

Data-Driven Spatial Modeling for Quantifying Network-Wide Resilience in the Aftermath of Hurricanes Irene and Sandy

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1 DATA-DRIVEN SPATIAL MODELING FOR QUANTIFYING NETWORK-WIDE 2 **RESILIENCE IN THE AFTERMATH OF HURRICANES IRENE AND SANDY**

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1 ABSTRACT

- 2
- 3 In recent years, New York City metropolitan area was hit by two major hurricanes, Irene and
- 4 Sandy. These extreme weather events had major impacts on the transportation infrastructures,
- 5 including road and subway networks. As an extension of our recent research on this topic, this
- 6 study explores the spatial patterns of infrastructure resilience in New York City using taxi and
- 7 subway ridership data. Neighborhood Tabulation Areas (NTAs) are used as units of analysis.
- 8 The recovery curve of each NTA is modeled using the logistic function to quantify the resilience
- 9 of road and subway systems. Moran's *I* tests confirm the spatial correlation of recovery patterns
- 10 for taxi and subway ridership. To account for this spatial correlation, citywide spatial models are
- estimated, and found to outperform linear models. Factors such as the percentage of area 11
- 12 influenced by storm surges, the distance to the coast and the average elevation are found to affect
- 13 the infrastructure resilience. The findings in this study provide insights into vulnerability of
- 14 transportation networks and can be utilized for more efficient emergency planning and 15 management.
- 16
- 17 Keywords: Hurricane, recovery curve, resilience, spatial analysis, taxi and subway data
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- 21

1 INTRODUCTION

2

3 Hurricanes are one of the biggest natural disaster threats in the Northeast Corridor of the United

- 4 States. New York City (NYC), which is located in the vulnerable area of Northeast Corridor,
- 5 experienced two major hurricanes in recent years. On August 2011, Hurricane Irene made
- 6 landfall in Brooklyn, NYC. One year later, Hurricane Sandy landed in New Jersey, south of
- 7 NYC. As shown in Figure 1, both hurricanes caused inundation of coastal areas of NYC and
- 8 brought different levels of impact on city's transportation services and infrastructures: Hurricane
- 9 Irene led to inland flooding and temporary suspension of city-wide public transit. Since most of
- 10 infrastructures were intact, public transit was back to normal one day after the landfall. Hurricane
- Sandy, however, turned into one of the most costly natural disasters in the recent history of NYC. Unlike Hurricane Irene, several subway stations and tunnels were flooded, especially the
- 12 NYC. Unlike Hurricane Irene, several subway stations and tunnels were flooded, especially the 13 ones located in Lower Manhattan and Coney Island (1). Although Metropolitan Transportation
- Authority (MTA) restored half of the major service within a week after the landfall, it took
- 15 several months for stations seriously damaged to be fully functional, due to the mass erosion of
- 16 power supply and tube structure by salt water. Both hurricanes also caused disruption and
- 17 destruction of the highway network. Major bridges and tunnels were closed, and several tunnels
- 18 were flooded during Hurricane Sandy.
- After suffering from disruption and devastation of hurricanes, researchers started to show an increasing interest in strengthening the city infrastructure to avoid, or at least to mitigate
- 21 the impact of future coastal storms. Therefore, it is necessary to evaluate the resilience of
- 22 roadway and transit networks in terms of vulnerability to storm surge. Current 6-category
- evacuation zone system based on NYC's hurricane contingency plan identifies possible impact to the city districts. A recent study by the authors of this paper (2-4) explored the recovery patterns
- 25 of highway and subway networks, and developed multi-layer models for evacuation zones in
- 26 NYC(2). In this paper, logistic curves, which is frequently used for evacuation demand
- 27 modeling, was used for recovery modeling. The results showed a clear relationship between
- 28 recovery patterns and evacuation zone characteristics, and it seemed plausible that the road
- 29 network has better resilience than the subway system. However, since zones of the same
- 30 category are widely distributed, it is hard to quantify different levels of impact on areas in the
- 31 same category, and it is not trivial to distinguish damage caused to highway or subway networks 32 separately.
- 32 separately.33 As a follow-up to our previous paper wh
 - As a follow-up to our previous paper where the analysis is done in terms of evacuation zones of NYC (2), the goal of this study is to model the resilience of roadway and transit systems
 - 35 in terms of individual neighborhoods of NYC, and conduct statistical spatial analysis to explore
 - 36 inter-correlation of zonal resilience. Besides, this study explores the resilience of the same
 - 37 network for two different events, namely, Hurricanes Sandy and Irene. Compared with previous
 - 38 models based on evacuation zones, our new models can better reveal the spatial distribution of
 - 39 recovery characteristics, and make it possible to predict resilience of highway and transit
 - 40 networks based on the geographical location and hurricane intensity.



Figure 1 Areas influenced by storm surges during Hurricanes Irene and Sandy in NYC (5).

5 LITERATURE REVIEW 6

7 Transportation infrastructure, including road networks, subway stations and tunnels alike, are

8 faced with disruptions due to natural disasters like hurricanes. In recent years, researchers started
9 to show interests in ability of transportation systems to withstand and recover from the

10 disruptions, and the concept of resilience is introduced. Heaslip *et al.* (6) pointed out to two key

factors of resilience: How can the system maintain demonstrated level of service (LOS), or how

12 long it takes for system to restore to demonstrated LOS. Similarly, Bruneau *et al.* (7) introduced

13 "Resilience Triangle", to quantify three key issues of resilience, that is possibility of failure,

14 severity of outcome, and duration of recovery. They defined the area of the triangle as Loss of

Resilience (LoR), which can be mathematically represented in Equation (1):

17
$$LoR = \int_{t_0}^{t_1} [100 - Q(t)]dt$$
 (1)

18

2 3 4

19 where Q(t) is the time-dependent quality of infrastructure (7). Therefore, the LoR can be 20 determined by depth of initial disruption and speed of quality restoration, as the key issues stated 21 above.

21 uc 22

23 Transportation System Resilience

24

25 Testa et al. (8) measured the resilience of highway network of metropolitan area of NYC by

26 testing the topological graph properties under various scenarios of link removal. According to

27 Donovan and Work (9), NYC taxi data set can be used to measure roadway resilience of NYC

- 28 during Hurricane Sandy by measuring the deviation of normalized travel times between four
- 29 different regions of the city.

Hosseini and Barker (10) utilized Bayesian Network approach to quantify resilience as a function of adaptive and restorative capacities, and the model was demonstrated in a case of inland waterway ports. Adjetey-Bahun *et al.* (11) developed a simulation-based model to quantify resilience of the mass railway transit system in Paris. The model evaluates system resilience during perturbation by quantifying passenger delay and load. Simulation results

6 indicated resiliency of the system, which is consistent with observation. D'Lima and Medda (12)
7 utilized a mean-reverting stochastic model to explore daily fluctuations of London Underground
8 in terms of subway lines.

Logistic functions, as first proposed by Belgian mathematician Pierre Francois Verhulst
in 1838 to analyze population growth in Belgium (13), was widely used in pre and post-hurricane
studies. The concept of S-Curve was introduced by Lewis (14) to describe evacuation pattern
before hurricanes. Hobeika *et al.* (15) suggested the use of logistic curve based on behavior
research. Fu *et al.* (16) used post-hurricane Floyd survey of South Carolina to model the
evacuation response curve. Same models are proved to be effective to estimate evacuation

15 demand in Hurricane Andrew. Li and Ozbay (17) used traffic count data of Cape May County,

16 New Jersey during Hurricane Irene to build empirical response curve, which showed better fit

17 with logistic function. Logistic function was also used as a demand generation approach by

- 18 Ozbay and Yazici (18).
- 19

20 Spatial Analysis of Transportation Networks

21

22 Spatial analysis is widely used in safety assessment of transportation networks. Tasic and Porter

23 (19) built an area-wide model for Chicago to evaluate spatial association of safety issues and

24 multi-model transportation infrastructure, and found strong relationship of crashes and

25 availability of transportation service. Xie et al. (20) developed an incident duration model for

26 Hurricane Sandy and confirmed spatial dependencies of durations of neighboring incidents.

Spatial error and spatial lag models are further developed to indicate factors affect duration ofincident.

In this paper, previously proposed methods of resilience quantification and logistic modeling are used for NYC by sub-dividing NYC into small units based on Neighborhood Tabulation Areas. Then, factors affecting recovery patterns and resilience are identified and analyzed. Based on results of this highly detailed spatial resilience modeling approach, spatial dependence tests and further statistical modeling efforts are made to study resilience

characteristics for roadway and subway systems of NYC for two different Hurricanes.

35

36 **DATA**

37

38 In order to analyze resilience of NYC's highway and transit networks, two types of dataset are

39 used. One is NYC taxi trips data, which was made available by NYC Taxi & Limousine

40 Commission (TLC) (9, 21). The dataset contains taxi trips from year 2010 to 2013. Each trip

41 record includes time and location information of pick-ups and drop-offs. The other is subway

42 ridership data obtained from data the feed of Metropolitan Transportation Authority (MTA) in

- 43 terms of turnstile dataset (22), which is stored in individual weekly text files containing hour by
- 44 hour counts along with other related spatio-temporal information. Each row in the weekly file
- 45 contains a record of entry and exit counts, and the remote unit (station) and control area
- 46 (turnstile) that the counter belongs to. In normal situations, counter readings of each turnstile are47 recorded every four hours, but the time of reading differs among stations. In order to get the

1 ridership for each subway station, it's necessary to convert counter readings to turnstile ridership

- 2 by subtracting last and first reading of a day, and then calculate sum of all turnstiles. Although
- the subway dataset has fields of Staten Island Railway, insufficient records are found in study
 periods, therefore, transit network of Staten Island is excluded from the analysis.
- Since we aimed to track recovery patterns, for both hurricanes, 12 days after landfall were chosen as the study periods. Specifically, Aug 28 to Sep 8, 2011 for Hurricane Irene, and Oct 29 to Nov 10, 2012 for Hurricane Sandy. For comparison purpose, datasets of same periods of previous years are used. Since traffic in NYC has significant day-of-the-week pattern, we find days closest to days of week in the study period.
- Both datasets of taxi and subways include noisy and erroneous records, and it is crucial to select appropriate part and filter the data. For taxi trips, according to (9), there are significant amounts of error in taxi dataset, including missing or unrealistic coordinates, impossible travel times or speeds. For subway trips, errors including extremely low or high ridership values, which is caused by counter reset due to maintenance need to be filtered out.
- 15 Other datasets used in this study include socioeconomic demographic (SED) data of 16 NYC obtained from US Census Bureau¹, surge area data of both hurricanes from FEMA (5), 17 elevation data of NYC (23). For modeling purpose, these datasets were further featured into 18 levels of Neighborhood Tabulation Area (NTA) (24). Table 1 presents the description and
- 19 descriptive analysis of key variables. The explanatory variables are grouped into three
- categories: geographical, socio-economical and transportation. The computation of dependent
 variables listed in Table 1 is introduced in the next section.
- 22 23

	Description	Mean	SD
Dependent			
Variable			
TI_LoR	LoR for the taxi system during Irene	0.447	0.393
SI_LoR	LoR for the subway system during Irene	0.855	0.597
TS_LoR	LoR for the taxi system during Sandy	0.858	1.189
SS_LoR	LoR for the subway system during Sandy	4.787	2.063
Geographical			
Near_Dist	Distance to coast (10^3 feet)	5.617	4.251
Elevation	Average elevation (feet)	78.970	36.367
Pct_Surge	Percentage of area influenced by storm surges	0.107	0.192
Manhattan	1 if in Manhattan, 0 otherwise	0.149	0.357
Brooklyn	1 if in Brooklyn, 0 otherwise	0.262	0.441
Queens	1 if in Queens, 0 otherwise	0.297	0.458
Bronx	1 if in Bronx, 0 otherwise	0.195	0.397
Socio-economical			
Population	Total population in 2010 (10^3)	42.047	22.484
Edu_Bac	Population with bachelor's degree or higher (10^3)	9.704	10.117
Avg_Income	Average income $(10^3 \$)$	73.994	35.890
Employment	Number of the employed (10^3)	19.371	11.457
Schools	Number of schools	14.056	10.011

¹ Source: http://factfinder.census.gov

Length of roadways (mile)	48.083	28.431
Number of families with private vehicles (10^3)	6.992	3.903
Subway access time (min)	16.771	16.749
Number of bus stops	66.323	41.630
	Length of roadways (mile) Number of families with private vehicles (10 ³) Subway access time (min) Number of bus stops	Length of roadways (mile)48.083Number of families with private vehicles (103)6.992Subway access time (min)16.771Number of bus stops66.323

MODELING NEIGHBORHOOD-BASED RECOVERY PATTERNS

The main objective of this subsection is to propose recovery models and identify coefficients for all neighborhoods, and then find spatial correlations of model parameters. Travel modes and weather events are modeled separately.

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6

Using NTAs as Units of Analysis

10 First, the processed datasets of taxi trips and subway ridership will be mapped into subareas of

- 11 NYC thus it is necessary to determine unit of study from the very beginning. In this paper,
- 12 neighborhoods of NYC in terms of NTAs are used as the geographical units of modeling. NTA is
- 13 a set of polygons created by New York City Department of City Planning, and used for
- 14 representing data from Census and American Community Survey (24). There are overall 195
- 15 NTAs in NYC and each NTA corresponds to one Neighborhood with unique ID and name.
- 16 Figure 2 (b) demonstrates NTAs of NYC, colored by Borough the NTA belongs to. Compared
- 17 with evacuation zones shown in Figure 2 (a) (2), there are two advantages of selecting NTAs.
- 18 First, the sizes of NTAs are appropriate for the analysis, especially for subway data because
- 19 these areas are neither too big that they may cover more than one category of evacuation zones,
- 20 nor too small that they may not include even one subway station. Second, as mentioned above,
- 21 unlike Traffic Analysis Zones (TAZs) or Census Tracts, each NTA also has a familiar name, so
- 22 it's much easier to follow the travel patterns using NTAs.



(a) Evacuation zones based on TAZs

(b) Neighborhoods based on NTAs



1 Outputs of first step are daily taxi trips and subway ridership of each NTA for each 2 hurricane and study period, which were later converted into time-dependent recovery rates. The 3 rate of recovery is defined as the quotient of trips during a certain hurricane period divided by 4 trips during a corresponding normal (control) period. Recovery rates of 12-day period for all 5 NTAs are calculated. Then recovery rates are processed to conform to satisfy prerequisites of the 6 logistic model: Firstly, values greater than one are rounded to one. Also, if the recovery rate 7 reaches one, we assume that the area has already been recovered, then recovery rate is kept as 8 one for the remaining portion of the study period.

9 For NYC, majority of taxi trips are located in Manhattan, Downtown Brooklyn, densely
10 populated areas in Queens and Bronx and major airports. For other neighborhoods farther away
11 from these areas, taxi trips are much fewer. Also, subway service is not available in all of NTAs.

12 Therefore, NTAs with no data availability for specific travel modes are filtered out.

13

14 Modeling Resilience for each NTA

15

16 This section briefly describes the functional form used for modeling recovery rates for each

- NTA, performance of model calibration efforts, and definition of zonal resilience. More detaileddiscussion about this specific methodology are given in (2).
- Basic logistic function is used for modeling evacuation curves, as shown in Equation(2):
- 21

22
$$P_t = \frac{1}{1 + e^{-\alpha(t-H)}}$$
 (2)

23

24 where P_t represents recovery rate of area by time t, α is the factor affecting slope of the recovery 25 rate, H is half recovery time, in other words the time system reaches half of the service capacity. 26 Therefore, α and H can determine the shape of S-curve, which reflects recovery behavior and 27 resilience for each NTA.

Nonlinear Least Square Error (LSE), as shown in Equation (3) is used to fit the model
by comparing difference between modeled function and empirically obtained data points.

31
$$LSE = \sum_{t=t_0}^{t_1} (y_t - P_t)^2$$
 (3)

32

33 where y_t is the observed recovery rate of day t, P_t is logistic function (Equation (2)). The values 34 of t_0 and t_1 are 0 and 11. The objective is to minimize LSE, the difference between observed and 35 estimated recovery rates. For subway and taxi trips each NTA, distinct pairs of model parameters 36 (α and H) are calibrated to minimize S.

Another critical factor that need to be identified is LoR, which can be calculated using abovementioned model in Equation (1) (7). By using logistic function P_t to replace Q_t , Equation (1) can be rewritten as:

40

41
$$LoR = \int_{t_0}^{t_1} \left[1 - \frac{1}{1 + e^{-\alpha(t-H)}}\right] dt$$
 (4)

42

1 where LoR is the loss of resilience from the time original hurricane impact, which is the area 2 enclosed by the logistic function, y axis and line x=1 (100%).

The logistic functions are built for most of areas, except following situations: 1. Recovery rates of entire study period are one. In this case, the area was not affected by the storm surge, and LoR is zero. 2. Recovery rates of period are zero, which happened in transit network of certain NTAs in Hurricane Sandy, in which subway restoration takes longer than the study period, therefore the LoR is maximum, the value is 11 in this case.

8

9 Empirical Analysis of Resilience

10

11 Since this study covers four recovery patterns of two networks for two distinct weather events,

12 and each of them contains sub-models of most NTAs, it is not practical to show this multi-layer

13 model in a table format. Instead, recovery characteristics are visualized on a map of NYC in

14 terms of NTAs to show four abovementioned critical factors (α , *H* LSE and LoR), and each

15 figure has maps of four recovery scenarios (Irene Highway (Taxi), Sandy Highway (Taxi), Irene

16 Subway and Sandy Subway). To be able to provide a side-by-side comparison, subplots are

17 created using the same scale for four scenarios. Another point worth mentioning is the selection

18 of color gradient. Plots of all four terms use green and red gradient, but colors of start and end

19 points varied among terms, and greener plot always stand for better recovery situation or

20 goodness of fit of models. For α , since higher value stands for steeper slope of recovery function,

21 greener colors are used for higher values. For *H*, gradient is from green to red, since *H* is an

indicator of recovery time. LSE and LoR plots use red to show higher values, which stand for

worse curve fit of empirical data, and higher loss of resilience, respectively. As mentioned
 above, Neighborhoods with no data input are excluded from modeling, as shown in grey in the

24 above, Ive 25 figures.

26 Parameter α from the modeling results are visualized in Figure 3. From this Figure 3, it 27 can be seen that both highway and transit networks have higher α values for Hurricane Irene 28 compared with Hurricane Sandy, which implies faster speed (lower travel time) in the aftermath 29 of Hurricane Irene. in Figure 3 (a) and Figure 3 (b), α values of Manhattan and coastal 30 neighborhoods are lower than those from inland neighborhoods. In addition, for Hurricane Irene, 31 most of inland areas in Brooklyn and Queens are green colored, while only a small proportion of 32 these areas is shown in green for Hurricane Sandy. It can be inferred that magnitude of 33 disruption of highway network based on taxi data is greater for Hurricane Sandy. Compared with 34 the recovery of highway network, α values of subway network is relatively lower for both 35 hurricanes. Also, in Figure 3 (c), α values of most areas are similar, with the exception of few

36 NTAs, in which subway stations or depots suffered from storm surge. The values for Hurricane

37 Sandy for the entire city are significantly low, as shown from the wide range of red colored

38 zones in Figure 3 (d).

39



Figure 3 Parameter Alpha (slope of recovery rate).



Figure 4 Parameter H (the time recovery reaches half of service capacity).

4 Parameter H is shown on Figure 4. As mentioned above, H stands for the time that 5 network recovery reaches half of service capacity, therefore, a lower value of H implies shorter 6 recovery time. Based on Figure 4 (a) and Figure 4 (c), during Hurricane Irene, H values are 7 below 1 and nearly identical for most of NTAs. That means that both highway and transit 8 networks were back to full capacity in two days after the landfall of Hurricane Irene, due to the 9 limited impact of that storm. For Hurricane Sandy, as expected, subway network has much 10 higher H values. However, highway network in some neighborhoods tend to have lower H values than Hurricane Irene. Particularly, certain NTAs in Bronx has negative H values. The 11 12 negative value of *H* means that the initial recovery rate of the NTA is already greater than 50%. 13 One possible reason for this outcome is that these areas were not impacted by the hurricane.

1 According to Figure 1 (a), however, due to the suspension of subway service, more travel

2 demand might have been diverted to taxi mode.





4 5 6

Figure 5 Least square errors (LSE) of model results.

Figure 5 shows calibration results of estimated models. It can be seen that the LSEs for
taxi data based models are much lower than subway data based models, which implies better fit
of taxi models.

10 The hurricane induced LoRs are shown in Figure 6. It can be observed that both 11 networks were quite resilient during Hurricane Irene, compared with high LoRs in Hurricane

12 Sandy. The overall LoRs for taxi data tend to be lower than the ones based on subway data,

- 13 which was given in the conclusion section of the previous study (2). Besides, distribution of
- LoRs appears to be more spatially correlated for the highway network. As shown in Figure 6 (b),

- 1 neighborhoods located in Bronx are found to be more resilient than the ones in Manhattan and
- 2 Brooklyn. Also, from uptown to downtown Manhattan, LoRs gradually increase. The south tip of
- 3 Manhattan has highest LoRs, which is consistent with the map of Sandy surge zones presented in
- 4 Figure 1. Unlike taxi trips, the resilience of subway ridership is not as correlated spatially.
- 5 However, LoR for zones with damaged critical subway infrastructure is still significantly higher,
- 6 such as the ones in Lower Manhattan or Coney Island.



SPATIAL ANALYSIS OF HURRICANE-INDUCED LOSS OF RESILIENCE

2 3

1

Spatial Dependence Test for Loss of Resilience (LoRs)

From Figure 6, spatial clustering of LoR can be visually observed. To quantitatively analyze the
spatial dependence of loss of resilience (LoR), the Moran's *I* test proposed by Moran (1948)
(26) was conducted. Given its simplicity and intuitiveness, the Moran's *I* test has been widely
used to measure the spatial autocorrelation of continuous observations (27-30). The Moran's *I*test was used in a recent study by Xie et al. (20) to measure the spatial dependence of highway
incident durations. The Moran's *I* in matrix form is defined as (20) in Equation (5):

11

12	$I = (N / S_0)(\mathbf{d'Wd} / \mathbf{d'd})$	(2
14	$I = (IV / S_0)(U V U / U U)$	

- 13
- 14 where **d** is the vector of deviations of the LoRs from the mean, **W** is the spatial weights matrix
- 15 between each pair of NTAs, N is the total number of NTAs, and S_0 is the aggregation of spatial
- 16 weights $\sum_{i=1}^{N} \sum_{j=1}^{N} w_{ij}$. If the distance between the centroids of NTAs *i* and *j* is less than the
- 17 threshold distance, the spatial weight W_{ij} is defined by the inverse distance between them.
- 18 Otherwise, the spatial weight w_{ii} is set to be 0. The minimum threshold distance which could
- 19 ensure all the NTAs have at least one neighbor was used (31).

20 The pseudo p-value obtained from permutation test is recommended to assess the

21 significance of Moran's *I* (32). Pseudo p-value is defined as $\frac{M+1}{S+1}$, where M is the number of

22 instances with Moran's *I* equal to or greater than that of the observed data and S is the total

number of permutations. A total of 999 permutations were performed to compute the pseudo p value.

The results of Moran's I tests for highway and subway systems during Hurricanes Irene and Sandy are presented in Table 2. Please refer to Xie et al. (20) for definitions of statistics E[I], SD[I] and z_I . It is found that all the pseudo p-values are less than 0.05, and thus the spatial dependence of LoR can be confirmed. If spatial dependence is neglected in estimating LoR, it will result in biased statistical inferences.

30 31

	Tal	ble 2 Results of M	loran's Tes	ts		
	I	E[I]	SD[I]	Z_I	Pseudo p-value	
TI_LoR	0.1176	-0.0052	0.0035	3.5273	0.0070	_
TS LoR	0.1138	-0.0052	0.0310	3.8025	0.0060	
SILoR	0.3184	-0.0052	0.0345	9.3733	0.0010	
SS LoR	0.0093	-0.0052	0.0209	4.7621	0.0050	

³²

33 Interpolating Missing Loss of Resilience (LoRs)

34

35 In order to build the spatial model, the missing values in the input data has to be interpolated.

36 The main task is to estimate the missing LoRs in subway data. Typically, if there is no subway

of station are directly related to the distance. Therefore, for NTA without direct subway service,
its resilience could be represented by the ones of all nearby stations. Inverse Distance Weighting

- 4 (IDW) method is used to interpolate missing LoR data. The function of IDW is specified in
- 5 Equation (6).
- 6

7
$$f(x) = \frac{\sum_{i}^{i} w_i(x) y_i}{\sum_{i}^{i} w_i(x)}, w_i(x) = \left(\frac{1}{\|x - x_i\|}\right)^p$$
 (6)

- 8 where x_i are points with LoR values y_i . The default value of exponent p is 2, however, to avoid
- 9 bulls-eye effect (value near data point has sharp increase, as shown in
- 10 Figure 7 (b), where interpolation results are not so smooth as Figure 7 (a)), a value of 1 is used.





24 Please refer to Xie et al. (20) for more details on model specification and estimation.

- 25
- 26 Linear Model
- 27

1 A linear relationship is assumed between LoR and explanatory variables. In matrix form, it can 2 be expressed as:

$$\begin{array}{l}
3 \\
4 \\
\epsilon \sim N(0, \sigma^2 \mathbf{I})
\end{array}$$
(7)

5

6 where \mathbf{y} is the vector of LoRs, \mathbf{X} is the vector of explanatory variables such as surge

7 percentage, average elevation and population, β is the vector of regression coefficients to be

8 estimated and **I** represents the identity matrix. In the linear model, the error term $\boldsymbol{\varepsilon}$ is assumed 9 to be independent and identically distributed with mean zero and a constant variance.

10

Spatial Error Model
 Spatial Error Model

In the spatial error model, spatial dependence is captured via spatial error correlation (omitted variables at one site can affect the dependent variable of itself and its neighboring sites). The spatial error model in matrix form can be specified as:

17
$$\begin{aligned} \mathbf{y} &= \mathbf{X}\boldsymbol{\beta} + \lambda \mathbf{W}\mathbf{u} + \boldsymbol{\varepsilon} \\ \boldsymbol{\varepsilon} &\sim N(0, \sigma^2 \mathbf{I}) \end{aligned}$$
(8)

18

16

19 In the spatial error model, the overall error is represented by two components, namely, $\boldsymbol{\varepsilon}$ is a 20 spatially uncorrelated error term and **u** is a spatially dependent error term.

22 Spatial Lag Model

23

21

In the spatial lag model, spatial dependence is captured through both spatial error correlation
effects and spatial spillover effects (observed variables at one site can affect the dependent
variable of itself and its neighboring sites). The spatial lag model in matrix form can be specified
as:

29
$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \rho \mathbf{W}\mathbf{y} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon} \sim N(0, \sigma^2 \mathbf{I})$$
(9)

30

31 where ρWy is a spatially lagged dependent variable, ρ is a spatial autoregressive parameter, 32 and the rest notation is as before. The assumption of error term ε is the same as the one in the 33 linear model.

34

35 Model Assessment

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 R^2 is generally used to measure goodness-of-fit of model (33). However, since residuals of spatial models are not independent to each other, it is not appropriate to compare spatial models

- 39 using R^2 . Instead, criteria based likelihood estimation methods can be used, such as maximum
- 40 likelihood and Akaike Information Criterion (AIC) developed by Akaike (34) or Bayesian

2

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6

12 13 Information Criterion (BIC) first proposed by Schwarz (35). Equation (10) and (11) specify term of AIC and BIC:

$$4 \qquad AIC = -2LL_{\max} + 2k \tag{10}$$

$$BIC = -2LL_{\max} + k\ln(N) \tag{11}$$

where LL_{max} is the maximum of log-likelihood that can be obtained according to Xie *et al.* (20), 7

8 k is the parameter number and N is the sample size. If the AIC and BIC differences between 9 two models are greater than 4, then the two models can be regarded as considerably different; if 10 the differences are greater than 10, it provides a strong evidence that the model with a lower AIC 11 and *BIC* should be favored (36, 37).

14 **Results of LoR Models**

15 Results of three modeling strategies in terms of R^2 , AIC and BIC are displayed in Table 3. 16

According to Table 3, both spatial error and spatial lag models have greater R^2 values compared 17

with classic linear modeling. However, as mentioned above, due to dependence of residuals, R^2 18

19 should be used with caution. The likelihood based criteria of AIC and BIC are presented as well.

20 For scenarios of Highway (Taxi) Irene, Subway Irene and Subway Sandy, differences of BIC are

greater than 4, which means the spatial error model is considerably better than spatial lag model. 21

22 It indicates that the spatial autoregressive process occurs mainly in the error term. It can be seen

23 from Table 3 that models estimating LoR during Sandy yield better performance than those of 24 Irene. It can be revealed that the spatial correlation of LoR is stronger in Hurricane Sandy than

25 Irene. In addition, modeling results of taxi network are also better than subway. Overall, the

26 behavior of each model is consistent with the findings of the empirical analysis presented in the

27 paper.

28 Table 4 shows the modeling results of spatial error and spatial lag models. The autoregressive parameters λ in the spatial error model and ρ in the spatial lag model are also 29 reported. The selected factors for modeling vary in four occasions, and Pct Surge is found to be 30 31 the major contributor for all four scenarios. The spatial error model is used to evaluate effects of 32 variables. For the interpretation of signs of coefficients in Table 4, a positive sign implies an 33 expected increase in LoR, while a negative sign suggests an expected decrease. The exponents of 34 coefficients can be used to measure percentage change in dependent variable with one unit

35 change of explanatory variables, according to Tavassoli Hojati et al. (38).

36 According to spatial error and spatial lag models shown in Table 4, in all four 37 occasions, the LoRs of Taxi during Hurricane Irene is positively related to Pct Surge, that is 38 because the human activity and service status of infrastructure was directly affected due to

39 landfall. As shown in Table 4(a), the LoRs of taxi in Hurricane Irene is also positively

40 determined by TAT, this probably because in areas far away from transit service, people relies

41 more on taxi service, then lack of alternative modes cause less resiliency in service recovery. The

42 signs of zones in Queens and Brooklyn are negatively related to the LoRs, which implies that

43 taxi ridership of two boroughs was more resilient during Hurricane Irene. But this conclusion

44 only applies to Hurricane Irene, considering limited impact they had on two boroughs.

45 According to Table 4(b) the values of LoR for the highway network during Hurricane Sandy is positively related to Avg_Income and Roads_Mi. This is an interest finding that shows LoRs are also related to the zonal income level and density of roadway. Normally, the areas of higher average income in NYC are located either in uptown Manhattan, or areas in other boroughs with considerably lower density (such as Dyker Heights, Brooklyn), where residents prefer to use taxi for travel, so an extreme event could have more significant impact on taxi trips in such areas. The way Avg_Income affects LoRs can be explained by the fact that hurricane might cause

7 greater disruption to areas with longer mileage of roadway.

8 The LoRs of subway network in Irene is found to be negatively related to Near Dist, 9 Elevation and Population, which can be seen in Table 4(c). The first two are direct indicators of 10 vulnerability of storm surge. If the area is near the shore or if the elevation of an area is low, it's prone to hurricane landfall. Also the model reveals that NTAs with higher population tend to 11 12 have higher transit resilience, probably due to high priority of system recovery. According to 13 Table 4(d), transit resilience after Hurricane Sandy is positively related to Employment and 14 negatively related to Near Dist and Veh Own. The relation between LoR and Employment 15 shows the relationship between resilience and land use. Hurricane Sandy did have significant impact on commercial areas such as Lower Manhattan, and caused severe disruptions of business 16 17 activities. In addition, the subway network resilience is also related to auto ownership, as areas 18 with higher auto ownership are also more resilient in terms transit, which is partially due to the 19 fact that residents don't have to rely on public transit or to the insignificance of public transit as 20 an alternative mode of travel. It is noticeable the Veh Own for Hurricane Irene is positively 21 related to LoR. The main reason for the inconsistency may be because Hurricane Irene actually 22 didn't cause much damage to the system so the system is immediately restored in the aftermath 23 of the hurricane. It is reasonable to conclude that auto ownership affects LoR in a negative way. 24 Table 3 Model Comparisons 25

		Table	5 Milluci Col	mpai isons		
		TI_LoR			SI_LoR	
	Linear	Spatial Error	Spatial Lag	Linear	Spatial Error	Spatial Lag
R-Squared	0.138	0.162	0.152	0.115	0.118	0.115
AIC	169.344	165.726	169.085	252.666	252.293	254.632
BIC	185.709	182.091	188.723	278.03	277.657	283.166
		TS_LoR			SS_LoR	
	Linear	Spatial Error	Spatial Lag	Linear	Spatial Error	Spatial Lag
R-Squared	0.348	0.354	0.359	0.290	0.292	0.292
AIC	548.389	547.062	547.715	653.923	653.522	655.615
BIC	568.027	566.699	570.626	669.776	669.374	674.638

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28 29

Table 4 Modeling Results

(a) Irene Highway (Taxi) (TI_LoR)							
	Sp	Spatial Error			Spatial Lag		
	Coefficient	Std.Error	p-value	Coefficient	Std.Error	p-value	
Constant	0.4766	0.0367	< 0.0001	0.5897	0.0912	< 0.0001	
Pct_Surge	0.4119	0.3067	0.1792	0.4097	0.3318	0.2169	
Queens	-0.3145	0.0518	< 0.0001	-0.3656	0.0750	< 0.0001	
Brooklyn	-0.1558	0.0504	0.0202	-0.1875	0.0675	0.0055	
Sub Time	0.0052	0.0014	0.0002	0.0061	0.0018	0.0006	

λ	-0.3170	0.1663	0.0566	-	-	-
ρ	-	-	-	-0.2335	0.1578	0.1390
	((b) Sandy Hi	ghway (Tax	i) (TS_LoR)		
	Sp	oatial Error		S	Spatial Lag	
	Coefficient	Std.Error	p-value	Coefficient	Std.Error	p-value
Constant	-0.1539	0.2758	0.5768	-0.2051	0.2623	0.4343
Pct_Surge	1.0737	0.4244	0.0114	1.0173	0.4099	0.0131
Near_Dist	-0.0238	0.0194	0.2474	-0.0193	0.0179	0.2827
Population	-0.0097	0.0032	0.0026	-0.0089	3.176e-06	0.0052
Avg_Income	0.0153	0.0021	< 0.0001	0.0143	0.0021	< 0.0001
Roads_Mi	0.0061	0.0026	0.0169	0.0050	0.0025	0.0469
λ	0.1421	0.1435	0.3221	-	-	-
ρ	-	-	-	0.1689	0.1169	0.1486

	(c) Irene Subway (SI_LoR)					
	SI	oatial Error		Spatial Lag		
	Coefficient	Std.Error	p-value	Coefficient	Std.Error	p-value
Constant	0.9619	0.1431	< 0.0001	0.9340	0.2111	< 0.0001
Pct_Surge	0.7172	0.5383	0.1828	0.5893	0.5420	0.2769
Near_Dist	-0.0108	0.0089	0.2222	-0.0113	0.0093	0.2273
Elevation	-0.0026	0.0011	0.0158	-0.0025	0.0012	0.0355
Veh_Own	0.0443	0.0142	0.0018	0.0447	0.0148	0.0025
Roads_Mi	-0.0032	0.0017	0.0544	-0.0032	0.0017	0.0613
Population	-0.0036	0.0026	0.1587	-0.0037	0.0027	0.1591
Bus_Stop	0.0019	0.0011	0.0740	0.0018	0.0011	0.0965
λ	-0.1281	0.1837	0.4854	-	-	-
ρ	-	-	-	0.0326	0.1634	0.8418

4 5

	(d) Sandy Subway (SS_LoR)						
	Sp	oatial Error		S	Spatial Lag		
	Coefficient	Coefficient Std.Error p-value			Std.Error	p-value	
Constant	4.6873	0.4150	< 0.0001	4.3525	0.8037	< 0.0001	
Pct_Surge	4.4688	0.7079	< 0.0001	4.3400	0.7374	< 0.0001	
Elevation	-0.0028	0.0039	0.4760	-0.0024	0.0038	0.5211	
Veh_Own	-0.1491	0.0417	0.0004	-0.1444	0.0421	0.0006	
Employment	0.0393	0.0146	0.00707	0.0378	0.0145	0.0093	
λ	0.0941	0.1676	0.5743	-	-	-	
ρ	-	-	-	0.0690	0.1448	0.6336	

CONCLUSION

9 In this study, a NTA based statistically robust spatial model is proposed to identify

10 characteristics of the recovery patterns for highway and subway networks in NYC. One major

11 contribution of this study is the introduction of the notion of spatial dependence, which

12 complements the empirical analysis of recovery patterns presented in our previous paper (2).

1 Also, the estimated recovery models were built to represent the spatio-temporal recovery patterns

using the logistic function with two parameters, with which Loss of Resilience (LoR) of each
NTA can be calculated. Compared with evacuation zone based modeling, neighborhood based

4 models can provide more detailed information about the variations in recovery behaviors.

5 Moreover, instead of six logistic functions estimated for six evacuation zones in Zhu *et al.* (2),

6 the improved spatio-temporal model has 195 NTA's and corresponding recovery curves for both

7 hurricanes. This new approach makes it possible to conduct a comprehensive spatial analysis.

8 Empirical analysis of modeling results demonstrated that values of estimated model parameters 9 α , *H* and LoR varied greatly by individual storms, transport modes, and spatial locations. Highe

9 α , *H* and LoR varied greatly by individual storms, transport modes, and spatial locations. Higher 10 spatial clustering of resilience is observed during Hurricane Sandy, which has greater intensities.

The spatial dependence of LoR is also explored quantitatively in this study. By using 11 12 Moran's I test, it is confirmed that the LoRs are spatially correlated. Linear, spatial error and 13 spatial lag models were used to estimate the LoRs using geographical, socio-economical and 14 transportation features. The spatial error models outperform the others by presenting smaller AIC 15 and BIC values. Results indicate that the spatial autoregressive process occurs mainly in the error 16 term. Omitted variables are the major cause of spatial correlation. Factors such as the percentage 17 of area influenced by storm surges, the distance to the coast and the average elevation are found 18 to affect the infrastructure resilience with respect to hurricanes. It is likely that contributing 19 factors to the infrastructure resilience when confronting other disruptions such as earthquakes 20 and tornadoes would be different.

As a result of the introduction of a smaller modeling unit for the zones and the study of spatial dependence, this paper is able to provide a deeper insight into the vulnerability of highway and transit networks in NYC compared with previous studies (2-4). The spatial error and lag models for LoR can be used as an estimation tool of vulnerability assessment in response to future storms, by using socio-economic and projected surge zone information. These models can also be useful for government agencies and policy makers dealing with emergency management.

However it should be emphasized that the results presented in this paper may not be directly transferrable to other cities, considering the uniqueness and complexity of the transportation network in NYC. To predict recovery performance of post-hurricane recovery in other regions, this model needs to be re-calibrated using empirical data or simulated data from regional multi-modal network models.

The future improvement and calibration of this proposed methodology may consider other factors related critical corridors, especially additional factors from highway and subway lines, since their recovery patterns may resemble within a common corridor. Another future research direction is to investigate the factors contributing to the infrastructure resilience when faced with other types of natural disasters.

38

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40

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