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hydraulics of BRIDGE WATERWAYS

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HYDRAULICS OF BRIDGE WATERWAYS

Hydraulic Design Series No. 1

By the Division of Hydraulic Research

Bureau of Public Roads

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PREFACE

This publication is the first of a proposed series on hydraulic design of highway drainage structures. With the exception of chapter VII, which is new, nearly all the information herein has appeared previously in preliminary drafts entitled *Bridge Waterway Design,* issued in January 1957, and *Computation of Backwater Caused by Bridges,* issued in October 1958. The material was given fairly wide circulation and has been used extensively by State highway departments and consulting engineering firms as well as by the Bureau of Public Roads. It is now superseded by this publication.

Attention is directed to Chapter IX, "Limitations of Data." Research is continuing to extend knowledge of hydraulics of bridges and channels on wide flood plains, spur dikes, and scour phenomena.

The methods here presented for computing backwater caused by bridges are based almost entirely on model tests conducted by Colorado State University for the Bureau of Public Roads. The experimental data supporting the empirical curves contained in this publication are reported fully in the project report (reference 9 of the bibliography). That report also includes prototype measurements of the drop in water surface across bridge approach embankments, recorded by the U.S. Geological Survey, which were used in checking the validity of the computational methods presented herein.

The reader is invited to communicate to the Division of Hydraulic Research, Bureau of Public Roads, Washington 25, D.C., suggestions for improvement of the procedures given in this publication. Further simplification is admittedly desirable.

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Tables 2–6 and figures 25–31, not listed above, are parts of the examples in Chapter VIII, "Illustrative Problems."

Chapter I.—INTRODUCTION

1.1 General Structural designers are well aware of economies which can be attained in the structural design of a bridge of a given overall length. The role of hydraulics in establishing what the length and vertical clearance of a bridge should be and even where it should be placed is less well understood. Confining the flood water unduly may cause excessive backwater with resultant damage to upstream land and improvements and overtopping of the roadway or may induce excessive scour endangering the bridge itself. Too long a bridge may cost far more in added capital investment than can be justified by the benefits obtained. Somewhere in between is the design which will be the most economical to the public over a long period of years. Finding that design is the ultimate goal of the bridge designer.

This publication is intended to provide, within the limitations described in chapter IX, a means of computing the effect of a given bridge upon the flow in a stream. It does not prescribe criteria as to amount of backwater or frequency of the design flood. These depend upon policy which in turn will take into account traffic, flood damage, and other factors as discussed briefly in section 9.2. Likewise this publication does not eliminate the need for careful study in evaluating the conditions at a particular site. Rather, it will serve to draw attention to details which should be given consideration.

1.2 Waterway studies In recognition of the need of dependable hydraulic information, the Bureau of Public Roads initiated a cooperative research project with Colorado State University in 1954 which culminated in the investigation of several features of the waterway problem. These included a study of bridge backwater (*9*),[1] scour at abutments and piers, and the effect of scour on backwater. Previously and concurrently with this work, the Iowa State Highway Commission and the Bureau of Public Roads sponsored studies of scour at bridge piers (*8*) and scour at abutments (*7*) at the Iowa Institute of Hydraulic Research at Iowa City. In 1957 the State Highway Departments of Mississippi and Alabama in cooperation with the Bureau of Public Roads sponsored a project at Colorado State University to study means of reducing scour under a bridge by the use of spur dikes (*4*) (elliptical-shaped earth embankments placed upstream from a bridge).

This combination of laboratory studies in which hydraulic models served as the principal research tool has now been completed. Much remains to be accomplished in the collection of field data to substantiate the model results and extend the range of application.

1.3 Bridge backwater An account of the testing procedure, a record of basic data, and an analysis of results on the bridge backwater studies are contained in a comprehensive report (*9*) issued by Colorado State University. Results of research described in that comprehensive report were drawn upon for this publication, which deals only with that part of the waterway problem that pertains to the nature and magnitude of backwater produced by bridges constricting streams. It is prepared specifically for the designer in a practical form, containing design charts, procedures, examples, and a text limited principally to describing the proper use of the information.

1.4 Nature of bridge backwater It is seldom economically feasible or necessary to bridge the entire width of a stream as it occurs at flood flow. Where conditions permit, approach embankments are extended out onto the flood plain to reduce costs, recognizing that, in so doing, the embankments will constrict the flow of the stream during flood stages. This is an acceptable practice. When carried to extremes, however, constriction of the flow can result in damage to bridges, costly maintenance, backwater damage suits, or even contribute to the complete loss of the bridge or the approach embankments.

The manner in which flow is contracted in passing through a channel constriction where the bed resists scour is illustrated in figure 1. The flow bounded by each adjacent pair of streamlines is the same (1,000 c.f.s.). Note that the channel constriction appears to produce practically no alteration in the shape of the streamlines near the center of the channel. A very marked change is evidenced near the abutments, however, since the momentum of the flow from the contracted portion of the channel must force the advancing central portion of the stream over to gain entry to the constriction. Upon leaving the constriction the flow gradually expands (5 to 7 degrees per side) until normal conditions in the stream are again reestablished.

Constriction of the flow produces loss of energy, the greater portion occurring in the reexpansion downstream. The loss of energy is reflected in a rise in the water surface and in the energy line upstream from the bridge. This is best illustrated by a profile along the center of the stream, as shown in figure 2A. The normal stage of the stream for a given discharge, before constricting the channel, is represented by the dash line labeled "normal water surface." (Water surface is abbreviated as "W.S." in the figures.)

[1] Italic numbers in parentheses refer to publications listed in the bibliography, p. 53.

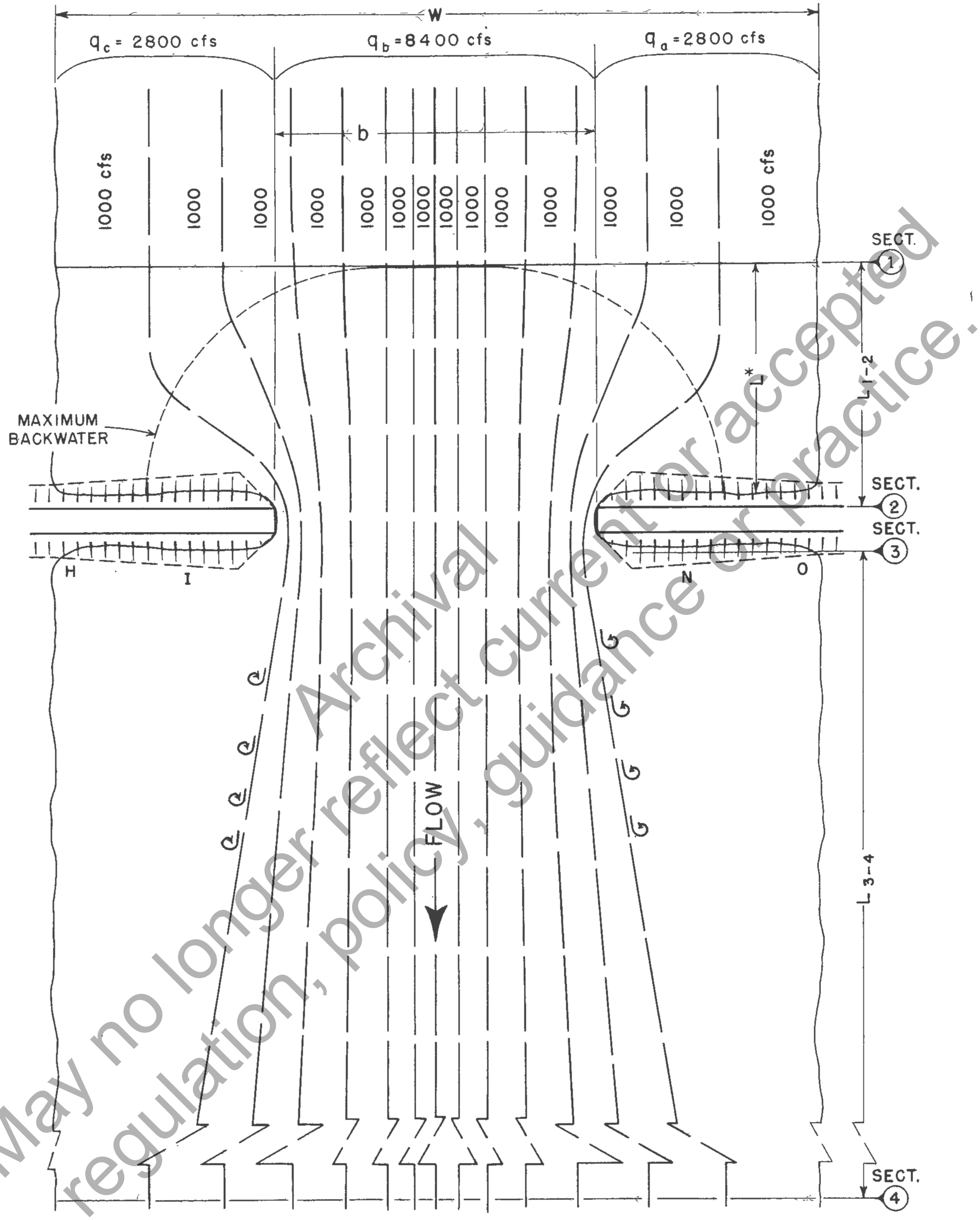

Figure 1.—Flow lines for typical normal crossing.

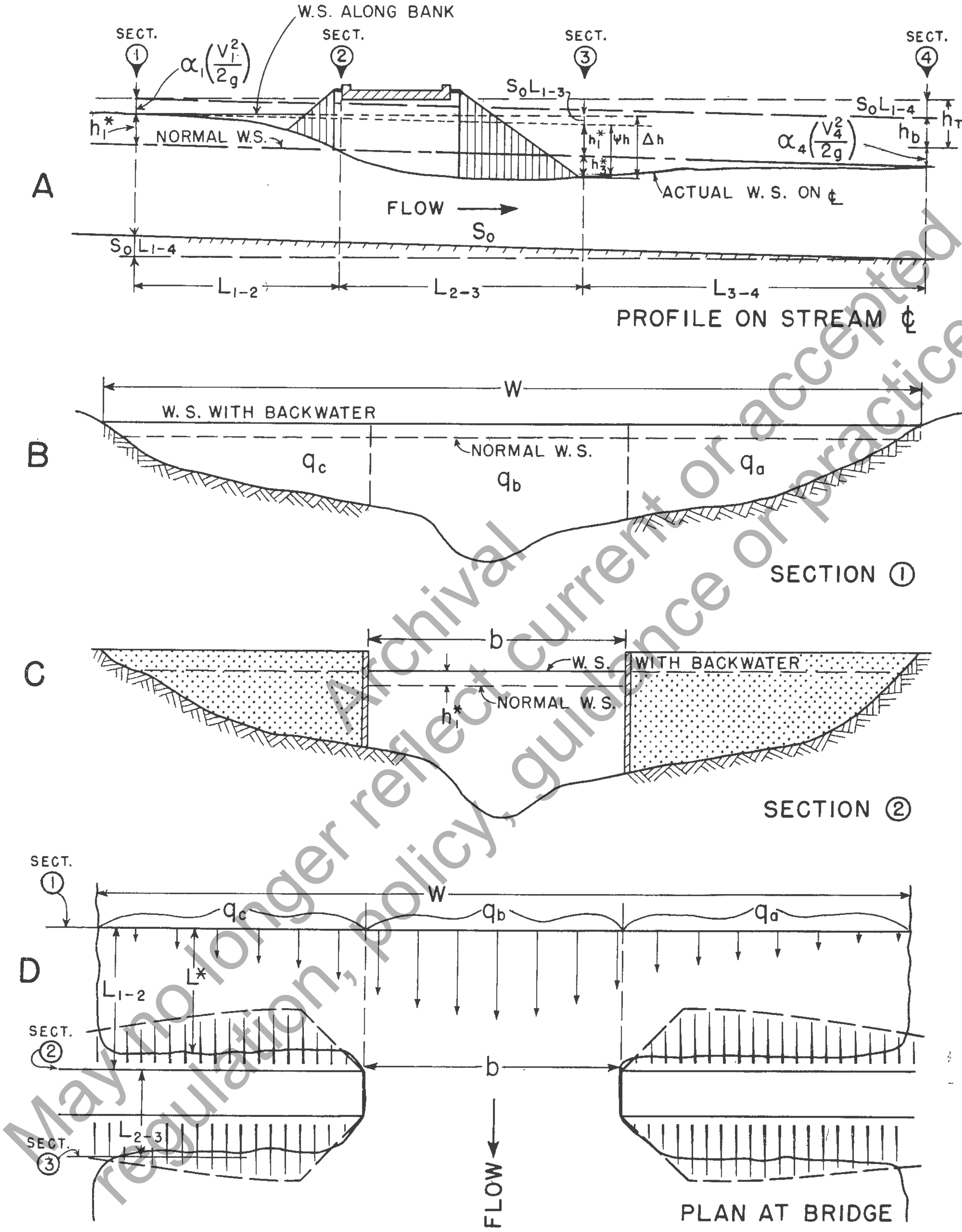

Figure 2.—Normal crossing: Wingwall abutments.

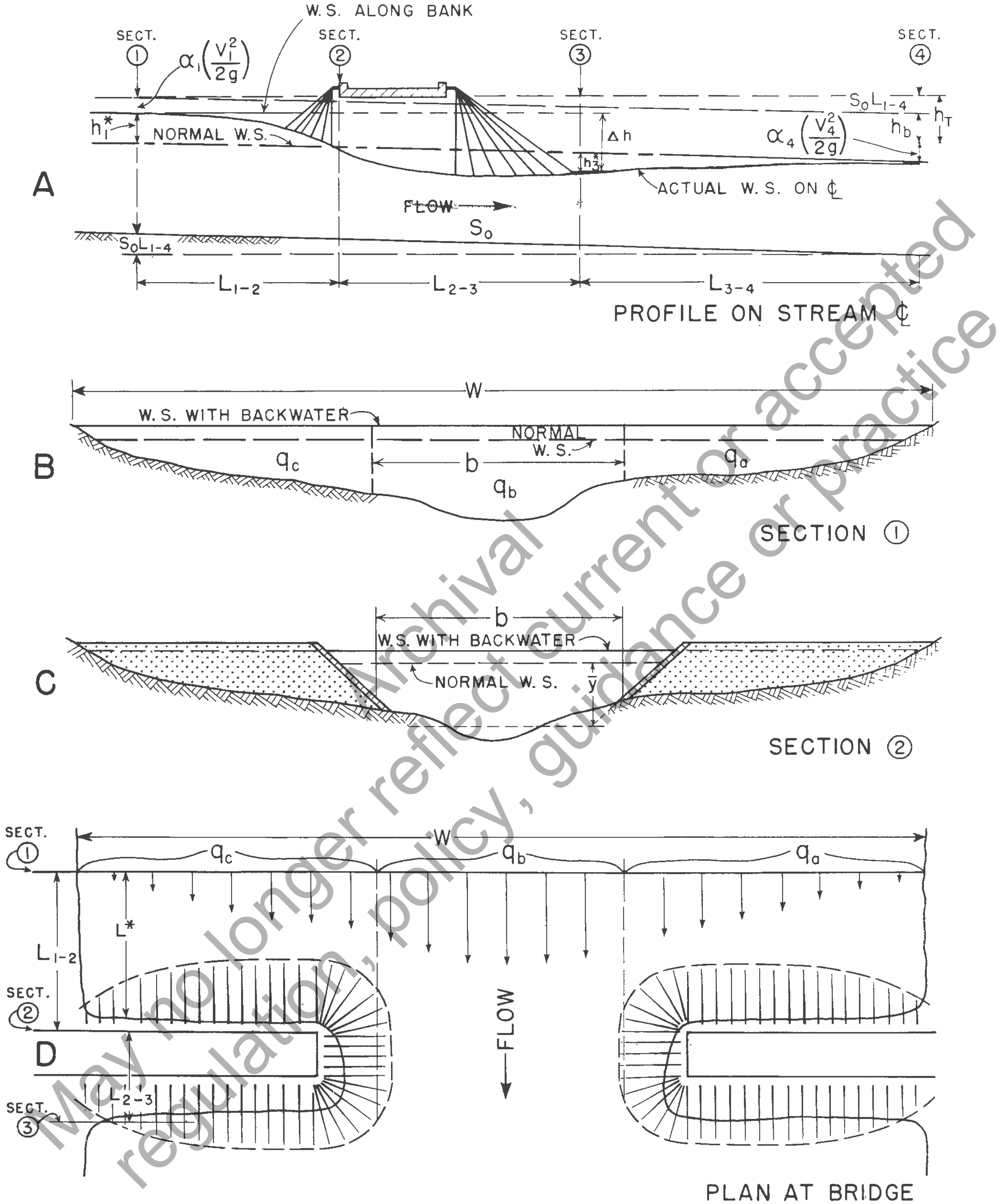

Figure 3.—Normal crossing: Spillthrough abutments.

The nature of the water surface after constriction of the channel is represented by the solid line, "actual water surface." Note that the water surface starts out above normal stage at section 1, passes through the normal stage close to section 2, reaches minimum depth in the vicinity of section 3, and then returns to normal stage a considerable distance downstream, at section 4. Determination of the rise in water surface at section 1, denoted by the symbol h_1^* and referred to as the bridge backwater, is the primary objective of this publication.

The Colorado laboratory model represented the ideal case since the testing was done principally in a rectangular, fixed bed, sloping flume, 8 feet wide by 75 feet long. Although bridge backwater was investigated with both supercritical and subcritical flow in the constriction, the results reported apply strictly to subcritical flow; i.e., flow at velocity less than critical velocity. The very real problem of scour in the constriction was avoided in the initial tests by the use of rigid boundaries. Ignoring scour is safe insofar as the computation of backwater is concerned, but scour must be considered for the safety of abutments and piers. As the water area in the constriction is increased due to scour, the backwater may be appreciably less than that for a streambed that resists scour. The effect of scour on backwater will be considered in chapter VII.

1.5 Verification of model results For the purpose of verifying the design charts used in this publication, the Geological Survey made available field measurements of flood flows for a number of bridges. Verification was accomplished by computing the backwater and related information from the design charts for each individual bridge and then comparing the result with the prototype measurements. The comparisons made in this way, although limited to bridges up to 220 feet in length and flood plains 0.5 mile wide, were considered quite satisfactory. The results of these comparisons are on record in the comprehensive report (*9*) and in other publications (*1*, *3*). Further verification on longer structures and wider flood plains is not only desirable but necessary.

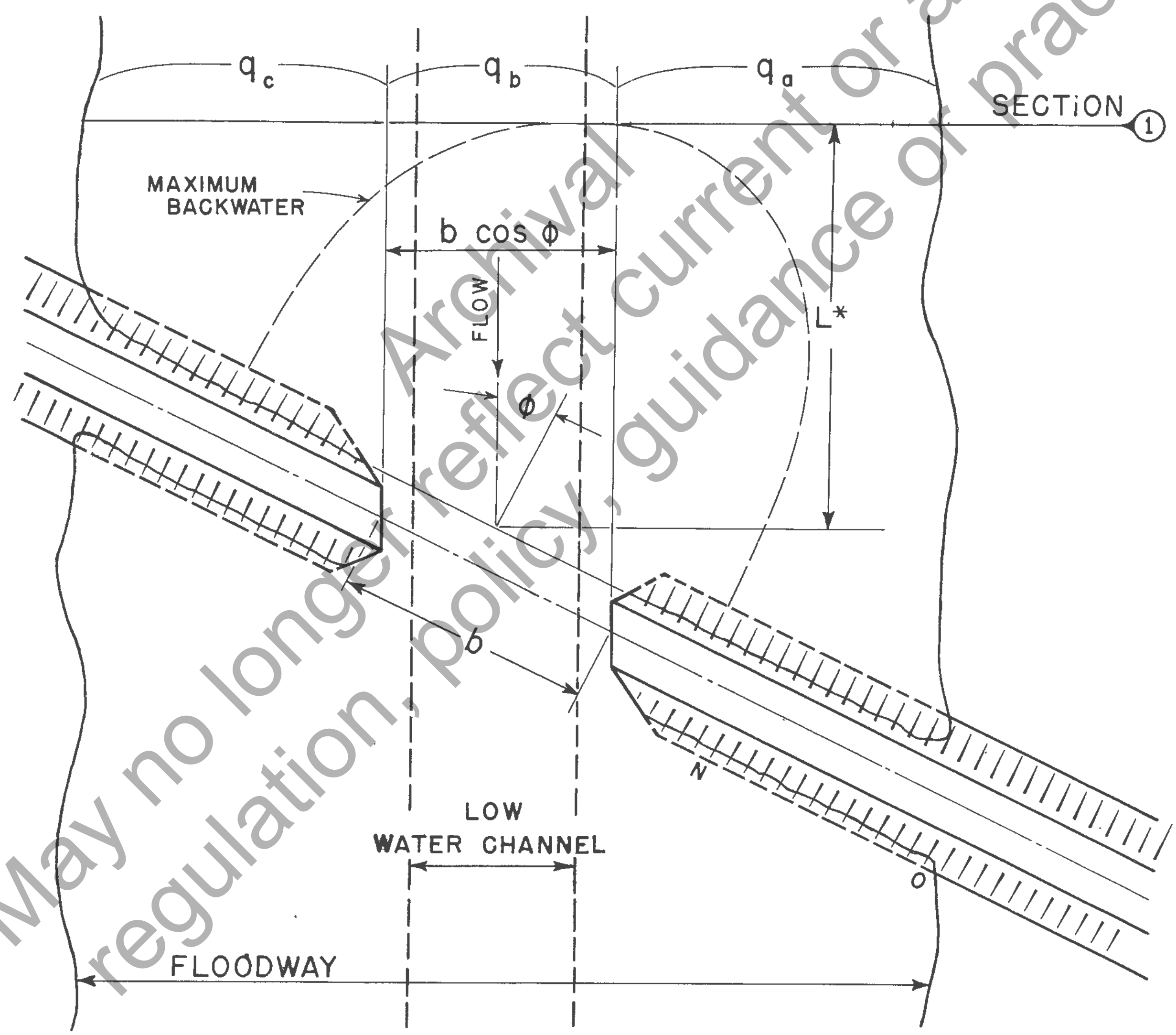

Figure 4.—Skew crossing.

1.6 Definition of symbols The symbols used in the ensuing text, figures, and illustrative examples, most of which can be understood more clearly by inspection of figures 1–4 (and others as cited), are as follows:

A_1 = Area of flow including backwater at section 1 (fig. 2B) (sq. ft.).

A_{n1} = Area of flow below normal water surface at section 1 (fig. 2B) (sq. ft.).

A_{n2} = Area of flow in constriction below normal water surface at section 2 (fig. 2C) (sq. ft.).

A_4 = Area of flow at section 4 at which normal water surface is reestablished (fig. 2A) (sq. ft.).

A_p = Projected area of piers normal to flow (between normal water surface and stream bed) (sq. ft.).

a = Area of flow in a subsection of a channel (fig. 2B) (sq. ft.).

b = Width of constriction (figs. 2C, 3C, and sec. 1.7) (ft.).

$D_b = h_b^*/h_3^*$ = Differential level ratio.

e = Eccentricity $= (1 - q_c/q_a)$ where $q_c < q_a$, or $(1 - q_a/q_c)$ where $q_a < q_c$ (fig. 8).

g = Acceleration of gravity = 32.2 (ft./sec.²).

h_T = Total energy loss between sections 1 and 4 (fig. 2A) (ft.).

$h_b = h_T - S_0 L_{1-4}$ = Energy loss caused by constriction (fig. 2A) (ft.).

h_1^* = Total backwater or rise above normal stage at section 1 (fig. 2A) (ft.).

h_3^* = Vertical distance from water surface on downstream side of embankment to normal water surface at section 3 (fig. 2A) (ft.).

h_b^* = Backwater computed from base curve (ft.).

h_d^* = Backwater produced by dual bridges measured at section 1 (fig. 16).

$\Delta h = h_b^* + h_3^* + S_0 L_{1-3}$ = Difference in water surface elevation across roadway embankment (fig. 14) (ft.).

$J = A_p/A_{n2}$ = Ratio of area obstructed by piers to gross area of bridge waterway below normal water surface at section 2 (fig. 7).

K_b = Backwater coefficient from base curve (figs. 5 and 6).

ΔK_p = Incremental backwater coefficient for piers (fig. 7).

ΔK_e = Incremental backwater coefficient for eccentricity (fig. 8).

ΔK_s = Incremental backwater coefficient for skew (figs. 9 and 10).

$K^* = K_b + \Delta K_p + \Delta K_e + \Delta K_s$ = Total backwater coefficient.

k_b = Conveyance of portion of channel within projected length of bridge at section 1 (figs. 2B and 2C and sec. 1.8).

k_a, k_c = Conveyance of that portion of the natural flood plain obstructed by the roadway embankment (subscripts refer to left and right side, facing downstream) (figs. 2B and 2C and sec. 1.8).

K_1 = Total conveyance at section 1 (sec. 1.8).

L_{1-4} = Distance from point of maximum backwater to reestablishment of normal water surface downstream, measured along centerline of stream (fig. 2A) (ft.).

L_{1-3} = Distance from point of maximum backwater to water surface on downstream side of roadway embankment (fig. 2A) (ft.).

L_{1-2} = Distance from point of maximum backwater to upstream face of bridge deck (fig. 2A) (ft.).

L^* = Distance from point of maximum backwater to water surface on upstream side of roadway embankment, measured parallel to centerline of stream (fig. 11) (ft.).

L_d = Distance between centerlines of dual parallel bridges (fig. 16) (ft.).

M = Bridge opening ratio (sec. 1.9).

n = Manning roughness coefficient (table 1).

p = Wetted perimeter of a subsection of a channel (ft.).

q_b = Flow in portion of channel within projected length of bridges at section 1 (fig. 2B) (c.f.s.).

q_a, q_c = Flow over that portion of the natural flood plain obstructed by the roadway embankments (fig. 2B) (c.f.s.).

$Q = q_a + q_b + q_c$ = Total discharge (c.f.s.).

$r = a/p$ = Hydraulic radius of a subsection of flood plain or main channel (ft.).

S_0 = Slope of channel bottom or normal water surface.

$V_1 = Q/A_1$ = Average velocity at section 1 (ft./sec.).

$V_4 = Q/A_4$ = Average velocity at section 4 (ft./sec.).

$V_{n2} = Q/A_{n2}$ = Average velocity in constriction for flow at normal stage (ft./sec.).

w_p = Width of pier normal to direction of flow (fig. 7) (ft.).

W = Surface width of stream including flood plains (fig. 1) (ft.).

y_n = Normal flow depth (ft.).

$\bar{y}$ = Height of trapezoid having equivalent cross section area of constriction (spillthrough abutments, fig. 3C).

$\alpha_1 = \dfrac{\Sigma(qv^2)}{QV_1^2}$ = Coefficient applied to velocity head to account for nonuniform velocity distribution over a flow section (sec. 1.10) (Greek letter alpha).

$\eta = h_d^*/h_1^*$ = Backwater multiplication factor for dual bridges (Greek letter eta).

σ = Multiplication factor for influence of M on incremental backwater coefficient for piers (fig. 7B) (Greek letter sigma).

$\psi h_{3B} = h_d^* + h_{3B}^*$ = Term used in computing difference in water surface elevation across two embankments (dual crossings) (fig. 18) (Greek letter psi).

$\xi = \psi h_{3B}/\psi h$ = Differential level multiplication factor for dual bridges (sec. 5.3) (Greek letter xi).

1.7 Definition of terms Specific explanation is given below with respect to the concept of several of the terms and expressions frequently used throughout the discussion:

Normal stage.—Normal stage is the normal water surface elevation of a stream at a bridge site, for a particular discharge, prior to constricting the stream (see fig. 2A). The profile of the water surface is essentially parallel to the bed of the stream.

Abnormal stage.—Where a bridge site is located upstream from but relatively close to the confluence of two streams, high water in one stream can produce a backwater effect extending for some distance up the other stream. This can cause the stage at a bridge site to be abnormal, meaning higher than would exist for the tributary alone. An abnormal stage may also be caused by a dam, another bridge, or some other constriction downstream (again assuming tranquil flow). The water surface with abnormal stage is no longer parallel to the bed (fig. 19).

Normal crossing.—A normal crossing is one with alinement at approximately 90° to the general direction of flow during high water (as shown in fig. 1).

Eccentric crossing.—An eccentric crossing is one where the main channel is not in the middle of the flood plain (fig. 8).

Skew crossing.—A skew crossing is one that is other than 90° to the general direction of flow during flood stage (fig. 4).

Dual crossing.—A dual crossing refers to a pair of parallel bridges, such as for a divided highway (fig. 16).

Width of constriction, b.—No difficulty will be experienced in interpreting this dimension for abutments with vertical faces since b is simply the horizontal distance between abutment faces. In the more usual case involving spillthrough abutments, where the cross section of the constriction is irregular, it is suggested that the irregular cross section be converted to a regular trapezoid of equivalent area as shown in figure 3C. Then the bridge opening can be interpreted as:

$$b=\frac{A_{n2}}{\bar{y}}$$

1.8 Conveyance Conveyance is a measure of the ability of a channel to transport flow. In streams of irregular cross section it is necessary to divide the water area into smaller but more or less regular subsections, assigning an appropriate retardance factor to each and computing the discharge for each subsection separately. According to the Manning formula for open channel flow, the discharge in a subsection of a channel is:

$$q=\frac{1.49}{n}\,ar^{2/3}S_0^{1/2}$$

By rearranging:

$$\frac{q}{S_0^{1/2}}=\frac{1.49}{n}\,ar^{2/3}=k$$

where k is the conveyance of the subsection. Conveyance can therefore be expressed either in terms of flow factors or strictly geometric factors. In bridge waterway computations, conveyance is used as a means of approximating the distribution of flow in the natural river channel upstream from a bridge. The method will be demonstrated in the examples. Total conveyance K_1 is the summation of the conveyances of the subsections.

1.9 Bridge opening ratio The bridge opening ratio M defines the degree of stream constriction involved. It is defined as the ratio of the flow which can pass unimpeded through the bridge constriction to the total flow of the river. Referring to figures 2B and 3B:

$$M=\frac{q_b}{q_a+q_b+q_c}=\frac{q_b}{Q} \qquad (1)$$

Or, considering the specific case shown in figure 1:

$$M=\frac{8{,}400}{14{,}000}=0.60$$

Because of the irregular cross section common in natural streams and the variation in boundary roughness within any cross section, the discharge is not uniform across a river but varies as might be indicated by the stream tubes in figure 1. The bridge opening ratio M is most easily explained in terms of discharges, but it is usually determined from conveyance relations. Since conveyance is proportional to discharge, assuming all subsections to have the same slope, M can be expressed also as:

$$M=\frac{k_b}{k_a+k_b+k_c}=\frac{k_b}{K_1} \qquad (2)$$

1.10 Kinetic energy coefficient As the velocity distribution in a river varies from a maximum at the deeper portion of the channel to essentially zero along the banks, the average velocity head, computed as $(Q/A)^2/2g$ for the stream at section 1, does not give a true measure of the kinetic energy of the flow. A weighted average value of the kinetic energy is obtained by multiplying the average velocity head, above, by a kinetic energy coefficient α_1, defined as:

$$\alpha_1=\frac{\Sigma(qv^2)}{QV_1^2} \qquad (3)$$

Where

v = average velocity in a subsection.
q = discharge in same subsection.
Q = total discharge in river.
V_1 = average velocity in river or Q/A_1.

The method of computation will be further illustrated in the examples in chapter VIII.

Chapter II.—COMPUTATION OF BACKWATER

2.1 Expression for backwater This chapter presents a practical method for computing the backwater caused by bridge constrictions in channels where scour is not a factor. Development of the backwater expression, analysis of the losses involved, or a detailed explanation of the experimental results will not be considered here as these aspects are discussed in detail in the comprehensive report (*9*). A practical expression for backwater has been formulated by applying the principle of conservation of energy between the point of maximum backwater upstream from the bridge, section 1, and a point downstream from the bridge at which normal stage has been reestablished, section 4 (fig. 2A). The expression is reasonably valid if the channel in the vicinity of the bridge is essentially straight, the cross-sectional area of the stream is reasonably uniform, the gradient of the bottom is approximately constant between sections 1 and 4, there is no appreciable erosion of the bed in the constriction due to scour, and the flow is in the tranquil range.

The expression for computation of backwater upstream from a bridge constricting the flow, which is developed in the comprehensive report (*9*), is as follows:

$$h_1^* = K^* \frac{V_{n2}^2}{2g} + \alpha_1 \left[\left(\frac{A_{n2}}{A_4} \right)^2 - \left(\frac{A_{n2}}{A_1} \right)^2 \right] \frac{V_{n2}^2}{2g} \quad \text{----(4)}$$

Where

h_1^* = total backwater (ft.).

K^* = total backwater coefficient.

α_1 = as defined in expression 3, sec. 1.10.

A_{n2} = gross water area in constriction measured below normal stage (sq. ft.).

V_{n2} = average velocity [2] in constriction or Q/A_{n2} (f.p.s.).

A_4 = water area at section 4 where normal stage is reestablished (sq. ft.).

A_1 = total water area at section 1 including that produced by the backwater (sq. ft.).

To compute backwater by expression (4), it is necessary to obtain the approximate value of h_1^* by using the first part of expression (4):

$$h_1^* = K^* \frac{V_{n2}^2}{2g} \quad \text{-----------------(4a)}$$

The value of A_1 in the second part of expression (4), which depends on h_1^*, can then be determined:

$$\alpha_1 \left[\left(\frac{A_{n2}}{A_4} \right)^2 - \left(\frac{A_{n2}}{A_1} \right)^2 \right] \frac{V_{n2}^2}{2g} \quad \text{----------(4b)}$$

[2] The velocity V_{n2} is not an actual measurable velocity, but represents a reference velocity readily computed for both model and field structures.

This part of the expression represents the difference in kinetic energy between sections 4 and 1, expressed in terms of the velocity head $V_{n2}^2/2g$. Expression (4) may appear cumbersome, but it was set up as shown to permit omission of the second part when the difference in kinetic energy between sections 4 and 1 is small enough to be insignificant in the final result.

To permit the designer to readily recognize cases in which the kinetic energy term may be ignored, the following guides are provided:

$$M > 0.7;$$

$$V_{n2} < 7 \text{ f.p.s.; and}$$

$$K^* \frac{V_{n2}^2}{2g} < 0.5 \text{ foot.}$$

If values in the problem at hand meet all three conditions, the backwater obtained from expression (4a) can be considered sufficiently accurate. Should one or more of the values not meet the conditions set forth, it is advisable to use expression (4) in its entirety. The use of the guides will be further demonstrated in the examples.

2.2 Backwater coefficient The value of the overall backwater coefficient K^*, which was determined experimentally, varies with—

1. Stream constriction as measured by the bridge opening ratio M;
2. Type of bridge abutment—wingwall, spillthrough, etc.;
3. Number, size, shape, and orientation of piers in the constriction;
4. Eccentricity, or asymmetric position of bridge with the flood plains; and
5. Skew (bridge crosses flood plain at other than 90° angle).

It will be demonstrated in succeeding paragraphs that the overall backwater coefficient K^* consists of a base curve coefficient K_b, to which is added incremental coefficients to account for the effect of piers, eccentricity, and skew. The value of K^* is primarily dependent on the degree of constriction of the flow but also changes to a limited degree with the other factors.

2.3 Effect of M and abutment shape (base curves) Figure 5 shows the base curve for backwater coefficient K_b, plotted with respect to the opening ratio M, for several wingwall abutments and a vertical-wall type. Note how the coefficient K_b increases with channel constriction. The several curves represent different angles of wingwall

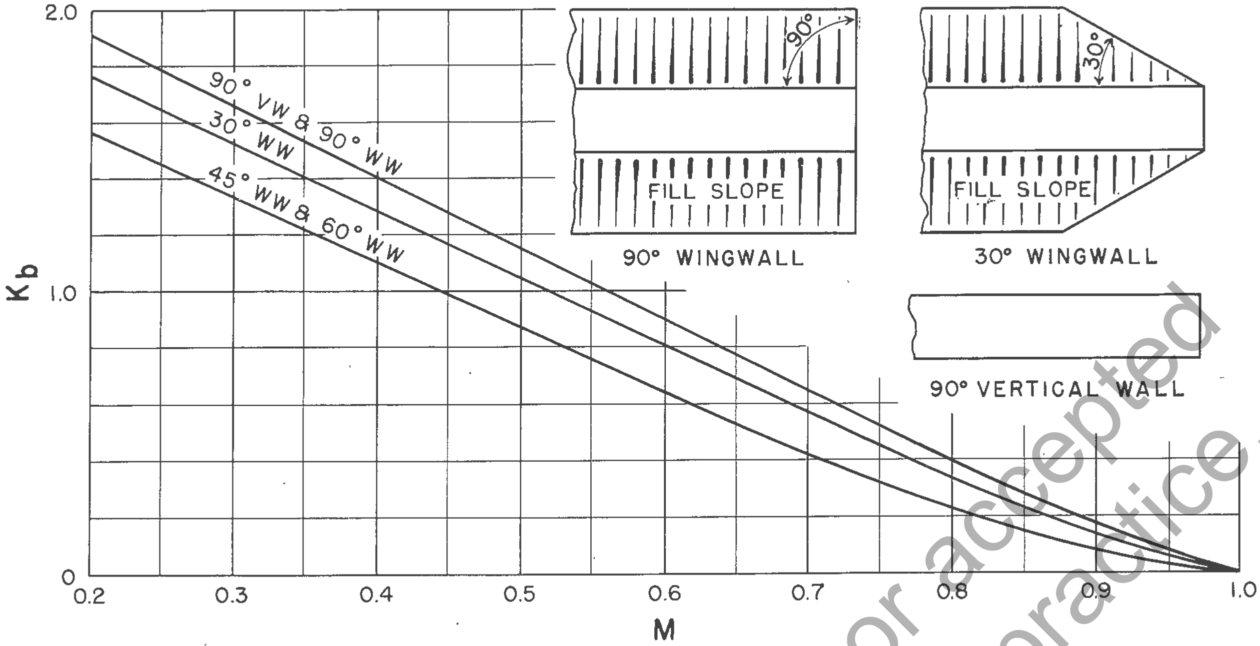

Figure 5.—Base curves for wingwall abutments.

as can be identified by the accompanying sketches; the lower curves, of course, represent the better hydraulic shapes.

Figure 6 shows the relation between the backwater coefficient K_b and M, for spillthrough abutments, for three embankment slopes. A comparison of the three curves indicates that the coefficient is little affected by embankment slope. Figures 5 and 6 will be designated "base curves" and K_b will be referred to as the "base curve coefficients." The base curve coefficients apply to normal crossings for specific abutment shapes, but do not include the effect of piers, eccentricity, or skew.

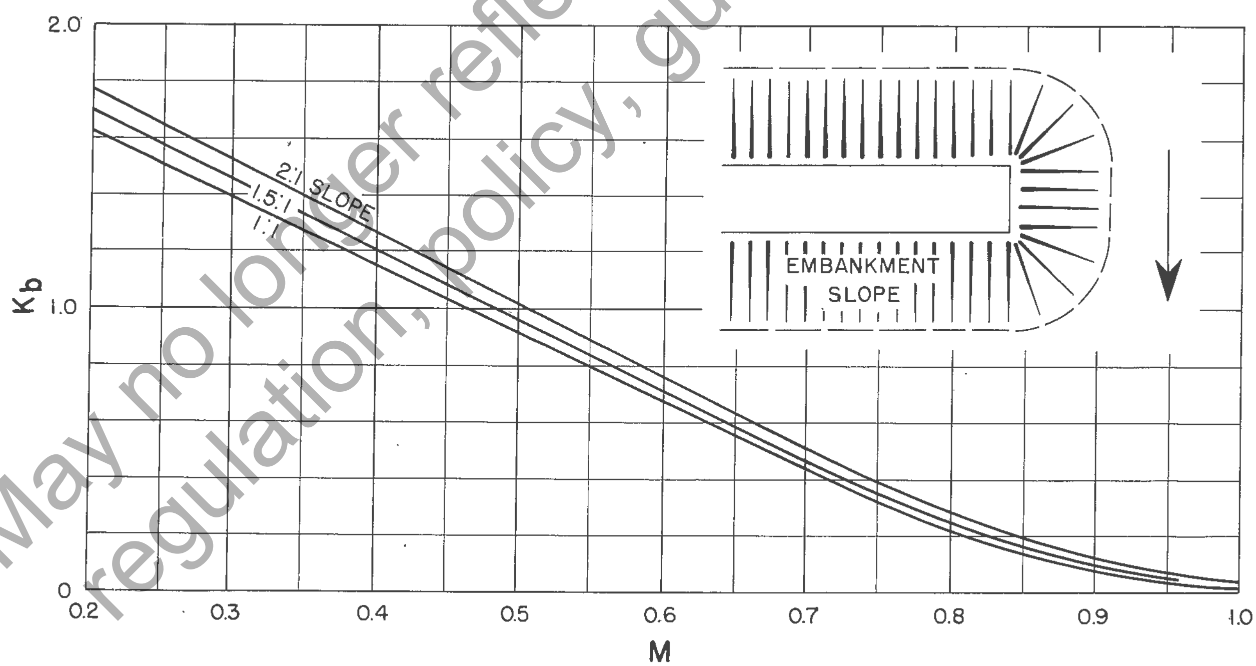

Figure 6.—Base curves for spillthrough abutments.

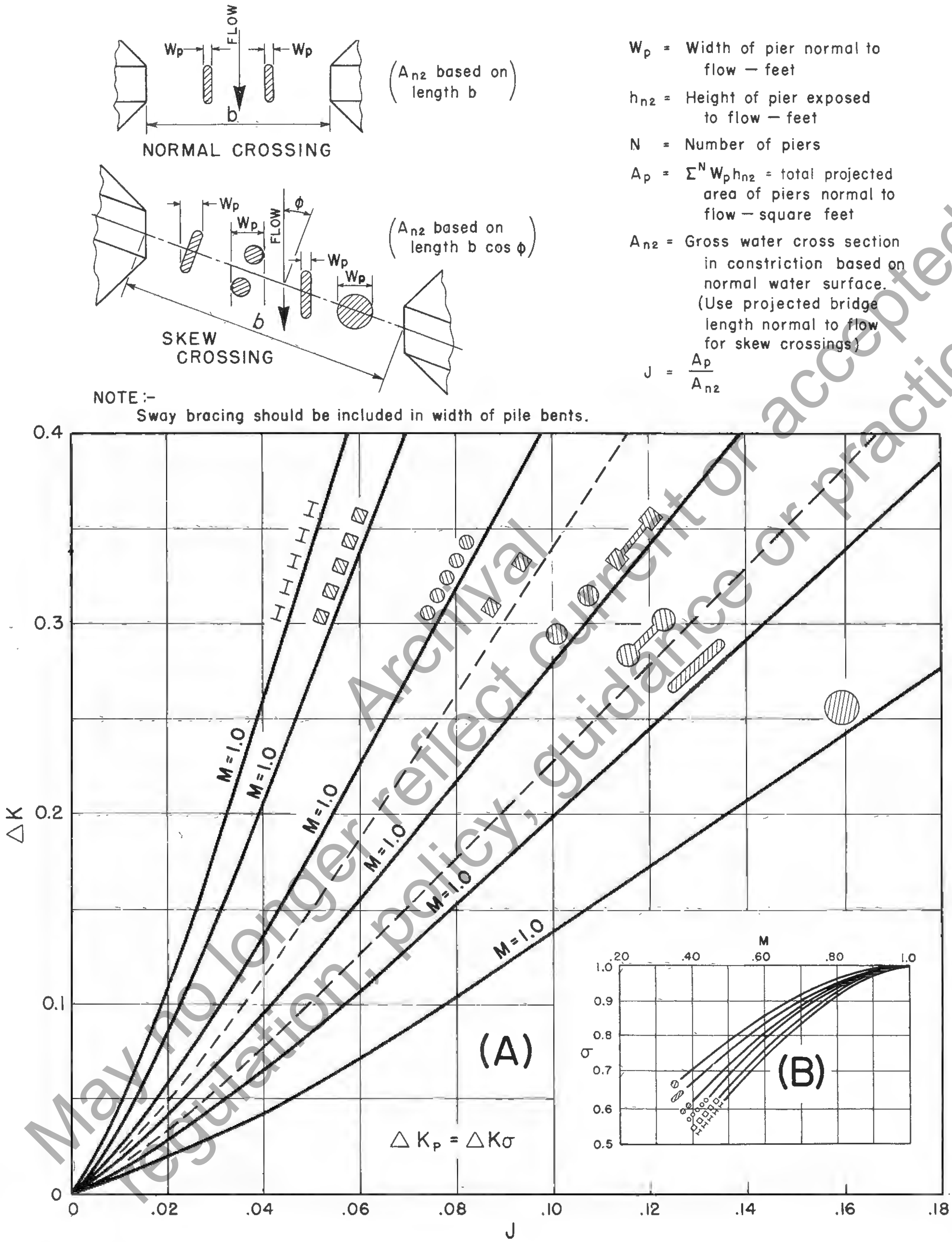

Figure 7.—Incremental backwater coefficient for piers.

2.4 Roadway widths Roadway widths ranging from a single traffic lane to as many as six lanes were tested by models for both wingwall and spillthrough abutments. It was found that variation of the width of abutment over the range tested produced a negligible effect on the value of K_b, so this factor has been omitted in this presentation. The effect of roadway width on K_b can be observed in the comprehensive report (*9*) on the model studies.

2.5 Effect of piers (normal crossings) The effect produced on the backwater by introduction of piers in a bridge constriction has been treated as an incremental backwater coefficient designated ΔK_p, which is added to the base curve coefficient when piers are a factor. The value of the incremental backwater coefficient ΔK_p is dependent on the ratio that the area of the piers bears to the gross area of the bridge opening, the type of piers (or piling in the case of pile bents), the value of the bridge opening ratio M, and the angularity of the piers with the direction of flood flow. The ratio of the water area occupied by piers A_p to the gross water area of the constriction A_{n2}, both based on the normal water surface, has been assigned the letter J. In computing the gross water area A_{n2}, the presence of piers in the constriction is ignored. The incremental backwater coefficient for the more common types of piers and pile bents can be obtained from figure 7. The procedure is to enter chart A with the proper value of J and read ΔK and obtain the correction factor σ from chart B for opening ratios other than unity. The incremental backwater coefficient is then:

$$\Delta K_p = \Delta K \sigma$$

The incremental backwater coefficients for piers can, for all practical purposes, be considered independent of diameter, width, or spacing, but should be increased if there

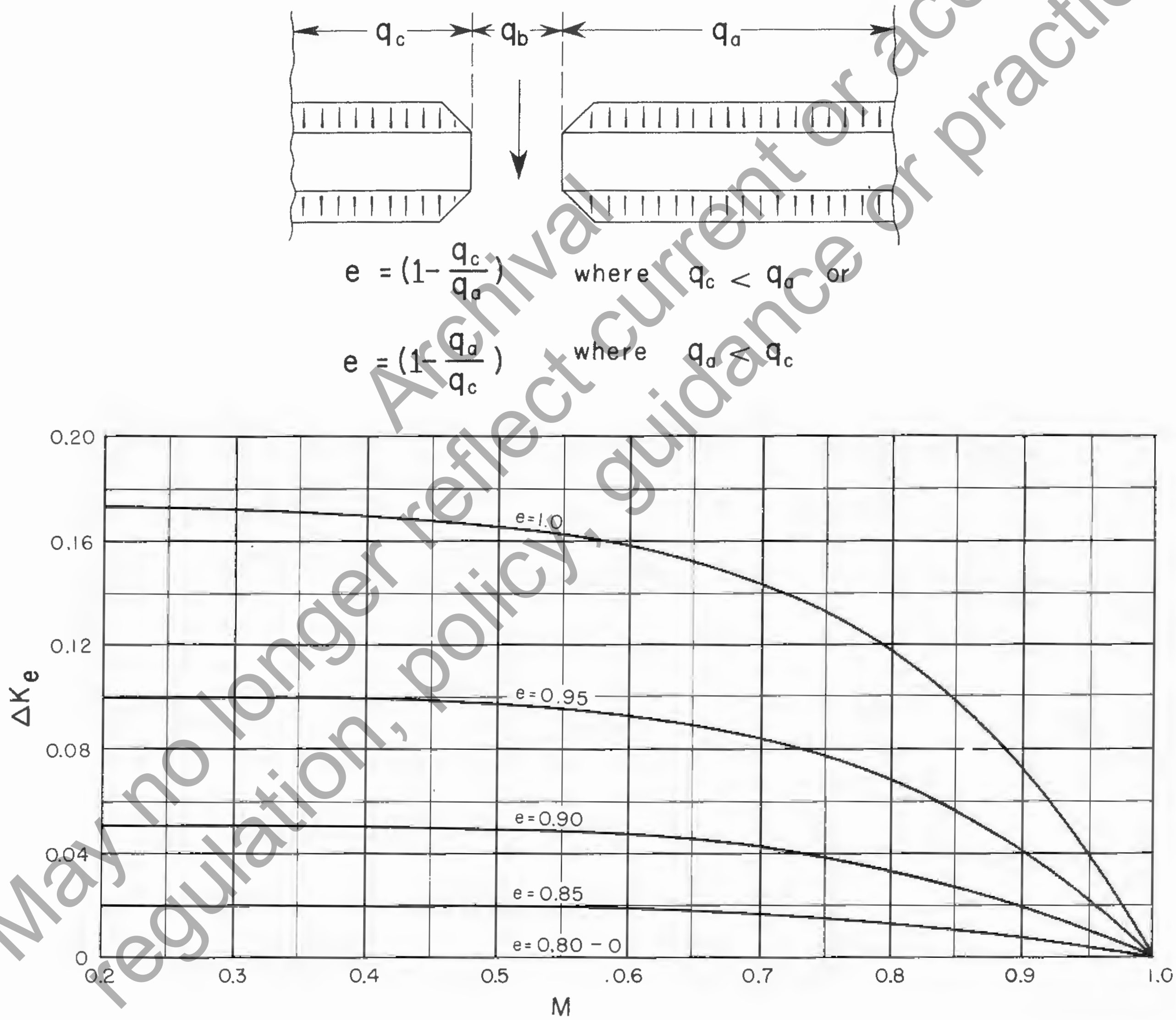

Figure 8.—Incremental backwater coefficient for eccentricity.

are more than 5 piles in a bent. A bent with 10 piles should be given a value of ΔK_p about 20 percent higher than those shown from bents with 5 piles. If there is a possibility of trash collecting on the piers, it is advisable to use a value greater than the pier width to include the trash. For a normal crossing with piers, the total backwater coefficient becomes:

$$K^* = K_b \text{ (figs. 5 or 6)} + \Delta K_p \text{ (fig. 7)}$$

2.6 Effect of piers (skew crossings) In the case of skew crossings, the effect of piers is treated as explained for normal crossings (sec. 2.5) except for the computation of J, A_{n2}, and M. The pier area for a skew crossing, A_p, is the sum of the individual pier areas normal to the general direction of flow, as illustrated by the sketch in figure 7. Note how the width of pier w_p is measured when the pier is not parallel to the general direction of flow. The area of the constriction A_{n2}, for skew crossings, is based on the projected length of bridge, $b \cos \phi$ (fig. 4). Again, A_{n2} is a gross value and includes the area occupied by piers. The value of J is the pier area A_p divided by the projected gross area of the bridge constriction, measured normal to the general direction of flow. The computation of M for skew crossings is also based on the projected length of bridge, which will be further explained (sec. 2.8).

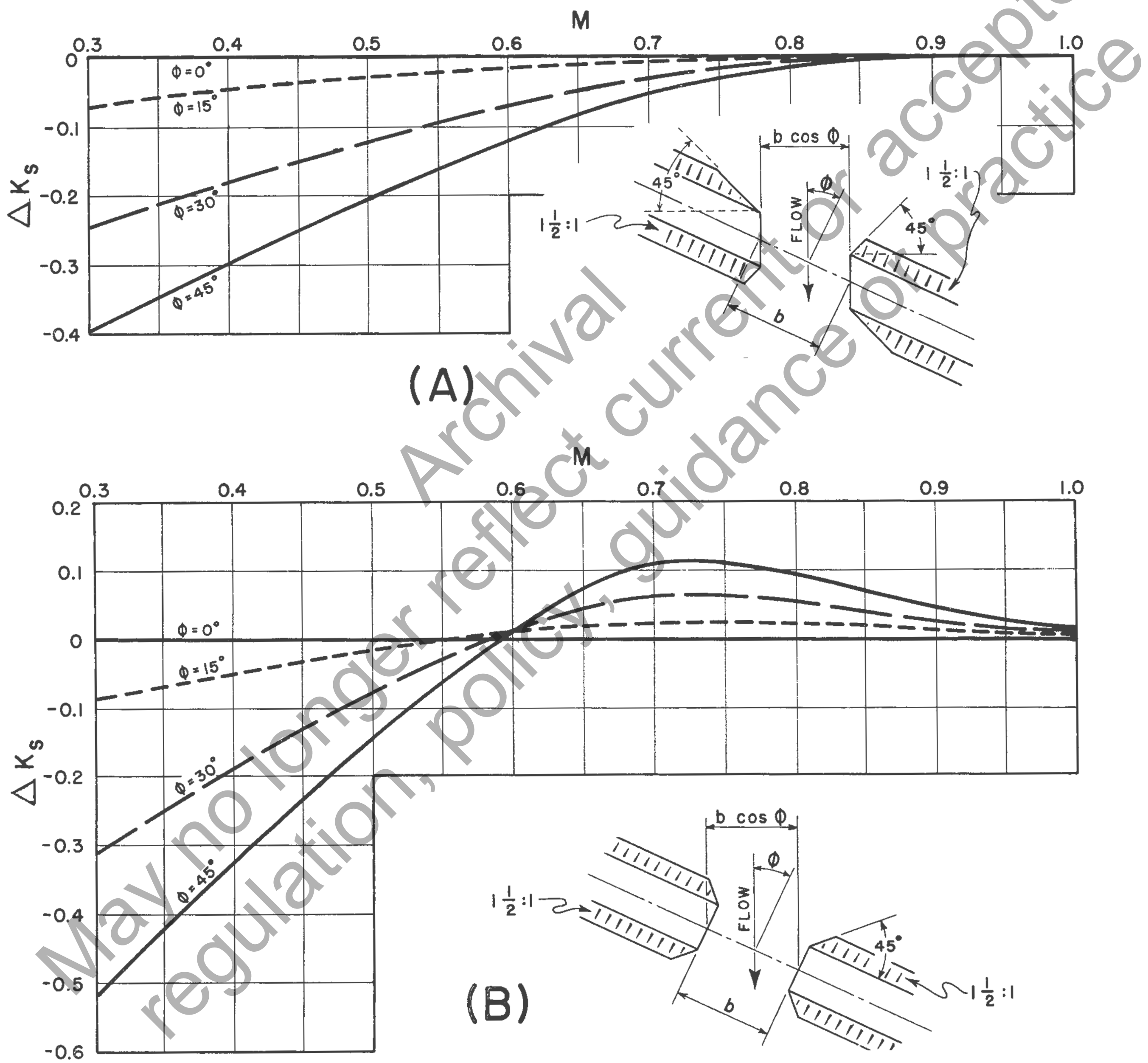

Figure 9.—Incremental backwater coefficient for skew, wingwall abutments.

2.7 Effect of eccentricity Referring to the sketch in figure 8, it can be noted that the symbols q_a and q_c at section 1 were used to represent the portion of the discharge obstructed by the approach embankments. If the cross section is extremely asymmetrical so that q_a is less than 20 percent of q_c, or vice versa, the backwater coefficient will be somewhat larger than for comparable values of M shown on the base curves. The magnitude of the incremental backwater coefficient ΔK_e, accounting for the effect of eccentricity, is shown in figure 8. Eccentricity e is defined as 1 minus the ratio of the lesser to the greater discharge outside the projected length of the bridge, or:

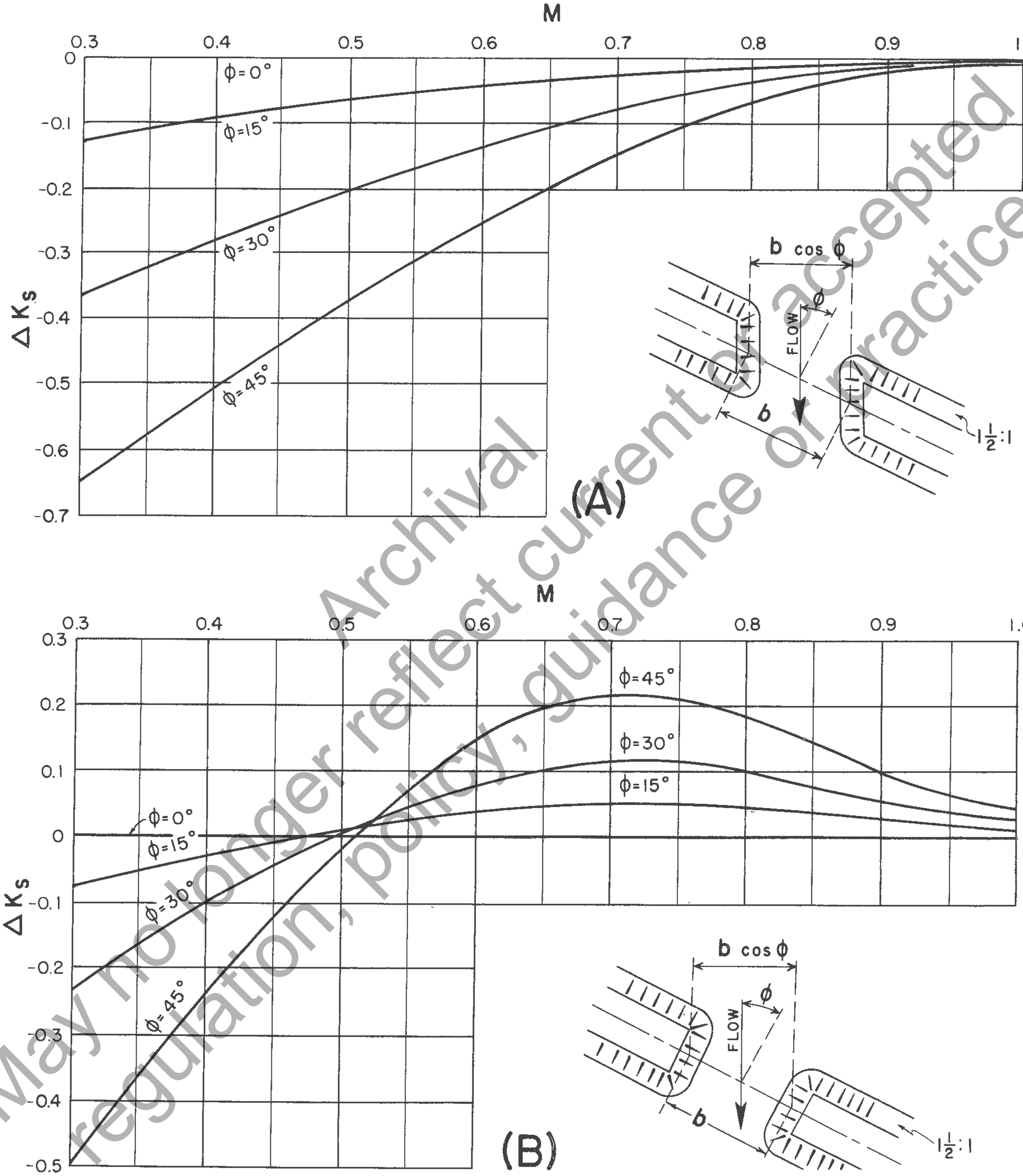

Figure 10.—Incremental backwater coefficient for skew, spillthrough abutments.

$$e = \left(1 - \frac{q_c}{q_a}\right) \text{ where } q_c < q_a$$

or:

$$e = \left(1 - \frac{q_a}{q_c}\right) \text{where } q_a < q_c \quad \text{............} (5)$$

Reference to the sketch in figure 8 will aid in clarifying the terminology. For instance, if $q_c/q_a = 0.05$, the eccentricity $e = (1 - 0.05)$ or 0.95 and the curve for 0.95 in figure 8 would be used for obtaining ΔK_e. The largest influence on the backwater coefficient due to eccentricity will occur when a bridge is located adjacent to a bluff where a flood plain exists on only one side and the eccentricity is 1.0. The overall backwater coefficient for an extremely eccentric crossing with wingwall abutments and piers will be:

$$K^* = K_b \text{ (fig. 5)} + \Delta K_p \text{ (fig. 7)} + \Delta K_e \text{ (fig. 8)}$$

2.8 Effect of skew The method of computation for skew crossings differs from that of normal crossings in the following respects: The bridge opening ratio M is computed on the projected length of bridge rather than on the full length. The length is obtained by projecting the bridge opening upstream parallel to the general direction of flood flow as illustrated in figure 4. By general direction of flow is meant the direction of flood flow as it existed previous to the placement of embankments in the stream. The length of the constricted opening is $b \cos \phi$, and the area A_{n2} is based on this length. The velocity head $V_{n2}^2/2g$, to be substituted in expression (4) (sec. 2.1) is based on the projected area A_{n2}. The method will be further illustrated in example 3.

Figures 9 and 10 show the incremental backwater coefficient ΔK_s for the effect of skew, for wingwall and spillthrough type abutments, respectively. The incremental coefficient varies with the opening ratio M, the angle of skew of the bridge ϕ, with the general direction of flood flow, and the alinement of the abutment faces, as indicated by the sketch accompanying each figure. Note that the incremental backwater coefficient ΔK_s can be negative as well as positive. These values are to be added algebraically to K_b, obtained from the base curves. The negative values result from the method of computation and do not necessarily indicate that the backwater will be reduced by employing a skew crossing. The total backwater coefficient for a skew crossing with spillthrough abutments and piers would be:

$$K^* = K_b \text{ (fig. 6)} + \Delta K_p \text{ (fig. 7)} + \Delta K_s \text{ (fig. 10)}$$

It was observed during the testing that crossings with skew up to an angle of 20° produced no particularly objectionable results for any of the four abutment shapes investigated. As the angle increased above 20°, however, the flow picture deteriorated; flow concentrations at abutments produced large eddies, reducing the efficiency of the waterway and increasing the possibilities for scour. The above statement should be qualified so as not to include cases where a bridge spans practically an entire valley and there is little constriction of the flow.

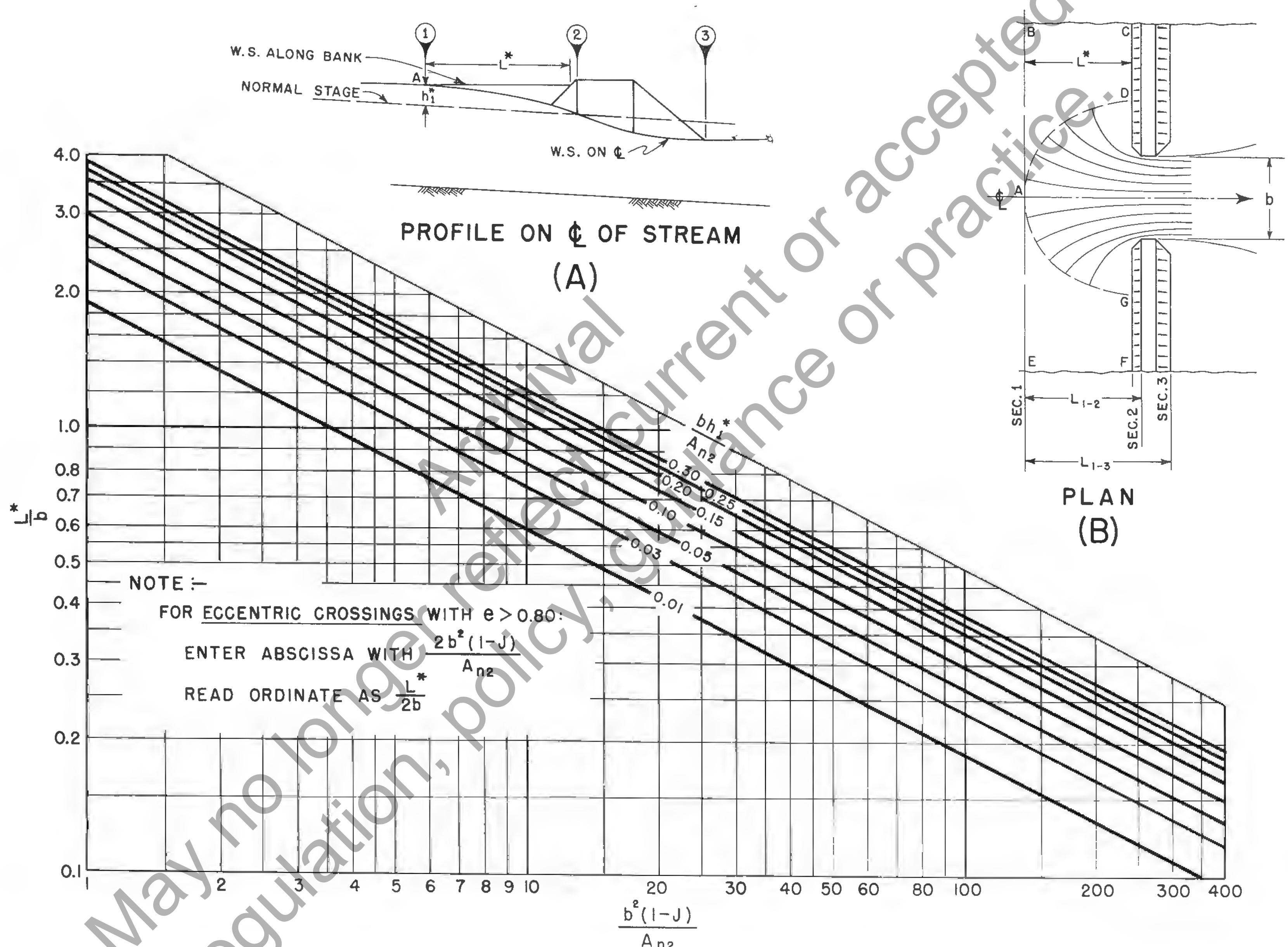

Figure 11.—Distance to maximum backwater.

Chapter III.—EXTENT OF BACKWATER

3.1 Distance to point of maximum backwater In backwater computations, it will be found necessary in some cases to locate the point of maximum backwater with respect to the midpoint of the bridge in order to establish elevation of the water surface in the upstream pool. The maximum backwater along the centerline of the channel for a bridge occurs at point A (fig. 11B), this point being a distance L^* from the waterline on the upstream side of the embankments. For streams of moderate width, where flood plains are inundated and the embankments constrict the flow, the elevation of the water surface throughout areas ABCD and AEFG will, for all practical purposes, be the same as at point A where the backwater measurement was made. This characteristic was borne out by field observations made available by the U.S. Geological Survey on bridges up to 220 feet in length. The comprehensive report (*9*) contains further discussion of this feature. Where borrow pits or ditches exist along the upstream side of long embankments, a noticeable gradient may exist along the upstream side, modifying the essentially level ponding described above.

3.2 Normal crossings To obtain the distance to maximum backwater L^*, for a normal crossing, enter figure 11 with the proper values of $b^2(1-J)/A_{n2}$ and bh_1^*/A_{n2} and read off the corresponding value of L^*/b from the ordinate scale. The distance L^* is then the product of the ordinate value and b. For all practical purposes consider $L^*=L_{1-2}$. If the backwater computation is based on the design discharge for normal stage at the bridge, which is usually the case, the water surface elevation at section 1 (also throughout areas ABCD and AEFG) will be:

Normal stage at bridge $+h_1^*+S_0L_{1-2}$ ---------------- (6)

3.3 Eccentric crossings Eccentric crossings with extreme asymmetry perform much like one-half of a normal symmetrical crossing with a marked contraction of the jet on one side and essentially no contraction on the other. Where the value of e exceeds 0.80, a more realistic value for the distance to point of maximum drawdown can be obtained by using the same chart but interpreting the co-

Figure 12.—Flow concentration along upstream side of embankment.

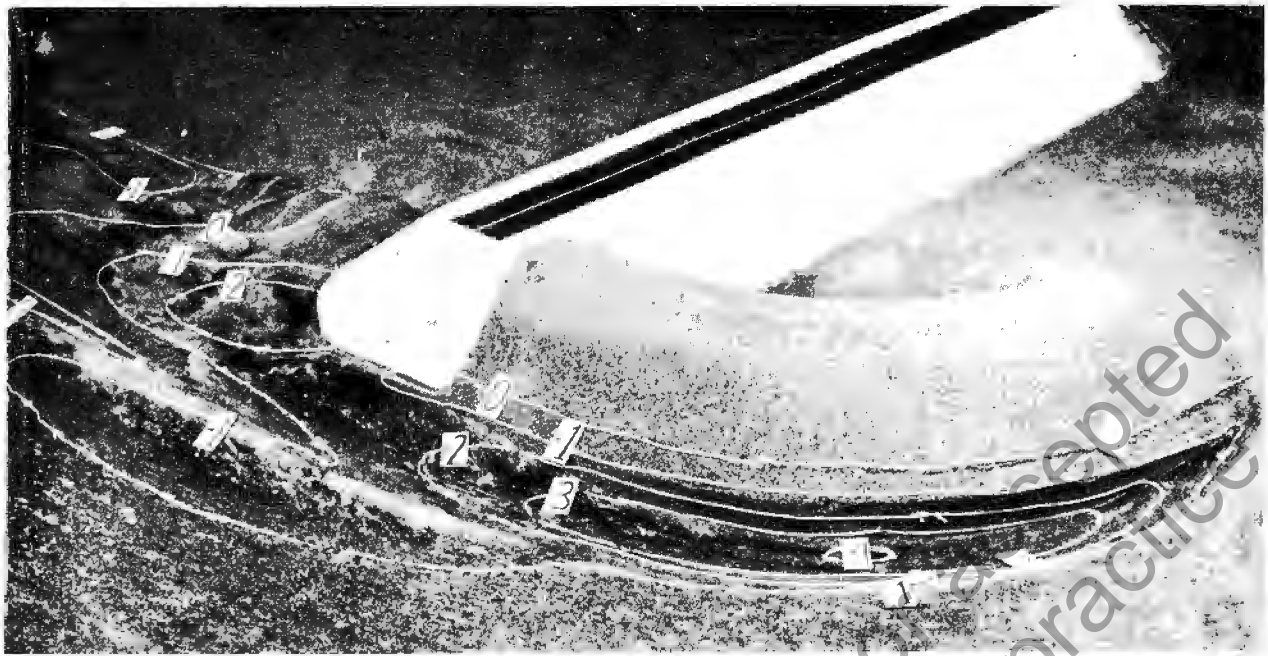

Figure 13.—Method of reducing flow gradient along embankment (model).

ordinates as $2b^2(1-J)/A_{n2}$ for the abscissa and $L^*/2b$ for the ordinate.

3.4 Skew crossings In the case of skew crossings, the water surface elevations along opposite banks of a stream are usually different than at point A; one may be higher and the other lower depending on the angle of skew, the configuration of the approach channel, and other factors. To obtain the approximate distance to maximum backwater L^* for skew crossings (fig. 4), the same procedure is recommended as for normal crossings using the full length of bridge b. The result, of course, will be approximate.

3.5 Wide crossings Where crossings are wide, involving long embankments on the flood plain, or where borrow pits or ditches exist along the upstream side of long embankments, the simple condition of essentially level ponding depicted by figure 11B may be considerably altered. Flow from the flood plain can utilize this channel of low resistance along the embankment, and enter the constriction as a concentrated jet normal to the direction of flow in the main channel. In so doing the severity of the contraction is increased at the abutment, the effective length of opening is reduced, the backwater is increased, and the possibility of scour at the junction of the two jets is great. This action is illustrated in the aerial photograph, figure 12. The concentration of flow is from right to left along the upstream side of the embankment; the flow is from top to bottom. The low water channel is to the left of the photograph. Note the violent mixing action where the side jet and the main flow converge, the ineffectiveness of the first span, and also witness that scour has been responsible for the loss of a portion of the bridge.

This condition can be alleviated to some extent on new bridges by prohibiting borrow pits on the upstream side of embankments and forbidding the cutting of trees back of the toe of the fill slope. For cases where channeling along an embankment is already present or cannot be avoided, the situation can usually be remedied by constructing a spur dike curving upstream and tangent to the abutment face as shown in the model in figure 13. A spur dike serves to reduce the gradient and velocity of flow along the embankment, to direct the flow in such a way as to utilize the entire waterway under the bridge, to reduce the backwater to normal or even less, and to alleviate scour under the bridge. Information to aid in the proportioning of spur dikes can be obtained from reference (*4*) of the bibliography. No data are yet available on the exact effect which spur dikes may have on backwater.

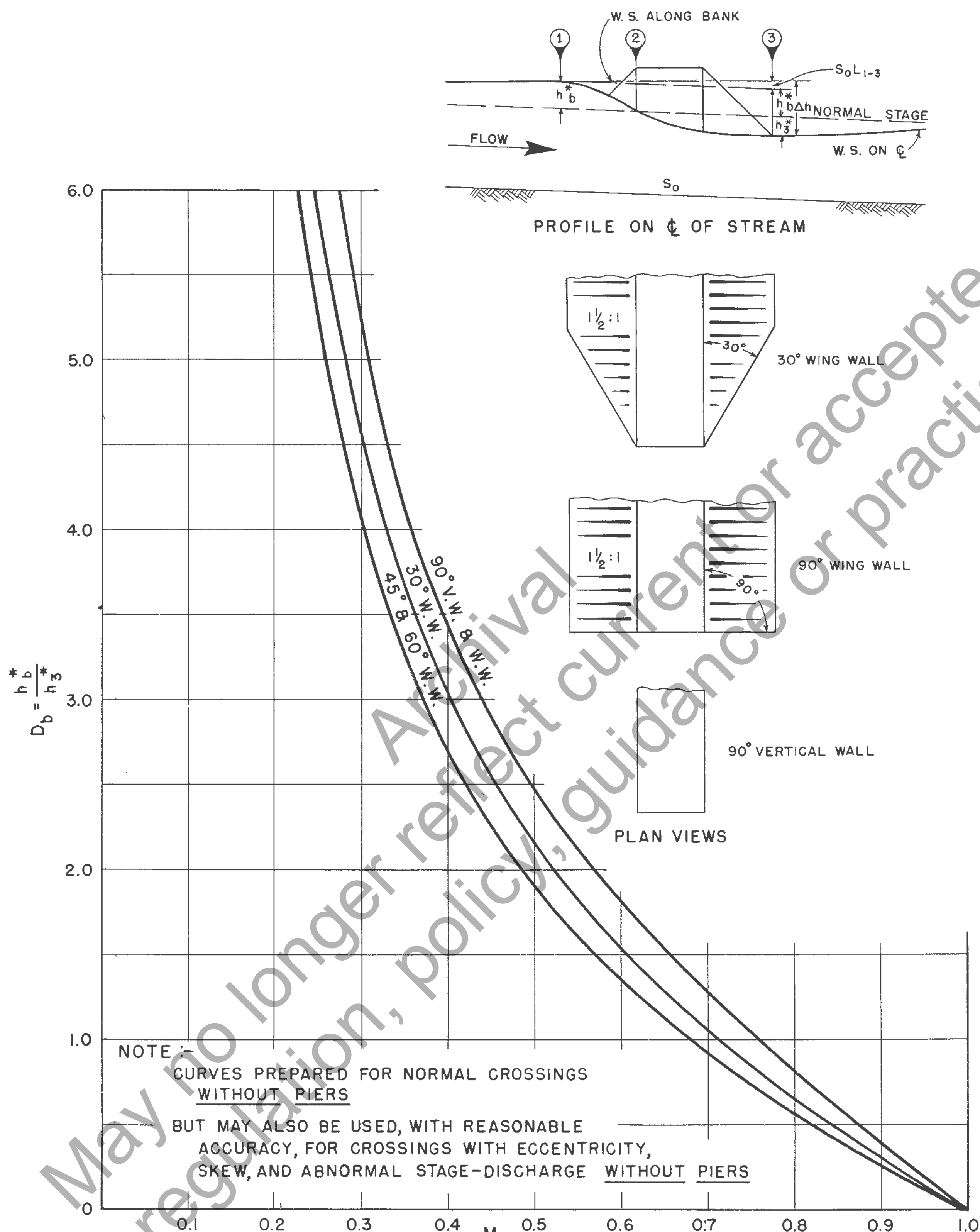

Figure 14.—Differential level ratio for wingwall abutments.

Chapter IV.—DIFFERENCE IN LEVEL ACROSS APPROACH EMBANKMENTS

4.1 Significance The difference in water surface elevation between the upstream and downstream side of bridge approach embankments Δh has often been interpreted in court testimony as the backwater produced by a bridge. This is not true, as an inspection of figure 2A will indicate. The water surface at section 3, measured along the downstream side of the embankment, is invariably lower than normal stage by amount h_3^*. The difference in level across an embankment, Δh, is always larger than the backwater h_1^* by the sum of h_3^* and S_0L_{1-3}.

The difference in level, Δh, is significant in the determination of backwater at a field structure since Δh is the only reliable head measurement that can be made. This difference in level is also of concern where approach embankments are designed to function as emergency spillways for flows exceeding the design flood (*1, 10*). An approach roadway simulates a broad-crested weir where the capacity is dependent on the length, depth of flow over roadway, and the degree of submergence. The water level along the downstream side of the embankment is needed to determine the submergence. Fortunately, the backwater and Δh bear a definite relationship to one another for any particular structure. Thus if one is known the other can be determined.

4.2 Base curves Base curves are also utilized for determining downstream levels. The ratio h_b^*/h_3^* is plotted with respect to the opening ratio M for several types of wingwall abutments in figure 14. The numerator h_b^* represents the backwater computed from the base curve coefficient K_b, from figure 5, and h_3^* is the difference in level between normal stage and the water surface on the downstream side of the embankment at section 3. Reference to the sketch in figure 14 should aid in defining these terms. The water surface depicted at section 3 represents the level along the downstream side of the embankments (from H to I and N to O in fig. 1) and does not necessarily represent the water surface in the constriction, which is often irregular. The ordinate h_b^*/h_3^* in figure 14 will be referred to as the differential level ratio, to which the symbol D_b has been assigned.

A similar set of curves for spillthrough abutments is included in figure 15. Figures 14 and 15 are for normal symmetrical crossings (without piers) and are considered base curves. Assuming the backwater h_b^* has already been computed for a normal crossing without piers, eccentricity, or skew, the water surface on the downstream side of the embankment is obtained by entering the appropriate curve in figure 14 or 15 with the opening ratio M and reading off the differential level ratio D_b; then:

$$h_3^* = h_b^*/D_b \qquad (7)$$

The elevation of the water surface on the downstream side of the embankment is simply normal stage at bridge less h_3^* (see sketch in fig. 14).

4.3 Effect of piers As piers were introduced in the bridge constrictions in the model, it was found that the backwater increased while the value of h_3^* showed no measurable change regardless of the value of J. Therefore, if the problem is the same as above except that piers are involved, the procedure for determining h_3^* is exactly as explained in section 4.2.

4.4 Effect of eccentricity In the case of severe eccentric crossings, the difference in level across the embankment considered here applies only to the side of the river having the greater flood plain discharge. In plotting the experimental differential level ratios with respect to M for eccentric crossings, without piers, it was found that the points fell directly on the base curves (figs. 14 and 15). The individual values of h_b^* and h_3^* for eccentric conditions are different than for symmetrical crossings, but the ratio of one to the other, for any given value of M, remains unchanged. Thus, figures 14 and 15 can also be considered applicable to eccentric crossings if used correctly. To obtain h_3^* for an eccentric crossing, with or without piers, enter the proper curve in figure 14 or 15 with value of M and read off D_b as before. In this case:

$$h_3^* = \frac{h_b^* + \Delta h_e^*}{D_b} \qquad (8)$$

where $\Delta h_e^* = \Delta K_e V_{n2}^2/2g$.

4.5 Drop in water surface across embankment (normal crossing) Having computed h_3^* as described above, and knowing the total backwater h_1^* (computed according to the procedure in chapter II), the difference in water surface elevation across the embankment (fig. 2A) is:

$$\Delta h = h_3^* + h_1^* + S_0L_{1-3} \qquad (9)$$

where h_1^* is total backwater including the effect of piers and eccentricity and S_0L_{1-3} is the normal fall in stream bed from section 1 to section 3.

4.6 Water surface on downstream side of embankment (skew crossing) The differential level across roadway embankments for skew crossings is naturally different for opposite sides of the river, the amount depending on the configuration of the stream, bends in the vicinity of the crossing, the degree of skew, etc. These factors can be so variable that a generalized model study can shed little light on the subject. The experimental information for the right embankment or side extending farthest upstream (see fig. 4) was not reliable as the flow impinged against the right wall of the flume downstream from the bridge, producing an unnatural condition; thus the test results from the left embankment have been omitted. The results for the left embankment, or side farthest downstream, gave consistent results and these are included on the basis that an indication or trend is preferable to a complete lack of information. The experimental points for the left embankment (without piers) fell slightly to both sides of the base curves (figs. 14 and 15) for both wingwall and spillthrough abutments, respectively.

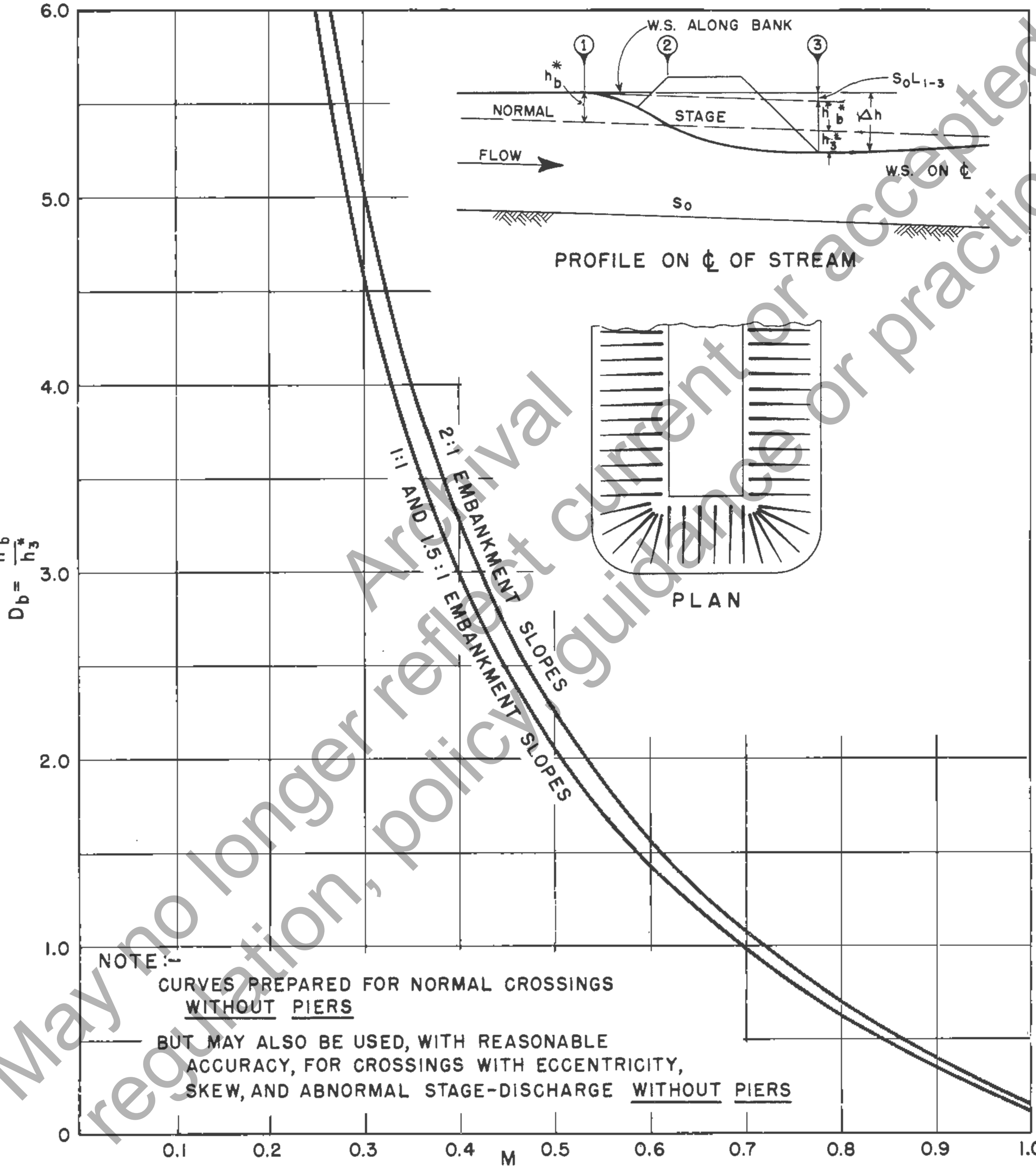

Figure 15.—Differential level ratio for spillthrough abutments.

Individual values of h_1^* and h_3^* for skew crossings again differ from those for symmetrical crossings, but the differential level ratio can be considered the same as for normal crossings for any given value of M. Thus it is again possible to use figures 14 and 15 for skew crossings. The differential level ratio D_b is obtained by entering the proper chart with the abutment shape and the opening ratio M. For the embankment located farthest downstream (with or without piers):

$$h_3^* = \frac{h_b^* + \Delta h_s^*}{D_b} \qquad (10)$$

where $\Delta h_s^* = \Delta K_s V_{n2}^2/2g$. The computation of Δh in this case has little meaning. Data for support of the above procedures can be found in the comprehensive report (*9*).

Chapter V.—DUAL BRIDGES

5.1 Arrangement With the advent of divided highways, dual bridges of essentially identical design, placed parallel and only a short distance apart, are now common. The backwater produced by dual bridges is naturally larger than that for a single bridge, yet less than the value which would result by considering the two bridges computed separately. As the combinations of dual bridges encountered in the field are legion, it was necessary to

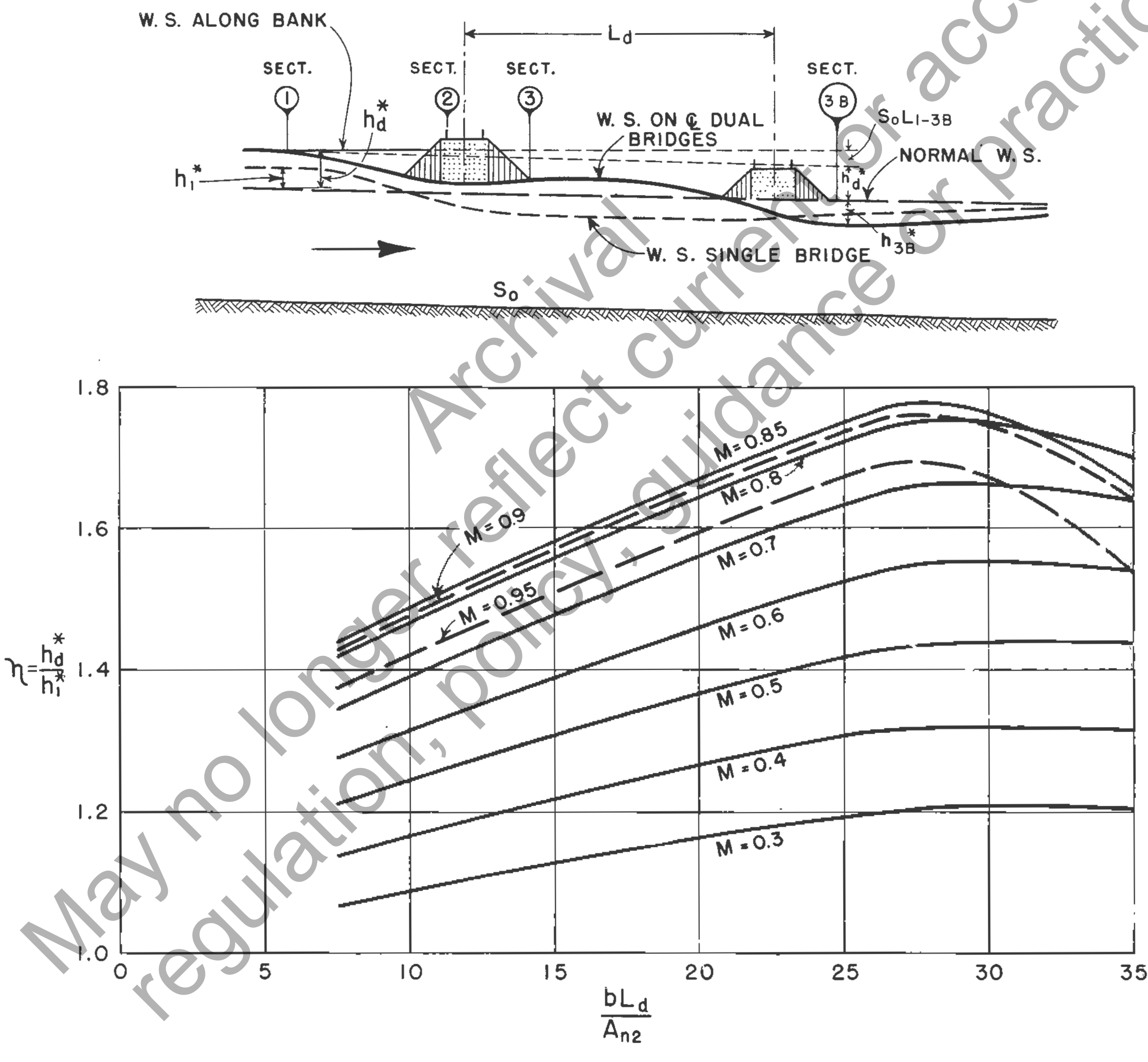

Figure 16.—Backwater multiplication factor for dual parallel bridges.

restrict model tests to the simplest arrangement; namely, identical parallel bridges crossing a stream normal to the flow (see sketch in fig. 16). The tests were made principally with 45° wingwall abutments, but also included a limited number of the spillthrough type, both having embankment slopes of 1½:1. The distance between bridges was limited by the range permissible in the model.

5.2 Backwater determination The method of testing consisted of establishing normal flow conditions, then placing one bridge constriction in the flume and measuring the backwater h_1^*. A second bridge constriction, identical to the first, was next placed downstream and the backwater for the combination h_d^* measured upstream from the first bridge. The ratio h_d^*/h_1^*, as thus obtained, is plotted with relation to the parameters bL_d/A_{n2} and M in figure 16, where L_d is the distance between center lines of the two bridges and b is the common width of each constriction. The curves were established from tests made with and without piers and can be considered applicable for both wingwall and spillthrough abutments. The ratio h_d^*/h_1^*, which is assigned the symbol η, increases as the bridges are moved apart, apparently reaching a limit as bL_d/A_{n2} approaches 30, whereupon the value of η then decreases as the distance is further lengthened between bridges. With the bridges in close proximity to one another, the flow pattern is little different than for a single bridge. As the bridges are spaced farther apart,

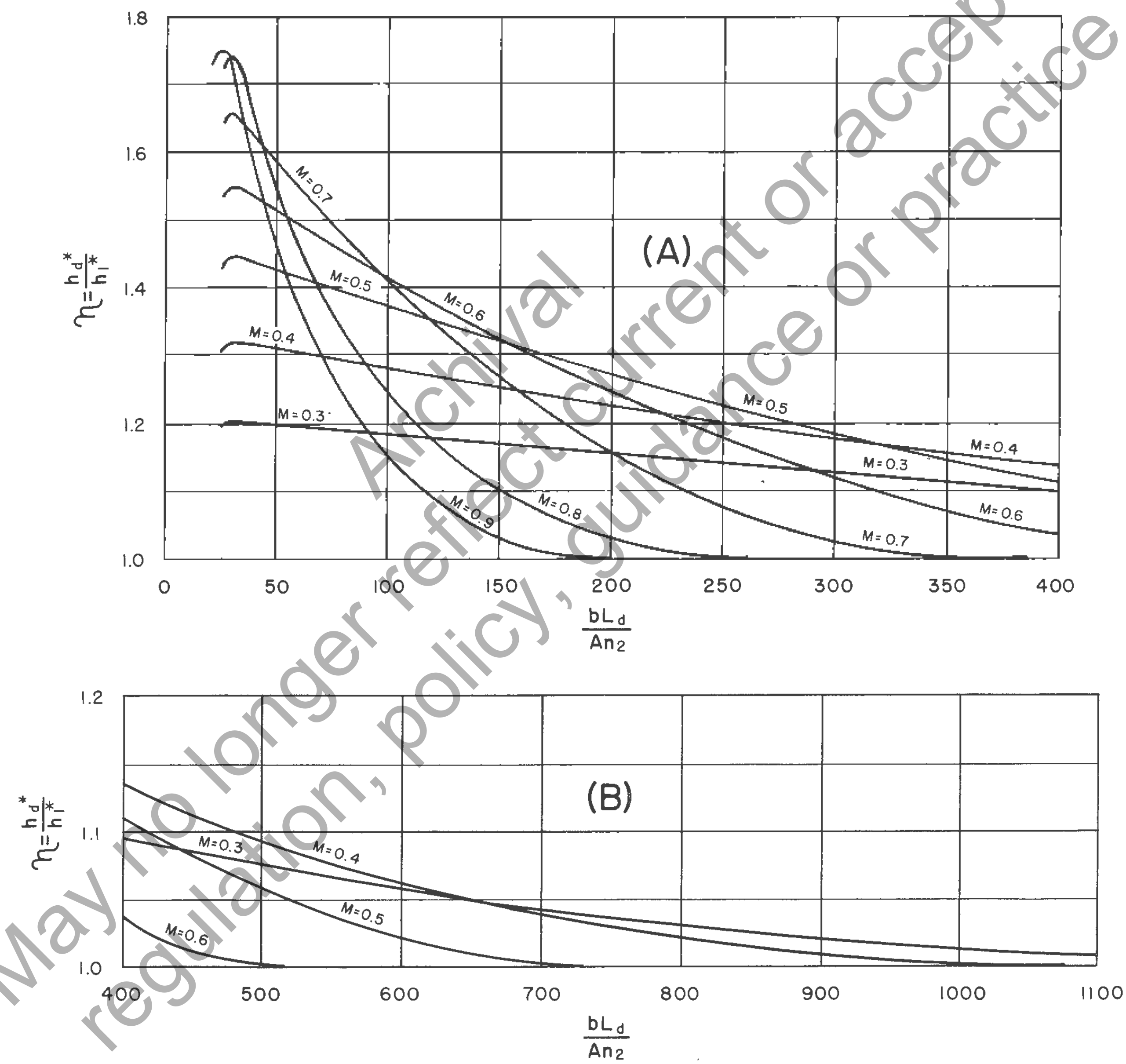

Figure 17.—Extension of backwater multiplication factor for dual parallel bridges.

the second bridge interferes with the expanding jet from the first, producing additional turbulence and loss of energy.

To determine backwater for dual bridges meeting the above specifications, it is necessary first to compute the backwater h_1^* for a single bridge, as previously outlined in chapter II. The backwater for the dual combination, measured upstream from the first bridge, is then:

$$h_d^* = h_1^* \eta \quad (11)$$

Should the value of bL_d/A_{n2} exceed the limit of the model tests, an approximate value of η can be obtained from figure 17, which was prepared by a process of mathematical extrapolation.

5.3 Drop in water surface across embankments In the case of identical dual bridges, the designer may wish to know the water surface elevation on the downstream side of the roadway embankment of the first bridge, or the water surface elevation on the downstream side of the embankment of the second bridge. Fluctuations in the water surface between bridges, due to turbulence and surging, caused the measurements to be so erratic that it was thought inadvisable to include the results here. These

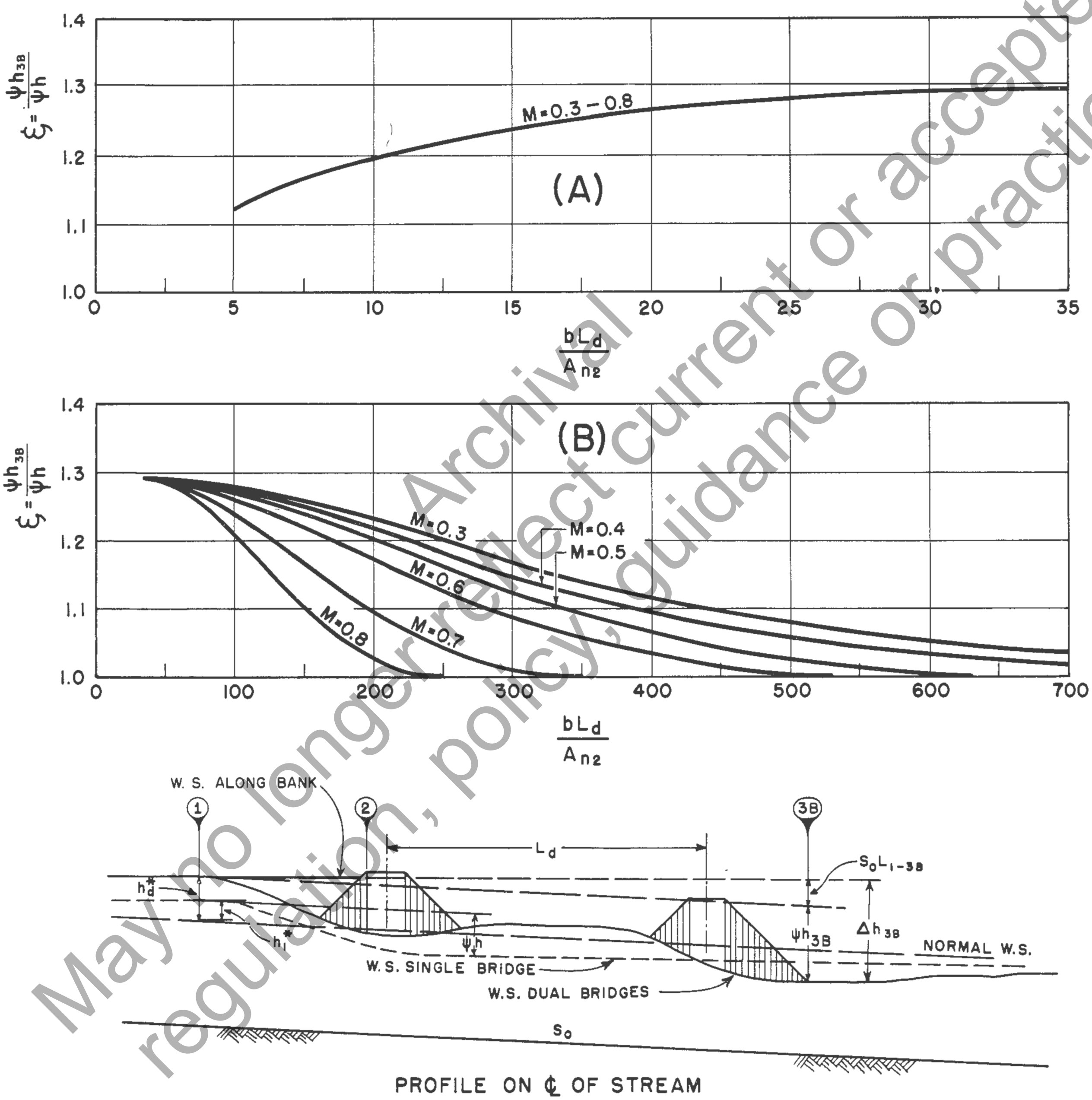

Figure 18.—Differential level multiplication factor for dual parallel bridges.

data are available in the comprehensive report (*9*). A characteristic to be noted in this connection, however, is that the water surface between bridges stands above normal stage (see sketch in fig. 16). The water surface downstream from the second bridge, on the other hand, was quite stable, permitting accurate measurement.

The procedure for computing the water surface elevation immediately downstream from the second bridge embankment or section 3B (see sketch in fig. 18) consists of first computing h_1^* and h_3^* for a single bridge as outlined in chapters II and IV, respectively. For convenience, the sum $h_1^*+h_3^*$ for the single bridge is assigned the symbol ψh. To obtain a corresponding drop in level across the two bridges, which is designated ψh_{3B}, it is only necessary to multiply ψh for the single bridge by a factor ξ. This factor is the ratio $\psi h_{3B}/\psi h$ and can be obtained by entering figure 18A with the proper value of bL_d/A_{n2} and reading from the ordinate. Then $\psi h_{3B}=\psi h\xi$. The difference in water surface across the two embankments (see fig. 18) is:

$$\Delta h_{3B}=\psi h_{3B}+S_0L_{1-3B} \qquad (12)$$

Should the water surface on the downstream side of the second embankment be desired relative to normal stage at section 3B:

$$h_{3B}^*=\psi h_{3B}-h_d^* \qquad (13)$$

Should the value of bL_d/A_{n2} exceed the range of the model tests, an approximate value of ξ can be obtained from figure 18B.

Chapter VI.—ABNORMAL STAGE-DISCHARGE CONDITION

6.1 Definition Up to this point the discussion has concerned streams flowing at normal stage; i.e., the natural flow of the stream has been influenced only by the slope of the bed and the boundary resistance along channel bottom and flood plains. Sometimes the stage at a bridge site is not normal but is increased by flood conditions from downstream. A general backwater curve is produced beginning at the confluence of the tributary and main stream or at a dam, and may extend a considerable distance upstream if the gradient of the tributary is flat. Where bridges are placed close to the confluence of two streams, abnormal stage-discharge conditions can be of importance in design. For example, if a stream can always be counted on to flow at abnormal stage during flood at a particular bridge site, the increased waterway area, for a given backwater and with adequate clearance beneath the superstructure, will permit a shorter bridge than would be possible under normal stage conditions. To take advantage of this opportunity, the length of the bridge would be determined on the basis of the minimum abnormal stage expected, which would produce the most adverse backwater condition. Estimating the design stage at a bridge site under abnormal conditions can be a complicated process requiring much individual judgment; thus the approach to the computation of backwater in this case has been treated strictly as an approximate solution. This is a case where it is more important to understand the problem than to attempt precise computations.

6.2 Backwater determination Tests were made by first establishing normal flow in the test flume as usual, without a constriction. The tailgate was then adjusted to increase the depth of flow by, say, 10 percent for the same discharge, after which a centerline profile was obtained. The resulting water surface is labeled "abnormal stage" in figure 19. Abutments were then placed in the flume and a second center line profile made of the water surface. The difference between the final water surface measurement and the previous one at abnormal stage, both made at section 1, is defined as the backwater h_{1A}^*. Similar backwater measurements were made for other degrees of bridge constriction and for depths of flow up to 40 percent greater than normal stage. Since the backwater analysis as developed is based on flow at normal stage, expression (4) (sec. 2.1) is, strictly speaking, not valid for abnormal stage-discharge conditions. The results described in this chapter apply specifically to a model on approximately a 1:40 scale with channel slope of 0.0012 and a Manning roughness factor of 0.024. The results do shed some light on this phase of the backwater problem, and an approximate solution may in some cases be preferable to none.

6.3 Backwater expression The experimental backwater coefficients for abnormal stage discharge (without piers, eccentricity, and skew) was computed according to the expression:

$$K_{bA}=\frac{h_{1A}^*}{V_{2A}^2/2g} \quad \text{-------------}(14)$$

where h_{1A}^*=backwater measured above *abnormal* stage at section 1 and $V_{2A}=Q/A_{2A}$ where $A_{2A}=b_Ah_A$ or gross area of constriction based on abnormal stage (see fig. 19).

The subscript A has been added throughout to signify that this is a special case, not to be confused with other expressions which precede or follow. Actually, expression (14) is a modification of expression (4a). Backwater coefficients computed according to expression (14) were found to plot slightly above the base curve for 45° wingwall abutments (fig. 5) and on both sides of the base curve for spillthrough abutments (fig. 6). The test results, which appear in the comprehensive report (*9*), plot in no particular order with regard to the degree of abnormality or difference in stage y_A-y_n (see fig. 19).

As the method of computation chosen results in backwater coefficients approximating those of the base curves, it is further assumed that the curves for incremental backwater coefficients, previously established for piers, eccentricity, and skew, may be reasonably applicable to abnormal stage-discharge conditions. If this is permissible the expression for the computation of backwater for abnormal stage discharge would then read:

$$h_{1A}^*=K^*\frac{V_{2A}^2}{2g} \quad \text{-------------}(15)$$

where $K^*=K_b$ (fig. 5 or 6) $+\Delta K_p$ (fig. 7) $+\Delta K_e$ (fig. 8) $+\Delta K_s$ (fig. 9 or 10). Thus the method and sources used to obtain the overall backwater coefficient remain unchanged. The one and important difference in expression (15) is insertion of the velocity head for *abnormal stage* rather than normal stage.

6.4 Drop in water surface across embankments The experimental points for the differential level ratio for abnormal stage discharge (without piers) were found to agree well with the base curve for the 45° wingwall abutment (fig. 14), but fell slightly above the base curve for the 1½:1 spillthrough abutment (fig. 15). The plotted information is included in the comprehensive report (*9*). Again the points plotted in no particular order with regard to the degree of abnormality or value of y_A-y_n. Thus to obtain the water surface along the downstream side of the roadway embankment for abnormal stage discharge, figures 14 and 15 are considered applicable. The method of computation is similar to that explained in chapter IV; the principal difference lies in the manner in which the backwater is computed for abnormal stage conditions. Other symbols involved in the abnormal stage-discharge computation also bear the subscript A to avoid confusion, so the differential level ratio:

$$D_b = \frac{h^*_{bA}}{h^*_{3A}} \quad \text{(16)}$$

where:

D_b = differential level ratio from base curve, figure 14 or 15 (no adjustment is needed for eccentricity or skew);

h^*_{bA} = backwater above abnormal stage (without piers);

h^*_{3A} = vertical distance from water surface to abnormal stage at section 3 (this dimension will be the same with or without piers).

The above procedures for abnormal stage will be further demonstrated in example 5.

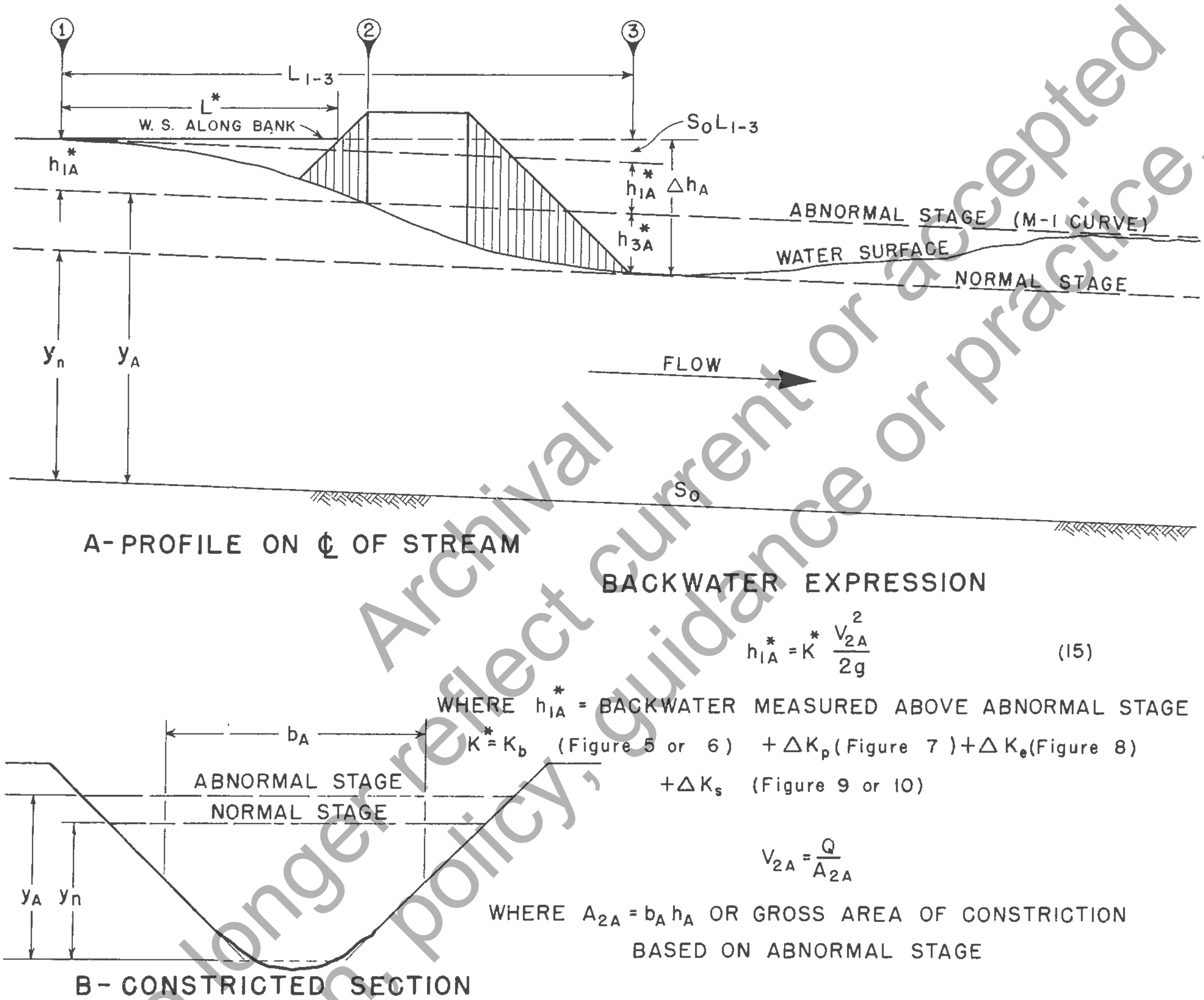

Figure 19.—Backwater with abnormal stage discharge.

Chapter VII.—EFFECT OF SCOUR ON BACKWATER

7.1 General Thus far the discussion of backwater has been limited to the case where the bed of a stream, in the vicinity of a bridge constriction, is rigid or immovable and does not degrade with introduction of embankments, abutments, and piers. It was necessary to obtain the initial experimental data under these more or less ideal conditions before introducing the further complication of a movable bed. In actuality the bed is usually composed of much loose material, some of which will move out of the constriction during flood flows. Nature wastes little time in attempting to restore the former regime, or the stage-discharge relation which existed prior to constriction of the stream. For within-bank flows nothing changes, but for flood flows there exists an altered regime, with a potential to enlarge the waterway area of the constriction.

Bearing in mind that during floods a stream is usually transporting sediment, the process might be described as follows, with the aid of figure 20: Constriction of a stream produces backwater at flood flows; backwater is indicative of an increase in potential energy upstream. This makes possible higher velocities in the constriction, thus increasing the transporting capacity of the flow to above normal in this reach. The greater capacity for transportation results in scouring of the bed in the vicinity of the constriction; the removed material is usually carried a short distance downstream and dropped as the velocity decreases. As the scouring action proceeds, the waterway area under the bridge enlarges, the velocity and resistance to flow decreases, and a reduction in the amount of backwater results. If the bed is composed of alluvial material, free to move, and a flood persists for a sufficient period of time, degradation under the bridge may approach a state of equilibrium; e.g., the scour hole will reach such proportions that the rate of transport out of the hole is essentially reduced to the rate of transport to the hole from upstream. Upon reaching this state of equilibrium it will be found that the stream has been practically restored to its former regime so far as stage discharge is concerned and the backwater has all but disappeared. This state could be fully realized in the model operating under controlled conditions.

Seldom is it possible to reach this extreme state in the field where cohesive, compacted, and cemented soils are encountered together with boulders and vegetation which materially retard the scouring process. Nevertheless, now that information is available to aid in determining the extreme case of equilibrium scour (*7, 8*), prediction of this should be of value in the lesser scour at field structures. In cases where abutments and piers can be keyed into bedrock, it may be advisable to encourage scour in the interest of utilizing a shorter bridge. This same objective is sometimes attained in another way by enlarging the waterway area under a bridge with excavation machinery during construction. In such cases, it is desirable to be able to determine the amount of backwater to be expected after localized enlargement of the waterway.

7.2 Nature of scour It is advisable to mention a few of the characteristics of scour, as observed during the model experiments, prior to considering the effect of scour on backwater. Where the depth of flow is essentially uniform and the bed is composed of a narrow gradation of clean sand, as was the case in the model, scour was greatest in the vicinity of the abutments, as shown in figure 20B, and little was evidenced in the center of the constriction unless the scour holes overlapped. This is better illustrated by a photograph of the model in figure 21 which shows the nature of scour around a 45° wingwall abutment and at two circular piers after a test run. The zero contour line represents normal elevation of the sandbed before placing the embankment in the flume. The remainder of the contour lines, which are at 0.2-foot intervals, define the resulting scour hole produced by initially constricting the channel 38 percent with the embankment. This photograph was included to demonstrate that scour did not occur uniformly across the constriction, but was greatest at points where concentration of flow occurs. It can be noted that scour around the two circular piers is minor compared to scour at the abutment. Figure 22 is a cross section of the same scour hole, measured along the upstream side of the bridge. The normal flow depth was 0.52 foot in this case, while the maximum equilibrium scour at the abutment amounted to twice this value. A word of caution is advanced here: The pattern of scour experienced in the model is not necessarily indicative of that which will occur in a stream.

It is not only difficult to predict the magnitude of scour but it is equally difficult to predict the location of scour at field structures since the depth of flow from flood plain to main channel can differ widely as well as the direction and concentration of flow; in the model the greatest concentration occurred at the abutments, while in the field the deeper scour may occur in the main channel as indicated in figure 20C. Should the main flow or a secondary current be directed toward an abutment during flood, or should a concentration of flow exist parallel to an embankment as was demonstrated by figure 12, the area adjacent to the abutment is definitely vulnerable to scour. It was not the intention here to go into detail on the vagaries of scour, since this would require much illustrative matter and explanation, but merely to point out a few features fundamental to understanding the effect of scour on backwater. References *4*, *7*, *8*, and *18* are recommended for the study and prediction of scour at bridge abutments and piers.

7.3 Backwater determination From the foregoing it has been established that any means of increasing the waterway area under a bridge can be effective in reducing the backwater. It is by no means a simple task to measure backwater in a model with a bed that is free to move where the formation of sand dunes, which advance slowly down the channel, tend to alter the initial conditions of flow. The majority of tests were made in a flume of rectangular cross section 8 feet wide by 150 feet long in which the former rigid bed was replaced by a layer of sand. Normal flow was first established for a given discharge, then the abutments were placed in the flume and the flow allowed to continue uninterrupted until a stable condition of scour was established. At this time final measurements were taken of the backwater, the difference in level across embankments, and the cross section of the scoured bed under the bridge. The resulting backwater and the differential level across embankments, with scour, were then compared with the backwater and differential level, respectively, for an immovable bed operating under similar conditions of flow and geometry. The values used for the rigid bed were computed according to the methods outlined in chapters II and IV. Holding all factors the same for any test,

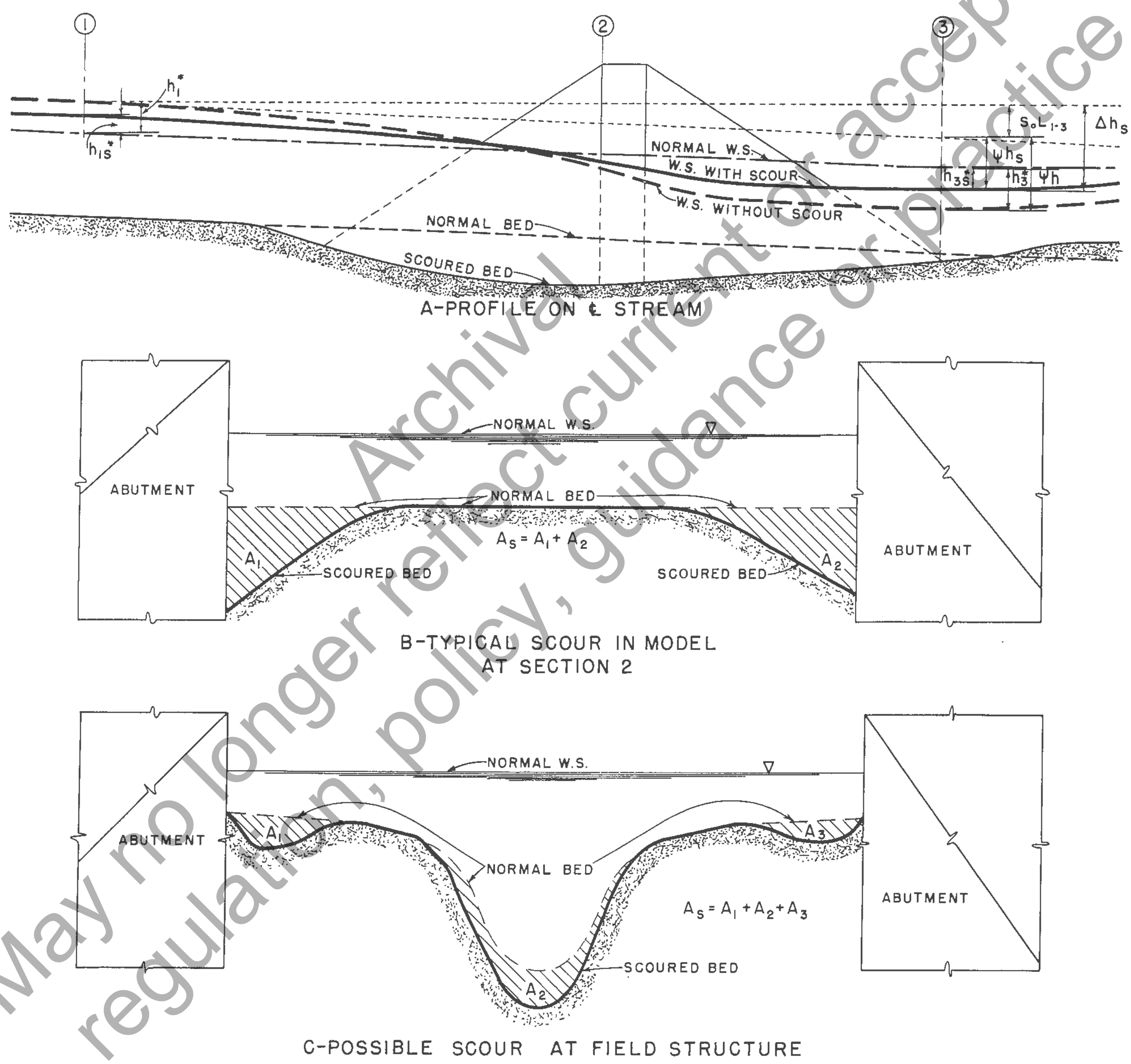

Figure 20.—Effect of scour on bridge backwater.

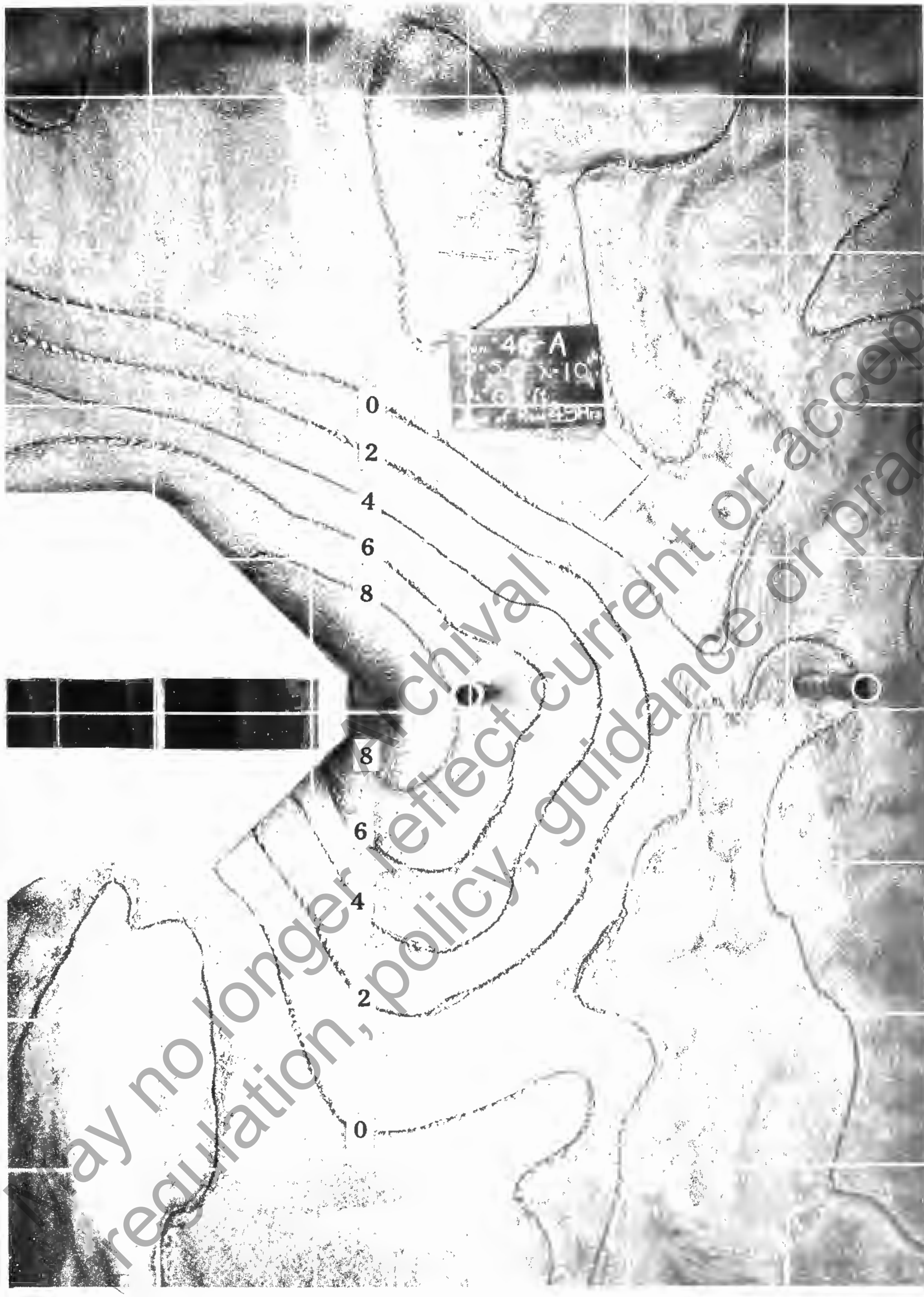

Figure 21.—Scour at wingwall abutment and single circular piers (model).

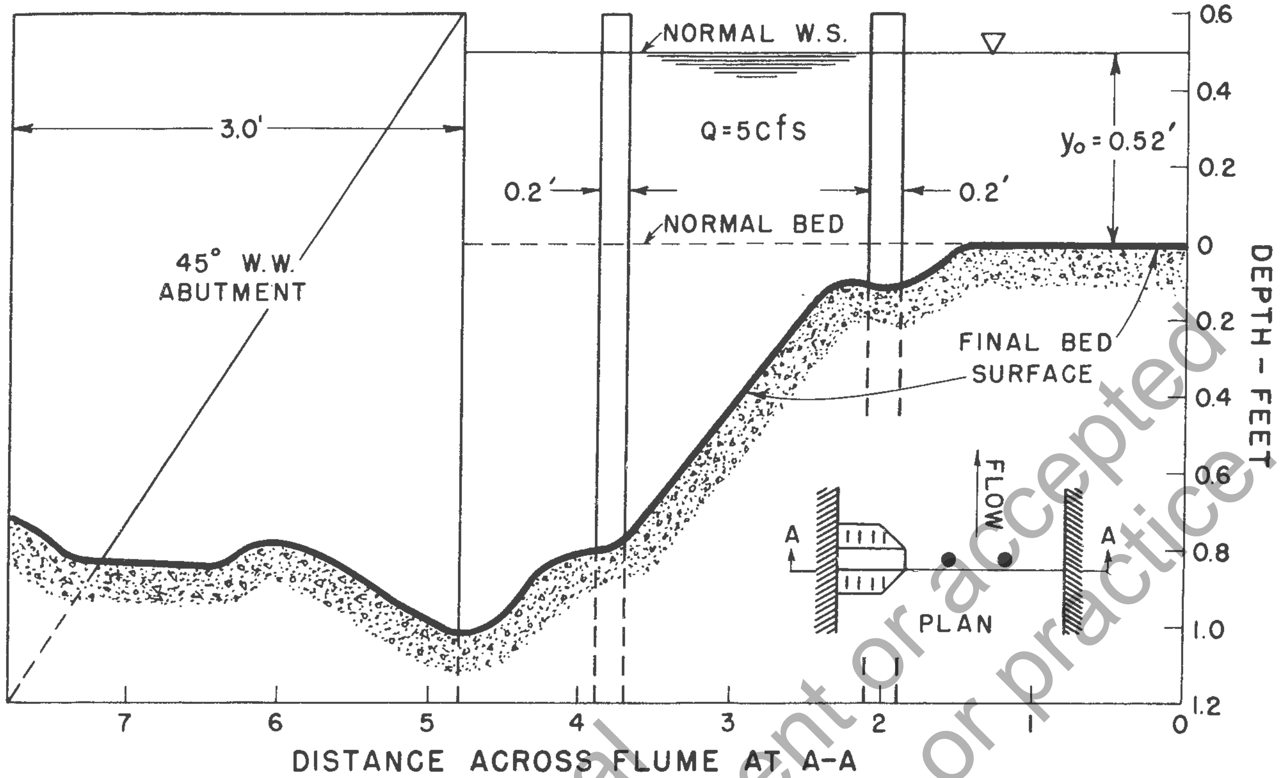

Figure 22.—Cross section of scour at upstream side of bridge (model).

except that for scour, the reduction in backwater was related directly to the area of scour. Scour and velocity are usually measured from the downstream side of a bridge, since this is the most practical way of obtaining these measurements during flood flows. Also the effective area of scour, so far as the computation of backwater is concerned, will more likely correspond to the scour at the downstream side than that at the upstream side of a bridge. Thus the area of scour measured at the downstream side, denoted as A_s, will be used for the computation of backwater. The model tests showed the scour at the downstream side to average about 75 percent of that at the upstream side of the bridge.

A design curve derived from the model experiments is included as figure 23. The ratio h^*_{1s}/h^*_1 is plotted with respect to A_s/A_{n2}, where the terms bearing the subscript s designate values with scour; those not bearing this subscript represent the same values computed without scour. Supposing the backwater at a given bridge was 1 foot, with no scour; it would be reduced to 0.52 foot were scour to enlarge the waterway area by 50 percent, or it would be reduced to 0.31 foot should the waterway area be doubled. The same reduction applies equally well to the ratios h^*_{3s}/h^*_3 and $\psi h_s/\psi h$ (see fig. 20A) so one curve suffices for all three. Thus to obtain backwater and related information for bridge sites where scour is to be encouraged, where scour cannot be avoided, or where the waterway is to be enlarged during construction, it is first necessary to compute the backwater and other quantities desired according to the method outlined in chapters II and IV for a rigid bed, using the original cross section of the stream at the bridge

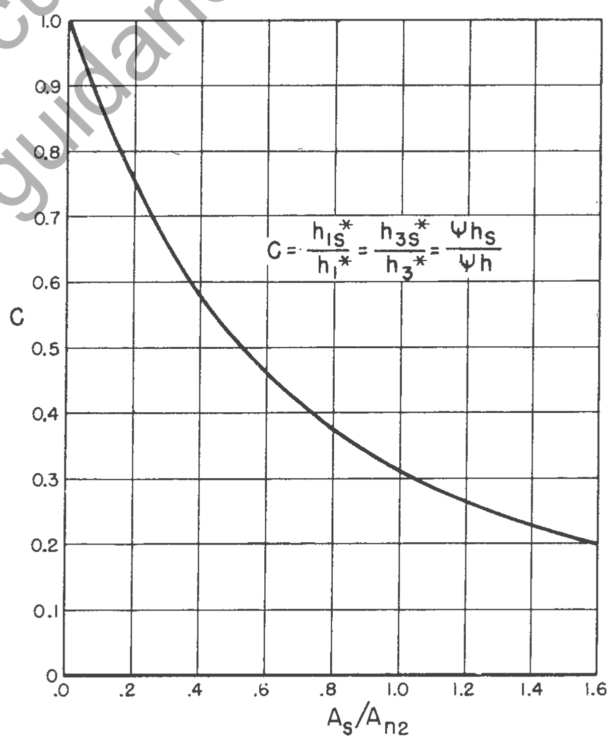

Figure 23.—Correction factor for backwater with scour.

site; these values are then multiplied by a common coefficient from figure 23 as follows:

$$h_{1s}^{*} = Ch_{1}^{*} \tag{17}$$

$$h_{3s}^{*} = Ch_{3}^{*} \tag{18}$$

$$\psi h_{s} = C\psi h \tag{19}$$

7.4 Enlarged waterways The designer will probably be reluctant to depend on scour as a means of enlarging the opening and thereby reducing backwater. If the waterway is enlarged by excavation there is little to gain by excavating much beyond the limits (upstream or downstream) of the abutments, as figure 21 attests. If additional volume is removed upstream or downstream, the channel may simply refill by deposition. Any enlargement of the cross section should be maintained to prevent reduction of area by the growth of willows and similar vegetation. Field surveys of existing bridges where channel enlargements had once been made should reveal worthwhile information on this question of the permanence of enlarged waterways.

Chapter VIII.—ILLUSTRATIVE PROBLEMS

8.1 Flood frequency Before proceeding with a backwater computation, certain basic field information is needed, such as the magnitude and frequency of floods at the bridge site, the river stages at which these floods will occur, and the distribution of flow across the stream and flood plains. Unless such information is known with reasonable accuracy, the computation of backwater may not be warranted.

A complete discussion of the problem of estimating flood frequency is beyond the scope of this publication, but sources of data will be cited. The frequency and magnitude of floods are best determined from gaging station records if available on the river in question. In the absence of such records, a regional flood frequency study may be made or may already be available from studies made by the U.S. Geological Survey. The Survey is preparing, under cooperative agreement with about 30 State highway departments, regional studies on magnitude and frequency of floods. Figure 24 has been included with the permission of the USGS to show the status of this work as of February 1960. (Numbers refer to parts used in issuance of annual Water Supply Papers.) Flood frequency reports are available for 16 States and parts of others, reports for other States are pending publication, and still others are in the process of preparation. In addition, the USGS may have other flood frequency data available and inquiry should be made of the District Engineer, Surface Water Branch, of the State involved.

The Bureau of Public Roads is in the process of making flood frequency studies according to physiographic areas. To date, studies have been completed for two physiographic areas in New England;[3] the Allegheny Cumberland Plateau (*11*); the glaciated shale and sandstone areas of New York, Pennsylvania, and Ohio (*11*); the Piedmont Plateau;[3] and the western slope of Colorado.[3]

[3] Available in mimeographed form from regional office of Bureau of Public Roads.

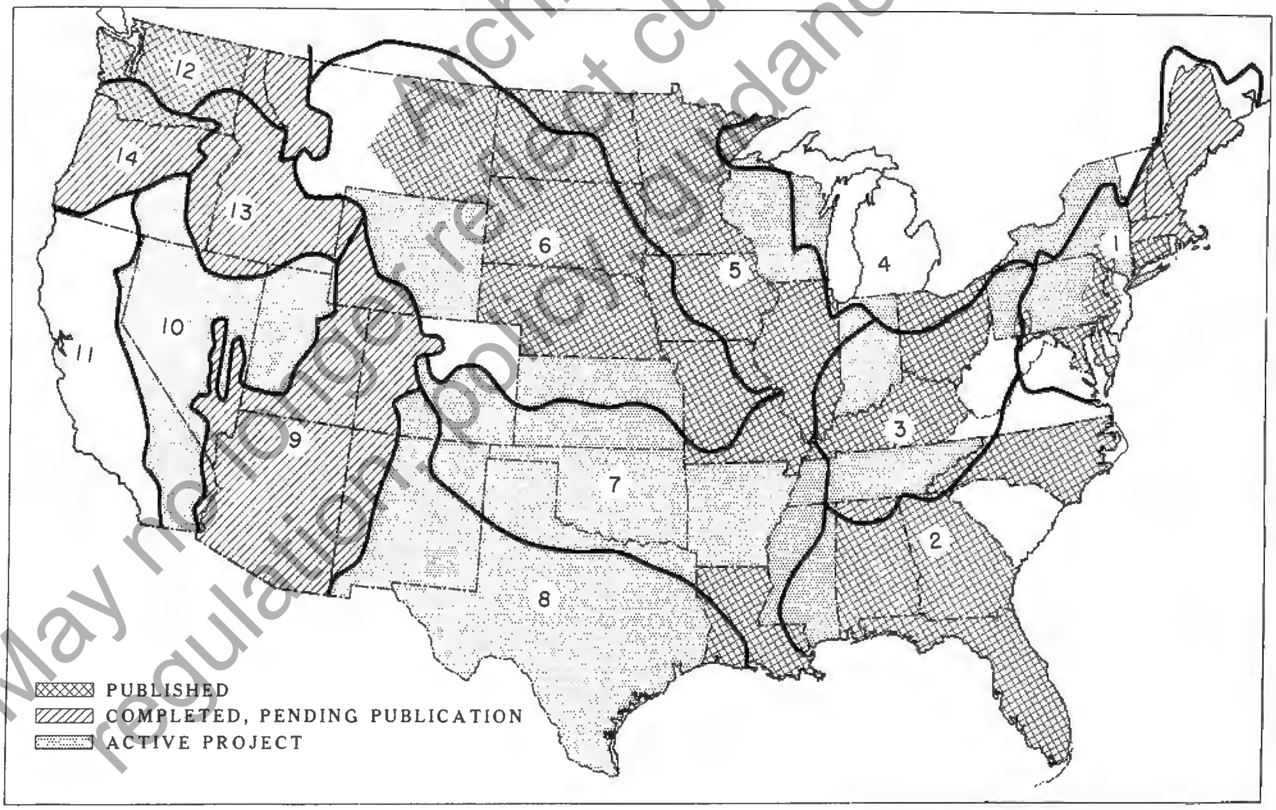

Figure 24.—Status of U.S. Geological Survey flood frequency reports as of February 1960.

8.2 Stage discharge It is important that the normal stage of a river for the design flood discharge be determined as accurately as possible at the bridge site. This may be accomplished in several ways, but where possible it is most desirable to establish it from a stage-discharge rating curve based on previous stream-gaging records in the vicinity of the bridge site. Such records are the most reliable; some are available in the files of the U.S. Geological Survey. A typical stage-discharge curve, figure 28, accompanies example 4. The scale at the top of the graph also shows flood recurrence interval. Where stage-discharge records are lacking for the stream in question, the usual procedure is to locate high-water marks of floods by consulting people who live in the vicinity of the bridge site. Flood information supplied by local residents is often inaccurate, but may be considered as reliable if confirmed by a number of other residents.

It is then necessary to find a means of relating stage to discharge. This can be done by the slope-area method, a simplified variation of which will be found illustrated in examples 1 and 6. Extreme care must be exercised in both the collection of field data and the manner in which it is processed if glaring discrepancies are to be avoided in the final result. In many cases where records are lacking, it is advisable to arrange for the installation and maintenance of a temporary stream gage at or near the bridge site several years in advance of construction. Even a single reliable point at an intermediate stage can be of inestimable value in the preparation of a stage-discharge curve.

Table 1.—Manning roughness coefficient for natural stream channels [1]

	Manning's n range
A. **Minor streams** (surface width at flood stage<100 ft.): [2]	
1. Fairly regular section:	
a. Some grass and weeds, little or no brush	0.030–0.035
b. Dense growth of weeds, depth of flow materially greater than weed height	0.035–0.05
c. Some weeds, light brush on banks	0.035–0.05
d. Some weeds, heavy brush on banks	0.05–0.07
e. Some weeds, dense willows on banks	0.06–0.08
f. For trees within channel with branches submerged at high stage, increase all above values by	*0.01–0.02*
2. Irregular section, with pools, slight channel meander; channels (a) to (e) above, increase all values about	*0.01–0.02*
3. Mountain streams, no vegetation in channel, banks usually steep, trees and brush along banks submerged at high stage:	
a. Bottom of gravel, cobbles, and few boulders	0.04–0.05
b. Bottom of cobbles with large boulders	0.05–0.07
B. **Flood plains** (adjacent to natural streams):	
1. Pasture, no brush:	
a. Short grass	0.030–0.035
b. High grass	0.035–0.05
2. Cultivated areas:	
a. No crop	0.03–0.04
b. Mature row crops	0.035–0.045
c. Mature field crops	0.04–0.05
3. Heavy weeds, scattered brush	0.05–0.07
4. Light brush and trees: [3]	
a. Winter	0.05–0.06
b. Summer	0.06–0.08
5. Medium to dense vegetation: [3]	
a. Winter	0.07–0.11
b. Summer	0.10–0.16
6. Dense willows, summer, not bent over by current	0.15–0.20
7. Cleared land with tree stumps, 100–150 per acre:	
a. No sprouts	0.04–0.05
b. With heavy growth of sprouts	0.06–0.08
8. Heavy stand of timber, a few down trees, little undergrowth:	
a. Flood depth below branches	0.10–0.12
b. Flood depth reaches branches (n increases with depth) [4]	0.12–0.16
C. **Major streams** (surface width at flood stage>100 feet): Roughness coefficient is usually less than for minor streams of similar description on account of less effective resistance offered by irregular banks or vegetation on banks. Values of n may be somewhat reduced. Follow general recommendations [1] if possible. The value of n for larger streams of mostly regular section, with no boulders or brush, may be in the range	0.028–0.33

[1] For calculations of stage or discharge in natural stream channels, it is recommended that the designer consult the local District Office of the U.S. Geological Survey to obtain data regarding values of n applicable to streams of any specific region. Where the recommended procedure is not followed, the table values may be used as a guide.
With channel of alinement other than straight, loss of head by resistance forces will be increased. A small increase in value of n may be made to allow for the additional loss of energy.
With steep slopes, depth of flow will generally be greater than computed by the usual methods for open channels due to air entrainment and additional resistance offered by air in contact with the high velocity flow. An approximate depth may be calculated by increasing n for the chute material involved by 20 to 30 percent.

[2] The tentative values of n cited are principally derived from measurements made on fairly short but straight reaches of natural streams. Where slopes calculated from flood elevations along a considerable length of channel, involving meanders and bends, are to be used in velocity calculations by the Manning formula, the value of n must be increased to provide for the additional loss of energy caused by bends. All values in the table must be so increased. The increase may be in the range of perhaps 3 to 15 percent.

[3] The presence of foliage on trees and brush under flood stage will materially increase the value of n. Therefore, roughness coefficients for vegetation in leaf will be larger than for bare branches. For trees in channel or on banks, and for brush on banks where submergence of branches increases with depth of flow, n will increase with rising stage.

[4] For important work and where accurate determination of water profiles is necessary, the designer is urged to consult references (*7–9*) to select n by comparison with specific conditions.

8.3 Channel roughness A matter of prime importance in bridge backwater or slope-area computations is the ability to evaluate properly the roughness of the main channel and the flood plains; both are subject to extreme variations with vegetal growth and depth of flow. As a guide, values of the Manning roughness coefficient n, as commonly encountered in practice, are tabulated for various conditions of channel and flood plain in table 1. Since the practicing engineer in this country is familiar with the Manning roughness coefficient, the Manning equation has been chosen for use here. In interpreting roughness coefficients from table 1, it should be kept in mind that the value of n, for a small depth of flow, especially on a flood plain covered with grass, weeds, and brush, can be considerably larger than that for greater flow depths over the same terrain (*12, 13*). On the other hand, as the stage rises in a stream with an alluvial bed, sand waves develop which can increase the value of n (*2*). It is therefore suggested that the notes accompanying table 1 be carefully considered along with the tabulation.

8.4 Design procedure The following is a brief step-by-step outline for determination of backwater produced by a bridge constriction:

1. Determine the magnitude and frequency of the discharge for which the bridge is to be designed from sources cited (sec. 8.1).
2. Determine the stage of the stream at the bridge site for the design discharge (sec. 8.2).
3. Plot representative cross section of stream for design discharge at section 1, if not already done under step 2. If stream channel is essentially straight and cross section substantially uniform in the vicinity of the bridge, the natural cross section of the stream at the bridge site may be used for this purpose.
4. Subdivide above cross section according to marked changes in depth of flow and roughness. Assign values of Manning roughness coefficient n to each subsection (table 1). Careful judgment is necessary in selecting these values.

5. Compute conveyance and then discharge in each subsection (method is demonstrated in examples).

6. Determine value of kinetic energy coefficient α_1 (method is illustrated in examples).

7. Plot natural cross section under proposed bridge based on normal water surface for design discharge, and compute gross water area (including area occupied by piers).

8. Compute bridge opening ratio M (sec. 1.9), observing modified procedure for skewed crossings (sec. 2.8).

9. Obtain value of K_b from appropriate base curve in figures 5 or 6 for symmetrical normal crossings.

10. If piers are involved, compute value of J (sec. 2.5) and obtain incremental coefficient ΔK_p from figure 7 (note method outlined for skewed crossings, sec. 2.6).

11. If eccentricity is severe, compute value of e (sec. 2.7) and obtain incremental coefficient ΔK_e from figure 8.

12. If a skewed crossing is involved, observe proper procedure in previous steps, then obtain incremental coefficient ΔK_s for proper abutment type from figures 9 or 10.

13. Determine total backwater coefficient K^*, by adding incremental coefficients to base curve coefficient K_b.

14. Compute backwater by expression (4) (sec. 2.1).

15. Determine distance upstream to maximum backwater from figure 11 and convert backwater to water surface elevation at section 1 if computations are based on normal stage at bridge.

Examples.—A clear understanding of the procedures for computing bridge backwater can be obtained from the illustrative examples which follow.

Example 1 comprises what is termed a simple normal crossing; the steps closely follow the outline of design procedure listed above.

Example 2 treats example 1 as a dual crossing.

Example 3 should help clarify the procedure recommended for skew crossings.

Example 4 demonstrates how backwater computations may be systematized for a typical bridge waterway problem where a range in bridge length and in flood discharge is to be studied. This example serves to demonstrate that the length, and hence the cost, of a bridge at a given site varies within wide limits depending on the amount of backwater considered tolerable.

Example 5 is included to demonstrate an approximate calculation for backwater at bridge sites where abnormal stage-discharge conditions prevail.

Example 6 illustrates how scour under a bridge affects the backwater.

Example 1

8.5 Example 1: Normal crossing *Given.*—The channel crossing shown in figure 25 with the following information: Cross section of river at bridge site showing areas, wetted perimeters, and values of Manning n; normal water surface for design = El. 115.0 ft. at bridge; average slope of river in vicinity of bridge $S_0 = 2.2$ ft./mi. (=0.00042 ft./ft.); cross section under bridge showing area to normal water surface; width of roadway = 40 ft.; and elevation of roadway = 123.0 ft.

The stream is essentially straight, the cross section relatively constant in the vicinity of the bridge, and the crossing is normal to the general direction of flow.

The problem is to investigate the following:

1. Conveyance at section 1.
2. Discharge of stream for stage 115.0 ft.
3. Velocity head correction coefficient α_1.
4. Bridge opening ratio M.
5. Backwater produced by bridge.
6. Water surface elevation on upstream side of roadway embankment.
7. Water surface elevation on downstream side of roadway embankment.

Computation (1a) Under the conditions stated, it is permissible to assume that the cross-sectional area of the stream at section 1 is the same as that at the bridge. This assumption made, the approach section is divided into subsections at abrupt changes in depth or channel roughness as shown in figure 25. The conveyance of each subsection is computed as shown in columns 1 through 8 of table 2, and the summation of the individual values in column 8 represents the overall conveyance of the stream or $K_1 = 342{,}000$. Note that the water interface between subsections is not included in the wetted perimeter.

Computation (1b) Since the slope of the stream is known (2.2 ft./mi.) and the cross-sectional area is essentially constant throughout the reach under consideration, it is permissible to solve for the discharge by what is known as the slope-area method or:

$$Q = K_1 S_0^{1/2} = 342{,}000 \times 0.00042^{1/2} = 7{,}000 \text{ c.f.s.}$$

It should be noted that the procedure in examples 3 and 4 conforms more nearly to what is usually required in practice.

Computation (1c) To compute the kinetic energy coefficient (sec. 1.10), it is first necessary to complete columns 9, 10, and 11 of table 2; then, using expression (3) (sec. 1.10):

$$\alpha_1 = \frac{\Sigma q v^2}{Q V_{n1}^2} = \frac{77{,}880}{7{,}000 \left(\frac{7{,}000}{2{,}680}\right)^2} = 1.64$$

where $\Sigma q v^2$ is the summation of column 11, and V_{n1} represents the average velocity for normal stage at section 1.

Computation (1d) The sum of the individual discharges in column 9 must equal 7,000 c.f.s. The factor M, as stated in section 1.9, is the ratio of that portion of the discharge approaching the bridge in width b to the total discharge of the river; using expression (1) (sec. 1.9):

$$M = \frac{q_b}{Q} = \frac{525 + 2{,}275 + 395}{7{,}000} = 0.46.$$

Computation (1e) Entering figure 6 with $M = 0.46$ for 1.5:1 spillthrough abutments, the base curve coefficient

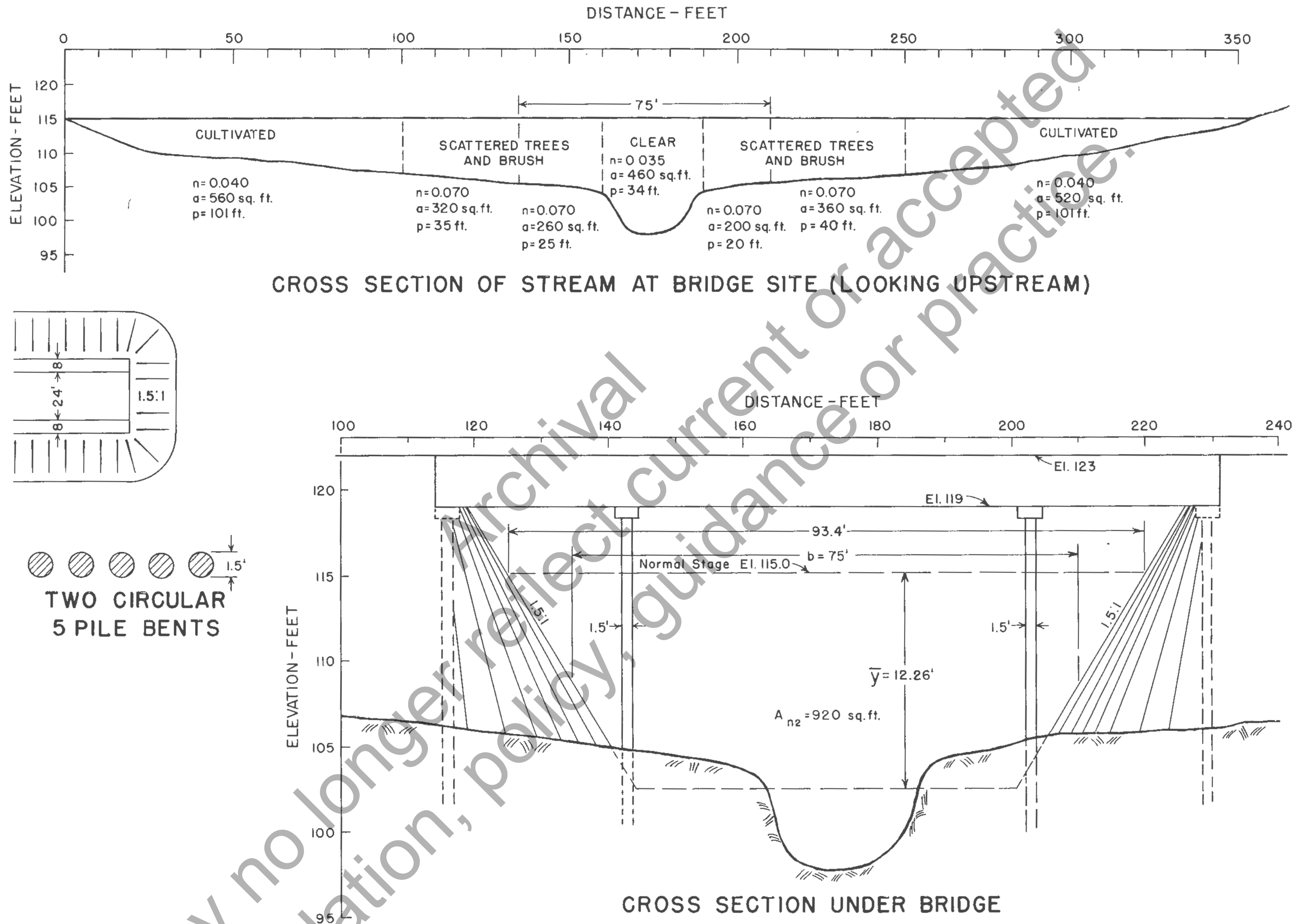

Figure 25.—Example 1: Plan and sections of normal crossing.

Table 2.—Example 1, sample computations: Properties of natural stream

Subsection	Computation (1a)							Computation (1c)		
	n	$\frac{1.486}{n}$	a	p	$r=\frac{a}{p}$	$r^{2/3}$	k	$q=Q\frac{k}{K_1}$	$v=\frac{q}{a}$	qv^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
			sq. ft.	*ft.*	*ft.*			*c.f.s.*	*f.p.s.*	
q_c 0–100	0.040	37.2	560	101	5.55	3.14	65,500	1,340	2.40	7,720
q_c 100–135	.070	21.2	320	35	9.15	4.38	29,800	610	1.90	2,200
q_b 135–160	.072	20.6	261	25	10.40	4.76	25,600	525	2.02	2,140
q_b 160–190	.035	42.5	460	34	13.50	5.67	111,000	2,275	4.95	55,800
q_b 190–210	.070	21.2	200	20	10.00	4.64	19,700	395	1.98	1,550
q_a 210–250	.070	21.2	360	40	9.00	4.33	33,000	675	1.88	2,390
q_a 250–350	.040	37.2	520	101	5.15	2.98	57,700	1,180	2.27	6,080
Total			$A_{n1}=2,680$				$K_1=342,000$	7,000		$\Sigma qv^2=77,880$

$K_b=1.06$. As the bridge is supported by two circular five-pile bents, the incremental coefficient for this effect will next be determined as described in section 2.5. Referring to figure 25, the gross water area under the bridge for normal stage A_{n2} is 920 sq. ft. and the area obstructed by the two circular pile bents A_p is 30 sq. ft., so:

$$J=\frac{A_p}{A_{n2}}=\frac{30}{920}=0.033.$$

Entering figure 7A with $J=0.033$ for circular five-pile bents, read from ordinate $\Delta K=0.11$. This value is for $M=1.0$. Now enter figure 7B with $M=0.46$ and obtain the correction factor σ for circular pile bents, which reads 0.64. The incremental backwater coefficient for the two circular pile bents $\Delta K_p=\Delta K\sigma=0.11\times0.64=0.07$.

The overall backwater coefficient:

$$K^*=K_b+\Delta K_p=1.06+0.07=1.13;$$

$$V_{n2}=\frac{Q}{A_{n2}}=\frac{7,000}{920}=7.60 \text{ f.p.s.; and}$$

$$\frac{V_{n2}^2}{2g}=0.90 \text{ ft.}$$

The approximate backwater will be, using expression (4a) (sec. 2.1):

$$K^*\frac{V_{n2}^2}{2g}=1.13\times0.90=1.02 \text{ ft.}$$

Inspection of pertinent values at this point show the following:

$$M=0.46;\ V_{n2}=7.60 \text{ f.p.s.; and } K^*\frac{V_{n2}^2}{2g}=1.02 \text{ ft.}$$

All three above values exceed those given by the guides (in sec. 2.1) so it is advisable to recompute the backwater, this time including the difference in kinetic energy between sections 1 and 4, using expression (4) (sec. 2.1):

$$h_1^*=K_b^*\frac{V_{n2}^2}{2g}+\alpha_1\left[\left(\frac{A_{n2}}{A_4}\right)^2-\left(\frac{A_{n2}}{A_1}\right)^2\right]\frac{V_{n2}^2}{2g}$$

$A_1=A_{n1}+1.02\ W$, where W is the surface width at section 1.

$A_1=2,680+360=3,040$ sq. ft.

Assuming that $A_{n1}=A_4$, which is not always the case:

$$h_1^*=1.02+1.63\left[\left(\frac{920}{2,680}\right)^2-\left(\frac{920}{3,040}\right)^2\right]0.90$$
$$=1.02+0.04=1.06 \text{ ft.}$$

Note that even in this case the error involved by omitting the second part of expression (4) is less than 4 percent.

Computation (1f) The statement was made (in sec. 3.1) that the water surface on the upstream side of the roadway embankment will be essentially the same as that at section 1. Thus, to determine the backwater elevation it is first necessary to locate the position of section 1, which is accomplished with the aid of figure 11.

From preceding computations:

$A_{n2}=920$ sq. ft.; $h_1^*=1.06$ ft.; $J=0.033$; and $S_0=0.000417$.

$$b=\frac{A_{n2}}{\bar{y}}=\frac{920}{12.26}=75 \text{ ft.},$$

where $\bar{y}$ is the depth of flow in an equivalent trapezoidal section for spillthrough abutments (see sec. 1.7). Then:

$$\frac{b^2(1-J)}{A_{n2}}=\frac{(75)^2(0.967)}{920}=5.90;$$

and

$$\frac{bh_1^*}{A_{n2}}=\frac{75\times1.06}{920}=0.086.$$

Entering figure 11 with the above values, $L^*/b=1.19$ and $L^*=1.19\times75=89$ ft. As noted in section 3.2, let $L_{1-2}=L^*$. The drop in the channel gradient between sections 1 and 2 (which can usually be ignored on the shorter bridges) is $S_0L_{1-2}=0.00042\times89=0.04$ ft.

The water surface elevation at section 1 and along the upstream side of the roadway embankment will be:

$$\text{El. } 115.0+S_0L_{1-2}+h_1^*=115.0+0.04+1.06=\text{El. } 116.1 \text{ ft.}$$

Computation (1g) The first step in determining the water surface elevation at section 3 is to compute the backwater for the bridge in question as though there were no piers, as explained in chapter IV:

$$h_b^* = K_b \frac{V_{n2}^2}{2g} = 1.06 \times 0.90 = 0.95 \text{ ft.}$$

Entering figure 15 with $M=0.46$ for a 1½:1 spill-through embankment slope, the differential level ratio for the bridge (without piers):

$$D_b = \frac{h_b^*}{h_3^*} = 2.35; \text{ and}$$

$$h_3^* = \frac{0.95}{2.35} = 0.40 \text{ ft.}$$

The placing of piers in a waterway results in no change in the value of h_3^* provided other conditions remain the same (sec. 4.3), so h_3^* (with pile bents) also equals 0.40 ft. The water surface elevation on the downstream side of the roadway embankment will be essentially El. 115.0 − 0.40 = 114.6 ft. The drop in water surface across the embankment is then 116.1 − 114.6 = 1.5 ft.

Example 2

8.6 Example 2: Dual bridges *Given.*—A second bridge, identical to that of example 1, which is to be constructed parallel and 300 feet downstream from the first bridge. The stream is essentially straight and of uniform cross section throughout this reach. Assuming no erosion at the constriction, compute the following:

1. The backwater upstream from the first bridge for a flood of 7,000 c.f.s.
2. The water surface elevation along upstream side of roadway embankment of first bridge.
3. The water surface elevation along downstream side of roadway embankment of second bridge (assuming elevation of roadway the same for both bridges).

Computation (2a) From example 1, $M=0.46$, $h_1^*=1.06$ ft., $J=0.033$, $S=0.00042$, $b=75$ ft., $A_{n2}=920$ sq. ft., $A_{n1}=2{,}680$ sq. ft., and $h_3^*=0.40$ foot. The value of the following parameter is required:

$$\frac{bL_d}{A_{n2}} = \frac{75 \times 300}{920} = 24.4.$$

Entering figure 16 with the above value and $M=0.46$, the backwater multiplication factor $\eta=1.37$. The backwater upstream from the first bridge for the combination is then:

$$h_d^* = \eta h_1^* = 1.37 \times 1.06 = 1.45 \text{ ft.}$$

Computation (2b) With normal stage of El. 115.0 ft. given at site of upstream bridge, it is necessary to determine drop in channel between section 2 and a new section 1. The value of the abscissa (fig. 11):

$$b^2 \frac{(1-J)}{A_{n2}} = 5.90$$

remains the same as for example 1. The other parameter is now:

$$\frac{bh_d^*}{A_{n2}} = \frac{75 \times 1.45}{920} = 0.12.$$

Entering figure 11 with the above values, and using h_d^* in place of h_1^*:

$$\frac{L^*}{b} = 1.30;$$

$$L_{1-2} = L^* = 1.30 \times 75 = 98 \text{ ft.}$$

The fall in the channel between sections 1 and 2:

$$S_0 L_{1-2} = 0.00042 \times 98 = 0.04 \text{ ft.},$$

the same as in example 1. The magnitude is unimportant in this case; the computation merely demonstrates the procedure. The water surface elevation at section 1 and along the upstream side of the roadway embankment of the first bridge will be:

$$\text{El. } 115.0 \text{ ft.} + S_0 L_{1-2} + h_d^* = 115.0 + 0.04 + 1.45 = \text{El. } 116.5 \text{ ft.}$$

Computation (2c) Entering figure 18A with $bL_d/A_{n2} = 24.4$, the multiplication factor $\xi = \psi h_{3B}/\psi h = 1.28$.

For the single bridge with pile bents in example 1:

$$\psi h = h_1^* + h_3^* = 1.06 + 0.40 = 1.46 \text{ ft.}$$

For the dual bridges:

$$\psi h_{3B} = \xi \psi h = 1.28 \times 1.46 = 1.87 \text{ ft.};$$

$$L_{1-3B} = 98 + 20 + 300 + 20 + 1.5 \times 8 = 450 \text{ ft.}; \text{ and}$$

$$S_0 L_{1-3B} = 0.00042 \times 450 = 0.19 \text{ ft.}$$

The approximate water surface elevation on the downstream side of the roadway embankment of the second bridge will be:

$$\text{El. } 116.5 - \psi h_{3B} - S_0 L_{1-3B} = 116.5 - 1.87 - 0.19 = \text{El. } 114.4 \text{ ft.}$$

Example 3

8.7 Example 3: Skew crossing *Given.*—A proposed skew crossing with wingwall abutments shown in figure 26, with the following information: The cross section of river showing areas, wetted perimeters, and values of Manning n for the several subsections chosen; normal stage at the bridge site and the projected waterway area at the bridge;

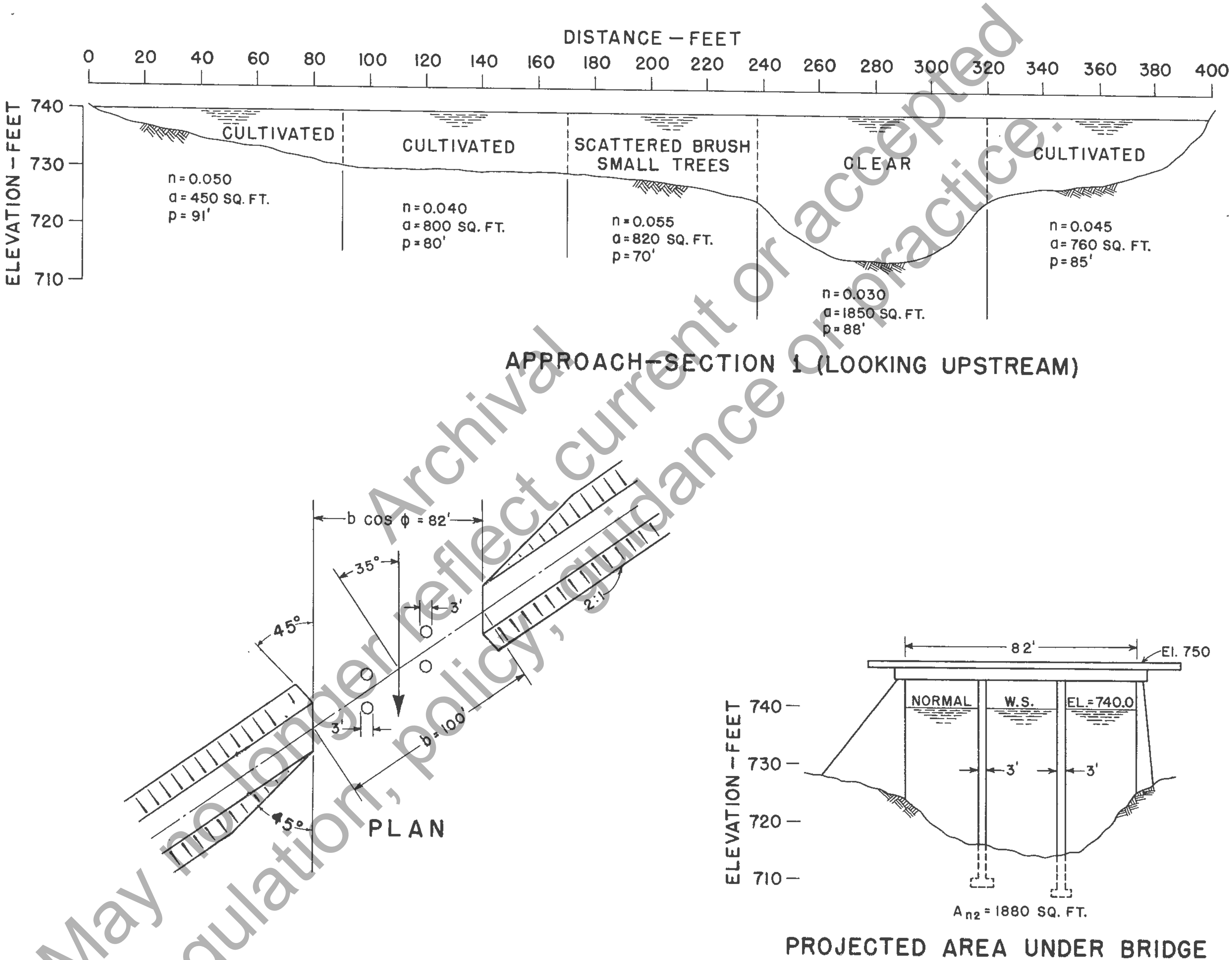

Figure 26.—Example 3: Plan and sections of skew crossing.

Table 3.—Example 3, sample computations: Properties of natural stream

Q=25,000 c.f.s.; Approach section $\phi=35°$

Subsection (1)	n (2)	$\frac{1.486}{n}$ (3)	a (4)	p (5)	$r=\frac{a}{p}$ (6)	$r^{2/3}$ (7)	$k=\frac{1.486}{n}ar^{2/3}$ (8)	$q=Q\frac{k}{K_1}$ (9)	$v=\frac{q}{a}$ (10)	qv^2 (11)
			sq. ft.	*ft.*	*ft.*			*c.f.s.*	*f.p.s.*	
q_c 0–90	0.05	29.7	450	91	4.95	2.90	38,900	890	1.98	3,490
q_c 90–170	.04	37.2	798	80	9.97	4.63	137,500	3,140	3.93	48,500
q_c 170–238	.055	27.0	820	70	11.71	5.16	114,200	2,610	3.18	26,400
q_b 238–320	.03	49.5	1,850	88	21.00	7.61	697,500	15,900	8.60	1,176,000
q_a 320–400	.045	33.0	760	85	8.94	4.31	108,000	2,460	3.24	25,800
Total			A_{n1}=4,678				K_1=1,096,100	25,000		Σqv^2=1,280,190

a design discharge of 25,000 c.f.s.; and the average slope of the river $S_0=2.6$ ft./mi. or 0.00049. The problem is to compute:

1. The backwater for the design discharge.
2. The approximate water surface elevation in midpoint of channel at section 1 for above conditions, assuming no scour.

Computation (3a) In this case both the design discharge and normal stage at bridge site are known. The same procedure demonstrated in example 1 is followed, with exceptions as noted. First, the general direction of flow in the river at the bridge site for the design flood, without constriction, is determined. Next, the position and extent of roadway embankments and the type of abutment are superimposed on the stream as illustrated in figure 4. The angle of skew is measured, which is 35° in this case; then the bridge opening is projected upstream, normal to the direction of flow, to section 1. The conveyance of each subsection is next computed for the full width of stream, as shown in columns 1 through 8 of table 3.

Checking the slope of the river from the conveyance computations:

$S_0=\left(\frac{25{,}000}{1{,}096{,}100}\right)^2=0.00052$ or 2.75 ft./mi. as compared to the given average slope of 2.6 ft./mi. Should the computed slope differ by more than ±10 percent from the average slope of the river at the bridge site, the values of n have not been chosen properly or there is an arithmetical error. If the computed slope does not meet this criterion and no error is found, the values of n should be reestimated and the computation for K_1 repeated. For the problem at hand, the computed slope is about 6 percent greater than the average slope given for the river, thus the computations through column 8 are considered satisfactory.

Columns 9 through 11 are next completed. Then:

$$\alpha_1=\frac{\Sigma qv^2}{QV_{n1}^2}=\frac{1{,}280{,}190}{(25{,}000)\,(5.35)^2}=1.79.$$

The contraction ratio:

$$M=\frac{q_b}{Q}=\frac{15{,}900}{25{,}000}=0.64.$$

Consulting the base curve in figure 5 for 45° wingwalls and $M=0.64$, $K_b=0.55$.

Entering figure 9A for the effect of skew with $M=0.64$, and $\phi=35°$, $\Delta K_s=-0.06$.

The water cross section occupied by piers is:

$A_p=2\times3\times25=150$ sq. ft.;

$A_{n2}=1{,}880$ sq. ft. (projected area under bridge); and

$$J=\frac{A_p}{A_{n2}}=\frac{150}{1{,}880}=0.080.$$

Entering figure 7A with $J=0.080$ for round double shaft piers, $\Delta K=0.22$. From figure 7B, $\sigma=0.85$ for $M=0.64$. Then $\Delta K_p=0.22\times0.85=0.19$.

Checking for eccentricity:

$$e=\left(1-\frac{q_a}{q_c}\right)=\left(1-\frac{2{,}460}{890+3{,}140+2{,}610}\right)=0.63$$

Since $e<0.80$, eccentricity is not a factor in this problem.

The total backwater coefficient:

$$K^*=K_b+\Delta K_s+\Delta K_p=0.55-0.06+0.19=0.68;$$

$$V_{n2}=\frac{Q}{A_{n2}}=\frac{25{,}000}{1{,}880}=13.3 \text{ f.p.s.; and}$$

$$\frac{V_{n2}^2}{2g}=2.75 \text{ ft.}$$

The above velocity is based on the projected area of the constriction.

Then the approximate backwater:

$$h_1^*=K^*\frac{V_{n2}^2}{2g}=0.68\times2.75=1.87 \text{ ft.}$$

Since M, V_{n2}, and $K^*\frac{V_{n2}^2}{2g}$ do not meet the conditions set forth in the guides (sec. 2.1), the backwater computation will be repeated according to expression (4):

$$h_1^*=K^*\frac{V_{n2}^2}{2g}+\alpha_1\left[\left(\frac{A_{n2}}{A_4}\right)^2-\left(\frac{A_{n2}}{A_1}\right)^2\right]\frac{V_{n2}^2}{2g}$$

$$A_1=A_{n1}+1.87W=4{,}680+1.87\times400=5{,}428 \text{ sq. ft.}$$

$$h_1^*=1.87+1.79\left[\left(\frac{1880}{4680}\right)^2-\left(\frac{1880}{5428}\right)^2\right]2.75$$

$$=1.87+0.20=2.1 \text{ ft. (backwater).}$$

Computation (3b) Computing the distance to maximum backwater:

$$\frac{b^2(1-J)}{A_{n2}}=\frac{100^2(1-0.080)}{1880}=4.9; \text{ and}$$

$$\frac{bh_1^*}{A_{n2}}=\frac{100\times2.1}{1880}=0.11.$$

Entering figure 11 with the above values:

$$\frac{L^*}{b}=1.40;$$

$$L^*=1.40\times100=140 \text{ ft.; and}$$
$$S_0L^*=140\times0.00049=0.07 \text{ ft.}$$

The approximate backwater elevation in the center of the channel at section 1 will be:

El. $740.0+S_0L^*+h_1^*=740.0+0.07+2.1=$ El. 742.2 ft.

Example 4

8.8 Example 4: Eccentric crossing This example is intended to show how computations may be tabulated systemically for alternate conditions, leading to a general solution for backwater at a given site for wide ranges in flood discharge and bridge length.

Given.—A representative cross section of the river and flood plain at a bridge site looking in the upstream direction, shown in figure 27, and the following information: The river is straight in the vicinity of the bridge, and has an average slope, for some 10 miles upstream and down-

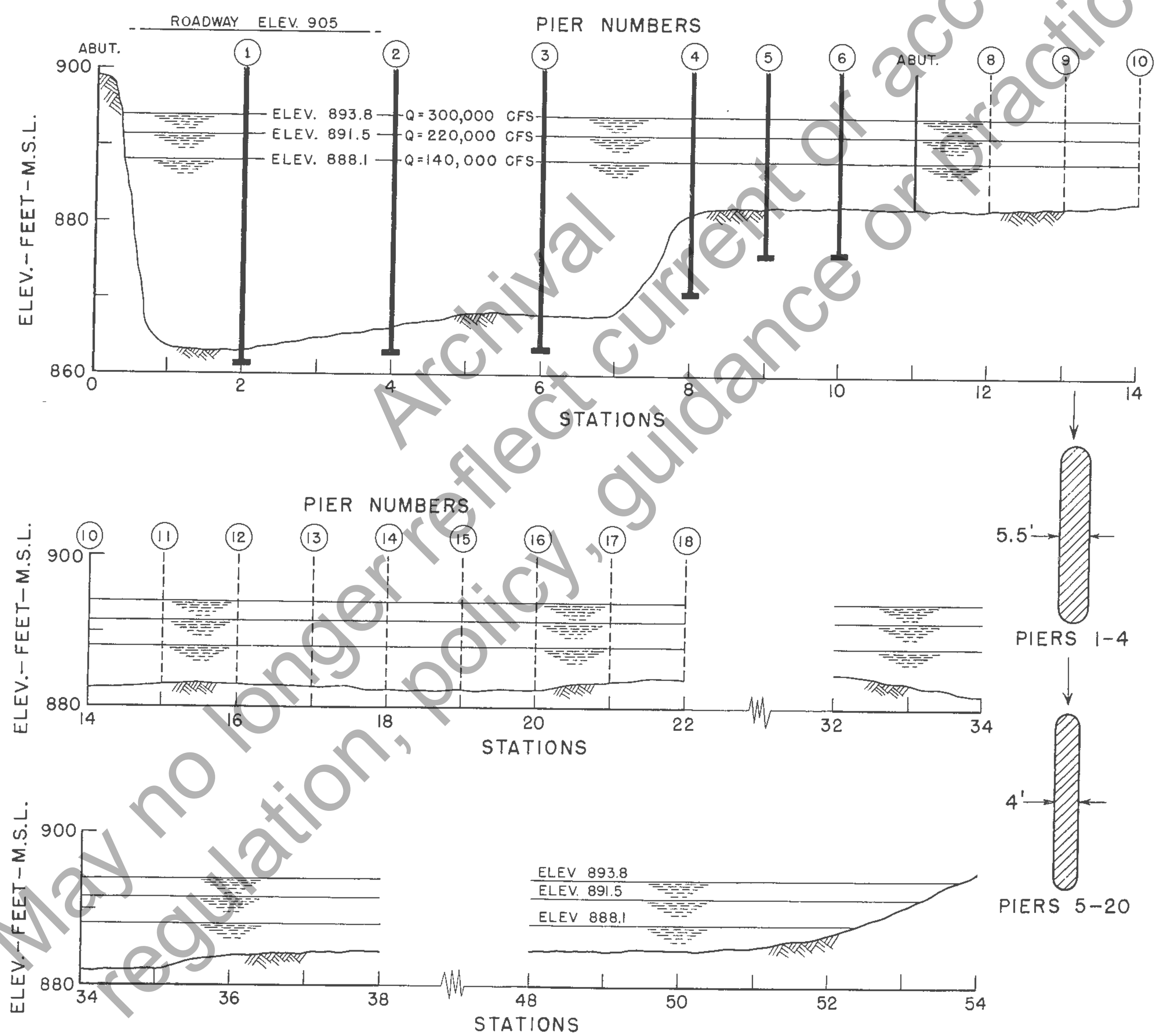

Figure 27.—Example 4: Eccentric crossing, section of river at bridge (facing upstream).

stream, of 2.2 ft./mi. or $S_0=0.00042$. The bluff on the left is limestone. The bed of the river and the flood plain consist of sand and loam overlying a limestone base. The abutments will be 45° wingwall type with embankment slopes of 2:1. The following computed information also has been supplied in connection with a bridge site survey:

Figure 28.—A stage discharge curve for the river at the bridge site with flood frequency scale superimposed.

Figure 29A.—Curves giving cumulative water areas across the river at the bridge site for discharges of 140,000, 220,000, and 300,000 c.f.s.

Figure 29B.—Curves showing cumulative conveyance across the river at the site for discharges of 140,000, 220,000, and 300,000 c.f.s.

Figure 29C.—A curve giving the relation between the kinetic energy coefficient and the discharge.

The problem in this case is to prepare a hydraulic design chart showing backwater related to discharge for bridge lengths from 1,100 to 2,500 feet and for flood frequencies ranging from approximately 10 to 100 years, assuming that no scour or erosion will occur under the bridge. (Actually there would be scour at the piers, the left abutment, and in the main channel; but to avoid complicating the problem any more than necessary at this stage, scour will not be considered.)

The computation is begun with four 200-ft. spans plus three 100-ft. spans or 1,100 feet of bridge (see fig. 27). Each span to be added will be 100 feet in length. Twelve conditions will be investigated for the problem at hand: The backwater will be computed for discharges of 300,000, 220,000, and 140,000 c.f.s. for bridge lengths of 1,100, 1,500, 2,000, and 2,500 feet. With an understanding of

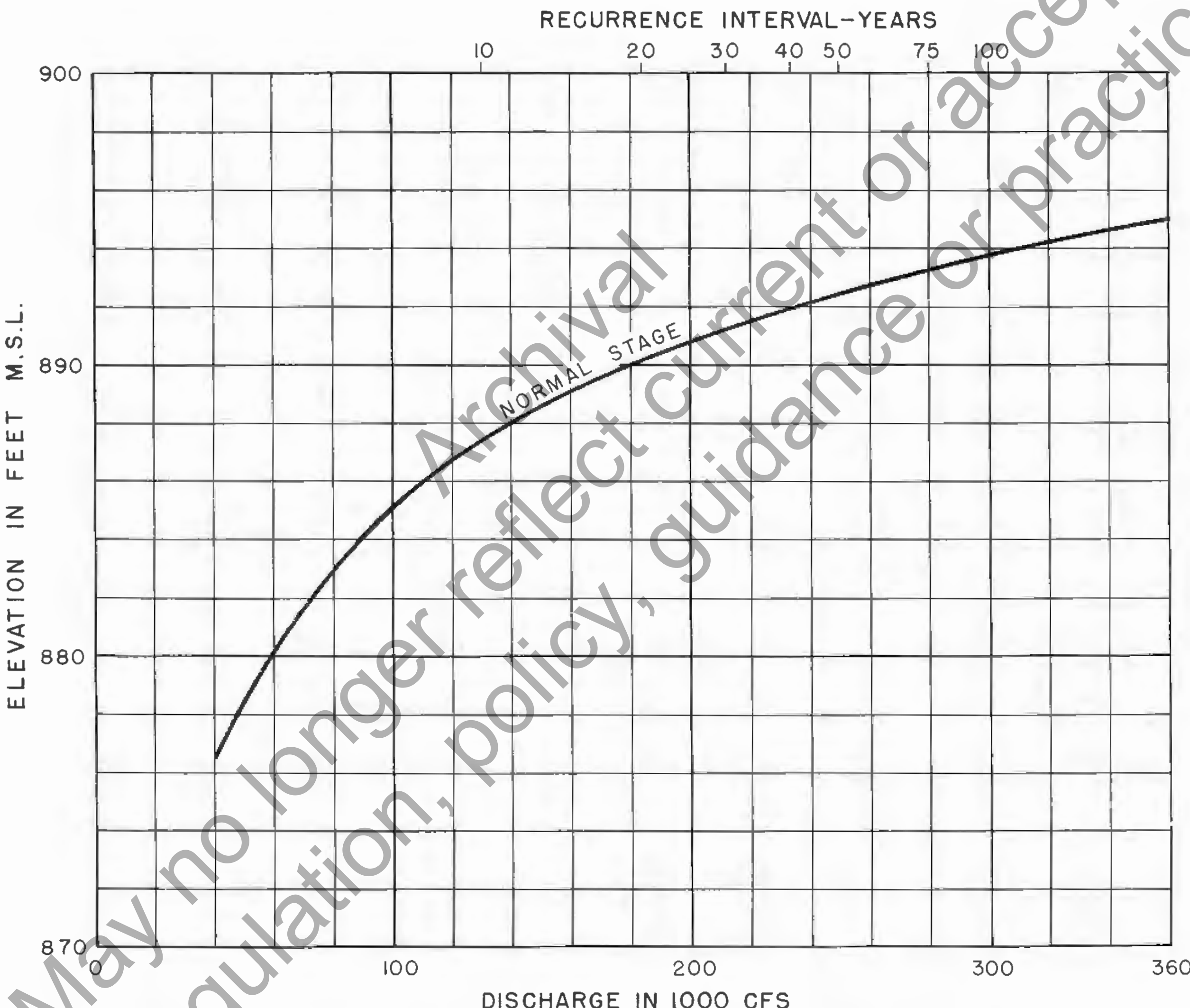

Figure 28.—Example 4: Stage-discharge curve for river at bridge site.

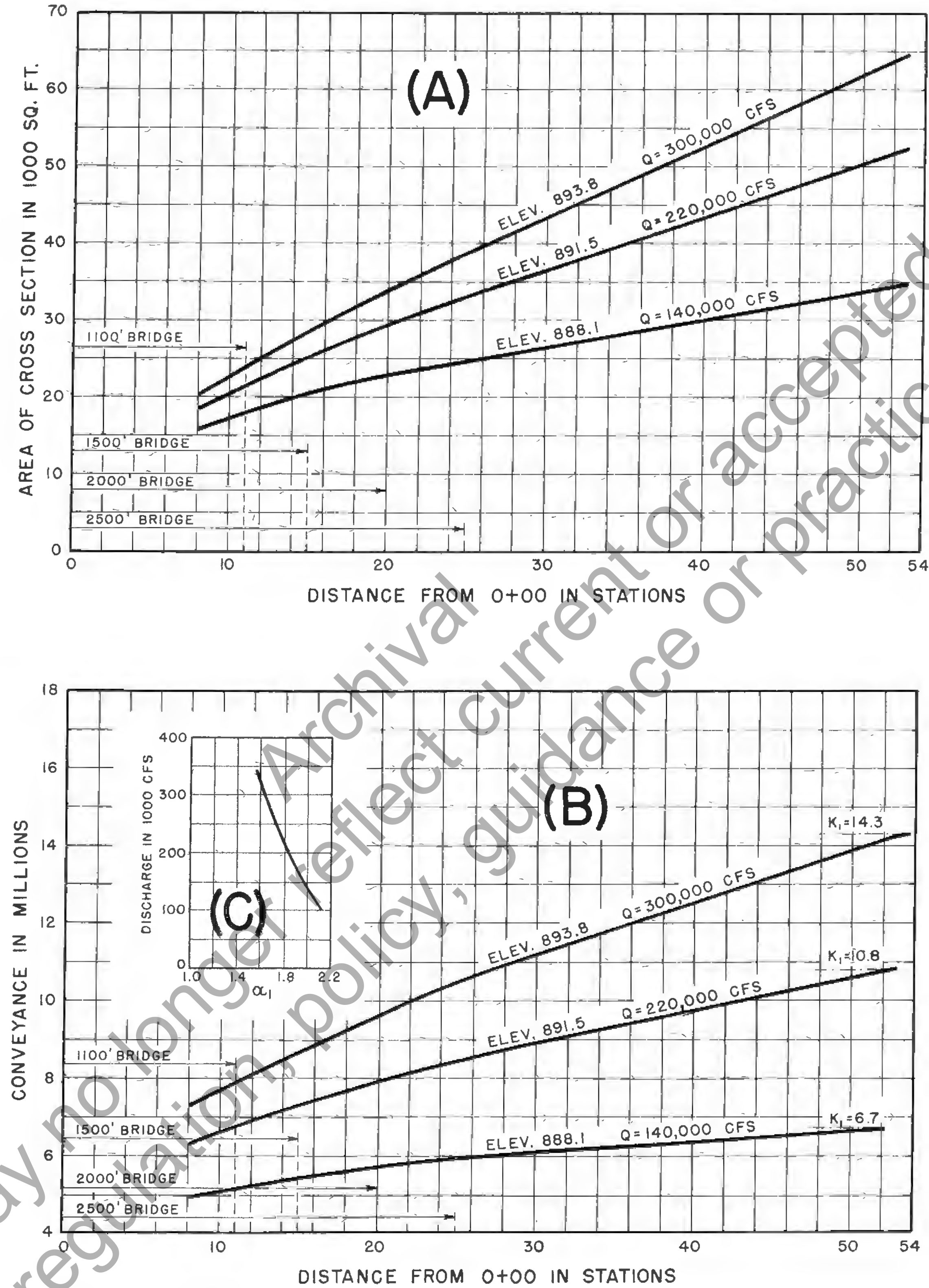

Figure 29.—Example 4: Area, conveyance, and velocity head coefficient (facing upstream).

Table 4.—Example 4: Constricted section computation

Q (1)	Normal stage (fig. 28) (2)	b (3)	Subsection (station) (4)	k_b (fig. 29B) (5)	K_1 (fig. 29B) (6)	$M = k_b/K_1$ (7)	Piers (fig. 27) (8)	w_p Width (9)	Σd_n Height (10)	$A_p = w_p \Sigma d_n$ (11)	A_{n2} (fig. 29A) (12)	$J = A_p/A_{n2}$ (13)
c.f.s.	*ft.*	*ft.*						*ft.*	*ft.*	*sq. ft.*	*sq. ft.*	
300,000	893.8	1,100	0–11	7,88			1–4	5.5	94	517	20,136	
							5–6	4.0	26	104	3,690	
			Total	7,880	14,264	0.55				621	23,826	0.026
		1,500	0–11	7,880			1–6			621	23,826	
			11–15	696			7–10	4.0	50	200	4,615	
			Total	8,576	14,264	.60				821	28,441	.029
		2,000	0–15	8,576			1–10			821	28,441	
			15–20	1,005			11–15	4.0	60	240	5,500	
			Total	9,581	14,264	.67				1,061	33,941	.031
		2,500	0–20	9,581			1–15			1,076	33,941	
			20–25	830			16–20	4.0	54	216	4,900	
			Total	10,411	14,264	.73				1,292	38,841	.033
220,000	891.5	1,100	0–11	6,711			1–4	5.5	85	466	18,380	
							5–6	4.0	21	84	3,000	
			Total	6,711	10,792	.62				550	21,380	.026
		1,500	0–11	6,711			1–6			550	21,380	
			11–15	480			7–10	4.0	41	164	3,695	
			Total	7,191	10,792	.67				714	25,075	.028
		2,000	0–15	7,191			1–10			714	25,075	
			15–20	680			11–15	4.0	48	192	4,350	
			Total	7,871	10,792	.73				906	29,425	.031
		2,500	0–20	7,871			1–15			906	29,425	
			20–25	529			16–20	4.0	42	168	3,750	
			Total	8,400	10,792	.78				1,074	33,175	.032
140,000	888.1	1,100	0–11	5,142			1–4	5.5	71	390	15,796	
							5–6	4.0	15	60	1,980	
			Total	5,142	6,659	.77				450	17,776	.025
		1,500	0–11	5,142			1–6			450	17,776	
			11–15	232			7–10	4.0	27	108	2,385	
			Total	5,374	6,659	.81				558	20,161	.028
		2,000	0–15	5,374			1–10			558	20,161	
			15–20	297			11–15	4.0	32	128	2,650	
			Total	5,671	6,659	.85				686	22,811	.030
		2,500	0–20	5,671			1–15			686	22,811	
			20–25	186			16–20	4.0	26	104	2,000	
			Total	5,857	6,659	.88				790	24,811	.032

the previous examples, the following procedure should be more or less self-explanatory.

The characteristics of the undisturbed river can be obtained from figures 28 and 29. Attention is called to the manner in which the curves in figures 29A and B are plotted—the areas and conveyances are accumulated from left to right looking upstream. For example, the water area between stations 30+00 and 20+00 for 220,000 c.f.s. would be (fig. 29A) 37,000−29,000 or 8,000 sq. ft. In a like manner the conveyance for the same subsection and discharge would be (fig. 29B) $8.9\times10^6-7.9\times10^6$ or 1,000,000.

Table 4 is prepared for the contracted bridge section for the three discharges and the four bridge lengths chosen. The computations in this table center around the determination of M, A_{n2}, and J, columns 7, 12, and 13, respectively. The values of conveyance (cols. 5 and 6) were read from figure 29B, and the waterway areas (col. 12) were obtained from figure 29A. Note that M is computed on the basis of conveyance.

The backwater computations are tabulated in table 5. Columns 1–13 embody the computations required to solve the first part of backwater expression (4a), and columns 14–22 represent computation of the second part (4b), where required (sec. 2.1). The sum of the two is listed in column 23.

A composite hydraulic design chart, plotted from the information tabulated in columns 1–23, is included in figure 30A. The designer can read from this chart the length of bridge required to pass various flows with a given backwater. A scale of bridge cost can also be added on the righthand side as shown. For convenience the recurrence interval is indicated at the top of the chart. To illustrate use of the resulting chart, suppose it is decided to design

Table 5.—Example 4: Backwater computation

$e=1.0$

Q (1)	Normal stage (2)	b (3)	M (table 4) (4)	J (table 4) (5)	K_b (fig. 5) (6)	ΔK_e (fig. 8) (7)	ΔK_p (fig. 7) (8)	$K^*=K_b+\Delta K_e+\Delta K_p$ (9)	A_{n2} (table 4) (10)	$V_{n2}=Q/A_{n2}$ (11)	$\frac{V_{n2}^2}{2g}$ (12)	$K^*\frac{V_{n2}^2}{2g}$ (13)
c.f.s.	*ft.*	*ft.*							*sq. ft.*	*ft./sec.*	*ft.*	*ft.*
300,000	893.8	1,100	0.55	0.026	0.76	0.16	0.03	0.95	23,826	12.55	2.45	2.33
		1,500	.60	.029	.65	.16	.04	.85	28,441	10.55	1.73	1.47
		2,000	.67	.031	.48	.15	.04	.67	33,941	8.85	1.22	.82
		2,500	.73	.033	.36	.14	.04	.54	38,841	7.73	.929	.50
220,000	891.5	1,100	.62	.026	.60	.16	.03	.79	21,380	10.30	1.65	1.30
		1,500	.67	.028	.48	.15	.04	.67	25,075	8.78	1.20	.80
		2,000	.73	.031	.36	.14	.04	.54	29,425	7.48	.870	.47
		2,500	.78	.032	.26	.12	.05	.43	33,175	6.65	.688	.30
140,000	888.1	1,100	.77	.025	.28	.13	.04	.45	17,776	7.89	.968	.44
		1,500	.81	.028	.21	.12	.04	.37	20,161	6.95	.751	.28
		2,000	.85	.030	.15	.10	.04	.29	22,811	6.14	.586	.17
		2,500	.88	.032	.11	.08	.05	.24	24,811	5.65	.496	.12

Q (1)	Normal stage (2)	b (3)	$A_{n1}=A_4$ (fig. 29A) (14)	W (15)	Cols. 13×15 (16)	$A_1=$cols. 14+16 (17)	$\left(\frac{A_{n2}}{A_4}\right)^2$ (18)	$\left(\frac{A_{n2}}{A_1}\right)^2$ (19)	$Y=$cols. 18−19 (20)	α_1 (fig. 29C) (21)	$\alpha_1\frac{V_{n2}^2}{2g}$ (22)	h_1^*: cols. 13+22 (23)
c.f.s.	*ft.*	*ft.*	*sq. ft.*	*ft.*	*sq. ft.*	*sq. ft.*					*ft.*	*ft.*
300,000	893.8	1,100	64,745	5,330	12,400	77,145	0.136	0.096	0.040	1.63	0.16	2.49
		1,500	64,745	5,330	7,850	72,595	.193	.153	.040	1.63	.11	1.58
		2,000	64,745	5,330	4,380	69,125	.275	.240	.035	1.63	.07	.89
		3,000										.50
220,000	891.5	1,100	52,564	5,285	6,860	59,424	.165	.129	.036	1.79	.11	1.41
		1,500	52,564	5,285	4,220	56,784	.228	.195	.033	1.79	.07	.87
		2,000										.47
		3,000										.30
140,000	888.1	1,100										.44
		1,500										.28
		2,000										.17
		2,500										.12

Q (1)	Normal stage at bridge (2)	b (3)	h_1^* (23)	A_{n2} (10)	J (5)	$\frac{bh_1^*}{A_{n2}}$ (24)	$\frac{2b^2(1-J)}{A_{n2}}$ (25)	$\frac{L^*}{2b}$ (fig. 11) (26)	L^* (27)	S_0L_{1-2} (28)	$h_1^*+S_0L_{1-2}$ (29)	W.S. elev. (sec. 1) cols. 2+29 (30)
c.f.s.	*ft.*	*ft.*	*ft.*	*sq. ft.*					*ft.*	*ft.*	*ft.*	*ft.*
300,000	893.8	1,100	2.49	23,826	0.026	0.11	99	0.310	682	0.29	2.8	896.6
		1,500	1.58								1.9	895.7
		2,000	.89								1.2	895.0
		2,500	.50	38,841	.033	.04	312	.143	715	.30	.8	894.6
220,000	891.5	1,100	1.41	21,380	.026	.07	110	.265	584	.25	1.7	893.2
		1,500	.87								1.1	892.6
		2,000	.47								.7	892.2
		2,500	.30	33,175	.032	.03	366	.123	615	.26	.6	892.1
140,000	888.1	1,100	.44	17,776	.025	.03	132	.205	450	.19	.6	888.7
		1,500	.28								.5	888.6
		2,000	.17								.4	888.5
		2,500	.12	24,811	.032	.01	490	.086	430	.18	.3	888.4

the bridge for a 50-year recurrence interval. If 1.5 feet of backwater can be tolerated, the bridge can be 1,250 feet long at a cost of $600,000; while if the backwater must be limited to 0.5 foot, the bridge length required would be 2,250 feet at a cost of $970,000, or $370,000 more. Thus an arbitrary decision to stay within a certain limiting rise of water surface can mean a relatively large increase in the length and cost of a bridge. A hydraulic design chart of this type is very useful for conveying information to others who are responsible for making decisions.

If the water surface along the upstream side of the embankment is desired, the drop in channel gradient between sections 1 and 2 will be required since normal stage was given at the bridge. The computational procedure was explained in the previous examples, where it was found that the magnitude of this drop proved to be insignificant for the short bridges considered. For longer bridges, such as the one in this example, the drop in channel gradient cannot be ignored as will be evident from the computations in columns 24–30 of table 5. In this case the drop ranges between 0.18 and 0.30 foot, column 28. The water surface elevation at section 1, or along the upstream embankment, is tabulated in column 30. The stage on the upstream side of the bridge embankment is plotted in figure 30B. Should it be desired to set an approach roadway to be overtopped for flows greater than a certain specified discharge, a chart of this type is of value.

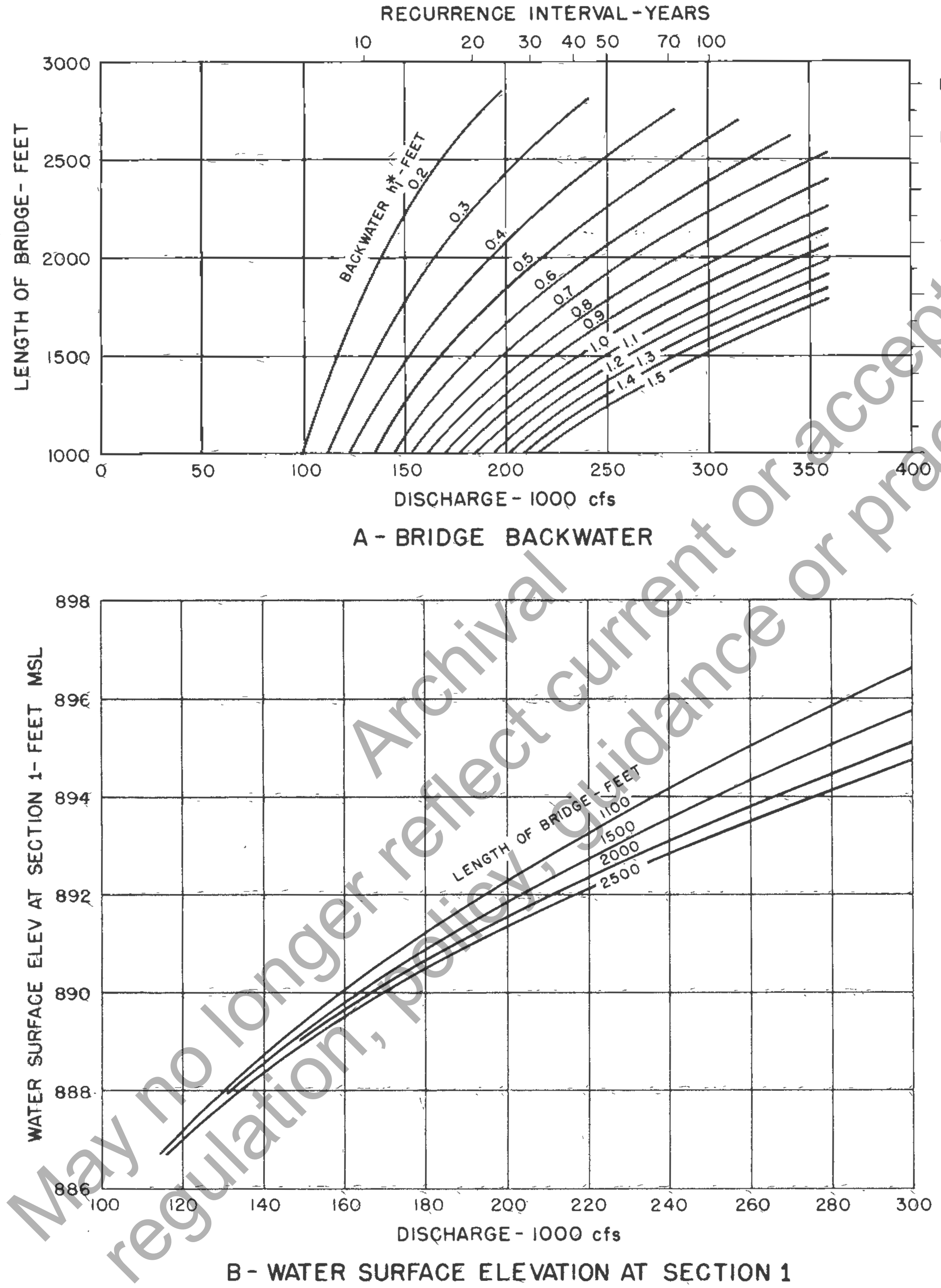

Figure 30.—Example 4: Composite backwater curves derived from computations.

Example 5

8.9 Example 5: Abnormal stage discharge To avoid misunderstandings in the computation of backwater for other than a normal stage-discharge relation for a stream, the method will be illustrated by an example.

Given.—The stream crossing used in example 1 (fig. 25) in which normal stage, roughness factors, discharge, and all dimensions remain the same except for an abnormal condition originating downstream which has increased the stage at the bridge site by 2 feet to elevation 117.0.

The problem here is to determine for this abnormal condition (assuming no scour):

1. The approximate backwater which will be produced by the bridge constriction.
2. The approximate water surface differential which can be expected to occur across the embankments.

Computation (5a) From the results of example 1 (sec. 8.5):

Normal stage at bridge = 115.0 ft.;

$Q = 7{,}000$ c.f.s.; $M = 0.46$; $b = 75$ ft.;

$A_{n2} = 920$ sq. ft.; $V_{n2} = 7.60$ f.p.s.;

$A_p = 30$ sq. ft.; $J = 0.033$;

$h_1^* = 1.06$ ft.;

$h_3^* = 0.40$ ft.;

$K^* = 1.13$; and $D_b = 2.35$.

For a stage 2 feet higher than the normal of example 1, the pertinent quantities are (see fig. 19):

Stage at bridge = 117.0 ft.;

$Q = 7{,}000$ c.f.s.; $M = 0.46$; $b = 78$ ft.;

$A_{2A} = 1{,}113$ sq. ft.; $V_{2A} = 6.27$ f.p.s.;

$A_p = 36$ sq. ft.; and $J = 0.033$.

The backwater in this case will be computed according to expression (15) (sec. 6.3), using the same value of K^* as in example 1:

$$h_{1A}^* = K^* \frac{V_{2A}^2}{2g}$$

The approximate backwater for the abnormal stage will be:

$$h_{1A}^* = 1.13 \frac{(6.27)^2}{2g} = 0.69 \text{ ft., or}$$

65 percent of the value computed for normal stage in example 1.

Computation (5b) To obtain the differential level ratio it will first be necessary to recompute the backwater (excluding the effect of piers):

$$h_{bA}^* = K_b \frac{V_{2A}^2}{2g} = 1.06 \times 0.611 = 0.65 \text{ ft.}$$

$D_b = 2.35$ from example 1. Then:

$$h_{3A}^* = \frac{h_{bA}^*}{D_b} = \frac{0.65}{2.35} = 0.28 \text{ ft.}$$

If it is assumed that the drop in channel gradient is the same as in example 1:

$S_0 L_{1-3} = 0.000417 \times 150 = 0.06$ ft.; then the approximate difference in water level across the embankment:

$$\Delta h_A = h_{1A}^* + h_{3A}^* + S_0 L_{1-3} = 0.69 + 0.28 + 0.06 = 1.0 \text{ ft.}$$

These computations are approximate at best.

Example 6

8.10 Example 6: Backwater with scour The following is an unusual but actual case involving scour under a bridge during flood for which reliable field data were obtained by the U.S. Geological Survey. This bridge site was chosen for the example as it effectively illustrates the marked effect scour can produce on backwater.

Given.—The cross section of the stream measured 170 feet upstream from the bridge, as shown in figure 31A; the cross section under the bridge showing normal water surface, initial bed surface, normal water area, and extent and area of scour during peak flow (fig. 31B); and the profile of the stream at the bridge (fig. 31C). The streambed consists of sand underlain with gravel and shale. At the peak of flood essentially all loose material was flushed out of the constriction. The pile bents and abutments are embedded in concrete foundations which are keyed into the hardpan as shown in figure 30B. The average slope of the stream in this reach is 11 feet to the mile, $S = 0.00208$, and the discharge, measured by current meter during the peak of the storm, was 9,640 c.f.s. No flow occurred over the road.

The problem is to compute the drop across the embankment and the water surface elevations expected upstream and downstream (with scour), as outlined in chapter VII, for the peak discharge of 9,640 c.f.s.

The procedure will involve the following steps:

1. Determine normal stage for the natural stream for a discharge of 9,640 c.f.s. by slope-area method.
2. Determine the backwater h_1^* which would exist without scour.
3. Determine the value of h_3^* that would exist without scour.
4. Compute the value of the backwater h_{1s}^* (with scour).
5. Compute the value of h_{3s}^* (with scour).
6. Compute water surface elevation on upstream and downstream side of embankment and Δh_s, the drop in water surface across the embankments (with scour).
7. Compare computed values with measured values.

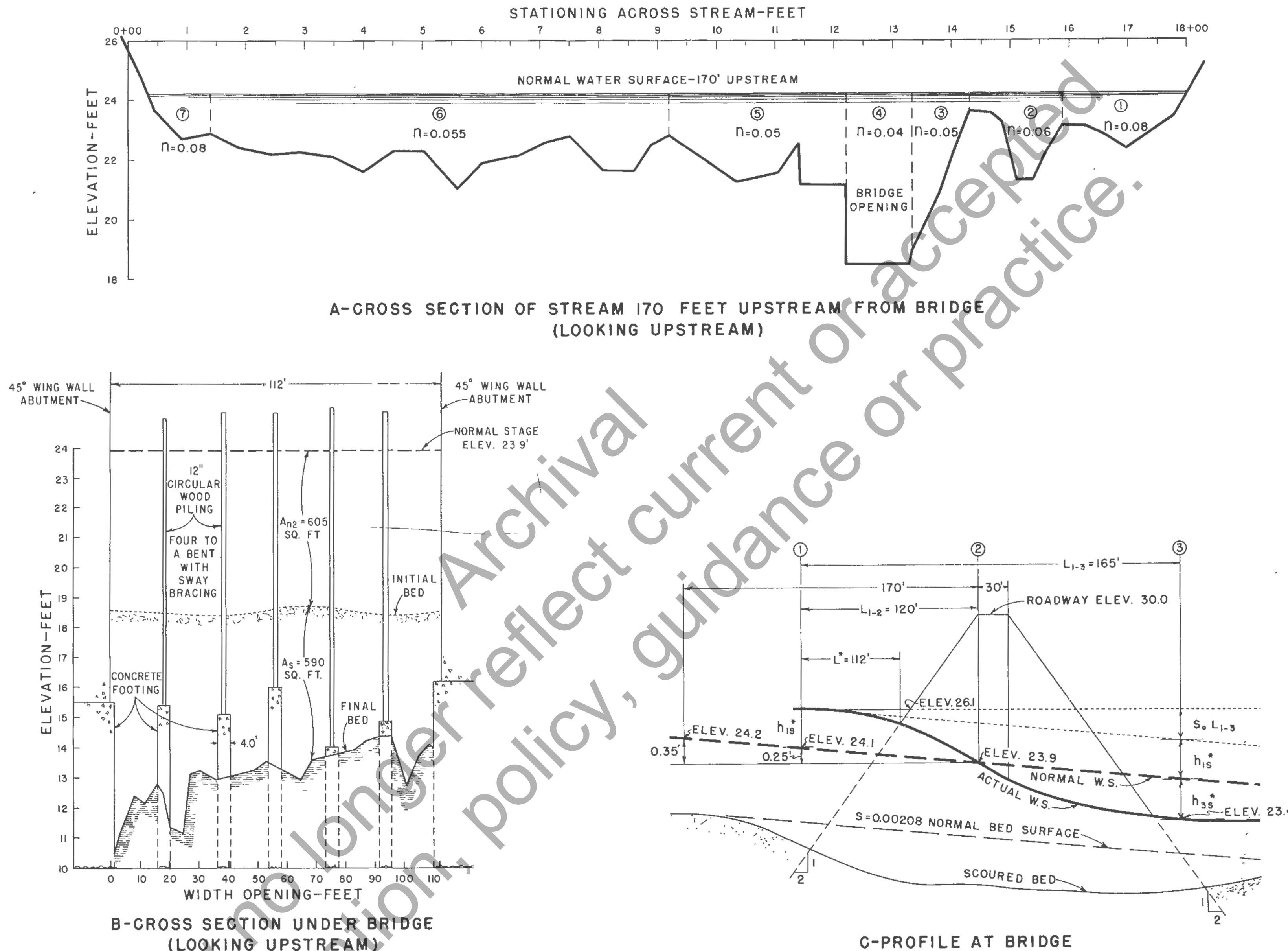

Figure 31.—Example 6: Backwater with scour.

Table 6.—Example 6, sample computations: Properties of natural stream

Q=9,640 c.f.s.; Measured S_0=0.00208; Normal stage elevation=24.2 ft.

Subsection	Computation (6a)							Computation (6b)		
	n	$\frac{1.486}{n}$	a	p	$r=\frac{a}{p}$	$r^{2/3}$	k	q	v	qv^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
			sq. ft.	*ft.*	*ft.*			*c.f.s.*	*f.p.s.*	
q_a 1	0.08	18.6	268	222	1.21	1.14	5,690	259	0.97	244
q_a 2	.06	24.8	267	159	1.68	1.41	9,340	425	1.59	1,072
q_a 3	.05	29.7	354	108	3.28	2.21	23,200	1,056	2.98	8,890
q_b 4	.04	37.1	555	121	4.59	2.76	56,900	2,590	4.67	56,200
q_c 5	.05	29.7	750	290	2.59	1.89	42,000	1,912	2.55	12,420
q_c 6	.055	27.0	1,636	780	2.10	1.64	72,400	3,295	2.01	13,300
q_c 7	.08	18.6	118	110	1.07	1.05	2,300	103	.87	78
Total			A_{n1}=3,948				K_1=211,830			Σqv^2=92,204

Computation (6a) Normal stage is determined by trial. The river cross section, taken 170 feet upstream from the bridge, is representative of the stream for several miles upstream and downstream. This is divided into subsections as shown in figure 31A and an appropriate value of n is assigned to each subsection. Normal stage is assumed, the overall conveyance of the water cross section determined, and the resulting longitudinal slope computed. Should the computed slope not agree with the measured slope of 11 feet per mile, it is then necessary to assume another stage and repeat the process until the computed and measured slopes agree. This has already been done and the final conveyance computations, the result of several trials, are tabulated in columns 1–8 of table 6.

Checking for the slope:

$$S_0=\left(\frac{Q}{K_1}\right)^2=\left(\frac{9,640}{211,830}\right)^2=0.00208,$$

which agrees with the measured value. Thus it will be considered that normal stage is elevation 24.2 feet at a point 170 feet upstream from the bridge (see fig. 31C).

Computation (6b) Columns 9, 10, and 11 are now completed and the velocity head correction coefficient and the value of M may be determined:

$$\alpha_1=\frac{92,204}{9,640\left(\frac{9,640}{3,948}\right)^2}=1.61;\text{ and}$$

$$M=\frac{q_b}{Q}=\frac{2,590}{9,640}=0.27.$$

Figure 31B shows the initial stream bed under the bridge at approximate elevation 18.5 feet, and figure 31C indicates that normal stage at the bridge is elevation 23.9 feet.

Assuming a pier width of 1.67 feet, to allow for sway bracing and trash:

$$A_p=45\text{ sq. ft.};\ A_{n2}=605\text{ sq. ft.; and}$$

$$J=\frac{A_p}{A_{n2}}=\frac{45}{605}=0.074.$$

Entering figure 5 for 45° wingwall abutments and $M=0.27$, $K_b=1.4$.

Referring to figure 7A with $J=0.074$ for circular pile bents, $\Delta K=0.30$.

From figure 7B, $\sigma=0.45$ and $\Delta K_p=0.30\times0.45=0.14$.

The overall backwater coefficient is then:

$K^*=K_b+\Delta K_p=1.4+0.14=1.54;$

$V_{n2}=\frac{Q}{A_{n2}}=\frac{9,640}{605}=15.9$ f.p.s.;

$\frac{V_{n2}^2}{2g}=3.94$ ft.; and an approximate value for the backwater, from expression (4a) (sec. 2.1):

$$h_1^*=K^*\frac{V_{n2}^2}{2g}=1.54\times3.94=6.1\text{ ft.}$$

$$A_1=A_{n1}+6.1\times2,100=3,948+12,810=16,758\text{ sq. ft.}$$

Completing expression (4) (sec. 2.1):

$$h_1^*=6.1+\alpha_1\left[\left(\frac{A_{n2}}{A_4}\right)^2-\left(\frac{A_{n2}}{A_1}\right)^2\right]\frac{V_{n2}^2}{2g}$$

or

$$6.1+1.61\left[\left(\frac{605}{3,948}\right)^2-\left(\frac{605}{16,758}\right)^2\right]3.94=6.1+0.12=6.2\text{ ft.},$$

which is the backwater to be expected without scour.

Computation (6c) Referring to figure 14 for 45° wingwall abutments and $M=0.27$:

$$D_b=\frac{h_b^*}{h_3^*}=4.60.$$

Ignoring pier effect (sec. 4.3):

$$h_b^*=K_b\frac{V_{n2}^2}{2g}=1.4\times3.94=5.52\text{ ft. (backwater without piers)}$$

and

$$h_3^*=\frac{5.52}{4.60}=1.2\text{ ft. (without scour).}$$

Computation (6d) From figure 31B the gross area of scour under the bridge (including piers) $A_s=590$ sq. ft. Since the piers are not of uniform width throughout, it is advisable to use net areas in computing the ratio A_s/A_{n2}.

Thus:

$$\frac{A_s}{A_{n2}}\text{ (net)}=\frac{590-60}{605-45}=\frac{530}{560}=0.95.$$

Entering figure 23 with this value:

$$\frac{h_{1s}^*}{h_1^*}=0.32.$$

The backwater with scour is then:

$$h_{1s}^*=0.32\times 6.2=1.98\text{ ft.}$$

Computation (6e) From the same figure and in like manner:

$$\frac{h_{3s}^*}{h_3^*}=0.32,\text{ and}$$

$$h_{3s}^*=0.32+1.2=0.38\text{ ft. (with scour).}$$

Computation (6f) Assuming that maximum backwater develops one bridge length upstream, which is perhaps the extent of the accuracy to be expected in this case, the elevation, with backwater, at section 1 and along the upstream face of the embankment will be (see fig. 31C): El. 24.1+1.98=26.1 ft.

The drop in water surface across the embankment: $\Delta h_s=S_0L_{1-3}+h_{1s}^*+h_{3s}^*=0.35+1.98+0.38=2.7$ ft., and the water surface along the downstream side of the embankment will be El. 26.1−2.7=23.4 ft.

Comparison The following tabulation shows a comparison of the computed values with those determined by measurement in the field:

	Measured	*Computed*
Δh_s ft.	2.6	2.7
Elevation upstream	25.8	26.1
Elevation downstream	23.2	23.4

The agreement between measured and computed values is so good as to raise suspicion that the figures have been adjusted, but that is not the case. The field measurements were used exactly as reported. While one example is not enough to prove the case, this example does support the reasonableness of the conclusions drawn from the model experiments in the laboratory.

Obviously the bridge as built was too short since the backwater which would have occurred, had not scour taken place, would have been excessive.

The extent to which the designer might count on scour reducing backwater is subject to field verification.

Chapter IX.—LIMITATIONS OF DATA

9.1 Limitations of design charts The design charts and methods which have been presented are applicable to a wide variety of bridge backwater problems. Some of the procedures may appear more involved and lengthy than necessary; this was done to make clear each step of the computations. As familiarity with this material increases, the designer may find certain variables to be relatively insignificant, thus permitting innovations or short cuts in the procedure. In fact, it might be feasible to prepare special charts for specific standard bridge designs. Certain limitations should be pointed out to avoid misuse of the material presented (some of these limitations have already been mentioned while others have not):

1. The method of computing backwater as presented is intended to be used for relatively straight reaches of streams having approximately uniform cross section and slope. Field measurements indicate that there can be considerable variation from uniformity in cross section, however, without causing serious error in computing backwater.

2. The U.S. Geological Survey field measurements used to verify the application of the laboratory data to field conditions were limited to single bridges up to 220 feet in length, on streams up to about half-mile in width at flood stage. Just how well the method of computing backwater applies to flood plains of much greater width is unknown at this time. Small-scale models are not suitable for the study of streams with large width-to-depth ratios. This phase must of necessity be studied in the field.

3. As the length of a bridge is increased, it stands to reason that the type or shape of abutment should have less effect on the backwater; so far, data are lacking to evaluate this effect.

4. The design information applies specifically to the normal stage-discharge condition, although one exception was made in demonstrating an approximate solution for a particular type of abnormal stage in example 5. In cases where the slope of the water surface is either much flatter or much steeper than the slope of the bed (abnormal or subnormal stage discharge), it is suggested that the method developed by the U.S. Geological Survey (*5, 14*) for indirect flow measurement be tried. The reason for this suggestion is the fact that the U.S. Geological Survey performed their model tests under conditions more nearly approaching nonuniform flow, while in the Bureau of Public Roads tests uniform flow was always established before the channel was constricted (with the exception of the tests described in chapter VI).

The Geological Survey method was developed for the express purpose of utilizing bridge constriction as flow measuring devices. By knowing the stream and bridge cross sections and measuring the drop across the embankment Δh, the discharge occurring at the time can be computed directly but the computation of backwater requires a trial solution. The Bureau of Public Roads method described in this publication permits a direct solution for backwater but requires a trial solution for discharge. It is evident by now that some backwater solutions are sufficiently complex without involving a trial solution. The differences in the two methods are outlined in a discussion by C. F. Izzard (*14*, p. 1008).

5. Plausible questions will arise in connection with the manner in which the foregoing design information was presented. For example, why was the gross rather than the net area used for determining the contraction ratio and the normal velocity under the bridge for cases where piers were involved? Why were skew crossings treated as they were? Are the incremental backwater coefficients applicable to very short bridges with wide piers? Any one of several methods could have been presented with the same accuracy; the choice made in each case was simply the one appearing the most logical and straightforward to the research staff of the Bureau of Public Roads. What must be borne in mind is that the empirical curves for various coefficients were derived by treating the model data in certain ways. It follows that exactly the same process must be used in reverse if one expects to come back to the original data. The methods for computation of backwater at proposed bridges therefore must follow the instructions faithfully and intelligently if correct answers are to be obtained.

6. For the case where a high flow concentration parallels an embankment, such as depicted in figure 12, the water surface along the upstream side will have a falling characteristic and the drop across the embankment will vary depending on where the measurement is taken. The backwater as computed is likely to be less than that actually existing, since a portion of the waterway under the bridge may be ineffective. Repeating what has been said previously, this is a condition of flow to be avoided whenever possible. It is important to avoid digging borrow pits or to allow channeling of any kind adjacent to the upstream side of bridge embankments. Clearing of the right-of-way beyond the toe of the embankment should not be permitted as trees and brush act most effectively to deter channeling. Where channeling is already present, the situation can be corrected by the use of spur dikes.

7. Questions will arise as to the permissible amount of backwater which can be tolerated under various situations. This is principally an economic consideration. For ex-

ample, if backwater produced by a bridge threatens flooding of improved property, the estimated damage from this source over the expected life of the bridge should be weighed against the initial cost of a longer or shorter bridge. Figure 30A illustrates the costliness of reducing backwater beyond a certain economic limit.

Should the bridge be located in open country where backwater damage is of little or no concern, a shorter bridge may serve the purpose but there is still a practical limit to the permissible backwater. Model tests indicate that the mean velocity at section 3 is essentially proportional to:

$$\left(h_1^* + \alpha_1 \frac{V_1^2}{2g}\right)^{1/2}$$

Assuming $V_1 = 3$ f.p.s. and $\alpha_1 = 1.0$, a backwater of one foot would produce an approximate velocity:

$$V_3 = [2g\ (1.0 + 0.14)]^{1/2} = 8.5 \text{ f.p.s.}$$

Holding upstream conditions the same, 2 feet of backwater would produce a velocity of approximately 12 f.p.s. and 3 feet of backwater about 14 f.p.s. For bridge sites where scour is not to be encouraged, 1 foot of backwater would certainly be an upper limit. On the other hand, for sites with stable river channels the backwater can be increased accordingly. Also, in cases where the bed is of a movable nature but foundation conditions are favorable, there is considerable latitude in the initial backwater that can be allowed, as was demonstrated in example 6. In the latter two cases the stability of the material composing or protecting the abutments will most likely govern the velocity and thus the backwater that can be tolerated, since the abutments will be most vulnerable to erosion.

8. Streams with extremely sinuous channels on wide flood plains introduce a special case for which the present design procedure may prove inadequate, partly because of undertainty regarding flow distribution at any cross section.

9. For cases where islands or other major obstructions occur in the main channel at or upstream from a bridge, the procedure will require some modification. If these obstructions extend under the bridge it may be possible to treat them in the same manner as piers.

10. For the computation of backwater where the flow of a stream is divided between two or more multiple bridges, the methods described in this publication are valid for each bridge provided the flow is divided properly between bridges. This is a subject on which the U.S. Geological Survey has completed an extensive research program.[4]

9.2 Hydraulic design as related to bridge design The design information presented herein on bridge backwater is of limited value in itself. It constitutes only one of the tools to be used in the design of a bridge. Recent improvement in methods of dealing with magnitude and frequency of flood peaks, experimental information on scour, and on the computation of backwater provide the steppingstones to a more scientific approach to the bridge waterway problem. The result should be greater safety, with fewer bridge failures because of underdesign, and increased economy due to a reduced tendency toward overdesign. Bridge design has suffered because of the lack of reliable hydraulic and hydrologic information on the waterway. In days past, this may not have been of great importance, but today traffic volumes have become so great on Interstate and primary roads that bridge failures, or even bridges out of service for any length of time, cause severe economic loss to the public. On the other hand, overdesign of the waterway, making modern bridges longer than necessary, can materially add to the initial cost, especially when dual or four- or six-lane bridges are involved.

A recent trend has been toward constructing bridges longer and embankments higher than in the past. From the hydraulic and long-range economic points of view, this practice may or may not be sound. Only a reliable engineering economic analysis, in which all factors of importance are considered, can lead to the correct answer for any one site. Young *(15)* discusses some of the economic factors which come into play during floods; much remains to be done in compiling data on flood damage costs, magnitude and frequency of floods, scour data, and flood risk factors, and in perfecting a sound and acceptable method of economic analysis. Since backwater is reflected in one way or another in practically every phase of the bridge waterway problem, it is hoped that the information contained in this publication will signal a significant forward step toward attaining the ultimate goal in bridge design, as stated in the opening paragraph on page 1.

[4] Report pending; presumably will appear in the form of a U.S. Geological Survey Circular.

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