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ABSTRACT

Snow can cause dangerous driving conditions by reducing the pavement friction and covering the road surface markings. Salt is widely used by highway maintenance managers in the U.S. for reducing the impact of snow or ice on traffic. To develop long-term plans especially for the next winter season, it is essential to know what are the factors affecting salt usage and to determine sufficient amount of salt needed in each depot location. This can be done by estimating statistically robust models for salt usage prediction. In this study, historical data regarding storm characteristics and salt usage of New Jersey Turnpike (NJT) and Golden State Parkway (GSP) are used to estimate those models. The linear models, the hierarchical linear (HL) models and the hierarchical linear models with varying dispersion (HLVD) are developed to predict the salt usage of these highways. Results show that districts with higher average snow depth, longer storm duration and lower average temperature are associated with greater salt usage. The HLVD models are found to have the best predictive performance by including random parameters to account for unobserved spatial heterogeneity and by including fixed effects in the dispersion term. In addition, by estimating case-specific dispersion based on storm characteristics, the HLVD models could be used appropriately to estimate the upper bounds of salt usage, which are not extremely large and could satisfy the salt demand in most cases. The findings of this paper can provide highway authorities with valuable insights into the use of statistical models for more efficient inventory management of salt and other maintenance materials.

INTRODUCTION

Snow storms remain as one the most disrupting events to highway systems. Snow on roads can cause dangerous driving conditions by reducing the pavement friction and covering the road surface markings. Black ice, caused by the refreezing of melting snow on roads, is difficult to be detected while driving, and thus increases the risk of traffic accidents. Salt is generally used by highway maintenance managers in the U.S. for reducing the impact of snow or ice on traffic. Since salt lowers the freezing point of water it comes into contact with, scattering salt on roads can help prevent icing and accelerate the melting process of snow. Stromberg (1*)* states that an "estimated 22 million tons of salt are scattered on the roads of the U.S. annually-about 137 pounds of salt for every American."

Having enough salt stored in each depot location is of utmost importance before and during snowfalls. Sufficient salt should be replenished in advance so that the maintenance operations would not be delayed during the snow storm. One of the challenges faced by highway authorities is to determine the sufficient amount of salt needed in each maintenance district. Underestimation of salt usage could slow down the snow or ice clearing process and place drivers in danger. Conversely, overestimation of salt usage could increase the storage cost and leave insufficient space for other maintenance materials. Hence, in-depth understanding of the factors affecting salt usage and an appropriate method for salt usage estimation are necessary tasks for more efficient inventory management.

This study proposes a statistically robust method to estimate the salt usage as a function of snow storm characteristics. Two tolled highways managed by New Jersey Turnpike Authority (NJTA), namely, New Jersey Turnpike (NJT) and Golden State Parkway (GSP) are selected as a case study. A web-based tool called WeatherEVANT (Real-time Weather related Event Visualization and ANalytics Tool) *(2)* is developed by the research team and it is being currently used by the NJTA maintenance department to assist the real-time management of traffic operations. WeatherEVANT extracts information from NJTA's snow operations database, which is updated frequently by the operators during the snow storms, and summarizes data on its web-based interface integrated with Google Maps©. Historical and live information on salt usage and storm conditions can be extracted from WeatherEVANT for analysis. WeatherEVANT also provides various visualizations of this real-time data and can also automatically generate a variety of performance reports for the use by decision makers. Active users of WeatherEVANT vary from maintenance clerks to the upper management of the authority.

This paper begins with introduction, literature review and data description. In the methodology section, novel models developed in the hierarchical framework are proposed to account for the unobserved heterogeneity of salt usage among different maintenance districts. The proposed models are used to predict the means and upper bounds of salt usage. This paper ends with summary and conclusions.

LITERATURE REVIEW

Emergency management in response to adverse weather events is gaining increasing attention recently *(3-7)*. Efficient management of maintenance materials is one of the essential tasks. Salt is the most widely used material for road maintenance in winter. In common practice, two forms of salt are applied: rock salt and salt brine (usually a 23 percent salt solution, derived from rock salt) *(8)*. Both forms have similar melting characteristics, while salt brine is typically more efficient *(8)*.

The action that removes snow or ice after the snowfalls is regarded as de-icing. Numerous studies were conducted to explore factors that affects effectiveness of salt in de-icing process. Gerbino-Bevins (9*)* explored the performance of multiple de-icing materials under varied temperatures and road surfaces, and indicated that salt brine became less effective and eventually stopped working when temperature goes lower. Besides, with similar application rate of salt, melting speed of snow is typically faster on asphalt concrete than cement concrete.

Besides deicing, anti-icing is another common action conducted before the occurrence of snowfalls in preparation for snow and ice. According to Cuelho and Harwood (10*)*, anti-icing can reduce the efforts to clear snow from pavement. The performance of anti-icing is related to temperature and duration of snow event *(11)*. Fu*et al.* (11*)* developed a statistical model based on the results of lab and field tests, and depicted that anti-icing became less effective when pavement temperature is below 14°F. They also pointed out that anti-icing should be favored in light snow events.

Regarding the salt inventory management, Roelants and Muyldermans (12*)* developed a stock management system based on an (R-S) Inventory Policy, where R is reorder points and S stands for the target stock. Both parameters vary spatially and temporally. Based on (R-S) Policy, Ciarallo*et al.* (13*)* constructed a strategy that met local salt inventory guideline based on a weather regression model, and the developed approach was able to determine the amount of salt need and time to make the order. Shi*et al.* (14*)* discussed the decision making process of using chloride-based products for winter maintenance under asset management framework, which provides a new prospective for all stakeholders.

Most of the previous studies focus on the effectiveness of de-icing/anti-icing materials under different situations, and the management of their inventory. Studies on salt usage prediction based on weather-related factors are rare. Ciarallo*et al.* (13*)* developed regression models to estimate the amount of salt needed at the city/county level, using predictors such as amount of snow, days of snow, and temperature. However, they assume a linear relationship between salt usage and its contributing factors, which may not result in reliable estimates under more complicated situations. This study aims to add to the literature by proposing a more robust method for salt usage prediction.

There are also several studies *(15-18)* conducted in the area of inventory management of different types of commodities for intelligent transportation systems and emergency operations. One of the major problems in these studies is the lack of real-world data that can be used to calibrate and validate developed models. This paper is unique in that sense because it has extensive amount of salt usage data obtained from real world problems.

DATA DESCRIPTION

Maintenance activities along NJT and GSP are distributed to multiple districts. There are 12 maintenance districts at NJT and 9 maintenance districts (composed of 15 sub-districts) at GSP as shown in Figure 1. Each district is responsible for the maintenance of the assigned roadway segments. The equipment for storm maintenance such as plow trucks and salt spreaders is stationed and salt used in the storms is stored in the tanks at each maintenance district. Regarding the practical application and management, salt usage of each maintenance district during a storm is what we try to estimate.

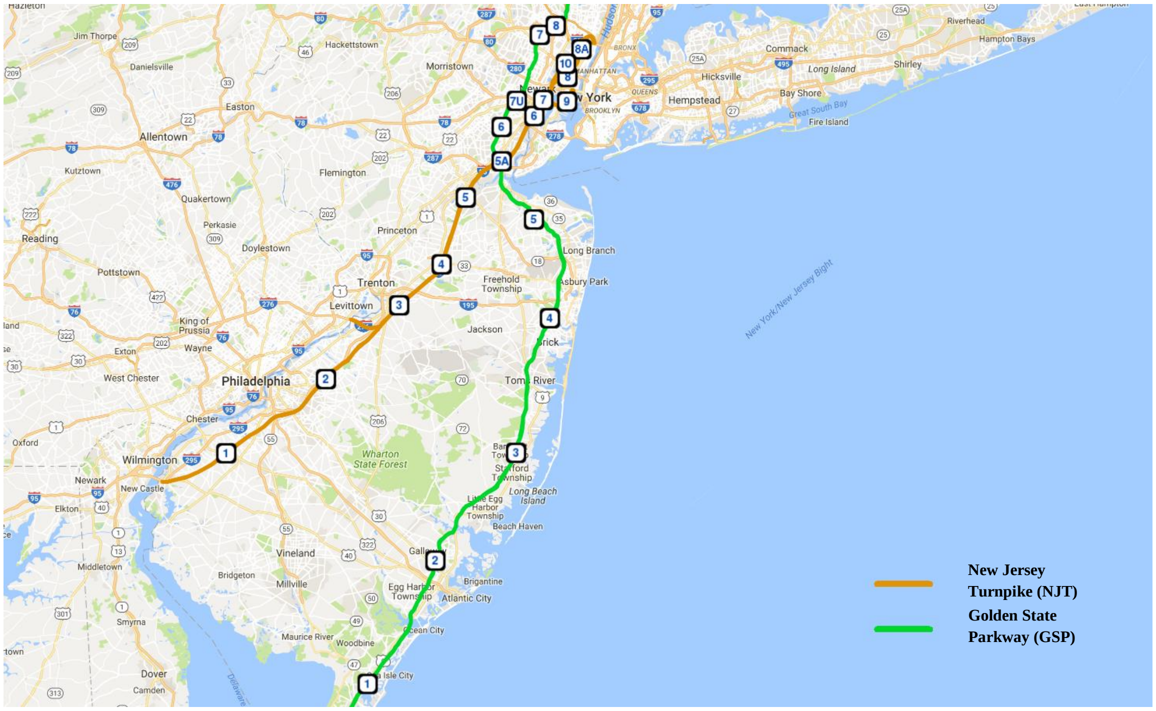


Figure 1 Maintenance districts at New Jersey Turnpike (NJT) and Golden State Parkway (GSP). *(19)*

WeatherEVANT' was used to obtain historical salt usage by querying the database at the district or event levels. The salt usage of each maintenance district during each storm event was obtained. We also used WeatherEVANT to extracts storm characteristics including average temperature, average snow depth, and storm duration from a NJTA's database called SPEAR. As shown in Figure 2, maintenance districts with lower average temperature, higher average snow depth and longer storm duration are associated with more salt usage. Salt usage and storm data from the 2011-2012, 2012-2013 and 2013-2014 winter seasons (in-sample dataset) is used to develop salt usage models, and the data from the 2014-2015 winter season (out-of-sample dataset) is used for model validation. There are a total of 44 storm events during the study period. The description and descriptive statistics of variables are presented in Table 1.

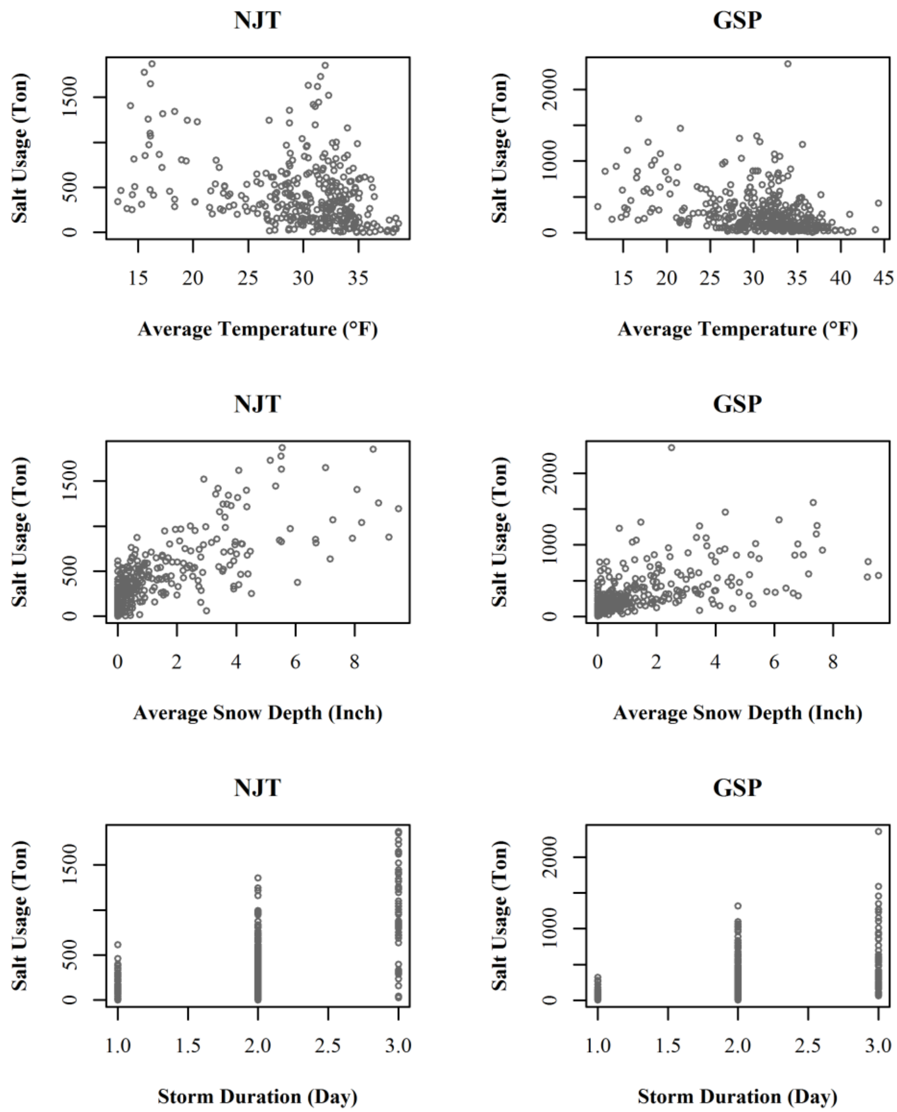


Figure 2 Factors affecting salt usage at New Jersey Turnpike (NJT) and Golden State Parkway (GSP).

Table 1 Descriptions and Descriptive Statistics of Key Variables for New Jersey Turnpike (NJT) and Golden State Parkway (GSP)

Variable	Description	NJT (490 samples)		GSP (618 samples)	
		Mean	S.D.	Mean	S.D.
Salt usage	The amount of salt used in a maintenance district during a storm event (ton)	428.98	370.28	286.7	300.33
Average temperature	Average temperature in a maintenance district during a storm event (°F)	29.74	5.47	30.6	5.68
Minimum temperature	Minimum temperature in a maintenance district during a storm event (°F)	26.38	7.07	26.07	6.95
Average snow depth	Average snow depth in a maintenance district during a storm event (inch)	1.34	1.94	1.22	1.85
Maximum snow depth	Maximum snow depth in a maintenance district during a storm event (inch)	2.34	3.42	2.26	3.44
Storm duration	the lasting time of a storm (day)	2.02	0.54	2.06	0.53
October	1 for storms occurring in October; 0 for others	0.12	0.32	0.12	0.33
November	1 for storms occurring in November; 0 for others	0.04	0.19	0.04	0.20
December	1 for storms occurring in December; 0 for others	0.16	0.37	0.15	0.36
January	1 for storms occurring in January; 0 for others	0.27	0.44	0.25	0.43
February	1 for storms occurring in February; 0 for others	0.30	0.46	0.31	0.46
March	1 for storms occurring in March; 0 for others	0.12	0.32	0.12	0.33
District	Categorical variable which indicates maintenance district; 12 levels for NJT and 15 levels for GSP	-	-	-	-

METHODOLOGY

Model Specification

To estimate the salt usage, the linear model, the hierarchical linear (HL) model and the hierarchical linear model with varying dispersion (HLVD) are proposed, in this section. The specifications of those models are presented in the following subsections. It should be noted that salt usage models are developed for NJT and GSP separately, considering the heterogeneity between them. Ordinary least squares (OLS) method *(20)* is used to estimate the coefficients of the linear model. Extended quasi likelihood method *(21)* is used for the estimation of the HL and the HLVD models.

<u>Model 1: linear model</u>

The linear model is based on the assumption that salt usage is independent from each other. Its specification is given by equation (1):

$$\log(y_{ij}) = \alpha_0 + \sum_{p=1}^{P} \alpha_p \quad {}_{pij} + \varepsilon_{ij} \tag{1}$$

where y_{ij} denote the amount of salt used during i^{th} storm event at j^{th} maintenance district. X_{pij} are explanatory variables such as average temperature and average snow depth. α_p ($p = 0,1,...,P$, P is the number of explanatory variables) are the regression coefficients to be estimated. The error term ε_{ij} is assumed to follow a normal distribution with mean 0 and variance σ_{ε}^2.

<u>Model 2: hierarchical linear (HL) model</u>

The independence assumption of the linear model could be violated by possible spatial heterogeneity of salt usage data. The spatial heterogeneity can be attributed to district-specific unobserved factors such as road surface area, road priority and traffic volume. Those unobserved factors can not only affect salt usage directly but also the effects of explanatory variables (e.g. average temperature and storm duration) on salt usage. To account for the potential heterogeneity across homogeneous groups, hierarchical models, which allows coefficients to vary across different groups, have be used in previous studies *(22-25)*. The hierarchical modeling framework is used in this study due to its two advantages: 1) able to account for the spatial heterogeneity of salt usage across maintenance districts; and 2) able to make more reliable estimation when samples are not enough to develop a model for each maintenance district *(26)*. The HL model can be specified as:

$$\log(y_{ij}) = \alpha_{0j} + \sum_{p=1}^{P} \alpha_{pj} \quad {}_{pij} + \varepsilon_{ij} \tag{2}$$

$$\alpha_{pj} = \alpha_p + \kappa_j \tag{3}$$

α_{pj} ($p = 0,1,...,P$, P is the number of explanatory variables) are the random parameters to be estimated. Different from α_p in equation (1), random parameters α_{pj} are allowed to vary across maintenance district as shown in equation (3). κ_j is a normally distributed term with mean 0 and variance σ_{κ}^2. The error term ε_{ij} is assumed to follow a normal distribution with mean 0 and variance σ_{ε}^2.

<u>Model 3: hierarchical linear model with varying dispersion (HLVD)</u>

Different from the previous linear model and the HL model, error term ε_{ij} in the HLVD model is assumed to follow a normal distribution with mean 0 and $\sigma_{\varepsilon ij}^2$, where the dispersion (or residual variance) $\sigma_{\varepsilon ij}^2$ is allowed to vary across observations. In the HLVD model, fixed effects are included in the dispersion term:

$$\log(\sigma_{\varepsilon ij}^2) = \beta_0 + \sum_{q=1}^{Q} \beta_q Z_{qij} \tag{4}$$

where Z_{qij} are the variables having effects on the dispersion $\sigma_{\varepsilon ij}^2$ during ith storm event at jth maintenance district. β_q ($q = 0,1,...,Q$, Q is the total number of variables affecting the dispersion) are the regression coefficients to be estimated. Equations (2)-(4) construct the HLVD model.

Model Assessment

R-squared and its modified version adjusted R-squared that consider the number of explanatory variables used are usually used to measure the goodness-of-fit of models *(27)*. Additionally, another two measures, Mean Absolute Deviance (MAD) and Mean Squared Predictive Error (MSPE), are used to assess models' predictive performance *(22)*. MAD and MSPE are expressed as:

$$MAD = \frac{1}{N}\sum_{\forall i,j} \left| y_{ij} - \hat{y}_{ij} \right| \tag{10}$$

$$MSPE = \frac{1}{N}\sum_{\forall i,j} (y_{ij} - \hat{y}_{ij})^2 \tag{11}$$

where $\hat{y}_{ij}$ is the estimated amount of salt used during i^{th} storm event at j^{th} maintenance district, and N is the number of samples. Models associated with less MAD and MSPE have better predictive performance. MAD/Mean ratio, which is the MAD divided by the mean of salt usage $\frac{1}{N}\sum_{\forall i,j} y_{ij}$, is used to show the relative prediction errors.

MODELING RESULTS

The linear models, HL models and HLVD models specified in the methodology section were used to estimate the salt usage at NJT and GSP separately. To conduct effective comparisons, all the explanatory variables included in the six models were kept the same. In the HL and HLVD models, only if the estimated standard deviation (SD) of a random parameter was significantly positive and the inclusion of this random parameter would lead to better predictive performance, the parameter was allowed to vary randomly across maintenance districts. Consequently, the intercepts and the coefficients of average temperature, average snow depth and storm duration in the HL and HLVD models were set to be random parameters. The data from the 2011-2012, 2012-2013 and 2013-2014 winter seasons (in-sample) is used to calibrate the salt usage models, and the data from the 2014-2015 (out-of-sample) winter season is used to assess model's predictive performance. Models were compared using in-sample and out-of-sample testing, with results reported in Table 2.

Table 2 Comparisons of Model Performance

	NJT			GSP		
	Linear	HL	HLVD	Linear	HL	HLVD
R-squared	0.590	0.741	0.823	0.426	0.699	0.737
Adjusted R-squared	0.570	0.729	0.814	0.403	0.687	0.727
In-sample testing						
MAD	166.357	133.329	115.157	131.675	100.318	95.696
MAD/Mean	0.388	0.311	0.268	0.459	0.350	0.334
MSPE	56077	35400	24250	51643	27091	23646
Out-of-sample testing						
MAD	258.812	215.550	196.353	181.340	129.316	127.366
MAD/Mean	0.518	0.432	0.393	0.690	0.492	0.485
MSPE	132967	85633	76892	84610	45429	40351

According to the R-squared and adjusted R-squared shown in Table 2, the HL models (for both NJT and GSP) show substantial improvement over the linear models in terms of goodness-of-fit by allowing the intercepts and the coefficients of the logarithm of average temperature, average snow depth and storm duration to vary across maintenance districts. The goodness-of-fit get further improved in the HLVD models (for both NJT and GSP) by allowing the dispersion $\sigma^2_{\varepsilon ij}$ to be case-specific (can be estimated with average temperature, average snow depth and storm duration). In addition, the smallest values of MAD, MAD/Mean and MSPE of both in-sample testing and out-of-sample testing indicate that the HLVD models (for both NJT and GSP) have the best predictive performance.

The estimation results of the linear, HL and HLVD models are shown in Table 3, Table 4 and Table 5, respectively. To test the significance of explanatory variables, a widely used statistic indicator p-value was used. Most coefficients of explanatory variables were found to be statistically significant at 95% level (p-values<0.05) except the categorical variable *month* in the models for GSP (i.e., Table 3b, Table 4b and Table 5b). Compared with the linear model for GSP (Table 3b), the significance of the variable *month* gets improved in the HL (Table 4b) and HLVD (Table 5b) models when *average temperature*, *average snow depth* and *storm duration* are included as random parameters. According to the coefficient estimates in Table 3 , Table 4 and Table 5, it is found that *average snow depth* and *storm duration* are positively correlated with salt usage, while *average temperature* is negatively correlated with salt usage. This finding is consistent with the patterns presented in Figure 2. The quantitative impacts of those variables can be interpreted. For example, in the Table 5a, the coefficient of the logarithm of *average temperature* is -0.2796, which indicates that 1% increase of *average temperature* would lead to 0.2796% decrease in salt usage. In the Table 5b, the coefficient of *October* is -0.2469, implying that the salt usage in October is expected to be 21.88% ($1-e^{-0.2469}$) less than the salt usage in other months. In addition, as shown in Table 5, the effects of variables *average temperature*, *average snow depth* and *storm duration* on dispersion are found to be statistically significant. The dispersions in the linear (Table 3) and HL (Table 4) models are constant, and the dispersions (0.6150 and 0.3884) of the HL models are smaller than the dispersions (0.8602 and 0.7266) of the linear models, since over-dispersion is partially accounted for by random parameters in the HL models.

Table 3 Estimation Results of the Linear Models

(a) New Jersey Turnpike (NJT)

	Estimate	Std. Error	t value	p-value
Intercept	7.9047	0.8215	9.6230	< 0.0001
log(Average temperature)	-0.6160	0.2361	-2.6090	0.0095
log(Average snow depth)	0.2971	0.0240	12.3830	< 0.0001
log(Storm duration)	0.3844	0.1901	2.0230	0.0439
Month				
November	-0.8263	0.2549	-3.2420	0.0013
March	-0.4358	0.1556	-2.8010	0.0054
Others	0.0000	-	-	-
Dispersion σ_ε^2	0.8602	-	-	-

(b) Golden State Parkway (GSP)

	Estimate	Std. Error	t value	p-value
Intercept	6.9683	0.8424	8.2720	< 0.0001
log(Average temperature)	-0.5329	0.2417	-2.2050	0.0280
log(Average snow depth)	0.2701	0.0197	13.7350	< 0.0001
log(Storm duration)	0.7665	0.1431	5.3570	< 0.0001
Month				
October	-0.1564	0.1427	-1.0960	0.2740
November	-0.0964	0.2017	-0.4780	0.6330
January	-0.1300	0.1372	-0.9480	0.3440
February	-0.1415	0.1155	-1.2250	0.2210
March	-0.0842	0.1430	-0.5890	0.5570
Others	0.0000	-	-	-
Dispersion σ_ε^2	0.7266	-	-	-

Table 4 Estimation Results of the HL Models

(a) New Jersey Turnpike (NJT)

	Estimate	Std. Error	t value	p-value
Intercept	8.1247	0.7581	10.7170	< 0.0001
SD of parameter distribution=0.0232				
log(Average temperature)	-0.7051	0.2194	-3.2140	0.0014
SD of parameter distribution=0.0798				
log(Average snow depth)	0.2925	0.0353	8.2860	0.0000
SD of parameter distribution=0.0436				
log(Storm duration)	0.4085	0.1771	2.3070	0.0217
SD of parameter distribution=0.0160				
Month				
November	-0.8383	0.2348	-3.5710	0.0004
March	-0.4217	0.1429	-2.9510	0.0034
Others	0.0000	-	-	-
Dispersion σ_ε^2	0.6150	-	-	-

(b) Golden State Parkway (GSP)

	Estimate	Std. Error	t-value	p-value
Intercept	6.7551	0.7355	9.1840	< 0.0001
SD of parameter distribution=0.0232				
log(Average temperature)	-0.4825	0.2119	-2.2770	0.0234
SD of parameter distribution=0.0744				
log(Average snow depth)	0.2731	0.0276	9.9070	< 0.0001
SD of parameter distribution=0.0381				
log(Storm duration)	0.8458	0.1280	6.6080	< 0.0001
SD of parameter distribution=0.0238				
Month				
October	-0.2469	0.1250	-1.9760	0.0489
November	-0.3054	0.1762	-1.7340	0.0838
January	-0.1564	0.1200	-1.3030	0.1935
February	-0.1473	0.1005	-1.4650	0.1437
March	-0.1383	0.1257	-1.1000	0.2722
Others	0.0000	-	-	-
Dispersion σ_{ε}^{2}	0.3884	-	-	-

Table 5 Estimation Results of the HLVD Models

(a) New Jersey Turnpike (NJT)

	Estimate	Std. Error	t-value	p-value
Intercept	6.3699	0.3097	20.5720	<0.0001
SD of parameter distribution=0.0361				
log(Average temperature)	-0.2796	0.0870	-3.2140	0.0015
SD of parameter distribution=0.1079				
log(Average snow depth)	0.2820	0.0394	7.1570	< 0.0001
SD of parameter distribution=0.0471				
log(Storm duration)	0.8002	0.1452	5.5120	< 0.0001
SD of parameter distribution=0.0444				
Month				
November	-0.4920	0.1698	-2.8980	0.0040
March	-0.4268	0.0997	-4.2790	<0.0001
Others	0.0000	-	-	-
Dispersion Effects (link=log)				
Intercept	-10.4109	1.4498	-7.1809	<0.0001
log(Average temperature)	2.7833	0.4169	6.6762	<0.0001
log(Average snow depth)	-0.3249	0.0397	-8.1839	<0.0001
log(Storm duration)	-0.7204	0.3043	-2.3674	0.0090

(b) Golden State Parkway (GSP)

	Estimate	Std. Error	t-value	p-value
Intercept	6.8874	0.6082	11.3250	<0.0001
SD of parameter distribution=0.0246				
log(Average temperature)	-0.5449	0.1773	-3.0740	0.0023
SD of parameter distribution=0.0815				
log(Average snow depth)	0.2684	0.0280	9.5880	< 0.0001
SD of parameter distribution=0.0357				
log(Storm duration)	1.0012	0.1320	7.5870	< 0.0001
SD of parameter distribution=0.0241				
Month				
October	-0.2783	0.1181	-2.3570	0.0189
November	-0.2222	0.1632	-1.3620	0.1740
January	-0.2918	0.1170	-2.4940	0.0131
February	-0.1709	0.0926	-1.8460	0.0658
March	-0.1956	0.1151	-1.7000	0.0899
Others	0.0000	-	-	-
Dispersion Effects (link=log)				
Intercept	-3.9839	1.3247	-3.0074	0.0013
log(Average temperature)	0.9567	0.3891	2.4588	0.0070
log(Average snow depth)	-0.1274	0.0374	-3.4064	0.0003
log(Storm duration)	-0.7994	0.2804	-2.8509	0.0022

As mentioned previously, intercepts and coefficients of variables *average temperature*, *average snow depth* and *storm duration* in the proposed HLVD (Table 5) models are not fixed but follow certain distributions across maintenance districts. For example, the coefficient of the logarithm of the *average temperature* in the HLVD model for NJT is assumed to follow a normal distribution with mean -0.2796 and standard deviation (SD) 0.1079 *(see* Table 5a*).* The distributions of all the random parameters in the HLVD models are depicted in Figure 3. It is found that the 95% confidence intervals (CI) of those random variables do not cover 0, indicating that in most cases, the effects of those random variables are unidirectional (either positive or negative). For instance, an increase in average snow depth would lead to greater salty usage in most of maintenance districts. Among all the random parameters, the coefficients of the logarithm of the *average temperature* have the greatest variation (SDs of parameter distribution are 0.1079 in Table 5a and 0.0815 in Table 5b).

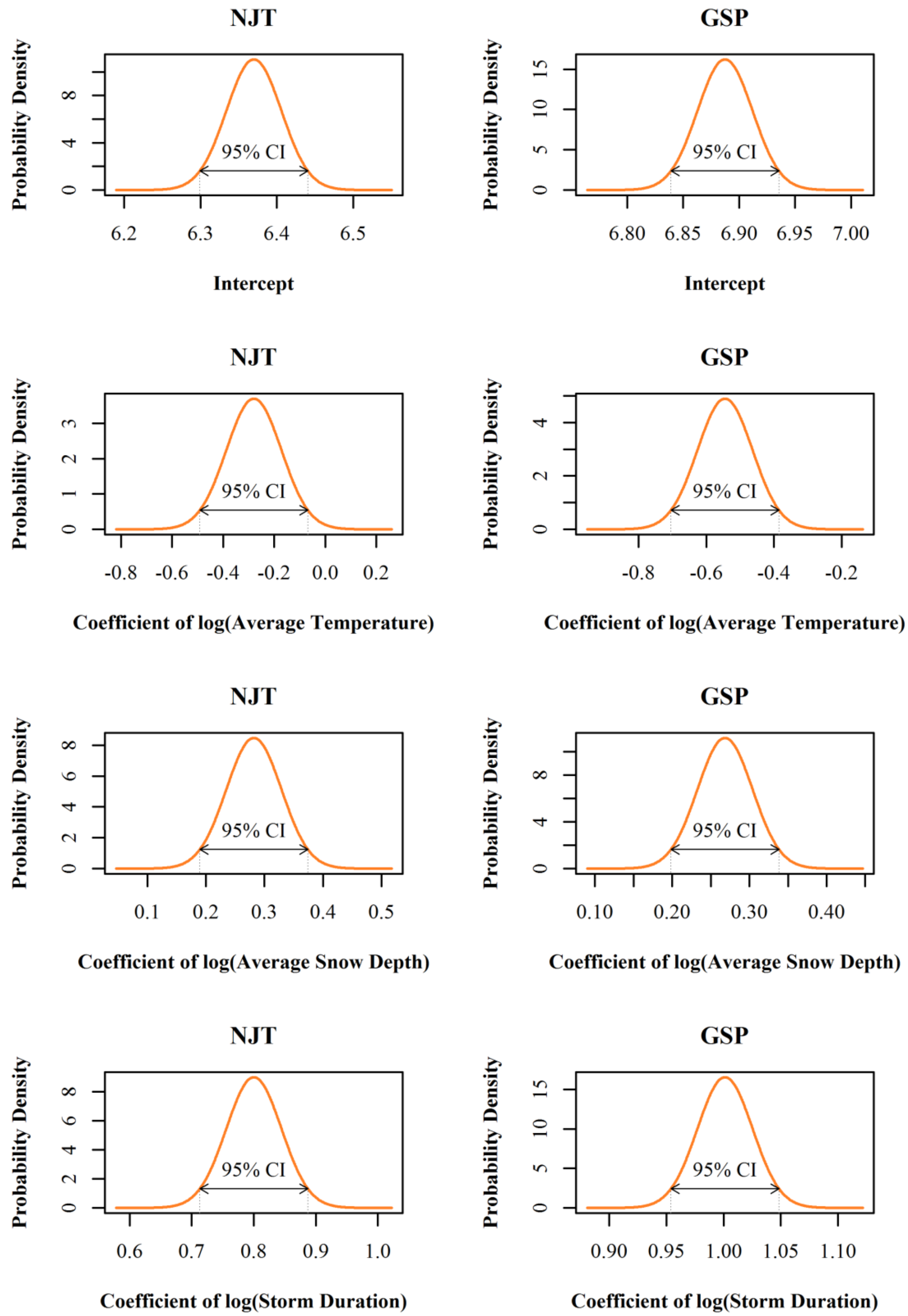


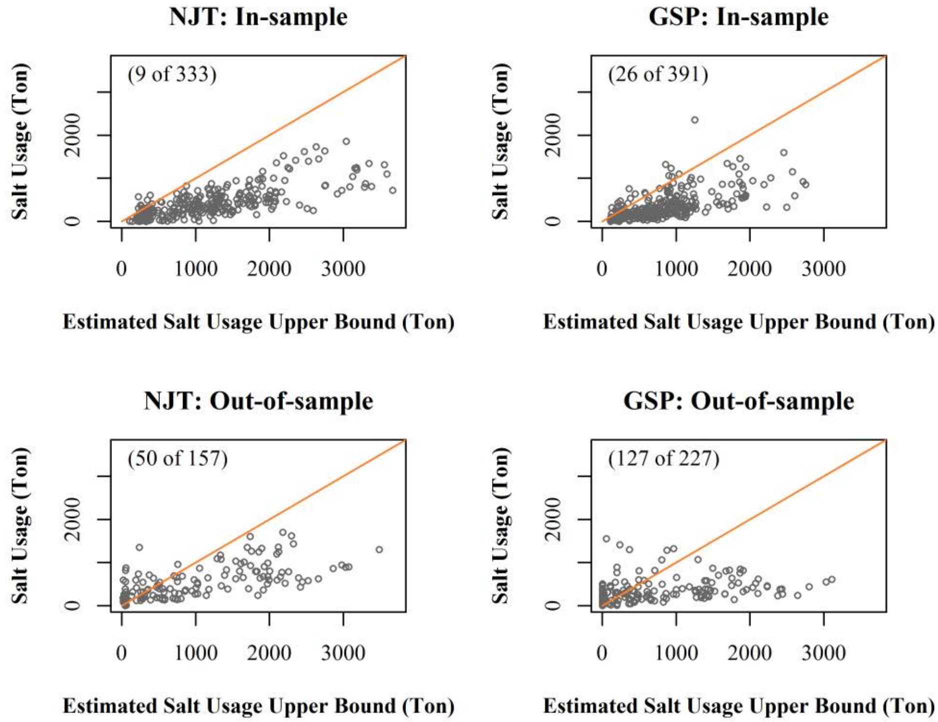
Figure 3 Probability densities and 95% confidence intervals (CI) of random parameters in the HLVD models.

PREDICTION OF THE UPPER BOUNDS OF SALT USAGE

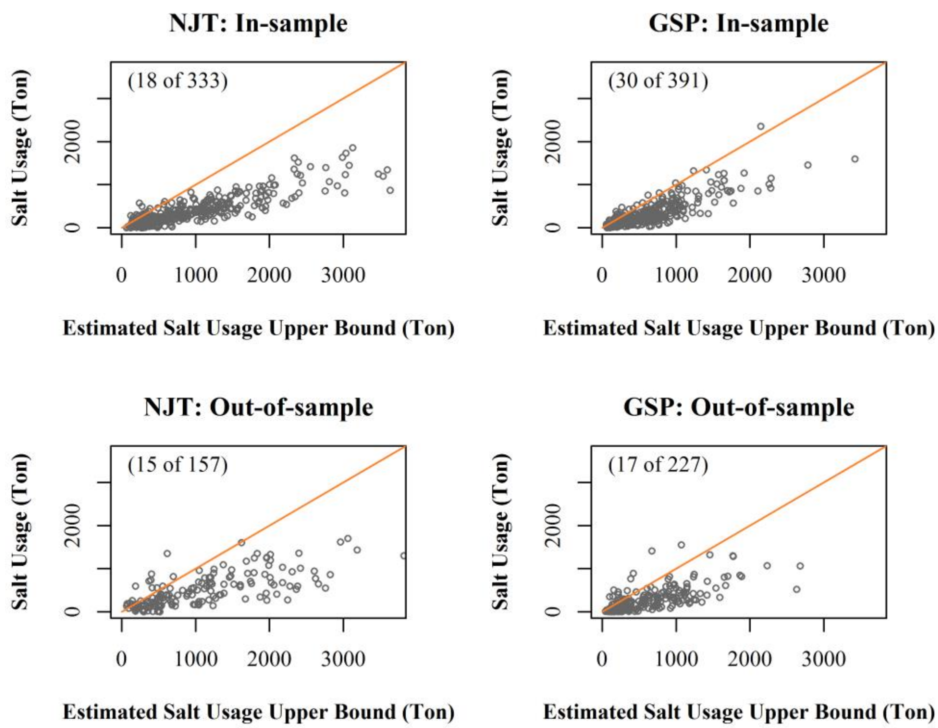
To ensure that the salt stored is sufficient in most cases, determination of the upper bound of salt usage is of great interest. The HLVD models, which can provide case-specific estimation of salt usage mean and dispersion (residual variance), was used to predict the upper bounds of salt usage. An $100(1-\alpha)\%$ upper confidence bound of salt usage is $y_{ij} + z_{\alpha}\sigma_{\varepsilon ij}$, where z_{α} is the z-score of a standard normal distribution, and y_{ij} and $\sigma_{\varepsilon ij}$ can be estimated in equations (2) and (4), respectively. In this study, we choose $\alpha = 0.1$ as an example to ensure that the estimated upper bound of salt usage is greater than the actual demand in 90% of the cases. For comparison purpose, the linear models and the HL models were also used to estimate the salt usage upper bounds from the equation $y_{ij} + z_{\alpha}\sigma_{\varepsilon}$, where σ_{ε} is constant for different cases.

In Figure 4, the horizontal axis of each subplot denotes the upper bound of salt usage estimated from salt usage models, and the vertical axis represents the actual salt usage. In each subplot, the data points above the diagonal line are cases when estimated salt usage upper bound is smaller than the actual salt use, and the number of those cases and the total number of cases are labelled in the parentheses at the top-left corner. For example, in the subplot "NJT: In-sample" of Figure 4a, the estimated salt usage upper bound is smaller than the actual salt usage in 9 out of 333 cases.

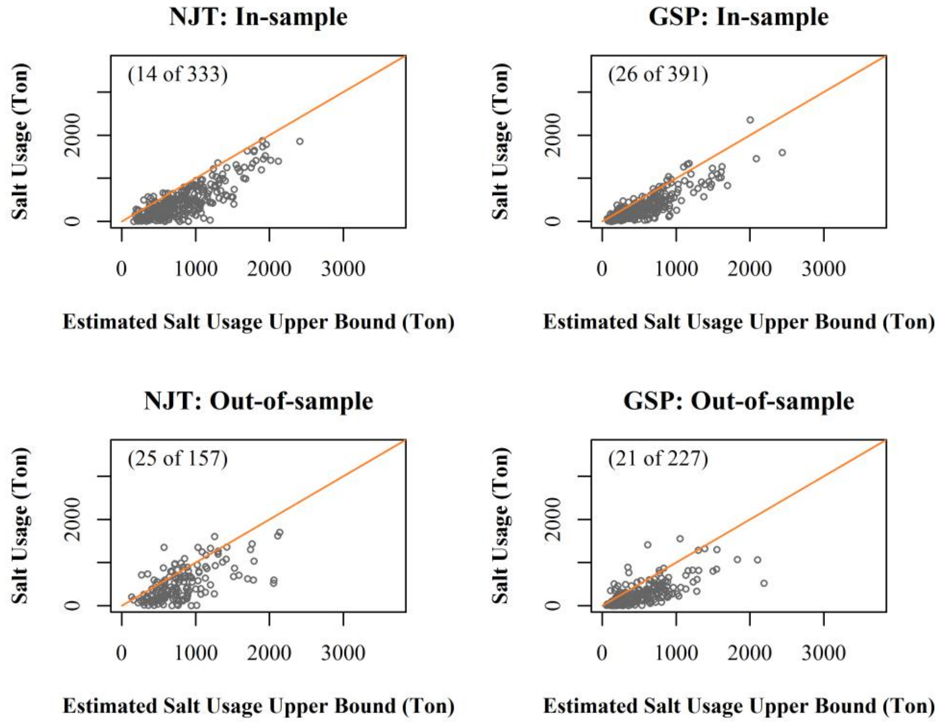
A good salt usage model should provide estimates of salt usage upper bounds which satisfy the demand of salt usage in most cases. However, as shown in Figure 4a, in the out-of-sample testing, the salt usage upper bounds estimated by the linear models are less than the actual salt usage in as many as 50 cases out of 157 for NJT and 127 cases out of 227 for GSP. In contrast, the salt usage upper bounds estimated by the HL models and the HLVD models could satisfy the demand in most cases for both in-sample and out-of-sample testing. On the other hand, a good salt usage model should avoid extremely large estimates of salt usage upper bounds. In Figure 4a and Figure 4b, both the linear models and the HL models provide estimates of salt usage upper bounds greater than 3000 tons, while the maximum of salt usage recorded for one district during one storm is about 2000 tons. However, as shown in Figure 4c, the HLVD models could prevent those extremely large estimates by adjusting the dispersion $\sigma_{\varepsilon ij}^{2}$ in specific cases according to the storm characteristics.



(a) Linear models



(b) HL models



(c) HLVD models

Figure 4 Estimated salt usage upper bound versus actual salt usage.

SUMMARY AND CONCLUSIONS

This study proposes novel salt usage prediction models which can account for the unobserved spatial heterogeneity and allow dispersion of residuals to vary. It can serve as a useful complement to the literature, since studies on salt usage prediction based on weather-related factors are rare. The proposed method can provide appropriate estimates of the means as well as the upper bounds of salt usage at the maintenance district level.

New Jersey Turnpike (NJT) and Golden State Parkway (GSP) are selected as a case study. Historical data on salt usage and storm characteristics is extracted from WeatherEVANT (Real-time Weather related Event Visualization and ANalytics Tool) *(2)* developed by the research team. The data from the 2011-2012, 2012-2013 and 2013-2014 winter seasons is used to estimate salt usage models, and the data from the 2014-2015 winter season is used for model validation. The linear models, the hierarchical linear (HL) models and the hierarchical linear models with varying dispersion (HLVD) are developed to predict the salt usage at NJT and GSP separately. HL models show substantial improvement over the linear models in terms of both in-sample and out-of-sample predictive performance by including random parameters which can vary across maintenance districts. The predictive performance of the HLVD models gets further improved by including fixed effects in the dispersion term. Results show that districts with higher average snow depth, longer storm duration and lower average temperature are associated with greater salt usage. In addition, the effects of variables average temperature, average snow depth and storm duration on dispersion are found to be statistically significant. The 95% confidence intervals of random variables included in the HLVD models do not cover 0, indicating that the effects of those random variables are unidirectional (either positive or negative) in most of the maintenance districts. Among all the random parameters, the coefficients of the logarithm of average temperature have the greatest variation.

Compared with the linear models, both the HL models and the HLVD models could give estimates of the upper bounds of salt usage that could satisfy salt usage demand in most cases for both in-sample and out-of-sample testing. Moreover, it is found that the HLVD models could prevent extremely large estimates of the upper bounds of salt usage by estimating case-specific dispersion based on storm characteristics.

The transferability of the proposed models can be tested once the data from other highways becomes available. It is likely that the proposed models couldn't achieve the same prediction accuracy for other highways, since the relationship between salt usage and contributing factors are location-specific. The effects of variables such as average temperature and average snow depth can vary greatly when confronting totally different environments. It is highly recommended to re-estimate the HLVD models to capture the local characteristics of other highways. However, this study identifies variables affecting salt usage and identifies best model specification for this type of data. Thus, other agencies can use these findings to estimate their site specific models without having to go all the steps we went through when estimating the models presented in the paper.

The findings of this paper can provide highway authorities in-depth understanding of the factors affecting salt usage and a robust method for salt usage estimation. Material replenishment decisions in the future snow storms can be made based on the expected means and upper bounds of salt usage. For the future study, additional variables affecting salty usage will be collected to improve the model performance, such as the roadway length, number of lanes and pavement condition. Furthermore, the potential of developing hierarchical nonlinear models that have greater flexibility to accommodate the data for salt usage prediction could be explored, when additional data is collected in the future. For more applications, real-time information on storm conditions and salt usage can be leveraged to assist agencies in developing more active, efficient and cooperative strategies in inventory management.

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