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DESIGN OF DEPRESSED INVERT CULVERTS

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16. Abstract

The hydraulic characteristics of a depressed invert culvert were studied. Also, a design procedure for depressed invert culverts is outlined. The hydraulic characteristics were studied by reviewing pertinent literature and by the use of a hydraulic model. The design procedure is similar to that already used by state hydrologists.

Formulas for determining the geometric properties of a depressed invert culvert are presented. The hydraulic model was used to determine the discharge coefficients for a depressed invert culvert flowing under inlet control and set flush to a vertical headwall. A literature review was performed which examined velocity profiles, flow over permeable beds, and flow resistance in culverts and over rough beds.

The design procedure is applicable to depressed invert culverts flowing under nonsubmerged conditions and set flush to a vertical headwall. The design procedure can be used as an outline for the development of a comprehensive design manual for depressed invert culverts.

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IMPLEMENTATION

This report discusses the hydraulic performance of depressed invert culverts with inlet control and set flush to a vertical headwall. The models presented in this report may be used to calculate hydraulic capacity, barrel losses and velocity profiles within the culvert. However, the designer must keep in mind that the models are based on specific conditions that may not be applicable to all situations. Additional research should be conducted with different culvert inlet configurations, outlet control conditions and culvert bed materials typical of what is used in Alaska in order to refine the presented modeling.

Research is currently being conducted on the swimming abilities of fish passing through culverts. This research, in conjunction with the ability to predict the hydraulic capacity and velocity profiles, will allow the designer to determine the suitability of a depressed invert culvert for fish passage.

Michael Travis
Project Manager
Research Section

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ACKNOWLEDGMENTS

The authors are particularly indebted to Dean Griggs of the Department of Transportation and Public Facilites (DOT&PF) for suggesting the original concept of this project and for continuing suggestions during its conduct. The review comments received from Mark Miles and Mike Travis are appreciated.

1.0 INTRODUCTION

Background

Culverts are usually installed with the invert at the same elevation as the stream channel bottom. In an attempt to improve fish passage and to accommodate other design constraints, culverts are sometimes installed with buried inverts. The desired effect is to produce lower flow velocities, thus presenting less resistance to fish moving upstream. These lower velocities are achieved by the wider bottom and increased roughness which result from the placement of riprap into the bottom of the depressed invert culvert (DIC).

The design procedures for conventional culverts are well established but do not include appropriate methods for the hydraulic design of DICs. Thus, ad hoc methods must be used in the design of DICs.

Objectives

The project objective is to develop an improved hydraulic design procedure for DICs. The design procedure is based on the basic principles of fluid mechanics, established methods for determining culvert flow and research related to flow over rough boundaries.

The initial analysis identified the specific topics to be studied which led to the development of a design procedure. The individual topics were studied as allowed by the time and budget constraints of the project.

Also, the design procedure is presented in a manner usable to hydrologists and design engineers.

Methods

The approach taken in this study follows closely to the procedures Bodhaine (1968) used in the USGS manual for predicting peak flows through culverts. The initial analysis identified culvert geometry, inlet losses and barrel losses as areas requiring further investigation.

Exact relationships are used to describe the geometry and the occurrence of critical depth in DICs.

A laboratory model was used to study the inlet losses for a DIC set flush to a vertical headwall. The model study examined the effects of invert depth and bed roughness.

A literature review of flow resistance over rough and permeable beds was conducted. The review can be used for developing a procedure for determining the barrel losses in a DIC.

Results

The project has developed a design procedure for DICs. The method is similar to that for conventional culverts as presented by Bodhaine (1968). The design procedure can be used as a guideline for future research and for the development of a comprehensive design manual for DICs.

The design procedure enables the hydraulic characteristics of a DIC to be estimated with greater confidence than previous design methods allowed. The design procedure considers such features as culvert geometry, discharge coefficients, barrel losses and velocity profile.

Estimates of the depth of flow and velocity profile enables the designer to better evaluate the suitability of a DIC for fish passage.

2.0 FLOW THROUGH CULVERTS

General Culvert Flow

The material presented in this section is primarily adapted from Bodhaine (1968).

The flow through a culvert can be separated into five regions: approach channel, inlet, barrel, outlet and tailwater. A general description of the culvert flow regions is given in Figure 2.1. To avoid the possibility of the approach section being within the drawdown region, the division between the approach section and culvert inlet is located one culvert width upstream of the culvert entrance.

Culvert flow can be generally classified as inlet control or outlet control. Inlet control occurs when the water surface elevation passes through critical depth at the culvert inlet and is therefore controlled by conditions upstream of that point. Outlet control is characterized by either critical depth occurring at the culvert outlet or when the flow is influenced by tailwater conditions. Outlet control is a function of the approach channel, inlet, barrel, outlet and tailwater conditions.

For convenience in computation, culvert flow has been further classified into six types on the basis of the location of the control section and the relative heights of the headwater and tailwater elevations. The culvert's discharge for each type of flow can be determined by application of the continuity equation and the energy equations. Descriptions of each flow type, the conditions under which it occurs, and the appropriate discharge equations are given in Figures 2.2 and 2.3. Type 1 and 5 flows are inlet control while type 2, 3, 4 and 6 flows are outlet control. A detailed description of each of these types of flow is given by Bodhaine (1968).

When the cross-sectional area in the approach section is sufficiently large so that the velocity, V_1, is negligibly small, then it is appropriate to let V_1 equal zero.

In those situations when the flow through the culvert is not described by one of these six types of flow, routing methods should be used.

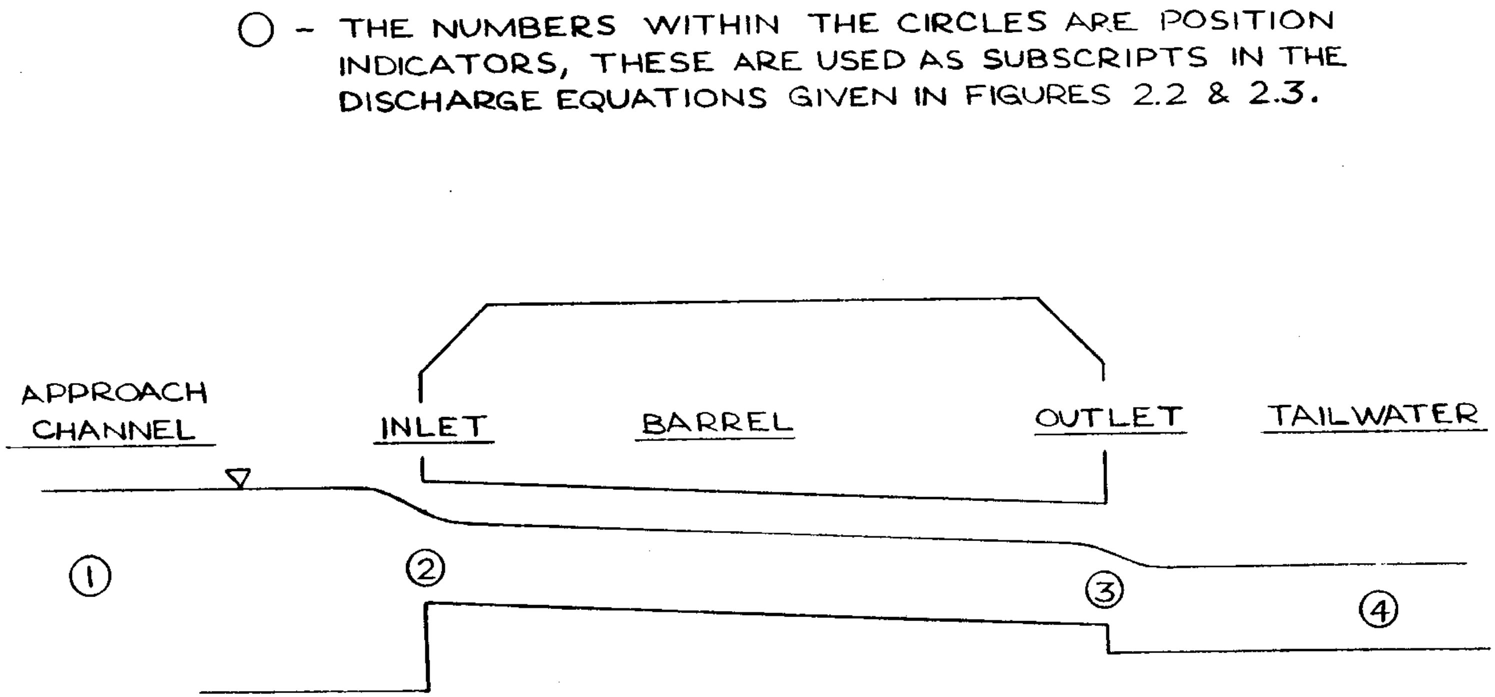


Figure 2.1. Flow sections of a culvert (after Bodhaine, 1968).

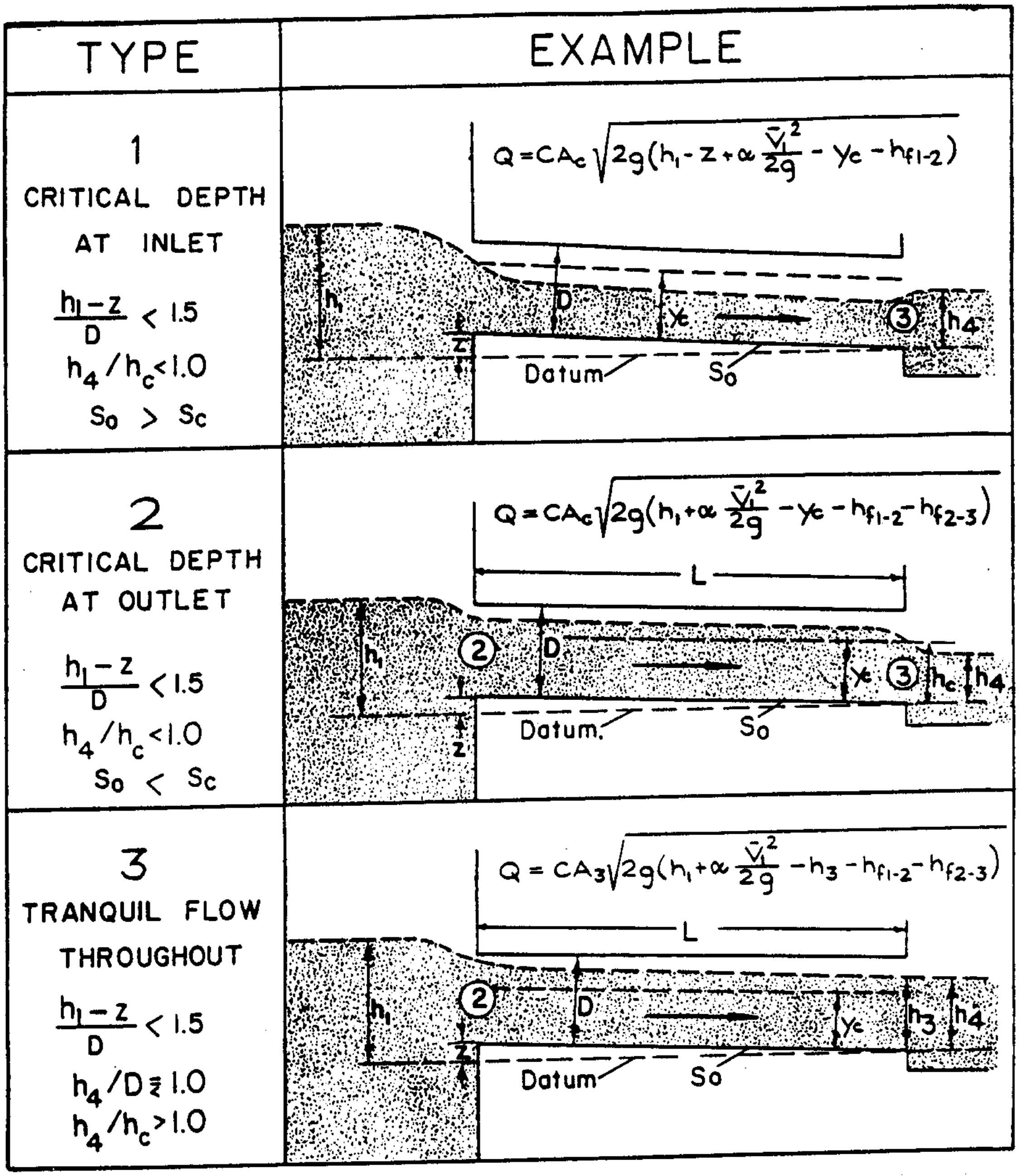


Figure 2.2. Classification of culvert flow, types 1–3 (after Carter, 1975).

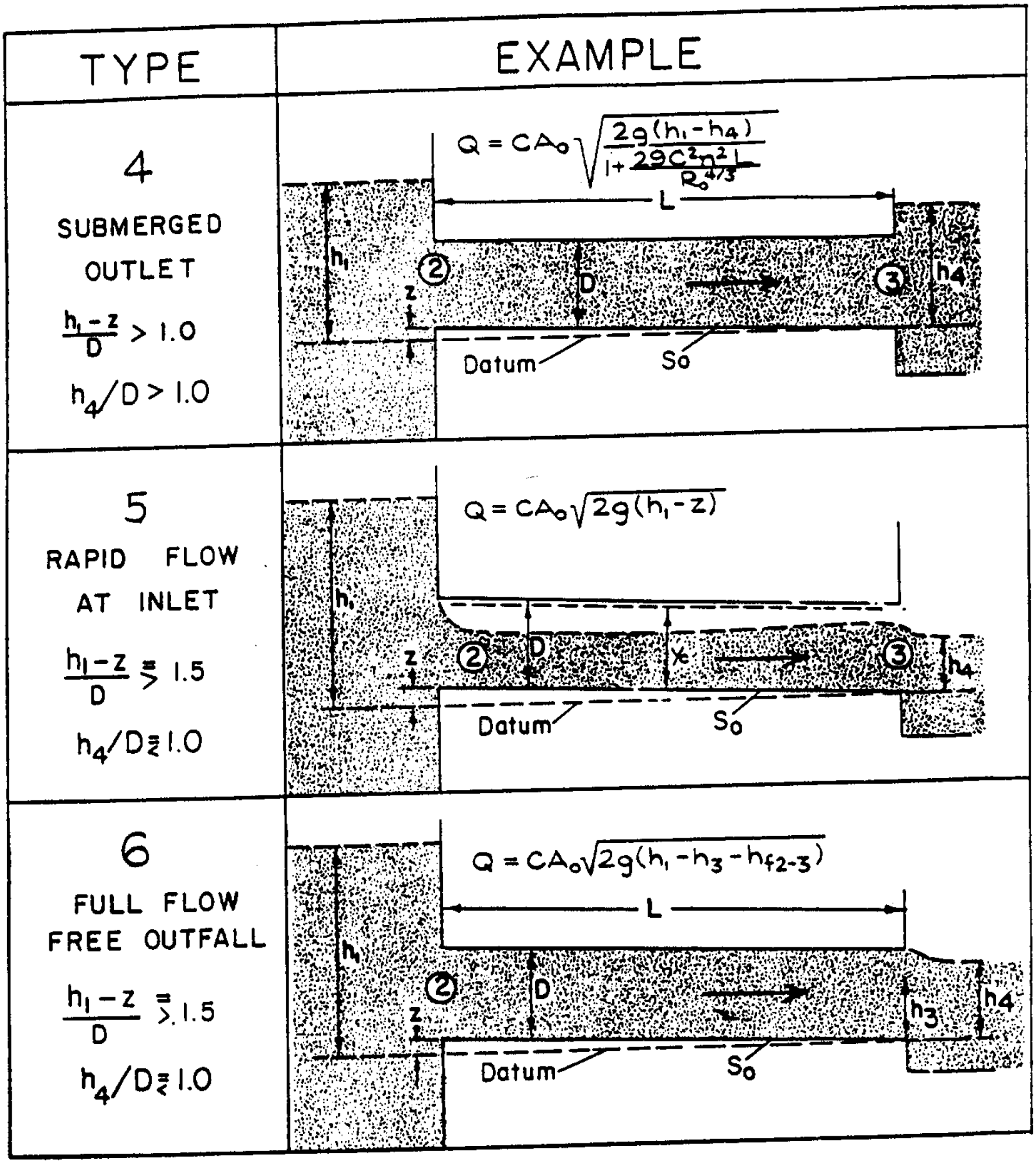


Figure 2.3. Classification of culvert flow, types 4-6 (after Carter, 1975).

The terms used in the equations associated with Figures 2.2 and 2.3 are either illustrated in the figures or are defined as follows:

Q = discharge

C = discharge coefficient

A_c = cross-sectional area at critical depth

A_o = cross-sectional area of culvert entrance

g = acceleration of gravity

α_i = kinetic energy correction factor

$\bar{V}_i$ = average velocity at section i

y_c = critical depth

h_{fi-j} = head loss due to friction between sections i and j

n = Manning's roughness coefficient

R_o = hydraulic radius

Depressed Invert Culverts

A depressed invert culvert (DIC) is defined as a circular culvert in which the invert is buried below the stream bed, thus allowing the barrel to be partially filled during the time of installation. The fill materials are either natural stream bed material or riprap. A sketch of a DIC is shown in Figure 2.4.

The discharge equations for the six types of flow are applicable to all culverts regardless of the cross-sectional shape. The evaluation of culverts with circular or square cross-sections is well established. Bodhaine (1968), Alaska Department of Highways (N.D.) and U.S. Department of Transportation, Federal Highway Administration (1985) are several of the many documents available relating to the hydraulic design of culverts.

Application of the six discharge equations to a DIC requires an analysis which accounts for the unique properties of the DIC.

GEOMETRY: the cross-sectional shape of a DIC is neither rectangular or circular, thus mathematical expressions for the various geometric properties of interest need to be developed. Also, the appropriate equations for describing the occurrence of critical flow should be determined.

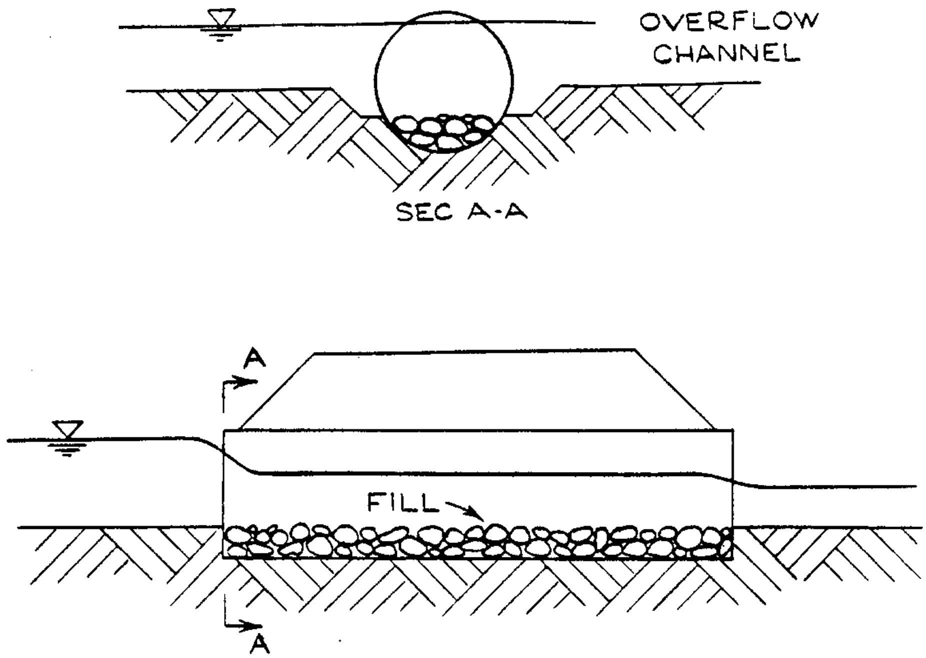


Figure 2.4 A depressed invert culvert.

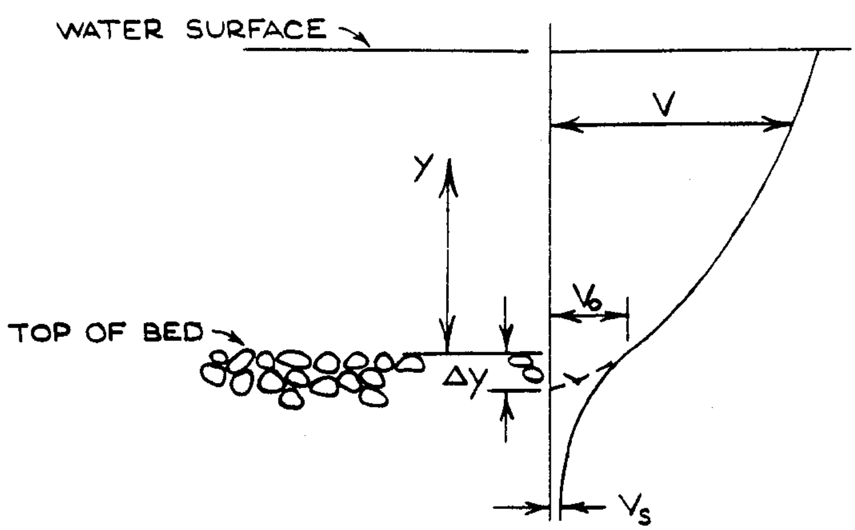


Figure 2.5. Velocity profile over a permeable bed.

INLET LOSSES: the discharge coefficient (C) is used to account for the inlet losses in a culvert. The discharge coefficient is a function of the channel contraction and the geometry of the culvert entrance (Bodhaine, 1968). Laboratory studies are used to determine the discharge coefficient for DICs.

BARREL LOSSES: to determine the barrel losses of a DIC, a composite Manning roughness coefficient (n) or composite Darcy-Weisbach friction factor (f) must be determined.

Channel Flow Over a Permeable Bed

When the fill for a DIC is riprap or other highly permeable material, the flow within the culvert is characterized as channel flow over a permeable bed.

Paudyal (1982) with reference to Figure 2.5 gives the following description of the velocity profile for free surface flow over a permeable bed.

1. Within the main channel normally occurring flow characteristics prevail. However, due to seepage that occurs in the permeable bed the velocity profile does not necessarily tend to zero at the bed surface, but there is a finite slip velocity, V_o.

2. Within the bed at some distance below the interface, normal pressure seepage occurs at a velocity V_1, which can be computed from the hydraulic gradient.

3. Between the interface and the pressure seepage zone the fluid is not only acted upon by the seepage gradient, τ, but also by shear stress, derived from the mechanical fluid shear stress at bed level, τ_o. The fluid shear stress in the permeable bed decreases very rapidly with the depth below the surface, as a result of transfer to the bed particles.

The studies showed that the boundary resistance of free surface flow over a permeable bed is greater than that of an impermeable boundary having a similar roughness.

Whether or not a coarse bed material in a DIC behaves as a permeable boundary will depend upon if the bed remains open after the initial installation. Simons et al. (1979) using field and laboratory studies showed that a bed load of sand particles was able to cover a normally gravel channel bed, thus the sand became the primary mechanism in producing flow resistance.

3.0 GEOMETRY OF A DEPRESSED INVERT CULVERT

Geometric Relationships

The geometric properties of the cross-section of a DIC are readily described by exact mathematical relationships. Once determined, the geometric properties can be substituted into the appropriate discharge equation or used in routing techniques.

The symbols representing the culverts cross-sectional dimensions are shown in Figure 3.1. They are

D = culvert diameter,

d = depth of bed,

y_o = depth of water above bed,

ϕ = angle describing width of bed (radians), and

θ = angle describing the water's free surface (radians).

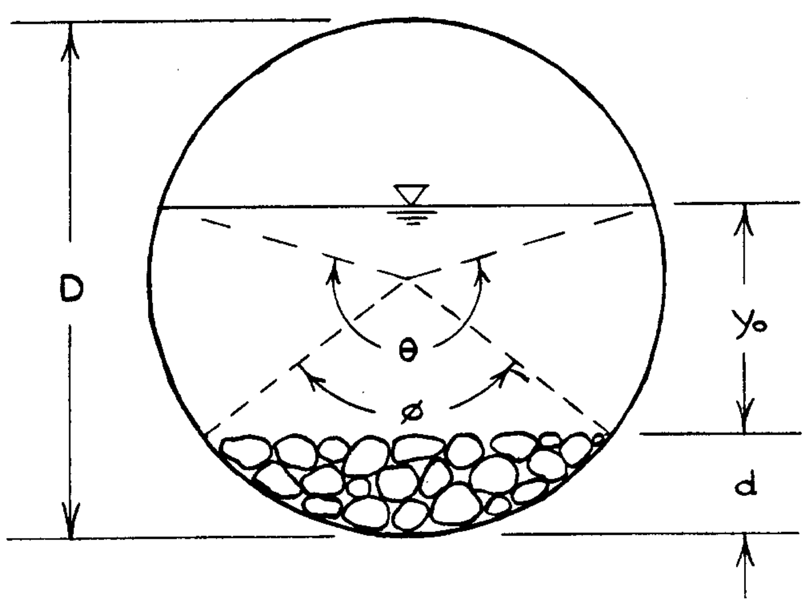


Figure 3.1. Definition of symbols for a DIC.

The angles ϕ and Θ are related to the other culvert dimensions by

$$\phi = 2\cos^{-1}(1 - \frac{2d}{D}) \tag{3.1}$$

and

$$\Theta = 2\cos^{-1}(1 - \frac{2(y_0+d)}{D}) \tag{3.2}$$

The total area above the bed is given by

$$A_o = D^2(0.7854 - \frac{\phi-\sin\phi}{8}) \tag{3.3}$$

The area of the water above the bed (wetted area) is

$$A_w = \frac{D^2}{8}((\Theta-\sin\Theta) - (\phi-\sin\phi)) \tag{3.4}$$

The portion of the wetted perimeter contributed by the culvert wall is

$$P_c = \frac{D}{2}(\Theta-\phi) \tag{3.5}$$

The portion of the wetted perimeter contributed by the bed is

$$P_b = D\sin(\frac{\phi}{2}) \tag{3.6}$$

The hydraulic radius for a DIC is

$$R = \frac{A_w}{P_c + P_b}$$

or

$$R = \frac{D((\Theta-\sin\Theta) - (\phi-\sin\phi))}{4((\Theta-\phi + 2\sin(\frac{\phi}{2}))} \tag{3.7}$$

Selected values of R/D as a function of y_o/D and d/D are given in Table 3.1.

The depression ratio for a DIC is defined by

$$\lambda = 100\frac{d}{D} \tag{3.8}$$

The term λ is used both as a percent or a fraction within the report.

The equivalent diameter is a useful concept when comparing DICs of different diameters and depression ratios. The equivalent diameter is the diameter of a circle whose area is equal to the total area (A_o) above the bed of a DIC. The equivalent diameter can be determined from

$$D_e = \left(\frac{4A_o}{\pi}\right)^{1/2} \tag{3.9}$$

Combining equations 3.3 and 3.9 yields

$$D_e = D\left(1 - \frac{\phi - \sin\phi}{2\pi}\right)^{1/2} \tag{3.10}$$

Critical Depth

At this point it is appropriate to develop a method for determining critical depth, y_c, in a DIC. Critical depth is the point where the flow has its minimum specific energy. The specific energy of a fluid is a function of the flow geometry and discharge.

In determining y_c the concepts of water surface width, T, and hydraulic depth, y_n, are used. The water surface width is defined by Equation 3.11. The hydraulic depth is determined by dividing the wetted area, A_w, by the water suface width, Equation 3.12.

$$T = D\sin\left(\frac{\Theta}{2}\right) \tag{3.11}$$

$$y_h = \frac{A_w}{T} \tag{3.12}$$

TABLE 3.1. Values of R/D for DICs as a function of y_o/D and d/D.

	d/D					
y_o/D	0.0	0.1	0.2	0.3	0.4	0.5
0.1	0.0635	0.0803	0.0837	0.0845	0.0841	0.0823
0.2	0.1216	0.1410	0.1462	0.1465	0.1437	0.1378
0.3	0.1709	0.1904	0.1946	0.1921	0.1845	0.1709
0.4	0.2142	0.2304	0.2311	0.2237	0.2084	0.1825
0.5	0.2500	0.2610	0.2563	0.2411	0.2140	----
0.6	0.2776	0.2818	0.2692	0.2423	----	----
0.7	0.2962	0.2915	0.2669	----	----	----
0.8	0.3042	0.2867	----	----	----	----
0.9	0.2980	----	----	----	----	----

R/D = hydraulic radius/culvert diameter

y_o/D = depth of flow/culvert diameter

d/D = depth of bed/culvert diameter

At critical depth

$$y_c = y_o$$

and

$$A_c = A_w$$

where A_c is the cross-sectional area of the water above the bed at critical depth.

At critical depth

$$Q^2 = \frac{gA_c^3}{T} \tag{3.13}$$

where Q is the volumetric discharge and g is the local gravitational acceleration, Bodhaine (1968) and French (1985).

By combining equations 3.4, 3.11 and 3.13 yields

$$Q = \frac{g^{0.5}D^{2.5}((\Theta-\sin\Theta) - (\phi-\sin\phi))^{1.5}}{22.63(\sin(\frac{\Theta}{2}))^{0.5}} \tag{3.14}$$

Rewriting equation 3.14 in terms of D and Q gives

$$\frac{22.63Q}{g^{0.5}D^{2.5}} = \frac{((\Theta-\sin\Theta) - (\phi-\sin\phi))^{1.5}}{(\sin(\frac{\Theta}{2}))^{0.5}} \tag{3.15}$$

A graphical solution to the right hand side of equation 3.15 in terms of D, λ and y_c is given in Figure 3.2.

When D, d and y_c are known, Q can be determined directly from equations 3.1, 3.2 and 3.15. When D, d and Q are known, and y_c is desired then either Figure 3.2 or a numerical solution to equation 3.15 should be used.

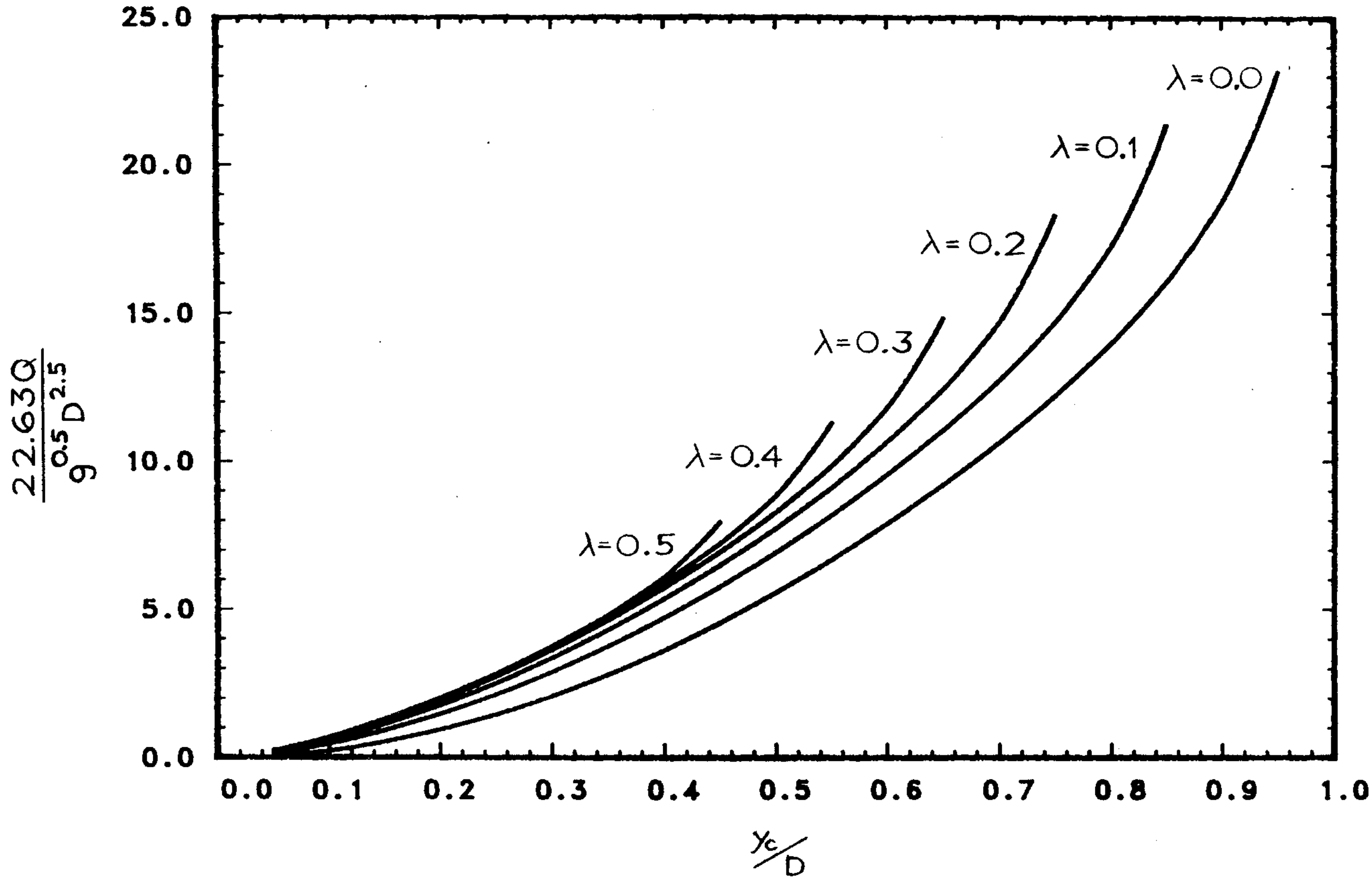


Figure 3.2. Critical depth, y_c, as a function of Q, D and d.

Dimensional Analysis

To calculate the Froude number, the appropriate characteristic depth is the hydraulic depth as defined in equation 3.12. The Froude number is defined by

$$N_{Fr} = \frac{\overline{V}}{(gy_h)^{1/2}} \qquad (3.16)$$

The Froude number can be interpreted as the ratio between the inertial or acceleration forces and gravitational forces acting on a fluid particle.

Two additional dimensionless terms are used in describing the relationships between flow and culvert geometry. H/D_e is the ratio between the headwater depth, H, to the equivalent diameter, D_e. The headwater depth is defined as $H = h_1 - z$.

A modified form of the Froude number is used in the dimensionless performance curves presented in Section 4. The modified form of the Froude number is

$$N_{Fr} = \frac{Q}{g^{0.5} D_e^{2.5}} \qquad (3.17)$$

4.0 INLET LOSSES

Discharge Coefficient and Inlet Losses

Inlet losses are a result of fluid contraction and subsequent expansion at the culvert entrance. These losses are accounted for in the aforementioned discharge equations by the use of the discharge coefficient, C. The discharge coefficient is related to the entrance losses by

$$h_e = \left(\frac{1}{C^2} - 1\right)\frac{\overline{V}_i}{2g} \qquad (4.1)$$

where $\overline{V}_i$ is the average velocity at the control or terminal section (Bodhaine, 1968).

The following summary describing the nature of the discharge coefficient is adapted from Bodhaine (1968).

The discharge coefficient is a function of channel contraction and inlet geometry. Laboratory tests indicate that the discharge coefficient is independent of the proximity of the culvert invert relative to the stream bed level at the culvert entrance. These observations indicate that for types 1, 2 and 3 flow the geometry of the entrance sides determines the value of C, similarly, in types 4, 5 and 6 flow the value of C varies with the geometry of the top and sides of the entrance. Thus, for a given geometry the discharge coefficient for types 1, 2 and 3 flow should be identical, also the discharge coefficient for types 4 and 6 flow will be the same and type 5 flow will have a unique discharge coefficient.

Culvert entrance geometry can be separated into four general categories: flush setting in a vertical headwall, wingwall entrance, projecting entrance and mitered pipe set flush with a sloping embankment.

Experimental Approach

A hydraulic model was used to study the inlet losses of a DIC with a square edged entrance, set flush to a vertical headwall. The study was conducted in the Department of Civil Engineering's hydraulics laboratory at the University of Alaska-Fairbanks. A more detailed description of the laboratory studies and data analysis is given by Jordan (1987).

Experimental Apparatus

The model consisted of a headbox, outlet pipe, approach plane and a set of baffles (see Figure 4.1). A series of inserts were placed inside the pipe to simulate the culvert bed. The approach plane was adjustable, thus allowing it to be set at the same level as the inserts. The baffles reduced the turbulence caused by the upwelling effect of the fluid entering through the inlet located in the bottom of the headbox.

Two outlet pipe diameters, 4" and 6", were used to simulate the culvert barrel. The use of these two model sizes allowed for a full range of H/D ratios to be tested. Comparing these model sizes to prototype DICs of 6' through 14' diameters, the range of scale ratios is 1:12 to 1:42.

The first series of experiments were conducted using the unmodified surface of the plexiglass inserts and approach plane. The flow resistance of plexiglass is given as n = 0.009 (Sharp, 1981).

A second series of experiments were conducted after a single layer of crushed rock was glued to the surface of the inserts and approach plane. The median size of the rock was d_{50} = 0.19 in, the particle-size distribution curve is given in the Appendix. Using this rock size results in medium rock diameter to culvert diameter ratios of

	d_{50}/D
4" model:	0.048
6" model:	0.032

The outlet flow from the model was collected in a tank and circulated with a pump. Inline with the pump were a (globe) valve for adjusting the flow and a venturi meter for determining the discharge.

ITEM	DIMENSIONS	MATERIAL
HEADBOX	30' WIDE, 36' LONG, 24' DEEP	SHEET METAL
PIPE	4' & 6' DIAMETERS, 48' LONG	PVC
INSERTS	48' LONG, WIDTH VARIED	PLEXIGLASS
ROUGHNESS MATERIAL SIZE - 0.19' DIA		CRUSHED ROCK

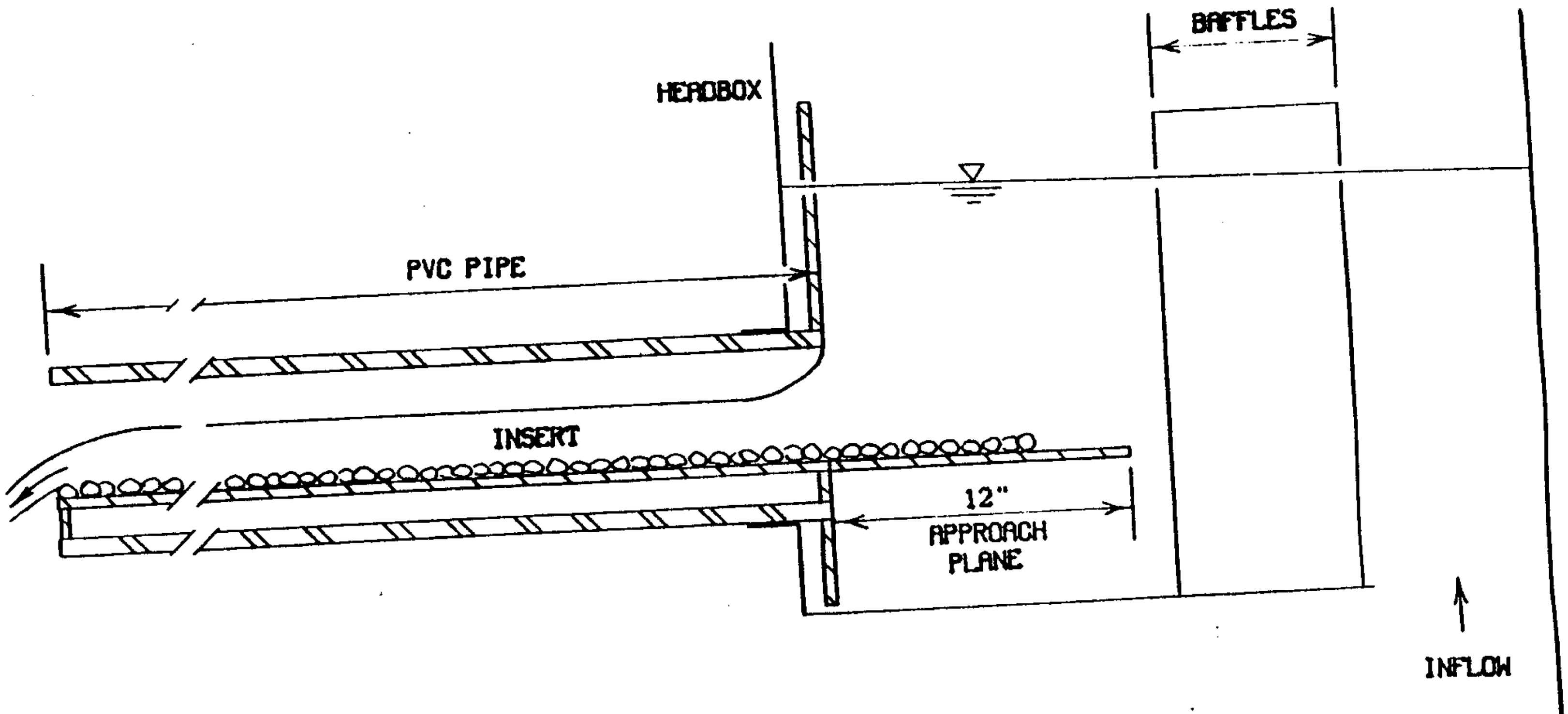


Figure 4.1. Experimental apparatus.

Experimental Design

The current design criterion of the Alaska DOT&PF for culvert installations at highway crossings is for the headwater-diameter ratio, H/D, to be in the range of 1.0 to 1.5. Thus, it is expected that culvert flow will be predominately types 1 to 3.

The major portion of the experimental work concentrated on studying type 1 flow. Also, a small amount of data was collected for type 5 flow. The experimental apparatus was not appropriate for studying types 4 or 6 flow.

The following variables were measured for each experimental run:

- culvert diameter (D),
- depth of bed (d),
- bed slope (S_o),
- bed type (smooth or rough),
- headwater depth (H), and
- discharge (Q).

In reference to Fgiures 2.2 and 2.3 the headwater depth, H, is defined as $H = h_1 - z$.

An experimental run consisted of establishing a series of discharges for a particular setup. At each discharge, time was allowed for the system to come to equilibrium before the appropriate measurements were recorded.

The parameters for each of the experimental runs are given in Table 4.1. For these experiments, the range of discharge and head were 0.055 to 0.38 cfs and 0.15 to 0.82 feet, respectively. The range for the depression ratio, λ, was 0 to 45 percent for a smooth bed, and 0 to 50 percent for a rough bed.

Analysis

The initial step in the analysis was to plot head versus discharge for each experimental run. The plots were used to look for general trends and data entry errors.

TABLE 4.1. Parameters for experimental runs.

Run number	Diameter (in)	Depressed (%)	Bed type	Slope (%)
13	4.0	0	Smooth	2.1
15	4.0	9	Smooth	2.1
17	4.0	19	Smooth	2.1
18	4.0	30	Smooth	2.3
30	4.0	37	Smooth	2.1
19	4.0	45	Smooth	2.8
20	6.0	0	Smooth	2.2
21	6.0	21	Smooth	2.1
22	6.0	30	Smooth	2.1
23	6.0	42	Smooth	2.1
24	6.0	25	Rough	2.3
25	6.0	34	Rough	2.3
26	6.0	45	Rough	2.3
27	4.0	50	Rough	2.2
28	4.0	35	Rough	2.2
29	4.0	23	Rough	2.2

TABLE 4.2. Grouping of experimental runs by model diameter and bed type.

	Smooth bed	Rough bed
	Group I	II
6" diameter model	Run No. 20,21,22,23	20,24,25,26
	III	IV
4" diameter model	13,15,17,18,19,30	13,27,28,29

The experimental runs were divided into four groups as shown in Table 4.2.

For each group, plots of head versus discharge and H/D_e versus Fr were made, see Figures 4.2 and 4.5. The terms H/D_e and Fr are based on the equivalent diameter as defined in Chapter 3.

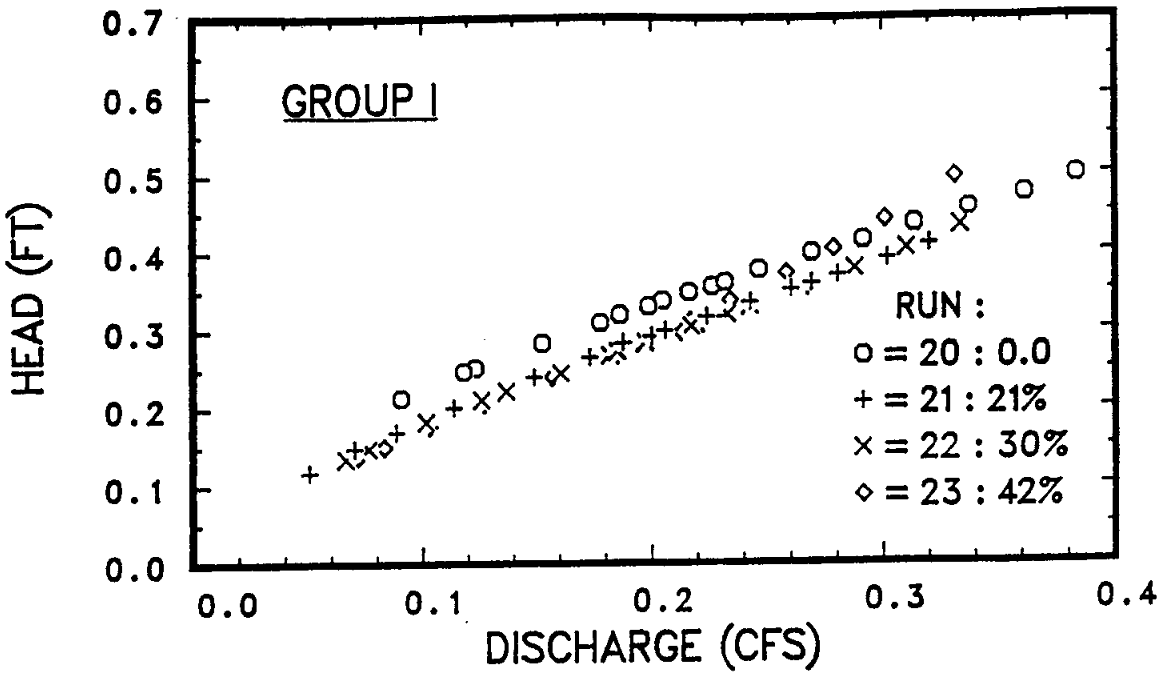
The plots of H/D_e versus Fr show a well defined trend which is repeated for all four groups. The data points plot as parallel curves with a slight shift toward higher Froude numbers for a greater depression ratio. This trend indicates that the use of D_e explains only a portion of the flow relationship among head, discharge and percent depressed.

The complete relationships between head, discharge and culvert geometry are given by the discharge equations presented in Chapter 2. These equations can be rewritten to solve for the discharge coefficient. For type 1 flow, neglecting the approach channel velocity head and losses

$$C = \frac{Q}{A_c(2g(h_1-z-y_c))^{1/2}} \tag{4.2}$$

and for type 5 flow

$$C = \frac{Q}{A_o(2g(h_1-z))^{1/2}} \tag{4.3}$$



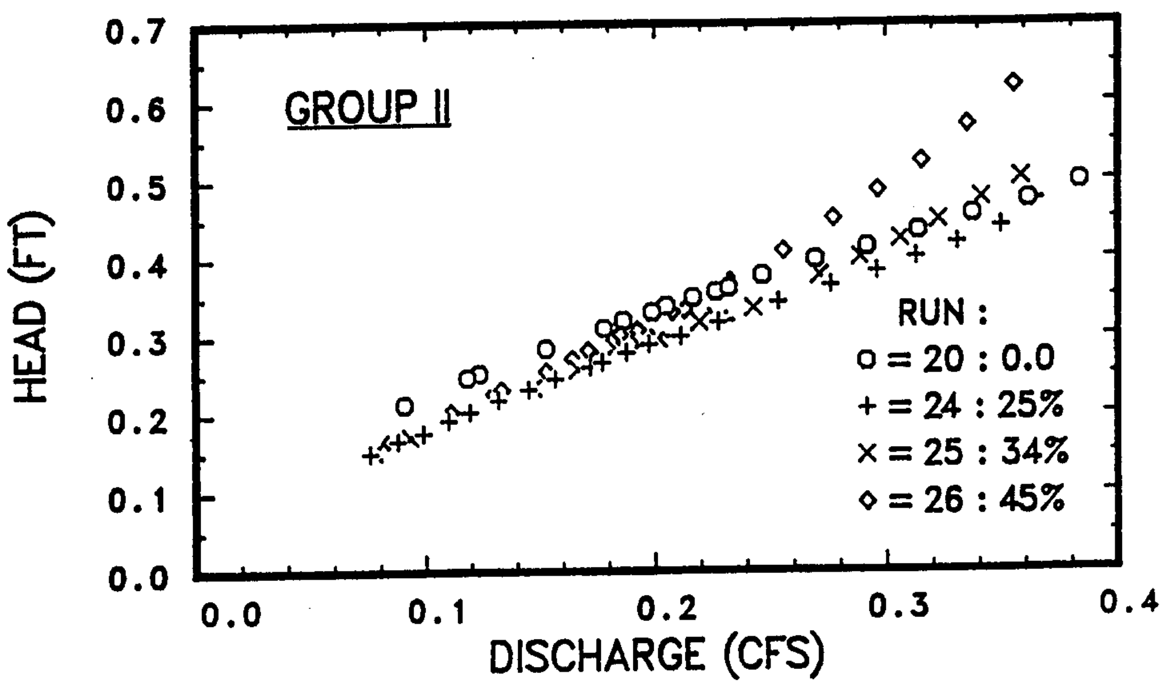
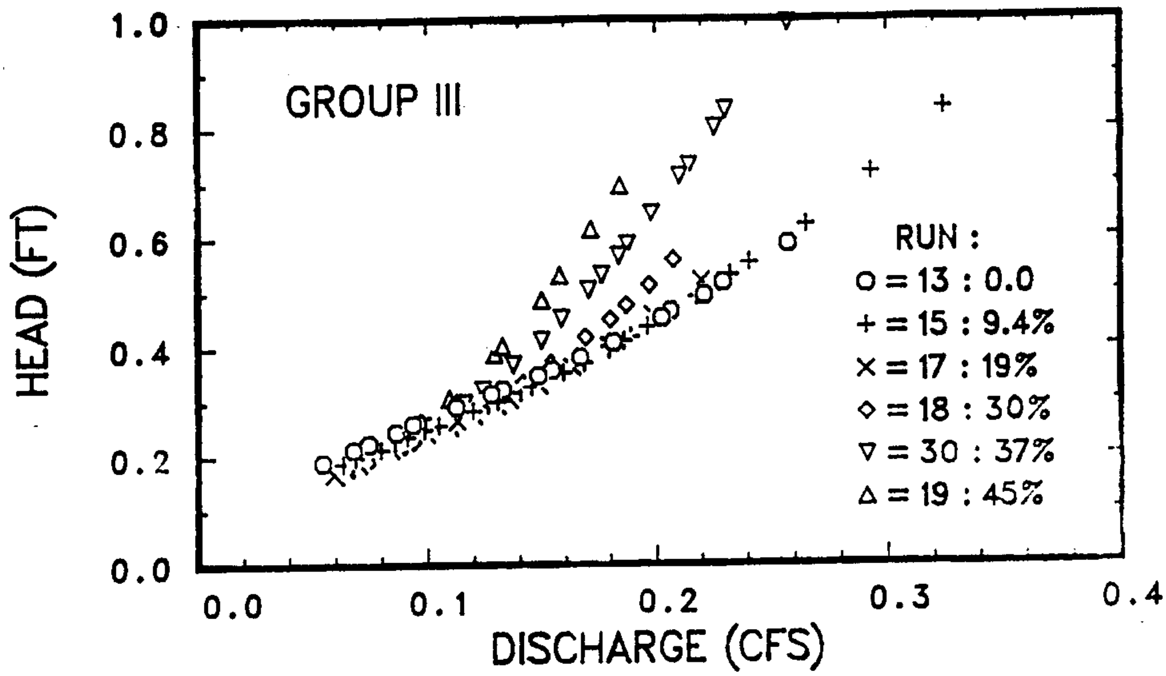


Figure 4.2. Measured performance curves for experimental runs of depressed invert culvert model, groups I and II.



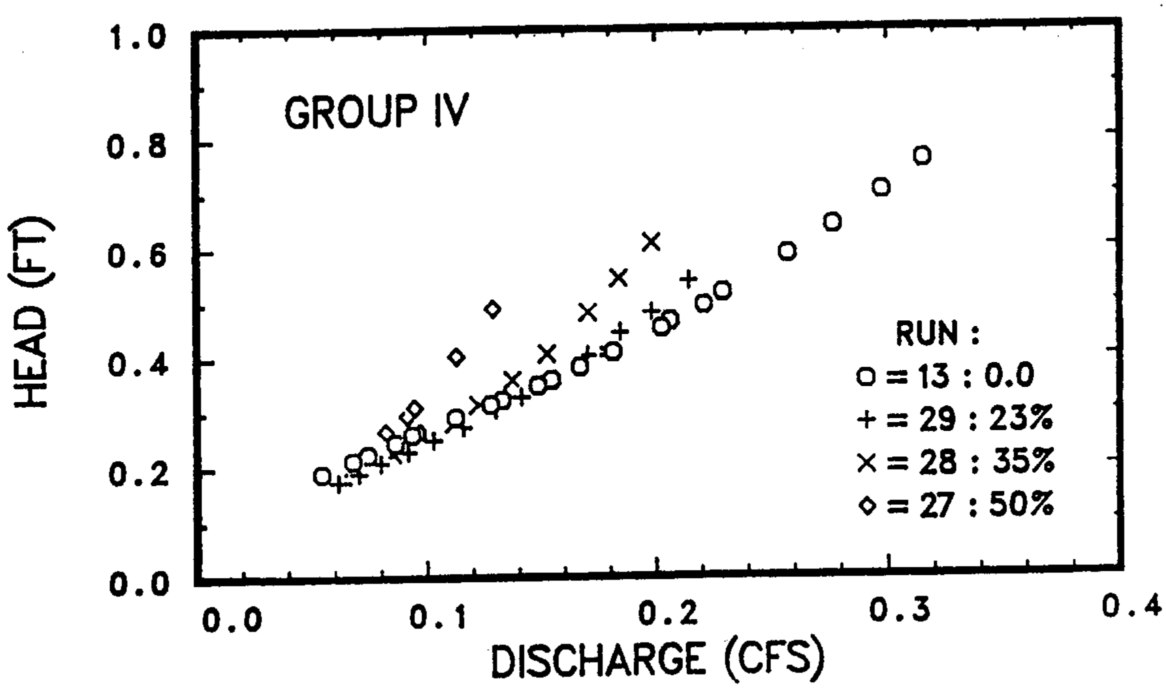
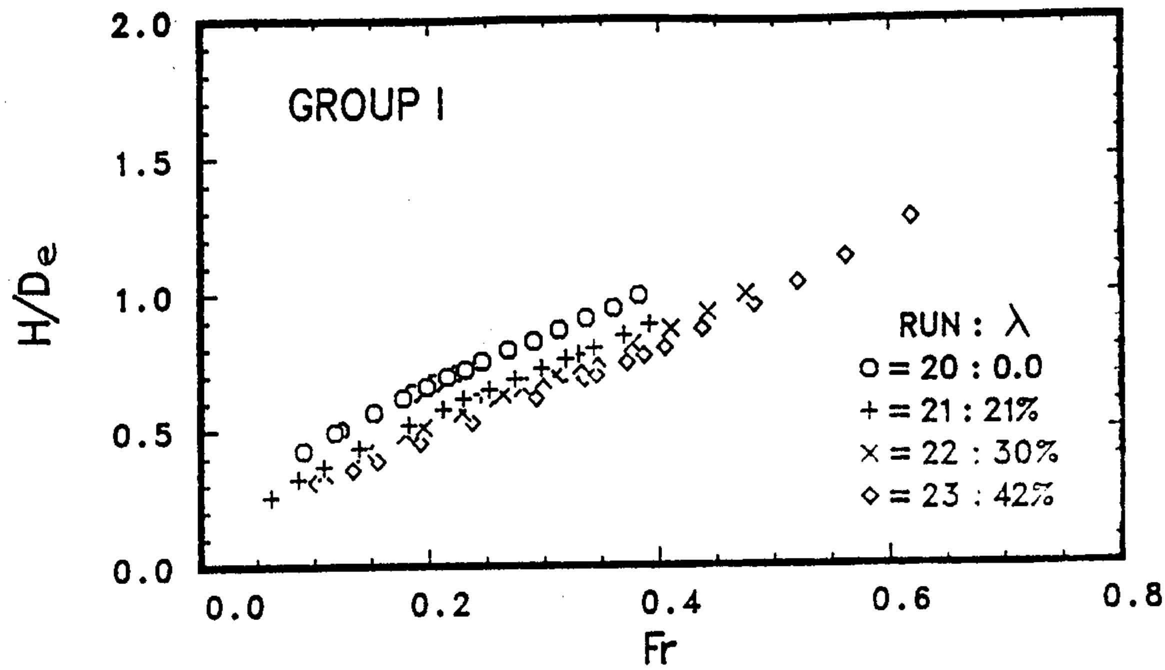


Figure 4.3. Measured performance curves for experimenal runs of depressed invert culvert model, groups III and IV.



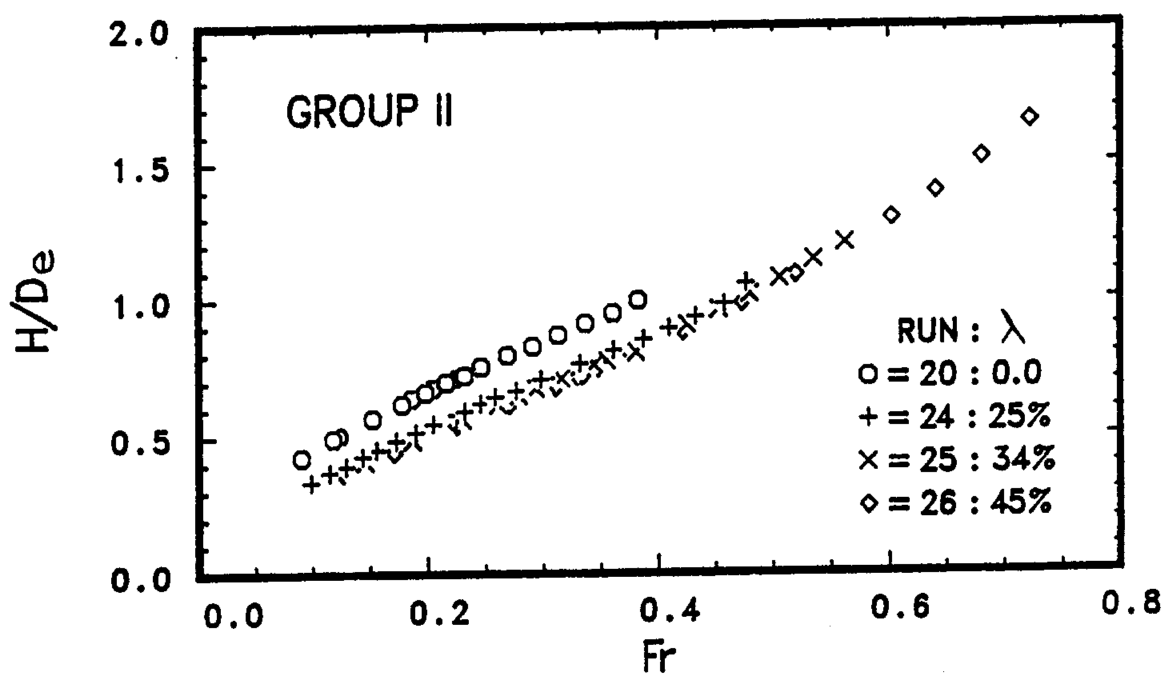


Figure 4.4. Dimensionless performance curves for experimental runs of DIC model, groups I and II.

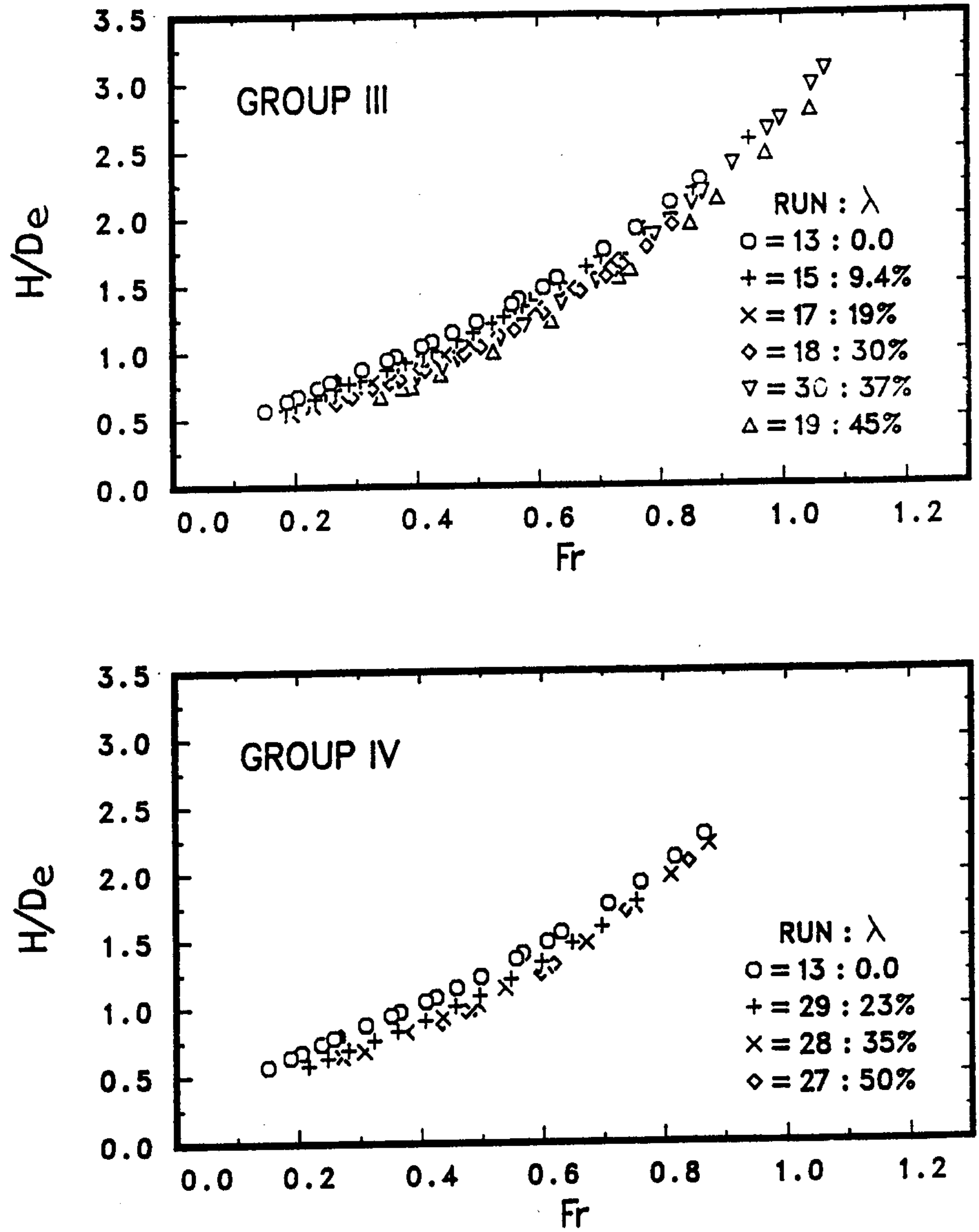


Figure 4.5. Dimensionless performance curves for experimental runs of DIC model, groups III and IV.

For the experimental runs, all terms on the right hand side of equations 4.2 and 4.3 were either measured or can be calculated from the relationships developed in Chapter 3.

Experimental runs 13 and 20 are the baseline case for this study, that is, the case where the invert is not depressed, $\lambda = 0$. The term $C(0)$ is used to identify the discharge coefficient for the baseline case. Linear regression techniques were used to develop a relationship between $C(0)$ and H/D.

The term $C(\lambda)$ identifies the discharge coefficient for a specific depression ratio, λ. A new coefficient relating $C(\lambda)$ to $C(0)$ is given as

$$\eta(\lambda) = \frac{C(\lambda)}{C(0)} \tag{4.4}$$

Linear regression techniques were used to develop expressions for η as a function of the type of flow, inlet geometry, H/D_e and λ.

The equations describing $C(0)$ and η which result from the regression analysis are presented below.

Deviations to Analysis

No statistically significant relationship between η, λ and H/D_e was found for non-submerged flow over a smooth bed. Two distinct trends were apparent in the plot of C versus H/D_e. The data associated with these two trends are distinctly separated by their depression ratios. For each trend, regression curves as a function of H/D_e were developed. These are presented below.

Type 1 Versus Type 5 Flow

Type 1 flow resembles weir flow, while type 5 flow is similar to flow through a sluice gate. Flow through a culvert under inlet control and low inlet head (h_1) will be type 1 flow. As the inlet head increases, type 5 flow will eventually be established. There is no clear physical break point between the two types of flow. Bodhaine (1968) uses the criteria of

type 1 flow: $\frac{h_1 - z}{D} < 1.5$

type 5 flow: $\frac{h_1 - z}{D} > 1.5$

For this study, the dividing point between the two types of flow is based on D_e and is stated as

type 1 flow: $\frac{h_1 - z}{D_e} < 1.5$

type 5 flow: $\frac{h_1 - z}{D_e} > 1.5$

Certain conditions will favor the establishment of type 6 flow rather than type 5 flow. These conditions are shallow culvert slopes or high barrel losses. The determination of barrel losses is discussed in Section 5.

Results: Type 1 Flow

The following regression equations are for type 1 flow through a squared edged circular culvert set flush to a vertical headwall.

For a circular culvert $\lambda = 0$

$$C(0) = 0.9057 + 0.0289(H/D_e) - 0.0819(H/D_e)^2 \tag{4.5}$$

$r^2 = 0.97$

$N = 35$

Range: $0.40 < h/D_e < 1.50$

where N is the number of observations used in the regression analysis.

The function, η, relating $C(\lambda)$ to $C(0)$ for a rough bed is

$$\eta = 1.014 - 0.0222(H/D_e) - 0.3676(\lambda)^2 \qquad (4.6)$$
$$r^2 = 0.90$$
$$N = 84$$
Range: $0.1 < \lambda < 0.50$, $0.4 < H/D_e < 1.5$

Two separate regression equations are used to relate C to H/D_e for type 1 flow over smooth bed. When λ is less than 0.30

$$C(\lambda) = 0.8856 + 0.05129(H/D_e) - 0.05178(H/D_e)^2 \qquad (4.7)$$
$$r^2 = 0.95$$
$$N = 81$$
Range: $0.09 < \lambda < 0.30$, $0.40 < H/D_e < 1.50$

and for λ greater than 0.37.

$$C(\lambda) = 0.9505 - 0.04584(H/D_e) - 0.07906(H/D_e)^2$$
$$r^2 = 0.96$$
$$N = 30$$
Range: $0.37 < \lambda < 0.45$, $0.4 < H/D_e < 1.5$

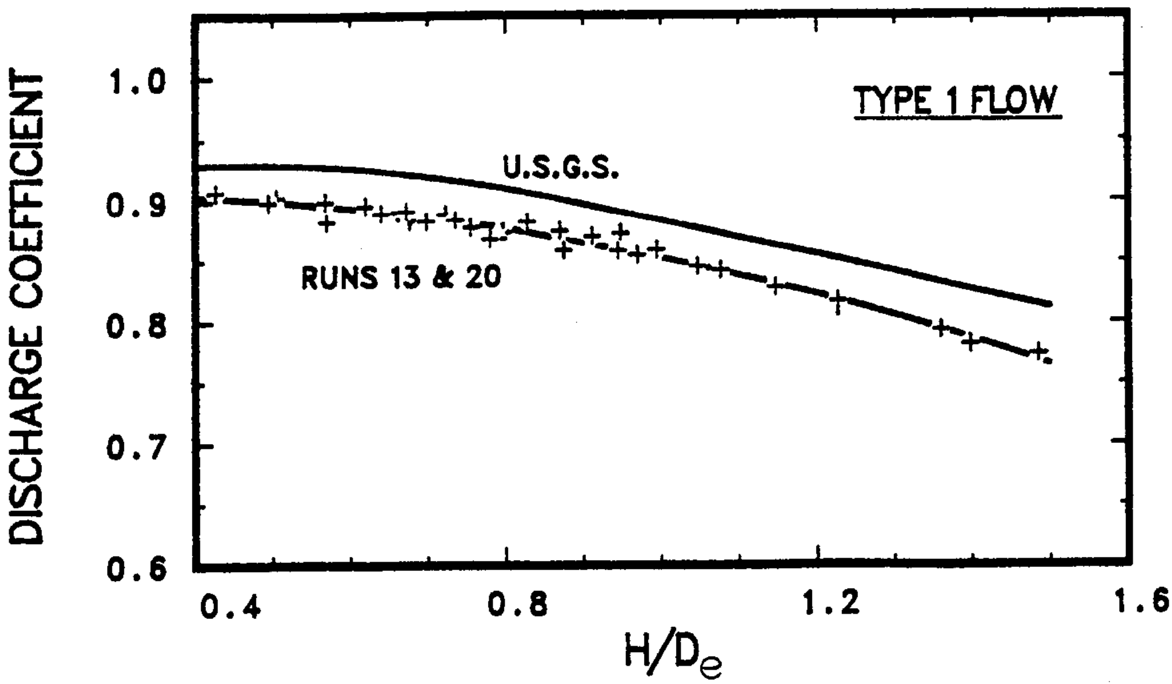
Figure 4.6 compares C as given by equation 4.4 to that published by Bodhaine (1968). Design curves for C as a function of λ and H/D_e for rough and smooth beds are given in Figure 4.7.

Results: Type 5 Flow

The following regression equations are for type 5 flow through a squared-edged circular culvert set flush to a vertical headwall.

For a circular culvert, $\lambda = 0$.

$$C(0) = 0.1207 + 0.3046(H/D_e) - 0.05747(H/D_e)^2 \qquad (4.8)$$
$$r^2 = 0.999$$
$$N = 5$$
Range: $1.4 < H/D_e < 2.2$



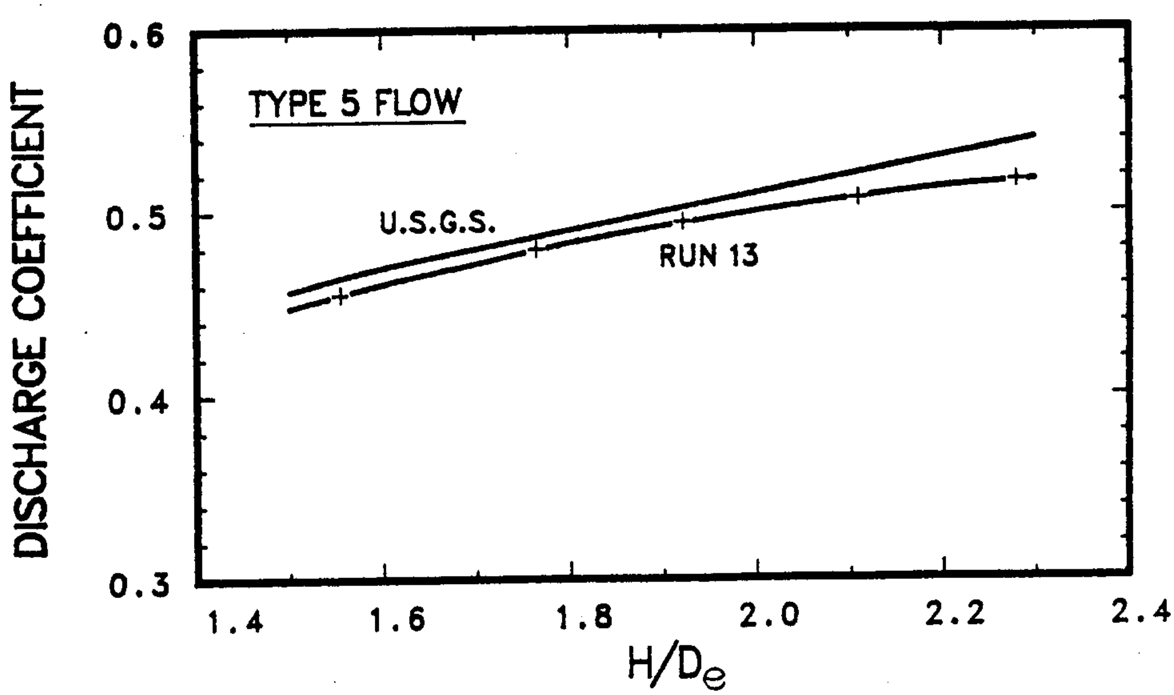


Figure 4.6. A comparison of the discharge coefficient, C, between that published by Bodhaine (1968) and the experimental data for $\lambda = 0$.

+ indicate experimental data points.

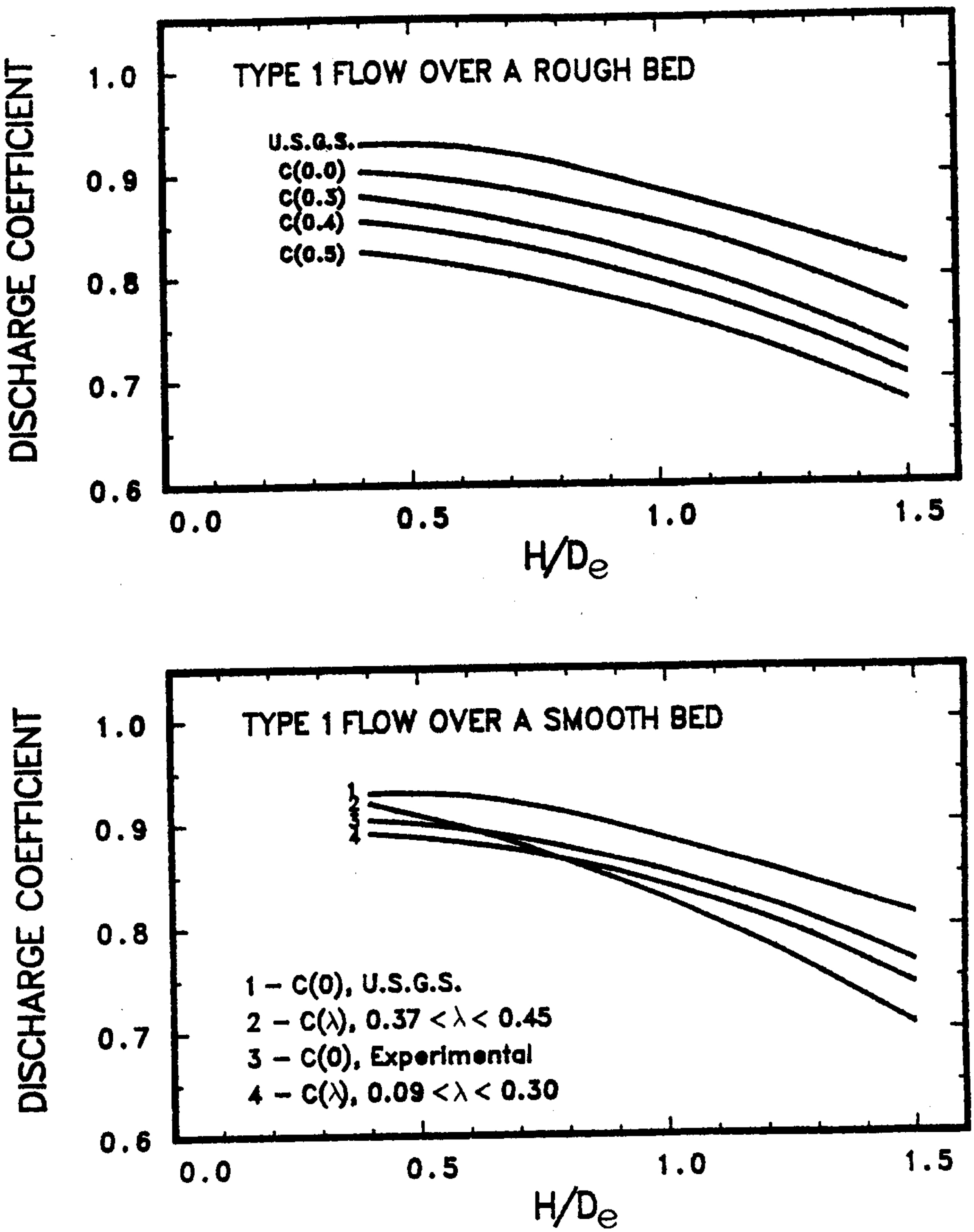


Figure 4.7. Coefficient of discharge (C) for types 1, 2 and 3 flow in a DIC, smooth and rough beds.

The function, η, relating $C(\lambda)$ to $C(0)$ for a rough bed is

$$\eta = 1.248 - 0.101(H/D_e) \qquad (4.9)$$

$r^2 = 0.84$

$N = 11$

Range: $0.20 < \lambda < 0.50$, $1.4 < H/D_e < 2.2$

The function, η, relating $C(\lambda)$ to $C(0)$ for a smooth bed is

$$\eta = 1.181 + 0.2694(\lambda) - 0.1026(H/D_e) \qquad (4.10)$$

$r^2 = 0.85$

$N = 25$

Range: $0.1 < \lambda < 0.50$, $1.4 < H/D_e < 2.2$

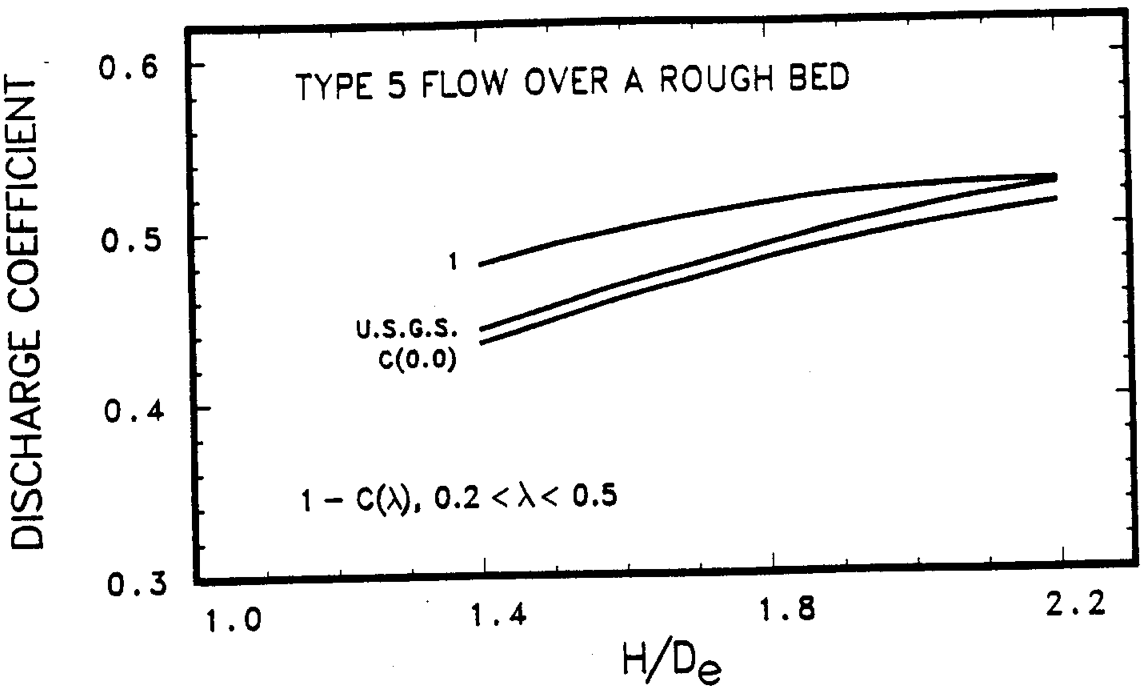
A comparison of the curve for C as defined by equation 4.8 to that published by Bodhaine (1968) is given in Figure 4.6. Design curves for C as a function of λ and H/D_e for rough and smooth beds are given in Figure 4.8.

Discussion

When comparing the values for C given by Bodhaine (1968) to those determined by this study the results are very good. The experimental curves for type 1 and 5 flows lie nearly parallel to the curves given by Bodhaine. The difference between these two curves for type 1 flow is approximately $\Delta C = 0.03$, and for type 5 flow the difference between the two curves is approximately $\Delta C = 0.01$. The curves for C for type 1 and 5 flows as given by Bodhaine are included in the design curves. They are labeled "USGS".

Type 1 flow over a rough bed. There is a well defined relationship among C, λ and H/D_e. As either λ or H/D_e increases, C decreases.

Type 1 flow over a smooth bed. Two curves are used to describe the discharge coefficient for type 1 flow over a smooth bed. Each curve is for a specific range of depression ratios and is expressed as a function of H/D_e.



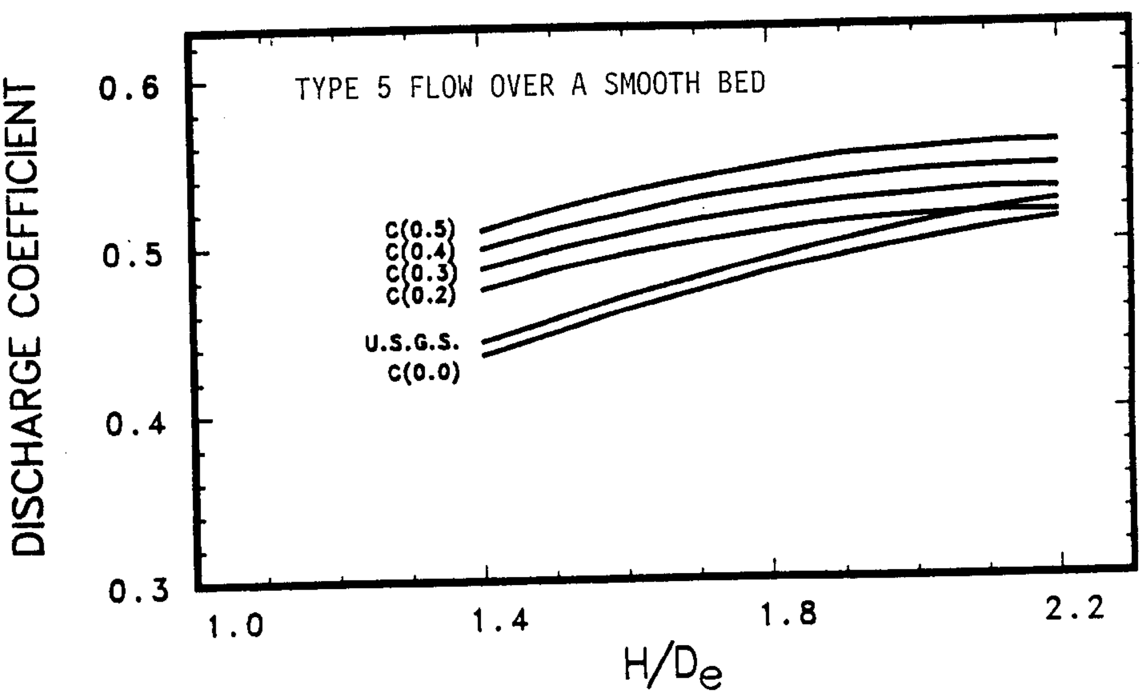


Figure 4.8. Coefficient of discharge (C) for type 5 flow in a DIC, smooth and rough beds.

Type 5 flow over a rough bed. Within the range of $0.20<\lambda<0.50$, C is only a function of H/D_e. No experimental data were collected in the range of $0.0<\lambda<0.20$. Therefore, no relationship for C is given for within this range.

Type 5 flow over a smooth bed. The relationship among C, λ and H/D_e is well defined. The value of C increases with increasing values of λ and H/D_e.

Slope Effects

The culvert slope has a slight effect on the discharge coefficient for type 1 - 3 flows (French, 1955). The significance of this effect needs to be determined through futher model tests.

5.0 FLOW RESISTANCE

General

To determine the barrel losses for a DIC, the flow resistance associated with both the culvert walls and bed material must be accounted for. Since the roughness properties of the culvert and bed material are independent of each other, it is desirable to be able to estimate the flow resistance of each, and then combine them into an overall flow resistance factor.

There are two commonly used measures of flow resistance: the Darcy-Weisbach friction factor (f) and the Mannings roughness coefficient (n). The Darcy-Weisbach friction factor is defined by

$$f = \frac{8gRS_f}{\overline{V}^2} \tag{5.1}$$

where S_f is the average water slope, $\overline{V}$ is the average velocity for the cross section, and the remaining terms are as previously defined.

The Mannings roughness coefficient is defined as

$$n = \frac{\phi R^{2/3} S_f^{1/2}}{\overline{V}} \tag{5.2}$$

where ϕ = 1.0 for SI units and ϕ = 1.49 for English units.

The two terms can be related by

$$n = \phi\left[\frac{f}{8g}\right]^{1/2} R^{1/6} \tag{5.3}$$

The Chezy coefficient, C, is occasionally used as a measure of flow resistance. The Chezy coefficient is related to the Darcy-Weisbach friction factor by

$$C = \left(\frac{8g}{f}\right)^{1/2} \tag{5.4}$$

The measures of flow resistance are influenced by relative roughness, the geometry of the roughness elements, the geometry of the channel, the mobility of the bed and suspended sediments.

The material presented within this report will be restricted to rigid boundary channels.

Testing for Rigid Boundary Channels

The following material is adapted from Henderson (1966) and is presented in the form used by Griffiths (1981).

The criterion for distinguishing a rigid boundary (vanishingly small rates of bedload transport) from mobile boundary conditions is the Shields Entrainment Function, F_s. For fully turbulent flow, F_s is given as

$$F_s = \frac{RS_f}{(S_s-1)d_{50}} \qquad (5.5)$$

where S_s is the specific gravity of the bed particles, usually taken to equal 2.65. The experimental work by Shields (Henderson, 1966) for turbulent flow shows that a rigid bed will be maintained when F_s is less than 0.056.

Equation 5.5 is applicable for particle Reynolds numbers, Re*, which are greater than 400. The particle Reynolds number, and friction velocity, V_*, are defined by

$$Re^* = \frac{V_* d_{50}}{\nu} \qquad (5.6)$$

where ν is the kinematic viscosity, and

$$V_* = (gRS)^{1/2} \qquad (5.7)$$

Using $F_s \leq 0.056$, equation 5.5 can be written to give the criterion for a rigid bed as

$$d_{50} \geq 11RS_f \qquad (5.8)$$

Resistance Factors for Culverts

The parameters that influence the flow resistance of culverts are barrel diameter and shape, corrugation size, pattern and helix angle, type of construction (riveted versus multiplate) and flow rate (U.S. Dept. of Transportation, 1980).

Numerous laboratory and field tests have been performed on a variety of culvert types to determine their resistance factors or coefficients. A comprehensive study on the hydraulic resistance factors for corrugated metal conduits was performed by the U.S. Department of Transportation (1980). Individual charts are presented for determining both f and n. These charts are a function of diameter, corrugation type, flow rate and full or partial flow conditions. In addition, the U.S. Department of Transportation (1985) and Bodhaine (1968) give selected values of n.

Resistance Factors for Rough Beds

The flow resistance relationships developed for gravel bed rivers are used to approximate the flow resistance for the bed material in DICs.

In summarizing several existing equations used for predicting the roughness coefficient for river beds, Bray (1979) presents the following.

$$n = 0.034 d_{50}^{1/6} \tag{5.9}$$

$$n = 0.031 d_{90}^{1/6} \tag{5.10}$$

$$n = \frac{0.113 y_m^{1/6}}{1.16 + 2.00 \log\left(\frac{y_m}{d_{84}}\right)} \tag{5.11}$$

where y_m is the mean depth.

Equation 5.9, commonly known as the Strickler equation, is from Chow (1959). Equation 5.10 is from Simons and Senturk (1976), and equation 5.11 is from Limerinos (1970).

Based on a data set composed of natural gravel bed rivers, Bray (1979) developed several best fit relationships for both the Mannings roughness coefficient and Darcy-Weisbach friction factor.

$$n = \frac{y_m^{1/6}}{9.66 + 19.5 \log(\frac{y_m}{d_{84}})} \quad (5.12)$$

$$\frac{1}{f^{1/2}} = 1.36(\frac{y_m}{d_{50}})^{0.281} \quad (5.13)$$

$$\frac{1}{f^{1/2}} = 0.248 + 2.36\log(\frac{y_m}{d_{50}}) \quad (5.14)$$

Griffiths (1981) used river data from New Zealand and the United States to develop the relationship

$$\frac{1}{f^{1/2}} = 0.760 + 1.98\log(\frac{R}{d_{50}}) \quad (5.15)$$

for the range of $1 < R/D_{50} < 200$. Griffiths also recommends that for low values of relative roughness, $R/D_{50} < 5$, the studies by Bathurst (1978), Judd and Peterson (1969), Simons et al. (1979), and Thompson and Campbell (1979) should be consulted.

There are several problems associated with applying equations 5.9 through 5.15 to DICs. These problems are related to the differences in geometry between typical gravel bed rivers and DICs.

Most of the river reaches used in the studies by Bray (1979) and Griffiths (1981) have width to depth ratios of 20 to 80, while for DICs this ratio is estimated to be 0.5 to 0.9. River width is taken as the surface width, and the depth is the mean depth.

In equations 5.11 through 5.14, the mean depth, y_m, is defined as the average cross-sectional flow area divided by the average water surface width. It is inappropriate to use this definition for a DIC. As a culvert approaches full flow conditions, the surface width becomes relatively small, so dividing width into area can result in mean depths greater than the culvert diameter.

In wide shallow rivers, the mean depth and hydraulic radius are essentially equal. Thus, it makes little difference which measure is used when developing a relationship for predicting the flow resistance. For a DIC, there is a substantial difference between depth of flow and hydraulic radius.

When applying equations 5.13 through 5.15 to DICs, using the hydraulic radius in place of the mean depth gives a more conservative estimate of the flow resistance. Thus, we recommend that the hydraulic radius be used to estimate the flow resistance.

Friction Factors for Permeable Beds

In general, research has shown that for fully rough, turbulent flow over a permeable bed that: (1) the flow resistance is higher for the permeable bed than for a nonpermeable bed of similar roughness; (2) the Darcy-Weisbach friction factor increases with increasing Reynolds number (Zagni and Smith, 1976; Gupta and Paudyal, 1985).

Zagni and Smith (1976) give the following description of the flow mechanisms which occur at the bed interface.

> "From observations and theoretical work carried out so far, however, a certain pattern of behavior is apparent as far as boundary friction is concerned. Because there is a finite slip velocity at the interface, it would appear logical that boundary friction should be less than for flow over an impermeable boundary of the same surface roughness or rugosity. However, energy is dissipated within the transition zone in the porous medium, and it also appears that fluid and momentum exchange across the interface is translated back into the main flow, resulting in additional energy

loss. The net effect of the permeable boundary might, therefore, be to increase boundary friction as compared with the impermeable boundary case, but this would depend on the relative magnitudes of the effects previously considered."

The flume study performed by Zagni and Smith (1976) used permeable beds made from spheres ranging in d_{50} from 2.1 mm to 17.0 mm. From this study they calculated the friction factor for a hydraulically rough bed. To calculate the friction factor as a function of the bed the Vanoni-Brooks method (see Frech, 1985) was used to eliminate the effects of the side walls. The Vanoni-Brooks method determines both the friction factor associated with the bed, f, and the hydraulic radius associated with the bed, R_b. The equation Zagni and Smith developed is

$$\frac{1}{f^{1/2}} = 2.03\log\frac{12.27R_b}{K_s} \tag{5.16}$$

where K_s is determined from

$$K_s = 4.466(d_{65}K_P^{1/2})^{1/2} \tag{5.17}$$

where K_P is the intrinsic permeability.

Gupta and Paudyal (1985) present several equations for determining the Darcy-Weisbach friction factor for free surface flow over a permeable bed. Based on the Prandtl-Von Karman velocity distribution law, the following relationship was developed.

$$\frac{1}{f^{1/2}} = \frac{1}{2.83\kappa}(\ln(\frac{y_o+\Delta y}{K_s}) -1) + \frac{1}{2.83}\frac{V_o}{V_*} \tag{5.18}$$

where K_s is taken to equal d_{50}, $\kappa = 0.28$, V_o is the slip velocity at the bed, and the term V_o/V_* can be evaluated from equation 6.10.

Using the relationship Beavers and Joseph (1967) found for the velocity gradient at the porous bed boundary, Gupta and Paudyal (1985) also developed the expression

$$\frac{1}{f^{1/2}} = \frac{0.35}{\kappa}\left(\ln\left(\frac{y_o+\Delta y}{\Delta y}\right) + \frac{K_p^{1/2}}{\alpha \Delta y} - 1\right) \qquad (5.19)$$

where α is a constant characterizing the structure of the bed material.

They evaluated the constants for equation 5.19 based on a flume study using a permeable gravel bed with the properties of d_{50} = 9.8 mm and $K_p = 846 \times 10^{-6}$ cm^2. The final equation was

$$\frac{1}{f^{1/2}} = 1.26\ln\left(1 + \frac{y_o}{0.32 d_{50}}\right) + 24.6 K_p^{1/2} - 1.26 \qquad (5.20)$$

The dimensional analysis for free surface flow over a permeable bed performed by Gupta and Paudyal (1985) yielded

$$\frac{1}{f^{1/2}} = \mathrm{fct}\left[\frac{R}{y_o}, \frac{y_o}{d_{50}}, \frac{K_p^{1/2}}{y_o}, \frac{h}{y_o}, \frac{\bar{V}}{(g y_o)^{1/2}}, \frac{\bar{V} y_o}{\nu}\right] \qquad (5.21)$$

The equations presented in this section are not recommended for use in estimating the friction factors for DICs. These equations are based on the results of studies which used laboratory flumes, they may not include all the terms necessary or the appropriate constants for application to full-scale culvert installations. They are presented here as a partial review of the literature describing free surface flow over a permeable bed. This review is meant to be used as background material for future studies investigating the hydraulic characteristics of free surface flow over a permeable bed.

Composite Flow Resistance

Once the flow resistance for the sides and bottom of a DIC has been determined, it is necessary to determine a combined flow resistance.

In describing several methods for combining channel sections of different roughness, Chow (1959) presents equations 5.22 through 5.24. In applying these equations, the channel is divided into N parts -- each with a specified wetted perimeter, P_i and Manning's roughness coefficient, n_i.

$$n = \left(\frac{\sum_{i=1}^{N} (P_i n_i^{3/2})}{P} \right)^{2/3} \tag{5.22}$$

$$n = \left(\frac{\sum_{i=1}^{N} (P_i n_i^{2})}{P} \right)^{1/2} \tag{5.23}$$

$$n = \frac{PR^{5/3}}{\sum_{i=1}^{N} \left(\frac{P_i R_i^{5/3}}{n_i} \right)} \tag{5.24}$$

The U.S. Department of Transportation (1985) recommends using equation 5.22 for determining a composite n for culverts. Bodhaine (1968) suggests equation 5.25 for this purpose.

$$n = \frac{\sum_{i=1}^{N} (P_i n_i)}{P} \tag{5.25}$$

By using the same assumptions that were used in deriving equation 5.22, an expression for determining a composite friction factor is given as

$$f = \frac{\sum_{i=1}^{N} (P_i f_i)}{P} \tag{5.26}$$

Barrel Losses

Under steady-state, uniform flow, the barrel losses for a DIC can be determined from the geometry of the channel and the composite flow resistance.

When using n as the measure of flow resistance, the concept of conveyance is used for determining the barrel losses. The conveyance of a channel is determined at a given cross section and is defined by

$$K_j = \frac{\phi}{n_j} A_j R_j^{2/3} \tag{5.27}$$

where ϕ = 1.0 for metric units and ϕ = 1.49 for English units. From the conveyance at the inlet, K_2, and outlet, K_3, of a DIC, the barrel losses are determined from

$$h_{f2-3} = \frac{LQ^2}{K_2 K_3} \tag{5.28}$$

where L is the length of the culvert barrel.

In those cases where f is used as the measure of flow resistance, the barrel losses are determined from

$$h_{f2-3} = f\,\frac{L\overline{V}^2}{8gR} \tag{5.29}$$

For nonuniform flow, routing techniques are required for determining the barrel losses.

6.0 VELOCITY DISTRIBUTION

Average Velocity

In designing DICs, both the average velocity and velocity profile are likely to be of interest.

The average velocity can be determined from the Mannings equation

$$\bar{V} = \frac{\phi}{n} R^{2/3} S_f^{1/2} \tag{6.1}$$

or the Darcy-Weisbach equation

$$\bar{V} = \left(\frac{8g}{f} R S_f\right)^{1/2} \tag{6.2}$$

Velocity Profile

The boundary conditions for turbulent, fully developed free surface flow can be classified as hydraulically smooth or rough. For hydraulically smooth boundary conditions the velocity profile is described by

$$\frac{V}{V_*} = \frac{1}{\kappa} \ln\left(\frac{yV_*}{\nu}\right) + A \tag{6.3}$$

The velocity profile for a hydraulically rough surface is given as

$$\frac{V}{V_*} = \frac{1}{\kappa} \ln\left(\frac{y}{y_o}\right) + A \tag{6.4}$$

(Streeter and Wylie, 1975).

Values for the terms κ and A have been studied by numerous researchers, several of these are listed in Figure 6.1. The term κ is described by Gupta and Paudyal (1985) as a measure of the velocity gradient or the rate of dissipation of mean flow energy.

TABLE 6.1. Values for the coefficients, κ and A from equations 6.4 and 6.5.

Source	κ	A	Notes
Streeter & Wylie (1975)	0.417	5.84	flat plates (smooth)
Streeter & Wylie (1975)	0.40	5.5	smooth wall pipes, after Nikuradse
Streeter & Wylie (1975)	0.40	8.48	sand roughened pipes, after Nikuradse
Nezu & Wolfgang (1986)	0.412	5.29	hydraulically smooth open channel flow
*	0.40	8.5	hydraulically rough open channel flow
Zagni & Smith (1976)	0.40	9.09	permeable bed of graded spheres
Gupta & Paudyal (1985)	0.28	8.21	permeable bed of gravel, $d_{50} = 9.88$ mm

* The values for κ and A for hydraulically rough open channel flow are commonly taken as those for sand roughened pipes (Chow, 1959; French, 1985).

In a study that examined 49 sites throughout Alaska, velocity profiles were measured in the culvert barrels near the outlet (Kane and Wellen, 1985). Based on these velocity profiles, Mountjoy (1986) developed the following method for predicting velocity profiles in culverts.

$$V = B_1 \log\left(\frac{y}{y_o}\right) + B_2 \qquad (6.5)$$

where

$$B_1 = 0.828\overline{V} - 0.429 \qquad (6.6)$$

$$B_2 = 1.32\overline{V} - 0.089 \qquad (6.7)$$

<u>Velocity Profile Over a Permeable Bed</u>

From the general nature of flow over a permeable bed as described in Chapter 2, the velocity distribution can be written as

$$\frac{V}{V_*} = \frac{1}{\kappa}\ln\left(\frac{y+\Delta y}{d_{50}}\right) + A \qquad (6.8)$$

where Δy is the depth below the bed interface to the zero velocity plane.

From their work with graded spheres Zagni and Smith (1976) give the following equation and general description

$$\frac{V}{V_*} = \frac{1}{0.40}\ln\left(\frac{y}{d_{50}} + 0.68\right) + 9.09 \qquad (6.9)$$

> ... it is clear that Δy is of the order of a particle diameter ... the zero velocity plane approaches the nominal bed surface as the grading becomes wider, and thus, permeability decreases. Accordingly for material with a small median diameter and a large standard deviation the zero plane displacement would be negligible and the bed behavior would be similar to an impermeable boundary, but with large median diameter materials of narrow grading, the effect could be very significant, especially in the case of shallow flows.

In obtaining a best fit relationship to their data, Gupta and Paudyal (1985) found the velocity distribution to be described by

$$\frac{V}{V_*} = \frac{1}{0.28} \ln\left(\frac{y}{d_{50}} + 0.32\right) + 8.21 \tag{6.10}$$

From their work and in reviewing other research, Gupta and Paudyal (1985) concluded that the appropriate values for the Karman constant (κ) are 0.28 for permeable boundaries and 0.38 for nonpermeable boundaries.

7.0 SAMPLE CALCULATIONS

Introduction to Sample Calculations

A hypothetical example of a DIC will be evaluated to demonstrate the use of the various equations which have been presented in this report. A culvert performance curve will then be developed to compare the hypothetical DIC to a conventional culvert installation of the same size.

In applying the various equations required to determine culvert flow, many interactions between variables exist. The numerous interactions usually required that iterative techniques be used in the analysis.

A designer has a number of possibilities to choose from when deciding how to approach the analysis of flow through culverts. Generally, the analysis requires initial assumptions, a series of calculations based on the known quantities, then verification of the initial assumptions.

In this example the following approach is used for developing the culvert performance curve. The approach was selected to minimize the number of iterative calculations in the design process.

1. Select the normal depth of flow, y_n, within the culvert barrel, determine the composite friction factor and the required discharge, Q, to maintain y_n.

2. Based on Q and the DIC's geometry, determine the critical depth, y_c. Comparing y_n to y_c indicates whether the flow is supercritical or subcritical.

3. Select the most likely flow type.

   - If $y_n < y_c$ and the barrel flows only partly full, then inlet control occurs.
   - If $y_n > y_c$ or the barrel flows full, then outlet control occurs.

4. For a culvert flowing under outlet control, determine the barrel losses, h_{f2-3}. If the actual depth of flow, y_o is approximately equal to y_n then $h_{f2-3} = z$, if $y_o \neq y_n$ then the barrel losses are determined by routing the flow through the culvert.

5. Determine the approach channel head, h_1, required to produce the flow calculated in step 1. h_1 is determined from the appropriate discharge equations as given in Figures 2.2 and 2.3.

6. Determine the head to equivalent diameter, H/D_e, ratio and verify the assumed flow type from step 3.

The culvert performance curve is developed by selecting a series of values for y_n at Step 1. Steps 1 to 6 are repeated for each value of y_n. The culvert performance curve is constructed by plotting discharge, Q, versus headwater depth, H.

Parameters for Sample Problem

Culvert size

$D = 10.0$ ft

$d = 2.0$ ft

$L = 100.0$ ft

$S_o = 0.02$

Approach Channel

Sufficiently wide such that

$$\frac{\bar{V}_1^{\,2}}{2g} \approx 0.0 \text{ and } h_{f1-2} \approx 0$$

Inlet Conditions

Square-edged entrance set flush to a vertical headwall.

Corrugation/Culvert Construction

6 x 1 in annular corrugated metal pipe culvert.

Tailwater

Sufficient channel width and grade so as not to cause a backwater effect at all flow levels.

Bed Material

Rock size, $d_{50} = 0.50$ ft

Impermeable bed, $K_p = 0.0$

Initial Value

$y_n = 5.0$ ft

Calculation: Step 1. For $y_n = 5.0$ ft, determine f and Q

$$\phi = 2.0\cos^{-1}(1.0 - \frac{2.0d}{D}) \tag{3.1}$$

$$\phi = 2.0\cos^{-1}(1.0 - \frac{2.0 \times 2.0}{10.0})$$

$$\phi = 1.85 \text{ rad}$$

$$\Theta = 2.0\cos^{-1}(1.0 - \frac{2.0(y+d)}{D}) \tag{3.2}$$

$$\Theta = 2.0\cos^{-1}(1.0 - \frac{2.0(5.0 \times 2.0)}{10.0})$$

$$\Theta = 3.96 \text{ rad}$$

$$A_o = D^2(0.7854 - \frac{\phi - \sin\phi}{8.0}) \tag{3.3}$$

$$A_o = (10.0)^2(0.7854 - \frac{1.85-\sin(1.85)}{8.0})$$

$$A_o = 67.4 \text{ ft}^2$$

$$A_w = \frac{D^2}{8.0}((\Theta - \sin\Theta) - (\phi - \sin\phi)) \tag{3.4}$$

$$A_w = \frac{(10.0)^2}{8.0}((3.96 - \sin(3.96)) - (1.85 - \sin(1.85)))$$

$$A_w = 47.5 \text{ ft}^2$$

$$P_c = D(\Theta - \phi)/2.0 \tag{3.5}$$

$$P_c = (10.0)(3.96 - 1.85)/2.0$$

$$P_c = 10.6 \text{ ft}$$

$$P_b = D\sin(\phi/2.0) \tag{3.6}$$

$$P_b = 10.0\sin(1.85/2.0)$$

$$P_b = 8.0 \text{ ft}$$

$$R = \frac{A_w}{P_c + P_b} \tag{3.7}$$

$$R = \frac{47.5}{10.6 + 8.0}$$

$$R = 2.55 \text{ ft}$$

$$D_e = D(1.0 - \frac{\phi - \sin\phi}{2\pi})^{1/2} \tag{3.10}$$

$$D_e = (10.0)(1.0 - \frac{1.85 - \sin(1.85)}{6.2834})^{1/2}$$

$$D_e = 9.26 \text{ ft}$$

$$z = (\text{slope})(\text{length})$$
$$z = (0.02)(100.0)$$
$$z = 2.0 \text{ ft}$$

The friction factor for an impermeable bed will be estimated from equation 5.15.

$$\frac{1}{f^{1/2}} = 0.760 + 1.98\log(\frac{R}{d_{50}}) \tag{5.15}$$

$$\frac{1}{f_b^{\ 1/2}} = 0.760 + 1.98\log(\frac{2.55}{0.50})$$

$$f_b = 0.214$$

From the U.S. Department of Transportation (1980)

$$f_c = 0.045$$

The composite friction factor is

$$f = \frac{P_c f_c + P_b f_b}{P_c + P_b} \tag{5.26}$$

$$f = \frac{(10.6)(0.045) + (8.0)(.214)}{10.6 + 8.0}$$

$$f = 0.118$$

Rewriting equation 5.29 gives

$$Q = A_w(\frac{8gRS}{f})^{1/2}$$

$$Q = 47.5(\frac{8.0 \times 32.2 \times 2.55 \times 0.02}{0.118})^{1/2}$$

$$Q = 501 \text{ ft}^3/\text{sec}$$

<u>Calculation</u>: <u>Step 2</u>. Find critical depth.

$$\lambda = \frac{d}{D} = \frac{2.0}{10.0}$$

$$\lambda = \frac{d}{D} = 0.20$$

$$\frac{22.63Q}{g^{0.5}D^{2.5}} = \frac{22.63(501)}{(32.2)^{0.5}(10)^{2.5}}$$

$$\frac{22.63Q}{g^{0.5}D^{2.5}} = 6.32$$

From Figure 3.2 at d/D = 0.20 and $\frac{22.63Q}{g^{0.5}D^{2.5}} = 6.32$

$$\frac{y_c}{D} = 0.44$$

$$y_c = (10.0)(0.44)$$

$$y_c = 4.4 \text{ ft.}$$

Comparing y_n = 5.0 ft to y_c = 4.4 ft indicates the flow is subcritical.

$$\Theta_c = 2.0\cos^{-1}(1.0 - \frac{2.0(4.4+2.0)}{10.0}) \qquad (3.2)$$

$$\Theta_c = 3.71 \text{ rad}$$

$$A_c = \frac{(10.0)^2}{8.0}((3.71-\sin(3.71)) - (1.85-\sin(1.85))) \qquad (3.4)$$

$$A_c = 42.0 \text{ ft}^2$$

Calculation: Step 3. Assume type 2 flow occurs.

Calculation: Step 4. Determine the barrel losses. The velocities in the approach channel are negligible so

$$h_{f1-2} \simeq 0$$

and

$$\alpha_1 \frac{V_1^2}{2g} \simeq 0$$

For uniform flow, the barrel losses will be equal to the elevation drop, z.

$$h_{f2-3} = 2.0 \text{ ft}$$

By routing the flow through the barrel, the losses were calculated to be

$h_{f2-3} = 2.1$ ft

Calculation: Step 5. Determine the approach channel head, h_1.

Make an initial estimate of C. Then calculate h_1.

First trial, C = 0.88

$$Q = C\,A_c(2g(h_1 - y_c - h_{f2-3}))^{1/2} \qquad \text{(Figure 2.2)}$$

$$501 = (0.88)(42.0)(2.0(32.2)(h_1-4.4-2.1))^{1/2}$$

$$h_1 = 9.35$$

Check the estimated value of C.

$$\frac{H}{D_e} = \frac{h_1 - z}{D_e} = \frac{9.35 - 2.0}{9.26}$$

$$\frac{h_1 - z}{D_e} = 0.79$$

From Figure 4.7 at $H/D_e = 0.79$ and $\lambda = 0.20$, .

C = 0.86

Second trial, C = 0.86

$$501 = (0.86)(42.0)(2.0(32.2)(h_1-4.4-2.1))^{1/2}$$

$$h_1 = 9.49$$

$$\frac{h_1 - z}{D_e} = \frac{9.49 - 2.0}{9.26}$$

$$\frac{h_1 - z}{D_e} = 0.81$$

from Figure 4.9, C = 0.86. OK!

Calculation: Step 6. Check assumption for type 2 flow.

$$\frac{h_1 - z}{D_e} = 0.81 < 1.5$$

and $y_n > y_c$. Therefore, the assumption of type 2 flow is OK!

Performance Curves

The performance curves shown in Figure 7.1 compare a hypothetical DIC to a conventional culvert of the same size. The performance curves were developed from the same parameters as were used in the preceeding sample problem. The one exception is for the conventional culvert d = 0; thus $\lambda = 0$.

The performance curve for the DIC continues beyond the point shown in Figure 7.1. The upper end point of the curve indicates the approximate point of transition from type 2 flow to type 6 flow.

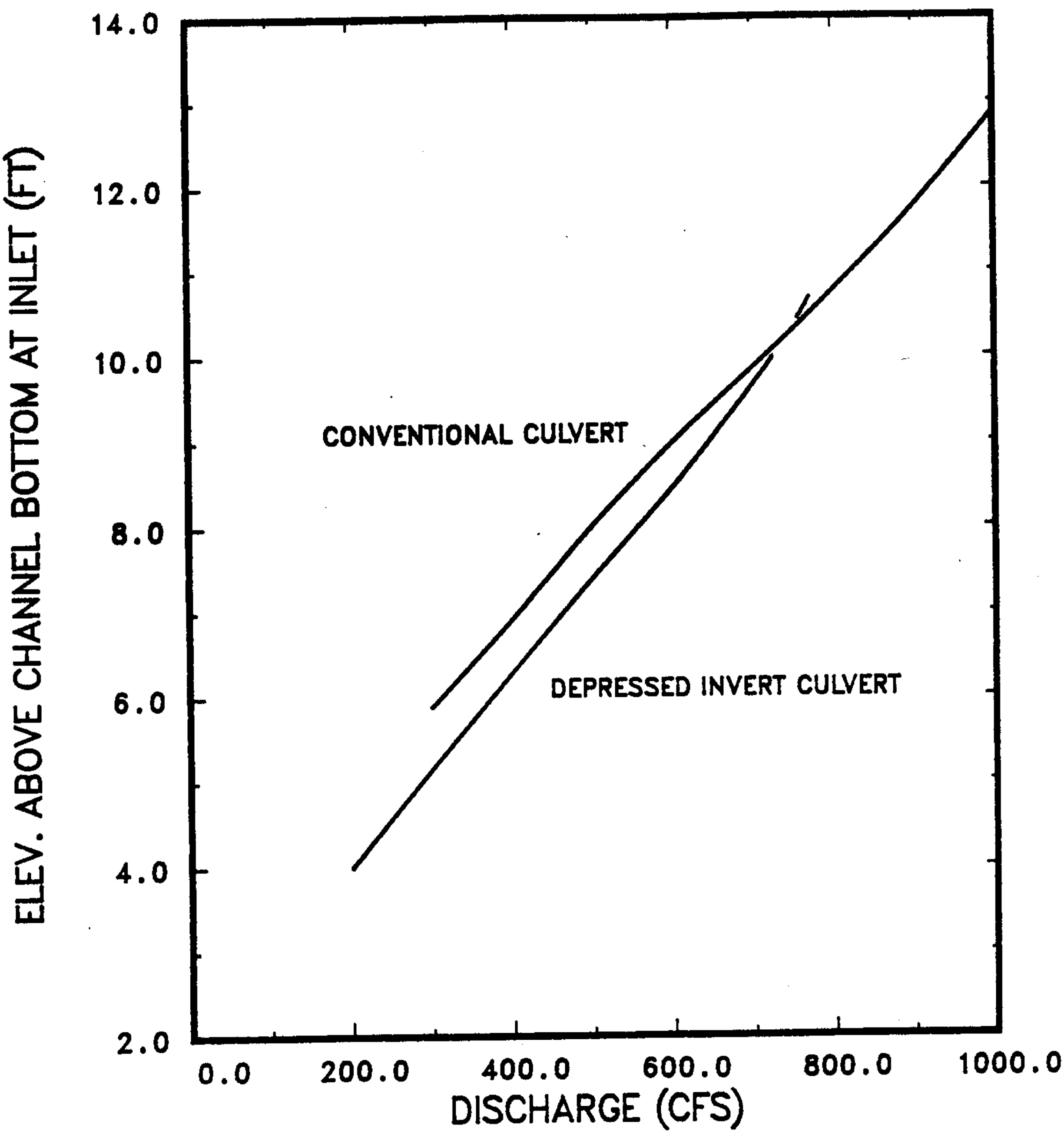


Figure 7.1. Performance curves for sample culverts.

8.0 CONCLUSIONS

General

This report gives an overall description of the hydraulic phenomenon associated with DICs. The information contained herein will allow a designer to calculate the hydraulic properties of a DIC under inlet control and set flush to a vertical headwall.

The methods and information given in this report can be used as a guideline for the additional research required to develop a complete design manual.

Recommendations for Future Research

1. Inlet Losses

This study examined the discharge coefficients for a DIC set flush to a vertical headwall. Additional studies should be conducted to determine the discharge coefficient of other commonly used entrance types.

The effect of culvert slope on the discharge coefficient for types 1 - 3 flow need to be studied. In addition, the entrance losses for type 4 and 6 flows should be investigated for all commonly used entrance types.

The results of this study show that the entrance losses are influenced by bed roughness. Unfortunately, no clear relationship is evident. Studies should be conducted to develop a relationship between the size (roughness) of the bed material and entrance losses.

2. Barrel Losses

The results from gravel bed river studies were used to estimate f and n of the bed material within a DIC. Several problems with this approach were discussed in Chapter 5. Studies should be conducted using a DIC model to determine f and n for bed material of various sizes.

Several methods for combining f or n in composite channels were presented in Chapter 5. Their applicability to DICs should be verified by actual tests.

The occurrence and properties of free surface flow over a permeable bed within a DIC should be further investigated.

3. <u>Velocity Profiles</u>

As the flow within a culvert approaches full flow conditions (pipe flow), a one-dimensional, velocity profile model for free surface flow will be inadequate. A two-dimensional velocity profile model should be developed and tested for its applicability to DICs. In addition, a velocity profile model that accounts for the slip velocity associated with a permeable bed should be investigated.

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APPENDIX

PARTICLE SIZE DISTRIBUTION CURVE
OF ROUGHNESS ELEMENTS

PARTICLE SIZE DISTRIBUTION CURVE

OF ROUGHNESS ELEMENTS