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Final Report

Project Title:

Route Choice in Congested Grid Networks

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Project Abstract

The objective of this research project was to model the traffic performance networks in which travelers have heterogeneous preferences for aspects of their trip. In real networks, drivers make choices of which route to use based on many criteria including the travel time and the variability of travel time. This research project focused on developing equilibrium models that recognize the differences in preferences among travelers. The resulting model related the performance of links in a network to the equilibrium distribution of routes selected by drivers. The equilibrium method was based on an analytical approach that has applications to many transportation problems in which individuals make travel choices based on multiple criteria. The resulting approach has applications to systems of car traffic and transit use, as well as to the development of advanced traveler information systems that account for diverse travel preferences among users.

Introduction

Rapid growth in the demand for travel in the urban networks has given rise to the critical need for the conscious plans and policies that can maintain and improve the quality of service by making efficient use of available resources in the transportation network. In this respect, success of the plans and management policies in fulfilling their objectives crucially depends on accuracy of the approximation of travel behavior of users in transportation networks. To demonstrate the behavior of the users in the transportation system, it is reasonable to consider that rational users generally tend to follow their own interests by making the decisions that maximize their utilities in the network. Nevertheless, the discrepancy in the resulting decisions of the users in comparable situations in the network can be attributed to the heterogeneity exists in the preferences of the users due to the dissimilarities in their socioeconomic backgrounds, personalities, and motivations. Hence, it is of great importance to recognize the heterogeneity in preferences of the users as an aspect of their decision-making procedure that should be taken into account in modeling and optimizing transportation systems.

Modeling Choices of Heterogeneous Travelers

In the transportation network, users make different decisions regarding the starting time, destination, mode and route of their trips. For each of these decisions, users consider a variety of different factors in their decision-making procedures, while the relative importance of these factors varies with the purpose of their trips among the heterogeneous users. Rational users seek for the choices that minimize the cost of their trips according to their own preferences. The cumulative result of the individual decisions of the heterogeneous users is the user equilibrium condition in which no one can reduce his or her cost by changing his or her decisions in the network. The objective of this research is to account for heterogeneity in the preferences of users in modeling the user equilibrium and system optimum conditions of the network. For this purpose, we adapt the concept of the efficient frontier from portfolio theory (Markowitz, 1952) in modern finance to represent the equilibrium condition of the bicriterion choice problems with heterogeneous user preferences in the transportation network.

In bicriterion choice problems, users seek to minimize the disutility or cost of their own choices by balancing the trade-off between the two types of costs associated with different possible choices. In this trade-off, the relative importance between these costs plays an important role in optimizing this balance. In this respect, the general cost of the users in the of the bicriterion choice problems in transportation network can be expressed as a linear combination of two costs, weighted by their relative importance as follows:

$$C_{p,i} = C_i^1 + \alpha_p C_i^2 \tag{1}$$

where, C_i^1 and C_i^2 represent the components of the cost associated with the choice i, while the coefficient α_p denotes the relative importance of these components for users in the preference group p. To include the heterogeneity of the preferences among the users, we may assume that the value of α_p is represented by a discrete or continuous distribution over the demand. $C_{p,i}$ is the resulting general cost of choice i for the users in preference group p. With this definition of the general cost function, rise in α_p corresponds to preferences of the users that give more importance to the second component of the cost. Heterogeneous users with different values for α_p find different choices optimal according to their own preferences. As a result, the concept of the efficient frontier can be employed to represent the dominant choices set for users with heterogeneous values for α_p.

Characterizing Equilibrium with the Efficient Frontier

The concept of the efficient frontier was introduced in Portfolio Theory by Markowitz (1952) to represent the cumulative result of individual bicriterion decisions of heterogeneous investors in a free market. The free market offers the investor a variety of assets with different levels of risk and return, so heterogeneous investors select different assets according to their own preferences. More conservative investors prefer assets with a lower level of risk in spite of the lower profit they return while risk-takers invest in assets with higher levels of return even though their investments are subject to higher level of risk. Fig. 1a illustrates the efficient frontier (green points) of dominant assets that is desirable to a population of heterogeneous investors while no one will invest in assets that are dominated (red points) by an alternative asset with a lower level of risk and higher level of return.

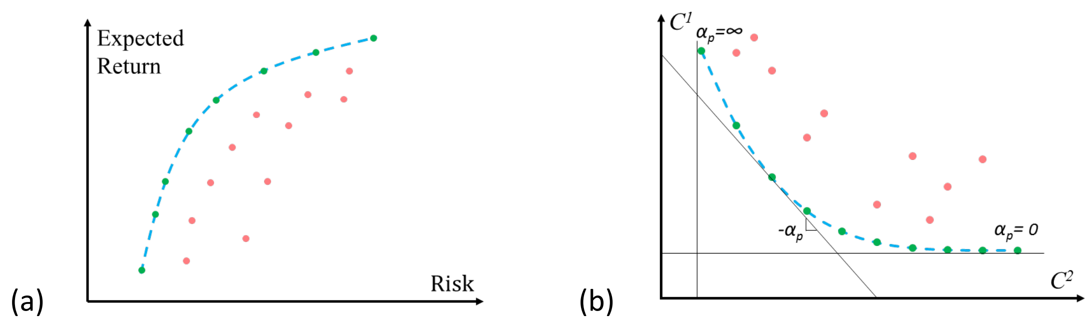


Fig. 1. The efficient frontier of (a) the dominant assets in a free market (b) the choices with heterogeneous preferences

The analogy between the bicriterion problems in the finance and transportation disciplines makes it possible to adapt the theory by substituting the concepts of the risk and return with the components of the users' generalized cost in transportation problems. Rational users, who tend to minimize their costs by seeking the choice with the lowest combined cost (1), make a variety of different choices due to heterogeneity of their preferences described by distribution of α_p. As illustrated in Fig. 1b, there is a set of choices (green points), each of which are associated with a minimum generalized cost for some positive value of α_p. The other choices (pink points) are dominated and will not be optimal for anyone. Accordingly, the cumulative result of individual decisions with heterogeneous preferences can be represented by the lower envelope of the choice set in the C^2-C^1 plane that entails the equilibrium choices with the minimum general cost for the users with heterogeneous preferences, which is call the efficient frontier. The efficient frontier can be shown to be always a non-increasing convex function with a specific geometric property that relates the shape of the efficient frontier to the distribution of the preferences. In this research, we make a use of these primary properties of the efficient frontier to model the equilibrium condition of different bicriterion choice problems with heterogeneous user preferences in the transportation network.

Choosing the starting time of a trip is one of the important decisions that users need to make ahead of their trips according to their own preferences. The trip scheduling problem, introduced in Vickrey (1969), addresses a single bottleneck problem with a time-dependent demand and fixed capacity. Insufficient capacity of the bottleneck to meet the demand results in the formation of a queue, which causes users to experience a combination of queueing delay and schedule deviation in their commutes. Rational users tend to minimize the generalized costs of their own trip by adjusting their arrival times to the bottleneck. However, the relative importance of the components of the cost may vary among commuters, and such heterogeneity can be represented by a set of probability distributions over the population of users. Competition between the users to minimize their own cost eventually leads to the user equilibrium condition in which no one can reduce his or her cost by changing his or her own arrival time to the bottleneck. In this research, we employ the concept of the efficient frontier to propose an analytical solution to the morning commute problem as a bicriterion choice problem for heterogeneous users with a general distribution of the schedule preferences over time and a joint distribution of schedule penalty preferences over the population of the commuters. We also demonstrate how the proposed model can be inversely used to approximate the probability distribution of schedule penalty preferences of the users using empirical arrival time data from the network. Research shows that charging a dynamic price equals to the difference between the generalized cost of users in the user equilibrium and system optimum condition, as illustrated in Fig. 2, can keep the system optimized over time by incentivizing the users to adjust their arrival times to the bottleneck. Thus, we use of the result to propose a dynamic pricing strategy that can minimize the total cost of the system by avoiding the formation of a queue in the bottleneck, while the result can be also extended for designing dynamic cordon pricing strategies on a network level. As an example, the approach is employed to derive a closed form solution when the probability distribution of the preferences is uniform. A numerical example is also presented using the proposed model to compare the solutions for different distributions of the schedule deviation penalty preferences.

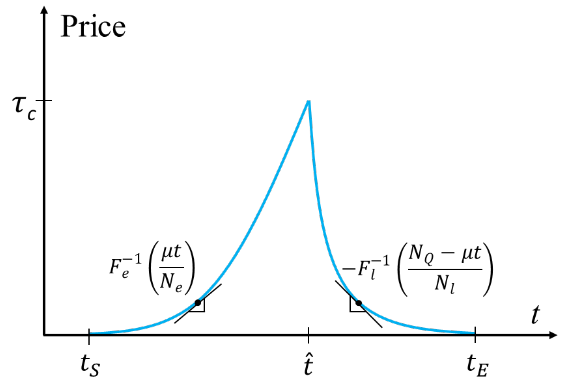


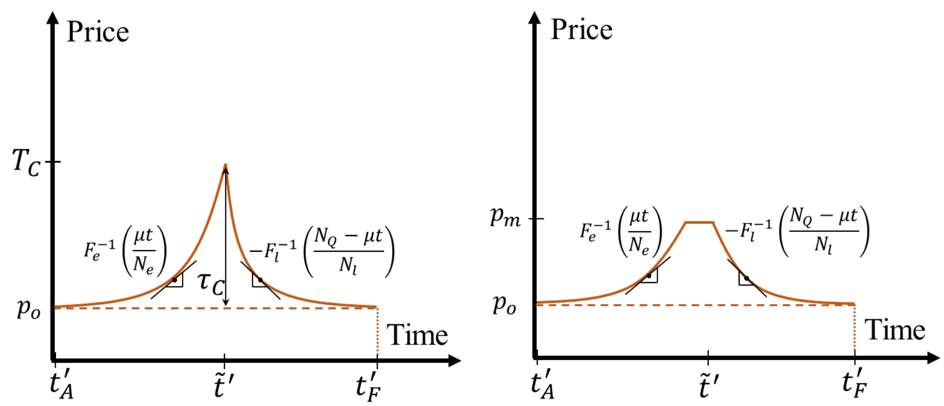
Fig. 2. The system optimal dynamic pricing of the bottleneck with heterogeneous demand

Equilibrium with Transit

The congestion theory proposed by Vickrey (1969) can also have applications in analyzing other transportation systems with state-variable capacity and time-dependent demand that can be modeled as a queueing system. In this respect, we show that a demand responsive transit (DRT) service can be modeled as a queueing system with limited capacity and time-dependent demand. Accordingly, Vickrey's congestion model is employed to analyze and optimize the operation of the DRT system, while the analytical solution derived for the morning commute problem can be extended to account for the heterogeneity in preferences of the DRT service users. The operating cost of a DRT system strictly depends on the quality of service that it offers to its users. The agency seeks to minimize the operating costs, meanwhile keeping the quality of service high for the users. In this research, the operation cost of running a DRT system with dynamic demand is approximated as a linear combination of the fleet cost, total vehicle miles traveled, and total vehicle hours traveled in the network based on an analytical model presented in Daganzo (1978). In addition, the total delay and schedule deviation that users experience in the system is also approximated by combining the congestion theory with classic analytical models of DRT. In this respect, the approach makes use of Vickrey's (1969) congestion theory to model the dynamics of the DRT system in the equilibrium condition and approximates the generalized cost for users when the operating capacity is inadequate to serve the time-dependent demand over the peak period without excess delay. The efficiency of the DRT system can be improved by optimizing the parameters that define the agency's operating decision:

1) the operating capacity of the system,
2) the number of passengers that have requested a pick-up and are awaiting service, and
3) the distribution of requested times for service from the DRT system.

The DRT system can be also optimized by charging a dynamic price (or subsidizing the service) equals to the difference between the generalize cost of the users in the user equilibrium and system optimum conditions, as illustrated in Fig. 3a, in order to flatten the demand of the DRT system over the peak period. However, in practice there is usually an upper bound for the price the operator can charge as the fare for the service. In this case, the constrained (second-best) pricing strategy can be derived by restricting the most effective (first-best) pricing pattern to the upper limit, as depicted in Fig. 3b. On this basis, a schedule management policy and dynamic pricing strategies are presented that can be implemented to manage demand and to reduce the total cost of the DRT system by keeping the number of waiting requests optimized over the peak period. Results of different optimization scenarios are also compared in a numerical example.



(a) Optimum Pricing Strategy (b) Constrained Pricing Strategy

Fig. 3. Dynamic pricing strategies for the DRT system with heterogeneous user preferences.

Equilibrium of Route Choice

Route choice is another important decision that users make for their trips in a network. Rational users tend to reduce the costs of their trips by choosing the routes with the minimum travel cost. Thus, the cumulative result of the individual decisions leads to the user equilibrium condition in which no one can reduce his or her cost by switching to another route. The duration of the trip is one of the important components of the travel cost, which is also correlated with number of other components of the cost like fuel consumption. Conventional traffic assignment models simplify the route choice problem by making the assumption that travel time is the only influential factor in route choice behavior of the users, which can be precisely predicted by travelers in the network. However, research shows that travelers can just have estimations of the average travel time (t) and its variations (s) for different routes based on their previous experiences in the network, while the relative importance of these factors varies among the heterogeneous users with different trip purposes. Accordingly, the route choice decision-making behavior of the travelers can be addressed as a bicriterion choice problem with heterogeneous user preferences. In the last part of the research, we study the route choice behavior of the heterogeneous users in the network under travel time variability, while there is a heterogeneity associated with risk sensitivity of the users. For this purpose, we employ the concept of the efficient frontier to represent the equilibrium solution of the route choice problem as represented in Fig. 4. On the basis of the primary properties of the efficient frontier, we propose a mathematical formulation of the route choice problem under travel time variability. A solution algorithm is also designed that uses the primary characteristics of the equilibrium condition to assign the heterogeneous demand to the network. The efficiency of the proposed solution method is also comparted with a classic smoothing assignment method in a numerical example. The proposed model can also have broader applications in modeling decision making procedure of the travelers in the transportation network.

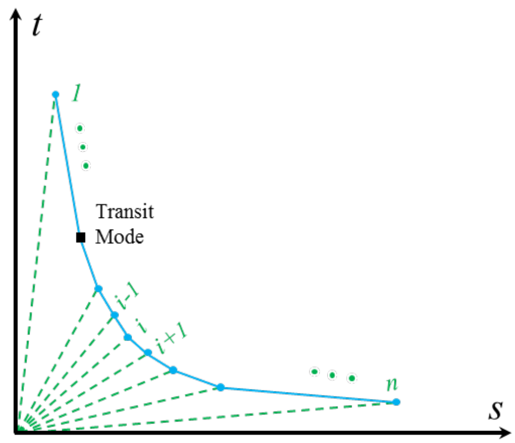


Fig. 4. The efficient frontier of the mode and route choices.

Conclusion

The objective of this research is to account for the heterogeneity in preferences of the users in modeling bicriterion choice problems in transportation network, and for that purpose, we have adapted the concept of the efficient frontier from portfolio theory in finance. However, the concept of the efficient frontier can be extended to include multiple factors in the disutility function in modeling the general decision making process for heterogeneous users. The general efficient frontier is expected to have equivalent properties, which can be used in modeling and solving the multicriterion problems. Nonetheless, multidimensionality of the problem naturally complicates the solution as well. This project resulted four peer-reviewed journal publications (Amirgholy and Gonzales, 2016; 2017a; 2017b; Amirgholy et al., 2017).

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