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16. Abstract

Three aspects of slurry wall construction are considered, These are (1) the alteration of the initial stress state in the soil adjacent to the wall by the trench excavation and support process, (2) the further alteration of the initial stress condition for a cast-in-place wall by placement of the high density concrete slurry and subsequent shrinkage of the slurry upon set-up, and (3) the modification of the soil structure interface due to the impregnation of slurry upon set-up, and (3) the modification of the soil adjacent to the wall and the formation of a slurry cake between the soil and the wall.

Analytical methods, including finite element analysis and associated fields, for modeling slurry wall construction are reviewed and evaluated. The problems associated with insufficient field data, behavioral models, failure criteria, yield criteria, initial conditions, and boundary conditions are described. A research program for improving slurry trench technology is proposed.

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INTRODUCTION

The slurry trench cutoff and wall construction technique developed and popularized in Europe is finding increased usage in the United States. It seems to be particularly suited for application in construction of excavation support walls (52)(53) and permanent walls for basements of large buildings, underground structures and tunnels (52)(53)(82)(73). Interestingly, the slurry wall has been developed largely through the innovative ideas of contractors and construction engineers. The realization of the system has, in fact, preceded the development of consistent procedures to analytically model the system.

The need for modeling procedures lies in our need to predict the performance of a slurry wall and the adjacent soil medium and to more fully understand the mechanisms behind their behavior. By the nature of the problems for which the slurry wall is adapted, predictions are particularly critical. For example, advance knowledge that small movements may be expected behind a slurry wall can result in elimination of underpinning of adjacent buildings, yielding substantial construction economies. Alternatively, predictions of movement patterns can help design underpinning where large movements are expected.

Probably the most predominate approach to prediction of the performance of the slurry wall system is through conventional limit theory and empirical data collected on similar systems. Recently more sophisticated approaches such as the finite element method have also been adapted to the slurry wall (14)(93). Like marriage, the more sophisticated methods are not inherently better than other alternatives. Lambe (1973) has demonstrated that simpler techniques have sometimes yielded more accurate predictions. However the fact is that only more sophisticated methods can predict deformations for slurry wall systems with a minimum of simplifying assumptions, and in view of the importance of deformations to slurry wall performance the sophisticated methods deserve detailed study. It is also likely that results from such studies will generate improvements in the simpler and more accessible techniques. Thus this paper will be devoted primarily to a discussion of the problems of applying the finite element method to the slurry wall and of means for solution of these problems. A brief review is also provided of another promising analytical technique, the associated fields approach (51)(95), which has some of the strengths of the finite element method and should be of value after it is more fully developed.

THE SLURRY WALL PROBLEM

Before considering analytical methods for the slurry wall, it is useful to review the nature of the slurry wall problem and define those areas where special difficulties arise which are unique to the slurry wall.

The slurry wall is usually built following the sequence shown in Fig. 1. A trench about 10 to 15 feet long is excavated to the full depth of the wall with a bentonite slurry or bentonite grout used to keep the trench open. Next a reinforcing cage is dropped into the slurry if the wall is to be cast-in-place, or alternatively a finished precast wall panel may be dropped into the slurry. For the cast-in-place wall, concrete is subsequently tremied into the trench to force the slurry out and form the wall panel. In the meantime a second trench has been or is being excavated one panel length away. This wall panel is then constructed; the panel in-between is not built until the concrete or bentonite grout sets up in the first two panels. This process is repeated to form the entire length of the wall.

There are many variations as to the means utilized to connect the panels together. Several are shown in Fig. 2. In some cases preaugered H-piles or prefabricated "T" or "H" sections are set at the ends of the trenches so that the wall panels are keyed into the flanges of these elements. Other methods use tongue and groove paneling or removable forming elements which shape the ends of the panels so as to receive the ends of the alternately built panels.

Three facets of this construction process serve to complicate the analysis of a slurry wall: (1) the alteration of the

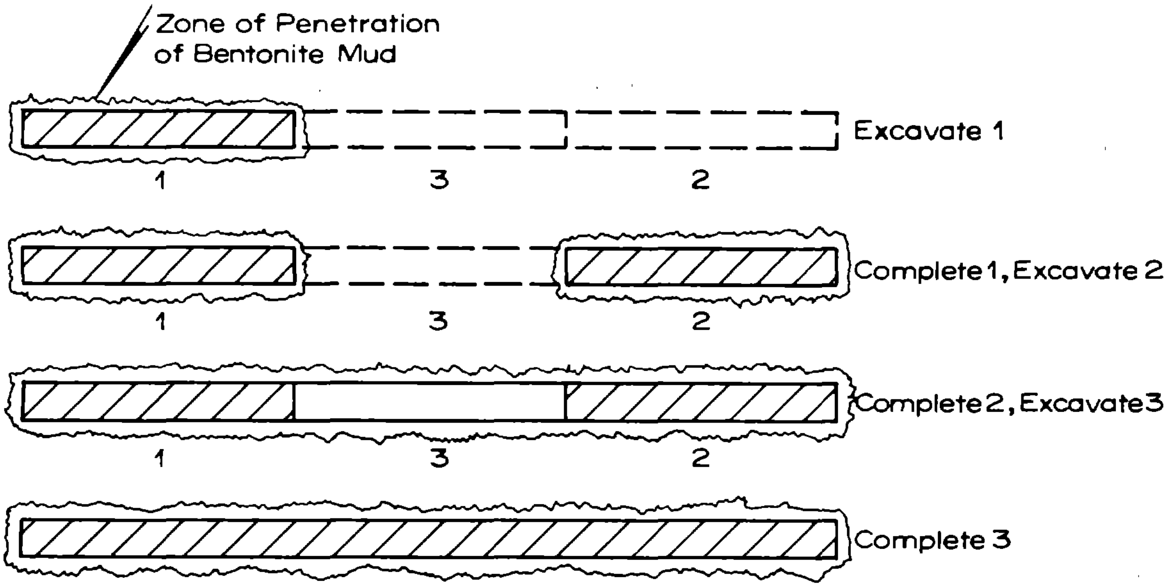


Figure 1. Alternate Panel Slurry Wall Construction

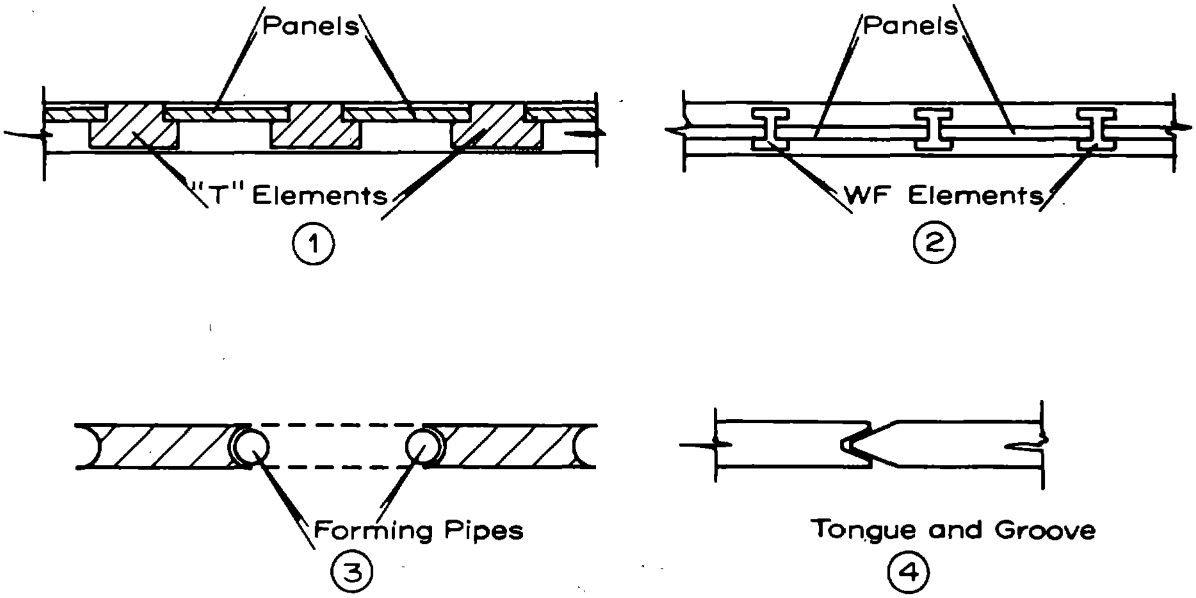


Figure 2. Alternate Methods for Connecting Adjacent Panels

initial stress state in the soil adjacent to the wall by the trench excavation and support process; (2) the further alteration of the initial stress condition for a cast-in-place wall by placement of the high density concrete slurry and the subsequent shrinkage of the slurry upon set-up; and, (3) the modification of the soil-structure interface due to the impregration of slurry in the soil adjacent to the wall and the formation of a slurry cake between the soil and the wall (see Fig. 1). The significance of these factors will be discussed in a subsequent section of this paper.

With the completion of the slurry wall the intended excavation may be completed in front of the wall. The walls may support the excavation without additional reinforcement, or internal bracing or tie-backs may be added as the excavation proceeds. Subsequently the walls may be incorporated as a part of the structure or abandoned.

Analyses of the slurry wall problem should be addressed to the questions of trench stability, wall design and design of additional wall support systems. In this regard primary design criteria are: (1) factor of safety against trench failure; (2) maximum moments and shears in the wall; (3) stresses in bracing elements; (4) factor of safety against anchorage failure for tie-backs; and, (5) deformations of the wall and the soil retained behind the wall.

ANALYTICAL MODELING TECHNIQUES - A REVIEW

Conventional methods used for analysis of the slurry wall attack the design of each wall problem almost independently. For example limit theory is used to determine the slurry trench stability (45)(54)(65)(69); moments and shears in the wall and stresses in the braces are determined from assumed pressure diagrams (72) or beam on elastic foundation theory (5); anchorage stability is analyzed as separate problem by limit theory (39)(92); and soil deformations are estimated on the basis of experience, elastic theory or one-dimensional consolidation theory (72). If the walls are subsequently incorporated into a tunnel or underground structure the pressures on the structure are generally estimated from past experience with similar types of problems (37).

The value of these approaches lies in their simplicity and accessability. They will undoubtedly continue to serve as the primary design vehicle for many systems. However this does not mean that improvements in design methods are not forthcoming. Advances have recently been made in the application of lower limit theory (64), slab on elastic foundation theory (41)(88), associated fields theory (51)(75)(80) and finite element theory (11)(70)(93) to retaining structures. Of these newer methods the associated fields approach and the finite element method yield the most information and have the potential to provide a single, integrated approach to design of the slurry wall.

Associated Stress and Velocity Fields

The concepts involved in this approach have only recently been developed and it is not ready for use in design. Ongoing research, however, is being devoted to the method, primarily at

Cambridge University, with the aim of overcoming present practical and technical difficulties. Applications thus far have been limited to simple retaining structures (80)(51). The method offers to the geotechnical engineer one means for obtaining detailed information on the stress and strain distribution in the soil mass and on the stress distribution on the structure.

The basis for the method has been described by Roscoe (1970), Wroth (1972), James, et.al. (1972), and Serrano (1972). The approach utilizes finite difference techniques to develop stress and strain fields which are everywhere compatible with specified stress-strain relationships, assumed relationships for development of soil strength, and assumed failure criteria if failure occurs. The stress fields are developed for specified structural loadings in accordance with a modified Sokolovski (1965) technique. Strain fields are calculated from the principal stress directions indicated by the stress field, the stress-strain relationship, and velocity field equations originally developed by Shield (1953). In order to develop compatible stress and strain fields an iterative solution technique is employed. This process must be repeated for each increment of structural loading.

The method would appear to have potential on the study of the slurry wall, but a number of fundamental questions remain to be resolved; e.g., how to treat non-homogeneous soil conditions, how to suitably define boundary conditions in complex problems to allow the Sokolovski stress field technique to be used, and how to account for effects of construction sequence. In addition the question of practicality of the method remains to be answered. A knowledge of some rather esoteric theoretical tools is required, including the Sokolovski stress field technique, the Cambridge approach to plasticity of soils, the upper limit velocity field concept of Shield. Thus it would appear that accessibility will be limited but this does not rule out future use of the method as a research tool for study of slurry walls.

The Finite Element Method

The finite element method is further developed than the associated fields approach and is already finding widespread application in analysis of soil-structure interaction problems. A description of the published applications is shown in Table 1 from which it can be seen that the problems range from simple retaining walls to navigation locks and braced and anchored walls. Slurry wall analyses have been reported by Wong (1971) and Clough and Tsui (1973).

Theoretical bases for the finite element method are well established (15)(96) and will not be reviewed here. The fundamental theory for the finite element method is that embodied in the stiffness approach to indeterminate structures a theory widely studied in Civil Engineering. Thus the basics of the method are accessible to most Civil Engineers.

The primary advantage of the finite element method in modeling a slurry wall is that detailed information is obtained on both stresses

TABLE 1

Soil-Structure Interaction Problems To Which The Finite Element Method Has Been Applied

Type of Problem	Reference(s)
Simple retaining structures	(6)(12)(17)(68)(81)
Navigation locks	(11)
Braced walls	(3)(14)(70)(90)(93)
Anchored walls	(13)(33)
Bulkheads	(3)
Bridge piers	(67)
Cut and cover tunnels	(14)

and deformations in the wall and the soil medium. This information may be obtained using modeling procedures which can realistically simulate the wall construction process and complex soil conditions and behavior. The use of the method in analysis of a slurry wall problem is demonstrated in the following section.

USE OF THE FINITE ELEMENT METHOD FOR MODELING THE SLURRY WALL

The procedure which must be followed before a finite element analysis may be conducted is shown in Fig. 3 in the form of a flow chart. Each step in this procedure represents an idealization of the actual problem and introduces assumptions into the analysis.

The first two steps are common to all analytical modeling and involve development of a conception of the problem and the idealized soil profile and groundwater condition. Next behavioral models are chosen for the soil, structure and interface between the structure and soil. These models idealize the behavior into some mathematically expressable form, i.e., elastic, elasto-plastic, etc. The soil model and the interface model are the most difficult to define, a problem which is discussed in depth in the next section of the paper.

When the models are defined parameters must be selected, a selection process always difficult for the soil media because of inherent scatter in test results. Then initial stress conditions must be defined for all of the media, a task which is more difficult

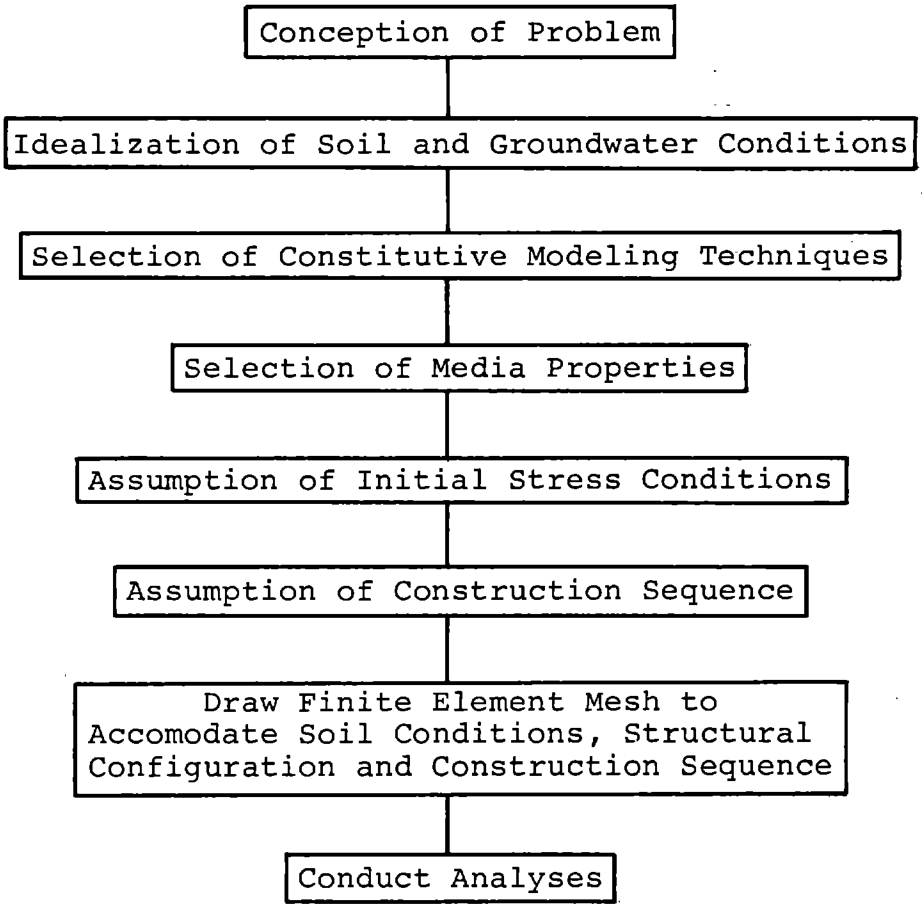


FIGURE 3. PROCEDURES LEADING TO FINITE ELEMENT ANALYSIS.

for overconsolidated soils and clay-shales than other soil types. Finally the construction sequence to be modeled is assumed, a finite element mesh is drawn and the analyses may be undertaken. The last two steps deserve further comment.

Modeling a Construction Sequence. The construction sequence can have an important influence on the predicted behavior. Terzaghi (1936) demonstrated this influence in the braced excavation problem showing that if the sequence were not modeled in the analysis erroneous earth pressures and deflections would be predicted. More recently Clough and Woodward (1967) have shown that backfill settlements are directly related to the construction sequence and Lambe (1970) has described the importance of modeling the effects of dewatering in predicting braced excavation performance.

Simulation of construction procedures in a finite element analysis is best accomplished by dividing the loading sequence into small increments, analyzing for the effects of each increment in sequence, and superimposing the results of each increment in the preceding results to obtain the resultant stress and displacement conditions. The construction sequence for a slurry wall involves trench excavation with slurry replacement, concrete tremie for cast-in-place walls, concrete and/or bentonite grout set-up, excavation in front of the wall possibly accompanied by dewatering and recharging and installation of internal bracing or tiebacks. A schematic representation of the construction sequence followed on building a cut and cover tunnel which employs slurry trench walls is shown in Fig. 4. Techniques for simulation of all of these effects in finite element analyses have been reported (11)(13)(70)(93). However questions remain as to the effects of concrete tremie for cast-in-place walls and subsequent set-up of the concrete. For example during tremie does the concrete partially set-up or does the full hydrostatic pressure of liquid concrete act over the entire slurry trench depth?

Constructing a Finite Element Mesh. The finite element mesh should reflect the soil conditions, the structural configurations and the construction sequence. The mesh must be refined in areas of expected stress concentrations and enough elements must be included in the structural elements so that adequate flexibility is assured. An example of a mesh for an analysis of the cut and cover tunnel is shown in Fig. 5.

Only one-half the problem is represented on the mesh because the problem is symmetrical about the tunnel centerline. It can be seen that the mesh is drawn to accomodate all of the structural features which eventually enter the problem. However in the simulation process the elements which eventually become structural components, such as those representing the top of the tunnel, follow a change in material properties to reflect the different roles required in the construction simulation. In the first instance the elements have properties reflecting soil stiffnesses, subsequently they are excavated and are assigned very low stiffnesses, and finally they are reassigned concrete properties when the simulation involves

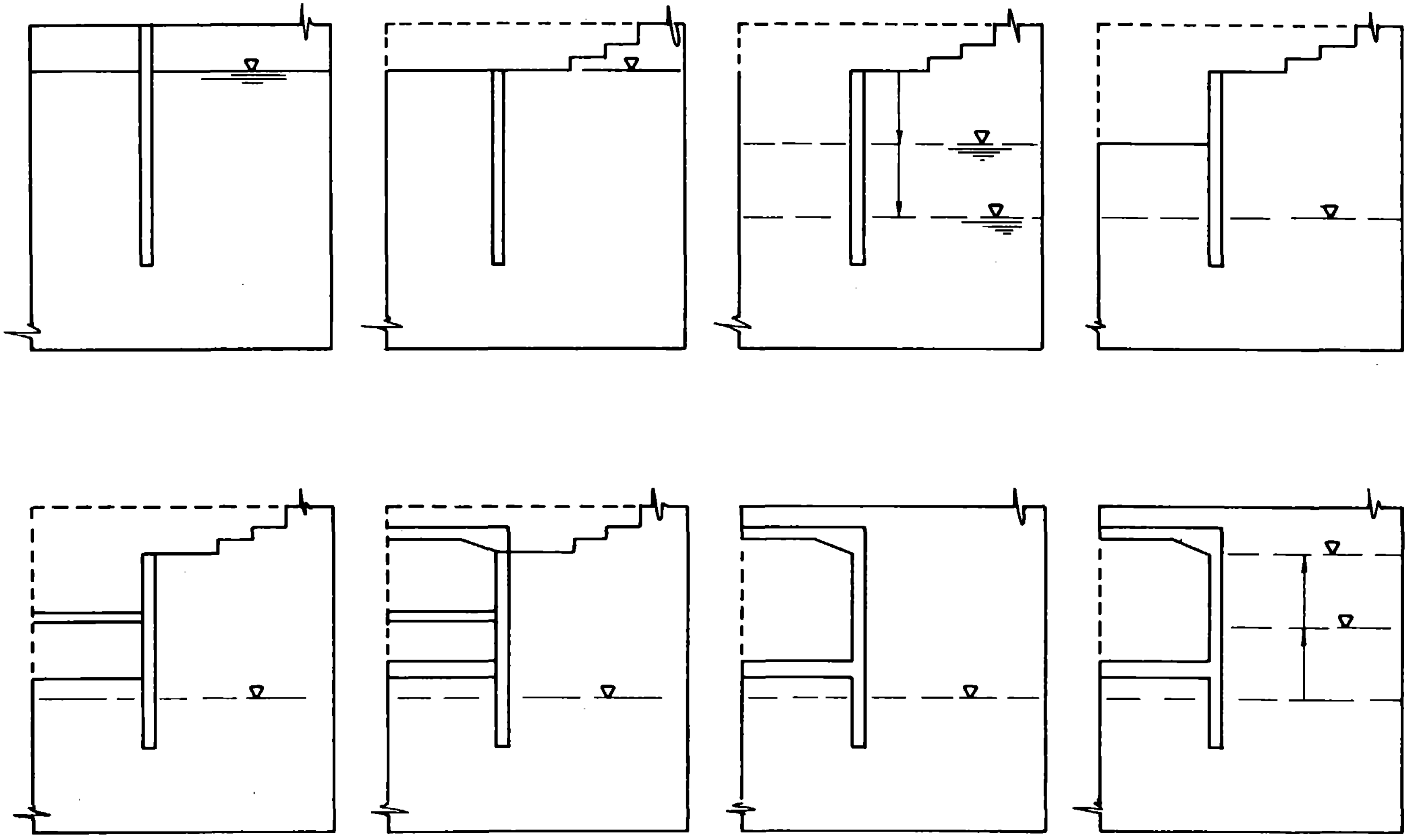


Figure 4. Construction Sequence for Cut and Cover Tunnel

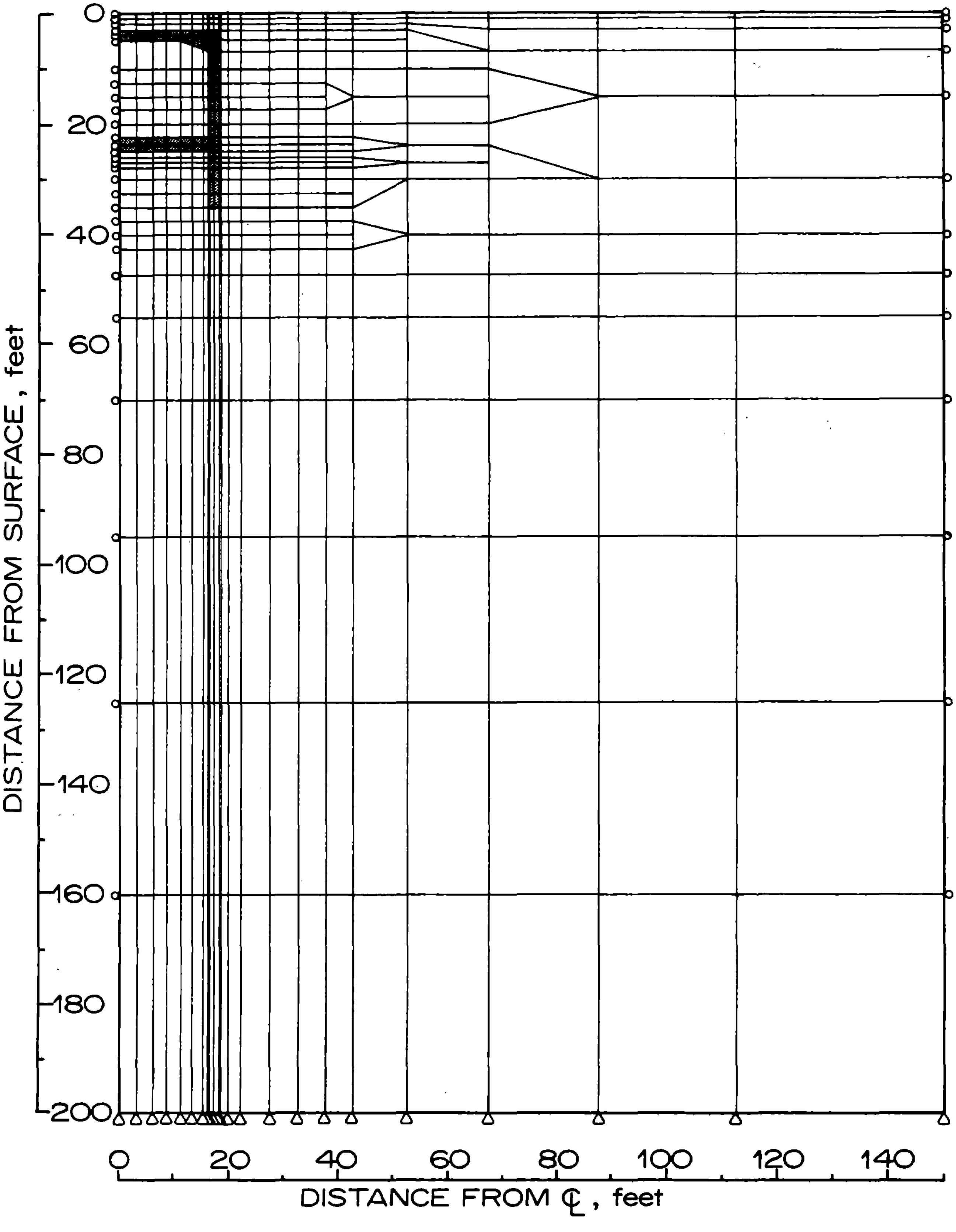
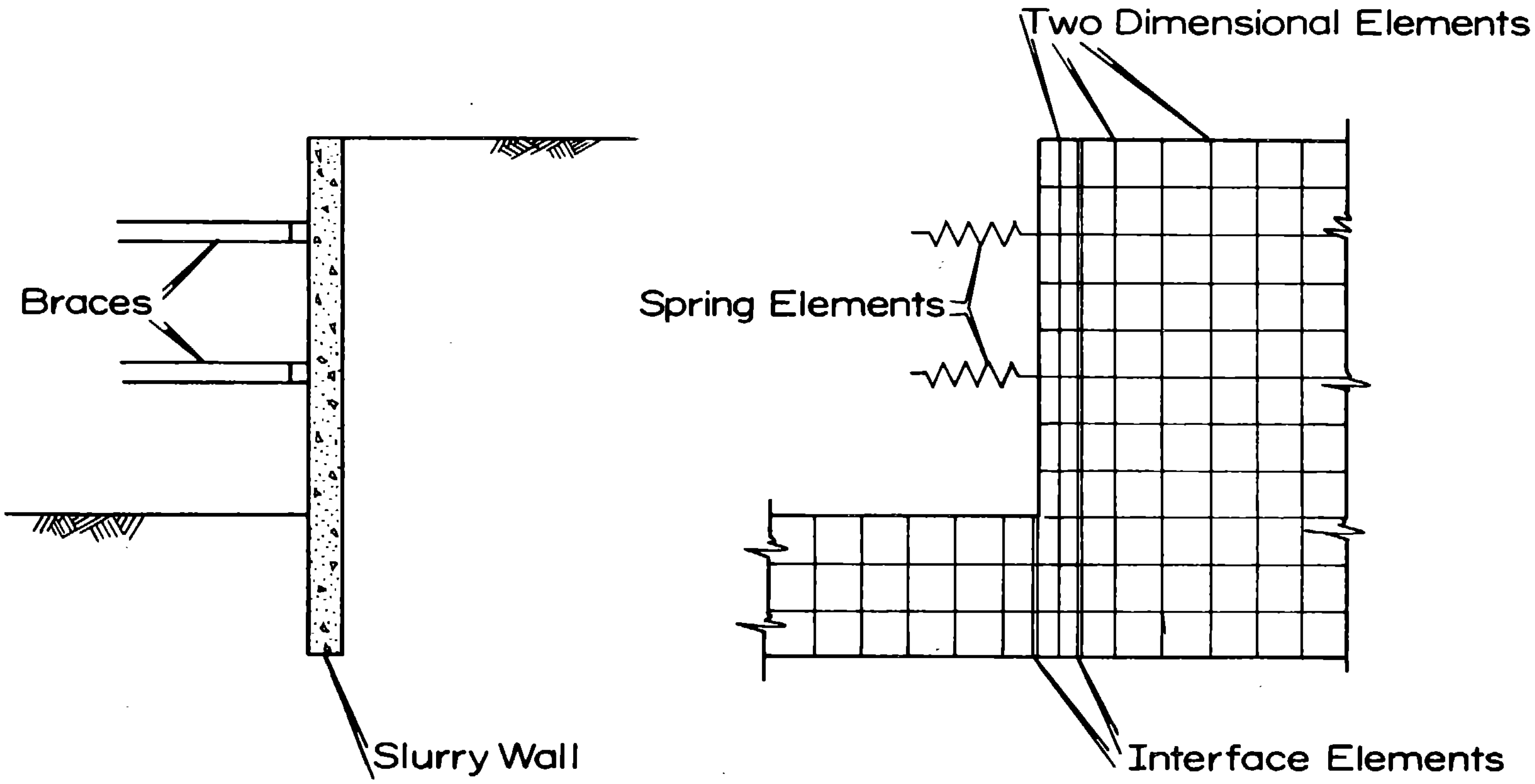


Figure 5. Finite Element Mesh for Cut and Cover Tunnel Analysis

building the top of the tunnel.

Types of Elements. The mesh shown in Fig. 5 contains two different types of finite elements and during simulation of the presence of the internal brace a third is added. The three element types are shown in Fig. 6; two of the elements are one-dimensional while the third is two-dimensional. The two dimensional elements in the mesh are used to represent the media, one dimensional slip elements on either side of the wall are used to represent the interface between the wall and the soil, and a one dimensional truss element is used to represent the presence of the brace. The use of different types of elements adds greatly to flexibility in simulation of the different aspects of the problem.

In the case of the two-dimensional elements many different versions have been derived (15)(26)(35)(96). Earlier elements used simple assumptions for strain distribution within the element, such as the constant strain element first developed by Clough (1960). More recently linear strain, quadratic strain and quintic strain distributions have been used (35)(96) all of which give an element an increasing degree of flexibility, a most desirable characteristic in modeling soil and structural behavior. Unfortunately the higher order elements have correspondingly larger degrees of freedom and require greater computer costs for a solution. Most often therefore the higher order elements have not been used; however, an attractive compromise to this problem has been reported by Doherty, Wilson and Taylor (1969). These investigators developed an element, entitled



BRACED SLURRY WALL

FINITE ELEMENTS-Used to
Simulate Slurry Wall Problem

Figure 6. Types of Finite Elements Used in Slurry Wall Analysis

the QM5, which has a completely linear strain distribution within the element but a constant strain distribution along the element boundaries. Although there is an incompatibility between the two assumptions the violations of compatibility were carefully selected so that the response of the element is in most cases far superior to other lower order elements. Further, no additional degrees of freedom are needed for the element over the lower order elements and thus the computing costs are not increased.

One-dimensional slip elements are essential in order to allow for controlled relative movements on the interface between the wall and the soil. The nature of the interface behavior to be modeled by the slip element can be determined from a direct shear test where the soil is sheared over the wall material. Results for such a test between sand and concrete are shown in Fig. 7 plotted as shear stress on the interface, τ, versus relative displacement, Δ_s. The curves are nonlinear, depend on the value of normal stress, and show a hysteresis effect on unloading.

For the one-dimensional slip element the slip response is characterized by the equation

$$\tau = k_s \Delta_s$$

in which k_s is the shear stiffness, or the slope of the curves shown in Fig. 7. Obviously there is no one k_s value for the curves in Fig. 7 and in order to realistically model the observed behavior a nonlinear response must be derived. Clough and Duncan (1971) report an analytical formulation for this purpose and demonstrate

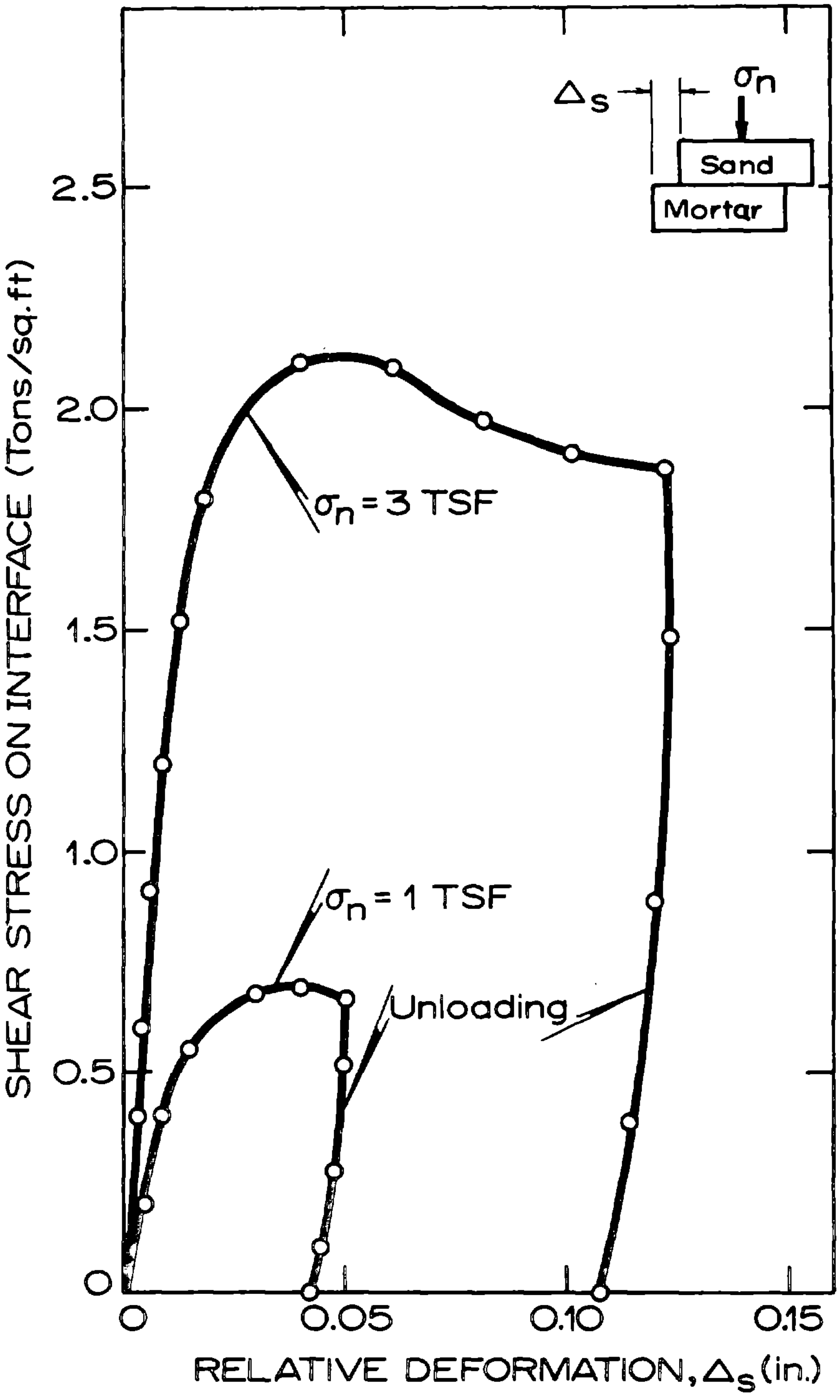


Figure 7. Interface Test Results, Sand on Mortar

the successful simulation of interface behavior using it. For the slurry wall the problem of interface representation lies in our lack of knowledge of the deformation behavior of the slurry wall-soil interface separated by a slurry cake.

Example. In Figs. 4 and 5 the finite element mesh and construction sequence for a slurry wall cut and cover tunnel was shown. The results of the analysis of this example, reported by Clough and Tsui (1973), may be used to illustrate the type of information obtained from the finite element method. The analysis was performed using a nonlinear elastic soil model similar to that described by Duncan and Chang (1970) in which the tangent modulus is calculated from a hyperbolic stress-strain curve during primary loading and a straight line unload-reload curve should the shear stresses in an element decrease during an increment of loading. The properties for the sand are shown in Table 2.

No attempt was made to simulate the excavation of the slurry trench and instead a simple at-rest condition with $k_0 = 0.5$ was assumed with the wall in place. The interface between the wall and the soil was assumed to be influenced by the presence of slurry left between the wall and the sand, so that its friction angle was one-half that of the adjacent sand and its shear stress - deformation correspondingly modified.

Deflections predicted for the wall during various phases of construction of the tunnel are shown in Fig. 8. Most of the deflection occurred during excavation down to the brace level. At that stage the wall deflection was enough to reduce the wall pressures

TABLE 2

Soil Properties Reported for Sand Subsoil in Cut and Cover Problem

Unit Weight Above Water Table - 125 lbs./cu. ft.

Friction Angle - 35 Degrees

Poisson Ratio - 0.2

Coefficient of Earth Pressure At-Rest - 0.5

Cohesion - o

* Modulus Exponent, n - 0.5

* Modulus Number, K - 280

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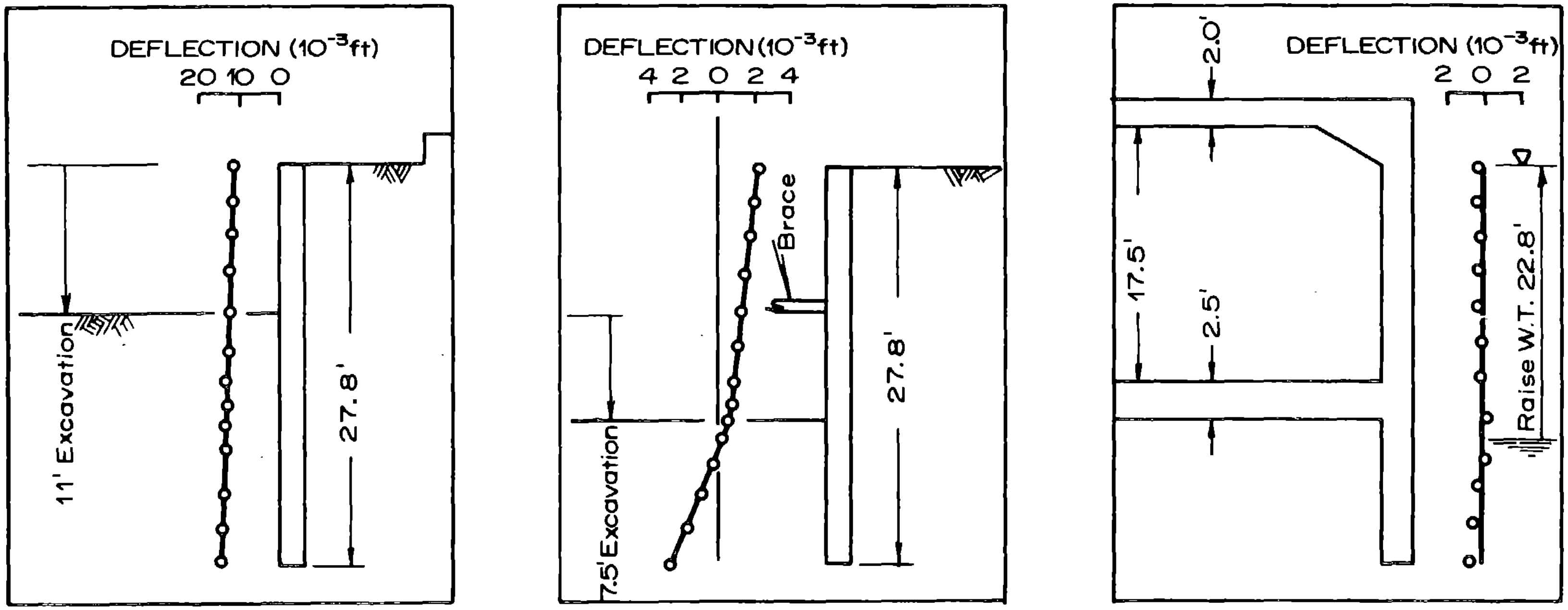
*Initial tangent modulus, E_i, assumed to vary with minor principal stress, σ_3 as: $E_i = K(\sigma_3)^n$.



Wall Deflection Due to Excavation to Brace Level

Wall Deflection Due to Excavation Below Brace Level

Wall Deflection Due to Raising Water Table With Top and Bottom Slab In-Place

Figure 8. Incremental Wall Deflection Patterns for Cut and Cover Tunnel [After Clough and Tsui (1973)]

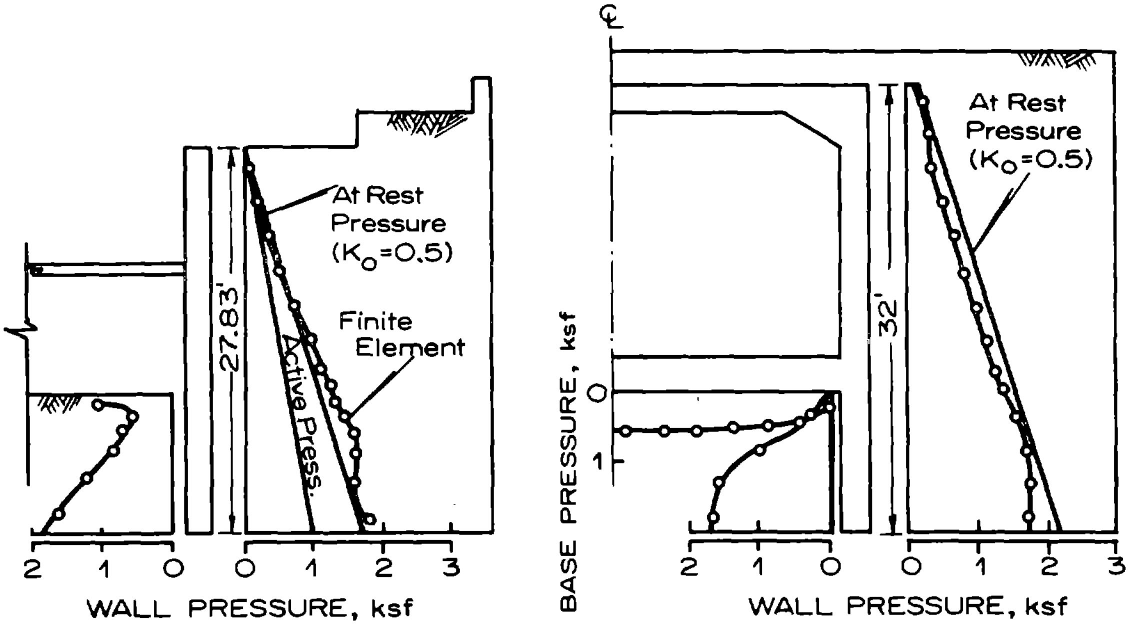
to a near active condition for the portion of the wall above the brace level (see Fig. 9). Although the later wall deflections of the wall were small it can be seen the modes for the subsequent behavior were reasonably defined, e.g., for excavation after placement of the brace the wall rotated around the brace point with only the bottom moving away from the soil being retained.

Effective earth pressures on the wall and base slab are shown in Fig. 9 for various construction stages. The variations in wall pressure are consistent with the wall deflections. Note the decrease in effective wall earth pressures caused by bringing the water table back to its original level.

The base pressures are particularly interesting in that after completion of construction the resultant of these pressures are less than the weight of the overlying material. The difference between these forces lies in the friction developed along the slurry walls during the build-up of the slab and top cover. In essence the slurry walls are acting as piles for the tunnel.

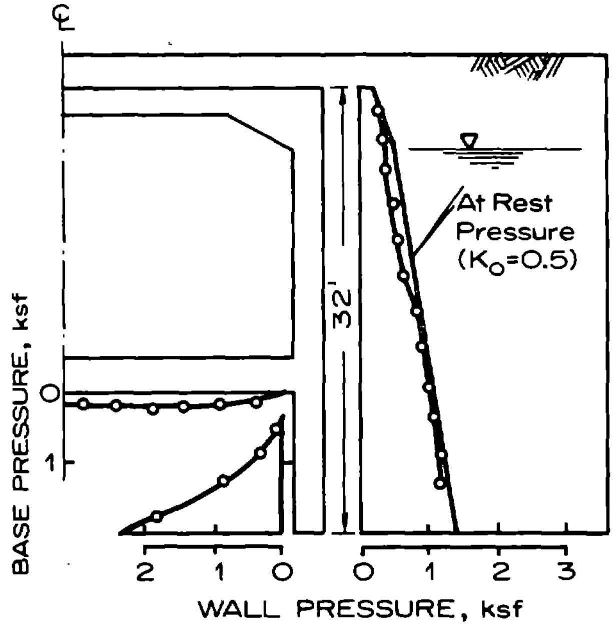
PROBLEMS IN ANALYTICAL MODELING OF THE SLURRY WALL

The preceding example demonstrates that much of the technology exists to apply the finite element method to the slurry wall problem. However many assumptions were made in order to conduct the analysis and it is primarily in these assumptions that the problems



Effective Earth Pressures at Completion of Excavation

Effective Earth Pressures at Completion of Structure and Backfill Above Structure



Effective Earth Pressures Following Re-establishment of Water Table

Figure 9. Effective Earth Pressures Acting on Cut and Cover Tunnel For Various Constructed Stages [After Clough and Tsui (1973)]

of modeling the slurry wall lie. These problems may be generally stated as follows:

1. Lack of evidence that existing soil behavioral models can adequately simulate soil behavior in the slurry wall problem.
2. Lack of data on the effects of slurry wall construction on the initial stress condition (assuming that adequate information on the original initial stress condition can be obtained).
3. Lack of information as to the deformation and strength behavior of the slurry modified interface between the structure and the soil.
4. Lack of proof that a plane strain analysis accurately simulates a braced excavation.
5. Lack of information as to the magnitude of the effect of environmental factors which are not included in the analysis.

The following discussion sets these problems in the context of their relation to the finite element method, but development of solutions to them is important for even the simplest analytical approach.

Behavioral Models for Soil

A behavioral model involves specification of a failure criterion, a yield criterion, and constitutive equations. Each of these items has been subjected to intensive research particularly in the past 10 or 15 years; it is beyond the scope of this paper to review these efforts in detail. Instead the interested reader is advised to consult Bishop (1966), Scott and Ko(1969), Roscoe (1970) and Wroth (1972).

One fact which can be said with certainty is that none of the problems of behavioral modeling of soils has been completely solved at least from a theoretical standpoint. However, this has not prevented employment of useful approximate behavorial models in finite element analyses which in many cases yield reasonable predictions of performance. The following paragraphs therefore discuss the problems of behavioral modeling with the idea of identifying certain areas where major problems exist which if solved could provide acceptable practical answers for analysis of slurry walls.

Failure Criteria. There are many fundamental questions to be answered before a correct failure criterion is developed for soils. Areas not fully understood are: (1) the effect of the intermediate principal stress (4)(38)(79); (2) anisotropy (1)(29)(83); (3) long term loss of strength (47)(82). However, most evidence suggests that for practical problems the Mohr-Coulomb failure criterion serves as a satisfactory model, particularly if the Mohr-Coulomb strength parameters are normalized from test results appropriate to the problem at hand (4)(38). Thus of all the aspects of soil behavior, failure criteria would seem to be the most well established and require the least research in order to be adapted to the slurry wall problem.

Yield Criteria. Initially, in the introduction of the theory of plasticity into soil mechanics yield criteria and failure criteria were thought to be the same. Subsequent research has demonstrated that this is not true (30)(78) and has led to the introduction of separate yield and failure criteria for soils. The definition of a yield surface is important in that it establishes the boundary between widely different modes of behavior, that of the plastic and elastic regions.

One of the more popular yield surfaces is the so-called capped surface, an example of which is shown in Fig. 10. The yield surface in Fig. 10 was determined from the work of Roscoe and Burland (1968) and represents a yield criterion for the drained behavior of a clay. Similar surfaces have been proposed for sands (78)(81). Capped yield surface models have been employed in finite element analyses by Christian (1966), Smith (1971) and Simpson and Wroth (1972).

The capped yield surface offers two advantages: (1) Plastic strains can occur even under hydrostatic stress increases as per actual soil behavior; and, (2) Volumetric strains predicted when the normality principle is assumed to apply are reasonable. Detailed discussion of these concepts is beyond the scope of this paper; interested readers should consult Drucker, Gibson and Henkel (1957), Schofield and Wroth (1968), Roscoe and Burland (1968), Roscoe (1970) and Scott and Ko (1969).

Unfortunately the established validity of the capped yield surface concept is somewhat limited. Most of the experimental evidence has been of a rather simple type wherein the loading and unloading is carried out on the same stress path. Even for this case Roscoe and Burland (1968) note that if unloading proceeds to less than 50 percent of the previous loading then subsequent deformations

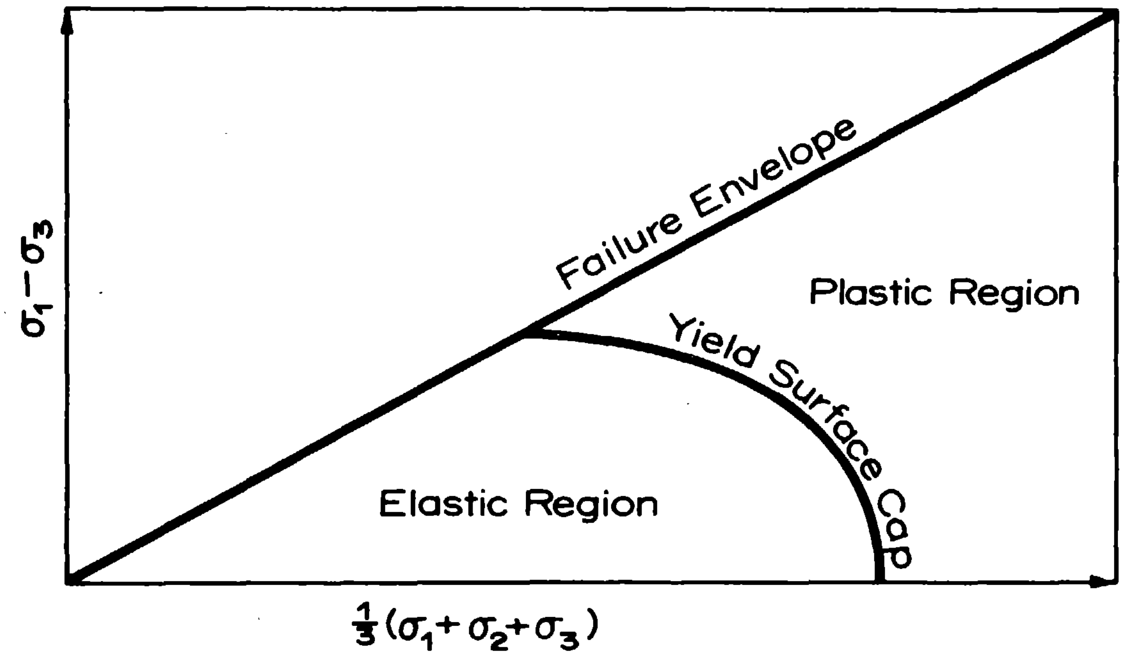


Figure 10. Capped Yield Surface Predicted by Modified Cam Clay Model [After Roscoe and Burland (1968)]

will not be elastic and the yield surface must be redefined. For stress paths in which the reloading does not follow the unloading path Ko and Scott (1968) and Audibert (1971) have shown that clearly inelastic behavior occurs beneath the capped yield surface.

Obviously problems remain in the definition of a yield surface for soils. These are significant for slurry wall modeling in that the stress paths followed during its construction and the subsequent excavation are complex and often result in various forms of unloading and reloading. An example is shown in Fig. 11, a plot of stress paths for various elements in a finite element analysis of a tied back slurry trench wall. For most of these elements there is a cyclic type of stress path where loading occurs during excavation and unloading along a new stress path occurs during prestressing the anchors. If dewatering should also occur additional complexities would be added. A suitable yield surface concept must be developed to ascertain where plastic and elastic behavior occurs under such conditions.

Time Independent Constitutive Relations. New constitutive models have proliferated with the introduction of the finite element method into geotechnical engineering. A compilation of recent models is shown in Table 3. Eighteen of these models have been proposed since 1967, and few have been subjected to close scrutiny. Most often the model predictions are checked against the results of simple monotonic loading triaxial tests and even then comparisons as to volumetric strain predictions are not made. Validation for complex

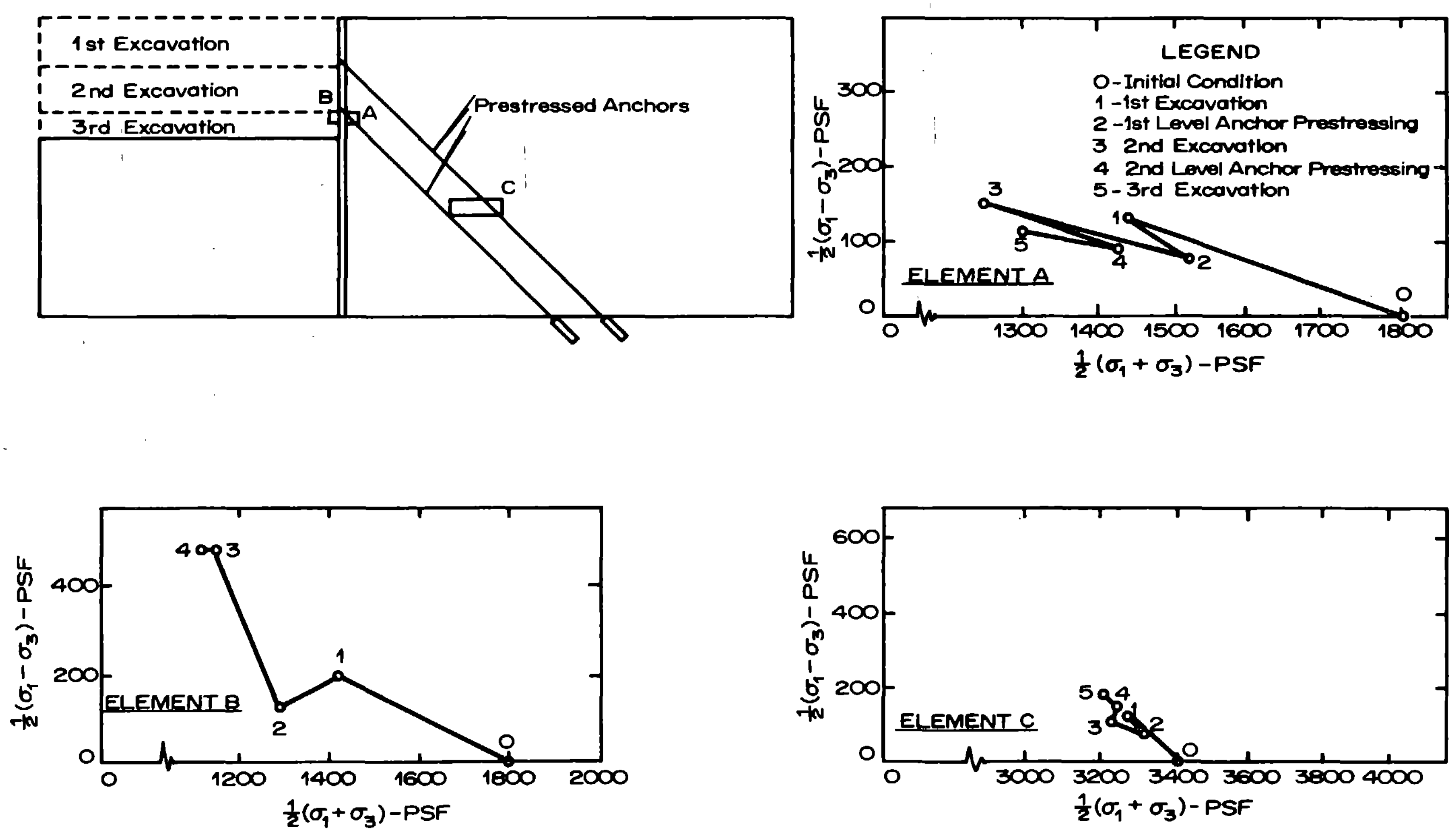


Figure 11. Stress Paths for Selected Elements in an Anchored Slurry Wall

TABLE 3

Proposed Models for Soil Behavior

Author(s)	Date	Type of Analytical Model
Drucker & Prager	1952	Elasto-plastic material
Kondner	1963	Nonlinear elastic hyperbolic fitting of stress-strain curves.
Kondner & Zelasko	1963	
Wroth & Bassett	1965	Stress-dilatancy model.
Christian	1966	(1) Elasto-plastic material (Drucker Prager yield criterion), (2) Strain hardening material, elliptical cap yield surface.
Clough & Woodward	1967	Pseudoelastic model (bulk and deviatoric moduli).
Holubec	1968	Anisotropic elastic model for unloading
Dunlop, Duncan & Seed	1968	Bilinear elastic model.
Girijavallabhan & Reese	1968	Psuedoelastic model using secant values of E and ν.
Roscoe & Burland	1968	Strain hardening material, elliptical yield surface cap (clays only).
Ellison	1968	Quilinear elastic model (E, ν).
Dibaj & Penzien	1969	Elasto-plastic, strain hardening Drucker Prager model.
Domaschuk & Wade	1969	Pseudoelastic model (bulk and deviatoric moduli).
DiMaggio & Sandler	1970	Elasto-plastic, strain hardening material, elliptical cap yield surface (sands only).
Duncan & Chang	1970	Pseudoelastic model, hyperbolic fit to stress-strain curves (E, ν).
Weidler & Paslay	1970	Elasto-plastic model, Mises yield criterion.

Table 3 (continued)

Author(s)	Date	Type of Analytical Model
Smith & Kay	1971	Stress-dilatancy model (sands only).
Hagmann	1971	(1) Elasto-plastic Drucker Prager material (dilatant); (2) non-dilatant elasto-plastic material; (3) strain hardening material, elliptical cap yield surface.
Zienkiewicz & Naylor	1971	Strain hardening material, elliptical cap yield surface, followed by strain softening (special yield surface in the super critical region).
Desai	1971	Psuedoelastic model using spline fit to stress-strain curve (E, ν).
Coon & Evans	1971	Hypoelastic material, unloading only.
Höeg	1972	Elasto-plastic material with strain softening (clays only).
Simpson & Wroth	1972	Elasto-plastic material, strain hardening, capped yield surface (sands only).

stress paths is uncommon and little to no information is available on model performance under a plane strain or true three dimensional stress state. The description of the models in Table 3 also indicates large differences exists in model capabilities. For example, only two consider strain-softening behavior (42)(97) and few explicitly consider unload-reload effects.

Documentation of the models in analyses of practical problems is also rare. Nonlinear elastic models have been employed in most of the finite element analyses where field data could be compared to predicted data (8)(11)(13)(16)(21)(34)(56)(66)(93). Fewer of the plasticity models have been so used because suitable plasticity models have only recently been developed (42)(81). Unfortunately none of these analyses were coordinated beforehand with the field problem that was investigated and thus the data observed could not be directly used to validate the modeling procedures.

The problems facing the user in choosing a suitable constitutive model therefore are as follows:

(1) The multiplicity of existing models which claim to accurately model soil behavior.

(2) The lack of information validating the models for (a) simple plane strain conditions; (b) stress paths common in the slurry wall problem; and (c) three dimensional stress conditions.

(3) The absence of a true field evaluation of any of the models for slurry walls.

Time Effects. In soils both creep and consolidation can produce significant time effects. Consideration of both phenomena has been accomplished in finite element analyses (10)(44)(71)(76)(77) although the costs of the analyses are high.

Models for creep in soils have been proposed by many investigators and a review has been provided by Scott and Ko (1969). Use of existing creep models in finite element analyses however has not resulted in accurate predictions of observed behavior in soils (10)(71). It would appear that additional research into modeling of creep behavior is needed but first documentation of the significance of the problem to the slurry wall should be provided through study of instrumented slurry walls. It is clear that for many problems such as slurry walls constructed in sands that creep will not be a significant factor.

Significant settlements behind a slurry wall or any braced wall can be caused by consolidation (59). However the need to model the actual consolidation process only exists if it is desired to obtain information about the time history of the settlements, because the use of drained or long-term deformation parameters in an analysis will yield estimates of the long-term settlements. Thus extension of the consolidation capability to slurry walls appears not to be of immediate significance unless a knowledge of the time-history of the settlements is desired.

Initial Conditions

Definition of the initial stress conditions for an analysis of a slurry wall is as important as choosing suitable soil modeling procedures. It is well known that simply determining initial at-rest pressures is sometimes difficult and in the slurry wall the construction of the wall in a narrow trench results in changes in these pressures making the determination of initial wall pressures even more difficult. In the subsequent discussion the effects of this process are discussed; the problems of determining the initial at-rest pressures are to be covered in another session of this conference.

The degree of the effect of wall construction can be judged from Fig. 12 where the results of a finite element analysis are shown in which the excavation of the slurry trench and the placement of a cast-in-place wall was simulated. The mesh shown in Fig. 5 was employed in the analyses. Because this mesh assumes symmetry about the leftmost boundary of the mesh the construction simulation assumes two walls 32 feet apart are being constructed simultaneously. The concrete slurry was assumed to be placed in a fully liquid condition and the initial stresses in the soil before opening the trench were calculated assuming a coefficient of earth pressure at rest of 0.5 with a water table at a depth of 6 feet below the ground surface.

The lateral pressures exerted by the concrete slurry can be seen to be substantially higher than the initial lateral pressures in the soil. If the pressures were not relieved by concrete shrinkage the wall would have initial lateral pressures exerted on it equivalent to that produced by a coefficient of earth pressure at-rest of 1.5. Concrete shrinkage will modify this situation somewhat

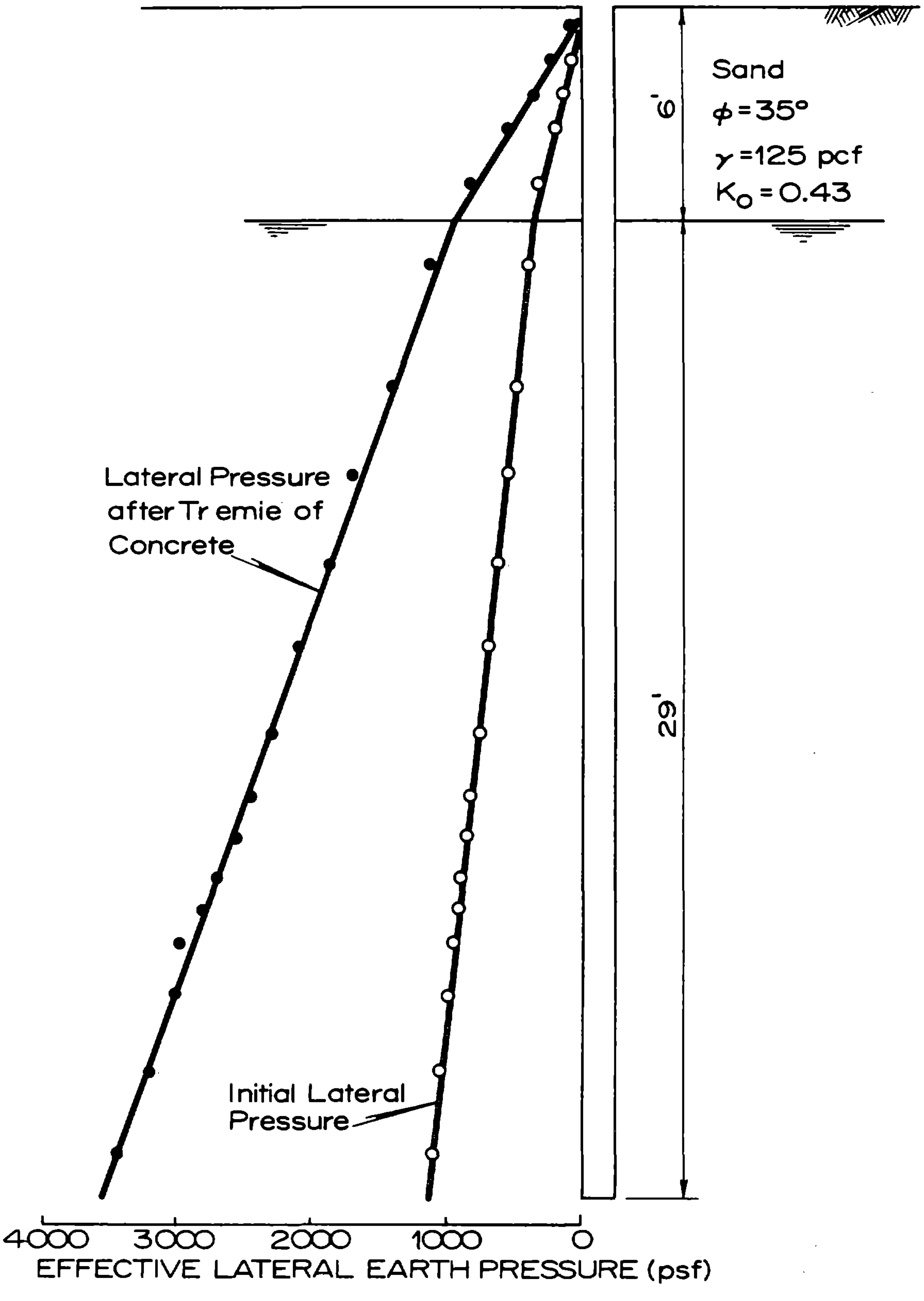


Figure 12. Effects of Placement or Liquid Concrete in Slurry Trench on Initial Lateral Pressures as Predicted by Finite Element Analysis

but based on the magnitude of possible shrinkage movements this effect should be small. Thus the initial lateral pressures on a cast-in-place wall could be substantially greater than the initial lateral stresses in the soil.

Some field measurements of the pressures caused by a concrete slurry have been made by Dibagio and Roti (1972) and Dibagio and Myrvoll (1972). These measurements suggest that the pressure exerted on the sides of the trench are not equivalent to that caused by the full hydrostatic load of liquid slurry, but is somewhat less. Similar effects occur in wall forms filled with liquid concrete. The ACI code (46) suggests that the pressures are dependent upon the rate of pour, the temperature and the maximum height of fresh concrete in the form. It would obviously be desirable to deliniate these relationships for slurry walls so that the initial lateral wall pressures can be correctly included in an analysis.

Boundary Conditions

There are two important problem areas in representing boundary conditions in a finite element analysis of a slurry wall: (1) The interface between the soil and the wall; and, (2) The three dimensional effects of internal bracing or tie-backs.

Interface Conditions. When bentonite slurry is used to stabilize the trench a slurry cake has been observed to form in the soil along the trench walls (19)(69)(89) as shown in Fig. 1 . The degree of penetration of the soil by the slurry varies with the soil type and presumably the head exerted by the slurry. The effects of the slurry cake on interface shear stress-deformation behavior and strength will apparently depend upon the type of slurry used. For example, pull tests on cast-in-place piles which were cast in trenches filled with (1) a bentonite slurry and (2) a silica based slurry have shown that the resistance to pulling in the silica based slurry was 40 percent greater than that for the bentonite slurry (32). Greater differences may be expected if a bentonite grout is used as opposed to a simple bentonite slurry.

The interface can be accurately modeled in finite element analyses using the technique described in the example analysis of a slurry wall. However the properties of the interface must be accurately defined and this includes both the deformation behavior as well as the strength. Information of this type needs to be developed for the different slurry wall interface conditions.

Three-Dimensional Effects. If internal bracing or tie-backs are used to support the slurry wall the stress distribution behind a slurry wall becomes three-dimensional. The degree of this effect will be a function of support spacing, wall rigidity, and soil stiffness. However in all cases it would be preferable to model the system as a plane strain problem because the cost of a true three-dimensional finite element analysis would be much greater.

Three dimensional finite element programs are available and have been utilized for analyses of earth dams (63). It would be desirable to develop such a program for slurry walls and analyze some typical cases to determine if the three dimensional effects cause significant differences in behavior relative to plane strain analysis of the same problems.

Effects of Environmental Factors

Some environmental factors play an important role in performance of a slurry wall but are not amenable to simulation in an analysis, e.g., frost penetration of the soil which causes increased bracing loads (22) or different brace to wale connection procedures which result in large differences in wall deformations (61). Because these factors are not considered in analytical modeling a clear understanding of the degree of their effects must be available so that at least qualitative adjustments may be made in the predicted answers. Unfortunately these factors are random by nature and no one instrumentation site could provide enough data to define the range of the effects. Instead careful compilation of results of a number of jobs needs to be made where analytical modeling has been attempted. Areas where environmental factors have had an influence should be identified and documented.

Summary

A condensation of the problems in analytical modeling of the slurry wall is shown in Fig. 13 in the form of a block diagram. Research programs to attack the problems are discussed in the next section of the paper.

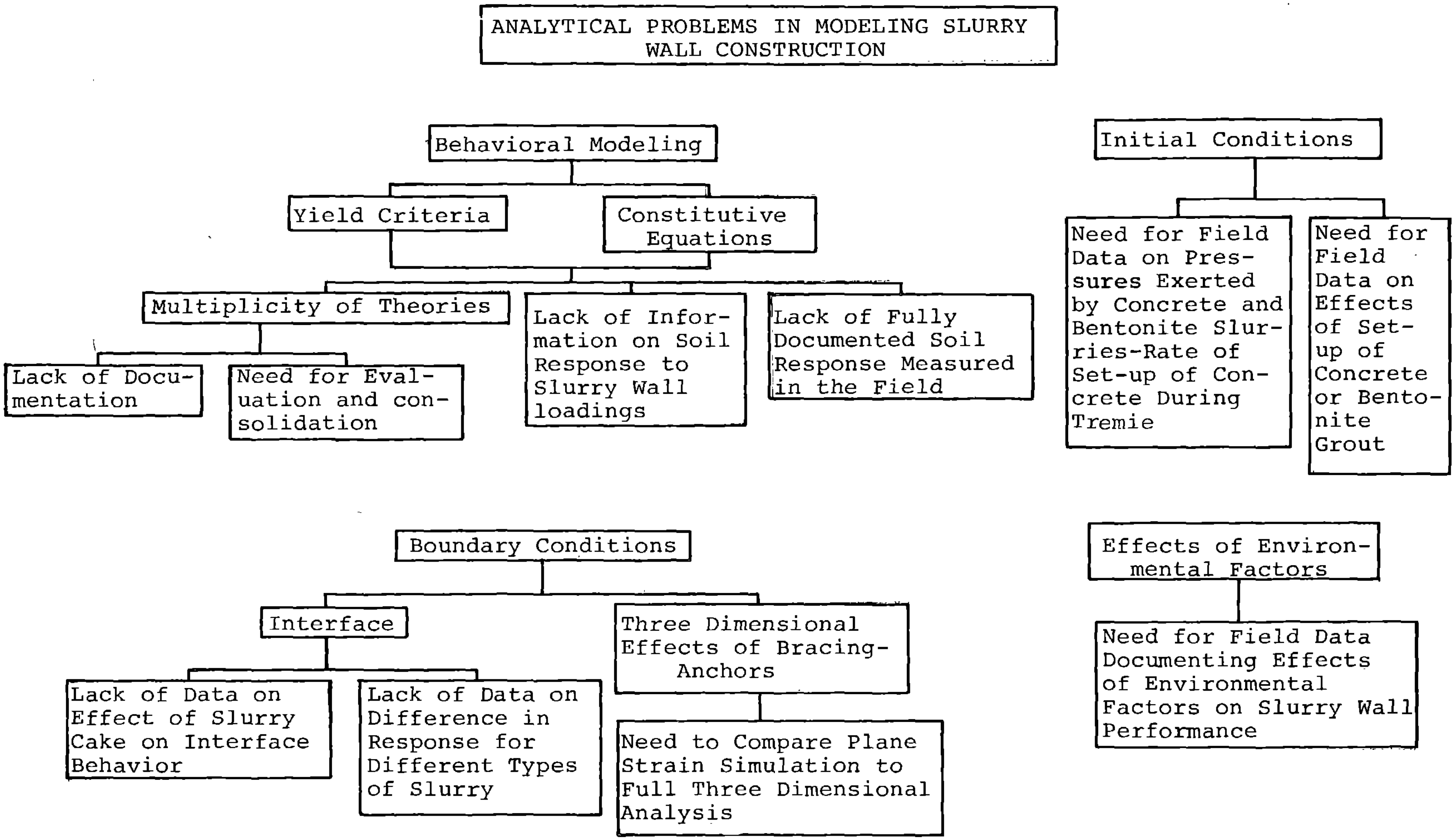


Figure 13. Analytical Problems in Modeling Slurry Wall Construction

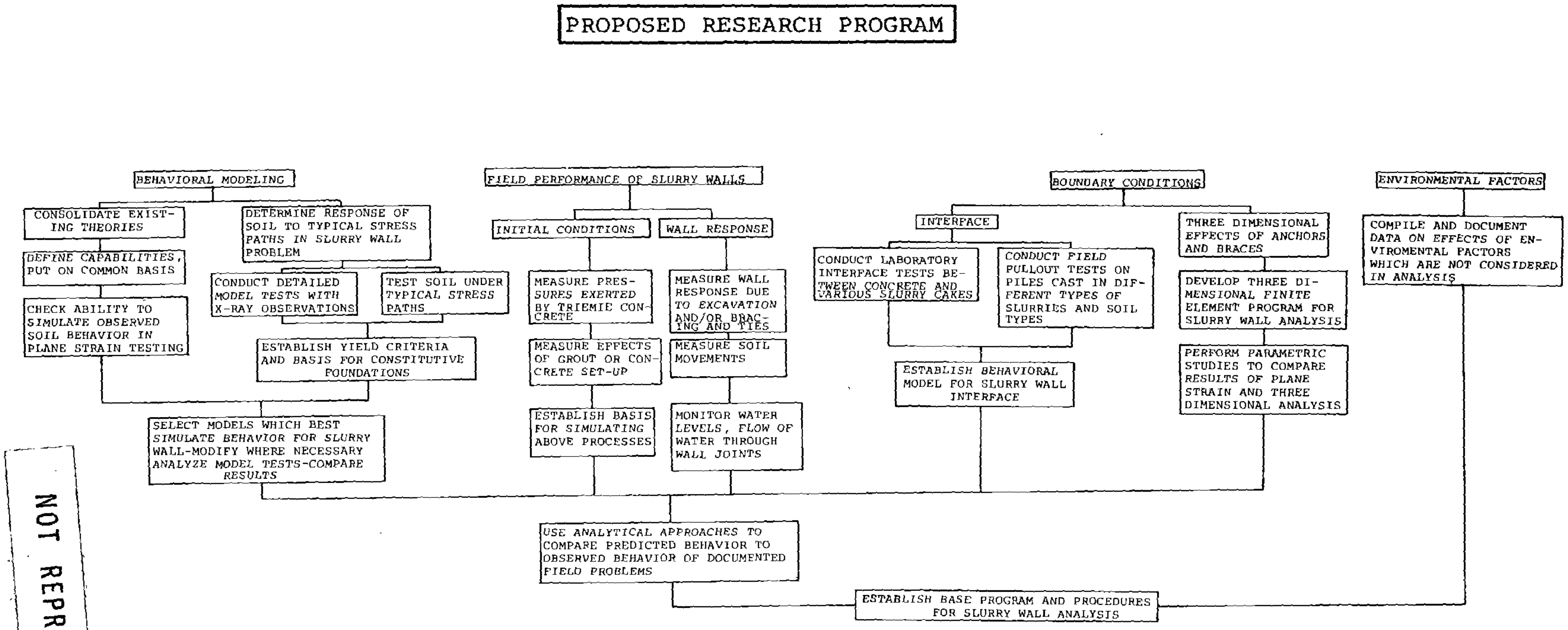
PROPOSED RESEARCH PROGRAM

A research program which is designed to help solve the problems of analytical modeling of the slurry wall is shown in block diagram form in Fig. 14. The program is broken into pieces which can be handled separately so that various aspects of the problem may be studied simultaneously.

Behavioral Modeling

There is a danger in undertaking studies into the constitutive behavior of soils if the goals are ill-defined. The problems remaining in constitutive modeling are so substantial that no complete solution may be forthcoming for decades if it is found at all. Thus the proposed research program constricts these studies to relate closely to modeling soil behavior in connection with typical slurry walls.

Existing Theories. As a first step it is suggested that the numerous existing constitutive theories be catalogued and put into a common format. Capabilities must be defined and the utility in practical analyses evaluated. Basic evaluation of the ability of each model to simulate soil behavior should begin by comparisons of predictions with observed soil behavior in a sophisticated, but simple test. For example, tests conducted in a plane strain device where the intermediate principal stresses could be measured and the major and minor principal stresses could be varied independently would provide data to evaluate the ability of the models to predict stress-strain and volumetric strain behavior and intermediate principal



NOT REPRODUCIBLE

Figure 14. Proposed Research Program

stresses under stress paths similar to those occurring during slurry wall construction.

Soil Response Under Slurry Wall Loadings. One of the facets of soil response investigation should be the plane strain testing program described in the preceding paragraphs. A more sophisticated testing program could be undertaken using a three dimensional loading apparatus. Several of these devices are in existance in the United States and because of the expense of fabrication it would seem advisable to utilize one of the existing facilities for this purpose.

In addition to these testing programs, well documented model studies of slurry walls could be very useful in determining the response of a soil to the construction of a slurry wall. The tests conducted at Cambridge University on simple retaining walls elegantly demonstrate the utility of such data (49)(50)(51). One of the strengths of the Cambridge tests was the utilization of x-ray techniques to monitor strain fields in the soil. These data allow careful study of the mobilization of soil strength and the response of various portions of the soil to loading. They also provide a data base which can be used to test analytical tools. Simpson and Wroth (1972) for example report finite element studies of the Cambridge model tests which were used to evaluate a proposed soil model for sands. Successful use of model tests of this type have been also reported in the United States (2)(48).

Selection of Modeling Techniques. The best of the soil modeling procedures selected during the first phases of the study should be tested for their ability to model the soil response data developed in the soil tests and the model tests. These comparisons may establish areas where the models need to be modified and improved, or develop a need for completely new modeling techniques.

Boundary Conditions

Problems in the boundary condition category are much easier to define than those in behavioral modeling and the solutions obtained should be more definitive.

Interface Behavior. Information as to both deformation and strength behavior of the interface between a slurry wall and the adjacent soil needs to be obtained. Pullout tests conducted on cast-in-place and precast piles which are set into holes held open by various slurries would appear to be the most direct means of obtaining data on the strength of the different types of interfaces. Documentation could also be obtained on the depth of slurry penetration and the nature of the cake formation. However, pullout tests cannot be used to determine deformation behavior for the interface because deformations measured during pullout of the pile include not only those due to movements on the interface between the pile and the soil, but also those caused by strains developed in the surrounding soil. The deformations contributed by the soil may in fact be larger than those contributed by the interface.

For this reason it would appear advisable to conduct direct shear tests in the laboratory between representative slurry impregnated soils and concrete surfaces characteristic of cast-in-place and precast walls. The slurry could be varied to simulate the common types employed in slurry wall construction. From these data and those of the pull-out test behavioral models could be developed for the slurry wall interface and incorporated into a finite element program so as to control the behavior of the interface elements in the program.

Three Dimensional Effects. The degree of the three-dimensional effects introduced by anchors and braces could most easily be studied by comparing true three-dimensional finite element analyses and plane strain analyses of typical slurry wall problems. Basic three-dimensional programs are available (67) but would require some modification in order to be adapted to slurry wall analysis. The modifications are not extensive however and can follow from similar changes incorporated for plane strain finite element programs (14)(70)(93).

Initial Conditions

Deliniation of the problems associated with the effects of excavation, concrete tremie and concrete or bentonite grout set-up on the initial stress conditions can most logically be determined through field testing of the type described by Dibagio and Myrvoll (1972). In this case, movements around the trench during tremie and pressures exerted by the grout and concrete were measured. In future testing it would be desirable to vary the rate of the concrete tremie within reasonable limits to determine if this factor is important to the observed pressures and deflection. Also strains within the concrete during set-up should be measured to determine the amount of vertical and horizontal shrinkage occurring during set-up.

With information of this type a rational basis for simulating the observed behavior could be incorporated into finite element analyses and other analytical techniques as they are developed. These field tests need not be conducted as a separate program but could be incorporated in a larger program where the wall response due to excavation and bracing is also measured, as described in the following paragraphs.

Fully Documented Field Tests

Documentation of the field performance of slurry walls will provide data to serve as a final testing ground for proposed modeling techniques as well as to guide development of better modeling techniques. While such an assertion can be easily made actually achieving the desired result is difficult. The instrumentation programs need to be designed to test the assumptions made in the modeling process.

For example, not only gross boundary movements should be measured but also distributions of these movements within the soil mass. Knowledge of the variation of vertical and horizontal movements would be valuable in checking to see if the predicted source of major movements is the same as that observed. Such movement patterns could be observed by use of embedded horizontal and vertical movement plates. Additionally, data could be obtained on the initial and boundary stress conditions by careful design of the monitoring system.

The linkage between the analytical modeling program and the documentation of field performance cannot be stressed too much. Many expensive instrumentation efforts in the past have failed in their purpose because the data obtained was not what was needed. The program needs to be designed to insure mutual cooperation between those individuals involved in the modeling effort and those individuals most knowledgeable about the field instrumentation. In later cases the modeling procedures should be used to predict behavior of instrumented walls, to serve as a true test of the methods developed.

Effects of Environmental Factors

The effects of those factors which cannot be explicitly accounted for in an analysis need to be documented. This type of information and the knowledge of the results predicted by a reliable analytical modeling process will provide a complete data base from which engineering judgements may be made.

The data on the effects of environmental factors could be obtained by compilation of instrumentation results on slurry walls and by discussions with field engineers and slurry wall contractors.

Careful examinations of the variations in performance at one site may lead to deliniation of external factors which were their cause. Studies of data from different sites is also important because some environmental factors may present problems only in certain parts of the world as is the case with frost penetration. Finally discussions with those individuals who actually construct the walls could provide data on items such as lack of wall plumbness and effects of labor quality.

ESTIMATED EFFORT AND TIME FOR FACETS OF RESEARCH PROGRAM

All of the research phases are of equal importance, however the amount of time and effort needed to achieve the objectives will vary, as discussed in the following paragraphs.

Effort

The amount of effort required for each of the research phases is in part related to the complexity of the research objective. In this regard the investigation of behavioral modeling and the development of a monitoring program for field performance of selected slurry walls should require the greatest effort, followed by the investigation of boundary conditions and finally the documentation of the environmental factors influencing slurry wall performance.

Time

The time required for any part of the proposed research program will vary with both the complexity of the research objectives and the level of funding committed. For example, the technology for the investigation of boundary conditions is in existence and could be mobilized relatively quickly. By funding each independent aspect of the program so that it could be undertaken simultaneously the problems could be resolved in two or three years. On the other hand the development of a soil model is more difficult and even if funding of each level of associated research was provided simultaneously progress may be much slower and time-frame goals should be flexible. Monitoring performance of slurry walls will require extra time because of the physical scope of the effort involved, and, in fact, this effort should be long range so that a range of walls in various soil conditions can be observed. An estimate of the time involved in the various research phases is as follows:

Behavioral Modeling	3-5 years
Monitoring Field Performance	4-6 years
Boundary Conditions	2-3 years
Environmental Factors	1-2 years

SUMMARY

Lambe (1973) has admirably demonstrated the significant role analytical predictions play in Civil Engineering. This paper has attempted to define a research program which will lead to an analytical tool for predicting the performance of a slurry wall which yields results useful to the design engineer. Means are also suggested to develop information concerning those problems which cannot be included in the analysis so as to develop as complete as possible a data base from which the engineer can make his design judgements. The suggested research program is shown in block diagram form in Fig. 14. It is hoped that the program in Fig. 14 will provide a base for discussion and evolvement of a program which will result in an improved design technology for slurry walls.

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