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16. Abstract

Experimental determinations have shown that the resistance factors for conduits manufactured of corrugated metal vary over a wide range for each of the different corrugation forms now available. Hydraulic design procedures require a reliable determination of the resistance factor applicable to each specific form. Methods are developed, using all available data, whereby such resistance factors can be determined within close tolerances in terms of either the Darcy f or the Manning n. Variables considered include conduit size and shape, corrugation form, flow rate, and flow depth. Design charts and geometric tables are presented for the commonly available conduits of five corrugation forms, and examples illustrating use of the charts and procedures are given.

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HYDRAULIC FLOW RESISTANCE FACTORS FOR CORRUGATED METAL CONDUITS

Introduction

Corrugated metal sheets having a variety of corrugation forms are used to fabricate circular pipes and pipe-arch conduits commonly used as highway drainage structures. On the basis of early, limited hydraulic test results, a fixed coefficient, usually a Manning n value of 0.024, was often used to define the hydraulic resistance of all such conduits, regardless of size, shape, corrugation form, flow depth, flow rate, or method of manufacture. More recent experimental data (*1, 2, 3, 4, 5*)[1] on *standard* corrugated metal pipe (C.M.P.) with a 2⅔-inch pitch and a ½-inch depth have shown that these variables do, in fact, affect the resistance coefficient to different degrees and that use of a constant coefficient to define hydraulic resistance is not good practice.

Recent hydraulic model studies of conduits manufactured from steel structural plate with 6- by 2-inch corrugations (*6*) have contributed much to current knowledge of resistance to flow in corrugated metal conduits. These model studies have also provided resistance factors for 3- by 1-inch C.M.P. However, hydraulic test information of this type is not available for two new corrugation forms in use at present, or for other forms that may appear on the market in the future.

In hydraulic studies in which resistance coefficients have been determined by head-loss measurements for a range of flow rates, resistance data have been obtained, almost exclusively, by tests of circular pipes flowing full. Therefore, a method is needed not only to estimate the resistance coefficients for full flow in circular pipes made of the untested corrugation types, but also to determine, with equal reliability, resistance factors in pipe-arch sections and for partly full flow conditions in circular and pipe-arch conduits of all corrugation forms.

As will be shown later, the predominant characteristic that determines the C.M.P. resistance factor is relative roughness in terms of conduit size and depth of corrugation. In selecting a conduit-size dimension for a tabular or graphic presentation of resistance factors, either the diameter or the hyraulic radius can be used, as $D=4R$ for full circular pipes.

Because the more reliable hydraulic tests were performed on circular pipes flowing full, pipe diameter, rather than hydraulic radius, has been used as the conduit-size dimension in reports of these investigations. For most engineering applications, it is more convenient to use pipe diameter.

[1] Numbers in parentheses refer to corresponding references in Appendix A.

Accordingly, in this report, the actual inside diameter is given for the dimensions of circular pipes, and values of the Darcy f and Manning n are related to the diameters. The decrease in resistance factor as pipe size increases is significant, and indications are that it cannot be considered good practice to use a single value as an approximation for all available sizes of pipes having a particular corrugation type.

Similarly, resistance factors for corrugated metal pipe-arch sections, though determined from the respective hydraulic radii, also can be related to $4R$ for full flow conditions, in which $4R$ can be considered the effective diameter of the pipe-arch that corresponds to a circular pipe of equal resistance. Because of this relation, the same charts, in which f or n is plotted against circular-pipe diameter, can be used for pipe-arches.

Except for very shallow relative depths of flow (d/D) below about 0.4, in all approximate flow calculations for C.M.P. or pipe-arches it would be satisfactory in a given section shape and size to disregard the variation of resistance factor with depth of flow and to apply some common factor to all depths of flow. The degree of error in such a simplified procedure is defined later. When more exact computations are required, the charts and examples of chart use in this report provide for direct and more precise determination of resistance factors.

Design information on hydraulic resistance of five forms of corrugations is presented. This information can be used to select a culvert or storm-drain size for a given rate of flow and conduit slope, or to determine the depth of flow occurring in a long conduit of a given size. The usual methods available to hydraulic engineers can be applied for these purposes. The presentation of complete design solutions were not considered essential to the purpose of this publication.

Although the resistance factors derived for this publication differ somewhat from those in Public Roads Hydraulic Engineering Circulars No. 5 (*7*) and 10 (*8*), they were based on a more thorough study of the same basic data; but the differences are minor and revision of those circulars is not considered necessary.

The information given here applies explicitly to annular corrugations only; information is too scarce at this time to generalize solutions for helically corrugated metal pipe. However, the data that do exist indicate that the resistance of helical corrugations is less than that of annular corrugations; therefore, it is safe to use the resistance factors in this publication for helically corrugated metal pipe.

Background Information

Most experimental investigations of resistance factors in corrugated metal conduits have dealt with standard 2⅔- by ½-inch corrugations. Notable among these studies are those of the U.S. Army Corps of Engineers at the North Pacific Division Hydraulic Laboratory, (*1*), formerly Bonneville Hydraulic Laboratory, and the earlier work of Straub and Morris at the St. Anthony Falls Hydraulic Laboratory (*2*). These tests were concerned with corrugated pipes from 1.5- to 7-feet in diameter. Other tests on smaller standard corrugated pipes were conducted by C. R. Neill (*3*) on 15-inch pipe and by Chamberlain (*4*) and Garde (*5*) on 12-inch pipe, although the data from these experiments are more erratic than those of the North Pacific Division and St. Anthony Falls hydraulic laboratories, possibly owing to the greater relative roughnesses of the smaller diameter pipes.

Recognizing the errors that might result from applying the standard 2⅔- by ½-inch C.M.P. results to other corrugation types, especially to 6- by 2-inch structural-plate corrugated pipe, the U.S. Army Engineer Waterways Experiment Station (WES) in 1958 began hydraulic model studies of corrugated pipes with a 3:1 pitch-to-depth ratio. Relative corrugation depths corresponding to 2-inch-deep corrugations in 5-, 10-, and 20-foot diameter pipes were investigated. These tests, sponsored by both the U.S. Army, Office, Chief of Engineers, and the U.S. Department of Commerce, Bureau of Public Roads (BPR), resulted in a report published in 1966.

In addition to the results of model tests on corrugations with a 3:1 pitch-to-depth ratio, the WES report (*6*) also included the results of studies on a 1:4 scale model of a 5-foot-diameter standard 2⅔- by ½-inch C.M.P. These data differed from full-scale test results (*1*) both in velocity distribution and resistance coefficient. A possible explanation for this deviation is that it is difficult to reproduce precisely the ½-inch corrugations, which are only ⅛-inch deep when modeled at a 1:4 scale ratio. Also, plate 1 of the WES report indicates that the model may have had more sharply peaked corrugations than the full-size pipe. Therefore, the WES model studies of the 5-foot-diameter standard 2⅔- by ½-inch C.M.P. are excluded from the analysis here, and only the full-size standard C.M.P. results are considered.

Two separate hydraulic studies have been conducted on full-size 6- by 2-inch structural plate C.M.P. Neill (*3*) performed two series of tests on full-size 60-inch structural plate C.M.P., and Bauer Engineering, Inc. (*9*), tested a 14-foot-diameter power-plant cooling-water intake pipe in Baileytown, Indiana. These studies produced several resistance factors at various Reynolds numbers for the two pipe sizes, which will be discussed later. Data points for these factors are not illustrated in the accompanying figures, but the discussion will show that they generally verify the analysis methods of this report.

Some hydraulic flow tests were performed at low Reynolds numbers by A. H. Gibson (*10*) on a 1.8-inch-diameter corrugated copper pipe and by Rolf Kellerhals (*11*) on a 3.6-inch plastic model of the 60-inch structural plate C.M.P. tested by Neill. In both series of tests, the Reynolds-number range was below the practical limit for highway drainage-design use and in the area of sharply rising f values. In addition, the copper pipe tested by Gibson had a pitch-to-depth ratio of 4:1, which is an intermediate ratio between standard and structural plate C.M.P. Therefore, these test results were not included in the discussion and analysis reported here.

Objectives

As the aforementioned tests comprise all known data on C.M.P. resistance factors, it was considered desirable to devise a method by which the existing results on full, circular 2⅔- by ½-inch standard C.M.P. (pitch-to-depth ratio, $c/k=5.33$) and on 6- by 2-inch structural plate C.M.P. ($c/k=3.00$) could be correlated and systematized to estimate the hydraulic resistance of various conduits of other corrugation types. Such untested corrugations include 3- by 1-inch C.M.P. ($c/k=3.00$), 9- by 2½-inch structural plate C.M.P. ($c/k=3.60$), and 6- by 1-inch C.M.P. ($c/k=6.00$).

The pitch-to-depth ratios of the untested corrugation types are nearly the same as those of the tested types. Consequently, interpolation between the known values or extrapolation from them can be performed with little resultant error, if the difference in f owing to the corrugation form is not very large at the same relative roughness—that is, the same ratio of corrugation depth to hydraulic radius or diameter.

Hydraulic Resistance Factors

The hydraulic resistance factor, or coefficient, applicable to a conduit can be used to determine the rate of energy loss (rate of slope of the total head line) under a given condition of flow rate, conduit size, and depth of flow. The resistance factor also determines the hydraulic capacity, or flow rate, when the other conditions are fixed. Although in the design of unimportant conduits, the usual simplified practice has been to assume that the resistance factor (commonly the Manning n) is determined by the material forming the walls alone, and does not vary with pipe size or other factors, this assumption is not actually valid.

The experimental determinations of resistance factors for C.M.P. previously outlined indicate that resistance factors depend not only on the type of corrugation, but also on the pipe diameter. In addition, resistance varies to a lesser degree with the flow rate, although for some corrugation forms, this effect may be neglected for typical drainage discharge rates.

The fact that the resistance factor decreases as the pipe diameter increases indicates that resistance is significantly affected by the ratio of corrugation depth to the hydraulic radius of the pipe, $R=A/P$, or relative roughness.

As velocity distribution measurements, relating the increase in local velocity to the distance from the pipe wall, were obtained in some of the previously mentioned experimental studies on C.M.P., a usual method of fluid mechanics can be used. The velocity distributions can be expressed in terms of equations wherein the ratio of local velocity to shear velocity $(RS_f g)^{1/2}$ is related to the distance from the pipe wall. Subsequent integrations of such equations over the conduit flow areas give expressions for total flow rates in terms of shear velocity, which can be transformed into resistance factors. According to the form of these equations, the resistance factor is a function of relative roughness, in terms of the ratio of either pipe radius or hydraulic radius to corrugation depth (page 16 of the WES report (*6*) and Appendix E).

In material presented here, as in the WES report (*6*), the pipe diameter, $D=4R$ for full flow, is used as a more convenient dimension of conduit size, so that relative roughness can be stated as k/D, where k is the corrugation depth, in feet. To aviod the small decimal values that result from k/D, the graphical solutions for C.M.P. resistance factors are plotted against the reciprocal of relative roughness—D/k.

Because the experimental determinations of C.M.P. resistance factors show that both the Darcy f and the Manning n have maximum values for the smallest pipes of each corrugation form, and that the resistance decreases as the diameter increases, it follows that partly-full, uniform flow in a conduit of any given diameter will result in a series of resistance factors that correspond to the relative depths of flow. Methods for determining the resistance factor over a range of flow depths from full to 0.2 D in circular pipes and in the various shapes of corrugated metal pipe-arches are presented here. The variation of resistance factor with flow rate is accounted for, as necessary, in the methods presented for determining either f or n.

The relative effect of variations in either f or n on the flow capacity of a corrugated metal conduit is evident from the velocity, or discharge, equations for a given energy line slope, S_f. Using the Manning resistance factor n,

$$Q = AV = A \frac{1.486}{n} R^{2/3} S_f^{1/2},$$

the discharge varies inversely as n, and a 3 percent reduction in n results in a 3 percent increase in flow capacity. To use the Darcy resistance factor f, the usual form of the equation

$$S_f = \frac{h_f}{L} = \frac{f}{D}\frac{V^2}{2g}$$

can be modified to express flow velocity in terms of the hydraulic radius,

$$V = \left(\frac{2g\ 4R\ S_f}{f}\right)^{1/2}$$

Therefore, the flow rate can also be expressed in terms of the Darcy f,

$$Q = AV = A \frac{16.04}{f^{1/2}} R^{1/2} S_f^{1/2},$$

which means that a 6-percent reduction in f results in about 3 percent increase in flow capacity—comparable to the effect of a 3 percent reduction in n. As indicated by the above equation, n varies as $f^{1/2}$ for any particular conduit and flow depth.

These demonstrations of the effects of variations in the resistance factors on the computed discharge capacity of a conduit indicate that some error in estimating these factors is acceptable, especially since there are many variables connected with the flow conditions in C.M.P. In general, any resistance-factor-determination method resulting in errors of less than 6 percent in f or 3 percent in n can be considered adequately reliable for design computations.

Systematization of Available Data

The available experimental results emphasize the dependence of the C.M.P. resistance factor on the pipe Reynolds number, $N_R = VD/\nu$, in which V is the mean velocity of flow, D is the pipe diameter, and ν is the kinematic viscosity of water. In figure 11 of the report by the North Pacific Division Hydraulic Laboratory (*1*), and in plate 22 of the WES Report (*6*), it is shown that f initially increases as the Reynolds number is increased, reaching a peak for each pipe size. The pipe Reynolds number at the peak increases progressively as the pipe diameter is increased. Moreover, a comparison of the two illustrations indicates that in pipes of equal diameter, the Reynolds number for peak f is somewhat larger for 2-inch corrugations than for ½-inch corrugations.

However, it was determined that the use of a wall Reynolds number, N_{Rw}, in place of the pipe Reynolds number, N_R, aided to systematize the data, as the maximum value of the Darcy f would occur at the same N_{Rw} for all pipe sizes with a given corrugation depth.

The wall Reynolds number is defined as:

$$N_{Rw} = \frac{v^*k}{\nu} = \frac{k(RS_fg)^{0.5}}{\nu} \quad \text{------------------------} (1)$$

Where,

v^* is the mean shear velocity, ft./sec., = $(RS_fg)^{0.5}$,

R is the hydraulic radius, ft., = $D/4$ for full flow in circular pipes,

S_f is the friction slope—slope of the total energy line, equal to the slope of the hydraulic grade line in pipes flowing full,

g is the gravitational acceleration, 32.16 ft./sec.^{2},

k is the corrugation depth, ft., and

ν is the kinematic viscosity, ft.^{2}/sec.

In addition, by plotting the Darcy f against the ratio of N_{Rw} to corrugation depth, k, the f for either the ½-inch-deep or the 2-inch-deep corrugations will peak at about the same N_{Rw}/k for all pipe sizes of either corrugation type. This fact is helpful in the following interpolation methods, although its physical significance is unknown at this time.

In figure 1, these plots of N_{Rw}/k and f are shown for the experimental data from the previously named sources (*1, 2, 3, 4, 5, 6*) and apply to both standard 2⅔- by ½-inch C.M.P. and 6- by 2-inch structural plate C.M.P. The results for full-size 6- by 2-inch structural plate C.M.P. (*3, 9*) are neglected at this point, but are discussed later.

The curves of figure 1 are based on wall Reynolds numbers computed from the true kinematic viscosity, ν, of the water during each test. To apply the relationships of figure 1 to general solutions, it is convenient to use an average kinematic viscosity for water, at a temperature of 60°F., for which,

$$\nu = 1.217 \times 10^{-5} \text{ ft.}^2\text{/sec.}$$

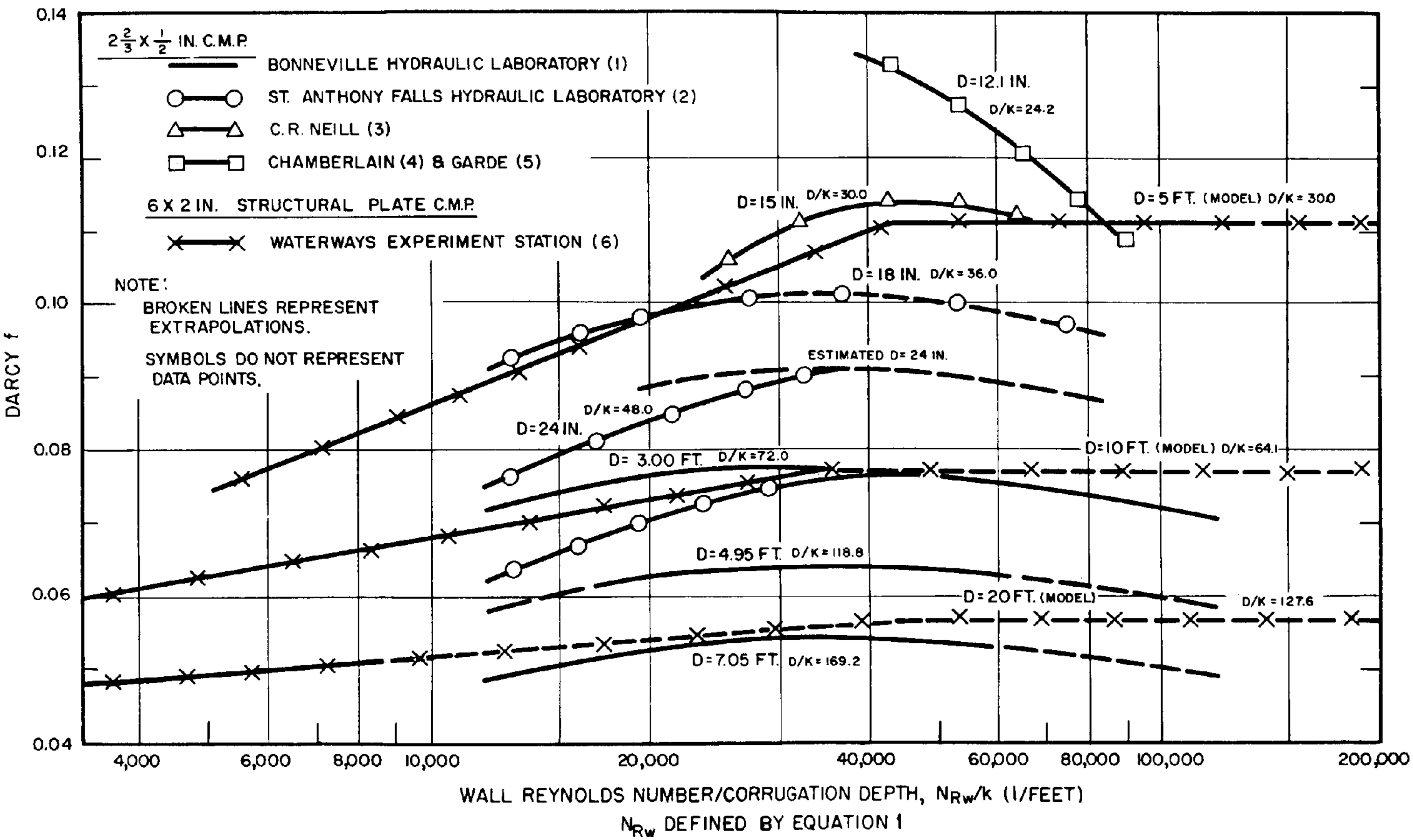

Figure 1.—Wall Reynolds number divided by corrugation depth plotted against Darcy f.

This is permissible as it can be shown that rather wide differences in water temperature ($\pm 10°F.$) affect resistance factors by insignificant amounts.

The curves for standard C.M.P. (2⅔- by ½-inch) seem to reach a peak f in the vicinity of $N_{Rw}/k = 30{,}000$–$40{,}000$ after which they decrease as the Reynolds number increases. This premise is supported by the North Pacific Division Hydraulic Laboratory data (*1*) on 3-, 5-, and 7-foot diameter pipe.

The same phenomenon may occur in the structural plate corrugated pipe (6- by 2-inch), but the paucity of data in the high Reynolds number range precludes any definite determination. Therefore, for the structural plate C.M.P., it was decided to estimate conservatively that f remains constant after peaking. Support for this assumption is given in Appendix E.

Although in figure 1 the curves for similar relative roughnesses—in terms of pipe diameter and corrugation depth—have unlike shapes on either side of their peaks, the peak f values of corrugated pipes with unlike pitch-to-depth ratios (c/k) seem to be related. To further investigate this relation, the solid curves of figure 2 were drawn to relate the peak f values to the reciprocal of relative roughness, D/k.

The lower curve of figure 2, curve No. 1, is based on an integration of the semi-logarthmic form of the velocity-distribution equation obtained from measurements of velocity in scale models of structural plate C.M.P. having a 3:1 pitch-to-depth ratio (*6*). Curve No. 1 applies to 6- by 2-inch structural plate C.M.P. excluding bolt resistance, and to 3- by 1-inch C.M.P. The number of bolts in structural plate C.M.P. differs with the number of longitudinal seams as well as with pipe diameter. The absence of bolts in the 6- by 2-inch corrugation models facilitates comparisons with standard 2⅔- by ½-inch C.M.P. on the basis of relative roughness alone.

To define a similar peak f — D/k curve for standard 2⅔- by ½-inch C.M.P., peak f values were selected from figure 1 and plotted in figure 2, as indicated by the circled points. Although these points do exhibit scatter, all except the point for 18-inch C.M.P. fall above the curve for the 3:1 pitch-to-depth ratio, indicating that f increases as pitch-to-depth ratio increases. This trend is contrary to the one that might be intuitively expected, but it may be caused by a different type of vortex shed in the more widely spaced corrugations.

Some of the curves of figure 1 were based on more consistent data than others, as indicated by the curve analysis in table 1. Based on this analysis, the peak f values for 12.1- and 18-inch-diameter C.M.P. were given little consideration in locating a peak f—D/k curve for the 2⅔- by ½-inch C.M.P. The remaining circled data points in figure 2 can be represented well by a line, curve No. 2, drawn parallel to curve No. 1 at an f increment of 0.004. Use of a constant f increment facilitates interpolation and extrapolation for corrugations of other pitch-to-depth ratios, and according to the available data, more refined curve-fitting methods are not necessary. Thus curves No. 1 and 2 in figure 2 represent peak f—D/k plots for the tested corrugation types, with 3:1 and 5.33:1 pitch-to-depth ratios. Such a large difference in corrugation form produces little difference in the resistance factor for a given relative roughness.

Table 1.—Analysis of data for f-N_{Rw}/k curves of figure 1, 2⅔- by ½-inch. C.M.P.

D	$\frac{D}{k}$	Data scatter	Peak f definition	Reference
12.1 in.	24.2	Wide	Uncertain	(*4, 5*)
15 in.	30.0	Medium	±0.003	(*3*)
18 in.	36.0	Narrow	Estimated	(*2*)
24 in.	48.0	do	±0.002	(*2*)
[1] 3.00 ft.	72.0	do	±0.002	(*1, 2*)
4.95 ft.	118.8	do	±0.001	(*1*)
7.05 ft.	169.2	do	±0.001	(*1*)

[1] Best curve to define declining curvature.

Full-Size Hydraulic Tests of Structural Plate C.M.P.

In Neill's studies of full-size 60-inch structural plate C.M.P. (*3*), most of the tests were performed under partly full flow conditions, making an accurate resistance factor determination difficult. All tests in the first series were in a free-surface condition; however, two tests in the second series were in a full-flow condition owing to the submerged outlet. The resistance factors, in terms of the Darcy f, averaged about 0.14, which is high compared to the WES model results and analyses (*6*) for 5-foot structural plate C.M.P., which included increases in f to account for the assembly bolts.

In Bossy's discussion (*12*) of Neill's paper, the following three suggestions were given to explain the possible overestimation of the resistance coefficients.

- The nominal diameter (5.0 ft.) was used in resistance coefficient calculations rather than the actual diameter (4.93 ft.), which should be used.
- The weir coefficient used in determining flow rate may be too low, resulting in an underestimation of Q.
- The free surface determinations of n include inlet and outlet effects that increase the apparent slope of the water surface profile.

Neill, in his closure (*13*), presents revised resistance coefficients based on the true pipe diameter. The following f values were computed for full flow tests S2 and S3, including bolt effects:

Test	Velocity	N_{Rw}	N_{Rw}/k	Revised f
S2	6.91	13,000	78,000	0.130
S3	7.60	14,400	86,400	0.132

The above tests are in the range of wall Reynolds numbers for which a constant value of f is indicated. Later it will be shown that bolts contribute an increment, Δf, of about 0.0085 to resistance in a 6- by 2-inch structural plate pipe with a 4.93-foot diameter. The WES model tests results applied to a pipe of this size produce an f value of 0.1115, which must be increased by the bolt Δf to 0.120 for comparison with the above f values.

Therefore, the revised f values presented by Neill (*13*) are about 0.01 higher than those of the WES model results. An underestimation of flow rate owing to the weir coefficient could explain this difference, as suggested by Bossy (*12*).

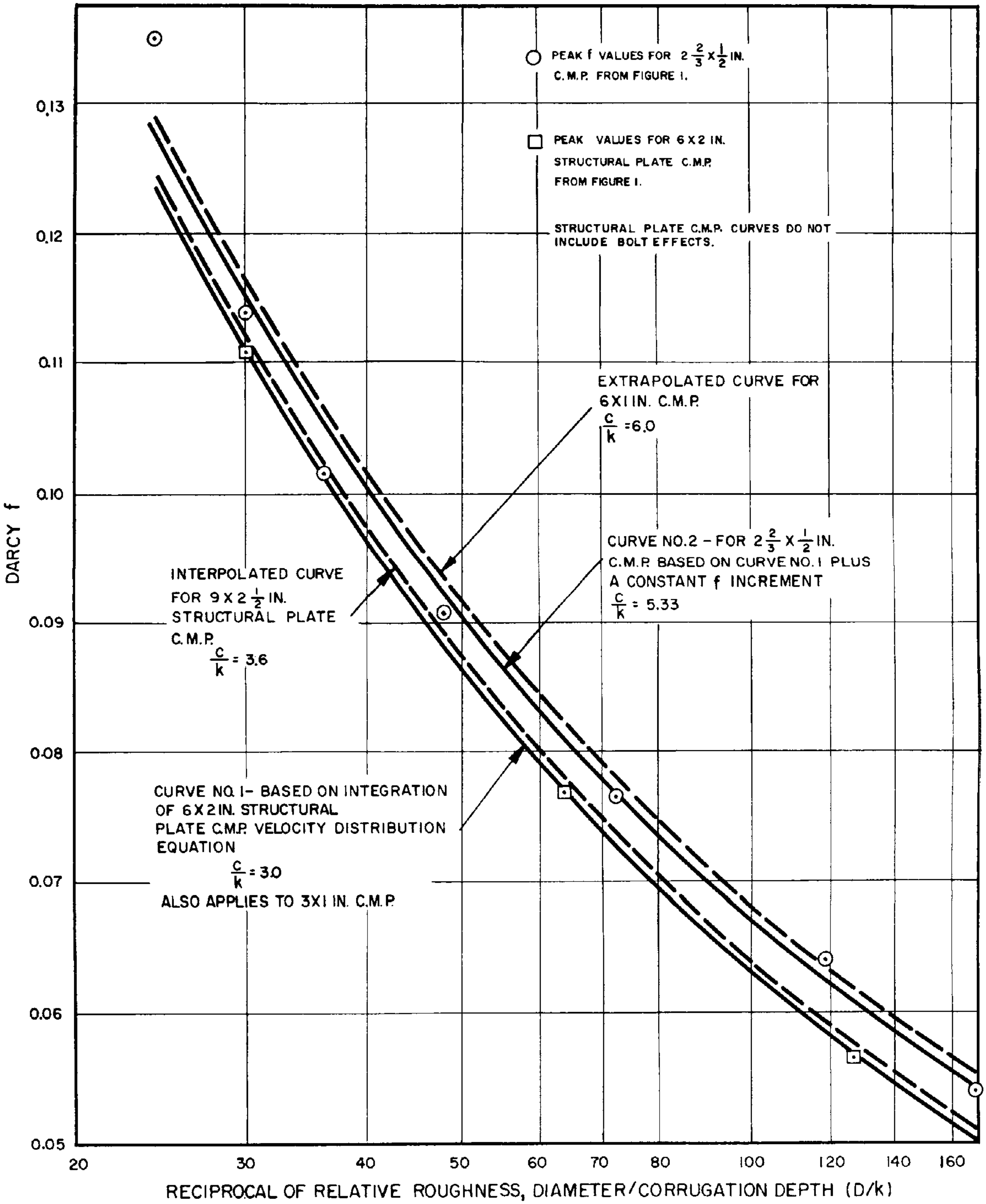

Figure 2.—Relative roughness plotted against Darcy f for corrugated metal pipe of various cycle length-to-depth ratios.

The Bauer Engineering tests (*9*) were conducted on a completely submerged full-size 6- by 2-inch structural plate C.M.P., 1,526 feet long and 14 feet in diameter. Two flow rates, based on the capacity of the power plant intake pumps, were studied. The flow rates were determined from velocity-distribution measurements obtained from both horizontal and vertical scans for the lower flow rate and from a horizontal scan only for the higher flow rate. Most velocity-distribution measurements were derived from current meter readings, but a pitot tube was also used in the horizontal scans as a check.

The total head loss, including pipe friction as well as minor inlet, bend, and outlet losses, was determined by measuring the difference between the water levels upstream and downstream of the pipe. The Bauer analysis of the results produced f values for the low and high flow tests that were significantly lower than the value for a similar pipe presented in the WES report (*6*).

The WES report value is based on an integration of the velocity-distribution equations for 6- by 2-inch structural plate C.M.P. (See Appendix E.) The resultant f value of 0.0675 is then increased by 0.004 to account for the bolt resistance increment (Δf), producing a total f value of 0.0715. This total f applies to the constant f range above the peak, as shown in figure 1.

As the flow rates in the Bauer tests were not sufficient to attain the constant f range (N_{Rw}/k greater than 30,000–40,000), the peak f stated above must be reduced somewhat for comparison. Based on the rising portion of the WES structural plate C.M.P. curves in figure 1, the peak f value should be decreased by 0.0045 for the lower flow rate and by 0.0020 for the higher flow rate, resulting in f values of 0.0670 and 0.0695, respectively. These latter values represent the resistance factors that would be derived by the methods of the WES report at the lower Reynolds numbers of the field tests.

A subsequent analysis of the Bauer data by the Public Roads staff produced f values nearly identical with those of the WES results. The main modification in the reanalysis was in the evaluation of the minor loss velocity head coefficients, which appeared to have been overestimated in the Bauer report. The overestimation of the minor losses caused an undestimation of the pipe friction head loss, producing a low f value. The revised f values computed by the Public Roads staff were 0.0675 for the low flow tests and 0.0650 for the high flow test.

The revised test values are within 1 percent of the WES f values for the low flow tests and within 6.5 percent for the high flow test. The high flow test results were somewhat less reliable as only one velocity scan was made. An additional reason to question the reliability of the high flow test is that the Bauer f values exhibit a downward trend with increasing N_{Rw}/k in the range below $N_{Rw}/k = 30{,}000$, whereas all other data in figure 1 indicate an opposite trend.

These field tests of full-size 6- by 2-inch structural plate C.M.P. (*3*, *9*) are necessarily limited because of the large size of the conduits, which required high flow capacities. However, the test results largely substantiate the analyses of this report and verify the resistance factors presented in figures 8 and 12, which were derived from the WES model investigations. Therefore, it is recommended that C.M.P. design be based on these figures and the methods set forth herein. Additional tests of large diameter structural plate pipes would be desirable but are not considered essential to an adequate design method.

Corrugation Types Considered

The corrugation types discussed here, including the aforementioned types for which hydraulic test results are available, 2⅔- by ½-inch and 6- by 2-inch, are shown in figure 3. Resistance factors for 3- by 1-inch C.M.P. can be obtained, at a given D/k, from curve No. 1 in figure 2, which also applies to 6- by 2-inch structural plate corrugated pipes without bolts. The 6- by 1-inch and 9- by 2½-inch corrugations (6.0:1 and 3.6:1 pitch-to-depth ratios, respectively) must be estimated from the results of the tested corrugation types.

On the basis of similar pitch-to-depth ratios, it is assumed that the 6- by 1-inch corrugations have a resistance coefficient that varies with the Reynolds number in the same manner as that of 2⅔- by ½-inch corrugations—increasing to a peak and then decreasing with increasing wall Reynolds number. In addition, because the pitch-to-depth ratios of the 9- by 2½-inch, the 3- by 1-inch, and the 6- by 2-inch corrugations are comparable, it was estimated that their resistances behave similarly—increasing to a peak and then remaining constant with increasing wall Reynolds number.

The corrugation types considered in this report and their classification according to pitch–depth ratio and variation of f with Reynolds number are as follows:

Corrugation types that peak and then remain constant with increasing Reynolds number:

Pitch–depth	Ratio (c/k)
6 : 2	3.00
3 : 1	3.00
9 : 2½	3.60

Corrugation types that peak and then decrease with increasing Reynolds number:

Pitch–depth	Ratio (c/k)
2⅔ : ½	5.33
6 : 1	6.00

Methods of Estimating Resistance Factors for Untested Corrugation Types

To temporarily neglect Reynolds-number effects, a straight line interpolation and extrapolation was performed between peak f values only, as shown in figure 4. The interpolation indicates that an f increment of 0.001 should be added to curve No. 1 in figure 2 to obtain the peak f—D/k curve for 9- by 2½-inch structural plate corrugated pipe, and the extrapolation shows that an f increment of 0.0052 should be added to curve No. 1 to obtain a similar curve for 6- by 1-inch C.M.P. This latter increment is equivalent to an increment of 0.0012 added to curve No. 2. In neither the interpolation nor the extrapolation is the difference between the base and estimated curve of any significant magnitude—of the order of 1 or 2 percent of the total f. These increments

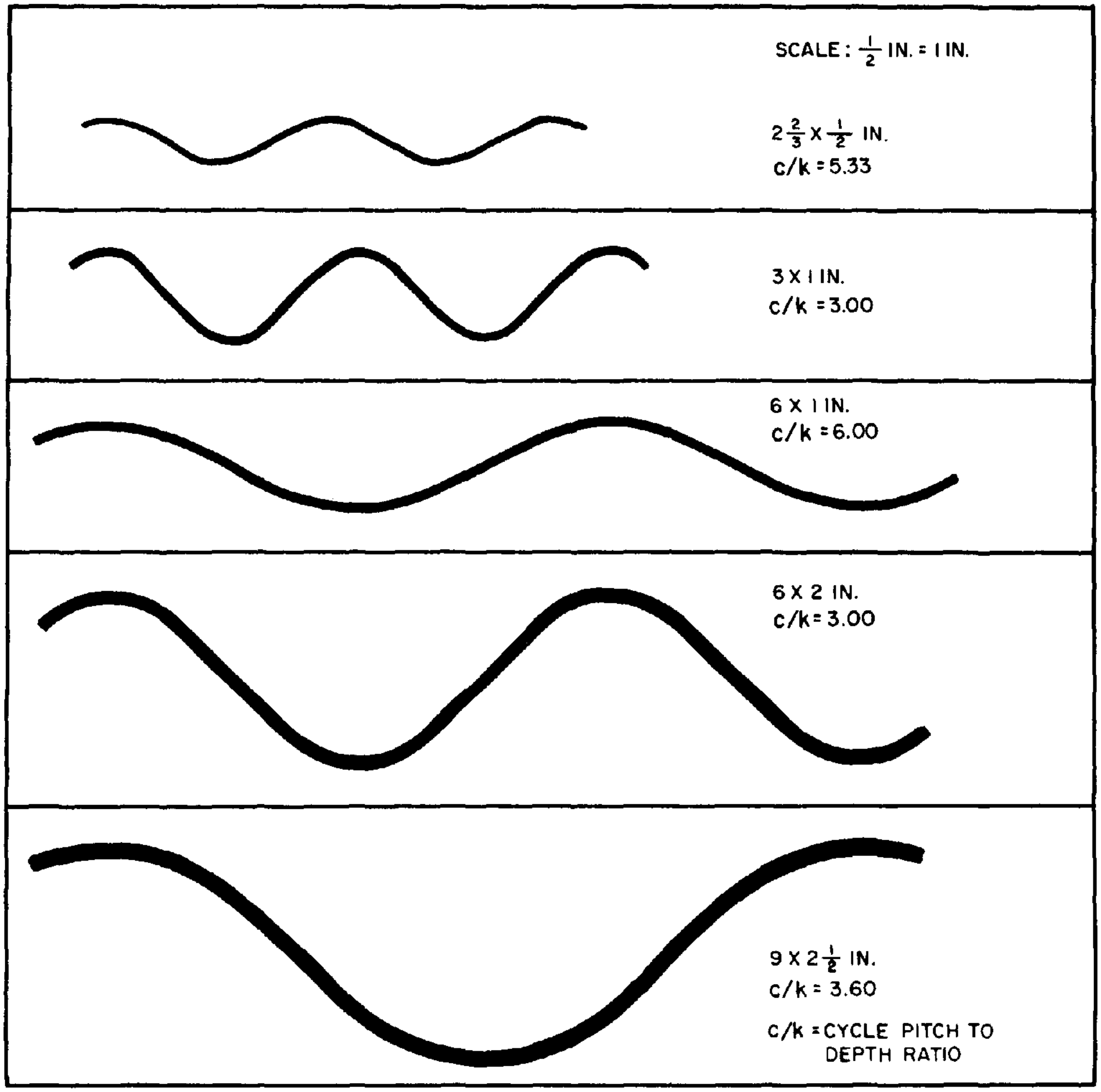

Figure 3.—Types of corrugations investigated.

were used to draw the dashed curves in figure 2 for the untested corrugation types and thereby produce peak f—D/k curves for each corrugation type pictured in figure 3.

The Reynolds-number effect was considered next. In general, highway storm drains operate at a flow, Q, of about 2.0 $D^{2.5}$, and culverts at a higher flow rate of about 4.0 $D^{2.5}$. Because the f values do not change rapidly with changes of Reynolds number, as shown by figure 1, it was decided to base design tables and graphs on these two flow rates and to interpolate resistance factors for other flows, which can be done with sufficient accuracy.

For 6- by 2-inch, 3- by 1-inch, and 9- by 2½-inch corrugations, the f of which is assumed to peak and then remain constant with increasing wall Reynolds number, the only determination necessary is whether the hydraulic properties produce an N_{Rw} large enough to create an f within the constant range at its peak value. If so, Reynolds-number effects can be ignored for these corrugation types.

It is shown in equation (1) that

$$\frac{N_{Rw}}{k} = \frac{v^*}{\nu} = \frac{(RS_f g)^{1/2}}{\nu}$$

From the basic form of the Darcy equation, $h_f = f\,\frac{L}{D}\frac{V^2}{2g}$, where h_f is the friction head loss, feet; L is the length of conduit considered, feet; and V is the mean flow velocity, feet/second; the equation $S_f = \frac{f}{4R}\frac{V^2}{2g}$ can be evolved by substituting $S_f = h_f/L$ and $4R = D$. If this equation for S_f is introduced into equation (1) the following relation results:

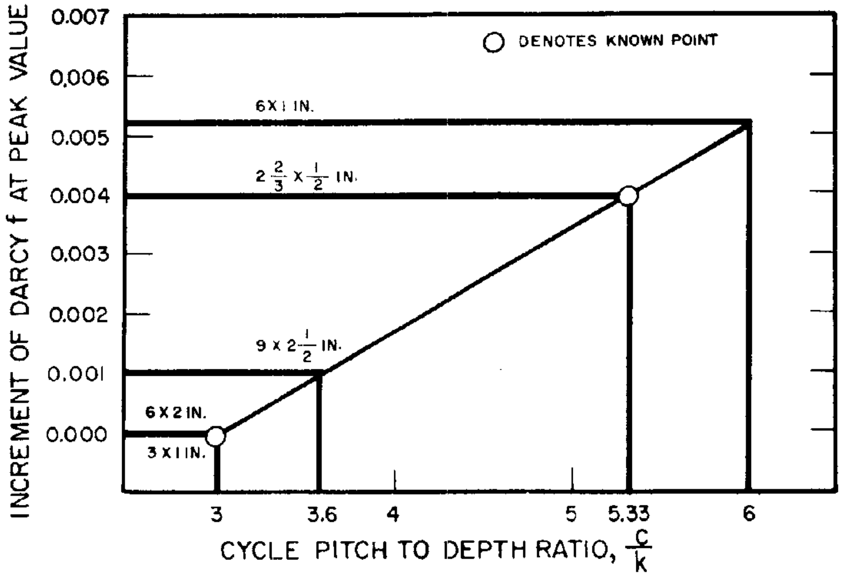

Figure 4.—Determination of f increment at peak value.

$$\frac{N_{Rw}}{k}=\frac{f^{0.5}V}{2.828\nu}=\frac{f^{0.5}Q}{2.828A\nu} \qquad (2a)$$

or, in dimensionless terms:

$$\frac{N_{Rw}}{k}=\frac{(f)^{0.5}\,(Q/D^{2.5})\,(D)^{0.5}}{2.828\ (A/D^2)\quad \nu} \qquad (2b)$$

Where, A is the area of flow.

According to figure 1, the peak f is always reached at a N_{Rw}/k of 40,000 or slightly more for the 6- by 2-inch structural plate C.M.P. As shown in equation (2b), the lowest N_{Rw}/k will be reached when $Q/D^{2.5}$ and D are minimum and A/D^2 is maximum. ν is assumed constant at 1.217×10^{-5} (for water at 60°F.), and f can be considered at its peak and within the constant f range. Then, if the N_{Rw}/k for a full flow Q of $2.0\,D^{2.5}$ is more than 40,000 for the smallest diameter pipes made of 6- by 2-inch, 3- by 1-inch and 9- by 2½-inch corrugations, the Reynolds-number effect need not be considered for these corrugations. Partly full flow, of course, results in a lower A/D^2 than full flow and, therefore, a higher N_{Rw}/k for the same flow rate $(Q/D^{2.5})$.

As shown in table 2, the above hydraulic conditions produce an N_{Rw}/k greater than 40,000 for all three corrugation types. Accordingly, Reynolds-number effects can be ignored for these corrugation types in highway drainage use, meaning that f does not vary with the rate of flow in a pipe of given size for discharges of general interest—greater than $2.0\ D^{2.5}$. Some lesser discharges will be considered later when shallow flow depths, less than one-half full, are discussed.

Table 2.—Lowest N_{Rw}/k at $Q = 2\ D^{2.5}$ for corrugated metal pipes with assumed constant f after peaking (T = 60°F.)

Pitch-depth	Smallest diameter available	$\frac{D}{k}$	f (from fig. 2)	$\frac{N_{Rw}}{k}$
Inches	*Feet*			
3 by 1	3.0	36.0	0.1012	40,800
6 by 2	4.93	29.6	[1] 0.1117	54,900
9 by 2½	6.42	30.8	[1] 0.1103	62,300

[1] Not including bolt resistance.

Conduits that are made of 2⅔- by ½-inch corrugated metal and have a larger pitch-to-depth ratio have an f that peaks and then decreases with further increases in Reynolds number, or rate of discharge, as described earlier. (See *Systematization of Available Data.*) The f—N_{Rw}/k curves of figure 1 for the 2⅔- by ½-inch C.M.P. tests are somewhat inconsistent as the peak f values are reached at N_{Rw}/k values that vary within a limited range, and the curvatures of the plots for different diameters differ, particularly for the 12.1-inch and 15-inch pipes. The first step in dealing with these corrugations was to systematize the curves of figure 1 by defining a common peak location, in terms of N_{Rw}/k, as well as a common curvature. The most complete data describing the downswing portion of the f—N_{Rw}/k curve beyond the peak were the North Pacific Division Laboratory (*1*) data on the 3.00-foot diameter pipe (D/k = 72.0). In the vicinity of the peak, these data are substantiated by the St. Anthony Falls Hydraulic Laboratory data (*2*) on a pipe of the same size.

The 3-foot diameter, 2⅔- by ½-inch C.M.P. has a relative roughness that is about average for all the pipes studied, and the curvature of the declining portion of its f—N_{Rw}/k curve fits the curves for other pipe diameters fairly well, being flatter than the curves of figure 1 for smaller D/k ratios and steeper than the corresponding curves for larger D/k ratios. For these reasons, it was decided to use the shape and horizontal position of the f—N_{Rw}/k curve for the 3-foot pipe to represent all other relative roughnesses. The location of the peak f on this curve, at an N_{Rw}/k of about 30,000 is slightly lower than the value that might be con-

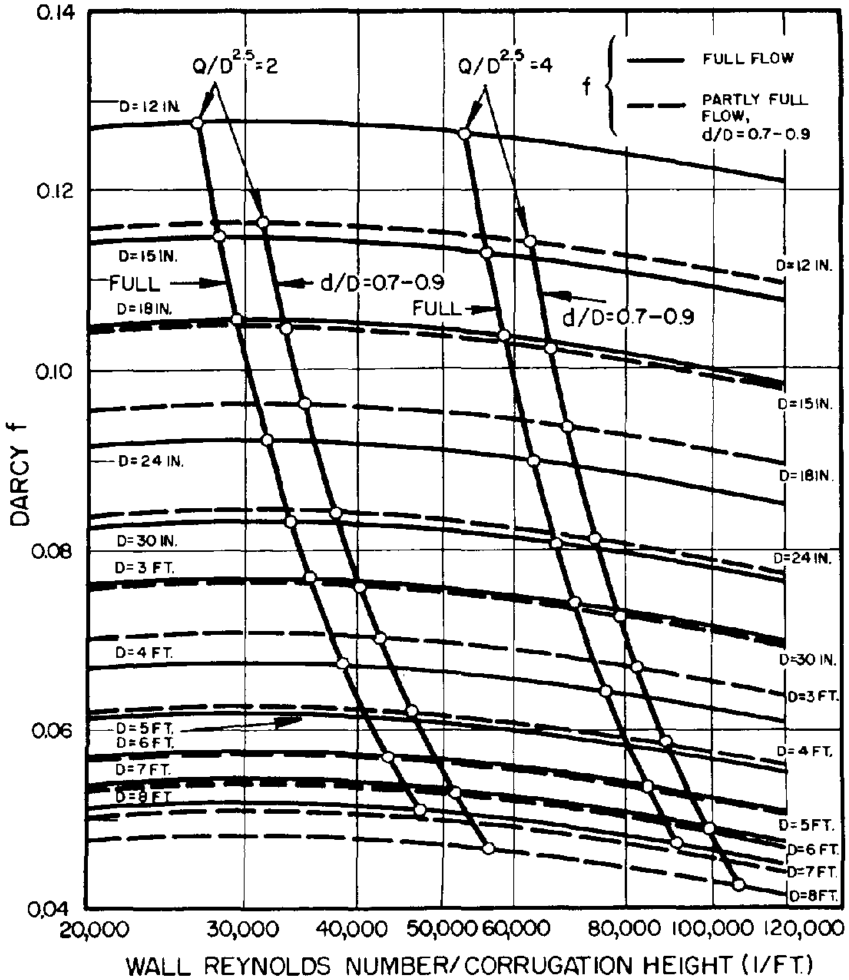

Figure 5.—Darcy f plotted against N_{Rw}/k for $2\frac{2}{3}$- by $\frac{1}{2}$-inch corrugated metal pipe.

sidered an average for all curves, about 35,000. However, this is not critical, as the f value changes very slightly over a large N_{Rw}/k range near the peak.

Using peak f values from curve No. 2 of figure 2 for standard $2\frac{2}{3}$- by $\frac{1}{2}$-inch C.M.P., and the curvature and peak location of the 3-foot pipe f—N_{Rw}/k curve from figure 1, the f—N_{Rw}/k curves of figure 5 were constructed. Curves were prepared for both full flow and partly full flow. The partly full flow curves were derived for circular pipes flowing three-quarters full, but because hydraulic radius varies only slightly with relative-depth variations in the range 0.7–0.9, they may be applied directly to relative depths within this range for circular pipes or pipe-arches.

Effective circular diameter in terms of hydraulic radius, $4R$, where R is the hydraulic radius of the three-quarters full circular prism, was used to construct the partly full flow curves. To determine the peak f from figure 2 for the partly full flow curves, $4R/k$ was used as the value of D/k, and peak f values were read from the curve for the appropriate corrugation type.

Metal conduits with 6- by 1-inch corrugations can be classed as one of the larger pitch-to-depth ratios, for which f will peak and then decrease with further increases in Reynolds number. Therefore, the f—N_{Rw}/k curves of figure 6 for 6- by 1-inch C.M.P., full and partly full, were derived similarly to those of figure 5 for $2\frac{2}{3}$- by $\frac{1}{2}$-inch C.M.P. using peak f values from figure 2 and the same curvature and peak location as in figure 5.

From the f—N_{Rw}/k curves of figures 5 and 6, the values of f for different pipe diameters flowing full and partly full at $Q/D^{2.5}$ values of 2.0 and 4.0 can be determined by equation (2b), which requires a trial and error procedure in which N_{Rw}/k is estimated, f is computed, and the resulting point is compared with the N_{Rw}/k curve for the particular diameter. The steps are then repeated until the desired accuracy is achieved. The values obtained by this process are connected by the steeply sloped lines, labeled $Q/D^{2.5}$ = 2 and 4, FULL, and d/D = 0.7–0.9, in figures 5 and 6. The intercepts of the two curves, flow and diameter, are the source of the f—diameter curves of figure 7.

Representations of Darcy Resistance Factors

Figures 7 through 9 are plots of Darcy f and pipe diameter for conduits having the five types of corrugations in-

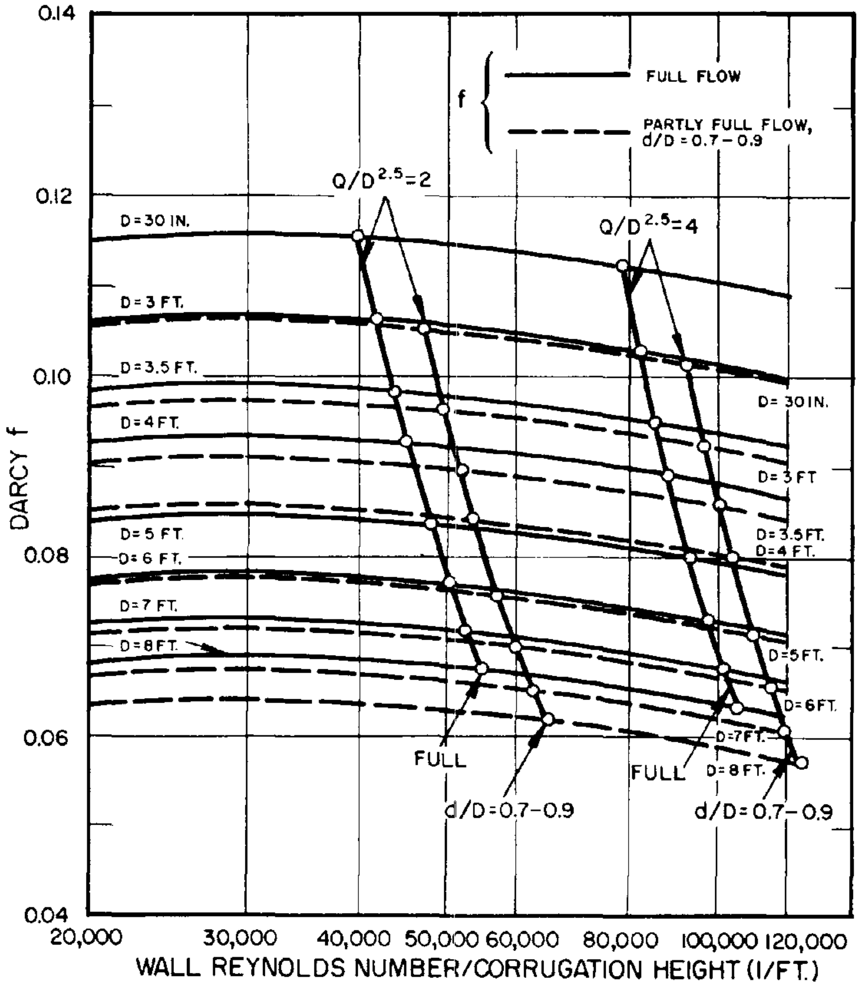

Figure 6.—Darcy f plotted against N_{R_w}/k for 6- by 1-inch corrugated metal pipe.

vestigated. Figure 7 represents the resistance of the corrugated metal pipes for which Reynolds number (flow rate) is a factor—2⅔- by ½-inch and 6- by 1-inch. These f—D curves are obtained from the intercepts of the diameter and flow rate curves of figures 5 and 6. The plots of figures 8 and 9 apply to the corrugated metal pipes for which flow rate is assumed to have no effect—3- by 1-inch (shop fabricated) and 6- by 2-inch and 9- by 2½-inch (field assembled). The resistance caused by the method of joining the seams in shop-fabricated pipe is estimated to be negligible. However, the total resistance coefficient for the field-assembled structural plate C.M.P. is assumed to be made up of two parts: conduit wall resistance and resistance caused by bolt heads or nuts. In figures 8 and 9, two sets of curves are presented for the structural plate pipes, one depicting the wall resistance only and the other representing the total resistance including bolt effects. The wall resistance, excluding bolt effects, is obtained directly from figure 2, and for the 3- by 1-inch, shop-fabricated C.M.P., the f value for the particular relative roughness is the total resistance as no bolts are present in this type of pipe. The bolt resistance increment, Δf, for the other structural plate pipes is computed as explained in the next section.

Curves for full and partly full flow (d/D = 0.7–0.9) are presented in figures 8 and 9. As previously mentioned, the partly full flow curves are based on d/D = 0.75, but they can also be used for the range d/D = 0.7–0.9 because R varies only slightly in this range of relative depths. The curves for structural plate C.M.P. are based on actual pipe diameters, as presented in Appendix C. Nominal diameters are represented by tick marks at the tops of these graphs.

Bolt Resistance in Structural Plate Corrugated Pipes

The resistance of bolt heads or nuts on the inside crests of corrugations must be considered for the structural plate pipes having 6- by 2-inch and 9- by 2½-inch corrugations. It was assumed that such obstructions in corrugation troughs do not affect resistance. The methods presented by Bossy in Appendix A of the WES report (6) were used in computing the Darcy resistance increment, Δf, caused by these isolated roughness elements, which must be added to the wall resistance to obtain the total f value. Bossy evaluates the resistance increment by the formula:

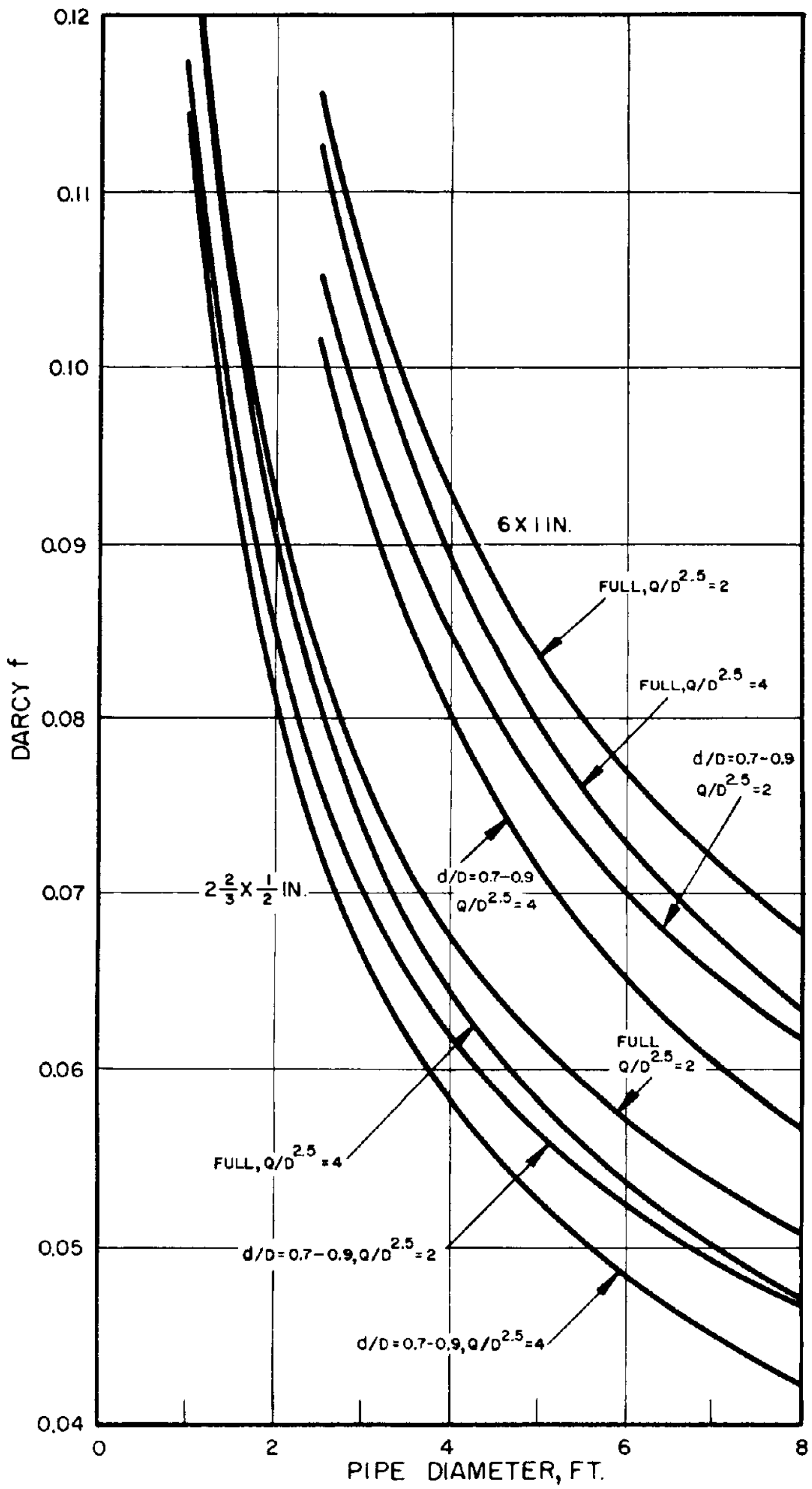

Figure 7.—Darcy f plotted against diameter for 6- by 1-inch and 2⅔- by ½-inch corrugated metal pipe.

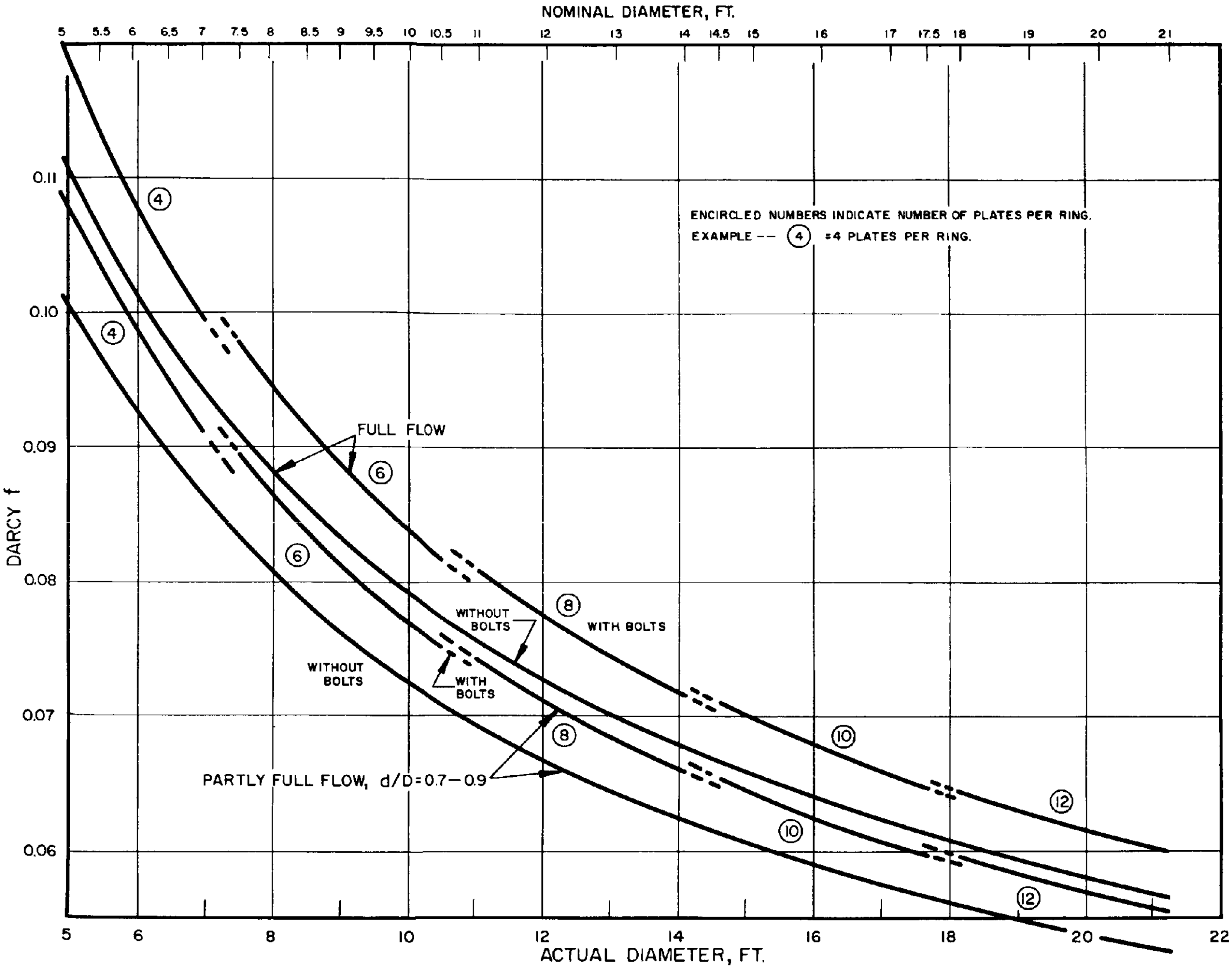

Figure 8.—Darcy f plotted against diameter for 6- by 2-inch structural plate corrugated pipe, with and without bolts.

$$\Delta f = \frac{C_D\ N\ a\left(\frac{v}{V}\right)^2}{\frac{L}{D}\quad A} \qquad \text{(Full flow)} \text{ -----------(3a)}$$

$$= \frac{C_D\ N\ a\left(\frac{v}{V}\right)^2}{\frac{L}{4R}\quad A} \qquad \text{(Partly full flow)} \text{ -----(3b)}$$

Where,

Δf is the incremental Darcy resistance factor.

C_D is the coefficient of drag, esimated to equal 1.1.

N is the average number of bolts per length L.

a is the projected area of one nut normal to flow.

v is the velocity near the wall at mid-height of a nut located on the crest of a corrugation.

L is the length of pipe being considered.

R is the hydraulic radius.

A is the flow area.

V is the mean flow velocity.

(Lengths are in feet, areas in square feet, and time is in seconds).

In the main part of the WES report (*6*), page 14, it is shown that for 6- by 2-inch structural plate corrugated pipes, the local velocity remains nearly constant inward from the crests for a distance of 0.7 times the corrugation depth, which is much greater than the height of a bolt head or nut and which has the value:

$$v = 5.5\ v^* \text{ --(4)}$$

Where,

v is the local velocity.

v^* is the shear velocity.

Also, as the resistance factors for structural plate corrugated pipes without bolts have already been determined, the following relation can be used:

$$\frac{V}{v^*} = \sqrt{\frac{8}{f}} \text{ --(5)}$$

Where,

V is the mean flow velocity.

These two equations permit derivation of the following relation between the local velocity at the projecting nut and the mean velocity based on f (without bolts):

$$\left(\frac{v}{V}\right)^2 = 3.78f \text{ --(6)}$$

For lack of better information, it was assumed that equation (4), and therefore equation (6), applies to the local velocity in 9- by 2½-inch structural plate C.M.P., as well as to 6- by 2-inch structural plate C.M.P. Although this is probably not exactly true, it should be close enough for estimation of bolt resistance effects.

Equation (6) can be combined with equation (3b), as follows, to arrive at a general equation for bolt resistance:

$$\Delta f = \frac{C_D\, N\, a\ (3.78f)}{\frac{L}{4R}\quad A} \text{ ------------------------------(7a)}$$

$$= \frac{15.12\ C_D\ N\ a\ (R/D)f}{(A/D^2)\ D\ L} \text{ ----------------------(7b)}$$

For the 6- by 2-inch structural plate corrugated pipes with nuts on the inside crests of both longitudinal and circumferential seams, the average number of crest bolts in a length, L, equal to the diameter, D, was computed. The average number of bolts in a length equal to D was determined from the total number at the inside crests in a length of 102 feet made up of twelve 8-foot plates and one 6-foot plate, producing a total of 13 circumferential joints.

For partly full flow, it was necessary to determine the number of bolt heads or nuts on corrugation crests that were actually submerged by the flow depth, $d = 0.75D$, used here to represent a usual range of partly full flow depths. At points where one of the longitudinal seams might or might not be submerged, depending on the orientation of the pipe, an average was used, which resulted in a fractional number of seams. This analysis was based on an equal spacing of longitudinal joints as occurs in the optimum pipe sections with maximum area per number of circumferential plates.

In figure 8, the f—D curves are shown for the 6- by 2-inch structural plate C.M.P., with and without bolt resistance included. The discontinuities in the curves for pipes with bolts indicate changes in the number of plates used to fabricate the particular pipe.

The procedure used to determine bolt resistance for the 9- by 2½-inch structural plate C.M.P. was slightly different owing to assembly differences between this pipe and the 6- by 2-inch structural plate C.M.P. First, the 9- by 2½-inch structural plate C.M.P. has its circumferential-seam bolts in the inside corrugation troughs, so these bolts are neglected. Also, each longitudinal seam has two bolts on each inside crest, instead of the single bolt used in the 6- by 2-inch structural plate C.M.P. Either aluminum or steel bolts and nuts can be used. The steel bolts are the same size as those used in the 6- by 2-inch structural plate C.M.P., but the aluminum nuts are shorter than the steel nuts, 11/16-inch compared to 13/16-inch. The dimensions of the aluminum fasteners were used in the computation of the Δf for figure 9; the bolt Δf should be increased by a small amount of about 0.0005, for steel nuts.

The curve discontinuities in figure 9 for 9- by 2½-inch structural plate C.M.P. are also due to changes in the number of plates used to construct the particular size pipe. One minor exception was made for partly full flow in the 14.59- and 15.10-foot (true diameter) pipes. The 14.59-foot pipe has four joints submerged at $d = 0.75D$ whereas the 15.10 foot pipe has only two joints submerged. Rather than plot individual points for each of these pipes, an average number of bolts was used for both, resulting in the smooth curve designated ⑤.

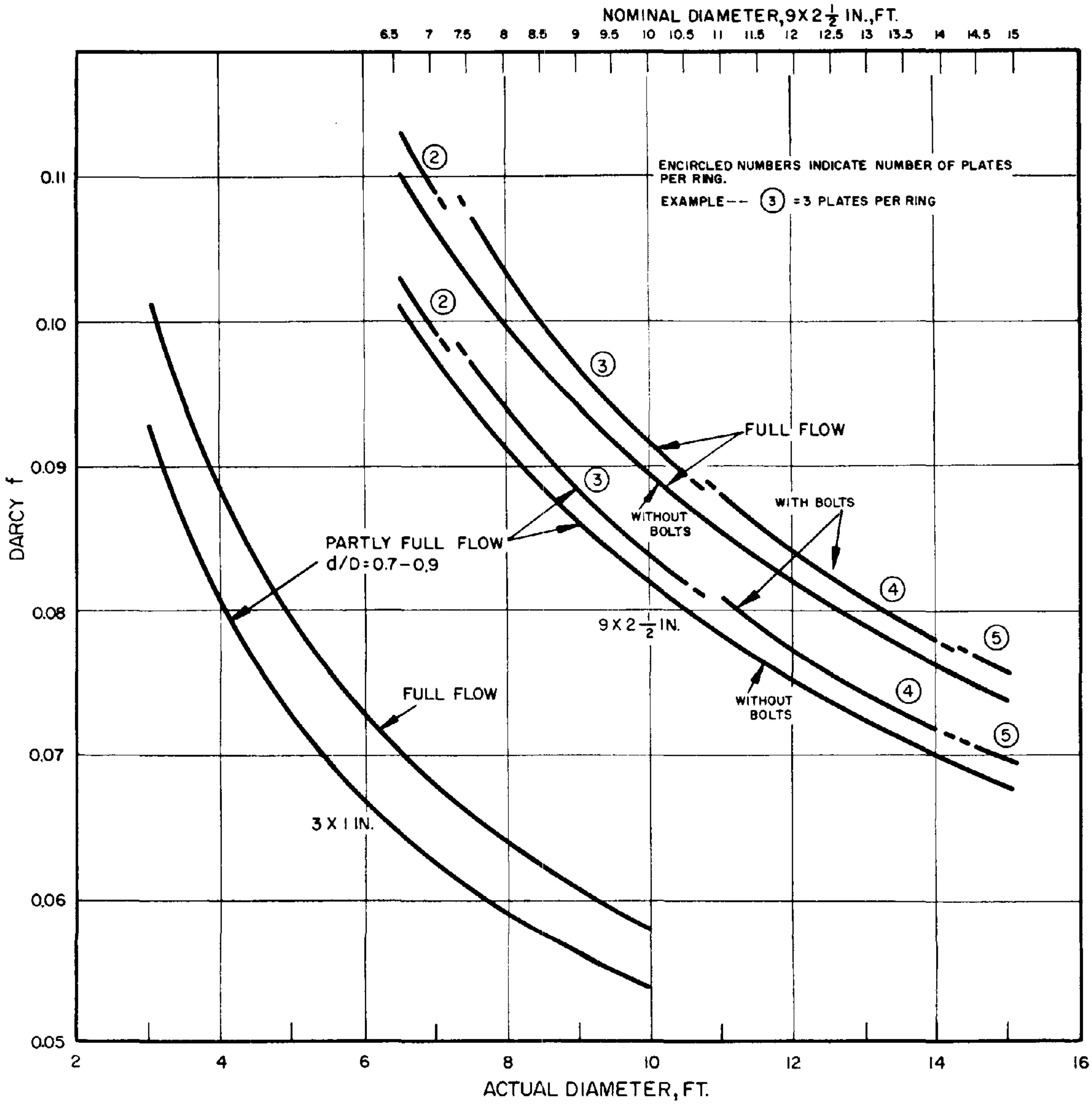

Figure 9.—Darcy f plotted against diameter for 3- by 1-inch corrugated metal pipe and 9- by 2½-inch structural plate corrugated pipe.

Representations of Manning Resistance Factors

For the convenience of designers who prefer to use Manning's equation, the *f—D* curves of figures 7 through 9 are also presented in the form of Manning *n—D* curves in figures 10 through 13 for full and partly full (d/D = 0.7–0.9) flow. Only the curves that include bolt resistance are presented for the structural plate C.M.P. in figures 12 and 13. These figures are based on actual pipe diameters (Appendix C) and have nominal diameters represented by tick marks at the tops of the graphs.

Conversion of the Darcy f to the Manning n was accomplished by use of the equation:

$$n = 0.0926\ (R)^{1/6}\ (f)^{1/2} \qquad (8)$$

Where,

R is the hydraulic radius, in feet.

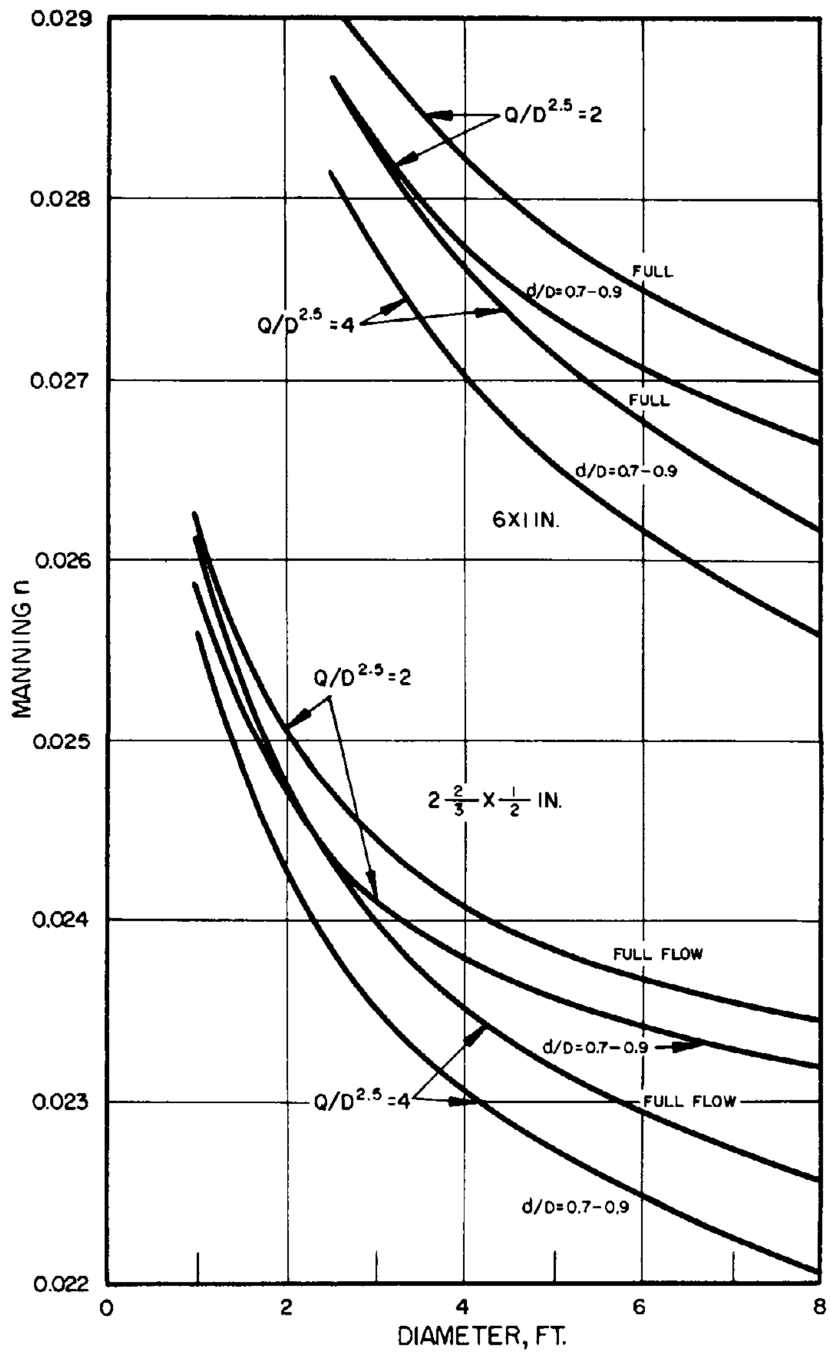

Figure 10.—Manning n plotted against diameter for 2⅔- by ½-inch and 6- by 1-inch corrugated metal pipe.

This formula can be easily derived by equating the Manning equation:

$$V = \frac{1.486}{n} R^{2/3} S_f^{1/2} \text{ --------------------------- } (9)$$

to the Darcy equation in the following form (See *Hydraulic Resistance Factors*):

$$V = \frac{16.04}{f^{1/2}} R^{1/2} S_f^{1/2} \text{ --------------------------- } (10)$$

and solving for n.

Helically Corrugated Metal Pipes

Corrugated metal pipe manufactured by the lock seam process, known as helical C.M.P. is available in the same range of sizes as the riveted or spot welded C.M.P. with annular corrugations and seams. Hydraulic tests on helical pipes are extremely limited, and results are confined to small diameter pipes.

The handbook (*14*) of the American Iron and Steel Institute (AISI) presents a range of f values for different pipe diameters that were obtained from flow tests in which

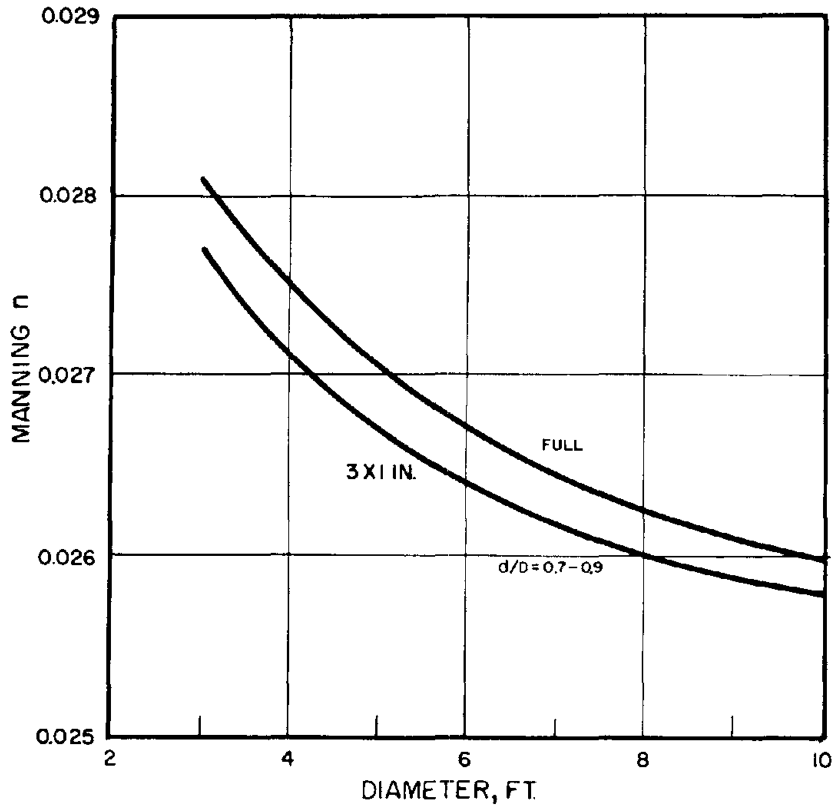

Figure 11.—Manning n plotted against diameter for 3- by 1-inch corrugated metal pipe.

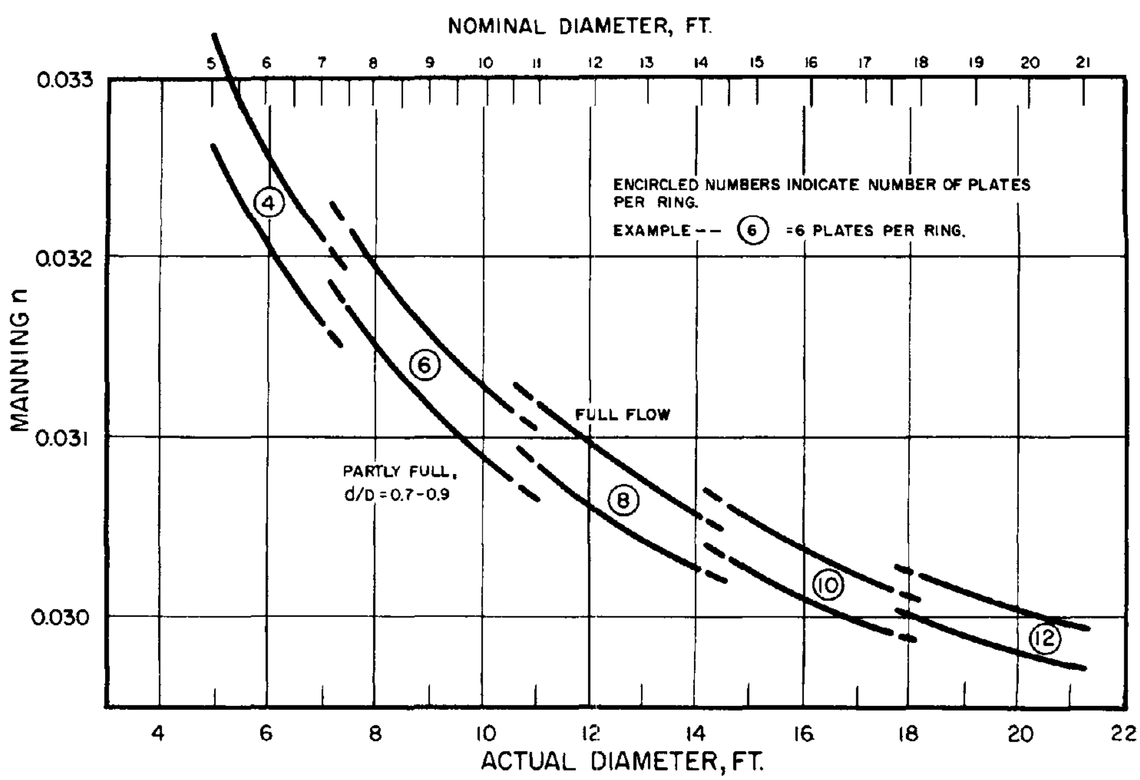

Figure 12.—Manning n plotted against diameter for 6- by 2-inch structural plate corrugated pipe, including bolt resistance.

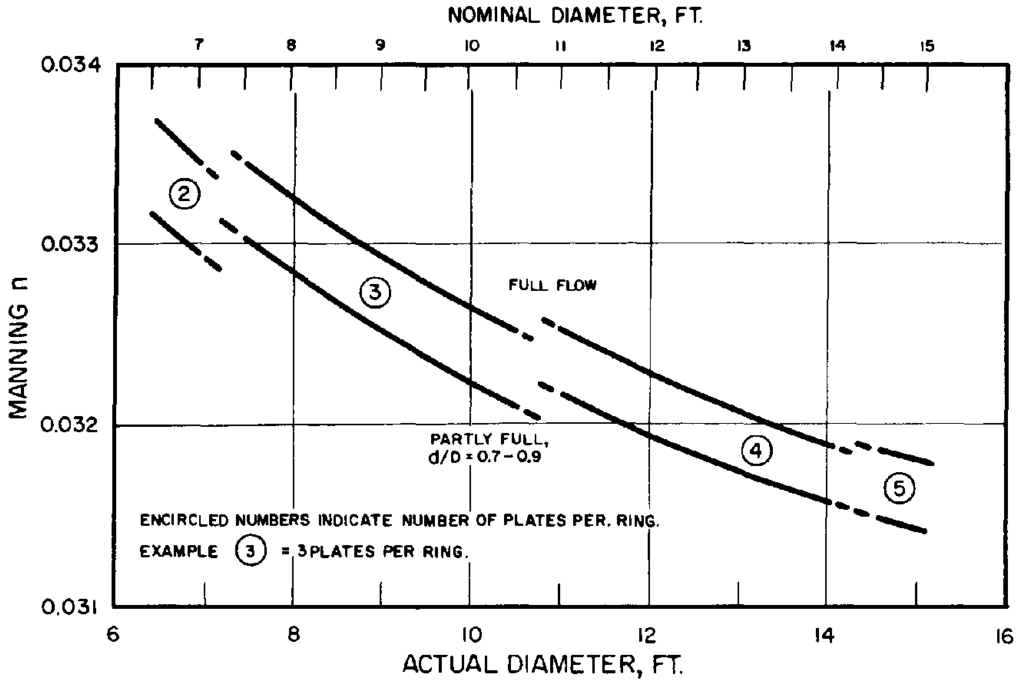

Figure 13.—Manning n plotted against diameter for 9- by 2½-inch structural plate corrugated pipe, including bolt resistance.

air was used. However, the reason for the range of values is not explained and no indication of the Reynolds numbers of the tests is presented. Chamberlain (*4*) tested 12-inch helically corrugated metal pipe with 2- by ½-inch corrugations in conjunction with his sediment transport studies. No systematic variation with Reynolds-number changes was detected, and the mean f value was determined to be 0.040. Rice (*15*) conducted flow tests on 8-inch and 12-inch helically corrugated metal pipe with 1½- by ¼-inch and 2- by ½-inch corrugations respectively, and a decline in f with increasing Reynolds number was detected in the 8-inch pipe. Some results from the above tests are plotted in figure 14 along with corresponding curves for full flow in standard C.M.P. with annular corrugations.

According to figure 14, the helical corrugations result in a three-fold reduction in f for small pipe sizes. In the small pipes, the helix angle measured from the pipe axis is about 66 degrees and tends to induce a shell of spiral flow around the conduit periphery. However, as the pipe diameter increases, the helix angle also increases, and as the helix angle approaches 90 degrees, the pipe must behave as a C.M.P. with annular corrugations.

For a partly full flow condition in a helically corrugated metal pipe in which spiral flow cannot be maintained, it is presumed that even a small helix angle would cause little reduction in resistance and that the same resistance coefficient as that for standard C.M.P. should be used.

There is need to test further helically corrugated metal pipe, especially the larger sizes. At present, the use of a reduced resistance coefficient is indicated only for the small diameters, 2 feet or less, and then only under full flow conditions. In figure 14, there must be a transition curve between the small diameter helically corrugated metal pipe with the smaller helix angles and the large pipe with a helix angle approaching 90 degrees. This curve is undefined at present, although the magnitude of the reduction does not seem large for the intermediate pipe sizes. The best course for conservative design, pending further test results, is to use the annular C.M.P. resistance coefficients for helically corrugated pipe.

Use of Resistance Factor–Diameter Curves

As noted in the introduction, resistance factors for partly full flow in a conduit of a given size and shape can be approximated by applying the full flow resistance coefficient for that conduit to any depth of flow from full to about $0.4D$. Errors in the determination of f inherent in this procedure are moderate, in the 10 percent vicinity, as will be shown.

For better precision with little additional effort, the curves of figures 7 through 13 can be used to determine the resistance coefficient in terms of either f or n for corrugated metal pipes or pipe-arches flowing from one-half full to full. The errors induced by the necessary approximations will seldom exceed 5 percent for f determinations, or 2½ percent for n.

Resistance factors, f or n, for circular corrugated metal pipes of the corrugation types studied, flowing full or partly full ($d/D = 0.7$-$d/D = 0.9$), can be read directly from the appropriate curves of figures 7 through 13. These factors should be adequate for many design problems.

Determination of more precise f or n values for flow depths between $d/D = 1.0$ and $d/D = 0.9$, and from $d/D = 0.7$ to $d/D = 0.5$ requires interpolation between the full and partly full curves, because of the inverse relation of f to

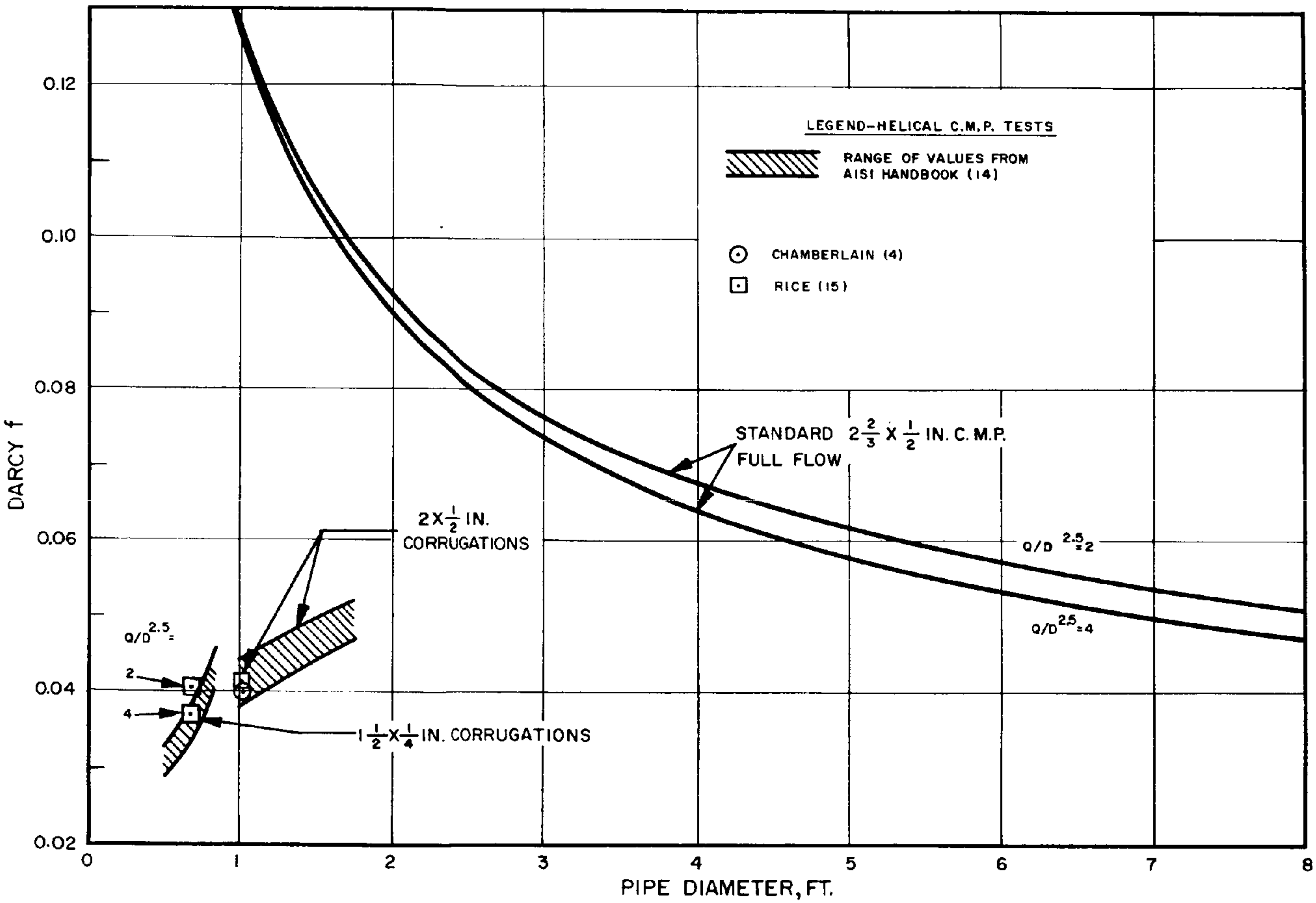

Figure 14.—Darcy f plotted against diameter for helically-corrugated metal pipe.

the hydraulic radius at different depths in a given conduit. For simplicity, it is assumed that a straight line interpolation can be performed between the full and partly full flow curves, downward for the range $d/D = 1.0$–$d/D = 0.9$, and upward for the range $d/D = 0.7$–$d/D = 0.5$.

The hydraulic radius is the same for both one-half full flow and full flow; thus, in conduits having the corrugation types affected only by relative roughness, the resistance factor is also the same for both one-half full flow and full flow.

In the aforementioned method of using the resistance factor–diameter curves, Reynolds-number effects have been accounted for in the full and partly full (0.7–0.9) curves of figures 7 and 10 for the 2⅔- by ½-inch and 6- by 1-inch corrugation types. However, Reynolds-number effects would be ignored in making interpolations between 1.0 and 0.9 and between 0.7 and 0.5, which will result in some estimation error because the resistance coefficient for one-half full flow is not the same as that for full flow in these conduits. Therefore, as the relative depth approaches one-half full, the resistance coefficient obtained by the above methods will be a few percent higher than the true resistance coefficient. The error caused by neglecting Reynolds-number effects for depths between $0.9D$ and full is insignificant.

The above methods of using the curves of figures 7–13 perform extremely well for the conduits in which resistance is considered to be independent of Reynolds-number effects—that is, velocity changes related to relative depth of flow. Such conduits include the 3- by 1-inch, 6- by 2-inch, and 9- by 2½-inch corrugation types. For the 6- by 2-inch and 9- by 2½-inch structural plate corrugated conduits, bolt resistance must be considered. In circular structural plate pipes, the bolt resistance amounts to less than 9 percent of the total f, and can be satisfactorily assessed by interpolating between the resistance coefficient curves, with bolts, of figures 8, 9, 12, and 13. It is assumed for this method that the total resistance, wall resistance plus bolt resistance, varies inversely with changes in the hydraulic radius of the flow prism. Although this assumption is not exactly true because the number of submerged crest bolts changes abruptly, trial calculations such as those in Appendix D have shown little resultant error.

Because pipe-arches have no linear dimension corresponding to the diameter of a circular pipe, the effective diameter, D_e, must be determined to make use of figures 7–13. This effective diameter is assumed to be four times the hydraulic radius of the *full* pipe-arch section. Its use in place of the hydraulic radius was discussed in the introduction.

Also, to account for the flow rate in pipe-arches manufactured of 2⅔- by ½-inch or 6- by 1-inch corrugated metal, a flow factor other than $Q/D^{2.5}$ must be used. An equivalent parameter is $Q/BD_a^{1.5}$, where B is the pipe-arch span and D_a is the pipe-arch rise. Use of this factor to enter graphs based on $Q/D^{2.5}$ results in an error of less than 2 percent in the determination of f. Tables of these hydraulic factors for circular pipes and pipe-arches are included in Appendix C of this report.

The relative hydraulic radii—ratios of R-partly full to R-full—of the pipe-arch sections flowing partly full vary with relative depth, d/D_a, much the same as the relative hydraulic radii of circular pipe sections flowing partly full vary with d/D. Thus, the same interpolation procedures can be used for pipe-arches as for circular pipes, as follows: having determined D_e, read directly for $d/D_a = 1.0$ and $d/D_a = 0.9$–0.7, and use a straight line interpolation for d/D_a from 1.0 to 0.9 and for d/D_a from 0.7 to 0.5.

Trial calculations, similar to those in Appendix D, have shown that the bolt resistance of the full structural plate pipe-arch can be estimated to be the same as the bolt resistance of the equivalent ($D_e = 4R$) circular pipe. Also, the variation of bolt resistance with relative depth in pipe-arches is similar to the variation of bolt resistance with relative depth in circular pipes. Accordingly, it is recommended that the total resistance (wall resistance plus bolt resistance) factor for structural plate pipe-arches, $D = D_e$, be read from the resistance factor curves, with bolts, of figures 8, 9, 12, and 13, interpolating in the ranges $d/D_a = 1.0$–0.9 and $d/D_a = 0.7$–0.5.

Determining resistance factors for circular pipes or pipe-arches flowing *less than one-half full* requires procedures other than those described above, and the curves of figures 7–13 cannot be used for this purpose. The methods that are available for shallow flow depths are discussed later; however, the range of relative depths less than one-half full is generally of minor importance, as most flow problems fall in a higher depth range.

Summary of Methods for Use of Figures 7–13

To use the curves of figures 7 through 13 to determine the resistance factors for circular pipes or pipe-arches flowing full to one-half full, perform the following steps:

1. From the tables of Appendix C, determine the true diameter for circular pipes or the effective diameter, $D_e = 4R_{\text{full}}$, for pipe-arches.
2. For conduits having 2⅔- by ½-inch or 6- by 1-inch corrugations, for which flow rate (Reynolds number) is a factor, determine $Q/D^{2.5}$ for circular pipes or $Q/BD_a^{1.5}$ for pipe-arches. Determine $D^{2.5}$ from table 1, 3, or 4, or $BD_a^{1.5}$ from table 5, 7, 8, or 10 of Appendix C.
3. To determine the resistance factor for *full flow* in a pipe or pipe-arch, select the appropriate figure, 7, 8, or 9 to obtain f, or 10, 11, 12, or 13 to obtain n and read the resistance factor from the full flow curve. Use the curve that includes bolt effects for structural plate pipe. For the long-cycle corrugations, 2⅔- by ½-inch and 6- by 1-inch, use the most nearly appropriate $Q/D^{2.5}$ curve, or interpolate.
4. If an estimate of the resistance factor for three-quarters full flow, $d/D = 0.7$–0.9, will serve for the problem under consideration, read the resistance factor from the curve for $d/D = 0.7$–0.9 of the appropriate chart, using a procedure similar to that of step 3.
5. When the depth of flow can be estimated or a series of depths are to be used, as in a non-uniform flow computation, select the desired relative depths of flow, in terms of d/D for pipes or d/D_a for pipe-arches.
6. For relative depths, d/D or d/D_a from 1.0 to 0.9 and from 0.7 to 0.5, read f or n from the appropriate chart by interpolating between the depth curves at the pipe diameter or at D_e for pipe-arches. Use straight-line interpolation,

downward from 1.0 to 0.9 and upward from 0.7 to 0.5 (the full flow curves also represent the resistance at $d = 0.5D$, very closely for the short-cycle corrugations and approximately for long-cycle corrugations).

7. For 3- by 1-, 6- by 2- and 9- by 2½-inch corrugations, step 6 requires only the interpolation described, and the effects of bolts submerged in the partly full flow are included by using the curves for conduits with bolts.

8. For 2⅔- by ½- and 6- by 1-inch corrugations, step 6 requires interpolation for relative depth between the $Q/D^{2.5}$ curves, full and partly full, that are most nearly representative of the discharge rate. For intermediate flow rates, the depth interpolation can first be made at Q equals 2.0 and 4.0 $D^{2.5}$. A second interpolation can then be made for the particular discharge rate. The two interpolations for flow rate and relative depth can be performed in either sequence.

Chart Use for Resistance in Full to One-Half Full Flow

The following examples demonstrate the use of figures 7–13 to determine resistance factors for circular or pipe-arch corrugated metal conduits at flow depths from full to one-half full. Where the independent resistance determinations of Appendix D indicate a significant error in the simple interpolation processes used, the magnitude of that error is stated.

1.—*Circular C.M.P., resistance affected by relative roughness only.*

Given: Six-foot (actual diameter) C.M.P. with 3- by 1-inch corrugations.

Required: Resistance factors f and n for full flow and partly full flow. Assume that the partly full flow depth will vary between $d/D = 0.7$ and $d/D = 0.9$.

d/D	f 3- by 1-in. corrugations (Fig. 9)	n 3- by 1-in. corrugations (Fig. 11)
1.00	0.0727	0.0267
0.9–0.7	0.0668	0.0264

For one-half full flow, f would be the same as for full flow, 0.0727. For flow at $d/D = 0.95$ or at $d/D = 0.60$, f would be halfway between the full and partly full curves, equal to 0.0698, and n would be 0.0266. Since bolt resistance and Reynolds-number effects are absent in this conduit, the derived values have little or no error.

The error in applying the full flow f to the partly full flow condition of this example would be about 9 percent.

2.—*Circular structural plate C.M.P., resistance affected by relative roughness and bolt effects.*

Given: Twelve-foot (nominal diameter) structural plate C.M.P. with 6- by 2-inch corrugations.

Required: Resistance factors f and n for full flow and partly full flow at $d/D = 0.75$, 0.6, and 0.5.

From table C–3, actual diameter = 12.06 feet.

d/D	f 6- by 2-in. with bolts (Fig. 8)	n 6- by 2-in. with bolts (Fig. 12)
1.00	0.0774	0.03095
0.75	0.0710	0.0306
0.60	[1] 0.0742	[1] 0.0308
0.50	0.0774	0.03095

[1] By straight line interpolation. From independent determinations in Appendix D, the error in f caused by estimating bolt effects is less than 1 percent.

Application of the full flow, f, to the 0.75 full flow results in an error of about 9 percent.

3.—*Circular C.M.P., resistance affected by relative roughness and Reynolds-number effects (rate of discharge).*

Given: Four-foot (actual diameter) C.M.P. with 2⅔- by ½-inch corrugations. Flow estimated to be 64 cubic feet per second (c.f.s.).

Required: Resistance factors f and n for full flow and partly full flow at $d/D = 0.75$, 0.6, and 0.5.

From table C–1, $D^{2.5} = 32.00$.

Therefore, $Q/D^{2.5} = \dfrac{64}{32.00} = 2.0$.

d/D	f 2⅔- by ½-in. $Q/D^{2.5} = 2.0$ (Fig. 7)	n 2⅔- by ½-in. $Q/D^{2.5} = 2.0$ (Fig. 10)
1.00	0.0675	0.0241
0.75	0.0618	0.0238
0.60	[1] 0.0646	[1] 0.02395
0.50	0.0675	0.0241

[1] By straight line interpolation. From independent determinations, the errors in f caused by Reynolds-number effects are about 2.5 percent at $d/D = 0.60$ and less than 5 percent at $d/D = 0.50$. A 5 percent error in f results in about a 2.5 percent error in computing V or n. (See Appendix D for the true values.)

If the full flow f value is applied at $d/D = 0.75$, a 9 percent error results.

4.—*Corrugated metal structural plate pipe-arch, resistance affected by relative roughness and bolt effects.*

Given: Structural plate corrugated metal pipe-arch with 6- by 2-inch corrugations. Nominal size = 12 feet 10 inches by 8 feet 4 inches. Corner radius = 18 inches.

Required: Resistance factors f and n for full flow and partly full flow at $d/D_a = 0.75$, 0.6, and 0.5.

From table C–7, $R = 2.461$ feet.

Therefore, $D_e = 4(2.46) = 9.84$ feet (based on full flow).

d/D_a	f 6- by 2-in. with bolts (Fig. 8)	n 6- by 2-in. with bolts (Fig. 12)
1.00	0.0847	0.03135
0.75	0.0778	0.03095
0.60	[1] 0.0812	[1] 0.03115
0.50	0.0847	0.03135

[1] By straight line interpolation. Error in f caused by estimating bolt and shape effects is about 1 to 2 percent, from independent determinations in Appendix D.

The error in applying the full flow f to flow at $d = 0.75\ D$ is about 9 percent.

5.—*Corrugated metal pipe-arch, resistance affected by relative roughness and Reynolds-number effects.*

Given: Corrugated metal pipe-arch with 2⅔- by ½-inch corrugations. Nominal size = 36 by 22 inches. Flow = 25 c.f.s.

Required: Resistance factors f and n for full flow and partly full flow at $d/D_a = 0.75$, 0.6, and 0.5.

From table C-5, $R = 0.564$ and $BD_a^{1.5} = 7.566$.

Therefore, $D_e = 4R = 4\ (0.564) = 2.256$ feet and

$$Q/BD_a^{1.5} = \frac{25}{7.57} = 3.3 \approx Q/D^{2.5}$$

d/D_a	f 2⅔- by ½-in. $Q/BD_a^{1.5} = 3.3$[1] (Fig. 7)	n 2⅔- by ½-in. $Q/BD_a^{1.5} = 3.3$[1] (Fig. 10)
1.00	0.0858	0.02465
0.75	0.0777	0.0242
0.60	[2] 0.0818	[2] 0.0244
0.50	0.0858	0.02465

[1] Interpolate between $Q/D^{2.5} = 2.0$ and $Q/D^{2.5} = 4.0$.

[2] By a second straight line interpolation for relative depth. From independent determinations, the errors in f caused by the combination of Reynolds number and shape effects are about 2.5 percent at $d/D_a = 0.60$ and less than 5 percent at $d/D_a = 0.50$. (See Appendix D).

The error in applying the full flow f to flow at $d = 0.75\ D$ is more than 10 percent.

Resistance Factors for Conduits Flowing Less Than One-Half Full

Short-cycle corrugations

The hydraulic resistance of C.M.P. and pipe-arches is controlled primarily by the relative roughness, that is, the relation of the corrugation depth to the hydraulic radius of the flow area. As previously noted, this is evidenced by figures 7–13, which exhibit rapidly increasing values of the Darcy f or the Manning n as the pipe diameter decreases. For conduits formed of the short-cycle corrugations, 3- by 1-inch, 6- by 2-inch, and 9- by 2½-inch, the relative roughness is the only factor that determines resistance, provided that hydraulic conditions (conduit size, corrugation depth, relative flow depth, and flow rate) are sufficient to produce a wall Reynolds number within the constant f range. (See figure 1.)

Therefore, for circular pipes of 3- by 1-inch corrugations or of either of the two forms of structural plate, 6- by 2-inch and 9- by 2½-inch, the resistance factor at *any depth* can be determined in the following manner. First, compute an effective diameter, D_e, equal to 4 times the hydraulic radius of the flow prism at the required depth. Then, using D_e as the pipe diameter D, read f from the appropriate full flow curve of figure 8 or 9, or n from the equivalent full flow curve of figure 11, 12 or 13. The same method is applicable to pipe-arches, using the hydraulic radius determined for the depth of flow in the pipe-arch.

This procedure is not recommended for relative depths of less than 0.25 D owing to the high relative roughnesses encountered, which are generally outside of the range of available data. (See figure 2.) Moreover, because of the extreme deviation of the flow prism shape from the shape of a circular conduit, the effective diameter concept ($D_e = 4R$) for the very shallow depth may not be reliable.

As relative roughness alone determines the resistance factor for a conduit constructed from one of the three short-cycle corrugation forms mentioned above, it follows that the method used for shallow flow depths can also be applied to pipe or pipe-arches with flow depths from full to one-half full. However, the procedure given previously to determine resistance factors for conduits flowing full to one-half full is more advantageous in the higher depth range because the hydraulic radius of each desired relative flow depth need not be determined. The resistance factor can be read from the appropriate figure at the actual pipe diameter, D, or for pipe-arches, at the equivalent diameter, $D_e = 4R_{full}$, using a previously described straight-line interpolation procedure for relative depths between the two curves for full flow and $d/D = 0.7$–0.9.

Thus, for the more common designs involving flows deeper than one-half full, either of the two methods can be used to determine resistance factors for conduits with the three forms of short-cycle corrugations: (1) interpolation between the relative depth curves at the actual conduit diameter, or effective diameter, or (2) computation of effective diameter for each flow depth and use of only the curve for full flow. Only the second method is applicable to conduits flowing less than one-half full.

Of course, the second procedure, using D_e equal to 4 times the hydraulic radius of the flow prism, has a further limitation when it is applied to the smaller conduit sizes. The values of D_e for some shallow depths of flow may be less than the minimum diameter shown in figures 8, 9, 11, 12, or 13.

The assumption that conduits of 3- by 1-inch, 6- by 2-inch and 9- by 2½-inch corrugated metal are free of Reynolds-number effects has been shown to be valid for normal drainage applications with flow rates above $Q = 2D^{2.5}$. These flow rates result in a wall Reynolds number-to-k ratio, N_{Rw}/k, greater than 30,000–40,000 (see table 2) in the range in which f is constant. Through a combination of flow rate and flow depth, small rates of flow at depths less than one-half full may occasionally produce N_{Rw}/k values below

30,000 in the declining portion of the f—N_{Rw}/k curves of figure 1 for 6- by 2-inch structural plate C.M.P. However, this is a rare occurrence, as in most design applications the flow rate will be high enough to produce an N_{Rw}/k above 30,000. The results obtained by assuming a constant f at a lower N_{Rw}/k will be conservative, as they will be moderately higher than the true f. Also, in checking minimum flow conditions at shallow depths, extreme precision usually is not necessary.

Generally, if the average flow velocity in the partly full flow prism is more than 3.5 f.p.s., it is valid to assume a constant resistance coefficient for the short-cycle corrugations, which can be derived from equation (2a)

$$\frac{N_{Rw}}{k} = \frac{f^{0.5}\ V}{2.828\ \nu} \qquad \text{(2a)}$$

by setting N_{Rw}/k equal to 30,000 and ν equal to 1.217×10^{-5} (for water at 60°F.), thus obtaining:

$$f^{0.5}V = (30{,}000)\ (2.828)\ (1.217 \times 10^{-5}) = 1.03.$$

Then, for an f value of 0.12, the minimum velocity producing the required N_{Rw}/k is 2.97 f.p.s.; whereas for an f value of 0.06, the minimum velocity is 4.21 f.p.s. This represents the range of minimum velocities, the average of which is about 3.5 f.p.s.

Bolt resistance in the structural plate 6- by 2-inch and 9- by 2½-inch conduits flowing partly full can be adequately accounted for by assuming that the bolt resistance of the partly full flow prism is the same as that of the equivalent ($D_e = 4R$) circular pipe—one with correspondingly fewer bolts exposed to the flow. Thus, the total f or n can be read directly from the full flow curves, with bolts, of figures 8, 9, 12, and 13.

Examples of shallow flow calculations for short-cycle corrugations

1.—*Partly-full circular C.M.P., resistance affected by relative roughness only.*

Given: Six-foot (actual diameter) C.M.P. with 3- by 1-inch corrugations.
Required: Resistance factors f and n for partly-full flow at d/D = 0.50 and 0.30.

d/D	R/D (Table C–2)	$R = (R/D)D$	$D_e = 4R$	f Full flow curve (Fig. 9)	n Full flow curve (Fig. 11)
0.50	0.2500	1.500	6.00	[1] 0.0727	[1] 0.0267
0.30	0.1709	1.025	4.10	0.0872	0.0275

[1] These same results at d/D = 0.50 could be obtained by the interpolation process described earlier.

Differences between the resistance factors produced by this method and by the complete computation process described in Appendix D are due to curve plotting and reading errors alone. Except for these small random errors, the results from the two methods should be exactly the same for this pipe in which resistance is affected only by relative roughness.

If the flow rate, Q, is at least 50 c.f.s. at d/D = 0.50 or 25 c.f.s. at d/D = 0.30, the wall Reynolds number-to-k ratio will be greater than 30,000, and the range of constant f values will be attained. This flow rate is based on a minimum velocity of 3.5 f.p.s. and the areas of the respective flow prisms.

It is evident that the errors in applying the full flow resistance factor to flow depths less than one-half full are large and often unacceptable. In this example the full flow f equals 0.0727, and if this is assumed to be the f for flow at d/D = 0.30, the error is almost 20 percent.

2.—*Partly-full circular structural plate C.M.P., resistance affected by relative roughness and bolt effects.*

Given: Twelve-foot (nominal diameter) structural plate C.M.P. with 6- by 2-inch corrugations.
Required: Resistance factors f and n for partly full flow at d/D = 0.50 and 0.30.

From table C–3, actual diameter = 12.06 feet.

d/D	R/D (Table C–2)	$R = (R/D)D$	$D_e = 4R$	Total f Full flow curve with bolts (Fig. 8)	Total n Full flow curve with bolts (Fig. 12)
0.50	0.2500	3.015	12.06	[1] 0.0774	[1] 0.03095
0.30	0.1709	2.061	8.24	0.0929	0.03185

[1] These same results at d/D = 0.50 could be obtained by the interpolation process described earlier.

Differences between the resistance factors produced by this method and the complete computation process described in Appendix D are due to errors in estimating bolt effects, as well as to the graphical errors described in example 1. Such errors in f amount to less than 0.5 percent at d/D = 0.30. Errors in n are negligible, as they are approximately one-half of the corresponding errors in f.

If the flow rate is more than 200 c.f.s. at d/D = 0.50, or 100 c.f.s. at d/D = 0.30, the constant f range, above N_{Rw}/k = 30,000, will be attained. These Q values are based on V = 3.5 f.p.s.

3.—*Corrugated metal structural plate pipe-arch, resistance affected by relative roughness and bolt effects.*

Given: Structural plate corrugated metal pipe-arch with 6- by 2-inch corrugations. Nominal size = 12 feet 10 inches by 8 feet 4 inches. Corner radius = 18 inches.
Required: Resistance factors f and n for partly full flow at d/D_a = 0.50 and 0.30.

From table C–7, D_a = 8.31 feet.

d/D_a	R/D_a (Table C–9)	$R = (R/D_a)D_a$	$D_e = 4R$	Total f Full flow curve with bolts (Fig. 8)	Total n Full flow curve with bolts (Fig. 12)
0.50	0.306	2.543	10.17	0.0832	0.03125
0.30	0.211	1.753	7.01	0.0992	0.0321

The results from this simplified method are low by 2 percent at d/D_a = 0.50 and by 4 percent at d/D_a = 0.30, in

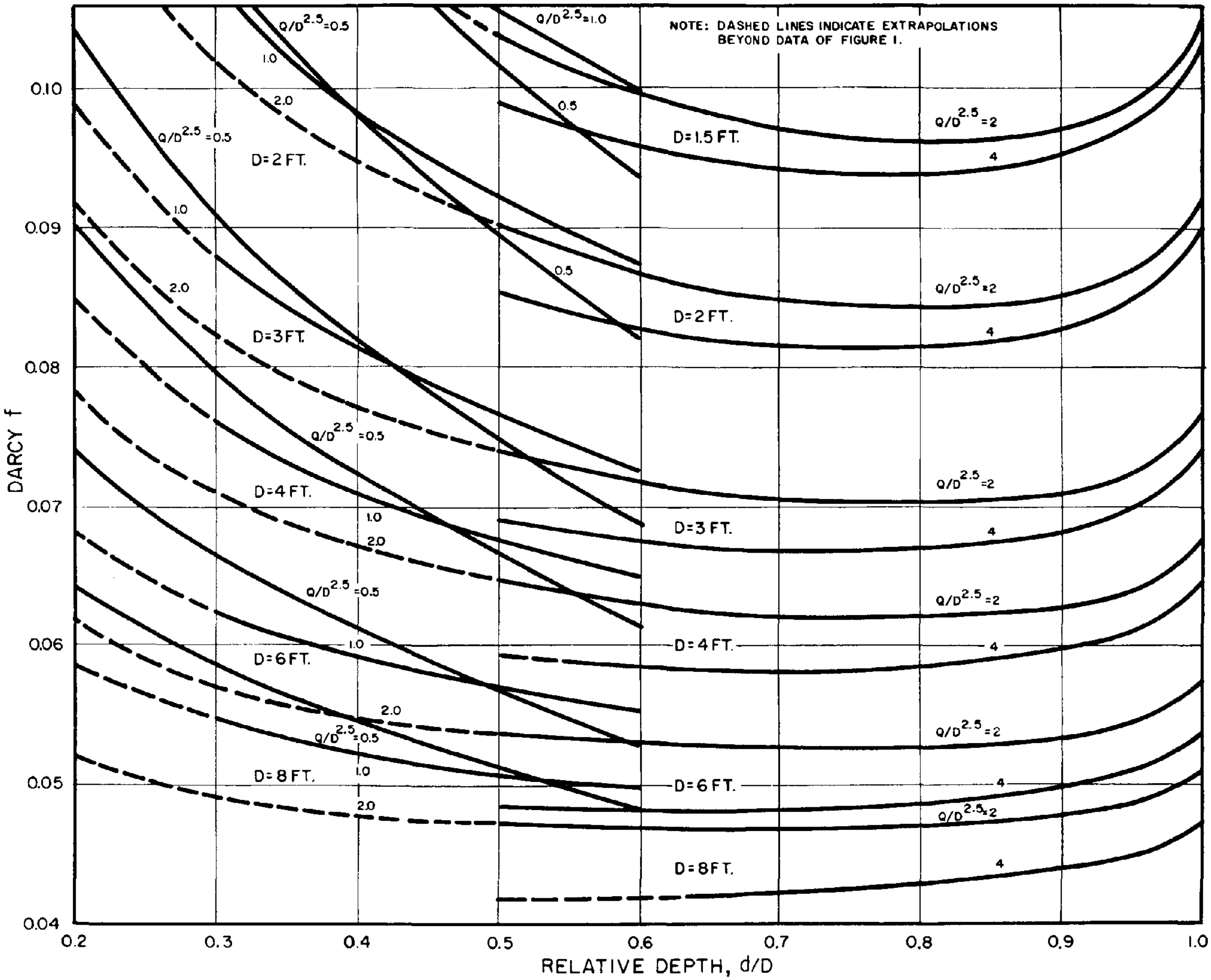

Figure 15.—Darcy f plotted against relative depth for a range of sizes of 2⅔- by ½-inch circular corrugated metal pipe under various flow rates.

terms of f. Errors in n are about one-half of this magnitude. These errors are due to the relatively larger number of longitudinal joints, and therefore bolts, submerged by shallow flow depths in structural plate pipe-arches, as opposed to circular pipes.

In this example, the results at $d/D_a = 0.50$ differ from those of the interpolation procedure described earlier for flow depths one-half full and up. This is due to the noncircular shape of the structure, meaning that the hydraulic radius of the one-half full flow prism is not equal to the full flow hydraulic radius.

The flow must be at least 170 c.f.s. at $d/D_a = 0.50$ or 90 c.f.s. at $d/D_a = 0.30$ to produce an N_{Rw}/k greater than 30,000 in the constant f range. These flows are based on the flow prisms of the partly full pipe-arch and a minimum velocity of 3.5 f.p.s.

Long-cycle corrugations

For conduits formed of corrugated metal having long-cycle corrugations, 2⅔- by ½-inch and 6- by 1-inch, the primary factor controlling resistance is again the relative roughness. Because in any given conduit the hydraulic radius increases as depth decreases from full downward to almost $0.75D$, the resistance factor will decrease in this range of relative depths. As is shown by figure 5 or 6, for a given flow rate, the change in wall Reynolds number and, therefore, its effect on f is small in this range of depths. Thus, the interpolation methods described earlier for obtaining f from figure 7, or n from figure 10, are reliable in this range of depths. Trial determinations have shown that errors from the use of figures 7 or 10 are also small for flow depths down to $0.6D$.

For relative depths of flow less than 0.6, the wall Reynolds number increases significantly owing to the higher velocities for a given flow rate and decreased area. Thus, its effect in reducing f below the value determined by relative roughness alone becomes increasingly large. The overestimation of f at $d/D = 0.5$ through use of the previously described interpolation method using figure 7 approaches, and may exceed, 10 percent. An error of 10 percent in f or 5 percent in n is not unacceptably large, considering the convenience of interpolating in figures 7 and 10, using the full flow curve to represent resistance factors for one-half full flow.

A different method is required for depths less than one-half full, one that can also be used to obtain a more precise value of f at $d/D = 0.5$. Because increasing the wall Reynolds number decreases the value of f (from that of figure 2 on the basis of relative roughness alone or from the full-flow curves of figure 7 on the basis of effective diameter, $D_e = 4R$), the previous methods are not applicable to shallow relative depths of flow. In fact, the reliability of the f in the range of relative depths from 0.6–0.5 can be increased significantly by use of the following procedure for shallow depths. As was noted, this method is essential for relative depths less than $0.5D$ in conduits having the long-cycle corrugations, 2⅔- by ½-inch and 6- by 1-inch.

Where resistance factors vary with wall Reynolds number as well as with relative roughness, as for the two corrugation forms under consideration, diagrams of f against N_{Rw} must be constructed and f determined by a trial and error procedure using equation (2b), as described in connection with figures 5 and 6. A detailed description of the method used is given in Appendix D. A diagram of results would be more complex than figure 5 or 6 because f first decreases as d/D decreases and then increases as d/D drops below about 0.75.

The difficulties of using such complex diagrams can be avoided by resorting to a different type of chart. The procedure described in detail in Appendix D produces figure 15 for 2⅔- by ½-inch corrugations and figure 16 for 6- by 1-inch corrugations.

The value of f for circular corrugated metal pipes having one of the two long-cycle corrugation forms can be read directly at the desired relative depth for the pipe diameters shown, using the applicable $Q/D^{2.5}$ value. Note that lesser flow rates are shown for the depths below $0.6D$ than were considered necessary in the figures applicable only to depths from one-half full to full. Where the desired flow rate or pipe size is not included in the figure, a value of f is obtained by interpolation, first for $Q/D^{2.5}$ at the relative depth, and then for the pipe diameter.

Darcy f values for pipe-arch conduits can also be obtained from these figures in a similar manner, using d/D_a to enter the relative depth scale and taking $D_e = 4R_{full}$ as the equivalent of the pipe diameter D. A discharge value of $Q/BD^{1.5}$ is used for selection of the $Q/D^{2.5}$ curve, as described in previous examples. The use of figure 15 or 16 will reveal that the full flow curves of figure 7 overestimate the f for one-half full flow, in the interest of describing a simple method.

If a value of the Manning n is desired, equation (8) can be used to perform the conversion.

It should be apparent that curves similar to figures 15 and 16 could be constructed for any corrugation type, including the short-cycle corrugations, and for any range of relative depths. However, the method is most advantageous for the long-cycle corrugations, which cause f to vary with changes of Reynolds number, owing to their inherent calculation complexities.

Examples of shallow flow calculations for long-cycle corrugations

1.—*Partly full circular C.M.P., resistance affected by relative roughness and Reynolds-number effects.*

Given: Four-foot (actual diameter) C.M.P. with 2⅔- by ½-inch corrugations. Flow estimated to be 16 c.f.s.

Required: Resistance factors f and n for partly-full flow at $d/D = 0.50$ and 0.30.

From table C–1, $D^{2.5} = 32.00$.

$$\text{Therefore, } Q/D^{2.5} = \frac{16}{32.0} = 0.5.$$

d/D	f (Fig. 15)	n (Equation (8))
0.50	0.0665	0.0239
0.30	0.0794	0.0245

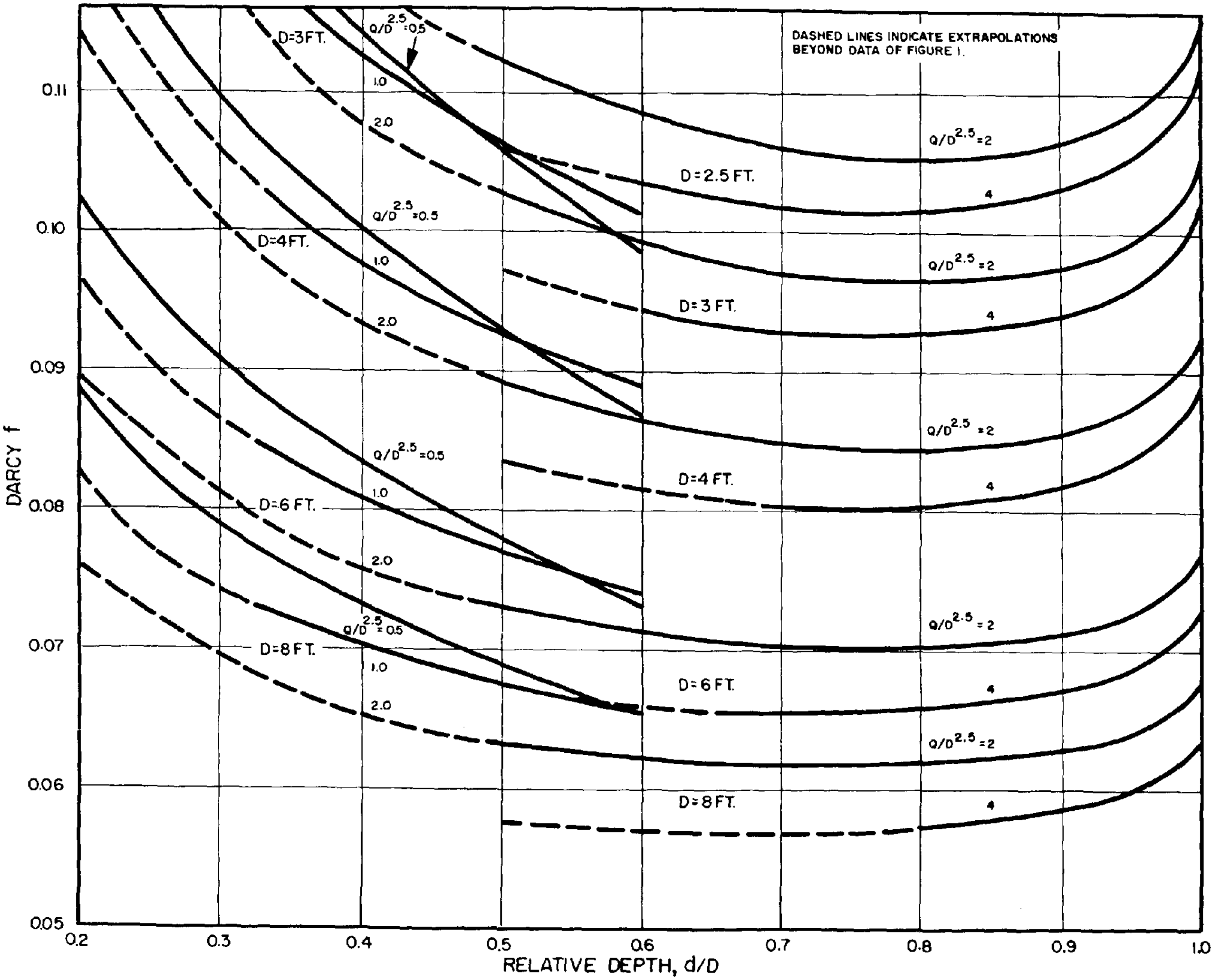

Figure 16.—Darcy f plotted against relative depth for a range of sizes of 6- by 1-inch corrugated metal pipe under various flow rates.

These values are the same as the results from the complete computation process of Appendix D.

2.—*Corrugated metal pipe-arch, resistance affected by relative roughness and Reynolds-number effects.*

Given: Corrugated metal pipe-arch with 2⅔- by ½-inch corrugations. Nominal size = 36 by 22 inches. Estimated flow is 5 c.f.s.

Required: Resistance factors f and n for partly-full flow at $d/D_a = 0.50$ and 0.30.

From table C–5.

Nominal size	D_a	R	$BD_a^{1.5}$
36 by 22 in.	1.85	0.564	7.566

$D_e = 4R_{full} = 4(0.564) = 2.26$ feet.

$Q = 5$ c.f.s.; $Q/BD_a^{1.5} = \dfrac{5}{7.57} = 0.66.$

Then, assuming the following parameters to be equivalent:

Circular pipe	*Pipe-arch*
d/D	d/D_a
D	$D_e = 4R_{full}$
$Q/D^{2.5}$	$Q/BD_a^{1.5}$

the f values below are read from figure 15, interpolating first for flow rate and then for conduit size, D_e:

d/D_a	R/D_a (Table C–6)[1]	$R = (R/D_a)D_a$[1]	$Q/BD_a^{1.5}$	f (Fig. 15)	n (Equation (8))
0.50	0.319	0.590	0.66	0.0866	0.0249
0.30	0.322	0.411	0.66	0.1040	0.0258

[1] Needed to compute n from equation (8).

Differences in f between this solution and the complete computation procedure of Appendix D are about 4 percent. Such errors are due to the fact that, in the shallow flow depths, the areas and hydraulic radii of pipe-arches differ significantly from the like properties of circular sections.

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(*14*) *Handbook of Steel Drainage and Highway Construction Products*, American Iron and Steel Institute, 150 East 42d St., New York, New York, 1967, p. 317.

(*15*) Rice, Charles E., "Friction Factors for Helical Corrugated Pipe," *ARS 41–119*, U.S. Dept. of Agriculture, Agricultural Research Service, February 1966.

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APPENDIX B — DEFINITIONS OF TERMS

A = area of flow, feet^2.

a = average projected area of an obstruction, such as a bolt head or nut, normal to flow, feet^2.

B = pipe-arch span, feet.

C_D = coefficient of drag for structural plate bolts, estimated to equal 1.1.

c = pitch of corrugation, feet.

D = pipe diameter, feet.

D_a = pipe-arch rise, feet.

D_e = equivalent circular pipe diameter, based on $4R$, feet.

d = depth of flow, feet.

f = Darcy resistance factor.

g = gravitational acceleration = 32.16 feet/second^2.

h_f = friction head loss, feet.

k = depth of corrugation, feet.

L = length of conduit, feet.

N = number of structural plate bolts per length L.

N_R = Pipe Reynolds number = $VD/\nu = 4VR/\nu$.

N_{Rw} = wall Reynolds number = $v^* k/\nu$.

n = Manning resistance factor.

P = perimeter of conduit, feet.

Q = flow rate, feet^3/second.

R = hydraulic radius = $A/P = D/4$ for full flow in circular pipes, feet.

r = distance measured from a pipe axis outward, feet.

r_o = radius of a circular pipe, feet.

S_f = friction slope—slope of total energy line, equal to slope of the hydraulic grade line in pipes flowing full.

T = temperature, °F.

V = mean velocity, Q/A, feet/second.

v = local flow velocity at a point within a conduit, feet/second.

v^* = mean shear velocity, $(RS_f g)^{0.5}$, feet/second.

w = specific weight of water = 62.37 pounds/foot^3 at 60°F.

y = distance measured from a conduit wall inward, feet. For C.M.P., the origin is at a corrugation crest.

Δf = incremental Darcy resistance factor resulting from structural plate bolts.

ν = kinematic viscosity = 1.217×10^{-5} feet^2/second for water at 60°F.

ρ = mass density of water = w/g = 1.939 pound second^2/feet^4 for water at 60°F.

τ_o = unit shear stress in a fluid at the conduit wall, pound/feet^2.

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APPENDIX C — DIMENSIONAL, GEOMETRIC, AND HYDRAULIC FACTORS FOR CORRUGATED METAL CONDUITS

This appendix contains tables of geometric and hydraulic properties for circular and pipe-arch corrugated metal conduits in terms of the actual dimensions rather than the nominal ones. Manufacturing standards govern actual dimensions, and small tolerances are to be allowed and should be expected. Information on some corrugation types and conduit shapes covered in the main text are not included here for one of two reasons. First, actual dimensions for some conduits, such as 6- by 1-inch pipe-arches and 3- by

Table C–1.—$D^{2.5}$ values for a range of pipe diameters

True diameter	True diameter	$D^{2.5}$
Inches	*Feet*	
12	1.0	1.000
15	1.25	1.747
18	1.5	2.756
21	1.75	4.051
24	2.0	5.657
30	2.5	9.882
36	3.0	15.59
42	3.5	22.92
48	4.0	32.00
54	4.5	42.96
60	5.0	55.90
66	5.5	70.94
72	6.0	88.18
78	6.5	107.72
84	7.0	129.64
90	7.5	154.0
96	8.0	181.0
102	8.5	210.6
108	9.0	243.0
114	9.5	278.2
120	10.0	316.2

Table C–2.—Geometric factors for circular conduits, full or partly full
[d = Depth of flow, D = Pipe diameter, R = Hydraulic radius, and A = Area of flow]

$\frac{d}{D}$	$\frac{R}{D}$	$\frac{A}{D^2}$	$\frac{d}{D}$	$\frac{R}{D}$	$\frac{A}{D^2}$
1.00	0.2500	0.7854	0.50	0.2500	0.3927
0.95	0.2865	0.7707	0.45	0.2331	0.3428
0.90	0.2980	0.7445	0.40	0.2142	0.2934
0.85	0.3033	0.7115	0.35	0.1935	0.2450
0.80	0.3042	0.6736	0.30	0.1709	0.1982
0.75	0.3017	0.6319	0.25	0.1466	0.1535
0.70	0.2962	0.5872	0.20	0.1206	0.1118
0.65	0.2882	0.5404	0.15	0.0929	0.0739
0.60	0.2776	0.4920	0.10	0.0635	0.0409
0.55	0.2649	0.4426	0.05	0.0325	0.0147

Table C–3.—Dimensions and $D^{2.5}$ values for 6- by 2-inch structural plate corrugated circular pipes, full-flow condition

Nominal diameter	True diameter	Plates per ring	$D^{2.5}$
Feet	*Feet*	*Number*	
5.0	4.93	4	53.97
5.5	5.43	4	68.71
6.0	5.94	4	85.99
6.5	6.45	4	105.66
7.0	6.97	4	128.26
7.5	7.48	6	153.0
8.0	7.98	6	179.9
8.5	8.49	6	210.0
9.0	9.00	6	243.0
9.5	9.51	6	278.9
10.0	10.02	6	317.8
10.5	10.53	6	359.8
11.0	11.04	8	405.0
11.5	11.55	8	453.4
12.0	12.06	8	505.1
12.5	12.57	8	560.2
13.0	13.08	8	618.8
13.5	13.58	8	679.6
14.0	14.09	8	745.2
14.5	14.60	10	814.5
15.0	15.11	10	887.5
15.5	15.62	10	964.3
16.0	16.13	10	1044.9
16.5	16.64	10	1130.0
17.0	17.15	10	1218.0
17.5	17.66	10	1310.8
18.0	18.17	12	1407.0
18.5	18.67	12	1506.0
19.0	19.18	12	1611.0
19.5	19.69	12	1720.0
20.0	20.21	12	1836.0
20.5	20.72	12	1954.0
21.0	21.22	12	2074.0

Table C–4.—Dimensions and $D^{2.5}$ values for 9- by 2½-inch structural plate corrugated circular pipes, full flow condition

Nominal diameter	True diameter	Plates per ring	$D^{2.5}$
Feet	*Feet*	*Number*	
6.5	6.42	2	104.4
7.0	6.93	2	126.4
7.5	7.44	3	151.0
8.0	7.96	3	178.8
8.5	8.46	3	208.2
9.0	8.97	3	241.0
9.5	9.48	3	276.7
10.0	9.99	3	315.4
10.5	10.50	3	357.2
11.0	11.01	4	402.2
11.5	11.52	4	450.4
12.0	12.04	4	503.0
12.5	12.52	4	554.6
13.0	13.05	4	615.2
13.5	13.57	4	678.3
14.0	14.08	4	743.9
14.5	14.59	5	813.1
15.0	15.10	5	886.0

Table C–5.—Dimensions and hydraulic properties of 2⅔- by ½-inch corrugated metal pipe-arches, full-flow condition

Nominal size		True size		Hydraulic		
Span B	Rise D_a	Span B	Rise D_a	radius R	Area A	$BD_a^{1.5}$
Inches	*Inches*	*Feet*	*Feet*	*Feet*	*Feet²*	
18	11	1.51	0.92	0.282	1.11	1.338
22	13	1.81	1.11	0.338	1.59	2.106
25	16	2.11	1.29	0.394	2.17	3.099
29	18	2.41	1.48	0.451	2.83	4.329
36	22	3.01	1.85	0.564	4.42	7.566
43	27	3.61	2.22	0.676	6.37	11.93
50	31	4.22	2.59	0.789	8.67	17.53
58	36	4.82	2.96	0.902	11.3	24.49
65	40	5.42	3.33	1.014	14.3	32.88
72	44	6.02	3.70	1.127	17.7	42.78
79	49	6.62	4.06	1.240	21.4	54.27
85	54	7.23	4.43	1.352	25.5	67.48

Table C–6.—Geometric factors for 2⅔- by ½-inch corrugated metal pipe-arches, full or partly full flow—mean values for all standard sizes

[d = Depth of flow, D_a = Rise of pipe-arch, R = Hydraulic radius, A = Area of flow, and B = Span of pipe-arch]

$\frac{d}{D_a}$	$\frac{R}{D_a}$	$\frac{A}{B\ D_a}$	$\frac{d}{D_a}$	$\frac{R}{D_a}$	$\frac{A}{B\ D_a}$
1.00	0.305	0.795	0.50	0.319	0.459
0.95	0.346	0.783	0.45	0.300	0.412
0.90	0.361	0.762	0.40	0.277	0.363
0.85	0.369	0.736	0.35	0.252	0.315
0.80	0.372	0.704	0.30	0.222	0.264
0.75	0.370	0.668	0.25	0.189	0.214
0.70	0.365	0.632	0.20	0.154	0.165
0.65	0.358	0.592	0.15	0.117	0.117
0.60	0.348	0.549	0.10	0.076	0.069
0.55	0.335	0.505	0.05	0.037	0.028

Table C–7.—Dimensions and hydraulic properties of 6- by 2-inch structural plate corrugated metal pipe-arches with 18-inch corner radius, full-flow condition

Section No.	Nominal size		Plates per ring	Span B	Rise D_a	Hydraulic radius R	Area A	$BD_a^{1.5}$
	Span (B)	Rise (D_a)						
	ft.–in.	*ft.–in.*	*Number*	*Feet*	*Feet*	*Feet*	*Feet²*	
1	6–1	4–7	5	6.08	4.58	1.299	22.09	59.60
2	6–4	4–9	5	6.33	4.76	1.353	24.09	65.77
3	6–9	4–11	5	6.77	4.91	1.405	26.14	73.66
4	7–0	5 1	5	7.02	5.09	1.460	28.39	80.59
5	7 3	5–3	6	7.25	5.27	1.515	30.60	87.72
6	7–8	5–5	6	7.70	5.42	1.567	32.92	97.17
7	7–11	5–7	6	7.93	5.60	1.622	35.39	105.07
8	8–2	5–9	6	8.15	5.78	1.677	37.95	113.28
9	8–7	5–11	7	8.62	5.92	1.726	40.40	124.1
10	8–10	6–1	7	8.83	6.11	1.781	43.10	133.3
11	9–4	6–3	7	9.32	6.26	1.832	45.83	146.0
12	9–6	6–5	7	9.52	6.44	1.887	48.70	155.6
13	9–9	6–7	7	9.72	6.62	1.940	51.64	165.5
14	10–3	6–9	7	10.22	6.77	1.989	54.51	180.0
15	10–8	6 11	7	10.70	6.91	2.037	57.46	194.3
16	10–11	7 1	7	10.92	7.09	2.092	60.70	206.2
17	11–5	7–3	7	11.40	7.24	2.142	63.87	222.1
18	11–7	7–5	8	11.62	7.42	2.196	67.23	234.8
19	11–10	7–7	8	11.82	7.61	2.250	70.68	248.1
20	12–4	7–9	8	12.32	7.75	2.298	74.05	265.8
21	12–6	7–11	8	12.52	7.93	2.352	77.64	279.6
22	12–8	8–1	8	12.70	8.12	2.406	81.34	293.9
23	12–10	8–4	8	12.87	8.31	2.461	85.20	308.3
24	13–5	8–5	9	13.40	8.44	2.507	88.74	328.6
25	13–11	8–7	9	13.93	8.58	2.555	92.55	350.1
26	14–1	8–9	9	14.12	8.77	2.608	96.53	366.7
27	14 3	8–11	9	14.28	8.96	2.664	100.73	383.0
28	14–10	9–1	9	14.82	9.10	2.713	104.75	406.8
29	15–4	9–3	9	15.33	9.23	2.758	108.65	429.8
30	15–6	9–5	10	15.53	9.42	2.813	113.1	449.0
31	15–8	9–7	10	15.70	9.61	2.866	117.5	467.7
32	15–10	9–10	10	15.87	9.80	2.922	122.2	486.4
33	16–5	9–11	10	16.42	9.93	2.968	126.4	513.8
34	16–7	10–1	10	16.58	10.12	3.023	131.2	533.7

1-inch pipe-arches, and 9- by 2½-inch structural plate pipe-arches, flowing partly full, are not available at present. Use of nominal dimensions given in manufacturers' catalogs for these conduits probably will produce no significant errors in the determination of resistance coefficients. Secondly, some circular conduits, notably the riveted 2⅔- by ½-inch, 3- by 1-inch, and 6- by 1-inch C.M.P., have actual diameters equal to the nominal diameters. For these conduits, standard tables and formulas can be used to determine geometric and hydraulic properties. (See tables C–1 and C–2.)

The $D^{2.5}$ values for circular pipes and $BD_a^{1.5}$ values for pipe-arches, presented in the accompanying tables, are for use in computing conduit flow factors in terms of either $Q/D^{2.5}$ or $Q/B\,D_a^{1.5}$. This flow factor is required to determine resistance coefficients for the 2⅔- by ½-inch and 6- by 1-inch corrugated metal conduits. Notice that the flow factor would be dimensionless if divided by $g^{0.5}$, a constant.

Pipe-arches, in general, are not geometrically similar, and the area and hydraulic radius of each pipe-arch section, flowing full or partly full, must be determined individually from its dimensions. However, it has been found that for

Table C–8.—Dimensions and hydraulic properties of 6- by 2-inch structural plate corrugated metal pipe-arches with 31-inch corner radius, full-flow condition

Section No.	Nominal size		Plates per ring	Span B	Rise D_a	Hydraulic radius R	Area A	$BD_a^{1.5}$
	Span (B)	Rise (D_a)						
	ft.–in.	*ft.–in.*	*Number*	*Feet*	*Feet*	*Feet*	*Feet²*	
1	13–3	9–4	8	13.28	9.36	2.715	98.30	380.3
2	13–6	9–6	8	13.52	9.53	2.764	102.00	397.8
3	14–0	9–8	8	13.97	9.68	2.811	106.00	420.8
4	14–2	9–10	8	14.22	9.87	2.857	110.88	441.0
5	14–5	10–0	8	14.40	10.04	2.927	115.28	458.1
6	14–11	10–2	9	14.88	10.19	2.967	119.6	484.0
7	15–4	10–4	9	15.35	10.34	3.031	124.0	510.4
8	15–7	10–6	10	15.58	10.52	3.093	129.0	531.6
9	15–10	10–8	10	15.80	10.71	3.139	133.8	553.8
10	16–3	10–10	10	16.28	10.85	3.187	138.0	581.8
11	16–6	11–0	10	16.50	11.03	3.242	143.0	604.4
12	17–0	11–2	10	16.97	11.18	3.296	148.0	634.3
13	17–2	11–4	10	17.18	11.36	3.348	153.1	657.8
14	17–5	11–6	10	17.40	11.54	3.400	158.5	682.1
15	17–11	11–8	10	17.88	11.69	3.446	163.4	714.7
16	18–1	11–10	10	18.10	11.87	3.492	168.0	740.3
17	18–7	12–0	10	18.58	12.01	3.558	174.0	772.9
18	18–9	12–2	10	18.78	12.20	3.600	179.0	800.2
19	19–3	12–4	10	19.28	12.34	3.646	184.7	835.8
20	19–6	12–6	11	19.50	12.52	3.696	190.0	863.8
21	19–8	12–8	11	19.70	12.71	3.755	196.2	892.6
22	19–11	12–10	11	19.88	12.89	3.818	202.4	920.0
23	20–5	13–0	12	20.40	13.03	3.866	207.8	959.4
24	20–7	13–2	12	20.58	13.22	3.919	214.0	989.3

Table C–9.—Geometric factors for 6- by 2-inch structural plate corrugated metal pipe-arches with 18-inch or 31-inch corner radius, full or partly full flow—mean values for all sizes, both corner radii

[d = Depth of flow, D_a = Rise of pipe-arch, R = Hydraulic radius, A = Area of flow, and B = Span of pipe-arch]

$\frac{d}{D_a}$	$\frac{R}{D_a}$	$\frac{A}{B\ D_a}$	$\frac{d}{D_a}$	$\frac{R}{D_a}$	$\frac{A}{B\ D_a}$
1.00	0.294	0.788	0.50	0.306	0.443
0.95	0.336	0.775	0.45	0.286	0.393
0.90	0.349	0.754	0.40	0.264	0.346
0.85	0.356	0.726	0.35	0.239	0.295
0.80	0.358	0.693	0.30	0.211	0.246
0.75	0.357	0.657	0.25	0.179	0.197
0.70	0.353	0.618	0.20	0.144	0.148
0.65	0.345	0.577	0.15	0.107	0.101
0.60	0.335	0.534	0.10	0.068	0.056
0.55	0.321	0.489	0.05	0.030	0.020

all pipe-arch sections, the dimensionless ratios for these properties, R/D_a and $A/B\,D_a$ at a given relative depth, d/D_a, deviate little from an average value. Mean values of these dimensionless ratios for 2⅔- by ½-inch pipe-arches are given in table C–6 and similar average ratios for 6- by 2-inch structural plate pipe-arches are given in table C–9. Table C–9 is to be used for both the 18-inch-corner-radius and 31-inch-corner-radius 6- by 2-inch pipe-arches, as the averages of the R/D_a and $A/B\,D_a$ ratios for conduits of both corner radii are about equal at any given relative depth of flow. The 6- by 2-inch structural plate pipe-arches with 18-inch corner radii comprise a large range of sizes and thus deviate from the mean value more than the arches with 31-inch corner radii. The errors involved in using means are still less than 5 percent for determination of A or R from full flow down to a relative depth of 0.25.

Similar average dimensionless ratios R/D_a and $A/B\,D_a$ were computed from available data for 9- by 2½-inch structural plate corrugated pipe-arches with 28.8 inch corner radius flowing partly full. The values so determined at various relative depths d/D are nearly identical with those of table C–9 for 6- by 2-inch arches with the two different corner radii. The values of table C–9 can be used for determination of resistance factors without introducing significant error.

Table C–10.—Dimensions and hydraulic properties of 9- by 2½-inch structural plate corrugated metal pipe-arches with 28.8-inch corner radius, full-flow condition

Section No.	Nominal size: Span (B)	Nominal size: Rise (D_a)	Plates per ring	Span B	Rise D_a	Hydraulic radius R	Area A	$BD_a^{1.5}$
	ft.–in.	*ft.–in.*	*Number*	*Feet*	*Feet*	*Feet*	*Feet²*	
1	5–11	5–4	2	5.91	5.32	1.415	25.16	72.52
2	6–3	5–5	2	6.28	5.46	1.472	27.37	80.13
3	6–8	5–7	2	6.65	5.60	1.533	29.73	88.11
4	6–11	5–9	2	6.96	5.76	1.590	32.11	96.19
5	7–4	5–11	2	7.34	5.91	1.646	34.55	105.48
6	7–8	6–1	2	7.65	6.08	1.704	37.13	114.67
7	8–0	6–2	3	8.04	6.19	1.760	39.75	123.8
8	8–4	6–4	3	8.33	6.34	1.816	42.48	132.9
9	8–7	6–6	3	8.64	6.54	1.874	45.35	144.5
10	9–0	6–8	3	9.04	6.64	1.924	48.09	154.7
11	9–4	6–10	3	9.32	6.82	1.982	51.13	166.0
12	9–9	6–11	3	9.73	6.94	2.026	53.90	177.9
13	10–0	7–1	3	10.03	7.11	2.085	57.13	190.2
14	10–5	7–3	3	10.45	7.24	2.141	60.38	203.6
15	10–9	7–5	3	10.73	7.41	2.193	63.61	216.4
16	11–2	7–6	3	11.15	7.54	2.243	66.85	230.8
17	11–5	7–8	3	11.44	7.71	2.296	70.29	244.9
18	11–8	7–10	3	11.69	7.84	2.360	74.14	256.6
19	12–2	8–0	3	12.15	8.01	2.392	77.06	275.4
20	12–5	8–2	3	12.40	8.15	2.451	80.90	288.5
21	12–10	8–3	4	12.91	8.39	2.517	85.11	313.7
22	13–1	8–5	4	13.09	8.42	2.541	87.97	319.8
23	13–7	8–7	4	13.57	8.58	2.601	92.13	341.0
24	13–10	8–9	4	13.81	8.73	2.657	96.23	356.2
25	14–3	8–10	4	14.28	8.88	2.699	99.90	377.8
26	14–6	9–0	4	14.55	9.06	2.760	104.4	396.8
27	14–9	9–2	4	14.77	9.16	2.790	107.8	409.4
28	15–3	9–4	4	15.20	9.26	2.838	111.9	428.3
29	15–6	9–6	4	15.52	9.52	2.916	117.3	455.8
30	16–0	9–7	4	15.97	9.64	2.954	121.2	478.0
31	16–2	9–9	4	16.22	9.80	3.005	125.7	497.6
32	16–8	9–11	4	16.70	9.92	3.045	129.8	521.7
33	16–11	10–1	4	16.90	10.04	3.080	133.8	537.6

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APPENDIX D — COMPREHENSIVE COMPUTATION PROCEDURE FOR PRECISE DETERMINATIONS OF RESISTANCE COEFFICIENTS IN CORRUGATED METAL CONDUITS

The complete calculation process used to derive the flow resistance curves of figures 7–13 and 15–16 of the main text is presented here together with examples of the process. Usually the approximate procedures set forth in the main text will provide resistance coefficients of sufficient precision with less effort, but at times it may be desirable to obtain the most precise results possible for conditions—size, relative depth, flow rate—not specifically given in the main text or for extreme conditions of conduit size, flow rate, or flow depth. As shown in the main text, errors in the approximate methods are largest for the extreme conditions, such as for the largest or smallest conduit available or for shallow flow depths. For these determinations, it will be necessary to follow the methods described here.

The initial steps of the calculation procedure are the same both for conduit corrugation forms affected by Reynolds number, which include 2⅔- by ½-inch and 6- by 1-inch corrugations, and for those free of Reynolds-number effects, which include 3- by 1-inch, 6- by 2-inch, and 9- by 2½-inch corrugations. However, the corrugation types affected by Reynolds number require additional operations to adjust their resistance coefficients for these effects.

Bolt resistance must also be considered in the structural plate conduits. All structural plate conduits dealt with in this report are assembled of corrugated metal that has corrugations free of Reynolds-number effects. The number of bolts exposed to flow per linear foot of conduit full or partly full, can be obtained readily from manufacturers' design manuals in which the structural plate sizes used to fabricate the conduits are given.

As indicated in the main-text section *Bolt Resistance in Structural Plate Corrugated Pipes*, the number of longitudinal joints submerged by a given relative depth of flow in a circular structural plate C.M.P. depends on the orientation of the pipe and the widths of the plates used to assemble the pipe. For simplicity, it was assumed that the longitudinal joints were spaced evenly about the circumference of the pipe. Also, when a given depth of flow might or might not submerge one of the longitudinal joints, depending on orientation, an average number of joints was used, resulting in a fractional number of seams. These fractional seams are seen in example 2 of this appendix.

Of course, in pipe-arches in which joint positioning is fixed, the number of longitudinal seams submerged by a certain flow depth is known. Also, if the actual positioning of the joints in a circular pipe is known, bolt resistance effects can be estimated more precisely.

The steps required to determine the resistance coefficients for full or partly full flow in conduits having 3- by 1-inch, 6- by 2-inch, or 9- by 2½-inch corrugations (no Reynolds-number effects) are as follows:

1. Determine the effective diameter, D_e, of the conduit in feet. For full circular pipes, this is merely the true diameter. For partly full circular pipes or for pipe-arches, full or partly full, D_e equals four times the hydraulic radius of the flow prism.

2. Determine the inverse of the relative roughness, D_e/k, where k is the particular corrugation depth in feet.

3. Enter figure 2 with the D_e/k ratio and read f from the curve for the appropriate corrugation type. This is the peak f value, which is the total f for 3- by 1-inch corrugations and the wall f, excluding bolt resistance, for the 6- by 2-inch and 9- by 2½-inch structural plate corrugations.

4. Determine the bolt resistance, Δf, for the structural plate pipe or pipe-arch, based on the number of crest bolts (do not include bolt heads or nuts in inside troughs of corrugations) submerged by the particular relative depth, using either equation (7a) or (7b). The sum of the wall f from step 3 and bolt resistance is the total f for the structural plate conduits.

5. Convert f to n by use of equation (8), if desired. The need for this step can be eliminated by using the Darcy equation in the form of equation (10) to compute the mean velocity of flow:

$$V = \frac{16.04}{f^{0.5}} R^{0.5} S^{0.5} \qquad (10)$$

rather than the Manning equation. Note that in the above equation, the square root of R appears rather than the ⅔ power of R as in the Manning equation, thus simplifying the design calculations to a degree.

The procedure for determination of the resistance coefficients for full or partly full flow in the conduit types affected by Reynolds number is similar to that for the other short-corrugation conduit types, but additional steps are required to account for Reynolds-number effects, which in general, cause a decrease in the resistance coefficient:

1. Determine D_e (same as for short corrugations).
2. Compute D_e/k (same as for short corrugations).
3. Enter figure 2 with D_e/k and read the peak f value from the appropriate curve (same as for short corrugations).
4. The peak f from step 3 must now be adjusted for Reynolds-number effects. On a figure similar to figure 5 for 2⅔-by ½-inch corrugations, or to figure 6 for 6- by 1-inch corrugations, draw a curve for the one or more relative depths of flow involved, parallel to the curves for the various pipe diameters with its peak f value, as computed in step 3, at $N_{R_{4R}}/k = 30{,}000$.
5. Based on equation (2a), non-circular conduits, or equation (2b), circular conduits, determine the relation between $N_{R_{4R}}/k$ and f. (Q, conduit size, relative depth, A, and ν are either known or have been estimated).
6. By a trial and error procedure (demonstrated in the following examples) using the relation derived in step 5, determine the f and $N_{R_{4R}}/k$ values that intersect on the particular relative depth curve constructed in step 4. This is the desired f value for the specific corrugation type, conduit shape, flow rate, and depth of flow.
7. Convert f to n by use of equation (8), if desired. Again the modified form of the Darcy equation (equation (10) of the main text) can be used in the design calculations rather than the Manning equation, thus eliminating the need for this step.

The trial and error procedure to determine resistance coefficients for conduits of the corrugation types affected by Reynolds number (2⅔- by ½-inch and 6- by 1-inch) is tedious and time consuming, as will be illustrated in the following examples. For this reason, the two graphs, figures 15 and 16 of the main text, were developed for circular pipes having these corrugation types using the precise calculation methods. Figure 15 is for 2⅔- by ½-inch corrugations and figure 16 for 6- by 1-inch corrugations. In both figures, the resistance coefficient is presented as a function of flow rate, $Q/D^{2.5}$, and relative depth, d/D, for the range of circular conduit sizes commonly available. Use of these curves was discussed in the last section of the main text.

Examples of determinations of the resistance coefficients of C.M.P. using the comprehensive computation procedure

1.—*Circular C.M.P., resistance affected by relative roughness only.*

Given: Six-foot (actual diameter) C.M.P. with 3- by 1-inch corrugations (k = 1 inch = 0.0833 feet).
Required: Resistance coefficients f and n at a range of relative depths.

d/D	R/D (Table C–2)	$R = (R/D)D$	$D_e = 4R$	D_e/k	f (Fig. 2, curve 1)	n Equation (8)
1.00	0.2500	1.500	6.00	72.0	0.0727	0.0267
0.75	0.3017	1.810	7.24	86.9	0.0668	0.0264
0.60	0.2776	1.666	6.66	79.9	0.0692	0.0265
0.50	0.2500	1.500	6.00	72.0	0.0727	0.0267
0.30	0.1709	1.025	4.10	49.2	0.0871	0.0274

2.—*Circular structural plate C.M.P., resistance affected by relative roughness and bolt effects.*

Given: Twelve-foot (nominal diameter) structural plate C.M.P. with 6- by 2-inch corrugations (k = 2 inches = 0.1667 feet).
Required: Resistance coefficients f and n at a range of relative depths.

From table C–3, actual diameter = 12.06 feet, 8 plates per ring; assume equal spacing of longitudinal joints around circumference.

d/D	R/D (Table C–2)	$R = (R/D)D$	$D_e = 4R$	D_e/k	f (Fig. 2, curve 1, no bolts)
1.00	0.2500	3.015	12.06	72.4	0.0727
0.75	0.3017	3.638	14.55	87.3	0.0666
0.60	0.2776	3.348	13.39	80.3	0.0691
0.50	0.2500	3.015	12.06	72.4	0.0727
0.30	0.1709	2.061	8.24	49.4	0.0869

Bolt resistance (Δf) computations:

d/D	Longitudinal seams submerged[1]	Longitudinal bolts/102 ft.[1]	Circumferential bolts/102 ft.	Total bolts/102 ft.[1]	A/D^2 (Table C–2)
1.00	8.0	1,632	520	2,152	0.7854
0.75	[2] 5.5	1,122	347	1,469	0.6319
0.60	[3] 4.5	918	294	1,212	0.4920
0.50	4.0	816	260	1,076	0.3927
0.30	3.0	612	192	804	0.1982

[1] Determined from manufacturer's design manual.
[2] Could be 5 or 6, depending on orientation of pipe.
[3] Could be 4 or 5, depending on orientation.

From equation (7b) of the main text:

$$\Delta f \text{ (Bolts)} = \frac{15.12\ (C_D)\ (N)\ (a)\ (R/D)\ (f)}{(A/D^2)\quad (D)\qquad (L)} \text{--------(7b)}$$

Where,
C_D = 1.1.
a = 0.0070 feet².
N = number of bolts per 102-foot length (see above).
D = 12.06 feet (table C–3).
L = 102 feet (12 8-foot-long plates plus one 6-foot-long plate, 13 circumferential joints).
f = Darcy f excluding bolts.

d/D	f (No bolts)	Δf (Bolts)	Total f	Total n (Equation (8))
1.00	0.0727	0.0047	0.0774	0.0310
0.75	0.0666	0.0044	0.0710	0.0306
0.60	0.0691	0.0045	0.0736	0.0307
0.50	0.0727	0.0047	0.0774	0.0309
0.30	0.0869	0.0057	0.0926	0.0317

The main complication in this example is the determination of bolt resistance (Δf), which is only a small percentage of the total f.

3.—*Circular C.M.P., resistance affected by relative rougness and Reynolds-number effects.*

Given: Four-foot (actual diameter) C.M.P. with 2⅔- by ½-inch corrugations (k = ½ inch = 0.0417 feet).

Required: Resistance coefficients f and n at the following relative depths and flow rates:

d/D	Q (c.f.s.)	$(Q/D^{2.5})$[1]
1.00	64	2.0
0.75	64	2.0
0.60	64	2.0
0.50	64, 16	2.0, 0.5
0.30	16	0.5

[1] From table C–1, $D^{2.5} = 32.00$.

d/D	R/D (Table C–2)	$R = (R/D)D$	$D_e = 4R$	D_e/k	Peak f (Fig. 2, curve 2)
1.00	0.2500	1.000	4.00	96.0	0.0679
0.75	0.3017	1.207	4.83	115.9	0.0629
0.60	0.2776	1.110	4.44	106.6	0.0649
0.50	0.2500	1.000	4.00	96.0	0.0679
0.30	0.1709	0.684	2.74	65.8	0.0799

Reynolds-number effects must now be considered:

$$\frac{N_{Rw}}{k} = \frac{(f)^{0.5}\ (Q/D^{2.5})\ (D)^{0.5}}{2.828\ (A/D^2)\ \nu} \qquad \text{(2b)}$$

D = 4.0 feet.

$\nu = 1.217 \times 10^{-5}$ feet²/second (at 60°F)

d/D	$Q/D^{2.5}$	A/D^2 (Table C–2)	N_{Rw}/k (Equation (2b))
1.00	2.0	0.7854	$(1.48 \times 10^5)f^{0.5}$
0.75	2.0	0.6319	$(1.84 \times 10^5)f^{0.5}$
0.60	2.0	0.4920	$(2.36 \times 10^5)f^{0.5}$
0.50	2.0	0.3927	$(2.96 \times 10^5)f^{0.5}$
0.50	0.5	0.3927	$(7.40 \times 10^4)f^{0.5}$
0.30	0.5	0.1982	$(1.47 \times 10^5)f^{0.5}$

Next, on a figure similar to figure 5, curves must be drawn parallel to the curves for the different pipe diameters and with their peak f values positioned at N_{Rw}/k = 30,000. There should be one curve for each relative depth, as each relative depth has a different peak f based on relative roughness. This is demonstrated in figure D–1. Then, a trial and error procedure is performed to determine the f value at each relative depth that satisfies the above relations with N_{Rw}/k and intersects on the appropriate curve in figure D–1. For example, at d/D = 0.60, the peak f value = 0.0649 and $N_{Rw}/k = (2.36 \times 10^5)\ f^{0.5}$ for $Q/D^{2.5}$ = 2.0.

Trial	Estimated N_{Rw}/k	$f^{0.5}$	Estimated f		True[1] f
1	4.4×10^4	0.1864	0.0347	$<$	0.0642
2	6.0×10^4	0.2542	0.0646	$>$	0.0629
3	5.93×10^4	0.2513	0.0631	$\approx$	0.0630

[1] From curve with peak f = 0.0649 on figure D–1, at the estimated N_{Rw}/k value.

By repetition of the same procedure, the following f values are determined:

d/D	$Q/D^{2.5}$	f	n (Equation (8))
1.00	2.0	0.0675	0.0241
0.75	2.0	0.0620	0.0238
0.60	2.0	0.0630	0.0236
0.50	2.0	0.0645	0.0235
0.50	0.5	0.0665	0.0239
0.30	0.5	0.0794	0.0245

The above trial and error procedure could also be performed by estimating the f value, computing the N_{Rw}/k and comparing the point with the appropriate curve in figure D–1. In actuality, the process is a series of comparisons, in which the results of each preceding step are used to adjust the next estimated values.

4.—*Corrugated metal structural plate pipe-arch, resistance affected by relative roughness and bolt effects.*

Given: Structural plate corrugated metal pipe-arch with 6- by 2-inch corrugations (k = 0.1667 feet). Nominal size = 12 feet 10 inches by 8 feet 4 inches. Corner radius = 18 inches.

Required: Resistance coefficients f and n at a range of relative depths.

From table C–7:

Section No.	Nominal size	Plates per ring	B	D_a	R	A	$BD_a^{1.5}$
23	12 ft. 10 in. by 8 ft. 4 in.	8	12.87	8.31	2.461	85.20	308.3

BD_a = (12.87) (8.31) = 106.9

d/D_a	R/D_a (Table C–9)	A/BD_a (Table C–9)	$R = (R/D_a)D_a$	$A = (A/BD_a)BD_a$	$D_e = 4R$
1.00	0.294	0.788	[1] 2.443	[1] 84.24	[1] 9.77
0.75	0.357	0.657	2.967	70.23	11.87
0.60	0.335	0.534	2.784	57.08	11.14
0.50	0.306	0.443	2.543	47.36	10.17
0.30	0.211	0.246	1.753	26.30	7.01

[1] To demonstrate that the average ratios presented in table C–9 are not exactly correct, but close. For full flow, true values should be used; therefore R = 2.461, A = 85.20, and D_e = 9.84.

d/D_a	D_e	D_e/k	f (Fig. 2, curve 1, no bolts)	Δf Bolts[1]	Total f	Total n (Equation (8))
1.00	9.84	59.0	0.0798	0.0058	0.0856	0.0315
0.75	11.87	71.2	0.0730	0.0050	0.0780	0.0308
0.60	11.14	66.8	0.0752	0.0058	0.0810	0.0313
0.50	10.17	61.0	0.0786	0.0064	0.0850	0.0315
0.30	7.01	42.1	0.0939	0.0092	0.1031	0.0326

[1] Bolt resistance (Δf) calculations follow.

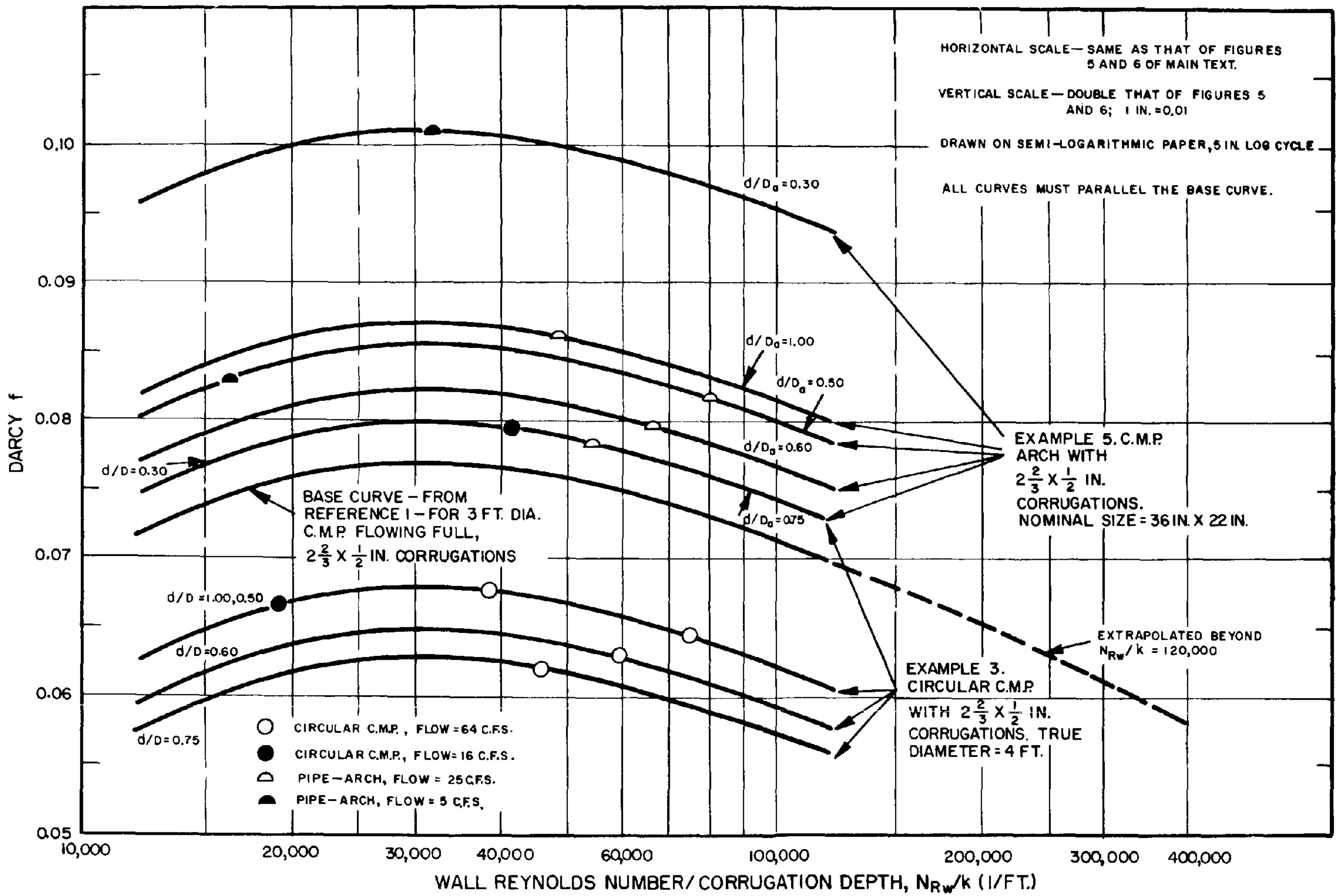

Figure D-1.—Wall Reynolds number divided by corrugation depth plotted against Darcy f—for example determinations of resistance coefficients of corrugated metal pipe.

Bolt resistance (Δf) computations:

d/D_a	Long. seams submerged [1]	Longitudinal bolts/102 ft.[1]	Circumferential bolts/102 ft.[1]	Total bolts/102 ft.[1]
1.00	8	1,632	572	2,204
0.75	5	1,020	390	1,410
0.60	5	1,020	351	1,371
0.50	5	1,020	312	1,332
0.30	5	1,020	260	1,280

[1] Determined from manufacturer's design manual.

From equation (7b) of the main text:

$$\Delta f\ (\text{Bolts}) = \frac{15.12\ (C_D)\ (N)\ (a)\ (R)\ (f)}{(A)\ (L)} \text{---from (7b)}$$

Where,

$C_D = 1.1$.

N = No. of bolts submerged/length L.

$a = 0.0070$ feet².

$f = f$ (no bolts).

L = length studied, feet = 102 feet (12 8-foot-long plates plus one 6-foot-long plate, 13 circumferential joints).

$\Delta f\ (\text{Bolts}) = 0.001142\ (N)\ (R)\ (f)/A$

d/D_a	N	R	f (no bolts)	A	Δf (bolts)
1.00	2,204	2.461	0.0798	85.20	0.0058
0.75	1,410	2.967	0.0730	70.23	0.0050
0.60	1,371	2.784	0.0752	57.08	0.0058
0.50	1,332	2.543	0.0786	47.36	0.0064
0.30	1,280	1.753	0.0939	26.30	0.0092

The main difficulty in this example is the computation of bolt resistance effects. Notice that these effects become quite large (about 9 percent) in the shallow depths owing to the large number of seams submerged by these depths in structural plate pipe-arches as compared with structural plate circular pipes.

5.—*Corrugated metal pipe-arch, resistance affected by relative roughness and Reynolds-number effects.*

Given: Corrugated metal pipe-arch with 2⅔- by ½-inch corrugations ($k = 0.0417$ feet). Nominal size = 36 by 22 inches).

Required: Resistance coefficients f and n at the following relative depths and flow rates:

d/D_a	Q(c.f.s.)	$Q/BD_a^{1.5}$
1.00	25	3.3
0.75	25	3.3
0.60	25	3.3
0.50	25, 5	3.3, 0.66
0.30	5	0.66

From table C–5:

Nominal Size	B	D_a	BD_a	R	A	$BD_a^{1.5}$
36 in. by 22 in.	3.01	1.85	5.57	0.564	4.42	7.566

d/D_a	R/D_a (Table C–6)	A/BD_a (Table C–6)	$R = (R/D_a)D_a$	$A = (A/BD_a)BD_a$	$D_e = 4R$
1.00	----	----	0.564	4.42	2.26
0.75	0.370	0.668	0.684	3.72	2.74
0.60	0.348	0.549	0.644	3.06	2.58
0.50	0.319	0.459	0.590	2.56	2.36
0.30	0.222	0.264	0.411	1.47	1.64

d/D_a	Q	D_e	D_e/k	Peak f (Fig. 2, curve 2)	N_{Rw}/k
1.00	25	2.26	54.2	0.0871	$(1.64 \times 10^5)f^{0.5}$
0.75	25	2.74	65.8	0.0799	$(1.95 \times 10^5)f^{0.5}$
0.60	25	2.58	61.9	0.0822	$(2.37 \times 10^5)f^{0.5}$
0.50	25	2.36	56.6	0.0855	$(2.84 \times 10^5)f^{0.5}$
0.50	5	2.36	56.6	0.0855	$(5.67 \times 10^4)f^{0.5}$
0.30	5	1.64	39.4	0.1010	$(9.88 \times 10^4)f^{0.5}$

Reynolds-number effects:

$$\frac{N_{Rw}}{k} = \frac{(f)^{0.5}\ (Q)}{2.828\ (\nu)\ (A)} \text{-------------------------(2a)}$$

$\nu = 1.217 \times 10^{-5}$ feet²/second for water at 60°F.

On a figure similar to figure 5, curves for each relative depth must be drawn parallel to the curves for the various diameters with their peak f values positioned at N_{Rw}/k = 30,000. This is shown in figure D–1 in which one such curve is drawn for each relative depth of flow.

Then, a trial and error procedure is used to determine the f value that satisfies the above relationship between f and N_{Rw}/k and intersects on the appropriate depth curve in figure D–1. For example, at $d/D = 0.60$ with a flow of 25 c.f.s., $N_{Rw}/k = (2.37 \times 10^5)f^{0.5}$ and the peak $f = 0.0822$.

Trial	Estimated N_{Rw}/k	$f^{0.5}$	Estimated f		True f[1]
1	6×10^4	0.2532	0.0641	<	0.0802
2	7×10^4	0.2954	0.0873	>	0.0795
3	6.7×10^4	0.2827	0.0799	≈	0.0798

[1] From curve with peak f = 0.0822 on figure D–1, at the estimated N_{Rw}/k value.

By repetition of the same procedure, the following f values are derived, and n is computed from f:

d/D	Q	f	n (Equation (8))
1.00	25	0.0861	0.0246
0.75	25	0.0784	0.0243
0.60	25	0.0798	0.0243
0.50	25	0.0818	0.0242
0.50	5	0.0830	0.0244
0.30	5	0.1009	0.0254

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APPENDIX E — APPRAISAL OF MODEL TESTS OF CORRUGATED METAL PIPES

As reported in Technical Report No. 2–715 by the Waterways Experiment Station (WES), Corps of Engineers (*6*), the investigations to determine the hydraulic resistance of structural plate corrugated steel pipe were sponsored jointly by the Office of Chief Engineers, U.S. Army, and the Bureau of Public Roads (BPR), Department of Transportation[1]. Representatives of both agencies evaluated the laboratory work as it progressed and proposed various methods of analysis for consideration by the author, Mr. John L. Grace, Jr., and by others of the WES laboratory technical staff.

The results of hydraulic tests in a 1.25-foot-diameter fiber glass model of a 5-foot-diameter standard C.M.P. with 2⅔- by ½-inch corrugations are described in the WES report. These initial tests were performed to establish that such models, scaled at 1:4, were satisfactory for determining hydraulic resistance in the prototypes. On page 24 of the report, it is concluded that corrugated metal pipes can be adequately simulated by geometrically similar fiber glass models.

In plate 34 of the WES report, a peak f value of 0.068 is given for the 1.25-foot-diameter standard C.M.P. model, whereas a peak f value of 0.064 was obtained from a full-sized test series on a 4.95-foot diameter pipe (*1*). (See figure 1 of this report.) At least in part, this 6 percent error is due to the greater relative roughness of the model and to the shorter radius of the corrugation crests, as indicated in plate 1 of the WES report. Larger models, such as the 1:2.2 scale model of a 5-foot diameter structural-plate pipe with 2-inch-deep corrugations, facilitate more accurate reproductions of the prototype corrugations. Also, larger corrugations are less difficult to model, even at the same scale ratio.

The experimental procedures used for the 2.27-foot-diameter model of a 5-foot-diameter structural plate C.M.P. with 6- by 2-inch corrugations differed somewhat from those for the 1.25-foot-diameter model of a 10-foot-diameter structural plate C.M.P. The total flow rate in the 2.27-foot model was obtained solely from point velocity measurements that were applied to a series of annular rings, without the advantage of independent venturi meter-flow measurements that were available for the smaller 1.25-foot-diameter model. The flow rate computations for the 2.27-foot model were independently verified by BPR personnel who obtained very good confirmations.

Two conclusions in the WES report (*6*) pertaining to 6- by 2-inch structural plate C.M.P. seemed to require verification. First, it was stated on page 13 that the resistance coefficient, f, is constant over some range of wall Reynolds numbers larger than 8,000 (plate 23, of the WES report). Secondly, it was concluded that a common velocity-distribution equation, relating the local flow velocity to distance from the pipe wall, had been obtained for the range of wall Reynolds numbers in which f was assumed to be constant. This common velocity-distribution equation would be applicable to models of any scale having 3 to 1 corrugation pitch-to-depth ratios, and therefore, to prototype pipes of any diameter with the same corrugation form.

Regarding the first conclusion, it was definitely established in plates 22 and 23 of the WES report that resistance factors, f, determined from tests of the 2.27-foot model of the 5-foot-diameter structural plate C.M.P., do not continue into the higher range of Reynolds numbers the same rising trends established in the lower Reynolds number range. From these plates, there appears to be a tendency for the resistance factor of the 5-foot-pipe model to become constant beyond some Reynolds number, or wall Reynolds number. The available flow capacity did not permit extension of the tests on the 1.25-foot model of the 10-foot pipe into a similarly high range of wall Reynolds numbers. For the 5-foot pipe model, a peaking of f followed by a decline as the Reynolds number rises may be obscured by the spread of data points in the wall Reynolds number range above 8,000, although a peak and decline was definitely established for pipes with 2⅔- by ½-inch corrugations. However, a sufficiently high Reynolds number was not attained in the tests to establish this fact. Therefore, for conservative design requirements, a constant f value should be assumed to represent the data in the upper Reynolds number range, pending possible future determinations of lesser f values at higher velocities of flow.

Three factors support the validity of assuming the constant f value. First, the data spread for the 5-foot pipe model could be represented by a straight line as well as by a slightly curved line. In part, the data spread could have been caused by the fact that some data points were based on test runs in which the velocity distribution was measured only in two quadrants of the model pipe, whereas, all other data were derived from traverses in four quadrants. Most of the points representing lower than average f values were from the two quadrant scans.

Secondly, if the line should in fact be curved, the variation of f in this peaking area is small over a wide Reynolds number range, and therefore, a straight line should be a

[1] When these investigations were conducted, Public Roads was a Bureau of the Department of Commerce.

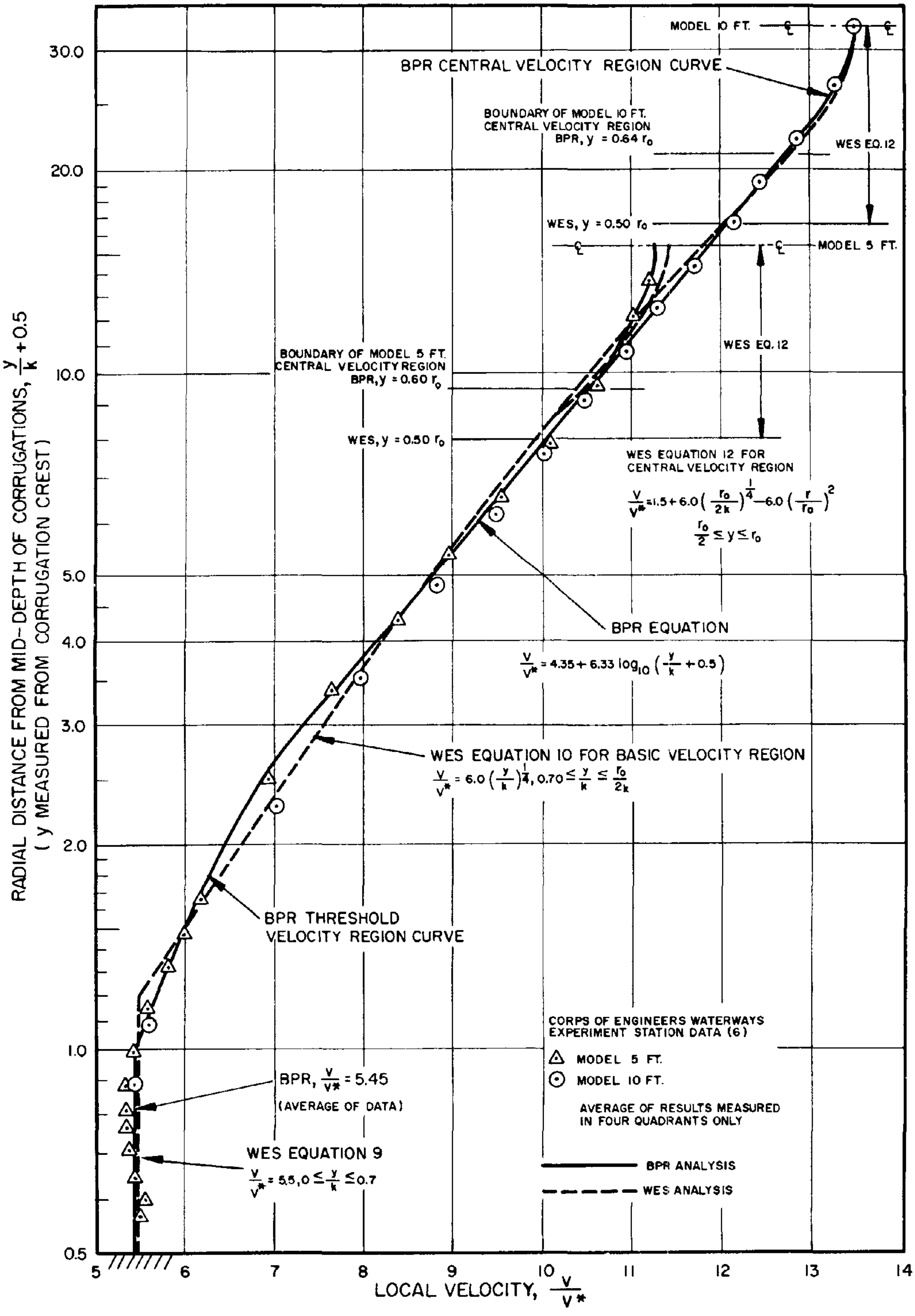

Figure E-1.—Velocity distribution in models of 6- by 2-inch structural plate corrugated pipe.

satisfactory average approximation. An average value is probably an adequate representation, considering the data scatter in the high N_{Rw} range.

Lastly, the range of wall Reynolds numbers, in which f is assumed constant, encloses the full-flow discharge rates of $Q/D^{2.5} = 2.0$ and 4.0. These are the discharges of most interest in highway drainage, as explained in the main text under the heading *Methods of Estimating Resistance Factors for Untested Corrugation Types.*

The straight line representations of f, at wall Reynolds numbers larger than 8,000, shown in plates 22 and 23 of the WES report for the three modeled pipe sizes, 5 feet, 10 feet, and 20 feet, were derived from an integration of the form of the velocity distribution equation advocated by equation 20, p. 16, of the WES report. A discussion of the velocity distribution equations formulated by both the WES and BPR follows.

Velocity Distribution in 6- by 2-inch Structural Plate C.M.P.

The local velocity traverses made in the model of the 5-foot structural plate C.M.P. in the range of assumed constant f values, above a wall Reynolds number of 7,000–8,000, permit the derivation of velocity distribution equations. These equations relate the local point velocities, v, to distances from the pipe wall, y. The local velocity is expressed as a ratio of the shear velocity, v^*, and the distances from the pipe wall are in terms of the corrugation depth, k, for C.M.P. Thus, the velocity distribution equation for conduits in which resistance is affected primarily by wall roughness, are of the general form:

$$\frac{v}{v^*} = \text{a function of } \left(\frac{y}{k}\right).$$

In the main text, the shear velocity v^* was defined as being equal to $(RS_f g)^{0.5}$. More generally, v^* is a function of the unit shear in the fluid at the condit wall, τ_0, and the mass density of the fluid, ρ. The following development,

$$v^* = \left(\frac{\tau_0}{\rho}\right)^{0.5} = \left(\frac{g\tau_0}{w}\right)^{0.5} = \left(\frac{g\ w\ A\ L\ S_f}{w\ P\ L}\right)^{0.5} = (RS_f g)^{0.5},$$

where w is the specific weight of water, P is the perimeter of the conduit, and the other terms are as previously defined, demonstrates the relationships involved in v^*.

The subsequent integration of the velocity distribution equations over the circular flow prism produces a relation between the total flow in the conduit, Q, and the relative roughness, in terms of the pipe diameter, radius, or hydraulic radius and corrugation depth. The resultant integration can be manipulated into the form,

$$\frac{Q}{Av^*} = \frac{V}{v^*} = \text{a function of conduit relative roughness.}$$

As $v^* = (RS_f g)^{0.5}$, and $S_f = \dfrac{f}{4R}\dfrac{V^2}{2g}$ (see page 8 of the main text),

$$\frac{V}{v^*} = \frac{V}{(RS_f g)^{0.5}} = \frac{V(8)^{0.5}\ R^{0.5}\ g^{0.5}}{R^{0.5}\ g^{0.5}\ f^{0.5}\ V} = \sqrt{\frac{8}{f}} \text{-------(E-1)}$$

The above derivation of the resistance factor from the velocity distribution equation is a commonly used method of fluid mechanics, and in most textbooks is applied to the equations for *smooth* and *rough* pipe velocity distribution.

There seem to be three regions of velocity distribution indicated by measurements in 6- by 2-inch structural plate C.M.P. (Plate 24 of the WES report), which are identified on page 14 of the WES report as: (a) threshold velocities (near the wall), (b) basic velocities (main body of flow), and (c) central velocities (near the axis of the pipe).

It was determined that velocity distribution measurements of the basic velocities from the 5-foot pipe model, in the range of constant resistance above $N_{Rw} = 8{,}000$, could be well represented either by a power equation:

$$\frac{v}{v^*} = p\left(\frac{y}{k}\right)^m$$

or by a semi-logarithmic (exponential) equation:

$$\frac{v}{v^*} = b\ \log\left(\frac{y}{k}\right) + s$$

where p, m, b, and s, are slope and intercept constants. This is generally true for these equation forms, as best-fit analyses usually indicate that when one form fits the particular data well, the other will also fit reasonably well.

The WES chose power equations to represent the data in the three regions of flow, and these equations and their curves are shown in figure E–1. The data points plotted in figure E–1 are the same as those in plate 24 of the WES report, except that the point velocities obtained from measurements in fewer than four quadrants are eliminated. The WES equation for the central velocity region indicates that the central region effect of reducing the local velocity extends outward from the axis a constant distance of one-half the pipe radius, regardless of relative roughness.

It was the opinion of BPR personnel that the semi-logarithmic form of the velocity distribution equation would provide a better data fit over a larger part of the velocity distribution data than the power form. Also trial determinations indicated that the best fit would be obtained by considering the origin of the curve to be at the mid-depth of the corrugation, rather than at the crest. The resultant equation and its curve, also shown in figure E–1, traces the data from a point at a distance of twice the corrugation depth from the crest to a point at a distance from the pipe axis that depends on relative roughness.

Near the conduit wall and in the vicinity of the pipe axis, it was recognized that the velocity distribution data deviated from the BPR velocity distribution equation for the basic region. Near the wall, the velocities are higher than indicated by the formula, whereas in the central region, they are lower. However, the effect on the total flow would be small and use of a single equation to represent the majority of the data is advantageous. The total Q, and thus the mean velocity, derived by integrating the BPR semi-logarithmic velocity distribution equation for the basic region over the circular area bounded by the corrugation crest, should therefore be adjusted by an increment near the wall and by a decrement near the pipe center. The magnitude of the increment and decrement was determined to be related to

Table E–1.—Structural Plate Corrugated Pipe Resistance Factors
[Comparison of Waterways Experiment Station (WES) values and Bureau of Public Roads (BPR) values, as computed by the equations shown]

$$\text{WES: } \frac{V}{v^*} = \frac{\sqrt{8}}{\sqrt{f}} = 0.188 + 4.96\left(\frac{r_o}{2k}\right)^{1/4} + 1.56\frac{k}{r_o} \quad \text{------- Eq. (E–2)}$$

$$\text{BPR: } \frac{V}{v^*} = \frac{\sqrt{8}}{\sqrt{f}} = -1.68 + 2.7\frac{k}{D} + 6.33\left(1+\frac{k}{D}\right)^2 \log\left(\frac{D}{k} + 1\right) \quad \text{------- Eq. (E–3)}$$

Diameter	$\frac{D}{k}$	Δf Bolts	WES f No bolts	WES f Total	BPR f No bolts	BPR f Total
Model results						
Feet			*Eq. E–2*		*Eq. E–3*	
2.269	29.96	-----	0.1107	-----	0.1110	-----
1.25	64.1	-----	0.0774	-----	0.0769	-----
1.25	127.6	-----	0.0556	-----	0.0567	-----
Full-size pipe						
4 plates per ring:						
4.93	29.6	0.0085	0.1115	0.1200	0.1117	0.1202
5.43	32.6	0.0075	-----	-----	0.1064	0.1139
5.94	35.7	0.0068	0.1022	0.1090	0.1019	0.1087
6.45	38.7	0.0061	-----	-----	0.0978	0.1039
6.97	41.8	0.0056	0.0949	0.1005	0.0943	0.0999
6 plates per ring:						
7.48	44.8	0.0068	-----	-----	0.0911	0.0979
7.98	47.9	0.0063	0.0890	0.0953	0.0882	0.0945
8.49	51.0	0.0058	-----	-----	0.0857	0.0915
9.00	54.0	0.0054	0.0841	0.0895	0.0833	0.0887
9.51	57.0	0.0051	-----	-----	0.0811	0.0862
10.02	60.1	0.0048	0.0799	0.0847	0.0792	0.0840
10.53	63.2	0.0045	-----	-----	0.0774	0.0819
8 plates per ring:						
11.04	66.2	0.0052	0.0763	0.0815	0.0756	0.0808
12.06	72.4	0.0047	0.0731	0.0778	0.0727	0.0774
13.08	78.4	0.0043	-----	-----	0.0700	0.0743
14.09	84.6	0.0039	0.0679	0.0718	0.0676	0.0715
10 plates per ring:						
14.60	87.6	0.0044	-----	-----	0.0666	0.0710
15.11	90.6	0.0042	0.0656	0.0698	0.0656	0.0698
16.13	96.8	0.0039	0.0635	0.0674	0.0636	0.0675
17.15	102.9	0.0037	0.0617	0.0654	0.0619	0.0656
17.66	106.0	0.0035	-----	-----	0.0611	0.0646
12 plates per ring:						
18.17	109.0	0.0039	0.0600	0.0639	0.0604	0.0643
19.18	115.1	0.0037	0.0584	0.0621	0.0591	0.0628
20.21	121.2	0.0035	0.0570	0.0605	0.0577	0.0612
21.22	127.4	0.0033	-----	-----	0.0566	0.0599

relative roughness, and in terms of the ratio of mean velocity to shear velocity, to equal approximately $+6.0\ k/D$ and $-0.6\ k/D$, respectively. These corrections can be incorporated in the integrated equation for mean velocity to produce the adjusted BPR equation E–3, shown at the top of table E–1.

The final form of the WES equation, relating the resistance factor (or ratio of mean velocity to shear velocity, as shown by equation E–2) to the conduit relative roughness, is also presented at the top of table E–1. In both the BPR and WES formulas bolt effects are excluded.

A comparison of the results of the two methods in the columns of f values with no bolts reveals that differences between the two procedures are negligible. The nearly identical results obtained by the two methods is considered to reinforce the conclusion that experimental data based primarily on the model of a 5-foot-diameter pipe is adequate to predict resistance factors for full size pipes.

According to the velocity distribution measurements from the 5-foot-pipe model at wall Reynolds numbers near, but below, the peak f location (below 8,000), there was no abrupt change in the measurements, and the equations and curves above the peak also fitted well into the data just below the peak. Therefore, it follows that if the same velocity distribution equations apply to larger pipe sizes, the equations should fit the 10-foot-pipe model data at the highest obtainable wall Reynolds numbers, which appear to be just below the peak f. This, in fact, is shown by the velocity distribution data for the 10-foot-pipe model in figure E–1. The BPR semi-logarithmic equation fits the data from both models about equally well, except for a slight deviation from the 10-foot model results at a distance from the corrugation crest of one to two times the corrugation depth.

The velocity distribution equations, as well as the resistance factor equations derived from them, are thus good representations of the peak, or constant, range of f values for both structural plate C.M.P. models and, therefore, apply to a range of prototype conduit sizes. The constant f values shown in figure 1 of the main text for 6- by 2-inch structural plate C.M.P. result from BPR equation E–3, shown at the top of table E–1.

Correlation With Other Hydraulic Results

In both figure 1 of this report and plate 23 of the WES report, the horizontal lines representing the constant f values intercept the rising portion of the f—N_{Rw} (or N_{Rw}/k) plots near a common N_{Rw} value. This phenomenon has been observed in other types of pipe that can be broadly classified as *rough*—having projections of various shapes that protrude from the walls and pierce or otherwise disrupt the laminar sublayer present in *smooth* flow.

The Nikuradse resistance experiments, conducted on pipes roughened with uniform sand grains, and reported by Rouse (*16*), reach a uniform *rough pipe* f at a constant value of $N_R f^{1/2}/\ (r_o/k)$, for all relative roughnesses. (N_R is the pipe

Reynolds number, VD/ν.) This factor can be shown to be directly related to the wall Reynolds number as follows:

$$\frac{N_R f^{1/2}}{r_o/k} = \frac{VD}{\nu}\frac{f^{1/2}}{r_o/k} = \frac{4V}{\nu}\frac{R}{2R/k}\frac{f^{1/2}}{} = \frac{2V\ f^{1/2}\ k}{\nu}$$

From equation (2a) of the main text:

$$N_{Rw} = \frac{V\ f^{1/2}\ k}{2.828\ \nu}$$

Therefore: $N_{Rw} = 0.177\ \dfrac{N_R\ f^{1/2}}{r_o/k}$

Nikuradse's experiments were conducted on a series of pipe sizes and sand grain diameters. However, if the grain size had been held constant and only the diameter changed, the k in the above equations could have been transposed to form the N_{Rw}/k ratio used in the main text. Similarly, in corrugated pipe of a given corrugation form, the corrugation depth is constant and the range of pipe diameters produces a change in relative roughness.

Just as the Nikuradse rough-pipe experiments resulted in an f that peaked at a common wall Reynolds number, the wall Reynolds number in C.M.P. also brings the peak f location to a common value, as can be seen by comparing plates 33 and 34 of the WES report for standard 2⅔- by ½-inch C.M.P. In plate 33, a plot of f against the pipe, or central, Reynolds number shows that the peak f location occurs at higher N_R values as the relative roughness (k/D) decreases. In plate 34 however, in all test results except those for the small diameters with their high relative roughnesses and erratic data, the peak f falls within a narrow range of N_{Rw} values. A similar situation is also apparent in figure 1 of the main text, where the majority of the peak f values fall within an N_{Rw}/k range of 30,000–40,000.

Because the rough pipe in the Nikuradse tests and the standard 2⅔- by ½-inch C.M.P. in the various other tests reached their peak, or constant, f values at a common N_{Rw}, or N_{Rw}/k value for all sizes of each type of pipe, it can be concluded that the 6- by 2-inch structural plate C.M.P. of any diameter should reach its peak, or constant, f at a common N_{Rw}/k.

The horizontal lines in plate 23 of the WES report and in figure 1 of this report represent constant f values for 6- by 2-inch structural plate C.M.P. The intersections of these lines with the projections of the rising f—N_{Rw} curves for lower flow rates occur at nearly the same N_{Rw} for the 10- and 20-foot pipe models as for the 5-foot pipe model. It must be remembered that the horizontal lines represent f values resulting from integrations of the velocity distribution equations. The rising portion of each f—N_{Rw} curve is at a rather flat slope, and any significant change in the vertical position (f) of the corresponding horizontal line would shift the intercept of the two lines a considerable distance and change the wall Reynolds number accordingly. In other words, if the velocity distribution equations derived earlier would not produce an appropriate f, the peaks would not occur at common N_{Rw} values, and the hydraulic results from other conduits generally classified as *rough* would be contradicted. The occurrence of the peaks at common N_{Rw} values verifies the validity of applying the velocity distribution equations to a range of model sizes, and thus, to a range of prototype pipe diameters.

Velocity Distribution Measurements in Standard C.M.P.

The velocity distribution equations derived from models of 6- by 2-inch structural plate C.M.P., can be further validated by comparing them with some similar equations derived by BPR personnel from the results of tests conducted by the North Pacific Division Hydraulic Laboratory (*1*) on standard 2⅔- by ½-inch C.M.P. The equations applicable to standard C.M.P. are based on velocity distribution measurements near the peak f values, and were developed independently from the equations for 6- by 2-inch structural plate C.M.P. Only the equations for the most important region of basic velocities, which cover the major part of the pipe area, will be compared. As would be expected, the velocity distributions produced by the two corrugation forms do change near the wall, but the resultant f differences are of secondary importance.

In the region from a height above the corrugation crests of 4.5 times the corrugation depth, the following equation from the North Pacific Division Hydraulic Laboratory data was found to fit fairly well:

$$\frac{v}{v^*} = 4.00 + 6.44\ \log_{10}\left(\frac{y}{k} + 0.5\right) \text{-----------------(E–4)}$$

For comparison, the equation advocated by the BPR for the basic velocity region in the 6- by 2-inch structural plate C.M.P. (figure E–1) is:

$$\frac{v}{v^*} = 4.35 + 6.33\ \log_{10}\left(\frac{y}{k} + 0.5\right) \text{-----------------(E–5)}$$

The similarities of the slopes, 6.44 and 6.33, and of the intercepts, 4.00 and 4.35, of the two equations are apparent.

There is enough spread in the velocity distribution data from the standard 2⅔- by ½-inch C.M.P. tests to justify some adjustment of equation E–4. The slope term can be modified to match that of the structural plate C.M.P. equation, 6.33, thus producing the following revised equation for standard C.M.P.

$$\frac{v}{v^*} = 4.09 + 6.33\ \log_{10}\left(\frac{y}{k} + 0.5\right) \text{-----------------(E–6)}$$

If the threshold and central velocity deviations are neglected, intergrations of equations E–5 and E–6 and their subsequent solutions to obtain f over a range of relative roughnesses, demonstrate the fact that the larger intercept term in equation E–5, coupled with an equal slope term, will result in a lower f than that produced by equation E–6, and that the average difference over a range of relative roughnesses is close to the 0.004 value suggested in the main text (see fig. 4). The absolute f values that would be derived by this simplified procedure are higher, at a given relative roughness, than the peak f values plotted in figure 2. However, inclusion of the local velocity increment near the wall in the velocity distribution integrations would tend to reduce the absolute values to levels that would be at or near the values shown in figure 2.

The marked similarity between the velocity distribution equations for the two different corrugation forms emphasizes the fact that relative roughness is the prime factor in the hydraulic resistance of corrugated metal pipe. This similarity also supports the validity of the velocity distribution and resistance equations, within the constant f range, derived for 6- by 2-inch structural plate C.M.P. from the WES model results.

Conclusions

In the 6- by 2-inch structural plate corrugated pipes, it is satisfactory to assume that the f reaches a peak, after which it remains constant as the Reynolds number or wall Reynolds number is increased. If the f for this pipe does, in fact, peak and then decrease, the error caused by assuming a constant f is negligible, unless very high flow rates—higher than 6.0 $D^{2.5}$— are to be encountered. For such flow rates, a constant f would produce a conservative result, in terms of conduit capacity or required slope.

In general, the wall Reynolds numbers on either end of the assumed constant f range, for which data are available from the WES study, include the normal discharge rates found in highway drainage—from about 2 $D^{2.5}$ to 4 $D^{2.5}$.

Sufficient evidence exists to support the general application of the velocity distribution equations from the WES model study to prototype 6- by 2-inch structural plate C.M.P. in the upper range of Reynolds numbers. Because they apply to such prototype pipes, integrations of these equations will yield wall resistance values. The total resistance factor then can be obtained by adding a bolt resistance increment to the wall resistance factor.

Results of these models tests on 6- by 2-inch structural plate C.M.P., coupled with results from the more abundant tests on standard 2⅔- by ½-inch C.M.P., provide a solid basis for the estimation of resistance coefficients for any corrugation form, conduit shape, flow depth, or flow rate, as presented in the main text of this paper.

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