Meta-Analysis for Explaining the Variance in Public Transport Demand Elasticities in Europe

PETER NIJKAMP
GERARD PEPPING
Free University

ABSTRACT

Results from past studies on transport demand elasticities show a large variance. This paper assesses key factors that influence the sensitivity of public transport users to transport costs in Europe, by carrying out a comparative analysis of the different elasticity values of demand for transport that are being used in some of the different Member States. Our empirical base is elasticity studies in Norway, Finland, the Netherlands and the United Kingdom. The paper identifies a set of potential factors causing variances between results of different studies. An indepth rough set analysis of causes of variances between elasticity values across the four countries is presented. Our analysis supports the literature, which indicates that the difference between aggregated, empirical-based research methods and the use of disaggregated choice models, as well as model assumptions, explain the variance in elasticity values across studies. It also appears that the country involved, the number of competitive modes, and type of data collected are important factors in accounting for the size of elasticities.

Professor Peter Nijkamp, Department of Regional Economics, Free University, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. Email: pnijkamp @econ.vu.nl.

INTRODUCTION

Public authorities in many countries have an increasing interest in the financial side of the transport system. Various agencies involved with the provision of transport infrastructure are faced with a mismatch between supply and demand. On the one hand, we observe endless traffic jams on main arteries and in urban areas, but on the other hand we also witness empty motorways in more peripheral areas. It is increasingly recognized that the price mechanism is not properly used to ensure a balance between supply and demand. However, the introduction of market principles in transport policy does not mean an automatic panacea for all friction in the transport systems under all circumstances, as we have different types of travelers, different (competing and complementary) modes, different travel motives, different goods, different time horizons, different distances to be bridged, and different (site-specific) travel conditions. So we need to have more insight into behavioral responses.

Easily the most important parameter for understanding how pricing policies will affect transport demand is the price elasticity of demand. This elasticity expresses the change in demand induced by a change in price. More precisely, it is defined as the ratio of the relative change in demand and the relative change in price. Public transport operators use price elasticities to assess the behavioral implications of a change in the fare system. It is also used by fiscal authorities to estimate the financial revenue consequences of a rise of gasoline taxes in the private transport sector. Furthermore, it is used to make assessments of the sensitivity of car drivers to a toll system (bridges, tunnels, toll roads). More recently, price elasticity has gained much popularity in the context of road pricing proposals in many countries, through which not only the private but also the social costs of surface transport might be incorporated in the travelers' decisions.

In the past years, several studies in European countries have assessed price elasticities of demand in the transport sector. There is a great diversity of empirical results. Clearly, most investigations have been made on a noncontrolled basis, so that the comparability of the results of these studies is rather feeble. Nevertheless, it makes sense to analyze the differences in statistical results more carefully, in order to identify commonalities and site-specific differences more precisely. This would also allow for more transferability of results under certain conditions.

In this context, meta-analysis may play an important role. Meta-analysis has been developed as a tool for comparing and synthesizing results from different studies with a similar goal in the natural sciences, and has increasingly found its position in the social sciences (e.g., experimental psychology and economics). (For more details, see Van den Bergh et al 1997.) This methodological tool offers opportunities for comparing different findings and will hence also be used for a comparative statistical exercise on cost elasticities in the transport sector in different European countries.

The aim of this paper is to assess key factors that influence the sensitivity of public transport travelers to transport costs in Europe, by carrying out a comparative analysis of the different elasticity values of demand for transport that are being used in some of the different Member States.[1] This comparative analysis is based on a meta-approach called rough set analysis.

The paper is organized as follows. The next section covers earlier reviews of elasticity studies and their main results. These reviews tell us the extent of the knowledge on elasticity values for European countries. Next, an indepth analysis of possible causes of variances between elasticity values across four European countries (Norway, Finland, the Netherlands and the UK) is presented. This section is followed by an introduction to the meta-analytic method used in our study, rough set analysis. The application of this technique to our European database takes place in the next section. Finally, the main conclusions and recommendations following from our analysis are presented.

EXISTING ELASTICITY REVIEWS

In the past, numerous studies have been carried out that aimed to assess values of transport elasticities. Many methods have been used in these studies. The European Commission (1996) provides the

[1] The data used for this empirical application have been obtained from the EXTRA research project (1996–97) in the Transport Programme of the European Commission.

following list of the different sources where elasticity values can be found, or the different methodologies for the estimation of elasticity values.

- Before and after surveys: assessment of elasticities by comparing demand before and after a price change.
- *Aggregated time series analysis:* use of econometric models based on monthly, quarterly, or annual data.
- *Aggregated cross-sectional data:* use of data collected for a single time period.
- *Aggregated time series and cross-sectional analysis:* use of pooled time series and cross-sectional data.
- *Disaggregated cross-sectional analysis:* use of data collected from economic subjects like households and individuals.
- *Hypothetical market research:* inference of elasticities from expressed travel behavior.
- *Model-based elasticities:* derivation of elasticities from (computer) models of travel behavior with respect to price change.

Extensive literature reviews have been undertaken by Goodwin (1988, 1992) and Oum et al (1990, 1992). Between them they probably cover most of the work up until the time of their reviews. Two other important literature reviews are a report commissioned by the Department of Transport in Britain (Halcrow Fox 1993) and a report of the European Commission (1996). Also, Luk and Hepburn (1993) provided a useful addition, while a more recent review is Espey (1996). Although none of these review studies explicitly focus on public transport, they face the same problem as in the underlying study.

In these historical reviews, elasticities have been summarized in various ways, generally without discussing the different ways the elasticities have been estimated, although the authors of the reviews cited here are aware of these problems. For example, Goodwin (1988, 1992) lumps various estimates of the same mode to calculate a total mean fairly uncritically, taking all results by equal merit, apart from a few studies that were omitted due to "incomprehensibility or absurdity." He then subdivides the elasticities according to data type and length of period. Oum et al (1990, 1992) do not calculate means of elasticities, but list the whole range of estimates. In their World Bank working paper, they present subjective "most likely" ranges of elasticity values of demand for various travel modes.

Across the studies, there is much diversity in modes included, type of data, and methods of estimation. In addition, there is great variety in the geographical diversity. In view of the discussion in Oum et al, the estimated elasticities are not directly comparable. Even though mode choice elasticities may be distinguished from market demand elasticities, the various mode choice elasticities may not be comparable due to the inclusion of different alternative modes. For example, bus as an alternative to car may be different according to frequency, comfort, and speed. Estimated mode choice elasticities would then differ.

Therefore, generalizing the value of an estimated elasticity to different circumstances is a dubious practice. The same can be said about calculating the mean of elasticities from different studies.

On the other hand, when numerous studies are carried out with different data, and models give similar values for an elasticity, the result may be regarded as robust. The conclusion of Oum et al (1992) that the demand for car usage and urban transit are unambiguously inelastic is therefore strongly supported. If, however, the choice of policy in a particular situation is dependent on precise estimates of elasticities, estimation of elasticities should be undertaken.

Oum et al (1992) identify a number of issues that can cause different elasticity estimates, which they believe warrant attention. The most important of these are the presence of intermodal competition, the use of different functional forms, and different locations. It is concluded that: "While some generalisations, particularly on demand elasticities of car usage and public transit are possible, across-the-board generalisations about transport demand elasticities are impossible."

Most of the elasticities in these reviews are derived from empirical studies. An alternative way of estimating elasticities is by using disaggregated travel demand models, which is the case for some of the surveys in the applied meta-analysis presented in this paper. Such models can produce estimate of elasticities for not only mode-specific elasticities for different purposes but also for different segments of the population. However, it should be noted that values from travel demand models are often very different from those in empirical studies.

Halcrow Fox (1993) concludes that those literature values, i.e., based on empirical studies, are 50% to 200% higher than model elasticities. The main reason for this is that the results from models depend on a limited number of variables and thus do not allow for all the many causes of variation that exist in reality. Empirical values, however, are more likely to include the system effects, within a specific timeframe. This leads Halcrow Fox to conclude that model elasticities should be treated as minimum values.

POTENTIAL CAUSES OF VARIANCE BETWEEN ELASTICITIES IN DIFFERENT REGIONS

In this section, we focus on the fact that choices regarding different aspects of research on transport demand elasticities may have impacts on the size of the estimated elasticities. From theory, we can derive criteria that can be used to evaluate the results of different elasticity studies. Such a checklist of criteria can be used to evaluate the differences between similar elasticities by means of meta-analysis. In this section, these criteria are systematically described. But first we turn to the fundamental issues that determine the definition of the elasticity.

Definition of the Elasticity

This criterion relates to the type of elasticity measured. Various types of transport elasticities exist. The most important distinctions are own-price versus cross-price elasticities, regular versus mode-choice elasticities, and the definitions of the dependent and the independent variables.

- *Own- versus cross-price elasticities.* Instead of a demand for travel in general, demand for specific modes may be studied. Mode-specific demand leads to mode-specific elasticity values. The elasticity of one mode of transport with respect to its own price is called own-price elasticity. The elasticity of one mode of transport with respect to the price of another is called cross-price elasticity. Since a price increase for a mode will tend to reduce demand, the own-price elasticity values are negative. The cross-price elasticity values are normally positive. An increase in the price of one mode of transport will transfer some of the demand to the other modes.
- *Regular versus mode-choice elasticities.* For mode-specific elasticities, it is important to distinguish between mode-choice and regular demand elasticities. Mode-choice elasticities express the change in demand for one mode given a fixed demand of traffic for all modes. They do not take into account the change in price on the aggregated volume of traffic. Mode-choice elasticities are therefore a lower limit to regular demand elasticities.
- *The dependent variable.* Travel demand may be defined as travel volume (e.g., number of trips), modal choice, route choice, etc.
- *The independent variable.* In principle, three explaining variables can be used on the basis of which elasticities can be calculated: travel cost, travel time, and income. These variables show a high level of heterogeneity (e.g., perceived travel time in a bus is different from waiting at a bus stop; the issue of generalised costs).

In this paper, we restrict ourselves to own-price regular elasticities, where the dependent variable is travel volume and the independent variable is travel cost.

Nature of the Elasticity

Important aspects of the nature of the elasticity are:

- *Ordinary versus compensated elasticities.* Of theoretical interest is the difference between Marshallian or ordinary and Hicksian or compensated elasticities. In the first case, no compensation is given for a price rise. In the case of compensated elasticities, compensation is given so that the utility level is constant. No direct compensation is usually given in real life, though in the case of tax increases, indirect compensation can take place in the shape of better roads, etc.
- *Demand measurement unit.* A distinction must be made between measurement of demand in number of trips, distance traveled, etc.
- *The specific market segment.* The transport demand market can be segmented to different

population classes with different sensitivities to policy measures. Also, a distinction can be made between travel motives. The purpose of travel may have an influence on elasticities. Travel to work is expected to be less elastic than travel for leisure purposes, since the latter can be canceled more easily. Elasticities for rush-hour travel (peak) should ideally therefore be distinguished from off-peak elasticities.

Size of Choice Possibilities

In general, the level to which substitution is possible is an important potential factor influencing the size of a given type of elasticity. Substitution can be defined as a change in choice behavior (e.g., modal choice and route choice) in order to maintain the existing activity pattern as much as possible. If substitution is possible, it may be expected that the resulting elasticities are higher than when no substitution is possible. Important aspects are:

- *Level of aggregation over alternatives.* The level of aggregation over alternatives is important for the evaluation of the size of the elasticities. The higher the level of aggregation, the less the number of substitutes. We then expect a lower elasticity. In addition, aggregation will lead to averaging out the underlying variation in the elasticities, as no allowance is made for the heterogeneity of the alternatives to be chosen. For example, price elasticities may differ between the train and bus modes. For an effective differentiated price policy, it is necessary to have insight into the underlying elasticities.
- *Time horizon.* The possibilities to react to changing transport conditions will in general be larger in the long run than in the short run, because in the long run variations in location choice and asset holding resulting from changing transport conditions may also take place. Therefore, long-term elasticities are expected to be higher than short-term elasticities.
- *Travel distance*. It is plausible that there are differences in the sensitivities to price change between short trips and long-distance trips. Therefore, the geographical coverage of mobility surveys is an important factor.
- *Choice possibilities.* An important reason for the existence of low elasticity values in various studies is that many people do not have a choice possibility, implying that the share of these travelers in the sample investigated co-determines the size of the elasticities.
- *Other factors.* From economic theory we may derive several other factors that influence the estimated size of elasticities. For example, there is the hypothesis that travelers often have incomplete information on the real costs and travel times. These uncertainties imply that people not only react on the basis of true travel costs and travel times, but also on expected travel costs and travel times, and the associated risk that their expectation is wrong.

Model Specification

From the type of research methodology, we may derive criteria for the evaluation of elasticities. The important ones are:

- *Point versus arc elasticities.* The elasticity defined by the product of the derivative and the ratio of price demand at a point on the demand function is called a point elasticity. An elasticity can also be estimated by the change in demand induced by a finite change in price. This is an arc elasticity. Both elasticities may differ from each other when the demand curve shows a changing elasticity value. In general, arc elasticities are more suitable when one wants to know the consequences of a relatively large change in price.
- *Aggregated and disaggregated models.* The most important criterion is likely to be whether the model used is an aggregated or a disaggregated model. Aggregated models do not make an allowance for individuals who make choices based on specific circumstances. Therefore, problems related to methods of aggregation may cause significant biases in the elasticities. In most cases, this will lead to lower elasticities resulting from aggregated models in comparison with disaggregated models. In addition, aggregated models do not make an allowance for the large variation in mobility behavior of individuals, even within groups with similar characteristics. In other words, the use of aggregated models is based on a low level of variation in (aggregated) behavior, which causes a less precise estimation of model parameters.

- *Number of competitive modes taken up in the model.* In addition to real-world choice possibilities, the number of modes included in (choice) models when they are estimated can have an influence on the elasticity values. Of course, this is a matter of proper modeling, but usually there are discrepancies between real-world choice possibilities and those represented in a model.
- *Control for other factors.* In many cases, two situations are compared and it is concluded that a policy measure has led to a certain change in mobility behavior. However, other (external) developments may have had an impact on the dependent variables, and it is, therefore, important to verify this.
- *The functional form of the model.* The functional form of the model used can, in a number of respects, influence the size of elasticities. Different model types may also generate different elasticity types. A model may allow for a distinction between generation and substitution effects (gross and net substitution). Some models yield higher elasticities when changes in the independent variables, like transport price, are higher. Dynamic models allow for an explicit modeling of short-term and long-term effects.

Criteria Derived from the Data

- *Type of data source.* Various data types exist: cross-section, time series, panel, and stated preference data. The use of these data has different consequences for the size of the estimated elasticities. For example, it appears that elasticities based on cross-section data are often higher than elasticities based on time series data. Also, it appears that elasticities based on stated preference data are higher than cross-section data, unless they are re-scaled. Therefore, a proper recording of the data source from which elasticities are calculated is of great importance. In addition, other aspects related to the data source are important.
- *The operationalization of the variables.* Even slight differences in definitions of income, transport price, and travel times (e.g., monetary or generalized travel costs) may cause significant differences in the estimated elasticities.
- *Actual level of travel costs.* As already mentioned, the level of travel demand may show a relationship to travel cost with a changing elasticity value on this curve. It is plausible that travel demand becomes more sensitive to changes in transport costs when these costs are already relatively high.
- *General problems with the data source.* If there are general problems related to the data source, this should be properly recorded. For instance, results from panel data may be influenced by selectivity in panel attrition.
- *Year of collection of the data.* In general, the sensitivity to price change is likely to vary over time, especially when there are large time periods between measurements (more than 10 years). In past decades, the role of transport has rapidly increased in the whole society.

QUALITY OF RESEARCH

In addition to the theory used, the model specification, and the data used, it is important that the research from which elasticities are derived meets some quality standards. We take into consideration here:

- *Statistical techniques used.* It is important to verify that appropriate methods of estimation, given the nature of the data and model structure, have been used for the determination of parameter values, and whether chosen techniques are correctly applied.
- *Sample size.* The size of the sample determines the level of representativeness of the results of the study for the population investigated.

From the considerations set out above, it should be clear that elasticities estimated with different methods under different circumstances are not necessarily comparable. We may formulate from this a list of items on the basis of which we will apply a meta-analysis. The aim is to assess the most important aspects responsible for the variation of elasticity estimates between the different studies in the countries investigated.

META-ANALYSIS IN THE CONTEXT OF COMPARING EUROPEAN DEMAND ELASTICITIES

We have noted in the previous sections that results from past studies on transport elasticities vary strongly, and we have explored potential factors that may cause these differences. Knowledge of these factors may be useful for harmonization of (future) international research on the sensitivity of transport demand to prices.

Our empirical data come from 12 elasticity studies in 4 European countries: Norway, Finland, the Netherlands, and the United Kingdom. We are dealing here with a data set consisting of a limited number of observations (i.e., elasticity study results), thus we are facing a high level of uncertainty. Therefore, our indepth analysis of these causes of variance is based on a meta-analytic approach. Such an approach can be used to extract lessons from a limited set of different research studies.

Meta-analysis is a statistical procedure for combining and comparing research findings from different studies focusing on similar phenomena (see Hedges and Olkin 1985; Light and Pillemer 1984; and Wolf 1986). Meta-analysis is particularly suitable in cases where research outcomes are to be judged or compared (or even transferred to other situations) when there are no controlled conditions. In the past, a variety of meta-analytical methods were developed (see e.g., Hunter et al 1982; Rosenthal 1991). Most meta-analytical techniques are designed for sufficiently large numbers of case studies, so that statistical probability statements can be inferred (e.g., Espey 1996). In this respect, meta-analysis has demonstrated its validity and usefulness as a methodological tool for comparative study in the social sciences. In conclusion, meta-analysis is not a single technique, but rather an analytical approach to comparative study that may comprise a multiplicity of different methods and techniques, which are often statistical in nature.

Especially in the case of quasi-controlled or non-controlled comparative experimentation, the level of information is often not cardinal, but imprecise (e.g., categorical, qualitative, fuzzy). In recent years, rough set theory has emerged as a suitable analytical tool for dealing with “soft” data. Rough set theory, proposed in the early 1980s by Pawlak (1982; 1991), aims to classify data measured on any information level by manipulating the data in such a way that a range of consistent and feasible cause-effect relationships can be identified, while at the same time eliminating redundant information. It has proven to be a useful tool for a large class of qualitative or fuzzy multi-attribute decision problems, and can deal with problems of explanation and prescription of a decision situation where knowledge is imperfect.

ROUGH SET ANALYSIS

Rough set analysis is essentially a nonparametric statistical method that is able to handle a diverse and less immediately tangible set of factors. It provides a formal tool for transforming a data set, such as a collection of past examples or a record of experience, into structured knowledge, in the sense that it can classify objects having distinctive patterns of attributes. Using such an approach, it is not always possible to distinguish objects on the basis of available information (descriptors). This imperfect information causes indiscernibility of objects through the values of the attributes describing them and prevents them from being unambiguously assigned to a given single set. In this case, the only sets that can be precisely characterized in terms of values of ranges of such attributes are lower and upper approximations of the set of objects. We will now set out the basic principles of this method (for more details, see also Pawlak 1991; Van den Bergh et al 1997; Slowinski and Stefanowski 1994; and Greco et al 1995).

With reference to a certain finite set of objects U, it is assumed possible to perceive the differences existing between them by observing some information associated with each of them. A finite set Q of *attributes* is identified, which serves to identify and characterize these objects. As the rough set theory aims to classify and distinguish data on the basis of different values their attributes assume with reference to each object, each attribute $q \in Q$ must be able to assume different values in its domain U_q. There must be, therefore, at least two of these values in order for the attribute to be a significant basis for the required characterization. If an attribute is quantitative, its domain is, in practice, partitioned into a suitable number of sub-intervals, which give a good description of the phenomenon studied, so as to avoid ending up with a distribution of values with a high number of modalities, which would not be useful for the analysis intended. The difficult choice of the bounds (called *norms*) used to define these sub-intervals is important to ensure a correct application of this approach and that too much information is not lost in the translation of original quantitative attribute-values into qualitative coded values.

At this point, to every object $x \in U$ may be associated a vector whose components are the distinct evaluations of x with respect to every attribute of Q and called *description* of x in terms of attribute-values from set Q. The table containing the descriptions of every $x \in U$ by means of the attributes of the set Q is known as the *information table.* It is also possible to obtain a description of $x \in U$ in terms of any one subset of attributes $P \subseteq Q$.

A fundamental concept of rough set theory is that of the binary relation of *indiscernibility*, denoted I_P. Two objects $x, y \in U$ are said to be P-indiscernible by means of the set of attributes $P \subseteq Q$ if and only if they have the same description. Naturally, the binary relation I_P is reflexive, symmetric, and transitive (equivalence relation); its classes, that is, the subsets of U containing all the objects having the same description in terms of the attributes from subset P, and only these, are called *P-elementary sets.* If all the attributes of Q are considered, the Q-elementary sets are called atoms. The P-elementary sets, $P \subseteq Q$, generate a partition of U, in that every object $x \in U$ belongs to one and only one P-elementary set.

For the definition of rough set, it is necessary to introduce two other key concepts. Let $P \subseteq Q$ be a subset of attributes and $X \subseteq U$ a subset of objects of U. We define as *P-lower approximation* of X, denoted with P_LX, the subset of U having as its elements all the objects belonging to the P-elementary sets contained in the set X, and only these. In other words, the elements of P_LX are all the elements of U belonging to all the classes generated by the indiscernibility relation I_P and contained in X, and only these.

We define as the *P-upper approximation* of X, denoted with P_UX, the subset of U having as its elements all the objects belonging to the P-elementary sets having at least one element in common with the set X, and only these. In other words, the elements of P_UX are all the elements of U belonging to all the classes generated by the indiscernibility relation I_P that have at least one representative belonging to X, and only these.

The difference between these sets is known as *P-boundary* of X, denoted with $Bn_P(X) = P_UX – P_LX$. Therefore, $P_LX \subseteq X \subseteq P_UX$ results and, consequently, if an object x belongs to P_LX, it is also an element of X; if x belongs to P_UX, it may belong to the set X; $Bn_P(X)$, therefore, constitutes the “doubtful region” (with reference to its elements, nothing can be said with certainty about its belonging to the set X). The indiscernibility classes generated by I_P, therefore, constitute the basic instrument of the rough set theory used to obtain a better knowledge of reality. This knowledge is intended as a family of partitions of U, generated by the indiscernibility relation I_P on U, $P \subseteq Q$.

A *P-rough set* is the family of all subsets of U that have the same lower and upper P-approximations. The intention is thus to approximate a set X, $X \subseteq U$, by means of a pair of sets associated with it, called lower approximation, P_LX, and upper approximation P_UX, of X, that can be then considered as a particular case of interval set. Only if $P_UX = P_LX$ does X prove to be equal to the union of a certain number of P-elementary sets and is called *P-definable*. Clearly, in this case (and only in this case), it is possible to affirm with certainty whether x, $x \in U$, belongs to X, $X \subseteq U$, using the set of attributes P. Moreover the *accuracy* of the approximation of X, equal to

$$\frac{\text{card}\ (P_LX)}{\text{card}\ (P_UX)}$$

will be at the maximum value (i.e., equal to 1). In general, therefore, the aim of the rough set analysis is to establish whether x is an element of X on the basis of the lower and upper approximations of X, rather than directly by means of a specific characteristic function.

Let $\mathbf{Y} = (Y_1, Y_2, ..., Y_n)$ be a certain classification of U. With reference to the classification $\mathbf{Y}$, we denote as P-lower approximation and P-upper approximation respectively, the sets having as their elements the P-lower and P-upper approximations of its classes, that is $P_L\mathbf{Y} = (P_LY_1, P_LY_2, ..., P_LY_n)$ and $P_U\mathbf{Y} = (P_UY_1, P_UY_2, ..., P_UY_n)$. An indicator of the quality of the approximation of the partition **Y** by means of the set of attributes P, notation $\gamma_P(\mathbf{Y})$, is given by the ratio between the total number of P-correctly classified objects (i.e., belonging to the P-lower approximations of Y_i, i = 1, 2, ..., n), and the total number of objects considered. This is called the *quality of the classification*. This index will assume its maximum value (equal to one) if, and only if, each of the classes Y_i of Y prove P-definable, that is, if each of them is given by the union of P-elementary sets.

Another fundamental concept is that of attribute *reduction* (i.e., given a classification **Y** of the objects of U, the search for a minimal set of (independent) attributes R that supplies the same quality of classification as the original set of attributes P). The minimal subset $R \subseteq P \subseteq Q$ such that $\gamma_R(\mathbf{Y}) = \gamma_P(\mathbf{Y})$ is called **Y**-reduct of P and denoted $RED_Y(P)$. (Note that a single information table may have more than one reduct.) The intersection of all the **Y**-reducts is known as **Y**-core of P, that is, $CORE_Y(P) = \cap\ RED_Y(P)$. Naturally the *core* contains all the attributes from P which are considered of greatest importance in the information table (i.e., the most relevant for a correct classification of the objects of U).

In other words, in order to analyze the information table, it is sufficient to use any one of the attribute reducts $R \subseteq Q$, that is, the classification **Y** of the objects of U may be characterized without losing any information using only the attributes from R, while the information supplied by the attributes of Q–R prove redundant for this purpose. On the other hand, none of the attributes belonging to the core may be neglected without deteriorating the quality of the classification considered, that is, if any one attribute belonging to the core is eliminated from the information table, it will not be possible to obtain the highest quality of approximation with the remaining attributes.

APPLICATION OF ROUGH SET ANALYSIS

As mentioned in the previous section, rough set theory is essentially a classification method devised for non-stochastic information. This also means that ordinal or categorical information (including dummies) may be taken into consideration. This makes rough set analysis particularly useful as a meta-analytical tool in the case of incomplete, imprecise, or fuzzy information. We can expect the following results from the rough set analysis:

- evaluation of the relevance of particular condition attributes;
- construction of a minimal subset of variables ensuring the same quality of description as the whole set (i.e., reducts of the set of attributes);
- intersection of those reducts giving a core of attributes that cannot be eliminated without disturbing the quality of description of the set of attributes; and
- elimination of irrelevant attributes.

The application of rough set analysis on transport elasticity values in different countries proceeds in two successive steps: the construction of an information survey, and the classification of information contained in the survey.

Information survey. In our case, the information survey consists of a series of public transport elasticity studies based on surveys in four European countries. Included are both aggregated and disaggregated elasticity studies. The total number of studies considered is limited, in order to eliminate, as much as possible, differences in definitions of transport costs and elasticities. The information survey contains site- and study-specific characteristics (attributes) of these studies. Because of the limited number of observations, we selected variables from the criteria listed in the previous section. The set of chosen variables is based on maximizing the extent to which elements of other variables are captured in these. Details of the information survey are in table 1.[2]

[2] It should be stated that the combination of 12 observations (in casu, studies/surveys) with 8 explaining attributes leaves us only few degrees of freedom.

TABLE 1 Concise Survey Table for Meta-Analysis of Transport Elasticities for Public Transport in Four European Countries

		1 Country	2 Year of data collection	3 Level of aggre-gation	4 Indicator of transport demand	5 Geo-graphical coverage	6 Number of com-petitive modes	7 Data type	8 Model type	Elasticity value
1	Helsinki	Finland	1988	Bus, tram, metro, train	Trips	Urban	2	Cross-section	Nested logit	–0.48
2	Helsinki	Finland	1995	Bus, tram, metro, train	Trips	Urban	3	Cross-section	Logit	–0.56
3	Sullström, 1995	Finland	1966–90	Bus, tram, metro, train	Person-km	Urban, interurban	1	Repeated cross-section	Linear demand OLS	–0.75
4	Netherlands	Netherlands	1984–85	Bus, tram, metro	Trips	Urban, semi-urban	2	Panel	Linear demand OLS	–0.35/ –0.40
5	BGC, 1988	Netherlands	1980–86	Bus, tram, metro	Trips	Urban, semi-urban	2	Time series	Linear demand OLS	–0.35/ –0.40
6	Roodenburg, 1983	Netherlands	1950–80	Bus, tram, metro	Person-km	Urban, semi-urban	1	Time series	Linear demand OLS	–0.51
7	Fase, 1986	Netherlands	1965–81	Bus, tram, metro	Person-km	Urban	1	Time series	Linear demand OLS	–0.53/ –0.80
8	Gunn, 1987	Netherlands	1986	Train	Person-km	Semi-urban	2	Cross-section	Discrete choice	–0.77
9	Oum, 1992	Netherlands	1977–91	Bus, tram, metro	Person-km	Urban, semi-urban	2	Time series	Translog utility function	–0.74
10	Oslo	Norway	1990–91	Bus, tram, metro, train	Trips	Urban	3	Cross-section	Multinomial logit	–0.40
11	Norway	Norway	1991–92	Bus	Trips	Interurban	5	Cross-section	Multinomial logit	–0.63
12	UK	UK	1991	Bus, tram, metro, train	Trips	Urban, interurban	4	Cross-section	Nested logit	–0.15

Note: Studies referred to by a city or country name were part of the EXTRA project. The other studies result from a literature review.

Classification of Information

The rough set approach can effectively handle quantitative data, but this data must first be converted into qualitative or categorical data by means of an adequate codification. This is done by means of a set of thresholds called norms, which discretize the measurement scales by which the quantitative data are expressed. This applies to both categorical and ratio information. The observations or objects are classified into various categories for each attribute separately. From the researcher's viewpoint, the introduction of the thresholds could mean a methodological advantage, because the discretization of the measurement scale for quantitative attributes should represent the researcher's perception of the analyzed phenomenon that can be represented and analyzed in a form that is understandable to the researcher. However, this step is one of the most problematic issues in the application of rough set analysis.

First, the use of thresholds implies some loss of information. Second, thresholds are chosen subjectively. For example, the thresholds are often those that produce some satisfactory approximation of the considered categories. This is the case in our survey, for both the attribute variables and the elasticity value range. In general, some sensitivity analysis on the classification used is meaningful, as a balance needs to be found between homogeneity and class size. This classification exercise leads then to a decision table, in which all objects are subdivided into distinct categories for each relevant attribute. The categories used are listed in table 2.

The resulting coded information table is in table 3. (When we speak of respectively high or low values of the elasticity size, we refer to the *absolute* value of the elasticity.)

TABLE 2 Categorization of Variables Investigated

Elasticity value

1 Lower than –0.40
2 –0.40 to –0.50
3 –0.50 to –0.60
4 Higher than -0.60

Explanatory variables

1 Country (COU)

1 Finland
2 Norway
3 Netherlands
4 UK

2 Year of data collection (YEA)

1 1985 and before (including studies using data periods over 10 years of which the median year was before 1986)
2 1986 and after (including studies using data periods over 10 years of which the median year was after 1986)

3 Level of aggregation (AGG)

1 Bus, tram, metro, train
2 Bus, tram, metro
3 Bus
4 Train

4 Indicator of transport demand (IND)

1 Number of trips
2 Number of person-km

5 Geographical coverage (GEO)

1 Urban
2 Urban and semi- or interurban
3 Interurban

6 Number of competitive modes (CMD)

1 One
2 Two
3 Three
4 Four and more

7 Data type (DAT)

1 Time series
2 Survey, cross-section
3 Survey, panel

8 Model/estimation type (MOD)

1 Basic OLS (linear demand models)
2 Discrete choice (probit/logit)
3 Other types

TABLE 3 Coded Table for Meta-Analysis of Transport Elasticities for Public Transport

Case/ attribute	COU	YEA	AGG	IND	GEO	CMD	DAT	MOD	Elasticity value
1	1	2	1	1	1	2	2	2	2
2	1	2	1	1	1	3	2	2	3
3	1	1	1	2	2	1	2	1	4
4	3	1	2	1	2	2	3	1	2
5	3	1	2	1	2	2	1	1	2
6	3	1	2	2	2	1	1	1	3
7	3	1	2	2	1	1	1	1	3
8	3	2	4	2	2	2	2	2	4
9	3	1	2	2	2	2	1	3	4
10	2	2	1	1	1	3	2	2	2
11	2	2	3	1	3	4	2	2	4
12	4	2	1	1	2	4	2	2	1

Applying this classification to the samples of elasticity studies within the four investigated European countries, four main sets of indicators and outputs can be calculated.

(1) The *reducts*, that is, all combinations of explanatory or independent variables that can completely determine (or explain) the variation in the dependent variable, without needing other explanatory variables. The reducts are given in table 4. There appear to be, on the basis of the chosen set of characteristics and classification of these characteristics, two competitive theories for explaining the variance in the estimated elasticity values. The first is that this variance is completely determined by the combination of the country of data collection, the number of competitive modes, the type of data collected, and the type of model used. The second theory is that this variance is completely determined by the country, the indicator for transport demand, the number of competitive modes, and the type of data collected.

(2) The *core*, that is, the set of variables that are in all reducts as discussed under (1), or that are part of all theories. The core consists of the country, number of competitive modes, and type of data collected. Without these characteristics, it is impossible to classify the results of the elasticity studies according to the considered categories. This means that these three variables strongly influence the elasticity size. In conclusion, in addition to the practical findings mentioned earlier on the differ-

TABLE 4 Reducts and Core

Reduct	Set no. 1 {COU, CMD, DAT, MOD}
	Set no. 2 {COU, IND, CMD, DAT}
Core	{COU, CMD, DAT}

TABLE 5 Accuracy and Quality of the Classification of the Elasticity Value

Elasticity value class	Accuracy	Lower approximation	Upper approximation
Lower than –0.40	1	1	1
–0.40 to –0.50	1	4	4
–0.50 to –0.60	1	4	4
Higher than –0.60	1	3	3
Accuracy of classification:	1		
Quality of classification:	1		

Note: The accuracy for each class is the lower divided by the upper approximation.

ence between empirical-based research methods and the use of disaggregated choice models, country differences also have a major influence on the elasticity value.

(3) The *lower and upper approximation*, and derived accuracy of relationships for each value class of the decisional variable. The latter is the lower divided by the upper approximation of each class. Accuracy and quality of classification can also be derived from this (i.e., choice of thresholds). The results are shown in table 5. For all classes of the elasticity value, the accuracy is 1. Also, the accuracy and quality of classification are equal to 1. This value is the maximum value in all these cases. This means that on the basis of the chosen characteristics the studies in our sample are fully discernible regarding the four classes of the elasticity value. This strengthens the conclusions on the other indicators from the rough set analysis.

(4) *Rules*, that is, exact or approximate relationships between explanatory variables and dependent variables. These may be considered "if . . . then . . . " statements. A rule may be exact (or deterministic), or approximate (or non-deterministic). An exact rule guarantees that the values of the decision attributes correspond to the same values of the condition attributes (same conditions, same decisions); an approximate rule, on the other hand, states that more than one value of the decision attributes corresponds to the same values of the condition attributes (same conditions, different decisions). Therefore, only in the case of exact rules, using the information contained in the decision table, is it always possible to state with certainty if an object belongs to a certain class of the decision variable. An exact rule, therefore, offers a sufficient condition of belonging to a decision class; an approximate rule (only) admits the possibility of this. Table 6 shows the rules that can be generated from our data set. The support of rules by cases is also a useful indicator. If a rule is supported by more objects, then it is more important, for instance, in summarizing the different single study results.

We see from the decision algorithm in table 6 that all rules generated in the elasticity study information survey, using the classes of table 3, are deterministic. Some statements may then be derived on the influence of the variables occurring in this algorithm, but we should take into account that some of these rules are supported by only one

TABLE 6 Rules Generated by the Rough Set Analysis

Classes of dependent attributes	Implied class of elasticity size
COU=UK	Lower than –0.40
COU=Finland, IND=trips, CMD=2	–0.40 to –0.50
COU=Norway, IND=trips, CMD=3	–0.40 to –0.50
COU=Netherlands, IND=trips, CMD=2, DAT=panel	–0.40 to –0.50
AGG=bus/tram/metro, CMD=1	–0.50 to –0.60
COU=Finland, AGG=bus/tram/metro/train, CMD=3	–0.50 to –0.60
COU=Netherlands, AGG=bus/tram/metro, IND=trips, CMD=2, DAT=time series	–0.50 to –00.60
IND=person-km, GEO=urban and interurban	Higher than –0.60
AGG=bus	Higher than –0.60

observation (e.g., the case of where the country (UK) implies a relatively low elasticity value). Nevertheless, within the limits of our small data set we may derive some interesting information from these rules. A rule supported by more observations is that when the area covered is a mixture of the urban, semi-urban and interurban level, the elasticity value is relatively high.

With the limitation of having only few degrees of freedom, our analysis leads to some prudent findings. First, conclusions from past elasticity study reviews on the importance of the difference between aggregated, empirical-based research methods and the use of disaggregated choice models, as well as the model assumptions, seem to be to a certain extent supported by this application of meta-analysis. In this analysis, there appear to be, on the basis of a chosen classification of study characteristics, two competitive theories for explaining the variance in the estimated elasticity values. The first is that this variance is completely determined by the combination of the country of data collection, the number of competitive modes, the type of data collected, and the type of model used.

The second theory is that this variance is completely determined by the country, the indicator for transport demand, the number of competitive modes, and the type of data collected. Thus, it appears that the variables of country, number of competitive modes, and type of data collected are important factors in accounting for the elasticity size. The result of the meta-analysis is, therefore, that in addition to the practical findings on the difference between empirical-based research methods and the use of disaggregated choice models, country differences also have a major influence on the elasticity value. *This means that, even when the estimation method is the same in terms of data used and the model specification, the elasticities for the different European countries should be looked at very carefully.* The situations between the countries may differ to a large extent. For example, in the Netherlands bicycles are a relatively important mode in comparison with the other countries, primarily because of the relatively short travel distances, the flat surface, and the good infrastructure provided for the bicycle. The public transport elasticity of those who are dependent on public transport (e.g., young people) is, therefore, quite high in comparison with other countries. The short travel distances in the Netherlands (looking at both urban and interurban trips) also enlarge substitution possibilities between other modes.

Further reasons for the high impact of the country on elasticity values can be found in the cultural differences between the countries. Differences in the infrastructure and the quality of public transport also determine the level of competitiveness between the transport modes.

CONCLUSIONS

The goal of this paper has been to assess the factors that influence the sensitivity of travelers to public transport travel costs in Europe, by carrying out a comparative analysis of elasticity values of transport demand resulting from studies in various countries. We have made use of a rather limited data set containing 12 studies/surveys on demand elasticities with 8 site- and study-specific characteristics. Because of this, we had only a few degrees of freedom. By applying meta-analysis, this comparative study has still led to some interesting conclusions.

The main findings from existing reviews of elasticity studies assessing causes of variances— namely the importance of the difference between aggregated, empirical-based research methods and disaggregated choice models, as well as the model assumptions—seem to be reasonably supported by our indepth analysis of a set of potential factors of influence by means of rough set analysis. It appears that from our set of variables, country, number of competitive modes, and type of data collected have the strongest explanatory power for the elasticity size. The result of our meta-analytic application is that in addition to the practical findings on the difference between empirical-based research methods and the use of disaggregated choice models, country-specific factors also play a large role. This means that care should be taken when comparing elasticities for the different European countries, even when estimation methods are the same (i.e., data used and the model specification). Relevant country-specific characteristics like natural circumstances and travel distances may mean that certain modes are favored (e.g., the bicycle in the Netherlands). Cultural differences and differences in the quality of public transport are also important, as these determine the level of competitiveness between the transport modes.

The findings above on the importance of country-specific factors that determine the price sensitivity of travelers imply that the formulation of a common transport price policy at the European level, in terms of harmonizing prices, is a difficult task, and will probably not lead to a first-best solution to the rising negative transport externalities in Europe. Instead, pricing policies for public transport should be adapted to local situations in order to be able to derive optimal effects.

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