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Rehabilitation Project  
Selection and Scheduling in  
Transportation Networks



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# **REHABILITATION PROJECT SELECTION AND SCHEDULING IN TRANSPORTATION NETWORKS**

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## ABSTRACT

Highway project selection and scheduling are traditionally treated as two separate problems in the literature. It is critical to investigate how to select and schedule M&R projects in a way that can maximize their benefit or effectiveness while minimizing the traffic impacts of work zones across project development phases. There is a pressing need to develop an integrated framework for simultaneous selection and scheduling of multiple M&R projects at the network level. Among various types of M&R projects, road capacity expansion is the one that requires massive resources and takes a long time to complete. Therefore, this study focuses on the project selection and scheduling for road capacity expansion projects. In this study, we introduce time dimension into the traditional discrete network design problem (DNDP) to explicitly consider the impact of road construction work and adopt an overtime policy to add flexibility to construction duration. We address the problem of selecting road-widening projects from several candidate projects in an urban road network, determining the optimal link capacity and designing the schedules of the selected projects simultaneously. A time-dependent DNDP (T-DNDP) model is developed with the objective of minimizing total weighted net user cost during the entire planning horizon. An active-set algorithm is applied to solve the model. To demonstrate the practicability of the proposed model, two case studies are developed to demonstrate the necessity of considering the construction process in T-DNDP and to illustrate the trade-off between the construction impact and the benefit realized through capacity extension.

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# 1. INTRODUCTION

Road infrastructure in the United States is aging rapidly as many roads are approaching or exceeding their design life. As a result, transportation agencies need to allocate more resources to maintenance and rehabilitation (M&R) activities. The National Highway System (NHS) spent 48.5% of its total capital 2008 spending in system rehabilitation, the highest percentage since 2000 (FHWA, 2010). On the other hand, stringent budgets provide insufficient funding to support all needed M&R projects. Decision makers have to prioritize and select projects based on their tangible benefits to the transportation system. Meanwhile, traffic congestion across the country has been on the rise over the past 30 years by every measure (TTI, 2012). The problem is further exacerbated by an increasing number of M&R projects performed on already congested roads. Work zones are estimated to account for nearly 24% of non-recurring delay on freeways (USDOE, 2002). Hence, M&R project selection and scheduling are not only essential to restore and maintain a reasonable level of service on existing roads, but also have a profound impact on congestion mitigation.

Highway project selection and scheduling are traditionally treated as two separate problems in the literature. It is critical to investigate how to select and schedule M&R projects in a way that can maximize their benefit or effectiveness while minimizing the traffic impacts of work zones across project development phases. There is a pressing need to develop an integrated framework for simultaneous selection and scheduling of multiple M&R projects at the network level.

This goal of this study is to develop a systems approach for selecting and scheduling M&R projects simultaneously. The proposed modeling framework will accomplish the following two objectives:

1. Explicitly capture the impacts of the presence of multiple M&R projects on travelers' route choice behavior.
2. Strategically select and schedule M&R projects in a transportation network over a finite planning horizon to maximize social benefit.

Among various types of M&R projects, road capacity expansion is the one that requires massive resources and takes a long time to complete. Therefore, this study focuses on the project selection and scheduling for road capacity expansion projects. That being said, the modeling framework and solution algorithm developed in this study are capable of modeling the selection and scheduling of other types of M&R projects.

The selection of road capacity expansion projects in a transportation network is usually referred to in the literature as the network design problem (NDP). Over the past few decades, NDP has been widely studied. Most of the literature related to NDP has focused either on modeling or new algorithms for network design models. However, these early studies regarded road construction work as a one-time event and did not consider the gradual improvement of the network until researchers introduced the time dimension to the traditional NDP (Friesz et al., 1994, 1996; Lo and Szeto, 2004). Lo and Szeto (2004) claimed that the road network is improved yearly before the completion of the improvement project, which makes the NDP model more realistic. Nevertheless, even though they considered network improvement to be gradual in their model, they still assumed the construction process to be a one-off procedure. Actually, capacity expansion work usually involves work zones and lane closures, which may reduce the current link capacity during construction and result in congestion and delays for road users. Furthermore, road infrastructure construction generally lasts for months or even years, and the impact of construction may greatly affect planners' decisions. For example, when multiple projects are simultaneously underway, planners may choose to adjust the schedule of some projects to avoid excessive delays in a region. Therefore, the impact of construction work should not simply be ignored.



This study explicitly considers construction impact in conjunction with the benefits brought about by capacity expansion as the two primary factors that govern the network design problem. Furthermore, in light of the fact that the construction process may have a tremendous impact on the road network, shortening the construction period represents a possible method for mitigating the impact. Thus, the proposed model also allows the construction period to be flexible, which means the planners can choose to speed up construction to shorten its duration by paying overtime to construction personnel.

Compared with existing NDP models, the proposed model has the following advantages:

- 1) The construction impact is clearly evaluated so that the selection and schedule of road infrastructure projects will be optimized.
- 2) This model adopts an overtime policy in the candidate projects, which allows planners to choose whether or not to accelerate a project by paying overtime. Thus, the construction duration of the candidate projects is flexible.
- 3) This model is able to address the problem of selecting road-widening projects from several candidate projects, simultaneously determining the optimal amount of increased capacity and designing the optimal schedule for the chosen projects.

## 2. BACKGROUND

### 2.1 Network Design Problem

The transportation NDP aims to achieve certain objectives, such as reducing traffic congestion, energy consumption, and environmental pollution, by choosing improvements or additions to an existing network (Abdulaal and LeBlanc, 1979). A common methodology used to formulate the NDP is bi-level programming. The upper level is the system level, which optimizes the system benefits subject to limited resources, while the lower level is the users' level, which models users' route choice behavior in the network. The upper level can be formulated with different decision variables and objective functions. The decision variables can be merely continuous or discrete, or can contain both continuous and discrete elements. Based on the types of decision variables, network design problems are generally divided into three categories. The network design problem with only continuous variables is called the continuous network design problem (CNDP) (Dantzig et al., 1979; Aashtiani and Magnanti, 1981; Suwansirikul et al., 1987; Friesz et al., 1992; Meng et al., 2001; Meng and Yang, 2002). In road network design problems, continuous variables are usually introduced in order to simplify computation. For example, the capacity expansion of a roadway can be continuous (Lo and Szeto, 2004; Yin and Lawphongpanich, 2007). However, continuous variables do not necessarily indicate the changes that are practical, because road capacity is normally measured by the number of lanes. Hence, despite the fact that it may be more computationally expensive, the discrete network design problem (DNDP) with solely discrete variables (see, Steenbrink, 1974; Leblanc, 1975; Chen and Alfa, 1991; Lee and Yang, 1994; Drezner and Wesolowsky, 1997, 2003; Poorzahedy and Abulghasemi, 2005; Gao et al., 2005; Meng and Khoo, 2008), and the mixed network design problem (MNDP) with both continuous and discrete variables (Cantarella et al., 2006; Cantarella and Vitetta, 2006; Gallo et al., 2010; Luathep et al., 2011) are still worth investigating.

Previous studies have made substantial contributions to the understanding and applications of DNDP. Some have studied various applications associated with DNDP. For instance, Drezner and Wesolowsky (1997) formulated a DNDP for the purpose of selecting the best distribution of one-way and two-way routes in a road network. Lam and KS (2005) solved the DNDP of choosing the location of pedestrian-only streets in a multi-modal network. Song et al. (2015) developed a DNDP model that settled the problems of selecting locations for high-occupancy vehicle (HOV) and high-occupancy toll (HOT) lanes and determining toll rates on HOT lanes. Liu and Song (2018a) proposed a DNDP model to determine the deployment of dynamic charging lanes for hybrid electric trucks. Miandoabchi and Farahani (2011) determined street orientations and expansions, as well as lane allocations, based on the reserve capacity concept in a DNDP model. The problem of deploying autonomous vehicle and autonomous vehicle/toll lanes is also formulated as a DNDP model in Liu and Song (2018b). Others have developed different kinds of approaches to solve DNDP. It is well known that solving a bi-level network design problem is very difficult because the problem is NP-hard and non-convex. After LeBlanc (1975) proposed a branch-and-bound algorithm to solve this bi-level problem, many researchers began to seek better approaches to assess the trade-off between computation of speed and solution accuracy. For example, Dantzig et al. (1979) transformed the non-convex programming problem to a convex problem using system equilibrium flow to replace user equilibrium flow. Poorzahedy and Turnquist (1982) utilized approximation to transform the bi-level problem into a single-level problem. Solanki et al. (1998) decomposed the highway network design problem in a sequence of small sub-problems and limited the search using heuristics to reduce computation time. Poorzahedy and Abulghasemi (2005) adapted meta-heuristic algorithms to solve NDP for the Sioux Falls network. Poorzahedy and Rouhani (2007) improved the meta-heuristic algorithm and designed the hybrid meta-heuristic algorithm. A genetic algorithm is also widely used (Drezner and Wesolowsky, 2003; Yin, 2000; Jeon et al., 2006). Gao et al. (2005) transformed the upper-level programming of the traditional DNDP to a nonlinear problem based on the support function concept.

Zhang et al. (2009) developed the active-set algorithm, which eliminates complementary constraints in the DNDP by assigning initial values and solving binary knapsack problems. Farvaresh and Sepehri (2013) revised the branch-and-bound algorithm proposed by LeBlanc (1975) for bi-level DNDP.

## 2.2 Time-Dependent Network Design Problem

In recent years, the time varying evolution of road networks began to gain interest in transportation network design problems. Different time scales were studied in the literature, ranging from the smallest day-to-day dynamics (Friesz et al., 1994, 1996; Friesz and Shah, 2001) to network upgrades spanning many years (Szeto and Lo, 2006, 2008; O'Brien and Yuen, 2007. Lo and Szeto (2004) introduced the time dimension to CNDP and built a comprehensive and practical model that considered not only user equilibrium (UE), but also travel demand and land-use patterns as time dependent. In conjunction with other researchers, they further studied a series of time-dependent NDP problems, including the following:

- budget sensitivity analysis among users, private toll road operators, and the government (Hong and Szeto, 2003)
- the trade-off between the social and financial aspects of three possible network improvement strategies under demand and the value of time uncertainty (Szeto and Lo, 2005)
- the trade-off between social benefit and intergeneration equity (Szeto and Lo, 2006)
- cost recovery issues over time (O'Brien and Yuen, 2007; Lo and Szeto, 2009)
- land-use transport interaction over time (Szeto et al., 2010)
- sustainability with land-use transport interaction over time (Szeto et al., 2015)
- health impacts attributable to network construction (Jiang and Szeto, 2015)
- a multi-objective time-dependent model to determine the sequence of link expansion projects and link construction projects (Miandoabchi et al., 2015)

Time dimension was also introduced in other studies. For instance, Kim et al. (2008) formulated a time-dependent DNDP framework to address the project scheduling problem, Ukkusuri and Patil (2009) developed a multi-period flexible network design model with demand uncertainty and demand elasticity, and Hosseininasab and Shetab-Boushehri (2015) integrated project selection and scheduling into a single time-dependent DNDP model.

However, in the literature referenced above, the road network is optimized for a certain future time without considering the construction impact. In practice, modifications to a network are gradual processes rather than one-off events. Hence, the construction process, which results in a negative impact to traffic, should also be considered. The construction process is explicitly modeled in this study.

### 3. BASIC CONSIDERATIONS

This study considers the problem of simultaneously determining the selection and scheduling of road expansion projects for a transportation network. The evaluation of a design is based on system performance throughout a given planning horizon, which includes the construction process. Below, we summarize our basic considerations and assumptions for the modeling and analysis of the construction process of road expansion projects.

1. Within the planning horizon, a road segment has at most one expansion project. This consideration is not overly restrictive, as we can always divide a road segment into several parallel links and assign each link with a project.
2. The construction procedure of an expansion project spans a continuous period of time.
3. Throughout the planning horizon, the route choice behaviors of drivers in the network follow the UE principle. Considering the construction process, the traffic network will change, as will the UE pattern.
4. The potential demand growth over time is known.
5. The interest and inflation rates are constant within the planning horizon.

For the convenience of readers, below we list some notations frequently used in the study.

#### Sets

$N$	Set of nodes
$L$	Set of links
$L_1$	Set of links with a potential expansion project
$L_2$	Set of links without a potential expansion project
$W$	Set of O-D pairs

#### Parameters

$a$	Link $a = (i, j) \in L$
$w$	O-D pair $w \in W$
$M$	The total number of unit time intervals for the planning horizon
$M^C$	The total number of unit time intervals for the construction time window
$m$	Time interval $m \in \{1, 2, \dots, M\}$
$d_m^w$	Travel demand between O-D pair $w \in W$ in time interval $m \in \{1, 2, \dots, M\}$
$D_a^0$	Fixed time cost for the expansion project on link $a \in L_1$
$D_a^1$	Variable time cost per additional lane for the expansion project on link $a \in L_1$
$c_a$	Average cost per time interval during construction for the expansion project on link $a \in L_1$ without overtime work

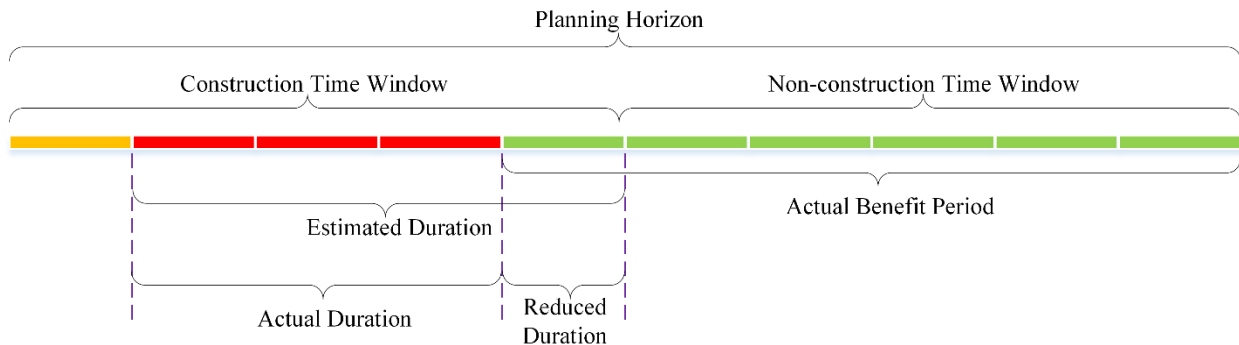
## Variables

$x_{a,m}^w$	Traffic flow on link $a$ for O-D pair $w \in W$ in time interval $m \in \{1,2,\dots,M\}$
$v_{a,m}$	Aggregate traffic flow on link $a \in L$ in time interval $m \in \{1,2,\dots,M\}$
$t_{a,m}$	Travel time of link $a \in L$ in time interval $m \in \{1,2,\dots,M\}$
$C_{a,m}$	Capacity of link $a \in L$ in time interval $m \in \{1,2,\dots,M\}$
$y_{a,m}$	A binary variable, representing whether link $a \in L_1$ is under construction in time interval $m \in \{1,2,\dots,M\}$ . If yes, $y_{a,m} = 1$ ; otherwise, $y_{a,m} = 0$
$z_{a,m}$	A binary variable, representing whether construction has been finished on link $a \in L_1$ in time interval $m \in \{1,2,\dots,M\}$ . If yes, $z_{a,m} = 1$ ; otherwise, $z_{a,m} = 0$
$S_{a,m}$	A binary variable, representing whether time interval $m \in \{1,2,\dots,M\}$ is the start date of construction on link $a \in L_1$ . If yes, $S_{a,m} = 1$ ; otherwise, $S_{a,m} = 0$
$E_{a,m}$	A binary variable, representing whether time interval $m \in \{1,2,\dots,M\}$ is the end date of construction on link $a \in L_1$ . If yes, $E_{a,m} = 1$ ; otherwise, $E_{a,m} = 0$
$l_a$	Number of newly added lanes on link
$D_a^e$	The estimated construction duration for the expansion project on link $a \in L_1$ without overtime work
$D_a^r$	Reduced construction duration for the expansion project on link $a \in L_1$ through overtime work

## 4. PROBLEM FORMULATION

Consider a general transportation network  $G(N, L)$ , where  $N$  and  $L$  are the set of nodes and the set of directed links, respectively. The latter are represented as a node pair  $(i, j)$ , where  $i, j \in N$  and  $i \neq j$ , or a single letter  $a$ . There are two types of links in the network: the links with a potential road-widening project, and the links without a potential project, denoted as  $L_1$  and  $L_2$ , respectively. In this study, the planning horizon  $[0, T]$  is equally divided into  $M$  unit intervals. The unit interval could be a month, a season, or another reasonable time interval. Note that the unit interval is the unit of measurement of the time cost of the construction process. The planning horizon includes a construction time window and a non-construction time window. All construction projects are supposed to be completed within the construction time window; the non-construction time window is designed to evaluate the continuing benefits realized through the finished road expansion projects. Planners determine the lengths of these two time windows. Approximately, the duration of the non-construction time window represents the service life of the improved roads before requiring extensive renovation. For an individual project, the benefit period begins immediately after the completion of the project. Therefore, the benefit period should be at least as long as the non-construction time window. Let  $M^C$  denote the number of intervals in the construction time window,  $M^C < M$ .

Figure 4.1 shows an example of the timeline of one road expansion project. In this example, the planning horizon is divided into 10 intervals, among which the former five intervals belong to the construction time window, and the latter five intervals belong to the non-construction time window. This project is scheduled to start at the beginning of the second time interval, and the estimated construction duration is four unit intervals. The planner decides to shorten the construction duration by one interval through overtime work. Therefore, the actual construction duration is reduced to three unit intervals, and the benefit lasts for six unit intervals (the detailed description of the flexible construction duration will be presented in the following model).



**Figure 4.1** An Example of the Timeline of a Road Expansion Project

### 4.1 Time-Dependent Traffic Assignment Constraints

#### 4.1.1 Feasible Region

To describe the feasible flow distributions of a network, let  $A$  be the node-arc incidence matrix associated with the network, and  $E^w$  be an “input-output” vector indicating the origin and destination of O-D pair  $w$ .  $E^w$  has exactly two non-zero components: one has the value 1 corresponding to the origin node of the O-D pair  $w$ , and the other’s value is -1, corresponding to the destination node. For all other nodes in this O-D pair,  $E^w$  equals 0. The flow distributions are said to be feasible if and only if the following constraints hold for  $x_m^w$ :

$$\begin{aligned}
Ax_m^w &= E^w d_m^w & \forall w \in W, m \in \{1, 2, \dots, M\} & \quad (1) \\
x_m^w &\geq 0 & \forall w \in W, m \in \{1, 2, \dots, M\} & \quad (2) \\
v_m &= \sum_w x_m^w & \forall m \in \{1, 2, \dots, M\} & \quad (3)
\end{aligned}$$

where  $x_m^w \in R^{|L|}$  is a vector whose components,  $x_{a,m}^w$ , represent a link flow on link  $a$  for O-D pair  $w$  in interval  $m$ , and  $v_m$  is a vector whose components,  $v_{a,m}$ , represent an aggregate link flow on link  $a$  in interval  $m$ .  $d_m^w$  represents the travel demand between O-D pair  $w$  in interval  $m$ . For simplicity, the travel demand of each O-D pair is assumed to be increasing at a constant rate. For an O-D pair  $w \in W$ , given the travel demand in the first interval, i.e.,  $d_1^w$ , the demand in interval  $m \in \{1, 2, \dots, M\}$  is calculated as:

$$d_m^w = d_1^w \cdot (1 + \varepsilon^w)^{m-1} \quad \forall w \in W, m \in \{1, 2, \dots, M\} \quad (4)$$

where  $\varepsilon^w$  is the growth factor of demand between O-D pair  $w$ .

To make the subsequent expressions more easily discernable, we introduce a set  $V_m^F$  for each period  $m$  to cover all of the feasible flow distributions:

$$V_m^F = \left\{ v_m : v_m = \sum_w x_m^w, Ax_m^w = E^w d_m^w, x_m^w \geq 0, \forall w \in W \right\} \quad \forall m \in \{1, 2, \dots, M\} \quad (5)$$

#### 4.1.2 Time-Dependent Link Capacity

Within the planning horizon, if a link is selected for expansion, its capacity will be time-dependent. During construction, the capacity of a link may be reduced due to the impact of construction; after construction, the capacity of a link will be improved due to added lanes. Two binary variables,  $y_{a,m}$  and  $z_{a,m}$ , are introduced to indicate the status of a link  $a \in L_1$  with a potential widening project in time interval  $m \in \{1, 2, \dots, M\}$ .  $y_{a,m}$  represents whether link  $a \in L_1$  is under construction in time interval  $m \in \{1, 2, \dots, M\}$ . If yes,  $y_{a,m} = 1$ ; otherwise,  $y_{a,m} = 0$ .  $z_{a,m}$  represents whether construction has been finished on link  $a \in L_1$  in time interval  $m \in \{1, 2, \dots, M\}$ . If yes,  $z_{a,m} = 1$ ; otherwise,  $z_{a,m} = 0$ . Note that if link  $a \in L_1$  is not selected for expansion, there will be no construction process on link  $a$ , and  $y_{a,m} = 0, z_{a,m} = 0, \forall m \in \{1, 2, \dots, M\}$ . The time-dependent capacity function can be formulated in equations (6)-(8):

$$C_{a,m} = C_a^0 - y_{a,m} \cdot C_a^r + z_{a,m} \cdot l_a \cdot C_a^1 \quad \forall a \in L_1, m \in \{1, 2, \dots, M\} \quad (6)$$

$$C_{a,m} \leq C_a^{max} \quad \forall a \in L_1, m \in \{1, 2, \dots, M\} \quad (7)$$

$$C_{a,m} = C_a^0 \quad \forall a \in L_2, m \in \{1, 2, \dots, M\} \quad (8)$$

where  $C_a^0$ ,  $C_a^r$ ,  $C_a^1$ , and  $C_a^{max}$  are the initial capacity, the reduced capacity during construction, the capacity of a single lane, and the maximum allowable capacity of link  $a$ , respectively.  $l_a$  denotes the number of lanes added after construction, which is a decision variable to be optimized in our model.  $l_a$  is an integer variable. Equation (7) restricts the capacity of a link to be less than its maximum allowable capacity.

### 4.1.3 Travel Time

In this study, the Bureau of Public Roads (BPR) function is used to define the link travel time. The travel time of an existing link in each period,  $t_{a,m}$ , is determined by the link travel flow,  $v_{a,m}$ , and the link capacity,  $C_{a,m}$ .

$$t_{a,m} = t_a^0 \left[ 1 + 0.15 \left( \frac{v_{a,m}}{C_{a,m}} \right)^4 \right] \quad \forall a \in L, m \in \{1, 2, \dots, M\} \quad (9)$$

where  $t_a^0$  is the free flow travel time on link  $a$ .

### 4.1.4 User Equilibrium Assignment

For each time interval  $m \in \{1, 2, \dots, M\}$ , the user's route choice behavior is assumed to follow Wardrop's first principle (Wardrop, 1952), which is ensured by:

$$\begin{aligned} & \text{Min}_{(v_m)} \quad \sum_{a \in L} \int_0^{v_{a,m}} t_{a,m}(\omega) d\omega, \\ & \text{s. t} \quad \quad \quad v_m \in V_m^F \\ & \quad \quad \quad \text{Definitional constraints (6), (8), (9)} \end{aligned} \quad (10)$$

The KKT conditions of this user equilibrium model are shown as follows:

$$t_{i,j,m}(v_{i,j,m}, y_{i,j,m}, z_{i,j,m}, l_{i,j}) - (\rho_{i,m}^w - \rho_{j,m}^w) \geq 0 \quad \forall w \in W, (i,j) \in L, m \in \{1, 2, \dots, M\} \quad (11)$$

$$x_{i,j,m} [t_{i,j,m}(v_{i,j,m}, y_{i,j,m}, z_{i,j,m}, l_{i,j}) - (\rho_{i,m}^w - \rho_{j,m}^w)] = 0 \quad \forall w \in W, (i,j) \in L \quad (12)$$

where the multipliers  $\rho_{i,m}^w$  and  $\rho_{j,m}^w$  are associated with equation (1) and are called "node potentials" (Ahuja, 2017).

## 4.2 Time-Dependent Construction Constraints

### 4.2.1 Design Constraints with Flexible Construction Duration

In practice, for each expansion project, the workload can be estimated based on the planner's experience. We assume the normal working hours per day are fixed, for example, eight hours, and the work efficiency of a crew team is stable. The construction duration for a project can then be roughly estimated according to the workload of that project. The estimated construction duration, denoted as  $D_a^e$ , can be expressed as a function of the number of newly added lanes  $l_a$ , given by:

$$D_a^e = f^a(l_a)$$

In this model, we assume that  $D_a^e$  is linearly related to  $l_a$  for simplicity. Other functional forms can be adopted in our model framework without difficulty:

$$D_a^e = D_a^0 + D_a^1 \cdot l_a \quad \forall a \in L_1 \quad (13)$$

$$l_a \in \mathbb{Z} \quad \forall a \in L_1 \quad (14)$$



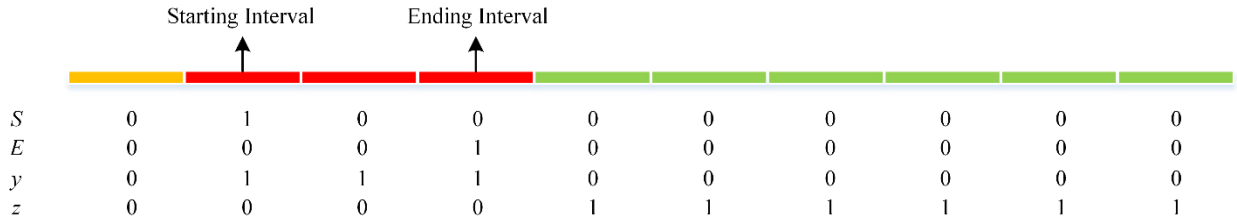
where  $D_a^0$  represents the fixed time cost of the project on link  $a$  regardless of how many lanes are added, e.g., the required time for construction preparation and quality control, and  $D_a^1$  denotes the extra time cost for each additional lane.

In practice, planners may choose to pay extra money for overtime work to accelerate a project if necessary. In this study, we introduce an integer variable,  $D_a^r$ , to denote the reduced component of the construction duration. The actual duration for the project on link  $a$  should then be  $D_a^e - D_a^r$ . Even though overtime work can speed up the process, project duration cannot be infinitely shortened. Let  $D_a^{max}$  denote the maximum allowable shortened duration for a project on link  $a$ .

$$D_a^r \leq D_a^{max} \quad \forall a \in L_1 \quad (15)$$

$$D_a^r \in \mathbb{Z} \quad \forall a \in L_1 \quad (16)$$

Within the planning horizon, the construction process on link  $a \in L_1$  should be a continuous period of unit intervals. To properly model the timeline of the construction process, we introduce two additional binary variables,  $S_{a,m}$  and  $E_{a,m}$ .  $S_{a,m} = 1$  implies that the construction process on link  $a$  starts at the beginning of interval  $m$ , and  $S_{a,m} = 0$  otherwise.  $E_{a,m} = 1$  implies that the construction process on link  $a$  ends by the end of interval  $m$ , and  $E_{a,m} = 0$  otherwise. Note that if a link  $a \in L_1$  is not selected for expansion, there will be no construction process on link  $a$ , and  $S_{a,m} = 0, E_{a,m} = 0, \forall m \in \{1, 2, \dots, M\}$ . Moreover, there should be only one starting time and one ending time for each chosen project. As shown in Figure 4.2, we use the same road expansion project used in Figure 4.1 to illustrate the values of  $S_{a,m}, E_{a,m}, y_{a,m}, z_{a,m}$  throughout the entire planning horizon.



**Figure 4.2** An Example to Illustrate the Values of  $S_{a,m}, E_{a,m}, y_{a,m}, z_{a,m}$  throughout the Entire Planning Horizon

Variables  $S_{a,m}, E_{a,m}, y_{a,m}, z_{a,m}$  are not mutually independent. Based on their definitions and the fact that they are all binary variables, the relationships among them can be specified by a series of conditional constraints. Let the construction time window be  $[1, M^C]$ . Subsequently, the non-construction time window is  $[M^C + 1, M]$ . This yields the following constraints:

$$S_{a,m}, E_{a,m}, y_{a,m}, z_{a,m} \in \{0, 1\}, \quad \forall a \in L_1, m \in \{1, 2, \dots, M\} \quad (17)$$

$$S_{a,m}, E_{a,m}, y_{a,m} = 0 \quad \forall a \in L_1, m \in \{M^C + 1, \dots, M\} \quad (18)$$

$$z_{a,m} = z_{a, M^C + 1} \quad \forall a \in L_1, m \in \{M^C + 2, \dots, M\} \quad (19)$$

Equation (17) requires variables  $S_{a,m}, E_{a,m}, y_{a,m}$  and  $z_{a,m}$  to be binary. Equation (18) ensures that no project can start, end, or be under construction in the non-construction time window. Equation (19) ensures that the completion status of the potential expansion project on link  $a \in L_1$  will not change in the non-construction time window.

The logical relationship between  $S_{a,m}$  and  $y_{a,m}$  can then be given by the following conditional constraints:

$$S_{a,m} \leq y_{a,m} \quad \forall a \in L_1, m \in \{1,2, \dots, M^C\} \quad (20)$$

$$S_{a,m} \leq 1 - y_{a,m-1} \quad \forall a \in L_1, m \in \{2,3, \dots, M^C\} \quad (21)$$

$$S_{a,m} \geq y_{a,m} - y_{a,m-1} \quad \forall a \in L_1, m \in \{2,3, \dots, M^C\} \quad (22)$$

Equation (20) ensures that if the project on link  $a$  is to start at time interval  $m$ , i.e.,  $S_{a,m} = 1$ , this project must be under construction at interval  $m$ , i.e.,  $y_{a,m} = 1$ . Equation (21) guarantees that if link  $a$  is under construction at time interval  $m - 1$ , i.e.,  $y_{a,m-1} = 1$ , it cannot start at time interval  $m$ , i.e.,  $S_{a,m} = 0$ . Equation (22) ensures that if the project on link  $a$  is not under construction at interval  $m - 1$  and is under construction at time interval  $m$ , i.e.,  $y_{a,m} = 1, y_{a,m-1} = 0$ , then interval  $m$  must be the starting time of the project, i.e.,  $S_{a,m} = 1$ . These three equations cover all possible relationships between  $S_{a,m}$  and  $y_{a,m}$ .

Similarly, the relationships between  $E_{a,m}$  and  $y_{a,m}$  are specified by the following constraints:

$$E_{a,m} \leq y_{a,m} \quad \forall a \in L_1, m \in \{1,2, \dots, M^C\} \quad (23)$$

$$E_{a,m} \leq 1 - y_{a,m+1} \quad \forall a \in L_1, m \in \{1,2, \dots, M^C - 1\} \quad (24)$$

$$E_{a,m} \geq y_{a,m} - y_{a,m+1} \quad \forall a \in L_1, m \in \{1,2, \dots, M^C - 1\} \quad (25)$$

Equation (23) means that if the project on link  $a$  is to end at time interval  $m$ , i.e.,  $E_{a,m} = 1$ , this project must be under construction at interval  $m$ , i.e.,  $y_{a,m} = 1$ . Equation (24) ensures that if the project on link  $a$  is to end at time interval  $m$ , i.e.,  $E_{a,m} = 1$ , this project cannot be under construction at time interval  $m + 1$ , i.e.,  $y_{a,m+1} = 0$ . Additionally, equation (25) guarantees that if the project on link  $a$  is under construction at interval  $m$  and is no longer under construction at time interval  $m + 1$ , i.e.,  $y_{a,m} = 1, y_{a,m+1} = 0$ , then interval  $m$  must be the ending time of the project, i.e.,  $E_{a,m} = 1$ .

Likewise, the logical relationships between  $E_{a,m}$  and  $z_{a,m}$  are given as follows:

$$E_{a,m} \leq z_{a,m+1} \quad \forall a \in L_1, m \in \{1,2, \dots, M^C\} \quad (26)$$

$$E_{a,m} \leq 1 - z_{a,m} \quad \forall a \in L_1, m \in \{1,2, \dots, M^C\} \quad (27)$$

$$E_{a,m} \geq z_{a,m+1} - z_{a,m} \quad \forall a \in L_1, m \in \{1,2, \dots, M^C\} \quad (28)$$

Equation (26) ensures that if time interval  $m$  is the ending time of the project on link  $a$ , i.e.,  $E_{a,m} = 1$ , then in the next interval  $m + 1$ , the project must have been finished, i.e.,  $z_{a,m+1} = 1$ . Equation (27) indicates that if in time interval  $m$  the project on link  $a$  has already been finished, i.e.,  $z_{a,m} = 1$ , then time interval  $m$  cannot be the ending time, i.e.,  $E_{a,m} = 0$ . Equation (28) ensures that if in time interval  $m + 1$  the project on link  $a \in L_1$  has already been finished, i.e.,  $z_{a,m+1} = 1$ , but in interval  $m$  the project has not been finished, i.e.,  $z_{a,m} = 0$ , then interval  $m$  must be the ending time of the project, i.e.,  $E_{a,m} = 1$ .

Moreover,  $S_{a,m}$  and  $E_{a,m}$  should satisfy the following constraints:

$$\sum_{m=1}^{M^C} S_{a,m} \leq 1 \quad \forall a \in L_1 \quad (29)$$

$$\sum_{m=1}^{M^C} E_{a,m} \leq 1 \quad \forall a \in L_1 \quad (30)$$

$$\sum_{m=1}^{M^C} S_{a,m} = \sum_{m=1}^{M^C} E_{a,m} \quad \forall a \in L_1 \quad (31)$$

Equations (29)-(31) ensure that the potential expansion project on link  $a \in L_1$  either has no starting time and no ending time, i.e., the project is not selected, or has exactly one starting time and one ending time.

Finally, the following two constraints must also hold:

$$\sum_{m=1}^{M^C} y_{a,m} = (D_a^e - D_a^r) \cdot \sum_{m=1}^{M^C} S_{a,m} \quad \forall a \in L_1 \quad (32)$$

$$\sum_{m=1}^{M^C} y_{a,m} + \sum_{m=1}^M z_{a,m} = M \cdot \sum_{m=1}^{M^C} S_{a,m} - \sum_{m=1}^{M^C} (S_{a,m} \cdot (m-1)) \quad \forall a \in L_1 \quad (33)$$

The value of  $\sum_{m=1}^{M^C} S_{a,m}$  can represent whether the potential expansion project on link  $a \in L_1$  is selected. If the project is selected,  $\sum_{m=1}^{M^C} S_{a,m} = 1$ , and  $\sum_{m=1}^{M^C} S_{a,m} = 0$  otherwise. Therefore, equation (32) guarantees that if the potential expansion project on link  $a \in L_1$  is selected, i.e.,  $\sum_{m=1}^{M^C} S_{a,m} = 1$ , the total length of the time intervals under construction equals the actual construction duration. Equation (33) ensures that if the potential expansion project on link  $a \in L_1$  is selected, i.e.,  $\sum_{m=1}^{M^C} S_{a,m} = 1$ , the total duration of the construction period and benefit period is equal to the planning horizon, subtracting the duration before construction. Note that  $\sum_{m=1}^{M^C} (S_{a,m} \cdot (m-1))$  can represent the duration before construction if the potential expansion project on link  $a \in L_1$  is selected.

These above constraints, i.e., equations (13)-(33), ensure that if the potential expansion project on link  $a \in L_1$  is selected, different phases of the project (i.e., before construction, under construction and after construction) occur in correct sequence.

In order to reduce the complexity of our model and improve computational speed, the nonlinear constraint (32), together with constraints (7) and (15), can be equivalently replaced by the following linear constraints:

$$\sum_{m=1}^{M^C} y_{a,m} = D_a^0 \cdot \sum_{m=1}^{M^C} S_{a,m} + D_a^1 \cdot l_a - D_a^r \quad \forall a \in L_1 \quad (34)$$

$$l_a \cdot C_a^1 \leq (C_a^{max} - C_a^0) \cdot \sum_{m=1}^{M^C} S_{a,m} \quad \forall a \in L_1 \quad (35)$$

$$D_a^r \leq D_a^{max} \cdot \sum_{m=1}^{M^C} S_{a,m} \quad \forall a \in L_1 \quad (36)$$

We briefly prove the equivalence by examining both the selected and unselected projects. If the potential expansion project on link  $a \in L_1$  is not selected, i.e.,  $\sum_{m=1}^{M^C} S_{a,m} = 0$ . Equation (14) and equation (35) imply that  $l_a = 0$ . Equation (16) and equation (36) imply that  $D_a^r = 0$ . Consequently, equation (34) implies that  $\sum_{m=1}^{M^C} y_{a,m} = 0 = (D_a^e - D_a^r) \cdot \sum_{m=1}^{M^C} S_{a,m}$ , which is identical to equation (32). If the potential expansion project on link  $a \in L_1$  is selected,  $\sum_{m=1}^{M^C} S_{a,m} = 1$ . Equation (35) is reduced to equation (7), and equation (36) is reduced to equation (15). Equation (13) and equation (34) imply that  $\sum_{m=1}^{M^C} y_{a,m} = D_a^e - D_a^r = (D_a^e - D_a^r) \cdot \sum_{m=1}^{M^C} S_{a,m}$ , which is identical to equation (32).

#### 4.2.2 Budget and Resource Constraints

We assume that the government allocates a certain amount of construction budget  $B_\tau$  at the beginning of time interval  $\tau$ . This time interval  $\tau$  does not have to be the same as the predefined time interval  $m$ . We introduce a conversion factor  $\beta$  to express the ratio between  $\tau$  and  $m$ . For example, if the unit of  $m$  is month, and the unit of  $\tau$  is year, then  $\beta$  should be 12. In this study, we assume that the remaining budget in period  $\tau$  is available for use in period  $\tau + 1$ . Similar assumptions were employed in Lo and Szeto (2004). Apart from the budget limitation, we should also consider other resource limitations, e.g., the limitation of construction personnel and the limited number of specialized construction equipment. For the sake of simplicity, we only consider the construction personnel limitation in this study. We assume that all construction teams have the same construction capability, and each ongoing project requires one construction team. The total number of available construction teams is limited and denoted by  $R^{max}$ . The budget and construction personnel constraints are then given as follows:

$$TC_1 + RB_1 = B_1 \quad (37)$$

$$TC_\tau + RB_\tau = RB_{\tau-1} + B_\tau \quad \forall \tau > 1, \tau \in \{1, 2, \dots, M^C/\beta\} \quad (38)$$

$$\sum_{a \in L_1} y_{a,m} \leq R^{max} \quad \forall m \in \{1, 2, \dots, M^C\} \quad (39)$$

where  $TC_\tau$  is the total construction cost generated in period  $\tau$ , and  $RB_\tau$  is the cumulative remaining budget in period  $\tau$ . Equations (37)-(38) represent the budget constraints.  $M^C/\beta$  converts the construction time to the same time unit as  $\tau$ . If the government allocates the entire budget at the beginning of the planning horizon, the budget constraints will be reduced to equation (37). Equation (39) specifies that for each time interval  $m \in \{1, 2, \dots, M^C\}$ , the total construction teams at work should not exceed the number of available teams.

The total construction cost of a project consists of two components: basic costs to complete the project (e.g., equipment cost, material cost, and labor cost) and extra costs for overtime work. Based on the previous assumption that a construction team works a fixed number of hours per day under normal conditions, we assume that without overtime work, each construction time interval for the expansion project on link  $a \in L_1$  includes an identical and fixed basic cost, denoted as  $c_a$ .  $c_a$  includes all of the wages for workers and other costs (e.g., material costs, equipment costs) needed in a normal construction time interval. If the planner wants to shorten the duration of a project, workers may choose any period to work overtime as long as they meet the time limit requirement. To simplify our model, we assume the overtime cost will be placed in the starting time interval of any project (Figure 4.3). The example in Figure 4.3 corresponds to the examples in Figure 4.1 and Figure 4.2. The expansion project originally lasts for four months, and it is reduced to three months through overtime. Therefore, both the basic cost in the fourth construction interval and the additional wages attributable to overtime work are allocated to the first construction interval when applying overtime work.

The total construction cost in period  $\tau$  can be formulated as:

$$TC_\tau = \left[ \sum_{m=\beta(\tau-1)+1}^{\beta\tau} \sum_{a \in L_1} y_{a,m} \cdot c_a + \sum_{a \in L_1} OC_{a,\tau} \right] \cdot (1 + \theta_1)^{\tau-1} \quad \forall \tau \in \{1, 2, \dots, M^C/\beta\} \quad (40)$$

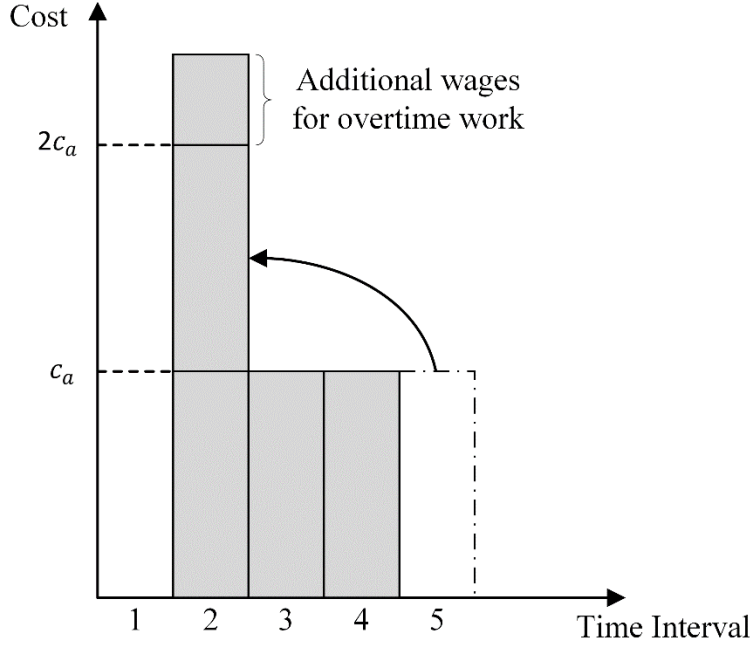
where  $OC_{a,\tau}$  is the overtime cost for the project on link  $a$  in period  $\tau$ .  $\theta_1$  represents the inflation rate.

$$\sum_{\tau=1}^{M^C/\beta} OC_{a,\tau} = \lambda \cdot c_a \cdot (1 + \mu) \cdot D_a^r + (1 - \lambda) \cdot c_a \cdot D_a^r \quad \forall a \in L_1 \quad (41)$$

$$OC_{a,\tau} \geq 0 \quad \forall a \in L_1, \tau \in \{1, 2, \dots, M^C/\beta\} \quad (42)$$

$$OC_{a,\tau} \leq \sum_{m=\beta(\tau-1)+1}^{\beta\tau} S_{a,m} \cdot Q \quad \forall a \in L_1, \tau \in \{1, 2, \dots, M^C/\beta\} \quad (43)$$

where  $\lambda$  denotes the percentage of the workers' salary in the total construction cost,  $\mu$  is the increased rate of overtime salary, and  $Q$  is a large constant value. The first term on the right-hand side of equation (41) represents the salary portion of the overtime cost, and the second term represents the remaining portion. Equations (41)-(43) ensure that the overtime cost for the expansion project on link  $a \in L_1$  is placed in the starting period.



**Figure 4.3** Illustration of Construction Costs for a Project

### 4.3 Objective Function

As aforementioned, during the construction period, the system performance may deteriorate due to work zones or lane closures. Different planners may have different preferences when selecting road expansion projects. Some planners may focus more on future benefits of the projects, while others may consider more about reducing the adverse impacts of the projects during construction. To provide a flexible model, the objective function is to minimize the weighted sum of the total travel time during construction period and the total travel time during benefit period. This can be stated as:

$$NPV = \alpha_1 \sum_{\tau=1}^{M^c/\beta} \frac{TT_{\tau}}{(1 + \theta_2)^{\tau-1}} + \alpha_2 \sum_{\tau=\frac{M^c}{\beta}+1}^{M/\beta} \frac{TT_{\tau}}{(1 + \theta_2)^{\tau-1}} \quad (44)$$

where  $\alpha_1$  and  $\alpha_2$  are the weighting factors for the construction period and benefit period, respectively,  $TT_{\tau}$  is the total travel time occurring in period  $\tau$ ,  $\theta_2$  is the discount rate, and  $(1 + \theta_2)^{\tau-1}$  represents the discount factor for period  $\tau$ .  $TT_{\tau}$  is given in equation (45):

$$TT_{\tau} = \sum_{m=\beta(\tau-1)+1}^{\beta\tau} \sum_{a \in L} t_{a,m} v_{a,m} \quad \forall \tau \in \{1, 2, \dots, M/\beta\} \quad (45)$$

#### 4.4 Uncertainty Set of the Robust Model

Based on the above notations, the time-dependent discrete network design problem considering construction impact and flexible duration can be formulated as the following mathematical program P1.

P1:

$$\min NPV = \alpha_1 \sum_{\tau=1}^{M^c/\beta} \frac{\sum_{m=\beta(\tau-1)+1}^{\beta\tau} \sum_{a \in L} t_{a,m} v_{a,m}}{(1 + \theta_2)^{\tau-1}} + \alpha_2 \sum_{\tau=\frac{M^c}{\beta}+1}^{M/\beta} \frac{\sum_{m=\beta(\tau-1)+1}^{\beta\tau} \sum_{a \in L} t_{a,m} v_{a,m}}{(1 + \theta_2)^{\tau-1}}$$

$$s.t. \quad v_m \in V_m^F \quad \forall m \in \{1, 2, \dots, M\}$$

Time-dependent definitional constraints, equations (6), (8), (9);

Traffic assignment constraints, equations (11)-(12);

Design constraints, equations (13), (14), (16)-(31), (33)-(36);

Budget and resource constraints, equations (37)-(43).

This formulation involves two integer variables (i.e.,  $l_a$  and  $D_a^r$ ). As is generally known, it is much more difficult to solve optimization problems with integer variables, especially for large-scale networks. Hence, we introduce two sets of binary variables,  $p_a^{b1}$  and  $q_a^{b2}$ , to replace  $l_a$  and  $D_a^r$ , as follows:

$$l_a = \sum_{b1=1}^{B1} 2^{(b1-1)} p_a^{b1} \quad \forall a \in L_1 \quad (46)$$

$$D_a^r = \sum_{b2=1}^{B2} 2^{(b2-1)} q_a^{b2} \quad \forall a \in L_1 \quad (47)$$

According to equation (46), the number of newly built lanes  $l_a$  can take the value 0 to  $(2^{B1} - 1)$ . For example, if we use three binary variables to represent  $l_a$ , i.e.,  $B1 = 3$ , then  $l_a = p_a^1 + 2p_a^2 + 4p_a^3$ , ranging from 0 to 7. Similarly, the reduced value of construction interval  $D_a^r$  can range from 0 to  $(2^{B2} - 1)$ . Note that the binary variables can be written in the form of complementarity constraints so that the binary variable can be treated as continuous variables, as follows:

$$0 \leq p_a^{b1} \leq 1 \quad \forall a \in L_1 \quad (48)$$

$$p_a^{b1}(1 - p_a^{b1}) = 0 \quad \forall a \in L_1 \quad (49)$$

$$0 \leq q_a^{b1} \leq 1 \quad \forall a \in L_1 \quad (50)$$

$$q_a^{b1}(1 - q_a^{b1}) = 0 \quad \forall a \in L_1 \quad (51)$$

Then,  $l_a$  and  $D_a^r$  in previous equations can be replaced by  $\sum_{b1=1}^{B1} 2^{(b1-1)} p_a^{b1}$  and  $\sum_{b2=1}^{B2} 2^{(b2-1)} q_a^{b2}$ , respectively. Equations (6), (13), (34), (35), (36), (41) are replaced by equations (52), (53), (54), (55), (56), (57), respectively.

$$C_{a,m} = C_a^0 - y_{a,m} \cdot C_a^r + z_{a,m} \cdot \sum_{b1=1}^{B1} 2^{(b1-1)} p_a^{b1} \cdot C_a^1 \quad \forall a \in L, m \in \{1,2, \dots, M\} \quad (52)$$

$$D_a^e = D_a^0 + D_a^1 \cdot \sum_{b1=1}^{B1} 2^{(b1-1)} p_a^{b1} \quad \forall a \in L_1 \quad (53)$$

$$\sum_{m=1}^{M^C} y_{a,m} = D_a^0 \cdot \sum_{m=1}^{M^C} S_{a,m} + D_a^1 \cdot \sum_{b1=1}^{B1} 2^{(b1-1)} p_a^{b1} - \sum_{b2=1}^{B2} 2^{(b2-1)} q_a^{b2} \quad \forall a \in L_1 \quad (54)$$

$$\sum_{b1=1}^{B1} 2^{(b1-1)} p_a^{b1} \cdot C_a^1 \leq (C_a^{max} - C_a^0) \cdot \sum_{m=1}^{M^C} S_{a,m} \quad \forall a \in L_1 \quad (55)$$

$$\sum_{b2=1}^{B2} 2^{(b2-1)} q_a^{b2} \leq D_a^{max} \cdot \sum_{m=1}^{M^C} S_{a,m} \quad \forall a \in L_1 \quad (56)$$

$$\sum_{\tau \in \mathbb{1}}^{M^C/\beta} OC_{a,\tau} = \lambda \cdot c_a \cdot (1 + \mu) \cdot \sum_{b2=1}^{B2} 2^{(b2-1)} q_a^{b2} + (1 - \lambda) \cdot c_a \cdot \sum_{b2=1}^{B2} 2^{(b2-1)} q_a^{b2} \quad \forall a \in L_1 \quad (57)$$

We also use the following complementarity constraints to replace equation (17) so that the binary variables  $S_{a,m}$ ,  $E_{a,m}$ ,  $y_{a,m}$ , and  $z_{a,m}$  can be treated as continuous variables:

$$0 \leq S_{a,m} \leq 1 \quad \forall a \in L_1, m \in \{1,2, \dots, M^C\} \quad (58)$$

$$S_{a,m}(1 - S_{a,m}) = 0 \quad \forall a \in L_1, m \in \{1,2, \dots, M^C\} \quad (59)$$

$$0 \leq E_{a,m} \leq 1 \quad \forall a \in L_1, m \in \{1,2, \dots, M^C\} \quad (60)$$

$$E_{a,m}(1 - E_{a,m}) = 0 \quad \forall a \in L_1, m \in \{1,2, \dots, M^C\} \quad (61)$$

$$0 \leq y_{a,m} \leq 1 \quad \forall a \in L_1, m \in \{1,2, \dots, M^C\} \quad (62)$$

$$y_{a,m}(1 - y_{a,m}) = 0 \quad \forall a \in L_1, m \in \{1,2, \dots, M^C\} \quad (63)$$

$$0 \leq z_{a,m} \leq 1 \quad \forall a \in L_1, m \in \{1,2, \dots, M^C + 1\} \quad (64)$$

$$z_{a,m}(1 - z_{a,m}) = 0 \quad \forall a \in L_1, m \in \{1,2, \dots, M^C + 1\} \quad (65)$$



Based on the above discussions, the base model can be reformulated as follows:

P2:

$$\min NPV = \alpha_1 \sum_{\tau=1}^{M^C/\beta} \frac{\sum_{m=\beta(\tau-1)+1}^{\beta\tau} \sum_{a \in L} t_{a,m} v_{a,m}}{(1 + \theta_2)^{\tau-1}} + \alpha_2 \sum_{\tau=\frac{M^C}{\beta}+1}^{M/\beta} \frac{\sum_{m=\beta(\tau-1)+1}^{\beta\tau} \sum_{a \in L} t_{a,m} v_{a,m}}{(1 + \theta_2)^{\tau-1}}$$

$$s.t. \quad v_m \in V_m^F \quad \forall m \in \{1, 2, \dots, M\}$$

(48)-(65);

Time-dependent definitional constraint, equations (8), (9);

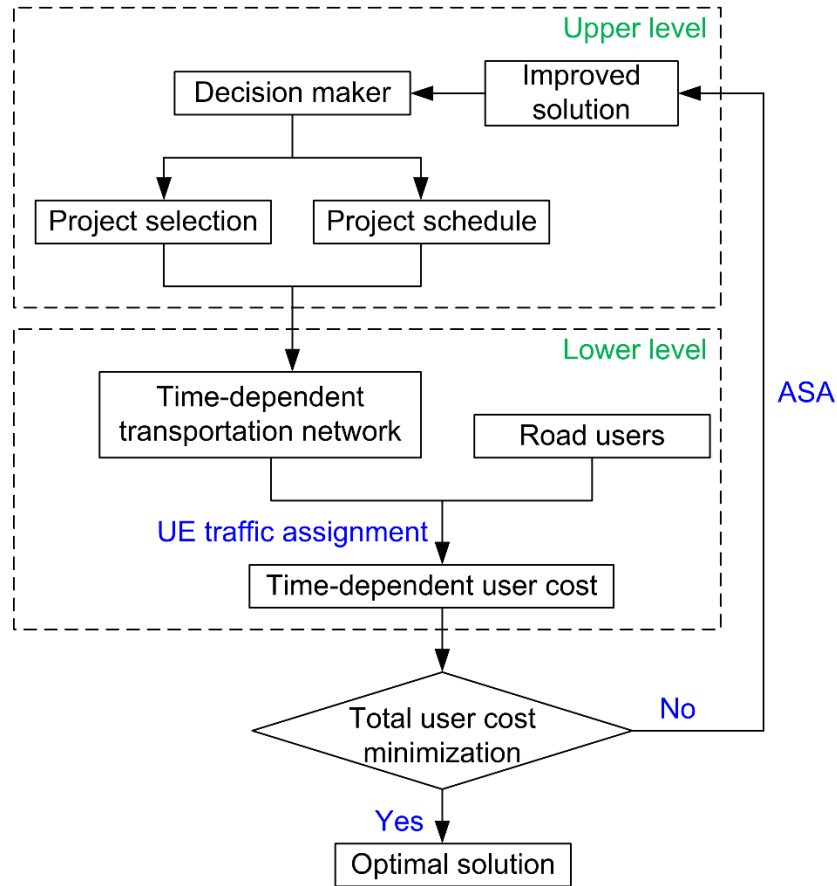
Traffic assignment constraints, equations (11)-(12);

Design constraints, equations (18)-(31), (33);

Budget and resource constraints, equations (37)-(40), (42)-(43).

## 5. SOLUTION ALGORITHM

P2 is a mathematical program with complementarity constraints (MPCC). It is well known that MPCC problems are difficult to solve because the feasible region of an MPCC problem is not convex, and the Magasarian-Fromovitz constraint qualification (MFCQ) fails to hold (Scheel and Scholtes, 2000). Several previous efforts have been undertaken to make the problem easier to solve (Bouza and Still, 2007, Raghunathan and Biegler, 2005). In this study, we extend the active-set algorithm (ASA) proposed by Zhang et al. (2009) to solve the MPCC problem. Figure 5.1 shows the fundamental concepts of our model and the conceptual solution procedure of the ASA.



**Figure 5.1** The Framework of the T-DNDP Model and ASA

Instead of solving an MPCC directly, the ASA solves two simpler problems sequentially. The first problem is a restricted version of P2, in which the variables  $p_a^{b1}$ ,  $q_a^{b1}$ ,  $S_{a,m}$ ,  $E_{a,m}$ ,  $y_{a,m}$ , and  $z_{a,m}$  have a set of predefined values. The initial set of values is only a feasible solution, not necessarily the best solution. Therefore, we need to make adjustments to find a better solution, which is realized through the second problem, a sub-problem. For each variable, i.e.,  $p_a^{b1}$ ,  $q_a^{b1}$ ,  $S_{a,m}$ ,  $E_{a,m}$ ,  $y_{a,m}$ , or  $z_{a,m}$ , there are only two possible values: 0 and 1. We divide all components of each variable into two active sets, where one set stores the components with a value of 0 and the other set stores the components with a value of 1.

Then equations (48)-(51) and equations (58)-(65) can be replaced by the following equations:

$$p_a^{b1} = 0 \quad \forall (a, b1) \in \Omega_{p,0}, b1 \in B1 \quad (66)$$

$$p_a^{b1} = 1 \quad \forall (a, b1) \in \Omega_{p,1}, b1 \in B1 \quad (67)$$

$$q_a^{b2} = 0 \quad \forall (a, b2) \in \Omega_{q,0}, b2 \in B2 \quad (68)$$

$$q_a^{b2} = 1 \quad \forall (a, b2) \in \Omega_{q,1}, b2 \in B2 \quad (69)$$

$$S_{a,m} = 0 \quad \forall (a, m) \in \Omega_{S,0} \quad (70)$$

$$S_{a,m} = 1 \quad \forall (a, m) \in \Omega_{S,1} \quad (71)$$

$$E_{a,m} = 0 \quad \forall (a, m) \in \Omega_{E,0} \quad (72)$$

$$E_{a,m} = 1 \quad \forall (a, m) \in \Omega_{E,1} \quad (73)$$

$$y_{a,m} = 0 \quad \forall (a, m) \in \Omega_{y,0} \quad (74)$$

$$y_{a,m} = 1 \quad \forall (a, m) \in \Omega_{y,1} \quad (75)$$

$$z_{a,m} = 0 \quad \forall (a, m) \in \Omega_{z,0} \quad (76)$$

$$z_{a,m} = 1 \quad \forall (a, m) \in \Omega_{z,1} \quad (77)$$

The restricted version of P2 can be formulated as:

P3:

$$\text{Min}_{(v,p,q,S,E,y,z,\rho)} NPV = \alpha_1 \sum_{\tau=1}^{M^C/\beta} \frac{TT_\tau}{(1+\theta_2)^{\tau-1}} + \alpha_2 \sum_{\tau=\frac{M^C}{\beta}+1}^{M/\beta} \frac{TT_\tau}{(1+\theta_2)^{\tau-1}}$$

$$\text{s.t. } v_m \in V_m^F \quad \forall m \in \{1,2, \dots, M\}$$

$$(52)-(57)$$

$$(66)-(77)$$

Time-dependent definitional constraint, equations (8), (9);

Traffic assignment constraints, equations (11)-(12);

Design constraints, equations (18)-(31), (33);

Budget and resource constraints, equations (37)-(40), (42)-(43).

Let  $\bar{v}_m$  denote the solution of the UE problem for time period  $m$  ( $m \in \{1,2, \dots, M\}$ ) for a given feasible design  $(\bar{p}, \bar{q}, \bar{S}, \bar{E}, \bar{y}, \bar{z})$ , and combine  $\bar{v}_m$  into one vector denoted as  $\bar{v}$  (i.e.,  $\bar{v} = \{\bar{v}_1, \bar{v}_2, \dots, \bar{v}_{m=M}\}$ ). According to Proposition 1.1, in a study conducted by Facchinei and Pang (2007), there must be a multiplier vector  $\bar{\rho}$  associated with Eq. (1) so that  $(\bar{p}, \bar{q}, \bar{S}, \bar{E}, \bar{y}, \bar{z}, \bar{v}, \bar{\rho})$  is also optimal to P3. Therefore, instead of solving P3 directly, we can obtain the optimal solution for P3 by solving a set of corresponding UE problems. Let  $\delta_a^{b1}$  and  $\gamma_a^{b1}$  denote the multipliers associated with equations (66) and (67), respectively, and let  $\sigma_{a,m}$ ,  $\xi_{a,m}$ ,  $\chi_{a,m}$ , and  $\varrho_{a,m}$  denote the multipliers associated with equations (74), (75), (76) and (77), respectively. The feasible design and the active sets can then be improved based on the information obtained from these multipliers. For example, if  $\delta_a^{b1} < 0$  for some specific  $(a, b1) \in \Omega_{p,0}$ , shifting the  $(a, b1)$  from  $\Omega_{p,0}$  to  $\Omega_{p,1}$  may reduce the objective function value. And if  $\gamma_a^{b1} > 0$  for some specific  $(a, b1) \in \Omega_{p,1}$ , it may be beneficial to shift the  $(a, b1)$  from  $\Omega_{p,1}$  to  $\Omega_{p,0}$ . Similarly, the multipliers  $\sigma_{a,m}$ ,  $\xi_{a,m}$ ,  $\chi_{a,m}$ , and  $\varrho_{a,m}$  provide information on updating  $\Omega_{y,0}$ ,  $\Omega_{y,1}$ ,  $\Omega_{z,0}$ , and  $\Omega_{z,1}$ , respectively. The switching process, however, may make the budget and crew constraints unsatisfactory.

For this reason, the following sub-problem is used to complete the switching process as well as prevent problem P3 from becoming infeasible.

SUB:

$$\begin{aligned}
& \sum_{(a,b1) \in \Omega_{p,0}} K^{b1} \delta_a^{b1} g_a^{b1} + \sum_{(a,m) \in \Omega_{y,0}} \sigma_{a,m} h_{a,m} + \sum_{(a,m) \in \Omega_{z,0}} \chi_{a,m} \eta_{a,m} \\
\text{Min}_{(g,h,\eta)} & - \sum_{(a,b1) \in \Omega_{p,1}} K^{b1} \gamma_a^{b1} g_a^{b1} - \sum_{(a,m) \in \Omega_{y,1}} \xi_{a,m} h_{a,m} - \sum_{(a,m) \in \Omega_{z,1}} \varrho_{a,m} \eta_{a,m}
\end{aligned}$$

s.t.

Design constraints, equations (17)-(31), (33), (52)-(57);

Budget and resource constraints, equations (37)-(40), (42)-(43), (57);

$$p_a^{b1} = g_a^{b1} \quad \forall (a, b1) \in \Omega_{p,0} \quad (78)$$

$$p_a^{b1} = 1 - g_a^{b1} \quad \forall (a, b1) \in \Omega_{p,1} \quad (79)$$

$$y_{a,m} = h_{a,m} \quad \forall (a, m) \in \Omega_{y,0} \quad (80)$$

$$y_{a,m} = 1 - h_{a,m} \quad \forall (a, m) \in \Omega_{y,1} \quad (81)$$

$$z_{a,m} = \eta_{a,m} \quad \forall (a, m) \in \Omega_{z,0} \quad (82)$$

$$z_{a,m} = 1 - \eta_{a,m} \quad \forall (a, m) \in \Omega_{z,1} \quad (83)$$

$$\sum_{b1=1}^{B1} g_a^{b1} \leq 1 \quad \forall a \in L_1 \quad (84)$$

$$g_a^{b1} \in \{0,1\} \quad \forall a \in L_1, b1 \in B1 \quad (85)$$

$$h_{a,m}, \eta_{a,m} \in \{0,1\} \quad \forall a \in L_1, m \in \{1,2, \dots, M^{CP}\} \quad (86)$$

$$q_a^{b2} \in \{0,1\} \quad \forall a \in L_1, b2 \in B2 \quad (87)$$

$$\begin{aligned}
& \sum_{(a,b1) \in \Omega_{p,0}} K^{b1} \delta_a^{b1} g_a^{b1} + \sum_{(a,m) \in \Omega_{y,0}} \sigma_{a,m} h_{a,m} + \sum_{(a,m) \in \Omega_{z,0}} \chi_{a,m} \eta_{a,m} \\
& - \sum_{(a,b1) \in \Omega_{p,1}} K^{b1} \gamma_a^{b1} g_a^{b1} - \sum_{(a,m) \in \Omega_{y,1}} \xi_{a,m} h_{a,m} - \sum_{(a,m) \in \Omega_{z,1}} \varrho_{a,m} \eta_{a,m} > \varphi
\end{aligned} \quad (88)$$

where binary variables  $g_a^{b1}$ ,  $h_{a,m}$ , and  $\eta_{a,m}$  are “switch” variables, indicating whether to move the corresponding design variable to the complementary set. Equation (84) ensures that only one digit of variable  $p_a^{b1}$  can be changed at a time to prevent too much fluctuation in iterations. Equation (88) gives a predetermined lower bound to the objective function value of the sub-problem. A vector of constant  $K^{b1}$  is introduced to ensure that changes are always made to the smallest digit possible, because the multipliers generated by the CONOPT solver (Drud, 1994) are linear in magnitude with respect to its digit  $b1$ . Note that although we only introduce “switch” variables for variables  $p_a^{b1}$ ,  $y_{a,m}$ , and  $z_{a,m}$ , due to the

dependency relationships among  $p_a^{b1}$ ,  $q_a^{b1}$ ,  $S_{a,m}$ ,  $E_{a,m}$ ,  $y_{a,m}$ , and  $z_{a,m}$ , variables  $q_a^{b1}$ ,  $S_{a,m}$ , and  $E_{a,m}$  will also be determined.

The procedure to solve the T-DNDP is as follows:

**Step 0:** Choose an initial feasible design  $(p_a^{b1}, q_a^{b1}, S_{a,m}, E_{a,m}, y_{a,m}, \text{ and } z_{a,m})$  and solve the UE problem. Initialize sets  $\Omega_{p,0}, \Omega_{p,1}, \Omega_{q,0}, \Omega_{q,1}, \Omega_{S,0}, \Omega_{S,1}, \Omega_{E,0}, \Omega_{E,1}, \Omega_{y,0}, \Omega_{y,1}, \Omega_{z,0}, \text{ and } \Omega_{z,1}$ .

**Step 1:** Solve P3 and denote the optimal objective function value as  $TT$ . Obtain multipliers  $\delta_a^{b1}, \gamma_a^{b1}, \sigma_{a,m}, \xi_{a,m}, \lambda_{a,m}, \text{ and } \varrho_{a,m}$ .

**Step 2:** Set  $\varphi = -\infty$  and let  $(\bar{p}_a^{b1}, \bar{q}_a^{b1}, \bar{S}_{a,m}, \bar{E}_{a,m}, \bar{y}_{a,m}, \bar{z}_{a,m})$  solve the SUB problem. Denote the optimal objective function value as  $\bar{\varphi}$ . If  $\bar{\varphi} = 0$ , stop, as  $(\bar{p}_a^{b1}, \bar{q}_a^{b1}, \bar{S}_{a,m}, \bar{E}_{a,m}, \bar{y}_{a,m}, \bar{z}_{a,m})$  is the best solution found. Otherwise, go to Step 3.

**Step 3:** Solve the UE problem with  $(\bar{p}_a^{b1}, \bar{q}_a^{b1}, \bar{S}_{a,m}, \bar{E}_{a,m}, \bar{y}_{a,m}, \bar{z}_{a,m})$ . If the total travel time associated with the UE distribution is greater than  $TT$ , set  $\varphi = \bar{\varphi} + \varepsilon$ , where  $\varepsilon > 0$  is sufficiently small, and return to Step 2. Otherwise, use  $(\bar{p}_a^{b1}, \bar{q}_a^{b1}, \bar{S}_{a,m}, \bar{E}_{a,m}, \bar{y}_{a,m}, \bar{z}_{a,m})$  to update the current design  $(p_a^{b1}, q_a^{b1}, S_{a,m}, E_{a,m}, y_{a,m}, z_{a,m})$  and sets  $\Omega_{p,0}, \Omega_{p,1}, \Omega_{q,0}, \Omega_{q,1}, \Omega_{S,0}, \Omega_{S,1}, \Omega_{E,0}, \Omega_{E,1}, \Omega_{y,0}, \Omega_{y,1}, \Omega_{z,0}, \text{ and } \Omega_{z,1}$ . Return to Step 1.

## 6. NUMERICAL STUDIES

In this section, two numerical examples are presented to demonstrate the proposed model and solution algorithm.

### 6.1 Example 1: Nguyen-Dupuis Network

To illustrate the usefulness and advantages of our model, we first solve it for the Nguyen-Dupuis network (Nguyen and Dupuis, 1984) with two different scenarios. As shown in Figure 6.1, the Nguyen-Dupuis network consists of 13 nodes, 19 links, and four O-D pairs. Table 6.1 reports the link characteristics of the network. The travel demand is given by Nguyen and Dupuis (1984):  $q_{1 \rightarrow 2} = 400 \text{ veh/h}$ ;  $q_{1 \rightarrow 3} = 800 \text{ veh/h}$ ;  $q_{4 \rightarrow 2} = 600 \text{ veh/h}$ ;  $q_{4 \rightarrow 3} = 200 \text{ veh/h}$ .

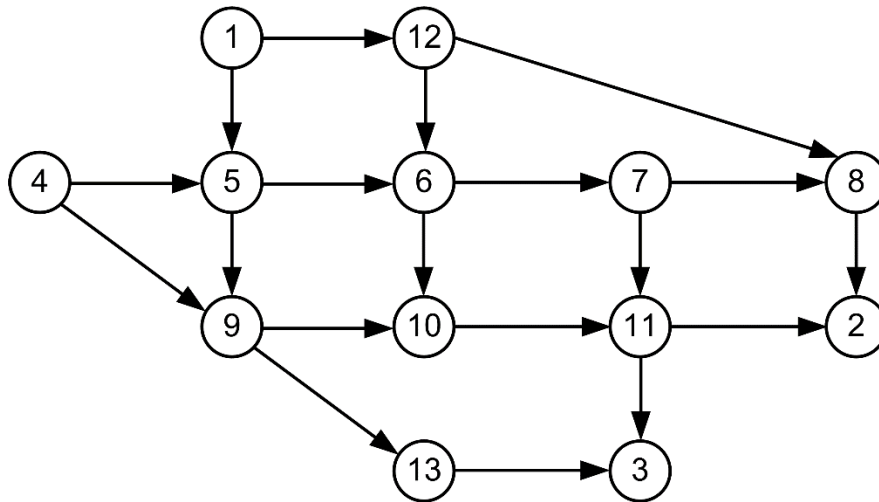


Figure 6.1 Nguyen-Dupuis Network

Table 6.1 Link Characteristics of the Nguyen-Dupuis Network

Link	Free flow travel time $t_a^0$ (min)	Initial capacity $C_a^0$	Link	Free flow travel time $t_a^0$ (min)	Initial capacity $C_a^0$
1-5	11.17	177	8-2	14.36	275
1-12	14.36	104	9-10	12.32	221
4-5	14.36	163	9-13	11.25	278
4-9	18.88	235	10-11	12.76	241
5-6	4.79	245	11-2	14.36	283
5-9	14.36	121	11-3	12.77	169
6-7	7.98	295	12-6	11.17	164
6-10	70.75	213	12-8	5.05	179
7-8	7.98	183	13-3	11.25	278
7-11	14.36	291			

#### 6.1.1 Scenario 1: Considering Construction Impacts During Project Selection

In this scenario, we show that our model can consider the construction impacts during the project selection process and thus provide better solutions than conventional methods (i.e., separately optimizing the selection and schedule of the road expansion project).

In order to clearly illustrate the results without being distracted by other factors, this scenario does not consider overtime policy. Assume there are six candidate road expansion projects on links (5,9), (7,11), (9,10), (10,11), (9,13), and (13,3). The parameters for the candidate projects are given in Table 6.2. Other parameters are given as follows:

- 1) Planning horizon: 20 years (the planning horizon is equally divided into 240 design periods, namely, 240 months); Construction period: 2 years; Benefit period: 18 years.
- 2) Weighting for construction period  $\alpha_1 = 0.5$ ; Weighting for benefit period  $\alpha_2 = 0.5$ .
- 3) Budget:  $B_1 = 20, B_2 = 20$ .
- 4) Number of available crew teams:  $R^{max} = 2$ .
- 5) Inflation rate  $\theta_1 = 0.01$ ; discount rate  $\theta_2 = 0.05$ .
- 6) Conversion factor:  $\beta = 12$ .

**Table 6.2** Parameters for Candidate Projects in the Nguyen-Dupuis Network

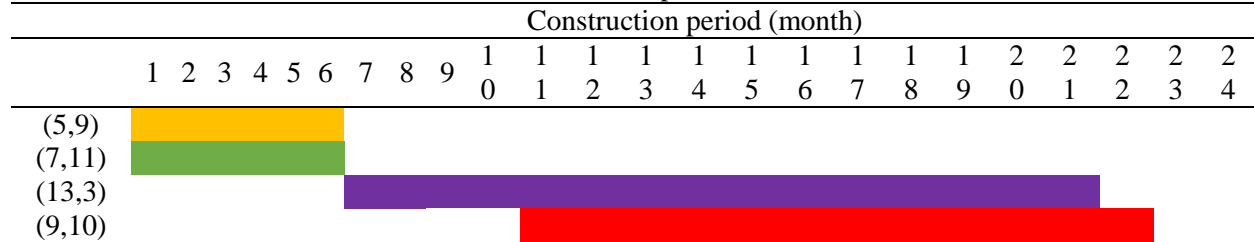
Candidate link	Initial capacity $C_a^0$	Lane capacity $C_a^1$	Maximum allowable capacity $C_a^{max}$	Number of closed lanes $k_a$	Fixed construction duration (month)	Extra duration for adding one lane $D_a^1$
5-9	121	121	242	0	3	3
7-11	291	291	582	1	4	2
9-10	241	241	482	0	6	6
9-13	278	278	556	1	7	8
10-11	241	241	482	1	5	7
12-6	164	164	328	0	3	5
13-3	278	278	556	0	6	9

The ASA solution procedure is implemented using GAMS (Rosenthal, 2012) and CONOPT solver (Drud, 1994) on a Dell computer with a 3.4 GHz processor and 16.0 GB RAM. It takes 23 minutes and 48 seconds to solve the model. The project selection and schedule results are shown in Table 6.3. To show the benefits of our model, we separately optimize the selection and schedule of road expansion projects. Table 6.4 presents the results with separate optimization, and Table 6.5 compares the system performance under the two different approaches. It can be observed that the joint optimization approach improves the overall system performance by 29.6%. Compared with the separate optimization approach, the joint optimization results have much better performance in the construction period and a little bit worse performance in the benefit period.

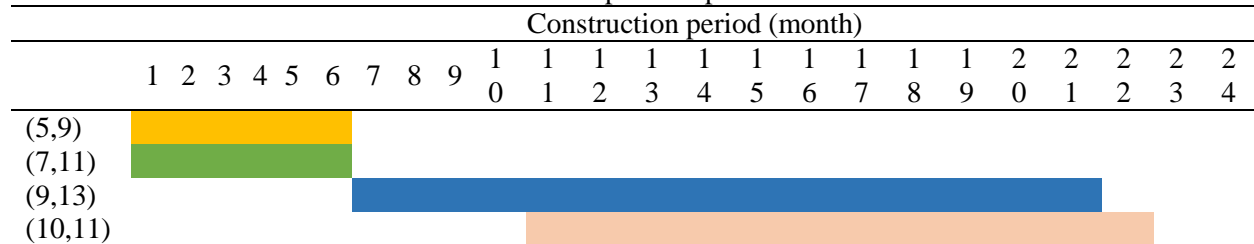
Through further comparison of the selected projects in the two approaches, we have the following observations: First, when other conditions remain the same, the project on link (9,13) will have the same benefit as the project on link (13,3); Second, when other conditions remain the same, the project on link (10,11) will have a little bit higher benefit than the project on link (9,10); Third, the projects on links (13,3) and (9,10) will have no adverse construction impact because they do not require lane closures, while the projects on links (9,13) and (10,11) will have severely adverse construction impacts because they both require lane closures. Based on these observations, the results of the joint and separate optimization approaches can be further analyzed. Because the separate optimization approach only considers the benefits but neglects the construction impacts when selecting road expansion projects, the projects on links (9,13) and (10,11) are selected. Nevertheless, because the joint optimization approach explicitly considers the potential construction impact during project selection and scheduling, the projects on links (13,3) and (9,10), which have better overall performance, are selected. Therefore, the proposed joint optimization approach has the potential to provide better solutions for planners.

Compared with the conventional planning approach that separately selects and schedules road expansion projects, the proposed time-dependent joint optimization approach can help planners choose the projects that not only have significant benefits after completion but also yield relatively fewer adverse impacts during construction. As shown in the above numerical experiment, this joint optimization approach is beneficial, especially when there are projects with similar potential benefits but quite different construction impacts.

**Table 6.3** Selection and Schedule Results with Joint Optimization for Scenario 1



**Table 6.4** Selection and Schedule Results with Separate Optimization for Scenario 1



**Table 6.5** System Performance Comparison for Scenario 1

	Net user cost in construction period	Net user cost in benefit period	Total weighted net user cost <i>NPV</i>
Separate optimization	\$105,952,239	\$132,304,481	\$119,128,360
Joint optimization	\$24,516,051	\$143,304,748	\$83,910,400
		Improvement	29.6%

### 6.1.2 Scenario 2: Focusing More On Future Benefits

In scenario 1, we consider a 20-year planning horizon with a two-year construction period and an 18-year benefit period and assume that the weightings for construction period and benefit period are the same (i.e.,  $\alpha_1 = \alpha_2 = 0.5$ ). This assumption is preferred for planners who focus on the near-term overall performance of a transportation network. For planners who focus more on future benefits of road-expansion projects, they can choose relatively higher weighting for the benefit period.

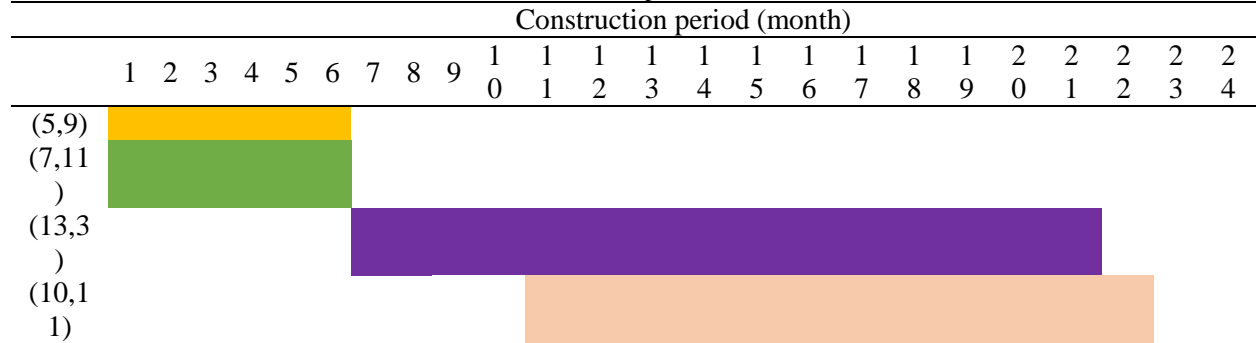
In this scenario, the weighting factors are given by  $\alpha_1 = 0.2$  and  $\alpha_2 = 0.8$  and other parameters are the same as scenario 1. Note that with these weighting factors, it is approximately equivalent to considering a 72-year benefit period. The new results from our joint optimization model are shown in Table 6.6. The selection and schedule results of the separate optimization approach will not change. Table 6.7 compares the system performance under the two different approaches. It can be observed that the joint optimization approach improves the overall system performance by 12.0%. Compared with the separate optimization approach, the joint optimization results have the same performance in the benefit period but have better performance in the construction period. Compared with scenario 1, this scenario selects the project on link (10,11) instead of the project on link (9,10) because the project on link (10,11) will lead to a better overall system performance. We should note that, because the system performance in the benefit period



for the two results are the same, the separate optimization approach may obtain the same optimal solution as the joint optimization approach under the best-case situation. However, because the separate optimization approach cannot consider the construction impact during project selection, it has a high chance of obtaining the less optimal solutions.

This scenario first shows the flexibility of our model in considering planners with different preferences. It also further demonstrates that the proposed time-dependent joint optimization approach can provide better solutions than the separate optimization approach because it considers construction impacts during project selection.

**Table 6.6** Selection and Schedule Results with Joint Optimization for Scenario 2



**Table 6.7** System Performance Comparison for Scenario 2

	Net user cost in construction period	Net user cost in benefit period	Total weighted net user cost <i>NPV</i>
Separate optimization	105952239	132304481	127034033
Joint optimization	29964650	132304481	111836515
		Improvement	12.0%

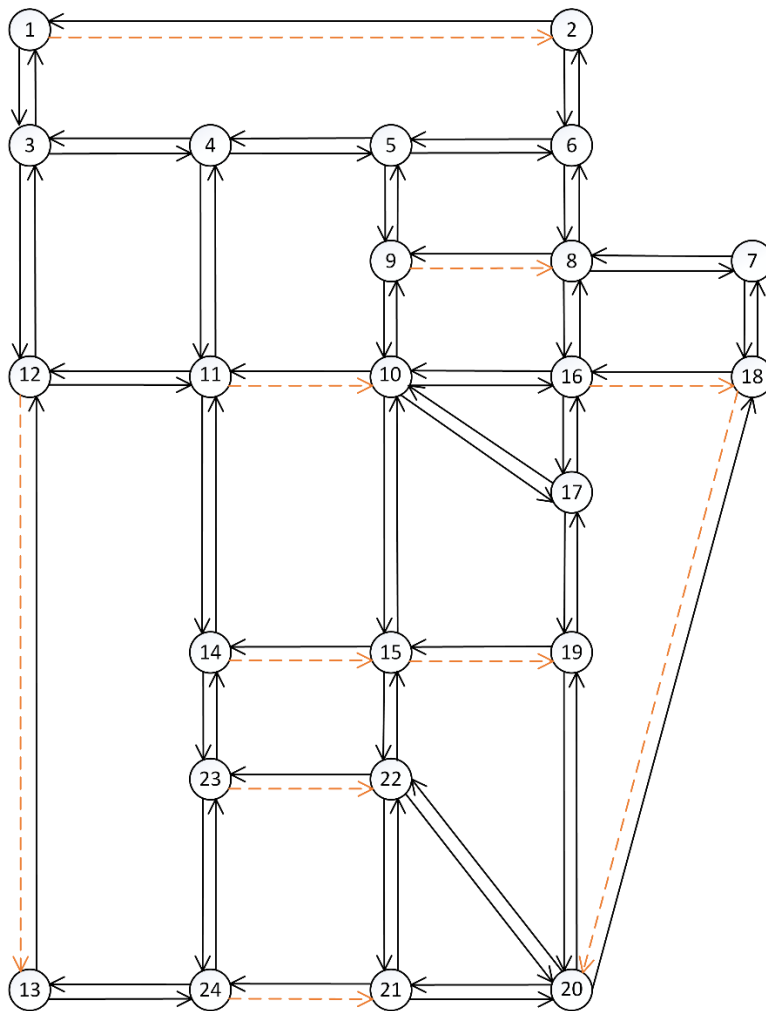
## 6.2 Example 2: Sioux Falls Network

To further demonstrate the real-world applicability of our model, we solve it for the transportation network of the City of Sioux Falls. Figure 6.2 shows the network of Sioux Falls. The yellow lines represent the links with candidate projects. The network data are derived from a study conducted by LeBlanc et al. (1975), and the attributes of all 10 candidate projects are given in Table 6.8. Other parameters are given as follows:

- 1) Planning horizon: 20 years (the planning horizon is equally divided into 240 design periods, namely 240 months); Construction period: 2 years; Benefit period: 18 years.
- 2) Weighting for construction period  $\alpha_1 = 0.5$ ; Weighting for benefit period  $\alpha_2 = 0.5$ .
- 3) Budget:  $B_1 = 15$ ,  $B_2 = 20$ .
- 4) Number of available crew teams:  $R^{max}=2$ .
- 5) Inflation rate  $\theta_1=0.01$ ; discount rate  $\theta_2=0.05$ .
- 6) Conversion factor:  $\beta=12$ .
- 7) Percentage of the workers' salary in the total construction cost:  $\lambda =0.1$
- 8) Overtime salary parameter:  $\mu=0.5$ .
- 9) Normal costs per period without overtime work:  $c_a=1$ .

**Table 6.8** Parameters for Candidate Projects in the Sioux Falls Network

Link	Lane capacity $C_a^1$	Maximum allowable capacity $C_a^{max}$	Number of closed lanes $k_a$	Maximum allowable shortened duration $D_a^{max}$	Fixed duration $D_a^0$	Extra duration for adding one lane $D_a^1$
(1,2)	13.0	40	1	4	8	8
(9,8)	3.0	12	1	2	3	3
(11,10)	5.0	15	1	1	3	3
(12,13)	13.0	50	1	2	4	4
(14,15)	3.0	9	1	1	2	2
(15,19)	8.0	32	1	1	2	2
(16,18)	10.0	40	1	1	2	3
(18,20)	12.0	36	1	2	4	4
(23,22)	2.4	8	1	0	1	1
(24,21)	2.4	8	1	0	1	1



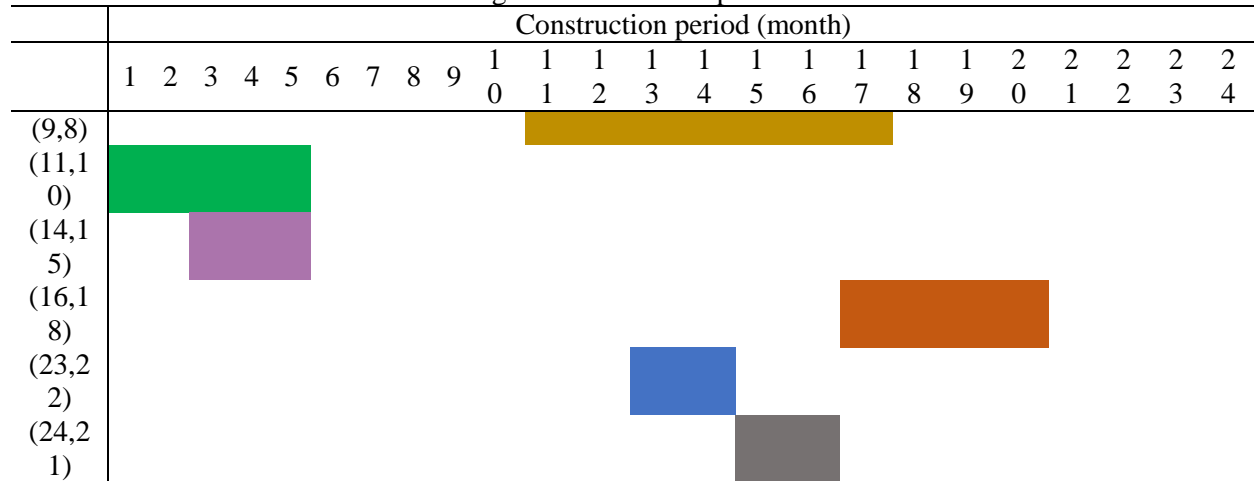
**Figure 6.2** Network of Sioux Falls

The ASA solution procedure is implemented using GAMS (Rosenthal, 2012) and CONOPT solver (Drud, 1994) on a Dell computer with a 3.4 GHz processor and 16.0 GB RAM. It takes 4 hours, 13 minutes, and 30 seconds to solve the model. The project selection and schedule results are provided in Table 6.9. To make the scheduling results more readable, Table 6.10 provides the graphical representation. We can observe that six projects are chosen, among which, four are chosen to be shortened by overtime work. The construction duration of the project on link (9,8) is shortened by two months, and for the other three projects on links (11,10), (14,15) and (16,18), the construction duration is shortened by one month. Total construction costs generated in the first and second year are 14.526 and 14.132, respectively, which are within the budget. According to the scheduling results, no more than two projects are under construction simultaneously. Hence, the resource constraint is also met. Without any road expansion projects, the total weighted net user cost will be  $5.662 \times 10^{10}$ . The selected road expansion projects will reduce the total weighted net use cost to  $4.274 \times 10^{10}$ . The overall system performance within the planning horizon is improved by 24.5%.

**Table 6.9** Selection and Schedule Results for Example 2

	Starting time	Ending time	Newly added lanes	Reduced construction duration
(9,8)	11	17	2	2
(11,10)	1	5	1	1
(14,15)	3	5	1	1
(16,18)	17	20	1	1
(23,22)	13	14	1	0
(24,21)	15	16	1	0

**Table 6.10** Illustration of the Scheduling Results for Example 2



## 7. CONCLUDING REMARKS

This study proposed a systems approach for selecting and scheduling M&R projects simultaneously. The primary significance of the model developed in this study is that it introduces a time dimension into the traditional NDP to consider the impact of road construction work and applies the overtime policy to further improve the design. The proposed model can solve the capacity expansion project selection and project scheduling problems simultaneously. The proposed T-DNDP model also allows for the addition of time-dependent resource constraints. We employ the active-set algorithm to solve this problem and test two numerical examples to demonstrate the effectiveness of the proposed model. The results show that the proposed T-DNDP model has the potential to provide better solutions than the conventional approach, which separately optimizes the selection and scheduling of road expansion projects. Note that, although this study focuses on the project selection and scheduling for one specific type of M&R project, i.e., road capacity expansion projects, the modeling framework and solution algorithm developed in this study can be easily modified to model the selection and scheduling of other types of M&R projects.

A number of research extensions can be considered in the future. For instance, the objective function of the proposed T-DNDP formulation only takes into account total system travel time. In future studies, we plan to integrate multiple objectives that are often considered by decision makers.

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