principles and relations and their engineering implications, and articulate the results of the report research and previous efforts in that framework.

## 1.3 ORGANIZING FRAMEWORK

Drawing from several traffic engineering texts (Pignataro, 1973; Garber and Hoel, 1988; May, 1990) and class notes from two graduate level transportation engineering courses taught at the University of Texas at Austin (Dr. Randy Machemehl's course - Advanced Traffic Engineering and Dr. Robert Herman's course - Advanced Theory of Traffic Flow), a framework for organizing bicycle-automobile traffic science and engineering topics was developed. This framework is shown in Figure 1.1. The level of detail in the framework outline is not intended to be complete nor is each level developed to the same degree of completeness. The intent is to provide at least a broad category for all topics, not to list all possible topics. Greater detail is given under selected categories for illustrative purposes. The framework can be used to place the work presented in this report, as well as past and future work that characterizes bicycle-automobile mixed-traffic.

I. Basic elements
  A. Bicycle/Rider Characteristics
    1. Static
      a. Length
      b. Width
      c. Weight
    2. Dynamic
      a. Height
    3. Operating
      a. Perception-reaction
      b. Speed
      c. Turning
      d. Braking
      e. Accelerating
  B. Road (Geometric Design)
    1. Cross section types
      a. Wide curb lane
        i. Width
      b. Bikelane
        i. Width
        ii. Markings
        iii. At intersections
      c. Separate paths
        i. Width
        ii. Markings
        iii. At intersections
    2. Vertical alinement
      a. Speed, grade, distance traveled relations
      b. Maximum grade relations
    3. Sight distance
      a. Stopping
        1. Tangents
        2. Vertical curves
        3. Horizontal curves
        4. Intersection
    4. Surface
      a. Pavement type
      b. Rumble strips
      c. Drain grates
      d. Detectors
      e. Railroad crossings

II. Traffic Flow
  A. Models
    1. Macroscopic
      a. Speed, flow, density relations
        i. Bike-bike
        ii. Bike-car
        iii. Bike-pedestrian
    2. Microscopic
      a. Vehicle following
        i. Bike-bike
        ii. Bike-car
        iii. Bike-pedestrian
  B. Shock waves
  C. Gap acceptance
  D. Queuing
III. Intersection Control
  A. At-grade
    1. Uncontrolled
    2. Yield
    3. Stop
    4. Signalized
      a. Signal timing
  B. Interchanges
IV. Highway Capacity and LOS
V. Intersection Capacity and LOS
VI. Networks
  A. Arterial progression systems
VII. Computer Models
  A. Simulation
  B. Optimization

Figure 1.1. Organizing framework for topics in bicycle-automobile mixed-traffic science and engineering.

Following is a brief but broad review of past work, placing it in context of the organizing framework. Past work that directly impacts the topics researched for this dissertation will be reviewed in greater detail in the appropriate subsequent chapters.

First is a brief overview of general references providing some information on bicycle traffic science, much of which is based on engineering judgment and experience rather than systematic scientific study. The Guide for the Development of Bicycle Facilities (AASHTO, 1991) contains design guidance pertaining to basic element operating characteristics (item I.A.3. in the organizing framework), sight distance computations (item I.B.3.) and cross section types (I.B.1.), along with a few other topics. Dutch research and experience has been documented in English (CROW, 1993). This treatise is somewhat more detailed than AASHTO's, but the degree of transfer to U.S. situations is unknown. Of particular interest, the Dutch have developed some macroscopic relations (II.A.1.) and a relation between speed and turning radius (I.A.3.c.). The Manual on Uniform Traffic Control Devices has a chapter on markings and signage for bicycle applications (FHWA, 1988). The Highway Capacity Manual has a short section on the impacts of bicycle traffic on capacity (TRB, 1985). Whitt and Wilson (1990) present some detailed research into basic elements, both static and operating, especially in the area of braking. Finally, several state design guides provide some unique guidance on bicycle traffic topics (North Carolina DOT, 1994; Oregon DOT, 1988; Florida DOT, 1982).

The following is a brief, non-exhaustive review of several individual studies whose results may not have been completely distilled and used in the general references reviewed above. Studies relating to the topics of this dissertation also fit this description, but they are reviewed later in Chapters 2, 3, and 4. In the geometric design area (I.B.1.), McHenry and Wallace (1985) used videotape to study bicycle-automobile mixed-traffic behavior in curb lanes of various widths. Navin (1994) has researched both macroscopic traffic models (II.A.1.) and highway capacity impacts from bicycle traffic (IV.). Researchers have also analyzed geometric design, capacity, and operating characteristics of bicycle interchanges (III.B.) in China (Liu et al., 1993; Wang and Wei, 1993).

### 1.4 OVERVIEW

Chapters 2, 3, and 4 present the research completed for this dissertation. This research is divided into three topics: gap acceptance behavior (Chapter 2), bicycle/rider unit behavior at the onset of yellow (Chapter 3), and progression (Chapter 4). Each of these three chapters is a self-contained study, with limited interdependence. The common link between the three is primarily thematic in that each deals with a specific aspect of bicycle-automobile mixed-traffic. Most essential introductory material on each topic is presented in the associated chapter. Finally, Chapter 5 summarizes the overall contributions of the work, major limitations, and directions for future work, while also providing some concluding comments.

# 2 GAP ACCEPTANCE BEHAVIOR

The focus of this chapter is the study of gap acceptance behavior in bicycle-automobile mixed-traffic. Among other applications, gap acceptance models are used to quantify and analyze intersection capacity and delay. The first section introduces this topic. The second section contains most of the background review and presents the theory underlying the model, the model structure, and the general specification of the gap acceptance behavior model. Next, the data collection process is explained. The fourth section discusses some qualitative observations that may or may not be reflected in the quantitative models. The next section presents important summary statistics. The sixth section presents restricted models of only the mean and variance of the critical gap distribution. The less restrictive final models, which incorporate attributes that systematically impact gap acceptance behavior and attempt to capture serial correlation, are presented in the next section. Finally, some concluding comments and a summary of contributions are presented.

## 2.1 INTRODUCTION

Consider a vehicle crossing or merging with traffic of higher priority. A *gap* is defined as the time between two successive vehicles in a traffic stream. A gap begins with the rear of the leading vehicle and ends with the front of the next vehicle. A gap is *accepted* when the vehicle operator on a *minor* street crosses or merges with the traffic on a *major* street. A gap is *rejected* if the vehicle operator does not cross or merge. A *lag* is defined by the moment the crossing or merging vehicle arrives at the intersection until the first vehicle arrives. The lag is also a gap. It is usually the first gap that a vehicle faces. Exceptions to this normal case are discussed in Section 2.3 on data collection. Each vehicle faces a *gap sequence* that usually begins with the lag and ends with the accepted gap. Gaps are numbered consecutively in the sequence, beginning with the lag as *gap number* 1. There is little prior published work analyzing gap acceptance for bicycle-automobile mixed-traffic. The situations illustrated in Figures 2.1, 2.2, 2.3, and 2.4 are of importance in this area.

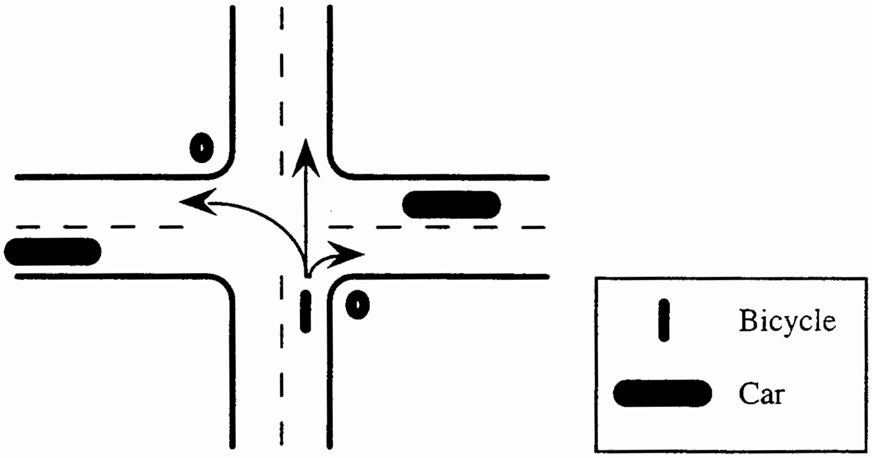

Figure 2.1. Lateral maneuvers (*lateral left-turn*, *lateral right-turn*, *and lateral straight movement*) at uncontrolled and two-way yield or stop intersections.

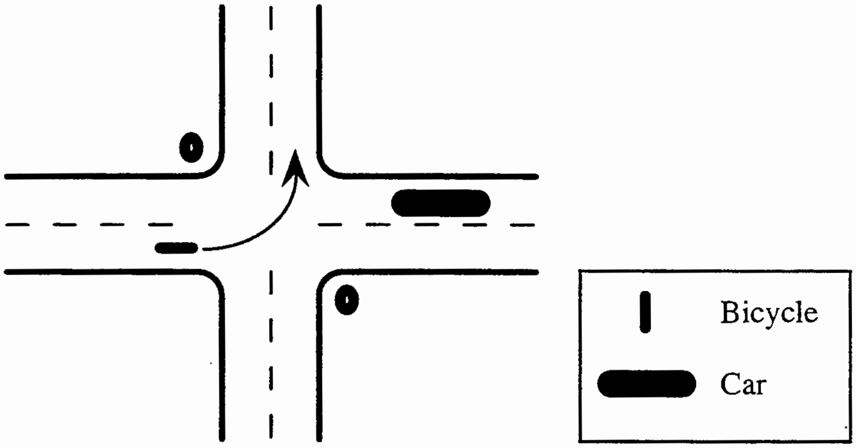

Figure 2.2. Frontal maneuver for left-turns without signal arrows (*frontal left-turn*).

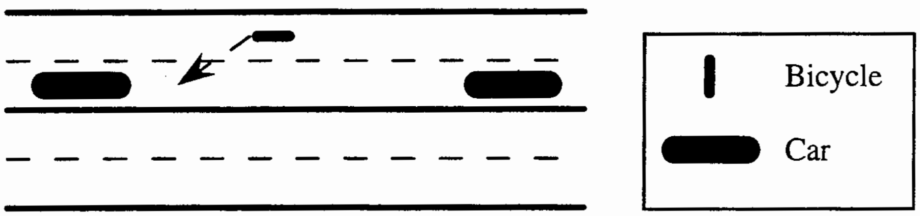

Figure 2.3. Lane change maneuver.

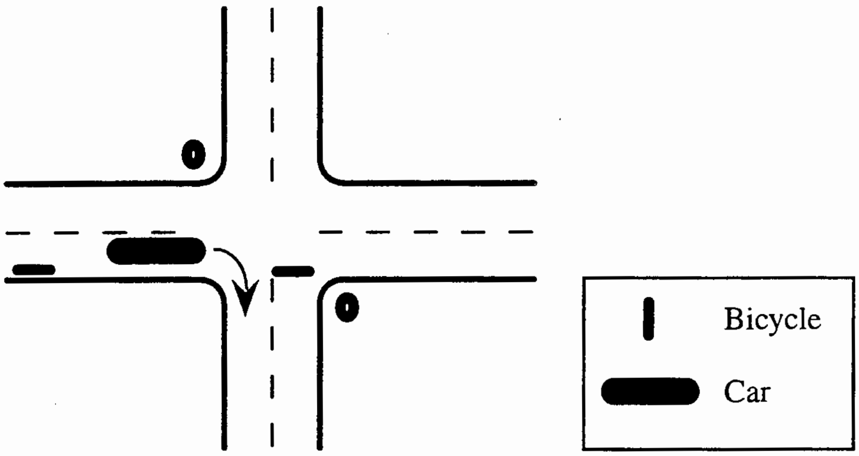

Figure 2.4. Parallel maneuver for right turning automobiles through bicycle traffic (*parallel right-turn*).

Similar situations to those shown in Figures 2.1, 2.2, and 2.3, exist for motor vehicles crossing or merging with bicycle traffic streams. More situations are possible when one considers the fact that the vehicles defining the gap may be one bicycle and one motor vehicle. In general, the vehicle attempting to cross or merge (the *acting vehicle*) can be either a bicycle or automobile and the vehicles defining the gap (the *gap vehicles*) may be any combination of bicycles and automobiles. Gaps defined by two automobiles are called *auto-auto* gaps, and those defined by two bicycles are called *bike-bike* gaps. Gaps defined by one bicycle and one automobile are either *bike-auto* or *auto-bike* gaps. In these terms, the first vehicle type mentioned is the leading or first gap vehicle, also termed the *opening* gap vehicle, because it "opens" the gap for the acting vehicle. The second vehicle type defines the end of the gap or "closes" the gap, and is therefore termed the *closing* gap vehicle. By definition, lags do not have an opening gap vehicle, only a closing one.

For lateral maneuvers (Figure 2.1) the gap vehicles can travel in either the *near* lanes, which are the lanes closest to the acting vehicle (in which gap vehicles travel from the acting vehicle's right to its left), or the *far* lanes (in which gap vehicles travel in the opposite direction). On multilane roadways, one identifies in which near lane (or far lane) the gap vehicle in question travels. Gaps are defined in terms of consecutive gap vehicles, regardless of their travel lane. In this study the major roadways all have only two lanes, one in each direction, so there is only one near and one far lane. Hereafter, these are referred to as the near and far lane, respectively.

Figure 2.4 illustrates a situation that does not have an analogous motor vehicle scenario, if driver behavior is such that the automobile movement is a right-turn through the bicycle traffic and not a lane change/merge into the bicycle stream and then a right-turn. An attempt was made in this dissertation to study this behavior, but only very limited data could be obtained.

Specific types of lane changing maneuvers on controlled access facilities (merging, diverging, and weaving) are also typically analyzed using the concept of gap acceptance. However, bicycle traffic in most countries is not sufficient to warrant these types of facilities, and speed differentials would make mixed-use of controlled access facilities quite unsafe. Therefore, these analyses are not pursued in this dissertation.

Detailed analysis will not be performed for parallel or lane changing gaps, due to the large amount of time it would take to collect enough data on these situations in Austin, Texas. The Dutch bicycle design guide (CROW, 1993) states that lane changing in mixed bicycle-automobile traffic "has not yet been theoretically or fundamentally researched". However, some guidelines are provided in that document.

The principal emphasis of this analysis is gap acceptance behavior while attempting lateral and frontal maneuvers. The primary analysis method employed is probit modeling of gap acceptance behavior. In addition to capturing the effect of gap duration, probit modeling provides a formal mechanism to identify and test the significance of other factors that might affect gap acceptance behavior. These other factors include the vehicle operator's sex, type of bicycle, use of a helmet, waiting time, number of rejected gaps, width of travel lanes, and presence or absence of bikelanes.

Data was collected by videotape at unsignalized intersections. Three different sites were used, allowing the study of factors that vary by site. Examples of these factors are intersection geometry, lane width, and presence or absence of bikelanes. To minimize the data collection time, the expected bicycle traffic volume also guided site selection.

The goal of this gap acceptance study is to begin to characterize gap acceptance behavior in bicycle-automobile mixed-traffic. This study is not large enough nor does it have enough diversity in sites and situations to be considered the “definitive” study. The magnitudes of all results must be treated with this in mind. Results are not "magic numbers" to be used in all mixed-traffic situations. They are, however, useful in comparison to the auto-only numbers resulting from previous studies. This comparison provides insights into mixed-traffic gap acceptance behavior. Finally, the process used to study mixed-traffic gap acceptance behavior, also considers other factors, such as cyclist's sex, presence or absence of bikelanes, and bicycle type, in terms of their significance in the decision to accept or reject gaps.

## 2.2 BACKGROUND REVIEW AND THEORY FORMULATION

While the lateral and frontal gap acceptance situations (Figures 2.1 and 2.2) are similar for bicycles and automobiles as the acting vehicle, the lane-changing situation (Figure 2.3) is fundamentally different. Automobiles normally change lanes when the speeds of the three significant vehicles (the acting vehicle and the two gap vehicles) are of similar magnitude or at least the vehicle changing lanes has the capability to match or exceed the speeds of the vehicles defining the gap. However, bicycles will probably be traveling slower than the vehicles defining the gap they are entering. In addition, there are vehicular differences (e.g. no side- and rear-view mirrors on most bicycles) and operator skill differences (such as in steering a straight line while checking behind and to the side of the bicycle).

Gap acceptance for bicycle lane changing maneuvers is important when analyzing left-turning situations for bicycles. Since bicycles typically travel on the far right-hand side of the curb lane or in a bikelane, they often need to merge into a gap, to properly position themselves to make a left turn in a vehicular manner. This occurs even if there is only one automobile lane for their direction of travel. Characterizing the behavior in these situations can lead to an estimate of the critical volume above which lane changing (and therefore, left-turning) becomes unsafe or uncomfortable for bicyclists and creates a safety hazard for the traffic stream as a whole. This lane-changing behavior may vary depending on the bicycle speed, motor vehicle speeds, and other factors. Finally, cyclist behavior may sometimes be fundamentally different than pure lane changing. In some cases it might be best described as lateral lane crossing.

The concept of a *critical gap* has proven to be very important in the study of gap acceptance. The prevailing notion of the critical gap is that it is the minimum gap duration which will be accepted by an individual vehicle/operator unit in a specific situation (Miller, 1971). This critical gap varies across the population of interest. The early literature has often suggested the use of some percentile of this distribution as the value for analysis and design purposes. In this study, this value will be called the *design critical gap.* In much of the early literature, some confusion can arise because the term “critical gap” was used to denote this design critical gap. The critical gap is a latent variable that cannot be measured directly, but must be inferred from discrete observations of whether a given gap was actually accepted or rejected by a vehicle operator.

Researchers have used various techniques to estimate the critical gap distribution and design critical gap values (Greenshields, 1947; Raff, 1949; Herman and Weiss, 1961; Solberg and Oppenlander, 1966; Wagner, 1966; Mahmassani and Sheffi, 1981b; Garber and Hoel, 1988). For the purposes of this study, it is not important to discuss the various estimation techniques. However, the resulting design critical gaps are of interest, because they can provide a comparative basis for the design critical gaps determined in this work. The design critical gap was usually chosen to be some measure of the center of the critical gap distribution. Since the critical gap distribution is approximately symmetric, design critical gaps determined by any reasonable measure (e.g. mean, median, or mode) are approximately the same. Another discrepancy arises because some researchers measure the gap and others measure the headway.

Design critical headways (measured as the median of the critical headway distribution) from a large number of studies have been synthesized in the Highway Capacity Manual (TRB, 1985). The manual gives basic design values dependent on the type of movement and the major street speed. Adjustments from these basic values are given for right-turning curb radius and angle, right-turn acceleration lane provision, the population of the area, and sight distance restrictions. Applying the adjustments, one can find the ranges for the passenger car design critical headways, assuming no restricted sight distances. They are:

(1) lateral right-turn: 4 to 6.5 seconds,

(2) lateral straight maneuver: 5 to 8 seconds,

(3) lateral left-turn: 5.5 to 8.5 seconds,

(4) frontal left-turn: 4.5 to 6 seconds.

For an average (2-lane) major street running speed of 30 mph in an urban area with population over 250,000, with no restricted sight distances and right-turn curb radii less than 50 ft, the design values given in the Highway Capacity Manual (TRB, 1985) are:

(1) lateral right-turn: 5.0 seconds,

(2) lateral straight maneuver: 5.5 seconds,

(3) lateral left-turn: 6.0 seconds,

(4) frontal left-turn: 4.5 seconds.

Results of individual studies show values close to these ranges. For instance, Solberg and Oppenlander (1966) found design critical gaps for right-turns to be 7.36 seconds, 7.82 seconds for left-turns, and 7.18 seconds for straight-through movement from two-way stop controlled minor streets intersecting 2-lane major streets in Indiana.

Smith (1976) reports the only published findings on motor vehicles accepting gaps in bicycle traffic. Collecting data at the intersections of two-lane streets (with widths varying between 28 and 34 feet) for motor vehicles entering or crossing bicycle traffic streams, the following design critical gap times were found:

(1) lateral straight maneuver[1]: 3.5 seconds,

(2) frontal left-turn: 2.6 seconds,

(3) parallel right-turn[2]: 2.0 seconds.

The design critical gap for crossing traffic (lateral straight maneuver) found in Smith's study (3.5 seconds) is less than those shown above from the Highway Capacity Manual (TRB, 1985) for crossing motor vehicle traffic streams, as expected. Bicycles travel slower than motor vehicles, so smaller gaps are probably acceptable to the acting drivers. In addition, bicycles offer a lower threat of accident damage (to the motor vehicle) should a collision occur. Therefore, it is probable that motor vehicle drivers do not feel as threatened by entering the same time gap in a bicycle traffic stream as they would in a motor vehicle traffic stream. The same arguments hold for the lower frontal left-turn design critical gap value found by Smith (2.6 seconds).

Smith (1976) also collected data for bicycles crossing motor vehicle traffic traveling on a two-lane street[3] (2 x 14-foot lanes). He distinguished between moving and stopped bicycles and reported the following design critical gap estimates:

(1) bicycles moving: 4.1 seconds,

(2) bicycles stopped: 4.6 seconds.

However, because of the difficulty in distinguishing between a moving and a stopped bicyclist (since few stopped completely) and the similarity of the moving and stopped data, Smith pooled the data together and computed a design critical gap of 4.4 seconds for bicycle crossing maneuvers.

This value is lower than the low design value in the Highway Capacity Manual, shown above for passenger cars (5.0 seconds), and is lower than all but one of the design motor vehicle critical gap values reviewed by Smith and reported in his study. Intuition would suggest that bicycle/rider units might desire longer gaps than car/driver units, due to vehicular acceleration capabilities and risk of injury in an accident. Smith offers no explanation for this seemingly counter-intuitive finding, nor does he compare it to the motor vehicle values reviewed in his study. One possible

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[1] It is not clear if this indicates a car crossing a single stream of bicycle traffic or two-way bicycle traffic, one-way in each lane of the two-lane street.

[2] Smith seems to treat this as a right turn through the gap and not a merging maneuver followed by a right turn (see Figure 2.4).

[3] It is not clear if this was two-way traffic or two lanes of one-way traffic.

explanation is that the motor vehicle traffic in his study might have been traveling very slowly (as might be the case on a college campus), though traffic speeds are not reported.

Opiela et al. (1980) analyzed the gap acceptance behavior of bicycles crossing two lanes (12 feet each) of one-way motor vehicle traffic. However, their results are not comparable to those reported above, since they did not use a standard definition of an accepted gap. They measured the accepted gap duration as the time elapsed from the instant the bicycle left the queuing point to when the crossing was completed, not as the time between the successive mainstream vehicles delineating the accepted gap. Rejected gaps were measured in the standard way. This may explain why their estimate of a design critical gap of 3.2 seconds is quite low. In addition, they noted that many cyclists accepted shorter gaps by not stopping before crossing.

One hypothesis examined in this research is that bicycle/rider units will have larger critical gaps than car/driver units for all the aforementioned maneuvers due to slower acceleration capabilities and the greater risk of injury in an accident. A variety of factors may affect the gap acceptance behavior of both motorists and cyclists and therefore, their critical gaps. These factors can be conveniently divided into four groups: (1) Inherent Gap Characteristics and Surrounding Events, (2) Intersection Environment, (3) Acting Vehicle Maneuver, and (4) Individual Vehicle/Operator Unit. There may also be interactive effects between the factors in different groups.

The *inherent gap characteristics and characteristics of surrounding events* group incorporates the factors that cause many of the possible differences between each gap acceptance situation. Inherent gap characteristics include all factors associated with the gap, such as the type of gap vehicles, the gap vehicle lane positions, and the speed of the gap vehicles. Also included in this group are attributes associated with surrounding events that take place during the duration of the gap, such as temporary sight distance blocks and any distraction of the vehicle operator. Other factors included in this group are:

(1) Pedestrian crossings in and around the intersection

(2) Lag versus other gap acceptance; found significant by Wagner (1966), but insignificant by Solberg and Oppenlander (1966)

(3) A vehicle on the opposite side of the intersection from the acting vehicle (for lateral maneuvers)

(4) Behavior of gap vehicles (accelerating, decelerating, turning, etc.)

The *intersection environment* group contains all the factors associated with the intersection and its environment that typically do not change for relatively long periods of time. A subset of the factors in this group may introduce serial correlation in a data set due to a persistent constant unexplained effect for all the gap acceptance decisions made in the same intersection environment. Examples of some of these factors include:

(1) Intersection geometry (grade, channelization, curb radii, etc.)

(2) Sight distance from the minor street

(3) Peak versus off-peak times; found significant by Wagner (1966)

(4) Speed and volume of traffic on major road

(5) Major street characteristics (grade, number of lanes, width, presence of bikelane, etc.)

(6) Minor street characteristics (grade, number of lanes, width, presence of bikelane, etc.)

(7) Weather conditions

(8) Minor street control (stop, yield, or none)

(9) Type of land-use surrounding the intersection

The *acting vehicle maneuver* group contains all the factors associated with the maneuver executed by the acting vehicle. Factors in this group can also cause serial correlation in the same manner as intersection environment factors. The factors included in this group are:

(1) Type (lateral right, lateral left, frontal left, etc.)

(2) Initial lane position of acting vehicle

(3) Final lane position of acting vehicle

The *individual vehicle/operator unit* group contains all the factors associated with a vehicle/operator unit. This group of factors may also give rise to serial correlation due to a persistent constant unexplained effect on each decision made by an individual vehicle operator. Factors in this group include:

(1) Individual driver characteristics (gender, age, driving experience, etc.)

(2) Acting vehicle characteristics (type, acceleration capabilities, etc.)

(3) Whether the acting vehicle waited in a queue before entering

(4) Presence or absence of a queue of vehicles behind the acting vehicle

(5) Waiting time or number of gaps rejected after arrival at the intersection

Discrete choice modeling procedures have been applied to gap acceptance situations by several researchers (Mahmassani and Sheffi, 1981b; Daganzo, 1981; and Palamarthy et al., 1994). As pointed out by Daganzo (1981), these procedures allow estimation of individual gap acceptance function parameters without the two problems Miller (1971) associated with most other methods: (1) only the average critical gap across the population is estimated and (2) data is lost, because only one gap per individual vehicle/operator unit is used to avoid overrepresentation of cautious vehicle operators who reject more gaps than the average.

Following is a development of the theory underlying the specification and estimation of discrete choice models of gap acceptance behavior. Simple models are specified for which the mean and variance of the critical gap distribution are the only parameters. Restricted models are specified under the assumption that all gap acceptance decisions are made independently, even those by the same vehicle/operator unit. Enhanced specifications are also shown which incorporate the systematic effects that some factors have on the critical gap and relax assumptions to recognize the serial correlation that is common in gap acceptance data sets.

### 2.2.1 Formal Discrete Choice Theory Development

Consider the critical gap (duration) $T_{gvmi}$ for an individual vehicle/operator unit v facing the $g^{th}$ gap in a sequence, while attempting a maneuver m in an intersection environment i. The index g reflects the sequential number of the gap in the gap sequence. Typically, g=1 is the lag, and $g=G_{vmi}$ is the accepted gap. Within a sample there are I intersection environments, and for each of these there are $M_i$ acting vehicle maneuvers. For each maneuver in each environment there are $V_{mi}$ vehicle/operator units facing $G_{vmi}$ gap acceptance decisions. One can also think in terms of a vehicle/operator unit v facing a gap acceptance decision *situation*, where the situation is defined by the intersection environment i, the acting vehicle maneuver m, and any factors associated with the gap g. Note that the four indices associate directly with the four groups of factors (or attributes) discussed above. Since $T_{gvmi}$ varies over gaps, vehicle/operator units, maneuvers, and intersection environments, it can be modeled as a random variable, as follows:

$$T_{gvmi} = \mu + \varepsilon_{gvmi} \qquad (2.1)$$

where $\mu$ is the mean critical gap over all vehicle/operator units and situations, and

$\varepsilon_{gvmi}$ is a stochastic disturbance (or error) term.

The gap acceptance decision rule is: if the actual gap duration $d_{gvmi}$ is greater than the critical gap then the gap is accepted, else it is rejected. An individual vehicle/operator unit's gap acceptance decision can then be modeled as follows:

$$Y_{gvmi} = (\mu + \varepsilon_{gvmi}) - d_{gvmi}. \qquad (2.2)$$

If $Y_{gvmi}<0$, the gap is accepted; otherwise, it is rejected.

Assuming the $\varepsilon_{gvmi}$ are normally distributed across gaps (and surrounding events), vehicle/operator units, acting vehicle maneuvers, and intersection environments, a *mean and variance (only)* model of the sample's critical gap distribution is specified by the following (probit) choice probability function:

$$\Pr(\text{accepting}) = \Pr(Y_{gvmi} \leq 0) = \Pr(\mu + \varepsilon_{gvmi} - d_{gvmi} \leq 0) =$$

$$\Pr(\varepsilon_{gvmi} \leq d_{gvmi} - \mu) = \Phi\left(\frac{d_{gvmi} - \mu}{\sigma}\right) = \Phi\left(\frac{1}{\sigma} d_{gvmi} - \frac{\mu}{\sigma}\right), \qquad (2.3)$$

where $\Phi(\,)$ is the standard cumulative normal distribution and $\sigma^2$ is the total variance.

**2.2.1.1 Systematic Effects and Components of Variation.** In the mean and variance only model specification (equation 2.3) no attempt was made to capture the effect of any of the previously discussed factors that may impact the gap acceptance decision, and thus the latent critical gap variable. Mahmassani and Sheffi (1981b) found that the "number of rejected gaps" systematically impacted car/driver unit critical gaps. Palamarthy et al. (1994) tested several other attributes for systematic effects, while modeling pedestrian gap acceptance behavior. Using the groups defined above, any systematic effects can be tested by including attributes in the specification, as follows:

$$T_{gvmi} = \beta_0 + \beta_{gap}X_{gap,gvmi} + \beta_{int}X_{int,i} + \beta_{man}X_{man,mi} + \beta_{vo}X_{vo,vmi} + \beta_{ia}X_{ia,gvmi} + \varepsilon_{gvmi} \quad (2.4)$$

where $\beta_0$ = the mean critical gap when all other attributes are zero,

$X_{gap}$ = vector of attributes characterizing the gap and surrounding events,

$X_{int}$ = vector of attributes characterizing the intersection environment,

$X_{man}$ = vector of attributes characterizing the acting vehicle maneuver,

$X_{vo}$ = vector of attributes characterizing the vehicle/operator unit,

$X_{ia}$ = vector of interactions between attributes from different groups, and

$\beta_{gap}$, $\beta_{int}$, $\beta_{man}$, $\beta_{vo}$, and $\beta_{ia}$ are vectors of parameters to be estimated.

Replacing $\mu$ in the choice probability expression (equation 2.3) by the above specification, the following probit gap acceptance probability function is obtained (with the indices dropped):

$$\Pr(\text{accepting}) = \Phi\left(\frac{d-(\beta_0+\beta_{gap}X_{gap}+\beta_{int}X_{int}+\beta_{man}X_{man}+\beta_{vo}X_{vo}+\beta_{ia}X_{ia})}{\sigma}\right). \quad (2.5)$$

Note that the model in equation 2.3 is a restricted version of this model, in which $\beta_0=\mu$ when all the other $\beta$ coefficients are set to zero.

The effects of any factors that impact the gap acceptance decision that are not captured in the systematic variation of the mean critical gap are reflected in the random disturbance $\varepsilon_{gvmi}$. To capture serial correlation it is necessary to separate the total variance $\sigma^2$ of this disturbance term into components by decomposing the disturbance, as follows:

$$\varepsilon_{gvmi} = \varepsilon_{gap,gvmi} + \varepsilon_{vo,vmi} + \varepsilon_{man,mi} + \varepsilon_{int,i}, \quad (2.6)$$

where $\varepsilon_{gap}$ reflects the unexplained variation across gaps and surrounding events,

$\varepsilon_{int}$ reflects the unexplained variation across intersection environments,

$\varepsilon_{man}$ reflects the unexplained variation across acting vehicle maneuvers, and

$\varepsilon_{vo}$ reflects the unexplained variation across vehicle/operator units.

The gap acceptance decision rule is then given by:

$$Y_{gvmi} = (\mu + \varepsilon_{gap,gvmi} + \varepsilon_{vo,vmi} + \varepsilon_{man,mi} + \varepsilon_{int,i}) - d_{gvmi}. \tag{2.7}$$

Each disturbance component reflects the unexplained variation caused by the unobserved factors in the corresponding group. There is one realization of $\varepsilon_{vo}$ per vehicle/operator unit, so it is constant across gaps in the sequence (within each individual vehicle/operator unit). Similarly, the maneuver disturbances are constant within each maneuver and the intersection environment disturbances are constant within each intersection environment. Each of the disturbance components is independent of the others, since the factors in each group are independent of the factors in the other groups.

It seems fairly reasonable to assume that $\varepsilon_{vo}$, $\varepsilon_{man}$, and $\varepsilon_{int}$ are normally distributed with means of zero and variances $\sigma_v^2$, $\sigma_m^2$, and $\sigma_i^2$, respectively. Daganzo (1981) assumed that any deviations from normality for $\varepsilon_{vo}$ are probably only of the second order. He did not explicitly consider $\varepsilon_{man}$ or $\varepsilon_{int}$, but they also should not have first order deviations from normality.

The across gap (and surrounding events) disturbance component, $\varepsilon_{gap}$, is assumed to be independently and identically normally distributed (iid normal) with a mean of zero and a variance of $\sigma_g^2$. The independence and normality assumptions seem reasonable. However, different variance magnitudes may be associated with different individual gap sequences, due to differences in variations of vehicle operator aggressiveness, mood swings, tolerance to distractions, etc. Using controlled experiments, Bottom and Asworth (1978), as reported in Daganzo (1981), found that the within individual variance was moderately correlated with the critical gap. Nevertheless, Daganzo (1981) assumed that deviations from the identity assumption are quite likely only second order effects.

Since there is no covariation between the disturbance components, the $\varepsilon_{gvmi}$ are normally distributed with a mean of zero and variance $\sigma^2$ given by:

$$\sigma^2 = \sigma_g^2 + \sigma_v^2 + \sigma_m^2 + \sigma_i^2 \tag{2.8}$$

where $\sigma_g^2$ is the variance across gaps (and surrounding events),

$\sigma_v^2$ is the variance across individual vehicle/operator units,

$\sigma_m^2$ is the variance across acting vehicle maneuvers, and

$\sigma_i^2$ is the variance across intersection environments.

Models specified to explicitly identify the variance components are called *components of variance* models. The formulation of a components of variance model of gap acceptance behavior is presented next.

**2.2.1.2 Components of Variance Model.** Both Daganzo (1981) and Palamarthy et al. (1994) have formulated components of variance models of gap acceptance behavior. Such models offer three main advantages in that they allow one to: (1) capture and test the significance of serially correlated effects, (2) interpret the sources of variability correctly, and (3) obtain efficient parameter estimates.

The following formulation allows one constant persistent unexplained effect for each intersection environment, just as it allows one constant persistent unexplained effect for each vehicle/operator unit. These effects are captured by $\sigma_i$ and $\sigma_v$, respectively. Because acting vehicle maneuvers are nested inside intersection environments, a different constant persistent unexplained effect is allowed for each m,i combination. This effect is captured by $\sigma_m$. Alternatively, one could nest intersection environments inside of acting vehicle maneuvers or one could allow constant persistent unexplained effects for m,i combinations only. Other more complex disturbance structures are also possible.

If a data set contains a sufficiently large number of gap decision observations associated with each vehicle/operator unit, maneuver, or intersection environment, then serial correlation will probably be present that can be captured by $\sigma_v$, $\sigma_m$, or $\sigma_i$, respectively. In data sets without sufficiently large numbers of gap decision observations associated with each vehicle/operator unit, maneuver, or intersection environment the associated components of variance cannot be identified nor is serial correlation significant, so it is reasonable to assume that all the disturbances, $\varepsilon_{gvmi}$, are independent. A model making this assumption is hereafter called an *independent decisions* model.

For example, if there are typically only one or two gap decision observations per vehicle/operator unit, then it will probably not be possible (or necessary) to estimate $\sigma_v$. In addition, the total number of different vehicle/operator units, maneuvers, and intersection environments must also be sufficiently large. The fewer intersection environments or maneuvers for which data is collected, the less likely it is that $\sigma_i$ or $\sigma_m$ can be identified, however it is more likely that any constant persistent effect associated with an intersection environment or a maneuver can be captured systematically.

Within the same intersection environment i, for the same maneuver m, the covariance matrix for individual vehicle/operator unit v is the covariance matrix for the vector of decision variables associated with the gap sequence for vehicle/operator unit v (i.e. $Y_{gvmi}$, for g=1, ... $G_{vmi}$):

$$\Sigma Y_{vmi} = \begin{bmatrix} \sigma^2 & & & & & \\ \sigma_v{}^2 & \sigma^2 & & & & \\ \sigma_v{}^2 & \sigma_v{}^2 & \sigma^2 & & & \\ \vdots & \vdots & \vdots & \ddots & & \\ \vdots & \vdots & \vdots & & \ddots & \\ \sigma_v{}^2 & \sigma_v{}^2 & \sigma_v{}^2 & \cdots\cdots & \cdots\cdots & \sigma^2 \end{bmatrix}. \qquad (2.9)$$

The number of rows and columns is equal to the number of gaps in the gap sequence for vehicle/operator unit v ($G_{vmi}$). It is derived by the relationship between the disturbance terms (for one i and one m):

$E[\varepsilon_{gv}\varepsilon_{hu}]$ $= \sigma^2 = (\sigma_g{}^2 + \sigma_v{}^2 + \sigma_m{}^2 + \sigma_i{}^2)$[4], if g = h and v = u,

$= \sigma_v{}^2$, if $g \neq h$ and $v = u$, because $\varepsilon_{vo}$ is the same for all gaps in the gap sequence for individual v, and

$= 0$, if $v \neq u$, because gap acceptance behavior is independent across vehicle/operator units, as long a vehicle operator is not following another or a cyclist does not pull up side by side with an auto or another cyclist making the same maneuver.

Next, one needs to obtain the covariance matrix for more than one maneuver in more than one intersection environment. Because $\varepsilon_{man}$ and $\varepsilon_{int}$ are persistent and constant across each maneuver (m=1,..$M_i$) and each intersection environment (i=1,..I), respectively, it can be shown that $E[\varepsilon_{gvmi}\varepsilon_{hunj}]$ yields the (I x I) covariance matrix

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[4] $\sigma_m{}^2$ and $\sigma_i{}^2$ are included for the more general case of more than one intersection i and maneuver m.

$$\Sigma Y=\begin{bmatrix} \Sigma Y_1 & & & & & \\ \sigma_i^2 & \Sigma Y_2 & & & & \\ \sigma_i^2 & \sigma_i^2 & \ddots & & & \\ \vdots & \vdots & \vdots & \Sigma Y_i & & \\ \vdots & \vdots & \vdots & & \ddots & \\ \sigma_i^2 & \sigma_i^2 & \sigma_i^2 & \ldots\ldots & \ldots\ldots & \Sigma Y_I \end{bmatrix}. \tag{2.10}$$

Where $\Sigma Y_i$ is the ($M_i$ x $M_i$) covariance matrix associated with each intersection environment i, given by:

$$\Sigma Y_i=\begin{bmatrix} \Sigma Y_{v1i} & & & & & \\ \sigma_m^2 & \Sigma Y_{v2i} & & & & \\ \sigma_m^2 & \sigma_m^2 & \ddots & & & \\ \vdots & \vdots & \vdots & \Sigma Y_{vmi} & & \\ \vdots & \vdots & \vdots & & \ddots & \\ \sigma_m^2 & \sigma_m^2 & \sigma_m^2 & \ldots\ldots & \ldots\ldots & \Sigma Y_{vM_ii} \end{bmatrix}. \tag{2.11}$$

It should be noted that the independent decisions model is a restricted version of the components of variance model where all the off-diagonal terms in the covariance matrix are restricted to zero, and all the variance components are confounded.

**2.2.1.3 Estimation of Components of Variance Model.** To estimate the parameters in equation 2.5 and the covariance matrix 2.10, it is important to note that the probability of an individual vehicle/operator unit accepting gap $G_{vmi}$ (while making a maneuver in an intersection environment) is

$$\Pr(Y_{1vmi}>0,\ Y_{2vmi}>0,\ Y_{3vmi}>0,\ \ldots\ldots,Y_{(Gvmi-1)vmi}>0,\ Y_{(Gvmi)vmi}\leq 0)$$

$$= \Pr([a_{gvmi}Y_{gvmi} \leq 0],\ g = 1,..\ G_{vmi})$$

$$= \prod_{g} \Pr(a_{gvmi}Y_{gvmi} \leq 0), \tag{2.12}$$

where $a_{gvmi}$= 1, if gap g is accepted by individual v making maneuver m in intersection environment i;

= -1, if rejected.[5]

Equation 2.12 is the likelihood function for the gap sequence. The likelihood function for the sample is

$$\prod_{i}\prod_{m}\prod_{v}\prod_{g} \Pr(a_{gvmi}Y_{gvmi} \leq 0).^{6} \tag{2.13}$$

For simplicity, the development will continue for the case of one maneuver in one intersection environment. This development also applies to the case where observations from more than one maneuver and intersection environment are present in the data, but $\sigma_m$ and $\sigma_i$ are confounded with $\sigma_v$ and $\sigma_g$. As proposed by Daganzo and Sheffi (1982), the probability of a gap acceptance sequence is equivalent to the probability of selecting the auxiliary alternative $U_0$ (=0) from a set of $(G_v + 1)$ alternatives where

$$U_{gv} = [a_{gv}Y_{gv}], \quad \text{for } g = 1,..\ G_v, \tag{2.14}$$

$$U_{gv} = U_{0v} = U_0 = 0 \quad \text{for } g = 0, \text{ and}$$

equation 2.12 can be rewritten as

$$\Pr([a_{gv}Y_{gv}] \leq U_0,\ g = 1,..\ G_v). \tag{2.15}$$

Using 2.15, the problem has been reduced to a decision problem with $(G_v+1)$ alternatives, where the chosen alternative is always $U_0$. In order to specify the model one also needs the covariance matrix of U, $\Sigma_U$. Dropping the index v, equation 2.15 can be written in matrix notation as

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[5] The covariance term is deactivated for gaps g where no decision is made, to insure that gap g does not influence the estimate. To do so, $a_{gvmi}$ is set to 0. This is done for blocks (defined in Section 2.3.2.1) and when a gap is accepted where g is less than the maximum gap number accepted in the sample.

[6] To handle gaps g where no decision is made $a_{gvmi}Y_{gvmi}$ is set to –999 so that gap g does not influence the likelihood value.

$$U = A^T Y, \text{ where } A^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ a_1 & 0 & 0 & \cdots & 0 \\ 0 & a_2 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & a_G \end{bmatrix} \text{ and } Y = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ \vdots \\ Y_G \end{bmatrix}, \qquad (2.16)$$

and

$$\Sigma_U = A^T \Sigma_Y A =$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & \cdots \\ 0 & a_1^2\sigma^2 & & & & \\ 0 & a_1a_2\sigma_v^2 & a_2^2\sigma^2 & & & \\ 0 & a_1a_3\sigma_v^2 & a_2a_3\sigma_v^2 & a_3^2\sigma^2 & & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ 0 & a_1a_G\sigma_v^2 & a_2a_G\sigma_v^2 & a_3a_G\sigma_v^2 & \cdots & a_G^2\sigma^2 \end{bmatrix}, \qquad (2.17)$$

The dimension of $\Sigma_U$ is (G + 1). Because vehicle/operator unit v accepts gap G, all elements in row (and column) (G + 1) are negative, except for the diagonal (and first) element. All elements (except for the first) in rows (and columns) 2 through G are positive (for rejected gaps). The diagonal elements, except for row one, are all $\sigma^2$ (= $\sigma_g^2 + \sigma_v^2$), because $a_g^2 = 1$, for all g.

**2.2.1.4 Estimation Process.** Because the strict concavity of a multinomial probit log-likelihood function cannot be established when the number of alternatives is more than two, there are no efficient algorithms that guarantee a global maximum for the log-likelihood function. To help obtain satisfactory estimates, a two-step process is used. First, an independent decisions model is estimated. Second, the resulting parameter estimates are used as initial values to estimate the components of variance model. The starting point found in step one should be in the general neighborhood of the global optima, so the local optima found in this neighborhood in step two is likely to be the global one.

In the two-step process, the SST software package (Dubin and Rivers, 1988) is used to initially estimate the independent decisions model and to explore the data and many possible attribute interactions, because it has a user-friendly interface for data manipulation. A maximum likelihood multinomial probit (MNP) estimation program, which uses Monte Carlo simulation and was developed at The University of Texas at Austin, is used to estimate the final independent decisions and components of variance models (Lam, 1991; Palamarthy, 1994).

## 2.3 DATA COLLECTION

Data was collected by videotape at three different unsignalized intersections in Austin, TX. Three different sites were used to allow the study of a variety of factors that vary by site, including intersection geometry and presence or absence of bikelanes. The expected volume of bicycle traffic also influenced site selection. The videotaping required two persons. One to operate the camera and one to watch traffic and signal when to start taping. Taping commenced only when a bicycle/rider unit was involved as either an acting or gap vehicle. Bicycle type, helmet use, and rider's sex were verbally recorded, because they are not easy to visually distinguish on the tape.

Most of the video collection was done in the summer of 1994. Because of a lack of data with bicycles as acting vehicles (only about 70 data points), more video collection was done in the summer of 1996. The same intersections were used and site conditions were virtually identical to those in 1994. The only identified changes were that a new pedestrian crosswalk was painted at one site and that a city bicycle helmet ordinance, requiring all cyclists to wear helmets, was instituted a few months prior to taping in 1996. However, only warnings were being given at the time of taping, not fines. Possible effects of these two changes are tested. The videotaping was done primarily between 9 a.m. and 1 p.m., with mostly sunny or partly cloudy skies and temperatures usually between 80 and 95 degrees. Winds were often present, but usually below 10 mph. Videotaping was never done in the rain or with wet pavement.

### 2.3.1 Site Descriptions

All intersections are located within a mile of the University of Texas campus and therefore, serve significant student bicycle traffic. Most of this traffic is assumed to be for transportation, not recreation. Diagrams of the three sites are given in Figures 2.5, 2.6, and 2.7. These sites will hereafter be referred to by the major land-use adjacent to them. Most of the key site characteristics are evident from these diagrams. Those that are not are discussed below.

The commercial site (Figure 2.5) is located in the middle of a large neighborhood about 1 mile north of the University of Texas. This neighborhood contains both homes and many two-story condominiums and apartments, housing both students and working adults. The residential site (Figure 2.6) is also located in this same neighborhood about a half mile north of the campus. Its major street is a main artery for bicycle traffic to the campus. There is a small possibility of traffic turning onto the major street from residences and other minor streets within close proximity to this intersection. This activity is not in the camera view. The only possible indication of this type of activity is if the vehicle is visibly accelerating. Besides this, neither of these two locations seems to have significant sight distance restrictions or unique upstream or downstream characteristics that may affect gap acceptance behavior. Both have fairly level grades. These two sites were videotaped at ground level. Therefore, the positioning of bicycles in the lanes on the major street is somewhat difficult to discern. This is one reason why most of the data was collected at the commercial/campus location.

The commercial/campus site is bordered on the north by the University campus, with a high-rise private dormitory and small shopping mall on the southwest corner and a small private ground level parking lot and parking garage on the southeast corner. Videotaping on the top floor of this garage provided the best view of the intersection. This location has several unique features that might impact gap acceptance behavior. First, the building on the southwest corner obstructs sight distance down the near lane. Vehicles must pull well into the crosswalk to see. Second, if a bus is parked in the intersection as indicated in Figure 2.7, then the effective width of that lane is reduced by about nine feet. This may impact the shared use of that lane by autos and bicycles and may impact lateral left-turning autos when a bicycle closes a gap.

Third, there is a bus stop about 200 feet upstream (west) of the intersection in the near lane. If a bus is parked at this stop, sight distance is further restricted. This bus stop is not visible on the videotape, so any impacts cannot be quantified. Fourth, there is a signal controlled intersection about 400 feet west of the intersection. If an automobile is traveling in the near lane it might have just turned onto the major street at this signal controlled intersection. Therefore, gaps of over about nine to ten seconds may not have even had a closing vehicle in view of the acting vehicle operator when the acceptance decision is made. This may be one reason why no gaps of over 10 seconds were rejected. Again, this situation cannot be seen on the videotape. It should have a negligible impact anyway, since it is expected that all gaps of this duration will be accepted.

Next, the crest of a small hill is located just west of the intersection. Vehicles traveling in the far lane are negotiating a short upgrade while those in the near lane a short downgrade. This does seem to impact bicycle speeds somewhat, but any difference was not quantified. Finally, bicyclists at this site usually use the motorcycle parking area for right turns. It seems to function almost like an acceleration lane (or a bikelane) for lateral right-turning bicycles.

No attempt was made to measure the average automobile speeds at any location. The three speed limits (not posted for the commercial/campus site, 30 mph for the residential site, and 35 mph for the commercial site) were not vastly different, and there did not seem to be much visible difference in automobile speeds.

Note that two of the intersections are T-intersections, so straight through auto traffic is impossible. However, some bicycles do travel straight to and from the sidewalk opposite the minor street at the commercial/campus site.

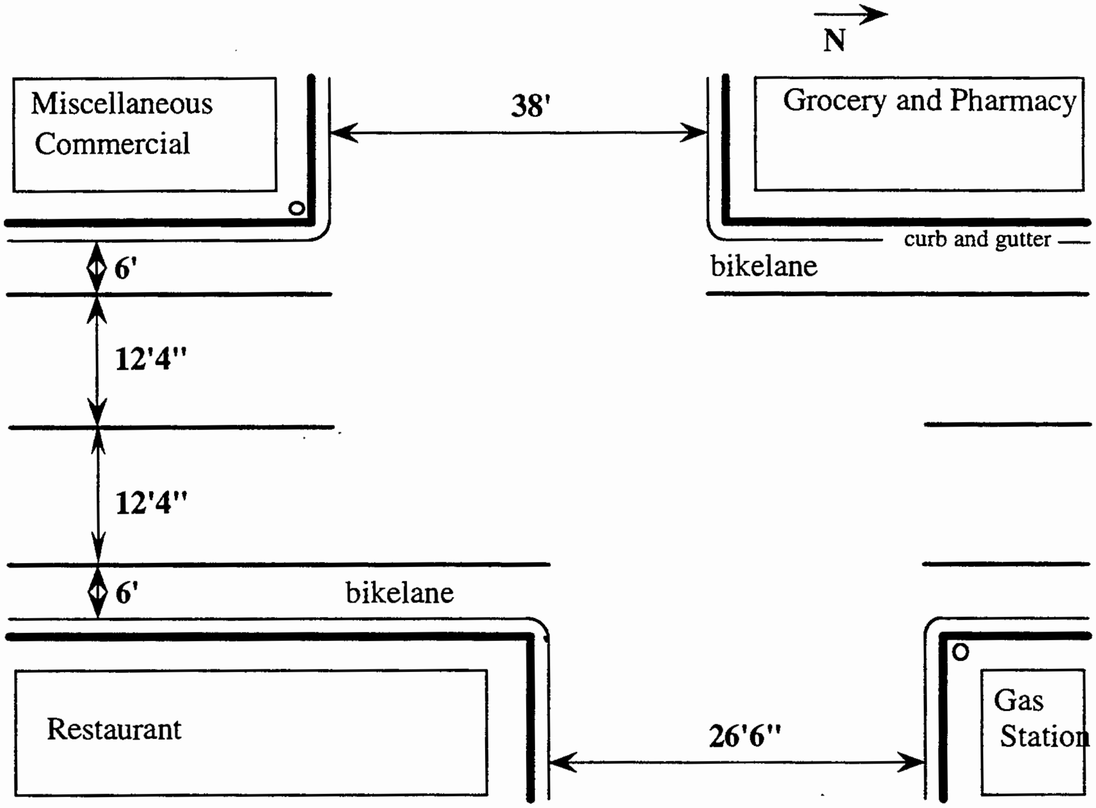


Figure 2.5. Commercial site used to collect gap data at the intersection of Duval and 43rd in Austin, TX.

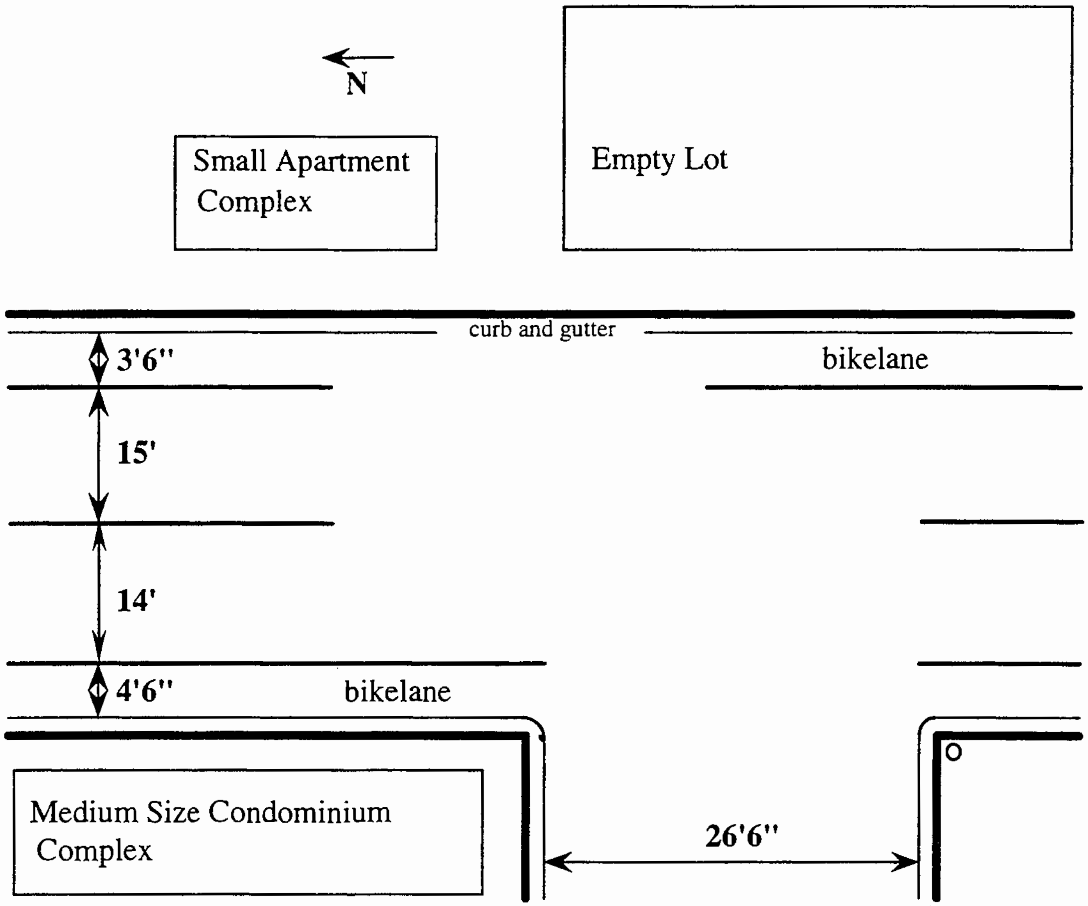


Figure 2.6. Residential site used to collect gap data at the intersection of Speedway and 34th in Austin, TX.

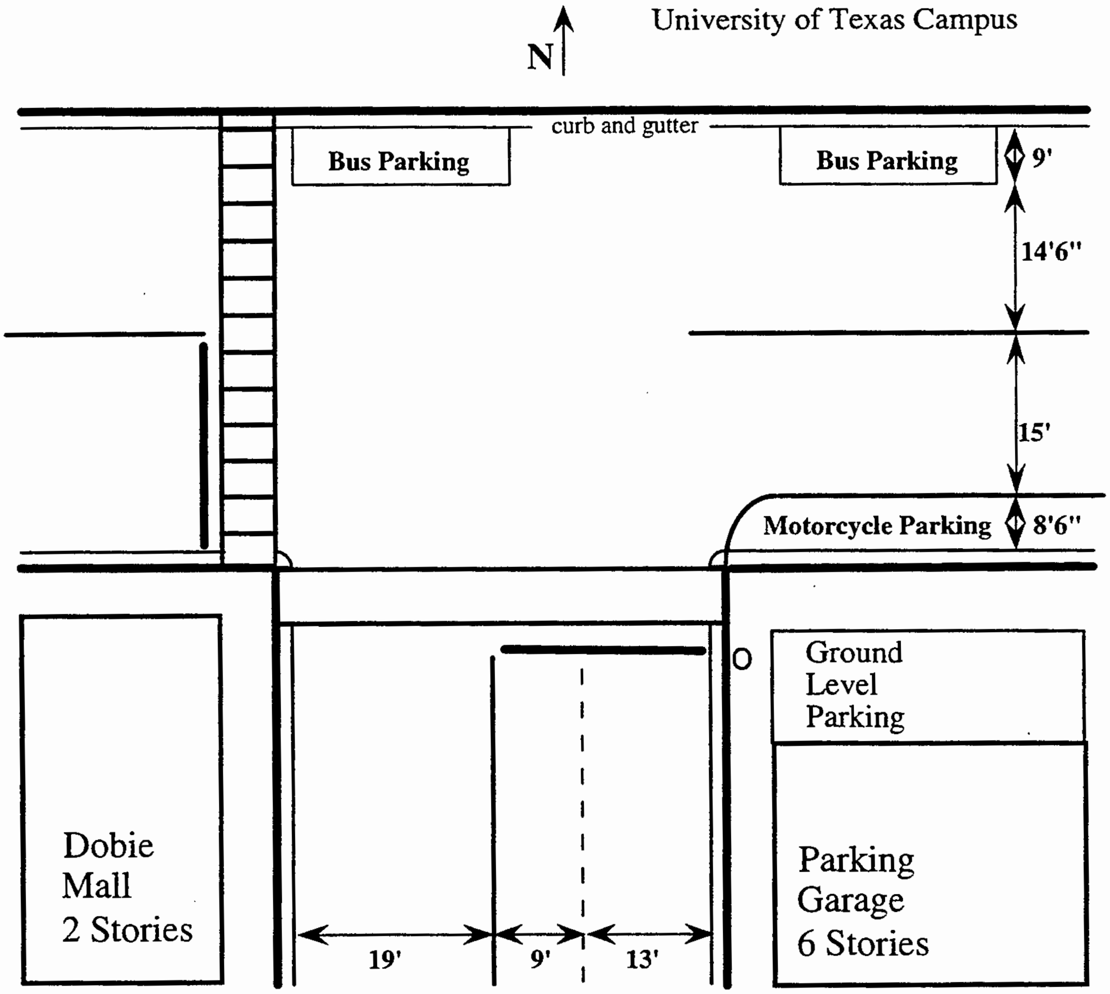


Figure 2.7. Commercial/campus site used to collect gap data at the intersection of 24th and Whitis in Austin, TX.

### 2.3.2 Data Reduction

Data was reduced from the videotape into a spreadsheet database. Two assistants performed this reduction, with the author providing training and help with individual data interpretation problems. One assistant reduced all the 1994 videotapes in 1995. The other assistant reviewed this data again in 1996, correcting for a few systematic errors. This assistant then reduced the 1996 video. This process should ensure a fairly error free set of data. Further error checking was conducted on the data set using spreadsheets and statistical software packages. This process found the data to be relatively free of errors of inconsistency.

Vehicles performing a maneuver not consistent with the gap acceptance scenarios under study (i.e., lateral and frontal maneuvers) were not used. For example, a few cyclists turned left from the left lane of the minor street onto the near lane of the major street. These cyclists were riding the wrong way on both streets and therefore, did not fit any of the scenarios under study.

The data set consists of *gap sequences.* Each gap sequence involves one acting vehicle and includes all gaps rejected by this vehicle. The accepted gap is included if it is less than 15 seconds. The gap sequence begins when the acting vehicle reaches the stop bar (and is first in the queue). Each gap within the gap sequence is a *gap decision observation*, with an associated *vector* of *data items*, explained below. A gap sequence is recorded if either the acting vehicle is a bicycle or at least one of the gap vehicles in the sequence is a bicycle. This produces a set of data that has a mixture of bike-bike, auto-auto, auto-bike, and bike-auto gaps, while ensuring that gaps involving bicycles as either gap vehicles or acting vehicles are well represented.

First, each data item in a gap decision observation vector is listed and explained. Then, the rules used to measure gap duration are presented.

**2.3.2.1 Data Items.** Following are the data items, organized according to the four categories of factors explaining gap acceptance behavior, associated with each gap acceptance decision observation:

(A) Data associated with the gap and surrounding events

  (1) Gap duration

  (2) Decision of the acting vehicle: accept or reject

  (3) Opening and closing gap vehicle characteristics:

    (a) type of vehicle: none, auto, bike, pedestrian, bus, motorcycle, or heavy truck

      None is only used for a lag, because there is no opening gap vehicle. Auto includes both cars and pickups.

    (b) vehicle position: left, center, right, or bikelane

      This data item is defined only for bicycles. If there is one lane, then *right* is considered to be right of an imaginary 13 foot section from the center line (but not in a bikelane). The 13 feet is the space assumed to be used by autos, with no interference from bicycles that ride on the right. Within this space, the rightmost 6.5 feet is *center* and the leftmost 6.5 feet is *left.*

      At the commercial/campus site there are data points where the initial facility has two lanes (see Figure 2.7). Here, the left lane is *left* and the rightmost 4 feet of the right lane is *right.* This leaves the other 9 feet of the right lane as *center.* These 9 feet are assumed to be used by autos with no interference from bicycles.

        - (c) location of the vehicle: near or far lane (only applicable for lateral left or lateral straight maneuvers)
        - (d) vehicle movement: left, right, or straight
        - (e) reason for slowing: pedestrians, congestion, turning, other, or not slowing

          Indicates whether or not the gap vehicle reduces its speed as it approaches the intersection. If the vehicle decelerates, the slowing is attributed to pedestrian interference, congestion, turning, or some other cause.
        - (f) is the gap vehicle visibly accelerating?
- (B) Data associated with the intersection environment
    - (1) Land-use around the intersection (residential, commercial, campus, or some combination)
    - (2) Posted speed limit on the major street
    - (3) Description of the initial and final facilities of the acting vehicle:
        - (a) presence of bike lanes
        - (b) number of auto lanes
        - (c) average auto lane width
    - (4) General characteristics: date and time
- (C) Data associated with the acting vehicle maneuver
    - (1) Movement direction: left, right, or straight
    - (2) Type: lateral, frontal, parallel, or block

      A *block* occurs when the acting vehicle cannot perform its desired maneuver because it is physically blocked from moving for a period of time. A block is timed and recorded as a gap decision observation, but is not used to estimate models, since there is no choice to be made. Some block scenarios include: (a) pedestrians in the crosswalk in front of the acting vehicle, (b) acting vehicle stopped before the crosswalk to allow a pedestrian to enter and cross the crosswalk, (c) a frontal left-turning vehicle has the right-of-way over an acting lateral straight or left-turning vehicle, or (d) the vehicle at the opposite stop sign at a four-way intersection arrive first, and therefore has the right-of-way.
    - (3) Initial and final facility positions of the acting vehicle: left, center, right, bikelane, or sidewalk (defined same as above for gap vehicle positions)
- (D) Data associated with the vehicle/operator unit
    - (1) Type of acting vehicle: bicycle, motorcycle, car (includes vans), or pickup truck (includes small trucks, SUVs, and other off-road vehicles of similar capacity)
    - (2) If the acting vehicle is a bicycle:
        - (a) cyclist gender

        (b) type of bicycle (either (1) thin tires, (2) wide tires, or (3) 1 to 3 speeds)
        (c) helmet use
    (3) Cumulative lag/gap/block time while the acting vehicle is stopped at the intersection as the first in the queue (seconds)

This cumulative time is only an approximation of the total time that the vehicle has been waiting. Any time spent in a queue (other than in the first position) is not included. However, there were not many instances of queues at the times the data was collected. Also, the time each gap vehicle takes to traverse its own distance is not included, since gap durations, not headways, are summed to obtain this value.

    (4) Cumulative number of gaps/lags/blocks the acting vehicle has rejected to this point
(E) Miscellaneous data
    (1) Did the acting vehicle ever come to a complete stop during the sequence?
    (2) Angle of attack for frontal left-turning bicycle accepting a lag: sharp (approximately 90 degrees), moderate, or flat (approximately 45 degrees)

**2.3.2.2 Rules for Measuring Gap Duration.** Gap times (or durations) are used in this study, not headways. The duration of a gap is measured from the time the rear of the opening gap vehicle passes the point of conflict (defined below) until the front of the closing gap vehicle. Gaps are shorter than headways by the time it takes a gap vehicle to traverse its length. Gaps are assumed to be more directly relevant than headways to the acting vehicle operator's decision process, especially for long gap vehicles, like buses, or slow moving cars (as in some of our locations). Gaps can also be adjusted to approximate headways, by using average vehicle speeds and lengths. For cars travelling at 30 mph and bicycles travelling at 10 mph gaps will be about 0.4 seconds shorter than headways.

The *point of conflict* is the intersection of the *typical* acting vehicle trajectory with the gap vehicle path. This typical trajectory can be thought of as an “average” trajectory of a random sample of vehicles (both autos and bikes) accepting very large gaps or the average "desired" trajectory of a vehicle operator if there were no conflicts or interference from other vehicles.

The reference point for measuring the gap (or lag) is normally the point of conflict between the acting vehicle and gap vehicle. It is assumed that this is closest to what the acting vehicle operator will perceive, especially in light of the dynamic nature of gap duration in traffic streams with non-uniform speeds, such as slow bikes with faster cars. The point of conflict reference is also critical for lag measurement accuracy, even if the traffic stream has uniform speeds and therefore, static gap durations.

At a specific intersection for a specific maneuver, the typical acting vehicle trajectory (and therefore, point of conflict) is the same for both autos and bikes. However, bicycles exhibit greater variability around this trajectory than autos. To account for variation in the initial positions of the acting vehicle (especially for bicycles), the point of conflict is determined relative to the actual initial acting vehicle position.

An acting vehicle's (auto or bike) deviation from the typical vehicle trajectory in a specific situation may reflect, in part, its operator's aggressiveness. The operator may be attempting to move the point of conflict to accept a "short" gap. Many other factors may also influence this deviation, such as operator habit, vehicle operating characteristics, and foreign objects on the roadway.

There is a need to define the starting reference points for measuring lag duration. If the acting vehicle is confronted with a stop sign, the starting reference point is either: (1) when stopped in a position to see gap vehicles and able to make the maneuver (i.e., point of adequate sight distance) or (2) if it does not stop, at the point of adequate sight distance where it should have stopped. If the acting vehicle is an auto turning right through a parallel bike gap, the starting reference point is where the auto begins to turn right. If the acting vehicle is making a frontal left-turn, the starting reference point is when the front of the acting vehicle enters the intersection (defined by extension of the curbface).

There is also a need to define the reference points when gap vehicles slow down as they enter the intersection but do not reach the point of conflict, e.g. to make a right or left turn. In this case, the reference point is when the gap vehicle's wheels start turning. This defines both the end of a gap (or lag) and the start of next gap.

## 2.4 QUALITATIVE OBSERVATIONS

Throughout the process of videotaping, reviewing video, and discussing mixed-traffic gap acceptance with colleagues assisting in the data collection, a few important qualitative observations were made. One observation was that cyclists' gap acceptance behavior was more “erratic” than that of motorists. Cyclists sometimes used non-standard lane positions to begin and end their movements, and their trajectories were much more varied.

This non-standard behavior seems to be due mostly to the fact that some cyclists do not obey the rules-of-the-road, but take the shortest path (in continuous space) or a trajectory that they perceive to be the safest. No attempt was made to quantify the proportion of cyclists that engaged in various categories of non-standard behavior. These behaviors seem to be the same as some of those which education and training programs, such as Effective Cycling (Forester, 1993), advise against.

These non-standard behaviors seem to fall into three broad categories. First, some behaviors are both illegal and dangerous, such as running stop signs and riding on the wrong side of the road. Second, some are just dangerous, such as turning and changing lanes without sufficiently looking for other vehicular traffic. Finally, some are just annoying to other vehicle operators, such as inconsistent lane positioning.

Bicyclists were rarely blocked by pedestrian traffic. In all but very large pedestrian group crossings, cyclists seemed able to safely maneuver through pedestrian gaps. This was not the case for auto drivers. Automobiles however, occasionally blocked cyclists.

While performing lateral left-turns, several (acting) automobiles arrived in their destination lane at the same time as the bicycle that was closing the gap in that lane, e.g. they arrived at the point of conflict at the same time. This was possible only because the bicycle was (or moved) far enough to the right for the auto and bicycle to briefly travel side by side. This did not seem to cause the bicycles any major inconvenience or safety problems, although we have no definitive way of ascertaining this. This allowed the autos to accept some very short gaps. The effect of a bicycle closing the gap is tested in the probit models discussed later. All three sites had major roadway space that seemed to effectively act as a bikelane, perhaps because of this, almost all cyclists ran the stop sign at fairly high speeds and did not appear to check very carefully for traffic when turning right.

## 2.5 SUMMARY STATISTICS

In this section, some of the more important summary statistics are presented. Statistics are based on either gap sequences (i.e. individual vehicle/operator units) or gap decision observations (i.e. gap acceptance decisions), whichever is most appropriate. First, some general statistics are given. Next, statistics relating to gaps where automobiles are the acting vehicles are presented, followed by those for bicycle acting vehicles.

### 2.5.1 General Statistics

The entire sample consists of 306 gap sequences, which contain a total of 618 gap decision observations. Of these, 555 gap decision observations were collected in 1994. Sixty-three additional gap decision observations on bicycle acting vehicles were collected in 1996. Five of the gap decision observations are for one gap sequence with a motorcycle acting vehicle and 48 are blocks, none of which are used to estimate gap acceptance models. This leaves 565 gap decision observations with which to estimate discrete choice models.

Table 2.1 shows the segmentation of the sample gap observations (and gap sequences) by acting vehicle and maneuver. The large number of lateral left-turns is due to the fact that two of the three sites are T-intersections, and the major flow at the intersection at which most data was collected is a lateral left-turn.

TABLE 2.1. SEGMENTATION OF GAP DECISION OBSERVATIONS BY GAP AND VEHICLE TYPE, WITH SEGMENTATION OF GAP SEQUENCES IN PARENTHESES.

| Maneuver | Acting Vehicle | | | | Total | |
|---|---|---|---|---|---|---|
| | Bicycle | | Automobile* | | | |
| Lateral left | 62 | (38) | 318 | (140) | 380 | (178) |
| Lateral right | 26 | (24) | 63 | ( 41) | 89 | ( 65) |
| Lateral straight | 11 | ( 8) | 19 | ( 3) | 30 | ( 11) |
| Frontal left | 40 | (29) | 19 | ( 15) | 59 | ( 44) |
| Parallel right | (n/a) | (n/a) | 7 | ( 7) | 7 | ( 7) |
| Block | 5 | (n/a) | 43 | (n/a) | 48 | (n/a) |
| Total | 144 | (99) | 469 | (206) | 613 | (305) |

* Automobile includes both cars and pickups.

Seventy-seven percent of all the gap decision observations were collected at the commercial/campus site, 13 percent at the commercial site, and 10 percent at the residential site. Finally, no gap greater than 9.4 seconds was rejected, and no gap less than 0.9 seconds was accepted.

### 2.5.2 Automobile Acting Vehicle

In this study, automobile refers to both cars and pickup trucks, as previously defined. The statistics in the section pertain to the major four maneuvers, whereas those presented later, in conjunction with the discrete choice models, are for lateral left-turns only.

Only seven parallel right gap decisions were collected. These were from seven different gap sequences, so each auto/driver unit accepted the lag. No parallel right gaps were rejected. The gap duration (in seconds) for each of the accepted gaps was 1.1, 4.6, 3.4, 2.9, 11.6, 13.2, and 4.3. As discussed earlier, this is a very interesting gap acceptance scenario, which has no counterpart in traffic made up entirely of automobiles. Unfortunately, not enough data was collected to analyze this situation. This would be an excellent topic for future research, since many in the field believe that numerous auto-bike accidents or near accidents occur in this situation. In fact, the 1.1 second accepted gap seems very short, especially if the cyclist was traveling at a relatively high speed or was not prepared for a potential right-turning automobile.

The rest of the data for automobile acting vehicles consists of 199 gap sequences, which contain 462 gap decision observations, 43 of which are blocks. The remaining 419 gap decision observations require a decision by the motorist to accept or reject; 39 percent are accepted. The average gap sequence contains only 2.1 gap decisions per vehicle/operator unit, and the longest sequence contains eleven gap decisions.

Table 2.1 shows that 318 of the gap decision observations (76 percent) are lateral left-turns, with 4.5 percent frontal left-turns, 15 percent lateral right-turns, and the remaining 4.5 percent are lateral straight movements. Lateral straight maneuvers are severely underrepresented due to the use of T-intersections. Lateral right-turns and frontal left-turns are underrepresented because of the flow patterns at the study intersections.

Figure 2.8 shows that the sample consists mostly of cars, with 323 cars (which include vans) and 96 pickups (which include small trucks and sport utility vehicles). Seventy-seven percent of the 419 gap decision observations were collected at the commercial/campus site, 10 percent at the commercial site, and 13 percent at the residential site. No gap greater than 9.4 seconds was rejected, and no gap less than 0.9 seconds was accepted.

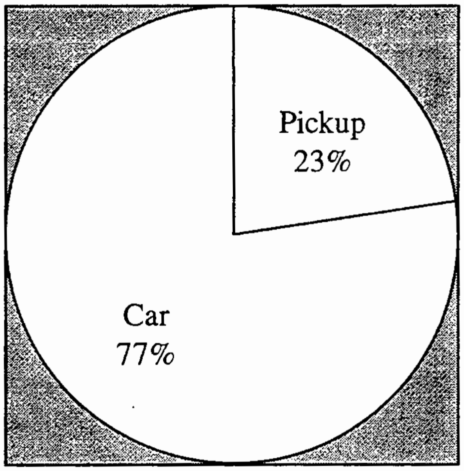


Figure 2.8. Automobile vehicle types (419 gap decisions).

Table 2.2 shows that the majority of the sample motorists stopped before making their maneuver, while the majority of cyclists did not. This agrees with the popular notion that cyclists tend to run stop signs far more often than motorists. However, motorist behavior is far from perfect, with about a quarter of all motorists disobeying the stop sign. Of course, one does not legally have to stop before making a frontal left-turn, so this statistic is meaningless from a compliance standpoint.

TABLE 2.2. PERCENTAGE OF GAP DECISIONS MADE BY VEHICLE OPERATORS WHO EVENTUALLY STOPPED. (NOTE: ONLY THREE MOTORISTS ATTEMPTED LATERAL STRAIGHT MANEUVERS.)

| Maneuver | Percent Stopping | |
|---|---|---|
| | Bicyclists | Motorists |
| Lateral left | 45% | 76% |
| Lateral right | 8% | 67% |
| Lateral straight | 55% | 100% |
| Frontal left | 13% | 21% |
| Total | 29% | 73% |

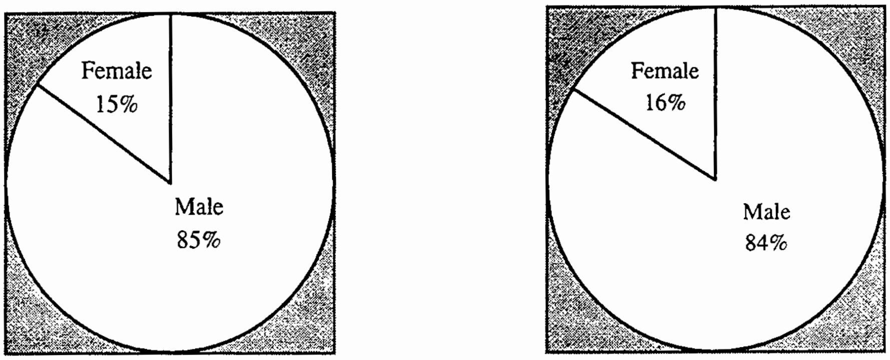
Additional statistics are presented later along with the discrete choice models of lateral left-turning motorist gap acceptance behavior.

### 2.5.3 Bicycle Acting Vehicle

The data for bicycle acting vehicles consists of 99 gap sequences, which contain a total of 144 gap decision observations, five of which are blocks. The remaining 139 gap decision observations, used to estimate the models presented in Section 2.7, require a decision by the cyclist to accept or reject; 61 percent are accepted. The average gap sequence contains only 1.4 gap decisions per bicycle/rider unit. The longest sequence contains eight gap acceptance decisions, but it is the only sequence with more than four.

Table 2.1 shows that 62 of the gap decision observations (45 percent) are lateral left-turns, with 29 percent frontal left-turns, 19 percent lateral right-turns, and the remaining 7 percent are lateral straight movements. However, the percentages of gap sequences of the three most prevalent maneuvers are more evenly distributed, due to the fact that a much higher percentage of cyclists accept lateral right-turn lags and a somewhat higher percentage accept frontal left-turn lags, than accept lateral left-turn lags. Lateral straight maneuvers are underrepresented due to the selection of T-intersections.

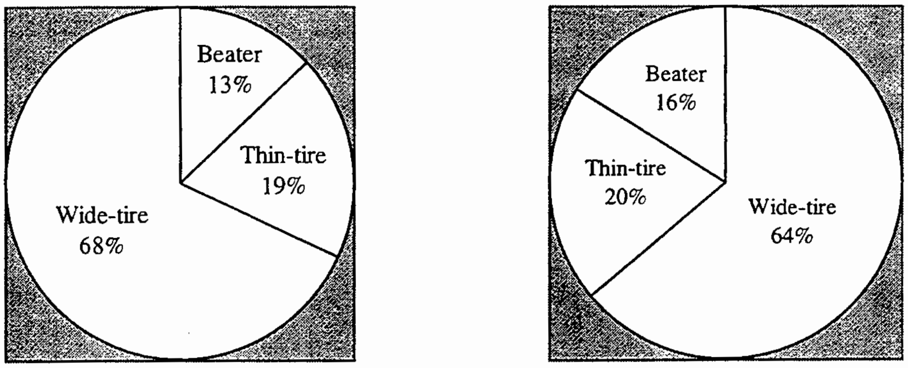
Figures 2.9 and 2.10 and Table 2.3 show that the sample is mostly male, most ride wide-tire mountain-type bicycles, and most do not wear helmets. These percentages are similar to those found in the study of bicyclist behavior at the onset of yellow, presented in Chapter 3.



Based on gap decisions

Based on gap sequences

Figure 2.9. Bicyclist gender.



Based on gap decisions

Based on gap sequences

Figure 2.10. Bicycle types. Wide-tire is a mountain bike, thin-tire is a thinner tire road or racing bike, and beater is a zero to three speed bike (e.g., stunt bike or old bike with no gears).

TABLE 2.3. HELMET USE. NOTE: IN 1994 NO ORDINANCE REQUIRED HELMET USE, BUT IN 1996 A CITY HELMET ORDINANCE HAD BEEN RECENTLY VOTED IN AND PUBLICIZED, BUT FINES WERE NOT BEING ISSUED FOR ANOTHER FEW WEEKS.

| | Based on Gap Decisions | | | Based on Gap Sequences | | |
|---|---|---|---|---|---|---|
| | 1994 | 1996 | Total | 1994 | 1996 | Total |
| Helmet | 28% | 20% | 25% | 28% | 22% | 25% |
| No helmet | 72% | 80% | 75% | 72% | 78% | 75% |

Seventy-three percent of the 139 gap decision observations were collected at the commercial/campus site, 24 percent at the commercial site, and three percent at the residential site. No gap greater than 6.7 seconds was rejected, and no gap less than 1.8 seconds was accepted.

Table 2.2 showed that a majority of the sample cyclists did not stop at the stop sign before making their maneuver. In addition, the initial and final lane positioning of cyclists making maneuvers is of interest. Figure 2.11 shows that the majority of cyclists (70 percent) quickly finish their maneuver in a perceived "safe" area (i.e., sidewalk, bikelane, or the right side of the curb lane). Many of the 30 percent in the left and center of the travel lane probably also moved to the right, but were out of the camera field before doing so.

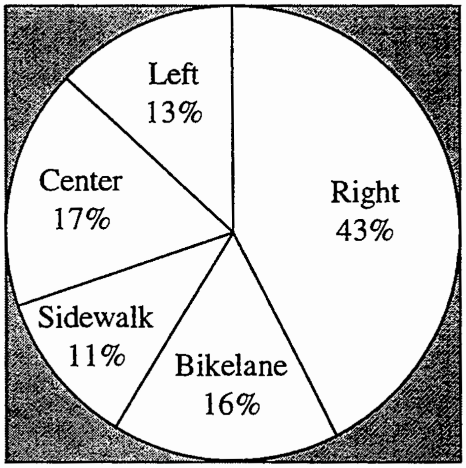


Figure 2.11. Final position of bicycle acting vehicles.

For the intersections studied, unless on the sidewalk, a cyclist cannot be in an illegal or unsafe lane position if intending to go straight (i.e., a cyclist can be left, right, center, or in a bikelane). However, making a lateral left-turn from the right is unsafe at all three sites and illegal at the commercial/campus site, where there is a left-turn lane. A bicyclist should merge to the left before turning left. Seventeen of the 38 lateral left-turns were made from this unsafe initial position. A similar proportion (10 of 29) of the frontal left-turns made by cyclists were also made from an unsafe position, in this case on the right side of the lane or from a bikelane. As expected, almost no lateral right-turns (1 of 24) were made from an unsafe initial position, since most cyclists prefer to travel on the right side of the curb lane. Finally, one cyclist started from a sidewalk. Sidewalks are often found to be an unsafe initial position (Forester, 1983). To conclude, a high percentage (40 percent) of cyclists making lateral and frontal left-turns put themselves in an unsafe initial position.

Of the 29 cyclists who attempted frontal left-turns, 20 of them accepted the lag. The trajectory of these 20 cyclists is of interest. By modifying their trajectories they can effectively move the point-of-conflict with oncoming traffic to accept shorter (or reject longer) gaps than if they had proceeded according to the typical vehicle trajectory. Figure 2.12 shows that 55 percent (11 of 20) did so possibly to accept shorter (than average) gaps, and five percent did so possibly to reject a longer (than average) gap, without having to stop. This data was not collected for automobiles, so no direct comparison is possible. Since bicycles are smaller and more maneuverable than automobiles, one could suspect that cyclists might take greater advantage of this behavior.

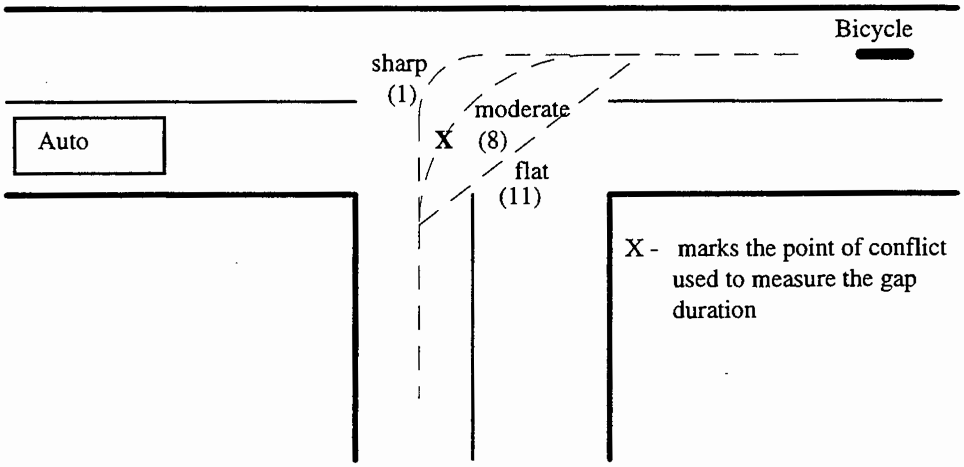


Figure 2.12. Number of cyclists (in parentheses) accepting frontal left-turn lags and their associated trajectories.

Additional statistics are presented later along with the discrete choice models of bicyclist gap acceptance behavior.

## 2.6 RESTRICTED MODELS – INDEPENDENT DECISIONS, MEAN AND VARIANCE ONLY

In this section, mean and variance only models are estimated for different population segments assuming independent decisions. Simultaneous estimation is performed using the following specification:

$$\text{Pr(accepting/I mutually exclusive segments)} = \Phi\left(\frac{g_{jn} - \sum_{i=1}^{I}(\delta_i \mu_i)}{\sum_{i=1}^{I}(\delta_i \sigma_i)}\right), \qquad (2.18)$$

where $\delta_i$ = 1, if the observation is in population segment i;

= 0, if not,

I = the total number of mutually exclusive segments,

$\sigma_i$ = the standard deviation of the critical gap distribution for segment i, and

$\mu_i$ = the mean of the critical gap distribution for the segment i.

Because all gap decisions are assumed independent, only the total variance $(\sigma_i)^2$ is estimated for each segment i. The following segmentation variables define the 16 segments for which estimation was attempted:

(1) Acting vehicle stops or does not stop (rolls),

(2) Acting vehicle maneuver: lateral left, lateral right, lateral straight, or frontal left, and

(3) Acting vehicle type: bicycle or automobile.

Seven segments were not included in the final model, due to low numbers of gap decision observations.

Models are estimated simultaneously to allow one to test for significant differences between segments, using the log-likelihood ratio test by estimating both restricted and unrestricted models. Because of the restricted nature of these models and the small number of gap decision observations, they are intended primarily for exploratory purpose. Inferences from these tests or the magnitudes of the parameter estimates are only suggestive. Nevertheless, they allow initial order-of-magnitude comparisons to previous study results and provide a contrasting basis to the less restrictive models presented later.

Consider the operator's decision to stop or not (roll). First, one should note that rolling is legal for frontal left-turns, but illegal for all lateral maneuvers. For this reason it would be equally (or maybe more) appropriate to estimate the frontal left model with "stops" and "rolls" combined. However, in this sample the low number of "stops" would probably not have much impact.

Second, as expected, bicycles are much more likely to disobey the stop sign. Third, rolling behavior obviously occurs most often on the lag. Occasionally the lag is short enough that the behavior persists to the next (second) gap and sometimes even to the third and fourth gaps.

The implemented data reduction scheme marks all gap decisions in the sequence as "stopped", if the vehicle ever stopped. The vehicle might have been rolling during the first few gaps, but was forced to stop because it could not accept an early gap, not because it wished to obey the stop sign. (No vehicle in the sample rolled for more than the first three gaps before either accepting the next gap or stopping.) This illustrates one way in which stopping behavior is not independent of the gap duration and other attributes associated with the lag and early gaps. Stopping behavior is a decision that depends not only on the operator's aggressiveness and adherence to traffic law, but also on some of the same factors as the gap acceptance decision, and is therefore an endogenous variable. This variable is also subject to selectivity bias, because some vehicle operators will be forced into stopping against their wishes. If included in a model, it would cause endogeneity bias. By segmenting based on this variable, endogeneity bias is circumvented.

### 2.6.1 Analysis and Discussion

Table 2.4 and Figure 2.13 present the estimates of the means and standard deviations (only) for the critical gap distributions for different population segments, estimated over all the gap acceptance situations present in the segment's data. Table 2.5 presents models for the same segments for lag acceptance situations only.

TABLE 2.4. ESTIMATED MEANS AND STANDARD DEVIATIONS OF THE CRITICAL GAP DISTRIBUTIONS ASSOCIATED WITH VARIOUS SEGMENTS.

| Acting Vehicle Maneuver | Vehicle Type | Stop or Roll | Mean (sec) | t-statistic[1] | Standard Deviation (seconds) | t-stat.[1] | # of obs[3] |
|---|---|---|---|---|---|---|---|
| Lateral left | Auto | Roll | **3.5** | 6.8 | **3.0** | 5.8 | 76 |
| Lateral left | Auto | Stop | **6.4** | 7.6 | **2.1** | 5.7 | 242 |
| Lateral left | Bike | Roll | **4.5** | 6.4 | **1.4** | 8.6 | 34 |
| Lateral left | Bike | Stop | **6.0** | 6.1 | **0.9** | 7.0 | 28 |
| Lateral right | Auto | Roll | **3.0** | 5.5 | **1.2** | 8.9 | 21 |
| Lateral right | Auto | Stop | **5.4** | 5.2 | **1.7** | 4.6 | 42 |
| Lateral right[2] | Bike | Roll | **2.1** | 5.2 | **1.0** | 5.3 | 24 |
| Frontal left[2] | Auto | Roll | **3.2** | 7.2 | **0.7** | 7.0 | 15 |
| Frontal left[2] | Bike | Roll | **4.6** | 5.9 | **0.8** | 5.9 | 35 |

Log-likelihood (final) = -114.21

[1] Some t-statistics may be overestimated due to the relative flatness of the log-likelihood function around the maximum.

[2] Only two lateral right-turning cyclists stopped. For frontal left-turns, only five cyclists and four motorists stopped. Therefore, none of these three segments are included.

[3] In this case, observation (obs) denotes a gap acceptance decision.

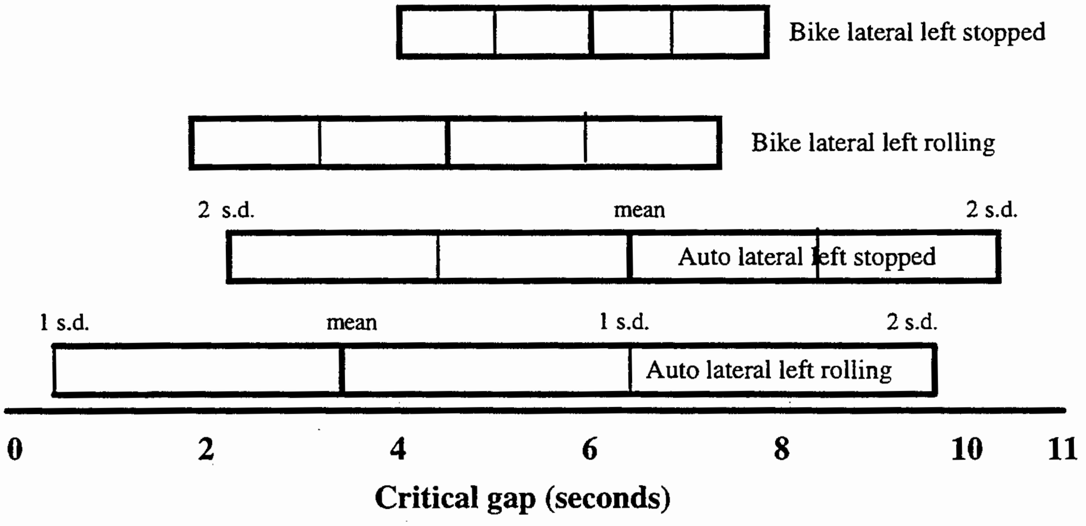


Figure 2.13. Estimated critical gap distributions associated with lateral left-turn segments, showing the intervals containing the central 95 percent of the critical gaps in each segment (except for auto lateral left rolling).

The estimated standard deviations seem reasonable relative to the estimated means and the summary statistics from the sample. For example, in the motorist sample, all gaps less that 0.9 seconds were rejected and all greater than 9.4 seconds were accepted. From this, the range of "indecision" would be 8.5 seconds wide, which agrees well with the estimated range of critical gaps for lateral left-turning[7] motorists who stop shown in Figure 2.13.

One possible systematic effect which can easily be (and often is) accounted for is the difference in the mean critical lag and the mean critical gaps for the rest of the gaps in a sequence. As expected, comparing Tables 2.4 and 2.5 gives evidence that the mean critical lag may be greater than the rest of the mean critical gaps for the same segment, reflecting the fact that operators require time to become familiar with the intersection environment. However, the general nature of the conclusions in this section will not change if they are based on all gaps or just lags, with the exception of the variances for automobile lateral right-turns. Therefore, only the distributions over the entire population of gap acceptance situations (Table 2.4) will be discussed further.

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[7] By far the most prevalent motorist maneuver.

TABLE 2.5. ESTIMATED MEANS AND STANDARD DEVIATIONS OF THE CRITICAL LAG DISTRIBUTIONS ASSOCIATED WITH VARIOUS SEGMENTS.

| Acting Vehicle Maneuver | Vehicle Type | Stop or Roll | Mean (sec) | t-statistic[1] | Standard Deviation (seconds) | t-stat.[1] | # of obs[3] |
|---|---|---|---|---|---|---|---|
| Lateral left | Auto | Roll | **4.7** | 7.7 | **2.7** | 4.3 | 54 |
| Lateral left | Auto | Stop | **8.5** | 8.6 | **1.6** | 7.7 | 58 |
| Lateral left | Bike | Roll | **5.2** | 6.8 | **2.1** | 9.9 | 28 |
| Lateral left | Bike | Stop | **7.3** | 7.3 | **0.2** | 2.7 | 9 |
| Lateral right | Auto | Roll | **2.9** | 5.2 | **0.9** | 6.9 | 20 |
| Lateral right | Auto | Stop | **6.4** | 4.1 | **0.6** | 5.6 | 20 |
| Lateral right[2] | Bike | Roll | **2.3** | 9.6 | **0.5** | 4.2 | 22 |
| Frontal left[2] | Auto | Roll | **3.2** | 8.7 | **1.0** | 7.6 | 12 |
| Frontal left[2] | Bike | Roll | **4.6** | 6.4 | **1.5** | 8.0 | 26 |

[1] Some t-statistics may be overestimated due to the relative flatness of the log-likelihood function around the maximum.

[2] Only two lateral right-turning cyclists stopped. For frontal left-turns, only five cyclists and four motorists stopped. Therefore, none of these three segments are included.

[3] In this case, observation (obs) denotes a gap acceptance decision.

Of interest is how the models compare with previous models and a priori theories. The automobile mean critical gap estimates are comparable to the critical gap design values given in the Highway Capacity Manual (TRB, 1985), summarized in Section 2.2. The estimated mean critical gaps for automobiles fit into the lower end of the Highway Capacity Manual ranges, as expected for low speed streets. In addition, the lateral left-turn mean critical gap for automobiles is greater than that for a frontal left-turn, which is in turn greater than that for a lateral right-turn. As expected, this trend is similar for bicycles.

The fact that the estimated automobile mean critical gaps while facing mixed-traffic are on the low side of the standard ranges, also agrees with Smith's (1976) findings (reported earlier) that autos require lower critical gaps when facing bicycle-only traffic than when facing auto-only traffic. The estimated mean critical gaps of rolling vehicles are less than the estimated means of vehicles that stop. This is in agreement with Smith (1976), Opiela et al. (1980), and the a priori theory.

A log-likelihood ratio test shows a highly significant difference between lateral left-turning automobiles that roll or stop. The final log-likelihood value for the restricted model (rolling and stopping parameters restricted to be equal) is –125.26, so the chi-squared test statistic is 22.1

with one degree of freedom. The statistic is computed from (-2 x [-125.26 + 114.21]). A chi-squared statistic of 3.84 is significant at the 0.05 level. Lateral right-turning automobiles show a marginal significant difference (at the 0.10 level) in the mean critical gap of rolling versus stopped vehicles. There is no significant difference between the means for rolling and stopped lateral left-turning bicycles.

The estimated mean critical gap for lateral left-turning bicycles which stop is comparable to (and not statistically different than) that for automobiles. The difference for rolling vehicles is marginally statistically different (0.10 level). This is preliminary evidence against the hypothesis that bicycles require longer gaps than automobiles. However, as discussed earlier for similar results found in limited studies by Smith (1976) and Opiela et al. (1980) this could be due to the fact that auto speeds are very low. It also may also be attributed partly to the familiarity of the cyclists with the intersections under study or that most of the cyclists are college students and are at a risk-taking time in their lives. In addition, bicyclists around college campuses may feel overly protected (compared to their feelings in other environments), because of special treatment or large numbers. Finally, cyclists may take advantage of the fact that on streets with wide lanes, they can pause in the middle of the street much as pedestrians do on a median when crossing wide streets. Even if this result is true for the study intersections and cycling sample, it may not apply to all intersections or cyclists.

Though not significantly different, the lower mean critical gap for rolling bicycles (versus rolling automobiles) making lateral right-turns may be due to the presence of a bikelane or other free space in which to turn. Finally, the higher mean critical gap for frontal left-turning cyclists relative to motorists is expected (and significantly different at a 0.10 level). This difference may be magnified (or caused) by the subset of cyclists who turn left from the right side of the lane.

Results of the significance tests may be due only to the low number of observations. Also, only the seven comparable segment pairs were tested. For example, there is no good reason to test the difference in frontal left rolling autos and lateral left stopped bicycles. Pair-wise comparisons were made only within the same maneuver, where one variable (either vehicle type or stopping behavior) was the same.

Further log-likelihood ratio tests were performed for models estimated as specified in equation 2.18 with eight segments, one for each combination of acting vehicle type and maneuver (rolling and stopping observations combined). There was no significant difference in the mean critical gaps for autos and bikes for lateral left, lateral straight, or frontal left maneuvers. This may be caused by the greater proportion of rolling cyclists compared to rolling motorists, or the reasons discussed previously (e.g. bicyclist youth). The lateral right-turn mean critical gap for cyclists was significantly less than that for autos. The estimated difference was 1.9 seconds, which, if true, is probably due to the fact that bicycles have a bikelane (or its equivalent) and therefore, do not have to merge directly in front of the closing gap vehicle.

Finally, two additional models were specified. The first included two separate mean critical gaps, one for all rolling bicycles and one for all stopped bicycles, with all maneuvers combined. The second was a comparable but restricted model estimating the mean critical gap for all bicycles (rolling and stopping). The log-likelihood ratio test showed a highly significant difference ($p<0.0005$) between the mean critical gaps of stopped and rolling bicycle/rider units. The estimate for rolling cyclists is about four seconds, and the estimate for stopped cyclists is about six seconds.

Because the statistical tests of comparable mean critical gap estimates with the higher number of observations show significant differences and the rest of the mean critical gap differences can be explained or fit a priori expectations, the model is presented with all segments estimated independently. This also allows the readers to make their own preliminary judgements.

As expected, the variance (or standard deviation) of the critical gap distributions is less for lateral left-turns when the vehicle/operator unit stops before making its decision. Since a lateral left-turn decision is fairly complex, it makes sense that stopped vehicle operators might make choices with less variation. For (automobile) lateral right-turns this relation is reversed, which could be due to less complexity in the decision making process. However, even in this sample the relationship is reversed when one considers only lags (see Tables 2.4 and 2.5).

It is interesting that, except for frontal left maneuvers, estimated bicycle variances are less than estimated automobile variances. Many would probably have guessed the reverse. Possible substantive reasons include the following.

(1) The relative (to motorists) homogeneity of the cyclist sample, as most cyclists are male college-age students and cyclists can more easily (than motorists) avoid intersections with which they are not comfortable.

(2) Most cyclists probably use the intersections often and are very familiar with them. Accentuating this was the fact that the video taping always took place at approximately the same times in the mornings and early afternoons.

(3) Automobile acceleration potential might allow more variation in behavior than the acceleration potential of a bicycle.

(4) Bicyclists can use their other senses, especially hearing, to a greater degree to detect major street traffic.

This result also may not hold for larger or different samples or when less restrictive specifications account for various systematic effects. The reverse result for frontal left-turns may be due to the fact that some cyclists turn from the right side of the lane, thereby having to cross an extra stream of traffic from the rear.

Log-likelihood ratio significance tests were performed for the four comparable (i.e. same maneuver) variances for automobiles versus bicycles. The frontal left and lateral right rolling variances were not statistically different. On the other hand, the lateral left rolling variance was significantly less for bicycles, as was the lateral left stopped variance.

### 2.6.2 Closure

As discussed, all the conclusions and discussion in this section are tenuous, because of the small number of observations and the restrictions of the model specifications and assumptions. They require verification and refinement using repeated studies with larger and richer data sets and less restrictive models, such as those presented in the next section. However, the automobile mean critical gap estimates compare well to values estimated in previous studies for low speed streets, and bicycle estimates do not severely disagree with the limited previous studies on bicycle gap acceptance. In addition, most estimates agree fairly well with a priori expectations. The two main exceptions were that (1) most bicycle mean critical gap estimates were similar to or less than the comparable automobile estimates and (2) most bicycle critical gap distributions had less variance than the comparable automobile distributions. Possible substantive reasons for both of these exceptions were discussed.

## 2.7 LESS RESTRICTIVE MODELS

The models in this section introduce parameters that capture the systematic influence of certain variables on an individual vehicle/operator's critical gap. The models are specified by equation 2.5. The model of the gap acceptance behavior of automobile/driver units is estimated using only lateral left-turns, because the overwhelming majority of the observed motorist gap decisions were made while attempting this maneuver. The bicycle gap acceptance model is estimated using data containing lateral left-turns, lateral right-turns, lateral straight movements, and frontal left-turns, because not enough data was collected to estimate separate models.

There are not enough acting vehicle maneuvers or intersection environments present in the data to capture their constant persistent effects using components of variance, so the (base) covariance matrix is specified by equations 2.9, 2.10, and 2.11, with $\sigma_m$ and $\sigma_l$ set to zero. To capture serial correlation due to the constant persistent unexplained variation associated with all the gap decisions made by the same vehicle/operator unit, the automobile model was estimated using this covariance specification. Estimation results are given in Table 2.6.

TABLE 2.6. COMPONENTS OF VARIANCE MODEL FOR AUTOMOBILE LATERAL LEFT-TURNS.

| Variable | Number of true observations | Parameter Estimate (seconds) | t-statistic[1] |
|---|---|---|---|
| Mean critical gap when all variables are false (or 0) ($\beta_0$) | all | 6.85 | 4.20 |
| Closing gap (including lag) vehicle is a bus or heavy truck ($X_{gap}$) | 14 | 1.49 | 8.96 |
| Closing lag vehicle is a motor vehicle ($X_{gap}$) | 36 | 4.01 | 5.65 |
| Closing gap (not including lag) vehicle is a bicycle ($X_{gap}$) | 79 | -3.43 | -9.51 |
| Closing lag vehicle is a bicycle ($X_{gap}$) | 76 | -1.43 | -9.46 |
| Opening gap vehicle-bike in far lane; Closing gap vehicle-motor vehicle in near lane ($X_{gap}$) | 13 | -2.90 | -5.19 |
| Opening gap vehicle is a slowing automobile ($X_{gap}$) | 37 | 1.72 | 7.97 |
| Closing gap (including lag) vehicle is slowing in the far lane ($X_{gap}$) | 14 | -1.86 | -8.93 |
| **Across gap component of variance term ($\sigma_g$)** | - | **1.78** | 9.52 |
| **Across vehicle/operator unit component of variance term ($\sigma_v$)** | - | **-0.02** [2] | -0.89 |

Number of gap sequences = 140
Number of gap decisions = 318

Log-likelihood (*LL*) (initial) = -220.42
Log-likelihood (*LL*) (final) = -65.91

[1] The t-statistics may be overestimated.

[2] Over alternative specifications, the estimate of $\sigma_v$ was usually positive, as required by the theory, but was occasionally negative (and close to zero) like this estimate.

The across vehicle/operator unit component of variance term $\sigma_v$ was not significant, probably because the data consists of relatively short gap sequences. (There was an average of only 2.27 gap decisions per sequence.) This leads to the conclusion that possible serial correlation can be ignored and the model estimated assuming independence. This type of serial correlation can probably also be ignored in other data sets with relatively short gap sequences, which is much more convenient in practice. The final independent decisions model for automobiles is presented in Table 2.7. It, as well as the final bicycle model, is estimated assuming independent decisions by further restricting the covariance matrix, specified by equations 2.9, 2.10, and 2.11, by also setting $\sigma_v$ (in addition to $\sigma_m$ and $\sigma_l$) to zero.

TABLE 2.7. FINAL INDEPENDENT DECISIONS MODEL FOR AUTOMOBILE LATERAL LEFT-TURNS.

| Variable | # of true obs. | Parm. Est. (sec) | t-stat.[1] | Log-Likelihood Ratio Test of Associated Individual Variable | | |
|---|---|---|---|---|---|---|
| | | | | LL Restricted Model | Chi-squared (1 d.f.)[2] | Signif. level |
| Mean critical gap when all variables are false (or 0) ($\beta_0$) | all | 6.74 | 4.34 | - | - | - |
| Closing gap (including lag) vehicle is a bus or heavy truck ($X_{gap}$) | 14 | 1.45 | 4.15 | -67.21 | 3.04 | $p<0.10$ |
| Closing lag vehicle is a motor vehicle ($X_{gap}$) | 36 | 4.03 | 7.20 | -68.51 | 5.64 | $p<0.02$ |
| Closing gap (not including lag) vehicle is a bicycle ($X_{gap}$) | 79 | -2.95 | -4.41 | -81.63 | 31.88 | $p< 0.0005$ |
| Closing lag vehicle is a bicycle ($X_{gap}$) | 76 | -1.32 | -8.71 | -71.56 | 11.74 | $p<0.001$ |
| Opening gap vehicle-bike in far lane; Closing gap vehicle-motor vehicle in near lane ($X_{gap}$) | 13 | -3.24 | -5.67 | -69.35 | 7.32 | $p<0.01$ |
| Opening gap vehicle is a slowing auto ($X_{gap}$) | 37 | 1.82 | 9.85 | -68.59 | 5.80 | $p<0.02$ |
| Closing gap (including lag) vehicle is slowing in the far lane ($X_{gap}$) | 14 | -1.87 | -4.03 | -68.26 | 5.14 | $p<0.025$ |
| Standard deviation of the critical gap dist. ($\sigma$) | - | 1.78 | 9.76 | - | - | - |

Number of gap sequences = 140
Number of gap decisions = 318

Log-likelihood (LL) (initial) = -220.42
Log-likelihood (LL) (final) = -65.69

[1] The t-statistics may be overestimated.

[2] A chi-squared test statistic greater that 3.84 is significant at $p<0.05$.

One can use the estimation of an unrestricted model, which includes a variable, and a restricted model, which is identically specified except that it excludes the variable, to test the significance of the variable using the log-likelihood ratio test.[8] This test is more reliable than the

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[8] The **log-likelihood ratio** test statistic is computed as twice the absolute difference between the restricted and unrestricted model log-likelihood values, with the unrestricted model theoretically having a greater log-likelihood. This statistic is chi-squared distributed with one degree of freedom (when testing for the significance of a single parameter restriction).

corresponding t-test of individual parameter significance due to the possible numerical instabilities associated with the relative flatness of the log-likelihood function around convergence for this data.

The parameter estimates for the final independent decisions model are very similar to those for the components of variance model. Note that both the t-statistics and log-likelihood ratio test statistics for the same hypotheses (whether the coefficients are significantly different from zero) are given in the table, in case the t-statistic is overestimated. For example, the t-statistic for the "gap closed by bus or heavy truck" parameter is highly significant at 4.15, but the log-likelihood ratio test shows only marginal significance at the 0.10 level. This variable is included because of strong a priori and substantive reasons. All other parameters are highly significant using either statistical test and have strong substantive reasons for their inclusion, as do those in the final bicycle model.

Table 2.8 presents the results of the estimation of the final bicycle model. An independent decisions model is appropriate because of the low number of gap decisions per sequence and (effectively) all the variation is explained systematically.

TABLE 2.8. FINAL INDEPENDENT DECISIONS MODEL FOR BICYCLES, INCLUDING LATERAL LEFT, LATERAL STRAIGHT, LATERAL RIGHT, AND FRONTAL LEFT MANEUVERS.

| Variable | # of true obs. | Parm Est. (sec) | t-stat.[1] | Log-Likelihood Ratio Test of Associated Individual Variable | | |
|---|---|---|---|---|---|---|
| | | | | LL Restricted Model | Chi-squared (1 d.f.)[2] | Signif. level |
| Mean critical gap when all variables are false (or 0) ($\beta_0$) | all | 4.55 | 4.73 | - | - | - |
| Lateral right-turn at commercial/campus site ($X_{ia}$) | 14 | -3.50 | -9.92 | -19.71 | 38.40 | p< 0.0005 |
| Lateral left or straight lag is closed by a motor vehicle in the far lane ($X_{gap}$) | 20 | 2.45 | 8.30 | -18.46 | 35.90 | p< 0.0005 |
| Lateral left or right gap (including lag) is closed by a bus or heavy truck ($X_{gap}$) | 7 | 1.10 | 5.56 | -3.81 | 6.60 | p<0.02 |
| Frontal left-turn from right-side of lane ($X_{ia}$) | 16 | 1.03 | 5.38 | -8.92 | 16.82 | p< 0.0005 |
| Lateral left or straight gap (including lag) is closed by a bicycle in the near lane ($X_{ia}$) | 4 | -1.25 | -5.93 | -8.61 | 16.20 | p< 0.0005 |
| Standard deviation of the critical gap distribution ($\sigma$) | - | 0.01 | 2.65 | - | - | - |

Number of gap sequences = 99
Number of gap decisions = 137

Log-likelihood (LL) (initial) = -94.96
Log-likelihood (LL) (final) = -0.51

[1] The t-statistics may be overestimated.

[2] A chi-squared test statistic greater that 3.84 is significant at $p<0.05$.

### 2.7.1 Discussion, Interpretation, and Inference

It is interesting to compare the standard deviation estimates to those from mean and variance only models, estimated as specified in equation 2.3. For automobiles the estimated standard deviation dropped from 2.21 seconds (in the mean and variance only model) to 1.78 seconds, reflecting that the variables included in the final model "explain" a portion of the total variation. The standard deviation estimate from the bicycle mean and variance only model was 1.64 seconds, which is quite a bit more than the 0.01 second estimate in the final model. It is remarkable that almost all of the variation in the sample is explained by the five variables. This is partly a coincidence of this sample, so this specification could not be expected to explain as much of the variation in another set of data collected from the same (or other) intersection environments. Nonetheless, the included variables would still be expected to be significant in other samples.

The coincidence that this data set is so well explained was aided by the fact that it is relatively small (only 137 gap decision observations). For example, consider a small sample of only six gap decision observations, with rejected gaps of 1.1, 2.3, and 4.0 seconds and accepted gaps of 4.6, 5.6, and 8.0 seconds. An estimated mean critical gap anywhere between 4.0 and 4.6 seconds would yield perfect prediction of the sample and a standard deviation estimate of zero. The more observations the more likely it is that a gap less than 4.0 is accepted or that a gap greater than 4.6 is rejected, thereby precluding perfect prediction with only the mean critical gap. The logic behind this simple example can be easily extended to the case with multiple explanatory variables and a relatively small sample size.

All the parameters in the final models are of the correct sign and were robust in sign, magnitude, and significance in many alternative specifications. All variables are Boolean categorical variables (1-if true, 0-if false). Each parameter has the unit of seconds, and the interpretation is that a positive coefficient increases the mean critical gap, while a negative one decreases the mean critical gap for a specific gap situation.

All the variables in the automobile model can be derived from the following data items, which map to the group of attributes characterizing the gap and surrounding events $X_{gap}$ (see equation 2.4):

(1) gap number (either lag or “gap number two and greater”),

(2) closing gap vehicle characteristics (type, decelerating, or near/far lane), and

(3) opening gap vehicle characteristics (type, decelerating, or near/far lane).

The bicycle model includes a variable that reflects some of the characteristics of the vehicle/operator unit and therefore, captures some of the across vehicle/operator unit variation. This variable (“frontal left from right-side of lane”) belongs to the attribute vector $X_{vo}$. The variable, “lateral right at the commercial/campus site”, is an interaction ($X_{ia}$) of a variable that is a characteristic of the maneuver ($X_{man}$) with one that is a characteristic of the intersection environment ($X_{int}$). This variable captures some of the across maneuver variation and some of the across intersection environment variation. The remaining variables are interactions of maneuver ($X_{man}$) and gap ($X_{gap}$) attributes. See equation 2.4 for reference on these and all variable categories.

Using the variables in the models one can define a variety of gap acceptance situations and their associated critical gap distribution estimates. In the next subsection, gap acceptance behavior is compared over many of these situations. In the following subsection, inferences from and interpretation of the models' parameters are discussed.

**2.7.1.1 Comparison of Behavior in Various Gap Acceptance Situations.** Figures 2.14 and 2.15 compare the mean critical gaps for some of the gap acceptance situations that can be constructed using the automobile model and the bicycle model, respectively.

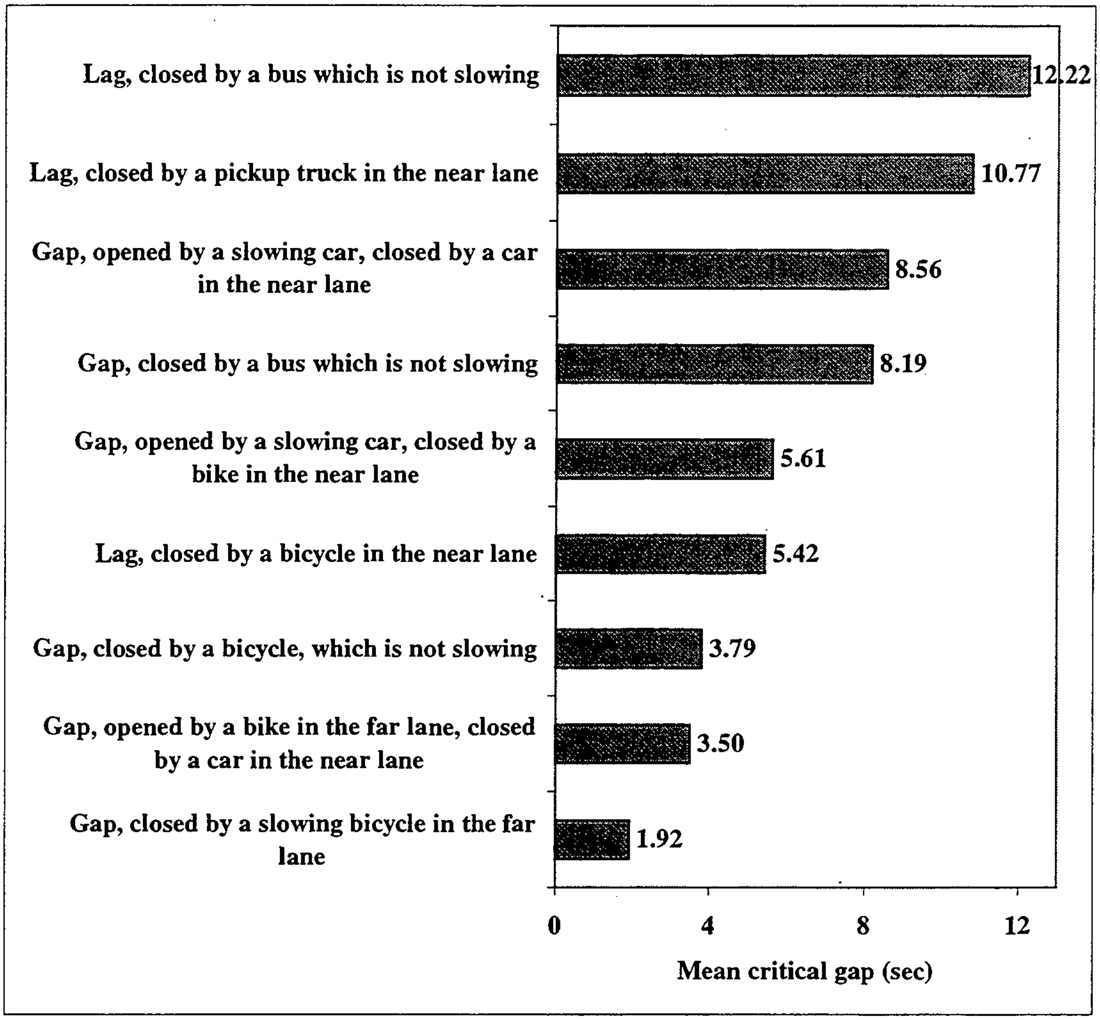

Figure 2.14. Comparison of a variety of possible lateral left-turning automobile gap acceptance situations.

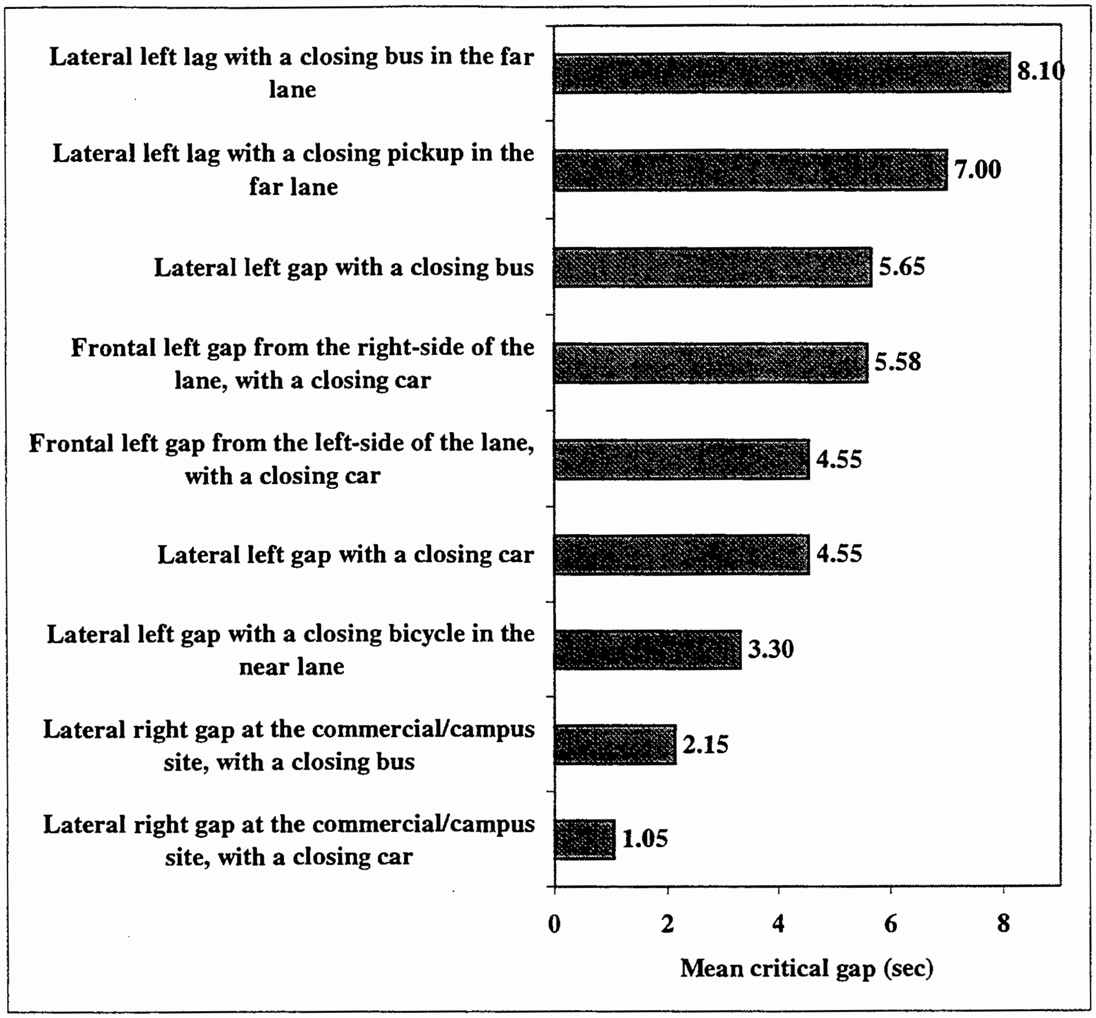

Figure 2.15. Comparison of a variety of possible gap acceptance situations for bicycles.

If all variables are zero, the predicted automobile mean critical gap is 6.74 seconds. "Gap number three in a sequence at the commercial site is opened and closed by cars proceeding at constant speed" is one of the numerous gap acceptance situations to which this mean critical gap estimate applies. If the gap is a lag and the closing lag vehicle is a bicycle in the near lane, the predicted mean critical gap for automobiles is 5.42 seconds (6.74 - 1.32). The means in all these situations are the means of normal distributions of the critical gaps for that situation. Each of the automobile situational distributions has an estimated standard deviation of 1.78 seconds, whereas the bicycle situational distributions are virtually spike functions (standard deviations of 0.01 seconds).

Situational critical gap estimations from models developed from small samples cannot be expected to be extremely accurate. However, the comparisons illustrate the range of critical gaps one might expect for a variety of gap acceptance situations and the limitations of relying on only the overall mean critical gap obtained from restricted models like those presented in Section 2.6. For automobiles, the mean critical gap varies from about two to 12 seconds and for bicycles, it varies from about one to eight seconds.

The less restrictive models give further indication that the hypothesis, that bicyclists would require longer gaps than motorists in the same situations, is incorrect. In fact, they provide evidence that motorists require longer gaps in comparable lateral left-turning situations. In Figure 2.16, specific comparisons of bicycle and automobile behavior in the same gap acceptance situation show a fairly large disparity (from two to almost four seconds) between the two. This difference may be exaggerated by the very small sample size for lateral left-turning bicycles, and needs to be verified with a more diverse sample, across both intersection environments and cycling populations.

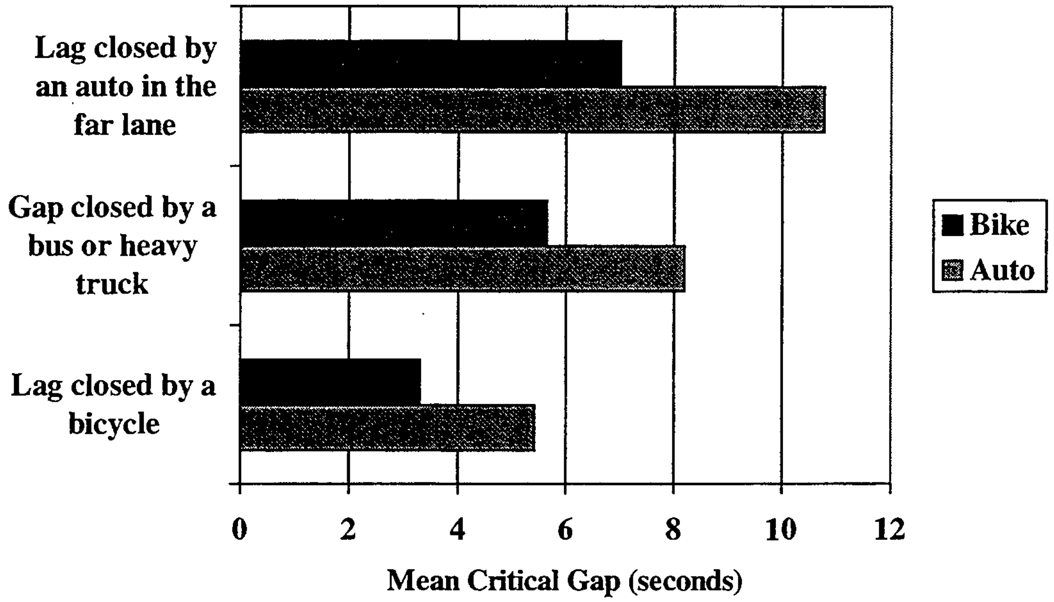

Figure 2.16. Comparison of mean critical gaps required by lateral left-turning bicycles and automobiles in the same situation.

Several plausible reasons for this were given earlier in Section 2.6 (e.g. cyclist youth and familiarity with intersection environments). It would be interesting to study one of these, i.e. cyclists' use of the middle of the street when crossing.

One must be careful in using the predicted situational mean critical gaps and any inferences from the model, since the largest data set (for the automobile model) contains only 318 gap decisions (by only 140 motorists) at only three low speed intersections. The main purpose of these models is to provide some indications of the directions and relative magnitudes of factors impacting the gap acceptance behavior of mixed bicycle-automobile traffic. Any inferences are most suited to college student cyclists at very low speed intersections near a campus with which they are familiar. In addition, any inferences are stronger for automobiles (versus bicycles) because of a higher number of gap decision observations (318 versus 137), fewer acting vehicle maneuvers (one auto compared to four bike), and stronger priors. These inferences are fully explored in the following subsection.

**2.7.1.2 Interpretation of and Inferences from the Estimated Parameters.** First, the difference in lag and gap acceptance behavior is discussed. Second, the important impacts of the closing gap vehicle type are explored. Then, the other significant automobile model parameters are discussed, followed by the other bicycle model parameters. Finally, a short discussion is given on variables found to be insignificant. To avoid repeating that an inference is suspect because the parameter estimate is based on a low number of observations for a particular gap acceptance situation, especially for bicycles, sometimes only the number of gap decision observations for the particular situation is given.

*2.7.1.2.1 Lag Versus "Other Gap" Acceptance Behavior.* As indicated by theory, previous auto-only study results, and the restricted models presented in Section 2.6, several parameter estimates indicate that for both cyclists and motorists the critical lag is significantly greater than other critical gaps. The likely reason is that when first arriving at an intersection a vehicle/operator needs time to become familiar with the current intersection environment.

For lateral left-turning motorists facing a lag acceptance situation with a closing motor vehicle, the magnitude of this effect is quite large (4.03 seconds) relative to other model parameters. For motorists facing a lag with a closing bicycle, the magnitude of the lag effect is quite a bit less at 1.63 seconds (-1.32 + 2.95). For cyclists, a lag effect (2.45 seconds) was found for lateral left and lateral straight maneuvers with a closing motor vehicle in the far lane.

It is of interest that there was not a significant effect on cyclist behavior for lags closed by motor vehicles in the near lane, with 19 gap decision observations for this situation. If true, the reason may be that cyclists only need to cross one lane to avoid these vehicles, and in an emergency, they (unlike motorists) have enough space in the center of the road to wait for an automobile coming in the far lane before completing the crossing. For cyclists, there was also no lag effect when the lag is closed by a bicycle or for lateral right or frontal left maneuvers, possibly due to the small number of observations.

*2.7.1.2.2 Effect of Closing Gap Vehicle Type.* A very important inference from these models is the effect of the type of vehicle that closes the gap on the gap acceptance behavior of lateral left-turning cyclists and motorists. As expected, the conclusion is that vehicle/operator units require shorter gaps when the gap is closed by a bicycle and longer gaps when the gap is closed by a bus or heavy truck (i.e. a large motor vehicle), relative to gaps closed by a car or pickup (i.e. a regular-sized motor vehicle).

There are several reasons why vehicle operators might accept smaller gaps when a bicycle closes the gap. First, if the closing bicycle is in the near lane traveling on the right, less time is required for the acting vehicle to clear the point of conflict than if the closing gap vehicle were a motor vehicle traveling in the middle of the near lane. If the bicycle is in the far lane and the lane has enough width (or a bikelane) to accommodate a bicycle and an automobile traveling side by side, as the test sites do, then little (or no) clearance is required for a turning automobile to merge into the far lane. (This case was observed during data collection and discussed previously in Section 2.4.) Next, bicycles pose a much reduced threat relative to automobiles in a collision, so vehicle operators probably feel more comfortable being in close proximity to closing bicycles, which could pose comfort and safety problems for the closing bicycle/rider units. Finally, a closing bicycle normally travels at a speed significantly less than a closing automobile. (Vehicular speeds were not measured, so this reason could not be explored further.)

Reasons for the large vehicle effect include the fact that vehicle operators probably fear a collision with a large heavy vehicle much more than with a vehicle the size and weight of a car or pickup. They might also believe that the braking capabilities of the larger vehicle are not as good.

Of these situations, the one with by far the most observations is for a lateral left-turning motorist facing a gap (or lag) with a closing bicycle. The magnitude of this effect (relative to a closing car or pickup) is quite large for gaps other than the lag (2.95 seconds). For lags, the (relative) effect is even larger at 5.35 seconds (4.03 + 1.32). The magnitudes of the other closing vehicle type effects are all between one and 1.5 seconds.

Each model is specified such that the large vehicle effect must be considered in combination with the variable capturing the effect of a lag closed by a motor vehicle. The effects are additive, so the large closing vehicle lag effect is even greater than the regular-sized closing vehicle lag effect. This additive effect seems reasonable, even though there were not enough situational observations for statistical verification.[9]

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[9] In the bicycle model, only three of the 20 motor vehicles closing a lag in the far lane were large (buses). In the automobile model, only two of the 36 motor vehicles closing a lag were large (buses).

Note that in the automobile model the large closing vehicle effect is only marginally significant (0.10 level). The variable is kept because of the strong substantive reasons presented above and because its sign and magnitude were robust over many alternative specifications.

For bicycle acting vehicles, the closing bicycle effect is only present when the closing bicycle is in the near lane. A further reason for this effect is that after crossing the point of conflict, the acting cyclist has almost all the near lane to use as a safety zone, for use similar to the way a median can be used by a crossing pedestrian. The lack of a far lane closing bicycle effect may be explained by the fact that the point of conflict is further away than for a closing automobile, or by a low number of observations.

Surprisingly, there are only four gap decision observations on which to base the significance test for the closing bicycle effect in the bicycle model.[10] Despite the low number of observations, the variable is included because of the high significance and strong substantive reasons.

The automobile model had enough observations to test a variety of hypotheses concerning different ways the variable "gap closed by a bicycle" might be segmented. The following segmentations were tested, as were the segmentation of each by near and far lane:

(1) Opening gap vehicle type: bike, auto, vehicle, pedestrian (block), or none (lag).

(2) Lane position of closing bicycle: bikelane, right, left, or center.

(3) Is the final lane a bicycle-friendly street (over 15 foot wide curb lane or bikelane) or not (less than 15 foot curb lane).[11]

(4) Lane position of the closing bicycle, either: (1) on the right side of a bicycle-friendly street or (2) positioned left or center or on a non-bicycle friendly street.

The above discussion for lateral left maneuvers probably also holds for lateral straight maneuvers, especially for closing gap vehicles in the near lane. The dynamics might change somewhat for closing gap vehicles in the far lane (especially when a bicycle closes the gap), because the acting vehicle must cross, not merge, with the closing gap vehicle.

The effect of a large closing gap vehicle on lateral right-turning cyclists is suspect because of the low number of observations for this situation (two) and somewhat weaker substantive reasons than for lateral left-turns, due to the fact that right-turning cyclists have effective bikelanes at all

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[10] Three of these are for lateral lefts and one is for a lateral straight. Three are on lags and the other is on a gap that is not a lag.

[11] Note that the only way the final lane was not considered bicycle-friendly was if a bus were parked in the far lane at the commercial/campus site. It is very possible that this arbitrary distinction is not reflective of a street with less than a 15 foot curb lane, especially since the bus would be parked before the point of conflict, allowing the cyclists to move farther to the right if necessary to avoid a collision. Therefore, all sites may actually be bicycle-friendly at all times.

the observed intersections. However, even with their own space, right-turning cyclists may feel that larger vehicles reduce the shy distance between the two to an uncomfortably small size.

The effects of closing bicycles on cyclists performing lateral right (four observations) and frontal left maneuvers (seven observations) were not significant. For frontal left-turns a closing bicycle might possibly increase the critical gap, because the point of conflict is farther away than for a closing automobile. However, the bicycle can use most of the lane as a safety zone before having to cross the path of a closing bicycle, which may negate the longer clearing distance. Bicycles performing a lateral right-turn onto a street with a wide curb lane or a bikelane have a greater chance of being in conflict with a closing bicycle than with a closing automobile. This may cause the impact of a closing bicycle to be insignificant or even positive compared to the impact of a closing automobile. Both of these effects require further investigation with larger, more diverse samples.

The finding, that lateral left-turning motorists and bicyclists accept significantly smaller gaps when the closing gap vehicle is a bicycle, is potentially very important for several reasons. It could mean that bicycles are subject to a safety hazard from being cut-off by automobiles. Whether a safety hazard or not, the magnitude of the effect is large enough that it and the effect of a large closing gap vehicle warrants consideration when estimating intersection capacity and delay. The capacity contribution of an additional bicycle on the major street would be less than that for an additional automobile, whereas the capacity contribution of a bus or heavy truck would be greater.

*2.7.1.2.3 Other Automobile Model Variables.* Three variables that reflect the effects of gap characteristics ($X_{gap}$) on lateral left-turning automobile gap acceptance behavior did not have a similar effect on bicycle acceptance behavior. They are discussed in the following three subsections.

*2.7.1.2.3.1 Closing Gap (including lag) Vehicle is Slowing in the Far lane.* As expected, if the closing gap or lag vehicle is decelerating in the far lane, most likely to turn, the acting motorist can perceive this and will therefore, accept a shorter gap.[12] This situation is shown in Figure 2.17.

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[12] Of the 14 slowing vehicles, six were cars or pickups, one was a bus, and seven were bicycles.

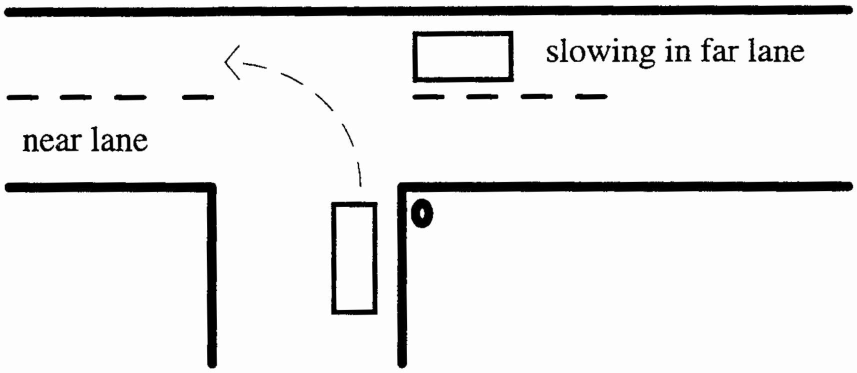

Figure 2.17. The situation where the closing gap (including lag) vehicle is slowing in the far lane is depicted.

Many (42) closing gap vehicles decelerated in the near lane, most likely to turn right. This parameter was also negative, but not significant, possibly because motorists more easily perceive deceleration in the far lane due to a better view and perspective. Also, the penalty for being wrong is less in the far lane. A collision in the far lane would probably be in the side or rear of the turning automobile, with both vehicles traveling in approximately the same direction, whereas, a near lane collision would be at a 90 degree angle into the driver's side. However, in the more general case, where the closing gap vehicles are not slowing, there was not a significant difference between near and far lane impacts.

*2.7.1.2.3.2 Opening Gap Vehicle is a Slowing Automobile.* The impact of an opening gap vehicle that is decelerating, probably to turn, is to increase the critical gap by about two seconds, all other situational factors being equal. This situation is depicted in Figure 2.18. The effect is not significantly different for the near or far lane.[13] An opening gap vehicle slowing to turn left from the far lane or right from the near lane, which are the predominant reasons for slowing in this sample, will block the acting motorist's view of the near lane, after the gap has been opened. Assume the opening gap vehicle travels at about 10 mph (15 ft/s) during its turn, after the gap has been opened.[14] To travel the 15 to 30 feet required to clear the motorist's sight line down the near lane would take one to two seconds, which is approximately the magnitude of this effect.

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[13] There were 32 occurrences in the near lane and five in the far lane. All of these opening gap vehicles were either cars or pickups.

[14] Remember that the gap is considered open when (1) the rear of the left-turning far lane vehicle passes the point of conflict or (2) the wheels of the right-turning near lane opening gap vehicle begin to turn.

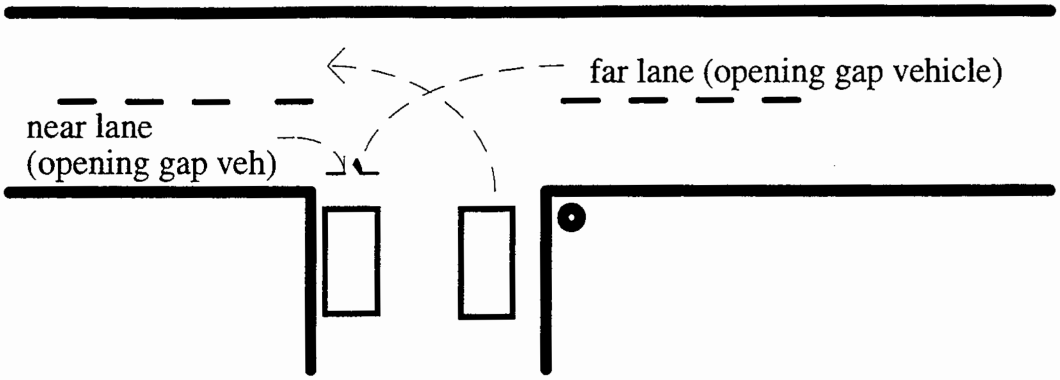

Figure 2.18. The situation where the opening gap vehicle is an automobile slowing to turn (from either the near or far lane) is depicted.

There are four bicycles that open gaps while decelerating, which is not enough to confidently estimate their impact. Since the theory does not hold for slowing bicycles, these four observations were not included in the final variable. Inclusion or exclusion did not greatly impact the magnitude or significance of the parameter estimate.

*2.7.1.2.3.3 Opening Gap Vehicle-Bike in Far Lane; Closing Gap Vehicle-Motor Vehicle in Near Lane.* A gap acceptance situation (pictured in Figure 2.19) in which the gap is opened by a bicycle in the far lane and closed by a motor vehicle in the near lane (far bike-near auto) has a mean critical gap 3.24 seconds less than other situations, all other factors being equal.[15] The main reason for this effect is probably that a bicycle opening a gap in the far lane allows the left-turning motorist to proceed well into the near lane and even into the far lane before the opening bicycle crosses the point of conflict. The car/driver unit might also retain momentum by timing the crawl out into the near (and far) lane so that it does not have to stop.

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[15] Ten of the closing motor vehicles were cars or pickups, two were buses, and one was a motorcycle.

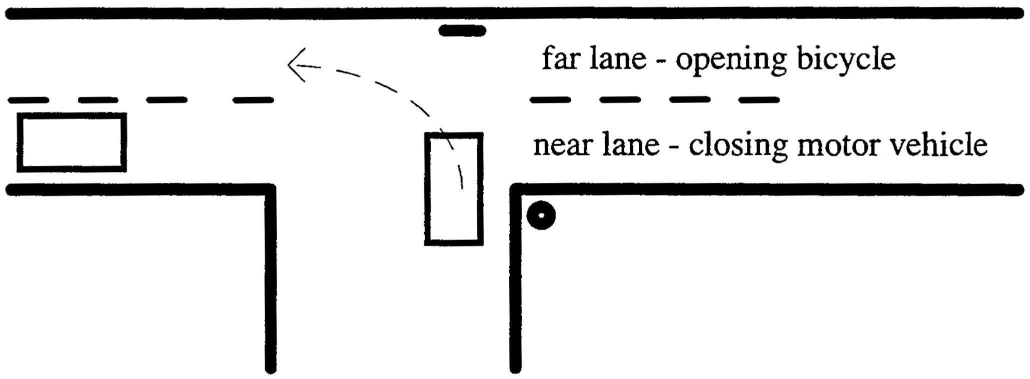

Figure 2.19. The situation in which the gap is opened by a bicycle in the far lane and closed by a motor vehicle in the near lane is depicted.

In addition, by proceeding into the near lane early, the acting vehicle is very visible to the motor vehicle closing the gap. Because of this visibility, the motorist may feel confident in accepting a shorter gap, assuming that the motor vehicle closing the gap has plenty of warning to slow down and even stop to avoid a collision. This reason might provide left-turning motorists a solution to the sight distance problem down the near lane at the commercial/campus site, where most of the data was collected, and the problem that, in general, lateral left-turners have less depth perspective down the near lane (than the far lane).

Because of the significance of this parameter, it was initially troubling to see that the far bicycle-far auto gap situation parameter (14 observations) was not significant. If truly significant, this may be due to the fact that when both the opening and closing gap vehicles are in the far lane, the acting motorist has a better perspective to judge the size of the gap, and may not feel compelled to enter the intersection early.

The far auto-near auto variable (13 gap observations) was also not significant, possibly due to the fact that the merging automobile cannot pull out as far into the intersection as for far bike-near auto gaps. Finally, the far auto-far auto variable was also not significant.

*2.7.1.2.4 Other Bicycle Model Variables.* Two variables that reflect interactive effects ($X_{ia}$) between acting vehicle maneuvers ($X_{man}$), bicycle/rider characteristics ($X_{vo}$), and intersection environments ($X_{int}$) on bicyclist gap acceptance behavior are discussed in the following two subsections.

*2.7.1.2.4.1 Frontal Left-Turn from Right-Side of Lane.* When a cyclist attempts to make a frontal left-turn from the right-side of the lane, the cyclist is in an "incorrect"[16] initial lane position for this maneuver. This situation is shown in Figure 2.20. From the right-side of the lane a cyclist not only has a longer distance to traverse to complete a frontal left maneuver, an extra automobile lane (the lane the cyclist is in) must be crossed, with conflicting automobile traffic coming from behind the cyclist. This is obviously a more difficult task than if the cyclist was in the center or left side of the lane and did not have to contend with traffic from the rear. As expected, the effect of this variable is to increase the gap duration required for acceptance (by 1.03 seconds). This result indicates that the bicycle capacity of the intersection could be increased and delay reduced, if a behavioral change could be achieved in the cycling population.

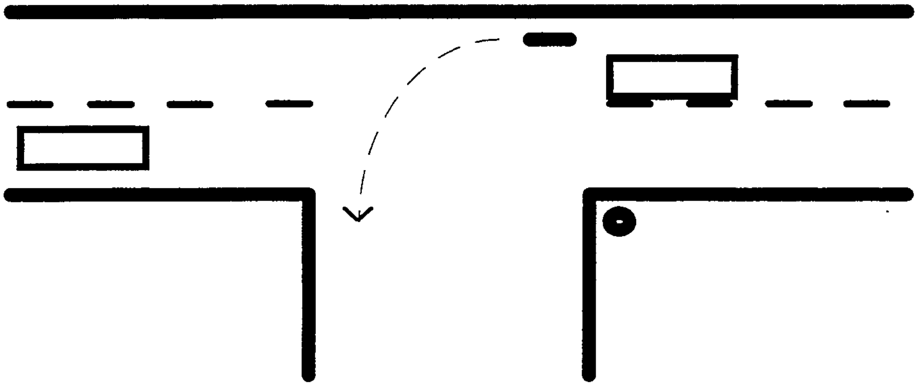

Figure 2.20. The situation where a cyclist attempts to make a frontal left-turn from the right-side of the lane is depicted.

This variable can be interpreted as an attribute of the vehicle/operator unit.[17] It separates the cyclist population into two groups: one that gets into what is usually considered the "correct" position for frontal left-turns and one that does not. Cyclists may not get into the "correct" position for several reasons, such as:

(1) They are risk-averse and feel unsafe merging into the auto lane before the turn.

(2) They have never been told that it is recommended in bicycle safety training courses and by bicycle transportation professionals that cyclists merge to the left before turning left.

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[16] Meaning incorrect according to most cycling transportation professionals and cycling training programs (Forester, 1993).

[17] It could also be interpreted as a different maneuver (than a frontal left from the left-side).

(3) If they have been told, they may not have been trained in how to check behind them to merge safely and effectively.

(4) Even if trained, they may not have practiced the merge maneuver often enough to become comfortable with it.

(5) They may not have found a lane-change gap large enough, however the traffic did not seem heavy enough during data collection for this to happen very often.

(6) They may have made a late decision to turn and did not have time to merge to the left.

There were also a large number of observed lateral left-turn gap decisions (27) made from an "incorrect" initial position for that maneuver (i.e. to the far right). Since there are substantive reasons why this position may also have a significant impact on gap acceptance behavior, it should be investigated further with a richer data set.

*2.7.1.2.4.2 Lateral Right-Turn at the Commercial/Campus Site.* The interpretation of this parameter is that cyclists making a lateral right-turn at the commercial/campus site will accept gaps that are 3.50 seconds less than for any other maneuver at any of the three intersections and lateral right-turns at the other two intersections, all other things being equal. The magnitude of this effect is quite large, leading to an estimated mean critical gap for lateral right-turns at the commercial/campus site of only about one second. The reasons for this effect seem to be threefold: (1) there is a larger turning radius for lateral right-turns at the commercial/campus intersection, (2) the intersection effectively has a very wide bikelane, in the form of a motorcycle parking area, and (3) the actual travel lane is also wide. Comparing Figure 2.7 to Figures 2.5 and 2.6 illustrates these three points.

The results indicate that cyclists making lateral right-turns at stop signs achieve great efficiency and little delay when provided with relatively large turning radii and space on the major road for them to occupy after turning. This may be desirable if conditions do not encourage cyclists to run the stop sign or sacrifice safety.

Even though an effect was not captured in this small sample, it is expected that cyclists at the other two intersections would also accept shorter gaps for lateral right maneuvers relative to other maneuvers, since they both have bikelanes. This warrants further testing with a larger sample of right turns into bikelanes at intersections with varying curve radii.

*2.7.1.2.5 Insignificant Variables.* Because of the small data sets, especially for bicycles, one cannot infer too much about whether the variables found insignificant actually do not impact gap acceptance behavior in mixed-traffic. In addition, the bicycle model explained in essence all of the sample variation, however it is expected that many other factors impact bicyclist gap acceptance behavior. For instance, it was (and still is) expected that helmet use, gender, and the type of bicycle might explain some of bicycle acceptance behavior. They should be investigated again in future work.

In larger and more diverse samples, different combinations of opening and closing gap vehicle lane positions may prove to have impacts on gap acceptance behavior that were not significant in this sample. Table 2.9 shows all possible combinations of these lane positions and the number of observations of each for this sample. All combinations were tested, along with any reasonable interactions with other combinations and numerous interactions with other variables, such as the type of gap vehicle.

TABLE 2.9. ALL COMBINATIONS OF OPENING AND CLOSING GAP VEHICLE LANE POSITIONS FOR THE LATERAL LEFT-TURN GAP DECISION SITUATIONS PRESENT IN THE DATA SET.

| **Lane** | | **Number of Observations** | |
|---|---|---|---|
| **Opening Gap Vehicle** | **Closing Gap Vehicle** | **Auto** | **Bike** |
| Near | Far | 37 | 2 |
| Near | Near | 56 | 10 |
| Far | Far | 40 | 10 |
| Far | Near | 37 | 5 |
| None (lag) | Far | 56 | 23 |
| None (lag) | Near | 56 | 22 |
| None (pedestrian block) | Far | 16 | 0 |
| None (pedestrian block) | Near | 15 | 0 |
| Near | None (pedestrian block) | 3 | 0 |
| Far | None (pedestrian block) | 2 | 0 |
| **Total** | | 318 | 72 |

Of note is the fact that variables (e.g., number of rejected gaps), reflecting the hypothesis that impatience, caused by waiting at the stop sign, tends to make motorists accept smaller gaps, were insignificant. An "impatience" factor was mentioned by Daganzo (1981) and was found significant by Mahmassani and Sheffi (1981b). One reason for the insignificance might be the lack of sufficiently long gap sequences.

Another variable found insignificant reflected whether the acting vehicle was a passenger car or a pickup truck. This variable was introduced, because a prior study in Oregon found some evidence that pickup trucks were involved in more auto-bike accidents than passenger cars (Oregon DOT, 1990). No significant difference was found in the gap acceptance behavior of the two types of vehicles. A difference might not have been discovered because small trucks were included in the pickup category.

### 2.7.2 Capturing Vehicle Operator Aggressiveness

One vehicle/operator unit characteristic (i.e. an $X_{VO}$ variable) that probably impacts its gap acceptance behavior is operator aggressiveness. This characteristic is hard to capture; however, the previously discussed endogenous variable indicating whether or not the acting vehicle ever came to a complete stop (stop or roll) during the gap sequence would seem to reflect it to some degree. The bicycle model has no more variation left to explain, so this variable is of use only in the automobile model.

A standard method of handling endogenous variables, the instrument variable approach (Mannering and Hensher, 1987), was not used for this preliminary analysis. The endogenous variable was introduced directly in the specification, presented in Table 2.10, which was estimated assuming independent decisions. The specification is essentially identical to the final automobile model presented in Table 2.7, with the addition of the endogenous interaction variable (highlighted in bold text in the table). This variable is a combination of the Boolean variables indicating that "the acting motorist did not stop and the closing lag vehicle is a bicycle" and that the "gap (not including lag) is closed by a bicycle".

TABLE 2.10. AUTOMOBILE LATERAL LEFT-TURN INDEPENDENT DECISIONS MODEL, WITH ENDOGENOUS AGGRESSIVENESS VARIABLE INCLUDED.

| Variable | Number of true observations | Parameter Estimate (seconds) | t-statistic [1] |
|---|---|---|---|
| Mean critical gap when all variables are false (or 0) ($\beta_0$) | all | 7.00 | 7.76 |
| Closing gap (including lag) vehicle is a bus or heavy truck ($X_{gap}$) | 14 | 1.14 | 8.11 |
| Closing lag vehicle is a motor vehicle ($X_{gap}$) | 36 | 4.05 | 4.15 |
| **Either (a) closing gap (not including lag) vehicle is a bicycle or (b) the acting motorist did not stop and the closing lag vehicle is a bicycle ($X_{ia}$)** | **127** | **-3.06** | **-4.82** |
| Opening gap vehicle-bike in far lane; Closing gap vehicle-motor vehicle in near lane ($X_{gap}$) | 13 | -2.94 | -4.07 |
| Opening gap vehicle is a slowing automobile ($X_{gap}$) | 37 | 1.47 | 7.65 |
| Closing gap (including lag) vehicle is slowing in the far lane ($X_{gap}$) | 14 | -1.50 | -8.53 |
| Standard deviation of the critical gap distribution ($\sigma$) | - | 2.06 | 7.75 |

Number of gap sequences = 140
Number of gap decisions = 318

Log-likelihood (*LL*) (initial) = -220.42
Log-likelihood (*LL*) (final) = -62.41

[1] The t-statistics may be overestimated.

The parameter estimates of the exogenous variables in Table 2.10 are of similar magnitude to those in the final (exogenous) model (Table 2.7). The log-likelihood value increased from –65.69 to –62.41, which is to be expected with the inclusion of an endogenous variable. Nonetheless, the estimated coefficient of the endogenous variable exhibits reasonable magnitude and direction (for an "aggressiveness" variable). Notwithstanding its bias, more aggressive drivers would be expected to have a greater tendency to roll and less aggressive ones would be more likely to stop. This would explain why the rolling drivers would accept smaller gaps (for situations where the closing lag vehicle was a bicycle), as indicated by the endogenous variable's parameter estimate (-3.06) compared to the previous final (exogenous) model (-1.32). In essence, this indicates that the lag effect is not present for aggressive drivers, for situations where the closing lag vehicle is a bicycle. However, it also may just reflect the fact that many motorists rolled through the stop sign because there was a closing bicycle instead of a closing automobile.

The lag closed by bicycle situations when the acting motorist stops are now captured in the constant term $\beta_0$.

There was no significant difference in the mean critical gap of rolling or stopping motorists for lags closed by motor vehicles.[18] The effect of stopping behavior on any gap acceptance situations other than lags was not investigated.

Though the model estimation results seem reasonable, the inference is only to suggest an aggressiveness effect, because the use of the endogenous stopping variable in this model specification is conceptually incorrect. However, the estimation results and the fact that a far greater percentage of motorists rolled through the stop sign when the lag was closed by a bicycle (48 of 76) than rolled when the lag was closed by a motor vehicle (6 of 36), suggest the importance of further research. This research should concentrate on more theoretically correct ways to capture an aggressiveness effect. For example, the binary stopping decision could be modeled. Using this model and the sample data, the (predicted) probability of stopping could be computed and used as an instrument variable in estimating the gap acceptance model (Mannering and Hensher, 1987). One might also measure the deceleration rates of the acting vehicles as they approach the stop sign and use this exogenous variable as a proxy for "aggressiveness". Another approach would be to estimate models conditioned on whether the acting vehicle stops or not. Finally, models of both the stopping decision and the gap acceptance decision could be estimated simultaneously, with the covariance matrix specified to explicitly recognize (probable) dependence between the decisions.

### 2.7.3 Closure

Several of the mean and variance only models presented in the last section (2.6) were expanded in this section by relaxing assumptions. The across intersection and across maneuver components of variance were (assumed) not significant because of the low number of different observations for each. The across vehicle/operator component was found not significant by estimation, so the final models were estimated assuming independent.decisions and only a total variance term was estimated.

Some of the unexplained variation in the mean and variance only models was explained by introducing systematic variables. The model for lateral left-turning automobiles seem to include several significant variables that may not have been investigated previously, including two variables that deal with whether the gap vehicles are decelerating and the variables reflecting the

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[18] However, only six acting motorists rolled when the "lag was closed by a motor vehicle", which may not be enough to get an accurate assessment.

type (and size) of the closing gap vehicle. Results indicate that motorists accept smaller gaps when bicycles close the gaps and require larger gaps when closed by large vehicles, relative to gaps closed by cars or pickups. The type of closing gap vehicle was found to have a similar effect on bicyclist gap acceptance behavior.

Some model results, though theoretically suspect because of the inclusion of an endogenous variable, indicate possible and potentially important behavior differences caused by the aggressiveness of motorists in accepting lags closed by bicycles. This warrants further investigation.

The data collection procedure may have introduced several possible model biases. First, models are subject to possible self-selectivity bias, especially for bicycle/rider units. Bicyclists, probably more so than motorists, seem to avoid intersections and gap acceptance situations with which they are uncomfortable. Second, the sample of cyclists, and possibly motorists, is biased towards university students. Finally, the data was all collected in the summer months in the South in the morning and early afternoon hours, which could also bias the vehicle/operator unit sample.

There was data associated with four different bicycle maneuvers in this sample. Readers should note that bicyclist gap acceptance behavior for lateral left-turn maneuvers was impacted by three of the model variables, lateral right and lateral straight maneuvers were each impacted by two of the variables, and one variable impacted frontal left-turns. This is a great amount of inference for a small amount of data. Even though the priors and substantive reasons were strong for all these inferences, they may not hold for larger samples.

## 2.8 CONCLUDING COMMENTS

This work is the most extensive study of gap acceptance behavior in bicycle-automobile mixed-traffic reported to date, with consideration of a greater variety of possible explanatory factors than previous studies. The study continues and broadens a stream of work that conceptualizes the critical gap at an individual vehicle/operator unit level distributed over the population of gap acceptance situations of interest. Another characteristic of this stream of work is the use of discrete choice models to estimate the critical gap distribution. Other works in this stream include those of Miller (1971), Mahmassani and Sheffi (1981b), Daganzo (1981), and Palamarthy et al. (1994). Both this study and that of Palamarthy et al. (1994) use a multinomial probit model structure to attempt to capture the serial correlation often present in gap acceptance data. Nonetheless, this study is very limited in its scope and applicability, primarily because of the small sample and focus on a single location.

The collection and reduction of detailed gap acceptance data is very time and labor intensive. It is especially so for bicycle-automobile mixed-traffic, because bicycle volumes are very low in many countries. For this reason, care should be taken to build upon the conclusions of this study, using engineering judgement as to which inferences may be applicable in different intersection environments for different cycling populations. The results of this study provide a useful basis to target the data collection efforts for subsequent studies.

This study illustrates the benefits of conceptualizing traffic to be of a mixed nature, including bicycles and motor vehicles with different operating characteristics. The concept was implemented by explicitly recognizing the operational differences between bicycles and different types of motor vehicles. Data was collected so that bicycle/rider units could be categorized based on gender, helmet use, and type of bicycle and motor vehicle/driver units could be categorized into passenger cars, pickup trucks, buses, and heavy trucks. Results showed that the closing gap vehicle type had significant impacts on the gap acceptance behavior of both bicyclists and motorists, which were different for a closing bicycle, a regular-sized closing motor vehicle (car or pickup), and a large closing motor vehicle (bus or heavy truck).

The gap acceptance behavior models make some important, but preliminary, contributions to bicycle-automobile mixed-traffic science and engineering, which fit into the organizing framework presented in Figure 1.1 within Topic II. Traffic Flow. The results that indicate both motorist and bicyclist gap acceptance behavior are impacted by the size of the vehicle closing the gap are basic insights into traffic behavior, and can therefore be considered a part of a traffic science. Compared to a passenger car or pickup closing the gap, both cyclists and motorists will accept significantly smaller gaps if the closing gap vehicle is a bicycle, and both require larger gaps if the closing vehicle is large (e.g., a bus or heavy truck). The fact that the initial lane position of frontal left-turning cyclists impacts their gap acceptance behavior, and the impacts, on the behavior of lateral left-turning motorists, of opening and closing gap vehicles that are decelerating to turn are also basic insights that fit into the traffic science area.

The relative homogeneity of bicyclist gap acceptance behavior (compared to motorists) is a surprising (preliminary) traffic science insight. The indication that cyclists' mean critical gaps are considerably less than motorists' for some lateral left-turning situations (and seem to be similar for other situations) is another basic insight that may surprise some professionals.

Mean critical gap estimates for a variety of gap acceptance situations and vehicle/operator populations are a contribution to traffic engineering. First, the estimates can be used to compute stop-controlled intersection capacity and delay for both bicycles and automobiles when bicycles are a significant portion of the traffic mix. This allows engineers to make decisions on intersection design, such as the need for a traffic signal. The estimates are also useful in intersection or network simulations. Finally, they may be useful in attempting to resolve the problem of an intersection that has a high bicycle accident rate (e.g., intersections where traffic conditions produce gaps that are too short for cyclist safety).

Future research should focus on collecting and analyzing mixed-traffic data over a greater variety of gap acceptance situations and vehicle/operator units. Specific consideration should be given to studying intersections where the major roadway has standard lane widths, since all the data collected herein was associated with facilities that had bikelanes or very wide curb lanes. In addition, a variety of curb lane and bikelane widths should be studied. Specific consideration should also be given to studying higher (medium) speed intersections and intersections not close to college campuses. Richer data on more general cycling populations are also required.

# 3 BICYCLE/RIDER UNIT BEHAVIOR AT THE ONSET OF YELLOW

This chapter investigates the behavior of bicycle/rider units at the onset of the yellow traffic signal indication when operating in mixed bicycle-automobile traffic, with attention to the computation of the time, between the signal change warning and the release of conflicting traffic, required for safe operation. This work focuses on the situation in which a vehicle is proceeding straight through the intersection, with conflicting traffic on the cross-street. No formal attention is given to other situations, such as those in which the conflicting traffic is turning left from the opposite approach or the vehicle approaching the yellow is turning.

Engineers must choose a *design clearance point* based on their knowledge and assumptions about the point in the intersection that cyclists must reach (or clear) before the onset of the cross-street green for comfort and safety at a particular intersection. The *clearance interval* is the time between being warned of an impending signal change and the onset of the cross-street green needed for a vehicle/operator unit to clear (or reach) the design clearance point (or any other *specified clearance point*). The design clearance point is one characteristic of the *design vehicle/operator unit* that is often chosen for use in determining a clearance interval that accommodates a certain percentage of vehicle/operator units at the intersection.

In current practice, it is common to refer to two portions of the clearance interval, the *yellow vehicle change interval* and the *all-red clearance interval.* It is not common to use the collective term anymore (Wortman and Fox, 1987). According to the Manual on Uniform Traffic Control Devices, the exclusive function of the yellow vehicle change interval is to warn traffic of an impending change in the right-of-way assignment. The optional all-red clearance interval may be used to permit traffic to clear the intersection before conflicting traffic is released (FHWA, 1988). The yellow vehicle change interval is normally designed to meet the requirements of U.S. traffic law, which states that a vehicle may enter the intersection (past the stopbar) under a yellow and that the cross-street traffic must yield to such a vehicle before entering the intersection (NCUTLO, 1992; TDPS, 1995).

Because this topic is examined here for the first time from the perspective of bicycles, it is more appropriate to use the more general terminology (i.e. clearance interval) as it includes both the yellow and all-red intervals in one term, thereby shortening numerous explanations. Furthermore, the more general terminology accommodates alternative signal change warnings, other than the current U.S. warning, the yellow signal indication. The latter is important, because it will be shown that it may be beneficial to provide separate signal change warnings for bicycles and automobiles at some intersections. Use of the term "clearance interval" also fits in with U.S. traffic law and current automobile-only methods for determining yellow and all-red intervals.

Instead of *clearing*, often it is more appropriate to speak in terms of reaching a certain point, the only difference being that *clearing* means the rear of the vehicle is past the point, while *reaching* means the front of the vehicle is at the point. A clearance interval can easily be adjusted to either case.

Over the years, traffic engineers and researchers have designed automobile clearance intervals to accommodate motorist behavior, for which the required (design) clearance point is the stopbar (Butler, 1983). Because of many characteristic differences between bicycle/rider units and car/driver units, especially size, speed, deceleration, and acceleration, it seems likely that cyclists would like to proceed farther into the intersection and may require longer clearance intervals than motorists.

Because a bicycle/rider unit is small and normally travels on the far right side of the road, it is less likely to be seen by a motorist entering an intersection from the cross-street than a car, which makes it more likely that the motorist will fail to yield. Some accidents probably occur because the bicyclist could have and should have stopped before the intersection. Others may occur simply because a bicycle/rider unit is caught in a position where it cannot come to a comfortable stop before the intersection, but the clearance interval is not long enough for it to clear its safety point before the cross-street traffic receives a green. These bicycle/rider units are those caught in a *dilemma zone* (Gazis et al., 1960). If caught in its dilemma zone, a bicycle/rider unit cannot make a correct decision.

Inadequate clearance interval duration is considered by Forester (1983) to be the "largest identified facility-associated cause of car-bike collisions". In addition, an Oregon study reports that "bicyclists disregarding signals" account for eight percent of all urban bicycle-motor vehicle accidents in that state. Oregon DOT's (1990) current policy to help alleviate this problem is to add loop detectors in bike lanes to extend the green phase if a bicyclist is in position to be caught by a clearance interval of inadequate length. Such "green extension" systems have been in place for the benefit of cars for over twenty years at high-speed isolated intersections (Parsonson, 1978).

By analogy to motorist behavior models (Sheffi and Mahmassani, 1981), the behavior of a bicycle/rider unit at the onset of a yellow traffic signal indication can be viewed as a binary choice to either stop or proceed. One can seek to represent further dimensions of this behavior. For example, if the choice is to proceed, the vehicle operator may also decide whether to accelerate. An understanding of these decision processes is essential when developing yellow change intervals and all-red clearance intervals (or alternative warnings of the signal change) to facilitate safe and efficient operation of mixed-traffic at signalized intersections.

Two approaches will be used to study this behavior. The first adapts the classic treatment by Gazis et al. (1960) that relies on a *deterministic* model to define the dilemma zone. The second is a *probabilistic* approach, which uses behavioral discrete choice models of vehicle/operator unit behavior at the onset of yellow to identify the factors influencing the unit's choice between stopping and proceeding.

Determining the appropriate clearance interval for automobiles has been the object of much study. Most studies have relied on the deterministic approach (see Wortman and Fox [1987] for a review of much of this work), while some have focused on the behavioral aspects of car drivers' decisions at the onset of yellow (Sheffi and Mahmassani, 1981; Mahalel and Zaidel, 1986). These analyses have shown that a clearance interval of three to five seconds is usually sufficient for autos, but they did not consider that bicycles might also use the road.

Prior to the present work, very limited information was available on determining clearance intervals for bicycles (Forester, 1983; AASHTO, 1991), and no behavioral studies had been performed. This chapter addresses an important gap in terms of understanding bicyclist behavior at the onset of yellow and methods for determining adequate bicycle clearance intervals. As such, it contributes to preventing the potentially dangerous situations associated with insufficient bicycle clearance intervals.

In the next two sections, respectively, the deterministic and the probabilistic approaches are discussed in more detail, and previous motor vehicle and bicycle research is reviewed. Behavioral differences and similarities for bicycle/rider and auto/driver units are hypothesized and discussed. These hypotheses are then tested using data collected from experiments and field observations. In the third section, an expression is derived for the probability of a vehicle being caught short of a specified clearance point when the cross-street traffic is released. In the next section, the results are synthesized with previous findings into suggestions for the design of signal change warning systems for bicycle-automobile mixed-traffic at intersections. Finally, significant results and contributions are summarized.

## 3.1 ADEQUATE CLEARANCE INTERVAL COMPUTATION USING A DETERMINISTIC APPROACH

The problem is to determine the duration of the clearance interval that will eliminate the dilemma zone for a particular design bicycle/rider unit intended to allow safe operation for a high percentage of actual bicycle/rider units. The characteristics of the design bicycle/rider unit are obtained by taking a certain percentile value from the respective distributions of various attributes of the bicycle/rider unit population, which include speed, deceleration, acceleration, and perception-reaction time.

Elementary kinematic laws are used to define a section of the intersection approach-street within which the design bicycle/rider unit cannot comfortably stop before the stopbar. If the design bicycle/rider unit is in this "cannot stop" area when the light turns yellow, the remaining distance is insufficient for it to comfortably stop. Another pertinent section is that where the design bicycle/rider unit "cannot go". If located in this section when the light turns yellow, the design bicycle/rider unit cannot clear the design clearance point before the cross-street green. The clearance interval determines the location of the "cannot go" section (relative to the design clearance point) but does not impact the length of the "cannot stop" section (which is relative to the stopbar).

The dilemma zone for a design clearance point chosen to be past the intersection is shown in Figure 3.1. If the clearance interval is too short, a dilemma zone exists such that no matter what the design bicycle/rider unit decides to do (stop or proceed) it will be caught short of the design clearance point. If the clearance interval is *adequate*, then the design bicyclist has the opportunity to make a correct decision, and there is, in theory, no physical dilemma zone. If the clearance interval is longer than adequate, an optional zone is introduced, in which either decision is acceptable.

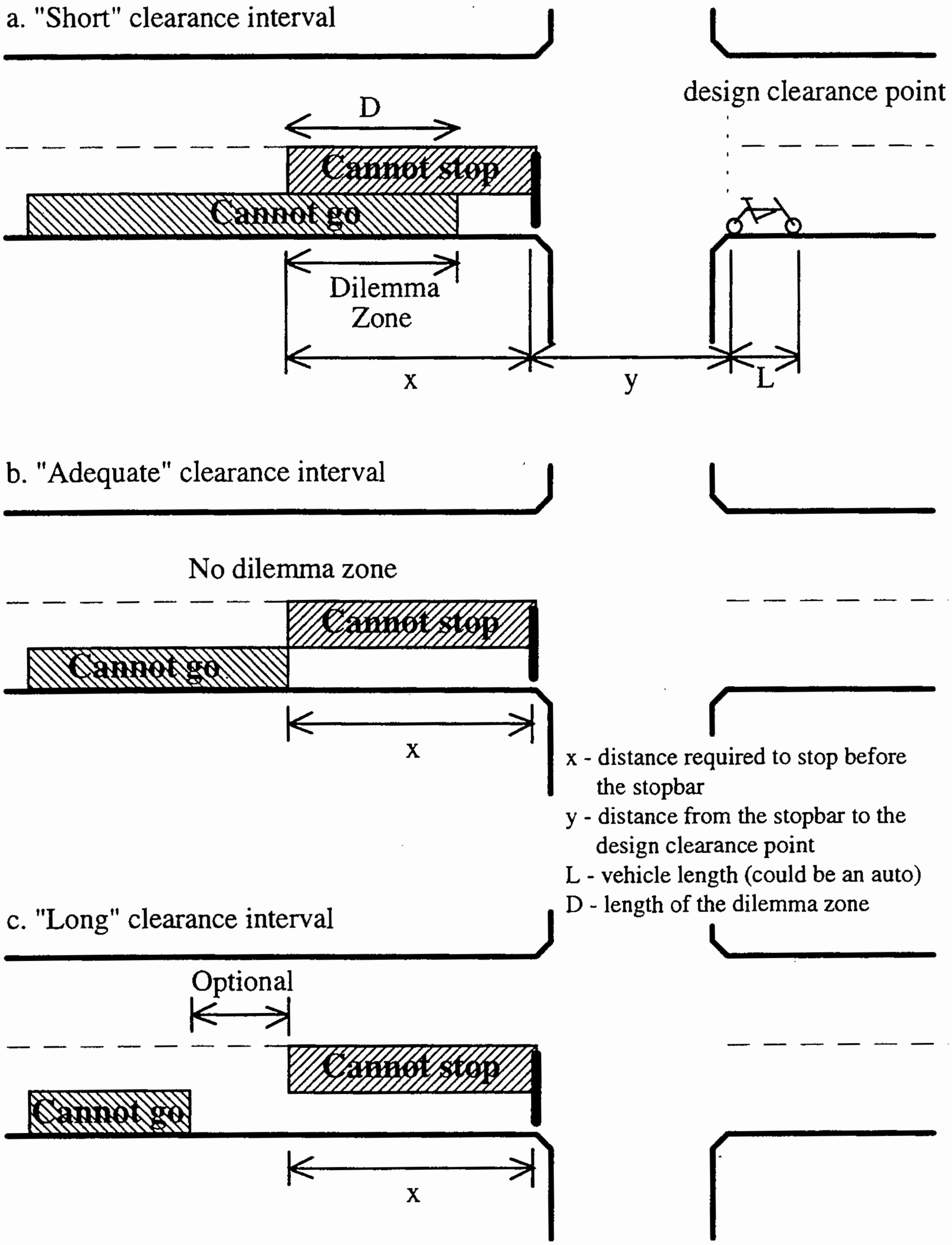

Figure 3.1. Implication of the clearance interval on the dilemma zone using an example in which the design clearance point is past the intersection

Bicyclist speed, acceleration, and deceleration measurements were taken and analyzed to determine the characteristics of the design bicycle/rider unit. The design values are used with a published theory to develop a methodology to compute adequate clearance intervals for bicycle-automobile mixed-traffic.

The remaining subsections are structured as follows. First, the deterministic theory for computing adequate clearance intervals is presented. Secondly, the conclusions from the data collection and analysis process are discussed, followed by an analysis of the applicability of the deterministic method for bicycle-automobile mixed-traffic and a recommended method for this computation. Finally, some concluding comments are made.

### 3.1.1 Deterministic Theory for Computing Adequate Clearance Intervals

A method for insuring comfort and safety is to provide a clearance interval such that vehicles that cannot comfortably stop before the intersection have enough time to (reach or) proceed past the design clearance point before the cross-street green. If the stopbar is chosen as the design clearance point (to reach), the results accommodate motorist behavior at many intersections, as well as the provisions of the Uniform Vehicle Code (NCUTLO, 1992), and therefore, the law in most U.S. states.

This situation was analyzed by Gazis et al. (1960) and presented in the Transportation and Traffic Engineering Handbook (ITE, 1982) as follows. With reference to Figure 3.1, let x denote the distance required for a vehicle to reach a complete stop upon the onset of the yellow indication, assuming a constant comfortable deceleration:

$$x = v \cdot t_{p\text{-}r} + v^2 / (2 \cdot d), \qquad (3.1)$$

where $x$ = distance required for stopping,

$t_{p\text{-}r}$ = perception-reaction time,

$v$ = approach speed, and

$d$ = comfortable deceleration.

If a vehicle at distance x from the intersection (when the clearance interval begins) can proceed just past the design clearance point before the clearance interval expires, then there is no dilemma zone (see Figure 3.1). A vehicle can do this without accelerating if the clearance interval (ci) is adequate.

$$ci_{adeq} = (x + y + L) / v = t_{p\text{-}r} + v / (2 \cdot d) + (y + L) / v,^{1} \qquad (3.2)$$

where $ci_{adeq}$ = adequate clearance interval without acceleration,

$y$ = distance from the stopbar to the design clearance point, and

---

[1] This equation can be formulated to explicitly include an adjustment for the start-up time required for cross-street vehicles or the start-up time can be implicitly incorporated into the design clearance point as assumed here.

L = vehicle length.

While the clearance interval computation for cars normally assumes that cars proceed at their prevailing speed, it may be appropriate to compute the adequate clearance interval for bicycles under the assumption that they will accelerate. The justification is that while cars are likely to be driving at about the speed limit, and hence would exceed the speed limit by accelerating, this is unlikely to be a problem for bicycles. The adequate clearance interval $ci_{adeq-a}$ assuming that the bicyclist will accelerate at a comfortable rate a is obtained by solving the following quadratic equation:

$$v \cdot ci_{adeq-a} + a \cdot (ci_{adeq-a} - t_{p-r})^2 / 2 = x + y + L \text{ or} \tag{3.3}$$

$$v \cdot ci_{adeq-a} + a \cdot (ci_{adeq-a} - t_{p-r})^2 / 2 = v \cdot t_{p-r} + v^2 / (2 \cdot d) + y + L;$$

yielding:

$$ci_{adeq-a} = (a \cdot t_{p-r} - v + (v^2 + 2 \cdot a \cdot (v^2 / (2 \cdot d) + y + L))^{1/2}) / a, \tag{3.4}$$

where $ci_{adeq-a}$ = adequate clearance interval with acceleration, and

a = comfortable acceleration.

Assuming no acceleration, the length of the dilemma zone D is given by:

$$D = v \cdot t_{p-r} + v^2 / (2 \cdot d) - v \cdot ci + y + L \tag{3.5}$$

where ci = clearance interval.

The formulation assuming acceleration would include an additional term, $-a \cdot (ci - t_{p-r})^2/2$.

### 3.1.2 Data Requirements

In order to analyze clearance intervals for bicycle/rider units, some required data are (1) normal cruising speeds (not fastest speeds), (2) comfortable accelerations for proceeding (not maximum accelerations in emergencies), and (3) comfortable decelerations (not maximum for emergency stops). The procedures to obtain these values are presented in the following two subsections.

**3.1.2.1 Measurement Procedures.** Unlike speed, it is not easy to measure acceleration and deceleration passively (without the bicyclist knowing it and hence possibly adjusting his or her behavior by virtue of being captured on record). Since bicycle speed measurements are available elsewhere, it was deemed more important to measure deceleration and acceleration, though speed was also measured. Attempts were made to make all measurements under similar conditions, with fairly negligible impact from wind, grades, traffic interference, and pavement conditions (e.g. pavement was dry and smooth). It should be noted that these measurements are likely to be location and situation dependent, in addition to being subject to participation (self-selection), experimental, and small-sample biases.

In the first measurement session, 12 subjects were observed (eight recruited voluntarily from city streets and four friends of the author). Each subject was instructed to make two rides on level pavement (see Figure 3.2). In Ride 1, they were asked to (1) accelerate up to their normal cruise speed (for travel on city streets in Austin, Texas) over a distance of about 210 ft and (2) remain at that speed over a distance of 150 ft, at the end of which (3) they were to accelerate (at a rate they would use to comfortably and safely make it through a yellow) over a distance of 165 ft. The time to traverse the cruise segment and the times to traverse both the first 60 ft and the first 150 ft of the acceleration section were measured.

A cruising distance of 150 ft was deemed sufficient, because if human error causes the cruise time measurement to be made over a distance of 150 plus or minus 7.5 ft only a five percent error in speed occurs. Acceleration over 150 ft was deemed sufficient, since it is close to the maximum distance a bicyclist would have to accelerate to clear a wide intersection.

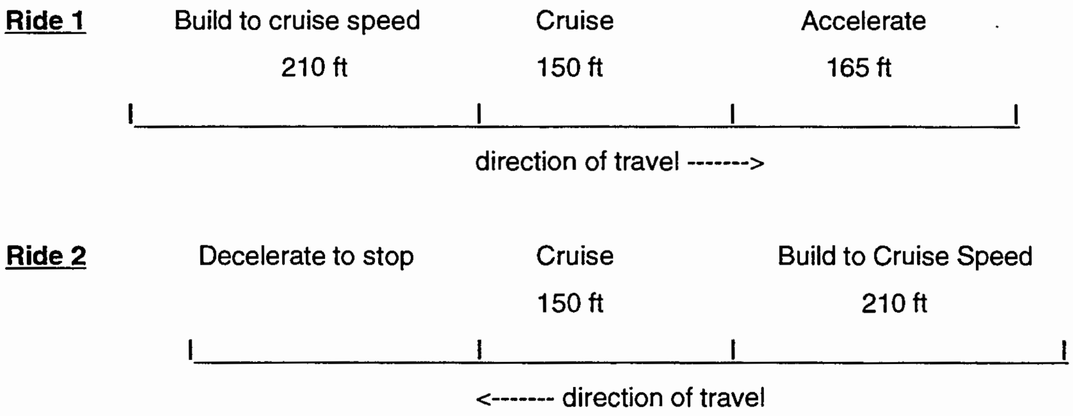

Figure 3.2. Diagram of the rides used to measure speed, acceleration, and deceleration

In Ride 2, they were again instructed to (1) accelerate up to their normal cruise speed, (2) remain at that cruise speed over a 150 ft segment, but then (3) comfortably decelerate to a stop by braking (never coasting), as if they had seen a yellow indication and wanted to be sure they stopped before the intersection. The average deceleration was then computed using the stopping distance and the stopping time (both measured from the start of braking).

To obtain a more reliable measure of deceleration additional experiments were performed. Fifteen new subjects were recruited (voluntarily from city streets) to perform Ride 2 only.

Though 23 of the subjects were tested on a rather narrow, somewhat busy street with cars parked along each side, they were held until the "coast was clear" before proceeding. Any remaining interference is considered negligible. The other four subjects were tested in head- and tail-winds of about 10 to 20 mph (estimated by the author). Their speeds exhibited significant differences for each direction of travel, however the two speed measurements were averaged to smooth out any wind effect. These four subjects' accelerations and decelerations were most likely affected, but they fell within the ranges of the other data and were not adjusted in any way to account for wind effects.

**3.1.2.2 Data Analysis.** Of the 27 subjects, five were female and 22 were male. Ages ranged from 19 to 55, with the average being 29. As expected, comfortable deceleration, comfortable acceleration, and normal cruise speed seem to be approximately normally distributed (see Figures 3.3, 3.4, and 3.5, respectively). Statistics on each quantity are given in Table 3.1.

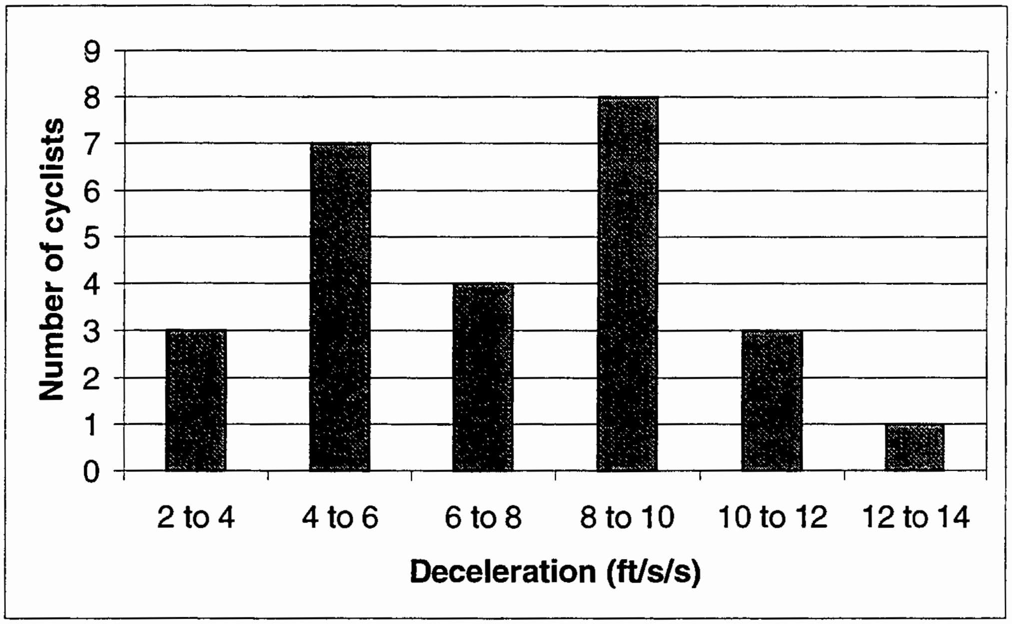

Figure 3.3. Frequency distribution of comfortable deceleration measurements.

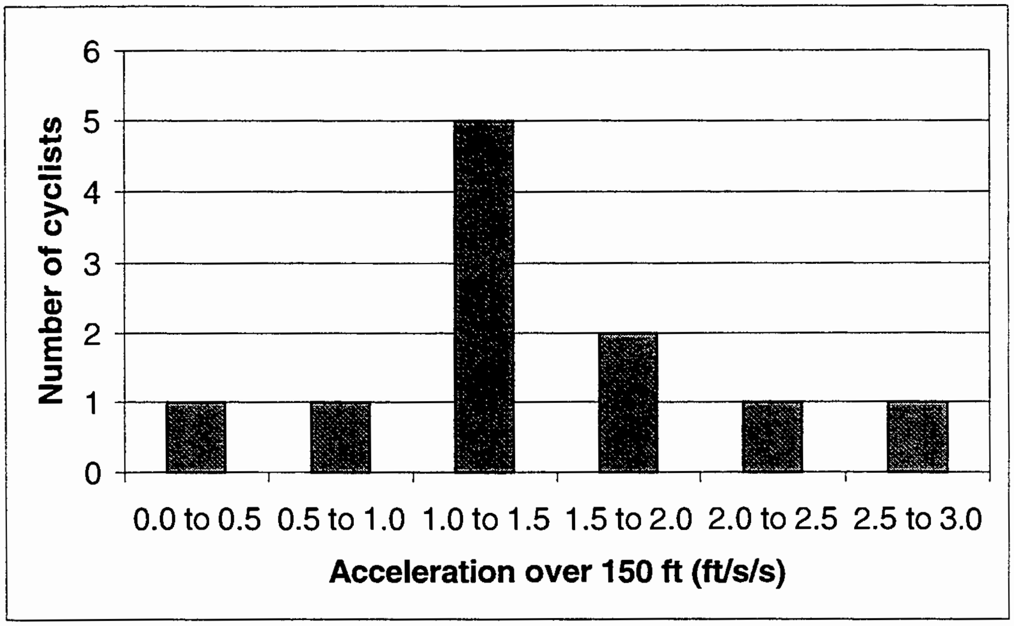

Figure 3.4. Frequency distribution of comfortable acceleration measurements.

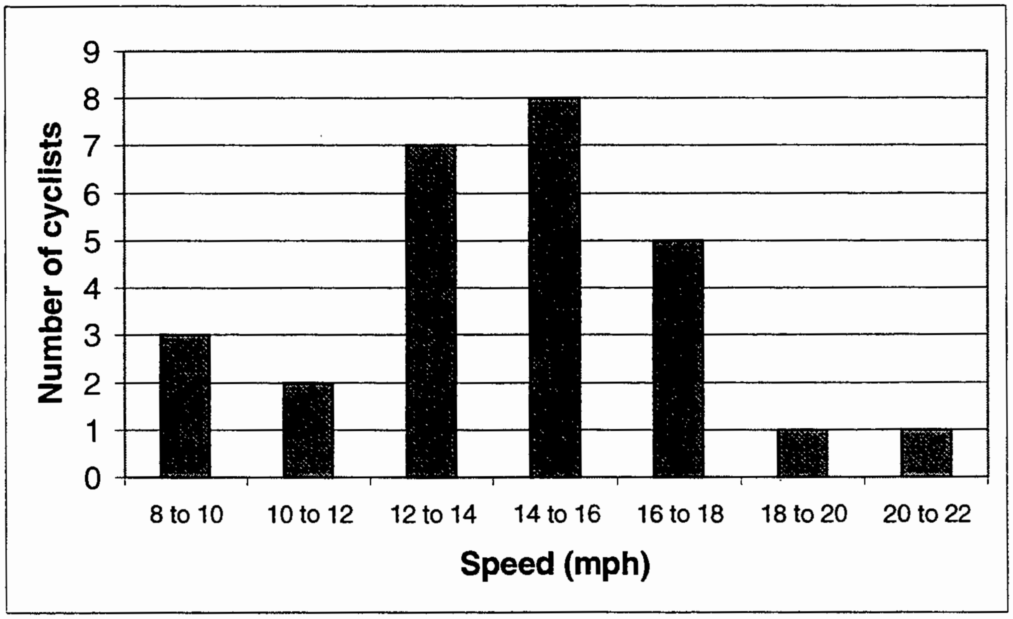

Figure 3.5. Frequency distribution of normal cruise speed measurements.

TABLE 3.1. STATISTICS FOR BICYCLE/RIDER UNIT MEASUREMENTS.

| Statistic | Normal Speed | Comfortable Deceleration | Comfortable Acceleration Over 150 ft | Comfortable Acceleration Over 60 ft |
|---|---|---|---|---|
| Low | 8.2 mph | 3.8 $ft/s^2$ | 0.1 $ft/s^2$ | 1.8 $ft/s^2$ |
| 15th percentile | 10.5 mph | 4.2 $ft/s^2$ | 0.9 $ft/s^2$ | -- |
| Mean | 14.1 mph | 7.5 $ft/s^2$ | 1.5 $ft/s^2$ | 3.8 $ft/s^2$ |
| Median | 14.3 mph | 7.7 $ft/s^2$ | 1.4 $ft/s^2$ | 3.6 $ft/s^2$ |
| 85th percentile | 16.6 mph | 9.7 $ft/s^2$ | 1.9 $ft/s^2$ | -- |
| High | 20.9 mph | 12.3 $ft/s^2$ | 3.0 $ft/s^2$ | 6.4 $ft/s^2$ |
| # of data points | 27 | 26 | 11 | 6 |

Note: Some statistics are missing (--) due to insufficient number of measurements for computation.

As has also been reported for automobiles (ITE, 1994), there is some evidence that as bicycle speed increases the comfortable deceleration also increases (see Figure 3.6). The correlation coefficient using all the data is 0.52, which is significant ($p<0.05$). However, if the four highest decelerations, which correspond to four of the five highest speeds (all greater that 16.8 mph) are removed, the correlation coefficient drops to 0.20, which is not significant. The strength of association between these two variables appears to be highly dependent on these higher speed observations. With this small sample, especially with limited high speed data, the existence or nature of this relationship cannot be determined.

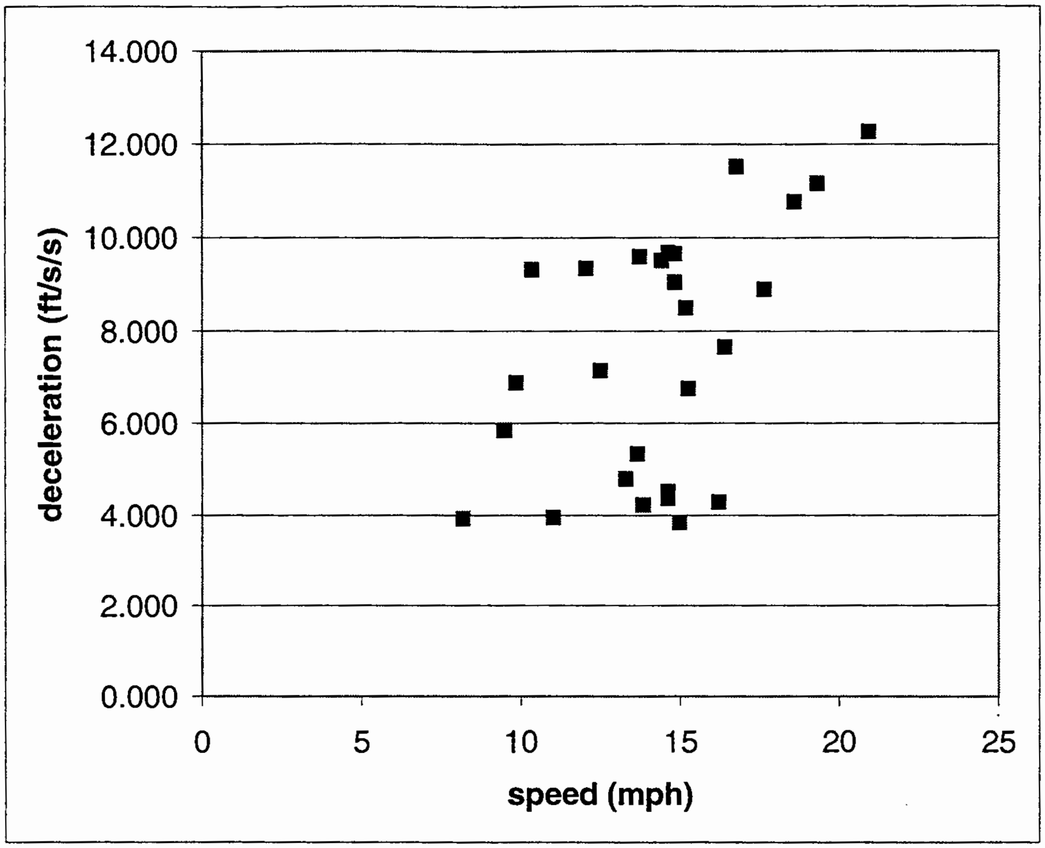

Figure 3.6. Scatter plot of bicycle comfortable deceleration versus speed.

It is interesting to compare these results with previously reported measurements. Data collected in 1972 show speeds of bicycle/rider units averaging about 10 to 11 mph and ranging from 7 to 15 mph (ITE, 1972). While these speeds are lower than those observed in this study, direct comparison is of limited value because the conditions under which the 1972 data were collected were not reported. Measurements in Mountain View, California (where bicycle commuting and club cycling are popular) reveal higher speeds, with a slowest speed of 12 mph, a median of 16 mph, and an 85th percentile of 18.5 mph (Forester, 1983). These are slightly higher than those observed in Austin, Texas, which can be similarly described as a city where bicycling for both transportation and recreation is popular. The Mountain View speeds were measured on a level street bikelane in the absence of wind during the entire morning commuting period (conditions comparable to those in this study, except that the riders in this study knew they were being timed).

Acceleration should vary according to the distance over which the cyclist accelerates, as indicated by the lower (constant) accelerations over 150 ft, relative to 60 ft. (This also indicates that the constant acceleration assumption is actually invalid.) Though no previous bicycle acceleration measurements were found in the transportation literature, comparisons to automobiles provide some insight. Acceleration available to cars can be estimated through Gazis' equation (Gazis et al., 1960):

$$a = 16.0 - 0.213 \cdot v, \tag{3.6}$$

where $a$ = acceleration (ft/s$^2$) and
$v$ = speed (mph).

At 14 mph (about the speed of an average bicyclist) a car would have the ability to accelerate at about 13 ft/s$^2$. As expected, this is greater than all the observed bicycle/rider units. Even at 40 mph, a car has the ability to accelerate at a higher rate (about 7.5 ft/s$^2$) than all the observed bicycle/rider units.

The deceleration measurements compare well with the Dutch design recommendation of 4.92 ft/s$^2$ (CROW, 1993), but are lower than those recommended by Forester (1983) of 15 ft/s$^2$ for adult transportation routes and 8 ft/s$^2$ for recreation and children routes. Neither source gives the basis or the measurement data for their choices.

Some theoretical calculations are also interesting. With tire braking, the maximum deceleration possible is 32.2 ft/s$^2$, or the acceleration due to gravity g, if the coefficient of friction between the tires and road surface is 1.0. Actually, the coefficient of friction for pneumatic-tired vehicles varies from about 0.8 (dry concrete) to 0.1 (wet ice), with a value between 0.4 and 0.7 for wet concrete or wet asphalt (Whitt and Wilson, 1990). Therefore, the maximum bicycle braking deceleration (under ideal conditions) is about 26 ft/s$^2$. On wet roads this can fall to about 13 ft/s$^2$ (for a coefficient of 0.4). Cyclists would probably brake at rates less than these to be safe and comfortable.

Unlike a bicycle/rider unit, the center of gravity and weight of a car/driver unit is such that there is no possibility of flipping over the front wheels while braking. Whitt and Wilson (1990) computed the maximum deceleration achievable by a crouched rider (using dropped handlebars) to be about 0.56g, or 18 ft/s$^2$. Again, one would expect cyclists to decelerate at a lower rate to avoid being thrown.

The above values hold only if an appropriate force is applied to the rim brake. Losses are introduced when brakes are not adjusted properly. Properly adjusted, the coefficient of friction between the brake block and the rim under dry conditions is around 0.95, and as such is not a major deceleration constraint. However, it may be as low as 0.05 under wet conditions using typical bicycle brake block material from 1971. Since 1971, wet brake coefficients of friction have probably improved to the 0.3 to 0.5 range (Whitt and Wilson, 1990).

The decelerations obtained in this experiment conform to what is suggested by theory. The maximum rate observed (12.3 $ft/s^2$) is less than the "over the handlebar" threshold of 18 $ft/s^2$, and the mean and 15th percentile rates (7.5 and 4.2 $ft/s^2$, respectively) are significantly less. This reflects the facts that bicyclists prefer a margin of safety and comfort and that some bicycle braking systems are out of adjustment.

Some experimental measurements on "hard" bicycle braking found that a 160 pound cyclist braking with both front and rear brakes simultaneously could perform a hard stop in about 18 feet if traveling at 20 mph, 12 feet at 15 mph, and 7 feet at 10 mph (Rice and Roland, 1970). These correspond to average decelerations of 25, 21, and 16 $ft/s^2$, respectively. Using just the rear brakes (or coaster brakes) the average hard decelerations were on the order of 10 $ft/s^2$. These values are consistent with the above theoretical discussion. In addition, they noted that "in over 200 hard braking stops no instance of pitchover occurred when the rider remained seated" (Rice and Roland, 1970). These braking decelerations are appropriate for emergency stops, but not for design, where comfort and safety are important considerations.

The statistics reported in Table 3.1 reflect not only the population sampled, but also possible small sample and measurement errors. Therefore, for simplicity, the rounded values below are used for analysis:

(1) Bicyclist normal cruise speed:

| | |
|---|---|
| 7.5 percentile | 10.0 mph |
| Mean/median | 14.0 mph |
| 92.5 percentile | 18.0 mph |

(2) Bicyclist comfortable acceleration:

| | |
|---|---|
| 15th percentile | 1.0 $ft/s^2$ |

(3) Bicyclist comfortable deceleration:

| | |
|---|---|
| 15th percentile | 4.0 $ft/s^2$ |
| Mean/median | 7.5 $ft/s^2$ |

Extreme caution must be exercised if any of these numbers are used in analysis or design outside of this study, since the numbers are based on a small sample of the cyclist population recruited around the University of Texas at Austin. In addition, all measurements were made using the same experimental technique, which required that the cyclists knew they were participating. Finally, adjustments to these design values will be required if grades are significant.

### 3.1.3 Clearance Intervals for Bicycle-Automobile Mixed-Traffic

A conservative design for mixed bicycle-automobile traffic might use the larger of the adequate clearance intervals computed for each vehicle type. While accepted design values for the parameters in the adequate clearance interval equations 3.2 and 3.4 are available for automobiles (ITE, 1982), this is not the case for bicycles.

The non-controversial design value is a bicycle length of 6 ft. The data analysis (in Section 3.1.2) suggests a design bicycle speed in the range of 10 to 20 mph and a design bicycle deceleration between 4.0 ft/s$^2$ (15th percentile) and 7.5 ft/s$^2$ (median). Without knowing the percentage of cyclists that accelerate if choosing to proceed, it is prudent to assume constant speed, which provides a margin of safety for bicyclists. Sensitivity to an acceleration of 1.0 ft/s$^2$ (85th percentile) is suggested by the measurements. Sensitivity to the following three bicycle design clearance points is investigated: 30 ft, 65 ft, and 100 ft past the stopbar. These are chosen to represent a broad array of possible clearance points (required for comfort and safety) for a wide range of intersection widths.

A design bicycle/rider unit perception-reaction time of 2.5 seconds is used, but sensitivity to lower values is analyzed. Though no published bicycle perception-reaction time measurements are available, this choice is consistent with AASHTO's (1991) bicycle design recommendation of 2.5 seconds and the Dutch recommendation of 2.0 seconds (CROW, 1993), but significantly higher than the 1.0 second recommendation of Forester (1983). Regardless of the fact that cyclists are in greater danger than motorists, which would seem to make them more alert (resulting in a lower perception-reaction time than motorists), the chosen bicycle design time is greater than that normally used for motorists. Motorist perception-reaction times range between about 0.5 and 4.0 seconds (Pignataro, 1973), with a design value of 1.0 seconds normally used in urban areas (Gazis et al., 1960; Olson and Rothery, 1962; Wu et al., 1983; Butler, 1983; Chang et al., 1985; and ITE, 1982), and 2.5 seconds used in rural areas (ITE, 1982). Reasons supporting an urban bicycle design time on the order of rural automobile design times include the fact that most cyclists do not seem to perceive a greater danger, as indicated by their general disregard for traffic control devices. In addition, cyclists probably have more distractions than motorists due to weather, debris, noise, shifting gears, etc. They also have a more difficult and varied response task, depending on the location of their hands in relation to the brake levers, and their heads are often tilted downward providing a poor view of the traffic signal.

Adequate clearance intervals computed using equations 3.2 and 3.4 for different design clearance point, speed, and acceleration assumptions are shown in Figure 3.7. The magnitudes of the increases in adequate clearance interval required for design clearance points further into the intersection and for no acceleration versus acceleration are easily determined from the figure. Note that the magnitude of the acceleration impact increases as the design clearance point is moved farther from the stopbar.

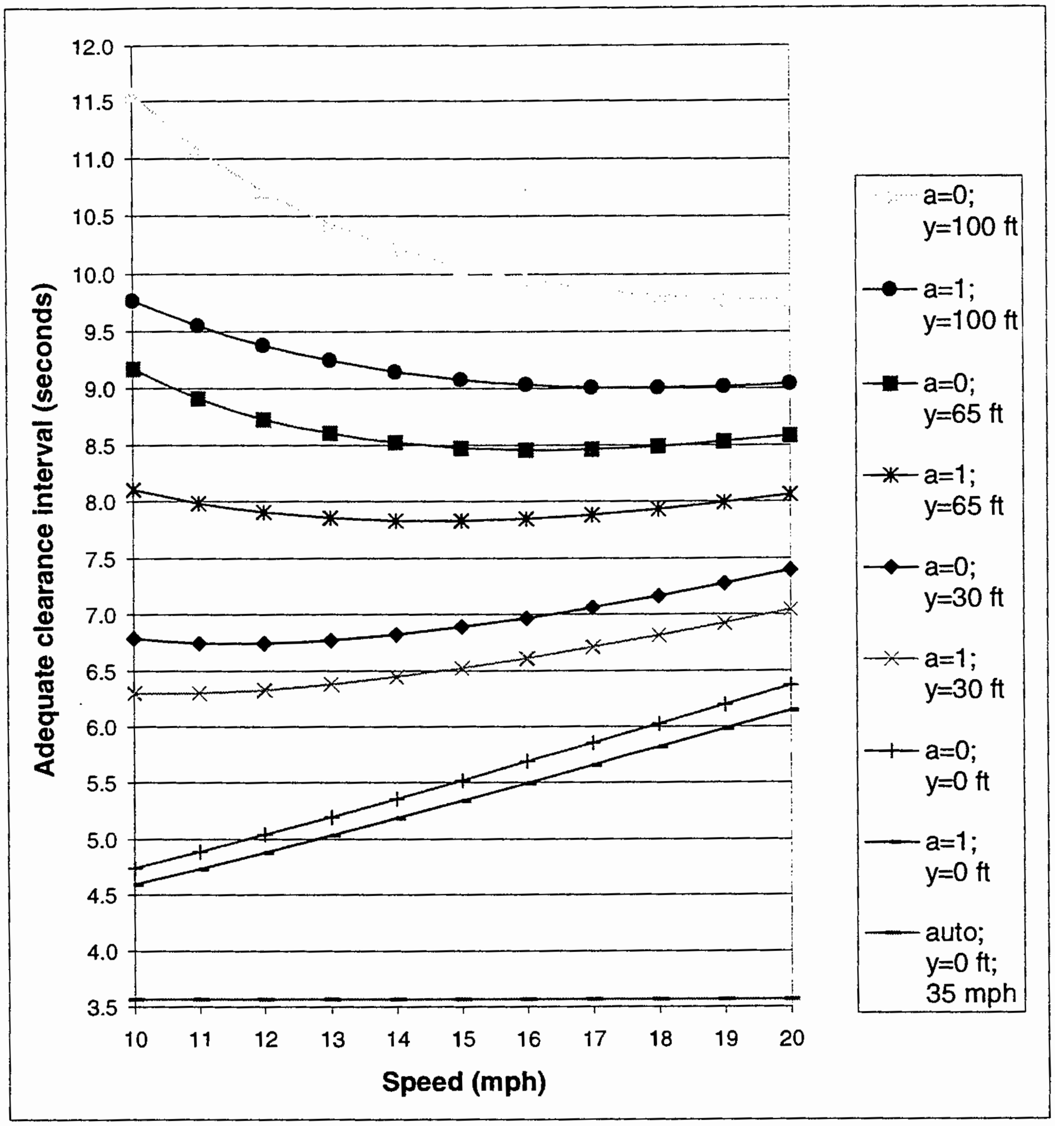

Figure 3.7. Adequate clearance intervals for bicycles traveling at speeds between 10 and 20 mph, for accelerations of zero (a=0) and 1 ft/s$^2$ (a=1) and four design clearance points (at a distance y from the stopbar) and a perception-reaction time of 2.5 seconds.

Figure 3.7 shows that the adequate clearance interval for automobiles travelling at 35 mph (3.6 seconds) is from one to eight seconds less than those for bicycles, depending on the acceleration, speed, and design clearance point assumptions.[2] The main conclusion is that if the bicycle design clearance point is much past the stopbar the bicycle adequate clearance interval will be significantly larger than the corresponding automobile adequate clearance interval. Even if the bicycle design clearance point is the stopbar, the bicycle adequate clearance interval is still from one to 2.5 seconds greater than that for automobiles.

Changes in perception-reaction time translate directly to equal changes to the adequate clearance interval. Thus, even for a 1.0 second bicyclist perception-reaction time, which is probably the minimum design value one might use, the bicycle adequate clearance interval is still considerably greater than that for automobiles if the design clearance point is much past the stopbar.

The sensitivity to deceleration is examined in Figure 3.8. Only the case where the design clearance point is 30 ft past the stopbar ($y = 30$ ft) is shown, because the magnitude of the differences (for the same speed) are identical for any design clearance point. For a deceleration increase from 4.0 to 7.5 ft/s$^2$, the adequate clearance interval decreases by 0.9 to 1.7 seconds, depending on the speed.

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[2] For further comparative purposes, the adequate clearance intervals for automobiles to clear design clearance points 30, 65, and 100 ft past the stopbar, are 4.5, 5.2, and 5.9 seconds, respectively, which are still about two to six seconds less than those for bicycles, respectively. All automobile adequate clearance intervals were computed using equation 3.2 with the following common automobile design values: $d = 10$ ft/s$^2$, $L = 19$ ft, $t_{p\text{-}r} = 1$ second.

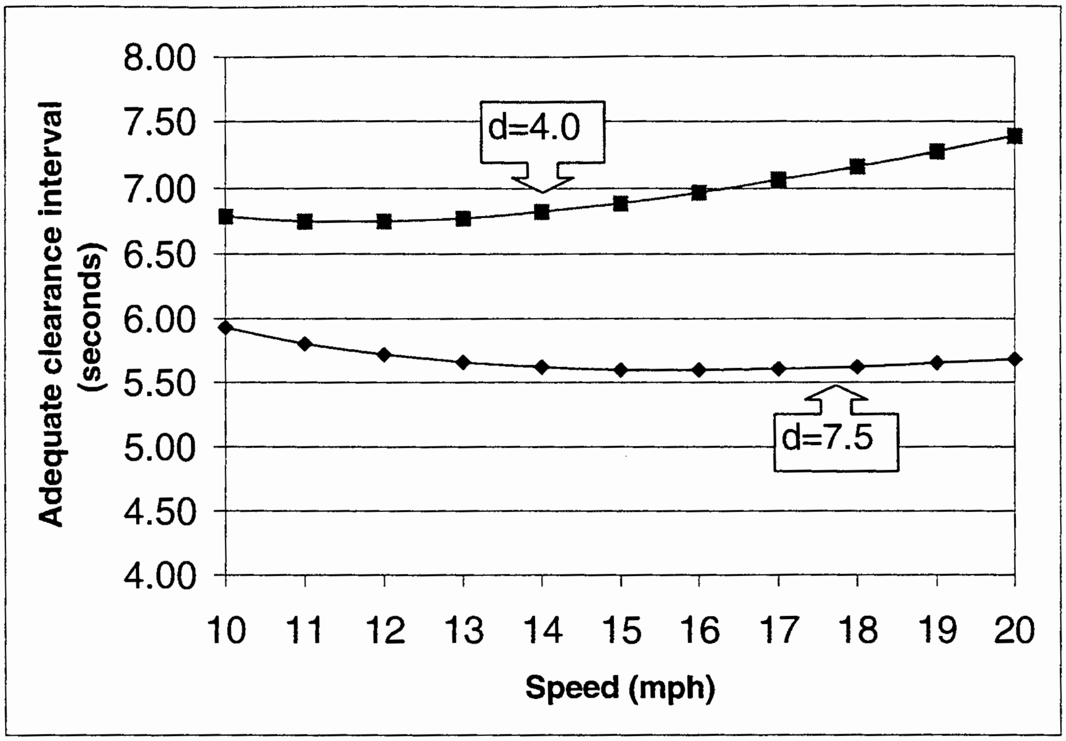

Figure 3.8. Sensitivity of adequate clearance intervals for bicycles, traveling at speeds between 10 and 20 mph, to a deceleration of 7.5 ft/s$^2$ (relative to 4.0 ft/s$^2$), assuming no acceleration, a design clearance point at a distance 30 ft from the stopbar, and a perception-reaction time of 2.5 seconds.

For different design clearance points, the highest adequate clearance interval within the speed range occurs at different values of the approach speed. For the no acceleration assumption, the largest adequate clearance interval occurs at a speed of 20 mph for the y=0 ft (stopbar) and y=30 ft cases, while at the two design clearance points farther into the intersection (65 and 100 ft), the largest occurs at a speed of 10 mph. Since the shape of these curves is convex with only one minimum, the largest adequate clearance interval (within a given speed range) occurs at either end-point of the speed range.[3] Therefore, it is more difficult to choose a single design speed, since a speed accommodating the same high percentage of bicycle/rider

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[3] This point can also be illustrated mathematically by differentiating equation 3.2 with respect to v to obtain the slope of these curves, $1/(2\cdot d) - (y + L)/v^2$. This slope is negative when v is small and becomes less negative as v increases. The minimum value of $ci_{adeq}$ occurs when the slope is zero or $v=\sqrt{2\cdot d\cdot(w+L)}$. For the no acceleration bicycle cases in Figure 3.7, these minimums occur at speeds of: 11.6 mph (y=30 ft), 16.2 mph (y=65 ft), and 19.9 mph (y=100 ft).

units might be chosen at either end of the speed range. It might be more convenient to choose a design speed range bounded by the bicycle population's lower $\alpha$ percentile speed and its upper $\alpha$ percentile speed to accommodate at least (the central) ($100 - 2\alpha$) percent of bicyclists. Obviously, the chosen design speed range will vary according to intersection location, topography, etc., but the data analysis in Section 3.1.2 indicates a range of about 10 to 18 mph will accommodate at least 85 percent of bicycle/rider units approaching relatively flat intersections.

Using this speed range, it can be seen in Figure 3.7 that the adequate clearance intervals for bicycle design clearance points more than about 33 ft (43 ft for acceleration) past the stopbar are greater than seven seconds. (The analysis is based on a combination of conservative assumptions, so adequate clearance intervals of seven seconds may not be required until the distance past the stopbar is somewhat greater than 33 ft.) Clearance intervals of this magnitude are often thought to encourage drivers to disregard signal indications and possibly increase rear-end collisions (ITE, 1976). This may not occur if the yellow is kept below five seconds and an all-red phase is used for the rest of the clearance interval; however, automobiles would then incur additional delay (ITE, 1982).

Considering the possible delay costs (if drivers do not change their behavior in response to a longer clearance interval) or safety costs (if some do change) associated with clearance intervals timed primarily for bicycles, a cost-effective solution may be to provide a separate warning signal for bicyclists. A precedent for this has been set by pedestrian signals.

### 3.1.4 Closure

A standard deterministic approach for computing adequate traffic signal clearance intervals was applied to bicycle-automobile mixed-traffic. The bicycle/rider unit design values for speed, deceleration, acceleration, perception-reaction time, and the design clearance point were based on a combination of new measurements, theories, and the literature. This analysis provided practical insights and considerations for determining clearance intervals to accommodate bicycle-automobile mixed-traffic.

As discussed above (in the chapter introduction), substantive reasoning and some accident study results support the notion that (for safety and comfort) bicycles need to proceed farther into the intersection than automobiles before the cross-street traffic is released. If this is true, the analysis indicates that bicycles are likely to require considerably longer clearance intervals than automobiles, especially at wide intersections. Other contributions of this work are the measurements of comfortable bicycle/rider unit deceleration and acceleration, which must nonetheless be used with caution, as they are based on a very small sample in a single experimental scenario.

The essence of the work on which this section is based has been published in a professional journal (Taylor, 1993). Subsequent to that publication, Wachtel et al. (1995) came to similar conclusions using the same deterministic analysis procedures.

The deterministic approach has three main weaknesses. First, factors other than the ones considered here are likely to affect bicyclist behavior. Second, use of a design bicycle/rider unit, chosen to accommodate some percentage of the population, may not adequately recognize the variability of stopping behavior and hence the clearance interval. Finally, the selection of the design clearance point for cyclists is rather arbitrary. The probabilistic approach used in the next section will help alleviate the first two weaknesses.

## 3.2 MODELING ACTUAL BEHAVIOR USING A PROBABILISTIC APPROACH

In the previous section, the decision situation faced by a bicyclist at the onset of the yellow signal indication was formulated and analyzed using a deterministic approach based on kinematic relations. In this section, the actual behavior of the bicycle/rider unit in the above situation is examined. The approach is to observe bicyclist behavior at the onset of yellow and model that behavior as the outcome of a discrete choice process, using probabilistic choice modeling techniques.

First, a brief background review is presented, followed by the formulation of the discrete choice model theory. Second, the data collection and reduction procedures are described. In the next subsection, some summary statistics are presented. The fourth subsection contains the behavioral analysis using discrete choice models. Finally, some concluding comments are made.

### 3.2.1 Background Review and Theory Formulation

The probabilistic approach seeks to model actual behavior in terms of the probability that an approaching vehicle at a given distance from some specified clearance point (which is often the stopbar) at the onset of yellow will stop. Early researchers developed probability of stopping curves from observations of actual traffic at intersections (Herman et al., 1963), depicting the "percent of drivers stopping" versus "distance from the stopbar". By the weak law of large numbers, "percent of drivers stopping" can be interpreted as "probability of stopping".

Sheffi and Mahmassani (1981a) developed an expression for the probability of stopping using discrete choice modeling techniques, from which probability of stopping curves and the dilemma zone can be determined. This method also allows one to statistically test the significance of various possible factors that may impact stopping behavior. Prashker and Mahalel (1989) used a similar approach to determine adequate clearance intervals.

In this approach, driver behavior at the onset of the yellow is viewed as a binary choice to either stop or proceed. Define T to be the perceived time for a specific bicycle/rider unit in a specific situation to clear (or reach) a specified clearance point in the intersection. The basic decision rule is that if T were greater than some individual and situation specific critical value (the *critical time*), $T_{cr}$, then the rider would stop, else the rider would proceed.

Sheffi and Mahmassani (1981a) stated that for automobile drivers "the critical time, $T_{cr}$, for a particular driver can be thought of as reflecting the driver's previous experience and expectancy as to the length of the yellow phase (acquired through the learning process associated with the development of driving skills and attitudes), the driver's perception of acceleration rates, as well as his aggressiveness." Also inherent in an individual's formation of a critical time is his or her perception of risk of an accident and injury in that accident. For this reason, an individual riding a bicycle may require a shorter $T_{cr}$ (to the same specified clearance point) than if driving an automobile in the same situation.

The critical time $T_{cr}$ is a latent variable that varies both across the population of individual vehicle/operator units and within each vehicle/operator over time and situations, as does the perceived time T. Because they are affected by many independent factors, it is reasonable to model them as normally distributed random variables as follows:

$$T_{ns} = t_{ns} + \xi_{ns}, \tag{3.7}$$

$$T_{cr,ns} = \mu + \varepsilon_{ns}, \tag{3.8}$$

where $T_{ns}$ is the perceived time for an individual bicycle/rider unit n in a specific situation s to clear (or reach) the specified clearance point,

$t_{ns}$ is the measurable component of the perceived time $T_{ns}$,

$T_{cr,ns}$ is the critical time defining whether an individual bicycle/rider unit n in a specific situation s will stop or proceed (if $T_{ns}$ is less than $T_{cr,ns}$ then the unit would proceed),

$\mu$ is the *mean critical time* over all individuals and situations, and

$\xi_{ns}$ and $\varepsilon_{ns}$ are normally distributed random variables $(\xi, \varepsilon) \sim MVN(0, \Sigma)$.

Dropping subscripts, for the specification of $T_{cr}$ in equation 3.8, the probability that a randomly chosen bicycle/rider unit will stop, $P_{stop}(T)$ is then given by the probit equation:

$$P_{stop}(T) = \Pr\{T > T_{cr}\} = \Phi\left(\frac{t-\mu}{\sigma}\right) \qquad (3.9)$$

where $\sigma = \sqrt{\sigma_\xi^2 + \sigma_\varepsilon^2 - 2\sigma_{\xi,\varepsilon}}$ ,

$\sigma_\xi^2$ and $\sigma_\varepsilon^2$ are the respective variances of $\xi$ and $\varepsilon$, and

$\Phi(\bullet)$ denotes the cumulative standard normal function.

As formulated, $\sigma$ represents the standard deviation of the distribution of the difference in the perceived time and the critical time. It can also be viewed as representing the standard deviation of the critical time distribution, if all perceptual factors are included in the critical time variable, and the perceived time is limited to its deterministic observable component.

It is desirable to investigate the dependence of $T_{cr}$ on variables that could affect time perception. For example, Sheffi and Mahmassani (1981a) specified $T_{cr}$ as a function of speed, v, as follows:

$$T_{cr,ns} = a + b \cdot v_{ns} + \varepsilon_{ns} \qquad (3.10)$$

where a and b are parameters to be estimated.

The specification of the critical time can readily include one or more explanatory (or systematic) variables; more generally,

$$T_{cr,ns} = \beta_0 + \beta X_{ns} + \varepsilon_{ns}, \qquad (3.11)$$

where $\beta_0$ is a constant term interpreted as the mean critical time when all other parameters are zero (i.e. the *base mean critical time*),

$\beta$ is a vector of parameters to be estimated, and

X is a vector of systematic variables, which can vary over individuals n and situations s.

By replacing $\mu$ in the choice probability expression (equation 3.9) with the general specification of $T_{cr}$ (equation 3.11), the following probit probability function is obtained (with the indices dropped):

$$P_{stop}(T) = \Phi\left(\frac{t-(\beta_0+\beta X)}{\sigma}\right). \qquad (3.12)$$

Note that the model in equation 3.9 is a restricted version of this model, in which $\beta_0=\mu$ when all the $\beta$ coefficients are set to zero.

The specification in equation 3.12 allows one to analyze various factors impacting cyclists' decisions to stop or proceed, namely attributes of the bicycle/rider unit and of the situation facing that unit. Attributes of the situation include characteristics of the intersection, such as the geometry, characteristics of the surrounding traffic, such as the extent of congestion and the type of vehicles involved, and characteristics of other surrounding events. Specific attributes of the bicycle/rider unit include the unit's speed at the onset of yellow, the bicyclist's gender, the bicycle type, and helmet use. Other factors in the categories defined in Section 2.2 for gap acceptance for the intersection environment and bicycle/rider unit apply here as well. If observations are obtained from a sufficiently large number of intersection environments then possible serial correlation can be captured with a component of variance model analogous to those presented for gap acceptance in Section 2.2.

Sheffi and Mahmassani (1981a) successfully used the time to reach the stopbar from the onset of the yellow, calculated assuming constant speed, as the value of $t_{ns}$ in equation 3.7. Any point on the roadway could be chosen as the clearance point reference for computing the $t_{ns}$. Initially, the stopbar is chosen in this analysis, because it will allow comparison to other studies and has legal meaning. Sheffi and Mahmassani (1981a) interpreted $\mu$ (relative to the stopbar) as the average driver's expectancy of the signal yellow change interval.

### 3.2.2 Data Collection

The plan followed to obtain data on the behavior of actual bicycle/rider units at the onset of yellow is described, including: a description of the collection site and the reduction procedure.

**3.2.2.1 Plan.** The target size was 150 observations, based on suggested values in Sheffi and Mahmassani (1981a). The required data includes instantaneous speeds and distances from the stopbar at the onset of the yellow for cyclists in and around their dilemma zones, and their decisions to stop or proceed. Instantaneous speed can be approximated by the average speed for traversing a section of the roadway just prior to the onset of yellow. A video camera was used to record about 300 feet of the roadway before the intersection. The section approximately 180 feet from the stopbar contains the possible bicyclist decision area; since it would take a fast cyclist (20 mph) about 6 seconds to traverse 180 feet, it is improbable that any cyclists will attempt to proceed from greater than 180 feet (and none did). Additional information that might explain bicyclist behavior is also collected.

All the data was collected using one video camera that was activated when a bicycle/rider unit approached the intersection and the yellow signal indication was imminent. The onset of yellow was marked audibly, in case it was not visible on the tape. Paint markings and visible features on or near the roadway were used as references for distance measurements. Two analysts were required to collect the data. One operated the video camera. The other kept track of the yellow and red indications, audibly marking their onsets on the tape, and kept track of the signal cycle to determine which cyclists might provide useful observations for this study (any cyclist within 180 feet of the intersection's stopbar at the onset of yellow). Only potential observations were taped. The recording of an observation usually began about 400 feet upstream of the stopbar and followed the bicycle until it traveled completely through the intersection.

The data was collected in the summer of 1994 at the intersection of 26th and Speedway in Austin, Texas. About 38 hours were required to obtain 150 observations.

**3.2.2.2 Site Description.** The collection site, diagrammed in Figures 3.9 and 3.10, is the intersection of 26th and Speedway in Austin, Texas, on the northern border of the University of Texas campus. The site is. Bicyclists were filmed, as they traveled south on Speedway, from the top of a six-story building on the northeast corner of the intersection. Proceeding towards the intersection from outside of 300 feet, Speedway slopes slightly downhill. From 300 to 180 feet it gradually levels out. The segment between 180 and 100 feet is fairly level, reaching the point of inflection at about 150 feet. From 100 feet to the intersection it rises slightly and finally levels out through the intersection. The southbound Speedway approach is one lane except at the intersection where there is also a left-turn only lane. There are angled parking spaces along the road, except for the 90 feet closest to the intersection. The width of the intersection, as bicyclists cross it for this study, is 66 feet measured from curbface to curbface.

The traffic signal for the approach under study had a pre-timed 75 second cycle, with a 42 second red, 29 second green, and a four second yellow during data collection (June and July 1994). The posted speed limit was 15 mph, and it appeared that most cars did not travel too much in excess of it. The roadway was marked with white paint at various intervals so that distance from the stopbar (or any specified clearance point) and speed measures could be recorded accurately from the videotape.

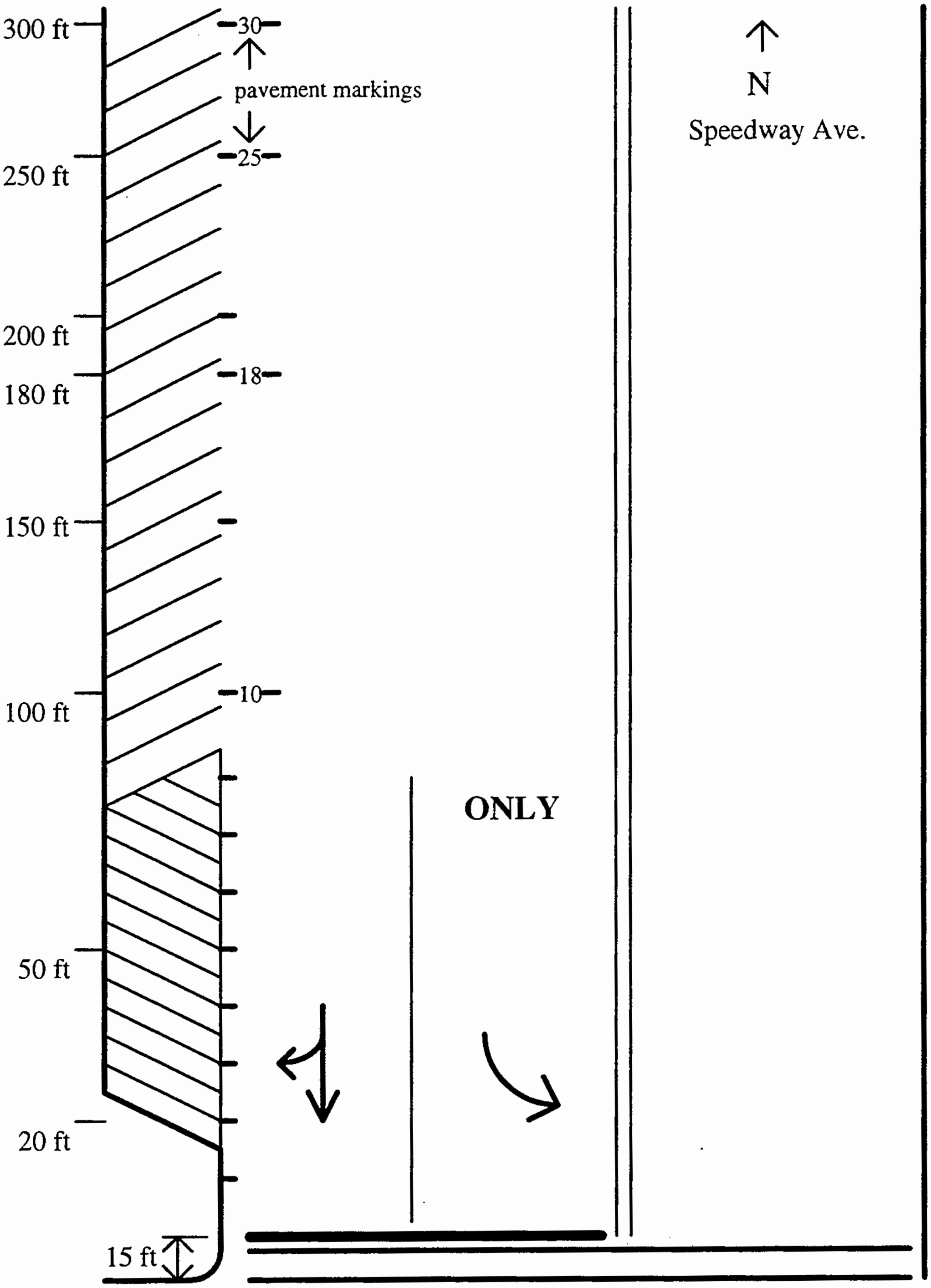

Figure 3.9. Diagram of the data collection site: the approach to the intersection of 26th and Speedway in Austin, Texas.

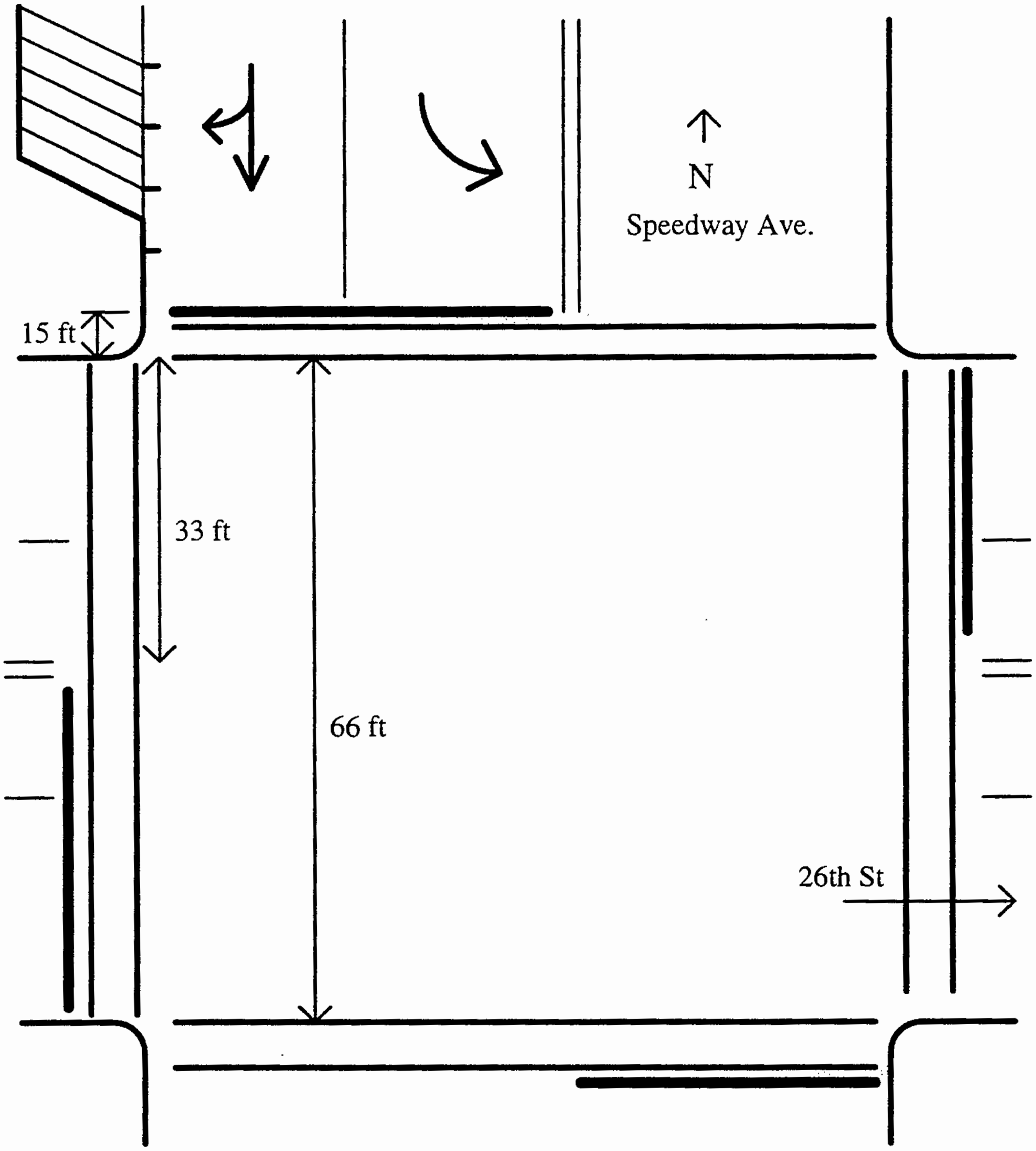

Figure 3.10. Diagram of the data collection site: the intersection of 26th and Speedway in Austin, Texas.

**3.2.2.3 Data Reduction.** For each observed bicycle/rider unit decision at the onset of yellow, the following data was recorded in spreadsheet format.

(1) Cyclist description
    a. Gender
    b. Bike type (i. Wide, ii. Thin, or iii. 0-3 gears)
    c. Wearing a helmet or not

(2) Cyclist's decision (Stop or Proceed)

(3) Distance from the stopbar at the onset of yellow

(4) Whether there was visible acceleration in first two seconds of yellow

(5) Whether there was visible acceleration in second two seconds of yellow

(6) Whether there was visible acceleration on red

(7) Whether there was visible deceleration on yellow, but the cyclist still decided to go

(8) Whether there was significant coasting in any of the following roadway segments (defined by their distance from the stopbar)

  a. 300 ft – 250 ft
  b. 250 ft –180 ft
  c. 180 ft – 100ft
  d. 100 ft – stopbar

(9) Position in intersection at the onset of red (stopbar, crosswalk, first half of intersection, second half, or completely through)

(10) Whether there were any unusual weather conditions (wind, rain, or wet pavement)

(11) Whether any congestion was present, and if so, what kind (car, bike, bus, or pedestrian) and the position of the congestion in relation to the cyclist

(12) What was the behavior of the cyclist just before the onset of yellow

  a. Coasting
  b. Pedaling at constant speed
  c. Visibly accelerating

(13) If bicyclist was traveling with a partner (at similar speeds)

  a. Distance between the pair

In addition to the above items, two average speed measurements were made, by timing each cyclist's traversal of two roadway sections defined by reference markings. To approximate the instantaneous speed at the onset of yellow, the first average speed measurement was made over a roadway section that ended as close as practical to the cyclist's position at the onset of the yellow. Eighty-eight percent of the first average speed measurements were made over distances greater than 100 ft. The second speed measurement was made using a roadway section farther from the intersection than the first. It is used to test whether a change in speed might indicate that a cyclist is preparing early (before the onset of yellow) to either stop or proceed. Seventy-five percent of the second speed measurements were made over distances greater than 100 ft. The roadway segments used for the two speed measurements never overlapped. Finally, each observed cyclist's time to the stopbar (or to any specified clearance point) at constant speed was computed using both the distance from the stopbar (or the point) at the onset of yellow and the first average speed measurement.

The cyclist's location at the onset of yellow could be obtained with accuracy using the frame by frame, stop motion playback option on the VCR when the signal change to yellow was visible on the tape. For the other observations (77.1 percent), the location of the cyclist had to be determined from the audio mark of the onset of yellow on the videotape, which leads to two inherent delays. First, there is a (*voice*) delay between the actual change to yellow and when the observer responds by saying, "yellow". Second, there is a (*reduction*) delay between perceiving the audible mark of yellow and noting the position of the cyclist, while reducing the data from the video. Hereafter, the total due to both sources of delay is referred to as the *voice-reduction delay*.

For each delay, no identification or judgement is required, the observer's expectancy is very high, and the tasks are very simple. So, one would expect the voice and reduction delays to be relatively small and approximately equal. Perception-reaction times measured in experiments conducted at the University of Texas from 1990 through 1995 (Lee, 1997) and engineering judgement provided the basis with which to estimate total voice-reduction delay. This yielded an assumption of 0.5 seconds and suggested sensitivity analysis between 0.3 and 0.7 seconds.

### 3.2.3 Summary Statistics

There were 144 observed bicyclist decisions. It was possible to identify the sex of 140 of the cyclists, and 81 percent were male, very close to the value found in the gap acceptance analysis in Chapter 2 (84 percent). As Figure 3.11 shows, a large majority of the sample rode wide tire or mountain type bikes (73 percent), the remaining either rode thin tire bikes (18 percent) or (beater) bikes with few or no gears (9 percent). These percentages are also similar to those found in the gap acceptance study.

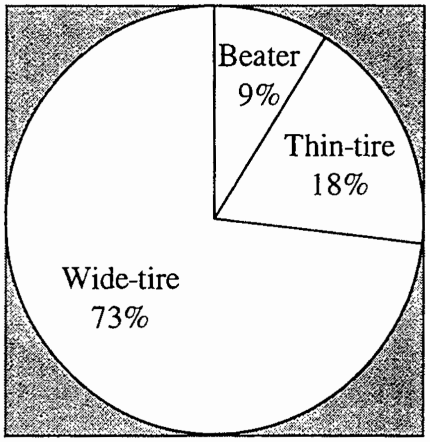

Figure 3.11. Proportions of different bicycle types. Wide-tire is a mountain bike, thin-tire is a thinner tire road or racing bike, and beater is a zero to three speed bike (e.g., stunt bike or old bike with no gears).

Only one out of four cyclists wore helmets. Cyclists on thin tire bikes wore helmets 39 percent of the time compared to 23 percent for cyclists on wide tire bikes and 15 percent for cyclists on beater bikes. Gender does not significantly affect helmet use. Of males, 24 percent wore helmets compared to 27 percent of females.[4]

There was no significant aggregate difference between the first and second set of average speed measurements. The average for each set was about 15 mph (one was 14.8 and the other 14.9). The minimum and maximum was 9.5 and 24.6 mph, respectively, for the first speed measurement, and 9.7 and 24.8 mph for the second. The standard deviation of each speed distribution was about three mph (2.8 and 2.6 mph, respectively). The (systematic) effect of a change in speed for each cyclist is tested in the discrete choice model.

As far as the cyclists' decisions are concerned, 51 percent decided to stop and 49 percent decided to proceed after the onset of yellow. Small accelerations and decelerations could not be accurately determined from the videotape. Only seven percent of the sample visibly accelerated within two seconds after the onset of the yellow, and 3.5 percent visibly decelerated after the onset of yellow and then (changed their minds and) proceeded through the intersection.

No observations were made in rain or on wet pavement. The only possible environmental difference was wind speed and direction, and any effect might be captured by its impact on the

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[4] The statistical test of the difference is not nearly significant.

cyclists' speed. Congestion in and around the intersection or its approach was present in only 13 percent of all observations, and never did the congestion appear to be very severe.

**3.2.3.1 Illustrations of Danger to Cyclists at the Onset of Red.** The possibility of danger to cyclists is illustrated by the fact that, of the 70 cyclists observed proceeding through the intersection after the onset of yellow, four did not make it past the stopbar before the onset of red, and another six were in the near-side crosswalk. Of those that did make it into the intersection, 27 were in the first half of the intersection at the onset of red, and six made it all the way through (see Table 3.5). When the cyclist's location was identified using the audible mark of the signal change to red, the cyclist did not proceed as far into the intersection as these statistics reflect, because no adjustment was made to account for the inherent voice-reduction delay. An estimated 0.6 percent of all cyclists approaching this intersection will be proceeding into the intersection while not having cleared the near-side crosswalk before the onset of red.[5] For some cyclists this may be their preference, but for others it may be the fault of a clearance interval that is too short.

The bicycle/rider unit positions at the onset of red give further insight into possible dangers to cyclists and help informally evaluate the comfortable bicycle decelerations found by experimentation in the deterministic approach (in Section 3.1). Table 3.5 maps the positions of the observed cyclists at the onset of red with the minimum constant deceleration required for them to stop before the stopbar. The boundaries of the deceleration ranges are determined using the comfortable bicycle decelerations found by experimentation (see table notes for details).

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[5] One-seventh of those observed proceeding through this intersection after being within about 150 feet of the stopbar at the onset of yellow did not make it into the intersection (past the crosswalk) before the onset of red. A representative cyclist traveling at 15 mph (the average speed of the sample) takes 6.8 seconds to traverse a 150 foot segment before the stopbar. Assuming that bicycle arrival times are independent of the onset of yellow times about nine percent (6.8/75 x 100) of all cyclists will be caught in this 150 foot section at the onset of yellow, with the 75 second cycle time. About 50 percent of the observed cyclists proceed (actually 70 of 144), so about 4.5 percent of all cyclists will proceed, with one-seventh of these short of the near-side crosswalk at the onset of red.

TABLE 3.5. BICYCLIST MINIMUM CONSTANT DECELERATIONS REQUIRED FOR STOPPING COMPARED WITH THE POSITIONS OF THE CYCLISTS AT THE ONSET OF RED.

| Cyclist Position at Onset of Red [1] | Number of Cyclists with a Constant Deceleration (ft/s$^2$) Required to Stop Before the Stopbar Between [2] | | | | | Total |
|---|---|---|---|---|---|---|
| | 0-4.2 | 4.2-7.7 | 7.7-10 | Over 10[3] | Had to Go[4] | |
| Stopbar (stopped) | 65 | 4 | 2 | 3 | | 74 |
| Before Stopbar (going) | 3 | 1 | | | | 4 |
| Near-side crosswalk (going) | | 6 | | | | 6 |
| 1st Half of Intersection | | 3 | 6 | 11 | 7 | 27 |
| 2nd Half of Intersection | | | | 7 | 20 | 27 |
| Through Intersection | | | | | 6 | 6 |
| Total | 68 | 14 | 8 | 21 | 33 | 144 |

[1] The statistics are conservative, as the positions of some cyclists at the onset of red are farther upstream than indicated, because voice-reduction delays are not incorporated.

[2] Decelerations required to stop are computed assuming a cyclist perception-reaction time of 1.5 seconds and a voice-reduction delay of 0.5 seconds. Comfortable cyclist decelerations found by experimentation in the deterministic approach (Section 3.1) form natural boundaries for some deceleration ranges. The 15th percentile was 4.2 ft/s$^2$ and the median was 7.7 ft/s$^2$.

[3] The 10 ft/s$^2$ boundary is a commonly accepted automobile design deceleration value. The over 10 category includes decelerations greater than the over the handlebar threshold (18 ft/s$^2$) and the theoretical threshold from the acceleration due gravity (32.2 ft/s$^2$).

[4] Some cyclists had to go, because they would be inside the stopbar before being able to react.

As expected, the percentage of cyclists stopping decreased as the required deceleration increased. Those with decelerations less than 4.2 ft/s$^2$, the 15th percentile measured comfortable rate, almost always stopped. The three who did proceed did not reach the stopbar before the onset of red, possibly because they have very low comfortable deceleration tolerances. Otherwise, they had a reasonable chance to stop, but may have made bad decisions, were very aggressive, were not paying attention (large perception time), or had bicycles with poor braking capabilities.

More cyclists requiring rates between the 15th percentile (4.2 ft/s$^2$) and median (7.7 ft/s$^2$) measured comfortable decelerations might have wished to stop, but proceeded because it would be too uncomfortable for them to stop. These cyclists, especially the seven that did not clear the near-side crosswalk, may have been placed in danger because of an inadequate clearance interval.

As expected, cyclists requiring decelerations of over the median measured comfortable rate (7.7 ft/s$^2$) seldom stopped, and all that proceeded made it past the near-side curbface before the onset of red. Finally, most of those cyclists who had to proceed, because a 1.5 second perception-reaction time put them past the stopbar before they could begin braking, were in the second half of or completely through the intersection at the onset of red.

Again, these statistics (in Table 3.5) are conservative, because the positions of some cyclists at the onset of red are farther upstream than indicated, because voice-reduction delays are not incorporated. Although the comparisons (in Table 3.5) do not rigorously or conclusively verify the comfortable deceleration measurements, they do support them, since the comfortable rates yield range boundaries that are reasonable. The indication is that bicycle/rider units with a comfortable deceleration between 4.2 ft/s$^2$ and 7.7 ft/s$^2$ may be put in a dangerous situation because the yellow change interval is inadequate (at four seconds) for this medium width intersection (66 feet).

**3.2.3.2 Insights into Cyclist Behavior.** Fourteen of the observed cyclists were accelerating prior to (and at) the onset of yellow, ten cyclists were accelerating in (approximately) the first two seconds of the yellow, ten were accelerating in (approximately) the second two seconds of the yellow, and six were accelerating during the red. These are not mutually exclusive. For example, some of the cyclists accelerating in the first two seconds of yellow were still accelerating in the second two seconds and are represented in both counts. This gives an indication of the percentage of cyclists who may accelerate to make their crossing safer or more comfortable. It also indicates some cyclist perception-reaction times are greater than 2.0 seconds.

It seems that traveling in pairs may impact the behavior of cyclists at the onset of yellow. Five pairs were observed and both members of each pair made the same decision. One pair proceeded through the intersection when they were 128 ft and 4.8 seconds from the stopbar at the onset of the yellow. Both members were at the stopbar at the onset of red. The trailing member of this pair almost stopped at the stopbar, before proceeding.

Five cyclists initially decelerated at the onset of yellow, but still proceeded, indicating that they might have wished to stop but were too close to the stopbar to stop comfortably. Their minimum decelerations required to stop before the stopbar were 4.7, 4.5, 4.0, 19.9, and 20.5 ft/s$^2$. The two with the decelerations of about 20 ft/s$^2$ would almost certainly be uncomfortable trying to stop, because they would have had to decelerate at a rate greater than the "over the handlebar" threshold of 18 ft/s$^2$ for crouched riders, which was discussed in Section 3.1.2. One (deceleration of 4.0) was the trailing member of the pair discussed above and probably proceeded because of that influence, not because of discomfort in stopping. So, this is visible evidence that four of the 70 proceeding cyclists wished to stop, but did not, because they could not do so comfortably.

No bicycle/rider unit stopped if it was within 2.2 seconds of the stopbar at the onset of yellow, and none proceeded if it was outside of 5.0 seconds. Between 2.2 and 5.0 seconds from the stopbar the observed cyclists sometimes stop and sometimes proceed. The corresponding range for the position of cyclists at the onset of yellow is between 51 and 152 feet from the stopbar. The cyclist's stopping probability function is derived in the next section using discrete choice modeling techniques.

### 3.2.4 Stopping Probability Models

The analysis of bicycle/rider unit behavior at the onset of yellow is performed using discrete choice models. First, the model specification and estimation results are presented, followed by interpretation of the estimated model parameters. Finally, modeling the decision based on the critical deceleration required to stop instead of the critical time to reach a specified clearance point is examined.

**3.2.4.1 Model Estimation Results.** Discrete choice model parameter estimates, as specified in equation 3.12, were recovered from confounded estimates obtained using the standard binary probit model estimation software SST (Dubin and Rivers, 1988). The true parameter estimates were recovered as illustrated in the following example, perhaps with a loss in estimation power relative to direct estimation. To estimate the model specified in equation 3.9, the following standard binary probit model is estimated.

$$P_{stop}(T) = \Phi\,(\alpha_1 t - \alpha_0). \qquad (3.13)$$

The mean critical time and standard deviation can be recovered by equating the coefficients of equations 3.9 and 3.13, yielding $\sigma = \dfrac{1}{\alpha_1}$ and $\mu = \dfrac{\alpha_0}{\alpha_1}$.

The time to reach a specified clearance point at constant speed is calculated (as the ratio of the distance from the point to the speed at the onset of yellow).[6] The estimation results for two alternative specifications are presented in Table 3.7. Both reflect the same insights into bicycle/rider unit behavior and explain the sample variation to the same degree. The only differences are the specified clearance point and the manner in which the speed variable is represented. The first model uses the stopbar as the clearance point and includes sensitivity to speed explicitly, while the other uses the center of the intersection as the clearance point and incorporates the effect of speed implicitly through the additional time required to reach that point. As discussed previously, cyclists may require (for safety) a design clearance point further into the intersection than motorists.

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[6] This time could also be computed assuming that cyclists accelerate. Even though cyclists can almost always accelerate without exceeding the speed limit, and some were observed accelerating at or after the onset of yellow, it is difficult to determine which cyclists would accelerate, at what rate, and in which situations they might do so.

TABLE 3.7. MODEL ESTIMATION RESULTS FOR BICYCLE/RIDER UNIT BEHAVIOR AT THE ONSET OF YELLOW FOR TWO DIFFERENT CLEARANCE POINTS.

| Parameter | # of true obs. | To Stopbar[3] | | To Center of Intersection (48 ft past stopbar)[3] | |
|---|---|---|---|---|---|
| | | Coef. Estimate | t-statistic[4] | Coef. Estimate | t-statistic[4] |
| Constant term (seconds) ($\beta_0$) | 136 | 0.92 | 0.82 | 6.10 | 5.63 |
| Bicyclist speed [1] (seconds/mph) ($\beta_1$) | - | 0.19 | 2.55 | - | - |
| Bicyclist coasting at the onset of yellow (sec) ($\beta_2$) | 12 | -2.23 | -2.93 | -2.17 | -2.77 |
| Auto partially blocking the intersection[2] (s) ($\beta_3$) | 4 | -1.23 | -1.90 | -1.23 | -1.89 |
| Bicyclist is a female and is wearing a helmet (sec) ($\beta_4$) | 7 | -1.74 | -1.71 | -1.80 | -1.75 |
| Standard deviation (sec) ($\sigma$) | - | 0.85 | 5.65 | 0.84 | 5.71 |
| Log-likelihood value | - | -24.99 | | -25.13 | |
| Number of observations | - | 136 | | 136 | |

[1] The speed parameter estimation is only valid for the sample speed range, about 10 to 20 mph.

[2] This is most likely from a left-turning motorist stopping in the intersection to turn left from the opposite approach or a right-turning motorist on the cyclists' approach.

[3] The voice-reduction delay assumption is 0.5 seconds.

[4] The t-statistic is for the associated confounded parameter estimate.

Consider first the critical time to reach the stopbar, which is the legal requirement in most states and a common automobile design clearance point. As Table 3.7 shows, the model has four explanatory variables and an estimated critical time standard deviation of 0.85 seconds. Model predictions, shown in Figure 3.12, cover the range of speeds for which the model is valid and include all three possible bicycle/rider unit and situation categories.

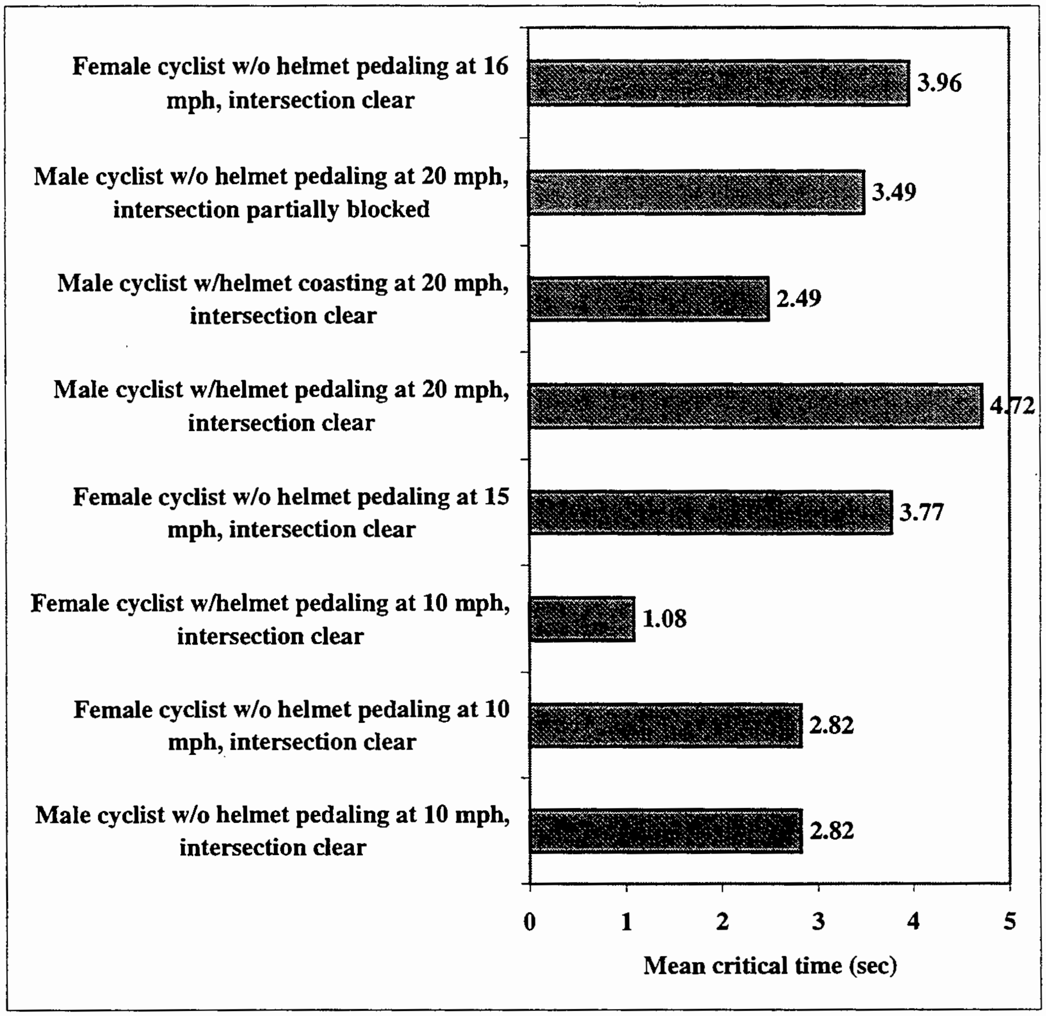

Figure 3.12. Mean critical time to the stopbar predicted by the final model for several bicycle/rider units and situations.

As expected, most mean critical times predicted by this model (relative to the stopbar) are less than the actual yellow change interval of 4.0 seconds (at the intersection where the data was collected), reflecting the influence of the length of the yellow on cyclists' behavior. Alternatively, one can view these mean critical times as the points where the probability of stopping is 0.5. A female bicyclist (who is not coasting or blocked) wearing a helmet and traveling at a speed of 10 mph would have a mean critical time of about 1.1 seconds relative to the stopbar. A male cyclist travelling at 20 mph who is not coasting or blocked would have the largest mean critical time (4.7

seconds), and hence the highest probability of proceeding. All cyclists traveling under 16.2 mph have mean critical times less than four seconds. The distribution around the mean has an estimated standard deviation of 0.85 seconds. The model results seem plausible in these and other predictive situations.

The second model specification in Table 3.7 defines the critical time relative to the middle of the intersection, considered safer and more comfortable for cyclists than just reaching the stopbar. The resulting mean critical times estimated under this definition are naturally larger than those relative to the stopbar that were estimated using the previous specification. Most cyclists would have a mean critical time of 6.1 seconds. Only three bicycle/rider unit categories or situations have different means and all are less than 6.1 seconds, with the smallest being 3.9 seconds for cyclists who are coasting at the onset of yellow.

Note that the second model explains almost the same amount of sample variation, but that speed was not significant, because its effect is now captured in the constant term. Finally, the standard deviation and all the categorical variables have nearly identical coefficient estimates and confounded parameter t-statistics.

The final model was found to be relatively insensitive to reasonable voice-reduction delay assumptions. As shown in Table 3.8, neither of the models estimated using the end points of the reasonable range of voice-reduction delays is much different from the final model which assumed a voice-reduction delay of 0.5 seconds. The maximum difference in a mean predicted critical time would be between 0.4 and 0.5 seconds for cyclists traveling at about the maximum bicycle speed of 20 mph in the model assuming a voice-reduction delay of 0.7 seconds.

TABLE 3.8. MODEL SENSITIVITY TO ALTERNATIVE VOICE-REDUCTION DELAY ASSUMPTIONS AT THE END POINTS OF THE REASONABLE RANGE

| Parameter | # of true obs. | To Stopbar with Voice-reduction delay of 0.3 seconds | | To Stopbar with Voice-reduction delay of 0.7 seconds | |
|---|---|---|---|---|---|
| | | Coef. Est. | t-statistic[3] | Coef. Est. | t-statistic[3] |
| Constant term (seconds) ($\beta_0$) | 136 | 0.85 | 0.77 | 1.00 | 0.87 |
| Bicycle/rider unit speed [1] (seconds/mph) ($\beta_1$) | - | 0.18 | 2.51 | 0.21 | 2.59 |
| Bicycle/rider unit coasting at the onset of yellow (sec) ($\beta_2$) | 12 | -2.23 | -2.95 | -2.23 | -2.91 |
| Automobile partially blocking the intersection [2] (sec) ($\beta_3$) | 4 | -1.20 | -1.90 | -1.25 | -1.90 |
| Bicyclist is a female and is wearing a helmet (sec) ($\beta_4$) | 7 | -1.65 | -1.71 | -1.83 | -1.70 |
| Standard deviation (sec) ($\sigma$) | - | 0.84 | 5.69 | 0.86 | 5.61 |
| Log-likelihood value | - | -25.29 | | -24.79 | |
| Number of observations | - | 136 | | 136 | |

[1] The speed parameter estimation is only valid for the sample speed range, about 10 to 20 mph.

[2] This is most likely from a left-turning motorist stopping in the intersection to turn left from the opposite approach or a right-turning motorist on the cyclists' approach.

[3] The t-statistic is for the associated confounded parameter estimate.

The parameter interpretations and inferences are discussed below. First, the speed and constant term are addressed, followed by the other three coefficients, each of which is associated with a Boolean categorical variable. Finally, the estimated standard deviation and selected variables found insignificant are also discussed.

As noted the only difference in the two model specifications (Table 3.7) is that the mean effect of speed is captured entirely by the constant term in the second specification (relative to the middle of the intersection). This difference can be illustrated by considering three individual cyclists, traveling at speeds of 10, 15, and 20 mph, respectively. The base mean critical times for these cyclists in the second model are all the same, 6.10 seconds, whereas in the first model (relative to the stopbar) the base mean critical times are the sum of the constant term and the speed effect, or 2.82, 3.77, and 4.72 seconds, respectively. The faster the cyclist is traveling the less time is required to traverse the distance (48 ft) from the stopbar to the middle of the intersection. Adding the time each of the three cyclists would take to traverse this distance gives an approximation of the base mean critical time relative to the middle of the intersection. These times are 6.09, 5.95, and 6.35 seconds, respectively, which are very close to the constant term in the model relative to the middle of the intersection.

Sheffi and Mahmassani (1981a) concluded that the effect of automobile speeds on motorist behavior at the onset of yellow was probably because speed is an indicator of motorist aggressiveness. More aggressive motorists have a greater propensity to proceed than less aggressive ones (for the same perceived time to the stopbar), all else being equal. Similarly, faster bicyclists exhibit greater preference for proceeding.

Another significant variable indicates that the cyclist was coasting right before and at the onset of yellow, instead of pedaling at constant speed or accelerating. The interpretation of the associated coefficient is that coasting cyclists have critical times that are about 2.2 seconds less, and hence are more likely to stop, than those who are not coasting, if all other variables are equal. The direction of this effect is as expected, and its magnitude is quite large compared to the other parameters. Possible explanations for this effect could include the following:

(1) the cyclist is risk-averse in general when approaching signalized intersections,
(2) the cyclist is ahead of schedule,
(3) the cyclist recognizes that the signal has been green for quite awhile,
(4) the cyclist noted that the pedestrian signal is flashing “don’t walk”,
(5) the cyclist is tired,
(6) the cyclists noticed some random situation at the intersection,
(7) the cyclist is taking advantage of the downgrade portion of the approach, or
(8) the cyclist desires a short break to recoup.

There is reason to believe that this effect might be different for the (slightly) downgrade and (slightly) upgrade sections of the approach. However, there were not enough observations of coasting cyclists to separate out this effect by type of grade.

Another significant variable indicates that the intersection was partially blocked by an automobile at the onset of yellow. In most cases, this occurred because an automobile from the opposite approach was stopped in the intersection waiting to turn left in front of the on-coming cyclist. It could also occur because an automobile from the same approach was turning right in front of the cyclist and was waiting for pedestrians to clear. The interpretation of the coefficient is that cyclists approaching a partially blocked intersection have critical times that are about 1.2 seconds less than those approaching a clear intersection, and hence are more likely to stop, all other things being equal. The direction of this effect is as expected, as cyclists are more likely to stop to avoid a potential conflict.

It was expected that female cyclists might be more risk-averse than males in similar mixed-traffic situations and that cyclists wearing helmets might also be more risk-averse. The bicycle gap acceptance model in Chapter 2 supported neither of these hypotheses, though the sample was rather small. On the other hand, the estimation results of the model of stopping behavior at the onset of yellow support the hypothesis that the combination of being female and wearing a helmet is an indication of risk-aversion. The magnitude of the estimated coefficient of this indicator variable suggests that female cyclists wearing helmets have critical times that are about 1.8 seconds less than either female cyclists who are not wearing helmets or male cyclists, all other things being equal.

Several variables were investigated and found insignificant, and the more important conclusions from those are mentioned here. First, all relevant interactions between gender, helmet use, and bicycle type were insignificant, except females wearing helmets. Second, wind speed had a marginal effect in many specifications, however the method used to record wind speed was somewhat unreliable and not consistently applied. There might be a wind effect that could be estimated if data collection was improved. Finally, the minimum constant deceleration required to stop was not significant if included directly (as an X variable) in the specification (equation 3.12). This conclusion was not sensitive to the assumed representative cyclist perception-reaction time (i.e. 0.7, 1.0, and 1.5 seconds).

Probability of stopping models can be used to evaluate the "adequacy" of implemented yellow change and all-red clearance intervals. However, no definitive conclusions can be drawn from model estimation results, because the design clearance point that is best for cyclist safety and comfort is unknown. One can only evaluate assumed design clearance points. For example, consider the yellow change interval of four seconds at the study intersection. The results indicate that most proceeding cyclists reach the stopbar by the onset of red. A cyclist traveling at 16.2 mph has a probability of stopping greater than 0.5 if over four seconds from the stopbar at the onset of yellow, so the probability of them being caught short of the stopbar at the onset of red is less than 0.5. Those cyclists traveling slower have greater probabilities of stopping, and the median speed of the sample is about 15 mph. Also, because of their lesser propensity to proceed, slower cyclists are more likely to be farther into the intersection at the onset of red (and therefore more visible to cross-street motorists). From this it would seem that it is the faster cyclists (if any) that warrant extra clearance time.

Finally, consider the critical gap distribution for the sample with a mean critical gap over all cyclists and situations ($\mu$=3.7 seconds) and standard deviation ($\sigma$=1.1 seconds), estimated as specified by equation 3.9. If the sample is representative of the cyclist population at the intersection under study, then evaluation of the implemented yellow change interval (four seconds) is possible. The probability that a cyclist four seconds from the stopbar at the onset of yellow will stop is 0.61, or the probability is 0.39 that the cyclist will proceed and be caught short of the stopbar at the onset of red. Perhaps this is considered too great a percentage to be in violation of the law, so a designer considers a five second clearance interval, which would lower the probability of a cyclist being caught short of the stopbar at the onset of red to 0.12.

Cyclists will be caught short of the stopbar only when the time to reach the stopbar at the onset of yellow is less than their situation specific critical time but greater than the yellow change interval. There are many times that a cyclist will have plenty of time to stop or plenty of time to reach the stopbar. Assuming that arrivals are independent of the onset of yellow, the probability that a cyclist, with an associated critical time greater than the clearance interval, is caught short of the stopbar can be computed. The proportion of cyclists caught short of the stopbar over the long-term is obtained by summing over all cyclists (and situations). A measure of risk is obtained that is analogous to that obtained in Section 3.3, using the deterministic approach. This measure can be used to further evaluate the adequacy of the implemented clearance interval.

**3.2.4.2 Critical Deceleration.** Consider that instead of modeling the decision based on an individual's critical time to the specified clearance point, one could model using the concept of a critical minimum constant deceleration required to comfortably stop before the stopbar. A model based on deceleration would appear to have several drawbacks. First, if individuals base their decisions on the minimum deceleration required to stop, their decision process can be summarized by: "if the deceleration required for me to stop is low enough, then I will stop". Inherent in this process seems to be a dominant desire to stop. A more appealing decision process would seem to be one summarized by: "if I am near enough to the intersection to proceed, then I will". Inherent in this process is a dominant desire to proceed, which is reflected by the assumption that the decision is based on the critical time.

Second, to compute the minimum deceleration required, the deceleration is assumed to be constant. For simplicity, constant deceleration is often assumed, but is probably not true. Third, critical times have the advantage of being directly comparable to clearance intervals, and are therefore useful in design and analysis. Finally, to compute the decelerations a representative bicycle/rider unit perception-reaction time must be assumed and the same value applied to each cyclist in the sample.

Even with the above problems, it is still not unreasonable to consider a model based on the minimum constant deceleration required for stopping ($D_{min}$). This value was computed for each cyclist, using several representative perception-reaction times within a reasonable range from:

$$D_{min} = -\frac{v^2}{2 \times \{s - [v \times (pr - vr)]\}}, ^{7} \qquad (3.14)$$

where v is the speed at the onset of yellow (ft/s$^2$),

s is the distance (ft) from the stopbar at the onset of yellow or the voice mark, whichever was used in data reduction,

pr is the bicycle-rider unit perception-reaction time (seconds), and

vr is the voice-reduction delay (seconds).

Simple models specified by equation 3.9 were estimated and the results are shown in Table 3.3. Sensitivities to the voice-reduction delay were also explored.

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[7] The denominator is adjusted to account for the voice-reduction delay for those observations where the traffic signal change is not captured on the videotape. A $D_{min}$ greater than zero indicates that the bicycle/rider unit could not begin its deceleration until after the stopbar, because it was too close to the intersection to perceive and react. In this case, it is necessary for estimation purposes to set $D_{min}$ to a very high rate (1000 ft/s$^2$), to reflect the fact that it is impossible for the cyclist to stop.

TABLE 3.3. RESULTS OF SIMPLE MODEL ESTIMATIONS TO EXAMINE WHETHER TO BASE THE MODEL ON CRITICAL TIME (TO STOPBAR) OR CRITICAL DECELERATION.

| V-R[3] Delay (sec) | **Critical Time to Stopbar Models** | | **Critical Deceleration Models** | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Perception-reaction time 0.7 sec | | Perception-reaction time 1.0 sec | | Perception-reaction time 1.5 sec | | Perception-reaction time 2.0 sec | |
| | *LL*[1] | Mean (sec) | *LL*[1] | Mean (ft/s$^2$) | *LL*[1] | Mean (ft/s$^2$) | *LL*[1] | Mean (ft/s$^2$) | *LL*[1] | Mean (ft/s$^2$) |
| 0.3 | -35.0 | 3.5 | -39.8 | -5.2 | -41.3 | -6.3 | -45.8 | -10.4 | 2 | 2 |
| 0.5 | -34.7 | 3.7 | -38.3 | -4.7 | -39.3 | -5.5 | -42.5 | **-8.3** | 2 | 2 |
| 0.7 | -34.5 | 3.9 | -37.1 | -4.4 | -37.9 | -5.0 | -40.2 | -7.0 | -45.3 | -12.9 |

[1] *LL* stands for log-likelihood value.

[2] Some models that are clearly not acceptable were not estimated.

[3] V-R stands for voice-reduction (delay).

Note: the comfortable bicycle/rider unit deceleration distribution determined by experimentation in the deterministic approach is described by: low-3.8 ft/s$^2$, 15th percentile-4.2 ft/s$^2$, median-7.7 ft/s$^2$, 85th percentile-9.7 ft/s$^2$, and high-12.3 ft/s$^2$.

The estimation results presented in Table 3.3 seem reasonable. The estimated mean critical times to the stopbar are on the order of four seconds, which is the implemented yellow change interval. Also, the estimated mean critical decelerations are close to the median comfortable deceleration of 7.7 ft/s$^2$ found by experimentation in the deterministic approach (in Section 3.1).[8]

In all cases, the critical time model explains the sample better than the critical deceleration model. The theoretically more appealing model also explains more of the sample variation, which leads to the conclusion that the decision concept of a critical time to the specified clearance point is best.

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[8] A perception-reaction time of 1.5 seconds seems to be the most representative of the sample, because its associated mean critical deceleration (8.3 ft/s$^2$) is closest to the median found by experimentation in Section 3.1 (7.7 ft/s$^2$).

### 3.2.5 Closure

A probabilistic approach using discrete choice models was used to characterize bicycle/rider unit behavior at the onset of yellow. Data was collected to estimate the models. Many variables and their interactions were investigated, and the following three Boolean categorical variables were found significant: (1) cyclist is female and wearing a helmet, (2) cyclist is coasting before and at the onset of yellow, and (3) an automobile is partially blocking the intersection. The associated inferences are preliminary because the estimation is based on a small sample of cyclists at only one intersection with very few observations associated with each variable and two of the three confounded parameter t-statistics are less than 2.0.

The model is not intended for accurate predictions that are transferable to other intersections or situations, but does provide some insight into the directions and relative magnitudes of factors impacting the critical times of university bound bicycle/rider units at low speed intersections with fairly high bicycle volumes. Any inferences are most suited to college students at intersections near campus with which they are very familiar.

One of seven proceeding cyclists ended up in a potentially dangerous position (before the near-side curbface) at the onset of the red and some (fairly weak) evidence indicated that four of the 70 proceeding cyclists wished to stop but could not do so comfortably. These suggest investigating design clearance points farther into the intersection than the stopbar to design for safety and comfort.

Probability of stopping models can be a useful tool to evaluate and design clearance intervals for bicycle-automobile mixed-traffic. However, no definitive conclusions can be drawn from model estimation results, since the design clearance point that is best for cyclist safety and comfort is unknown.

The presence or absence of traffic stopped or approaching on far-side cross-street lanes and the behavior of that traffic (e.g. "light-jumping") might impact a cyclist's decision, if the cyclist looked for it. A related factor might be the clear sight distance down those lanes for cyclists approaching the intersection. These variables were not available, but might be worth investigating in the future.

## 3.3 RISK ANALYSIS

If a longer clearance interval than that required by automobiles is required to accommodate bicycles, then signal timing must consider the trade-offs between the conflicting safety and efficiency objectives for bicycles and automobiles, especially in the presence of meaningful bicycle volumes. One measure of risk to consider is the number of bicyclists placed in danger (or discomfort) by a clearance interval of a certain length. This can be achieved by computing the proportion of cyclists that will be caught short of the design clearance point when the cross-street traffic is released. The average number of cyclists caught per hour is then obtained by multiplying this probability by the hourly bicycle traffic volume. A method for computing this proportion is developed based on the deterministic behavior modeling approach, and one could also be developed from the probabilistic modeling approach.

Using the deterministic behavior modeling approach one can compute the probability P that a representative cyclist will be in the dilemma zone at the onset of yellow, which is also the probability that the cyclist will be caught short of the design clearance point when the cross-street traffic is released. This probability is derived assuming that bicycles travel at constant speed and their arrival times are independent of the onset of yellow, which would not be appropriate if arrivals are systematically influenced upstream, e.g. by a traffic signal. Since the signal indication turns yellow once (for a given approach) in every cycle, and the bicyclist can travel the distance ($v \cdot C$) during the cycle, the bicyclist is equally likely to be at any point on the approach in the distance ($v \cdot C$) when the clearance interval begins. This distance ($v \cdot C$) includes the dilemma zone of length D, so the probability of a bicyclist being caught in the dilemma zone is simply the ratio of the length of the dilemma zone (D) to the distance that the bicyclist can travel during the signal cycle ($v \cdot C$).

$$P = D / (v \cdot C) \qquad (3.15)$$

where $P$ = probability of (1) being in the dilemma zone at the onset of yellow or (2) short of the design clearance point when the conflicting traffic is released,

$D$ = length of the dilemma zone (see equation 3.5),

$v$ = constant approach speed, and

$C$ = cycle length.

To verify equation 3.15, data was collected in 1992 at an intersection (26th and Speedway in Austin, Texas) that has considerable bicycle traffic and no systematic upstream influences on bicycle arrivals. The dilemma zone was computed using equation 3.5 for a representative bicycle/rider unit at this intersection with the following characteristics: $t_{p\text{-}r} = 1.5$ s, $d = 7.5$ ft/s$^2$, and $v = 12$ mph, determined from the earlier measurements (Section 3.1) and engineering judgment.

For this analysis the design clearance point is the far-side curbface. Measured parameters for the test intersection are cycle length, $C = 75$ s, intersection width $y = 66$ ft, and clearance interval $ci = 4$ s. The intersection width is measured from curbface to curbface, not stopbar to curbface. Therefore, it is assumed that the cyclist decision is either to stop before the near-side curbface, not the stopbar, or proceed past the far-side curbface. With bicycle length $L = 6$ ft, the theoretical dilemma zone size is 48.7 ft, and the theoretical probability that this representative cyclist is caught in the dilemma zone or intersection is 0.0368 (or 3.68 percent of all cyclists will be caught).

The position of each observed bicycle/rider unit at the onset of yellow was recorded, particularly whether or not they were in the dilemma zone. Also recorded was whether the bicycle/rider unit was in the intersection at the onset of red. A sufficient number of bicycle/rider units were observed so that the percentages caught in the dilemma zone (at the onset of yellow) and in the intersection (at the onset of red) could be statistically compared with the percentage predicted by the probability equation (3.15).

A total of 153 cyclists were observed traveling through this intersection during the day. Of these, six (or 3.92 percent) were in the computed dilemma zone at the onset of yellow, and four of these six were subsequently caught in the intersection at the onset of red. One bicyclist, travelling very slowly, stopped before the intersection, as he was not caught in the dilemma zone corresponding to his speed. Another cleared the intersection. None of the six cyclists appeared to accelerate in an attempt to clear the intersection. Seven cyclists (including the four already mentioned) or 4.58 percent were actually caught in the intersection at the onset of red. This indicates that three other cyclists were either caught in their individual dilemma zones, or they made "incorrect" decisions upon viewing the yellow.

The data seems to agree with the theory. Both the observed percentage of bicyclists caught in the dilemma zone (3.92) and the observed percentage of bicyclists caught in the intersection (4.58) are close to the predicted percentage (3.68). The sample is large enough to use the normal approximation to the binomial distribution in order to test the hypothesis that the true percentage of bicyclists being caught in this intersection (or in this dilemma zone) is 3.68, as predicted by the theory. Using a two-tailed test, one cannot reject this hypothesis or the theory at any reasonable level of significance, for either sample percentage (3.92 or 4.58). Since equation 3.5 (used in equation 3.15) embodies the logic of equation 3.2, the deterministic equation used to compute adequate clearance intervals for bicycle/rider units and the representative bicycle/rider unit characteristics are also supported by the statistical tests.

The essence of the work on which this section is based has been published (Taylor, 1993). After that publication, Wachtel et al. (1995) extended this expression of risk to consider not only the probability of a cyclist being caught in the intersection, but the duration that the cyclist remains in the intersection while the cross-street has a green.

### 3.4 SUGGESTIONS FOR ACCOMMODATING MIXED-TRAFFIC AT SIGNAL CHANGES

Warning automobile-only traffic of signal changes has been the topic of much study, discussion, and disagreement among traffic experts. The issue is even more complicated in the case of bicycle-automobile mixed-traffic, resulting in additional problems for design and implementation. Although the work presented in this chapter is preliminary, some suggestions can be made concerning how to handle potential problems. These suggestions address the situation where the vehicle is proceeding straight through the intersection, with the conflicting traffic on the cross-street. No explicit attention is given to other situations, such as when the conflicting traffic is turning left from the opposite approach or when the vehicle reacting to the signal change is itself turning.

The designer must first determine the clearance intervals that are adequate for each vehicle type, not considering the other vehicle type, at the intersection in question. The automobile clearance interval can be obtained using an accepted technique, and the bicycle clearance interval can be developed as discussed above in Sections 3.1 and 3.2.

Competing objectives may arise from the difference in clearance interval requirements for bicycles and automobiles. Because the warning of a signal change for cyclists may need to occur significantly before the warning for motorists, a single clearance interval (the combination of yellow and all-red intervals) timed for cyclists may create safety and efficiency problems for motorists. Efficiency reductions result from the loss of green time and safety problems from the introduction of a large optional zone, where a leading motorist may decide to stop and the following motorist decides to proceed, causing a rear-end collision. Conversely, a single interval timed for motorists may cause safety problems for cyclists, in addition to a possible reduction in comfort for cyclists. Other users, such as pedestrians, must also be considered as their delay and safety may be impacted.

The operating agency's objectives should also be considered, since the agency has a responsibility to provide for current roadway users, both cyclists and motorists. However, the agency may incur greater capital and maintenance costs if special provisions are made for cyclists. Finally, if additional warning signals or signs are used, roadway clutter and the dangers of over-signing are considerations.

Although there is no dominant solution that will resolve all the competing objectives, there are some designs that improve some of the objectives for some of the interested parties. The various partial solutions are categorized into (1) signals, (2) the intersection environment, and (3) modifying vehicle/operator unit behavior.

### 3.4.1 Possible Design Solutions Related to the Signal

Resolving the conflicts by judicious timing of a single traffic signal's yellow change and all-red clearance intervals would be the simplest solution. First, consider using the largest possible clearance interval for automobiles, which consists of about a five second yellow change interval and a two second all-red. Although the view is not universally accepted, Butler (1983) suggests using all-reds for automobiles following each yellow and claims they do not reduce intersection capacity. Chang et al. (1985) suggest that all-reds can be useful for automobiles when the intersection is wide and many motorists tend to enter under the latter part of the yellow.

According to the analyses in Sections 3.1 and 3.2, clearance intervals longer than the automobile design maximum (about seven seconds) may be required for cyclists. Therefore, if sufficient yellow and all-red is provided for cyclists, motorists may incur safety and efficiency impacts. One must also consider that cyclist behavior may change in response to a longer clearance interval, as some claim motorist behavior does. This cyclist issue has not received any study.

A reasonable compromise might be to use two different yellow change and all-red clearance interval pairs, implementing one during bicycle traffic peaks and the other (timed for automobiles) the rest of the time. Bicycle peaks often occur when motorist demand is low, so overall motorist delay and motorist behavior (and thus safety) may not change significantly due to the fact that motorists would infrequently experience the longer clearance intervals. All-reds longer than two seconds may work if implemented only during sufficiently short bicycle peaks.

Another technique to consider is to release the near-side cross-street traffic at the end of the automobile yellow (and possibly a short all-red), but delay the far-side cross-street traffic until the full duration of the bicycle clearance interval has passed. In this way only the motorists on the far-side of the cross-street experience increased delay and possible behavioral changes. Again, adverse impacts to motorists could be minimized if the plan is implemented only during bicycle peaks. Similarly, if leading left-turns are a reasonable option, one could begin by giving the near-side cross-street traffic a through green and protected left-turn green arrow. In this way, a cyclist only has to deal with conflicting near-side traffic and traffic merging into its lane for which it (and the turning motorist) has time to prepare.

One could also consider schemes using bicycle detection. The green for that approach could be extended (*green extension*) to allow bicycles in a predetermined dilemma zone to proceed safely, which can reduce the probability of conflict, but could not eliminate it. However, at moderate to low bicycle volumes, the probability of conflicts can probably be reduced sufficiently. Green extension does not pose safety impacts to motorists, because the yellow (and all-red) is not lengthened, and the increase in delay to cross-street motorists would be minimal.

A detection system could be implemented at all times, not just during bicycle peaks, providing greater efficiency to both motorists and cyclists than preset clearance intervals timed to accommodate bicycles. It would also be better as far as safety (from adverse behavioral changes) is concerned, because long clearance intervals would not occur.

The possible solutions involving a single warning signal can be summarized using two hierarchies. The first is a hierarchy of ways to *distribute* (among the green, yellow, and all-red) the longer time required for the bicycle to clear. This hierarchy is as follows:

(1) longer yellow, with no all-red,
(2) yellow and standard all-red (delay both near- and far-side cross-street greens by the same amount),
(3) extended green for a bicycle that is in danger (only with detection), and
(4) yellow and split all-red (delay far-side cross-street green longer than near-side).

The second, a hierarchy of *schedules* for implementing the bicycle timing plan, is as follows:

(1) at all times,
(2) only during bicycle peak times, and
(3) only upon detecting a bicycle in danger of not clearing the design clearance point.

Motorist safety and delay should improve as one moves down (with 1 being the top) either hierarchy, and any distribution can be used with any schedule, except for the green extension, which requires detection.

Instead of relying on just one signal, a separate warning signal for cyclists could be provided in a number of ways. One possibility is a warning signal upstream of the intersection programmed to begin flashing a light underneath a message such as "BICYCLES PREPARE TO STOP WHEN FLASHING", in coordination with the traffic signal. Any approaching bicyclist seeing the flashing light should stop or risk being caught short of the design clearance point at the onset of the cross-street green. The principle of this signal is the same as that for the "PREPARE TO STOP" signs currently used when stopping sight distance is insufficient for a signalized intersection.

In California, some (via internet discussion groups) have proposed timing the pedestrian signal to be an “unofficial” signal change warning for cyclists, which could have some impact on pedestrians. One could also provide an additional signal head, illuminating a yellow bicycle to warn cyclists of the upcoming signal change. Another option is to use a flashing green to warn cyclists, before the yellow indication warns motorists. These three options are very non-standard for U.S. practice and may have serious safety consequences, in addition to requiring some re-education of the motoring and cycling public.

Finally, where cyclist safety is an issue one could coordinate the signal with the immediate upstream signal(s) to systematically inhibit cyclists from arriving near the onset of yellow. Chapter 4 provides some insight into such coordination.

### 3.4.2 Possible Design Solutions Related to the Intersection Environment

The traffic engineer can also consider modifications to the intersection environment. A refuge or island could be provided and signed for use by bicycles in situations where it would be safer and more comfortable to stop in the middle of the intersection. Where feasible, this might be a cost-effective and efficient solution, since acceptable automobile-only clearance intervals might be sufficient for cyclists to safely and comfortably reach the middle of even very wide intersections, and even if some all-red is necessary, it may only be necessary for the near-side cross-street lanes. At minimum, the cyclists would be provided with two potential stopping points and a better opportunity to make a correct decision for their own comfort and safety.

Another possibility is to improve the sight angle for motorists stopped on the cross-street. This could be achieved by staggering the auto stopbars for each far-side lane, so the first automobiles in the farthest lanes have a better view of the crossing bicycle-rider unit. This sight angle could also be improved by installing a pedestrian crosswalk, even if pedestrian volumes do not require one. The latter solution would have the additional benefit of delaying the moment of conflict between the cyclist and the automobiles in every far-side lane. Finally, if the cross-street has a signal progression system, re-coordinating to prevent platoon arrivals at the onset of green should be considered.

### 3.4.3 Possible Design Solutions Related to Changing Behavior

The third and final category of solutions is to attempt to change vehicle/operator unit characteristics and behavior. For bicycle/rider units this entails reducing the perception-reaction time or increasing the comfortable deceleration. Theoretically, improvements in perception-reaction and deceleration are possible through education and training programs. However, current and past educational programs have not been very successful in changing the behavior of the majority of cyclists in traffic. A tip to teach cyclists would be to proceed through wider intersections in the middle or left-middle of the lane, if possible, to increase visibility to and distance from motorists in the far-side lanes of the cross-street.

Improvements may also be possible through the use of a simple warning sign at the intersection with a message similar to "DANGEROUS INTERSECTION: BIKES PREPARE TO STOP". Signs that repeat the obvious are usually not considered good engineering practice, but a precedent has been set with the general "bicycle warning" sign allowed by Manual on Uniform

Traffic Control Devices. This sign would provide specific guidance at those intersections where cyclist safety is in jeopardy from inadequate clearance.

Some type of pavement marking denoting the path of the crossing cyclists might be effective in increasing the awareness of cross-street motorists for clearing bicycles. In this regard, something similar to the bicycle symbol pioneered by the City of Denver, Colorado to denote the space for bicycles in widecurb lanes, or some sort of extension of a bicycle lane through the intersection may have some effect.

## 3.5 SUMMARY OF CONTRIBUTIONS AND CONCLUDING COMMENTS

Analysis of bicycle/rider unit behavior at the onset of a yellow signal indication and selection of adequate clearance intervals for bicycle-automobile mixed-traffic was approached in both a probabilistic and deterministic manner. Perhaps the primary advantage of the deterministic procedure is that it is more widely used and understood. The primary advantage of the probabilistic approach is that it is based on actual cyclist behavior, so a design based on it should accommodate the behavior of cyclists.

The main contributions from both approaches are summarized below and categorized according to whether they fall more into the area of traffic science or traffic engineering. Then, a few comments are made about the use of this research and future work in this area.

### 3.5.1 Traffic Science Contributions

The analysis of cyclist behavior at the onset of yellow suggests a design clearance point that is further into the intersection than that used for automobiles at sufficiently wide intersections. This study also provides insight into the extent to which cyclists might be placed in danger by clearance intervals determined primarily for automobile traffic. An expression, to estimate the number of cyclists caught in the intersection at the onset of the cross-street green was derived and validated with observations.

Both the deterministic and probabilistic approaches indicate that the required clearance interval for bicycles may be longer than that required for automobiles. The magnitude (or existence) of the difference depends on intersection width, the (assumed) design clearance points, as well differing operational characteristics, such as speed and deceleration.

A finding from the experimental measurements is that bicycle/rider units do not seem to be able to comfortably decelerate as quickly as car/driver units. Bicycle/rider unit deceleration is limited by both the "over the handlebar" threshold and the condition of the bicycle braking system. The acceleration bicycle/rider units might use to proceed through the intersection was also determined experimentally. The hypothesis that the perception-reaction times of bicycle/rider units are about one second longer than car/driver unit times could not be rejected, though the comparison tests were far from rigorous or conclusive.

Additional insights derived from the behavioral model include the finding that female cyclists wearing helmets and cyclists coasting at the onset of yellow seemed to be more risk-averse than the rest. Finally, and specifically with regard to bicycle-automobile mixed-traffic, partial blockage of the intersection by automobiles stopping or pausing, most likely to turn onto the cross-street, increases the bicyclist's mean critical time and hence the probability of stopping at the onset of yellow.

### 3.5.2 Traffic Engineering Contributions

The two analysis methods of bicycle stopping at the onset of yellow discussed in the previous sections were applied to analyze clearance intervals to accommodate bicycles. The probabilistic method can also be used to study cyclist (and motorist) behavior in response to changes in the clearance interval or to methods of warning cyclists of the impending signal change. This study is the first to analyze the deterministic expressions often used to determine adequate clearance intervals for automobile-only traffic for use with bicycle traffic and mixed-traffic. The contribution includes determining the required bicycle/rider unit design values for speed, deceleration, and acceleration. Finally, various suggestions concerning how to warn bicycle-automobile mixed-traffic of impending signal changes were discussed.

### 3.5.3 Applying this Research and Future Needs

This study is based on only one intersection and its associated bicycle users, and focuses only on the situation where the vehicle is proceeding straight through the intersection, with the conflicting traffic on the cross-street. As such, caution must be exercised in applying the results to similar, as well as different, situations. Further research is required to:

(1) more accurately quantify bicyclist perception-reaction time, deceleration, and acceleration,

(2) determine the likelihood that bicyclists will accelerate if proceeding,

(3) better relate bicycle speeds at intersections to various attributes of the intersection environment (e.g. grade),

(4) set a standard (in number of bicyclists caught short of the design clearance point per hour) for timing clearance intervals for bicycle-automobile mixed-traffic,

(5) give formal attention to other situations, such as when the conflicting traffic is turning left from the opposite approach or when the vehicle reacting to the yellow is turning,

(6) study (using the probabilistic choice model approach) a variety of intersection environments (especially different widths) for a variety of cycling populations, and

(7) examine recent accident studies to determine if conclusions such as Forester's (1983) (that inadequate clearance interval duration is the "largest identified facility-associated cause of car-bike collisions") are still valid. Researchers at the University of North Carolina are currently completing such a study and preliminary analysis indicates 42 of about 3000 bicycle accidents in their study occurred at intersections under a red indication, with cyclists entering the intersection under a green or yellow (Pein, 1996).

In addition to further theoretical and observational study, trial projects are likely to yield useful results regarding the effectiveness of different strategies to warn bicycle-automobile mixed-traffic of an impending signal change. After analyzing a sufficient number of trial projects, this work and that of others can be synthesized into accepted procedures and guidelines and placed in design manuals, such as the Transportation and Traffic Engineering Handbook (ITE, 1982) and the Guide for the Development of Bicycle Facilities (AASHTO, 1991).

# 4 PROGRESSION ALONG SIGNALIZED STREETS

The purpose of the study presented in this chapter is to investigate progression on streets with mixed automobile and bicycle traffic. Like automobile drivers, bicycle riders want fast trips (small delays) with limited stops. In fact, these may be more important to bicyclists. Bicycles are propelled by physical exertion, which is greatest when overcoming the inertia of a stop and accelerating back up to cruise speed, especially if stopped on an upgrade. Therefore, the disutility of a stop to a cyclist has a physical aspect as well as a similar psychological aspect to that for a motorist. Finally, stopped bicycles at intersections may receive a disproportionate share of the localized air pollution.

This study is guided by the impacts on the different vehicle types, both automobile and bicycle, and by the competing nature of the possible objectives, such as attempting to maximize bicycle progression while also attempting to maximize auto progression. Bicycle-automobile mixed-traffic progression schemes can be either *multimodal* (designed for both cars and bikes to have a progression band) or *unimodal* (designed for only one vehicle type to progress). For unimodal systems the main issues are the impacts to each vehicle type. For multimodal systems the main issue is the feasibility of providing simultaneous progression bands of sufficient widths for two vehicle platoons traveling at different speeds. Illustrating and explicitly dealing with the multiobjective nature of the situation handle these issues.

This work makes three contributions. First, the principal considerations for bicycle progression are identified; these do not appear to have been discussed elsewhere in the literature. Second, several concepts and techniques that provide alternative or better multiobjective solutions are proposed and analyzed. Third, a multiobjective formulation framework is proposed for solving the problem, formally incorporating the elements introduced as part of the first two contributions. Since there does not seem to be any literature dealing with these or other bicycle progression issues, this work lays down the conceptual foundation for analyzing bicycle progression in the context of mixed auto-bicycle traffic.

The first section provides some background for the study in the form of basic design elements of arterial progression and some analysis techniques developed for automobile progression. The following section examines some key considerations for bicycle progression that are different from those for automobile progression. The third section deals with one-way automobile-only progression systems and the next with one-way bicycle-only progression. The fifth section discusses ideal cases for one-way multimodal progression, while the next looks at more general cases. The seventh section briefly addresses two-way progression, and the eighth section discusses possible simultaneous provision of both bicycle progression and spillback prevention for automobiles. The next section comments on the impact to progression of the various on-street bicycle facility types (bikelane, wide curb lane, and standard shared lane). Finally, some concluding comments are made in the last section.

## 4.1 BACKGROUND

Much study has been directed at automobile progression on arterials. The main idea is to allow the platoon of vehicles entering the arterial section (that is timed for progression) at the first signal to proceed through subsequent signals without stopping (or with delay not greater than that caused by the critical intersection), thus reducing stops and delays on the arterial.

### 4.1.1 Basic Design Elements

Progression, as described above, normally entails offsetting the downstream green intervals from the first signal's green interval using the time that the platoon takes to travel to that downstream signal. This travel time determines the *offset* for that signal. For the progression to repeat over time, all signal *cycle lengths* must be an integer multiple of one another. In addition to selecting cycle lengths, red/green *splits* and *phasing patterns* must be chosen for all intersections.

One must consider the fact that only a limited range of automobile progression speeds is plausible in urban areas (perhaps 20 to 45 mph). One must also carefully consider the time period per cycle available for progressive travel (the *bandwidth*), as it determines the number of vehicles that can proceed through the arterial section without stopping. The bandwidth is limited by the smallest green interval on the arterial section coordinated for progression. This smallest green interval is typically required at the *critical intersection*, which usually has the greatest cross-street traffic demand, and therefore requires the largest cycle time to keep it undersaturated. Thus, other arterial intersections may have *excess* time, which can be allocated to either the arterial or the cross-street, as necessary.

Once suitable progression speeds are chosen (although this may not be straightforward for two-way progression) it is a simple matter to determine the offsets if one assumes a constant speed across all vehicles. For design purposes, it is common to assume a single constant speed (the progression speed) for all vehicles. This assumption is not quite as restrictive as it sounds, for what is really required is only that the travel time of all vehicles between intersections be consistent with the chosen speed. However, it is very restrictive in that it assumes this speed is constant over all vehicles. Some computer models have attempted to represent the effects of variability in automobile speeds that result in platoon dispersal. However, none have considered two different vehicle types with large differences in average speed.

Cycle lengths are determined by examining each intersection in isolation with the objectives of making sure they are undersaturated (in the long term) for their respective average (deterministic) demands and that they can effectively handle (in the short term) the variability in demand they commonly encounter. The principal trade-offs are between the extent to which stochastic effects are accommodated (mainly at the critical intersection), the progression bandwidth, and corresponding delays (mainly to cross-street vehicles). In practice, a single cycle time is normally chosen. However, there is a possible advantage in reducing total system delay (including cross-streets) by using different cycle lengths (Newell, 1989). Similar saturation and delay concerns are weighed when choosing splits and phasing patterns. This study will concentrate on two-phase timing.

Progression is complicated by the fact that vehicles turn onto and off of the arterial. It is common practice to allow for queues at intersections (from vehicles turning onto the arterial) to clear before the progressing platoon arrives by advancing the start of green accordingly. However, Newell (1989) makes sound arguments against this practice based on total delay. These arguments are discussed later.

### 4.1.2 Analysis Techniques

The main analysis technique used herein is the popular time-space diagram. Figure 4.1 shows an example. The bandwidth, offsets, cycle time, splits, and progression speeds are easily read or computed from these diagrams, which provide a visual aid for analyzing delays and stops to vehicles traveling on the arterial. Following Newell (1989), the time-space diagrams are drawn showing *effective* green (open space) and red (solid line) intervals. They do not specifically show the yellow change intervals, all-red clearance intervals, or the lost time due to switching.

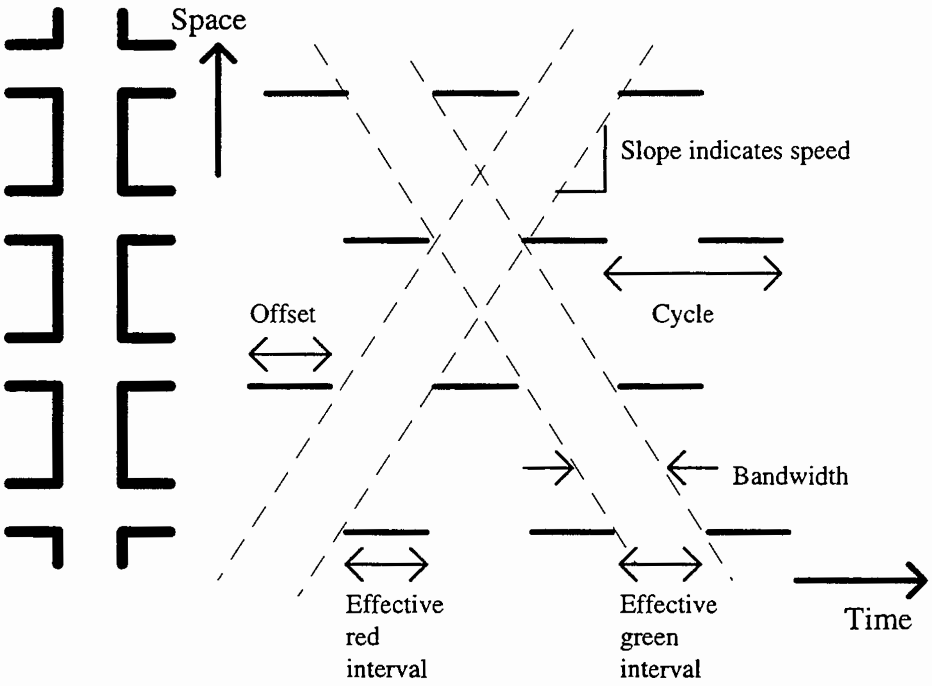

Figure 4.1. Example time-space diagram.

Criteria commonly used to analyze signal timing schemes are delay and number of stops. Delay is the difference in actual travel time and the desired travel time (travel distance divided by desired travel speed). The number of stops has various definitions. Some, such as Newell (1989), use the number of vehicles having to slow down from their desired speed. Others use the number of vehicles slowing below a certain (low) speed, which is typically less than five miles per hour. Several researchers have derived various expressions for delays and number of stops.

## 4.2 CONSIDERATIONS FOR BICYCLE PROGRESSION

In this section, key considerations for bicycle progression are introduced and discussed. In Section 4.6, these considerations are incorporated into a multiobjective formulation of the design problem.

### 4.2.1 Bicycle Speed Variability

Vehicular speed variability is probably the most important problem facing bicycle progression. The constant speed model does not seem to apply as well to bicycles as to platooning autos. Because of differences in rider desires, strength, physical fitness, and skill, as well as differences in bicycle technology and maintenance, bicycle speeds are usually approximately normally distributed between about 8 and 20 mph (Forester, 1983; Taylor, 1993;

ITE, 1976). Exact speed ranges vary according to location, differences in facilities, etc. In addition, bicycle volumes are typically low enough (in the U.S.) that platooning will probably not bring bicyclist speeds closer together. One possible solution to some of the aforementioned aspects of this problem would be to inform bicyclists of a design progression speed, selected so that a high percentage of them could maintain it. However, this brings another aspect of bicycle speed variability into the picture. That being to what extent bicyclists can maintain a target progression speed between intersections. Finally, the unpredictable aspect of a wind effect may also impact speeds.

Lacking information characterizing cyclists' abilities to learn to maintain a constant speed, one would assume that the oscillations around the progression speed would be normal, but the all important variance is unknown. This variance could be expected to shrink with practice, and there is anecdotal evidence that club and racing cyclists can maintain a very constant speed using pedal cadences and/or speedometers. It is unknown how long cyclists would take to learn (or if they could learn) to achieve variances within the tolerance needed for them to perceive some of the benefits of signals timed for bicycle progression. If it took too long, they might give up trying before any benefits were realized.

Even without any more information on bicycle speed variability, a few practical interventions are possible. First, bicyclists traveling "slightly" faster than the progression speed could obtain almost all the benefits of progression. They would arrive at an intersection "slightly" before the light turns green, but would presumably begin coasting (thus lowering their average speed) and might catch the green light while still maintaining forward motion.[1] This would suggest, posting a bicycle progression speed "slightly" higher than the progression design speed. Second, the wider the progression bandwidth, the less impact speed variability will have on the progressing cyclist, since this allows a greater probability that deviations on each side of the progression speed will "even out" before a cyclist is stopped (and delayed).

### 4.2.2 Grades

Grades have much more impact on bicycle speeds than auto speeds (Navin, 1994). Therefore, different bicycle progression speeds may be required for different street sections. Furthermore, the adverse impacts of stopping a cyclist on an upgrade must be considered. Starting on an upgrade is physically demanding for the cyclist and causes increased delay, above a level start, warranting special consideration for cyclist progression. Conversely, if progressing bicyclists must be stopped, a downgrade is preferable to a level grade.

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[1] The study of bicyclist behavior approaching a red signal indication would be of value here.

### 4.2.3 Bandwidth

The fairly low volumes of bicycle traffic in the U.S. suggest that it would be possible to design for fairly narrow bicycle progression bandwidths on most arterials, on the order of less than about 10 seconds, depending on bicycle following distances. Practically, narrow bandwidths may be problematic due to one aspect of bicycle speed variability -- maintaining the specified average speed between intersections required for progression. For example, at a bicycle progression speed of 12 mph it takes 56.8 seconds to traverse 1000 ft. If a bicyclist actually traveled at 11 mph, it would take 62.0 seconds or 52.5 seconds at 13 mph. There is a high probability that these fairly small travel speed deviations would cause a cyclist to miss a 10 second progression band in just one 1000 ft intersection spacing. The problem magnifies over larger spacing or more intersections if the speed deviations stay on the same side of the progression speed. The problem for autos is not nearly as pronounced. For similar percentage deviations in speed from 36 mph (33 to 39 mph), the travel time difference over the same distance is only one-third of what it was for bicycles, and auto progression bandwidths should be larger, due to greater auto trip volumes.

### 4.2.4 Platooning

The characteristics of bicycle "platooning" are also factors affecting the size of a bicycle progression bandwidth. Some platooning is obviously more likely if the bicycle progression speed is posted (or known). Furthermore, platooning is more likely if bicycles cannot easily pass one another. However, the extent or characteristics of bicycle platooning are unknown. For instance, vehicle headways under these conditions must be determined.

### 4.2.5 Clearing Wide Intersections

Due to the possible safety hazards of crossing wide intersections with yellow change intervals and red clearance intervals not long enough to provide safe clearance for bicycles (Taylor, 1993 and Wachtel, et al., 1995), special considerations for bicycle progression may be warranted. It may be desirable to either systematically stop progressing cyclists at sufficiently wide intersections or ensure that the bicycle progression bandwidth passes these intersections in the beginning and/or middle of their green intervals, thus avoiding the clearance interval safety problem.

### 4.2.6 Lane Changing and Left-Turning

If left-turning bicycles (off of the arterial) are significant, one needs to consider the impact of providing arterial progression for cars. Unless auto platoon dispersion is high and/or auto speeds are very low, automobile progression schemes will make lane changes into auto platoons practically impossible for bicyclists seeking to get into the left lane to make a left turn. One needs to consider making lane changing opportunities occur at the right times for left-turning cyclists. In addition, auto platoons might systematically eliminate the frontal gap in opposing traffic required for left-turning cyclists or systematically cause cyclists to just miss the left-turn phase, thus causing an additional wait time of almost one cycle.

### 4.2.7 Other Practical Considerations

It is possible that bicycle trips along the arterial might have different peak time periods than autos. This might allow unimodal bicycle and unimodal automobile progression schemes to be implemented at different times during the day. Also, if there are large bicycle flows on an automobile collector street, it might be coordinated for bicycle progression without too much delay to autos.

## 4.3 ONE-WAY AUTOMOBILE-ONLY PROGRESSION

In this section, one-way automobile-only progression is analyzed, mainly in terms of impacts to bicycles traveling on the same street. One-way refers to either a one-way street or a two-way street with only one direction designed for progression. This section starts with a review of the one-way automobile progression literature. In the next section (4.4), bicycle-only schemes are analyzed. These two discussions of unimodal vehicle schemes illustrate both the multiobjective nature of the situation and several techniques that can provide alternative or often better multiobjective solutions. In Sections 4.5 and 4.6, multimodal schemes are discussed, and this multiobjective nature is formally handled.

Most design guides present one-way automobile progression as a simple process of choosing a plausible progression speed and computing offsets by dividing the intersection spacing by that speed (ITE, 1976; USDOT, 1985; and ITE, 1992), with possible consideration of queue clearing before the platoon arrives. Newell (1989) critiques certain aspects of current practice and shows that significant total delay reductions (including cross-streets) are possible by operating intersections upstream of the critical intersection at one-half the cycle time of the critical intersection, though possibly increasing stops along the arterial. He also argues against the common practice of always clearing the initial queue (from traffic turning onto the arterial) before the progressing platoon reaches it. He points out that this causes the green to end earlier, thus cutting off the end of the platoon. The result is greater total delay to arterial vehicles, since the cut-off vehicles are delayed for the entire red interval versus the few seconds of delay saved for the initially queued vehicles. Also, some platoon spreading may be condensed by not clearing the queue, resulting in an increased saturation flow rate for that intersection. Thus, total delay can be decreased by ensuring that the last vehicle of a progressing platoon clears each intersection, even if the first few vehicles in the platoon must slow for the queued vehicles in front of them.

Newell (1989) also recognizes that the bandwidth, delay, and stops constraints for progressing vehicles are rooted at the critical intersection. If signal timing upstream of the intersection is such that a vehicle arrives at the critical intersection at or before it is scheduled to leave that intersection (i.e., the critical intersection green is fully utilized), then delay and stops to arterial vehicles are kept to a minimum for that particular critical intersection green interval. Stops can be dispersed over many intersections (not just the first one), while still keeping the number of stops and delay on the arterial at a minimum. There are many such solutions for upstream signal offsets. Delay and stops are not increased if downstream intersections (from the critical intersection) are timed to keep the entire platoon progressing (Newell, 1989).

Because the bandwidth is constrained by the critical intersection, Newell (1989) argues that there is little justification to allocate any "excess" green at non-critical intersections to the arterial. It may be best to give it to the cross-street to reduce overall delay. This is contrary to popular practice.

The use of vehicle actuation in arterial progression systems is not common. However, Newell (1989) presents a "risk-free" actuation strategy that is basically a "deviation from a pre-timed plan." He suggests using a pre-timed strategy providing for the average demand, then deviating from this strategy by:

(1) terminating the arterial green after the last platoon vehicle passes (not waiting for stragglers) and giving the excess time to the cross-street.

(2) possibly advancing the arterial green to clear queues caused by greater than average turning traffic (especially downstream of the critical intersection), but not if cross-street queues have not yet cleared. Also, do not end the arterial green any earlier if this is done.

This basically gives any (stochastic) excess time to the cross-street and allows for variations in turning traffic, while never delaying the arterial platoon more than a pre-timed strategy would. Newell estimates a 25 percent upper bound on delay reduction from this actuated scheme. However, only pre-timed plans are considered in this study.

In addition to considering the entire through bandwidth, one could look at other related objectives such as maximizing progression opportunities (Wallace and Courage, 1982). A progression opportunity being the number of successive green signals encountered at the design speed without stopping, even if they cover only a part of the entire progression section. An extension of this idea may have merit in mixed-traffic progression systems. One could envision a system with a continuous through bandwidth for automobiles, along with numerous progression opportunities for bicycles, especially to get cyclists over hills without having to stop. It is also possible to provide a continuous through bandwidth for bicycles, with numerous progression opportunities for autos.

While automobile-only progression schemes have documented benefits for automobiles, including reductions in stops and delay to arterial automobiles (ITE, 1976), the possible negative impacts on bicycles traveling on the same street do not appear to have been analyzed or reported. Besides the possibility of systematically inhibiting lane changing or left-turning cyclists (on two-way facilities), an auto progression scheme might systematically stop bicyclists at many of the arterial signals. Hereafter, the effects of the signal coordination on bicycle delay (and stops) are analyzed. In addition, the allocation of green time to either the arterial (to improve progression) or the cross-streets impacts the delay to vehicles traveling on the cross-streets.

Among others, one can envision the following three objectives for an arterial progression design: (1) maximize automobile progression, (2) maximize bicycle progression, and (3) minimize total cross-street delay. The objectives conflict with each other to a great extent. The following discussions will illustrate these conflicts, as well as some techniques that provide greater flexibility in forming solutions, thereby helping create alternative or often better multiobjective solutions.

### 4.3.1 Increase (Vary) Bicycle Progression Speed

In the worst case, an auto progression scheme could systematically stop a bicyclist traveling at a specific speed at every red interval, thereby inflicting the maximum number of stops and delay on the cyclist. This would occur, for example, for bicycles traveling at a speed ($V_b$) that is half the auto design speed ($V_a$) on a street with evenly spaced signals and a single alternate timing plan. Figure 4.2 shows this case for intersections up to and including the critical intersection ($I_{cr}$). The effective bicycle speed is only $V_a/3$, though the cyclist's actual travel speed is $V_a/2$. If bicycle volumes are at all significant (or one wishes them to be significant) this situation should probably be improved.

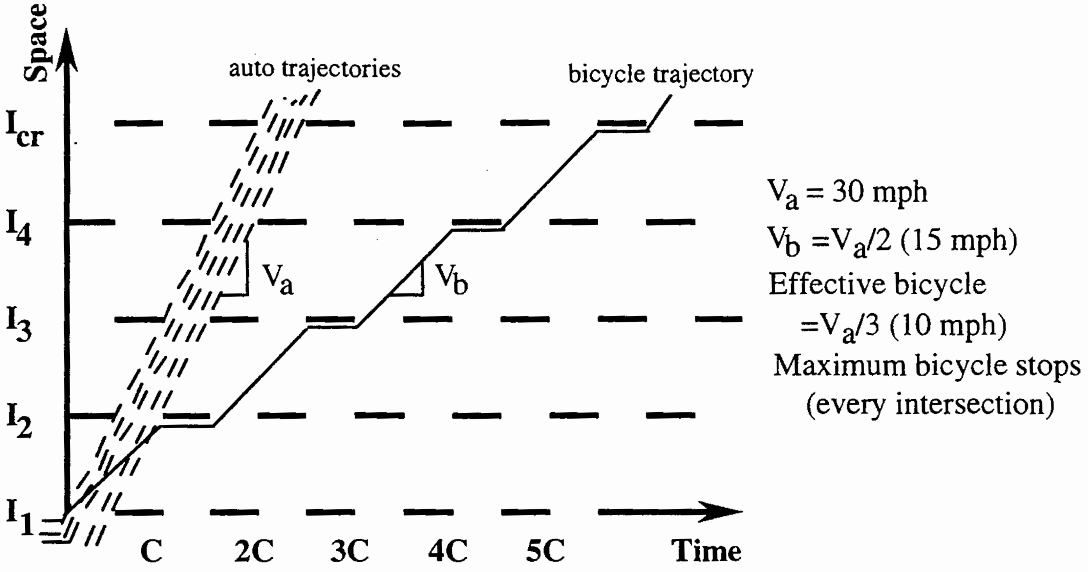
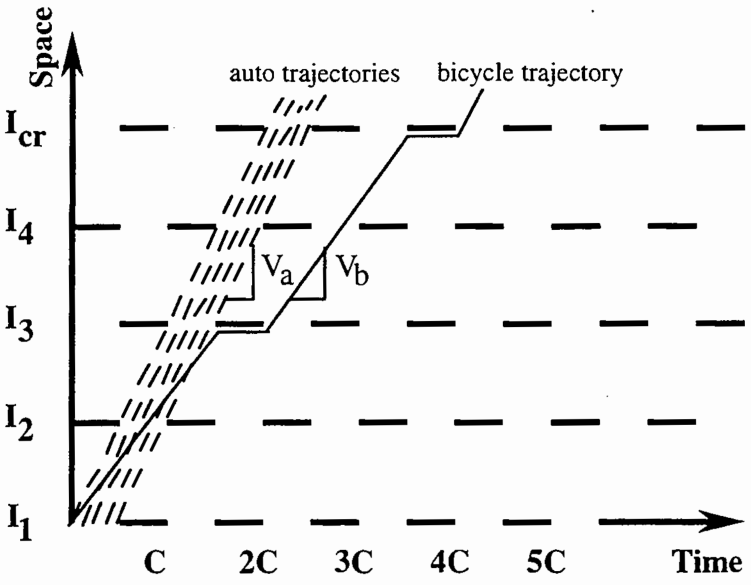


Figure 4.2. Example worst case for bicycle delay/stops from auto progression

The first thing one might consider is to try to increase the bicyclists' speeds by posting a progression speed. However, this may require a physical output neither possible nor comfortable for many cyclists. Therefore, the possible increase in bicycle speed is limited. Any increase in speed $V_b$ from $V_a/2$ up to $2V_a/3$ will result in the improvements diagrammed in Figure 4.3, for the pedagogical example. Bicyclists now achieve an effective speed of $V_a/2$ and a 50 percent reduction in stops (over Figure 4.2), but must pedal hard enough to travel faster than $V_a/2$. Theoretically, the cyclist need only travel at a speed slightly higher than $V_a/2$. However, speed variability and the fact that just making lights under the yellow may be dangerous argue for proceeding at a significantly higher speed than $V_a/2$. Increasing bicycle speeds to over $2V_a/3$ brings greater benefits. However, even if the auto progression speed is only 30 mph, $V_b$ would then need to be above 20 mph, to get those extra benefits.



$V_a = 30$ mph
$V_a/2 < V_b < 2V_a/3$
(15mph $< V_b <$ 20mph)
Effective bicycle speed
$= V_a/2$ (15 mph)
Bicycle stops = 1/2 max.
(every other int.)

Figure 4.3. Effect of increasing bicycle progression speed.

### 4.3.2 Allocation of Excess Green Time

One could also provide a (narrow) progression band for bicycles at speeds of $V_a/2$ by giving some of the excess time at non-critical intersections ($I_1$, $I_2$, $I_3$, $I_4$) to the arterial, as shown in Figure 4.4. This will cause total delay to autos (including side-streets) to be above the minimum, but should result in less total delay to autos than giving all the excess time to the arterial, as is commonly done. If one lengthens the arterial green at $I_2$ and $I_4$ by say, ten seconds, while shortening the red accordingly, a bicycle progression bandwidth of ten seconds is provided. This scheme presents a safety concern in that it increases the probability that cyclists will enter some intersections around signal changes.

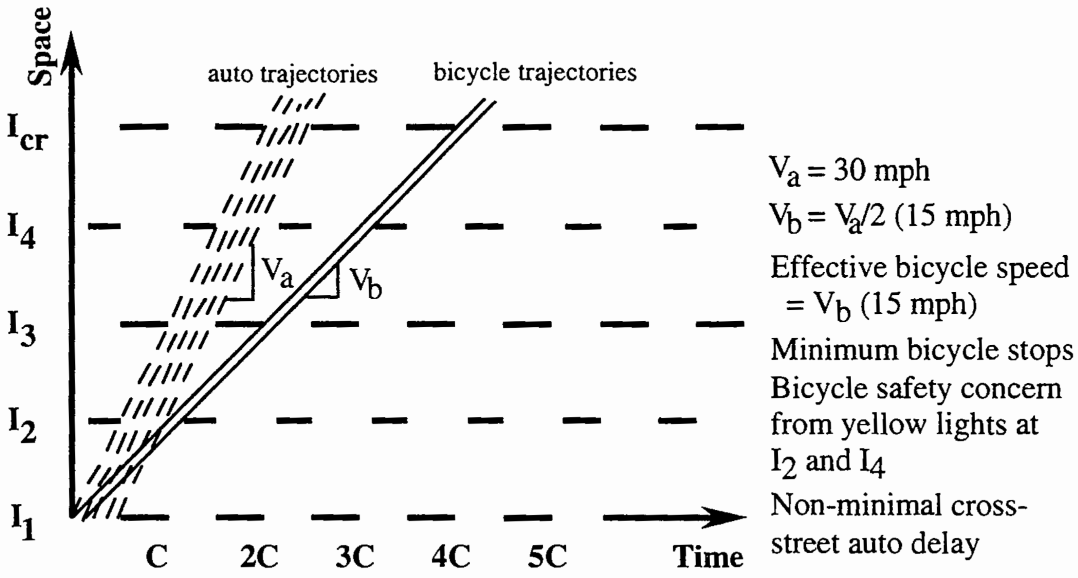
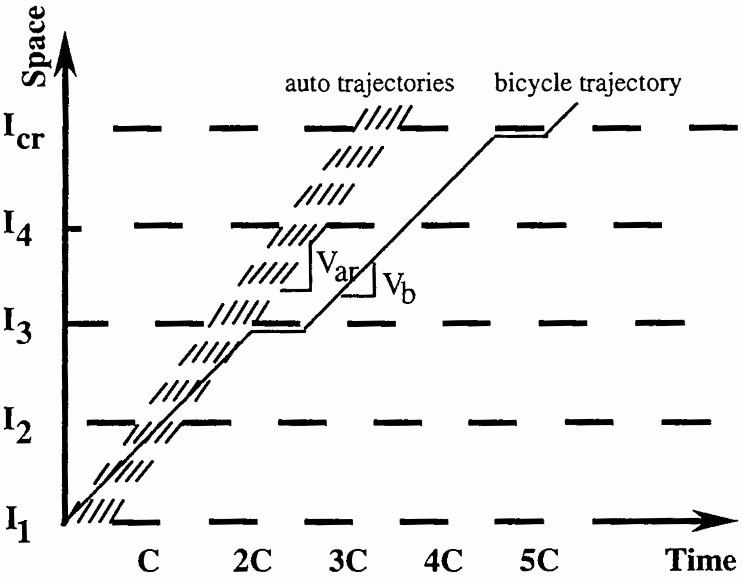


Figure 4.4. Allocation of excess green time to "sneak-through" a bicycle progression bandwidth.

This is only one special case showing a possible use for different allocations of excess green time. Allocating excess green time is a very general technique for trading-off increased cross-street delay with arterial progression benefits.

### 4.3.3 Lower (Vary) Automobile Progression Speed

Instead of, or in addition to, raising bicycle speeds, one could consider lowering (or raising) auto speeds to achieve delay and stops benefits for bicyclists. Lower auto speeds would also provide a more comfortable bicycling environment (FHWA, 1994). To see this, compare Figure 4.5 to the worst case example in Figure 4.2. In Figure 4.5, the auto progression speed has been reduced ($V_{ar}$) to two-thirds of the speed in Figure 4.2. Although conditions have improved for bicyclists, autos incur increased delay on the arterial due to the slower progression speed. In the limit of Figures 4.3 and 4.5, one would have autos and bikes progressing at the same speed. This is probably feasible only where the progression speed would be around 20 to 25 mph, which would require very fast bicyclists or a downgrade and very slow auto speeds.



Reduced auto speed, $V_{ar}$
= $2V_a/3$ (20 mph)
Original auto speed, $V_a$
= 30 mph (see Figure 2)
$V_b=V_a/2=3V_{ar}/4$ (15mph)
Effective bicycle speed =
$3V_{ar}/5$ (12 mph)
Bicycle stops =
1/2 of the maximum
Increased arterial auto
delay due to slower auto
speed

Figure 4.5. Reduced automobile progression speed.

Lowering automobile progression speeds on arterials is likely to encounter opposition by both motorists and traffic engineers. This option is probably most practical on collector streets, which can serve as arterials for bicyclists and are streets where high auto progression speed is not a very important objective.

### 4.3.4 Half-Cycle Times Upstream of Critical Intersection

As Newell (1989) points out, one could achieve, under certain conditions, the same delay for arterial autos by operating intersections upstream of the critical intersection ($I_{cr}$) at half the cycle time of $I_{cr}$. If the intersections are still undersaturated, the auto cross-street delay is cut approximately in half. Actually, any integer fraction is possible. However, cycle times less than one-half will probably be too short to be practical. Besides being beneficial to automobiles, this strategy may benefit bicycles in certain situations. Figure 4.6 illustrates improved overall delay conditions for autos (over Figure 4.2), though arterial stops increase, while also providing substantial benefits to bicycles. Bicycles now have a fairly wide progression band and do not systematically arrive at intersections at signal change points. However, twice as many non-critical intersection signal change points now exist. The impact of this technique does not extend downstream of $I_{cr}$.

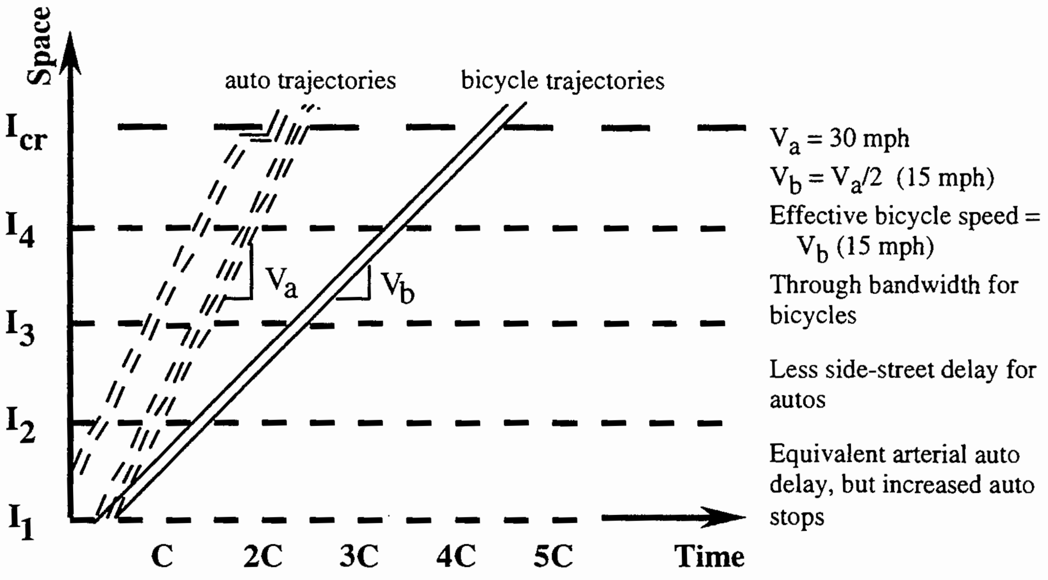
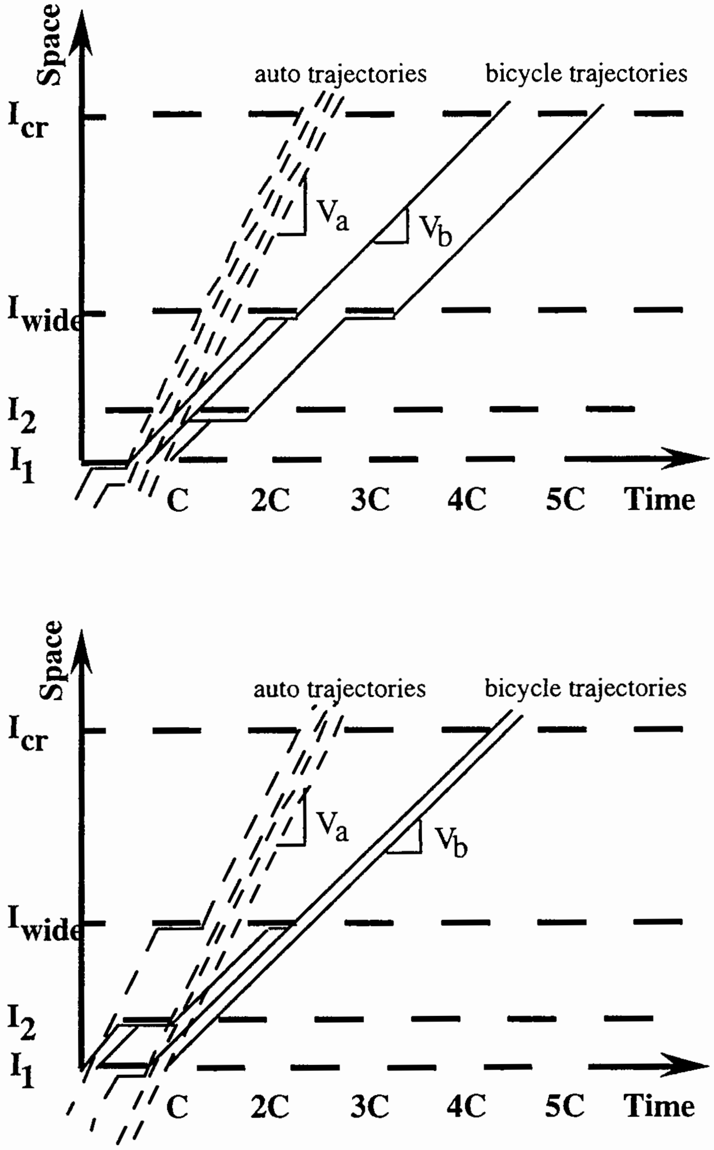


Figure 4.6. Half-cycles for non-critical upstream intersections.

### 4.3.5 Stop Dispersal

Newell's (1989) stop dispersal technique may also provide opportunities to improve bicycle progression, since there are numerous possible solutions for upstream (of $I_{cr}$) intersections that minimize arterial delay and stops for autos. Figure 4.7 shows one such possible instance that offers a substantial bicycle progression improvement over the base case. The key to Newell's method is that stopping caused by the critical intersection's minimum effective green interval will occur no matter what, so one must just assure that no additional stops or delay are caused. As for the half-cycle technique illustrated in Figure 4.6, the impact of this technique does not extend downstream of $I_{cr}$.



Base case: Maximum auto through bandwidth

$V_a = 30$ mph

$V_b = V_a/2$ (15 mph)

Possible safety problem, all bicycles stop at wide intersection, $I_{wide}$, which is on an upgrade

Stop dispersal: by advancing green at $I_1$ and $I_2$ 1/4 cycle

$V_a = 30$ mph

$V_b = V_a/2$ (15 mph)

Through bandwidth for bicycles.

Same delay and stops for autos.

75% reduction in bicycle stops at wide, upgrade intersection, $I_{wide}$.

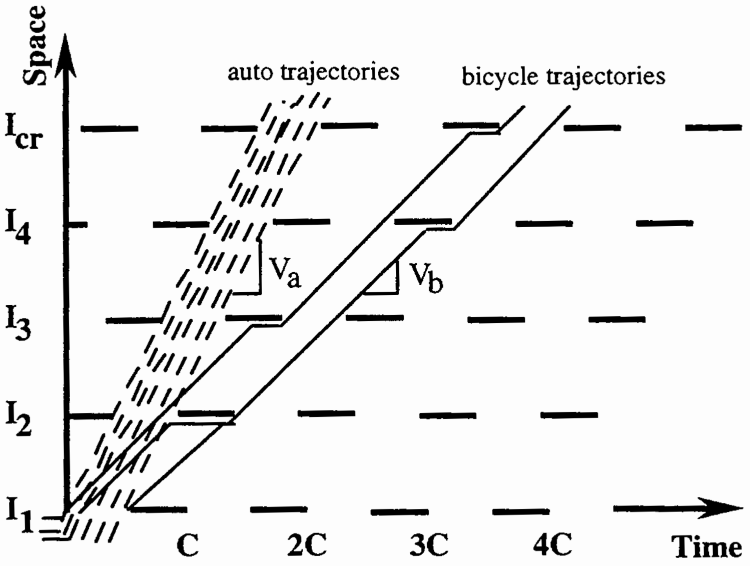
Fewer total bicycle stops and less total bicycle delay.

Figure 4.7. Newell's stop dispersal technique.

The technique consists of retarding greens (from the continuous through bandwidth) until vehicles leaving those intersections arrive at $I_{cr}$ just in time to keep $I_{cr}$ busy when the queue vanishes. Notice that in the base case of maximum auto through bandwidth, all the auto stops occur at the first intersection, $I_1$. The same number of auto stops occurs in the stop dispersal case, however they are dispersed over two intersections, $I_1$ and $I_{wide}$. Auto delay is also the same. While providing equivalent delay and stops to autos, using auto stop dispersal in this example yields several benefits to bicycles on the arterial: (1) a through bicycle bandwidth, (2) less bicycle delay, (3) fewer total bicycle stops, and (4) fewer bicycle stops at the wide, upgrade intersection, $I_{wide}$.

### 4.3.6 Increase (Vary) Cycle Time

Adjusting the critical intersection cycle time could also be used to improve conditions for bicycles. Figure 4.8 shows improvements possible over the base case (in Figure 4.2) by increasing the cycle time by 25 percent. It should be noted that in many cases only narrow ranges of cycle times may be tolerable, since most communities have an upper limit on acceptable cycle times. Consideration of such a strategy also requires careful assessment of the trade-offs between increased delay and progression benefits to bicycles.



$V_a = 30$ mph

$V_b = V_a/2$ (15 mph)

Fewer arterial bicycle stops

Less arterial bicycle delay

Increased total auto delay

Figure 4.8. Increased cycle time.

### 4.3.7 Reduce Automobile Bandwidth

If the previous techniques cannot produce a solution with adequate bicycle progression, then one can adjust specific signal offsets to create one. This will cause the auto through bandwidth to be reduced and a subsequent increase in arterial auto stops and delay. This is a trade-off of auto progression for bicycle progression.

### 4.3.8 Closure

The cases shown in Figures 4.2 through 4.8 are only meant to illustrate instances where one-way automobile progression schemes can negatively impact bicycles traveling on the same facility and present some concepts and techniques for reducing those impacts. The techniques discussed do not provide improvements for all cases of one-way progression. In Section 4.6, these techniques are incorporated into a multiobjective formulation of the design problem.

The problem of systematically inhibiting lane changing or left-turning cyclists adds much complexity when addressed in combination with delay and stops. The delay and stops concerns above should probably be addressed first. Then one can analyze (possibly with time-space diagrams) the relative positions of bicycles to progressing auto platoons when the bicycles are apt to begin a lane change, giving priority to those areas where left-turning bicycle volumes are high. It would be best if bicycles could make their lane changes just after an auto platoon passes. If they make their changes just before an auto platoon arrives, they might delay the auto platoon in the left-most lane.

It is also possible to determine the delay to left-turning cyclists caused by oncoming automobile platoons progressing on a two-way street. Probably the most that could be done here is to attempt to ensure that left-turn phasing (either leading or lagging) is most appropriate for bicycle and auto delay at high volume left turn signals. One could also consider the possibility that bicycles can make a left-turn as a pedestrian, which would also eliminate the lane change. However, this will add delay to the turn and is not the "favored" way for a vehicular cyclist to make a left turn. Therefore, some cyclists will not even consider this maneuver.

## 4.4 ONE-WAY BICYCLE-ONLY PROGRESSION

Methods and techniques developed to produce automobile only progression schemes are applicable to bicycle only schemes, assuming a constant bicycle speed. To keep deterministic and stochastic delays under control the demands of the most constraining vehicle type (probably the auto) should be used to determine required cycle times and green intervals. If the signals are coordinated for bicycles then cyclists receive stop, delay, and other benefits similar to motorists in car only schemes. However, now the negative impacts to the auto drivers must be considered. In addition, speed variability, safety, bicycle bandwidth sizing, and other key considerations for bicycles must be addressed.

The negative impacts to automobiles of providing progression for bicycles can be analyzed with time-space diagrams in a similar manner as was done for bicycles in the previous automobile-only section. The objective here would be to minimize the negative impacts to automobiles. To increase progression opportunities and decrease delay and stops for autos, while still maintaining sufficient bicycle progression, one can change the progression speeds for both vehicle types (within certain ranges), allocate excess green time at non-critical intersections to either the cross-street or arterial, use half-cycle times at all intersections upstream of the critical one, use stop dispersal while keeping the critical intersection as busy as possible, change the critical intersection cycle time, and/or reduce the bicycle bandwidth. It is then a trade-off as to whether the benefits of bicycle progression outweigh the negative impacts to autos, in terms of both progression on the arterial and delay on the cross-streets.

If detailed information on turning bicycles (onto and off of the progression section) is available, then it is possible to use unequal green split schemes to attempt to accommodate them. One should keep in mind the penalty of chopping off the end of an auto (or bicycle) progression platoon and avoid doing so.

## 4.5 ONE-WAY MULTIMODAL BICYCLE-AUTOMOBILE PROGRESSION - IDEAL CASE

Multimodal schemes would provide some benefits of progression to both bicycles and motor vehicles simultaneously. However, the feasibility of providing maximum progression for two different vehicle speeds simultaneously may be problematic for all but very special cases, such as equally spaced signals. These ideal cases are discussed in this section. In the next section, the more general case involving trade-offs between multiple objectives is analyzed.

Consider the ideal case of equally spaced signals and fairly constant grades, where it is assumed that bicycles can maintain a constant progression speed (though it may need to be posted for them to learn to maintain it). For equally spaced signals one-way (and two-way) progression bands can be simultaneously provided for several different speeds. It is very probable that some of these speeds can serve bicycles, while one serves cars. Table 4.1 shows the various bicycle progression speeds possible as fractions of the auto progression speed ($V_a$) for different timing schemes. It also gives examples of the bicycle progression speeds possible for an auto progression speed of 40 mph. Finally, it is possible to "sneak-through" a few narrow bicycle progression bands at different speeds using the same technique shown in Figure 4.4. These "sneak-through" speeds are also shown in Table 4.1. It should be noted that the bandwidths for double and triple alternate schemes are at most only one-half and one-third of the critical intersection effective green interval, respectively.

TABLE 4.1. IDEAL CASES FOR MULTIMODAL PROGRESSION - EQUALLY SPACED SIGNALS.

| | Simultaneous | Single Alternate | Double Alternate | Triple Alternate |
|---|---|---|---|---|
| Progression Speeds | $V_a$, $V_a/2$, $V_a/3$, $V_a/4$ | $V_a$, $V_a/3$, $V_a/5$, $V_a/7$ | $V_a$, $V_a/5$, $V_a/9$ | $V_a$, $V_a/7$ |
| Auto Progression Speed ($V_a$) (mph) | 40 | 40 | 40 | 40 |
| Bicycle Progression Speeds (mph) | 20, 13.3, 10 | 13.3, 8 | 8 | 5.7 |
| Bicycle "Sneak-Thru" Progression Speeds | $2V_a/3$, $2V_a/5$, $2V_a/7$ | $V_a/2$, $V_a/4$ | $V_a/2$, $V_a/4$ | $V_a/3$, $V_a/6$ |
| Bicycle "Sneak-Thru" Progression Speeds (mph) | 26.7,16, 11.4 | 20, 10 | 20, 10 | 13.3, 6.7 |

As Table 4.1 shows, equally spaced intersections can be timed for multimodal one-way (and two-way) bicycle-automobile mixed-traffic progression. It may even be possible to satisfy the speed desires of a variety of bicyclists simultaneously, solving several aspects of the bicycle speed variability problem. However, use of these schemes is further limited by the range of intersection spacing that yields reasonable cycle times (from about 30 to 90 seconds). Table 4.2 shows these ranges for auto progression speeds of 30 and 40 mph. Simultaneous schemes are only possible with wide intersection spacing. Single alternate schemes may be feasible for some reasonable block lengths, as long as fairly low speeds and cycle times are possible. Though double and triple alternate plans work for more realistic block lengths, they also have smaller progression bandwidths. One can see that the ranges for bicycle only progressions (speeds of 10 and 15 mph) are much less inhibiting, especially when one considers designs that allow cyclists' travel time between intersections to be an integer multiple of the cycle time. Then, intersection spacing greater than those in Table 4.1 is possible with the same cycle time.

TABLE 4.2. RANGES OF INTERSECTION SPACING THAT YIELD REASONABLE CYCLE TIMES FOR VARIOUS EQUALLY SPACED INTERSECTION TIMING PLANS.

| | | *Intersection Spacing (ft) for a Progression Speed of* | | | |
|---|---|---|---|---|---|
| *Timing* | *Cycle Time* | **40 mph** | **30 mph** | **15 mph** | **10 mph** |
| **Simultaneous** | **30 seconds** | 1760 | 1320 | 660 | 440 |
| | **60 seconds** | 3520 | 2640 | 1320 | 880 |
| | **90 seconds** | 5280 | 3960 | 1980 | 1320 |
| **Single alternate** | **30 seconds** | 880 | 660 | 330 | 220 |
| | **60 seconds** | 1760 | 1320 | 660 | 440 |
| | **90 seconds** | 2640 | 1980 | 990 | 660 |
| **Double alternate** | **30 seconds** | 440 | 330 | 165 | 110 |
| | **60 seconds** | 880 | 660 | 330 | 220 |
| | **90 seconds** | 1320 | 990 | 495 | 330 |
| **Triple alternate** | **30 seconds** | 293 | 220 | 110 | 73 |
| | **60 seconds** | 587 | 440 | 220 | 147 |
| | **90 seconds** | 880 | 660 | 330 | 220 |

## 4.6 ONE-WAY MULTIMODAL BICYCLE-AUTOMOBILE PROGRESSION - GENERAL CASE

The ideal case of equally spaced intersections, fairly constant grades, and intersection spacing within the feasible range covers only a small fraction of the streets one might want to time for multimodal progression. Therefore, a more general process is needed. This process must recognize the multiobjective nature of the situation, that objectives such as maximizing bicycle progression, maximizing auto progression, and minimizing total cross-street delay conflict with each other, as was clearly illustrated in the previous two sections. In non-trivial problems with multiple objectives, a solution cannot be chosen without incorporating the decision-maker's preferences. The "best" solution cannot be arrived at objectively. The *preferred* solution depends on how the decision-maker "weighs" the objectives (i.e., which are more important and by how much). In the extreme cases one would prefer a unimodal progression scheme, either auto or bicycle only.

Though no solution can be found using objective methods only, objective methods can be used to significantly aid the decision-maker. These methods have been formalized and are presented in a variety of texts, including Goicoechea et al. (1983) and Chankong and Haimes (1983). Consider a vector of objectives, $Z = [Z_1, Z_2, ....., Z_p]$, with each objective a function of a vector of decision variables, $x = [x_1, x_2, ......, x_n]$. One can then seek a preferred x vector, such that

$$\text{max-dominate } Z(x) = [Z_1(x), Z_2(x), \ldots\ldots, Z_p(x)] \qquad (4.1)$$

$$\text{subject to } \quad g_j(x) \leq 0, j = 1, .., m.$$

Max-dominate conveys the intent to identify the set of *nondominated* solutions. A solution is nondominated if there is no other feasible solution that performs better on every objective. The m constraint equations ($g_j(x) \leq 0$) define the *feasible region* of solutions. The set of nondominated solutions is often referred to as the *efficient frontier*. Figure 4.9 illustrates the feasible region and the efficient frontier in objective space for a hypothetical biobjective example. Notice that one cannot choose a solution objectively, since the ideal solution (where all objectives obtain their maximum values) is not feasible.

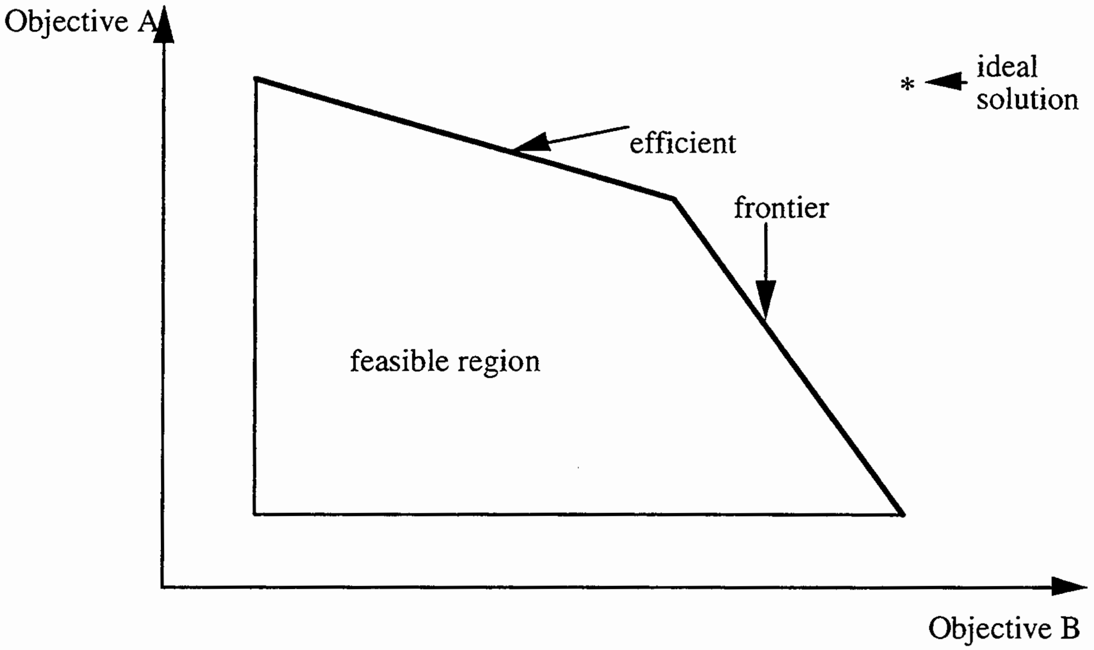


Figure 4.9. Illustration of an efficient frontier in biobjective space

Methods have been developed to generate the efficient frontier. These include the *weighting* and *epsilon-constraint* methods. In the weighting method, numerous single objective optimization problems are solved. Each of these single objectives is a differently weighted function of the entire vector of objectives Z(x). In the epsilon-constraint method, a single objective $Z_a(x)$ is maximized, subject to minimum achievement level constraints for all other objectives in Z(x). By varying the achievement levels, different nondominated solutions are found. These methods require minimal decision-maker preference information. The efficient frontier is generated objectively and is then presented to the decision-maker in a manner that illustrates the trade-offs involved in the decision. The decision-maker then uses this information as an aid in making the decision.

In this section, a framework is presented for a multiobjective decision making formulation for the problem of multimodal bicycle-automobile progression design, and it is illustrated with a fairly simple example. The formulation framework is simplified by considering only two-phase control and not addressing actuated signal control or turning traffic. Also, pedestrian crossing requirements are not discussed, though obviously they must be considered. In the first subsection, the general process for developing possible solutions is presented in narrative form. In the second subsection, the general process is developed into a multiobjective formulation framework. In the third subsection, a simplified math program is developed from the formulation framework, which allows us to illustrate its use in developing possible solutions and examining trade-offs between the chosen objectives.

### 4.6.1 General Process

Consider the following process to help resolve the competition between whatever appropriate objectives the decision-maker may wish to consider. It could be done in an add-hoc iterative manual way with time-space diagrams or programmed using optimization procedures to save time and effort. It is still assumed that fairly uniform bicycle speeds can be obtained from a combination of information (signing) and learning.

First, compute a cycle time and red/green split for the critical intersection necessary to keep it undersaturated and handle stochastic effects to the extent desired. With these as inputs along with signal spacing and non-critical intersection saturation and stochastic delay constraints, one can iterate until an acceptable solution is found, assuming one can find satisfactory trade-offs between the objectives.

In the iterations, both the automobile and bicycle speeds could be varied over appropriate ranges for the facility in question (somewhere between about 20 to 45 mph for autos and 10 to 15 mph for level-grade bicycles). One could adjust the amount of arterial green at all non-critical intersections (keeping them undersaturated in the long-run and with tolerable stochastic delay), trading-off delay to cross-street vehicles with progression opportunities for the arterial vehicles. The critical intersection cycle time could be varied. Both the possibility of operating upstream non-critical intersections at half the cycle length of the critical intersection and Newell's stop dispersal method could be considered. For safety, one could constrain progressing bicycles to prevent them from encountering the yellow signal at specified "wide" intersections. It would be preferable to attempt to ensure that bicycle progression opportunities are provided over all significant hills. One could also try to prevent lane changing and left-turning problems for cyclists.

Obviously, there are many feasible nondominated solutions to this design problem. As discussed earlier, the preferred solution depends on how the decision-maker "weighs" the objectives, with the unimodal progression schemes (i.e. auto or bike only) being the extreme cases.

### 4.6.2 Multiobjective Formulation Framework

It is obvious that manually iterating to an acceptable solution might be quite time consuming, and one would never know if a better solution had been missed. Therefore, a multiobjective math programming formulation might prove to be a significant aid to the design engineer. Before presenting the formulation framework, it should be noted that trade-offs between bicycle progression and auto progression have some similarities to trade-offs between different directions in two-way automobile-only progression. Therefore, a review of the graphical and programmable techniques developed to design two-way auto only progression schemes might provide some insights and ideas for the development of the multiobjective formulation framework. A brief review is provided next.

A graphical technique devised by Kell allows one to manually develop efficient coordinated two-way timing plans (ITE, 1976). The technique is based on the fact that progression with maximum bandwidths in both directions can be achieved if signal spacing is such that the inter-signal travel times are multiples of one-half the cycle length. A similar approach is detailed in the Traffic Control Systems Handbook (USDOT, 1985).

Manual techniques, such as Kell's, are limited by the time necessary to explore various solutions. This motivated the development of programmable techniques. Programs that generate time-space diagrams, but do not optimize solutions, are helpful. Sabra and Stockfisch (1995) review several of these. However, these may be most useful when used in conjunction with an optimizing program, to view and fine-tune the solution. Brook's interference algorithm and Little's optimized unequal bandwidth equation have been combined in the popular PASSER-II progression optimizing computer program (ITE, 1992; TTI, 1991; and Messer et al., 1973). Among other computer applications, a mixed-integer linear programming technique for progression optimization has been implemented in MAXBAND (Gartner et al., 1975a; Gartner et al., 1975b; and Little et al., 1981).

Most optimization programs maximize bandwidth. However, maximizing bandwidth does not necessarily minimize delay, stops, or other performance criteria. Nevertheless, bandwidth maximizing computer programs are the most popular method of obtaining progression timing plans (ITE, 1992). As discussed previously, one could also consider "maximizing" other progression measures, such as progression opportunities (Wallace and Courage, 1982).

Consider the following general framework for a multiobjective formulation. The philosophy behind this framework is to constrain (constraints 1, 2, and 3) both arterial and cross-street delay to remain below some predetermined tolerable level. Then values for the decision variables can be sought that achieve nondominated solutions to any progression problem involving n intersections. These nondominated solutions can be displayed and analyzed to aid the traffic engineer in making the trade-offs involved in selecting a solution.

For several reasons, this formulation is developed only as a framework. First of all, because of the uncertainty surrounding bicycle speed variability, it is probably not prudent to pursue a detailed formulation at this time. Secondly, there are many possible objectives, some of which may be more appropriate in different situations. Consideration of the relative merits of these is left to future work. Next, in an attempt to be comprehensive, all of the key considerations for bicycle progression and the concepts and techniques to improve the multiobjective solutions are incorporated in the framework. However, some of these add considerable computational burden and may not be appropriate in all cases. Decisions as to the best specific math program formulation will be case specific. Recommendations regarding which considerations to explicitly incorporate in mathematical form (at increased cost and complexity) or which to leave to the judgement of the engineer are left to future work. This formulation is intended as a broad framework to help guide and direct that future work.

Following are the objectives for the formulation framework:

(1) *Maximize auto progression.* This objective could be to maximize the auto progression through bandwidth, as is most commonly done. Alternatively, one could use other progression objectives, such as Wallace and Courage's (1982) progression opportunities, Gartner et al.'s (1991) multi-band objective or one could formulate an objective that could take advantage of stop dispersal, while keeping arterial delay and stops at the same level as for maximizing through bandwidth. There are probably numerous other ways to formulate this objective.

(2) *Maximize bicycle progression.* The same objective formulations are possible as discussed above for automobiles.

(3) *Minimize various delay components.* This objective could be to minimize all vehicular delay. It could be subdivided by vehicle type, bicycle and automobile, and/or street type, side-street and arterial.

(4) *Minimize various stop components.* One could minimize various stop objectives, which can be subdivided similarly to the delay objectives above.

(5) *Other possible objectives.* Any other relevant objectives that can be expressed mathematically can also be included.

Following are the decision variables for the formulation framework:

(1) $\mathbf{V_a}$: auto progression speed.

(2) $\mathbf{V_b}$: bicycle progression speed.

(3) $\mathbf{C_{cr}}$: critical intersection ($I_{cr}$) cycle time.

(4) $\mathbf{C_i}$: cycle time used at each intersection, i = 1 to n.

(5) $\mathbf{O_i}$: offsets for all intersections, i = 1 to n.

Defined to be a proportion of the cycle time, $\mathbf{C_i}$, $0 \leq O_i < 1$.

(6) $\mathbf{G_i}$: arterial effective green split at all intersections, i=1 to n.

Defined to be a proportion of the cycle time, $\mathbf{C_i}$, $0 \leq O_i < 1$.

Following are the constraints for the formulation framework:

(1) (Narrow) range on the critical intersection cycle time, $\mathbf{C_{cr}}$, based on saturation, stochastic effects, and delay. Note that one can limit $\mathbf{C_{cr}}$ to a constant value if desired.

$$C_{cr,min} \leq C_{cr} \leq C_{cr,max}$$

(2) An effective arterial green split (proportion) for the critical intersection, $\mathbf{G_{cr}}$, determined a priori based on arterial saturation and stochastic delay considerations. This is also the minimum effective green split for all other intersections, $\mathbf{G_{min}}$.

$$G_{cr} = G_{min},\ 0 < G_{cr} < 1$$

(3) Range for each intersection's effective arterial green split (proportion). The maximum effective arterial green split at each non-critical intersection ($\mathbf{G_{i,max}}$) is a limit used to keep cross-streets undersaturated in the long-run and keep negative stochastic delay effects below a chosen level.

$$G_{min} \leq G_i \leq G_{i,max},\ i \neq cr,\ 0 < G_i < 1$$

(4) All intersection cycle times must be constrained to be equal to or an integer fraction of the critical intersection cycle time, so that progression will repeat over time. In practice, only one-half or possibly one-third cycle times will be long enough to be feasible. Note: if fractional cycle times are used at any intersection, the range for that intersection's arterial green split (constraint 3) must be adjusted accordingly.

$$C_i = \frac{C_{cr}}{j},\ i \neq cr,\ j = 1, 2, 3, \ldots..$$

(5) A minimum bandwidth, $\mathbf{B_{min}}$, for the favored vehicle type (probably the auto). This is exclusionary screening of possible solutions and is not required.

$$B \geq B_{min}$$

(6) Constants for signal spacing according to the street section under analysis, defined by the distance, $\mathbf{D_i}$, between the first intersection and intersection i.

$\mathbf{D_i}$, for i=1 to n, with $\mathbf{D_1} = 0$

(7) Suitable auto progression speed range.

$\mathbf{V_{a,min}} \leq \mathbf{V_a} \leq \mathbf{V_{a,max}}$

(8) Suitable bicycle progression speed range; might require a different range for a different segment, if grades are significantly different.

$\mathbf{V_{b,min}} \leq \mathbf{V_b} \leq \mathbf{V_{b,max}}$

(9) For safety, consider constraints that do not allow the bicycle progression band at "wide" intersections to catch the end of green. Use of the stop dispersal technique could also help.

(10) Consider constraints ensuring that bicycle progression bands get cyclists over any significant hills without stopping. Use of the stop dispersal technique could also help.

(11) Consider constraints to enhance bicycle lane changing and left-turning maneuvers between auto platoons in segments where left-turning bicycle volumes are relatively high.

### 4.6.3 An Illustrative Example

For illustrative purposes, we present an explicit math program formulation developed from the above framework. This will help the reader to further understand the formulation framework and will illustrate how a multiobjective formulation can be used to examine the trade-offs between objectives. This math program uses the following two objectives: (1) maximize auto through bandwidth and (2) maximize bicycle through bandwidth. The selection of these two objectives is not intended to suggest that they are the most appropriate objectives or that this is the best way to formulate them. They are chosen and formulated as such only to provide an illustrative example that is fairly simple to understand.

The formulation was programmed using EXCEL (spreadsheet) Solver. The following simplifications were made. All cycle times were considered constant, therefore possible benefits gained by using half-cycle times upstream of the critical intersection were not possible, nor were any possible gains caused by slightly modifying the critical intersection cycle time. Arterial green splits were allowed to vary. However, without a cross-street delay constraint there is really no benefit to using anything less than the maximum arterial green. Of the techniques illustrated earlier, only varying progression speeds and bandwidth trade-offs between vehicle types are available to either improve the solution or provide an alternative nondominated solution. No minimum bandwidth was specified for either vehicle type. Constraints 9, 10, and 11, which address some of the additional considerations for bicycles, were not activated.

Following Brooks (Brooks; Messer et al., 1973), the auto through bandwidth is viewed as the difference in the maximum possible through bandwidth, as defined by the effective green interval at the first intersection (**$G_1$**) and the total interference to automobiles (**$I_a$**). The total interference can be computed as the sum of the maximum interference from the left and the maximum from the right, with left and right referring to the side of the through band as drawn on a time-space diagram. As illustrated in Figure 4.10, left interference accounts for stopping the beginning of a progressing platoon of vehicles and right interference accounts for chopping off the end of a progressing platoon. To maximize the through bandwidth one must minimize the total auto interference. This can be done as follows:

Minimize Auto Bandwidth Interference: **Min $I_a = I_{a,l,max} + I_{a,r,max}$**

where **$I_{a,l,max}$** = maximum interference from the left (see Figure 4.10)

= **$\max[I_{a,l,i}]$** for i=1 to n intersections,

**$I_{a,r,max}$** = maximum interference from the right (see Figure 4.10)

= **$\max[I_{a,r,i}]$** for i=1 to n intersections,

the subscript **a** stands for auto, **b** for bicycle, **l** for left, and **r** for right,

and the interference values at each individual intersection i are given by:

$$I_{a,l,i} = (O_i \times C_i) + C_i \times \text{Truncate}\left[\frac{\frac{D_i}{V_a} + (G_1 \times C_1) - (O_i \times C_i)}{C_i}\right] - \frac{D_i}{V_a} \quad ; (4.2)$$

if **$I_{a,l,i} < 0$**, then set **$I_{a,l,i} = 0$**,

and

$$I_{a,r,i} = \left[\frac{D_i}{V_a} + (G_1 \times C_1)\right] - \left\{(O_i \times C_i) + C_i \times \text{Truncate}\left[\frac{\frac{D_i}{V_a} + (G_1 \times C_1) - (O_i \times C_i)}{C_i}\right] + (G_i \times C_i)\right\} ; \quad (4.3)$$

if **$I_{a,r,i} < 0$**, then set **$I_{a,r,i} = 0$**.

**$O_i$**, **$C_i$**, **$D_i$**, **$V_a$**, and **$G_i$** are defined above.

Note that for simplicity, if the entire effective red interval at intersection i is contained within the maximum through progression bandwidth as defined by intersection 1, then the interference from the left actually contains some green time.

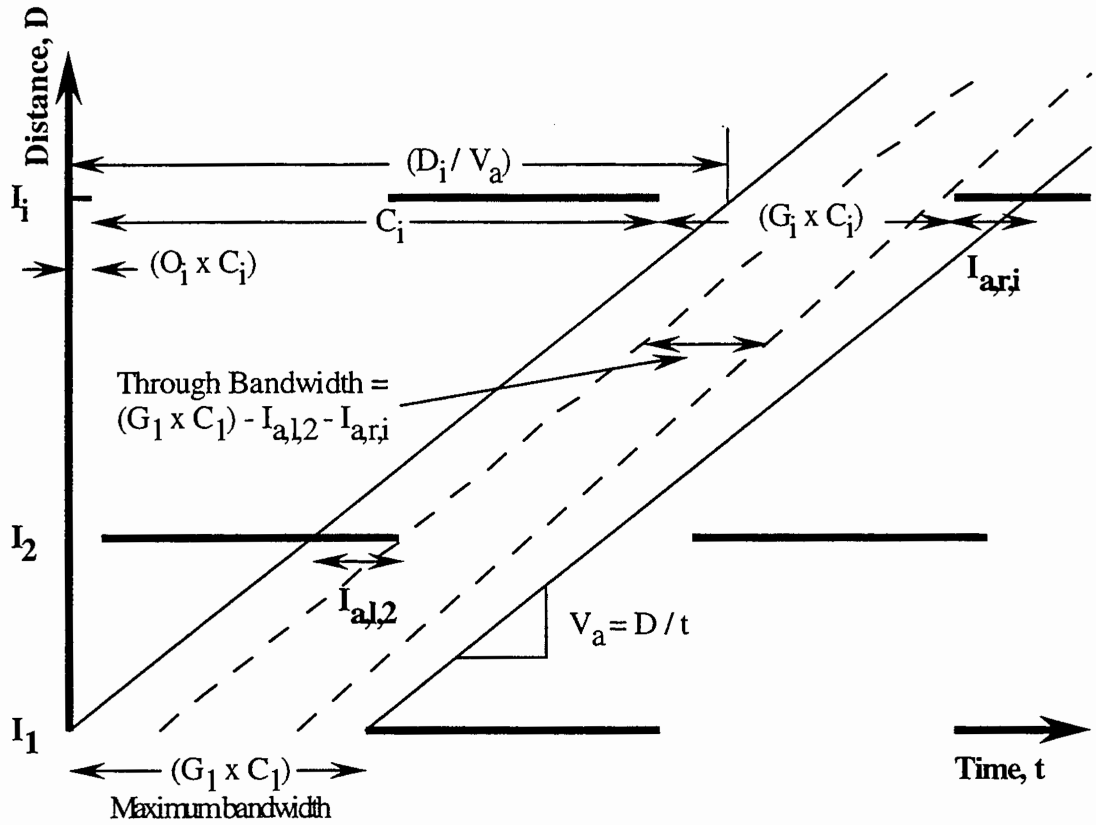


Figure 4.10. Relationships between the through auto bandwidth and interference from the left ($I_{a,l,i}$) and interference from the right ($I_{a,r,i}$).

Maximizing the bicycle bandwidth is accomplished using the same formulas as above for maximizing the auto bandwidth by replacing the auto progression speed ($\mathbf{V_a}$) with the bicycle progression speed ($\mathbf{V_b}$) and minimizing $\mathbf{I_b}$ instead of $\mathbf{I_a}$.

Though not done here, it would be easy to add constraints to this formulation that would help keep progressing cyclists from having problems with inadequate clearance intervals at "wide" intersections (constraint 9). This could be achieved by constraining the right bicycle interference value $\mathbf{I_{b,r,i}}$ at wide intersections to be less than $\mathbf{I_{b,r,i}}$ at at least one of the preceding intersections by an appropriate threshold.

Similarly, this math program lends itself to an easy formulation of constraint 10, which would allow progressing cyclists to climb hills without stopping. This could be accomplished by constraining both the left and right bicycle interference values, $\mathbf{I_{b,r,i}}$ and $\mathbf{I_{b,l,i}}$, for the intersections on the uphill grade so that cyclists will not have to stop once they begin up the hill.

The following are the hypothetical constraint data used for the example:

(1) Intersection IDs: i = 1, 2, 3, 4, or 5; for a total of five intersections in the segment.

(2) Critical intersection: $I_4 = I_{cr}$

(3) Cycle times: $C_i = C_{cr} = 100$ seconds, for all i

(4) Effective green splits:

$0.50 \leq G_1 \leq 0.70$;

$0.50 \leq G_2 \leq 0.65$;

$0.50 \leq G_3 \leq 0.65$;

$G_{min} = G_{cr} = G_4 = 0.50$;

$0.50 \leq G_5 \leq 0.70$

(5) Intersection spacing: $D_2 = 1000$ ft; $D_3 = 2000$ ft; $D_4 = 3000$ ft; $D_5 = 4000$ ft

(6) Progression speeds: 25 mph $\leq V_a \leq$ 35 mph; 10 mph $\leq V_b \leq$ 15 mph

The epsilon-constraint method was used to generate nondominated solutions to this problem (see Goicoechea et al. (1982) for a general reference on this method). The auto bandwidth interference, $I_a$, is constrained to be less than or equal to various values, and the bicycle bandwidth interference, $I_b$, is minimized. By choosing various values for the $I_a$ constraint and solving the problem for each, a different nondominated solution is found. Unfortunately, the objective function is not well behaved. This, in combination with the search algorithms used by EXCEL Solver, caused problems in finding optimum solutions to each problem. With experience, better solutions were sometimes found by visual inspection of the solution that Solver returned as "optimal". A better objective function formulation could probably be developed or a search algorithm better suited to the features of this objective function formulation might be found. However, for illustrative purposes this will suffice.

Drawbacks aside, many nondominated solutions were found, some by using multiple iterations of Solver, starting at different initial points and using visual inspection of the solutions and manual adjustments where necessary. One such solution is shown in the example spreadsheet in Table 4.3. This solution is for the problem that constrains $I_a$ to be less than or equal to five seconds. It yields an auto bandwidth of 45 seconds at an auto speed of about 27 mph and a bicycle bandwidth of 20 seconds at a bicycle speed of about 11 mph. Note that the solution includes the offsets and green splits at each intersection.

TABLE 4.3. ONE EXAMPLE OF A NONDOMINATED SOLUTION TO THE EXAMPLE PROBLEM, USING THE EXCEL SOLVER SPREADSHEET PROGRAM.

| | | | |
|---|---|---|---|
| **EPSILON CONSTRAINT** | | | |
| Auto bandwidth interference | 5 sec | | |
| **OBJECTIVES** | | | |
| Auto bandwidth interference | 5 sec | Auto through bandwidth | 45 sec |
| Bicycle bandwidth interference | 30 sec | Bicycle through bandwidth | 20 sec |
| **TIMING DECISION VARIABLES (fraction of cycle time)** | | | |
| Offset at intersection 1 | 0.00 | Arterial green split at int. 1 | 0.50 |
| Offset at intersection 2 | 0.25 | Arterial green split at int. 2 | 0.65 |
| Offset at intersection 3 | 0.30 | Arterial green split at int. 3 | 0.65 |
| Offset at intersection 4 | 0.75 | Arterial green split at int. 4 | 0.50 |
| Offset at intersection 5 | 0.00 | Arterial green split at int. 5 | 0.70 |
| **OTHER DECISION VARIABLES** | | | |
| Auto speed (kph) | 43.89 | Bicycle speed (kph) | 18.28 |
| Auto speed (mph) | 27.27 | Bicycle speed (mph) | 11.36 |

Unless the arterial section has a large bicycle demand, a 20 second bandwidth may be sufficient. One may want to trade-off bicycle bandwidth to gain more auto bandwidth. Other nondominated solutions that were generated are shown in Figure 4.11. For this example, the trade-off between bicycle and auto bandwidth is linear, with the sum of the two bandwidths equaling 65 seconds. It is evident how this type of plot can aid the engineer in making a decision on the arterial timing plan. It allows easy visualization of all possible solutions and the trade-offs involved.

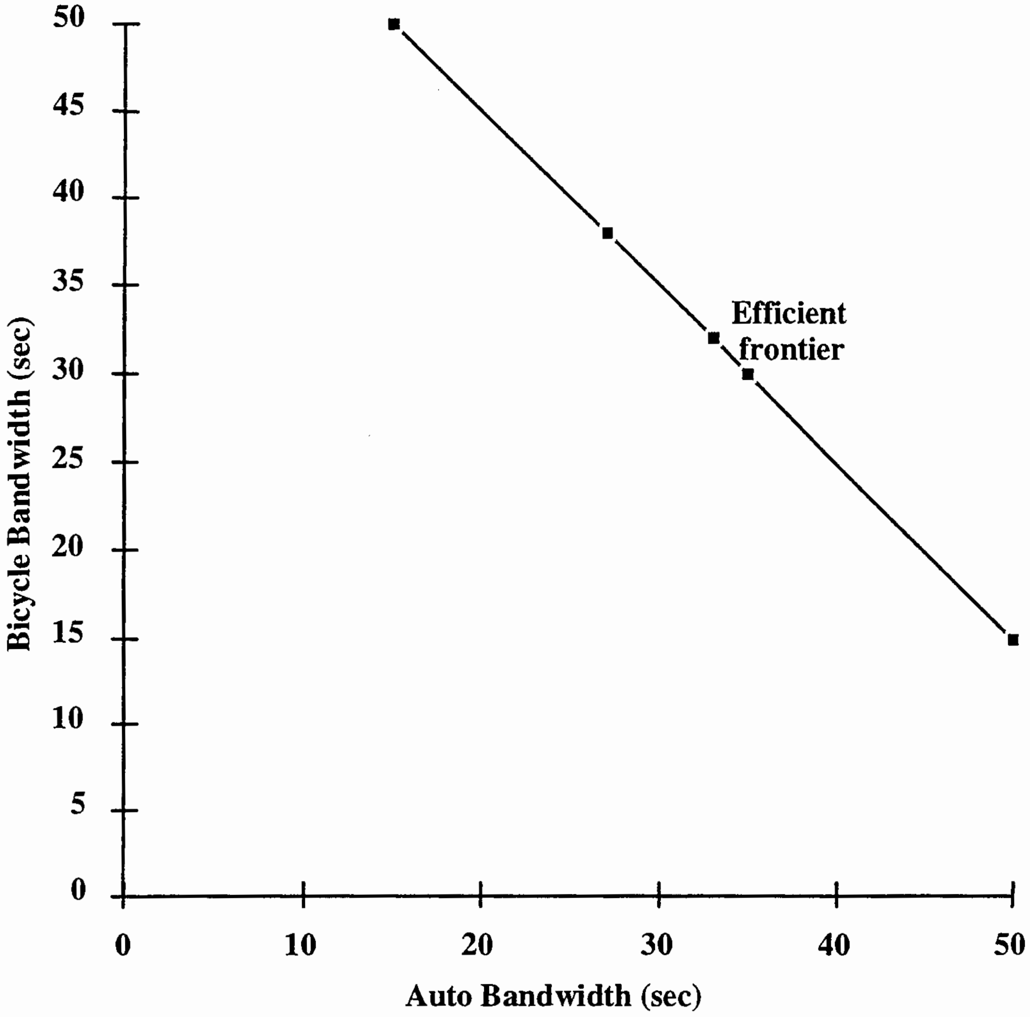


Figure 4.11. Plot of nondominated solutions to biobjective example formulation

The math program developed for this example could be improved by incorporating some of the additional considerations discussed in the formulation framework presented in the previous subsection. These include: allowing stop dispersal and half-cycle times, handling additional objectives such as minimizing total cross-street delay, and providing progression over hills for bicycles.

## 4.7 TWO-WAY PROGRESSION

In this section, two-way progression is analyzed. In these systems a two-way arterial has some progression bandwidth in each direction. This analysis of two-way progression is less thorough than the previous analysis of one-way progression for several reasons. First, much of the previous one-way discussion holds for two-way progression as well. Second, by its very nature two-way progression is much more complicated (and less feasible) than one-way, especially multimodal schemes. So, it may be prudent to first determine if one-way bicycle-auto progression systems can be beneficial enough to warrant their development and to see if the bicycle speed variability problem can be solved before delving too far into the complexities of two-way schemes.

First, automobile only schemes are analyzed, mainly for negative impacts to bicycles. Next, bicycle only schemes are analyzed. Finally, multimodal schemes are discussed.

### 4.7.1 Automobile Only

The literature on two-way automobile progression schemes is much richer than that for one-way progression. This is due mainly to the inherent conflict between progression in one direction versus the other. Except in a few idealized cases, progression in one direction limits the progression bandwidth in the other direction, and compromises are necessary.

Manual analytic techniques have been developed for a few ideal cases where bandwidths can be maximized in both directions (ITE, 1992 and Pignataro, 1973). These timing plans use either zero- or half-cycle offsets and are suitable mainly for equally spaced signals. These timing plans are referred to as:

(1) Simultaneous - zero offset used for several consecutive signals. Primarily used for short block lengths.

(2) Alternate - signals are "grouped" in sets of one or more throughout the progression length. A timing plan with half-cycle offsets is alternated with each group.

For the more general and common case with unequally spaced arterial signals, several graphical and programmable techniques have been proposed. These techniques attempt to maximize progression in both directions, while possibly weighing progression in one direction more heavily than the other. Several of these were reviewed previously.

It is fairly easy to assess the negative impacts of a two-way automobile only progression scheme on bicycles using time-space diagrams as shown in Figure 4.2. It is then possible to modify these time-space diagrams, as was done for one-way progression in Figures 4.3 through 4.8, to attempt to mitigate these negative impacts. However, a change improving conditions for bicycles in one direction may not only adversely impact autos in that direction, but both autos and bicycles in the opposite direction. The only variables that are "safe" to fine tune are the bicycle progression speeds and selectively increasing specific arterial green intervals (while keeping the corresponding side-streets undersaturated). However, to achieve two-way auto only progression much of the "excess" non-critical intersection green time may already be allocated to the arterial. Any changes in cycle time or offsets may impact autos and bikes in both directions. Therefore, the process of improving conditions for bicycles is much more complicated than for one-way auto progression and, in general, less improvement is possible.

### 4.7.2 Bicycle Only

Two-way bicycle progression schemes could be developed in a similar manner as two-way automobile schemes are now developed, while recognizing the greater speed variability of bicycles and the impact of the most constraining vehicle type on deterministic and stochastic queuing delays. However, one must consider the negative impacts to motor vehicles that operate on an arterial so coordinated.

Once the offsets and cycle times have been set to provide for bicycle progression, it is fairly easy to assess the negative impacts to automobiles. However, as discussed for auto only two-way schemes, reducing these negative impacts may be problematic. Here, in addition to varying auto progression speeds and increasing specific arterial green intervals, it may also be possible to reduce the bicycle bandwidths, since the bandwidths were probably driven by auto demand in the first place. Yet, the possible improvements here will, in general, not be as large as for one-way progression, since the initial bicycle only bandwidths will probably be much less than the maximum one-way bandwidth due to trade-offs between the competing directions.

### 4.7.3 Multimodal Bicycle-Automobile

As for one-way multimodal progression schemes, two-way multimodal schemes must be analyzed in terms of the relative benefits to each vehicle type. Only under a few ideal cases can maximum progression be provided for all four vehicle type/direction combinations.

Consider the ideal case of equally spaced signals (Table 4.1). Under this ideal case, simultaneous and single alternate timing plans provide maximum two-way auto progression bandwidths in both directions. Double and triple alternate schemes provide bandwidths of at most one-half and one-third of the critical intersection effective green, respectively. The various bicycle progression speeds shown in Table 4.1 also have continuous progression bandwidths of the same size in both directions. Under these ideal conditions, bicycles receive no negative impacts from two-way auto progression. If grades are significant, then bicycle progression speeds will be different in each direction. This difference may or may not be able to be captured by the different bicycle progression speeds provided by the same timing plan. For example, a single alternate timing plan with auto progression speeds of 40 mph in each direction could provide bicycle progression at 13.3 mph in the downgrade direction and 8 mph in the upgrade direction.

As discussed for multimodal one-way progression, these ideal conditions are not often present. Therefore, a more general process is required. The basics for a multiobjective formulation are the same as given previously for one-way multimodal progression. The formulation framework would need at least these basic changes:

(1) Four different competing progression objectives instead of two, to handle both vehicle types in each direction.

(2) Bicycle speed ranges could be different in each direction, if grades were significant.

(3) One may be able to serve non-critical intersection cross-street traffic in two cycle segments per one critical intersection cycle to better accommodate progression in opposite directions. This occurs when the platoons in opposite directions arrive at different times at a specific intersection (Newell, 1989).

## 4.8 AUTOMOBILE SPILLBACK PREVENTION

When arterials approach saturation, platoon spillback into upstream intersections becomes problematic. If it is possible to provide bicycle progression while simultaneously preventing auto spillback, such a scheme may provide the following dual benefits: (1) increase the "people moved per unit area of arterial" by better accommodating bicycles and (2) promote bicycle use by displaying free flowing bikes to stagnant car drivers. To prevent spillback, long auto platoons must be discouraged, so schemes that do not support progression are sought. Simultaneous timing plans are good for this purpose (Pignataro, 1973 and ITE, 1992).

As shown in Table 4.2, at reasonable cycle times and reasonable progression speeds, simultaneous timing schemes inhibit progression for block lengths where spillback usually occurs. Table 4.2 also shows that bicycle progression is very feasible for these block lengths. Therefore, a simultaneous timing plan may be ideal for preventing auto spillback while providing bicycle progression for equally spaced intersections.

It may also be fairly easy to design auto spillback prevention schemes under more general signal spacing by just accommodating bicycle progression to the greatest extent possible. This by its very intent should prevent auto progression to some extent. If necessary, one could then use some of the previously discussed techniques to further reduce the chances of spillback, by using them to impede auto progression instead of improve it.

## 4.9 BICYCLE FACILITY IMPACTS

In all the previous discussion it was assumed that bicycles and automobiles do not interfere with each other's progression to any significant extent. The only interference to progression was assumed to come from traffic signals and speed variability. However, there are several on-road bicycle facility types, each providing different operating conditions for bicycle-automobile mixed-traffic. For the "no interference" assumption to hold either a bikelane or wide curb lane facility must be used. If a standard shared lane is used, autos must slow and/or change lanes to pass bicycles. This would inhibit auto progression in the right-most lane.

There may even be some difference between wide curb lanes and bikelanes for bicycle progression. Bikelanes are required for bicycles to (legally) pass automobiles on the right (most likely while automobiles are queued at an intersection). In wide curb lanes, bicycles may be more forced to "platoon". Passing another bicycle may be difficult on these facilities, depending on the probability of a nearby automobile in the same lane. On the other hand, bikelanes (depending on their width) may provide enough space for bicycles to pass each other, thereby reducing the tendency to "platoon". There does not seem to be sufficient data available to characterize bicycle platooning on either facility type.

On one-way streets it is often legal for bicycles to travel on the left side of the left-most lane. However, the extent to which bicyclists are aware of this right and the extent to which they use it are not known. If they do so to a significant extent, then the points above also apply to the left-most lane on one-way streets. In that case, some form of bicycle accommodation in the left lane may be desirable. A wider lane would probably be the preferred choice, since a bikelane on the left side might encourage contra-flow use.

Auto spillback prevention/bicycle progression schemes require either a wide curb lane or bikelane, since bicycles will need to pass autos on the right. A bikelane is probably the preferred choice, since passing on the right in the same lane (even if it is a wide lane) is technically illegal and may be hazardous. Anecdotal evidence shows that many cyclists already pass autos on the right in the same lane, usually when autos are stopped. However, it is probably not desirable to encourage this behavior.

## 4.10 CONCLUDING COMMENTS

This study seems to be the first documented work investigating traffic progression systems that attempt to accommodate both bicycle and automobile progression on the same facility. Its overall contribution to bicycle-automobile mixed-traffic science is that it provides a general conceptual foundation for future work in this area. This foundation consists of the following three primary contributions.

First, the following principal considerations for bicycle progression were identified and discussed: (1) speed variability, (2) grades, (3) bandwidth sizing, (4) platooning, (5) clearing wide intersections, and (6) lane changing and left-turning. The most important of these is bicycle speed variability, which due to its unknown characteristics brings into question the feasibility of designing any timing scheme that can deliver significant bicycle progression benefits.

Second, the negative impacts of unimodal progression schemes on the other vehicle type were illustrated and ways of mitigating these impacts were discussed. These discussions clearly illustrated the multiobjective nature of the problem. Several concepts and techniques were presented to improve the multiobjective solutions (i.e., create solutions that dominate solutions generated without these concepts and techniques) or provide alternative nondominated solutions. These are: (1) vary auto progression speeds, (2) vary bicycle progression speeds, (3) allocation of excess green time, (4) use half-cycle times upstream of the critical intersection, (5) stop dispersal, (6) vary critical intersection cycle time, and (7) bandwidth trade-off between vehicles. Each of these applications of a technique to bicycle-automobile progression is a contribution in the area of traffic engineering.

Third, having to deal with the multiobjective nature of the multimodal progression problem led to the development of a solution generating process, which was formalized in a multiobjective formulation framework, itself a contribution in the area of traffic science. The framework is intended to be a guide to future work in this area, as it incorporates both the key considerations for bicycle progression and the concepts and techniques that help to generate improved or alternative multiobjective solutions. The framework assumes that sufficiently constant bicycle progression speeds can be maintained.

Guided by this general framework, a specific formulation was developed for a simplified situation and used to generate the efficient solution frontier for an example problem. This example illustrated the benefits of a multiobjective formulation: (1) generating and displaying nondominated solutions and (2) analyzing trade-offs between objectives. Specific formulations like this are a contribution to traffic engineering, since they allow engineers to design mixed-traffic progression systems.

The more complex problem of two-way progression was only touched upon. The two-way progression discussion mainly addressed the further complications (above those of one-way progression) of designing progression schemes for four competing vehicle/direction combinations. Finally, the impacts of the various bicycle facility types were addressed. The main conclusion of that discussion was that either a wide curb lane or bikelane is probably required for multimodal progression, because a standard lane would cause bicycles to impede auto platoons in the right-most lane.

It should be stressed that signal timing for bicycle progression should not be limited to arterials. In fact, collector streets may often be used by bicycles as their "arterials" and therefore, one would want to consider timing them for bicycle progression. It should also be noted that by judicious design of progression timing systems one can attempt to achieve various goals, such as increasing bicycle traffic while decreasing auto traffic on a specific street (probably a collector) or preventing spillback for autos while providing progression for bicycles.

The main limitations of this work are that it is purely theoretic and constant (average) bicycle speeds are assumed. Before much further theoretical work or implementation is done, it seems that bicycle speed variability must be studied. One approach would be to perform experiments with willing cyclists attempting to travel at a specified speed and quantify their speed variability. While this would provide further insight, there may be an effect from having the coordinated traffic lights "tracking" the speed the cyclist is trying to maintain. In addition, learning effects may be difficult to assess in such an experiment.

To effectively assess speed variability (and other factors) in the context of progression, a test project(s) is necessary. One could target facilities that would not significantly inhibit automobiles, such as collector streets with heavy bicycle traffic or street segments in need of auto spillback prevention. It would also probably be simpler to start with a one-way scheme, unless ideal two-way conditions exist. A test project would allow study of bicycle speed variability under actual conditions, in addition to allowing before and after observation of bicycle delay and stops. The benefits perceived by the users and the characteristics of bicycle "platooning" could also be assessed. It may be necessary to publicize the project and educate cyclists on their progression speed, in addition to posting it.

Without further study, "long" street sections with widely spaced intersections are not recommended for bicycle-only or multimodal progression, unless the impact to automobiles is minimal, since benefits to bicycles may not be realized due to speed variability. However, if the negative impacts to autos are minimal, little is lost by attempting to incorporate bicycle progression. One could then study the improvements to see what benefits are actually realized. It is both most desirable and most feasible to design "short" sections with closely spaced intersections for bicycle progression where bicycle volumes are high or to get bicycles over hills without stopping. The shorter the segment, the closer the intersections are spaced, and the larger the bandwidth, the less speed variability will impact bicycle progression.

# 5 SUMMARY AND CONCLUDING COMMENTS

Three objectives are accomplished in this concluding chapter. First, the work presented in this dissertation is placed in perspective relative to past work, and the most important contributions are briefly summarized. Second, the limitations of this work, the implications of these limitations, and directions for future work in the three study areas are discussed. Third, the mixed-traffic representation for the analysis and design of traffic systems is discussed in an attempt to direct future work in this area. Closing comments aimed at motivating additional work in this area are given in the last section.

## 5.1 RELATION TO PAST WORK AND IMPORTANT CONTRIBUTIONS

It is useful to place this work in perspective relative to previous studies of both bicycle traffic and automobile traffic, as well as to studies using similar analysis methodologies. With regard to bicycles, this study appears to be the most extensive and comprehensive to date for the three topics investigated, namely:

(1) It appears to be the first study to investigate signal coordination along streets to provide bicycle progression. It establishes a conceptual foundation and methodology for analysis and design of bicycle-automobile mixed-traffic signal coordination.

(2) It presents the first detailed analysis of bicyclist behavior at the onset of yellow and the related design of clearance intervals to accommodate bicycle-automobile mixed-traffic.

(3) It adds considerably to previous studies of cyclist and motorist gap acceptance behavior, both substantively and methodologically.

With regard to previous studies of the corresponding topics for automobile-only traffic, this study adds to the state-of-the-art in an incremental manner in the following respects:

(1) Consideration of additional competing objectives in signal setting, for coordination strategies to provide progression, as well as change and clearance intervals at individual intersections.

(2) Richer behavioral content through consideration of additional factors not previously investigated in models of motorist behavior at the onset of yellow and motorist gap acceptance.

The studies of gap acceptance and bicyclist behavior at the onset of yellow continue and extend the work performed in a limited number of previous studies that have applied a discrete choice modeling framework to analyze motorist as well as pedestrian (crossing) behavior. This modeling approach allowed insights to be derived from a rich data set of measurements made over the course of this study. Finally, this study continues a line of work by researchers at The University of Texas who have successfully used the multinomial probit estimation software developed at the university in a variety of similar and interesting applications involving discrete choice modeling (Lam, 1991; Palamarthy et al., 1994; Yen, 1994; Liu, 1997).

The individual contributions of this research were discussed in detail in each of the three analysis chapters. Only the most important among those are briefly summarized in this section. They are organized according to the research objectives presented in Section 1.2. The first objective was to study and characterize individual bicyclist (and sometimes motorist) choice behavior in bicycle-automobile mixed-traffic situations. The major contributions relating to this objective are to characterize, using discrete choice models, (1) bicycle/rider unit behavior at the onset of yellow and (2) the gap acceptance behavior of both cyclists and motorists in bicycle-automobile mixed-traffic.

The second objective was to investigate the implications for the design of traffic control systems for bicycle-automobile mixed-traffic. The major contribution in this regard is the development of a three-part theoretic foundation for the design and analysis of bicycle-automobile mixed-traffic progression on signalized streets. First, the principal considerations involved in bicycle-only progression are identified and explored. Second, several techniques are presented that can be used to improve the set of multiobjective solutions available to a decision-maker (i.e., create solutions that dominate solutions generated without these techniques) or provide alternative nondominated solutions. Third, a multiobjective math programming formulation was developed to coordinate traffic signals for bicycle-automobile mixed-traffic progression. It incorporates both the key considerations for bicycles and the techniques leading to improved or alternative multiobjective solutions.

The third objective was to develop techniques and procedures to help solve basic engineering problems for bicycle-automobile mixed-traffic. In addition to the multiobjective formulation for signal coordination for mixed-traffic, a deterministic procedure was developed to determine adequate clearance intervals for bicycle-automobile mixed-traffic, after a design clearance point is chosen. It was also shown that a probability of stopping model is useful for evaluating clearance intervals to accommodate bicycles based on the actual behavior of cyclists at the onset of yellow.

The fourth objective was to derive applicable traffic science laws and insights that contribute to the fundamental understanding of bicycle-automobile mixed-traffic. Important insights derived include:

(1) the impact of the type of vehicle that closes a gap: compared to a passenger car closing the gap, both motorists and bicyclists will accept significantly smaller gaps if the closing gap vehicle is a bicycle, and both require larger gaps if the closing vehicle is large (e.g. a bus),

(2) measurement/derivation of bicycle/rider unit comfortable deceleration rates in a transportation environment, consisting of the distribution of measured rates and a theoretical exploration of these distributions,

(3) the principal reasons (associated with the traffic reacting to the yellow) why standard automobile traffic signal change and clearance intervals may be inadequate for cyclists at sufficiently wide intersections, including: various characteristic differences between bicycle/rider units and car/driver units (e.g. speed and deceleration) and the fact that cyclists may need to proceed farther into the intersection than motorists (for safety), and

(4) in the context of signal progression systems for bicycle-automobile mixed-traffic, bicycle speed variability is the (unknown) factor that limits the possible benefits of progression to bicyclists.

The fifth, and least important, objective was satisfied in Section 1.3. An organizing framework for bicycle-automobile mixed-traffic science principles and relations and their engineering implications was developed (Figure 1.1), and the results of this dissertation research and some previous efforts were articulated in that framework.

Finally, a more general contribution of this dissertation is to illustrate the use of a mixed-traffic analysis approach to perform traffic studies that explicitly consider several vehicle classes in the traffic mix. The validity and utility of this approach have been demonstrated in the three individual studies presented in the dissertation. Of course, mixed-traffic is not limited to just bicycles and automobiles. In the gap acceptance and behavior at onset of yellow studies, several types of bicycle/rider units were considered, such as females riding standard 10-speed bikes and cyclists with helmets riding wide-tire mountain bikes. “Automobile” was refined into buses, heavy trucks, pickup trucks, motorcycles, and passenger cars. Further categories are possible and would be relevant to various types of studies, such as electric vehicles, emergency vehicles, and sport utility vehicles, as is the explicit inclusion of other members of the traffic stream, such as pedestrians and light-rail vehicles.

## 5.2 STUDY LIMITATIONS AND DIRECTIONS FOR FUTURE WORK

Because the limitations of the work performed herein suggest future work needed to overcome them, directions for future work in the three study areas are presented in conjunction with the limitations. First, the implications of these limitations on the use and interpretation of the results are discussed.

Because of the limitations (discussed next) all results from this work are preliminary and the analyses are primarily exploratory in nature. Therefore, the results are most suited to (1) providing preliminary insights into mixed-traffic behavior and (2) designing the follow-up research and trial projects necessary to verify and refine the insights, as well as the design methodologies that were developed. The results have not been verified or refined enough to use in practice (for design) without large doses of engineering judgement and sensitivity analysis. If so used, the gap acceptance and clearance interval design related results are most applicable to low-speed intersections near university campuses in urban settings frequented by mostly college-aged cyclists. The progression results are most suited to relatively short street (collector or arterial) segments with relatively close intersection spacing where the signal coordination for bicycles will not unduly impact automobile traffic, and where bicycle demand is high or bicyclists would benefit from traversing upgrades without having to stop.

The primary limitations of the study of gap acceptance behavior in mixed-traffic are the limited number of observed gap acceptance decisions and the limited variety of situations in which these decisions were made. Future work should be directed at increasing both the variety of situations and number of observations associated with each situation. Specifically, a limited number of intersection environments were studied, and they were all low-speed, two-lane, two-way stop controlled intersections near a university campus within an urban environment. Perhaps the most important intersection environment factors to study in the near term would be the presence (and absence) of bikelanes, various curb lane widths, and higher speed automotive traffic on the major street. While doing so, a number of other factors also require study, including number of traffic lanes, geometry, weather, time-of-day, and one-way street crossings.

The results of this gap acceptance study are based on a sample of mostly college-aged cyclists. Future work should focus on more general cycling populations, including children and older adults. Studies comparing novice and experienced cyclists would also be of importance for the design of intersections (and routes) that encourage new (novice) use.

Finally, this study lacked depth in observations for each gap acceptance maneuver. To develop a bicycle model all the data associated with four maneuvers (lateral left, lateral right, lateral straight, and frontal left) had to be combined. Future researchers should gather further observations of these maneuvers, as well as observations of bicycle lane changing maneuvers, to develop separate exploratory models for each. Only lateral left-turning automobile maneuvers were prevalent enough to develop a model with systematic explanatory variables. Future research should soon focus on performing similar initial exploratory modeling for all other motorist maneuvers (crossing and merging with mixed-traffic), including parallel right maneuvers.

The lack of sufficient variation in intersection environments and cycling populations is also a major limitation of the study of bicycle behavior at the onset of yellow. It is important for future research in this area to also focus on increasing this variation. The different cycling populations requiring further study are the same as those mentioned above for gap acceptance. As far as intersection environments are concerned, perhaps the most important factors to study are intersection width and the start-up behavior of cross-street motorists when proceeding cyclists are clearing the intersection (compared to when no cyclists are proceeding).

Other major limitations of the behavior on yellow work relate to the assumption that bicyclists may require longer clearance intervals than motorists. First, no overwhelming evidence was found that cyclists require a clearance interval longer than the automobile standard at the study (or any) intersection. However, the observational data and analyses did suggest the possibility. Second, if a designer implements a bicycle clearance interval significantly longer than the standard, it is not know if (or how) motorist and cyclist behavior might change. Therefore, there is a high priority for further work to determine if, and under what conditions, cyclists require clearance intervals significantly longer than the standard. After this (or concurrently), there is a need to study any implementations of "long" bicycle clearance intervals using the various ways to warn cyclists of the signal change (see Section 3.4), in conjunction with "before and after" evaluations of any changes in motorist or cyclist behavior.

A limitation of the deterministic design approach for computing adequate bicycle clearance intervals is the small number of deceleration measurements available to determine the design deceleration value. Similarly, the data collected was not rich enough to determine in what situations or at what rate cyclists might accelerate, if deciding to proceed after the onset of yellow. Finally, no attempt was made to measure bicyclist perception-reaction times for this situation. Future work focusing on measuring and analyzing all three of these cyclist characteristics to determine design values is of importance. This effort might be most successful if performed in conjunction with developing probability of stopping models from actual observations of cyclist behavior on yellow over a wide array of intersection environments and cycling populations.

The main limitations of the study of mixed-traffic progression are that it is only a theoretic study, with no attempt made to verify the theories with data and analysis, and it is assumed that bicycle/rider units can maintain sufficiently constant (average) speeds between intersections. The next step is to obtain observational (or experimental) data to verify and refine both the theory and constant speed assumption. Observational data from trial projects would seem to be the most reliable way to obtain this data, though experiments may also prove useful. In addition, trial projects offer the opportunity to determine to what extent bicyclists benefit, and to what extent they perceive the benefits, from signals coordinated for progression.

The work performed herein can also be of value in future work involving other types of vehicle/operator units in the traffic mix. For example, the conceptual framework for the analysis of bicycle-automobile mixed-traffic progression might be applied to other mixed-traffic, such as different automobile classes (e.g. low-speed, electric "city" cars and higher-speed, standard gasoline cars) or to light-rail vehicles traveling (at a different progression speed) through the same traffic signals as automobiles.

## 5.3 FUTURE WORK IN OTHER AREAS OF BICYCLE-AUTOMOBILE MIXED-TRAFFIC SCIENCE

All the bicycle-automobile mixed-traffic topics presented in the outline in Figure 1.1 require significant future work. Some work may have been performed that is not accessible to researchers in different countries or speaking different languages. Because of the scarcity of funding for bicycle research in the U.S. (and most other countries) it is a priority to obtain all previous research on each specific topic. Significant work on many of these topics may have been performed in countries like The Netherlands, Germany, China, and Japan but is not available to (English speaking) researchers in other countries. Even work published in one's own country or language is often forgotten or "lost", because it was performed long ago and was not continued. It would seem that a high priority should be placed on finding, reviewing, and organizing the prior work in bicycle traffic science, engineering, and operations. With such an organized review, research agendas can be better planned and a base of past work is provided for each new bicycle related research study.

Two of the main issues for bicyclists, cycling advocates, and bicycle planners are the encouragement of new bicycle transportation use and the safety of current and future cyclists. Some advocate extensive use of bikelanes to encourage new cyclists, while others argue that bikelanes are inherently more dangerous than wide curb lanes. However, most believe that there is a place for each type of facility, but are unsure of the specifics of what traffic environments (or what cycling populations) require (or desire) bikelanes and which are better served by wide curb lanes. The lack of consensus on the widths of bikelanes and curb lanes required in various different roadway environments adds further complication and confusion. Characterization of bicycle-automobile mixed-traffic operating on facilities with various curb lane and bikelane widths would be of great use in resolving these problems and is therefore, a topic of high priority.

Bicycle planners have problems designing bicycle routes (e.g. should a particular route be somewhat circuitous using low-speed, low-volume streets or should it be more direct using higher-speed, higher-volume streets). It would seem that modeling bicycle-automobile mixed-traffic flow in a variety of roadway and traffic environments could provide some of the insights required for planners to solve such route related problems.

There is ongoing work, such as that by Rouphail et al. (1997), to develop procedure modifications to accommodate bicycles in the Highway Capacity Manual (TRB, 1985). These procedures will require verification, refinement, and extension through future work, which has a high priority both in terms of need and continuing the momentum of this stream of work.

## 5.4 CLOSING COMMENTS

There are a variety of reasons that only limited bicycle-automobile mixed-traffic science research has been performed to date. In this section, a few comments are made in an attempt to motivate further work in this area. First, intellectual challenges increase when studying mixed-traffic. For example, consider the additional competing objectives that required thoughtful consideration when performing the study presented above on bicycle-automobile mixed-traffic progression. Researchers, who have become experts in specific automobile traffic topics, may find new challenges and rewards by applying their knowledge and expertise to mixed-traffic analysis. Furthermore, the study of mixed-traffic can only benefit the design of the transportation system as a whole, in addition to improving the methodologies and theories necessary for all types of traffic analyses.

One factor that appears to have dampened the interest of researchers and funding agencies in the study of mixed-traffic involving bicycles is the rather limited use of bicycles as a mode of transportation in the U.S. (and many other countries). However, one motivation for such study is that traffic engineers have a responsibility to provide for the safety and efficiency of bicycle travel, since cyclists (in the U.S.) have the legal right to use almost all the same facilities as motorists. In addition, it is U.S. DOT policy to encourage more bicycle transportation use (FHWA, 1994), and further study is needed to achieve and safely accommodate the new use.

Finally, another reason that researchers may be hesitant to study mixed-traffic is that they feel the addition of bicycle traffic will complicate the analysis beyond the point for which traffic can be characterized or modeled. This study provides an example that the additional complication remains tractable and can provide a stimulating (not frustrating) challenge. Sometimes when frustrated by such research, recalling the experience of one of the pioneers of automobile traffic science, the late Dr. Robert Herman, might help. In the early days of traffic research he told some of his colleagues in physics that he was going to study traffic; and they questioned his good judgement on the ground that traffic was totally chaotic, and that he would never be able to explain it to any significant extent. We now know that this is not the case, and it should also not be the case with bicycle-automobile mixed-traffic.

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