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Steel Bridge Design Handbook

Design Example 2B: Two-Span Continuous Straight Composite Steel Wide-Flange Beam Bridge

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FOREWORD

This handbook covers a full range of topics and design examples intended to provide bridge engineers with the information needed to make knowledgeable decisions regarding the selection, design, fabrication, and construction of steel bridges. Upon completion of the latest update, the handbook is based on the Seventh Edition of the AASHTO LRFD Bridge Design Specifications. The hard and competent work of the National Steel Bridge Alliance (NSBA) and prime consultant, HDR, Inc., and their sub-consultants, in producing and maintaining this handbook is gratefully acknowledged.

The topics and design examples of the handbook are published separately for ease of use, and available for free download at the NSBA and FHWA websites: http://www.steelbridges.org, and http://www.fhwa.dot.gov/bridge, respectively.

The contributions and constructive review comments received during the preparation of the handbook from many bridge engineering processionals across the country are very much appreciated. In particular, I would like to recognize the contributions of Bryan Kulesza with ArcelorMittal, Jeff Carlson with NSBA, Shane Beabes with AECOM, Rob Connor with Purdue University, Ryan Wisch with DeLong's, Inc., Bob Cisneros with High Steel Structures, Inc., Mike Culmo with CME Associates, Inc., Mike Grubb with M.A. Grubb & Associates, LLC, Don White with Georgia Institute of Technology, Jamie Farris with Texas Department of Transportation, and Bill McEleney with NSBA.

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16. Abstract

This design example presents an alternative design for that presented in the Steel Bridge Design Handbook Design Example 2A. Specifically, the design of a continuous steel wide-flange beam bridge is presented using a standard shape rolled I-beam, as an alternative to the plate girder design. The AASHTO LRFD Bridge Design Specifications are the governing specifications and all aspects of the provisions applicable to wide-flange beam bridge design (cross-section proportion limits, constructibility, serviceability, fatigue, and strength requirements) are considered. Furthermore, the optional moment redistribution specifications will be invoked. In addition to the beam design, the design of the concrete deck is also included. A basic wind analysis of the structure is also presented.

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1.0 INTRODUCTION

This design example presents an alternative design to that presented in Design Example 2A. Specifically, the design of a continuous steel I-beam bridge is presented using a rolled wide-flange beam, as an alternative to the preceding welded plate girder design. The Seventh Edition of the *AASHTO LRFD Bridge Design Specifications* [1], referred to herein as *AASHTO LRFD (7th Edition, 2014)*, is the governing specification and all aspects of the provisions applicable to I-beam design (cross-section proportion limits, constructibility, serviceability, fatigue, and strength requirements) are considered. Furthermore, the optional moment redistribution specifications given in Appendix B6 of *AASHTO LRFD* are invoked. In addition to the beam design, the design of the concrete deck is also included. A basic wind analysis of the structure is also presented.

2.0 DESIGN PARAMETERS

The purpose of this example is to illustrate the design of a tangent two-span continuous composite bridge having equal spans of 90.0 feet with rolled wide-flange beams using the moment redistribution procedures outlined in Appendix B6 of *AASHTO LRFD*. The bridge cross-section (see Figure 1) has four rolled wide-flange beams spaced at 10.0 feet with 3.5 foot overhangs providing for a 34.0 feet roadway width. The reinforced concrete deck is 8.5 inches thick, including a 0.5 inch integral wearing surface and a 2.0 inch haunch.

The framing plan for this design example (see Figure 2) has cross-frames spaced at 30 feet near the abutments and 15 feet near the pier. The spacing of the cross-frames is governed by constructibility requirements in regions of positive bending and moment redistribution requirements in regions of negative bending.

ASTM A709, Grade 50W is used for all structural steel and the concrete is normal weight with a 28-day compressive strength, f'_c, of 4.0 ksi. The concrete slab is reinforced with nominal Grade 60 reinforcing steel.

The design specifications are the *AASHTO LRFD Bridge Design Specifications,* Seventh Edition. Unless stated otherwise, the specific articles, sections, and equations referenced throughout this example are contained in these specifications.

The beam design presented herein is based on the premise of providing the same beam design for both the interior and exterior beams. Thus, the design satisfies the requirements for both interior and exterior beams. Additionally, the beams are designed assuming composite action with the concrete slab.

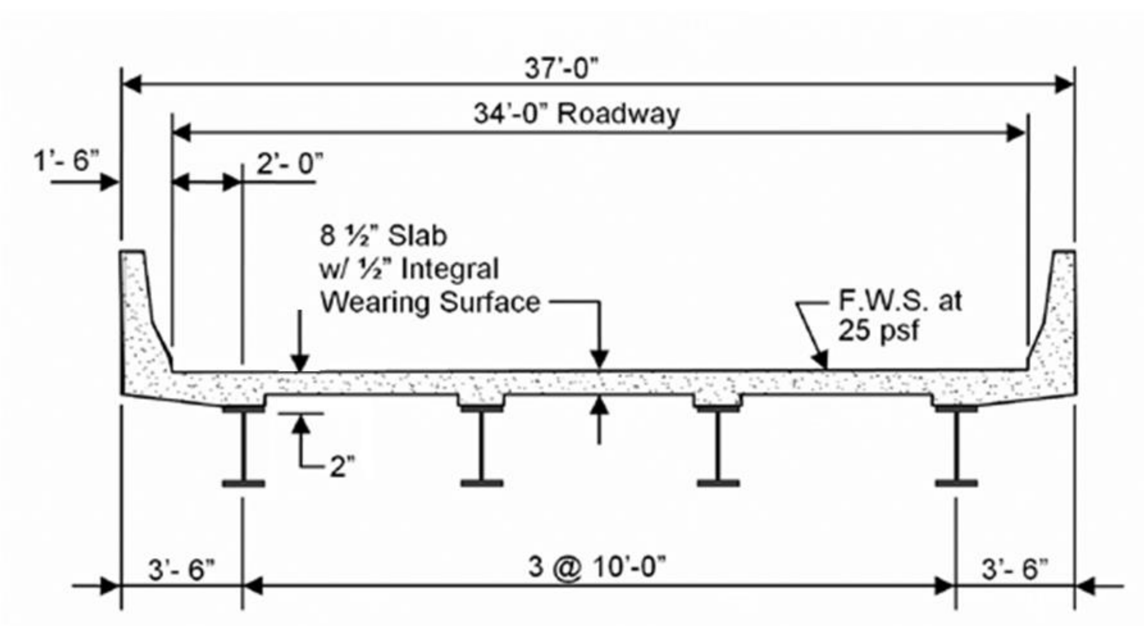

Figure 1 Sketch of the Typical Bridge Cross Section

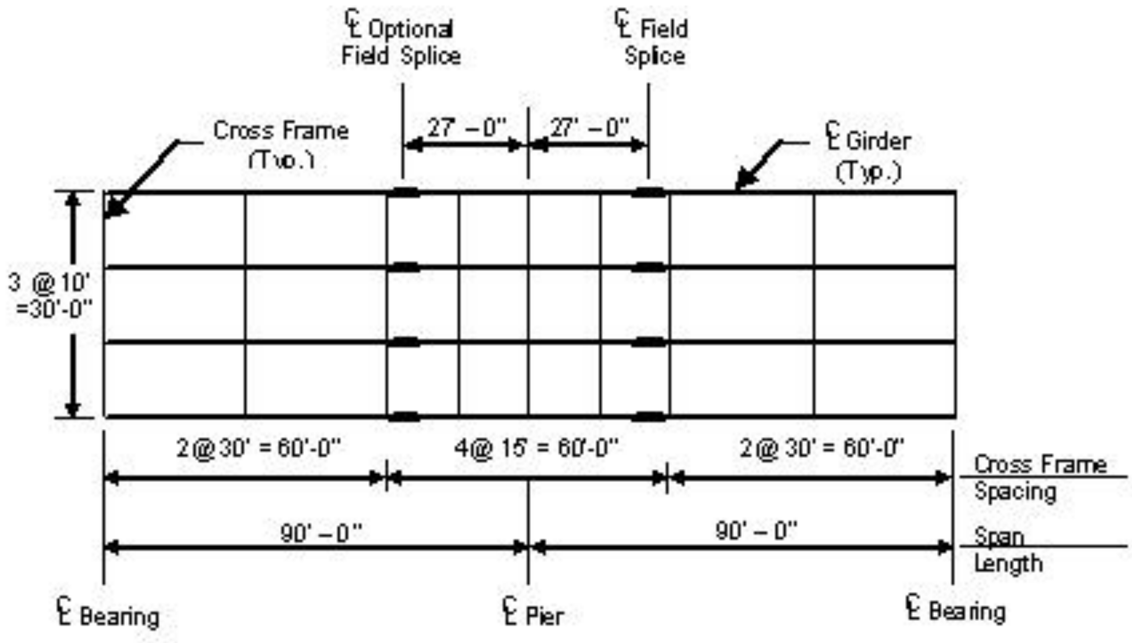

Figure 2 Sketch of the Superstructure Framing Plan

3.0 BEAM ELEVATION

The beam elevation, shown in Figure 3, has section transitions at 30% of the span length from the interior pier. The design of the beam from the abutment to a location 63.0 feet into each span is primarily based on positive bending moments; thus, these sections of the beam are referred to as either the "positive bending region" or "Section 1" throughout this example. Alternatively, the beam geometry at the pier is controlled by negative bending moments; consequently the region of the beam extending 27.0 feet on either side of the pier will be referred to as the "negative bending region" or "Section 2".

By iteratively selecting various rolled wide-flange beams from the standard shapes available, the selected rolled wide-flange section shown in Figure 3 was determined to be the most economical selection for this example.

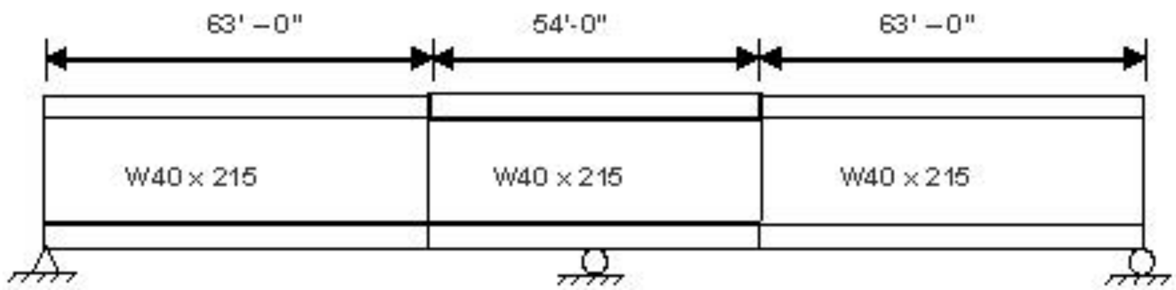

Figure 3 Sketch of the Beam Elevation

4.0 LOADS

This example considers all applicable loads acting on the superstructure including dead loads, live loads, and wind loads as discussed below. In determining the effects of each of these loads, the approximate methods of analysis specified in Article 4.6.2 are implemented.

4.1 Dead Loads

As discussed in the Steel Bridge Design Handbook Design Example 2A, the bridge dead loads are classified into two categories: dead load of structural components and non-structural attachments (DC), and dead load of wearing surface and utilities (DW).

Load factors of 1.25 and 1.00 are used for DC at the Strength and Service Limit States, respectively. For DW, a load factor of 1.50 is used at the Strength Limit State and a load factor of 1.00 is used at the Service Limit State.

4.1.1 Component Dead Load (DC)

As discussed in Example 2A, the component dead load is separated into two parts: dead loads acting on the non-composite section (DC1) and dead loads acting on the long-term composite section (DC2). DC1 is assumed to be carried by the steel section alone. DC2 is assumed to be resisted by the long-term composite section, which consists of the steel beam plus an effective width of the concrete slab when the beam is in positive bending, and the beam plus the longitudinal steel reinforcing within the effective width of the slab when the beam is in negative bending at the strength limit state. At the fatigue and service limit states, the concrete deck may be considered effective in both negative and positive bending for loads applied to the composite section if certain conditions are met.

DC1 includes the beam self-weight, weight of the concrete slab (including the haunch and deck overhang taper if present), deck forms, cross-frames, and stiffeners. The unit weight for steel (0.490 k/ft^3) used in this example is taken from Table 3.5.1-1, which provides approximate unit weights of various materials. Table 3.5.1-1 also lists the unit weight of normal weight concrete as 0.145 k/ft^3; the concrete unit weight is increased to 0.150 k/ft^3 in this example to account for the weight of the steel reinforcement within the concrete. The dead load of the stay-in-place forms is assumed to be 15 psf. To account for the dead load of the cross-frames, stiffeners and other miscellaneous steel details, a dead load of 0.015 k/ft is assumed. It is also assumed that these dead loads are equally distributed to all beams as permitted by Article 4.6.2.2.1 for the line-girder type of analysis implemented herein. Thus, the total DC1 loads used in this design are as computed below.

Slab = (8.5/12) x (37) x (0.150)/4	= 0.983 k/ft
Haunch = (2-1.22)(15.8)/144 x 0.150	= 0.013 k/ft
Overhang taper = 2 x (1/2) x (3.5-7.9/12) x (2/12) x 0.150/4	= 0.018 k/ft
Beam	= 0.215 k/ft

Cross-frames and misc. steel details = 0.015 k/ft

Stay-in-place forms = 0.015 x (30-3 x (15.8/12))/4 = 0.098 k/ft

Total DC1 = 1.342 k/ft

DC2 is composed of the weight from the barriers, medians, and sidewalks. No sidewalks or medians are present in this example and thus the DC2 weight is equal to the barrier weight alone. The barrier weight is assumed to be equal to 520 lb/ft. Article 4.6.2.2.1 specifies that when approximate methods of analysis are applied DC2 may be equally distributed to all beams or, alternatively, a larger proportion of the concrete barriers may be applied to the exterior beam which represents a more realistic distribution of these loads acting out on the deck overhangs. In this example, the barrier weight is equally distributed to all beams, resulting in the DC2 loads computed below.

Barriers = (0.520 x 2)/4 = 0.260 k/ft

DC2 = 0.260 k/ft

4.1.2 Wearing Surface Dead Load (DW)

Similar to the DC2 loads, the dead load of the future wearing surface is applied to the long-term composite section and is assumed to be equally distributed to each girder. A future wearing surface with a dead load of 25 psf is assumed. Multiplying this unit weight by the roadway width and dividing by the number of girders gives the following.

Wearing surface = (0.025) x (34)/4 = 0.213 k/ft

DW = 0.213 k/ft

4.2 Vehicular Live Loads

4.2.1 General Vehicular Live Load (Article 3.6.1.2)

The AASHTO vehicular live loading is designated as the HL-93 loading and is a combination of the design truck or tandem plus the design lane load. The design truck, specified in Article 3.6.1.2.2, is composed of an 8-kip lead axle spaced 14 feet from the closer of two 32-kip rear axles, which have a variable axle spacing of 14 feet to 30 feet. The transverse spacing of the wheels is 6 feet. The design truck occupies a 10 feet lane width and is positioned within the design lane to produce the maximum force effects, but may be no closer than 2 feet from the edge of the design lane, except for the design of the deck overhang.

The design tandem, specified in Article 3.6.1.2.3, is composed of a pair of 25-kip axles spaced 4 feet apart. The transverse spacing of the wheels is 6 feet.

The design lane load is discussed in Article 3.6.1.2.4 and has a magnitude of 0.64 klf uniformly distributed in the longitudinal direction. In the transverse direction, the load occupies a 10 foot width. The lane load is positioned to produce extreme force effects, and therefore, need not be applied continuously.

For both negative moments between points of contraflexure and interior pier reactions a special loading is used. The loading consists of two design trucks (as described above but with the magnitude of 90% the axle weights) in addition to 90% of the lane loading. The trucks must have a minimum headway of 50 feet between the two loads. The live load moments between the points of dead load contraflexure are to be taken as the larger of the moments caused by the HL-93 loading or the special loading.

Live load shears are to be calculated only from the HL-93 loading, except for interior pier reactions, which are to be taken as the larger of the reactions due to the HL-93 loading or the special loading.

The dynamic load allowance, which accounts for the dynamic effects of force amplification, is only applied to the truck portion of the live loading, and not the lane load. For the strength and service limit states, the dynamic load allowance is taken as 33 percent, and for the fatigue limit state, the dynamic load allowance is taken as 15 percent.

4.2.2 Optional Live Load Deflection Load (Article 3.6.1.3.2)

The loading for the optional live load deflection criterion consists of the greater of the design truck, or 25 percent of the design truck plus the lane load. A dynamic load allowance of 33 percent applies to the truck portions (axle weights) of these load cases. During this check, all design lanes are to be loaded, and the assumption is made for straight-girder bridges that all components deflect equally.

4.2.3 Fatigue Load (Article 3.6.1.4)

For checking the fatigue limit state, a single design truck with a constant rear axle spacing of 30 feet is applied.

4.3 Wind Loads

Article 3.8.1.2 discusses the design horizontal wind pressure, P_D, which is used to determine the wind load on the structure. The wind pressure is computed as follows:

$$P_D = P_B \frac{V_{DZ}^2}{10,000} \qquad \text{Eq. (3.8.1.2.1-1)}$$

where: P_B = base wind pressure of 0.050 ksf for beams (Table 3.8.1.2.1-1)

V_{DZ} = design wind velocity at design elevation, Z (mph)

In this example it is assumed the superstructure is less than 30 feet above the ground, at which the wind velocity is prescribed to equal 100 mph, which is designated as the base wind velocity, V_B. With V_{DZ} equal to the base wind velocity of 100 in Eq. 3.8.1.2.1-1 the horizontal wind pressure, P_D, is determined as follows.

$$P_D = 0.050 \frac{100^2}{10,000} = 0.050 \, \text{ksf}$$

4.4 Load Combinations

The specifications define four limit states: the service limit state, the fatigue and fracture limit state, the strength limit state, and the extreme event limit state. The subsequent sections discuss each limit state in more detail; however for all limit states the following general equation from Article 1.3.2.1 must be satisfied, where different combinations of loads (i.e., dead load, wind load) are specified for each limit state.

$$\eta_D \eta_R \, \eta_I \Sigma \gamma_i \, Q_i \leq \varphi R_n = R_r$$

where:

η_D = Ductility factor (Article 1.3.3)
η_R = Redundancy factor (Article 1.3.4)
η_I = Operational importance factor (Article 1.3.5)
γ_i = Load factor
Q_i = Force effect
φ = Resistance factor
R_n = Nominal resistance
R_r = Factored resistance

The factors relating to ductility and redundancy are related to the configuration of the structure, while the operational importance factor is related to the consequence of the bridge being out of service. The product of the three factors results in the load modifier, η, and is limited to the range between 0.95 and 1.00. In this example, the ductility, redundancy, and operational importance factors are each assigned a value equal to one. The load factors are given in Tables 3.4.1-1 and 3.4.1-2 of the specifications and the resistance factors for the design of steel members are given in Article 6.5.4.2.

When evaluating the strength of the structure during construction, the load factor for construction loads, for equipment and for dynamic effects (i.e. temporary dead and/or live loads that act on the structure during construction) is not to be taken less than 1.5 in the Strength I load combination (Article 3.4.2). Also, the load factor for any non-integral wearing surface and utility loads may be reduced from 1.5 to 1.25 when evaluating the construction condition. The load factor for wind may be reduced to not less than 1.25 when checking the Strength III load combination during construction (Article 3.4.2). Also, for evaluating the construction condition, the load factor for temporary dead loads that act on the structure during construction is not to be taken less than 1.25 and the load factor for any non-integral wearing surface and utility loads may be reduced from 1.5 to 1.25.

Article 3.4.2.1 further states that unless otherwise specified by the Owner, primary steel superstructure components are to be investigated for maximum force effects during construction for an additional load combination consisting of the applicable DC loads and any construction loads that are applied to the fully erected steelwork. For this additional load combination, the load factor for DC and construction loads including dynamic effects (if applicable) is not to be taken less than 1.4. For steel superstructures, the use of higher-strength steels, composite construction, and limit-states design approaches in which smaller factors are applied to dead load force effects than in previous service-load design approaches, have generally resulted in lighter members overall. To ensure adequate stability and strength of primary steel superstructure components during construction, an additional strength limit state load combination is specified for the investigation of loads applied to the fully erected steelwork (i.e., for investigation of the deck placement sequence and deck overhang effects).

5.0 STRUCTURAL ANALYSIS

The *AASHTO LRFD (7th Edition, 2014)* allows the designer to use either approximate (e.g., line beam) or refined (e.g., grid or finite element) analysis methods to determine force effects; the acceptable methods of analysis are detailed in Section 4 of the specifications. In this design example, the line beam approach is employed to determine the beam moment and shear envelopes. Using the line beam approach, vehicular live load force effects are determined by first computing the force effects due to a single truck or loaded lane and then multiplying these forces by multiple presence factors, live-load distribution factors, and dynamic load factors as detailed below.

5.1 Multiple Presence Factors (Article 3.6.1.1.2)

Multiple presence factors account for the probability of multiple lanes on the bridge being loaded simultaneously. These factors are specified for various numbers of loaded lanes in Table 3.6.1.1.2-1 of the specifications. There are two exceptions when multiple presence factors are not to be applied. These are when (1) distribution factors are calculated using the tabulated empirical equations given in Article 4.6.2.2 as these equations are already adjusted to account for multiple presence effects and (2) when determining fatigue truck moments, since the fatigue analysis is only specified for a single truck. Thus, for the present example, the multiple presence factors are only applicable when distribution factors are computed using the lever rule or the special analysis for the exterior girders at the strength and service limit states as demonstrated below.

5.2 Live-Load Distribution Factors (Article 4.6.2.2)

The distribution factors approximate the amount of live load (i.e., fraction of a truck or lane load) distributed to a given beam. These factors are computed based on a combination of empirical equations and simplified analysis procedures. Empirical equations are provided Article 4.6.2.2.1 of the specifications and are specifically based on the location of the beam (i.e. interior or exterior), the force effect considered (i.e., moment or shear), and the bridge type. These equations are valid only if specific parameters of the bridge are within the ranges specified in the tables given in Article 4.6.2.2.1. For a slab-on-stringer bridge, as considered in the present example, the following criteria must be satisfied: the beam spacing must be between 3.5 and 16.0 feet, the slab must be at least 4.5 inches thick and less than 12.0 inches thick, the span length must be between 20 and 240 feet, and the cross section must contain at least 4 beams. Because all of these requirements are satisfied, a refined analysis is not necessary and the computation of distribution factors using the approximate methods of Article 4.6.2.2 follows.

Distribution factors are a function of the beam spacing, slab thickness, span length, and the stiffness of the beam. Since the stiffness parameter depends on the beam geometry that is not initially known, the stiffness term may be assumed to be equal to one for preliminary design. In this section, calculation of the distribution factors is presented based on the beam geometry previously shown in Figure 3. It is noted that due to the uniform cross-section of the beam, the distribution factors are also uniform along the beam length. However, this is not always the case and separate calculations are typically required for the distribution factors for each unique cross-section.

5.2.1 Interior Beam - Strength and Service Limit State

For interior beams, the distribution factor at the strength and service limit states is determined based on the empirical equations given in Article 4.6.2.2.2. The stiffness parameter, K_g, required for the distribution factor equations is computed as follows.

$$K_g = n(I + Ae_g^2) \quad \text{Eq. (4.6.2.2.1-1)}$$

where:

- n = modular ratio = 8
- I = moment of inertia of the steel beam = 16,700 in.^{4} for the rolled beam
- A = area of the steel beam = 63.4 in.^{2} for the rolled beam
- e_g = distance between the centroid of the girder and centroid of the slab

The required section properties of the girder (in addition to other section properties that will be relevant for subsequent calculations) are determined as follows (refer to Figure 4):

$$e_g = 19.50 + (2 - 1.22) + 4 = 24.28 \text{ in.}$$

$$K_g = n(I + Ae_g^2) = 8(16{,}700 + 63.4(24.28)^2) = 432{,}604 \text{ in.}^4$$

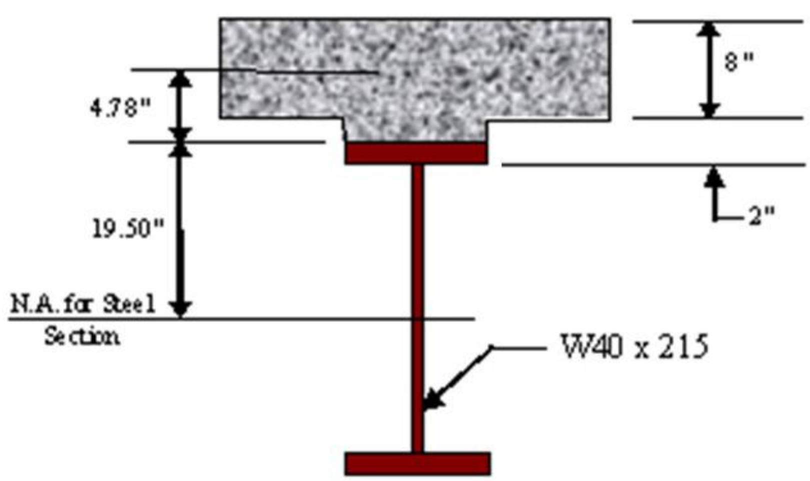

Figure 4 Rolled Beam Cross Section

5.2.1.1 Bending Moment

The empirical equations for distribution of live load moment at the strength and service limit states are given in Table 4.6.2.2.2b-1. Alternative expressions are given for one loaded lane and multiple loaded lanes, where the maximum of the two equations governs as shown below. It is noted that the maximum number of lanes possible for the 34 foot roadway width considered in this example is two lanes.

$$DF = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12Lt_s^3}\right)^{0.1} \text{ (for one lane loaded)}$$

where: S = beam spacing (ft)

L = span length (ft)

t_s = slab thickness (in.)

K_g = stiffness term ($in.^4$)

$$DF = 0.06 + \left(\frac{10}{14}\right)^{0.4}\left(\frac{10}{L}\right)^{0.3}\left(\frac{432,604}{12(90)(8.0)^3}\right)^{0.1} = 0.501 \text{ lanes}$$

$$DF = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12Lt_s^3}\right)^{0.1} \text{ (for two or more lanes loaded)}$$

$$\text{DF} = 0.075 + \left(\frac{10.0}{9.5}\right)^{0.6}\left(\frac{10.0}{90.0}\right)^{0.2}\left(\frac{432,604}{12.0(90.0)(8.0)^3}\right)^{0.1} = 0.723 \text{ lanes} \quad \text{(governs)}$$

5.2.1.2 Shear

The empirical equations for distribution of live load shear in an interior beam at the strength and service limit states are given in Table 4.6.2.2.3a-1. Similar to the equations for moment given above, alternative expressions are given based on the number of loaded lanes.

$$DF = 0.36 + \frac{S}{25} \text{ (for one lane loaded)}$$

$$DF = 0.36 + \frac{10}{25} = 0.760 \text{ lanes}$$

$$DF = 0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^2 \text{ (for two or more lanes loaded)}$$

$$DF = 0.2 + \frac{10}{12} - \left(\frac{10}{35}\right)^2 = 0.952 \text{ lanes} \qquad \text{(governs)}$$

5.2.2 Exterior Girder – Strength and Service Limit States

Distribution factors for the exterior beam at the strength and service limit states are based on the maximum of: (1) a modification of the empirical equations for interior beams given above, (2) the lever rule, or (3) a special analysis assuming the entire cross-section deflects and rotates as a rigid cross-section.

5.2.2.1 Bending Moment

Lever Rule:

As specified in Table 4.6.2.2.2d-1, the lever rule is one method used to determine the distribution factor for the exterior beam for the case of one-lane loaded. The lever rule assumes the deck is hinged at the interior beam, and statics is then employed to determine the percentage of the truck weight resisted by the exterior beam, i.e., the distribution factor, for one loaded lane. It is specified that the truck is to be placed such that the closest wheel is two feet from the barrier or curb, which results in the truck position shown in Figure 5 for the present example. The calculated reaction of the exterior beam is multiplied by the multiple presence factor for one lane loaded, m_1, to determine the distribution factor.

$$DF = \left(0.5 + 0.5\left(\frac{10-6}{10}\right)\right)m_1$$

m_1 = 1.20 (from Table 3.6.1.1.2-1)

DF = 0.7 x 1.2 = 0.840 lanes

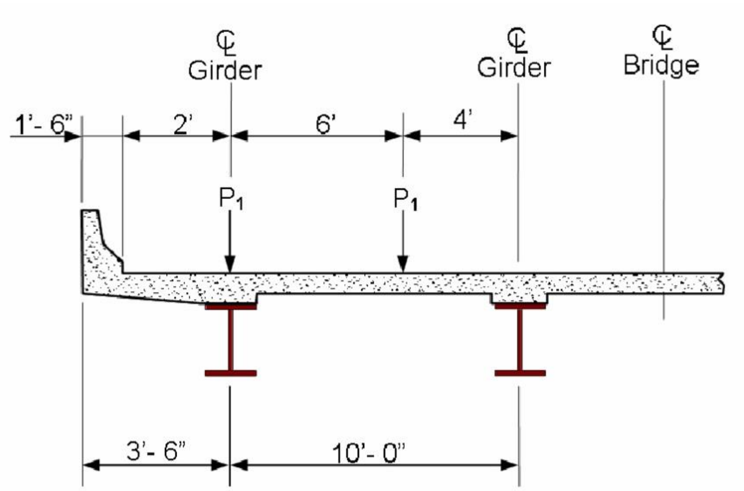

Figure 5 Sketch of the Truck Location for the Lever Rule

Modified of Interior Girder Distribution Factor:

For the case of two or more lanes loaded, Table 4.6.2.2.2d-1 gives a modification factor that is to be multiplied by the interior beam distribution factor to determine the exterior beam distribution factor. The modification factor for moment is given by the following equation.

$$e = 0.77 + \left(\frac{d_e}{9.1}\right)$$

where:

d_e = the distance between the exterior beam and the interior of the barrier or curb (ft.)

d_e = 2.0 ft

$$e = 0.77 + \left(\frac{2}{9.1}\right) = 0.990 < 1.0$$

Multiplying the modification factor by the interior beam distribution factor for two or more lanes loaded gives the following:

DF = 0.990(0.723) = 0.716 lanes

Special Analysis:
The special analysis assumes the entire bridge cross-section behaves as a rigid cross-section rotating about the transverse centerline of the structure and is discussed in the commentary of Article 4.6.2.2.2d. The reaction on the exterior beam is calculated from the following equation.

$$R = \frac{N_L}{N_b} + \frac{X_{ext}\sum^{N_L} e}{\sum^{N_b} x^2} \qquad \text{Eq. (C4.6.2.2.2d-1)}$$

where:

N_L = number of lanes loaded

N_b = number of beams or girders

X_{ext} = horizontal distance from center of gravity of the pattern of girders to the exterior girder (ft.)

e = eccentricity of a design truck or a design lane load from the center of gravity of the pattern of girders (ft.)

x = horizontal distance from the center of gravity of the pattern of girders to each girder (ft.)

Figure 6 shows the truck locations for the special analysis. It is shown that the maximum number of trucks that may be placed on half of the cross-section is two. Thus, the calculation of the distribution factors using the special analysis procedure proceeds as follows beginning with the calculations for one loaded lane (the appropriate multiple presence factors, MPF, that are applied in each case are shown):

DF = $m_1(R_1)$ (one lane loaded)

$$DF = 1.2\left(\frac{1}{4} + \frac{(15)(12)}{2(15)^2 + 5^2}\right) = 0.732 \text{ lanes (Note, MPF = 1.2)}$$

Similarly, for two loaded lanes the distribution factor is computed as follows:

DF = $m_2(R_2)$ (two lanes loaded)

$$DF = 1.0\left(\frac{2}{4} + \frac{(15)(12+0)}{2((15)^2 + 5^2)}\right) = 0.860 \text{ lanes (governs) (Note, MPF = 1.0)}$$

Comparing the four distribution factors computed above for moment in the exterior beam, it is determined that the controlling distribution factor is equal to 0.860 lanes, which is determined based on the special analysis procedure considering two lanes loaded. Compared to the interior beam distribution factor for moment, which was computed to be 0.723 lanes, it is shown that the exterior beam distribution factor is larger, and thus, the exterior beam distribution factor controls the bending strength design at the strength and service limit state.

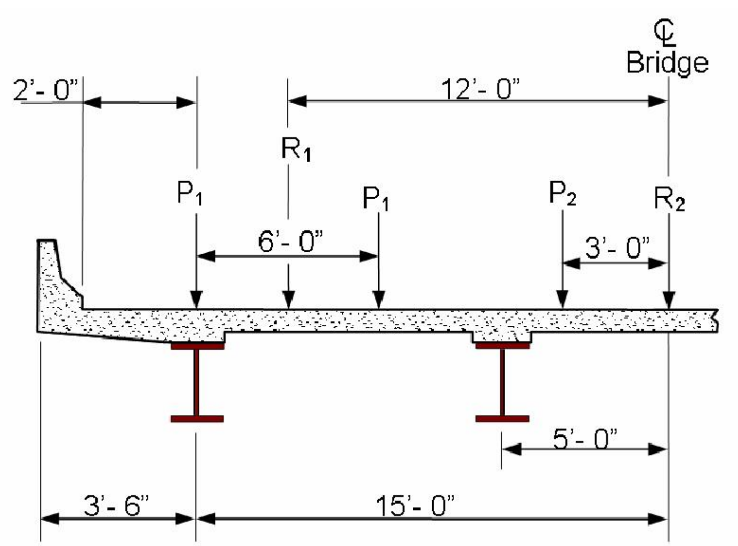

Figure 6 Sketch of the Truck Locations for the Special Analysis

5.2.2.2 Shear

The distribution factors computed above using the lever rule, approximate formulas, and special analysis are also applicable to the distribution of shear.

<u>Lever Rule:</u>
The above computations demonstrate that the distribution factor for shear for one-lane loaded is equal to 0.840 lanes based on the lever rule.

DF = 0.840 lanes

Modified of Interior Girder Distribution Factor:
For the case of two or more lanes loaded, the shear modification factor is computed using the following formula:

$$e = 0.60 + \frac{d_e}{10.0}$$

$$e = 0.60 + \frac{2}{10.0} = 0.800$$

Applying this modification factor to the previously computed interior beam distribution factor for shear for two or more lanes loaded gives the following:

DF = 0.800(0.952) = 0.762 lanes

Special Analysis:
It was demonstrated above that the special analysis yields the following distribution factors for one lane and two or more lanes loaded, respectively:

DF = 0.732 lanes for one lane loaded

DF = 0.860 lanes for two lanes loaded (governs)

Thus, the controlling distribution factor for shear in the exterior beam is 0.860 lanes, which is less than that of the interior beam. Thus, the interior beam distribution factor of 0.952 lanes controls the shear design.

5.2.3 Fatigue Limit State

As stated in Article 3.6.1.1.2, the fatigue distribution factor is based on one lane loaded, and does not include the multiple presence factor, since the fatigue loading is specified as a single truck load. Because the distribution factors calculated from the tabulated empirical equations incorporate the multiple presence factors, the fatigue distribution factors are equal to the strength distribution factors divided by the multiple presence factor for one lane loaded, as described subsequently.

5.2.3.1 Bending Moment

It was determined above that the governing distribution factor for moment at the strength and service limit states was equal to 0.840 lanes, which was based on one loaded lane (the lever rule). Dividing this value by the multiple presence factor gives the following distribution factor for fatigue moment.

$$DF = \frac{0.840}{1.20} = 0.700 \text{ lanes (exterior girder)}$$

5.2.3.2 Shear

From review of the shear distribution factors computed above for the strength and service limit states, it was determined that the maximum distribution factor for one-lane loaded was equal to 0.840 lanes, which was based on the lever rule. Thus, the distribution factor for fatigue shear is equal to 0.840 lanes divided by the multiple presence factor for one-lane loaded of 1.2.

$$DF = \frac{0.840}{1.20} = 0.700 \text{ lanes}$$

5.2.4 Distribution Factor for Live-Load Deflection

Article 2.5.2.6.2 states that all design lanes must be loaded when determining the live load deflection of the structure. In the absence of a refined analysis, for straight-girder bridges, an approximation of the live load deflection can be obtained by using a distribution factor computed assuming that all beams deflect equally with the appropriate multiple presence factor applied. The controlling case occurs when two lanes are loaded, and the calculation of the corresponding distribution factor is shown below.

$$DF = m\left(\frac{N_L}{N_b}\right) = 1.0\left(\frac{2}{4}\right) = 0.500 \text{ lanes}$$

The governing live load distribution factors based on the maximum distribution factors of the interior and exterior girder are provided in Table 1.

Table 1 Governing Live Load Distribution Factors

	Distribution Factor
Strength/Service Bending Moment	0.860
Strength/Service Shear	0.952
Fatigue Bending Moment	0.700
Fatigue Shear	0.700
Deflection	0.500

5.3 Dynamic Load Allowance

The dynamic effects of the truck loading are taken into consideration by the dynamic load allowance, IM. The dynamic load allowance, which is discussed in Article 3.6.2 of the specifications, accounts for the hammering effect of the wheel assembly and the dynamic response of the bridge. IM is only applied to the design truck or tandem, not the lane loading. Table 3.6.2.1-1 specifies IM equal to 1.33 for the strength, service, and live load deflection evaluations, while IM of 1.15 is specified for the fatigue limit state.

6.0 ANALYSIS RESULTS

6.1 Moment and Shear Envelopes

Figures 7 through 10 show the moment and shear envelopes for this design example, which are based on the data presented in Tables 2 through 8. The live load moments and shears shown in these figures are based on the controlling distribution factors computed above. For loads applied to the composite section, the envelopes shown are determined based on the composite section properties assuming the concrete deck to be effective over the entire span length.

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As previously mentioned, the live load in the positive bending region between the points of dead load contraflexure is the result of the HL-93 loading. In the negative bending region between the points of dead load contraflexure, the moments are the larger of the moments due to the HL-93 loading and the special negative-moment loading, which is composed of 90 percent of both the truck-train moment and lane loading moment.

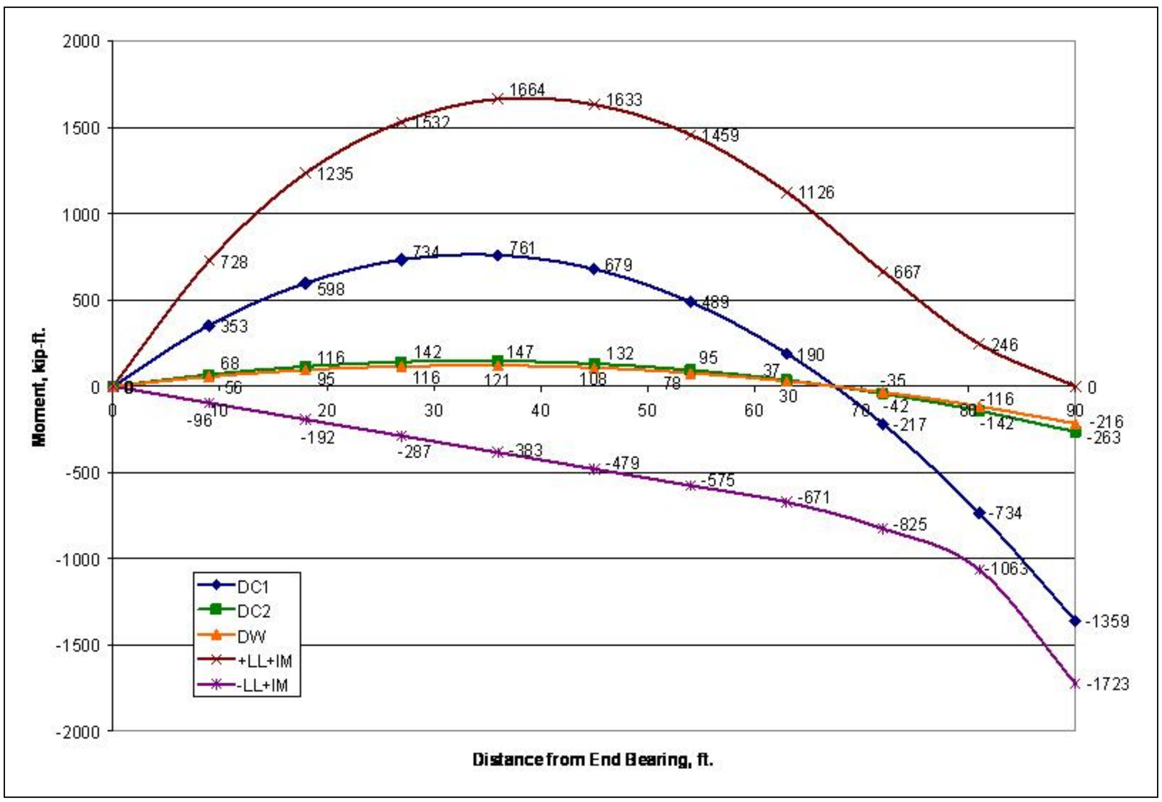

Figure 7 Dead and Live Load Moment Envelopes

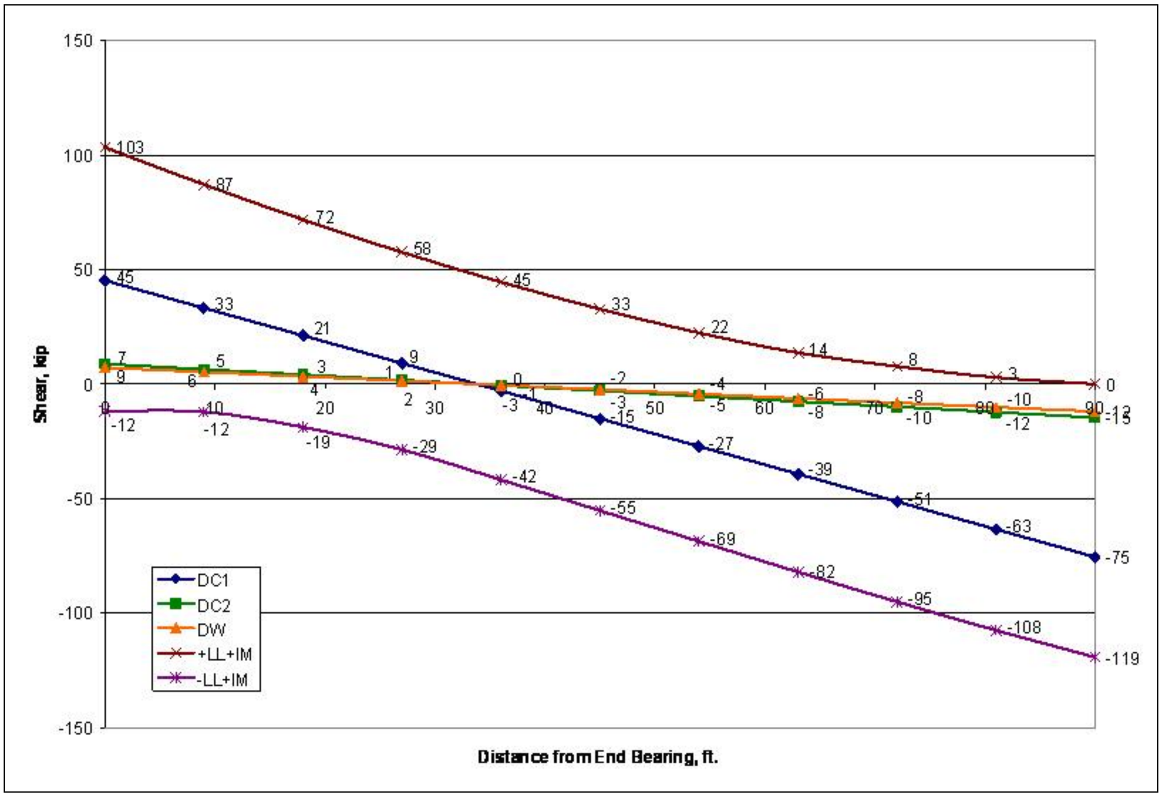

Figure 8 Dead and Live Load Shear Envelopes

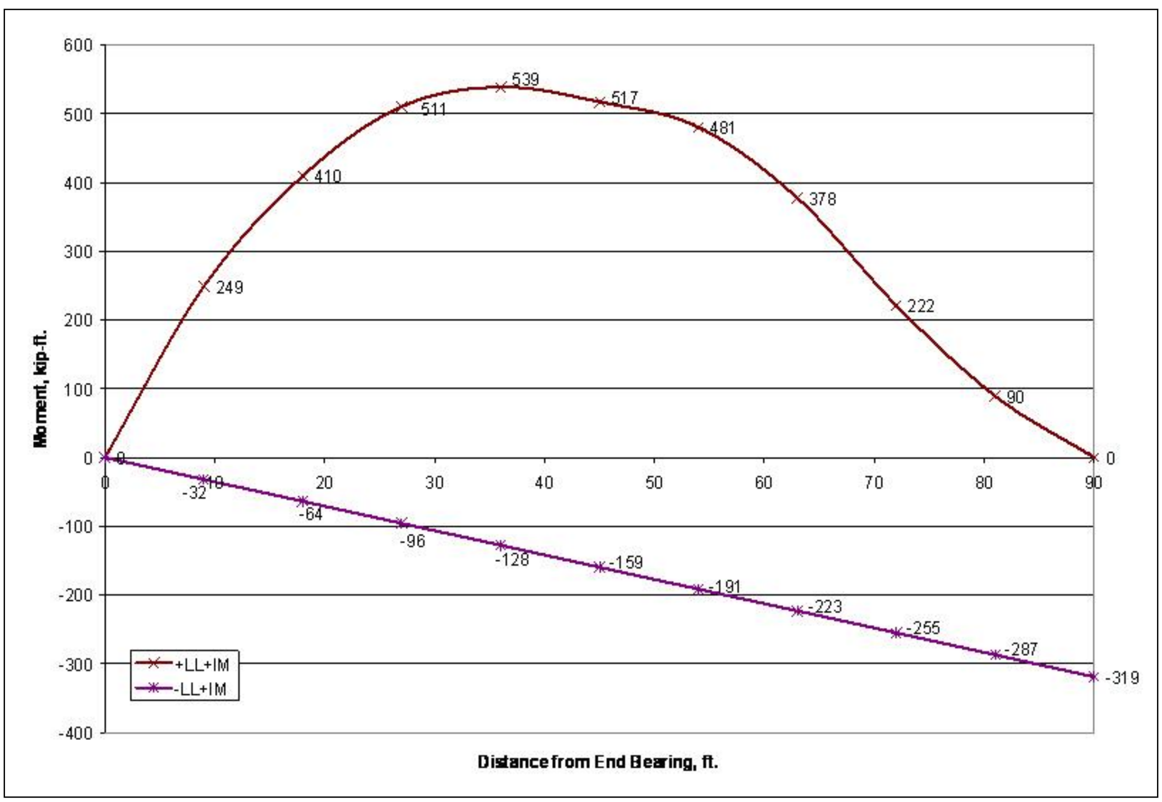

Figure 9 Fatigue Live Load Moments

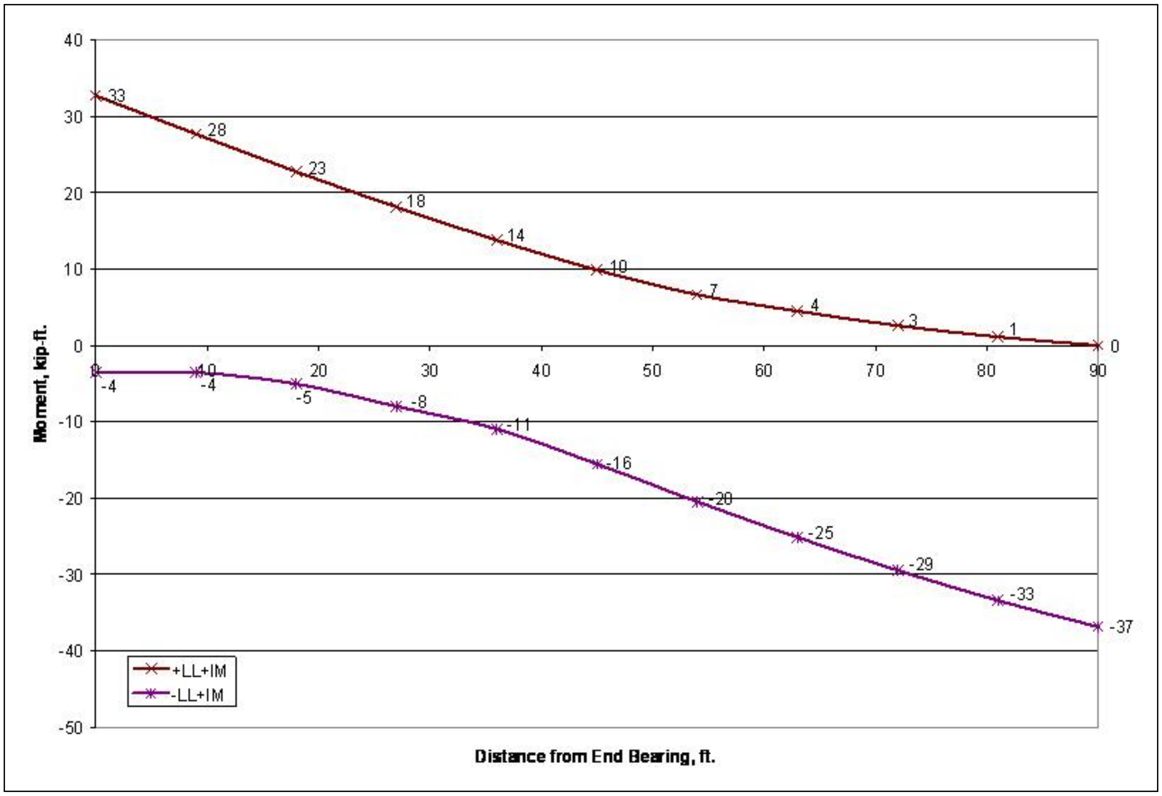

Figure 10 Fatigue Live Load Shears

Table 2 Unfactored and Undistributed Moments (kip-ft)

	Non-Com. Dead	Com. Dead	Wearing Surface	Truck Load		Lane Load		Tandem		Double Truck		Double Tandem	
Span 1	**DC1**	**DC2**	**DW**	**pos.**	**neg.**	**pos.**	**neg.**	**pos.**	**neg.**	**pos.**	**neg.**	**pos.**	**neg.**
0.00	0	0	0	0	0	0	0	0	0	0	0	0	0
0.10	353	68	56	486	-59	201	-32	381	-43	0	0	0	0
0.20	598	116	95	816	-119	350	-65	653	-86	0	0	0	0
0.30	734	142	116	1003	-178	447	-97	818	-130	0	0	0	0
0.40	761	147	121	1085	-238	492	-130	884	-173	0	0	0	0
0.50	679	132	108	1062	-297	486	-162	868	-216	0	0	0	0
0.60	489	95	78	954	-357	428	-194	781	-259	0	0	0	0
0.70	190	37	30	746	-416	318	-227	627	-302	0	0	0	0
0.80	-217	-42	-35	466	-475	156	-259	424	-346	0	-476	0	-607
0.90	-734	-142	-116	148	-535	30	-380	192	-389	0	-746	0	-682
1.00	-1359	-263	-216	0	-594	0	-648	0	-432	0	-1187	0	-758

Table 3 Unfactored and Undistributed Live Load Moments (kip-ft)

Span 1	Vehicle		Special	Standard	1.33 Vehicle + Lane	Distribution	LL+I	
	positive	negative	negative	negative	positive	Factors	Positive	Negative
0.00	0	0	0	0	0	0.86	0	0
0.10	486	-59	-29	-111	847	0.86	728	-96
0.20	816	-119	-58	-223	1436	0.86	1235	-192
0.30	1003	-178	-87	-334	1782	0.86	1532	-287
0.40	1085	-238	-117	-446	1935	0.86	1664	-383
0.50	1062	-297	-146	-557	1899	0.86	1633	-479
0.60	954	-357	-175	-669	1696	0.86	1459	-575
0.70	746	-416	-204	-780	1310	0.86	1126	-671
0.80	466	-475	-959	-891	775	0.86	667	-825
0.90	192	-535	-1236	-1092	286	0.86	246	-1063
1.00	0	-594	-2004	-1438	0	0.86	0	-1723

Table 4 Strength I Load Combination Moments (kip-ft)

Span 1	1.25 DC1	1.25 DC2	1.5 DW	1.75 (LL+IM)		Strength I	
				positive	negative	positive	negative
0.00	0	0	0	0	0	0	0
0.10	442	86	84	1274	-168	1885	444
0.20	747	145	142	2161	-335	3195	699
0.30	917	178	175	2681	-503	3951	766
0.40	951	184	181	2913	-671	4229	646
0.50	849	165	162	2857	-839	4033	337
0.60	611	118	116	2553	-1006	3399	-160
0.70	238	46	45	1971	-1174	2300	-845
0.80	-272	-53	-52	1167	-1444	790	-1820
0.90	-917	-178	-175	431	-1860	-839	-3129
1.00	-1698	-329	-323	0	-3016	-2351	-5367

Table 5 Service II Load Combination Moments (kip-ft)

Span 1	1.0 DC1	1.0 DC2	1.0 DW	1.3 (LL+IM)		Service II	
				positive	negative	positive	negative
0.00	0	0	0	0	0	0	0
0.10	353	68	56	947	-125	1424	353
0.20	598	116	95	1605	-249	2414	559
0.30	734	142	116	1992	-374	2984	619
0.40	761	147	121	2164	-498	3193	531
0.50	679	132	108	2123	-623	3041	296
0.60	489	95	78	1896	-747	2558	-86
0.70	190	37	30	1464	-872	1722	-615
0.80	-217	-42	-35	867	-1073	573	-1367
0.90	-734	-142	-116	320	-1382	-672	-2374
1.00	-1359	-263	-216	0	-2240	-1838	-4078

Table 6 Unfactored and Undistributed Shears (kip)

Span 1	Non-Com. Dead DC1	Com. Dead DC2	Wearing Surf. DW	Truck Load		Lane Load		Tandem	
				positive	negative	positive	negative	positive	negative
0.00	45	9	7	63	-7	25	-4	49	-5
0.10	33	6	5	54	-7	20	-4	42	-5
0.20	21	4	3	45	-10	15	-5	36	-11
0.30	9	2	1	37	-18	11	-7	30	-17
0.40	-3	-1	0	29	-26	8	-9	25	-23
0.50	-15	-3	-2	22	-34	5	-12	19	-28
0.60	-27	-5	-4	15	-42	3	-16	14	-34
0.70	-39	-8	-6	10	-50	2	-20	10	-38
0.80	-51	-10	-8	5	-56	1	-25	6	-43
0.90	-63	-12	-10	2	-62	0	-30	2	-46
1.00	-75	-15	-12	0	-67	0	-36	0	-49

Table 7 Unfactored and Undistributed Live Load Shears (kip)

Span 1	vehicle		1.33 V Vehicle + V Lane		Distribution Factors	V LL	
	positive	negative	positive	negative		positive	negative
0.00	63	-7	109	-12	0.952	103	-12
0.10	54	-7	92	-13	0.952	87	-12
0.20	45	-11	75	-20	0.952	72	-19
0.30	37	-18	60	-30	0.952	58	-29
0.40	29	-26	47	-44	0.952	45	-42
0.50	22	-34	34	-58	0.952	33	-55
0.60	15	-42	24	-72	0.952	22	-69
0.70	10	-50	14	-86	0.952	14	-82
0.80	6	-56	8	-100	0.952	8	-95
0.90	2	-62	3	-113	0.952	3	-108
1.00	0	-67	0	-125	0.952	0	-119

Table 8 Strength I Load Combination Shear (kip)

Span 1	1.25 DC1	1.25 DC2	1.5 DW	1.75 (LL+IM)		Strength I	
				positive	negative	positive	negative
0.00	57	11	11	181	-21	259	58
0.10	42	8	8	153	-21	210	36
0.20	26	5	5	126	-33	162	4
0.30	11	2	2	101	-50	116	-35
0.40	-4	-1	-1	78	-73	73	-78
0.50	-19	-4	-4	57	-97	31	-123
0.60	-34	-7	-6	39	-120	-8	-167
0.70	-49	-10	-9	24	-144	-44	-212
0.80	-64	-12	-12	13	-166	-75	-255
0.90	-79	-15	-15	5	-188	-105	-298
1.00	-94	-18	-18	0	-209	-131	-339

6.2 Live Load Deflection

As indicated in Article 3.6.1.3.2, control of live-load deflection is optional. Evaluation of this criterion is based on the flexural rigidity of the short-term composite section and consists of two load cases: deflection due to the design truck and deflection due to the design lane plus 25 percent of the design truck. The dynamic load allowance of 33 percent is applied to the design truck load only for both loading conditions. The load is distributed using the distribution factor of 0.500 lanes calculated earlier.

The maximum deflection due to the design truck is 1.114 inches. Applying the impact and distribution factors gives the following deflection for the design truck load case.

$$\Delta_{LL+IM} = 0.500 \text{ x } 1.33 \text{ x } 1.114 = 0.741 \text{ in.} \qquad \text{(governs)}$$

The maximum deflection due to the lane loading only is 0.578 inches. Thus, the deflection due to 25% of the design truck plus the lane loading is equal to the following:

$$\Delta_{LL+IM} = 0.500 \text{ (1.33 x 0.25 x 1.114 + 0.578)} = 0.474 \text{ in.}$$

Thus the governing deflection, equal to 0.741 inches, will subsequently be used to assess the beam design based on the live-load deflection criterion.

7.0 LIMIT STATES

As discussed previously, there are four limit states applicable to the design of steel I-girders. Each of these limit states is described below.

7.1 Service Limit State (Articles 1.3.2.2 and 6.5.2)

The intent of the Service Limit State is to ensure the satisfactory performance and rideability of the bridge structure by preventing localized yielding. For steel members, these objectives are intended to be satisfied by limiting the maximum levels of stress that are permissible. The optional live-load deflection criterion is also included in the service limit state and is intended to ensure user comfort.

7.2 Fatigue and Fracture Limit State (Article 1.3.2.3 and 6.5.3)

The intent of the Fatigue and Fracture Limit State is to control crack growth under cyclic loading. This is accomplished by limiting the stress range to which steel members are subjected. The permissible stress range varies for various design details and member types. The fatigue limit state also restricts the out-of-plane flexing of the web. Additionally, fracture toughness requirements are stated in Article 6.6.2 of the specifications and are dependent on the temperature zone.

7.3 Strength Limit State (Articles 1.3.2.4 and 6.5.4)

The strength limit state ensures the design is stable and has adequate strength when subjected to the highest load combinations considered. The bridge structure may experience structural damage (e.g., permanent deformations) at the strength limit state, but the integrity of the structure is preserved.

The suitability of the design must also be investigated to ensure adequate strength and stability during each construction phase. The deck casting sequence has a significant influence on the distribution of stresses within the structure. Therefore, the deck casting sequence should be considered in the design and specified on the plans to ensure uniformity between predicted and actual stresses. In addition, lateral flange bending stresses resulting from forces applied to the overhang brackets during construction should also be considered during the constructibility evaluation.

7.4 Extreme Event Limit State (Articles 1.3.2.5 and 6.5.5)

The extreme event limit state is to ensure the structure can survive a collision, earthquake, or flood. The collisions investigated under this limit state include the bridge being struck by a vehicle, vessel, or ice flow. This limit state is not addressed in this design example.

8.0 SAMPLE CALCULATIONS

This section presents the calculations necessary to evaluate the preliminary beam design for adequate resistance at the strength, service, and fatigue limit states. Adequate strength of the bridge in its final condition and at all stages of the construction sequence is verified. The optional moment redistribution specifications are utilized. Other design components presented include the concrete deck design. The moment and shear envelopes provided in Figures 7 through 10 are employed for the following calculations.

8.1 Section Properties

The section properties are first calculated as these properties will be routinely used in the subsequent evaluations of the various limit states. The structural slab thickness is taken as the slab thickness minus the thickness of the integral wearing surface (8 inches) and the modular ratio is taken as 8 in these calculations. Because the section is prismatic, the effective flange width and section properties are constant along the length of the beam. However, separate calculations are necessary for the computation of the plastic moment and yield moment depending on if the section is in negative bending or positive bending.

8.1.1 Effective Flange Width (Article 4.6.2.6)

Article 4.6.2.6 of the specifications governs the determination of the effective flange width of the concrete slab, where alternative calculations are specified for interior and exterior beams. The effective flange width for interior beams is one-half the distance to the adjacent girder on each side of the component.

For the interior beams in this example, b_{eff} is then computed as follows.

$$b_{eff} = \frac{120}{2} + \frac{120}{2} = 120.0 \text{ in.}$$

The effective flange width for exterior beams is determined as one-half the distance to the adjacent girder plus the full overhang width.

For the exterior beams, b_{eff} is then computed as.

$$b_{eff} = \frac{120}{2} + 42 = 102.0 \text{ in.}$$

Because the effective width is smaller and the moment distribution factor is greater for the exterior beam, the moment design is controlled by the exterior beam.

8.1.2 Elastic Section Properties

As discussed above, the elastic section properties that are to be considered in the analysis of the beam vary based on the loading conditions. The section properties for the steel section (beam alone) are used in the constructibility evaluation. In positive bending, live loads are applied to the full composite section, termed the short-term composite section, where the modular ratio of 8 is used in the computations. Alternatively, dead loads are applied to what is termed the long-term composite section. The long-term composite section is considered to be comprised of the full steel beam and one-third of the concrete deck to account for the reduction in strength that may occur in the deck over time due to creep effects. This is accounted for in the section property calculations through use of a modular ratio equal to 3 times the base modular ratio, or 24. The section properties for the short-term and long-term composite sections are thus computed below (Tables 9 through 11). In negative bending, the applicable section at the strength limit state consists of the steel beam in addition to the longitudinal steel reinforcement in the concrete deck.

The section properties of the W40x215 beam are as follows.

$I_{NA} = 16{,}700 \text{ in}^4$

$d_{TOP\ OF\ STEEL} = 19.50 \text{ in.}$ $\qquad S_{TOP\ OF\ STEEL} = \dfrac{16{,}700}{19.50} = 856.4 \text{ in.}^3$

$d_{BOT\ OF\ STEEL} = 19.50 \text{ in.}$ $\qquad S_{BOT\ OF\ STEEL} = \dfrac{16{,}700}{19.50} = 856.4 \text{ in.}^3$

Table 9 Short Term Composite (n) Section Properties

Component	A	d	Ad	Ad^2	I_o	I
Steel Section	63.4					16,700
Concrete Slab (8"x 102"/8)	102.0	24.28	2,477	60,142	544	60,686
	165.4		2,477			77,386
				-14.98(2,477) =		-37,105
						40,371 in⁴

$d_s = \dfrac{2{,}477}{165.4} = 14.98 \text{ in.}$

$d_{TOP\ OF\ STEEL} = 19.50 - 14.98 = 4.52 \text{ in.}$ $\qquad S_{TOP\ OF\ STEEL} = \dfrac{40{,}371}{4.52} = 8{,}932 \text{ in.}^3$

$d_{BOT\ OF\ STEEL} = 19.50 + 14.98 = 34.48 \text{ in.}$ $\qquad S_{BOT\ OF\ STEEL} = \dfrac{40{,}371}{34.48} = 1{,}171 \text{ in.}^3$

Table 10 Long Term Composite (3n) Section Properties

Component	A	d	Ad	Ad^2	I_o	I
Steel Section	63.4					16,700
Concrete Slab (8"x 102"/24)	34.0	24.28	825.5	20,044	181.3	20,225
	97.4		825.5			36,925
				-8.48(825.5) =		-7,000 in⁴
						29,925

$$d_s = \frac{825.5}{97.4} = 8.48 \text{ in.}$$

$$d_{TOP\ OF\ STEEL} = 19.50 - 8.48 = 11.02 \text{ in.} \qquad S_{TOP\ OF\ STEEL} = \frac{29{,}925}{11.02} = 2{,}716 \text{ in.}^3$$

$$d_{BOT\ OF\ STEEL} = 19.50 + 8.48 = 27.98 \text{ in.} \qquad S_{BOT\ OF\ STEEL} = \frac{29{,}925}{27.98} = 1{,}070 \text{ in.}^3$$

Table 11 Steel Section and Longitudinal Reinforcement Section Properties

Component	A	d	Ad	Ad^2	I_o	I
Steel Section	63.4					16,700
Top Long. Reinforcement	6.53	26.03	170.0	4,424		4,424
Bot. Long. Reinforcement	3.27	21.53	70.40	1,516		1,516
	73.2		240.4			22,640
				-3.28(240.4) =		-789
						21,851 in⁴

$$d_s = \frac{240.4}{73.20} = 3.28 \text{ in.}$$

$$d_{TOP\ OF\ STEEL} = 19.50 - 3.28 = 16.22 \text{ in.} \qquad S_{TOP\ OF\ STEEL} = \frac{21{,}851}{16.22} = 1{,}347 \text{ in.}^3$$

$$d_{BOT\ OF\ STEEL} = 19.50 + 3.28 = 22.78 \text{ in.} \qquad S_{BOT\ OF\ STEEL} = \frac{21{,}851}{22.78} = 959 \text{ in.}^3$$

The section modulus to the top layer of the longitudinal steel reinforcement is computed as follows.

$$S_{REIN.} = 21{,}851 / (26.03 - 3.28) = 960.5 \text{ in.}^3$$

8.1.3 Plastic Moment

8.1.3.1 Positive Bending

The plastic moment M_p is the resisting moment of an assumed fully yielded cross-section and can be determined using the procedure outlined in Table D6.1-1 as demonstrated below. The longitudinal deck reinforcement is conservatively neglected in these computations. The forces acting in the slab (P_s), compression flange (P_c), web (P_w), and tension flange (P_t) are first computed.

$P_s = 0.85f'_c b_s t_s = 0.85(4.0)(102.0)(8)$ = 2,774 kips

$P_c = F_{yc} b_c t_c = (50)(15.8)(1.22)$ = 964 kips

$P_w = F_{yw} D t_w = (50)(36.56)(0.65)$ = 1,188 kips

$P_t = F_{yt} b_t t_t = (50)(15.8)(1.22)$ = 964 kips

The forces within each element of the beam are then compared to determine the location of the plastic neutral axis (PNA). If the following equation is satisfied then the PNA is in the web.

$$P_t + P_w \geq P_c + P_s$$

$$964 + 1{,}188 \geq 964 + 2{,}774$$

$$2{,}152 \leq 3{,}738$$

Therefore, the PNA is not in the web and the following equation is evaluated to determine if the PNA is in the top flange.

$$P_t + P_w + P_c \geq P_s$$

$$964 + 1{,}188 + 964 \geq 2{,}774$$

$$3{,}116 \geq 2{,}774$$

Therefore, the plastic neutral axis is in the top flange and $\bar{y}$ is computed using the following equation.

$$\bar{y} = \left(\frac{t_c}{2}\right)\left[\frac{P_w + P_t - P_s}{P_c} + 1\right]$$

$$\bar{y} = \left(\frac{1.22}{2}\right)\left[\frac{1{,}188 + 964 - 2{,}774}{964}\right] = 0.22\text{in.}$$

The plastic moment is then calculated using the following equation.

$$M_p = \frac{P_c}{2t_c}\left[\bar{y}^2 + \left(t_c - \bar{y}\right)^2\right] + \left[P_s d_s + P_w d_w + P_t d_t\right]$$

The distances from the PNA to the centroid of the compression flange, web, and tension flange (respectively) are as follows.

$d_s = 0.22 + 8.0/2 + 2 – 1.22 = 5.00$ in.

d_w = 1.22 – 0.22 + 36.56/2 = 19.28 in.

d_t = 1.22 – 0.22 + 36.56 + 1.22/2 = 38.17 in.

Substitution of these distances and the above computed element forces into the M_p equation gives the following.

$$M_p = \frac{964}{2(1.22)}\left[(0.35)^2 + (1.22 - 0.35)^2\right] + \left[(2,557)(5.13) + (1,188)(19.15) + (964)(38.04)\right]$$

$$\mathrm{M_p} = \left(\frac{964}{2(1.22)}\right)\left[(0.22)^2 + (1.22 - 0.22)^2\right] + \left[(2,774)(5.00) + (1,188)(19.28) + (964)(38.17)\right]$$

M_p = 73,985 kip-in = 6,165 k-ft

8.1.3.2 Negative Bending

Similar to the calculation of the plastic moment in positive bending, Table D6.1-2 is used to determine the plastic moment for the negative bending section as demonstrated below. The concrete slab is neglected in the computation of the strength of the negative bending region due to the low tensile strength of concrete. The force acting in each element of the beam is first computed.

P_c	=	$F_{yc}b_ct_c$	=	(50)(15.8)(1.22)	=	964 kips
P_w	=	$F_{yw}D_{tw}$	=	(50)(36.56)(0.65)	=	1188 kips
P_t	=	$F_ytb_tt_t$	=	(50)(15.8)(1.22)	=	964 kips
P_{rb}	=	$F_{yrb}A_{rb}$	=	(60)(3.27)	=	196 kips
P_{rt}	=	$F_{yrt}A_{rt}$	=	(60)(6.53)	=	392 kips

As before, the relative forces in each member are used to determine the location of the plastic neutral axis. Because the following equation is satisfied, it is determined that the PNA is in the web.

$P_c + P_w \geq P_t + P_{rb} + P_{rt}$ = 964 + 1188 ≥ 964 + 196 + 392

2152 > 1552, therefore, the plastic neutral axis is in the web.

The plastic neutral axis location is then computed by the following equation.

$$\bar{y} = \left(\frac{D}{2}\right)\left[\frac{P_c - P_t - P_{rt} - P_{rb}}{P_w} + 1\right] = \left(\frac{36.56}{2}\right)\left[\frac{964 - 964 - 392 - 196}{1,188} + 1\right] = 9.23\text{in.}$$

M_p is then computed as follows.

$$M_p = \frac{P_w}{2D}\left[\bar{y}^2 + \left(D - \bar{y}\right)^2\right] + \left[P_{rt}d_{rt} + P_{rb}d_{rb} + P_t d_t + P_c d_c\right]$$

where:

d_{rt} = 9.23 + 2 + 8 – 2.25 = 16.98 in.

d_{rb} = 9.23 + 2 + 1.25 = 12.48 in.

d_t = 9.23 + 1.22/2 = 9.84 in.

d_c = 36.56 – 9.23 + 1.22/2 = 27.94 in.

$$M_p = \frac{1{,}188}{2(36.56)}\left[(9.23)^2 + \left(36.56 - 9.23\right)^2\right] + \left[(392)(16.98) + (196)(12.48) + (964)(9.84) + (964)(27.94)\right]$$

M_p = 59,042 kip-in = 4,920 k-ft

8.1.4 Yield Moment

8.1.4.1 Positive Bending

The yield moment, which is the moment causing first yield in either flange (neglecting flange lateral bending), is determined according to the provisions specified in Section D6.2.2 of the specifications. This computation method for the yield moment recognizes that different stages of loading (e.g. composite dead load, non-composite dead load, and live load) act on the beam when different cross-sectional properties are applicable. The yield moment is determined by solving for M_{AD} using Equation D6.2.2-1 (given below) and then summing M_{D1}, M_{D2}, and M_{AD}, where, M_{D1}, M_{D2}, and M_{AD} are the factored moments applied to the noncomposite, long-term composite, and short-term composite section, respectively.

$$F_{yf} = \frac{M_{D1}}{S_{NC}} + \frac{M_{D2}}{S_{LT}} + \frac{M_{AD}}{S_{ST}} \qquad \text{Eq. (D6.2.2-1)}$$

Due to the significantly higher section modulus of the short-term composite section about the top flange, compared to the short-term composite section modulus taken about the bottom flange, the minimum yield moment results when using the bottom flange section moduli.

Computation of the yield moment for the bottom flange is thus demonstrated below. First the known quantities are substituted into Equation D6.2.2-1 to solve for M_{AD}.

$$50 = 1.0\left[\frac{1.25(761)(12)}{856.4} + \frac{1.25(147)(12) + 1.50(120)(12)}{1{,}070} + \frac{M_{AD}}{1{,}171}\right]$$

M_{AD} = 38,145 k-in. = 3,179 k-ft.

M_y is then determined by applying the applicable load factors and summing the dead loads and M_{AD}.

M_y = 1.25(761) + 1.25(147) + 1.50(121) + 3179   Eq. (D6.2.2-2)

M_y = 4,496 k-ft

8.1.4.2 Negative Bending

The process for determining the yield moment of the negative bending section is similar to the process for the positive bending section. The one difference is that, since the composite short-term and the composite long-term bending sections are both composed of the steel section and the longitudinal reinforcing steel, the section modulus is the same for both the short-term and long-term composite sections.

As discussed above when computing the yield moment in positive bending, the yield moment is the minimum of the moment which causes yielding on the compression side and the moment which causes yielding on the tension side. Because, for negative bending, the section modulus values taken about the top and bottom of the beam are nearly equal to one another, it is not clear which yield moment value will control. Thus, the moments causing first yield in both compression and tension are computed below. The moment causing yielding in the compression flange is first computed based on Equation D6.2.2-1.

$$F_{yf} = \frac{M_{D1}}{S_{NC}} + \frac{M_{D2}}{S_{LT}} + \frac{M_{AD}}{S_{ST}} \qquad \text{Eq. (D6.2.2-1)}$$

$$(50) = \frac{(1.25)(1{,}359)(12)}{856.4} + \frac{(1.25)(263)(12) + (1.50)(216)(12)}{959} + \frac{M_{AD}}{959}$$

M_{AD} = 17,290 k-in. = 1441 k-ft.

M_{yc} = (1.25)(1,359) + (1.25)(263) + (1.50)(216) + 1441

M_{yc} = 3793 k-ft.   (governs)

The specifications indicate that for regions in negative flexure, M_{yt} is to be taken with respect to either the tension flange or the longitudinal steel reinforcement, whichever yields first. Therefore, compute M_{yt} for both and use the smaller value.

The moment which causes yielding in the tension flange is computed as follows:

$$50 = 1.0\left[\frac{1.25(1{,}359)(12)}{856.4} + \frac{1.25(263)(12) + 1.50(216)(12)}{1347} + \frac{\mathrm{M_{AD}}}{1347}\right]$$

M_{AD} = 27,454 k-in. = 2,228 k-ft

M_{yt} = (1.25)(1,359) + (1.25)(263) + (1.50)(216) + 2,228 = 4,639 k-ft

The moment which causes yielding in the longitudinal steel reinforcement is computed as follows. It is necessary to recognize that there is no non-composite moment acting on the longitudinal steel reinforcement, and that F_y should be taken as 60 ksi.

$$F_{yf} = \frac{M_{D1}}{S_{NC}} + \frac{M_{D2}}{S_{LT}} + \frac{M_{AD}}{S_{ST}} \qquad \text{Eq. (D6.2.2-1)}$$

$F_{yf} = F_y = 60$ ksi $\qquad M_{D1} = 0$ k-ft

$$60 = 1.0\left[0 + \frac{1.25(263)(12) + 1.50(216)(12)}{960.5} + \frac{M_{AD}}{960.5}\right]$$

$M_{AD} = 49{,}797$ k-in. = 4,150 k-ft

$M_{yt} = (1.25)(263) + (1.50)(216) + 4{,}150 = 4{,}803$ k-ft

Therefore, the top flange yields before the longitudinal reinforcement, and $M_{yt} = 4{,}639$ k-ft.

For the whole section, the compression flange governs, thus $M_y = M_{yc} = 3{,}793$ k-ft.

8.2 Exterior Beam Check: Section 2

This design example illustrates the use of the optional moment redistribution procedures given in Appendix B6 of the specifications, where moment is redistributed from the negative bending region to the positive bending region; therefore the negative bending region will be checked first in order to determine the amount of moment that must be redistributed to the positive bending region.

8.2.1 Strength Limit State (Article 6.10.6)

8.2.1.1 Flexure

The strength requirements for negative flexure are given by Article 6.10.8, Appendix A6, or Appendix B6 at the option of the Engineer. Article 6.10.8 limits the maximum capacity to the yield moment of the section. Alternatively, Appendix A6 permits beam capacities up to M_p and may be used for beams having a yield strength less than or equal to 70 ksi and a compact or non-compact web, which is defined by Equation A6.1-1. Appendix B6 utilizes the moment capacities predicted from either Article 6.10.8 or Appendix A6 and allows up to 20% of the moment at the pier to be redistributed to positive bending sections. It is demonstrated below that Appendix A6 is applicable for this example. Therefore, the moment capacity of the section is first computed based on these strength prediction equations as presented below.

8.2.1.2 Flexural Resistance (Appendix A6)

In order to evaluate the above flexural requirements, the flexural resistances based on buckling of the compression flange and yielding of the tension flange are evaluated in this section. The applicability of Appendix A6 for this design example is first evaluated below. The requirement that the nominal yield strength must be less than 70 ksi is easily evaluated.

$$F_{yf} = 50\ ksi \leq 70\ ksi \qquad \text{(satisfied)}$$

The web slenderness requirement is evaluated using Equation A6.1-1.

$$\frac{2D_c}{t_w} < 5.7\sqrt{\frac{E}{F_{yc}}} \qquad \text{Eq. (A6.1-1)}$$

As computed above the elastic neutral axis is located 22.78 in. from the bottom of the composite negative bending section. Subtracting the bottom flange thickness gives the web depth in compression in the elastic range (D_c) as computed below.

$$D_c = 22.78 - 1.22 = 21.56 \text{ in.}$$

Substituting the applicable values into Equation A6.1-1 shows that the equation is satisfied.

$$\frac{2(21.56)}{0.65} \leq 5.7\sqrt{\frac{29{,}000}{50}} = 66.34 \leq 137.27 \qquad \text{(satisfied)}$$

$$\frac{I_{yc}}{I_{yt}} \geq 0.3 \qquad \text{Eq. (A6.1-2)}$$

$$I_{yc} = I_{yt}$$

$$1 \geq 0.3 \qquad \text{(satisfied)}$$

Thus, Appendix A6 is applicable. Use of Appendix A6 begins with the computation of the web plastification factors, as detailed in Article A6.2 and calculated below.

$$\frac{2D_{cp}}{t_w} < \lambda_{pw(D_{cp})}, \qquad \text{Eq. (A6.2.1-1)}$$

where: $$\lambda_{pw(D_{cp})} = \frac{\sqrt{\frac{E}{F_{yc}}}}{\left(0.54\frac{M_p}{R_h M_y} - 0.09\right)^2} \leq \lambda_{rw}\left(\frac{D_{cp}}{D_c}\right) \qquad \text{Eq. (A6.2.1-2)}$$

The hybrid factor, R_h, is determined from Article 6.10.1.10.1, and is 1.0 for this example since the since the section is a homogeneous section. Therefore, $\lambda_{pw(Dcp)}$ is computed as follows:

$$\lambda_{pw(D_{cp})} = \frac{\sqrt{\frac{29000}{50}}}{\left(0.54\frac{59{,}042}{(1.0)(3793)(12)} - 0.09\right)^2} = 64.62 \leq 137.27\left(\frac{27.33}{21.56}\right) = 174.0$$

The web depth in compression at M_p is computed by subtracting the previously determined distance between the top of the web and the plastic neutral axis from the total web depth.

D_{cp} = (39.00 – 2(1.22)) – 9.23 = 27.33 in.

The web slenderness classification is then determined as follows.

$$\frac{2(D_{cp})}{t_w} = \frac{2(27.33)}{0.65} = 84.09 > \lambda_{pw(D_c)} = 64.62 \qquad \text{(not satisfied)}$$

As shown, the section does not qualify as compact. However, it was previously demonstrated, when evaluating the Appendix A6 applicability that the web does qualify as non-compact. Therefore, the applicable web plastification factors for non-compact web sections are used and are determined as specified by Eqs. A6.2.2-4 and A6.2.2-5:

$$R_{pc} = \left[1 - \left(1 - \frac{R_h M_{yc}}{M_p}\right)\left(\frac{\lambda_w - \lambda_{pw(D_c)}}{\lambda_{rw} - \lambda_{pw(D_c)}}\right)\right]\frac{M_p}{M_{yc}} \le \frac{M_p}{M_{yc}} \qquad \text{Eq. (A6.2.2-4)}$$

where $\lambda_{pw(D_c)}$ = limiting slenderness ratio for a compact web corresponding to $2D_c/t_w$

$$\lambda_{pw(D_c)} = \lambda_{pw(D_{cp})}\left(\frac{D_c}{D_{cp}}\right) \qquad \text{Eq. (A6.2.2-6)}$$

$$\lambda_{pw(D_c)} = (64.621)\left(\frac{21.56}{27.33}\right) = 50.98$$

$$R_{pc} = \left[1 - \left(1 - \frac{(1.0)(3793)(12)}{59042}\right)\left(\frac{66.34 - 50.98}{137.27 - 50.98}\right)\right]\frac{59042}{(3793)(12)} \le \frac{59042}{(3793)(12)}$$

R_{pc} = 1.244 ≤ 1.297

R_{pc} = 1.244

$$R_{pt} = \left[1 - \left(1 - \frac{R_h M_{yt}}{M_p}\right)\left(\frac{\lambda_w - \lambda_{pw(D_c)}}{\lambda_{rw} - \lambda_{pw(D_c)}}\right)\right]\frac{M_p}{M_{yt}} \le \frac{M_p}{M_{yt}} \qquad \text{(A6.2.2-5)}$$

$$R_{pt} = \left[1 - \left(1 - \frac{(1.0)(4639)(12)}{59042}\right)\left(\frac{66.34 - 50.98}{137.27 - 50.98}\right)\right]\frac{59042}{(4639)(12)} \le \frac{59042}{(4639)(12)}$$

R_{pt} = 1.050 ≤ 1.061

R_{pt} = 1.050

The flexural resistance based on the compression flange is determined from Article A6.3 and is taken as the minimum of the local buckling resistance from Article A6.3.2 and the lateral torsional buckling resistance from Article A6.3.3. To evaluate the local buckling resistance, the flange slenderness classification is first determined, where the flange is considered compact if the following equation is satisfied:

$$\lambda_f \leq \lambda_{pf}$$

where: $\lambda_f = \frac{b_{fc}}{2t_{fc}} = \frac{15.8}{2(1.22)} = 6.48$ Eq. (A6.3.2-3)

$$\lambda_{pf} = 0.38\sqrt{\frac{E}{F_{yc}}} = 0.38\sqrt{\frac{29,000}{50}} = 9.15$$ Eq. (A6.3.2-4)

$$\lambda_f = 6.48 \leq \lambda_{pf} = 9.15$$ (satisfied)

Therefore, the compression flange is considered compact, and the flexural capacity based on local buckling of the compression flange is governed by Equation A6.3.2-1.

$M_{nc} = R_{pc}\,M_{yc} = (1.244)(3,793) = 4,718$ k-ft Eq. (A6.3.2-1)

Similarly, to evaluate the compressive flexural resistance based on lateral-torsional buckling, the unbraced length must be first classified. Unbraced lengths satisfying the following equation are classified as compact.

$$L_b \leq L_p$$ Eq. (A6.3.3-4)

where: $L_b = (15.0)\,(12.0) = 180$ in.

$$L_p = r_t\sqrt{\frac{E}{F_{yc}}}$$

where: r_t = effective radius of gyration for lateral torsional buckling (in.)

$$r_t = \frac{b_{fc}}{\sqrt{12\left(1 + \frac{1}{3}\frac{D_c\,t_w}{b_{fc}\,t_{fc}}\right)}}$$ Eq. (A6.3.3-10)

$$r_t = \frac{15.8}{\sqrt{12\left(1 + \frac{1}{3}\frac{(21.56)(0.65)}{(15.8)(1.22)}\right)}} = 4.092\,\text{in.}$$

$$L_p = 4.092\sqrt{\frac{29,000}{50}} = 98.55$$

Therefore, $L_b > L_p$. (not compact)

Because the unbraced length does not satisfy the compact limit, the non-compact limit is next evaluated.

$$L_p < L_b \le L_r$$

where: L_r = limiting unbraced length to achieve the nominal onset of yielding in either flange under uniform bending with consideration of compression flange residual stress effects (in.)

$$L_r = 1.95 r_t \frac{E}{F_{yr}} \sqrt{\frac{J}{S_{xc}\, h}} \sqrt{1 + \sqrt{1 + 6.76\left(\frac{F_{yr}}{E}\frac{S_{xc}\, h}{J}\right)^2}} \qquad \text{Eq. (A6.3.3-5)}$$

where: F_{yr} = smaller of the compression flange stress at the nominal onset of yielding of either flange, with consideration of compression flange residual stress effects but without consideration of flange lateral bending, or the specified minimum yield strength of the web

J = St. Venant torsional constant

h = depth between the centerline of the flanges

$$F_{yr} = \min\left(0.7F_{yc}, R_h F_{yt}\frac{S_{xt}}{S_{xc}}, F_{yw}\right) > 0.5F_{yc}$$

$$S_{xt} = \frac{(4639)(12)}{50} = 1113.4\ \text{in}^3$$

$$S_{xc} = \frac{(3793)(12)}{50} = 910.3\ \text{in}^3$$

$$F_{yr} = \min\left(0.7(50), (1.0)(50)\frac{1113.4}{910.3}, 50\right) > 0.5F_{yc}$$

F_{yr} = min (35,61.2, 50) > 0.5(50)

F_{yr} = 35.0 ksi > 25.0 ksi (satisfied)

$$J = \frac{1}{3}\left(Dt_w^{\,3} + b_{fc}t_{fc}^{\,3}\left(1 - 0.63\frac{t_{fc}}{b_{fc}}\right) + b_{ft}t_{ft}^{\,3}\left(1 - 0.63\frac{t_{ft}}{b_{ft}}\right)\right) \qquad \text{Eq. (A6.3.3-9)}$$

$J = (1/3)\,[(36.56)(0.65)^3 + (15.8)(1.22)^3\,(0.951) + (15.8)(1.22)^3\,(0.951)] = 21.53\text{ in.}^4$

$$h = \frac{1.22}{2} + 36.56 + \frac{1.22}{2} = 37.78 \text{ in.}$$

$$L_r = 1.95(4.092)\frac{29{,}000}{35}\sqrt{\frac{21.53}{(910.3)(37.78)}}\sqrt{1+\sqrt{1+6.76\left(\frac{35(910.3)(37.78)}{(29{,}000)(21.53)}\right)^2}} = 408.9$$

$L_b = 180 \le L_r = 408.9$ (satisfied)

Therefore, the unbraced length is classified as non-compact and the lateral torsional buckling resistance is controlled by Eq. A6.3.3-2 of the Specifications.

$$M_{nc} = C_b\left[1-\left(1-\frac{F_{yr}S_{xc}}{R_{pc}M_{yc}}\right)\left(\frac{L_b-L_p}{L_r-L_p}\right)\right]R_{pc}M_{yc} \le R_{pc}M_{yc} \qquad \text{Eq. (A6.3.3-2)}$$

where: C_b = moment gradient modifier (discussed in Article A6.3.3)

$$C_b = 1.75 - 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \le 2.3 \qquad \text{Eq. (A6.3.3-7)}$$

where: M_1 = M_o when the variation in moment between brace points is concave

Otherwise:

M_1 = $2M_{mid} - M_2 \ge M_0$ Eq. (A6.3.3-12)

M_{mid} = major-axis bending moment at the middle of the unbraced length

M_0 = moment at the brace point opposite to the one corresponding to M_2

M_2 = largest major-axis bending moment at either end of the unbraced length causing compression in the flange under consideration

For the critical moment location at the interior pier, the applicable moment values are as follows.

M_2 = 5367 k-ft $\qquad$ $M_1 = 2M_{mid} - M_2 \ge M_0$

M_0 = 2126 k-ft $\qquad$ $M_1 = 2(3502) - (5367) = 1637 \le 2126$

M_{mid} = 3502 k-ft $\qquad$ M_1 = 2126 k-ft

$$C_b = 1.75 - 1.05\left(\frac{2126}{5367}\right) + 0.3\left(\frac{2126}{5367}\right)^2 = 1.38 \leq 2.3$$

$$M_{nc} = (1.38)\left[1 - \left(1 - \frac{(35.0)(910.3)}{(1.249)(3793)(12)}\right)\left(\frac{180 - 98.55}{410.7 - 98.55}\right)\right] \times (1.249)(3793) \leq (1.249)(3793)$$

$$\mathrm{M_{nc}} = (1.38)\left[1 - \left(1 - \frac{(35.0)(910.3)}{(1.244)(3793)(12)}\right)\left(\frac{180 - 98.55}{408.9 - 98.55}\right)\right](1.244)(3793) \leq 1.244(3793)$$

M_{nc} = 5,764 k-ft ≤ 4,718 k-ft

M_{nc} = 4,718 k-ft

If the computed M_{nc} had been less than $R_{pc}M_{yc}$ in this case, then the equations of Article D6.4.2 could have alternatively been used to potentially obtain a larger resistance. As previously stated, the flexural capacity based on the compression flange is the minimum of the local buckling resistance and the lateral torsional buckling resistance, which in this design example are equal.

M_{nc} = 4,718 k-ft

Multiplying the nominal moment capacity by the applicable resistance factor gives the following.

$\phi_f M_{nc}$ = (1.0)(4,718)

$\phi_f M_{nc}$ = 4,718 k-ft

The moment capacity is also evaluated in terms of the tensile moment capacity. For a continuously braced tension flange at the strength limit state, the section must satisfy the requirements of Article A6.1.4.

$M_u \leq \phi_f R_{pt} M_{yt}$ Eq. (A6.1.4-1)

Therefore, the factored moment resistance as governed by tension flange yielding is expressed by the following.

$\phi_f M_{nt} = \phi_f R_{pt} M_{yt}$ = (1.0)(1.050)(4,639) = 4,871 k-ft

8.2.1.3 Factored Moment

At the strength limit state, the design moment is equal to the sum of the factored vertical bending moments applied from forces such as dead loads and vehicular loads. In addition, one-third of the factored lateral bending moment induced by loads such as wind loads must also be added for discretely braced flanges. These design moments must be less than the moment resistance of the section. For the present design example, these requirements are expressed by Eqs. A6.1.1-1 and A6.1.4-1.

$$M_u + \frac{1}{3} f_\ell S_{xc} \le \phi_f M_{nc} \quad \text{(A6.1.1-1)}$$

$$M_u \le \phi_f R_{pt} M_{yt} \quad \text{(A6.1.4-1)}$$

Equation A6.1.1-1 requires that the vertical bending moment plus one-third of the lateral bending moment be less than the nominal moment resistance of the compression flange and is applicable for sections with discretely braced compression flanges (i.e., the bottom flange in Section 2). Equation A6.1.4-1 is intended to prevent yielding of the tension flanges and is applicable to continuously braced tension flanges (i.e. the top flange in Section 2), where lateral bending effects are not applicable.

Furthermore, at the Strength limit state there are five load combinations to consider. The three load combinations applicable to the superstructure elements in this design example are as follows:

Strength I = 1.25DC + 1.5DW + 1.75(LL+I)

Strength III = 1.25DC + 1.5DW + 1.4WS

Strength V = 1.25DC + 1.5DW + 1.35(LL+I) + 0.4WS

At the location of peak negative moment (e.g, the pier), the DC and DW moments are given in Table 2.

DC = -1359 – 263 = -1,622 k-ft

DW = -216 k-ft

From Table 3, the controlling LL+I moment is -1,723 k-ft.

LL+I = -1,723 k-ft

The horizontal pressure applied by the wind load loads was previously determined to be 0.050 ksf. It is assumed in this example that this pressure acts normal to the structure. The procedure given in Article C4.6.2.7.1 is then used to determine the force effects caused by the wind loading. It is required that the wind force per unit length of the bridge must exceed 0.3 kips/ft. Multiplying the design pressure by the exposed height of the superstructure, assuming a 42 in. parapet height, gives the following:

F_D = (0.050)(39 - 1.22 + 2 + 8.5 + 42)/12

F_D = 0.376 k/ft > 0.3

Therefore, the design wind pressure exceeds the minimum required design pressure. It may be assumed that the wind pressure acting on the parapets, deck, and top half of the beam is resisted by diaphragm action of the deck for members with cast-in-place concrete or orthotropic steel decks. The beam must then only resist the wind pressure on the bottom half of the beam. This force is expressed by Eq. C4.6.2.7.1-1.

$$W = \frac{\eta \gamma P_D d}{2} \qquad \text{Eq. (C4.6.2.7.1-1)}$$

where: η = 1.0

P_D = 0.050 ksf

d = beam depth = 39.0 in = 3.25 ft

γ = varies depending on limit state

$$W = \frac{(1.0)(\gamma)(0.050)(3.25)}{2} = 0.08125\gamma \text{ k/ft}$$

The maximum flange lateral bending moment is then computed according to Eq. C4-9.

$$M_w = \frac{WL_b^2}{10} = \frac{(0.08125\gamma)(15)^2}{10} = 1.828\gamma \text{ k-ft} \qquad \text{Eq. (C4.6.2.7.1-2)}$$

Consideration should also be given to increasing the wind load moments to account for second-order force effects, as specified in Article 6.10.1.6, through application of the amplification factor. However, no increase is required for tension flanges, so the amplification factor is negligible in this case. Lateral bending forces due to the wind loading are then determined by dividing M_w by the section modulus of the bottom flange.

$$f_l = \frac{M_w}{S_\ell} = \frac{(1.828\gamma)12}{(15.8)^2(1.22)/6} = 0.432\gamma \text{ ksi}$$

As specified in Article 6.10.1.6, the flange lateral bending stresses must not exceed 60% of the flange yield strength. Thus, for this example f_ℓ must be less than or equal to 30 ksi, which is easily satisfied for the above lateral stress of 0.432γ considering the maximum load factor is 1.4, which results in a maximum lateral bending stress of 0.60 ksi.

The controlling strength limit state can now be determined based on the above information. For the Strength I load combination, the design moments are as follows.

M_u =1.25(1622) + 1.5(216) + 1.75(1723) = 5,367 k-ft (governs)

$M_u + \frac{1}{3} f_\ell S_{xc}$ = 5,367 + 0 (wind loads not considered)

$M_u + \frac{1}{3} f_\ell S_{xc}$ = 5,367 k-ft (governs)

For the Strength III load combination, wind load is incorporated and the design moments are equal to the following:

M_u =1.25(1622) + 1.5(216) = 2,352 k-ft

$M_u + \frac{1}{3} f_\ell S_{xc}$ = 2352 + (1/3)(0.432)(1.4)(910.3)(1/12) = 2,367 k-ft

Lastly, the design moments computed using the Strength V load combination are equal to the following:

M_u = 1.25(1622) + 1.5(216) + 1.35(1723) = 4,678 k-ft

$M_u + \frac{1}{3} f_\ell S_{xc}$ = 4678 + (1/3)(0.432)(0.4)(910.3)(1/12) = 4,682 kip-ft

Reviewing the factored moments for each load combination computed above, it is determined that the Strength I moments govern for this example and that the design moment for both compression flange and tension flange resistances is equal to 5,367 k-ft.

Comparing this design moment to the moment capacities computed above shows that moment redistribution will occur as the governing resistance of the section (based on the compression flange) is less than the applied moment. Hence, the requirements of Appendix B6 are now used to evaluate the moment capacity of the negative bending section.

8.2.1.4 Moment Redistribution (Appendix B6)

Article B6.2 defines the applicability of the optional Appendix B6 provisions. Specifically, the provisions may only be applied to straight continuous span I-section members whose bearing lines are not skewed more than 10 degrees from normal and along which there are no staggered (or discontinuous) cross-frames. The specified minimum yield strength of the section must not exceed 70 ksi. In addition, the section must satisfy the web proportion (Article B6.2.1), compression flange proportion (Article B6.2.2), section transition (Article B6.2.3), compression flange bracing (Article B6.2.4), and shear (Article B6.2.5) requirements discussed below.

Equations B6.2.1-1, B6.2.1-2, and B6.2.1-3 specify the web proportion limits that must be satisfied.

$$\frac{D}{t_w} \leq 150 \qquad \text{Eq. (B6.2.1-1)}$$

$$\frac{D}{t_w} = \frac{36.56}{0.65} = 56.25 \leq 150 \qquad \text{(satisfied)}$$

$$\frac{2D_c}{t_w} \leq 6.8\sqrt{\frac{E}{F_{yc}}} \qquad \text{Eq. (B6.2.1-2)}$$

$$\frac{2(21.56)}{0.65} = 66.34 \leq 6.8\sqrt{\frac{29,000}{50}} = 163.8 \qquad \text{(satisfied)}$$

$$D_{cp} \leq 0.75D \qquad \text{Eq. (B6.2.1-3)}$$

$$D_{cp} = 27.33 \leq 0.75(36.56) = 27.42 \qquad \text{(satisfied)}$$

Section B6.2.2 requires that the following two compression flange proportion limits be satisfied.

$$\frac{b_{fc}}{2t_{fc}} \leq 0.38\sqrt{\frac{E}{F_{yc}}} \qquad \text{Eq. (B6.2.2-1)}$$

$$\frac{15.8}{2(1.22)} = 6.48 \leq 0.38\sqrt{\frac{29,000}{50}} = 9.15 \qquad \text{(satisfied)}$$

$$b_{fc} \geq \frac{D}{4.25} \qquad \text{Eq. (B6.2.2-2)}$$

$$b_{fc} = 15.8 \geq \frac{36.56}{4.25} = 8.60 \qquad \text{(satisfied)}$$

The compression flange bracing distance must satisfy:

$$L_b \leq \left[0.1 - 0.06\left(\frac{M_1}{M_2}\right)\right]\frac{r_t E}{F_{yc}} \qquad \text{Eq. (B6.2.4-1)}$$

$$L_b = 180.0 \leq \left[0.1 - 0.06\left(\frac{2,126}{5,367}\right)\right]\frac{(4.092)(29,000)}{50} = 180.9 \qquad \text{(satisfied)}$$

Additionally, the applied shear under the Strength I loading must be less than or equal to the shear buckling resistance of the beam as specified by.

$$V \leq \phi_v V_{cr} \qquad \text{Eq. (B6.2.5-1)}$$

where: V_{cr} = shear buckling resistance (kip)

V_{cr} = CV_p (for unstiffened webs) Eq.(6.10.9.2-1)

V_p = plastic shear force (kip)

V_p = $0.58\ F_{yw}Dt_w$ Eq. (6.10.9.2-2)

C = ratio of the shear buckling resistance to the shear yield strength determined as specified in Article 6.10.9.3.2, with the shear buckling coefficient, k, taken equal to 5.0

Equations are provided for computing the value of C based on the web slenderness of the beam. If the web slenderness satisfies the following equation, C is equal to 1.0.

$$\frac{D}{t_w} \le 1.12\sqrt{\frac{Ek}{F_{yw}}} = \frac{36.56}{0.65} = 56.25 < 1.12\sqrt{\frac{(29{,}000)(5)}{50}} = 60.31 \quad \text{(satisfied)}$$

C = 1.00

The shear buckling resistance is then computed as follows.

$$V_{cr} = CV_p = (1.00)(0.58)(50)(36.56)(0.65)$$

$$V_{cr} = 689\ \text{kips}$$

$$V = 339\ \text{kips} \le \phi_v V_{cr} = (1.0)(689) = 689\ \text{kips} \quad \text{(satisfied)}$$

The provisions of Article B6.2.1 through B6.2.6 are satisfied for this section. Therefore, moments may be redistributed in accordance with Appendix B6.

Once it is determined that Appendix B6 is applicable, the effective plastic moment is then determined in order to evaluate if the section satisfies the design requirements. The effective plastic moment (M_{pe}) may be determined based on either the equations given in Article B6.5 or the refined procedure given in Article B6.6. In either case, M_{pe} is a function of the geometry and material properties of the section.

When using the M_{pe} equations in Article B6.5, alternative equations are provided for beams that satisfy the requirements for enhanced moment rotation characteristics, i.e., beams classified as ultracompact sections. To be classified as ultracompact, the beam must either: (1) contain transverse stiffeners at a location less than or equal to one-half the web depth from the pier, or (2) satisfy the web compactness limit given by Eq. B6.5.1-1.

$$\frac{2D_{cp}}{t_w} \le 2.3\sqrt{\frac{E}{F_{yc}}} \quad \text{Eq. (B6.5.1-1)}$$

$$\frac{2(27.33)}{0.65} = 84.1 > 2.3\sqrt{\frac{29000}{50}} = 55.4 \quad \text{(not satisfied)}$$

Therefore, the section does not satisfy the web compactness limit and, because the section uses an unstiffened web, the beam does not satisfy the transverse stiffener requirement. Thus, the beam is not considered to be ultracompact and the applicable M_{pe} equation at the strength limit state is Equation B6.5.2-2.

$$M_{pe} = \left[2.63 - 2.3\frac{b_{fc}}{t_{fc}}\sqrt{\frac{F_{yc}}{E}} - 0.35\frac{D}{b_{fc}} + 0.39\frac{b_{fc}}{t_{fc}}\sqrt{\frac{F_{yc}}{E}}\frac{D}{b_{fc}} \right] M_n \le M_n \qquad \text{Eq. (B6.5.2-2)}$$

$$M_{pe} = \left[2.63 - 2.3\frac{15.8}{1.22}\sqrt{\frac{50}{29000}} - 0.35\frac{36.56}{15.8} + 0.39\frac{15.8}{1.22}\sqrt{\frac{50}{29000}}\frac{36.56}{15.8} \right] 4718 \le 4718$$

$M_{pe} = 5{,}042 \le 4{,}718$

$M_{pe} = 4{,}679$ k-ft

The redistribution moment, M_{rd}, for the strength limit state is taken as the larger of the values calculated from Equations. B6.4.2.1-1 and B6.4.2.1-2.

$$M_{rd} = |M_e| + \frac{1}{3} f_l S_{xc} - \phi_f M_{pe} \qquad \text{Eq. (B6.4.2.1-1)}$$

$$M_{rd} = |M_e| + \frac{1}{3} f_l S_{xt} - \phi_f M_{pe} \qquad \text{Eq. (B6.4.2.1-2)}$$

where: M_e = vertical bending moment at the pier due to the factored loads

Since the lateral bending stresses are negligible for this example, the previous equations reduce to the following equation.

$$M_{rd} = |M_e| - M_{pe}$$

If this redistribution moment is less than 20 percent of the elastic moment, as specified by Eq. B6.4.2.1-3, the strength requirements at the pier are satisfied.

$$0 \le M_{rd} \le 0.2|M_e| \qquad \text{Eq. (B6.4.2.1-3)}$$

Therefore, the redistribution moment is computed as follows, which is shown to satisfy the 20% limit.

$M_{rd} = |M_e| - \phi_f M_{pe} = 5{,}365 - (1.0)(4{,}718)$

$M_{rd} = 649$ k-ft = 12.1% $M_e \le 20\%\ M_e$

Therefore, the negative bending region of the beam satisfies the strength limit state requirements when the effective plastic moment equations given in Appendix B6 are used to evaluate the girder capacity..

It is noted that moment redistribution may also be utilized at the service limit state. However, as demonstrated below, the stress requirements at the service limit state are satisfied based on the elastic stresses, and therefore, moment redistribution is not employed at the service limit state in this design example.

8.2.1.5 Shear (6.10.6.3)

As computed above the shear resistance of the negative bending region is governed by Article 6.10.9.2 because the beam is comprised of an unstiffened web, i.e., no transverse stiffeners are provided. The shear resistance of the section was previously calculated to be.

$$V_n = V_{cr} = CV_p = 689 \text{ kips} \qquad \text{Eq. (6.10.9.2-1)}$$

The applied shear at the pier at the strength limit state was previously given in Table 8 as 339 kips. Thus, the shear requirements are satisfied.

$$V = 339 \text{ kips} \leq \phi_v V_{cr} = (1.0)(689) = 689 \text{ kips} \qquad \text{(satisfied)}$$

8.2.2 Constructibility (Article 6.10.3)

Article 2.5.3 requires that the engineer design bridge systems such that the construction is not difficult and does not result in unacceptable locked-in forces. In addition, Article 6.10.3 states the main load-carrying members are not permitted to experience nominal yielding or rely on post-buckling resistance during the construction phases. The sections must satisfy the requirements of Article 6.10.3 at each construction stage. The applied loads to be considered are specified in Table 3.4.1-1 and the applicable load factors are provided in Article 3.4.2.

The beams are considered to be non-composite during the initial construction phase. The influence of various segments of the beam becoming composite at various stages of the deck casting sequence is then considered. The effects of forces from deck overhang brackets acting on the fascia beams are also included in the constructibility checks.

8.2.2.1 Flexure (Article 6.10.3.2)

In regions of negative flexure, Eqs. 6.10.3.2.1-1, 6.10.3.2.1-2 and 6.10.3.2.2-1 specified in Article 6.10.3.2, which are to be checked for critical stages of construction, generally do not control because the sizes of the flanges in these regions are normally governed by the design checks at the strength limit state. Also, the maximum accumulated negative moments from the deck-placement analysis in these regions, plus the negative moments due to the steel weight, typically do not differ significantly from (or may be smaller than) the calculated DC1 negative moments ignoring the effects of the sequential deck placement. The deck-overhang loads do introduce lateral bending stresses into the flanges in these regions, which can be calculated and used to check the above equations in a manner similar to that illustrated later on in this example for Section 1. Wind load, when considered for the construction case, also introduces lateral bending into the flanges.

When applying Eqs. 6.10.3.2.1-1, 6.10.3.2.1-2 and 6.10.3.2.2-1 in these regions, the bottom flange would be considered to be a discretely braced compression flange and the top flange would be considered to be a discretely braced tension flange for all constructibility checks to be made before the concrete deck has hardened or is made composite. The nominal flexural resistance of the bottom flange, F_{nc}, for checking Eq. 6.10.3.2.1-2 would be calculated in a manner similar to that demonstrated below for Section 1. For the sake of brevity, the application of Eqs. 6.10.3.2.1-1, 6.10.3.2.1-2 and 6.10.3.2.2-1 to the construction case for the unbraced lengths adjacent to Section 2 will not be shown in this example.

Note that for sections with slender webs, web bend-buckling should always be checked in regions of negative flexure according to Eq. 6.10.3.2.1-3 for critical stages of construction. In this example, however, Section 2 is not a slender-web section.

8.2.2.2 Shear (Article 6.10.3.3)

The required shear capacity during construction is specified by Eq. 6.10.3.3-1. The unstiffened shear strength of the beam was previously demonstrated to be sufficient to resist the applied shear under the strength load combination. Therefore, the section will have sufficient strength for the constuctibility check.

$$V \le \phi_v V_{cr} \qquad \text{Eq. (6.10.3.3-1)}$$

8.2.3 Service Limit State (Article 6.10.4)

Permanent deformations are controlled under the service limit state. Service limit state checks for steel I-beam bridges are specified in Article 6.10.4.

Permanent deformations that may negatively impact the rideability of the structure are controlled by limiting the stresses in the section under expected severe traffic loadings. Specifically, under the Service II load combination, the top flange of composite sections must satisfy

$$f_f \le 0.95 R_h F_{yf} \qquad \text{Eq. (6.10.4.2.2-1)}$$

Because the bottom flange is discretely braced, lateral bending stresses are included in the design requirements for the bottom flange, which are given by Eq. 6.10.4.2.2-2 as follows:

$$f_f + \frac{f_l}{2} \le 0.95 R_h F_{yf} \qquad \text{Eq. (6.10.4.2.2-2)}$$

At the service limit state, the lateral force effects due to wind loads and deck overhang loads are not considered. Therefore, for bridges with straight, non-skewed beams such as the case in the present design example the lateral bending stresses are taken equal to zero and Eq. 6.10.4.2.2-2 reduces to Eq. 6.10.4.2.2-1.

For sections satisfying the requirements of Article B6.2, Appendix B6 permits the redistribution of moment at the service limit state before evaluating the above equations. As demonstrated previously, Section 2 satisfies the requirements of Article B6.2. Article B6.5.2 specifies the effective plastic moment to be used at the service limit state as follows:

$$M_{pe} = \left[2.90 - 2.3 \frac{b_{fc}}{t_{fc}} \sqrt{\frac{F_{yc}}{E}} - 0.35 \frac{D}{b_{fc}} + 0.39 \frac{b_{fc}}{t_{fc}} \sqrt{\frac{F_{yc}}{E}} \frac{D}{b_{fc}} \right] M_n \le M_n \qquad \text{Eq. (B6.5.2-1)}$$

$$M_{pe} = \left[2.90 - 2.3\frac{15.8}{1.22}\sqrt{\frac{50}{29000}} - 0.35\frac{36.56}{15.8} + 0.39\frac{15.8}{1.22}\sqrt{\frac{50}{29000}}\frac{36.56}{15.8} \right] 4737 \le 4737$$

$$M_{pe} = 6341 \le 4737$$

M_{pe} = 4,737 k-ft

M_{pe} = 4,737 k-ft > M_u = 4,078 k-ft

Because the effective plastic moment is greater than the maximum factored moment for the Service II load combination, it is assumed that there is no moment redistribution at this limit state. The elastic stresses under the Service II load combination are therefore computed using the following equation assuming no moment redistribution:

$$f_f = \frac{M_{DC1}}{S_{nc}} + \frac{M_{DC2} + M_{DW}}{S_{lt}} + \frac{1.3M_{LL+IM}}{S_{st}}$$

For members with shear connectors provided throughout their entire length that also satisfy the provisions of Article 6.10.1.7, and where the maximum longitudinal tensile stresses in the concrete deck at the section under consideration caused by the Service II loads are smaller than $2f_r$, Article 6.10.4.2.1 permits the concrete deck to also be considered effective for negative flexure when computing flexural stresses acting on the composite section at the service limit state. f_r is the modulus of rupture of the concrete specified in Article 6.10.1.7.

Separate calculations (not shown) were made to ensure that the minimum longitudinal reinforcement (determined previously) satisfied the provisions of Article 6.10.1.7 for both the factored construction loads and the Service II loads. Check the maximum longitudinal tensile stresses in the concrete deck under the Service II loads at Section 2. The longitudinal concrete deck stress is to be determined as specified in Article 6.10.1.1.1d; that is, using the short-term modular ratio *n* = 8. Note that only DC2, DW and LL+IM are assumed to cause stress in the concrete deck.

$$f_r = 0.24\sqrt{f'_c} = 0.24\sqrt{4.0} = 0.48 \text{ ksi}$$

$$f_{deck} = \frac{1.0[1.0(-263) + 1.0(-216) + 1.3(-1{,}723)](13.30)(12)}{40{,}371(8)} = 1.34 \text{ ksi} > 2f_r = 2(0.48) = 0.96 \text{ ksi}$$

Therefore, since the concrete deck may not be considered effective in tension at Section 2, the Service II flexural stresses will be computed using the section consisting of the steel girder plus the longitudinal reinforcement only for loads applied to the composite section.
The stress in the compression flange is thus computed as follows.

$$f_f = \frac{(1359)(12)}{856.4} + \frac{(263 + 216)(12)}{959} + \frac{1.3(1723)(12)}{959} = 53.06 \text{ ksi}$$

Then comparing this stress to the allowable stress shows that Equation 6.10.4.2.2-1 is not satisfied.

f_f = 53.06 ksi > $0.95R_hF_{yf}$ = 0.95(1.0)(50) = 47.5 ksi (**NOT** satisfied)**

*****Revision Note (2015).*** The check of the deck stress in tension at the service limit state against the limit of $2f_r$ above was not in the AASHTO LFRD Specifications when this design example was originally written. However, in accordance with the current *AASHTO LRFD (7th Edition)*, the deck stress in tension is to be checked against the limit of $2f_r$ at the service limit state. As shown above because the $2f_r$ limit is exceeded, the concrete deck may not be considered effective in tension at Section 2 at the service limit state when checking the flange stresses. The check of Equation 6.10.4.2.2-1 above shows that the factored stress exceeds the factored resistance by nearly 12 percent. This exceedance is **NOT** acceptable in design. Calculations not presented herein show that increasing the beam section to a W40x249 provides a cross-section that would satisfy this design check as well as the other design checks illustrated in this example. However, for the sake of continuity of this Steel Bridge Design Handbook design example as it relates to this current and past revisions, the W40x215 beam section is not changed, and all calculations herein are illustrated using this beam section. This particular design check and specification change also shows that the designer needs to be aware of specification changes and how they may affect a design and perhaps future load ratings.

Similarly, the computation of the stress in the tension flange is computed as follows.

$$f_f = \frac{(1359)(12)}{856.4} + \frac{(263 + 216)(12)}{1,347} + \frac{1.3(1723)(12)}{1,347} = 43.26\,\text{ksi}$$

Thus, it is demonstrated that Equation 6.10.4.2.2-1 is satisfied for the tension flange.

f_f = 43.26 ksi ≤ $0.95R_hF_{yf}$ = 0.95(1.0)(50) = 47.5 ksi (satisfied)

The compression flange stress at service loads is also limited to the elastic bend-buckling resistance of the web by Equation 6.10.4.2.2-4.

$$f_c \le F_{crw} \qquad \text{Eq. (6.10.4.2.2-4)}$$

where: f_c = compression-flange stress at the section under consideration due to the Service II loads calculated without consideration of flange lateral bending

F_{crw} = nominal elastic bend-buckling resistance for webs with or without longitudinal stiffeners, as applicable, determined as specified in Article 6.10.1.9

From Article 6.10.1.9, the bend-buckling resistance for the web is determined using the following equation.

$$F_{crw} = \frac{0.9Ek}{\left(\frac{D}{t_w}\right)^2} \leq \min\left(R_h F_{yc}, \frac{F_{yw}}{0.7}\right)$$ Eq. (6.10.1.9.1-1)

where: k = bend-buckling coefficient $= \frac{9}{\left(D_c / D\right)^2}$ Eq. (6.10.1.9.1-2)

As specified in Article D6.3.1, the depth of web in compression for composite sections in negative flexure where the concrete deck is not considered to be effective in tension at the service limit state is to be calculated for the section consisting of the steel girder plus the longitudinal reinforcement.

D_c = 22.78 – 1.22 = 21.56 in.

Therefore, k and F_{crw} are computed as follows.

$$k = \frac{9}{(21.56/36.56)^2} = 25.88$$

$$F_{crw} = \frac{0.9(29{,}000)(25.88)}{\left(\frac{36.56}{0.65}\right)^2} = 213.5 \text{ ksi} < R_h F_{yc} = 50 \text{ ksi}$$

It can then be demonstrated that Eq. 6.10.4.2.2-4 is satisfied as shown below.

f_c = |-53.06| ksi > F_{crw} = 50.0 ksi (**NOT** satisfied)**

*****Revision Note (2015).*** The web bend buckling factor resistance is exceeded by 6 percent, which is considered to be unacceptable . The reader should refer to the discussion earlier in this section regarding a change in the AASHTO LRFD specifications regarding the effective deck assumption and the cross-section properties used for computing the factored stress. In a true design, a larger beam section should be considered so that this check is satisfied. However, for the sake of continuity of this Steel Bridge Design Handbook design example as it relates to this current and past revisions, the W40x215 beam section is not changed, and all calculations herein are illustrated using this beam section.

8.2.4 Fatigue and Fracture Limit State (Article 6.10.5)

The fatigue and fracture limit state incorporates three distinctive checks: fatigue resistance of details (Article 6.10.5.1), which includes provisions for load-induced fatigue and distortion-induced fatigue, fracture toughness (Article 6.10.5.2), and a special fatigue requirement for webs (Article 6.10.5.3). The first requirement involves the assessment of the fatigue resistance of details as specified in Article 6.6.1 using the appropriate fatigue load combination specified in Table 3.4.1-1 and the fatigue live load specified in Article 3.6.1.4. The fracture toughness requirements in Article 6.10.5.2 are essentially material requirements. The special fatigue requirement for the web controls the elastic flexing of the web to prevent fatigue cracking. The factored fatigue load for this check is to be taken as the Fatigue I load combination specified in Table 3.4.1-1.

8.2.4.1 Load Induced Fatigue (Article 6.6.1.2)

Article 6.10.5.1 requires that fatigue be investigated in accordance with Article 6.6.1. Article 6.6.1 requires that the live load stress range be less than the nominal fatigue resistance. The nominal fatigue resistance, $(\Delta F)_n$, varies based on the fatigue detail category and is computed using Eq. 6.6.1.2.5-1 for the Fatigue I load combination and infinite fatigue life; or Eq. 6.6.1.2.5-2 for Fatigue II load combination and finite fatigue life.

$$(\Delta F)_n = (\Delta F)_{TH} \qquad \text{Eq. (6.6.1.2.5-1)}$$

$$(\Delta F)_n = \left(\frac{A}{N}\right)^{\frac{1}{3}} \qquad \text{Eq. (6.6.1.2.5-2)}$$

where: N = $(365)(75)n(ADTT)_{SL}$ Eq. (6.6.1.2.5-3)

A = constant from Table 6.6.1.2.5-1

n = number of stress range cycles per truck passage taken from Table 6.6.1.2.5-2

$(ADTT)_{SL}$ = single-lane ADTT as specified in Article 3.6.1.4

$(\Delta F)_{TH}$ = constant-amplitude fatigue threshold taken from Table 6.6.1.2.5-3

The fatigue resistance of the base metal at the weld joining the cross-frame connection plate to the flanges of the beam at the cross-frame located 15 feet from the pier is evaluated below. From Table 6.6.1.2.3-1, it is determined that this detail is classified as a fatigue Detail Category C′. The constant-amplitude fatigue threshold, $(\Delta F)_{TH}$, for a Category C′ detail is 12.0 ksi (see Table 6.6.1.2.5-3).

For this example, an $(ADTT)_{SL}$ of 800 trucks per day is assumed. Since this $(ADTT)_{SL}$ exceeds the value of 745 trucks per day specified in Table 6.6.1.2.3-2 for a Category C′ detail, the nominal fatigue resistance for this particular detail is to be determined for the Fatigue I load combination and infinite fatigue life using Eq. 6.6.1.2.5-1. Therefore:

$$(\Delta F)_n = (\Delta F)_{TH} = 12.00\text{ksi}$$

The applied stress range is taken as the stress range resulting from the fatigue loading (shown in Figure 9), with a dynamic load allowance of 15 percent applied, and distributed laterally by the previously calculated distribution factor for fatigue.

According to Article 6.6.1.2.1, for flexural members with shear connectors provided throughout their entire length and with concrete deck reinforcement satisfying the provisions of Article

6.10.1.7, flexural stresses and stress ranges applied to the composite section at the fatigue limit state at all sections in the member may be computed assuming the concrete deck to be effective for both positive and negative flexure. Shear connectors are assumed provided along the entire length of the girder in this example. Separate computations (not shown) were made to ensure that the longitudinal concrete deck reinforcement satisfies the provisions of Article 6.10.1.7. Therefore, the concrete deck will be assumed effective in computing all dead load and live load stresses and live load stress ranges applied to the composite section in the subsequent fatigue calculations.

The provisions of Article 6.6.1.2 apply only to details subject to a net applied tensile stress. According to Article 6.6.1.2.1, in regions where the unfactored permanent loads produce compression, fatigue is to be considered only if this compressive stress is less than the maximum tensile stress resulting from the Fatigue I load combination specified in Table 3.4.1-1. Note that the live-load stress due to the passage of the fatigue load is considered to be that of the heaviest truck expected to cross the bridge in 75 years. At this location, the unfactored permanent loads produce tension at the top of the girder and compression at the bottom of the girder. In this example, the effect of the future wearing surface is conservatively ignored when determining if a detail is subject to a net applied tensile stress.

At the bottom of the top flange the applied stress range is computed as follows:

$$\gamma(\Delta f) = (1.50)\left[\frac{(178)(12)(4.52-1.22)}{40{,}371} + \frac{|-267|(12)(4.52-1.22)}{40{,}371}\right]$$

$$\gamma(\Delta f) = 0.65 \text{ ksi} \leq (\Delta F)_n = 12.00 \text{ ksi} \qquad \text{(satisfied)}$$

At the top of the bottom flange:

$$f_{DC1} = \frac{(-389)(12)(19.50-1.22)}{16{,}700} = -5.11 \text{ ksi}$$

$$f_{DC2} = \frac{(-75)(12)(27.98-1.22)}{29{,}925} = -0.80 \text{ ksi}$$

$$\Sigma = -5.11 + -0.80 = -5.91 \text{ ksi}$$

$$f_{LL+IM} = \frac{1.5(178)(12)(34.48-1.22)}{40{,}371} = 2.64 \text{ ksi}$$

$$|-5.91 \text{ ksi}| > 2.64 \text{ ksi} \quad \therefore \text{ fatigue does not need to be checked}$$

Therefore, it is demonstrated that the applied stress range in the top and bottom flanges is acceptable.

8.2.4.2 Distortion Induced Fatigue (Article 6.6.1.3)

A positive connection is to be provided for all transverse connection-plate details to both the top and bottom flanges to prevent distortion induced fatigue.

8.2.4.3 Fracture (Article 6.6.2)

The appropriate Charpy V-notch fracture toughness, found in Table 6.6.2-2, must be specified for main load-carrying components subjected to tensile stress under the Strength I load combination.

8.2.4.4 Special Fatigue Requirement for Webs (Article 6.10.5.3)

Article 6.10.5.3 requires that the shear force applied due to the unfactored permanent loads plus the factored fatigue loading (i.e. the Fatigue I load combination) must be less than the shear-buckling resistance of interior panels of stiffened webs.

$$V_u \leq V_{cr} \qquad \text{Eq. (6.10.5.3-1)}$$

However, designs utilizing unstiffened webs at the strength limit state, as is the case here, automatically satisfy this criterion. Thus, Eq. 6.10.5.3-1 is not explicitly evaluated herein.

8.3 Exterior Girder Beam Check: Section 1

8.3.1 Strength Limit State

8.3.1.1 Flexure (Article 6.10.6.2)

For compact sections in positive bending, Equation 6.10.7.1.1-1 must be satisfied at the strength limit state.

$$M_u + \frac{1}{3} f_l S_{xt} \leq \phi_f M_n \qquad (6.10.7.1.1\text{-}1)$$

8.3.1.1.1 Flexural Resistance (6.10.7.1)

To calculate the flexural resistance at the strength limit state, the classification of the section must first be determined. The following requirements must be satisfied for a section in positive bending to qualify as compact:

$$F_y = 50 \text{ ksi} \leq 70 \text{ ksi} \qquad \text{(satisfied)}$$

$$\frac{D}{t_w} = \frac{36.56}{0.65} = 56.25 \leq 150 \qquad \text{(satisfied)}$$

$$\frac{2D_{cp}}{t_w} = \frac{2(0)}{0.4375} = 0 \leq 3.76\sqrt{\frac{E}{F_{yc}}} = 3.76\sqrt{\frac{29{,}000}{50}} = 90.55 \qquad \text{Eq (10.6.2.2-1) (satisfied)}$$

Therefore, the section is compact, and the nominal flexural resistance is based on Article 6.10.7.1.2, where the moment capacity of beams satisfying $D_p \leq 0.1D_t$ is given by Eq. 6.10.7.1.2-1 and by Eq. 6.10.7.1.2-2. for those beams violating this limit.

$$M_n = M_p \qquad \text{Eq. (6.10.7.1.2-1)}$$

$$M_n = M_p\left(1.07 - 0.7\frac{D_p}{D_t}\right) \qquad \text{Eq. (6.10.7.1.2-2)}$$

D_p is the distance from the top of the concrete deck to the neutral axis of the composite section at the plastic moment and is computed as follows. The plastic neutral axis was determined previously to be located 0.22 in. from the top of the top flange. Therefore:.

$$D_p = 8 + 2 - 1.22 + 0.22 = 9.00 \text{ in.}$$

The total depth of the composite beam, D_t, is equal to the following.

$$D_t = 8 + 2 + 36.56 + 1.22 = 47.78 \text{ in.}$$

Therefore, D_p is greater than 10% of D_t as computed below and the nominal flexural capacity is therefore determined using Equation 6.10.7.1.2-2.

$$D_p = 9.00 > 0.1D_t = 0.1(47.78) = 4.78 \qquad \text{(not satisfied)}$$

$$M_n = M_p\left(1.07 - 0.7\frac{D_p}{D_t}\right) \qquad \text{Eq. (6.10.7.1.2-2)}$$

$$M_n = 6165\left(1.07 - 0.7\frac{9.00}{47.78}\right) = 5{,}784 \text{ k-ft}$$

Since the span under consideration and all adjacent interior-pier sections satisfy the requirements of Article B6.2 (as determined previously), M_n is not limited to $1.3R_hM_y$ according to Eq. 6.10.7.1.2-3 in this case.

8.3.1.1.2 Factored Positive Bending Moment

In order to determine if the above determined moment resistance of 5,784 k-ft is adequate, the maximum value of $(M_u + f_lS_{xt}/3)$ must be determined, according to Eq. 6.10.7.1.1-2. Therefore the value of $(M_u + f_lS_{xt}/3)$ resulting from each of the three strength load combinations applicable to this design example is now computed. As previously discussed during the evaluation of the negative bending resistance of the beam, the load factors for each of the applicable load combinations are as follows:

Strength I = 1.25DC + 1.5DW + 1.75(LL+I)

Strength III = 1.25DC + 1.5DW + 1.4WS

$$\text{Strength V} = 1.25DC + 1.5DW + 1.35(LL+I) + 0.4WS$$

The location of the maximum positive moment is at 36 ft from the abutments. The DC and DW moments at this location are given in Table 2 and are equal to the following:

$$DC = 761 + 147 = 908 \text{ k-ft}$$

$$DW = 121 \text{ k-ft}$$

From Table 3, the controlling LL+I moment is 1664 k-ft.

$$LL+I = 1664 \text{ k-ft}$$

The unfactored moment ($f_\ell S_x$) due to wind load loads was previously determined to be 1.828 k-ft.

$$M_w = 1.828 \text{ k-ft}$$

Consideration should also be given to increasing the wind load moments to account for second-order force effects, as specified in Article 6.10.1.6, through application of the amplification factor. However, no amplification is required for tension flanges and the compression flange is continuously braced in positive bending, so the amplification factor is negligible in this case.

Lateral bending stresses due to the wind loading are then determined by dividing M_w by the section modulus of the bottom flange.

$$f_l = \frac{M_w}{S_\ell} = \frac{(1.828)12}{(15.8)^2(1.22)/6} = 0.432 \text{ ksi}$$

As specified in Article 6.10.1.6, the flange lateral bending stresses must not exceed 60% of the flange yield strength. Thus, for this example f_ℓ must be less than or equal to 30 ksi, which is easily satisfied by the above lateral bending stress. The maximum lateral bending stress is obtained by multiplying f_ℓ by the maximum wind load factor of 1.4, which is less than the allowable stress of 30 ksi.

$$(1.4)(0.432) = 0.6048 \text{ ksi} < 30 \text{ ksi} \quad \text{(satisfied)}$$

The controlling strength limit state can now be determined based on the above information. For the Strength I load combination, the design moments are as follows:

$$M_u = 1.25(908) + 1.5(121) + 1.75(1664) = 4{,}229 \text{ k-ft}$$

$$M_u + \frac{1}{3} f_\ell S_{xc} = 4{,}229 + 0 \qquad \text{(wind loads not considered)}$$

$$M_u + \frac{1}{3} f_\ell S_{xc} = 4{,}229 \text{ k-ft} \qquad \text{(governs)}$$

For the Strength III load combination, wind load is incorporated and the design moments are equal to the following:

M_u = 1.25(908) + 1.5(121) = 1,317 k-ft

$M_u + \frac{1}{3} f_\ell S_{xc}$ = 1317 + (1/3)(0.432)(1.4)(1,070)(1/12)

$M_u + \frac{1}{3} f_\ell S_{xc}$ = 1,335 k-ft

Lastly, the design moments computed using the Strength V load combination are equal to the following:

M_u = 1.25(908) + 1.5(121) + 1.35(1664) = 3,563 k-ft

$M_u + \frac{1}{3} f_\ell S_{xc}$ = 3,563 + (1/3)(0.432)(0.4)(1070)(1/12)

$M_u + \frac{1}{3} f_\ell S_{xc}$ = 3,568 k-ft

Reviewing the factored moments for each load combination computed above, it is determined that the Strength I moments govern for this example and that the factored design moment is equal to 4,229 k-ft.

8.3.1.1.3 Redistribution Moment

The redistribution moment must then be added to the above elastic moments. It was previously determined that the redistribution moment at the pier at the (governing) Strength I load combination is equal to 630 k-ft. Because the redistribution moment varies linearly from zero at the end-supports to a maximum at the interior pier, the redistribution moment at 36 ft from the abutment is simply computed as follows.

$M_{rd} = \left(\frac{36}{90}\right)(630) = 0.4(630) = 252$ k-ft.

The total design moment is then the sum of the redistribution moment and the elastic moment.

M_u = 4,229 + 252 = 4,481 k-ft.

8.3.1.1.4 Flexural Capacity Check

The design moment of 4,481 k-ft is then compared to the factored resistance of 5,784 k-ft, which shows that the positive bending capacity of the beam is sufficient.

M_u = 4,481 k-ft $\leq \phi_f M_n$ = (1.0)(5,784) = 5,784 k-ft (satisfied)

8.3.1.1.5 Ductility Requirement

Sections in positive bending are also required to satisfy Eq. 6.10.7.3-1, which is a ductility requirement intended to prevent premature crushing of the concrete slab.

$$D_p \leq 0.42D_t \qquad (6.10.7.3\text{-}1)$$

$$D_p = 9.00 \text{ in.} \leq 0.42(47.78) = 20.07 \text{ in.} \qquad \text{(satisfied)}$$

8.3.1.2 Shear (Article 6.10.3.3)

The shear requirements at the strength limit state were previously shown to be satisfied.

8.3.2 Constructibility (Article 6.10.3)

8.3.2.1.1 Deck Placement Analysis

In regions of positive flexure, temporary moments that the non-composite girders experience during the casting of the deck can sometimes be significantly higher than those which may be calculated based on the final conditions of the system. An analysis of the moments during each casting sequence must be conducted to determine the maximum moments in the structure acting on the non-composite girders in those regions. The potential for uplift during the deck casting should also be investigated. Wind load should not be considered in conjunction with the casting of the deck.

Figure 11 depicts the casting sequence assumed in this design example. As required in Article 6.10.3.4, the loads are applied to the appropriate composite sections during each casting sequence. For example, it is assumed during Cast One that all sections of the girder are non-composite. Similarly, the dead load moments due to the steel components are also based on the non-composite section properties. However, to determine the distribution of moments due to Cast Two, the short-term composite section properties are used in the regions of the girders that were previously cast in Cast One, while the non-composite section properties are used in the region of the girder where concrete is cast in Cast Two. The moments used in the evaluation of the constructibility requirements are then taken as the maximum moments that occur on the non-composite girder during any stage of construction, i.e., the sum of the moments due to the steel dead load and the first casting phase or the sum of the moments due to the steel dead load and both casting phases. Additionally, while not required, the dead load moment resulting from applying all dead load at once (i.e. without consideration of the sequential placement) to the non-composite section (DC1) is also considered.

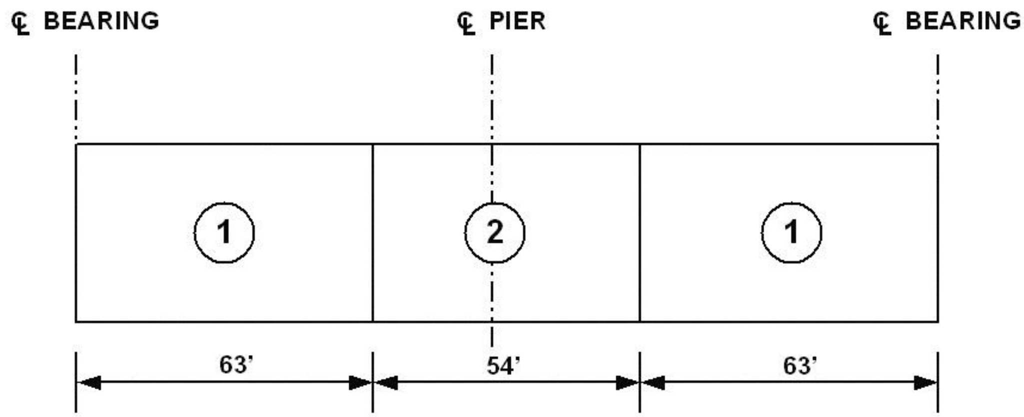

Figure 11 Deck Placement Sequence

The results of the deck placement analysis are shown in Table 12 where the maximum dead load moments in the positive bending region acting on the non-composite section is indicated by bold text. Note that because of the deck-casting sequence chosen for this particular example, the maximum positive bending moment acting on the non-composite section is not caused by the sequential deck placement (i.e. Cast One). Therefore, the DC1 moment of 761 kip-ft at Section 1, ignoring the effect of the sequential deck placement, will be used in the subsequent constructibility design checks.

Table 12 Moments from Deck Placement Analysis (kip-ft)

x/L	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Dist. (ft.)	0	9	18	27	36	45	54	63	72	81	90
Steel Wt.	0	62	106	131	137	124	93	44	-22	-104	-204
SIP Forms	0	27	45	56	58	53	40	19	-2	-45	-87
Cast 1	0	262	443	541	557	491	342	112	-150	-411	-672
Cast 2	0	297	511	643	693	661	547	350	84	-259	-678
Σ Cast 1	0	351	593	727	752	668	476	174	-173	-560	-964
Σ Cast 2	0	385	661	829	**888**	838	680	413	60	-408	-970
DC1	0	353	598	734	761	679	489	190	-217	-734	**-1359**

Because the shear requirements during construction are automatically satisfied for beams with unstiffened webs, only the evaluation of the flexural requirements is presented herein.

Article 6.10.1.6 states that when checking the flexural resistance based on lateral torsional buckling, f_{bu} is to be taken as the largest compressive stress in the flange under consideration, without consideration of flange lateral bending, throughout the unbraced length. When checking the flexural resistance based on yielding, flange local buckling or web bend buckling, f_{bu} is to be taken as the stress at the section under consideration. The maximum factored flexural stress due to the deck casting sequence is calculated below.

Because the section modulus with respect to the top flange is the same as the section modulus with respect to the bottom flange at this phase of construction, f_{bu} is the same for both flanges and is equal to the following:

For Strength I:

$$f_{bu} = \frac{1.0(1.25)(761)(12)}{856.4} = 13.33 \text{ ksi}$$

For the Special Load Combination (Article 3.4.1.2):

$$f_{bu} = \frac{1.0(1.4)(761)(12)}{856.4} = 14.93 \text{ ksi}$$

8.3.2.1.2 Deck Overhang Loads

The loads applied to the deck overhang brackets induce torsion on the fascia girders, which introduces flange lateral bending stresses. This section illustrates the recommended approach to estimate these lateral bending stresses.

The deck overhang bracket configuration assumed in this example is shown in Figure 12. Typically the brackets are spaced between 3 and 4 feet, but the assumption is made here that the loads are uniformly distributed, except for the finishing machine. Half of the overhang weight is assume to be carried by the exterior girder, and the remaining half is carried by the overhang brackets.

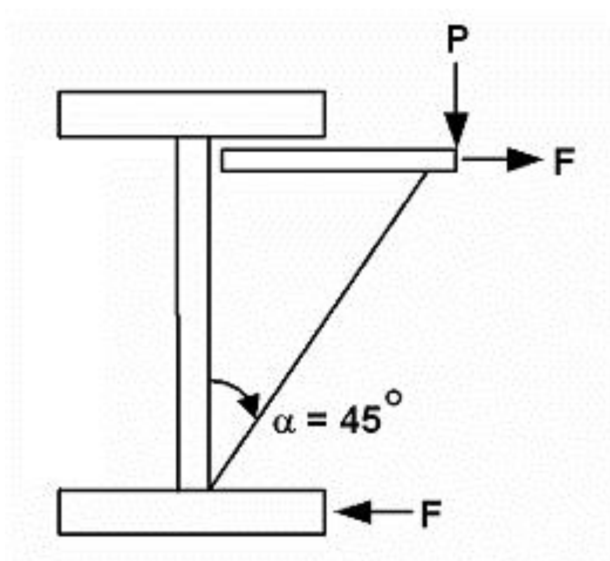

Figure 12 Deck Overhang Bracket Loads

The following calculation determines the weight of the deck overhang acting on the overhang brackets.

$$P = 0.5(150)\left[\frac{8.5}{12}(3.5) + \left[\frac{1}{12}\left(\frac{2.0}{2}\right)\left(3.5 - \frac{15.8/2}{12}\right)\right] + \frac{(2-1.22)}{12}\left(\frac{15.8/2}{12}\right)\right] = 207 \text{ lbs/ft}$$

The following is a list of typical construction loads assumed to act on the system before the concrete slab gains strength. The magnitudes of load listed represent only the portion of these loads that are assumed to be applied to the overhang brackets. Note that the finishing machine load shown represents one-half of the finishing machine truss weight.

Overhang Deck Forms:	P = 40 lb/ft
Screed Rail:	P = 85 lb/ft
Railing:	P = 25 lb/ft
Walkway:	P = 125 lb/ft
Finishing Machine:	P = 3,000 lb

The lateral force acting on the beam section due to the overhang loading is computed as follows:

$$F = P \tan \alpha$$

where: $\alpha = 45$ degrees

F = P tan 45

F = P

The equations provided in Article C6.10.3.4 to determine the lateral bending moment can be employed in the absence of a more refined method. From the article, the following equation determines the lateral bending moment for a uniformly distributed lateral bracket force:

$$M_l = \frac{F_l L_b^2}{12}$$

where: M_l = lateral bending moment in the top flange due to the eccentric loadings from the form brackets

F_l = statically equivalent uniformly distributed lateral force due to the factored loads

L_b = The unbraced length of the section under consideration = 15 ft (at the location of maximum negative bending)

Thus, the lateral moment due to the component (overhang) dead load is equal to the following.

$$M_l = \frac{(0.207)(30)^2}{12} = 15.53 \text{ ft - kips}$$

The flange lateral bending stresses due to the component dead load are then determined by dividing the lateral bending moment by the section moduli of the flanges, which in this case are equal for the top and bottom flanges.

$$f_l = \frac{M_\ell}{S_\ell} = \frac{15.53(12)}{1.22(15.8)^2/6} = 3.67\ ksi$$

Similarly, the lateral moment and lateral bending stress due to the construction dead loads are computed as follows.

$$M_l = \frac{(.275)(30)^2}{12} = 20.63 \text{ k-ft}$$

$$f_l = \frac{M_\ell}{S_\ell} = \frac{20.63(12)}{1.22(15.8)^2/6} = 4.88 \text{ ksi}$$

Note that the lateral moment contribution due to the construction loads and component loads is separated due to the alternative load factors applied to the different load types.

The equation which estimates the lateral bending moment due to a concentrated lateral force at the middle of the unbraced length is:

$$M_l = \frac{P_l L_b}{8}$$

where: P_l = statically equivalent concentrated force placed at the middle of the unbraced length

The unfactored lateral bending moment and lateral bending stress due to the finishing machine are then equal to the following.

$$M_l = \frac{(3)(30)}{8} = 11.25 \text{ k-ft}$$

$$f_l = \frac{M_\ell}{S_\ell} = \frac{11.25(12)}{1.22(15.8)^2/6} = 2.66 \text{ ksi}$$

For simplicity, the largest values of f_l within the unbraced length (computed above) will be used in the design checks, i.e., the maximum value of f_l within the unbraced length is conservatively assumed to be the stress level throughout the unbraced length.

Article 6.10.1.6 specifies the process for determining the lateral bending stress. The first-order lateral bending stress may be used if the following limit is satisfied.

$$L_b \le 1.2 L_p \sqrt{\frac{C_b R_b}{f_{bm}/F_{yc}}}$$ Eq. (6.10.1.6-2)

where: L_p = limiting unbraced length from Article 6.10.8.2.3 of the Specifications

C_b = moment gradient modifier

R_b = web load-shedding factor

F_{yc} = yield strength of the compression flange

C_b is the moment gradient modifier specified in Article 6.10.8.2.3. Separate calculations show that $f_{mid}/f_2 > 1$ in the unbraced length under consideration. Therefore, C_b must be taken equal to 1.0.

According to Article 6.10.1.10.2, the web load-shedding factor, R_b, is to be taken as 1.0 when checking constructibility.

Calculate L_p:

$$D_c = 19.50 - 1.22 = 18.28 \text{ in.}$$

$$r_t = \frac{b_{fc}}{\sqrt{12\left(1+\frac{1}{3}\frac{D_c t_w}{b_{fc} t_{fc}}\right)}} = \frac{15.8}{\sqrt{12\left(1+\frac{1}{3}\frac{(18.28)(0.65)}{15.8(1.22)}\right)}} = 4.15 \text{ in.}$$

$$L_p = 1.0 r_t \sqrt{\frac{E}{F_{yc}}} = 1.0(4.15)\sqrt{\frac{29{,}000}{50}} = 100.0 \text{ in.}$$ Eq. (6.10.8.2.3-4)

Thus, Eq. 6.10.1.6-2 is evaluated as follows.

$$L_b = 360 \text{ in.} > 1.2(100.0)\sqrt{\frac{(1.0)(1.0)}{|-14.93|/50}} = 219.6 \text{ in.}$$

Because Eq. 6.10.1.6-2 is not satisfied, Article 6.10.1.6 requires that second-order elastic compression-flange lateral bending stresses be determined. The second-order compression-flange lateral bending stresses may be determined by amplifying first-order values (i.e. $f_{\ell 1}$) as follows:

$$f_\ell = \left(\frac{0.85}{1-\frac{f_{bu}}{F_{cr}}}\right) f_{\ell 1} \ge f_{\ell 1}$$ Eq. (6.10.1.6-4)

or: $f_{\ell} = (AF)f_{\ell 1} \geq f_{\ell 1}$

where AF is the amplification factor and F_{cr} is the elastic lateral torsional buckling stress for the flange under consideration specified in Article 6.10.8.2.3 determined as:

$$F_{cr} = \frac{C_b R_b \pi^2 E}{\left(\frac{L_b}{r_t}\right)^2} \qquad \text{Eq. (6.10.8.2.3-8)}$$

$$F_{cr} = \frac{1.0(1.0)\pi^2(29{,}000)}{\left(\frac{30(12)}{4.15}\right)^2} = 38.04 \text{ ksi}$$

Note that the calculated value of F_{cr} for use in Eq. 6.10.1.6-4 is not limited to $R_b R_h F_{yc}$.

The amplification factor is then determined as follows:

For Strength I:

$$AF = \frac{0.85}{\left(1 - \frac{|-13.33|}{38.04}\right)} = 1.31 > 1.0 \quad \text{ok}$$

For the Special Load Combination specified in Article 3.4.2.1:

$$AF = \frac{0.85}{\left(1 - \frac{|-14.93|}{38.04}\right)} = 1.40 > 1.0 \quad \text{ok}$$

AF is taken equal to 1.0 for tension flanges.

8.3.2.1.3 Strength I

The lateral bending stresses for the Strength I load combination are computed as follows. As specified in Article 3.4.2.1, the load factor for construction loads and any associated dynamic effects is not to be taken less than 1.5 for the Strength I load combination.

Dead loads:

P = [1.25(207) + 1.5(40 + 85 + 25 + 125)] = 671.3 lbs/ft.

$F = F_{\ell} = P = 671.3$ lbs/ft.

$$M_\ell = \frac{F_\ell L_b^2}{12} = \frac{(0.6713)(30)^2}{12} = 50.35\ \text{kip} - \text{ft}$$

The flange lateral bending stresses due to the component dead load are then determined by dividing the lateral bending moment by the section moduli of the flanges, which in this case are equal for the top and bottom flanges.

$$f_\ell = \frac{M_\ell}{S_\ell} = \frac{50.35(12)}{1.22(15.8)^2/6} = 11.90\ \text{ksi}$$

Finishing machine load:

P = [1.5(3,000)] = 4,500 lbs.

F = P_ℓ = P = 4,500 lbs.

$$M_\ell = \frac{P_\ell L_b}{8} = \frac{(4.5)(30)}{8} = 16.88\ \text{kip} - \text{ft}$$

$$f_\ell = \frac{M_\ell}{S_\ell} = \frac{16.88(12)}{1.22(15.8)^2/6} = 3.99\ \text{ksi}$$

Total:

Top flange: f_ℓ = (11.90 + 3.99)(AF) = (11.90 + 3.99)(1.31) = 20.82 ksi

Bot. flange: f_ℓ = (11.90 + 3.99)(AF) = (11.90 + 3.99)(1.0) = 15.89 ksi

8.3.2.1.4 Special Load Combination (Article 3.4.2.1)

The computation of the lateral bending stresses for the special load combination specified in Article 3.4.2.1 is demonstrated below.

Dead loads:

$$P = [1.4(207 + 40 + 85 + 25 + 125)] = 674.8\ \text{lbs/ft}$$

$$F = F_\ell = P = 674.8\ \text{lbs/ft}$$

$$M_\ell = \frac{F_\ell L_b^2}{12} = \frac{(0.6748)(30)^2}{12} = 50.61\ \text{k} - \text{ft}$$

$$f_\ell = \frac{M_\ell}{S_\ell} = \frac{50.61(12)}{1.22(15.8)^2/6} = 11.96\ \text{ksi}$$

Finishing machine load:

P = [1.4(3,000)] = 4,200 lbs.

F = P_ℓ = P = 4,200 lbs.

$$M_\ell = \frac{P_\ell L_b}{8} = \frac{(4.2)(30)}{8} = 15.75\ \text{kip} - \text{ft}$$

$$f_\ell = \frac{M_\ell}{S_\ell} = \frac{15.75(12)}{1.22(15.8)^2/6} = 3.72\ \text{ksi}$$

Total:

Top flange: f_ℓ = (11.96 + 3.72)(AF) = (11.96 + 3.72)(1.40) = 21.95 ksi

Bot. flange: f_ℓ = (11.96 + 3.72)(AF) = (11.96 + 3.72)(1.0) = 15.68 ksi

According to Article 6.10.1.6, the lateral bending stresses (after amplification) must be less than 60 percent of the yield stress of the flange under consideration. It is shown above that the lateral bending stresses are highest in the top flange under the Special Load Combination, and highest in the bottom flange under the Strength I load combination. Thus, evaluation of Eq. 6.10.1.6-1 for the Strength I load combination is shown below.

$$f_l \leq 0.6F_y \qquad (6.10.1.6\text{-}1)$$

Top flange: f_ℓ = 21.95 ksi < $0.6F_{yf}$ = 30 ksi (satisfied)

Bottom flange: f_ℓ = 15.89 ksi < $0.6F_{yf}$ = 30 ksi (satisfied)

8.3.2.2 Flexure (Article 6.10.3.2)

During construction, both the compression and tension flanges are discretely braced. Therefore, Article 6.10.3.2 requires the non-composite section to satisfy Eqs. 6.10.3.2.1-1, 6.10.3.2.1-2, and 6.10.3.2.1-3, which ensure the flange stress is limited to the yield stress, the section has sufficient strength under the lateral torsional and flange local buckling limit states, and web bend buckling does not occur during construction, respectively.

First, determine if the non-composite section satisfies the noncompact slenderness limit as follows:

$$\frac{2D_c}{t_w} < 5.7\sqrt{\frac{E}{F_{yc}}} \qquad \text{Eq. } (6.10.6.2.3\text{-}1)$$

$$\frac{2(18.28)}{0.65} < 5.7\sqrt{\frac{29{,}000}{50}}$$

$56.25 < 137.27$ (satisfied)

The section is nonslender (i.e. the section has a compact or noncompact web). Therefore, Eq. 6.10.3.2.1-3 (web bend-buckling) need not be checked.

8.3.2.2.1 Compression Flange:

Flange nominal yielding:

$$f_{bu} + f_l \leq \phi_f R_h F_{yc} \quad \text{Eq. (6.10.3.2.1-1)}$$

Since the section under consideration is homogeneous, the hybrid factor, R_h, is 1.0, as stated in Article 6.10.1.10.1. Thus, Eq. 6.10.3.2.1-1 is evaluated as follows:

For Strength I:

$$13.33 + 20.82 \leq (1.0)(1.0)(50)$$

$$34.15 \text{ ksi} \leq 50 \text{ ksi} \quad \text{(satisfied)}$$

For the Special Load Combination (Article 3.4.2.1):

$$14.93 + 21.95 \leq (1.0)(1.0)(50)$$

$$36.88 \text{ ksi} \leq 50 \text{ ksi} \quad \text{(satisfied)}$$

Flexural Resistance:

$$f_{bu} + \frac{1}{3} f_l \leq \phi_f F_{nc} \quad \text{Eq. (6.10.3.2.1-2)}$$

As specified in Article 6.10.3.2.1, the nominal flexural resistance of the compression flange, F_{nc}, is to be determined as specified in Article 6.10.8.2. For sections in straight I-girder bridges with compact or noncompact webs, the lateral torsional buckling resistance may be taken as M_{nc} determined as specified in Article A6.3.3 (Appendix A6) divided by the elastic section modulus about the major axis of the section to the compression flange, S_{xc}. As mentioned in Article C6.10.3.2.1, this may be useful for sections in bridges with compact or noncompact webs having larger unbraced lengths, if additional lateral torsional buckling resistance is required beyond that calculated based on the provisions of Article 6.10.8.2. However, for this particular example, the increased lateral torsional buckling resistance obtained by using the provisions of Article A6.3.3 is not deemed to be necessary. Thus, the provisions of Article 6.10.8.2.3 will be used to compute the lateral torsional buckling resistance for this check.

First, calculate the local buckling resistance of the top (compression) flange. Determine the slenderness ratio of the top flange:

$$\lambda_f = \frac{b_{fc}}{2t_{fc}} \qquad \text{Eq. (6.10.8.2.2-3)}$$

$$\lambda_f = \frac{15.8}{2(1.22)} = 6.5$$

Determine the limiting slenderness ratio for a compact flange (alternatively, see Table C6.10.8.2.2-1):

$$\lambda_{pf} = 0.38\sqrt{\frac{E}{F_{yc}}} \qquad \text{Eq. (6.10.8.2.2-4)}$$

$$\lambda_{pf} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.2$$

Since $\lambda_f < \lambda_{pf}$,

$$F_{nc} = R_b R_h F_{yc} \qquad \text{Eq. (6.10.8.2.2-1)}$$

As specified in Article 6.10.3.2.1, in computing F_{nc} for constructibility, the web load-shedding factor R_b is to be taken equal to 1.0 because the flange stress is always limited to the web bend-buckling stress according to Eq. 6.10.3.2.1-3. Therefore,

$$(F_{nc})_{FLB} = (1.0)(1.0)(50) = 50.00 \text{ ksi}$$

For Strength I:

$$f_{bu} + \frac{1}{3} f_\ell \le \phi_f (F_{nc})_{FLB}$$

$$f_{bu} + \frac{1}{3} f_\ell = |-13.33| \text{ ksi} + \frac{20.82}{3} \text{ ksi} = 20.27 \text{ ksi}$$

$$\phi_f (F_{nc})_{FLB} = 1.0(50.00) = 50.00 \text{ ksi}$$

$$20.27 \text{ ksi} < 50.00 \text{ ksi} \quad \text{(satisfied)}$$

For the Special Load Combination specified in Article 3.4.2.1:

$$f_{bu} + \frac{1}{3} f_\ell \le \phi_f (F_{nc})_{FLB}$$

$$f_{bu} + \frac{1}{3}f_\ell = \left|-14.93\right| \text{ ksi} + \frac{21.95}{3} \text{ ksi} = 22.25 \text{ ksi}$$

$$\phi_f (F_{nc})_{FLB} = 1.0(50.00) = 50.00 \text{ ksi}$$

$$22.25 \text{ ksi} < 50.00 \text{ ksi} \quad \text{(satisfied)}$$

Next, determine the lateral torsional buckling resistance of the top (compression) flange within the unbraced length under consideration. The limiting unbraced length, L_p, was computed earlier to be 100.0 in. or 8.33 ft. The effective radius of gyration for lateral torsional buckling, r_t, for the non-composite section was also computed earlier to be 4.15 inches.

Determine the limiting unbraced length, L_r:

$$L_r = \pi r_t \sqrt{\frac{E}{F_{yr}}} \qquad \text{Eq. (6.10.8.2.3-5)}$$

where: $F_{yr} = 0.7F_{yc} \le F_{yw}$

$$F_{yr} = 0.7(50) = 35.0 \text{ ksi} < 50 \text{ ksi} \quad \text{ok}$$

F_{yr} must also not be less than $0.5F_{yc} = 0.5(50) = 25.0$ ksi ok.

Therefore: $L_r = \frac{\pi(4.15)}{12}\sqrt{\frac{29{,}000}{35.0}} = 31.27 \text{ ft}$

Since L_p = 8.33 feet < L_b = 30.0 feet < L_r = 31.27 feet,

$$F_{nc} = C_b\left[1 - \left(1 - \frac{F_{yr}}{R_h F_{yc}}\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right]R_b R_h F_{yc} \le R_b R_h F_{yc} \qquad \text{Eq. (6.10.8.2.3-2)}$$

As discussed previously, since $f_{mid}/f_2 > 1$ in the unbraced length under consideration, the moment-gradient modifier, C_b, must be taken equal to 1.0. Therefore,

$$F_{nc} = 1.0\left[1 - \left(1 - \frac{35.0}{1.0(50)}\right)\left(\frac{30.0 - 8.33}{31.27 - 8.33}\right)\right](1.0)(1.0)(50) = 35.83 \text{ ksi} < 1.0(1.0)(50) = 50 \text{ ksi} \quad \text{ok}$$

For Strength I:

$$f_{bu} + \frac{1}{3}f_\ell \le \phi_f (F_{nc})_{LTB}$$

$$f_{bu} + \frac{1}{3} f_{\ell} = |-13.33| \text{ ksi} + \frac{20.82}{3} \text{ ksi} = 20.27 \text{ ksi}$$
$$\phi_f (F_{nc})_{LTB} = 1.0(35.83) = 35.83 \text{ ksi}$$
$$20.27 \text{ ksi} < 35.83 \text{ ksi} \quad \text{(satisfied)}$$

For the Special Load Combination specified in Article 3.4.2.1:

$$f_{bu} + \frac{1}{3} f_{\ell} \leq \phi_f (F_{nc})_{LTB}$$

$$f_{bu} + \frac{1}{3} f_{\ell} = |-14.93| \text{ ksi} + \frac{21.95}{3} \text{ ksi} = 22.25 \text{ ksi}$$
$$\phi_f (F_{nc})_{LTB} = 1.0(35.83) = 35.83 \text{ ksi}$$
$$22.25 \text{ ksi} < 35.83 \text{ ksi} \quad \text{(satisfied)}$$

8.3.2.2.2 Tension Flange:

Flange Nominal Yielding:

$$f_{bu} + f_l \leq \phi_f R_h F_{yt} \qquad \text{Eq. (6.10.3.2.2-1)}$$

For Strength I:

$$13.33 + 15.89 \leq (1.0)(1.0)(50)$$

$$29.22 \text{ ksi} \leq 50 \text{ ksi} \qquad \text{(satisfied)}$$

For the Special Load Combination (Article 3.4.2.1):

$$14.93 + 15.68 \leq (1.0)(1.0)(50)$$

$$30.61 \text{ ksi} \leq 50 \text{ ksi} \qquad \text{(satisfied)}$$

8.3.3 Service Limit State (Article 6.10.4)

Service limit state requirements for steel I-girder bridges are specified in Article 6.10.4. The evaluation of the positive bending region based on these requirements follows.

8.3.3.1 Elastic Deformations (Article 6.10.4.1)

Since the bridge is not designed to permit pedestrian traffic, the live load deflection will be limited to L/800. It is shown below that the maximum deflection along the span length using the service loads and a line girder approach is less than the L/800 limit. It is noted, however, that the application of this requirement is optional.

$$\delta = 0.741 \text{ in.} < L/800 = (90 \times 12) / 800 = 1.35 \text{ in.}$$

8.3.3.2 Permanent Deformations (Article 6.10.4.2)

To control permanent deformations, flange stresses are limited according to Eq. 6.10.4.2.2-1 as follows:

$$F_f \leq 0.95 R_h F_{yf} \qquad (6.10.4.2.2\text{-}1)$$

where: $f_f = \frac{M_{DC1}}{S_{nc}} + \frac{M_{DC2} + M_{DW}}{S_{lt}} + \frac{1.3 M_{LL+IM}}{S_{st}}$

The stress in the compression flange at the critical positive bending location is then computed as follows based on the moment values given in Tables 2 and 3.

$$f_f = \frac{(761)(12)}{856.4} + \frac{(147+121)(12)}{2716} + \frac{1.3(1664)(12)}{8932} = 14.75\,\text{ksi}$$

Thus, the requirements of Eq. 6.10.4.2.2-1 are satisfied for the compression flange.

$$f_f = 14.75\,\text{ksi} \leq 0.95 R_h F_{yf} = 0.95(1.0)(50) = 47.50\text{ksi} \qquad \text{(satisfied)}$$

Similarly, the stress in the tension flange is as follows (f_ℓ is equal to zero in this case).

$$f_f = \frac{(761)(12)}{856.4} + \frac{(147+121)(12)}{1070} + \frac{1.3(1664)(12)}{1171} = 35.84\,\text{ksi}$$

Thus, the service requirements for the tension flange are also satisfied.

$$f_f = 35.84\,\text{ksi} \leq 0.95 R_h F_{yf} = 0.95(1.0)(50) = 47.50\text{ksi}$$

For composite sections in positive flexure, since the web satisfies the requirement of Article 6.10.2.1.1 (i.e. $D/t_w \leq 150$) such that longitudinal stiffeners are not required, web bend-buckling under the Service II load combination need not be checked at Section 1. Thus, all service limit state requirements are satisfied.

8.3.4 Fatigue and Fracture Limit State (Article 6.10.5)

8.3.4.1 Load Induced Fatigue (Article 6.6.1.2)

The fatigue calculation procedures in the positive bending region are similar to those previously presented for the negative bending region. In this section the fatigue requirements are evaluated for the flange welds of a cross-frame connection plate located 30 feet from the abutment.

From Table 6.6.1.2.3-1, it is determined that this detail is classified as a fatigue Detail Category C'. The constant-amplitude fatigue threshold, $(\Delta F)_{TH}$, for a Category C' detail is 12 ksi (see Table 6.6.1.2.5-3).

For this example, an $(ADTT)_{SL}$ of 800 trucks per day is assumed. Since this $(ADTT)_{SL}$ exceeds the value of 745 trucks per day specified in Table 6.6.1.2.3-2 for a Category C′ detail, the nominal fatigue resistance for this particular detail is to be determined for the Fatigue I load combination and infinite fatigue life using Eq. 6.6.1.2.5-1. Therefore:

$$(\Delta F)_n = (\Delta F)_{TH} = 12.00\text{ksi} \qquad \text{Eq. (6.6.1.2.5-1)}$$

Again, as discussed previously, the concrete deck will be assumed effective in computing all dead load and live load stresses and live load stress ranges applied to the composite section in the subsequent fatigue calculations.

At this location, the unfactored permanent loads produce compression at the top of the girder and tension at the bottom of the girder. In this example, the effect of the future wearing surface is conservatively ignored when determining if a detail is subject to a net applied tensile stress.

Bottom of Top Flange:

$$f_{DC1} = \frac{(743)(12)(19.50-1.22)}{16{,}700} = -9.76\text{ ksi}$$

$$f_{DC2} = \frac{(144)(12)(11.02-1.22)}{29{,}925} = -0.57\text{ ksi}$$

$$\Sigma = -9.76 + -0.57 = -10.33\text{ ksi}$$

$$f_{LL+IM} = \frac{1.5|-107|(12)(4.52-1.22)}{40{,}371} = 0.16\text{ ksi}$$

$$|-10.33\text{ ksi}| > 0.16\text{ ksi} \quad \therefore \text{ fatigue does not need to be checked}$$

Top of Bottom Flange:

$$\gamma(\Delta f) = (1.50)\left[\frac{(519)(12)(34.48-1.22)}{40{,}371} + \frac{|-107|(12)(34.48-1.22)}{40{,}371}\right]$$

$$\gamma(\Delta f) = 9.28\text{ ksi} \le (\Delta F)_n = 12.00\text{ ksi} \qquad \text{(satisfied)}$$

8.3.4.2 Special Fatigue Requirement for Webs (Article 6.10.5.3)

As discussed previously, the following shear requirement must be satisfied at the fatigue limit state:

$$V \le \phi_v V_{cr} \qquad \text{Eq. (6.10.5.3-1)}$$

However, this design utilizes an unstiffened web. Therefore, this limit does not control and is not explicitly evaluated. It has been demonstrated that the beam satisfies all the design requirements.

8.4 Deck Design

The following section will illustrate the design of the deck by the Empirical Deck Design Method specified in Article 9.7.2. This design process recognizes the strength gained by complex in-plane membrane forces forming an internal arching effect (see Commentary to Article 9.7.2.1).

To be able to use the Empirical Deck Design Method, certain design conditions must first be met, as specified in Article 9.7.2.4. It is also specified that four layers of minimum isotropic reinforcement are to be provided as specified in Article 9.7.2.5.

The Empirical Deck Design Method does not apply for the design of the deck overhang (see Article 9.7.2.2), which must be designed by traditional design methods.

8.4.1 Effective Length (Article 9.7.2.3)

For the Empirical Design Method, the effective length is taken equal to the distance between the flange tips, plus the flange overhang, taken as the distance from the extreme flange tip to the face of the web. The effective slab length must not exceed 13.5 feet. Figure 13 illustrates the effective slab length.

$$L_{eff} = (10.0)(12.0) - (12.0)\left(\frac{12 - 0.4375}{2}\right) = 112.78\text{in.} < 162.0\text{in.} \qquad \text{(satisfied)}$$

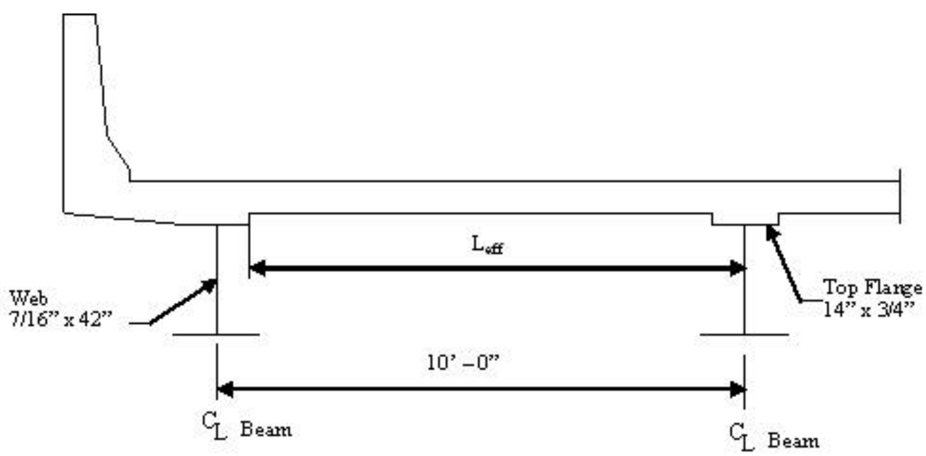

Figure 13 Effective Slab Length for Deck Design

8.4.2 Design Conditions (Article 9.7.2.4)

Specific design conditions must be met in order to use the Empirical Deck Design Method. The deck must be fully cast-in-place and water cured. The deck must also maintain a uniform cross section over the entire span, except in the locations of the haunches located at the beam flanges. Concrete used for the deck must have a specified 28-day compressive strength greater than or equal to 4.0 ksi. The supporting beams must be made of either steel or concrete, and the deck must be made composite with the beams. A minimum of two shear connectors at 24.0 inch centers must be provided in the negative moment regions of continuous steel superstructures. In addition the following requirement must be satisfied:

$$6.0 \le \frac{L_{eff}}{t_s} \le 18.0$$

where: L_{eff} = effective slab length (Article 9.7.2.3)

t_s = the structural slab thickness, which is the total thickness minus integral wearing surface (Article 9.7.2.6), and must be greater than 7 inches

$$t_s = 8.0 \text{ in.} > 7.0 \text{ in.} \qquad \text{(satisfied)}$$

$$\frac{112.78}{8.0} = 14.10 < 18.0 \qquad \text{(satisfied)}$$

The deck overhang beyond the centerline of the outside beam must be at least 5.0 times the depth of the slab.

$$(5.0)(8.0) = 40.0 \text{in.} < 42.0 \text{ in.} \qquad \text{(satisfied)}$$

The core depth of the slab must not be less than 4.0 inches. An illustration of the core depth is shown in Figure 14.

Assuming a 2-inch cover on the top and a 1-inch cover on the bottom of the slab

5.0 in. > 4.0 in. (satisfied)

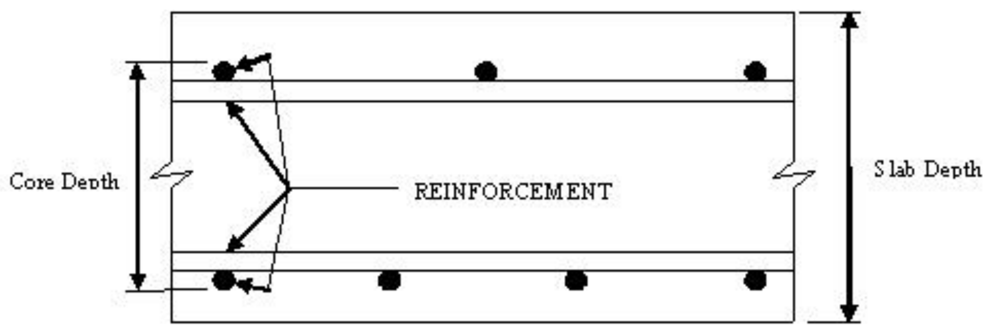

Figure 14 Core Depth of the Concrete Slab

8.4.3 Positive Flexure Reinforcement Requirements

Article 9.7.2.5 specifies that four layers of isotropic reinforcement be provided. The reinforcement is to be provided in each face of the slab, with the outermost layers placed in the direction of the effective length.

8.4.3.1 Top Layer (Longitudinal and Transverse)

The top layers are required to have a minimum reinforcement area of 0.18 in.^{2}/ft., with the maximum spacing permitted to be 18 inches.

Using No. 5 bars with a cross-sectional area of 0.31 in.^{2}, the required spacing is:

$$s = \frac{(0.31)(12)}{(0.18)} = 20.67\text{in.} > 18.0 \ (\text{max.})$$

Use a 12-inch spacing to match that of the negative flexure region as determined below.

8.4.3.2 Bottom Layer (Longitudinal and Transverse)

Bottom layers of reinforcement are required to have a minimum reinforcement area of 0.27 in.^{2}/ft., with the maximum spacing permitted to be 18 inches.

Using No. 5 bars with a cross-sectional area of 0.31 in.^{2}, the required spacing is:

$$s = \frac{(0.31)(12)}{(0.27)} = 13.78\text{in.} > 18.0 \ (\text{max.})$$

Therefore, use a 12-inch spacing in both of the bottom layers to match that of the negative flexure region as determined below.

8.4.4 Negative Flexure Reinforcement Requirements

Article 6.10.1.7 states that in regions of negative flexure the total cross sectional area of the longitudinal reinforcement shall not be less than 1 percent of the total cross sectional area of the concrete deck. The slab thickness is taken to be 8.0 inches; therefore, the minimum area of longitudinal reinforcement is:

Min. area of longitudinal reinforcement = (8.0)(0.01) = 0.08 in.^{2}/in.

The reinforcement used to satisfy this requirement shall have a minimum yield strength no less than 60 ksi and should have a size not exceeding No. 6 bars. The bars should be placed in two layers that are uniformly distributed across the deck width, with two thirds in the top layer and the remaining one third in the bottom layer. Bar spacing should not exceed 12.0 inches center-to-center.

8.4.4.1 Top Layer (Longitudinal)

$$\text{Minimum } A_{reinf} = \left(\frac{2}{3}\right)(0.08) = 0.05\text{in.}^2/\text{in.}$$

Use No. 6 bars (A=0.44 in.^{2}) at 12-0 inch spacing with No. 5 bars (A = 0.31 in.^{2}) at 12-inch spacing:

$$A_{reinf} = \frac{0.44}{12} + \frac{0.31}{12} = 0.06 \text{in.}^2/\text{in.} > 0.05 \text{in.}^2/\text{in.} \qquad \text{(satisfied)}$$

8.4.4.2 Bottom Layer (Longitudinal)

$$\text{Minimum } A_{reinf} = \left(\frac{1}{3}\right)(0.08) = 0.03 \text{in.}^2/\text{in.}$$

Use No. 5 bars (A=0.31 in.^{2}) at 12.0-inch spacing with No. 4 bars (A = 0.20 in.^{2}) at 12-inch spacing:

$$A_{reinf} = \frac{0.31}{12} + \frac{0.20}{12} = 0.04 \text{in.}^2/\text{in.} > 0.03 \text{in.}^2/\text{in.} \qquad \text{(satisfied)}$$

8.4.4.3 Top and Bottom Layer (Transverse)

The transverse reinforcing steel in both the top and bottom layers will be No. 5 bars at 12.0- inch spacing (Figure 15), the same as the positive flexure regions.

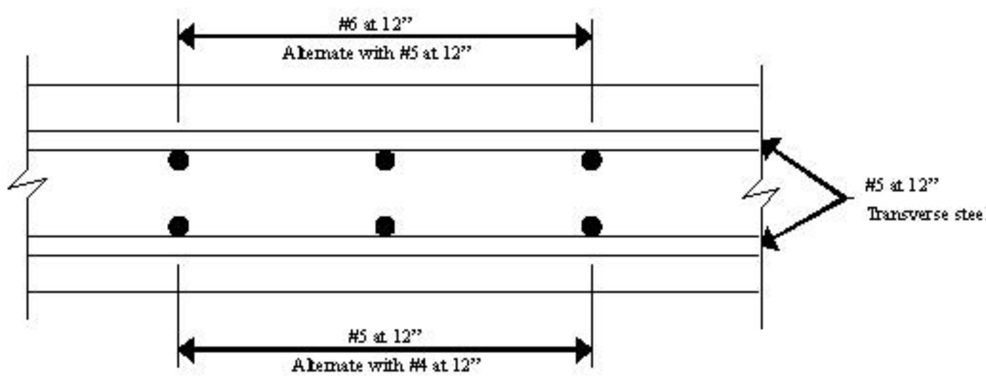

Figure 15 Deck Slab in Negative Flexure Region of the Beam

9.0 REFERENCES

1. AASHTO (2014). *AASHTO LRFD Bridge Design Specifications*, 7th Edition, American Association of State Highway and Transportation Officials, Washington, DC.