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BUS FLEET MANAGEMENT TECHNIQUES GUIDE

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16. Abstract

The contemporary problems faced by transit maintenance managers are more complex than those faced in the past. The recent escalation of costs is evidence that existing practices cannot efficiently tackle the fleet management problems of today. The purpose of the Bus Fleet Management Techniques Guide is to provide transit maintenance managers with methods that they can use to derive information for maintenance planning and fleet management. The Guide covers three basic areas: 1) statistical analysis of component and part failure mileages for use in maintenance planning, 2) life cycle economic analysis for component and bus replacement and procurement decision-making, and 3) non-technical methodologies for the planning of maintenance management information systems.

The Guide contains easy-to-follow examples derived from actual transit system maintenance records. All techniques can be done using inexpensive scientific calculators. A set of 19 work sheets is provided to facilitate the computations. Some chapters have study questions to further explain the methodologies.

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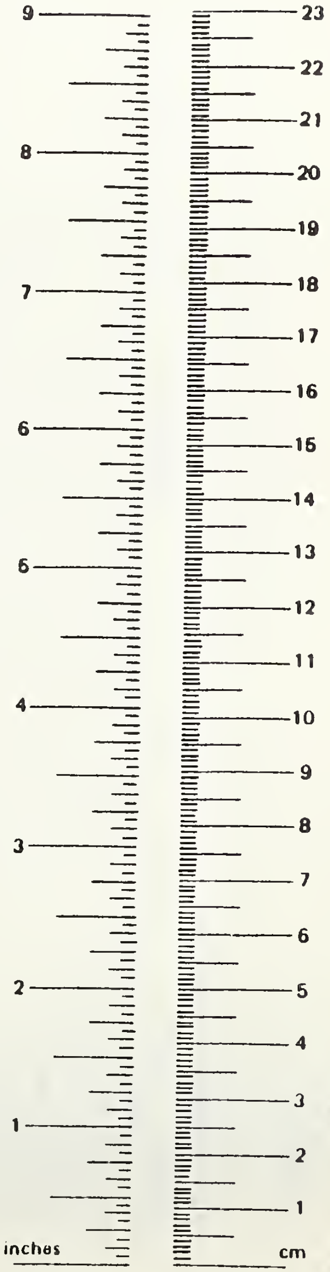
METRIC CONVERSION FACTORS

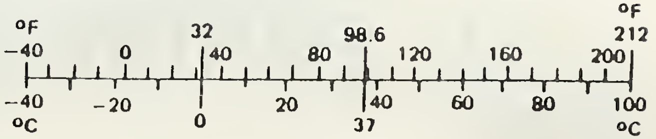
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
	AREA			
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
	MASS (weight)			
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
	VOLUME			
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
	TEMPERATURE (exact)			
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
	AREA			
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
	MASS (weight)			
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
	VOLUME			
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
	TEMPERATURE (exact)			
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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*1 in. = 2.54 cm (exactly). For other exact conversions and more detail tables see NBS Misc. Publ. 286. Units of Weight and Measures. Price $2.25 SD Catalog No. C13 10 286.

EXECUTIVE SUMMARY

All transit agencies are under political and financial pressure to reduce operating costs and improve system productivity. This has forced transit agencies to search for more economical ways to provide public transportation service, including more efficient fleet management practices. The Bus Fleet Management Techniques Guide presents easy-to-understand maintenance management techniques which can help fleet managers to understand the cost drivers in bus maintenance, get more out of their own maintenance records, better plan maintenance staffing and spare parts inventory requirements, and anticipate future problems.

Overview. The Guide contains fully documented practical examples derived from actual bus maintenance data obtained from leading transit agencies in the Southwest and throughout the United States. All of the techniques can be done using inexpensive scientific calculators. A set of 19 work sheets has been developed to facilitate the computations. Their use is demonstrated throughout the Guide and blank work sheets are provided. These techniques can be easily programmed on microcomputers, too. Some of the methods are accompanied by study questions to further explain and illustrate the techniques.

An introductory chapter to the Guide reviews the types of maintenance actions which may be taken to maintain a part or component. For example, for some parts and components it may be best to maintain them before they fail (preventive maintenance), and for others it may be best to maintain them after failure (corrective maintenance). The selection of the most suitable maintenance action is based on failure mileage patterns, maintenance cost information, and the expected life. With this information a decision diagram is used to select the best maintenance action.

The following five chapters of the Guide present techniques for the analysis of transit bus part and component failure patterns. The Guide then examines the entire bus and applies life cycle economic analysis techniques to bus replacement problems. In the last chapter, the Guide examines the entire maintenance system and it presents a technique to examine maintenance management information needs so that the whole maintenance system can be better managed.

Failure Pattern Analysis. The typical transit bus has thousands of parts of components. The maintenance of such complex vehicles is a challenging task. Improving the efficiency of maintenance operations starts with an understanding of the nature of failure mileage patterns of components and parts. Knowledge of the failure mileage patterns is a critical factor in deciding on the appropriate type of maintenance for each component and part and enables the development of an efficient maintenance policy.

For example, some parts and components have unpredictable lives and should be replaced after they fail. A fuse, for example, is as good as new until a short circuit occurs and the fuse fails. They are replaced when they fail and not much in the way of preventive maintenance can be done in the meantime. On the other hand, most mechanical parts and components show signs of wear and use. Sometimes they can be monitored to determine when they will fail. Other components that can not be monitored may have predictable failure intervals and they can be replaced in anticipation of a failure.

Chapters Two through Six detail procedures for collecting and tabulating component failure mileage statistics. Each chapter describes an analysis procedure and when all five chapters are taken together, they present methods that are applicable to all likely conditions. Each chapter describes how to develop survival and reliability curves for the component and then determine how many of the components in a bus fleet will fail during any time period, such as one year. This information can be used in inventory planning by predicting the average demand for parts and components by the maintenance shop and average repair loads.

One of the major problems with conducting component failure analysis is that conventional techniques generally assume that the entire generation of components has failed. However, waiting for all of the transmissions in a newly purchased bus fleet to fail may mean that important information on failure patterns is obtained too late. For this reason, the Guide demonstrates analysis techniques for incomplete data sets. Thus, components which have not yet failed can be accounted for in the analysis.

Failure pattern information can be used to help in selecting the optimal action for maintaining a part or component. There are four types of repair actions: 1) operate-until-failure where maintenance is conducted only when a failure occurs, 2) condition-based-maintenance where a part of component is replace, adjusted, or checked when a condition exceeds a specific measurable level (e.g. high concentrations of wear metals in oil, high oil consumption, worn brake shoes, etc.), 3) fixed-mileage-maintenance where a part of component is replaced, adjusted, or checked at a specific mileage interval, and 4) design-out maintenance where a part or component is not lasting as long as it should and the only way to alleviate this high maintenance cost problem is to alter the system. The Guide contains instruction and examples on how to determine which maintenance policy is best.

Component and Bus Replacement Decision-Making. Chapter Seven of the Guide provides a simple procedure for determining the optimal replacement interval for parts and components based on its mileage failure pattern identified in the previous chapters. The applications of life cycle costing to bus procurement and replacement problems are covered in Chapter Eight. This chapter reviews economic principles, introduces the concepts and applications of life cycle costing, and provides example applications. Work sheets are provided for easy application of the techniques. In addition to discussing vehicle procurement, this chapter demonstrates the use of life cycle costing in determing whether or not to replace old buses with new or rehabilitated buses.

Maintenance Management Information System Planning. The last category covered is a technique to be used in the conceptualizing and planning of computerized maintenance management information systems. It is generally accepted that maintenance managers can do a better job if they have easy access to maintenance information. Maintenance management information is best kept on computers and computerized systems have become a popular maintenance management tool.

The development of a maintenance information system is too important to the maintenance manager to allow consultants or in-house computer experts to make all the decisions regarding the system's performance. The maintenance manager should take a leading role in the development of performance specifications and system planning to make sure that the resulting system fits the maintenance manager's needs. However, it is not common for maintenance managers to be computer experts, themselves. Therefore, the maintenance manager should use a non-technical format to communicate maintenance department needs to the computer experts and to identify exactly what type of information the system will ultimately provide.

ACKNOWLEDGEMENTS

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There were several transit system staff and transit industry consultants who provided data used in the case study examples in the *Guide* and/or provided advice. Their assistance greatly enhanced the practicality of the *Guide*. The following individuals are acknowledged for their support and guidance: Mark Glandon, Director of Transportation for the Central Oklahoma Transportation and Parking Authority; Don Drassen, Chief of Maintenance, Central Oklahoma Transportation and Parking Authority; Dennis J. Belknap, Manager of Operations and Maintenance, City Transit Service of Fort Worth, Texas; Raymond Amsler, Equipment Maintenance Supervisor, Wichita (Kansas) Metropolitan Transit Authority; Randy Salmans, Manager of Maintenance, Topeka (Kansas) Metropolitan Transit Authority; Martin Judd, Superintendent-Maintenance Systems and Procedures, New Jersey Transit; Richard Golembiewski, Maintenance Manager, Sacramento (California) Regional Transit; Bob Babbit, former General Manager, Abilene (Texas) Transit; Timothy Lett, General Manager, Abilene (Texas) Transit; Lucas Montoya, Fleet Manager, City of Albuquerque (New Mexico) Transit Division; Sam Cox, Equipment Manager, Dallas Transit System; Phil Selinger, Manager of Maintenance Programs, Portland (Oregon) Tri-Met; Bill Lofgren, Director of Maintenance, Austin (Texas) Transit System; Elliot Gitten, LTK Management Services, Philadelphia; and John Spring, Superintendent of Maintenance Transit Management of Harvey, Inc., South Holland, Illinois. The assistance of Maggie Laird, Principal, Laird and Associates of Pearland, Texas, is particularly appreciated. She enthusiastically supported the development of the *Guide* and was responsible for the preparation and presentation of written material on semi-automatic fueling systems in Chapter Nine.

The Guide is intended not only to be a self instructional tool but also a text for short course instruction on bus fleet management techniques. As part of the project, a short course was prepared and presented at the Indiana Transportation Association Annual Meeting, South Bend, Indiana, July 30, 1985. This was made possible through the help of Dave Cyra, Director of the Office of State-Wide Transportation Programs, the University of Wisconsin at Milwaukee. Joseph ZaVisca, the organizer of the meeting and the General Manager of the South Bend Public Transportation Corporation, graciously assisted the researchers by allowing them to make their presentation during the Association's Annual Meeting.

Finally, the Guide was ably word-processed by Barbara Jones and Betty Craig, proficient technical text processors at the University. Joan Howeth of the University's Office of Grants and Contracts was responsible for the financial record-keeping.

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CHAPTER ONE

INTRODUCTION

Overview

All public transit agencies are under political and financial pressure to cut operating costs and improve system productivity. About one-third of the operating expenses of the typical bus transit operator are associated with vehicle fuel and maintenance. The Bus Fleet Management Techniques Guide presents easy-to-understand maintenance management techniques which can help busy transit planners and maintenance managers to understand the cost drivers in bus maintenance, get more use out of their own maintenance records, better plan maintenance staffing and spare parts inventory requirements, and anticipate future problems.

The Guide contains easy-to-follow examples derived from actual bus maintenance data obtained from leading transit agencies in the Southwest and throughout the United States. All techniques can be done using inexpensive scientific pocket calculators, the use of which is explained in Appendix B. A set of 19 work sheets has been developed to facilitate the computations. Their use is demonstrated throughout the Guide and blank work sheets are provided in Appendix A. These techniques can be easily programmed on microcomputers, too. Some of the chapters have study questions to further explain the methodologies.

The Bus Fleet Management Techniques Guide is intended for use by transit agency maintenance managers, transit agency planners and other interested personnel, officials of state and metropolitan transportation and planning agencies, university and college educators in business, industrial, and civil engineering. No knowledge of advanced mathematics is assumed and the Guide is entirely self-contained.

Cost Factors in Transit Bus Operations

Bus operating and maintenance expenses are significant elements in transit agency budgets. A 1983 Urban Mass Transportation Administration (UMTA) report estimated transit bus operating and maintenance costs as follows, based on 1981 Section 15 reports (9):

COST CATEGORY	PERCENT OF TOTAL
Operator Labor (wages, benefits):	46%
Vehicle Maintenance:	
Labor	15
Materials and Supplies	6
Fuel and Lubricants:	10
Other:	23
TOTAL:	100%

Those costs directly associated with the operation of transit vehicles, fuel and maintenance, were 31 percent of the total, and this amounted to an annual national expenditure of more than $1.3 billion in 1981.

Individual public transit agencies report figures similar to these national statistics. In fiscal year 1983 these costs amounted to about 34 percent of the total operating expenses for the Central Oklahoma Transportation and Parking Authority in Oklahoma City. Jones (4) cited fiscal year cost projections from 1981 to 1985 for Tri-Met in Portland, Oregon, of which about 27 percent was for maintenance and fuel costs. Peskin (7) projected that bus vehicle maintenance costs (including fuel) for Houston's Metropolitan Transit Authority would be 45.8 percent of the total operating costs in the year 2000.

Bus transit fleet managers are currently being pinched between two related forces: 1) shrinking Federal support for operating subsidies which are forcing transit operators to economize, and 2) the growing expense and complexity of modern rolling stock and maintenance equipment. Malec dramatically illustrated the problems of rising costs in his analysis of transit operating statistics (6). He found that in the late 1970's and early 1980's transit maintenance costs had risen five fold, an equivalent annual increase of about 20 percent per year.

As the availability of transit funds becomes tighter, transit management is naturally forced to look for more productivity in its operations through efficiency gains. In view of the disproportionate escalation of maintenance costs, maintenance is a logical candidate for improvement.

The Challenge in Maintenance Management

The contemporary problems faced by transit maintenance management are more complex than those faced in the past. The recent escalation of costs is evidence that existing practices can not efficiently tackle the fleet management problems of today. Old concepts of bus maintenance, such as "loving care" (the reliability of buses is directly proportional to the amount of maintenance they receive), or "operate until failure", may not be the most efficient strategies for economical maintenance.

Maintenance strategies must be efficiently adapted to meet the specific operating characteristics of a transit system. Blanket policies intended to cover all conditions or apply to every transit system simply will not efficiently deal with the specific bus operating environments and maintenance problems of individual transit agencies. Maintenance strategies should be adaptable to the specific system and its unique problems to achieve maximum efficiency. However, the tailoring of maintenance strategies is not a simple matter. The manager must analyze the options, study their consequences and select the most efficient course of action. This means that the manager cannot always rely on conventional wisdom or on knowledge of what seems to work some place else.

Research done on bus fleet management has found that the availability of maintenance information is the key to making efficient management decisions (8). Management decision-making information should include more than just the day-to-day records collected on daily work flows. It should also include information on expected work flows so that daily scheduling can be conducted, information on future work flows so that labor force requirements and parts inventory levels can be planned, and information on the distribution of component failures through time so that efficient repair policies can be planned.

Guide Objectives and Organization

The efficient planning of repair policies requires the use of state-of-the-art quantitative techniques. Unfortunately, the most effective methods which can be used to obtain this information are highly technical. Therefore, the principal objective of the *Bus Fleet Management Techniques Guide* is to provide practical instruction on these quantitative methods without requiring that the user have an in-depth background in mathematics. The *Guide* contains descriptions of several quantitative techniques and provides complete examples of each based on actual bus maintenance data examples.

The methods presented in the *Guide* can provide useful information for fleet management-decision making and methods to aid in the design of systems to collect information. The methods presented do not provide direct answers to fleet management problems, but they can help fleet managers to make more informed decisions by providing them with better information.

The text of the *Guide* is divided into nine chapters covering three categories of techniques. The first category, Chapters Two through Six, is the analysis of transit bus part and component failure patterns. In all likelihood, the greatest potential for better management and maintenance cost control exists through a better knowledge of failure patterns. This is because knowledge of component failure patterns permits the planning of maintenance and repair policies for daily use in managing bus fleets.

Life Cycle Costing and Computerized Information Systems also are common issues in bus fleet management. The second category of techniques, Chapters Seven and Eight, demonstrates the use of economic analysis in determining the optimal replacement mileage intervals for bus components and life cycle costing techniques in vehicle procurement and vehicle replacement decisions. The third category, Chapter Nine, is an aid to the better understanding of maintenance information needs. Chapter Nine demonstrates how to develop plans for computerizing maintenance management information systems.

Component Failure Patterns

The typical transit bus has thousands of parts and components. The maintenance of such complex vehicles is a challenging task. For example, Haenish and Miller estimated that bus mechanics at the Chicago Transit Authority regularly perform 1,800 different functions (3). Improving the efficiency of maintenance operations starts with an understanding of the nature of each component failure. This determines the appropriate type of maintenance for each component and enables the development of an effective maintenance policy.

For example, some parts and components have unpredictable lives and should be replaced after they fail. A fuse, for example, is as good as new until a short circuit occurs and the fuse fails. Electrical components in general show little or no deterioration until they fail. They are replaced when they fail and not much in the way of preventive maintenance can be done in the meantime.

Most mechanical components show signs of wear with use. Sometimes this wear can be monitored; brake shoe thickness can be measured periodically and the shoes replaced when they are worn to a minimum thickness. Other mechanical components, particularly those which are made up of many parts such as engines, transmissions, and air compressors, exhibit a different failure pattern because there are many potential causes of failures. Preventive maintenance, including changing the oil and filters, will extend the life of such components, but their condition is more difficult to monitor.

A first step in predicting future component failure patterns is to look at the distribution of component failures as a function of mileage. Figure 1.1, for example, is a bar chart of the mileages between wear outs of 68 brake shoes from Flxible 870 buses operated by the Dallas Transit System. One can tell a good deal about the brake shoe wear out mechanism from this bar chart.

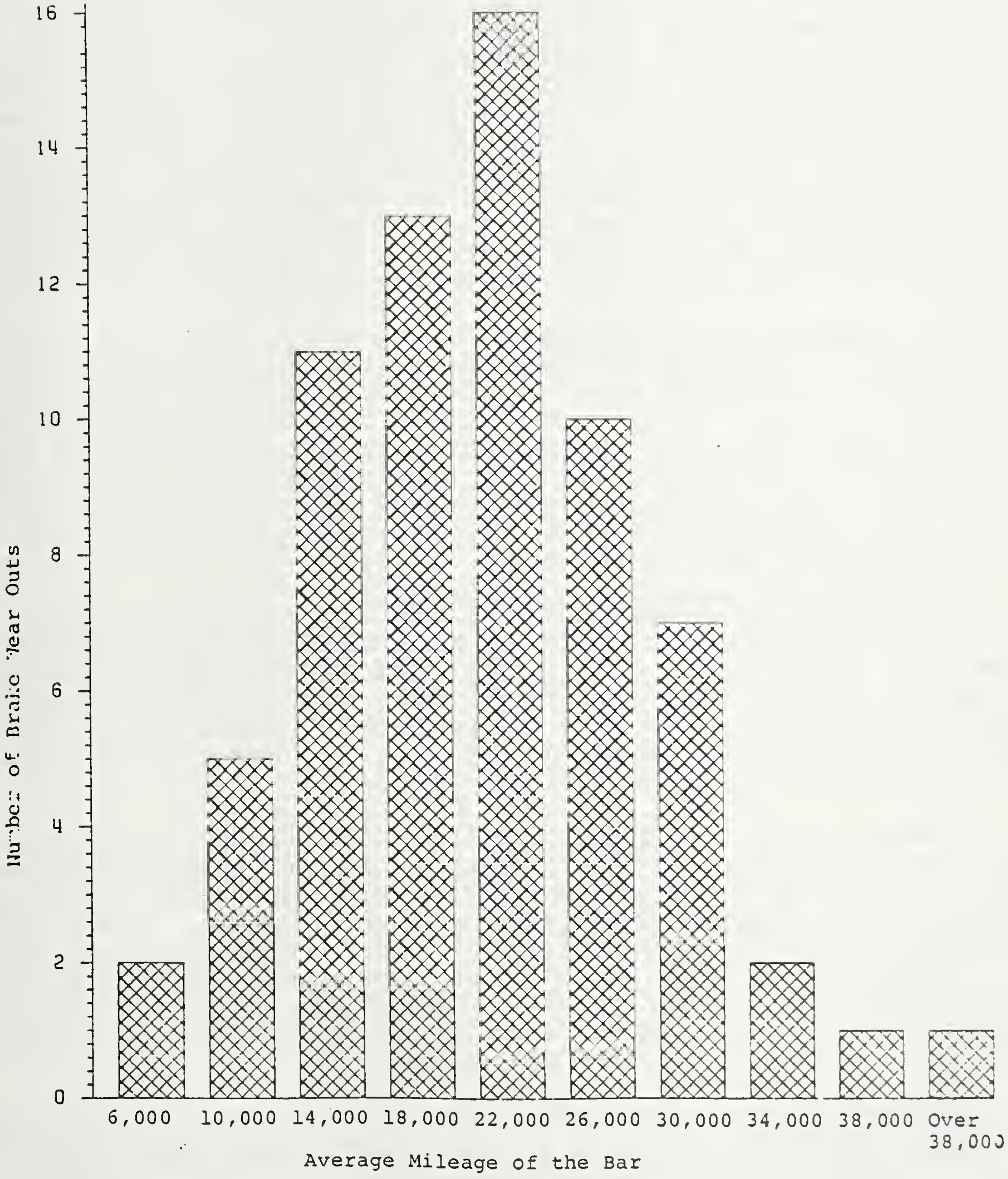


FIGURE 1.1

BRAKE SHOE WEAR OUT FREQUENCY BAR CHART

For example, wear out mileages range from less than 10,000 miles to more than 30,000 miles. The bar chart tends to be bell-shaped, with wear outs being most frequent at an average of about 22,000 miles, the "peak" of the bell.

Probability distributions are used to quantify and describe the failure pattern of parts and components. To quantify the pattern, a probability distribution curve, or equation, can be matched to the data. Generally, one of two families of curves will fit. In Figure 1.2 are shown characteristic members of each of the two curve families. One of the curves is shaped like a bell and this is the Normal distribution. The Normal distribution is more applicable for parts or components that wear out, like brake shoes. The other curve is shaped like a ski slope and it is called the Weibull distribution. The Weibull distribution is more applicable for parts and components that fail.

In Chapter Two on the Normal distribution, the mean mileage to failure, $\bar{X}$, and the standard deviation of the failure mileages, SD, are described and simple methods for their calculation are given. These two numbers, $\bar{X}$ and SD, determine the Normal equation and enable the Normal curve to match or fit any component failure bar chart that is bell-shaped. As indicated in Figure 1.2, the mean mileage to failure, $\bar{X}$, is the peak of the bell. The standard deviation, SD, describes the scatter of the mileages or the width of the bell.

In the Weibull chapters (Chapters Three, Four, Five and Six), simple techniques are given to determine the three Weibull constants, D, B, and T. These three numbers will be defined later in the Weibull chapters; they determine the Weibull equation that best fits bar chart data that resembles a ski slope in pattern. A mean ($\bar{X}$) and standard deviation (SD) can also be computed or estimated for Weibull failure data.

There are four chapters on Weibull distribution analysis in order to accommodate several different types of component failure data. If all of the components under study have failed, then the techniques of Chapter Three can be used. However, often it is desirable to obtain predictions of future failure problems without waiting for the failure of the entire generation of components under study, particularly if the component lasts a relatively long time (e.g., transmissions) which would require many years before the entire generation fails.

There are two types of unfailed components, those whose mileages exceed the mileage of any failed component and those whose mileages are exceeded by at least one failed component. Components in the first category are called "censored cases," and components in the second category are called "suspended cases." For example, consider a set of transmission failures where the largest failure mileage is 125,000 miles. An unfailed transmission with 130,000 miles is treated as a "censored case" while an unfailed transmission with 95,000 miles is considered a "sus-

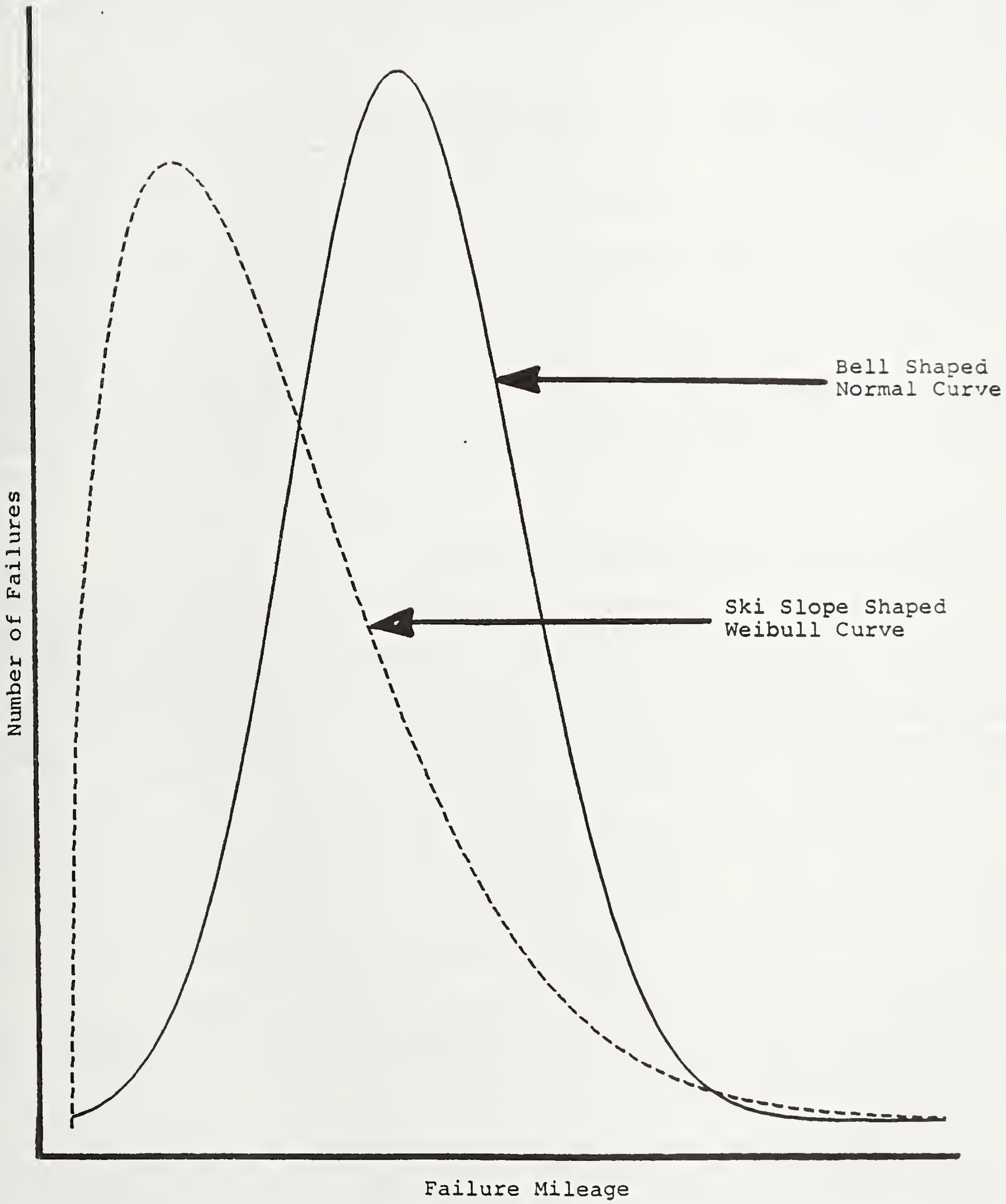


FIGURE 1.2

NORMAL AND WEIBULL PROBABILITY DISTRIBUTIONS pended case."

The Weibull distribution analysis is different if the data set contains censored or suspended cases, hence the need for four Weibull chapters. Each Weibull chapter handles the following data sets:

Chapter Three:	all components have failed,
Chapter Four:	the data set contains censored cases,
Chapter Five:	the data set contains suspended cases,
Chapter Six:	the data set contains both censored and suspended cases.

The different combinations of failed and unfailed component mileages are reviewed at the beginning of each chapter.

How this failure distribution information can effectively be used by the maintenance manager to develop the best maintenance policy for each major component is a principal objective of this *Guide*. Later in this chapter the selection of a maintenance policy for each component or part will be discussed. First, however, the different types of maintenance policies will be reviewed.

Preventive vs. Corrective Maintenance

There are two fundamental types of component maintenance, "preventive" and "corrective". Preventive maintenance is carried out at predetermined inspection intervals, typically based on accumulated mileage, or other prescribed criteria, such as when a monitored condition exceeds a tolerance level. This type of maintenance is intended to reduce the likelihood of the in-service failure of components by anticipating their failures.

During preventive maintenance inspections fluids and filters are changed and component checks and adjustments are made. A less common type of preventive maintenance is to preventively remove and replace parts or components in advance of their failure. For example, a transit agency might routinely overhaul all transmissions that haven't failed by a specified mileage even if they are still working properly. In such cases the agency has determined that the transmissions are close to the end of their design life. Chapter Seven of the *Guide* describes a technique for determining the optimal replacement mileage interval for such preventive replacements.

Corrective maintenance is carried out to repair a part or component after it has failed. There are trade-offs that must be made between the levels of corrective and preventive maintenance that are carried out. For example, suppose that a bus maintenance department performs only minimal preventive maintenance tasks and only changes fluids and filters at inspection intervals without making any other checks or adjustments. All other maintenance is executed on a corrective basis. In this situation, buses are run until something fails. Buses are then tied up in maintenance instead of being on the street. The maintenance department is in a situation largely of responding to failures instead of anticipating them. This limits the ability of the maintenance manager to plan and schedule maintenance activities. If the bus fleet is relatively low on spares, it places the maintenance department in a continual cycle of making emergency repairs to meet peak period demands for buses.

Continually responding to chance failures and making emergency repairs in order to get buses back in service is an unmanageable and costly situation. On the other hand, if the maintenance manager has the flexibility to schedule which buses are to be repaired first, it is possible to make tremendous efficiency gains. For example, experimentation has shown that the introduction of work load scheduling and simple maintenance job prioritization rules based on the expected number of man hours a job will take can decrease the average number of buses out of service for maintenance work by as much as 20 percent (1).

As preventive maintenance is increased, the amount of emergency repairs should decline, thereby increasing the efficiency of the maintenance operation and resulting in better control of costs. Figure 1.3 depicts the relationship between total maintenance cost, the cost of corrective and preventive maintenance and the amount of preventive maintenance conducted. As the level of preventive maintenance effort increases, corrective and total maintenance costs decrease. The dashed line indicates the optimum level of preventive maintenance where the total cost of preventive and corrective maintenance reaches a minimum. Past the minimum total cost point (indicated by the dashed line), additional preventive maintenance effort is not cost effective. The problem for the maintenance manager is to find the efficient level of preventive maintenance for each part and component which results in the minimum total maintenance cost.

The optimum level of preventive maintenance for each bus part or component depends on the following factors:

1. Component failure patterns. Some components and parts have failure rates that are related to the use and wear they have been exposed to (they are "age-dependent") while others are unrelated to use (they are "age-independent"). The previous examples of brake shoes and fuses are good illustrations of part failures that are age-dependent and age-independent, respectively. Preventive replacement is an appropriate policy for components which exhibit age-dependent failure patterns but not for those with age-independent patterns.

2. Repair costs. If it costs just as much to repair an item before it fails as it does after it fails, then the item should be replaced after it fails. For example, a burned out light bulb will require an equal amount of effort to replace before and after failure and it rarely disables the bus! Light

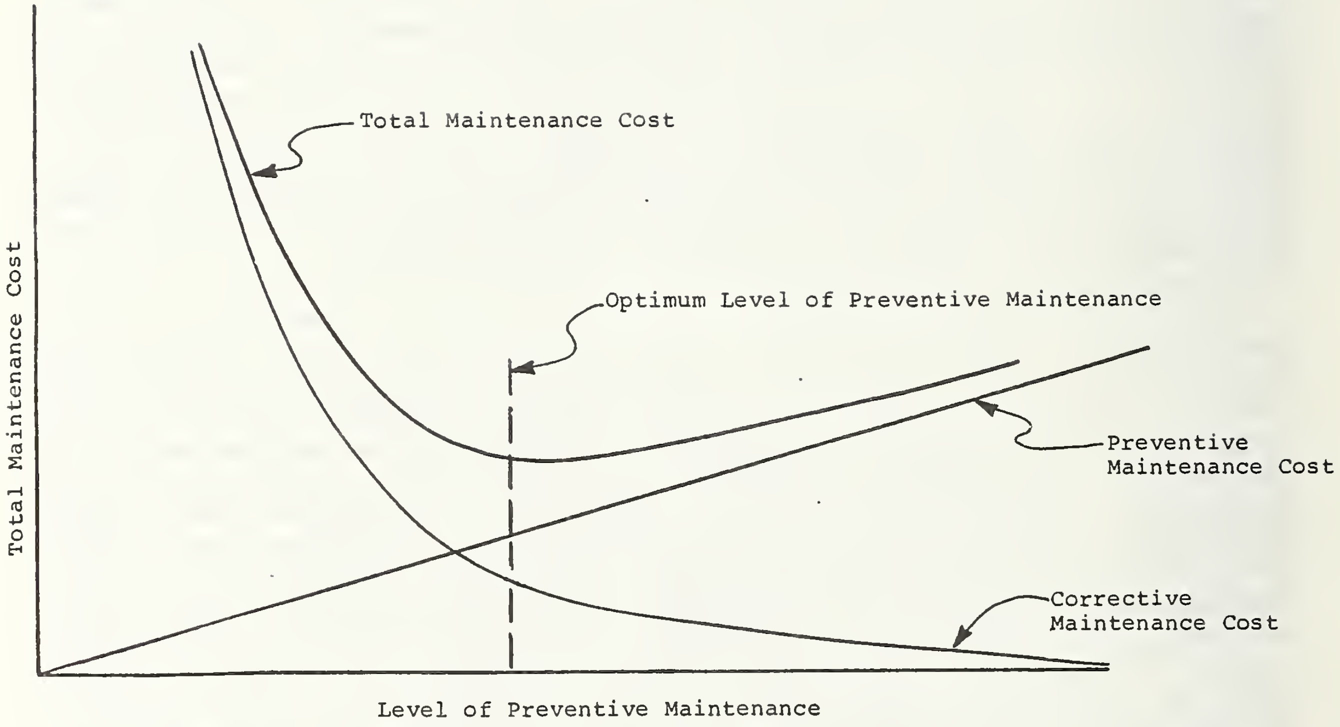


FIGURE 1.3

OPTIMAL LEVELS OF PREVENTIVE AND CORRECTIVE MAINTENANCE bulb failure is relatively unpredictable, anyway. Therefore, the bulb should be replaced only when it fails. The cost of repairing a component before it fails is just the direct cost of the repair.

The cost of repairing a component after it has failed must also include the damage caused by operating the component during and after the failure. If the failure disables the bus on the road, forcing passengers to transfer to another bus and a tow truck to be called, then a dollar penalty should be associated with the failure. One Southwest transit system assigns a one thousand dollar penalty for each mechanical failure where the bus is disabled even though in out-of-pocket terms it does not cost that much to trade the disabled bus for another. This subject is discussed further in Chapter Seven.

3. Vehicle spare ratio. Since preventive maintenance is conducted while the bus is still operative, the manager has some degree of flexibility over when the preventive maintenance is to be performed. In other words, the maintenance manager can remove buses from service for preventive maintenance when it is convenient to both the maintenance shop and the dispatcher. Corrective maintenance is required when an item randomly fails.

The flexibility to schedule preventive component replacements is a function of the number of spare buses available. If there are not enough spare buses then the dispatcher will be unable to meet peak period vehicle demands if too many buses are tied-up in maintenance. Corrective maintenance must be conducted promptly whenever a chance failure occurs. Failures may occur at inopportune times, thus requiring the transit agency to have more spare buses. Therefore, the transit agency with limited spares should attempt to schedule maintenance by repairing components preventively rather than waiting until they fail. However, if spares are plentiful, failure-based corrective maintenance will be more tolerable.

4. Ability to monitor the condition of the component. Preventive maintenance can be performed when a condition check indicates that the component is wearing out. Brake shoe inspections are a common example. Another is the monitoring of engine oil consumption. A bus with a higher than normal oil consumption rate should be brought in and inspected for engine problems.

Some components or parts may not be monitorable, however, and condition-based repairs may be impossible. For example, a relay will give no hint of its impending failure. Therefore, there is no justification for preventively replacing a relay.

5. Safety implications. The failure of certain parts or components, such as brakes, cannot be tolerated from a safety standpoint. For example, many transit systems set their interval

between preventive maintenance inspections equal to the maximum safe interval between brake inspections. In cases where safety is involved, safety rather than cost minimization should dictate preventive and corrective maintenance levels.

Maintenance Repair Policies

There are four ways to deal with the maintenance of a part or component according to Kelly (5). These are condition-based maintenance, fixed-mileage maintenance, operate-until-failure maintenance, and design-out maintenance.

Condition-Based Maintenance (CBM)

With condition-based maintenance, approaching failures are predicted when a monitorable condition reaches a level where it exceeds a tolerable limit. Previously cited examples of condition-based maintenance are brake shoe wear and engine oil consumption. Condition-based maintenance is appealing for several reasons:

1. CBM detects maintenance problems before the bus becomes inoperable, thus reducing the chances of an in-service failure. Also, CBM means early detection of a problem before it mushrooms into a catastrophic failure.
2. The part or component is repaired shortly before it ultimately fails. The part or component is used until nearly the end of its life, thereby getting maximal use out of the component while avoiding the consequences of failure.
3. Chance failures are nearly eliminated. This allows the manager to execute preventive repairs when they fit into the maintenance department's schedule.

There are three commonly performed types of condition-based monitoring of components: qualitative inspections, quantitative inspections, and trend monitoring.

Qualitative Inspections. These are periodic qualitative checks (visual, feeling or hearing) of a part or component's condition. For example, the visual inspection of the front tires of a bus for unusual wear patterns would be a qualitative check. Frequent qualitative inspections have been found to significantly reduce in-service failures. A study of bus maintenance records found that those transit systems that required their drivers to conduct qualitative inspections of their bus before hitting the streets have lower than normal road call rates (1).

Quantitative Inspections. These are quantitative checks which measure the condition of a part or component to see if it exceeds a tolerable level. A common example is the inspection of brake shoe thickness. If the shoe thickness is below a required minimum, the brakes are overhauled.

Another example of a condition check is engine oil analysis. This is a laboratory test to determine the levels of various foreign materials in the oil. Abnormal levels may indicate a problem. For example, engine oil analysis normally tests oil viscosity, water content, fuel dilution, anti-freeze content, silicon content, suspended solids and content of wear metals. Abnormally high concentrations of any of these factors may indicate a mechanical problem or an approaching component failure.

Figures 1.4 and 1.5 are plots of iron and tin concentrations in parts per million versus the mileage when the oil was drained from a Detroit Diesel Allison 8-V71 engine. Both plots show high concentrations of these metals at early mileages, which is expected during engine break-in. As mileage increases the levels of iron and tin subside to normal levels. Eventually, the metal levels sharply rise again, indicating abnormal wear and the likelihood of an impending engine failure.

This condition should trigger the maintenance manager to bring in the bus and have it thoroughly inspected and repaired if necessary. Needed repairs are made preventively before the bus becomes inoperable, thus allowing the maintenance manager to schedule the engine repair when convenient. If engines were failing prematurely the oil analysis results could be used as documentation in a warranty claim against the engine manufacturer.

Trend monitoring. These are quantitative measures of past experience used to set a norm. A common example is the monitoring of fuel and oil consumption rates and flagging buses that exhibit exceptional rates. An exceptional oil consumption rate, either high or low, may indicate a maintenance problem and the manager can schedule an inspection of the bus to diagnose the problem.

Fixed-Mileage-Maintenance (FMM)

These are maintenance actions that are carried out at regular mileage intervals. This type of maintenance is most applicable where there is a known relationship between the mileage travelled and the failure mechanism. For example, oil and oil filters are changed at specific intervals because they are known to deteriorate with use.

In general, condition-based maintenance is preferable to fixed-mileage-maintenance. This is because there is a degree of chance variation in the failure mileage of all parts and components, as illustrated in Figure 1.1. Condition-based maintenance permits the maximum use to be obtained from each part or compo-

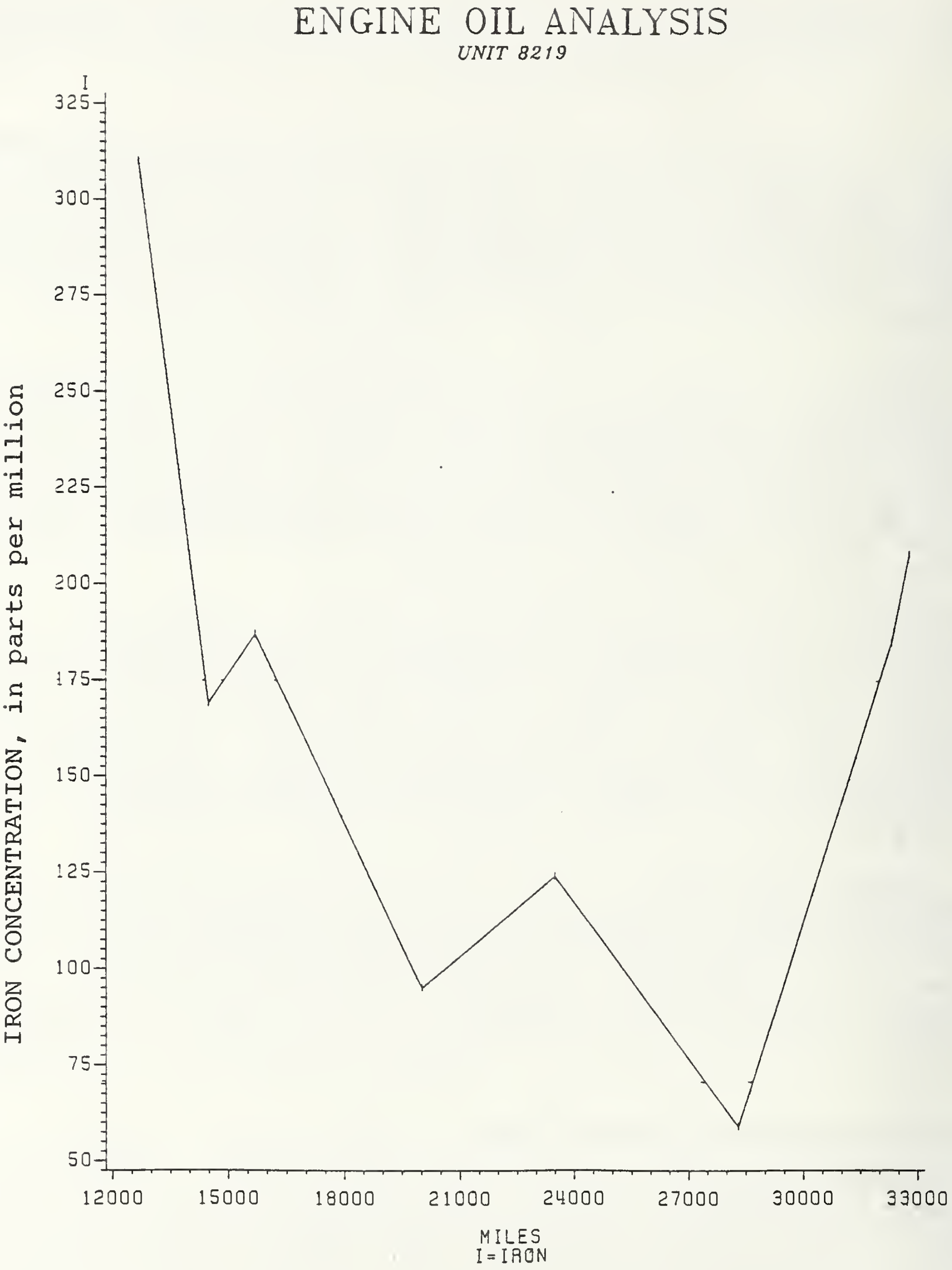


FIGURE 1.4

ENGINE OIL ANALYSIS OF IRON CONCENTRATION VERSUS MILEAGE

FIGURE 1.5

ENGINE OIL ANALYSIS OF TIN CONCENTRATION VERSUS MILEAGE nent without allowing the component to fail and makes it possible to schedule necessary repairs in advance. Fixed-mileage maintenance implies the establishing of blanket repair policies across all parts and components of a specific type. For example, consider a fixed-mileage maintenance policy which states that all air compressors on a specific bus model are to be overhauled every 84,000 miles if a failure has not already occurred before 84,000 miles. The problem with this policy is that a few air compressors will fail before 84,000 miles and require corrective repairs. Some, however, will be replaced long before they would ultimately fail and, thus, are prematurely overhauled.

Advantages. Fixed-mileage replacement, however, is preferable to operate-until-failure maintenance (next section) under some conditions. First, if the deterioration is time or mileage dependent and the failure pattern is known and predictable, then FMM is appropriate. Second, FMM is preferable if the costs of replacement after the failure has occurred are greater than the costs of replacement before a failure. The costs of replacement after a failure should include an assessment of the intangible (a penalty for a service disruption due to a disabled bus) and tangible costs of an in-service failure, the costs associated with having to make an unscheduled repair, and the costs that may result from a failure that would not be incurred if the system were overhauled before a failure. This topic is discussed in more detail in Chapter Seven.

Example. A Southwestern transit system overhauls its Detroit Diesel Allison V-730 transmissions at a constant interval of about 100,000 miles. FMM overhauls have resulted in the following benefits according to the transit agency:

1. The maintenance manager is able to schedule the rebuilding work in advance, thus keeping the work flow relatively constant. The maintenance manager has even more scheduling flexibility than with condition-based maintenance because the timing of when a transmission's condition exceeds a given tolerance level is also a chance event.

2. Since the removal and replacement of transmissions with FMM can be scheduled, buses can be brought in when it is convenient and when other corrective and preventive inspections can be scheduled. Several maintenance tasks can be performed while each bus is in the shop, thus reducing the time when the bus is unavailable for revenue service.

3. The maintenance manager has been able to dramatically reduce road calls that are a result of transmission failures. Although such road calls were rare, their occurrence had serious impacts on the ridership's impression of service reliability.

4. The maintenance manager has good information on the life characteristics of the transmissions in the fleet, similar to the presentation in Chapter Three, so few transmissions are overhauled much in advance of their ultimate failure.

Operate-Until-Failure Maintenance (OUF)

Operating a part or component until it fails implies that all maintenance will be corrective. Because failures are random events, a component or part that is not being monitored can fail without warning and maintenance work flows cannot be planned and scheduled. Generally, an operate-until-failure maintenance policy is the least preferable strategy for the maintenance of a part or component in terms of work flow management.

However, it is the most cost-effective policy under the following two conditions. First, if the condition of the part or component cannot be monitored and/or the failure mechanism is not mileage-dependent, then OUF is cost-effective. Second, OUF may be preferable if the total cost of repairing a part or component after a failure is equal to the cost of repairing it before a failure. The total cost of repair after a failure should account for the cost implications of conducting an unscheduled repair and the intangible cost implications of an in-service failure (if failure of the part or component would make the bus inoperable).

Design-Out-Maintenance (DOM)

The designing out of maintenance problems as a policy is different in objective than the three previously discussed policies because DOM seeks to remove the maintenance problem. The other policies seek to minimize maintenance problems. Sometimes designs are created which may appear feasible during the development, design and production stages of bus manufacture but are unsatisfactory in an actual operating environment. Similar problems may result from manufacturing flaws. In either case, if maintenance costs are excessive then the best solution may be for the manufacturer to redesign the component or for the transit agency to purchase an alternative component or system.

An example of a flaw due to poor design is the location of air conditioning systems on Advance Design Buses built in the late 1970s and early 1980s. The air conditioning system was located in the engine compartment where the heat, dust and dirt caused some of the air conditioners to fail prematurely. Air conditioning manufacturers have created retrofit units which are mounted on the upper rear of the bus away from the engine compartment and its adverse environment.

How to Select a Maintenance Policy

Information Requirements. The preceding sections discussed four policies that may be applied to the maintenance of bus parts and components. Each policy has advantages and disadvantages as noted above. The following information is required to select the appropriate a policy for a particular component:

1. Can the condition of the component be monitored in order to detect an approaching failure?
2. Are failures a result of a design or manufacturing flaw?
3. Is the failure mechanism mileage-dependent and predictable?

The maintenance manager can determine if it is feasible and economical to monitor the condition of the part or component (the first type of information required) based on the physical characteristics of the part or component. There are simple statistical tests that can be used to identify a design or manufacturing flaw and the predictability of failure, the second and third types of information required. These methods are described in Chapters Two through Six and involve the use of either the Normal or Weibull probability distributions.

Determination of Component Failure Mechanism. Table 1.1 can be used to determine the component failure mechanism using the values of the mean mileage to failure, $\bar{X}$, and the standard deviation, SD. If the Weibull distribution is employed, then B (the Weibull "shape factor") and D (the Weibull "minimum life term") are used. For example, consider a component with a Weibull failure pattern. If the mean mileage to failure, $\bar{X}$, is as expected and B is greater than 2, then Table 1.1. indicates that the failure mechanism is "mileage to failure predictable".

As another example, brake shoe wear outs follow the Normal distribution. The mean mileage between brake shoe wear outs of Figure 1.1 is 20,837 miles ($\bar{X}$ = 20,837 miles) and the standard deviation is 7,385 miles (SD = 7,385 miles). $(\bar{X}-D)/SD = 2.82$ (20,837/7,385 = 2.82). D is neglected in this computation since the failure pattern is Normal and not Weibull. Assuming that the mileage between wear outs is not below what is expected, Table 1.1 indicates that the mileage to failure (or wear out in this case) is predictable.

Component Maintenance Policy Selection. Once the failure mechanism has been determined using Table 1.1, the decision tree in Figure 1.6 is used to select a maintenance policy. As an example of the use of Figure 1.6, consider the above brake shoe wear out data where $\bar{X}$ conforms to expected performance. Brake shoe wear is detected by monitoring the thickness of the brake shoes, hence the failure occurrence is predictable. The mileage to failure is predictable as indicated above. The path through the decision tree in Figure 1.6 for this case is highlighted with

TABLE 1.1

IDENTIFICATION OF COMPONENT FAILURE MECHANISM

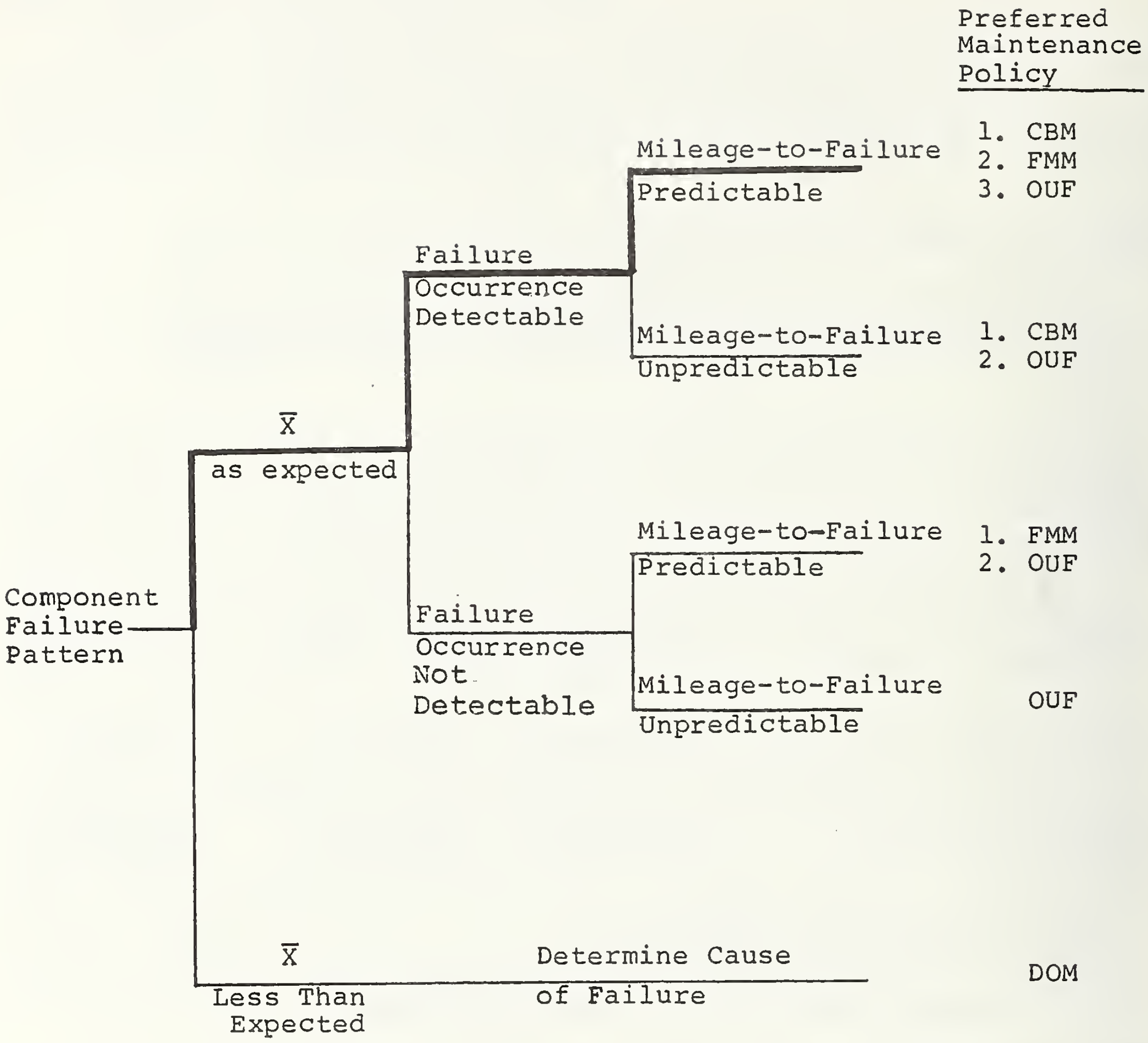
Component Failure Mechanism	Value of*		
	$\bar{X}$	$(\bar{X}-D)/SD$	B
Design or Manufacturing Flaw	Less Than Expected	Less Than 1	Less Than 1
Mileage to Failure Unpredictable	As Expected	Between 1 and 2	Between 1 and 2
Mileage to Failure Predictable	As Expected	Greater Than 2	Greater Than 2

* $\bar{X}$ = mean mileage to failure,

SD = standard deviation of failure mileages,

D = Weibull minimum life term,

B = Weibull shape factor.



$\overline{X}$: Mean Mileage-to-Failure

CBM: Condition-Based Maintenance

FMM: Fixed-Mileage Maintenance

OUF: Operate-Until-Failure

DOM: Design-Out Maintenance

FIGURE 1.6

DECISION DIAGRAM FOR SELECTION OF THE PREFERRED COMPONENT MAINTENANCE POLICY a bold line. The possible maintenance policies, in order of preference, are condition-based-maintenance (CBM), fixed-mileage-maintenance (FMM), and operate-until-failure (OUF). The preferred policy, CBM, is listed above FMM and OUF to indicate this preference. However, economic and safety factors also should be taken into account in the ultimate selection of a maintenance policy.

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STUDY QUESTIONS

(1) Consider the following 14 buses with the failure status of the original transmission indicated below:

Bus Number	Mileage at Time of Study	Failure Mileage of Original Transmission	Failure Status
8301	105,335	81,621	
8302	80,180	66,981	
8303	114,852	71,400	
8304	80,309	No failure yet	
8305	96,971	82,195	
8306	115,071	48,250	
8307	83,551	No failure yet	
8308	121,143	No failure yet	
8309	117,036	90,930	
8310	123,443	43,483	
8311	93,821	52,624	
8312	101,178	83,822	
8313	118,667	No failure yet	
8314	91,013	71,750	

Indicate the "failure status" of each transmission as follows: "failed," "censored," or "suspended."

(2) Engine failures were found to follow the Weibull distribution with the following statistics:

Mean mileage to failure, $\bar{X}$: 258,000 miles
Standard deviation, SD: 82,000 miles
Weibull minimum life term, D: 120,000 miles
Weibull shape factor, B: 1.75

Using Table 1.1 and Figure 1.6, determine the engine failure mechanism and the best maintenance policy. The mileage between failures is as expected and the condition of the engine is monitored in several ways, including fuel and oil consumption, and engine oil analysis.

ANSWERS TO STUDY QUESTIONS

(1) The transmission with the largest failure mileage is that of bus 8309, 90,930 miles. The failure status is indicated below for each transmission. Censored cases are unfailed transmissions with mileages exceeding 90,930 miles, while suspended cases have mileages less than 90,930 miles.

Bus Number	Failure Status
8301	Failed
8302	Failed
8303	Failed
8304	Suspended
8305	Failed
8306	Failed
8307	Suspended
8308	Censored
8309	Failed
8310	Failed
8311	Failed
8312	Failed
8313	Censored
8314	Failed

(2) $(\bar{X}-D)/SD$ = (258,000 miles - 120,000 miles)/(82,000 miles) = 1.68. In Table 1.1 either this number or B = 1.75 can be used. The engine failure mechanism is "mileage to failure unpredictable." From Figure 1.6, the best maintenance policies are, in order of preference, condition-based maintenance (CBM) and operate-until-failure (OUF).

CHAPTER TWO

COMPONENT FAILURE ANALYSIS USING THE NORMAL DISTRIBUTION: BRAKE SHOES EXAMPLE

This chapter is a self-contained analysis of brake shoe replacements to predict future replacement patterns. It demonstrates the use of the Normal distribution to determine the mean failure mileage, predict future failures per mileage interval, and predict future reliability over time. The topics are:

- How to collect brake shoe replacement data.
- How to tabulate the data.
- Fitting the Normal distribution to the brake shoe data.
- How many brake shoes will wear out during any mileage interval?
- How long will brake shoes survive?
- How reliable are the brake shoes?
- How often should brake shoes be inspected for wear?
- How many brake shoes will wear out during any time period?

As explained in the introductory chapter, the Normal distribution is used to model the failure and/or replacement of parts which wear out such as brake shoes. Brake shoes are replaced either when a periodic inspection finds that they are worn out or after a driver reports brake problems.

How to Collect Brake Shoe Replacement Data

Records of brake shoe replacements generally start with work orders. Many transit properties summarize work order information as individual vehicle histories. These histories often are the most convenient sources of repair information. Alternatively, one can collect the data directly from work order files.

The information to be collected is the number of miles accumulated by the bus between brake shoe replacements. Mileage-based statistics can be directly related to bus operating costs per mile.

Statistical analysis with the Normal distribution requires that at least 30 mileages are collected, but rarely should it be necessary to collect more than 50 mileages. Care must be taken in the data collection to make sure that all the brake shoes are of the same type and have experienced similar operating conditions. Front and rear brake shoes should be separately analyzed. Similarly, each data set should consist of just one bus model. Records for express buses, which experience fewer stops and starts, should be separated from buses which experience conventional duty cycles.

The information collected must be completely correct. There must be a record for each and every time the brake shoes on a bus were replaced and the exact mileage of the bus when the repair was made. Unfortunately, repairs made to a bus are occasionally omitted in the vehicle history summary. When a record is omitted, it makes that brake shoe appear as though it lasted an unusually long time ("super shoe"). Care must be taken to identify and throw out these "super shoes" because they will foul up the analysis. Usually, it is obvious from the data when a record is missing.

Shown below is the rear brake maintenance history for one bus from 1980 to 1983. The numbers in column 2 are the mileages when the rear brake shoes were replaced on the bus. In column 3 the mileage the bus traveled between brake shoe replacements is computed and tabulated. The first mileage between brake shoe replacements, 57,523 miles, looks suspiciously large, hence it is treated as a "super shoe" and deleted.

Date of Brake Shoe Replacement (1)	Mileage at Replacement (2)	Mileage Between Replacements (3)
1/10/80	153,303	
		57,523 ("super shoe")
5/21/81	210,826	
		18,154
10/26/81	228,980	
		26,343
5/14/82	255,323	
		30,505
1/03/83	285,828	
		8,684
3/06/83	294,512	

Similar information is gathered from other buses of the same model to obtain the 30 to 50 replacement mileages needed for the analysis.

How to Tabulate the Data

Class Data. Table 2.1 (Work Sheet 1) lists 31 mileage

TABLE 2.1

WORK SHEET 1

TABULATION OF FAILURE MILEAGE FREQUENCIES

Cost Driver Brake System Bus Model

Component Type Rear Brake Shoes Study Dates 1981-1983

Failure Mileage (1)	Failure Mileage (2)	Mileage Class		Tally (5)	Frequency, F (6)
		Lower (3)	Upper (4)		
23,972	30,952	5,000	10,000	//	2
18,833	8,684	10,000	15,000	////	4
24,642	16,831	15,000	20,000	///// ///	8
29,249	24,123	20,000	25,000	///// /////	10
19,456	22,105	25,000	30,000	/////	5
19,846	20,154	30,000	35,000	//	2
10,347					31
27,710					
18,154					
26,343					
30,505					
10,121					
24,563					
23,957					
21,266					
22,966					
14,363					
24,435					
29,506					
19,556					
13,245					
9,023					
19,304					
17,984					
26,459					

Number of failure mileages, N 31

Maximum mileage 30,952 miles Minimum mileage 8,684 miles intervals between rear brake shoe replacements taken from the vehicle maintenance histories of several buses. There is space for 50 mileages in the first two columns of the work sheet. In Table 2.1 the mileages are classed in order to simplify the analysis. Some information is lost by classing the mileages rather than working with actual mileages, however, the classed data are easier to interpret and the conclusions should be good enough.

Below are listed the guidelines which should be followed in constructing classes:

1. Use no less than six and no more than 15 classes.
2. Select classes that will include all the mileages.
3. Make sure that each mileage fits into one class.
4. Whenever possible, make the class intervals equal in size (each should contain the same number of miles).

Typically, about six classes should suffice. The number of classes and the range of mileages in each class are easily determined by working with the maximum and minimum observed replacement mileages. In the above example these are indicated at the bottom of Table 2.1, 30,952 miles and 8,684 miles respectively. If the difference between these two numbers, 22,268 miles, is divided by six, the result is 3,711 miles. This prompts the selection of 5,000 miles as the class interval, starting with 5,000 miles and going up to 35,000 miles, with six classes as indicated in columns 3 and 4 of Table 2.1.

The statistical analysis that follows would not substantially change if another class interval width were selected or if there were more classes. For example, a 4,000 mile class interval would work just as well, also resulting in six intervals ranging from 8,000 miles to 34,000 miles. With just 31 mileages in the data set, six class intervals is sufficient.

Class Frequency. Once the class intervals have been determined and entered into Table 2.1, the next step is to determine the number of mileages that fall within each class. Space is provided in the work sheet of Table 2.1 to do the tally work (column 5). The total number of mileages in each class interval is entered in column 6. These numbers are called the - "frequencies," the number of times a mileage falls within a particular class. As a check on the computations, the sum of the frequencies should equal the total number of mileages, N, which is 31 in this example.

If the frequency data in Table 2.1 are made into a graph, it makes the information easier to interpret. The bar chart in Figure 2.1 was derived from the frequency data in column 6 of Table 2.1. The bars are centered over the average mileage of each class and the height of each bar is determined by the frequency of the class. The taller the bar the more frequent were the mileages observed for that class.

This graph gives a picture of the variability in brake shoe life from one shoe to the next. Note the "bell-shape" pattern of the brake wear out mileages, which indicates Normally distributed data. Most wear outs are contained in the center bars and few in the lowest and highest classes.

Fitting the Normal Distribution to the Brake Shoe Data.

Mean. It is obvious from Figure 2.1 that all brake shoes do not wear out at the same mileage. But how does one summarize such variable numbers? One way is to calculate the average or mean" mileage.

The mean mileage is the sum of all the replacement mileages divided by the total number of mileages, N. In Table 2.1 the sum of all 31 mileages divided by 31 turns out to be 20,924 miles. Taking the average is a relatively simple task when using scientific calculators with statistics keys. However, even with a calculator adding up all those mileages can be tiring and may lead to mistakes.

A short cut method which uses the classed data of Table 2.1 is shown in Table 2.2 (Work Sheet 2) where the classes are listed in columns 1 and 2. The average of each mileage class (the lower plus upper mileages divided by 2), X, is entered in column 3 and multiplied by the class frequency, F, from column 4. The product, FX, is entered in column 5. The mean is found by adding all the numbers in column 5 (ΣFX) and dividing by the total number of mileages, N. There is room for this computation at the bottom of Table 2.2. The mean mileage by classes, 20,403 miles, differs a little from the actual mean of 20,924 miles, but this should not affect the results of the analysis.

Standard Deviation. Another way to summarize the different mileages is the "standard deviation," which measures the scatter or variability of the mileages, how spread out they are from the mean. Suppose that the brake shoes from two different manufacturers, for example, have the same mean mileage life. The first brand of shoe, however, tends to have a broad range of mileage lives (some wear out prematurely, others last far longer than the mean) while the mileages of the second brand tend to bunch closely about the mean. This might indicate that the first shoe brand is of less consistent quality than the second.

Scientific calculators are available which make it relatively easy to calculate the standard deviation. Special button functions do most of the work. However, even with a scientific calculator it can be tedious if there are a lot of mileages to enter.

A more convenient way to calculate the standard deviation is presented in Table 2.2 (Work Sheet 2) by building upon the computations done to determine the mean using classed data. First,

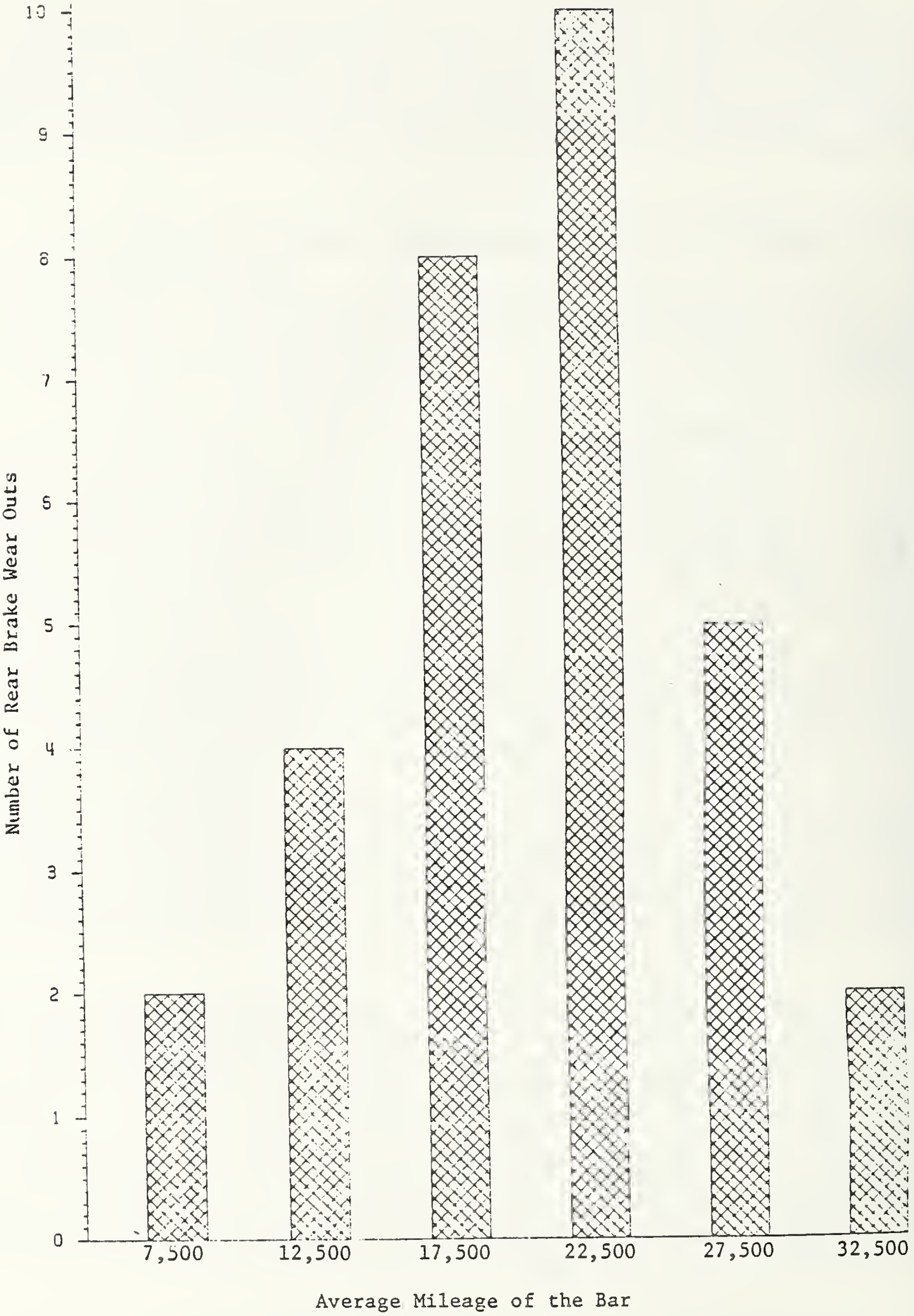


FIGURE 2.1

REAR BRAKE WEAR OUT FREQUENCY BAR CHART

TABLE 2.2

WORK SHEET 2

COMPUTATION OF MEAN AND STANDARD DEVIATION

Cost Driver Brake System Bus Model ________

Component Type Rear Brake Shoes Study Dates 1981-1983

Mileage Class		Class Average X (3)	Freq. F (4)		FX (5)	X^2 (6)	FX^2 (7)
Lower (1)	Upper (2)						
5,000	10,000	7,500	2	=	15,000	56,250,000	112,500,000
10,000	15,000	12,500	4	=	50,000	156,250,000	625,000,000
15,000	20,000	17,500	8	=	140,000	306,250,000	2,450,000,000
20,000	25,000	22,500	10	=	225,000	506,250,000	5,062,500,000
25,000	30,000	27,500	5	=	137,500	756,250,000	3,781,250,000
30,000	35,000	32,500	2	=	65,000	1,056,250,000	2,112,500,000
				=			
				=			
				=			
				=			
				=			
				=			
				=			
				=			
				=			
Sums, Σ			N= 31	ΣFX=	632,500		ΣFX^2= 14,143,750,000

Mean = $\bar{X} = \frac{\Sigma FX}{N}$ = 632,500 / 31 = $\bar{X}$ = 20,403 miles

$$\text{Standard Deviation} = SD = \sqrt{\frac{N\Sigma FX^2 - (\Sigma FX)^2}{N(N-1)}}$$

$$= \sqrt{\frac{31 \times 14{,}143{,}750{,}000 - 632{,}500 \times 632{,}500}{31 \times 30}} = SD = 6{,}426 \text{ miles}$$

the class averages, X, are squared and this number, X^2, is entered in column 6 for each class. Then column 6 is multiplied by the class frequency, F, with the result, FX^2, entered in column 7. The sum of column 7, ΣFX^2, is computed and entered at the bottom of the table. In this example, ΣFX^2 is 14,143,750,000.

The formula for the standard deviation is presented at the bottom of Table 2.1. The formula uses only three column sums: N, ΣFX, and ΣFX^2 from columns 4, 5, and 7 respectively. The work sheet provides blanks to fill in these numbers. The answer for the standard deviation, SD, is 6,426 miles. This result is only an approximation, but the correct standard deviation of 6,279 miles, obtained from a scientific calculator, is not greatly different. This error will not affect the results of this analysis.

Table 2.3 (Work Sheet 3) is a convenient summary sheet for the results of the above computations. Space is provided for the number of failure (replacement) mileages, the maximum and minimum mileages observed, the mean, and the standard deviation. The work sheet also includes a graph for plotting the bar chart of the frequencies (from Figure 2.1) upon which the mean can be indicated. The above results are summarized in Table 2.3 as an example.

The mean, $\bar{X}$, and the standard deviation, SD, can now be used to determine the best maintenance policy for brake shoes as explained in Chapter One. Dividing $\bar{X}$ by SD yields a value of 3.18 in Table 2.3. Based on Table 1.1 from Chapter One, the failure mechanism is "mileage to failure is predictable." The appropriate maintenance policy for brake shoe replacements, based on Figure 1.6, is "condition-based maintenance" (CBM) since brake shoe wear can be monitored.

Normal Distribution. Determining what is expected to happen is known as forecasting. The problem is to forecast how many wear outs will occur during any mileage interval, such as the first 10,000 miles. To make good forecasts, the brake wear out frequency distribution first is approximated with a smooth curve, the Normal distribution.

The bar chart of Figure 2.1 is drawn again in Figure 2.2 with a bell-shaped Normal curve added which approximates the shape of the bar chart. The formula for the Normal curve includes the mean and standard deviation of the mileage data; this is how the Normal curve is adapted to the data.

It is easier to work with the Normal curve if the frequencies are converted to percentages. In the brake wear out data this simply requires that all frequencies be divided by the total number of brake wear outs. For a frequency of 10, the percentage is 10/31 = 32.3 percent.

The Normal curve is shown again in Figure 2.3 but this time the vertical axis is scaled in percent. The horizontal axis is

TABLE 2.3

WORK SHEET 3

SUMMARY SHEET OF FAILURE MILEAGE DISTRIBUTION

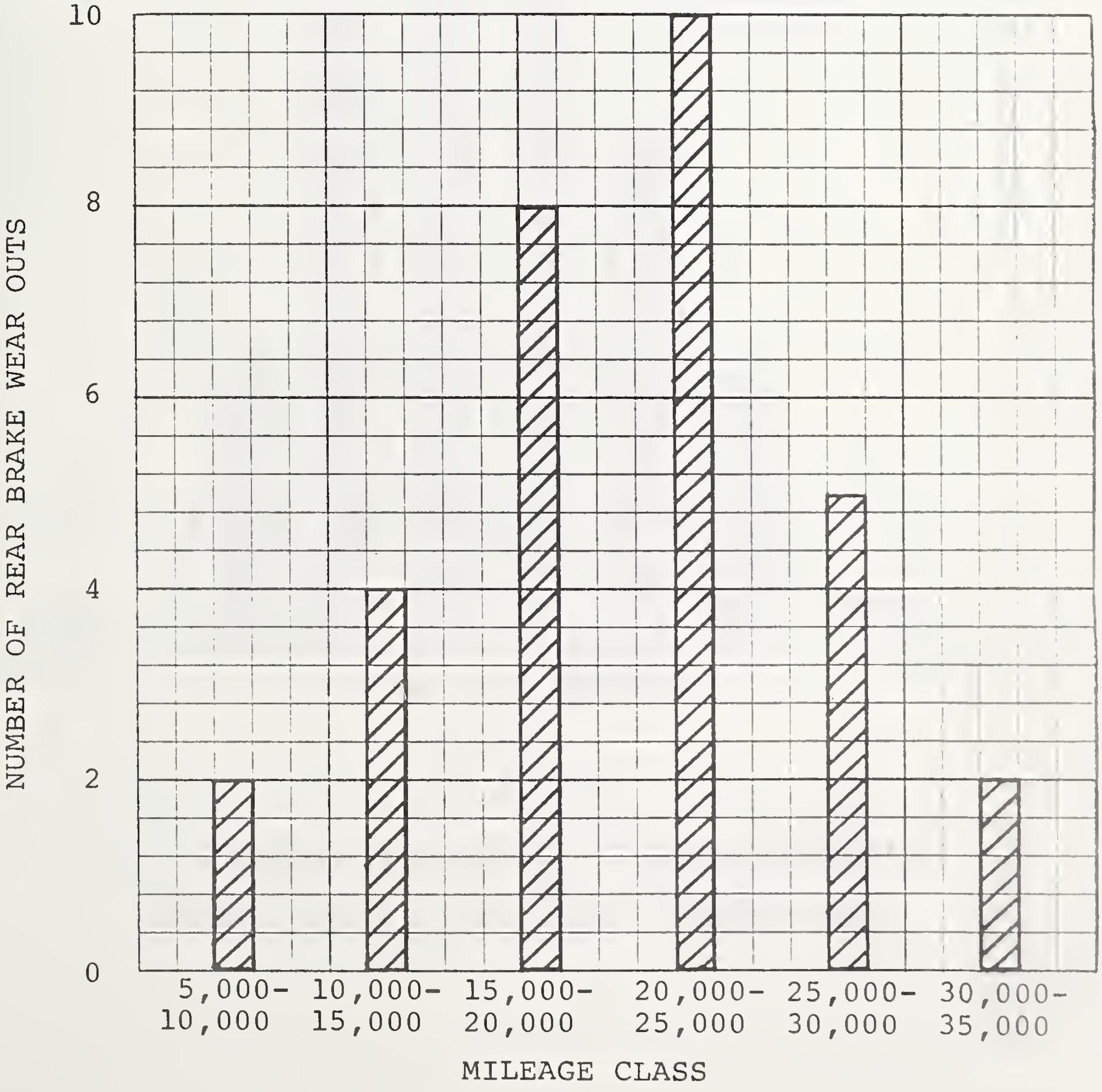
Cost Driver Brake System
Bus Model
Component Type Rear Brake Shoes
Study Dates 1981-1983

Number of cases, N 31
Number of failures, NF 31
Maximum mileage 30,952 miles
Minimum mileage 8,684 miles
Mean, $\bar{X}$ 20,403 miles
Std. deviation, SD 6,426 miles
$(\bar{X}-D)/SD$ 3.18

WEIBULL DISTRIBUTION PARAMETERS:

Minimum life term, D
Shape factor, B
Characteristic life factor, T

FAILURE MECHANISM: Mileage to failure is predictable



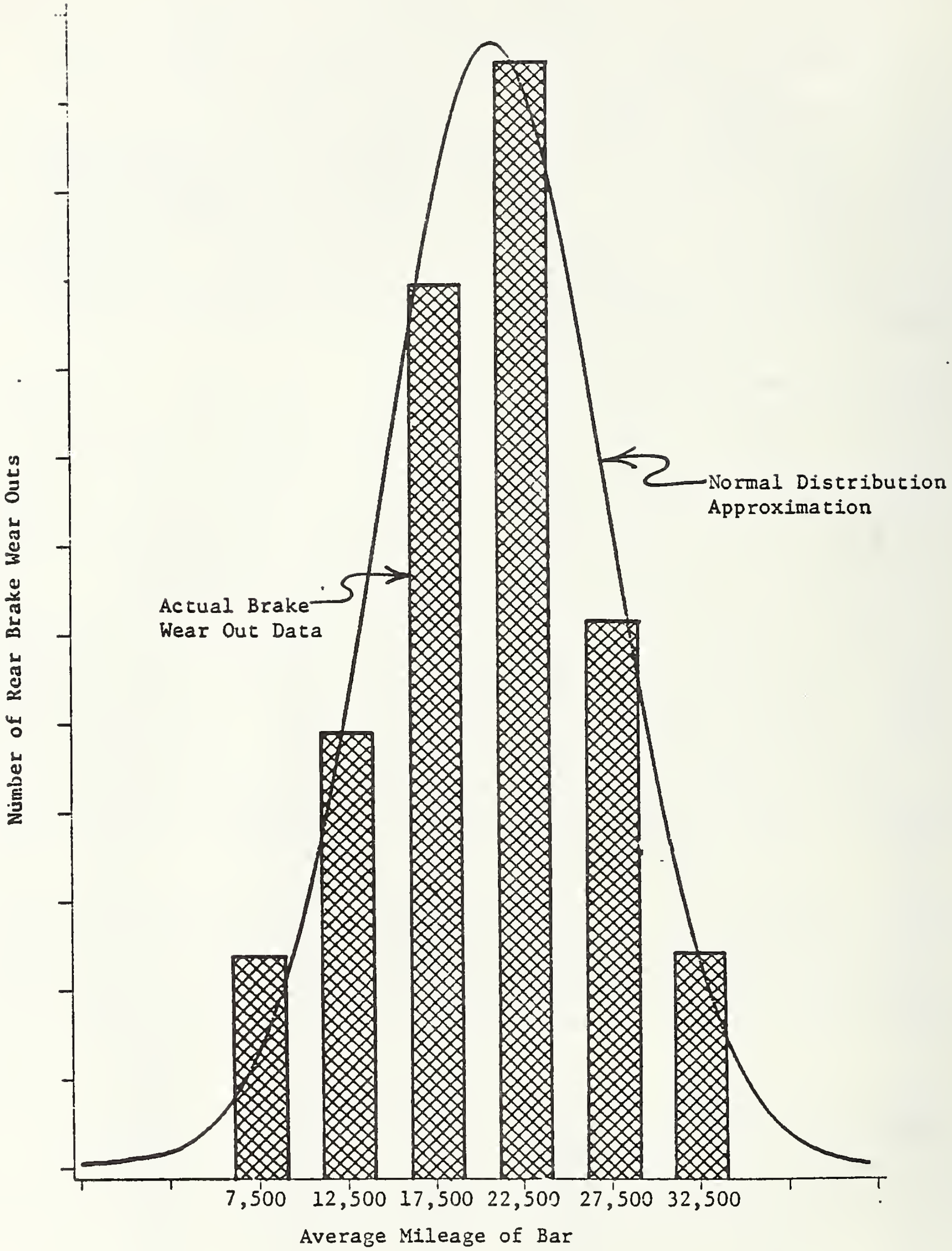


FIGURE 2.2

RELATIONSHIP BETWEEN ACTUAL BAR CHART DATA

AND NORMAL DISTRIBUTION APPROXIMATION

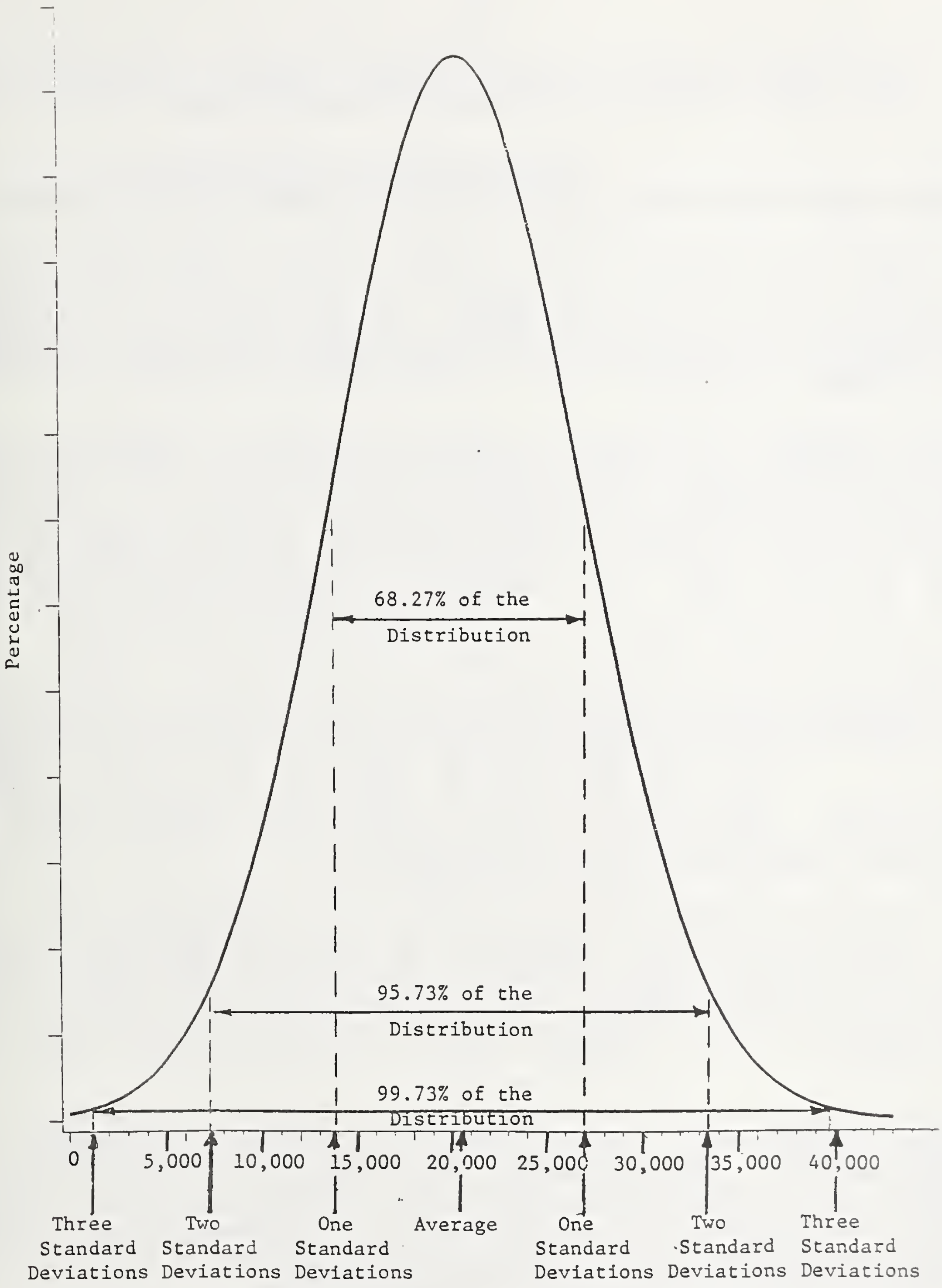


FIGURE 2.3

RELATIONSHIP BETWEEN STANDARD DEVIATIONS FROM THE MEAN AND PERCENT AREA UNDER THE NORMAL CURVE still in miles and scaled in standard deviations from the average. The mean mileage, 20,403 miles, falls exactly in the middle (the "peak") of the bell-shaped Normal curve.

How Many Brake Shoes Will Wear Out During any Mileage Interval?

Figure 2.3 shows the relationship between the percent of the brake wear outs included in a mileage interval and the width of an interval in standard deviations. In Figure 2.3, the percent of the brake wear outs (or percent of the total area under the curve) which is within one, two and three standard deviations from the mean is marked.

For example, Figure 2.3 shows that 68.27 percent of the brake wear outs will occur within one standard deviation above and below the average (26,829 miles to 13,977 miles). This percentage is the same for all data which is Normally distributed. Similarly, 95.73 percent of the wear outs are predicted to fall within plus or minus two standard deviations from the mean, 33,255 to 7,551 miles. This compares to the actually observed total mileage range (Table 2.3) of 30,952 to 8,684 miles.

Standard Normal Distribution. It is possible to use the above standard Normal curve relationships to predict brake shoe wear outs for any range of mileages. For example, what percentage of all the rear brakes can be expected to wear out before 10,000 miles? What percentage will wear out between 10,000 and 20,000 miles? The answer, in both cases, is the area under the bell curve within these mileage ranges.

Obviously, it would be difficult to measure these areas directly from Figure 2.3. Similarly, it would be tedious to mathematically compute the areas using the complex equation that expresses the Normal curve. Well, nobody does it this way and there is, in fact, a commonly used technique that easily solves the problem.

The technique is the "z-score." Figure 2.3 represents a specific Normal curve, the one that fits the rear brake data used in this chapter. What makes it specific is the values for the mean, $\bar{X}$, and standard deviation, SD. The "z-score" is used to convert this specific Normal curve to a "Standard Normal" curve. Then the areas under the Standard Normal curve can be found from a table and used to answer the above questions.

The z-score is computed as follows for any mileage, M:

$$z = \frac{M - \bar{X}}{SD}$$

For example, the z-score for 10,000 miles is:

$$z = \frac{10{,}000 \text{ miles} - 20{,}403 \text{ miles}}{6{,}426 \text{ miles}}$$

$$= -1.62$$

Table 2.4 presents z-scores ranging from -3.5 to +3.5. Each number in the table represents the proportion of the total area under the Standard Normal curve to the left of any mileage, M.

For example, the area to the left of 10,000 miles is shaded in Figure 2.4. A z-score for 10,000 miles has already been computed, -1.62. In Table 2.4 the number corresponding to a z-score of -1.62 is 0.0526. Multiplying this number by 100 results in 5.26 percent, the area of the shaded portion of Figure 2.4. This is the proportion of all the brakes which are predicted to wear out before 10,000 miles.

Given the original 31 brakes of Table 2.1, it is predicted that 0.0526 x 31 = 1.63, or about two of the brakes will wear out before 10,000 miles. In fact, inspection of the mileages in Table 2.1 indicates that, indeed, two brakes did wear out before 10,000 miles, at 8,684 miles and 9,023 miles.

One may well ask, why go to all this trouble when the answer could have been determined directly from the original data? And the answer to that is, what about the next 31 brakes, or the next 100? Statistically, it is more valid to make predictions based on a theory--the Standard Normal curve--than on raw data. As has already been seen, raw data is highly variable. One must first measure this variability, through the mean and standard deviation, and then it can be used for prediction. In other words, the above techniques get better answers. And it really isn't hard to do the necessary computations, is it?

Other Applications. Now suppose that the number of brakes predicted to wear out between 10,000 miles and 20,000 miles is of interest. This interval is shaded in Figure 2.5. Table 2.4 gives only total areas to the left of each z-score. Therefore, to determine the percent of wear outs in this interval, first the percent less than 20,000 miles must be determined. Then, the percent less than 10,000 miles is determined and subtracted from the percent less than 20,000 miles.

The z-score for 20,000 miles is -0.06, (20,000-20,420)/6,426 = -0.06. Using Table 2.4, a z value of -0.06 corresponds to 0.4761 or 47.61 percent. To derive the percent between 10,000 miles and 20,000 miles, the percent below 10,000 miles (5.26 percent) is subtracted from the percent below 20,000 miles (47.61 percent). The result, 42.35 percent (47.67 - 5.26 = 42.35) is the percent of rear brakes which are predicted to wear out between 10,000 and 20,000 miles of service. This means that

TABLE 2.4

CUMULATIVE AREAS UNDER THE STANDARD NORMAL CURVE

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
−3.4	0.00034	0.00033	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
−3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
−3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
−3.1	0.00097	0.00094	0.00090	0.00087	0.00085	0.00082	0.00079	0.00076	0.00074	0.00071
−3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0581
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Example: The proportion of the total area under the Standard Normal curve to the left of a z-score of -1.62 is 0.0526, or 5.26 percent. This is the number in the row corresponding to z = -1.6 and the column headed by z = 0.02.

TABLE 2.4 (continued)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
+0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
+0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
+0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
+0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
+0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6870
+0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
+0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
+0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
+0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8079	0.8106	0.8133
+0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
+1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
+1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
+1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
+1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
+1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
+1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
+1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
+1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
+1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
+1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
+2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
+2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
+2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
+2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
+2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
+2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
+2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
+2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
+2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
+2.9	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
+3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
+3.1	0.99903	0.99906	0.99910	0.99913	0.99915	0.99918	0.99921	0.99924	0.99926	0.99929
+3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
+3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
+3.4	0.99966	0.99967	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976
+3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983

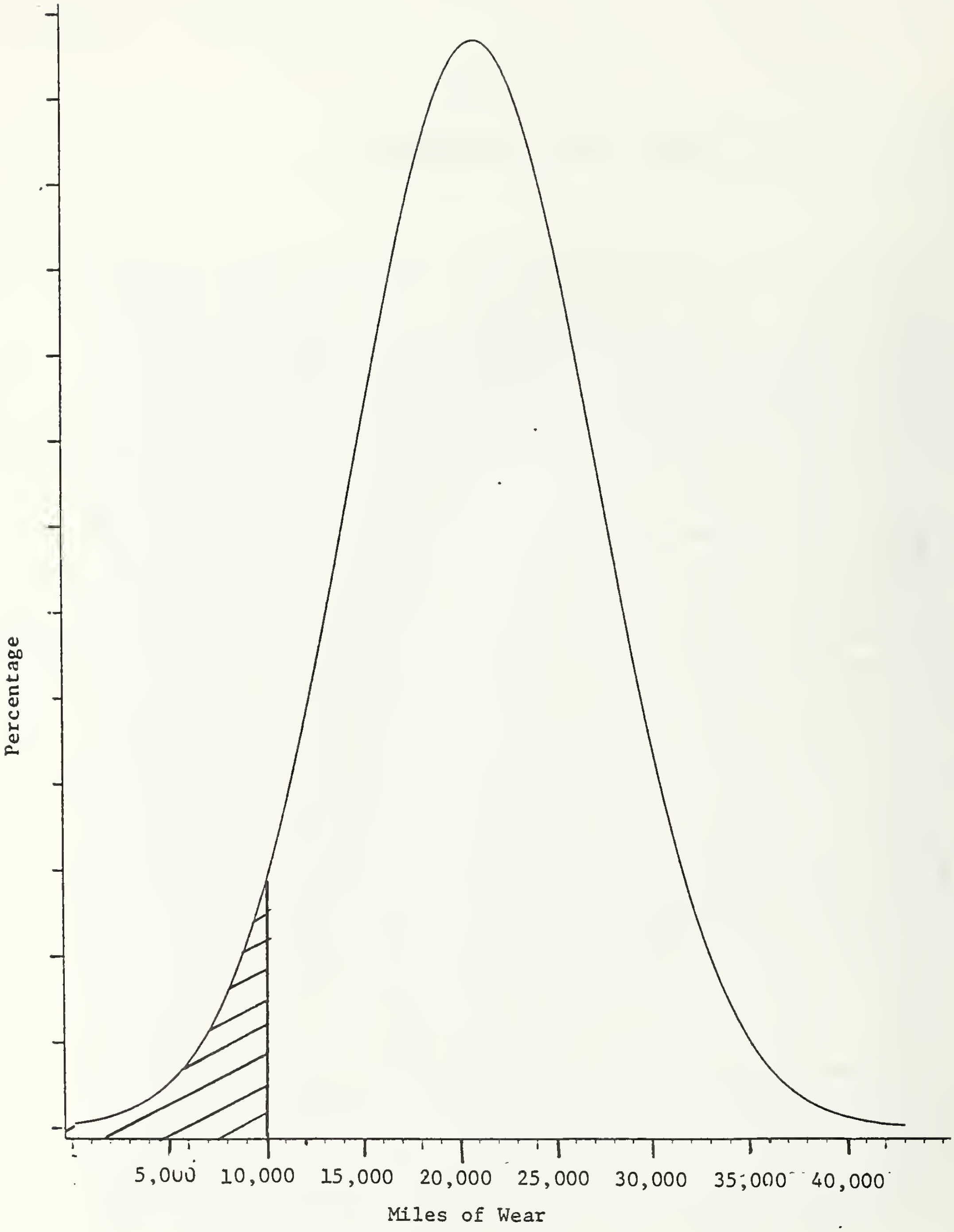


FIGURE 2.4

PERCENT OF REAR BRAKES WHICH WEAR OUT

BEFORE 10,000 MILES OF USE

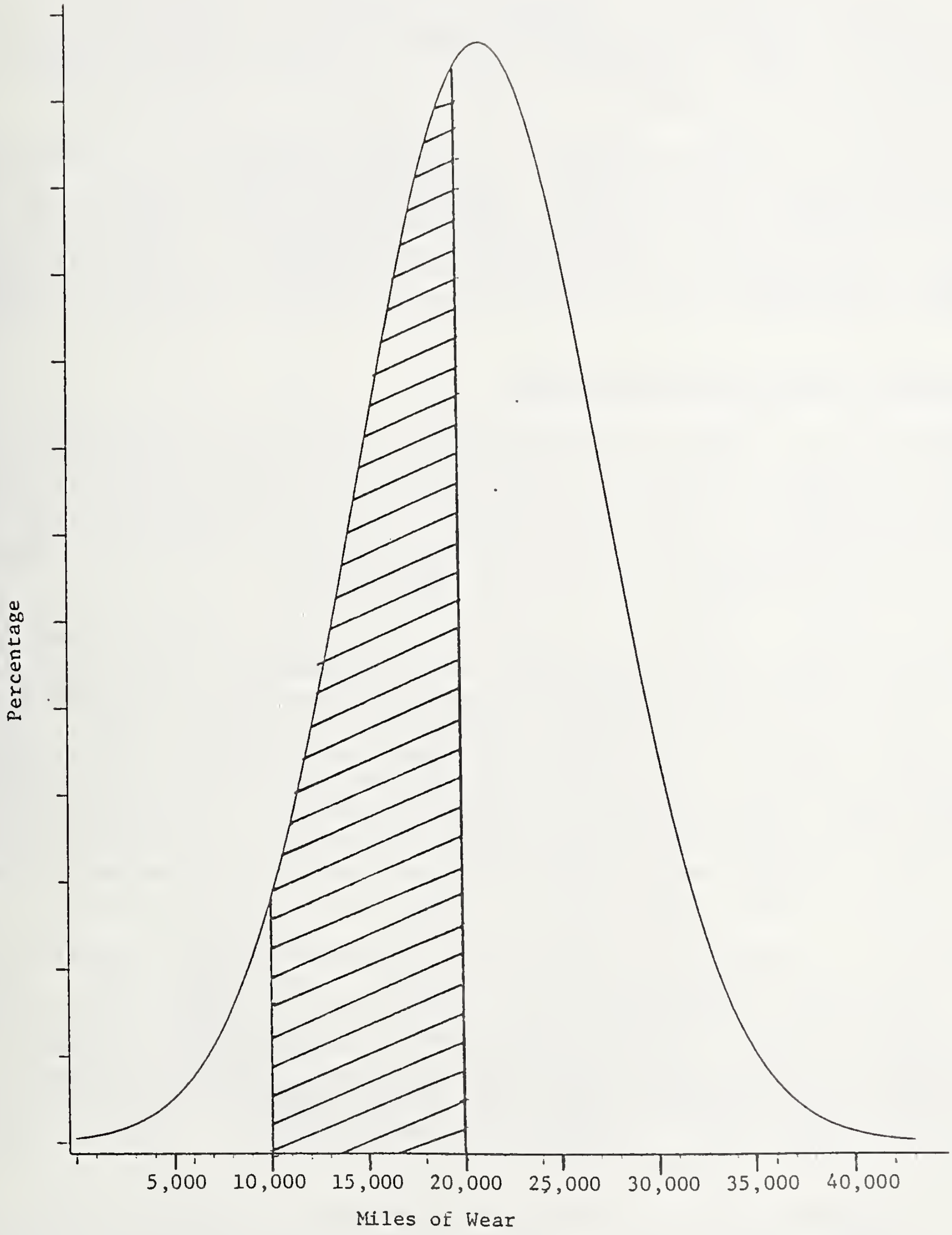


FIGURE 2.5

PERCENT OF REAR BRAKES WHICH WEAR OUT BETWEEN

10,000 AND 20,000 MILES OF USE

0.4235 x 31 = 13.13, or about 13 rear brakes are predicted to wear out with accumulated mileages of between 10,000 and 20,000 miles. In fact, based on Table 2.1, some 12 rear brakes actually did wear out in this interval. Again, a pretty good prediction has been made of wear outs.

As another example, consider a separate fleet of 60 buses of the same type as in the original brake wear out data set in Table 2.1. It is predicted that about three buses (5.26 percent of 60 buses = 3.16) will have worn out rear brakes by 10,000 miles of use. Some 29 buses (47.61 percent of 60 buses = 28.57) likely will suffer rear brake wear outs between 10,000 and 20,000 miles.

How Long Will Brake Shoes Survive?

Often, it is useful to know what proportions or numbers of the component are expected to survive until some point in service. For example, from the brake data it would be useful to know what percent will survive 10,000, 20,000 or 30,000 miles of wear. The technical term for the survival rate at each total mileage is the "reliability" of the component.

The original tabulation of the brake wear out information presented in Table 2.1 (Work Sheet 1) is presented again in Table 2.5 (Work Sheet 4) with some additional information. Column 4 of Table 2.5 contains the frequency converted to a percentage. Columns 5 and 6 contain the cumulative frequency and cumulative percent, respectively. The term "cumulative" means the total wear outs which have occurred up to a given mileage. For example, the cumulative frequency for the mileage interval, 15,000 to 20,000 miles, is 14. This means that by the end of 20,000 miles, 14 of the 31 original rear brakes, or 45.2 percent, have worn out.

Figure 2.6 contains a bar chart representing the cumulative percent of brake wear outs from column 6 of Table 2.5. The height of the bars increases going from left to right, with each height corresponding to the percent of the rear brakes which have worn out by the end of the mileage interval. The bar chart is overlaid with an S-shaped curve call the "survival curve". The curve is the cumulative form of the "bell-shaped" Normal distribution introduced in Figure 2.2.

The cumulative data is used to determine component survival rates, or their reliability. The portion of the brake shoes that have not worn out at the end of the mileage interval are those which have survived. Hence, the percent surviving is 100 percent minus the percent that have worn out. The percent surviving at the end of each interval is listed in column 7 of Table 2.5. For example, of the original 31 brake shoes in Table 2.1, 22.6 percent or 0.226 x 31 = 7.01 brake shoes have survived 25,000 miles of use.

TABLE 2.5

WORK SHEET 4

TABULATION OF FAILURE MILEAGE CUMULATIVE FREQUENCIES

Cost Driver Brake System Bus Model

Component Type Rear Brake Shoes Study Dates 1981-1983

Mileage Class		Frequency, F	Frequency, %	Cumulative Frequency		Percent Surviving
Lower (1)	Upper (2)	(3)	(4)	Cum F (5)	Cum % (6)	(7)
5,000	10,000	2	6.5 %	2	6.5 %	93.5 %
10,000	15,000	4	12.9 %	6	19.4 %	80.6 %
15,000	20,000	8	25.8 %	14	45.2 %	54.8 %
20,000	25,000	10	32.2 %	24	77.4 %	22.6 %
25,000	30,000	5	16.1 %	29	93.5 %	6.5 %
30,000	35,000	2	6.5 %	31	100.0 %	0.0 %
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
Sums:		N = 31	100.0%			

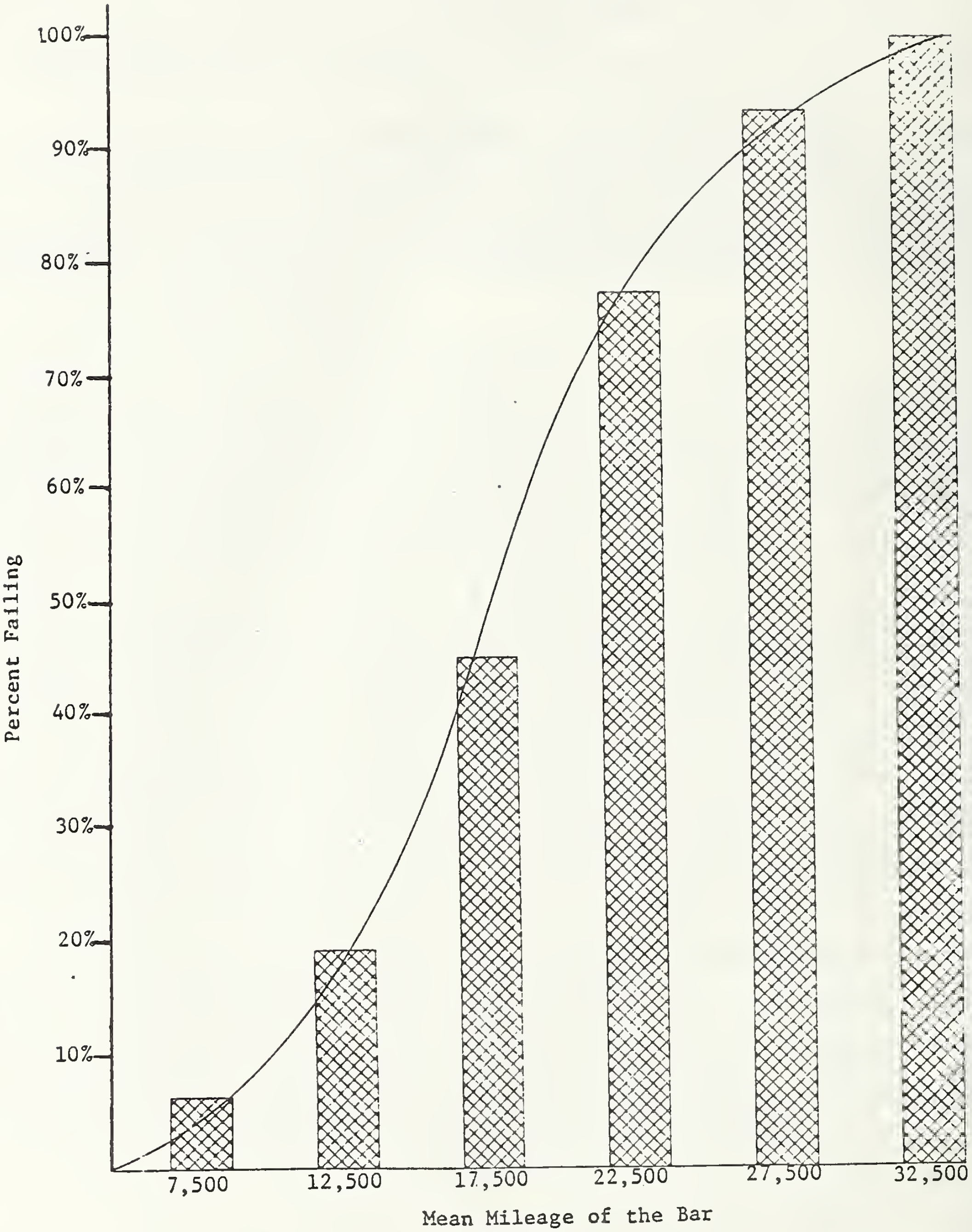


FIGURE 2.6

CUMULATIVE BAR CHART OF REAR BRAKE WEAR OUTS

WITH A CUMULATIVE NORMAL CURVE

How Reliable are the Brake Shoes?

Predictions of the durability of components can be made by comparing the reliability (survival rates) of different brands. The cumulative Standard Normal distribution is used to quantify the reliability (the percent surviving) at any mileage and compute a "reliability curve" for each component.

To illustrate, suppose that one wants to know how many brake shoes are likely to survive at 5,000 mile increments from 5,000 and 35,000 miles. First, the mileages are converted to z-scores based on the mean and standard deviation of the replacement mileages as was shown in an earlier section. Table 2.6 (Work Sheet 5) provides space for these calculations using the previous set of 31 rear brake shoe replacement mileages, whose mean and standard deviation were 20,403 and 6,426 miles, respectively.

The mileages (column 1) are converted to z-scores in column 2 of Table 2.6. The corresponding cumulative percentage is read directly from Table 2.4 and entered in column 3. The z-scores for 10,000 miles and 20,000 miles were calculated earlier in the chapter. As a reminder example, for 35,000 miles:

$$z = \frac{M-\bar{X}}{SD}$$

$$= \frac{35{,}000 \text{ miles} - 20{,}403 \text{ miles}}{6{,}426 \text{ miles}}$$

$$= 2.27$$

The Table 2.4 numbers entered in column 3 are multiplied by 100 to convert them to cumulative percentages in column 4.

Finally, the reliability of the brake shoes at each mileage is found by subtracting the percentage in column 4 from 100 percent. This number is entered in column 5 of the work sheet. Each percentage directly indicates the percentage of the original brake shoes predicted to last until that mileage. Thus, for example, 79.94 percent or about 25 of the original 31 brake shoes (0.7995 x 31 = 24.78) are predicted to survive 25,000 miles of use. Looking back at the original data in Table 2.1, 24 brakes actually survived that long. Another good prediction!

The percentages in column 5, when plotted as a smooth curve versus cumulative mileage (column 1) result in the reliability curve shown in Figure 2.7. As an example of how to read Figure 2.7, a dashed line is drawn upward from 22,500 miles. Where this line hits the curve another dashed line is drawn horizontally to the vertical axis. This line intersects the vertical axis at 46 percent. This means that 46 percent of the brake shoes are expected to survive until 22,500 miles, or about 14 brake shoes out

TABLE 2.6

WORK SHEET 5

COMPONENT RELIABILITY ANALYSIS

Cost Driver Brake System Bus Model

Component Type Rear Brake Shoes Study Dates 1981-1983

Cumulative Mileage, M (1)	z-score[1] (2)	Table 2.4 Number (3)	Cumulative Percentage[2] (4)	Reliability,[3] R (% Surviving) (5)	Percent Failing in Mileage Interval[4] (6)
0	---	---	0.00 %	100.00 %	%
5,000	-2.40	0.0082	0.82 %	99.18 %	%
10,000	-1.62	0.0526	5.26 %	94.74 %	%
15,000	-0.84	0.2005	20.05 %	79.95 %	%
20,000	-0.06	0.4761	47.61 %	52.39 %	%
25,000	0.72	0.7642	76.42 %	23.58 %	%
30,000	1.49	0.9319	93.19 %	6.81 %	%
35,000	2.27	0.9884	98.84 %	1.16 %	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%

[1] $z = \frac{M - \bar{X}}{SD}$, where M = mileage in column 1, $\bar{X}$ = mean = 20,403 mi. and SD = standard deviation = 6,426 miles

[2] The number in column 3 times 100.

[3] 100 percent less the number in column 4.

[4] Percent failing in mileage interval

$= R_1 - R_2$, where: R_1 = reliability (column 5) at previous cumulative mileage

R_2 = reliability at mileage in column 1

of 31 total.

How Often Should Brake Shoes be Inspected for Wear?

Different transit agencies have different practices with regard to the mileage interval between brake inspections. Some agencies inspect every 1,500 miles, some every 3,000 miles, and others as much as every 6,000 miles. What is the optimal interval for brake inspections? The answer lies in an inspection of the brake shoe reliability curve, Figure 2.7.

This figure suggests that there is little to be gained from inspecting buses with new brake shoes since it is highly unlikely that the shoes will wear out before 10,000 miles. In fact, as indicated in Figure 2.7, about 97 percent of the brake shoes will still be satisfactory after 10,000 miles of service. Only three percent (1.00 - 0.97 = 0.03) of the brake shoes will wear out this prematurely. Thus, as can be seen in the reliability curve in Figure 2.7, it is very unlikely that brake shoes will were out before 10,000 miles of use (point "A"). Therefore, early in the life of the brake shoes, brake inspection can be scheduled less frequently than later in the shoes' life.

It is assumed that brake shoes are replaced either at the time of inspection or after a driver complains of worn brakes. The next section tells how to predict the total number of brake shoe wear outs for specific mileage (or time) intervals, including replacement brake shoes. The optimal inspection interval is a function of how many brakes are predicted to wear out during the interval and the cost of allowing buses to remain on the road with bad brakes. This is something the transit agency must determine for itself, but the calculations in this chapter can provide a starting point by predicting the number of wear outs per selected inspection interval.

How Many Brake Shoes Will Wear Out During Any Time Period?

Knowledge of how many brake shoes will wear out in a particular time period, such as one month or every three months, for a given fleet of buses is useful information for the maintenance manager. This information helps to predict maintenance shop work loads and brake shoe inventory requirements.

In this section z-scores and the Standard Normal curve are used for getting this information. It is assumed that mileage can be translated into time. The following example uses the same Normal distribution, including mean and standard distribution, computed earlier.

Suppose that a fleet of 60 new buses is projected to average 40,000 miles of service per year. This means that every three months the buses receive 10,000 miles of brake wear. Also assume

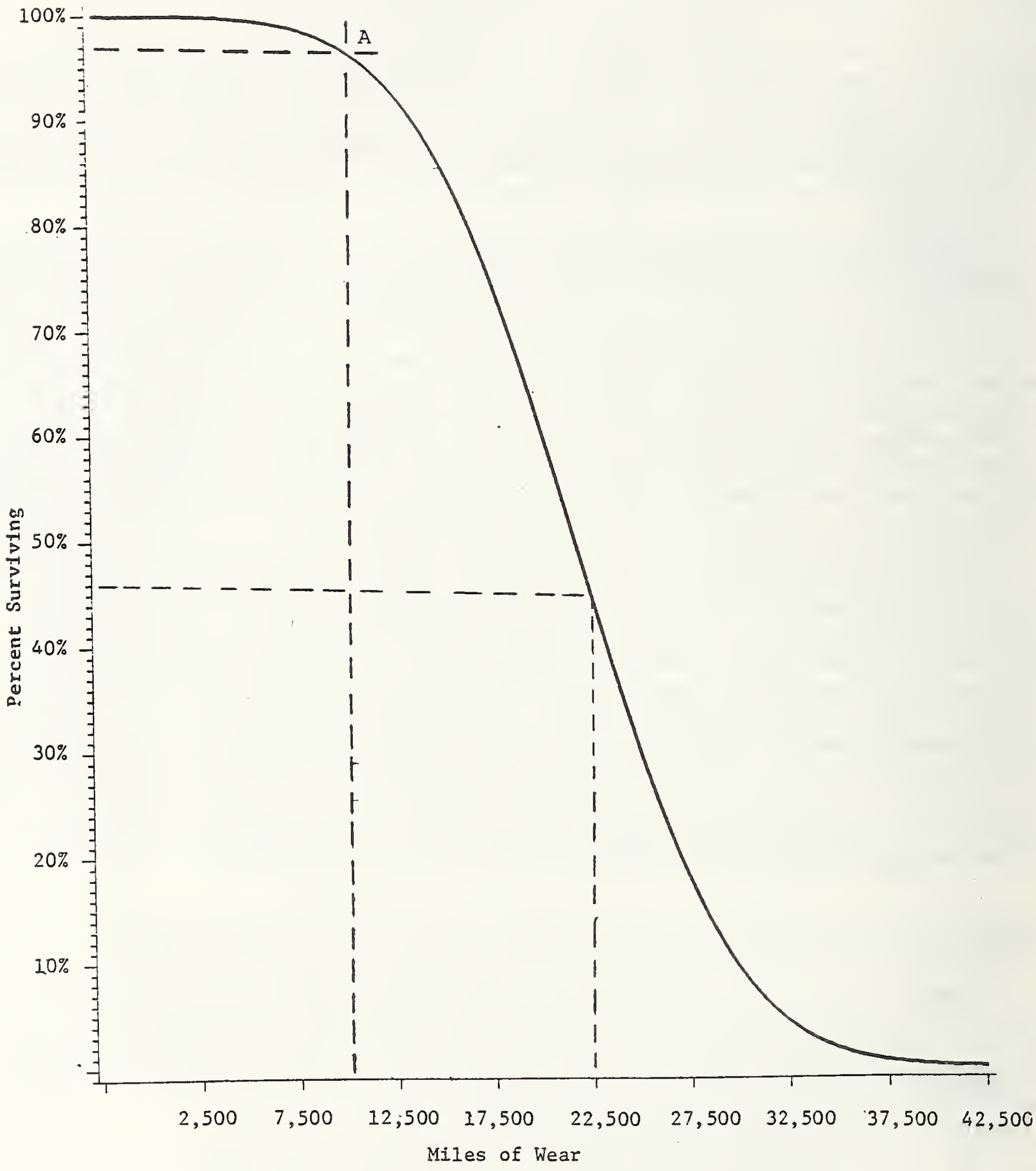


FIGURE 2.7

REAR BRAKE RELIABILITY CURVE

that these new buses were delivered to the transit agency and put into service at about the same time just prior to the first three month time interval.

Listed in Table 2.7 (Work Sheet 5 again) are the percent of the original rear brakes that are expected to wear out during 10,000 mile increments (or each three months) between zero and 40,000 miles. The percent of original brake shoe failures predicted per mileage interval (column 6) is determined from the reliabilities of column 5 using the following formula:

probability = $R_1 - R_2$, where: R_1 = mileage at previous cumulative mileage,

R_2 = reliability at cumulative mileage.

For example, in the interval between 10,000 and 20,000 cumulative miles:

probability = 94.74% - 52.30% = 42.35%

This means that 42.35 percent of all the original brake shoes are predicted to fail between 10,000 and 20,000 miles of service. Consideration of mileage intervals greater than 40,000 miles is neglected in the following analysis because the percent of survivors beyond 40,000 miles, 0.02 percent, is so small it will not affect the computations.

The percentages in column 6 of Table 2.7 are valid only for the original brake shoes. The wearing out of the replacement brake shoes has not been considered yet. In Table 2.8 (Work Sheet 6) forecasts are made of the total number of brake shoe wear outs, including replacements, which are predicted to occur in the first 6 three month periods.

In the first period, 5.26 percent of the original rear brakes wear out and are replaced, a total of 3.2 wear outs in the first three months. In the second period 42.35 percent of the original rear brakes wear out and another 5.26 percent of the replacement brake shoes wear out. A total of 25.6 brake shoe wear outs are thus predicted in the second three month period.

The forecast shows that brake wear outs will reach a peak during the third three month period when 30 brake shoes wear out. During period four, when the buses have been in service for 30,000 to 40,000 miles, the last of the original brake shoes are predicted to wear out. In the fifth and sixth periods, the forecasts reach a stable rate of about 25 wear outs per three month period.

In succeeding time periods this level of wear outs should continue to occur at about the same rate of 25 per period. This type of information is useful in predicting average demands for

TABLE 2.7

WORK SHEET 5

COMPONENT RELIABILITY ANALYSIS

Cost Driver Brake System Bus Model

Component Type Rear Brake Shoes Study Dates 1981-1983

Cumulative Mileage, M (1)	z-score[1] (2)	Table 2.4 Number (3)	Cumulative Percentage[2] (4)	Reliability,[3] R (% Surviving) (5)	Percent Failing in Mileage Interval[4] (6)
0	---	0	0.00 %	100.00 %	%
10,000	- 1.62	0.0526	5.26 %	94.74 %	5.26 %
20,000	- 0.06	0.4761	47.61 %	52.39 %	42.35 %
30,000	1.49	0.9319	93.19 %	6.81 %	45.58 %
40,000	3.05	0.9998	99.98 %	0.02 %	6.79 %
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%

[1] $z = \frac{M - \bar{X}}{SD}$, where M = mileage in column 1, $\bar{X}$ = mean = 20,403 mi. and SD = standard deviation = 6,426 miles

[2] The number in column 3 times 100.

[3] 100 percent less the number in column 4.

[4] Percent failing in mileage interval

= $R_1 - R_2$, where: R_1 = reliability (column 5) at previous cumulative mileage
R_2 = reliability at mileage in column 1

TABLE 2.8

WORK SHEET 6

Page 1 of 2

PREDICTION OF COMPONENT FAILURES OVER TIME

Cost Driver Brake System
Bus Model
Component Type Rear Brake Shoes
Study Dates 1981-1983

Cumulative Time (1)	Component State (2)	Number of Buses in State (3)		Component Failures Per Time Period (4)			Total Failures Per Time Period (5)
PERIOD 1:	Original shoes	60	x	5.26	%	=	3.2
0-3 months			x		%	=	3.2
			x		%	=	
PERIOD 2:	Original shoes	60	x	42.35	%	=	25.4
3-6 months	Shoes replaced		x		%	=	
	in Period 1	3.2	x	5.26	%	=	0.2
			x		%	=	25.6
			x		%	=	
PERIOD 3:	Original shoes	60	x	45.58	%	=	27.3
6-9 months	Shoes replaced		x		%	=	
	in Period 1	3.2	x	42.35	%	=	1.4
	Shoes replaced		x		%	=	
	in Period 2	25.6	x	5.26	%	=	1.3
			x		%	=	30.0
			x		%	=	
PERIOD 4:	Original shoes	60	x	6.79	%	=	4.1
9-12 months	Shoes replaced		x		%	=	
	in Period 1	3.2	x	45.58	%	=	1.5
	Shoes replaced		x		%	=	
	in Period 2	25.6	x	42.35	%	=	10.8
	Shoes replaced		x		%	=	
	in Period 3	30.0	x	5.26	%	=	1.6
			x		%	=	18.0
			x		%	=	
			x		%	=	

TABLE 2.8 (continued)

WORK SHEET 6

Page 2 of 2

PREDICTION OF COMPONENT FAILURES OVER TIME

Cost Driver Brake System Bus Model

Component Type Rear Brake Shoes Study Dates 1981-1983

Cumulative Time (1)	Component State (2)	Number of Buses in State (3)	Component Failures Per Time Period (4)	Total Failures Per Time Period (5)
PERIOD 5:	Shoes replaced		x % =	
12-15 mo.	in Period 1	3.2	x 6.79 % =	0.2
	Shoes replaced		x % =	
	in Period 2	25.6	x 45.58 % =	11.7
	Shoes replaced		x % =	
	in Period 3	30.0	x 42.35 % =	12.7
	Shoes replaced		x % =	
	in Period 4	18.0	x 5.26 % =	0.9
			x % =	25.5
			x % =	
PERIOD 6:	Shoes replaced		x % =	
15-18 mo.	in Period 2	25.6	x 6.79 % =	1.7
	Shoes replaced		x % =	
	in Period 3	30.0	x 45.58 % =	13.7
	Shoes replaced		x % =	
	in Period 4	18.0	x 42.35 % =	7.6
	Shoes replaced		x % =	
	in Period 5	25.5	x 5.26 % =	1.3
			x % =	24.3
			x % =	
			x % =	
			x % =	
			x % =	
			x % =	
			x % =	

parts and average repair work loads before a great deal of component use history has occurred.

CHAPTER THREE

INTRODUCTION TO WEIBULL DISTRIBUTION FAILURE ANALYSIS: TRANSMISSION EXAMPLE

This chapter is a self-contained analysis of original equipment bus transmission failures to predict future replacement patterns. It explains the use of the Weibull distribution to determine the mean mileage to failure, predict future failures as a function of mileage, and predict future reliability over time. The topics covered are:

- How to collect the transmission data.
- How to tabulate the transmission data.
- Fitting the Weibull distribution to the transmission data.
- How long will the transmissions survive?
- Predicting the future reliability of the transmissions.
- How many transmissions will fail during any time period?
- What about the replacement transmissions?

In this chapter every transmission in the study has failed. In later chapters, move advanced applications of the Weibull distribution will be discussed where some or most of the components under study have not yet failed.

How to Collect the Transmission Data

A bus transmission has failed when it must be replaced with a new or rebuilt transmission. The analysis in this chapter uses only the mileage accumulated by the transmission when it failed. A transmission is made up of many components and there are many possible reasons for a failure. Transmissions will experience environmental and operational factors -- passenger loads, grades, turns, stops per mile, -- which may vary from bus to bus. However, why the transmission failed is not of concern in this chapter, just the fact that it did fail at a particular mileage.

At least ten transmission failures are needed for the analysis. It is assumed that the transmissions under study are the same model and that all experienced similar use and maintenance. The possibility that some transmissions failed because of abuse is not accounted for, although the reader should separate such cases out.

In this chapter the example data set consists of original equipment transmission failures in 29 new General Motors RTS II buses owned by the Detroit Department of Transportation (DDOT). The transmissions, Detroit Diesel Allison V730 models, were among the first of this model manufactured. Some premature failures were experienced by DDOT, which prompted the collection of the data set. The mileages each had accumulated when they failed are entered in column 4 of Table 3.1 (Work Sheet 7). Since all of the transmissions have failed, columns 3, 5, and 6 are ignored.

How to Tabulate the Transmission Data

The mileages are plotted as a bar chart by first classing them into mileage intervals as shown in Table 3.2 (Work Sheet 1). The class intervals are 20,000 miles wide with a total of seven classes. The interval width of 20,000 miles was selected for convenience and because it resulted in at least six classes. Consult the previous chapter on the Normal distribution for more details on how to classify and tabulate the data.

The mean mileage in Table 3.1 is 53,759 miles but one new transmission (bus number 7925) failed after just 3,874 miles. In fact, more than half of the transmissions failed at mileages less than the mean. One reason for this is that the mean includes transmissions which proved to be relatively durable; as Table 3.1 indicates, the transmission from bus number 7904 lasted 126,273 miles. Recall that mileages for worn out brake shoes tended to occur in the center intervals and close to the mean.

The bar chart of transmission failure mileages is presented in Figure 3.1. This bar chart has quite a different pattern from the bar chart of brake shoe wear outs in the previous chapter (Figure 2.1), one that is not bell-shaped. Note that the tallest bar is to the left of the average or mean.

Bus components which fail, such as transmissions, typically have mileage-at-failure patterns which are different than components which wear out, such as brake shoes. In the previous chapter, the mileages accumulated by worn out brake shoes followed a bell-shaped normal distribution pattern when plotted in a bar chart. The mileage pattern bar chart for failed transmissions in Figure 3.1 looks more like a ski slope. A few transmissions fail prematurely but some last a very long time. This indicates that the Weibull distribution should be used to represent transmission failure patterns.

TABLE 3.1

WORK SHEET 7

FAILURE STATUS OF ORIGINAL COMPONENTS IN WEIBULL FAILURE ANALYSIS

Page 1 of 2

Cost Driver Transmission

Bus Model GM RTS II

Component Type Detroit Diesel Allison V730

Study Dates 1979-1981

Bus Number (1)	Component Number (2)	Current Bus Mileage (3)	Original Component Failure Mileage (4)	Status of Unfailed Component*		Comments (7)
				Censor (5)	Susp. (6)	
7901			58,844			
7902			33,181			
7903			28,664			
7904			126,273			
7905			48,027			
7906			41,194			
7907			34,612			
7908			36,767			
7909			56,852			
7910			24,065			
7911			32,806			
7912			90,771			
7913			56,436			
7914			33,950			
7915			79,996			
7916			30,536			
7917			80,827			
7918			119,060			
7919			64,113			
7920			51,513			

* CENSORED cases refer to surviving components which have accumulated more miles than any failed component in the data set.

SUSPENDED cases refer to surviving components which have accumulated less miles than at least one of the failed components.

TABLE 3.1 (continued)

WORK SHEET 7

FAILURE STATUS OF ORIGINAL COMPONENTS
IN WEIBULL FAILURE ANALYSIS

Page 2 of 2

Cost Driver Transmission

Bus Model GM RTS II

Component Type Detroit Diesel Allison V730

Study Dates 1979-1981

Bus Number (1)	Component Number (2)	Current Bus Mileage (3)	Original Component Failure Mileage (4)	Status of Unfailed Component*		Comments (7)
				Censor (5)	Susp. (6)	
7921			108,313			
7922			14,472			
7923			31,126			
7924			85,503			
7925			3,874			
7926			8,308			
7927			69,332			
7928			76,442			
7929			17,985			

* CENSORED cases refer to surviving components which have accumulated more miles than any failed component in the data set.

SUSPENDED cases refer to surviving components which have accumulated less miles than at least one of the failed components.

TABLE 3.2

WORK SHEET 1

TABULATION OF FAILURE MILEAGE FREQUENCIES

Cost Driver Transmission Bus Model General Motors RTS II

Component Type Detroit Diesel Allison V730 Study Dates 1979-1981

Failure Mileage (1)	Failure Mileage (2)	Mileage Class		Tally (5)	Frequency, F (6)
		Lower (3)	Upper (4)		
58,844	8,308	0	19,999	////	4
33,181	69,332	20,000	39,999	///// ////	9
28,664	76,442	40,000	59,999	///// /	6
126,273	17,985	60,000	79,999	///	3
48,027		80,000	99,999	////	4
41,194		100,000	119,999	//	2
34,612		120,000	139,999	/	1
36,767					29
56,852					
24,065					
32,806					
90,771					
56,436					
33,950					
79,996					
30,536					
80,827					
119,060					
64,113					
51,513					
108,313					
14,472					
31,126					
85,503					
3,874					

Number of failure mileages, N 29

Maximum mileage 126,273 miles Minimum mileage 3,874 miles

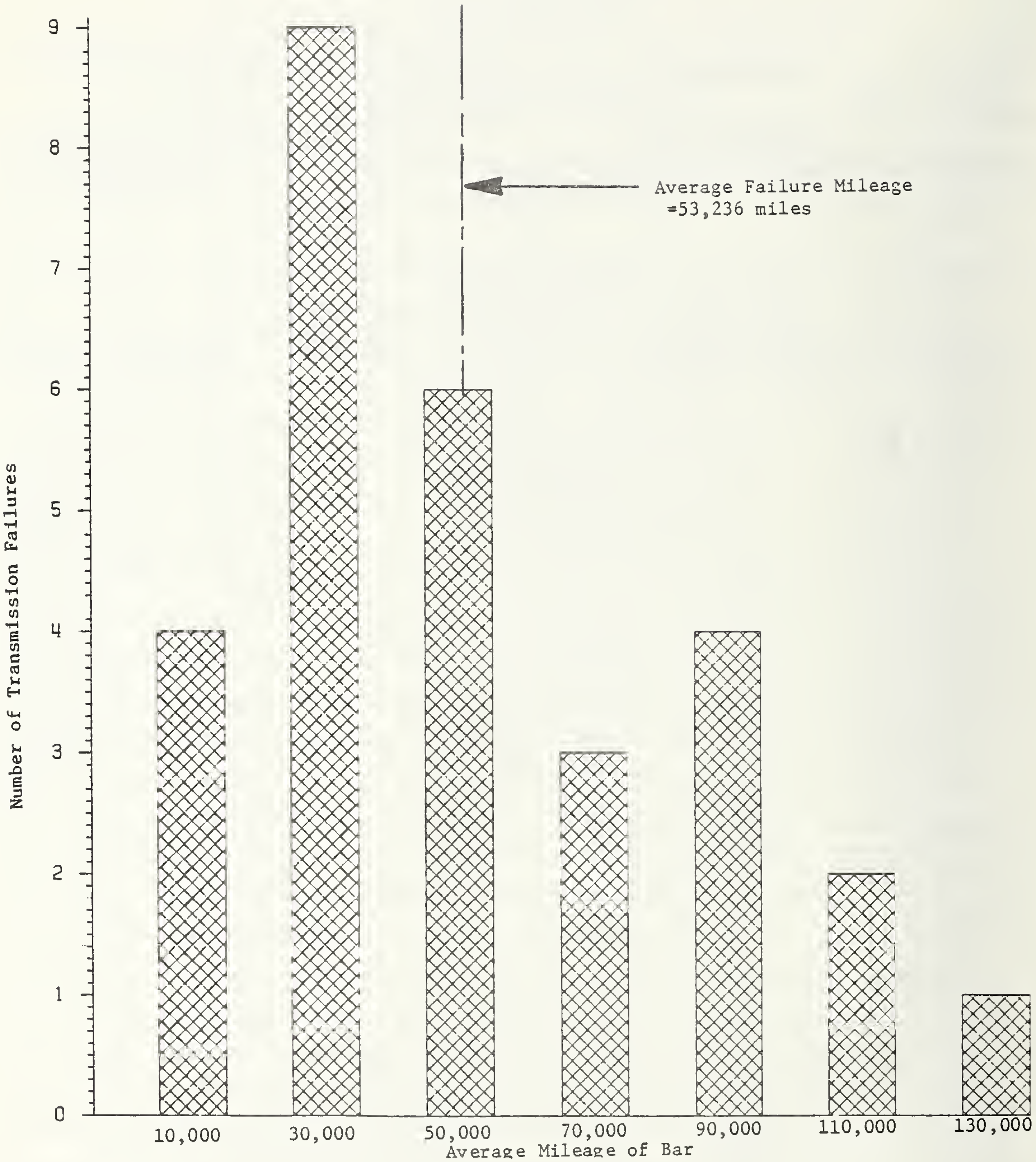


FIGURE 3.1

TRANSMISSION FAILURE MILEAGE BAR CHART

Fitting the Weibull Distribution to the Transmission Data

Calculating D, B, and T. The Weibull curve which best fits the original data is found by calculating three constants, D, B, and T. In order to determine good values for D, B and T, first the transmissions are listed in order of increasing mileage at failure in column 5 of Table 3.3 (Work Sheet 8). The ranked mileages are then transferred to column 2 of Table 3.4 (Work Sheet 9).

The first constant is D and it is called the "minimum life term." If it were likely that a component could fail without having accumulated any miles, then the "minimum life" would be zero. In fact, a good estimate of D is found by multiplying the lowest failure mileage by 90 percent. In Table 3.4, the lowest mileage is 3,874 miles (bus number 7825). Therefore, D equals 3,487 miles (3,874 x 0.90 = 3,487). There is room for this computation at the bottom of Work Sheet 9 in Table 3.4.

The other two constants are B and T*. B, the "shape factor," determines the shape of the curve and T, the "characteristic life factor," determines how far away the peak in the curve should be from zero miles. Before calculating B and T, two intermediate numbers, K and S, must be determined. These calculations are also shown in Table 3.4.

The minimum life term, D = 3,487 miles, is subtracted from each original failure mileage (column 3 minus column 2) to get the fourth column of numbers, which is a value of K for each failed transmission. The total of all the K's in column 4 is 1,442,719. The average K, K_{ave}, is 49,749 miles (1,442,719 divided by 29 = 49,749), a number which will be needed later.

The K values are then used to compute S as follows. First, compute the natural logarithm of the largest value of K, 122,786 miles, for the transmission of bus 7904. This number is called K_{max}. Obtaining the natural logarithm is simple when using a scientific calculator. Enter the number, 122,786, and then press the button marked "ln". The natural logarithm, 11.7182, will appear on the display.** The natural logarithms are rounded-off to only four digits to the right of the decimal point for simplicity (11.718198 becomes 11.7182) in Table 3.4. There is room for this computation at the bottom of Table 3.4. This number, $\ln(K_{max})$, is placed on every line of the fifth column of Table 3.4.

Next, the natural logarithm of each value of K, ln (K), is written in the sixth column. For example, in the first line of Table 3.4 (bus number 7925), ln (387) equals 5.9584.

---

* Method for calculating B and T was developed by Bain (1).

** For a discussion of the use of scientific calculators see Appendix B.

TABLE 3.3

WORK SHEET 8

Page 1 of 2

RANK ORDERING OF SURVIVING AND FAILED COMPONENTS IN WEIBULL FAILURE ANALYSIS

Cost Driver Transmission

Bus Model GM RTS II

Component Type Detroit Diesel Allison V730

Study Dates 1979-1981

Order Number (1)	Bus Number (2)	Component Number (3)	Failed, Suspended, or Censored (4)	Mileage When Failed, Suspended, or Censored* (5)
1	7925		Failed	3,874
2	7926		Failed	8,308
3	7922		Failed	14,472
4	7929		Failed	17,985
5	7910		Failed	24,065
6	7903		Failed	28,664
7	7916		Failed	30,536
8	7923		Failed	31,126
9	7911		Failed	32,806
10	7902		Failed	33,181
11	7914		Failed	33,950
12	7907		Failed	34,612
13	7908		Failed	36,767
14	7906		Failed	41,194
15	7905		Failed	48,027
16	7920		Failed	51,513
17	7913		Failed	56,436
18	7909		Failed	56,852
19	7901		Failed	58,844
20	7919		Failed	64,113

* Each component is listed in ascending order of the mileage entered in Column 5.

TABLE 3.3 (continued)

WORK SHEET 8

Page 2 of 2

RANK ORDERING OF SURVIVING AND FAILED COMPONENTS IN WEIBULL FAILURE ANALYSIS

Cost Driver Transmission

Bus Model GM RTS II

Component Type Detroit Diesel Allison V730

Study Dates 1979-1981

Order Number (1)	Bus Number (2)	Component Number (3)	Failed, Suspended, or Censored (4)	Mileage When Failed, Suspended, or Censored* (5)
~~1~~ 21	7927		Failed	69,332
~~2~~ 22	7928		Failed	76,442
~~3~~ 23	7915		Failed	79,996
~~4~~ 24	7917		Failed	80,827
~~5~~ 25	7924		Failed	85,503
~~6~~ 26	7912		Failed	90,771
~~7~~ 27	7921		Failed	108,313
~~8~~ 28	7918		Failed	119,060
~~9~~ 29	7904		Failed	126,273
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

* Each component is listed in ascending order of the mileage entered in Column 5.

TABLE 3.4

WORK SHEET 9

Page 1 of 2

COMPUTATION OF D, K, AND S FOR WEIBULL FAILURE DATA

Cost Driver Transmission

Bus Model GM RTS II

Component Type Detroit Diesel Allison V730

Study Dates 1979-1981

Bus Number (1)	Failure Mileage (2)	Minimum Life, D* (3)	K (4)	ln(K_{max})** (5)	ln(K) (6)	L*** (7)
7925	3,874 -	3,487	= 387	11.7182	- 5.9584	= 5.7598
7926	8,308 -	3,487	= 4,821	11.7182	- 8.4807	= 3.2375
7922	14,472 -	3,487	= 10,985	11.7182	- 9.3043	= 2.4139
7929	17,985 -	3,487	= 14,498	11.7182	- 9.5818	= 2.1364
7910	24,065 -	3,487	= 20,578	11.7182	- 9.9320	= 1.7862
7903	28,664 -	3,487	= 25,177	11.7182	- 10.1337	= 1.5845
7916	30,536 -	3,487	= 27,049	11.7182	- 10.2054	= 1.5128
7923	31,126 -	3,487	= 27,639	11.7182	- 10.2270	= 1.4912
7911	32,806 -	3,487	= 29,319	11.7182	- 10.2860	= 1.4322
7902	33,181 -	3,487	= 29,694	11.7182	- 10.2987	= 1.4195
7914	33,950 -	3,487	= 30,463	11.7182	- 10.3243	= 1.3939
7907	34,612 -	3,487	= 31,125	11.7182	- 10.3458	= 1.3724
7908	36,767 -	3,487	= 33,280	11.7182	- 10.4127	= 1.3055
7906	41,194 -	3,487	= 37,707	11.7182	- 10.5376	= 1.1806
7905	48,027 -	3,487	= 44,540	11.7182	- 10.7041	= 1.0141
7920	51,513 -	3,487	= 48,026	11.7182	- 10.7795	= 0.9387
7913	56,436 -	3,487	= 52,949	11.7182	- 10.8771	= 0.8411
7909	56,852 -	3,487	= 53,365	11.7182	- 10.8849	= 0.8333
7901	58,844 -	3,487	= 55,357	11.7182	- 10.9216	= 0.7966
7919	64,113 -	3,487	= 60,626	11.7182	- 11.0125	= 0.7057
Sums:		ΣK =			S =	

* Minimum life term, D = 0.90 x lowest failure mileage
= 0.90 x 3,874 mi. = D = 3,487 mi.

** $\ln(K_{max}) = \ln(\text{122,786}) = 11.7182$, where K_{max} is the largest value of K in column 4.

*** If the data set contains suspended failure cases, the L terms must be modified in Work Sheet 10a before computing S.

TABLE 3.4 (continued)

WORK SHEET 9

Page 2 of 2

COMPUTATION OF D, K, AND S FOR WEIBULL FAILURE DATA

Cost Driver Transmission

Bus Model GM RTS II

Component Type Detroit Diesel Allison V730

Study Dates 1979-1981

Bus Number (1)	Failure Mileage (2)	Minimum Life, D* (3)	K (4)	$\ln(K_{max})$** (5)	$\ln(K)$ (6)	L*** (7)
7927	69,332	– 3,487	= 65,845	11.7182	– 11.0951	= 0.6231
7928	76,442	– 3,487	= 72,955	11.7182	– 11.1976	= 0.5206
7915	79,996	– 3,487	= 76,509	11.7182	– 11.2452	= 0.4730
7917	80,827	– 3,487	= 77,340	11.7182	– 11.2460	= 0.4722
7924	85,503	– 3,487	= 82,016	11.7182	– 11.3147	= 0.4035
7912	90,771	– 3,487	= 87,284	11.7182	– 11.3769	= 0.3413
7921	108,313	– 3,487	= 104,826	11.7182	– 11.5601	= 0.1581
7918	119,060	– 3,487	= 115,573	11.7182	– 11.6577	= 0.0605
7904	126,273	– 3,487	= 122,786	11.7182	– 11.7182	= 0.0
		–	=		–	=
		–	=		–	=
		–	=		–	=
		–	=		–	=
		–	=		–	=
		–	=		–	=
		–	=		–	=
		–	=		–	=
		–	=		–	=
		–	=		–	=
		–	=		–	=
Sums:			ΣK = 1,442,719			S = 36.2082

* Minimum life term, D = 0.90 x lowest failure mileage
= 0.90 x 3,874 mi. = D = 3,487 mi.

** $\ln(K_{max}) = \ln($ 122,786 $) =$ 11.7182 , where K_{max} is the largest value of K in column 4.

*** If the data set contains suspended failure cases, the L terms must be modified in Work Sheet 10a before computing S.

The difference of the numbers in the fifth and sixth columns is written in the seventh and final column. The sum of all the numbers in column seven is 36.2082, which is the value for S.

Now it is possible to compute the two remaining Weibull constants, B and T. Table 3.5 (Work Sheet 10) outlines the computations for B and T and provides space for each intermediate result and the final answers.

To get the shape factor, B, Table 3.6 is used to first find the number which corresponds to the quantity of failed transmissions. In this case the number of failed transmissions is 29 and the corresponding number in Table 3.6 is 54.9903. B is this number divided by S from Table 3.4 (36.2082). Thus, B equals 1.5187 (54.9903 divided by 36.2082 = 1.5187).

To get the characteristic life factor, T, Table 3.7 is used to first find the number which corresponds to the value of B. First, round off B, which is 1.5187, to 1.5. The table number corresponding to B = 1.5 is 1.1077. T is calculated by multiplying this number by the average value of K, 49,749. Thus T is 55,107 (49,749 x 1.107 = 55,107).

Mean and Standard Deviation. The mean mileage to failure, $\overline{X}$, and the standard deviation, SD, are calculated in Table 3.8 (Work Sheet 2) in the same manner as discussed in Chapter Two. The mileage classes and frequencies developed in Table 3.2 are used again in Table 3.8. The estimated mean, using mileage classes, 52,759 miles, is close to the true mean, 53,236 miles. Similarly, the estimated standard deviation, 33,263 miles, compares to the actual standard deviation of 32,317 miles. Unlike the Normal distribution, these numbers are not parameters of the Weibull distribution, but they have several uses which will be discussed later.

Summary. The values for the Weibull distribution, D, B, and T, the mean mileage to failure and the standard deviation, can be entered in Table 3.9 (Work Sheet 3) along with a frequency bar chart as a summary of all the calculations. Dividing ($\overline{X}$ - D)by SD yields a value of 1.48 in Table 3.9. Based on Table 1.1 from Chapter One, the failure mechanism is determined to be unpredictable. The appropriate maintenance procedure for transmissions, based on Figure 1.6, is either condition-based maintenance or operate-to-failure.

The Weibull formula is plotted as a smooth curve in Figure 3.2 using the values for D, B, and T computed above. For purposes of comparison the bar chart of the actual transmission failure mileages (from Figure 3.1) has been added. Note that the Weibull curve closely follows the bar chart pattern and thus appears to be a good fit for the da-

TABLE 3.5

WORK SHEET 10

COMPUTATION OF B AND T FOR WEIBULL FAILURE DATA WHICH CONTAINS ONLY FAILED CASES

Cost Driver Transmission

Bus Model GM RTS II

Component Type Detroit Diesel Allison V730

Study Dates 1979-1981

INPUT NUMBERS:

N = 29
Number of failed components

D = 3,487 miles
Minimum life term
(from Work Sheet 9)

ΣK = 1,442,719 miles
(from Work Sheet 9)

S = 36.2082
(from Work Sheet 9)

SHAPE FACTOR, B:

$$B = \frac{54.9903}{\text{(Table 3.6 number for N)}} \Big/ \frac{36.2082}{S} = B = 1.5187$$

CHARACTERISTIC LIFE FACTOR, T:

$$K_{ave} = \frac{1{,}442{,}719}{\Sigma K} \Big/ \frac{29}{N} = K_{ave} = 49{,}749 \text{ miles}$$

$$T = \frac{1.1077}{\text{(Table 3.7 number for B)}} \times \frac{49{,}749}{K_{ave}} = T = 55{,}107$$

TABLE 3.6

NUMBERS USED TO FIND B

N	0	1	2	3	4	5	6	7	8	9
00				3.0876	4.5995	6.3244	8.1002	9.9118	11.8218	13.7554
10	15.6725	17.7056	19.6456	21.6365	23.6092	25.6919	27.7324	29.8325	31.8804	33.0061
20	36.1246	38.1515	40.3894	42.4824	44.6001	46.8757	48.9810	51.2179	53.2873	54.9903
30	57.6666	59.8223	61.9607	64.1437	66.2731	68.5366	70.8275	72.9437	75.2944	77.5128
40	79.7906	81.8898	84.2262	86.5053	88.7641	91.0807	93.2631	95.5749	97.6915	100.1198
50	102.4411	104.5045	106.4227	109.1620	111.1159	113.4846	116.1370	117.9392	119.7248	122.5984
60	125.6481	127.5384	129.8717	132.4961	135.1321	136.4222	139.5236	141.1700	143.3257	146.2944
70	148.2139	150.9364	153.0588	154.8250	157.4557	160.2575	162.3699	165.3072	166.7951	169.2090
80	172.0464	174.3130	177.3319	179.6447	180.9947	183.7294	186.6317	188.8208	191.1676	193.4308
90	195.8543	197.8507	200.9231	202.2633	205.2821	207.2669	209.9632	211.9442	215.1698	217.7204

Example: The number for N = 29 failures is 54.9903, which is the number in the final column ("9") of the third row ("20"). This is the coefficient for 20 + 9 = 29 data items.

TABLE 3.7

NUMBERS USED TO FIND T

B	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0			0.0083	0.1080	0.3009	0.4000	0.6646	0.7900	0.8826	0.9504
1.0	1.0000	1.0364	1.0631	1.0828	1.0972	1.1077	1.1154	1.1208	1.1245	1.1269
2.0	1.1284	1.1291	1.1288	1.1292	1.1281	1.1271	1.1259	1.1245	1.1215	1.1200
3.0	1.1199	1.1182	1.1165	1.1148	1.1131	1.1115	1.1098	1.1081	1.1065	1.1049
4.0	1.1033	1.1017	1.1002	1.0987	1.0972	1.0958	1.0944	1.0930	1.0917	1.0904
5.0	1.0891	1.0879	1.0857	1.0855	1.0843	1.0832	1.0821	1.0810	1.0800	1.0789
6.0	1.0779	1.0769	1.0760	1.0750	1.0741	1.0732	1.0723	1.0715	1.0706	1.0698
7.0	1.0690	1.0682	1.0675	1.0667	1.0660	1.0653	1.0646	1.0639	1.0632	1.0625
8.0	1.0619	1.0612	1.0606	1.0600	1.0594	1.0588	1.0582	1.0577	1.0571	1.0565
9.0	1.0560	1.0555	1.0550	1.0545	1.0540	1.0535	1.0530	1.0525	1.0529	1.0516

Example: The number for B = 1.5 is 1.1077, which is the number in the seventh column ("0.5") of the second row ("1.0"). This is the coefficient for B = 1.0 + 0.5 = 1.5.

TABLE 3.8

WORK SHEET 2

COMPUTATION OF MEAN AND STANDARD DEVIATION

Cost Driver Transmission

Bus Model GM RTS II

Component Type Detroit Diesel Allison V730

Study Dates 1979-1981

Mileage Class		Class Average X (3)	Freq. F (4)	FX (5)	X^2 (6)	FX^2 (7)
Lower (1)	Upper (2)					
0	19,999	10,000	4	= 40,000	100,000,000	400,000,000
20,000	39,999	30,000	9	= 270,000	900,000,000	8,100,000,000
40,000	59,999	50,000	6	= 300,000	2,500,000,000	15,000,000,000
60,000	79,999	70,000	3	= 210,000	4,900,000,000	14,700,000,000
80,000	99,999	90,000	4	= 360,000	8,100,000,000	32,400,000,000
100,000	119,999	110,000	2	= 220,000	12,100,000.000	24,200,000,000
120,000	139,999	130,000	1	= 130,000	16,900,000,000	16,900,000,000
				=		
				=		
				=		
				=		
				=		
				=		
				=		
				=		
Sums, Σ			N= 29	ΣFX= 1,530,000		ΣFX^2= 111,700,000,000

$$\text{Mean} = \bar{X} = \frac{\Sigma FX}{N} = \underline{1{,}530{,}000} \;/\; \underline{29} \quad = \quad \bar{X} = \underline{52{,}759 \text{ miles}}$$

$$\text{Standard Deviation} = SD = \sqrt{\frac{N\Sigma FX^2 - (\Sigma FX)^2}{N(N-1)}}$$

$$= \sqrt{\frac{\underline{29} \times \underline{111{,}700{,}000{,}000} - \underline{1{,}530{,}000} \times \underline{1{,}530{,}000}}{\underline{29} \times \underline{28}}} = SD = \underline{33{,}263 \text{ miles}}$$

TABLE 3.9

WORK SHEET 3

SUMMARY SHEET OF FAILURE MILEAGE DISTRIBUTION

Cost Driver Transmission

Bus Model GM RTS II

Component Type Det. D. Allison V730

Study Dates 1979-1981

Number of cases, N 29

Mean, $\bar{X}$ 52,759 miles

Number of failures, NF 29

Std. deviation, SD 33,263 miles

Maximum mileage 126,273 miles

$(\bar{X}-D)/SD$ 1.48

Minimum mileage 3,874 miles

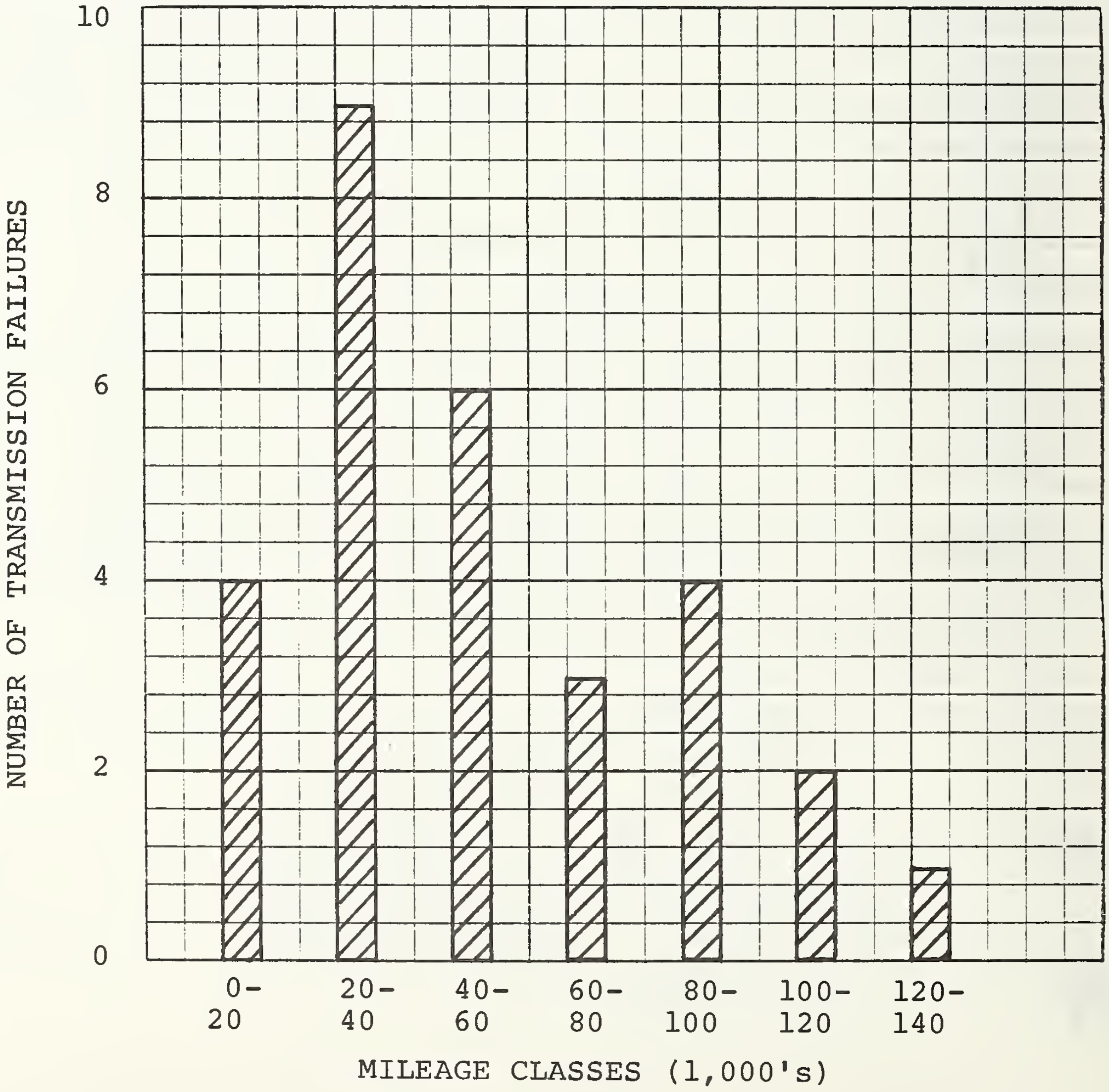
WEIBULL DISTRIBUTION PARAMETERS:

Minimum life term, D 3,487 miles

Shape factor, B 1.5187

Characteristic life factor, T 55,107 miles

FAILURE MECHANISM: Mileage to failure unpredictable



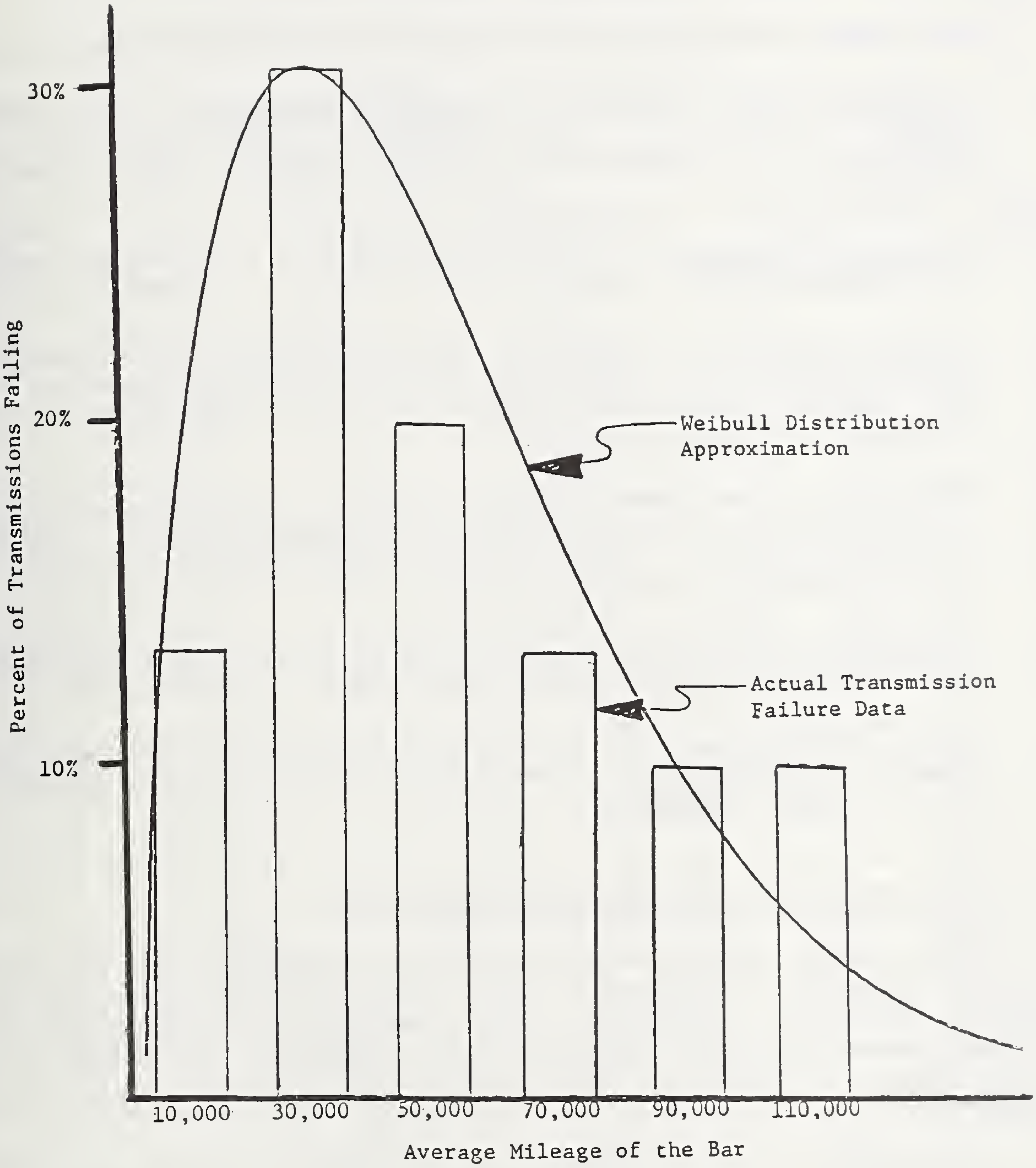


FIGURE 3.2

RELATIONSHIP BETWEEN ACTUAL BAR CHART DATA AND WEIBULL DISTRIBUTION APPROXIMATION ta. In general, a sample of at least ten similar transmissions is needed to compute a good curve. Using the Weibull formula so obtained, it is now possible to analyze the reliability of the transmissions being studied.

To recap the process, the steps to find D, B and T are:

1. Tabulate the component failure mileages using Work Sheet 1 (Table 3.2). The frequency distribution can be plotted as a bar chart in the summary Work Sheet 3 (Table 3.9).

2. List the component failure mileages in increasing order from smallest to largest in Work Sheet 8 (Table 3.3).

3. Use Work Sheet 9 (Table 3.4) to calculate the intermediate number, S, using the ordered failure mileages and D. The minimum life term, D, is 0.90 multiplied by the smallest failure mileage.

4. Determine the shape factor, B, and the characteristic life factor, T, by following the computational outline in Work Sheet 10 (Table 3.5). Input numbers are S, D, ΣK, and Tables 3.6 and 3.7.

5. Determine the mean mileage to failure, $\bar{X}$, and the standard deviation, SD, using Work Sheet 2 (Table 3.8).

6. Enter the values for D, B, T, $\bar{X}$, and SD in summary Work Sheet 3 (Table 3.9). The Weibull distribution computation is now complete.

How Long Will the Transmissions Survive?

This question is best answered by working with the cumulative percent of transmission failures that occurred before a specific mileage. The cumulative frequencies of transmission failure mileages are computed in Table 3.10 (Work Sheet 4) based on the failure data of Table 3.12 (Work Sheet 1).

In Figure 3.3 is shown a cumulative bar chart developed with the data in column 6 of Table 3.10. The first bar stands for the mileage interval 0 to 19,999 miles. Four transmissions had failed before accumulating 20,000 miles, or 13.8 percent (4 divided by 29 = 0.138) of the total. In the second interval, 20,000 to 39,999 miles, 9 more failed for a cumulative total of 13 (4 + 9 = 13), which is 44.8 percent (13 divided by 29 = 0.448) of the total. The remaining bars are similarly determined.

TABLE 3.10

WORK SHEET 4

TABULATION OF FAILURE MILEAGE CUMULATIVE FREQUENCIES

Cost Driver Transmission Bus Model GM RTS II

Component Type Detroit Diesel Allison V730 Study Dates 1979-1981

Mileage Class		Frequency, F	Frequency, %	Cumulative Frequency		Percent Surviving
Lower (1)	Upper (2)	(3)	(4)	Cum F (5)	Cum % (6)	(7)
0	19,999	4	13.8 %	4	13.8 %	86.2 %
20,000	39,999	9	31.0 %	13	44.8 %	55.2 %
40,000	59,999	6	20.7 %	19	65.5 %	34.5 %
60,000	79,999	3	10.3 %	22	75.8 %	24.2 %
80,000	99,999	4	13.8 %	26	89.6 %	10.4 %
100,000	119,999	2	6.9 %	28	96.5 %	3.5 %
120,000	139,999	1	3.4 %	29	99.9 %	0.1 %
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
Sums:		N = 29	100.0%			

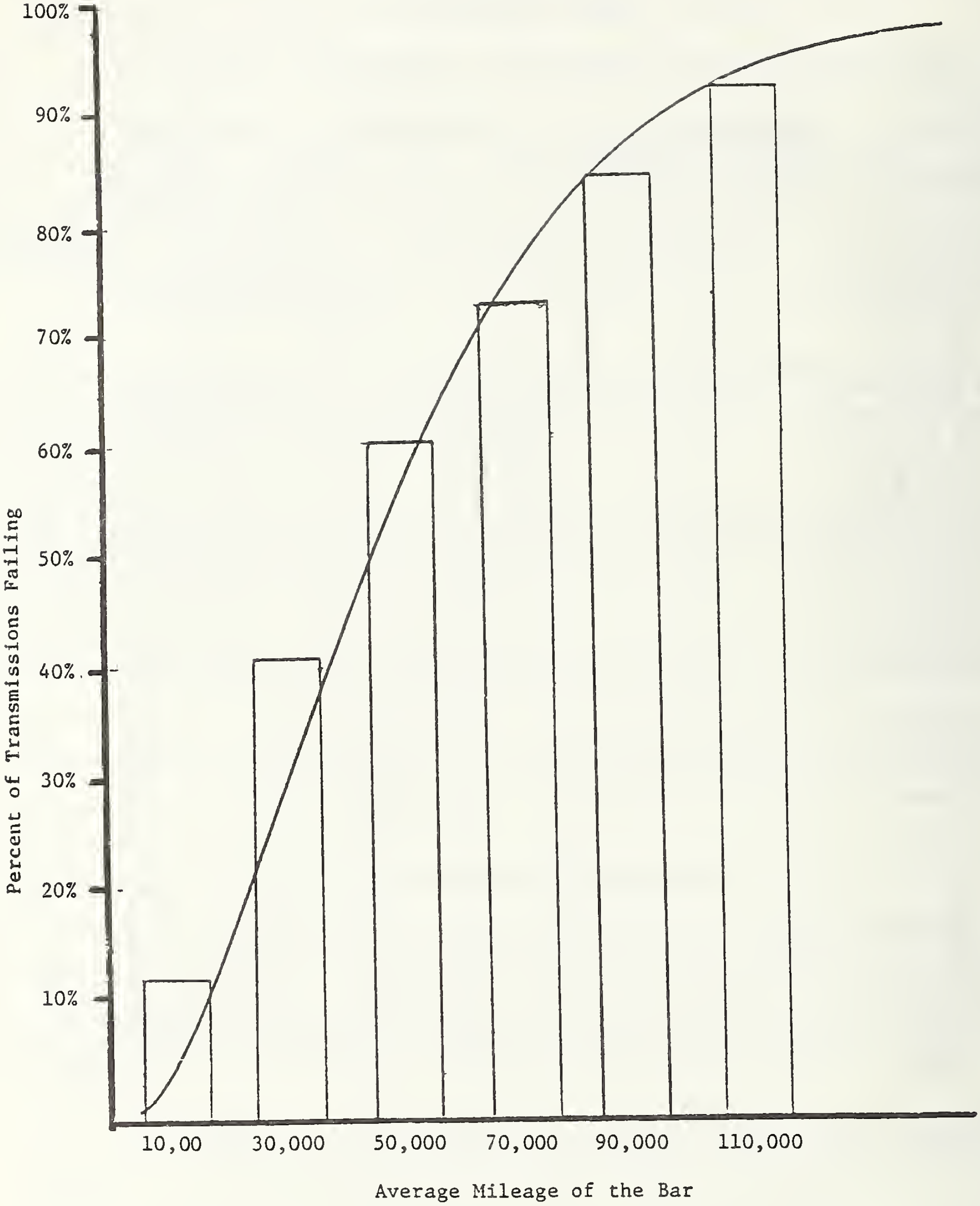


FIGURE 3.3

CUMULATIVE BAR CHART OF TRANSMISSION FAILURES

WITH A CUMULATIVE WEIBULL CURVE

Predicting the Future Reliability of the Transmissions.

The reliability curve based on the Weibull distribution for the failed transmissions is found by first calculating the cumulative Weibull probabilities. The formula used to calculate points along the Weibull curve is shown below.

$$\text{Cumulative Probability} = 1 - e^{\left(-\left[\frac{M - D}{T}\right]^{B}\right)}$$

Suppose one wants to estimate how many transmissions are predicted to fail by 30,000 miles. As determined in the previous section: D = 3,487, T = 55,107, and B = 1.5187. Thus, the proportion (cumulative probability) of transmissions expected to fail before 30,000 miles is:

$$\text{Cumulative Probability} = 1 - e^{\left(-\left[\frac{30{,}000-3{,}487}{55{,}107}\right]^{1.5187}\right)} = 0.2805$$

An explanation and pictures of how to work out this formula with a scientific calculator is shown in Appendix B.

From the above calculation it is thus predicted that 28.05 percent or about one fourth of the transmissions will fail by the time they accumulate 30,000 miles. In fact, six transmissions or 20.7 percent (6/29 = 0.207) did fail before 30,000 miles.

The cumulative probabilities for the mileage intervals used in Table 3.2 as computed by the above formula are entered in column 2 of Table 3.11 (Work Sheet 11). Note that the mileage intervals in column 1 start with the Weibull minimum life term, D = 3,487 miles, instead of 0 miles. Also, the last mileage interval is "greater than 120,000 miles," selected because only one of the 29 failed transmissions lasted more than 120,000 miles. The cumulative probability of failure for this last interval is, by definition, 1.0000 since it encompasses all the remaining failures.

The cumulative probabilities are converted to percentages in column 3 by multiplying the column 2 numbers by 100. In Figure 3.3 the cumulative bar chart of the transmission failures is overlaid with the S-shaped cumulative Weibull distribution curve derived from the numbers in column 3 of Table 3.11.

The reliability of the transmissions, or the percent

TABLE 3.11

WORK SHEET 11

COMPONENT RELIABILITY ANALYSIS FOR WEIBULL FAILURE DATA

Cost Driver Transmission

Bus Model GM RTS II

Component Type Detroit Diesel Allison V730

Study Dates 1979-1981

Cumulative Mileage, M (1)	Cumulative Probability of Failure[1] (2)	Cumulative Percentage[2] (3)	Reliability,[3] R (% Surviving) (4)	Percent Failing in Mileage Interval[4] (5)	Number Failing in Mileage Interval[5] (6)
3,487	0.0	0.0 %	100.0 %	%	
20,000	0.1482	14.9 %	85.1 %	14.9 %	4.3
40,000	0.4144	41.4 %	58.6 %	26.5 %	7.7
60,000	0.6462	64.6 %	35.4 %	23.2 %	6.7
80,000	0.8072	80.7 %	19.3 %	16.1 %	4.7
100,000	0.9039	90.4 %	9.6 %	9.7 %	2.8
120,000	0.9557	95.6 %	4.4 %	5.2 %	1.5
>120,000	1.0000	100.0 %	0.0 %	4.4 %	1.3
		%	%	——— %	———
		%	%	100.0 %	29.0
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	

[1] Cumulative Probability

$$= 1 - e^{-\left(\left[\frac{M - D}{T}\right]^{B}\right)}$$

$$= 1 - e^{\left(-\left[\frac{M - 3,487}{55,107}\right]^{1.5187}\right)}$$

[2] The number in column 2 times 100.

[3] 100 percent less the number in column 3.

[4] Percent failing in mileage interval

$= R_1 - R_2$, where: R_1 = reliability (column 4) at previous cumulative mileage
R_2 = reliability at mileage in column 1

[5] The total number of failures, N, times the decimal percentage in column 5.

surviving at any mileage, is computed in column 4 of Table 3.11 by subtracting the cumulative percentages of column 3 from 100 percent. The resulting reliability curve is depicted in Figure 3.4.

Finally, the reliabilities of column 4 are used to compute the predicted number of transmissions failing in each mileage interval in column 6 of Table 3.11. These numbers are relatively close to the actual frequencies (Table 3.2), indicating that the fitted Weibull distribution is satisfactory and can be used for predicting future reliability patterns, as demonstrated in the next section.

How Many Transmissions Will Fail During Any Time Period?

If a fleet of 60 new buses is purchased which will average 30,000 annual miles per bus, then based on the information from the previous section it is predicted that about 17 of the new transmissions will fail by the end of the first year (60 x 0.2805 = 16.83). Now suppose that one wants to determine how many of the original transmissions will fail during each six month period. If the buses travel 30,000 miles in one year (12 months), then in six months they should travel about 15,000 miles. Table 3.12 (Work Sheet 11) presents the cumulative probability of failure for six month intervals up to 72 months (6 years) for these buses, or 180,000 total miles per bus.

The expected number of transmissions failing in each six month interval is presented in the last column of Table 3.12. For example, five original transmissions (12.8 percent of the total) are predicted to fail between 30 and 36 months of life (75,000 to 90,000 miles). All of the original equipment transmissions are predicted to have failed after six years, or 180,000 miles, of use.

What About the Replacement Transmissions?

Table 3.12 predicts the number of first time failures that can be expected in each six month period but it does not account for the failures of the replacement transmissions. A satisfactory estimate of the total number of failures (including repeats) to expect during any period can be obtained as follows. Take the number of miles the buses will travel, divide it by the mean mileage to failure, and then multiply the result by the total number of buses. It is assumed that the replacement transmissions last as long as the originals.

For example, the transmissions in the above sections lasted an average of 53,759 miles (Table 3.9). The total number of transmission failures expected per year (assuming 30,000 miles per year and 60 buses) is estimated to be about

FIGURE 3.4

TRANSMISSION RELIABILITY CURVE

TABLE 3.12

WORK SHEET 11

COMPONENT RELIABILITY ANALYSIS FOR WEIBULL FAILURE DATA

Cost Driver Transmission Bus Model __________

Component Type __________ Study Dates __________

Cumulative Mileage, M (1)	Cumulative Probability of Failure[1] (2)	Cumulative Percentage[2] (3)	Reliability,[3] R (% Surviving) (4)	Percent Failing in Mileage Interval[4] (5)	Number Failing in Mileage Imterval[5] (6)
3,487	0.0	0.0 %	100.0 %	%	
15,000	0.0886	8.9 %	91.1 %	8.9 %	5
30,000	0.2805	28.1 %	71.9 %	19.2 %	11
45,000	0.4782	47.8 %	52.2 %	19.7 %	12
60,000	0.6462	64.6 %	35.4 %	16.8 %	10
75,000	0.7736	77.4 %	22.6 %	12.8 %	8
90,000	0.8624	86.2 %	13.8 %	8.8 %	5
105,000	0.9203	92.0 %	8.0 %	5.8 %	3
120,000	0.9557	95.6 %	4.4 %	3.6 %	2
135,000	0.9764	97.6 %	2.4 %	2.0 %	2
150,000	0.9879	98.8 %	1.2 %	1.2 %	1
165,000	0.9940	99.4 %	0.6 %	0.6 %	1
180,000	0.9971	99.7 %	0.3 %	0.3 %	--
205,000	0.9992	99.9 %	0.1 %	0.2 %	--
		%	%	%	
					60

[1] Cumulative Probability

$$= 1 - e^{-\left(\left[\frac{M - D}{T}\right]^{B}\right)}$$

$$= 1 - e^{\left(-\left[\frac{M - 3{,}487}{55{,}107}\right]^{1.5187}\right)}$$

[2] The number in column 2 times 100.

[3] 100 percent less the number in column 3.

[4] Percent failing in mileage interval

$= R_1 - R_2$, where: R_1 = reliability (column 4) at previous cumulative mileage
R_2 = reliability at mileage in column 1

[5] The total number of failures, N, times the decimal percentage in column 5.

34 (30,000 divided by 53,759 x 60 = 33.5).

When the 60 new buses are first put into service, the transmission failure rate (including repeats) will be low. Eventually the failure rate will increase to 34 transmissions per year, but when will the rate reach 34? A satisfactory estimate for this time is the mean mileage life of the transmissions converted into time. In the example, the buses will travel 30,000 miles per year and the mean mileage to failure of the transmissions is 53,759. Therefore, the failure rate will reach 34 per year at about 1.8 years (53,759 divided by 30,000 = 1.79).

The increase from zero failures per year to 34 per year is shown in Figure 3.5. A straight line between zero per year when the new buses are first put into service to 34 at 1.8 years (about one year and ten months) is good enough. This graph can be used to predict the demand for transmission repairs at any time during the bus life cycle for the new bus fleet.

List of References

Bain, L.J., Statistical Analysis of Relaiability and Life Test Models, Markel Dekker, Inc., New York, New York, 1978.

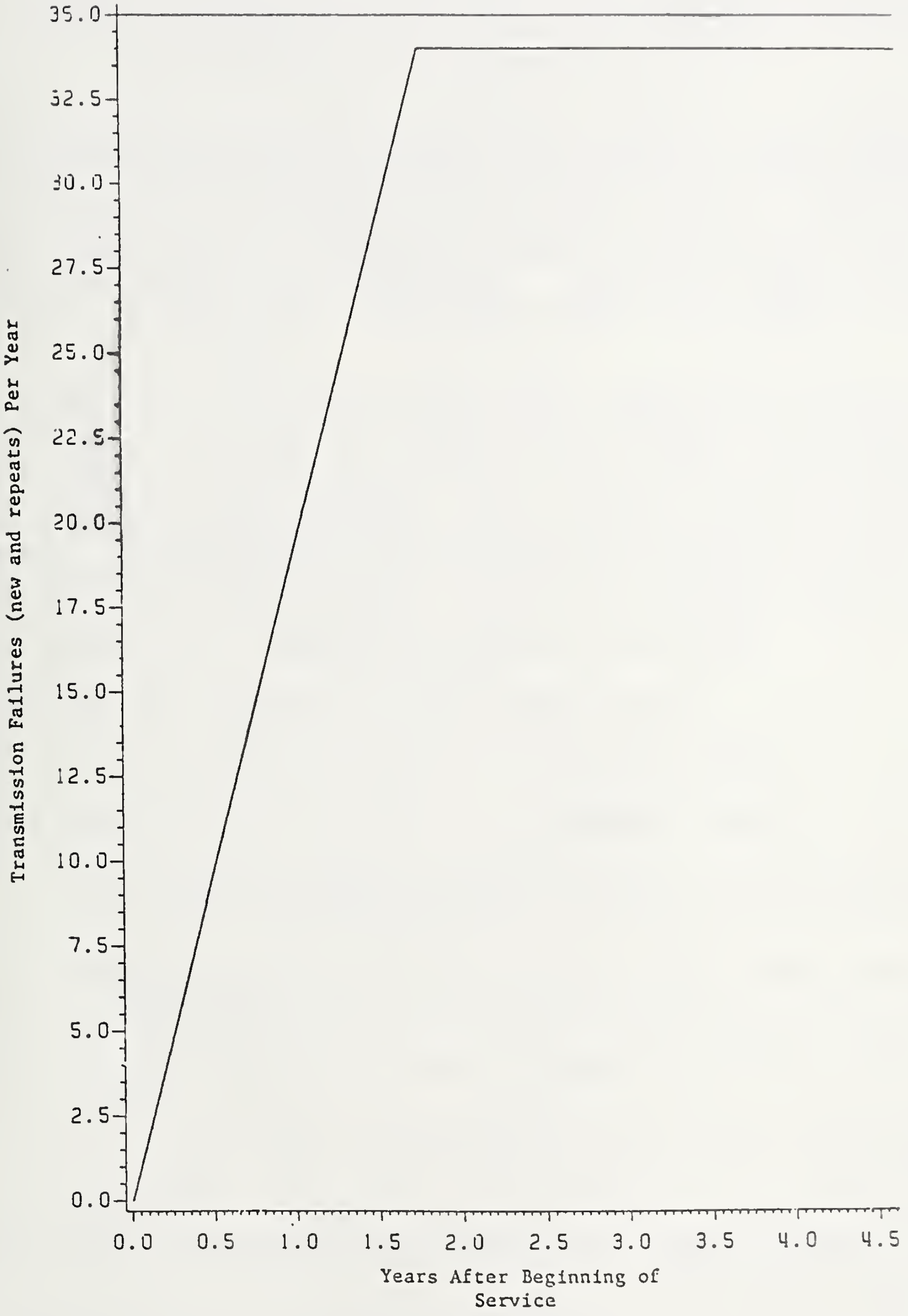


FIGURE 3.5

TOTAL PREDICTED TRANSMISSION FAILURES PER YEAR DURING THE SERVICE LIFE OF THE BUSES

CHAPTER FOUR

WEIBULL DISTRIBUTION FAILURE ANALYSIS WITH CENSORED SAMPLING: AIR COMPRESSOR EXAMPLE

This chapter is a self-contained analysis of original equipment bus air compressor failures to predict future failure patterns. The air compressor on a transit bus is used by several mechanical systems, including the brakes, doors, and windshield wipers. When the air compressor fails the bus shuts down, necessitating a road call and tow. The condition of the air compressor is difficult to monitor, hence it tends to fail with little or no warning.

Suppose that the bus maintenance manager has a fleet of buses purchased at about the same time. Each bus has experienced similar usage patterns and accumulated about the same mileage. At the time of the analysis some of the air compressors have failed and the manager wants to plan in advance for the remaining failures. The technique for helping the manager in this task is called Weibull distribution failure analysis with "censored" sampling, whereby all of the buses are included in the analysis even if their air compressors are still working.

The topics covered in this chapter are:

- How to collect the air compressor data.
- How to tabulate the air compressor data.
- Fitting the Weibull distribution with censored cases to the air compressor data.
- What is the appropriate maintenance policy for the air compressors?
- Predicting the future reliability of the air compressors.
- How many air compressors will fail during any time period?
- What about the replacement air compressors?

How to Collect the Air Compressor Data

The biggest payoff in analyzing component failure patterns comes from the ability to forecast future failures. These forecasts can be used to anticipate maintenance problems, and this can be important if some of the components under investigation show signs of premature failure. Forecasts also are useful in inventory and maintenance management planning.

If forecasts are desired to anticipate future problems then it frequently is necessary to do the failure analysis before most of the components of interest have failed. The previous chapter on transmission failures was based on a data set where all of the transmissions had failed. In contrast, consider the failures of the air compressors listed in Table 4.1 (Work Sheet 7). This table lists the current mileages (column 3) of 36 Grumman 870 buses owned and operated by the Dallas Transit System. Just 11 of the 36 air compressors have failed at the time of this study (failure mileages are indicated in column 4). The maintenance manager desires to predict the remaining failures by making effective use of all of the data in Table 4.1. This means using not only the failed air compressor mileages but also those of the remaining buses whose air compressors have not yet failed.

The data reduction technique for doing this analysis is known as "censored sampling." This means that the buses included in the study have both failed and yet-to-fail air compressors. Including all of the air compressors increases the accuracy of the failure predictions and ensures that the predictions will be accomplished in time to be useful in maintenance management planning.

Analysis with censored sampling has the following prerequisites. All of the buses should have been purchased at about the same time, accumulated similar mileages, and experienced similar operating conditions. It is assumed that the component of interest is the same model in each bus. At least twenty percent of the buses being studied should have recorded failures of the component of interest and there should be a minimum of ten failures. The air compressor analysis in this chapter is based on 11 failures out of a total fleet of 36 buses (31 percent).

Furthermore, all of the failed components should have accumulated mileages less than any yet-to-fail component. If this criterion is not met, it is necessary to use an alternative technique called censored sampling with suspended samples. This is covered in Chapters Five and Six.

How to Tabulate the Air Compressor Data

The analysis of censored samples begins by rearranging the bus mileages of Table 4.1 in ascending order by mileage. This is done in Table 4.2 (Work Sheet 8). The first 11 buses are those which have suffered a failed air compressor. The buses are ordered by increasing mileage until failure. The remaining 25 buses, which have not suffered an air compressor failure, also are listed by the increasing mileage.

Note that the largest failure mileage, 98,054 miles for bus number 614, is smaller than the least mileage bus with a surviving air compressor, bus number 609 with 99,283 miles. Thus, the prerequisite for censored sampling that the surviving components

TABLE 4.1

WORK SHEET 7

Page 1 of 2

FAILURE STATUS OF ORIGINAL COMPONENTS IN WEIBULL FAILURE ANALYSIS

Cost Driver Air Compressors

Bus Model Grumman 870

Component Type

Study Dates

Bus Number (1)	Component Number (2)	Current Bus Mileage (3)	Original Component Failure Mileage (4)	Status of Unfailed Component*		Comments (7)
				Censor (5)	Susp. (6)	
600		114,322		X		
601		109,562		X		
602		109,578		X		
603		101,473	97,863			
604		99,681	96,705			
605		102,043		X		
606		99,152	68,060			
607		108,877		X		
608		92,595	94,340			
609		99,283		X		
610		110,877		X		
611		102,271	76,366			
612		104,168		X		
613		99,433		X		
614		106,410	93,054			
615		109,981		X		
616		108,919		X		
617		107,467	95,328			
618		104,962	22,572			
619		110,115	84,021			

* CENSORED cases refer to surviving components which have accumulated more miles than any failed component in the data set.

SUSPENDED cases refer to surviving components which have accumulated less miles than at least one of the failed components.

TABLE 4.1 (continued)

WORK SHEET 7

Page 2 of 2

FAILURE STATUS OF ORIGINAL COMPONENTS IN WEIBULL FAILURE ANALYSIS

Cost Driver Air Compressors

Bus Model Grumman 870

Component Type

Study Dates

Bus Number (1)	Component Number (2)	Current Bus Mileage (3)	Original Component Failure Mileage (4)	Status of Unfailed Component*		Comments (7)
				Censor (5)	Susp. (6)	
620		108,240		X		
621		106,650		X		
622		103,825		X		
623		99,528		X		
624		107,773		X		
625		104,833	88,704			
626		108,940		X		
627		99,528		X		
628		100,198		X		
629		101,689		X		
630		114,038		X		
631		107,304		X		
632		110,775		X		
633		104,403		X		
634		97,906	86,745			
635		99,482		X		

* CENSORED cases refer to surviving components which have accumulated more miles than any failed component in the data set.

SUSPENDED cases refer to surviving components which have accumulated less miles than at least one of the failed components.

TABLE 4.2

WORK SHEET 8

Page 1 of 2

RANK ORDERING OF SURVIVING AND FAILED COMPONENTS IN WEIBULL FAILURE ANALYSIS

Cost Driver Air Compressors Bus Model Grumman 870

Component Type________ Study Dates________

Order Number (1)	Bus Number (2)	Component Number (3)	Failed, Suspended, or Censored (4)	Mileage When Failed, Suspended, or Censored* (5)
1	618		Failed	22,572
2	606		Failed	68,060
3	611		Failed	76,366
4	619		Failed	84,021
5	634		Failed	86,745
6	625		Failed	88,704
7	608		Failed	92,598
8	617		Failed	95,328
9	604		Failed	96,705
10	603		Failed	97,863
11	614		Failed	98,054
12	609		Censored	99,283
13	613		Censored	99,433
14	635		Censored	99,482
15	627		Censored	99,528
16	623		Censored	99,703
17	628		Censored	100,198
18	629		Censored	101,689
19	605		Censored	102,043
20	622		Censored	103,825

* Each component is listed in ascending order of the mileage entered in Column 5.

TABLE 4.2 (continued)

WORK SHEET 8

Page 2 of 2

RANK ORDERING OF SURVIVING AND FAILED COMPONENTS IN WEIBULL FAILURE ANALYSIS

Cost Driver Air Compressors Bus Model Grumman 870

Component Type ______ Study Dates ______

Order Number (1)	Bus Number (2)	Component Number (3)	Failed, Suspended, or Censored (4)	Mileage When Failed, Suspended, or Censored* (5)
~~1~~ 21	612		Censored	104,168
~~2~~ 22	633		Censored	104,403
~~3~~ 23	621		Censored	106,650
~~4~~ 24	631		Censored	107,304
~~5~~ 25	624		Censored	107,773
~~6~~ 26	620		Censored	108,240
~~7~~ 27	607		Censored	108,877
~~8~~ 28	616		Censored	108,919
~~9~~ 29	626		Censored	108,940
~~10~~ 30	601		Censored	109,562
~~11~~ 31	602		Censored	109,578
~~12~~ 32	615		Censored	109,981
~~13~~ 33	632		Censored	110,775
~~14~~ 34	610		Censored	110,877
~~15~~ 35	630		Censored	114,038
~~16~~ 36	600		Censored	114,322
17				
18				
19				
20				

* Each component is listed in ascending order of the mileage entered in Column 5.

have accumulated more mileage than any failed component has been met. Note that the pattern of failures in Table 4.2 indicates that the air compressors are starting to fail more frequently as mileages approach and exceed 100,000 miles. The average of the 11 failure mileages is 82,456 miles, but clearly this is too low a number. What is the true average failure mileage and how many more failures should the manager anticipate in the near future? Answers to these questions require the estimation of a failure distribution.

Fitting the Weibull Distribution with Censored Cases to the Air Compressor Data

The air compressor failures are modeled similarly to any complex component which fails rather than wears out. Thus, the incomplete set of air compressor failures is modeled with the Weibull distribution instead of the Normal distribution. Note that the transmission failures in Chapter Three also were modeled using the Weibull. The bar chart of the transmission failures (Figure 3.1) resembles the ski slope pattern of the Weibull. However, the air compressor failure bar chart will not, simply because most of them have not yet failed. Hence, the Weibull distribution is selected not because it resembles the failure pattern but instead because it is expected that it would if there were more data.

Calculating D, B, and T. Fitting the air compressor failure data to the Weibull distribution is similar to fitting the transmission failures but it is not quite the same. The same three Weibull constants, D, B, and T, have to be calculated, however, different tables are used. The computation begins by first listing the buses with failed air compressors in order of increasing mileage in the first column of Table 4.3 (Work Sheet 9).

The first constant is D and it is called the "minimum life term." If it were likely that a component could fail without having accumulated any miles, then the "minimum life" would be zero. In fact, a good estimate of D is 90 percent of the lowest failure mileage. In Table 4.3, the lowest mileage is 22,572 miles (bus 618) and therefore D equals 20,315 miles (22,572 x 0.90 = 20,315). There is space for this computation at the bottom of Work Sheet 9.

The other two constants are B and T*. B, the "shape factor," determines the shape of the curve (note the Weibull curve in Figure 1.2) and T, the "characteristic life factor," determines how far away the peak in the curve should be from zero miles. Before calculating B and T, two intermediate numbers, K and S, must be determined. These calculations also are shown in Table 4.3. Note that the following computations use different tables than were used for the transmissions analysis in Chapter Three.

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* Method for calculating B and T developed by Bain (1).

TABLE 4.3

WORK SHEET 9

COMPUTATION OF D, K, AND S FOR WEIBULL FAILURE DATA

Cost Driver Air Compressors Bus Model Grumman 870

Component Type ____________ Study Dates ____________

Bus Number (1)	Failure Mileage (2)	Minimum Life, D* (3)	K (4)	$\ln(K_{max})$** (5)	ln(K) (6)	L*** (7)
618	22,572 -	20,315 =	2,257	11.2611 -	7.7218 =	3.5393
606	68,060 -	20,315 =	47,745	11.2611 -	10.7736 =	0.4875
611	76,366 -	20,315 =	56,051	11.2611 -	10.9340 =	0.3271
619	84,021 -	20,315 =	63,706	11.2611 -	11.0620 =	0.1991
634	86,745 -	20,315 =	66,430	11.2611 -	11.1039 =	0.1572
625	88,704 -	20,315 =	68,389	11.2611 -	11.1330 =	0.1281
608	92,598 -	20,315 =	72,283	11.2611 -	11.1883 =	0.0728
617	95,328 -	20,315 =	75,013	11.2611 -	11.2254 =	0.0356
604	96,705 -	20,315 =	76,390	11.2611 -	11.2436 =	0.0175
603	97,863 -	20,315 =	77,548	11.2611 -	11.2587 =	0.0024
614	98,054 -	20,315 =	77,739	11.2611 -	11.2611 =	0.0
	-	=		-	=	
	-	=		-	=	
	-	=		-	=	
	-	=		-	=	
	-	=		-	=	
	-	=		-	=	
	-	=		-	=	
	-	=		-	=	
	-	=		-	=	
Sums:		ΣK =			S =	4.9667

* Minimum life term, D = 0.90 x lowest failure mileage
= 0.90 x 22,572 mi. = D = 20,315 miles

** $\ln(K_{max}) = \ln($ 77,739 $) =$ 11.2611 , where K_{max} is the largest value of K in column 4.

*** If the data set contains suspended failure cases, the L terms must be modified in Work Sheet 10a before computing S.

TABLE 4.4

WORK SHEET 12

COMPUTATION OF B AND T FOR WEIBULL FAILURE DATA WHICH CONTAINS CENSORED CASES

Cost Driver Air Compressors Bus Model Grumman 870

Component Type ______ Study Dates ______

INPUT NUMBERS:

$A1 = \underset{\text{(from Table 4.5)}}{\underline{0.3394}}$ $C1 = \underset{\text{(from Table 4.8)}}{\underline{-0.9869}}$

$A2 = \underset{\text{(from Table 4.6)}}{\underline{-1.1110}}$ $C2 = \underset{\text{(from Table 4.9)}}{\underline{-0.2238}}$

$A3 = \underset{\text{(from Table 4.7)}}{\underline{0.0566}}$ $C3 = \underset{\text{(from Table 4.10)}}{\underline{-0.0073}}$

$N = \underset{\text{(total number of buses)}}{\underline{36}}$

$S = \underset{\text{(from Work Sheet 9)}}{\underline{4.9667}}$

$K_{max} = \underset{\text{(from Work Sheet 9)}}{\underline{77{,}739 \text{ miles}}}$

SHAPE FACTOR, B:

$$M = \underset{A1}{\underline{0.3394}} + \underset{A2}{\underline{-1.1110}} / \underset{N}{\underline{36}} + \underset{A3}{\underline{0.0566}} / (\underset{N}{\underline{36}})^2 = \underline{0.3086}$$

$$B = \underset{M}{\underline{0.3086}} \times \underset{N}{\underline{36}} / \underset{S}{\underline{4.9667}} = B = \underline{2.2368}$$

CHARACTERISTIC LIFE FACTOR, T:

$$P = \underset{C1}{\underline{-0.9869}} + \underset{C2}{\underline{-0.2238}} / \underset{N}{\underline{36}} + \underset{C3}{\underline{-0.0073}} / (\underset{N}{\underline{36}})^2 = \underline{-0.9931}$$

$$\ln(T) = \underset{\ln(K_{max})}{\underline{11.2611}} - \underset{P}{\underline{-0.9931}} / \underset{B}{\underline{2.2368}} = \underline{11.7051}$$

$T = \underset{\text{miles}}{\underline{121{,}188}}$ ("inverse" of ln(T) on a scientific calculator)

TABLE 4.5

A1 NUMBERS

Number Failed N	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.10	0.1027	0.1131	0.1237	0.1344	0.1452	0.1560	0.1669	0.1779	0.1890	0.2001
0.20	0.2113	0.2225	0.2339	0.2453	0.2567	0.2682	0.2799	0.2916	0.3034	0.3153
0.30	0.3272	0.3394	0.3514	0.3637	0.3760	0.3884	0.4010	0.4137	0.4265	0.4393
0.40	0.4523	0.4655	0.4787	0.4920	0.5055	0.5192	0.5329	0.5468	0.5608	0.5750
0.50	0.5894	0.6039	0.6184	0.6334	0.6484	0.6636	0.6790	0.6946	0.7104	0.7265
0.60	0.7427	0.7592	0.7760	0.7930	0.8102	0.8278	0.8456	0.8638	0.8823	0.9011
0.70	0.9203	0.9398	0.9598	0.9802	1.0011	1.0224	1.0443	1.0668	1.0899	1.1137
0.80	1.1382	1.1635	1.1897	1.2169	1.2451	1.2745	1.3052	1.3373	1.3709	1.4063
0.90	1.4436	1.4830	1.5248	1.5691	1.6164	1.6668	-	-	-	-

Example: A1 for 0.31 (Number Failed/N = 11/36 = 0.31) is 0.3394, which is the number in the third row ("0.30") of the third column ("0.01").

TABLE 4.6

A2 NUMBERS

Number Failed N	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.10	-1.0271	-1.0292	-1.0319	-1.0350	-1.0385	-1.0422	-1.0461	-1.0501	-1.0541	-1.0581
0.20	-1.0622	-1.0663	-1.0704	-1.0746	-1.0788	-1.0831	-1.0875	-1.0919	-1.0965	-1.1012
0.30	-1.1060	-1.1110	-1.1161	-1.1214	-1.1269	-1.1325	-1.1383	-1.1443	-1.1505	-1.1568
0.40	-1.1634	-1.1702	-1.1771	-1.1843	-1.1917	-1.1994	-1.2072	-1.2154	-1.2238	-1.2325
0.50	-1.2415	-1.2508	-1.2605	-1.2706	-1.2811	-1.2920	-1.3034	-1.3152	-1.3276	-1.3405
0.60	-1.3540	-1.3681	-1.3829	-1.3984	-1.4147	-1.4317	-1.4496	-1.4685	-1.4883	-1.5092
0.70	-1.5313	-1.5547	-1.5795	-1.6060	-1.6342	-1.6645	-1.6970	-1.7321	-1.7701	-1.8115
0.80	-1.8567	-1.9063	-1.9609	-2.0212	-2.0882	-2.1627	-2.2459	-2.3390	-2.4435	-2.5608
0.90	-2.2929	-2.8417	-3.0095	-3.1988	-3.4125	-3.6536	-	-	-	-

Example: A2 for 0.31 (Number Failed/N = 11/36 = 0.31) is -1.1110, which is the number in the third row ("0.30") of the third column ("0.01").

TABLE 4.7

A3 NUMBERS

Number Failed N	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.10	0.0000	0.0004	0.0011	0.0038	0.0074	0.0113	0.0154	0.0194	0.0232	0.0267
0.20	0.0300	0.0330	0.0357	0.0382	0.0405	0.0427	0.0449	0.0471	0.0493	0.0516
0.30	0.0540	0.0566	0.0594	0.0623	0.0655	0.0689	0.0725	0.0763	0.0804	0.0846
0.40	0.0890	0.0936	0.0984	0.1034	0.1086	0.1140	0.1197	0.1255	0.1317	0.1382
0.50	0.1450	0.1522	0.1598	0.1679	0.1765	0.1857	0.1955	0.2060	0.2172	0.2292
0.60	0.2420	0.2557	0.2704	0.2861	0.3029	0.3209	0.3402	0.3609	0.3831	0.4071
0.70	0.4330	0.4612	0.4919	0.5257	0.5629	0.6043	0.6506	0.7028	0.7618	0.8290
0.80	0.9050	0.9945	1.0967	1.2150	1.3522	1.5117	1.6972	1.9132	2.1644	2.4565
0.90	2.7950	3.1899	3.6464	4.1746	4.7846	5.4877	-	-	-	-

Example: A3 for 0.31 (Number Failed/N = 11/36 = 0.31) is 0.0566, which is the number in the third row ("0.30") of the third column ("0.01").

The minimum life term D, 20,305 miles, is subtracted from each original mileage in Table 4.3 (column 2 minus column 3) to get the fourth column of numbers, which is a value of K for each failed air compressor.

The K values are then used to compute S as follows. First, compute the natural logarithm of the largest value of K, 77,739 miles for bus 614. Obtaining the natural logarithm is simple when using a scientific calculator. Enter the number, 77,739, and press the button marked "ln". The natural logarithm, 11.2611, will appear on the display.* The natural logarithms are rounded-off to only four digits to the right of the decimal point for simplicity (11.261112 become 11.2611) in Table 4.3. There is space for this calculation at the bottom of Work Sheet 9. This number is placed on every line of column 5 of Table 4.3.

Next, the natural logarithm of each value of K, ln (K), is written in the sixth column. For example, in the first line of Table 4.3 (bus number 618), ln (2,257) equals 7.7218.

The difference of the numbers in the fifth and sixth columns is written in the seventh and final column. The sum of all the numbers in column 7 is 4.9667, which is the value for S.

Now it is possible to compute the two remaining Weibull constants, B and T. Table 4.4 (Work Sheet 12) outlines the computations for B and T and provides space for each intermediate result and the final answers.

To get B requires the use of three tables, Tables 4.5, 4.6, and 4.7. First, divide the number of buses that experienced an air compressor failure, NF = 11, by the total number of buses in the study, N=36. The result is 0.31 (11 divided by 36 = 0.31 rounded to two decimal places). Next, find the number, A1, which corresponds to 0.31 in Table 4.5. This is 0.3394, found in the third row ("0.30") and column three (under "0.01"). The numbers A2 and A3 are similarly found from Tables 4.6 and 4.7, respectively. A2 equals -1.1110 and A3 equals 0.0566.

The values for A1, A2, and A3 are then plugged into the equation below:

$$M = A1 + \frac{A2}{N} + \frac{A3}{N^2} \quad \text{where N = the total number of buses.}$$

For the air compressor example:

$$M = 0.3394 + \frac{-1.1110}{36} + \frac{0.0566}{(36)^2} = 0.3086$$

---

* For a discussion of the use of scientific calculators see Appendix B.

B is then found by inserting M and S (from Table 4.3) into the equation below:

$$B = \frac{M \times N}{S}$$

For the air compressor example:

$$B = \frac{0.3086 \times 36}{4.9667} = 2.2368$$

There is room for the computation of M and B in Work Sheet 12.

Finally, Tables 4.8, 4.9, and 4.10 are used to get the remaining Weibull constant, the characteristic life factor, T. As before, the ratio of the number of failures to the total number of buses (0.31) is used to find C1, C2 and C3. First, Table 4.8 is used to look up the number C1 that corresponds to 0.31, which is -0.9869. The numbers C2 and C3 are similarly found from Tables 4.9 and 4.10, respectively. C2 equals -0.2238 and C3 equals -0.0073.

The values for C1, C2 and C3 are then plugged into the equation below:

$$P = C1 + \frac{C2}{N} + \frac{C3}{N^2}$$

For the air compressor example:

$$P = -0.9869 + \frac{-0.2238}{36} + \frac{-0.0073}{(36)^2} = -0.9931$$

T is then found by inserting P, B, and K_{max}, the largest value of K in Table 4.3, into the equation below:

$$\ln(T) = \ln\,(K_{max}) - \frac{P}{B}$$

For the air compressor example:

$$\ln(T) = \ln(77{,}739) - \frac{-0.9931}{2.2368} = 11.7051$$

$$T = 121{,}188$$

Obtaining T from the ln(T) is explained in Appendix B. For example, on scientific Texas Instrument calculators enter the number,

TABLE 4.8

C1 NUMBERS

Number Failed N	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.10	-2.2540	-2.0864	-1.9611	-1.8645	-1.7886	-1.7271	-1.6751	-1.6286	-1.5852	-1.5427
0.20	-1.4999	-1.4561	-1.4110	-1.3646	-1.3171	-1.2687	-1.2200	-1.1715	-1.1235	-1.0765
0.30	-1.0309	-0.9869	-0.9448	-0.9047	-0.8665	-0.8303	-0.7959	-0.7631	-0.7316	-0.7013
0.40	-0.6717	-0.6427	-0.6138	-0.5848	-0.5555	-0.5257	-0.4953	-0.4640	-0.4312	-0.3996
0.50	-0.3665	-0.3332	-0.2999	-0.2670	-0.2349	-0.2041	-0.1751	-0.1484	-0.1245	-0.1040
0.60	-0.0874	-0.0601	-0.0328	-0.0055	0.0218	0.0491	0.0764	0.1037	0.1310	0.1583
0.70	0.1856	0.2187	0.2536	0.2894	0.3249	0.3591	0.3908	0.4190	0.4430	0.4620
0.80	0.4759	0.4848	0.4897	0.4921	0.4946	0.5090	0.5164	0.5475	0.6032	0.6942
0.90	0.8340	1.0392	1.3294	1.7282	2.2634	2.9634	-	-	-	-

Example: C1 for 0.31 (Number Failed/N = 11/36 = 0.31) is -0.9869, which is the number in the third row ("0.30") of the third column ("0.01").

TABLE 4.9

C2 NUMBERS

Number Failed N	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.10	-0.5574	-0.5152	-0.4783	-0.4460	-0.4178	-0.3714	-0.3714	-0.3523	-0.3355	-0.3506
0.20	-0.3074	-0.2956	-0.2850	-0.2755	-0.2669	-0.2590	-0.2519	-0.2453	-0.2393	-0.2337
0.30	-0.2286	-0.2238	-0.2194	-0.2152	-0.2113	-0.2077	-0.2044	-0.2012	-0.1983	-0.1956
0.40	-0.1930	-0.1907	-0.1885	-0.1864	-0.1845	-0.1828	-0.1813	-0.1798	-0.1785	-0.1773
0.50	-0.1762	-0.1752	-0.1743	-0.1736	-0.1729	-0.1724	-0.1719	-0.1716	-0.1713	-0.1712
0.60	-0.1711	-0.1712	-0.1714	-0.1717	-0.1721	-0.1726	-0.1733	-0.1740	-0.1750	-0.1760
0.70	-0.1773	-0.1787	-0.1802	-0.1920	-0.1839	-0.1861	-0.1885	-0.1912	-0.1941	-0.1974
0.80	-0.2011	-0.2052	-0.2098	-0.2150	-0.2208	-0.2274	-0.2349	-0.2435	-0.2534	-0.2647
0.90	-0.2777	-0.2928	-0.3102	-0.3303	-0.3536	-0.3806	-	-	-	-

Example: C2 for 0.31 (Number Failed/N = 11/36 = 0.31) is -0.2238, which is the number in the third row ("0.30") of the third column ("0.01").

TABLE 4.10

C3 NUMBERS

Number Failed / N	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.10	-0.0720	-0.0640	-0.0565	-0.0496	-0.0434	-0.0378	-0.0328	-0.0285	-0.0248	-0.0216
0.20	-0.0189	-0.0166	-0.0147	-0.0131	-0.0119	-0.0108	-0.0092	-0.0093	-0.0087	-0.0081
0.30	-0.0077	-0.0072	-0.0068	-0.0064	-0.0060	-0.0055	-0.0051	-0.0047	-0.0042	-0.0038
0.40	-0.0034	-0.0029	-0.0025	-0.0022	-0.0019	-0.0016	-0.0014	-0.0012	-0.0010	-0.0010
0.50	-0.0009	-0.0009	-0.0009	-0.0010	-0.0011	-0.0011	-0.0012	-0.0012	-0.0012	-0.0012
0.60	-0.0011	-0.0010	-0.0007	-0.0004	-0.0000	0.0004	0.0009	0.0016	0.0022	0.0029
0.70	0.0037	0.0045	0.0052	0.0059	0.0066	0.0072	0.0078	0.0082	0.0085	0.0088
0.80	0.0089	0.0090	0.0091	0.0093	0.0098	0.0105	0.0118	0.0139	0.0171	0.0217
0.90	0.0283	0.0372	0.0491	0.0649	0.0852	0.1112	-	-	-	-

Example: C3 for 0.31 (Number Failed/N = 11/36 = 0.31) is -0.0073, which is the number in the third row ("0.30") of the third column ("0.01")

11.7052, and then press the following keys, "INV," and "lnX". There is room for the computation of P and T in Work Sheet 12.

Mean and Standard Deviation. The conventional techniques for finding the mean mileage to failure, $\bar{X}$, and the standard deviation, SD, that were used in Chapters Two and Three cannot be used in this chapter. The reason is simple: the two statistics are meaningful only when they describe a complete set of failures. This was true for the brake shoes and transmissions in the preceding two chapters. However, in this chapter just 11 out of 36 air compressors in the study have failed yet. And the primary objective of the study is to make predictions based on all 36 air compressors. Therefore, some way must be found to determine estimates of the $\bar{X}$ and SD where all 36 air compressors have failed.

The estimated mean mileage to failure, $\bar{X}$, is found by using the previously determined Weibull distribution factors, D, B, and T. Table 4.11 (Work Sheet 13) provides space for the following computations.

First, find the E1 number in Table 4.11 which corresponds to the value for B rounded off to just one decimal point. B is 2.2368, or 2.2 rounded off, and the corresponding number, E1, in Table 4.11 is 1.1288. E1 is used to find $\bar{X}$ in the following equation:

$$\bar{X} = \frac{T}{E1} + D$$

$$= \frac{121{,}188 \text{ miles}}{1.1288} + 20{,}315 \text{ miles}$$

$$\bar{X} = 127{,}675 \text{ miles}$$

Note that the estimated mean, $\bar{X}$ = 127,675 miles, is substantially higher than the average of the 11 failures alone, 82,456 miles. There is room for this computation in Table 4.11.

Estimating the standard deviation, SD, is accomplished in a similar manner using Table 4.13. First, find the number, E2, in Table 4.13 which corresponds to the value of B, which is 2.2. E2 is 0.1806. This number is then used in the following equation for the estimated standard deviation, SD:

$$SD = \sqrt{E2 \times T^2}$$

$$= \sqrt{0.1806 \times (121{,}188)^2}$$

$$= \sqrt{2{,}652{,}387{,}560}$$

$$SD = 51{,}501$$

TABLE 4.11

WORK SHEET 13

COMPUTING THE ESTIMATED MEAN MILEAGE TO FAILURE AND THE STANDARD DEVIATION FOR WEIBULL FAILURE DATA WITH CENSORED AND SUSPENDED CASES

Cost Driver Air Compressors Bus Model Grumman 870

Component Type ________ Study Dates ________

INPUT NUMBERS:

D = 20,315 miles
(minimum life term)

B = 2.2368
(Weibull shape factor)

T = 121,188 miles
(Weibull characteristic life factor)

E1 = 1.1288
(from Table 4.12)

E2 = 0.2360
(from Table 4.13)

ESTIMATED MEAN MILEAGE TO FAILURE, $\bar{X}$:

$$\bar{X} = \underset{T}{\underline{121{,}188\text{ mi.}}} \;/\; \underset{E1}{\underline{1.1288}} + \underset{D}{\underline{20{,}315\text{ mi.}}} = \bar{X} = \underline{127{,}675\text{ mi}}.$$

ESTIMATED STANDARD DEVIATION, SD:

$$SD = \sqrt{\underset{E2}{\underline{0.2360}} \times \underset{T}{\underline{(\;121{,}188\;)}}^{2}} = SD = \underline{51{,}501\text{ mi}}.$$

TABLE 4.12

E1 NUMBERS USED TO FIND ESTIMATED MEAN MILEAGE TO FAILURE

B	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0			0.0083	0.1080	0.3009	0.4000	0.6646	0.7900	0.8826	0.9504
1.0	1.0000	1.0364	1.0631	1.0828	1.0972	1.1077	1.1154	1.1208	1.1245	1.1269
2.0	1.1284	1.1291	1.1288	1.1292	1.1281	1.1271	1.1259	1.1245	1.1215	1.1200
3.0	1.1199	1.1182	1.1165	1.1148	1.1131	1.1115	1.1098	1.1081	1.1065	1.1049
4.0	1.1033	1.1017	1.1002	1.0987	1.0972	1.0958	1.0944	1.0930	1.0917	1.0904
5.0	1.0891	1.0879	1.0857	1.0855	1.0843	1.0832	1.0821	1.0810	1.0800	1.0789
6.0	1.0779	1.0769	1.0760	1.0750	1.0741	1.0732	1.0723	1.0715	1.0706	1.0698
7.0	1.0690	1.0682	1.0675	1.0667	1.0660	1.0653	1.0646	1.0639	1.0632	1.0625
8.0	1.0619	1.0612	1.0606	1.0600	1.0594	1.0588	1.0582	1.0577	1.0571	1.0565
9.0	1.0560	1.0555	1.0550	1.0545	1.0540	1.0535	1.0530	1.0525	1.0529	1.0516

Example: The number, E1, for B = 2.2 is 1.1288, which is the number in the third row ("2.0") of the fourth column ("0.2"). This is E1 for B = 2.0 + 0.2 = 2.2.

TABLE 4.13

E2 NUMBERS USED TO FIND THE ESTIMATED STANDARD DEVIATION OF FAILURE MILEAGES

B	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0					108.9553	20.0000	6.9968	3.4268	2.0397	1.3715
1.0	1.0000	0.7714	0.6197	0.5132	0.4351	0.3757	0.3292	0.2919	0.2614	0.2360
2.0	0.2146	0.1963	0.1806	0.1669	0.1548	0.1441	0.1347	0.1262	0.1185	0.1116
3.0	0.1053	0.0996	0.0944	0.0896	0.0852	0.0811	0.0773	0.0738	0.0705	0.0675
4.0	0.0647	0.0620	0.0595	0.0572	0.0550	0.0529	0.0510	0.0492	0.0474	0.0458
5.0	0.0442	0.0428	0.0414	0.0400	0.0388	0.0376	0.0364	0.0353	0.0343	0.0333
6.0	0.0323	0.0314	0.0305	0.0297	0.0289	0.0281	0.0274	0.0267	0.0260	0.0253
7.0	0.0247	0.0241	0.0235	0.0229	0.0224	0.0219	0.0214	0.0209	0.0204	0.0200
8.0	0.0195	0.0191	0.0187	0.0183	0.0179	0.0175	0.0171	0.0168	0.0164	0.0161
9.0	0.0158	0.0155	0.0152	0.0149	0.0146	0.0143	0.0141	0.0138	0.0136	0.0133

Example: The number, E2, for B = 2.2 is 0.1806, which is the number in the third row ("2.0") of the fourth column ("0.2").

There is room for this computation in Table 4.11.

Summary. The values for the Weibull distribution, D, B, and T, the estimated mean mileage to failure and the estimated standard deviation, can be entered in Table 4.14 (Work Sheet 3). A frequency bar chart of the air compressor failures is included with a note that only 11 failures had been recorded as yet among the original sample of 36 buses.

The steps required to find the Weibull constants, D, B, and T, and the estimated mean and standard deviation, are recapped as follows:

1. Tabulate the component failure mileages and the mileages accumulated by the buses with unfailed original components in Work Sheet 7 (Table 4.1). All of the unfailed components should have accumulated more miles than any of the failed components, which means they are "censored" cases.
2. List the component failed and unfailed mileages in increasing order from smallest to largest in Work Sheet 8 (Table 4.2).
3. Use Work Sheet 9 (Table 4.3) to calculate the intermediate number, S, using the ordered failure mileages and D. The minimum life term, D, is 0.90 multiplied by the smallest failure mileage.
4. Determine the shape factor, B, and the characteristic life factor, T, by following the computational outline in Work Sheet 12 (Table 4.4).
5. Determine the estimated mean mileage to failure, $\bar{X}$, and the estimated standard deviation, SD, using Work Sheet 13 (Table 4.11).
6. Enter the values for D, B, T, $\bar{X}$, and SD in summary Work Sheet 3 (Table 4.14). The Weibull distribution computation is now complete.

What is the Appropriate Maintenance Policy for the Air Compressors?

As in the previous chapters, a determination of the failure mechanism and the best maintenance policy for the air compressors can be made with the summary data of Table 4.14 (Work Sheet 3). Dividing ($\bar{X}$ - D) by SD yields a value of 2.08 in Table 4.14, which is greater than 2.0. B, of course, also is greater than 2.0. Based on Table 1.1 from Chapter One, the failure mechanism is determined to be "mileage to failure predictable."

Assuming that $\bar{X}$, the mean mileage to failure, is expected and noting that air compressor failures are relatively undetect-

TABLE 4.14

WORK SHEET 3

SUMMARY SHEET OF FAILURE MILEAGE DISTRIBUTION

Cost Driver Air Compressors   Bus Model Grumman 870

Component Type   Study Dates

Number of cases, N 36   Mean, $\bar{X}$ 127,675 miles

Number of failures, NF 11   Std. deviation, SD 51,501 miles

Maximum mileage not applicable $(\bar{X}-D)/SD$ 2.08

Minimum mileage 22,572 miles

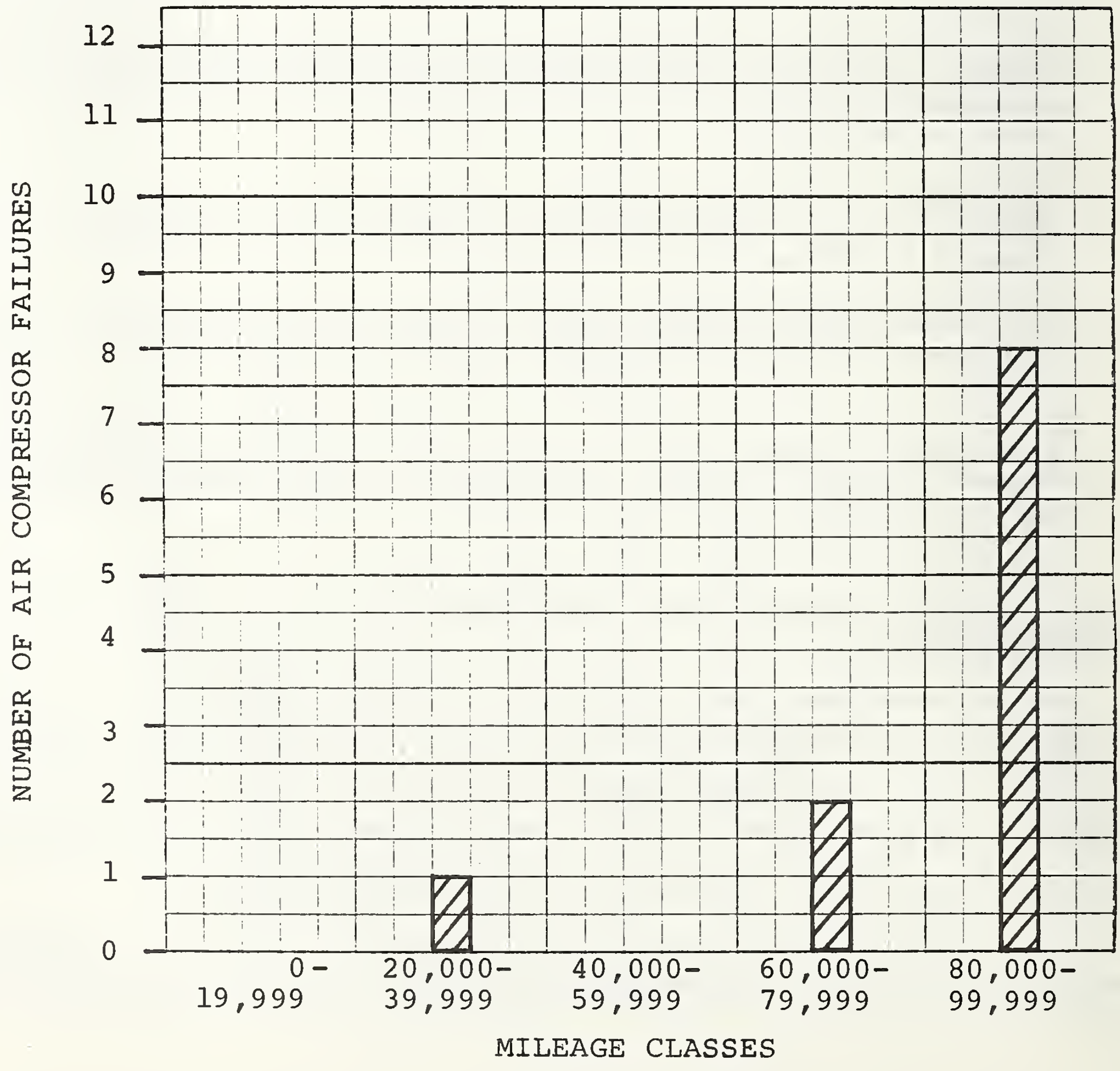
WEIBULL DISTRIBUTION PARAMETERS:

Minimum life term, D 20,315 miles

Shape factor, B 2.2368

Characteristic life factor, T 121,188 miles

FAILURE MECHANISM: "Mileage to Failure Predictable"



able, then the appropriate maintenance policy can be found in Figure 1.6. The preferred policy is "fixed-mileage maintenance," although "operate-until-failure" also is an acceptable maintenance policy, depending on safety and economic factors not discussed here.

Predicting the Future Reliability of the Air Compressors

The future reliability of the air compressors is determined by working with the cumulative percent of air compressor failures that occurred before a certain mileage. Figure 4.1 includes a cumulative bar chart of the first eleven failures. Each bar includes a 20,000 mile interval ranging from 20,000 to 100,000 miles. Table 4.15 (Work Sheet 1) tallies the frequency distribution for the 11 failures based on the ordered mileages of Table 4.2. There are so few failure mileages that determining the cumulative frequencies is a simple task:

Cumulative Mileage	Cumulative Frequency	Cumulative Percent
20,000	0	0.0
40,000	1	2.8
60,000	1	2.8
80,000	3	8.3
100,000	11	30.6

The cumulative frequencies are plotted as a bar chart of cumulative percentages in Figure 4.1. Since all 36 air compressors in the study eventually will fail, the cumulative percentages are obtained by dividing the frequencies by 36. For example, three air compressors have failed after 80,000 miles. The cumulative percentage is 8.3 percent (3 divided by 36 is 0.083). Similarly, 11 compressors have failed after 100,000 miles, or 30.6 percent (11 divided by 36 is 0.306).

The smooth curve in Figure 4.1 is the fitted Weibull distribution representation of the cumulative failure percentages. Note that although the bars end in the 80,000 to 100,000 mile interval, the smooth Weibull curve continues past 100,000 miles. Once past the bars the remainder of the cumulative Weibull curve forecasts future air compressor failures. The Weibull curve can now be used to forecast the reliability of the air compressors even though the analysis was done before any of the compressors had lasted as long as the predicted mean mileage to failure of 127,675 miles!

The reliability curve based on the Weibull distribution for the failed air compressors is found by first calculating the cumulative Weibull probabilities. The formula used to calculate points along the Weibull curve is shown below:

TABLE 4.15

WORK SHEET 1

TABULATION OF FAILURE MILEAGE FREQUENCIES

Cost Driver Air Compressors Bus Model Grumman 870

Component Type________ Study Dates________

Failure Mileage (1)	Failure Mileage (2)	Mileage Class		Tally (5)	Frequency, F (6)
		Lower (3)	Upper (4)		
22,572		0	19,999		
68,060		20,000	39,999	/	1
76,366		40,000	59,999		
84,021		60,000	79,999	//	2
86,745		80,000	99,999	///// ///	8
88,704					$\overline{11}$
92,598					
95,328					
96,705					
97,863					
98,054					

Number of failure mileages, N 11 of 36

Maximum mileage not applicable Minimum mileage 22,572 miles

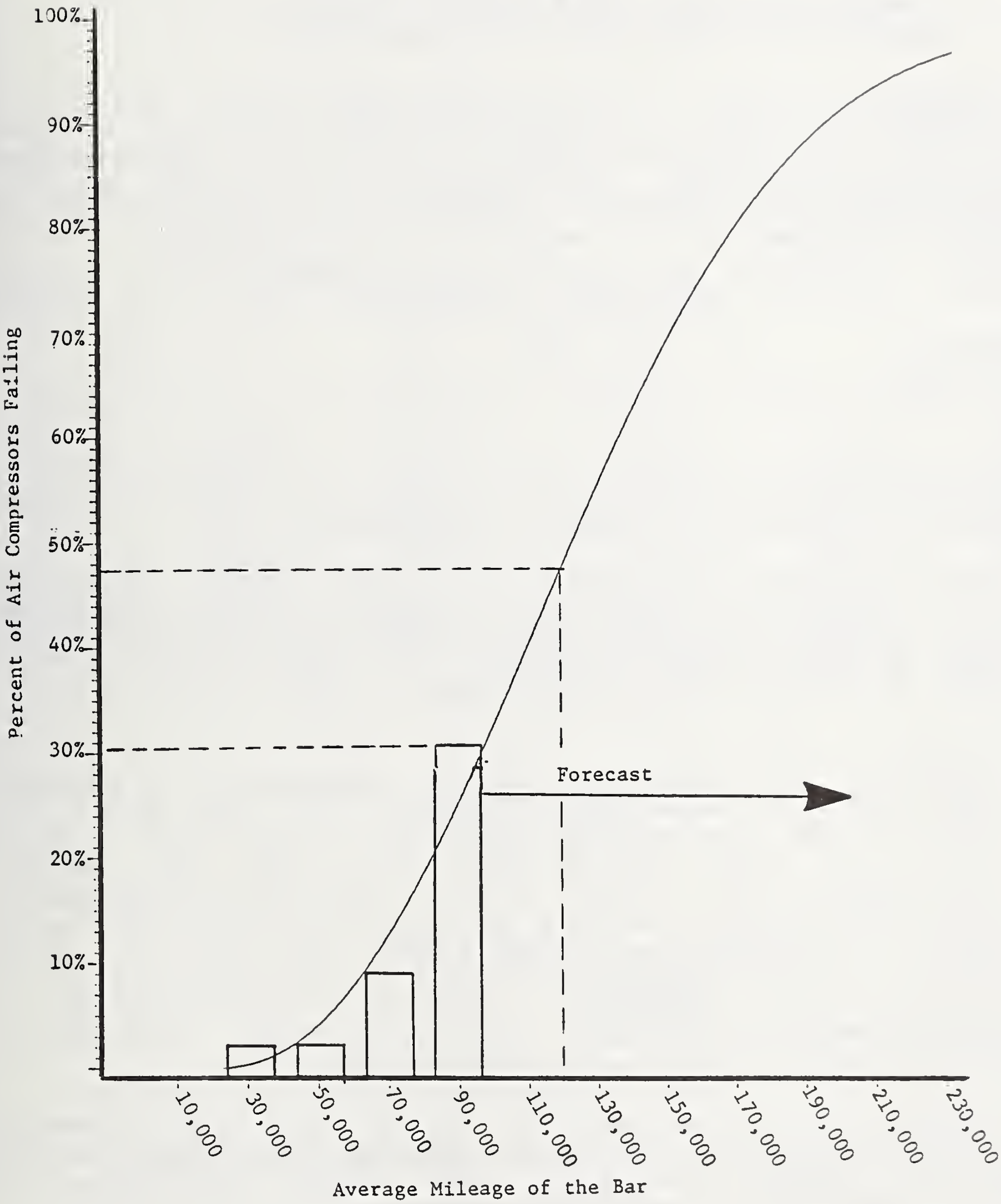


FIGURE 4.1

CUMULATIVE BAR CHART OF AIR COMPRESSOR FAILURES

WITH A CUMULATIVE WEIBULL CURVE

$$\text{cumulative probability} = 1 - e^{\left(-\left[\frac{M - D}{T}\right]^{B}\right)}$$

Suppose one wants to estimate how many of the 36 air compressors are likely to fail before 120,000 miles. As determined in the previous section: D = 20,315, T = 121,188, and B = 2.2368. Thus, the proportion (cumulative probability) of air compressors expected to fail before 120,000 miles is:

$$\text{cumulative probability} = 1 - e^{\left(-\left[\frac{120{,}000 - 20{,}315}{121{,}188}\right]^{2.2368}\right)} = 0.4759$$

An explanation and pictures of how to work out this formula with a scientific calculator is shown in Appendix B.

From the above calculation it is thus expected that 47.59 percent, or about half, of the air compressors will fail by the time they accumulate 120,000 miles. Thus, it is predicted that 17 of the original 36 air compressors (36 times 0.4759 = 17.13) will have failed by the time the buses reach 120,000 miles of use.

The reliability curve for the air compressors is computed from the cumulative percentages of Table 4.16 (Work Sheet 11). Cumulative mileage intervals of 30,000 miles were used in Table 4.16, starting with the minimum life, D = 20,315 miles, and extending to greater than 240,000 miles. The cumulative probabilities in column 2 are computed by using the above cumulative probability formula for the fitted Weibull distribution. These are converted to percentages in column 3. The reliability, or percent surviving, is computed in column 4 by subtracting the cumulative percentages of column 3 from 100 percent. The resulting reliability curve is depicted in Figure 4.2.

Finally, the reliabilities of column 4 are used to compute the predicted number of air compressors failing in each mileage interval in column 6 of Table 4.16. These predictions are compared to the actual frequencies below:

Cumulative Mileage	Actual Frequency	Predicted Frequency
0		
30,000	1	0.1
60,000	0	2.7
90,000	5	6.2

TABLE 4.16

WORK SHEET 11

COMPONENT RELIABILITY ANALYSIS FOR WEIBULL FAILURE DATA

Cost Driver Air Compressors Bus Model Grumman 870

Component Type ______ Study Dates ______

Cumulative Mileage, M (1)	Cumulative Probability of Failure[1] (2)	Cumulative Percentage[2] (3)	Reliability,[3] R (% Surviving) (4)	Percent Failing in Mileage Interval[4] (5)	Number Failing in Mileage Interval[5] (6)
20,315	0.0	0.0 %	100.0 %	%	
30,000	0.0035	0.4 %	99.6 %	0.4 %	0.1
60,000	0.0790	7.9 %	92.1 %	7.5 %	2.7
90,000	0.2518	25.2 %	74.8 %	17.3 %	6.2
120,000	0.4759	47.6 %	52.4 %	22.4 %	8.1
150,000	0.6877	68.8 %	31.2 %	21.2 %	7.6
180,000	0.8433	84.3 %	15.7 %	15.5 %	5.6
210,000	0.9344	93.4 %	6.6 %	9.1 %	3.3
240,000	0.9772	97.7 %	2.3 %	4.3 %	1.5
>240,000	1.0000	100.0 %	0.0 %	2.3 %	0.8
		%	%	%	———
		%	%	%	35.9
		%	%	%	
		%	%	%	
		%	%	%	

[1] Cumulative Probability

$$= 1 - e^{\left(- \left[\frac{M - D}{T}\right]^{B}\right)}$$

$$= 1 - e^{\left(- \left[\frac{M - 20{,}315}{121{,}188}\right]^{2.2368}\right)}$$

[2] The number in column 2 times 100.

[3] 100 percent less the number in column 3.

[4] Percent failing in mileage interval

$= R_1 - R_2$, where: R_1 = reliability (column 4) at previous cumulative mileage
R_2 = reliability at mileage in column 1

[5] The total number of failures, N, times the decimal percentage in column 5.

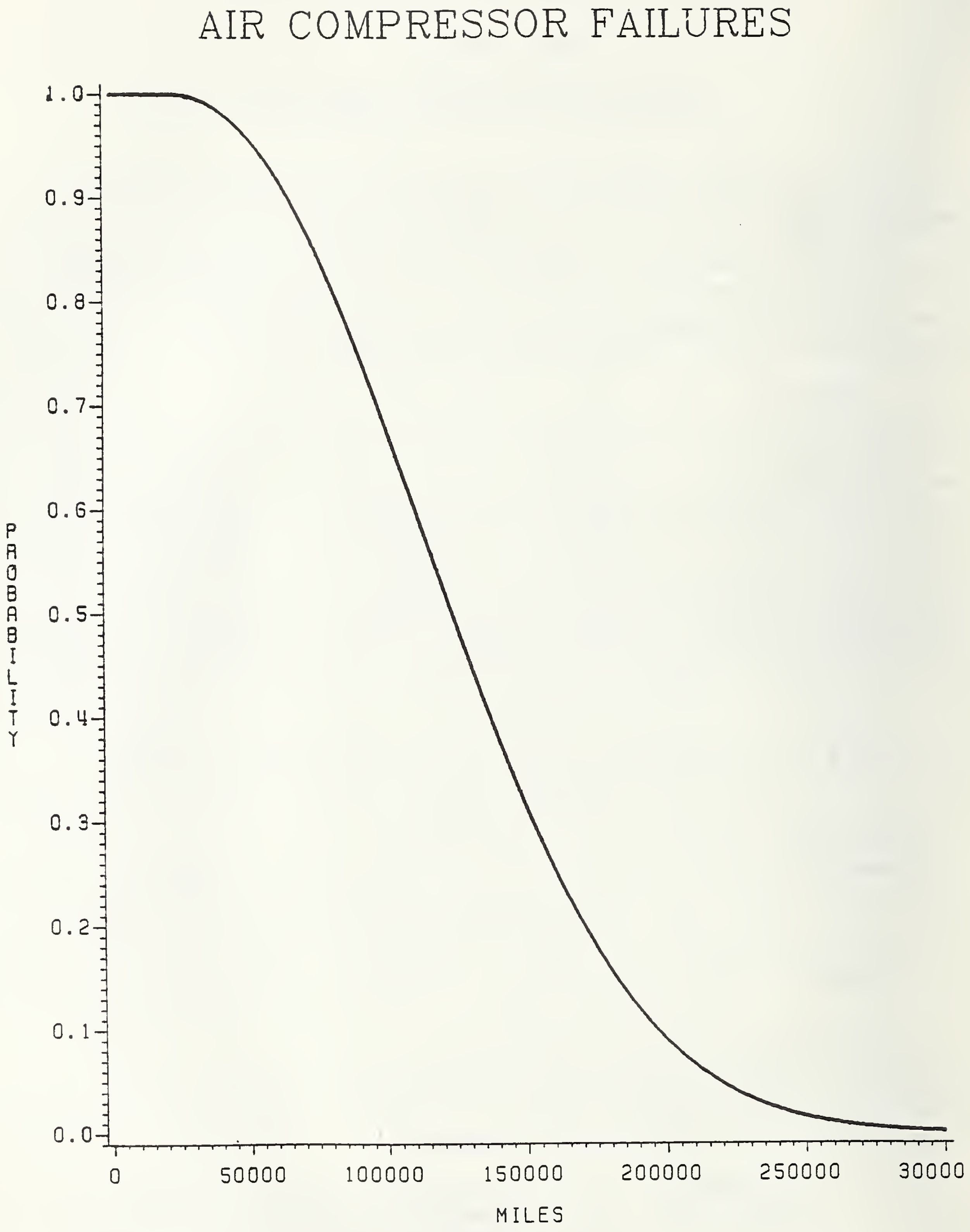


FIGURE 4.2

AIR COMPRESSOR RELIABILITY CURVE

The fitted Weibull distribution seems to be satisfactory and can be used to predict future reliability patterns, as demonstrated in the next section.

How Many Air Compressors Will Fail During Any Time Period?

If a fleet of 60 new buses is purchased which will average 30,000 annual miles per bus, then based on the information from the previous section it is predicted that about 29 of the new air compressors (60 times 0.4759 = 28.55) will have failed by the time the buses reach an average mileage of 120,000, which will occur at the end of the fourth year. Now suppose that one wants to determine how many of the original air compressors will fail during each year. The predicted percent failing in each 30,000 miles (or one year) interval up to 240,000 miles per bus, or eight years has already been computed in column 5 of Table 4.16. The number of original air compressor failures is found by multiplying these percentages by 60. For example, in the interval between the 90,000 and 120,000 miles per bus (between the third and fourth years) 13 original air compressors are expected to fail (0.224 times 60 = 13.4). All but one of them are predicted to have failed after eight years or 240,000 miles of use.

What About the Replacement Air Compressors?

Table 4.16 predicts the number of first time failures that can be expected each year but it does not account for the failures of the replacement air compressors. A satisfactory estimate of the total number of failures (including failures of the replacements) to expect during any period can be obtained as follows. Take the number of miles a bus will travel during the period, divide it by the estimated mean mileage until failure, and then multiply the result by the total number of buses. It is assumed that the replacement air compressors last as long as the originals.

To find the total number of failures (original and repeat replacements) during one year, first divide the total number of miles accumulated in a year (30,000 miles per bus per year) by the mean mileage until failure (127,675). The result is 0.2350 (30,000 divided by 127,675 = 0.2350). Then multiply the number of buses by 0.2350 to get the number of air compressors that are expected to fail during each year. If there are 60 buses, then about 14 are expected to fail each year (60 x 0.2350 = 14.1).

When the 60 new buses are first put into service, the air compressor failure rate (including repeats) will be low. Eventually the failure rate will increase and should stabilize at about 14 per year. A satisfactory estimate of how long it will take the rate to reach 14 per year is the mean mileage life of the air compressors converted into time. In this example, the mean life of the air compressors is predicted to be 127,675 miles. Therefore, the failure rate will reach 14 per year after about 4.25 years (127,675 divided by 30,000 = 4.2558).

The increase from zero failures per year to 14 per year is shown in Figure 4.3 as a straight line, which is good enough for estimating the annual failures until the fourth year. This graph can be used to predict the demand for air compressor repairs at any time during the bus life cycle for the new bus fleet.

Finally, the area under the graph between two points in time is an estimate of the total number of failed air compressors during that time interval. This information is useful for estimating annual maintenance costs and air compressor inventory requirements.

List of References

Bain, L.J., Statistical Analysis of Realibility and Life Test Models, Marcel Dekker, Inc., New York, New York, 1978.

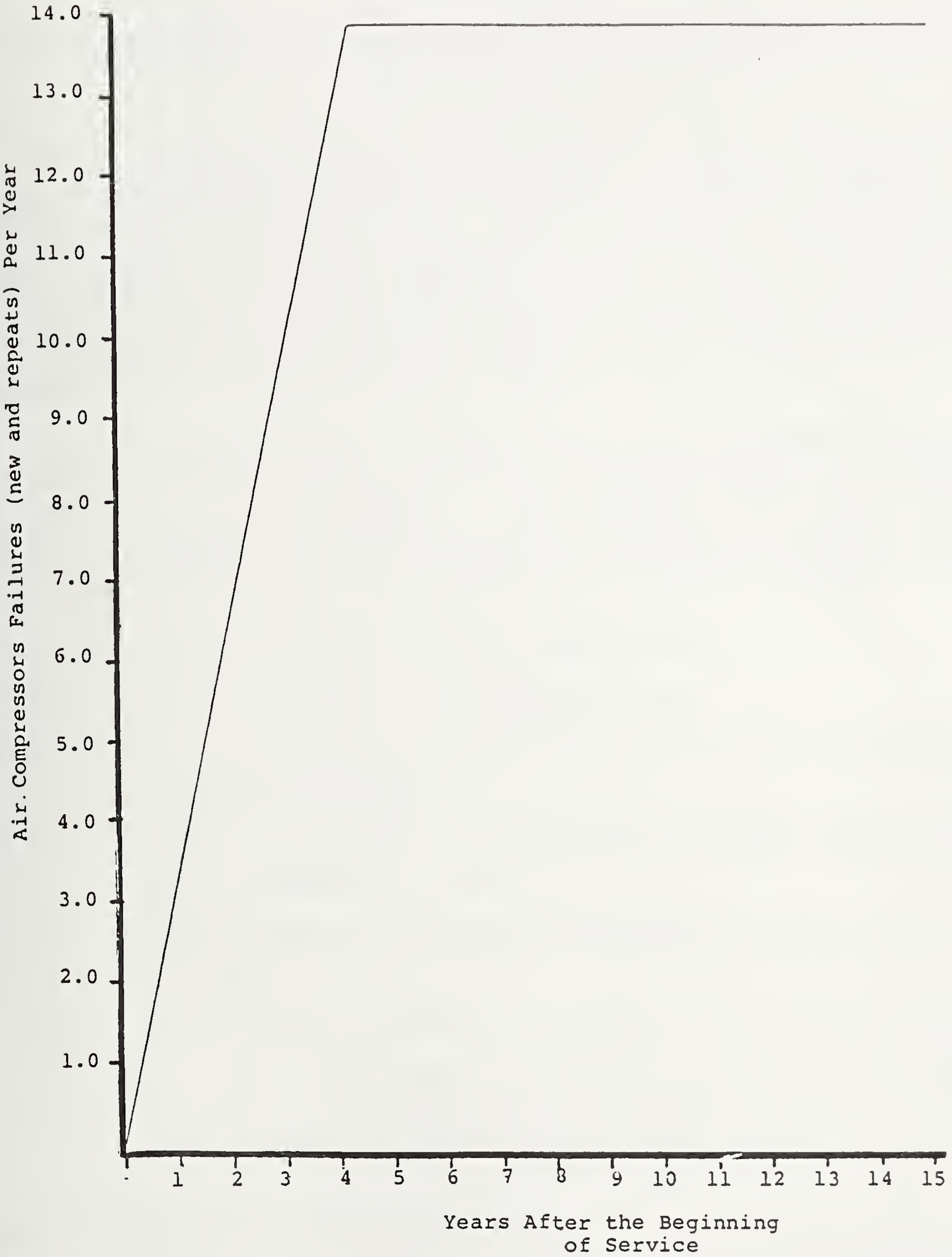


FIGURE 4.3

TOTAL PREDICTED AIR COMPRESSOR FAILURES PER YEAR DURING THE SERVICE LIFE OF THE BUSES

CHAPTER FIVE

WEIBULL DISTRIBUTION FAILURE ANALYSIS WITH SUSPENDED SAMPLING: GENERATOR EXAMPLE

This chapter is a self-contained analysis of original equipment bus generator failures in order to predict future failure patterns. The concept of "suspended sampling" is introduced and incorporated in the analysis. What should be done if the bus fleet includes original components which haven't failed yet, in part because they haven't accumulated as many miles of service as components which have failed? Should the analyst wait until all the components in the fleet have failed? Should the unfailed components be dropped from the study?

These unfailed cases should be included because the accuracy of the failure forecasts is improved by including as many buses as possible in the analysis. This chapter shows how to include the mileages of the unfailed bus components in the analysis. These bus components are treated as if they had been "suspended" from service and then are handled in a special manner.

The topics covered in this chapter are:

- How to collect the generator data.
- How to tabulate the generator data and account for the suspended cases.
- Fitting the Weibull distribution with suspended cases to the generator data.
- What is the appropriate maintenance policy for the generators?
- Predicting the future reliability of the generators.
- How many generators will fail during any time period?
- What about the replacement generators?

How To Collect the Generator Data

A bus generator has failed when it must be replaced with a new or rebuilt generator. This analysis assumes that the buses in the fleet under study all have experienced similar mileages, duty cycles, and maintenance. As with transmissions, it is assumed that none of the generators have been abused such as to bring about premature failure. Such cases should be removed from the analysis.

Ideally, all of the original generators in the fleet of buses under study should have failed. However, suppose that some working generators have accumulated fewer miles than at least one of the failed generators. If this is the case they cannot be treated as censored cases. Cases where the bus has a working component which has accumulated fewer miles than other buses with failed components are called "suspended cases".

Suspended cases may happen due to chance or perhaps for the following reasons:

1. By mistake, a failure mileage was not recorded and the only mileage available is a lower figure when the component was last known to be functioning.
2. Buses of the same model have accumulated different mileages. This might be because some of the buses were delivered months after the others or because of chance differences in use (e.g., a bus was out of service for a time because of an accident). Therefore, some buses without a failed component may have received fewer miles than other buses with failed components.

There should be at least 10 component failures for the following analysis and at least half of all the components should have failed. This chapter considers, as an example, original equipment generators from 22 new buses. The mileages accumulated by each bus at the time of the study are presented in Table 5.1 (Work Sheet 7). There are several suspended cases in Table 5.1. Three buses, 8103, 8111, and 8119, still have functioning original generators but each has accumulated less mileage than other buses which have experienced failures (e.g., 8104, 8105).

The generator on bus 8112 has failed but the mileage accumulated on the bus was not recorded on the repair work order. The original generator was still functioning at the 72,000 mileage inspection. Therefore, bus 8112 is considered "suspended" at 72,000 miles.

In summary, the analysis of component failures with suspended samples must meet the following prerequisites:

1. All buses should be the same type vehicles and they should have experienced similar operating conditions.
2. There should be at least 10 component failures.
3. No buses with a surviving component should have accumulated more miles than the failure with the greatest mileage.
4. No more than 50 percent of the data cases should be suspended.
5. Do not include failure data for replacement generators.

TABLE 5.1

WORK SHEET 7

Page 1 of 2

FAILURE STATUS OF ORIGINAL COMPONENTS IN WEIBULL FAILURE ANALYSIS

Cost Driver Generators Bus Model

Component Type Study Dates

Bus Number (1)	Component Number (2)	Current Bus Mileage (3)	Original Component Failure Mileage (4)	Status of Unfailed Component*		Comments (7)
				Censor (5)	Susp. (6)	
8100		114,707	93,604			
8101		101,234	67,807			
8102		112,926	99,126			
8103		122,963			X	
8104		150,474	150,474			
8105		142,948	138,547			
8106		121,912	101,520			
8107		121,462	111,676			
8108		105,427	94,260			
8109		131,242	62,807			
8110		126,667	47,619			
8111		132,372			X	
8112		82,544			X	Failure unrecorded
8113		92,838	89,196			
8114		134,173	107,196			
8115		139,761	135,338			
8116		127,382	59,056			
8117		124,961	35,397			
8118		108,021	47,488			
8119		107,279			X	

* CENSORED cases refer to surviving components which have accumulated more miles than any failed component in the data set.

SUSPENDED cases refer to surviving components which have accumulated less miles than at least one of the failed components.

TABLE 5.1 (continued)

WORK SHEET 7

Page 2 of 2

FAILURE STATUS OF ORIGINAL COMPONENTS
IN WEIBULL FAILURE ANALYSIS

Cost Driver Generators

Bus Model ____________

Component Type ____________

Study Dates ____________

Bus Number (1)	Component Number (2)	Current Bus Mileage (3)	Original Component Failure Mileage (4)	Status of Unfailed Component*		Comments (7)
				Censor (5)	Susp. (6)	
8120		96,719	61,113			
8121		105,934	42,183			

* CENSORED cases refer to surviving components which have accumulated more miles than any failed component in the data set.

SUSPENDED cases refer to surviving components which have accumulated less miles than at least one of the failed components.

The replacement on bus 8117, installed when the bus had recorded 35,397 miles, failed at 121,772 miles and this bus now has its third generator. Including this replacement would distort the results of the analysis.

How To Tabulate the Generator Data and Account for the Suspended Cases

Fitting generator failure mileage data which contains suspended cases to the Weibull distribution is similar to but not quite the same as the procedures used in Chapters Three and Four where the data set consisted entirely of failure mileages or just failures and censored cases.

The tabulation of the data with suspended cases begins by rearranging the bus mileages in Table 5.1 in ascending order by mileage. This is done in Table 5.2 (Work Sheet 8) with the order number indicated in column 1. The suspended cases are noted in column 4. The buses with generator failures are ordered according to their mileage at failure and the suspended cases are ordered with regard to their suspended mileage. For example, bus 8119 still had a functioning generator at 107,279 miles, and it is ranked 16th in Table 5.2. The bus with the unknown failure mileage, number 8112, is ordered as number 9 based on the last known functioning mileage, 72,000. Finally, note that the suspended cases are mixed in with the failed cases and that, by definition, the last bus in order in Table 5.2 is a failed case.

The next step in tabulating the data is to calculate "adjusted order numbers" using Table 5.3 (Work Sheet 14). The information in columns 1, 2, 3, and 4 is taken from Table 5.2 (Work Sheet 8). In column 5 of Table 5.3 the original order numbers of column 4 are reversed. In this example there are 22 cases, hence, the reverse order numbers listed in column 5 start at 22 for bus 8117 and decrease to 1 for bus 8104.

The next step is to calculate a "weight" for each bus. When calculating the weights the suspended cases are skipped. The weight equation is*:

$$\text{weight} = \frac{(\text{number of buses} + 1) - (\text{previous adjusted order number})}{1 + (\text{reverse order number})}$$

where the adjusted order number, explained shortly, is found in column 8.

As an example, for the first bus, number 8117:

22 = The total number of buses,
0 = the previous adjusted order number (it equals zero because it is the first case),
22 = the reverse order from column 5.

---

* The weighting system was developed by Johnson (1).

TABLE 5.2

WORK SHEET 8 Page 1 of 2

RANK ORDERING OF SURVIVING AND FAILED COMPONENTS
IN WEIBULL FAILURE ANALYSIS

Cost Driver Generators Bus Model

Component Type Study Dates

Order Number (1)	Bus Number (2)	Component Number (3)	Failed, Suspended, or Censored (4)	Mileage When Failed, Suspended, or Censored* (5)
1	8117		Failed	35,391
2	8121		Failed	42,183
3	8118		Failed	47,488
4	8110		Failed	47,619
5	8116		Failed	59,506
6	8120		Failed	61,113
7	8109		Failed	62,807
8	8101		Failed	67,807
9	8112		Suspended	72,000
10	8113		Failed	89,196
11	8101		Failed	93,604
12	8108		Failed	94,260
13	8102		Failed	99,126
14	8106		Failed	101,520
15	8114		Failed	107,193
16	8119		Suspended	107,279
17	8107		Failed	111,676
18	8103		Suspended	122,963
19	8111		Suspended	132,372
20	8115		Failed	135,338

* Each component is listed in ascending order of the mileage entered in Column 5.

TABLE 5.2 (continued)

WORK SHEET 8

Page 2 of 2

RANK ORDERING OF SURVIVING AND FAILED COMPONENTS IN WEIBULL FAILURE ANALYSIS

Cost Driver Generators

Bus Model

Component Type

Study Dates

Order Number (1)	Bus Number (2)	Component Number (3)	Failed, Suspended, or Censored (4)	Mileage When Failed, Suspended, or Censored* (5)
~~1~~ 21	8105		Failed	138,549
~~2~~ 22	8104		Failed	150,474
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

* Each component is listed in ascending order of the mileage entered in Column 5.

TABLE 5.3

WORK SHEET 14

Page 1 of 3

COMPUTING WEIGHTS AND ADJUSTED ORDER NUMBERS FOR WEIBULL FAILURE ANALYSIS WHEN CASES INCLUDE SUSPENDED CASES

Bus Number (1)	Failed, Suspended, or Censored (2)	Mileage When Failed, Suspended, or Censored (3)	Original Order Number (4)	Reverse Order Number (5)	Weight Calculation (6)	Weight (7)	Adjusted Order Number (8)
8117	Failed	35,391	1	22	$\frac{(22+1) - 0}{1 + 22}$	1 =	1
8121	Failed	42,183	2	21	$\frac{(22+1) - 1}{1 + 21}$	1 =	2
8118	Failed	47,488	3	20	$\frac{(22+1) - 2}{1 + 20}$	1 =	3
8110	Failed	47,619	4	19	$\frac{(22+1) - 3}{1 + 19}$	1 =	4
8116	Failed	59,506	5	18	$\frac{(22+1) - 4}{1 + 18}$	1 =	5
8120	Failed	61,113	6	17	$\frac{(22+1) - 5}{1 + 17}$	1 =	6
8109	Failed	62,807	7	16	$\frac{(22+1) - 6}{1 + 16}$	1 =	7
8101	Failed	67,807	8	15	$\frac{(22+1) - 7}{1 + 15}$	1 =	8
8112	Suspended	72,000	9	14		=	
8113	Failed	89,196	10	13	$\frac{(22+1) - 8}{1 + 13}$	1.0714=	9.0714

TABLE 5.3 (continued)

WORK SHEET 14

Page 2 of 3

COMPUTING WEIGHTS AND ADJUSTED ORDER NUMBERS FOR WEIBULL FAILURE ANALYSIS WHEN CASES INCLUDE SUSPENDED CASES

Bus Number (1)	Failed, Suspended, or Censored (2)	Mileage When Failed, Suspended, or Censored (3)	Original Order Number (4)	Reverse Order Number (5)	Weight Calculation (6)	Weight (7)	Adjusted Order Number (8)
8101	Failed	93,604	11	12	$\frac{(22+1)-9.0714}{1+12}$	1.0714 =	10.1428
8108	Failed	94,260	12	11	$\frac{(22+1)-10.1428}{1+11}$	1.0714 =	11.2142
8102	Failed	99,126	13	10	$\frac{(22+1)-11.2142}{1+10}$	1.0714 =	12.2856
8106	Failed	101,520	14	9	$\frac{(22+1)-12.2856}{1+9}$	1.0714 =	13.3570
8114	Failed	107,193	15	8	$\frac{(22+1)-13.3570}{1+8}$	1.0714 =	14.4284
8119	Suspended	107,279	16	7		=	
8107	Failed	111,676	17	6	$\frac{(22+1)-14.4284}{1+6}$	1.2245 =	15.6529
8103	Suspended	122,963	18	5		=	
8111	Suspended	132,372	19	4		=	
8115	Failed	135,338	20	3	$\frac{(22+1)-15.6529}{1+3}$	1.8368 =	17.4897

TABLE 5.3 (continued)

WORK SHEET 14

Page 3 of 3

COMPUTING WEIGHTS AND ADJUSTED ORDER NUMBERS FOR WEIBULL FAILURE ANALYSIS WHEN CASES INCLUDE SUSPENDED CASES

Bus Number (1)	Failed, Suspended, or Censored (2)	Mileage When Failed, Suspended, or Censored (3)	Original Order Number (4)	Reverse Order Number (5)	Weight Calculation (6)	Weight (7)	Adjusted Order Number (8)
8105	Failed	138,549	21	2	$\frac{(22+1)-17.4897}{1+2}$	1.8368 =	19.2365
8104	Failed	150,474	22	1	$\frac{(22+1)-19.2365}{1+1}$	1.8368 =	21.0733
						=	
						=	
						=	
						=	
						=	
						=	
						=	
						=	

$$\text{weight} = \frac{(22 + 1) - 0}{1 + 22} = 1$$

There is room for this computation in column 6 of Table 5.3. The weight is entered in column 7.

The next computation is for the adjusted order number of the bus. The adjusted order number, tabulated in column 8, is the sum of all the preceding weights from column 7. Thus, for bus 8117 the adjusted order number is the weight, 1, since it comes first.

For the second bus in Table 5.3, number 8121, the weight is:

$$\frac{(22 + 1) - 1}{1 + 21} = 1$$

The adjusted order number for bus 8121 is this weight, 1, plus the adjusted order number of bus 8117 (the preceding case), 1. The sum is 2, which is entered in column 8. Similar computations follow for each failure case until a suspended case is encountered.

The first suspended bus is number 8112, which is number 14 in reverse order. Weights and adjusted order numbers are not computed for suspended cases. Therefore, skip to the next bus, number 8113, which is 13th in reverse order. For bus 8113:

22 = the total number of buses
8 = the previous adjusted order number (for bus 8101)
13 = the reverse order

$$\text{weight} = \frac{(22 + 1) - 8}{1 + 13} = 1.0714$$

The adjusted order number continues to be the sum of the weights to that point. The adjusted order number for bus 8113 is 8 (the adjusted order number for bus 8101 since suspended bus 8112 has no order) plus 1.0714: 8 + 1.0714 = 9.0714. Note that for the first time in the tabulation the adjusted order number differs from the original order number, 10, for bus 8113.

As a fourth example, skip down to the 16th bus (number 8107) in order in Table 5.3, which also follows a suspended case. For bus 8107:

22 = the total number of buses
14.4284 = the previous adjusted order number
6 = the reverse order

$$\text{weight} = \frac{(22 + 1) - 14.4284}{1 + 6} = 1.2245$$

The adjusted order number for bus 8107 is: 14.4284 + 1.2245 = 15.6529.

This process of calculating the adjusted order numbers includes input from the suspended cases. Therefore, even though it is not known when the suspended samples will eventually fail, they are included in the analysis. This information can now be used to derive a Weibull failure rate distribution which includes the influence of the missing failures.

Fitting the Weibull Distribution with Suspended Cases to the Generator Data

Generator failures are modeled similar to any complex component which fails rather than wears out. Therefore, just as transmissions and air compressors were modeled using the ski-slope-shaped Weibull distribution in Chapters Three and Four, so too are generators. Normally, the ski-slope characteristic of generator failure mileages (some premature failures, most failing around an average mileage and some proving very durable) could be confirmed by plotting a bar chart of the failure mileages. However, if the data set includes suspended cases, which could be as many as half of the total number of buses, then a ski slope pattern may not be evident.

It is expected, however, that if the failure data were complete, the distribution of failure mileages would be ski-sloped. Therefore, a Weibull distribution is selected to fit the data without first drawing a bar chart of failure mileages.

Calculating D, B, and T. The Weibull curve that best fits the original generator data is determined by calculating the three Weibull constants, D, B, and T. However, different tables than in Chapters Three and Four are used when suspended sampling is involved. The analysis begins by listing in Table 5.4 (Work Sheet 9) the buses with generator failures in order of increasing failure mileage. The suspended buses are omitted from Table 5.4.

The first constant is D and it is called the "minimum life term." Should a component fail without having accumulating any miles, the "minimum life" would be zero. As before, D is estimated to be 90 percent of the lowest failure mileage. In Table 5.4 the lowest failure mileage is 35,391 for bus 8117. Therefore, D equals 31,852 miles (35,391 x 0.90 = 31,852). There is space for this computation at the bottom of the Work Sheet.

The other two constants are B and T. B, the "shape factor", determines the shape of the Weibull curve (a measurement of how bunched together or spread out the distribution is) and T, the

TABLE 5.4

WORK SHEET 9

COMPUTATION OF D, K, AND S FOR WEIBULL FAILURE DATA

Cost Driver Generators Bus Model __________

Component Type __________ Study Dates __________

Bus Number (1)	Failure Mileage (2)	Minimum Life, D* (3)	K (4)	$\ln(K_{max})$** (5)	$\ln(K)$ (6)	L*** (7)
8117	35,391 -	31,852 =	3,539	11.6837 -	8.1716 =	3.5121
8121	42,183 -	31,852 =	10,331	11.6837 -	9.2429 =	2.4408
8118	47,488 -	31,852 =	15,636	11.6837 -	9.6573 =	2.0264
8110	47,619 -	31,852 =	15,767	11.6837 -	9.6657 =	2.0180
8116	59,056 -	31,852 =	27,204	11.6837 -	10.2111 =	1.4726
8120	61,113 -	31,852 =	29,261	11.6837 -	10.2840 =	1.3997
8109	62,807 -	31,852 =	30,995	11.6837 -	10.3403 =	1.3434
8101	67,807 -	31,852 =	35,955	11.6837 -	10.4900 =	1.1937
8113	89,196 -	31,852 =	57,344	11.6837 -	10.9568 =	0.7269
8100	93,604 -	31,852 =	61,752	11.6837 -	11.0309 =	0.6528
8108	94,260 -	31,852 =	62,408	11.6837 -	11.0414 =	0.6423
8102	99,126 -	31,852 =	67,274	11.6837 -	11.1165 =	0.5672
8106	101,520 -	31,852 =	69,668	11.6837 -	11.1515 =	0.5322
8114	107,193 -	31,852 =	75,341	11.6837 -	11.2298 =	0.4539
8107	117,676 -	31,852 =	85,824	11.6837 -	11.3600 =	0.3237
8115	135,338 -	31,852 =	103,486	11.6837 -	11.5472 =	0.1365
8105	138,549 -	31,852 =	106,697	11.6837 -	11.5777 =	0.1060
8104	150,474 -	31,852 =	118,622	11.6837 -	11.6837 =	0.0
	-	=			-	=
	-	=			-	=
Sums:		ΣK =				S =

* Minimum life term, D = 0.90 x lowest failure mileage
= 0.90 x 35,391 mi. = D = 31,852 miles

** $\ln(K_{max})$ = ln(118,622) = 11.6837 , where K_{max} is the largest value of K in column 4.

*** If the data set contains suspended failure cases, the L terms must be modified in Work Sheet 10a before computing S.

"characteristic life factor", determines how far away the peak in the curve should be from zero miles. Before calculating B and T, three intermediate numbers, K, L, and S, must be calculated. These calculations also are shown in Table 5.4.

The minimum life term, D = 31,852 miles, is subtracted from each original failure mileage in Table 5.4 (column 2 minus column 3) to get column 4, which is a value called K for each failed generator. The largest value of K, for bus 8104 in the last row, is labeled K_{max}.

The K values are then used to calculate the values for L in column 7. First, compute the natural logarithm for K_{max}, 118,622 miles. Obtaining the natural logarithm is simple when using a scientific calculator. Enter the number, 118,622, and press the button marked "ln". The natural logarithm, 11.6837, will appear on the display.* The natural logarithms are rounded-off to four digits to the right of the decimal point for simplicity (11.683697 becomes 11.6837) in Table 5.4. This number is placed on every line of column 5 of Table 5.4.

Next, the natural logarithm of each value of K, ln (K), is written in column 6. For example, in the first line of Table 5.4 (bus 8117), ln (3,539) equals 8.1716. The difference of the numbers in columns 5 and 6 is L and this is written in the column 7.

Here, the computation for S diverges from Chapters Three and Four since the failed component set includes suspended samples. The L values must be modified using the adjusted order numbers of Table 5.3. Table 5.5 (Work Sheet 9a) is provided for these computations.

First, columns 1, 2, and 3 are filled in using the information from Tables 5.3 and 5.4. In column 4 the "row number" is already entered, starting with 1. There are 18 failures in this example, hence 18 rows. If there are more than 20 failure cases, then succeeding pages of Work Sheet 9a should re-number the rows as 21 to 40, 41 to 60, and so forth. There is room to do this in column 4.

Next, the adjusted order number in column 3 is divided by the row number in column 4 to get a ratio which is entered in column 5. In the final step re-enter the values of L from column 2 into column 6. Multiply the ratio by L (column 5 multipled by column 6) and enter the result in column 7. Sum all of the numbers in column 12. This sum is S, which is 19.6584.

Now it is possible to compute the two remaining Weibull con-

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* For a discussion of the use of scientific calculators see Appendix B.

TABLE 5.5

WORK SHEET 9a

MODIFICATION OF L FACTORS TO COMPUTE S FOR WEIBULL FAILURE DATA WITH SUSPENDED CASES

Cost Driver Generators Bus Model__________

Component Type__________ Study Dates__________

Bus Number (1)	L^* (2)	Adjusted Order No. (3)	Row Number (4)	Ratio (5)	L (6)	Modified L (7)
8117	3.5121	1	/ 1	= 1	x 3.5121	= 3.5121
8121	2.4408	2	/ 2	= 1	x 2.4408	= 2.4408
8118	2.0264	3	/ 3	= 1	x 2.0264	= 2.0264
8110	2.0180	4	/ 4	= 1	x 2.0180	= 2.0180
8116	1.4726	5	/ 5	= 1	x 1.4726	= 1.4726
8120	1.3997	6	/ 6	= 1	x 1.3997	= 1.3997
8109	1.3434	7	/ 7	= 1	x 1.3434	= 1.3434
8101	1.1937	8	/ 8	= 1	x 1.1937	= 1.1937
8113	0.7269	9.0714	/ 9	= 1.0079	x 0.7269	= 0.7326
8100	0.6528	10.1428	/ 10	= 1.0143	x 0.6528	= 0.6621
8108	0.6423	11.2142	/ 11	= 1.0195	x 0.6423	= 0.6548
8102	0.5672	12.2856	/ 12	= 1.0238	x 0.5672	= 0.5807
8106	0.5322	13.3570	/ 13	= 1.0275	x 0.5322	= 0.5468
8114	0.4539	14.4284	/ 14	= 1.0306	x 0.4539	= 0.4678
8107	0.3237	15.6529	/ 15	= 1.0435	x 0.3237	= 0.3378
8115	0.1365	17.4897	/ 16	= 1.0931	x 0.1365	= 0.1492
8105	0.1060	19.2365	/ 17	= 1.1316	x 0.1060	= 0.1199
8104	0.0	21.0733	/ 18	= 1.1707	x 0.0	= 0.0
			/ 19	=	x	=
			/ 20	=	x	=
Sums:						S = 19.6584

* From Work Sheet 9, Column 7.

stants, B and T. Table 5.6 (Work Sheet 15) outlines the computations for B and T and provides space for each intermediate result and the final answers.

To get B requires the use of Tables 5.7, 5.8 and 5.9. In Table 5.7 a value of F1 is found which corresponds to the total number of buses in the study, N = 22, including the suspended cases. F1 for a fleet size of 22 is 1.830. Similarly, F2 and F3 for 22 buses are found in Tables 5.8 and 5.9 and equal -0.033 and -0.0039.

The values for F1, F2 and F3 are then plugged into the following equation to find M:

$$M = F1 + F2 \times NS + F3 \times (NS)^2$$

For the generator example, there are four suspended cases. Hence, NS = 4 and:

$$M = 1.830 - 0.033(4) - 0.0039(4)^2 = 1.636$$

To get B insert M and S (from Table 5.5) into the equation below:

$$B = \frac{M \times N}{S}$$

For the generator example:

$$B = \frac{1.636 \times 22}{19.6584} = 1.831$$

There is room for the computation of M and B in Table 5.6 (Work Sheet 15).

Finally, the remaining Weibull constant, T, is found by using Tables 5.10 and 5.11. First, look up the constant G1 in Table 5.10 which corresponds to N = 22, which is 1.214. From Table 5.11, G2 is similarly found to be -0.157. The values for G1 and G2 are then plugged into the following equation:

$$P = G1 + G2 \times NS$$

For the generator example:

$$P = 1.294 - 0.157(4) = 0.666$$

To get T, enter P, B, K_{max} and its corresponding ratio value (from row 18, column 5 in Table 5.5), into the equation below:

$$\ln(T) = \ln\left(\frac{K_{max}}{\text{ratio}}\right) - \frac{P}{B}$$

TABLE 5.6

WORK SHEET 15

COMPUTATION OF B AND T FOR WEIBULL FAILURE DATA WHICH CONTAINS SUSPENDED CASES

Cost Driver Generators Bus Model ________

Component Type ________ Study Dates ________

INPUT NUMBERS:

F1 = 1.830 (from Table 5.7)

G1 = 1.294 (from Table 5.10)

F2 = - 0.033 (from Table 5.8)

G2 = 0.156 (from Table 5.11)

F3 = - 0.0039 (from Table 5.9)

N = 22 (total number of buses)

NS = 4 (total number of suspended cases)

S = 19.6584 (from Work Sheet 9a)

ratio = 1.1707 (from Work Sheet 9a, the ratio in column 5 corresponding to K_{max})

K_{max} = 118,622 miles (from Work Sheet 9)

SHAPE FACTOR, B

$$M = \underset{F1}{\underline{1.830}} + \underset{F2}{\underline{-\ 0.033}} \times \underset{NS}{\underline{4}} + \underset{F3}{\underline{-0.0039}} \times \left(\underset{NS}{\underline{4}}\right)^2$$

$$M = \underline{1.636}$$

$$B = \left(\underset{M}{\underline{1.636}} \times \underset{N}{\underline{22}}\right) / \underset{S}{\underline{19.6584}} = B = \underline{1.831}$$

CHARACTERISTIC LIFE FACTOR, T:

$$P = \underset{G1}{\underline{1.294}} - \underset{G2}{\underline{-0.157}} \times \underset{NS}{\underline{4}} = P = \underline{0.666}$$

$$\ln(T) = \ln\left(\underset{K_{max}}{\underline{118{,}622}} / \underset{ratio}{\underline{1.1707}}\right) - \underset{P}{\underline{0.666}} / \underset{B}{\underline{1.831}}$$

$\ln(T) = \underline{11.1624}$, $T = \underline{70{,}432}$ ("inverse" of ln(T) on a scientific calculator)

TABLE 5.7

F1 NUMBERS

N	0	1	2	3	4	5	6	7	8	9
10	1.568	1.607	1.642	1.666	1.678	1.713	1.724	1.748	1.763	1.791
20	1.794	1.815	1.830	1.834	1.853	1.856	1.864	1.882	1.891	1.906
30	1.910	1.922	1.929	1.936	1.942	1.951	1.964	1.968	1.971	1.982
40	1.985	1.992	1.994	2.006	2.014	2.020	2.025	2.038	2.032	2.036
50	2.043	2.048	2.050	2.053	2.056	2.060	2.065	2.070	2.073	2.076
60	2.080	2.084	2.088	2.091	2.095	2.099	2.102	2.107	2.112	2.114
70	2.116	2.118	2.120	2.122	2.125	2.129	2.132	2.135	2.138	2.140
80	2.142	2.144	2.146	2.150	2.152	2.154	2.156	2.159	2.163	2.165
90	2.167	2.169	2.171	2.173	2.175	2.176	2.178	2.180	2.182	2.185

Example: F1 for N = 22 is 1.830, which is the number in the second row ("20") of column 4 ("2").

TABLE 5.8

F2 NUMBERS

N	0	1	2	3	4	5	6	7	8	9
10	-0.073	-0.067	-0.062	-0.051	-0.047	-0.044	-0.038	-0.037	-0.036	-0.036
20	-0.035	-0.034	-0.033	-0.029	-0.028	-0.026	-0.025	-0.025	-0.024	-0.023
30	-0.022	-0.021	-0.020	-0.019	-0.018	-0.018	-0.017	-0.017	-0.016	-0.016
40	-0.016	-0.015	-0.015	-0.015	-0.015	-0.015	-0.015	-0.014	-0.014	-0.013
50	-0.013	-0.013	-0.013	-0.012	-0.012	-0.012	-0.012	-0.012	-0.011	-0.011
60	-0.011	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010
70	-0.010	-0.010	-0.009	-0.009	-0.009	-0.009	-0.008	-0.008	-0.008	-0.008
80	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008
90	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	-0.006	-0.006

Example: F2 for N = 22 is -0.033, which is the number in the second row ("20") of column 4 ("2").

TABLE 5.9

F3 NUMBERS

N	0	1	2	3	4	5	6	7	8	9
10	-0.0154	-0.0128	-0.0112	-0.0108	-0.0091	-0.0080	-0.0076	-0.0066	-0.0057	-0.0054
20	-0.0049	-0.0042	-0.0039	-0.0037	-0.0034	-0.0032	-0.0031	-0.0027	-0.0025	-0.0024
30	-0.0023	-0.0022	-0.0021	-0.0020	-0.0019	-0.0017	-0.0016	-0.0016	-0.0015	-0.0014
40	-0.0014	-0.0013	-0.0013	-0.0012	-0.0012	-0.0011	-0.0010	-0.0010	-0.0010	-0.0010
50	-0.0009	-0.0009	-0.0008	-0.0008	-0.0008	-0.0007	-0.0007	-0.0007	-0.0007	-0.0007
60	-0.0007	-0.0006	-0.0006	-0.0006	-0.0006	-0.0005	-0.0005	-0.0005	-0.0005	-0.0005
70	-0.0005	-0.0005	-0.0005	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004
80	-0.0004	-0.0004	-0.0004	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003
90	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003

Example: F3 for N = 22 is -0.0039, which is the number in the second row ("20") of column 4 ("2").

TABLE 5.10

G1 NUMBERS

N	0	1	2	3	4	5	6	7	8	9
10	1.006	1.050	1.069	1.117	1.140	1.163	1.178	1.212	1.223	1.238
20	1.259	1.282	1.294	1.305	1.317	1.342	1.347	1.360	1.367	1.381
30	1.390	1.410	1.406	1.427	1.424	1.434	1.444	1.454	1.456	1.470
40	1.467	1.478	1.485	1.498	1.503	1.508	1.514	1.516	1.523	1.523
50	1.533	1.541	1.543	1.548	1.546	1.561	1.560	1.565	1.571	1.576
60	1.577	1.584	1.585	1.592	1.595	1.597	1.601	1.607	1.609	1.612
70	1.617	1.620	1.625	1.629	1.630	1.636	1.637	1.642	1.643	1.647
80	1.648	1.651	1.654	1.655	1.657	1.660	1.664	1.668	1.670	1.673
90	1.676	1.678	1.680	1.681	1.683	1.685	1.688	1.690	1.693	1.696

Example: G1 for N=22 is 1.294, which is the number in the second row ("20") of column 4 ("2").

TABLE 5.11

G2 NUMBERS

N	0	1	2	3	4	5	6	7	8	9
10	-0.308	-0.290	-0.260	-0.253	-0.234	-0.222	-0.204	-0.198	-0.187	-0.180
20	-0.171	-0.166	-0.157	-0.152	-0.144	-0.142	-0.136	-0.132	-0.127	-0.124
30	-0.120	-0.118	-0.113	-0.111	-0.106	-0.104	-0.102	-0.099	-0.096	-0.095
40	-0.092	-.0090	-0.088	-0.086	-0.085	-0.083	-0.080	-0.078	-0.077	-0.075
50	-0.075	-0.074	-0.072	-0.071	-0.070	-0.069	-0.067	-0.066	-0.065	-0.064
60	-0.063	-.0062	-0.061	-0.060	-0.059	-0.059	-0.058	-0.057	-0.056	-0.055
70	-0.055	-0.054	-0.053	-0.053	-0.052	-0.051	-0.051	-0.050	-0.049	-0.049
80	-0.048	-0.048	-0.047	-0.046	-0.046	-0.046	-0.045	-0.044	-0.044	-0.044
90	-0.043	-0.043	-0.042	-0.042	-0.041	-0.041	-0.040	-0.040	-0.040	-0.039

Example: G2 for N=22 is -0.157, which is the number in the second row ("20") of column 4 ("2").

For the generator example:

$$\ln(T) = \ln\left(\frac{118{,}622}{1.1707}\right) - \frac{0.666}{1.831} = 11.1624$$

Hence, T = 70,432 (on a TI-35 calculator, press INV lnX for 11.1756). There is room for these computations in Table 5.6 (Work Sheet 15).

Mean and Standard Deviation. As in Chapter Four, the mean and standard deviation of the failed generator mileages cannot be directly computed because there are four suspended cases. In three of these cases the generators have not yet failed and in the fourth case the failure mileage is unknown (unrecorded). The estimated mean and standard deviation for all 22 generators is found by using the same technique as in Chapter Four.

Table 5.12 (Work Sheet 13) provides space for the computations. The estimated mean mileage to failure, $\bar{X}$, is found by using the previously determined Weibull distribution factors, D, B, and T. First, find the E1 number in Table 4.11 which corresponds to the value of B rounded off to just one decimal point. B is 1.831, or 1.8 rounded off, and the corresponding number, E1, in Table 4.11 is 1.1245. E1 is used to find $\bar{X}$ in the following equation:

$$\bar{X} = \frac{T}{E1} + D$$

$$= \frac{70{,}432 \text{ miles}}{1.1245} + 31{,}852 \text{ miles}$$

$$\bar{X} = 94{,}486 \text{ miles}$$

Note that the estimated mean, $\bar{X}$ = 94,486 miles, is slightly higher than the average of the 18 failure mileages in Table 5.4, 86,133 miles. The difference accounts for the suspended cases which will accumulate more miles before failing.

The estimated standard deviation, SD, is found by first finding the number, E2, in Table 4.13 which corresponds to B = 1.8. The value for E2 is 0.2614. This number is then used in the following equation to find the estimated standard deviation:

$$SD = \sqrt{E2 \times T^2}$$

$$= \sqrt{0.2614 \times (70{,}432)^2}$$

$$= \sqrt{1{,}296{,}718{,}256}$$

$$SD = 36{,}010 \text{ miles}$$

TABLE 5.12

WORK SHEET 13

COMPUTING THE ESTIMATED MEAN MILEAGE TO FAILURE AND THE STANDARD DEVIATION FOR WEIBULL FAILURE DATA WITH CENSORED AND SUSPENDED CASES

Cost Driver Generators

Bus Model ____________

Component Type ____________

Study Dates ____________

INPUT NUMBERS:

D = 31,852 miles
(minimum life term)

B = 1.831
(Weibull shape factor)

T = 70,432 miles
(Weibull characteristic life factor)

E1 = 1.1245
(from Table 4.12)

E2 = 0.2614
(from Table 4.13)

ESTIMATED MEAN MILEAGE TO FAILURE, $\bar{X}$:

$$\bar{X} = \underset{T}{\underline{70,432\text{ mi.}}} \; / \; \underset{E1}{\underline{1.1245}} + \underset{D}{\underline{31,852\text{ mi.}}} = \bar{X} = \underline{94,486\text{ mi.}}$$

ESTIMATED STANDARD DEVIATION, SD:

$$SD = \sqrt{\underset{E2}{\underline{0.2614}} \times \underset{T}{\underline{(\;70,432\;)}}^{2}} = SD = \underline{36,010\text{ mi.}}$$

This number compares to the standard deviation for the 18 failure cases alone of 34,161 miles.

Summary. The values for the Weibull distribution, D, B, and T, the estimated mean mileage to failure, $\bar{X}$, and the estimated standard deviation, SD, are entered in the summary Table 5.13 (Work Sheet 3). A frequency bar chart of the 18 generator failures is included.

The Weibull formula is plotted as a smooth curve in Figure 5.1 using the values for D, B, and T, computed above. Now that the Weibull distribution has been established for the generator example, it is possible to predict future failure patterns.

The steps required to find the Weibull constants, D, B, and T, and the estimated mean and standard deviation, are recapped as follows:

1. Tabulate the component failure mileages and the mileages accumulated by the buses with unfailed original components in Work Sheet 7 (Table 5.1). All of the unfailed components should have accumulated less miles than at least one of the failed components, which means they are "suspended" cases. If the failure mileage for a failed component was not recorded, then this case is considered suspended at the most recent inspection mileage when the component was still working. Furthermore, if the data set contains a mixture of censored and suspended cases, then the techniques of Chapter Six should be used instead.

2. List the failed and suspended component mileages in increasing order from smallest to largest in Work Sheet 8 (Table 5.2).

3. Compute the weights and adjusted order numbers for all of the components in Work Sheet 14 (Table 5.3).

4. Use Work Sheets 9 and 9a (Tables 5.4 and 5.5) to calculate the intermediate number, S, using the adjusted ordered failure mileages and D. The minimum life term, D, is 0.90 multiplied by the smallest failure mileage.

5. Determine the shape factor, B, and the characteristic life factor, T, by following the computational outline in Work Sheet 15 (Table 5.6).

6. Determine the estimated mean mileage to failure, $\bar{X}$, and the estimated standard deviation, SD, using Work Sheet 13 (Table 5.12).

7. Enter the values for D, B, T, $\bar{X}$, and SD in summary Work Sheet 3 (Table 5.13). The Weibull distribution computation is now complete.

TABLE 5.13

WORK SHEET 3

SUMMARY SHEET OF FAILURE MILEAGE DISTRIBUTION

Cost Driver Generators Bus Model

Component Type Study Dates

Number of cases, N 22 Mean, $\bar{X}$ 94,486 miles

Number of failures, NF 18 Std. deviation, SD 36,010 miles

Maximum mileage 150,474 miles $(\bar{X}-D)/SD$ 1.74

Minimum mileage 35,391 miles

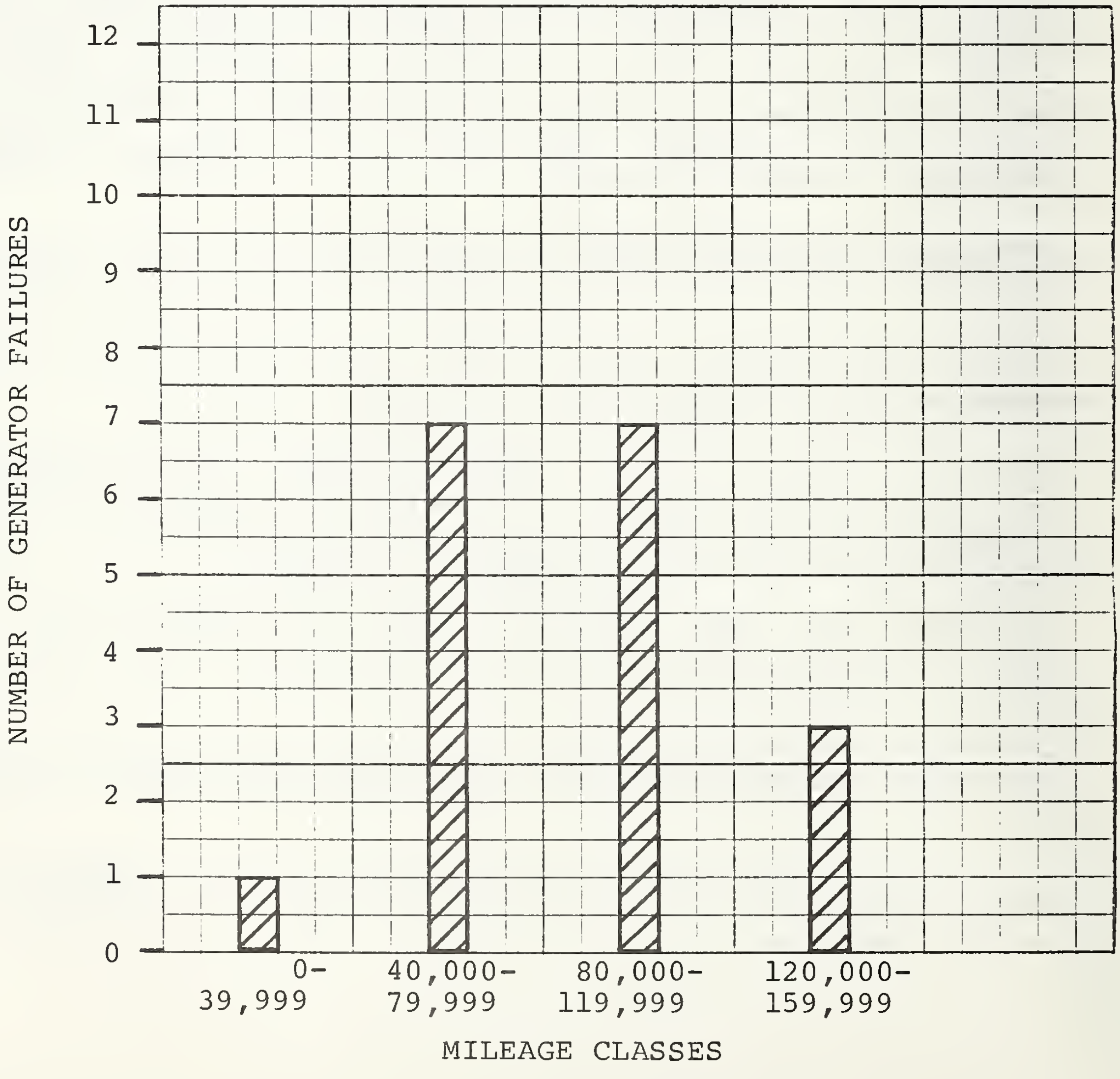
WEIBULL DISTRIBUTION PARAMETERS:

Minimum life term, D 31,852 miles

Shape factor, B 1.831

Characteristic life factor, T . 70,432 miles

FAILURE MECHANISM: "Mileage to Failure Unpredictable"



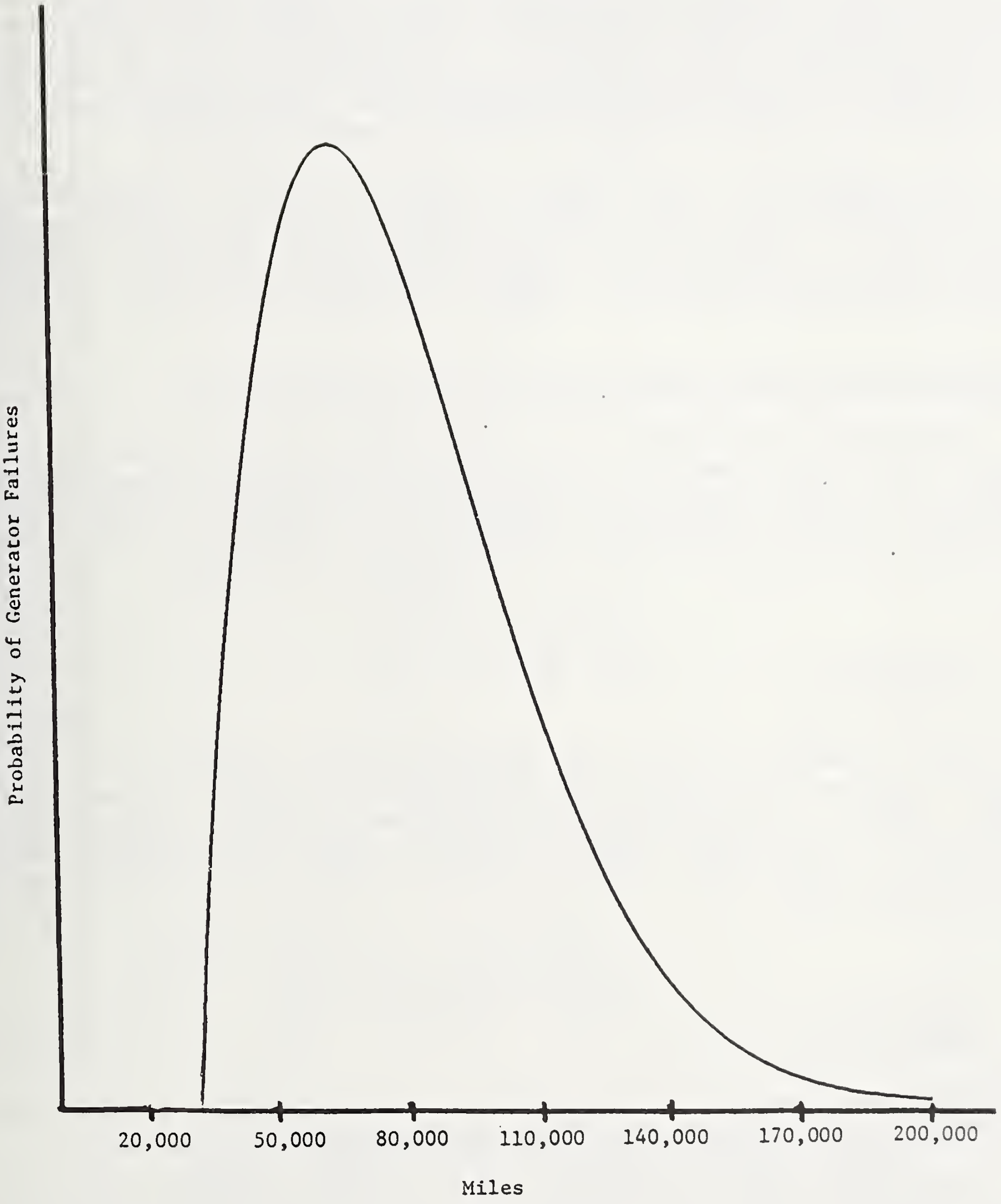


FIGURE 5.1

FITTED WEIBULL PROBABILITY CURVE FOR THE GENERATOR FAILURES

What is the Appropriate Maintenance Policy for the Generators?

A determination of the failure mechanism and the best maintenance policy for the generators can be made with the summary data of Table 5.13 (Work Sheet 3). Dividing ($\bar{X}$ - D) by SD yields a value of 1.74. B, of course, is 1.831, which also is less than 2.0. Based on Table 1.1 from Chapter One, the failure mechanism is determined to be "mileage to failure unpredictable".

The appropriate maintenance policy for the generators is found in Figure 1.6. The mean mileage to failure, $\bar{X}$, is assumed to be about as expected and the occurrence of generator failures is relatively undetectable. Based on Figure 1.6, then, the preferred maintenance policy is "operate-until-failure".

Predicting the Future Reliability of the Generators

The future reliability of the generators is determined by working with the cumulative percent of generators that are expected to fail by a certain mileage. The cumulative Weibull distribution for the generator example is plotted in Figure 5.2. The following formula is used to calculate the curve:

$$\text{cumulative probability} = 1 - e^{\left(-\left[\frac{M - D}{T}\right]^{B}\right)}$$

As an example of its use, determine the percent of original equipment generators which can be expected to fail before 60,000 miles. Using the previously determined Weibull values (D = 31,852 miles, B = 1.831, and T = 70,432 miles), the proportion or cumulative probability of generators expected to fail before 60,000 miles is:

$$\text{cumulative probability} = 1 - e^{\left(-\left[\frac{60{,}000 - 31{,}852}{70{,}432}\right]^{1.831}\right)} = 0.1701$$

An explanation and pictures of how to work out this formula with a scientific calculator are shown in Appendix B.

From the above calculation it is thus expected that 17.01 percent of the generators will fail by the time they accumulate 60,000 miles. This means that about three or four out of the 22 original generators (22 times 0.1701 = 3.74) are predicted to have failed by the time the buses have accumulated an average of 60,000 miles.

The reliability curve for the generators is computed from the cumulative percentages of Table 5.14 (Work Sheet 11). Cumulative mileage intervals of 15,000 miles were used in Table 5.14,

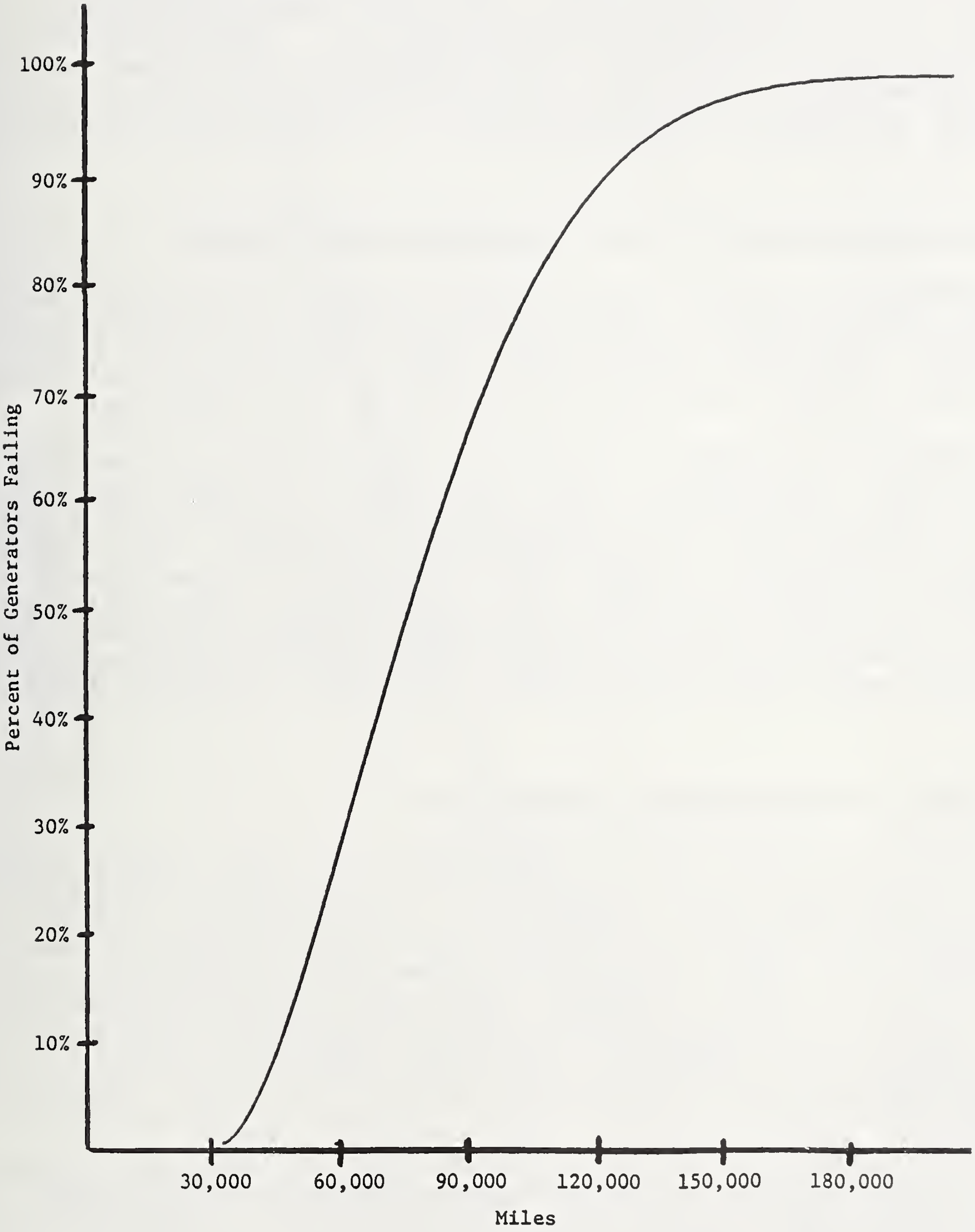


FIGURE 5.2

CUMULATIVE WEIBULL CURVE OF GENERATOR FAILURES starting with the minimum life, D = 31,852 miles, and extending to greater than 180,000 miles. The cumulative probabilities in column 2 are computed by using the above cumulative probability formula for the fitted Weibull distribution. These are converted to percentages in column 3. The reliability, or percent surviving, is computed in column 4 by subtracting the cumulative percentages of column 3 from 100 percent. The resulting reliability curve is depicted in Figure 5.3.

How Many Generators Will Fail During Any Time Period?

Consider a fleet of 60 new buses which will average 30,000 annual miles per bus. Based on the previous section it can be predicted that about ten new generators (60 times 0.1701 = 10.21) will have failed by the time the buses have accumulated 60,000 miles each, which will occur at the end of the second year.

Generator failures per year can be predicted assuming constant annual mileages per bus. In Table 5.14 each cumulative 15,000 mile increment is equivalent to six months of bus life. Hence, 180,000 miles is equivalent to six years. The expected number of generators failing in each mileage or time interval is presented in column 6. Note that no generators are predicted to fail in the first 31,852 miles of service (about one year), which is the minimum life term. All but one of the original generators are predicted to have failed after six years or 180,000 miles of use.

What About the Replacement Generators?

Table 5.14 predicts the number of first time failures that can be expected each year but it does not account for the failures of the replacement generators. A satisfactory estimate of the grand total number of failures (including failures of the replacements) to expect during any period can be obtained as follows. Take the number of miles a bus will travel during a period, divide it by the estimated mean mileage until failure, and then multiply the result by the total number of buses. It is assumed that the replacement generators last as long as the originals.

To find the total number of original equipment and replacement failures during one year, first divide the total number of miles accumulated by the average bus in a year (30,000) by the mean mileage until failure, $\bar{X}$ = 94,486 miles. This equals 0.3175 (30,000 divided by 94,486 = 0.3175). Then multiply the number of buses by 0.3175 to get the number of generators that are expected to fail during each year. If there are 60 buses, then about 19 are expected to fail each year (60 x 0.3175 = 19.05).

When the 60 new buses are first put into service, the generator failure rate will be low since the generators are new, too.

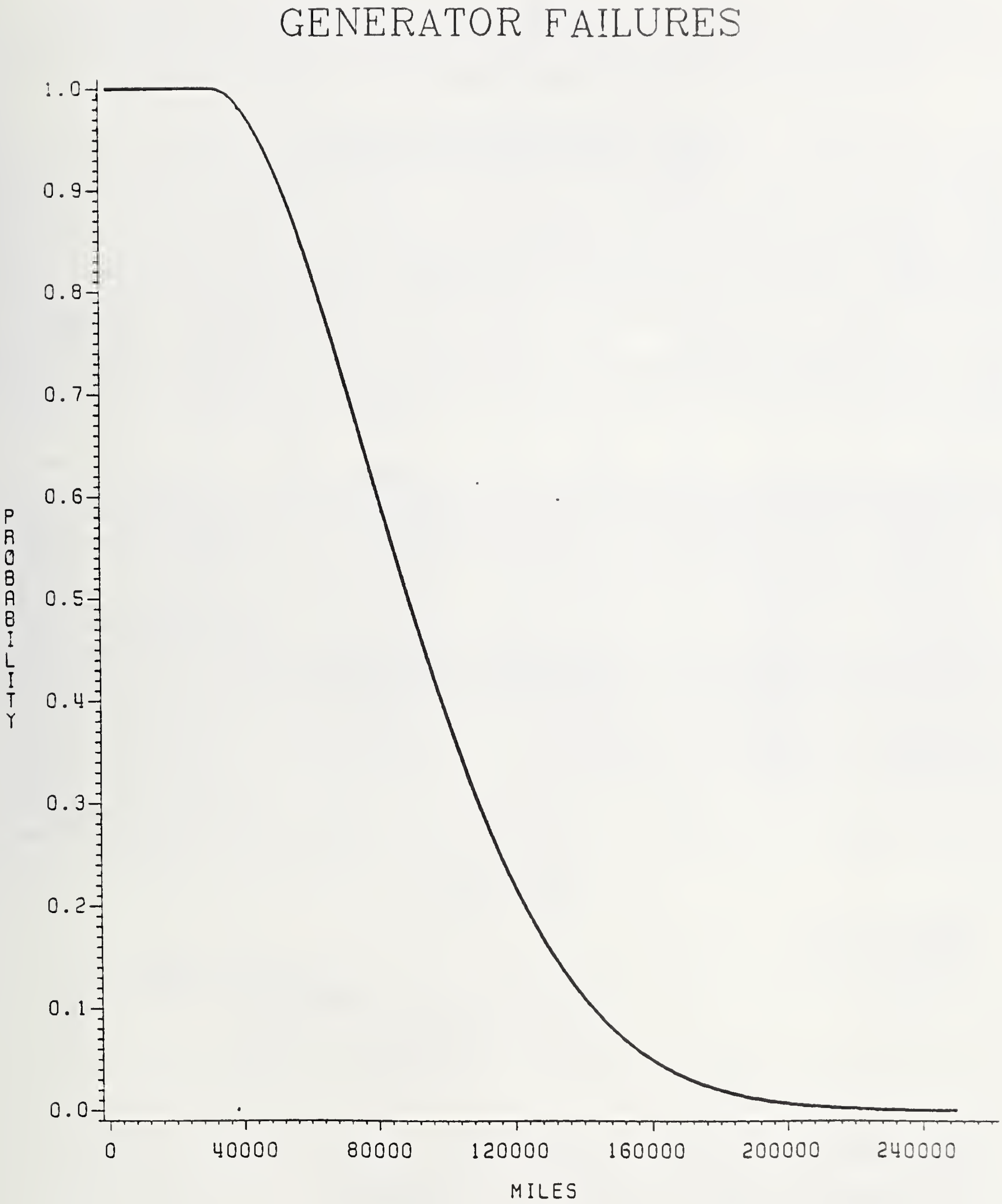


FIGURE 5.3

GENERATOR RELIABILITY CURVE

TABLE 5.14

WORK SHEET 11

COMPONENT RELIABILITY ANALYSIS FOR WEIBULL FAILURE DATA

Cost Driver Generators Bus Model ________

Component Type ________ Study Dates ________

Cumulative Mileage, M (1)	Cumulative Probability of Failure[1] (2)	Cumulative Percentage[2] (3)	Reliability,[3] R (% Surviving) (4)	Percent Failing in Mileage Interval[4] (5)	Number Failing in Mileage Interval[5] (6)
31,852	0.0	0.0 %	100.0 %	%	
45,000	0.0452	4.5 %	95.5 %	4.5 %	2.7
60,000	0.1701	17.0 %	83.0 %	12.5 %	7.5
75,000	0.3348	33.5 %	66.5 %	16.5 %	9.9
90,000	0.5054	50.5 %	49.5 %	17.0 %	10.2
105,000	0.6576	65.8 %	34.2 %	15.3 %	9.2
120,000	0.7787	77.9 %	22.1 %	12.1 %	7.3
135,000	0.8661	86.6 %	13.4 %	8.7 %	5.2
150,000	0.9241	92.4 %	7.6 %	5.8 %	3.5
165,000	0.9596	96.0 %	4.0 %	3.6 %	2.2
180,000	0.9798	98.0 %	2.0 %	2.0 %	1.2
>180,000	1.0000	100.0 %	0.0 %	2.0 %	1.2
		%	%	——— %	———
		%	%	100.0 %	60.1
		%	%	%	

[1]

$$\text{Cumulative Probability} = 1 - e^{\left(-\left[\frac{M - D}{T}\right]^{B}\right)}$$

$$= 1 - e^{\left(-\left[\frac{M - 31,852}{70,432}\right]^{1.831}\right)}$$

[2] The number in column 2 times 100.

[3] 100 percent less the number in column 3.

[4] Percent failing in mileage interval

$= R_1 - R_2$, where: R_1 = reliability (column 4) at previous cumulative mileage
R_2 = reliability at mileage in column 1

[5] The total number of failures, N, times the decimal percentage in column 5.

Eventually the failure rate will increase and should stabilize at the previously determined 19 failures per year. The time it takes for the failure rate to reach this steady-state can be estimated to be the same as the average life of a generator.

For example, if the average life of the generators is predicted to be 94,486 miles, then the time it takes for the failure rate to reach 19 per year is about 3.2 years (94,486 divided by 30,000 = 3.15). The increase in failure rate from zero to 19 per year is shown in Figure 5.4 as a straight line over time. This is good enough for estimating the failure rate as a function of time until 3.2 years is reached. Based on Figure 5.4, the generator failure rate for the 60 buses at the end of one year is about six per year, and at the end of two years about 12 per year. This graph can be used to predict the demand for generator repairs at any time during the bus life cycle for the new bus fleet.

Finally, the area under the graph between two points in time is an estimate of the total number of failed generators during that time interval. This information is useful for estimating annual maintenance costs and generator inventory requirements.

List of References

Johnson, L. G., The Statistical Treatment of Fatigue Experiments, Elsevier Publising Company, New York, 1964.

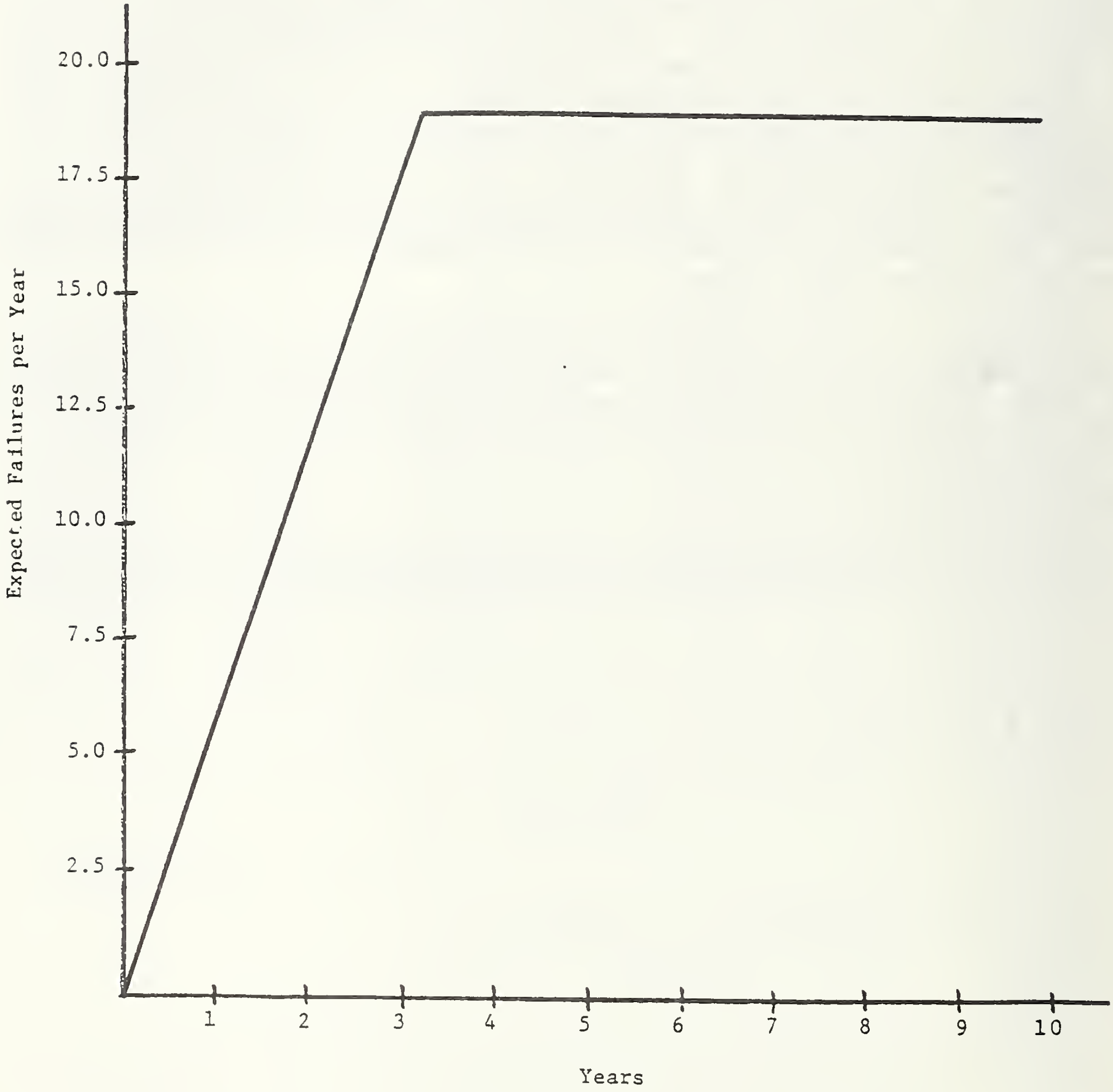


FIGURE 5.4

TOTAL PREDICTED GENERATOR FAILURES PER YEAR

DURING THE SERVICE LIFE OF THE BUSES

CHAPTER SIX

WEIBULL DISTRIBUTION FAILURE ANALYSIS WITH BOTH CENSORED AND SUSPENDED SAMPLING: TRANSMISSION EXAMPLE

This chapter is a self-contained analysis of original equipment bus transmission failures to predict future failure patterns. Not all of the bus transmissions in the fleet being studied have failed yet but the analyst desires to include every bus in the analysis. The buses with surviving transmissions fall into two categories:

1. Some surviving transmissions have accumulated more miles than any failed transmission in the fleet. This is data with censored cases: the buses with these transmissions have not been in service long enough for all to have failed. The analyst wishes to forecast failure patterns before the entire generation of transmissions has failed.

2. Some surviving transmissions have accumulated less miles than at least one of the failed transmissions. This is data with suspended cases: these buses have seen less service than those with failed transmissions.

Chapters Four and Five have treated censored and suspended sampling separately. This chapter considers the situation where the failure data includes both censored and suspended cases. The topics covered are:

- How to collect the transmission data.
- How to tabulate data which contains both censored and suspended cases.
- Fitting the Weibull distribution with censored and suspended cases to the transmission data.
- What is the appropriate maintenance policy for the transmissions?
- Predicting the future reliability of the transmissions?
- How many transmissions will fail during any time period?
- What about the replacement transmissions?

How to Collect the Transmission Data

As in the other chapters, it is assumed that the buses in the fleet under study were purchased at about the same time and the buses have experienced similar mileages and operating conditions. It is assumed that the component of interest is the same model in each bus. The significance of being able to account for both censored and suspended cases is that it gives the analyst more freedom when working with actual data. For example, the analyst naturally wishes to include as many new buses as possible in the study. However, suppose that the buses were delivered over a period of time, such as one year, so that the last buses delivered have accumulated fewer miles than the others. Alternatively, a bus may simply have accumulated less miles for a variety of reasons; perhaps it was in an accident. Such buses may become suspended cases. Finally, accounting for censored cases permits the analyst to forecast failure patterns without waiting for all of the original components to fail.

The bus fleet under study should have recorded at least ten original component failures, and the failed components should comprise at least twenty percent of the entire fleet under study. Do not include failures of component replacements in the analysis.

As an example, consider the fourteen Bluebird 30 foot buses placed in service by a transit agency in 1981 and listed in Table 6.1 (Work Sheet 7). For each bus the mileage at the time of the study and the mileage when the original transmission failed are recorded. Four out of the fourteen transmissions have not failed yet. Note that the failed transmission which accumulated the most miles before failure was from bus number 8109 with 90,930 miles. The transmission on bus 8107 has received less service (83,551 miles) and has not failed. Therefore, it should be treated as a suspended case. The transmission on bus 8113 has not failed but it has accumulated more miles (118,667 miles) than the highest mileage failure (bus number 8109, 90,930 miles). Therefore, it is a censored case. The data set thus includes both censored and suspended cases.

How to Tabulate Data Which Contains Both Censored and Suspended Cases

The analysis of the components with censored and suspended cases begins by listing the buses in order of increasing mileage when the original transmission failed, or the mileage at the time of the study if it is censored or suspended. This is done in Table 6.2 (Work Sheet 8). Whether the individual cases are failed, censored, or suspended is noted in column 4. Note the relationship of the censored and suspended cases:

1. The censored buses have higher mileages than all of the failed cases. They are ordered last in Table 6.2.

2. The suspended buses have lower mileages than some of the failed cases.

TABLE 6.1

WORK SHEET 7

FAILURE STATUS OF ORIGINAL COMPONENTS IN WEIBULL FAILURE ANALYSIS

Cost Driver Transmissions Bus Model Bluebird

Component Type Detroit Diesel Allison MT 643 Study Dates 1981-1983

Bus Number (1)	Component Number (2)	Current Bus Mileage (3)	Original Component Failure Mileage (4)	Status of Unfailed Component*		Comments (7)
				Censor (5)	Susp. (6)	
8101		105,335	81,621			
8102		80,180	66,981			
8103		114,852	71,400			
8104		80,309			X	
8105		96,971	82,195			
8106		115,071	48,250			
8107		83,551			X	
8108		121,143		X		
8109		117,036	90,930			
8110		123,443	43,483			
8111		93,821	52,624			
8112		101,178	83,822			
8113		118,667		X		
8114		91,013	71,750			

* CENSORED cases refer to surviving components which have accumulated more miles than any failed component in the data set.

SUSPENDED cases refer to surviving components which have accumulated less miles than at least one of the failed components.

TABLE 6.2

WORK SHEET 8

RANK ORDERING OF SURVIVING AND FAILED COMPONENTS IN WEIBULL FAILURE ANALYSIS

Cost Driver Transmissions

Bus Model Bluebird

Component Type Detroit Diesel Allison MT 643

Study Dates 1981-1983

Order Number (1)	Bus Number (2)	Component Number (3)	Failed, Suspended, or Censored (4)	Mileage When Failed, Suspended, or Censored* (5)
1	8110		Failed	43,483
2	8106		Failed	48,250
3	8111		Failed	52,624
4	8102		Failed	66,981
5	8103		Failed	71,400
6	8114		Failed	71,750
7	8104		Suspended	80,309
8	8101		Failed	81,621
9	8105		Failed	82,195
10	8107		Suspended	83,551
11	8112		Failed	83,822
12	8109		Failed	90,930
13	8113		Censored	118,667
14	8108		Censored	121,143
15				
16				
17				
18				
19				
20				

* Each component is listed in ascending order of the mileage entered in Column 5.

The next step in tabulating the data is to calculate "adjusted order numbers" using Table 6.3 (Work Sheet 14). The information in columns 1, 2, 3, and 4 is taken from Table 6.2 (Work Sheet 8). In column 5 of Table 6.3 the original order numbers of column 4 are reversed. In this example there are 14 buses, hence the reverse ordering goes from 14 to 1.

The next step is to calculate a "weight" for each failed bus. When calculating the weights the censored and suspended cases are skipped. The weight equation is:

$$\text{weight} = \frac{(\text{number of buses} + 1) - (\text{previous adjusted order number})}{1 + (\text{reverse order number})}$$

The previous adjusted order number is the sum of all the previous weights and it is found in column 8 of the immediately preceding failure case.

For example, the first bus (number 8110) has the following:

14 = the total number of buses, N
0 = the previous adjusted order number (it equals zero because it is the first case)
14 = the reverse order number from column 5.

Thus, the weight for bus 8110 is:

$$\text{weight} = \frac{(14 + 1) - 0}{1 + 14} = 1$$

The adjusted order number for bus 8110 is 1 since it is the first one in Table 6.3.

Similar calculations follow until the first suspended bus is encountered, number 8104, for which no weight or adjusted order number is computed. Next, consider the first bus after this suspended bus, number 8101. For bus 8101:

14 = the total number of buses

6 = the previous adjusted order number (for bus 8114)

7 = the reverse order

$$\text{weight} = \frac{(14 + 1) - 6}{1 + 7} = 1.1250$$

Thus, the adjusted order number for bus 8101 is 6 + 1.1250 = 7.1250.

As a third example, consider bus number 8112 which comes after the second suspended bus, number 8107. For bus 8112:

TABLE 6.3

WORK SHEET 14

Page 1 of 2

COMPUTING WEIGHTS AND ADJUSTED ORDER NUMBERS FOR WEIBULL FAILURE ANALYSIS WHEN CASES INCLUDE SUSPENDED CASES

Bus Number (1)	Failed, Suspended, or Censored (2)	Mileage When Failed, Suspended, or Censored (3)	Original Order Number (4)	Reverse Order Number (5)	Weight Calculation (6)	Weight (7)	Adjusted Order Number (8)
8110	Failed	43,483	1	14	$\frac{(14+1) - 0}{1 + 14}$	1 =	1
8106	Failed	48,250	2	13	$\frac{(14+1) - 1}{1 + 13}$	1 =	2
8111	Failed	52,624	3	12	$\frac{(14+1) - 2}{1 + 12}$	1 =	3
8102	Failed	66,981	4	11	$\frac{(14+1) - 3}{1 + 11}$	1 =	4
8103	Failed	71,400	5	10	$\frac{(14+1) - 4}{1 + 10}$	1 =	5
8114	Failed	71,750	6	9	$\frac{(14+1) - 5}{1 + 9}$	1 =	6
8104	Suspended	80,309	7	8		=	
8101	Failed	81,621	8	7	$\frac{(14+1) - 6}{1 + 7}$	1.1250 =	7.1250
8105	Failed	82,195	9	6	$\frac{(14+1) - 7.1250}{1 + 6}$	1.1250 =	8.2500
8107	Suspended	83,551	10	5		=	

TABLE 6.3 (continued)

WORK SHEET 14

Page 2 of 2

COMPUTING WEIGHTS AND ADJUSTED ORDER NUMBERS FOR WEIBULL FAILURE ANALYSIS WHEN CASES INCLUDE SUSPENDED CASES

Bus Number (1)	Failed, Suspended, or Censored (2)	Mileage When Failed, Suspended, or Censored (3)	Original Order Number (4)	Reverse Order Number (5)	Weight Calculation (6)	Weight (7)	Adjusted Order Number (8)
8112	Failed	83,822	11	4	$\frac{(14+1)-8.2500}{1 + 4}$	1.3500	= 9.6000
8109	Failed	90,930	12	3	$\frac{(14+1)-9.6000}{1 + 3}$	1.3500	= 10.9500
8113	Censored	118,667	13	2			=
8108	Censored	121,143	14	1			=
							=
							=
							=
							=
							=
							=

14 = the total number of buses
8.2500 = the previous order number (bus 8305)
4 = the reverse order number

$$\text{weight} = \frac{(14 + 1) - (8.2500)}{1 + 4} = 1.3500$$

The adjusted order number for bus 8112 is 8.2500 + 1.3500 = 9.6000.

This process of calculating the adjusted order numbers includes input from the suspended and censored buses even though it is not known when their original transmissions will fail. The adjusted order numbers are now used in the derivation of the Weibull distribution.

Fitting the Weibull Distribution with Censored and Suspended Cases to the Transmission Data

The transmission failures are modeled similar to any complex component which fails rather than wears out. Therefore, the incomplete set of transmission failures can be modeled with the Weibull distribution with its characteristic ski-slope shape. If all of the bus fleet data consisted of transmission failures, the bar chart of the failure mileages would resemble a ski slope pattern. However, when the data set is incomplete because some of the transmissions have not failed yet, the bar chart may not look like that. Instead, the Weibull distribution is selected because it is expected that the complete failure distribution pattern would resemble a ski slope.

Calculating D, B, and T. Fitting the transmission failure data with both censored and suspended cases is similar to previous chapters but not quite the same. To be able to calculate the Weibull curve that best fits the original data, the same three Weibull constants, D, B, and T, have to be calculated. However, different tables are used. First, the buses with transmission failures are listed in order of increasing failure mileage in Table 6.4 (Work Sheet 9). The suspended and censored cases are not included in this table.

The first constant is D and it is called the "minimum life term". Should a component fail without having accumulating any miles, the "minimum life" would be zero. As before, a satisfactory estimate of D is 90 percent of the lowest failure mileage. In Table 6.4, the lowest failure mileage is 43,483 miles (bus 8110) and, therefore, D equals 39,135 miles (43,483 x 0.90 = 39,135).

The other two constants are B and T. B, the "shape factor", determines the shape of the curve (a measurement of how bunched together or spread out the distribution is) and T, the "charac-

TABLE 6.4

WORK SHEET 9

COMPUTATION OF D, K, AND S FOR WEIBULL FAILURE DATA

Cost Driver Transmissions Bus Model Bluebird

Component Type Detroit Diesel Allison MT 643 Study Dates 1981-1983

Bus Number (1)	Failure Mileage (2)	Minimum Life, D* (3)	K (4)	$\ln(K_{max})$** (5)	ln(K) (6)	L*** (7)
8110	43,483	- 39,135 =	4,348	10.8550	- 8.3775 =	2.4775
8106	48,250	- 39,135 =	9,115	10.8550	- 9.1177 =	1.7373
8111	52,624	- 39,135 =	13,489	10.8550	- 9.5096 =	1.3454
8102	66,981	- 39,135 =	27,846	10.8550	- 10.2344 =	0.6206
8103	71,400	- 39,135 =	32,265	10.8550	- 10.3817 =	0.4733
8114	71,750	- 39,135 =	32,615	10.8550	- 10.3925 =	0.4625
8101	81,621	- 39,135 =	42,486	10.8550	- 10.6569 =	0.2011
8105	82,195	- 39,135 =	43,060	10.8550	- 10.6703 =	0.1847
8112	83,822	- 39,135 =	44,687	10.8550	- 10.7074 =	0.1476
8109	90,930	- 39,135 =	51,795	10.8550	- 10.8550 =	0.0
		- =			- =	
		- =			- =	
		- =			- =	
		- =			- =	
		- =			- =	
		- =			- =	
		- =			- =	
		- =			- =	
		- =			- =	
		- =			- =	
Sums:		ΣK =			S =	

* Minimum life term, D = 0.90 x lowest failure mileage
= 0.90 x 43,483 mi. = D = 39,135 miles

** $\ln(K_{max})$ = ln(51,795) = 10.8550 , where K_{max} is the largest value of K in column 4.

*** If the data set contains suspended failure cases, the L terms must be modified in Work Sheet 10a before computing S.

teristic life factor," determines how far away the peak in the curve should be from zero miles. Before calculating B and T, intermediate numbers must be calculated. These calculations are also shown in Table 6.4.

The minimum life term, D = 39,135 miles, is subtracted from each failure mileage in Table 6.4 (column 2 minus column 3) to get column 4, which is a value of K for each failed transmission.

The K values are then used to calculate the values for L in column 7. First, compute the natural logarithm of the largest value of K, K_{max}, which is 51,795 miles for bus 8109. Obtaining the natural logarithm is simple when using a scientific calculator. Enter the number, 51,795, and press the button marked "ln". The natural logarithm, 10.8550, will appear on the display.* The natural logarithms are rounded-off to only four digits to the right of the decimal point for simplicity (10.855049 becomes 10.8550) in Table 6.4. This number is placed on every line of the fifth column of Table 6.4.

Next, the natural logarithm of each value of K, ln (K), is written in the sixth column. For example, in the first line of Table 6.4 (bus 8110), ln (4,348) equals 8.3775. The difference of the numbers in the fifth and sixth columns is written in the seventh column. The values in the seventh column are the values of L.

Since the data set includes suspended samples, the L values must be modified using the adjusted order numbers of Table 6.3. Table 6.5 (Work Sheet 9a) is provided for these computations.

First, columns 1, 2, and 3 are filled in using the information from Tables 6.3 and 6.4. In column 4 the row numbers from 1 to 10 already are entered. Column 3 is divided by column 4 with the ratio result reported in column 5.

The final step is to rewrite the values of L from column 2 into column 6. The ratio is then multiplied by L (column 5 multipled by column 6) and the result is written in column 7. The sum of all the modified L's in column 7 equals S, which is 7.6692.

Now it is possible to compute the two remaining Weibull constants, B and T. Table 6.6 (Work Sheet 16) outlines the computations for B and T and provides space for each intermediate result and the final answers.

B and T are determined by looking up numbers H1, H2, H3, J1, J2, and J3 from Tables 6.7 through 6.54 for the total number of

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* For a discussion of the use of scientific calculators see Appendix B.

TABLE 6.5

WORK SHEET 9a

MODIFICATION OF L FACTORS TO COMPUTE S FOR WEIBULL FAILURE DATA WITH SUSPENDED CASES

Cost Driver Transmissions Bus Model Bluebird
Detroit Diesel Allison
Component Type MT 643 Study Dates 1981-1983

Bus Number (1)	L* (2)	Adjusted Order No. (3)	Row Number (4)	Ratio (5)	L (6)	Modified L (7)
8110	2.4775	1	/ 1	= 1	x 2.4775	= 2.4775
8106	1.7373	2	/ 2	= 1	x 1.7373	= 1.7373
8111	1.3454	3	/ 3	= 1	x 1.3454	= 1.3454
8102	0.6206	4	/ 4	= 1	x 0.6206	= 0.6206
8103	0.4733	5	/ 5	= 1	x 0.4733	= 0.4733
8114	0.4625	6	/ 6	= 1	x 0.4625	= 0.4625
8101	0.2011	7.1250	/ 7	= 1.0179	x 0.2011	= 0.2047
8105	0.1847	8.2500	/ 8	= 1.0313	x 0.1847	= 0.1905
8112	0.1476	9.6000	/ 9	= 1.0667	x 0.1476	= 0.1574
8109	0.0	10.9500	/ 10	= 1.0950	x 0.0	= 0.0
			/ 11	=	x	=
			/ 12	=	x	=
			/ 13	=	x	=
			/ 14	=	x	=
			/ 15	=	x	=
			/ 16	=	x	=
			/ 17	=	x	=
			/ 18	=	x	=
			/ 19	=	x	=
			/ 20	=	x	=
Sums:						S = 7.6692

* From Work Sheet 9, Column 7.

TABLE 6.6

WORK SHEET 16

COMPUTATION OF B AND T FOR WEIBULL FAILURE DATA WHICH CONTAINS BOTH CENSORED AND SUSPENDED CASES

Cost Driver Transmission Bus Model Bluebird

Component Type Detroit Diesel Allison MT 643 Study Dates 1981-1983

INPUT NUMBERS:

The following numbers are obtained from Tables 6.7 to 6.54:

H1 = 1.152 J1 = 0.360

H2 = - 0.052 J2 = - 0.235

H3 = - 0.0081 J3 = 0.0062

N = 14 (total number of buses) S = 7.6692 (from Work Sheet 9a)

NS = 2 (number of suspended cases) K_{max} = 51,795 miles (from Work Sheet 9)

NC = 2 (number of censored cases) ratio = 1.0950 (from Work Sheet 9a, the ratio in column 5 corresponding to K_{max})

(NC/N)x100 = 14.3 % (percent of censored cases)

SHAPE FACTOR, B:

$$M = \underset{H1}{\underline{1.152}} + \underset{H2}{\underline{-0.052}} \times \underset{NS}{\underline{2}} + \underset{H3}{\underline{-0.0081}} \times \left(\underset{NS}{\underline{2}}\right)^2$$

M = 1.016

$$B = \underset{M}{\underline{1.016}} \times \underset{N}{\underline{14}} \; / \; \underset{S}{\underline{7.6692}} = B = \underline{1.855}$$

CHARACTERISTIC LIFE FACTOR, T:

$$P = \underset{J1}{\underline{0.360}} + \underset{J2}{\underline{-0.235}} \times \underset{NS}{\underline{2}} + \underset{J3}{\underline{0.0062}} \times \left(\underset{NS}{\underline{2}}\right)^2$$

P = - 0.085

$$\ln(T) = \ln\left(\underset{K_{max}}{\underline{51,795}} \; / \; \underset{ratio}{\underline{1.0950}}\right) - \underset{P}{\underline{-0.085}} \; / \; \underset{B}{\underline{1.855}}$$

ln(T) = 10.7185 , T = 45,184 mi. ("inverse" of ln(T) on a scientific calculator)

TABLE 6.7

H1 NUMBERS FOR 10% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	1.202	1.219	1.234	1.248	1.261	1.273	1.284	1.294	1.303	1.311
20	1.318	1.325	1.331	1.336	1.341	1.345	1.348	1.352	1.355	1.357
30	1.360	1.362	1.364	1.366	1.368	1.370	1.372	1.374	1.376	1.377
40	1.379	1.380	1.382	1.383	1.385	1.386	1.387	1.388	1.390	1.391
50	1.392	1.393	1.394	1.395	1.396	1.397	1.397	1.398	1.399	1.400
60	1.400	1.401	1.402	1.402	1.403	1.403	1.404	1.404	1.405	1.405
70	1.406	1.406	1.407	1.407	1.407	1.408	1.408	1.408	1.409	1.409
80	1.409	1.410	1.410	1.410	1.411	1.411	1.411	1.412	1.412	1.412
90	1.413	1.413	1.413	1.414	1.414	1.414	1.415	1.415	1.416	1.416

TABLE 6.8

H2 NUMBERS FOR 10% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.061	-0.059	-0.057	-0.055	-0.053	-0.050	-0.048	-0.047	-0.045	-0.043
20	-0.041	-0.039	-0.038	-0.036	-0.035	-0.033	-0.032	-0.030	-0.029	-0.028
30	-0.027	-0.026	-0.025	-0.023	-0.022	-0.022	-0.021	-0.020	-0.019	-0.018
40	-0.017	-0.017	-0.016	-0.015	-0.015	-0.014	-0.014	-0.013	-0.013	-0.012
50	-0.012	-0.011	-0.011	-0.011	-0.010	-0.010	-0.010	-0.009	-0.009	-0.009
60	-0.009	-0.009	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.007	-0.007
70	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	-0.007	-0.006	-0.006	-0.006
80	-0.006	-0.006	-0.006	-0.006	-0.006	-0.005	-0.005	-0.005	-0.005	-0.004
90	-0.004	-0.004	-0.004	-0.003	-0.003	-0.003	-0.002	-0.002	-0.001	-0.001

TABLE 6.9

H3 NUMBERS FOR 10% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.0152	-0.0132	-0.0114	-0.0093	-0.0084	-0.0072	-0.0062	-0.0053	-0.0046	-0.0040
20	-0.0035	-0.0031	-0.0028	-0.0026	-0.0024	-0.0022	-0.0021	-0.0020	-0.0019	-0.0018
30	-0.0018	-0.0017	-0.0016	-0.0015	-0.0015	-0.0014	-0.0013	-0.0012	-0.0012	-0.0011
40	-0.0011	-0.0010	-0.0010	-0.0009	-0.0009	-0.0008	-0.0008	-0.0007	-0.0007	-0.0007
50	-0.0006	-0.0006	-0.0006	-0.0005	-0.0005	-0.0005	-0.0005	-0.0004	-0.0004	-0.0004
60	-0.0004	-0.0004	-0.0004	-0.0004	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003
70	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003
80	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
90	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002

TABLE 6.10

J1 NUMBERS FOR 10% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	0.299	0.316	0.332	0.347	0.360	0.371	0.381	0.390	0.398	0.405
20	0.410	0.416	0.420	0.424	0.427	0.430	0.432	0.434	0.436	0.438
30	0.440	0.442	0.444	0.446	0.447	0.449	0.451	0.452	0.454	0.456
40	0.457	0.459	0.460	0.461	0.463	0.464	0.465	0.466	0.468	0.469
50	0.470	0.471	0.472	0.473	0.474	0.475	0.476	0.477	0.478	0.478
60	0.479	0.480	0.481	0.482	0.482	0.483	0.484	0.484	0.485	0.486
70	0.486	0.487	0.487	0.488	0.488	0.489	0.489	0.490	0.490	0.491
80	0.491	0.492	0.492	0.492	0.493	0.493	0.494	0.494	0.494	0.495
90	0.495	0.495	0.496	0.496	0.496	0.497	0.497	0.497	0.498	0.498

TABLE 6.11

J2 NUMBERS FOR 10% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.277	-0.263	-0.250	-0.238	-0.226	-0.215	-0.205	-0.195	-0.186	-0.178
20	-0.170	-0.163	-0.156	-0.150	-0.144	-0.138	-0.133	-0.128	-0.124	-0.120
30	-0.116	-0.113	-0.109	-0.106	-0.104	-0.101	-0.098	-0.096	-0.094	-0.091
40	-0.089	-0.087	-0.085	-0.083	-0.081	-0.079	-0.077	-0.076	-0.074	-0.073
50	-0.071	-0.070	-0.068	-0.067	-0.065	-0.064	-0.063	-0.062	-0.061	-0.060
60	-0.059	-0.058	-0.057	-0.056	-0.055	-0.054	-0.053	-0.052	-0.052	-0.051
70	-0.050	-0.050	-0.049	-0.048	-0.048	-0.047	-0.047	-0.046	-0.046	-0.045
80	-0.045	-0.044	-0.044	-0.043	-0.043	-0.043	-0.042	-0.042	-0.041	-0.041
90	-0.041	-0.040	-0.040	-0.039	-0.039	-0.039	-0.038	-0.038	-0.037	-0.037

TABLE 6.12

J3 NUMBERS FOR 10% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	0.0036	0.0035	0.0033	0.0032	0.0031	0.0030	0.0028	0.0027	0.0026	0.0025
20	0.0024	0.0023	0.0022	0.0021	0.0020	0.0019	0.0018	0.0017	0.0016	0.0015
30	0.0014	0.0013	0.0012	0.0011	0.0011	0.0010	0.0009	0.0008	0.0008	0.0007
40	0.0006	0.0006	0.0005	0.0005	0.0004	0.0004	0.0003	0.0003	0.0003	0.0002
50	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001
60	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
70	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001
80	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
90	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001

TABLE 6.13

H1 NUMBERS FOR 20% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	0.955	0.970	0.984	0.996	1.009	1.017	1.026	1.034	1.041	1.047
20	1.052	1.056	1.060	1.063	1.066	1.069	1.071	1.073	1.074	1.076
30	1.078	1.079	1.081	1.082	1.084	1.085	1.087	1.088	1.089	1.090
40	1.092	1.093	1.094	1.095	1.096	1.097	1.098	1.099	1.100	1.101
50	1.102	1.103	1.104	1.105	1.105	1.106	1.107	1.108	1.108	1.109
60	1.110	1.110	1.111	1.111	1.112	1.112	1.113	1.113	1.114	1.114
70	1.115	1.115	1.115	1.116	1.116	1.116	1.117	1.117	1.117	1.117
80	1.117	1.118	1.118	1.118	1.118	1.118	1.119	1.119	1.119	1.119
90	1.119	1.119	1.119	1.119	1.119	1.119	1.119	1.120	1.120	1.120

TABLE 6.14

H2 NUMBERS FOR 20% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.064	-0.060	-0.057	-0.054	-0.051	-0.049	-0.046	-0.044	-0.041	-0.039
20	-0.038	-0.036	-0.034	-0.033	-0.031	-0.030	-0.029	-0.028	-0.027	-0.026
30	-0.025	-0.024	-0.023	-0.023	-0.022	-0.022	-0.021	-0.021	-0.020	-0.020
40	-0.019	-0.019	-0.018	-0.018	-0.017	-0.017	-0.017	-0.016	-0.016	-0.016
50	-0.015	-0.015	-0.015	-0.015	-0.014	-0.014	-0.014	-0.014	-0.013	-0.013
60	-0.013	-0.013	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012	-0.011	-0.011
70	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	-0.010	-0.010	-0.010	-0.010
80	-0.010	-0.010	-0.010	-0.010	-0.009	-0.009	-0.009	-0.009	-0.009	-0.009
90	-0.009	-0.009	-0.008	-0.008	-0.008	-0.008	-0.008	-0.008	-0.007	-0.007

TABLE 6.15

H3 NUMBERS FOR 20% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.0144	-0.0124	-0.0106	-0.0091	-0.0077	-0.0066	-0.0056	-0.0048	-0.0041	-0.0035
20	-0.0031	-0.0027	-0.0024	-0.0022	-0.0021	-0.0020	-0.0019	-0.0018	-0.0017	-0.0016
30	-0.0015	-0.0015	-0.0014	-0.0013	-0.0013	-0.0012	-0.0011	-0.0011	-0.0010	-0.0010
40	-0.0009	-0.0009	-0.0008	-0.0008	-0.0007	-0.0007	-0.0007	-0.0006	-0.0006	-0.0006
50	-0.0005	-0.0005	-0.0005	-0.0005	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0003
60	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003
70	-0.0003	-0.0003	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
80	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
90	-0.0002	-0.0002	-0.0002	-0.0002	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001

TABLE 6.16

J1 NUMBERS FOR 20% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	0.299	0.316	0.332	0.347	0.360	0.371	0.381	0.390	0.398	0.405
20	0.410	0.416	0.420	0.424	0.427	0.430	0.432	0.434	0.436	0.433
30	0.440	0.442	0.444	0.446	0.447	0.449	0.451	0.452	0.454	0.456
40	0.457	0.459	0.460	0.461	0.463	0.464	0.465	0.466	0.468	0.469
50	0.470	0.471	0.472	0.473	0.474	0.475	0.476	0.477	0.478	0.478
60	0.479	0.480	0.481	0.482	0.482	0.483	0.484	0.484	0.485	0.486
70	0.486	0.487	0.487	0.488	0.488	0.489	0.489	0.490	0.490	0.491
80	0.491	0.492	0.492	0.492	0.493	0.493	0.494	0.494	0.494	0.495
90	0.495	0.495	0.496	0.496	0.496	0.497	0.497	0.497	0.498	0.478

TABLE 6.17

J2 NUMBERS FOR 20% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.310	-0.293	-0.277	-0.262	-0.247	-0.234	-0.222	-0.210	-0.200	-0.190
20	-0.181	-0.172	-0.164	-0.157	-0.150	-0.144	-0.139	-0.134	-0.129	-0.125
30	-0.121	-0.117	-0.114	-0.110	-0.108	-0.105	-0.102	-0.100	-0.097	-0.095
40	-0.093	-0.090	-0.088	-0.086	-0.084	-0.082	-0.080	-0.079	-0.077	-0.075
50	-0.074	-0.072	-0.071	-0.070	-0.068	-0.067	-0.066	-0.065	-0.064	-0.062
60	-0.061	-0.060	-0.060	-0.059	-0.058	-0.057	-0.056	-0.055	-0.055	-0.054
70	-0.053	-0.053	-0.052	-0.052	-0.051	-0.050	-0.050	-0.049	-0.049	-0.048
80	-0.048	-0.047	-0.047	-0.046	-0.046	-0.045	-0.045	-0.044	-0.044	-0.043
90	-0.043	-0.042	-0.042	-0.041	-0.041	-0.040	-0.040	-0.039	-0.038	-0.038

TABLE 6.18

J3 NUMBERS FOR 20% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	0.0149	0.0137	0.0125	0.0114	0.0104	0.0095	0.0086	0.0079	0.0072	0.0065
20	0.0059	0.0054	0.0049	0.0045	0.0041	0.0038	0.0035	0.0033	0.0030	0.0028
30	0.0027	0.0025	0.0024	0.0022	0.0021	0.0020	0.0019	0.0018	0.0017	0.0016
40	0.0015	0.0014	0.0014	0.0013	0.0012	0.0012	0.0011	0.0010	0.0010	0.0009
50	0.0009	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007	0.0006	0.0006	0.0006
60	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
70	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
80	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004
90	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002

TABLE 6.19

H1 NUMBERS FOR 30% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	0.775	0.788	0.799	0.809	0.818	0.825	0.832	0.837	0.842	0.846
20	0.849	0.852	0.855	0.857	0.859	0.860	0.862	0.864	0.865	0.867
30	0.868	0.870	0.871	0.873	0.874	0.875	0.876	0.878	0.879	0.880
40	0.881	0.882	0.883	0.884	0.885	0.886	0.887	0.887	0.888	0.889
50	0.890	0.890	0.891	0.892	0.892	0.893	0.893	0.894	0.894	0.895
60	0.895	0.896	0.896	0.897	0.897	0.897	0.898	0.898	0.898	0.899
70	0.899	0.899	0.900	0.900	0.900	0.901	0.901	0.901	0.901	0.902
80	0.902	0.902	0.903	0.903	0.903	0.903	0.904	0.904	0.904	0.905
90	0.905	0.905	0.906	0.906	0.907	0.907	0.907	0.908	0.908	0.909

TABLE 6.20

H2 NUMBERS FOR 30% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.087	-0.079	-0.072	-0.065	-0.060	-0.055	-0.051	-0.047	-0.044	-0.042
20	-0.040	-0.038	-0.036	-0.035	-0.034	-0.033	-0.032	-0.031	-0.030	-0.029
30	-0.028	-0.027	-0.026	-0.025	-0.024	-0.024	-0.023	-0.022	-0.022	-0.021
40	-0.020	-0.020	-0.019	-0.019	-0.018	-0.018	-0.017	-0.017	-0.016	-0.016
50	-0.015	-0.015	-0.015	-0.014	-0.014	-0.014	-0.014	-0.013	-0.013	-0.013
60	-0.013	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012	-0.011	-0.011	-0.011
70	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	-0.010	-0.010	-0.010
80	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.009	-0.009
90	-0.009	-0.009	-0.009	-0.009	-0.009	-0.009	-0.008	-0.008	-0.008	-0.008

TABLE 6.21

H3 NUMBERS FOR 30% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.0121	-0.0104	-0.0088	-0.0076	-0.0064	-0.0055	-0.0047	-0.0041	-0.0035	-0.0030
20	-0.0027	-0.0025	-0.0022	-0.0021	-0.0019	-0.0019	-0.0018	-0.0016	-0.0016	-0.0014
30	-0.0014	-0.0014	-0.0012	-0.0012	-0.0012	-0.0010	-0.0010	-0.0010	-0.0009	-0.0009
40	-0.0008	-0.0008	-0.0007	-0.0007	-0.0007	-0.0006	-0.0006	-0.0005	-0.0005	-0.0005
50	-0.0005	-0.0005	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0003	-0.0003
60	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003
70	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
80	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
90	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001

TABLE 6.22

J1 NUMBERS FOR 30% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	0.019	0.035	0.050	0.063	0.074	0.084	0.093	0.101	0.107	0.113
20	0.118	0.123	0.127	0.130	0.133	0.136	0.139	0.142	0.144	0.147
30	0.149	0.152	0.154	0.156	0.158	0.160	0.162	0.163	0.165	0.167
40	0.168	0.169	0.171	0.172	0.173	0.174	0.175	0.176	0.177	0.178
50	0.179	0.180	0.180	0.181	0.182	0.182	0.183	0.183	0.184	0.184
60	0.185	0.185	0.185	0.186	0.186	0.187	0.187	0.187	0.188	0.188
70	0.188	0.189	0.189	0.189	0.190	0.190	0.190	0.191	0.191	0.192
80	0.192	0.193	0.193	0.194	0.195	0.195	0.196	0.197	0.198	0.199
90	0.200	0.201	0.202	0.203	0.204	0.206	0.207	0.209	0.210	0.212

TABLE 6.23

J2 NUMBERS FOR 30% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.295	-0.280	-0.265	-0.252	-0.239	-0.227	-0.216	-0.205	-0.195	-0.186
20	-0.177	-0.169	-0.161	-0.154	-0.147	-0.141	-0.136	-0.130	-0.126	-0.121
30	-0.117	-0.113	-0.110	-0.107	-0.104	-0.101	-0.098	-0.096	-0.094	-0.092
40	-0.090	-0.088	-0.086	-0.084	-0.082	-0.081	-0.079	-0.077	-0.076	-0.074
50	-0.073	-0.071	-0.070	-0.069	-0.068	-0.066	-0.065	-0.064	-0.063	-0.062
60	-0.061	-0.060	-0.059	-0.058	-0.057	-0.056	-0.055	-0.054	-0.054	-0.053
70	-0.052	-0.051	-0.051	-0.050	-0.050	-0.049	-0.048	-0.048	-0.047	-0.047
80	-0.046	-0.046	-0.045	-0.045	-0.044	-0.044	-0.044	-0.043	-0.043	-0.043
90	-0.042	-0.042	-0.041	-0.041	-0.041	-0.040	-0.040	-0.040	-0.039	-0.039

TABLE 6.24

J3 NUMBERS FOR 30% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	0.0130	0.0123	0.0117	0.0110	0.0105	0.0099	0.0093	0.0088	0.0083	0.0078
20	0.0074	0.0069	0.0065	0.0061	0.0057	0.0054	0.0050	0.0047	0.0044	0.0041
30	0.0038	0.0036	0.0033	0.0031	0.0029	0.0027	0.0025	0.0023	0.0022	0.0020
40	0.0019	0.0017	0.0016	0.0015	0.0014	0.0013	0.0013	0.0012	0.0011	0.0011
50	0.0010	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0008
60	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
70	0.0007	0.0007	0.0007	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006
80	0.0005	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004
90	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003

TABLE 6.25

H1 NUMBERS FOR 40% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	0.604	0.614	0.624	0.633	0.641	0.648	0.655	0.661	0.667	0.672
20	0.676	0.680	0.684	0.687	0.690	0.692	0.694	0.696	0.698	0.699
30	0.701	0.702	0.703	0.703	0.704	0.705	0.706	0.707	0.708	0.708
40	0.709	0.710	0.710	0.711	0.712	0.712	0.713	0.714	0.714	0.715
50	0.715	0.716	0.716	0.717	0.718	0.718	0.719	0.719	0.719	0.720
60	0.720	0.721	0.721	0.722	0.722	0.722	0.723	0.723	0.723	0.724
70	0.724	0.724	0.725	0.725	0.725	0.726	0.726	0.726	0.726	0.727
80	0.727	0.727	0.727	0.728	0.728	0.728	0.728	0.729	0.729	0.729
90	0.729	0.730	0.730	0.730	0.730	0.730	0.731	0.731	0.731	0.731

TABLE 6.26

H2 NUMBERS FOR 40% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.081	-0.076	-0.071	-0.066	-0.062	-0.058	-0.054	-0.051	-0.048	-0.045
20	-0.043	-0.040	-0.038	-0.036	-0.035	-0.033	-0.032	-0.031	-0.030	-0.029
30	-0.028	-0.027	-0.026	-0.026	-0.025	-0.024	-0.023	-0.023	-0.022	-0.022
40	-0.021	-0.020	-0.020	-0.019	-0.019	-0.018	-0.018	-0.018	-0.017	-0.017
50	-0.016	-0.016	-0.016	-0.015	-0.015	-0.015	-0.014	-0.014	-0.014	-0.014
60	-0.014	-0.013	-0.013	-0.013	-0.013	-0.013	-0.012	-0.012	-0.012	-0.012
70	-0.012	-0.012	-0.012	-0.012	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011
80	-0.011	-0.011	-0.011	-0.011	-0.011	-0.010	-0.010	-0.010	-0.010	-0.010
90	-0.010	-0.010	-0.010	-0.010	-0.009	-0.009	-0.009	-0.009	-0.009	-0.009

TABLE 6.27

H3 NUMBERS FOR 40% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.0097	-0.0083	-0.0071	-0.0060	-0.0051	-0.0044	-0.0038	-0.0033	-0.0029	-0.0026
20	-0.0024	-0.0022	-0.0020	-0.0019	-0.0018	-0.0017	-0.0016	-0.0015	-0.0014	-0.0013
30	-0.0012	-0.0012	-0.0011	-0.0010	-0.0010	-0.0009	-0.0009	-0.0008	-0.0008	-0.0007
40	-0.0007	-0.0007	-0.0006	-0.0006	-0.0006	-0.0006	-0.0005	-0.0005	-0.0005	-0.0005
50	-0.0005	-0.0005	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0003	-0.0003
60	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0002	-0.0002
70	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
80	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0001	-0.0001
90	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001

TABLE 6.28

J1 NUMBERS FOR 40% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.248	-0.236	-0.225	-0.214	-0.205	-0.196	-0.187	-0.180	-0.173	-0.166
20	-0.160	-0.155	-0.150	-0.145	-0.141	-0.138	-0.134	-0.131	-0.128	-0.126
30	-0.124	-0.122	-0.120	-0.118	-0.116	-0.115	-0.113	-0.112	-0.110	-0.109
40	-0.108	-0.107	-0.105	-0.104	-0.103	-0.102	-0.101	-0.100	-0.099	-0.098
50	-0.098	-0.097	-0.096	-0.095	-0.095	-0.094	-0.094	-0.093	-0.092	-0.092
60	-0.091	-0.091	-0.090	-0.090	-0.090	-0.089	-0.089	-0.088	-0.088	-0.088
70	-0.087	-0.087	-0.087	-0.086	-0.086	-0.086	-0.085	-0.085	-0.085	-0.084
80	-0.084	-0.083	-0.083	-0.083	-0.082	-0.082	-0.081	-0.081	-0.080	-0.080
90	-0.079	-0.079	-0.078	-0.077	-0.077	-0.076	-0.075	-0.074	-0.073	-0.073

TABLE 6.29

J2 NUMBERS FOR 40% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.337	-0.290	-0.254	-0.229	-0.211	-0.200	-0.192	-0.186	-0.180	-0.174
20	-0.168	-0.163	-0.158	-0.153	-0.148	-0.143	-0.139	-0.134	-0.130	-0.126
30	-0.122	-0.119	-0.115	-0.111	-0.108	-0.105	-0.102	-0.099	-0.096	-0.093
40	-0.091	-0.088	-0.086	-0.084	-0.081	-0.079	-0.077	-0.075	-0.074	-0.072
50	-0.070	-0.069	-0.068	-0.066	-0.065	-0.064	-0.063	-0.062	-0.060	-0.060
60	-0.059	-0.058	-0.057	-0.056	-0.056	-0.055	-0.054	-0.054	-0.053	-0.053
70	-0.052	-0.052	-0.051	-0.051	-0.050	-0.050	-0.050	-0.049	-0.049	-0.048
80	-0.048	-0.048	-0.047	-0.047	-0.046	-0.046	-0.045	-0.045	-0.044	-0.043
90	-0.043	-0.042	-0.041	-0.041	-0.040	-0.039	-0.038	-0.037	-0.036	-0.035

TABLE 6.30

J3 NUMBERS FOR 40% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	0.0378	0.0206	0.0126	0.0101	0.0097	0.0093	0.0089	0.0085	0.0082	0.0078
20	0.0075	0.0072	0.0069	0.0066	0.0063	0.0060	0.0057	0.0055	0.0052	0.0050
30	0.0048	0.0045	0.0043	0.0041	0.0039	0.0037	0.0035	0.0034	0.0032	0.0030
40	0.0029	0.0027	0.0026	0.0024	0.0023	0.0022	0.0021	0.0020	0.0019	0.0018
50	0.0017	0.0016	0.0015	0.0014	0.0014	0.0013	0.0012	0.0012	0.0011	0.0011
60	0.0010	0.0010	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008
70	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007
80	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006
90	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004

TABLE 6.31

H1 NUMBERS FOR 50% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	0.465	0.476	0.486	0.495	0.503	0.510	0.515	0.520	0.524	0.528
20	0.531	0.533	0.536	0.537	0.539	0.541	0.542	0.544	0.545	0.546
30	0.548	0.549	0.550	0.551	0.553	0.554	0.555	0.556	0.557	0.558
40	0.559	0.560	0.561	0.561	0.562	0.563	0.564	0.564	0.565	0.566
50	0.566	0.567	0.567	0.568	0.569	0.569	0.569	0.570	0.570	0.571
60	0.571	0.571	0.572	0.572	0.572	0.573	0.573	0.573	0.573	0.574
70	0.574	0.574	0.574	0.575	0.575	0.575	0.575	0.575	0.575	0.575
80	0.576	0.576	0.576	0.576	0.576	0.576	0.576	0.577	0.577	0.577
90	0.577	0.577	0.577	0.577	0.578	0.578	0.578	0.578	0.578	0.579

TABLE 6.32

H2 NUMBERS FOR 50% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.072	-0.068	-0.065	-0.062	-0.058	-0.056	-0.053	-0.050	-0.048	-0.046
20	-0.043	-0.041	-0.040	-0.038	-0.036	-0.035	-0.033	-0.032	-0.030	-0.029
30	-0.028	-0.027	-0.026	-0.025	-0.025	-0.024	-0.023	-0.023	-0.022	-0.022
40	-0.021	-0.021	-0.020	-0.020	-0.020	-0.019	-0.019	-0.019	-0.018	-0.018
50	-0.018	-0.017	-0.017	-0.017	-0.016	-0.016	-0.016	-0.016	-0.015	-0.015
60	-0.015	-0.015	-0.014	-0.014	-0.014	-0.014	-0.013	-0.013	-0.013	-0.013
70	-0.013	-0.012	-0.012	-0.012	-0.012	-0.012	-0.011	-0.011	-0.011	-0.011
80	-0.011	-0.011	-0.011	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010
90	-0.010	-0.010	-0.010	-0.009	-0.009	-0.009	-0.009	-0.009	-0.009	-0.009

TABLE 6.33

H3 NUMBERS FOR 50% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.0086	-0.0077	-0.0069	-0.0061	-0.0054	-0.0048	-0.0043	-0.0038	-0.0034	-0.0030
20	-0.0027	-0.0024	-0.0022	-0.0020	-0.0018	-0.0017	-0.0016	-0.0015	-0.0014	-0.0014
30	-0.0013	-0.0012	-0.0012	-0.0011	-0.0011	-0.0010	-0.0010	-0.0009	-0.0009	-0.0008
40	-0.0008	-0.0008	-0.0007	-0.0007	-0.0007	-0.0006	-0.0006	-0.0006	-0.0006	-0.0005
50	-0.0005	-0.0005	-0.0005	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0004	-0.0003
60	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0002
70	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
80	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
90	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001

TABLE 6.34

J1 NUMBERS FOR 50% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.544	-0.530	-0.517	-0.505	-0.495	-0.485	-0.476	-0.467	-0.460	-0.453
20	-0.447	-0.442	-0.437	-0.433	-0.429	-0.425	-0.422	-0.420	-0.417	-0.415
30	-0.413	-0.411	-0.409	-0.407	-0.406	-0.404	-0.403	-0.401	-0.400	-0.398
40	-0.397	-0.395	-0.394	-0.393	-0.392	-0.391	-0.389	-0.388	-0.387	-0.386
50	-0.385	-0.384	-0.384	-0.383	-0.382	-0.381	-0.380	-0.380	-0.379	-0.378
60	-0.378	-0.377	-0.376	-0.376	-0.375	-0.375	-0.374	-0.374	-0.373	-0.373
70	-0.372	-0.372	-0.372	-0.371	-0.371	-0.370	-0.370	-0.370	-0.369	-0.369
80	-0.369	-0.368	-0.368	-0.368	-0.367	-0.367	-0.366	-0.366	-0.366	-0.365
90	-0.365	-0.365	-0.364	-0.364	-0.363	-0.363	-0.363	-0.362	-0.362	-0.361

TABLE 6.35

J2 NUMBERS FOR 50% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	-0.290	-0.247	-0.222	-0.209	-0.201	-0.195	-0.188	-0.182	-0.176	-0.170
20	-0.164	-0.159	-0.154	-0.149	-0.144	-0.139	-0.135	-0.130	-0.126	-0.122
30	-0.118	-0.114	-0.111	-0.107	-0.104	-0.101	-0.098	-0.095	-0.092	-0.090
40	-0.087	-0.085	-0.082	-0.080	-0.078	-0.076	-0.074	-0.073	-0.071	-0.069
50	-0.068	-0.067	-0.065	-0.064	-0.063	-0.062	-0.061	-0.060	-0.059	-0.058
60	-0.057	-0.057	-0.056	-0.055	-0.055	-0.054	-0.054	-0.053	-0.053	-0.052
70	-0.052	-0.052	-0.051	-0.051	-0.050	-0.050	-0.050	-0.049	-0.049	-0.048
80	-0.048	-0.048	-0.047	-0.047	-0.046	-0.046	-0.045	-0.044	-0.044	-0.043
90	-0.042	-0.042	-0.041	-0.040	-0.039	-0.038	-0.037	-0.035	-0.034	-0.033

TABLE 6.36

J3 NUMBERS FOR 50% CENSORED

N	0	1	2	3	4	5	6	7	8	9
10	0.0145	0.0146	0.0146	0.0145	0.0143	0.0139	0.0135	0.0131	0.0125	0.0120
20	0.0114	0.0108	0.0102	0.0097	0.0091	0.0086	0.0081	0.0076	0.0072	0.0068
30	0.0064	0.0060	0.0056	0.0052	0.0049	0.0046	0.0043	0.0040	0.0038	0.0035
40	0.0033	0.0031	0.0029	0.0027	0.0025	0.0023	0.0022	0.0020	0.0019	0.0018
50	0.0017	0.0016	0.0015	0.0015	0.0014	0.0013	0.0013	0.0012	0.0012	0.0012
60	0.0012	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0012
70	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012	0.0013	0.0013
80	0.0013	0.0012	0.0012	0.0012	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010
90	0.0009	0.0009	0.0008	0.0007	0.0006	0.0005	0.0004	0.0004	0.0004	0.0004

TABLE 6.37

H1 NUMBERS FOR 60% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	0.397	0.399	0.402	0.404	0.406	0.408	0.410	0.411	0.413	0.414
30	0.415	0.416	0.417	0.418	0.419	0.420	0.421	0.421	0.422	0.423
40	0.423	0.424	0.425	0.425	0.426	0.426	0.427	0.427	0.428	0.428
50	0.429	0.429	0.430	0.430	0.431	0.431	0.431	0.432	0.432	0.432
60	0.433	0.433	0.433	0.434	0.434	0.434	0.435	0.435	0.435	0.435
70	0.436	0.436	0.436	0.436	0.436	0.437	0.437	0.437	0.437	0.437
80	0.438	0.438	0.438	0.438	0.438	0.439	0.439	0.439	0.439	0.439
90	0.440	0.440	0.440	0.440	0.441	0.441	0.441	0.441	0.442	0.442

TABLE 6.38

H2 NUMBERS FOR 60% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	-0.044	-0.042	-0.040	-0.039	-0.038	-0.036	-0.035	-0.034	-0.033	-0.032
30	-0.031	-0.030	-0.029	-0.028	-0.027	-0.027	-0.026	-0.025	-0.024	-0.024
40	-0.023	-0.022	-0.022	-0.021	-0.021	-0.020	-0.020	-0.019	-0.019	-0.018
50	-0.018	-0.017	-0.017	-0.017	-0.016	-0.016	-0.016	-0.015	-0.015	-0.015
60	-0.015	-0.014	-0.014	-0.014	-0.014	-0.014	-0.013	-0.013	-0.013	-0.013
70	-0.013	-0.013	-0.013	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012	-0.012
80	-0.012	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	-0.010
90	-0.010	-0.010	-0.010	-0.010	-0.009	-0.009	-0.009	-0.009	-0.009	-0.008

TABLE 6.39

H3 NUMBERS FOR 60% CENSORED.

N	0	1	2	3	4	5	6	7	8	9
20	-0.0023	-0.0021	-0.0019	-0.0017	-0.0015	-0.0014	-0.0013	-0.0012	-0.0011	-0.0010
30	-0.0009	-0.0009	-0.0008	-0.0008	-0.0007	-0.0007	-0.0007	-0.0006	-0.0006	-0.0006
40	-0.0006	-0.0006	-0.0005	-0.0005	-0.0005	-0.0005	-0.0005	-0.0004	-0.0004	-0.0004
50	-0.0004	-0.0004	-0.0004	-0.0004	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003	-0.0003
60	-0.0003	-0.0003	-0.0003	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
70	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
80	-0.0002	-0.0002	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
90	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001

TABLE 6.40

J1 NUMBERS FOR 60% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	-0.740	-0.750	-0.747	-0.744	-0.742	-0.739	-0.736	-0.734	-0.731	-0.729
30	-0.727	-0.725	-0.723	-0.721	-0.719	-0.717	-0.715	-0.713	-0.712	-0.710
40	-0.709	-0.707	-0.706	-0.704	-0.703	-0.702	-0.701	-0.700	-0.699	-0.698
50	-0.697	-0.696	-0.695	-0.694	-0.693	-0.693	-0.692	-0.691	-0.691	-0.690
60	-0.689	-0.689	-0.688	-0.688	-0.687	-0.687	-0.686	-0.686	-0.686	-0.685
70	-0.685	-0.684	-0.684	-0.684	-0.683	-0.683	-0.682	-0.682	-0.682	-0.681
80	-0.681	-0.680	-0.680	-0.680	-0.679	-0.679	-0.678	-0.678	-0.677	-0.676
90	-0.676	-0.675	-0.674	-0.674	-0.673	-0.672	-0.671	-0.671	-0.670	-0.669

TABLE 6.41

J2 NUMBERS FOR 60% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	-0.183	-0.172	-0.162	-0.153	-0.144	-0.137	-0.131	-0.125	-0.120	-0.115
30	-0.111	-0.108	-0.104	-0.102	-0.099	-0.097	-0.095	-0.093	-0.091	-0.089
40	-0.087	-0.085	-0.083	-0.082	-0.080	-0.079	-0.077	-0.075	-0.074	-0.073
50	-0.071	-0.070	-0.069	-0.067	-0.066	-0.065	-0.064	-0.063	-0.062	-0.061
60	-0.060	-0.059	-0.058	-0.057	-0.056	-0.055	-0.054	-0.054	-0.053	-0.052
70	-0.051	-0.051	-0.050	-0.049	-0.049	-0.048	-0.048	-0.047	-0.047	-0.046
80	-0.046	-0.045	-0.045	-0.044	-0.044	-0.043	-0.043	-0.043	-0.042	-0.042
90	-0.041	-0.041	-0.041	-0.040	-0.040	-0.039	-0.039	-0.039	-0.038	-0.038

TABLE 6.42

J3 NUMBERS FOR 60% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	0.0185	0.0164	0.0146	0.0130	0.0116	0.0104	0.0094	0.0086	0.0079	0.0073
30	0.0068	0.0064	0.0060	0.0058	0.0055	0.0053	0.0051	0.0049	0.0047	0.0045
40	0.0043	0.0042	0.0040	0.0038	0.0037	0.0035	0.0034	0.0033	0.0031	0.0030
50	0.0029	0.0028	0.0027	0.0026	0.0025	0.0024	0.0023	0.0022	0.0022	0.0021
60	0.0020	0.0020	0.0019	0.0018	0.0018	0.0017	0.0017	0.0016	0.0016	0.0016
70	0.0015	0.0015	0.0015	0.0014	0.0014	0.0014	0.0014	0.0013	0.0013	0.0013
80	0.0013	0.0012	0.0012	0.0012	0.0012	0.0012	0.0011	0.0011	0.0011	0.0011
90	0.0010	0.0010	0.0010	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0007

TABLE 6.43

H1 NUMBERS FOR 70% CENSORED

N.	0	1	2	3	4	5	6	7	8	9
20	0.278	0.280	0.283	0.285	0.287	0.289	0.290	0.291	0.292	0.293
30	0.294	0.295	0.2952	0.296	0.297	0.297	0.298	0.299	0.299	0.300
40	0.300	0.301	0.302	0.302	0.303	0.303	0.304	0.304	0.305	0.305
50	0.306	0.306	0.307	0.307	0.307	0.308	0.308	0.309	0.309	0.309
60	0.310	0.310	0.310	0.311	0.311	0.311	0.312	0.312	0.312	0.312
70	0.313	0.313	0.313	0.313	0.314	0.314	0.314	0.314	0.314	0.315
80	0.315	0.315	0.315	0.315	0.315	0.315	0.316	0.316	0.316	0.316
90	0.316	0.316	0.316	0.316	0.316	0.316	0.316	0.317	0.317	0.318

TABLE 6.44

H2 NUMBERS FOR 70% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	-0.045	-0.043	-0.041	-0.039	-0.037	-0.036	-0.034	-0.033	-0.031	-0.030
30	-0.029	-0.028	-0.027	-0.026	-0.025	-0.024	-0.024	-0.023	-0.022	-0.022
40	-0.021	-0.021	-0.021	-0.020	-0.020	-0.020	-0.019	-0.019	-0.019	-0.019
50	-0.018	-0.018	-0.018	-0.018	-0.017	-0.017	-0.017	-0.017	-0.016	-0.016
60	-0.016	-0.016	-0.015	-0.015	-0.015	-0.015	-0.014	-0.014	-0.014	-0.014
70	-0.014	-0.013	-0.013	-0.013	-0.013	-0.013	-0.012	-0.012	-0.012	-0.012
80	-0.012	-0.012	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	-0.010
90	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010

TABLE 6.45

H3 NUMBERS FOR 70% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	-0.0032	-0.0028	-0.0026	-0.0023	-0.0022	-0.0020	-0.0019	-0.0018	-0.0017	-0.0016
30	-0.0015	-0.0014	-0.0013	-0.0012	-0.0012	-0.0011	-0.0010	-0.0010	-0.0009	-0.0008
40	-0.0008	-0.0007	-0.0007	-0.0006	-0.0006	-0.0005	-0.0005	-0.0004	-0.0004	-0.0004
50	-0.0004	-0.0003	-0.0003	-0.0003	-0.0003	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
60	-0.0002	-0.0002	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
70	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
80	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
90	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001

TABLE 6.46

J1 NUMBERS FOR 70% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	-1.138	-1.122	-1.113	-1.109	-1.106	-1.104	-1.102	-1.100	-1.098	-1.097
30	-1.095	-1.093	-1.091	-1.090	-1.088	-1.087	-1.085	-1.084	-1.083	-1.081
40	-1.080	-1.079	-1.078	-1.077	-1.076	-1.075	-1.074	-1.073	-1.072	-1.071
50	-1.070	-1.069	-1.068	-1.068	-1.067	-1.066	-1.065	-1.065	-1.064	-1.063
60	-1.063	-1.062	-1.061	-1.061	-1.060	-1.060	-1.059	-1.058	-1.058	-1.057
70	-1.057	-1.056	-1.055	-1.055	-1.054	-1.054	-1.053	-1.052	-1.052	-1.051
80	-1.050	-1.050	-1.049	-1.048	-1.048	-1.047	-1.046	-1.045	-1.044	-1.043
90	-1.043	-1.042	-1.041	-1.040	-1.039	-1.037	-1.036	-1.035	-1.034	-1.033

TABLE 6.47

J2 NUMBERS FOR 70% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	-0.168	-0.162	-0.155	-0.150	-0.144	-0.138	-0.133	-0.128	-0.124	-0.119
30	-0.115	-0.111	-0.107	-0.103	-0.099	-0.096	-0.093	-0.090	-0.087	-0.084
40	-0.082	-0.079	-0.077	-0.075	-0.073	-0.071	-0.070	-0.068	-0.067	-0.065
50	-0.064	-0.063	-0.062	-0.061	-0.060	-0.059	-0.058	-0.057	-0.057	-0.056
60	-0.055	-0.055	-0.054	-0.054	-0.053	-0.053	-0.052	-0.052	-0.052	-0.051
70	-0.051	-0.050	-0.050	-0.049	-0.049	-0.048	-0.048	-0.047	-0.046	-0.046
80	-0.045	-0.044	-0.043	-0.042	-0.041	-0.040	-0.039	-0.038	-0.037	-0.036
90	-0.036	-0.035	-0.035	-0.034	-0.034	-0.034	-0.035	-0.035	-0.036	-0.037

TABLE 6.48

J3 NUMBERS FOR 70% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	0.0211	0.0197	0.0184	0.0171	0.0159	0.0148	0.0138	0.0128	0.0118	0.0110
30	0.0102	0.0094	0.0087	0.0080	0.0074	0.0069	0.0064	0.0059	0.0055	0.0051
40	0.0047	0.0044	0.0041	0.0039	0.0036	0.0034	0.0033	0.0031	0.0030	0.0029
50	0.0028	0.0027	0.0027	0.0026	0.0026	0.0025	0.0025	0.0025	0.0025	0.0025
60	0.0024	0.0024	0.0024	0.0024	0.0023	0.0023	0.0023	0.0022	0.0022	0.0022
70	0.0021	0.0021	0.0021	0.0020	0.0020	0.0019	0.0019	0.0018	0.0018	0.0018
80	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0015	0.0014	0.0014	0.0013
90	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010	0.0010

TABLE 6.49

H1 NUMBERS FOR 80% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	0.155	0.158	0.160	0.163	0.165	0.167	0.169	0.171	0.172	0.174
30	0.175	0.177	0.178	0.179	0.180	0.181	0.182	0.183	0.183	0.184
40	0.185	0.185	0.186	0.186	0.187	0.188	0.188	0.189	0.189	0.190
50	0.190	0.190	0.191	0.191	0.192	0.192	0.192	0.193	0.193	0.193
60	0.194	0.194	0.194	0.195	0.195	0.195	0.196	0.196	0.196	0.196
70	0.196	0.197	0.197	0.197	0.197	0.197	0.198	0.198	0.198	0.198
80	0.198	0.199	0.199	0.199	0.199	0.199	0.199	0.199	0.200	0.200
90	0.200	0.200	0.200	0.200	0.201	0.201	0.201	0.201	0.201	0.201

TABLE 6.50

H2 NUMBERS FOR 80% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	-0.043	-0.037	-0.033	-0.031	-0.031	-0.030	-0.030	-0.029	-0.029	-0.028
30	-0.028	-0.027	-0.027	-0.026	-0.026	-0.025	-0.025	-0.024	-0.024	-0.024
40	-0.023	-0.023	-0.022	-0.022	-0.022	-0.021	-0.021	-0.020	-0.020	-0.020
50	-0.019	-0.019	-0.019	-0.018	-0.018	-0.018	-0.017	-0.017	-0.017	-0.016
60	-0.016	-0.016	-0.015	-0.015	-0.015	-0.015	-0.014	-0.014	-0.014	-0.014
70	-0.013	-0.013	-0.013	-0.013	-0.013	-0.012	-0.012	-0.012	-0.012	-0.012
80	-0.012	-0.012	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011	-0.011
90	-0.011	-0.011	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010	-0.010

TABLE 6.51

H3 NUMBERS FOR 80% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	-0.0048	-0.0046	-0.0043	-0.0040	-0.0038	-0.0036	-0.0033	-0.0031	-0.0029	-0.0027
30	-0.0025	-0.0024	-0.0022	-0.0020	-0.0019	-0.0017	-0.0016	-0.0015	-0.0013	-0.0012
40	-0.0011	-0.0010	-0.0009	-0.0008	-0.0007	-0.0007	-0.0006	-0.0005	-0.0005	-0.0004
50	-0.0004	-0.0003	-0.0003	-0.0003	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
60	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
70	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
80	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0001	-0.0001	-0.0001	-0.0001
90	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001

TABLE 6.52

J1 NUMBERS FOR 80% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	-1.661	-1.650	-1.640	-1.631	-1.623	-1.617	-1.611	-1.605	-1.601	-1.597
30	-1.593	-1.590	-1.588	-1.585	-1.583	-1.581	-1.579	-1.577	-1.575	-1.574
40	-1.572	-1.570	-1.568	-1.567	-1.565	-1.564	-1.562	-1.561	-1.559	-1.558
50	-1.557	-1.556	-1.554	-1.553	-1.552	-1.551	-1.550	-1.549	-1.548	-1.547
60	-1.546	-1.545	-1.544	-1.543	-1.542	-1.541	-1.541	-1.540	-1.539	-1.538
70	-1.538	-1.537	-1.536	-1.536	-1.535	-1.535	-1.534	-1.533	-1.533	-1.532
80	-1.532	-1.531	-1.531	-1.530	-1.530	-1.529	-1.529	-1.528	-1.528	-1.527
90	-1.527	-1.526	-1.526	-1.525	-1.525	-1.524	-1.524	-1.523	-1.523	-1.522

TABLE 6.53

J2 NUMBERS FOR 80% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	-0.142	-0.139	-0.137	-0.134	-0.131	-0.128	-0.125	-0.122	-0.119	-0.116
30	-0.113	-0.110	-0.107	-0.104	-0.101	-0.098	-0.095	-0.092	-0.089	-0.087
40	-0.084	-0.082	-0.079	-0.077	-0.075	-0.073	-0.071	-0.069	-0.067	-0.066
50	-0.064	-0.063	-0.061	-0.060	-0.059	-0.058	-0.056	-0.055	-0.055	-0.054
60	-0.053	-0.052	-0.051	-0.051	-0.050	-0.050	-0.049	-0.049	-0.048	-0.048
70	-0.047	-0.047	-0.047	-0.046	-0.046	-0.046	-0.046	-0.045	-0.045	-0.045
80	-0.044	-0.044	-0.044	-0.044	-0.043	-0.043	-0.042	-0.042	-0.042	-0.041
90	-0.041	-0.040	-0.039	-0.039	-0.038	-0.037	-0.036	-0.035	-0.035	-0.033

TABLE 6.54

J3 NUMBERS FOR 80% CENSORED

N	0	1	2	3	4	5	6	7	8	9
20	0.0143	0.0140	0.0137	0.0134	0.0131	0.0127	0.0124	0.0121	0.0118	0.0115
30	0.0111	0.0108	0.0105	0.0101	0.0098	0.0095	0.0091	0.0088	0.0085	0.0082
40	0.0078	0.0075	0.0072	0.0069	0.0066	0.0063	0.0060	0.0058	0.0055	0.0052
50	0.0050	0.0048	0.0045	0.0043	0.0041	0.0039	0.0037	0.0036	0.0034	0.0033
60	0.0032	0.0030	0.0029	0.0028	0.0028	0.0027	0.0026	0.0026	0.0025	0.0025
70	0.0025	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024
80	0.0024	0.0024	0.0024	0.0024	0.0023	0.0023	0.0023	0.0023	0.0023	0.0022
90	0.0022	0.0021	0.0021	0.0020	0.0019	0.0019	0.0018	0.0016	0.0015	0.0014

buses in the study, N. To determine which tables to use, first determine what percent of the cases are censored. In the example, two cases are censored out of 14 or 14.3 percent (2 divided by 14 is 0.143). The tables give values for H1, H2, H3, J1, J2, and J3 which are 10, 20, 30, 40, 50, 60, 70, and 80 percent censored. In this example 14.3 percent falls between 10 and 20 percent. Therefore, look up the numbers for both 10 and 20 percent under N=14 and estimate the 14.3 percent number by using a technique called interpolation.

Below are listed the numbers from the 10 and 20 percent tables (Tables 6.7 to 6.18) for N=14.

Number	10% Censored	20% Censored
H1	1.261	1.008
H2	-0.053	-0.051
H3	-0.0084	-0.0077
J1	0.360	0.360
J2	-0.226	-0.247
J3	0.0031	0.0104

The next step is to determine the five numbers for 14.3 percent censored. To interpolate a number between the two numbers given, use the equation below:

$$\frac{\left(\begin{matrix}\text{Lower}\\ \text{Percentage}\end{matrix} - \begin{matrix}\text{Desired}\\ \text{Percent}\end{matrix}\right) \times \left(\begin{matrix}\text{Lower Percent}\\ \text{Number}\end{matrix} - \begin{matrix}\text{Higher Percent}\\ \text{Number}\end{matrix}\right)}{10.0} + \begin{matrix}\text{Lower Percent}\\ \text{Number}\end{matrix}$$

For H1:

Desired percent = 14.3
Lower percentage = 10
H1 for lower percent number = 1.261
H1 for higher percent number = 1.008

The interpolated number is:

$$H1 = \frac{(10 - 14.3) \times (1.261 - 1.008)}{10.0} + 1.261 = 1.152$$

Thus, the value of E1 for 14.3 percent censored data is 1.152. The calculations for the remaining four numbers are similar and are shown below:

$$H2 = \frac{(10 - 14.3) \times (-0.053 - -0.051)}{10.0} + -0.053 = -0.052$$

$$H3 = \frac{(10 - 14.3) \times (-0.0084 - -0.0077)}{10.0} + -0.0084 = -0.0081$$

$$J1 = \frac{(10 - 14.3) \times (0.360 - 0.360)}{10.0} + 0.360 = 0.360$$

$$J2 = \frac{(10 - 14.3) \times (-0.226 - -0.247)}{10.0} + -0.226 = -0.235$$

$$J3 = \frac{(10 - 14.3) \times (0.0031 - 0.0104)}{10.0} + 0.0031 = 0.0062$$

To calculate B insert the values for H1, H2, and H3 into the equation below:

$$M = \ = H1 + H2 \times NS + H3 \times NS^2$$

where NS = number of suspended cases

For the transmission example:

$$M = 1.152 + -0.052(2) + -0.0081(2)^2 = 1.016$$

To get B insert M and S (from Table 6.5) into the equation below:

$$B = \frac{M \times N}{S}$$

For the transmission example:

$$B = \frac{1.016 \times 14}{7.6692} = 1.855$$

To get the remaining Weibull constant, T, first determine P by entering the values for J1, J2 and J3 into the equation below:

$$P = J1 + J2 \times NS + J3 \times NS^2$$

For the transmission example:

$$P = 0.360 + -0.235(2) + 0.0062(2)^2 = -0.085$$

To get T the values for P, B, K_{max} and its corresponding ratio are entered into the equation below:

$$\ln(T) = \ln\left(\frac{K_{max}}{ratio}\right) - \frac{P}{B}$$

For the transmission example:

$$\ln(T) = \ln\left(\frac{51.795}{1.095}\right) - \frac{-0.085}{1.855} = 10.7185$$

T = 45,184 (on a TI-35 calculator, INV LN (10.7185) is 45,184).

Mean and Standard Deviation. As in Chapters Four and Five the mean and standard deviation of the failed transmission data cannot be directly computed because of the presence of censored and suspended cases. However, the same technique as used in those two chapters can be used to estimate the mean and standard deviation for the transmissions in this chapter.

Table 6.55 (Work Sheet 13) provides space for the computations. The estimated mean mileage to failure, $\bar{X}$, is found by using the previously determined Weibull distribution factors, D, B, and T. First, find the value of E1 in Table 4.12 which corresponds to the value of B rounded off to just one decimal point. B is 1.855, or 1.9 rounded off, and the corresponding number, E1, in Table 4.12 is 1.1269. E1 is used to find $\bar{X}$ in the following equation:

$$\bar{X} = \frac{T}{E1} + D$$

$$= \frac{45{,}184 \text{ miles}}{1.1269} + 39{,}135 \text{ miles}$$

$$\bar{X} = 79{,}231 \text{ miles}$$

The estimated mean, $\bar{X}$ = 79,231 miles, is slightly higher than the average of the ten failed transmission mileages in Table 6.1, 69,306 miles. The difference is the estimated impact on the mean when the censored and suspended cases eventually fail.

The estimated standard deviation, SD, is found by first finding the number, E2, in Table 4.13 which corresponds to B = 1.9. The value for E2 is 0.2360. This number is then used in the following equation to find the estimated standard deviation:

TABLE 6.55

WORK SHEET 13

COMPUTING THE ESTIMATED MEAN MILEAGE TO FAILURE AND THE STANDARD DEVIATION FOR WEIBULL FAILURE DATA WITH CENSORED AND SUSPENDED CASES

Cost Driver Transmissions
Bus Model Bluebird
Component Type Detroit Diesel Allison MT 643
Study Dates 1981-1983

INPUT NUMBERS:

D = 39,135 miles (minimum life term)

B = 1.855 (Weibull shape factor)

T = 45,184 miles (Weibull characteristic life factor)

E1 = 1.1269 (from Table 4.12)

E2 = 0.2360 (from Table 4.13)

ESTIMATED MEAN MILEAGE TO FAILURE, $\bar{X}$:

$$\bar{X} = \underset{T}{\underline{45{,}184\text{ mi.}}} \;/\; \underset{E1}{\underline{1.1269}} + \underset{D}{\underline{39{,}135\text{ mi.}}} = \bar{X} = \underline{79{,}231\text{ mi.}}$$

ESTIMATED STANDARD DEVIATION, SD:

$$SD = \sqrt{\underset{E2}{\underline{0.2360}} \times \left(\underset{T}{\underline{45{,}184}}\right)^{2}} = SD = \underline{21{,}950\text{ mi.}}$$

$$\begin{aligned} SD &= \sqrt{E2 \times T^2} \\ &= \sqrt{0.2360 \times (45{,}184)^2} \\ &= \sqrt{481{,}816{,}150} \\ SD &= 21{,}950 \text{ miles} \end{aligned}$$

This number compares to the standard deviation for the ten transmission failures of 15,493 miles.

Summary. The values for the Weibull distribution, D, B, and T, the estimated mean mileage to failure, $\bar{X}$, and the estimated standard deviation, SD, are entered in the summary Table 6.56 (Work Sheet 3). A frequency bar chart of the 10 transmission failures is included. The Weibull formula is plotted as a smooth curve in Figure 6.1 using the values for D, B, and T computed above.

The steps required to find the Weibull constants, D, B, and T, and $\bar{X}$ and SD, are recapped as follows:

1. Tabulate the component failure mileages and the mileages accumulated by the buses with unfailed original components in Work Sheet 7 (Table 6.1). The unfailed components should be classified as either censored or suspended cases. The following steps assume that the data set contains both types of cases.

2. List the failed and unfailed component mileages in increasing order by mileage in Work Sheet 8 (Table 6.2).

3. Compute the weights and adjusted order numbers for all of the components in Work Sheet 14 (Table 6.3).

4. Use Work Sheets 9 and 9a (Tables 6.4 and 6.5) to calculate the intermediate number, S, using the adjusted order failure mileages and D. The minimum life term, D, is 0.90 multiplied by the smallest failure mileage.

5. Determine the shape factor, B, and the characteristic life factor, T, by following the computational outline in Work Sheet 15 (Table 6.6).

6. Determine the estimated mean mileage to failure, $\bar{X}$, and the estimated standard deviation, SD, using Work Sheet 13 (Table 6.55).

7. Enter the values for D, B, T, $\bar{X}$, and SD in summary Work Sheet 3 (Table 6.56). The Weibull distribution computation is now complete.

What is the Appropriate Maintenance Policy for the Transmissions?

A determination of the failure mechanism and the best maintenance policy for the transmissions can be made with the summary

TABLE 6.56

WORK SHEET 3

SUMMARY SHEET OF FAILURE MILEAGE DISTRIBUTION

Cost Driver Transmissions Bus Model Bluebird

Component Type Det. Diesel A. MT643 Study Dates 1981-1983

Number of cases, N 14 Mean, $\bar{X}$ 79,231 miles

Number of failures, NF 10 Std. deviation, SD 21,950 miles

Maximum mileage not applicable $(\bar{X}-D)/SD$ 1.83

Minimum mileage 43,483 miles

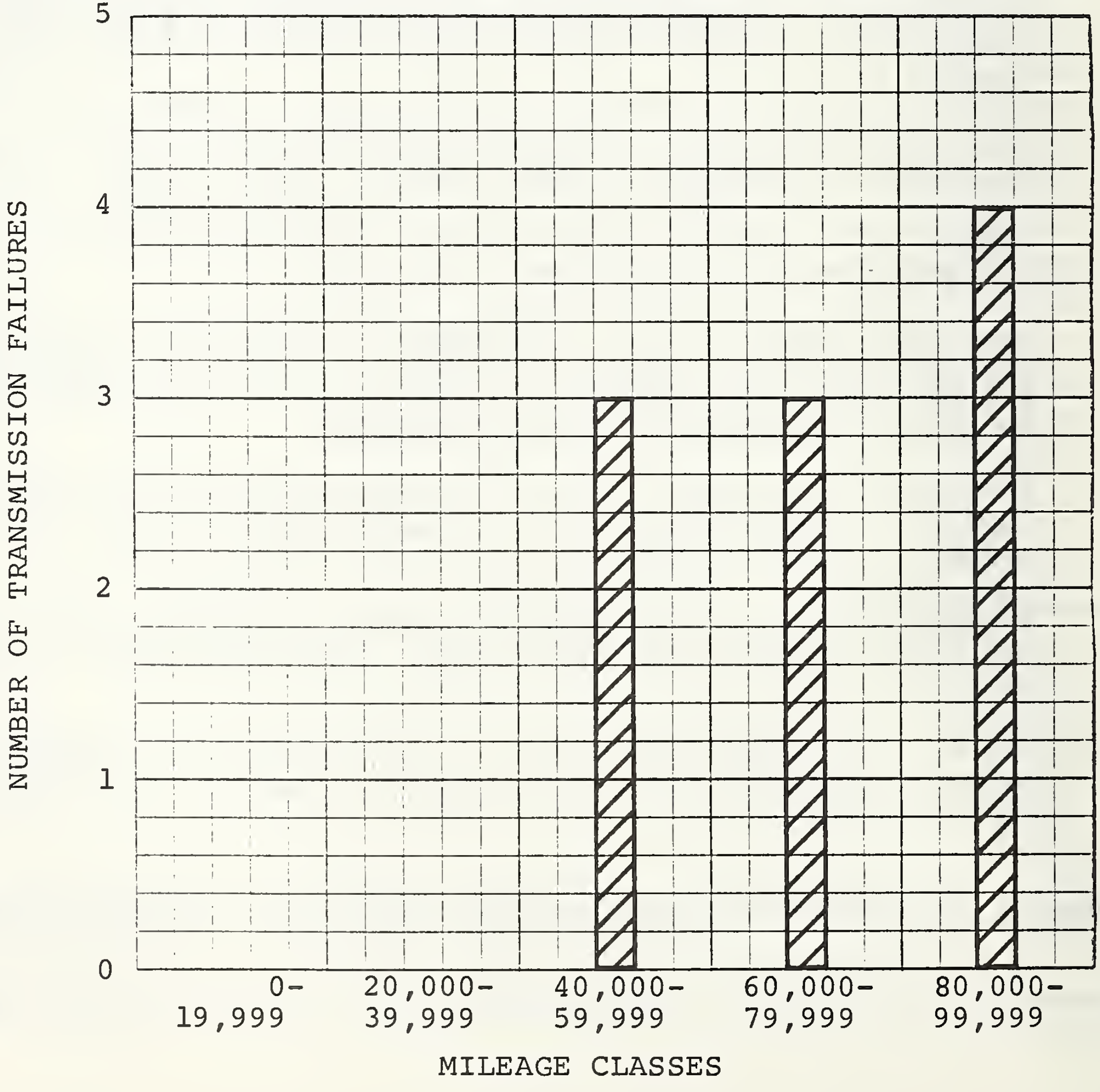
WEIBULL DISTRIBUTION PARAMETERS:

Minimum life term, D 39,135 miles

Shape factor, B 1.855

Characteristic life factor, T . 45,184 miles

FAILURE MECHANISM: "Mileage to Failure Unpredictable"



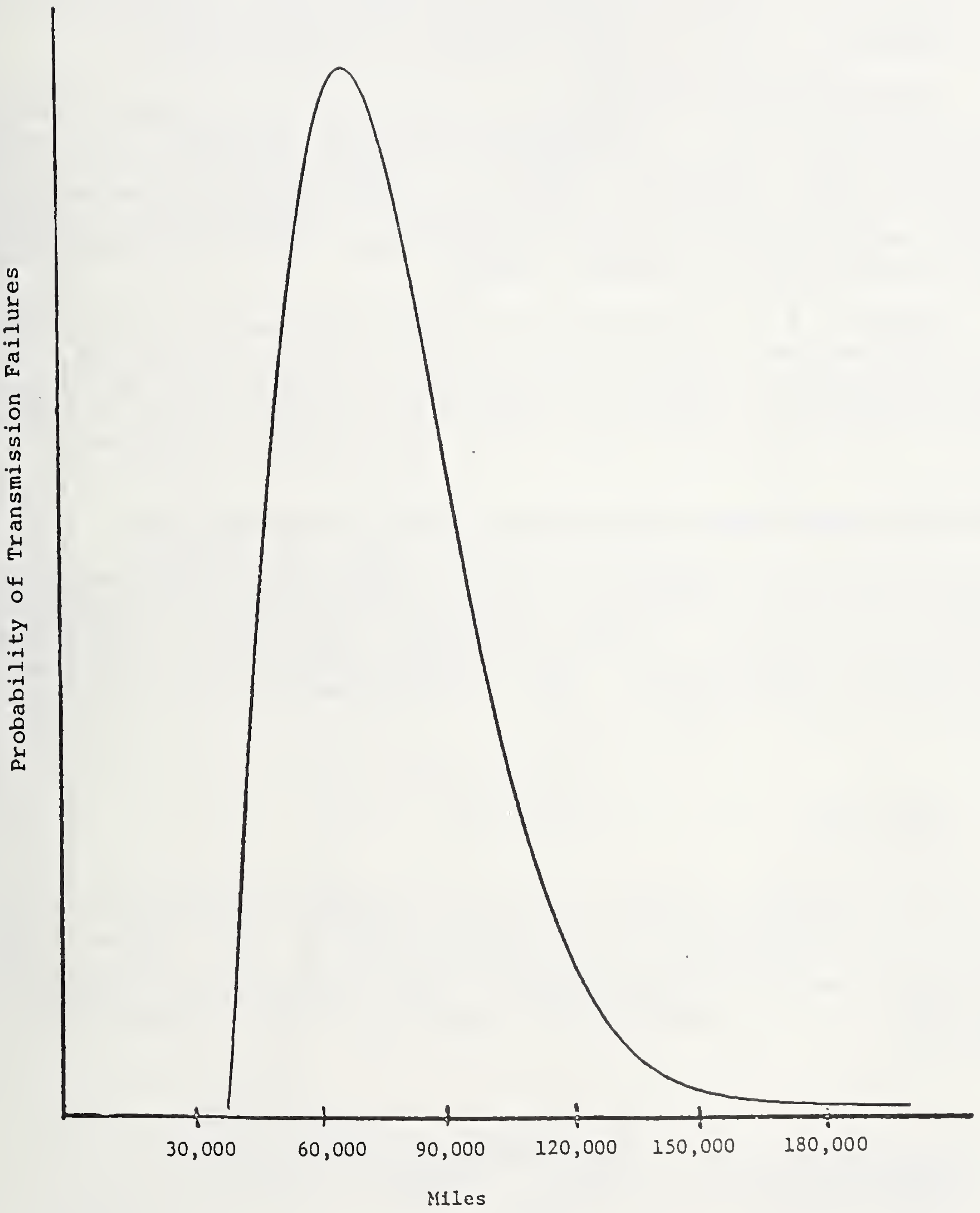


FIGURE 6.1

FITTED WEIBULL PROBABILITY CURVE FOR THE TRANSMISSION FAILURES data of Table 6.56 (Work Sheet 3). Dividing ($\bar{X}$ - D) by SD yields a value of 1.83. B, of course, is 1.855, which also is less than 2.0. Based on Table 1.1 from Chapter One, the failure mechanism is determined to be "mileage to failure unpredictable."

The appropriate maintenance policy for the transmissions is found in Figure 1.6. If the mean mileage to failure, $\bar{X}$, is about as expected and the occurrence of transmission failures is detectable, then, based on Figure 1.6, the preferred maintenance policy is "condition-based maintenance." "Operate-until-failure" is the second choice. This was the same conclusion reached for the transmissions of Chapter Three.

However, $\bar{X}$ is low relative to the performance of other types of transmissions. If $\bar{X}$ is treated as "less than expected," then the preferred maintenance policy based on Figure 1.6 is "design-out-maintenance." The data suggests that there should be further investigation into the cause of the transmission failures.

Predicting the Future Reliability of the Transmissions

The future reliability of the transmissions is determined by working with the cumulative percent of transmissions which are expected to fail by a certain mileage. The curve in Figure 6.2 is the fitted Weibull distribution representation of the cumulative failure percentages. The formula used to calculate points along the curve is shown below:

$$\text{cumulative probability} = 1 - e^{\left(-\left[\frac{M - D}{T}\right]^{B}\right)}$$

As an example of the use of this formula consider what percent of the transmissions can be expected to fail before 60,000 miles. As determined in the previous section: D = 39,135 miles, B = 1.855, and T = 45,184 miles. Thus, the portion (cumulative probability) of transmissions expected to fail before 60,000 miles is:

$$\text{cumulative probability} = 1 - e^{\left(-\left[\frac{60{,}000 - 39{,}135}{45{,}184}\right]^{1.855}\right)} = 0.2122$$

An explanation and pictures of how to work out this formula with a scientific calculator is shown in Appendix B. From the above calculation it is thus expected that 21.22 percent of the transmissions will fail by the time they accumulate 60,000 miles. In the example of this chapter, this means that three out of the 14 original transmissions (14 x 0.2122 = 2.97) are predicted to fail

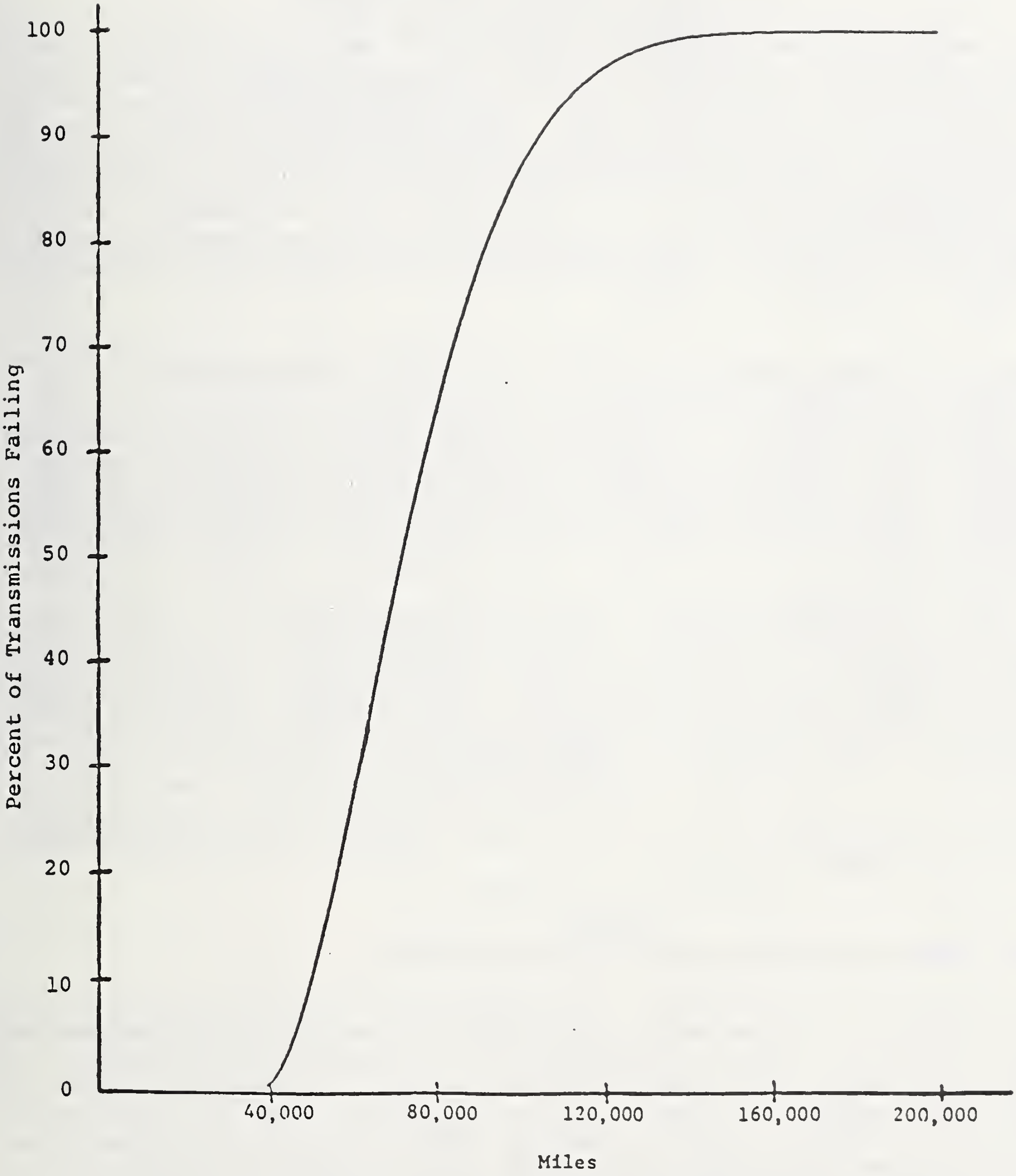


FIGURE 6.2

CUMULATIVE WEIBULL CURVE OF TRANSMISSION FAILURES by the time the buses have accumulated an average of 60,000 miles. In Table 6.1, note that three transmissions did fail before they accumulated 60,000 miles.

The reliability curve for the transmissions is computed from the cumulative percentages of Table 6.57 (Work Sheet 11). Cumulative mileage intervals of 15,000 miles were used in Table 6.57, starting with the minimum life, D = 39,135 miles, and extending to greater than 135,000 miles. The cumulative probabilities in column 2 are computed by using the above cumulative probability formula for the fitted Weibull distribution. These are converted to percentages in column 3. The reliability, or percent surviving, is computed in column 4 by subtracting the cumulative percentages of column 3 from 100 percent. The resulting reliability curve is depicted in Figure 6.3.

How Many Transmissions Will Fail During Any Time Period?

Consider a fleet of 60 new buses which will average 30,000 annual miles per bus. Based on the information from the previous section it is predicted that about 13 new transmissions (60 x 0.2122 = 12.73) will have failed by the time the buses have accumulated 60,000 miles each, which will occur at the end of the second year.

Now consider how many original transmissions will fail during each year. In Table 6.57 each cumulative 15,000 mile increment is equivalent to six months of bus life. Hence, 120,000 miles is equivalent to four years. The expected number of transmissions failing in each mileage or time interval is presented in column 6. Note that no transmissions are predicted to fail in the first 39,135 miles of service (about 18 months), which is the minimum life term. All but one of the transmissions are predicted to have failed after four years and six months or 135,000 miles of use.

What About the Replacement Transmissions?

Table 6.57 predicts the number of original transmission failures that can be expected each year but it does not include the failures of their replacements. A satisfactory estimate of the total number of transmission failures (including failures of the replacements) to expect during any period can be obtained as follows. Take the number of miles a bus will travel during a period, divide it by the estimated mean mileage to failure, and then multiply the result by the total number of buses. It is assumed that the replacement transmissions last as long as the originals.

To find the total number of transmission failures (originals and replacements) per year, first divide the total number of miles accumulated in a year (30,000 miles per bus per year) by

TABLE 6.57

WORK SHEET 11

COMPONENT RELIABILITY ANALYSIS FOR WEIBULL FAILURE DATA

Cost Driver Transmissions — Bus Model Bluebird

Component Type Detroit Diesel Allison MT 643 — Study Dates 1981-1983

Cumulative Mileage, M (1)	Cumulative Probability of Failure[1] (2)	Cumulative Percentage[2] (3)	Reliability,[3] R (% Surviving) (4)	Percent Failing in Mileage Interval[4] (5)	Number Failing in Mileage Interval[5] (6)
39,135	0.0	0.0 %	100.0 %	%	
45,000	0.0224	2.2 %	97.8 %	2.2 %	1.3
60,000	0.2122	21.2 %	78.8 %	19.0 %	11.4
75,000	0.4787	47.9 %	52.1 %	26.7 %	16.0
90,000	0.7123	71.2 %	28.8 %	23.3 %	14.0
105,000	0.8663	86.6 %	13.4 %	15.4 %	9.2
120,000	0.9473	94.7 %	5.3 %	8.1 %	4.9
135,000	0.9823	98.2 %	1.8 %	3.5 %	2.1
>135,000	1.0000	100.0 %	0.0 %	1.8 %	1.1
		%	%	——— %	———
		%	%	100.0 %	60.0
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	

[1] Cumulative Probability

$$= 1 - e^{\left(-\left[\frac{M - D}{T}\right]^{B}\right)}$$

$$= 1 - e^{\left(-\left[\frac{M - 39{,}135}{45{,}184}\right]^{1.855}\right)}$$

[2] The number in column 2 times 100.

[3] 100 percent less the number in column 3.

[4] Percent failing in mileage interval

$= R_1 - R_2$, where: R_1 = reliability (column 4) at previous cumulative mileage
R_2 = reliability at mileage in column 1

[5] The total number of failures, N, times the decimal percentage in column 5.

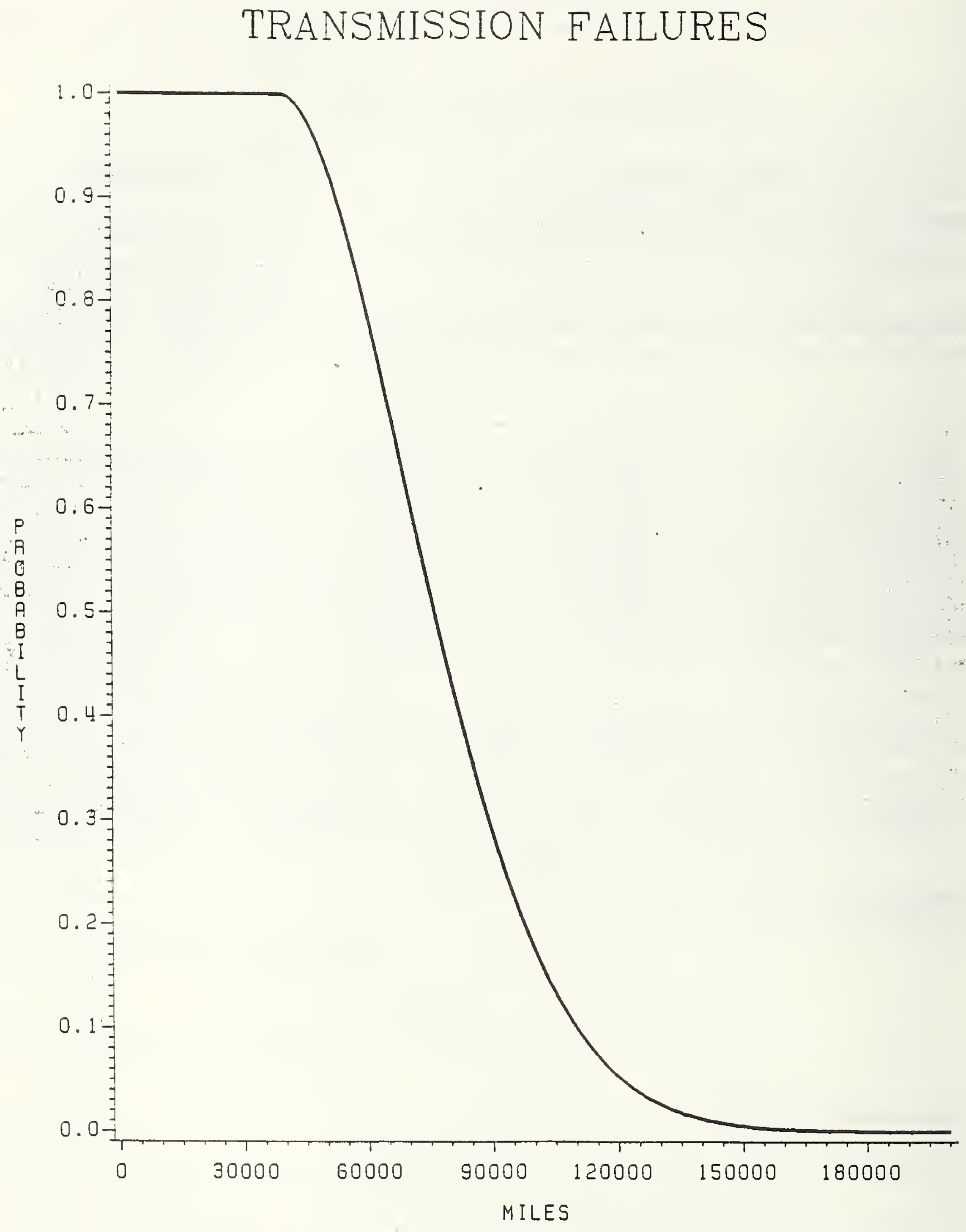


FIGURE 6.3

TRANSMISSION RELIABILITY CURVE

the mean mileage to failure, $\bar{X}$ = 79,231 miles, which equals 0.3786 (30,000 divided by 79,231 = 0.3786). Then multiply the number of buses by 0.3786 to get the number of transmissions that are expected to fail during each year. If there are 60 buses, then about 23 are expected to fail each year (60 x 0.3786 = 22.718).

When the buses are newly purchased, the failure rate (including replacements) will be low but eventually it will increase and stabilize at about 23 per year. A satisfactory estimate of how long it will take to reach the steady-state rate is the average lifetime of a transmission. In this example, the average life of the transmission is predicted to be 79,231 miles or 2.6 years (79,231 divided by 30,000 = 2.6410). The increase from zero failures per year to 23 per year is shown in Figure 6.4 as a straight line, which is good enough for estimating the annual failures until 2.6 years is reached.

The area under the failure line between any two times in Figure 6.4 is the prediction of the total number of transmission failures for the fleet during that time interval.

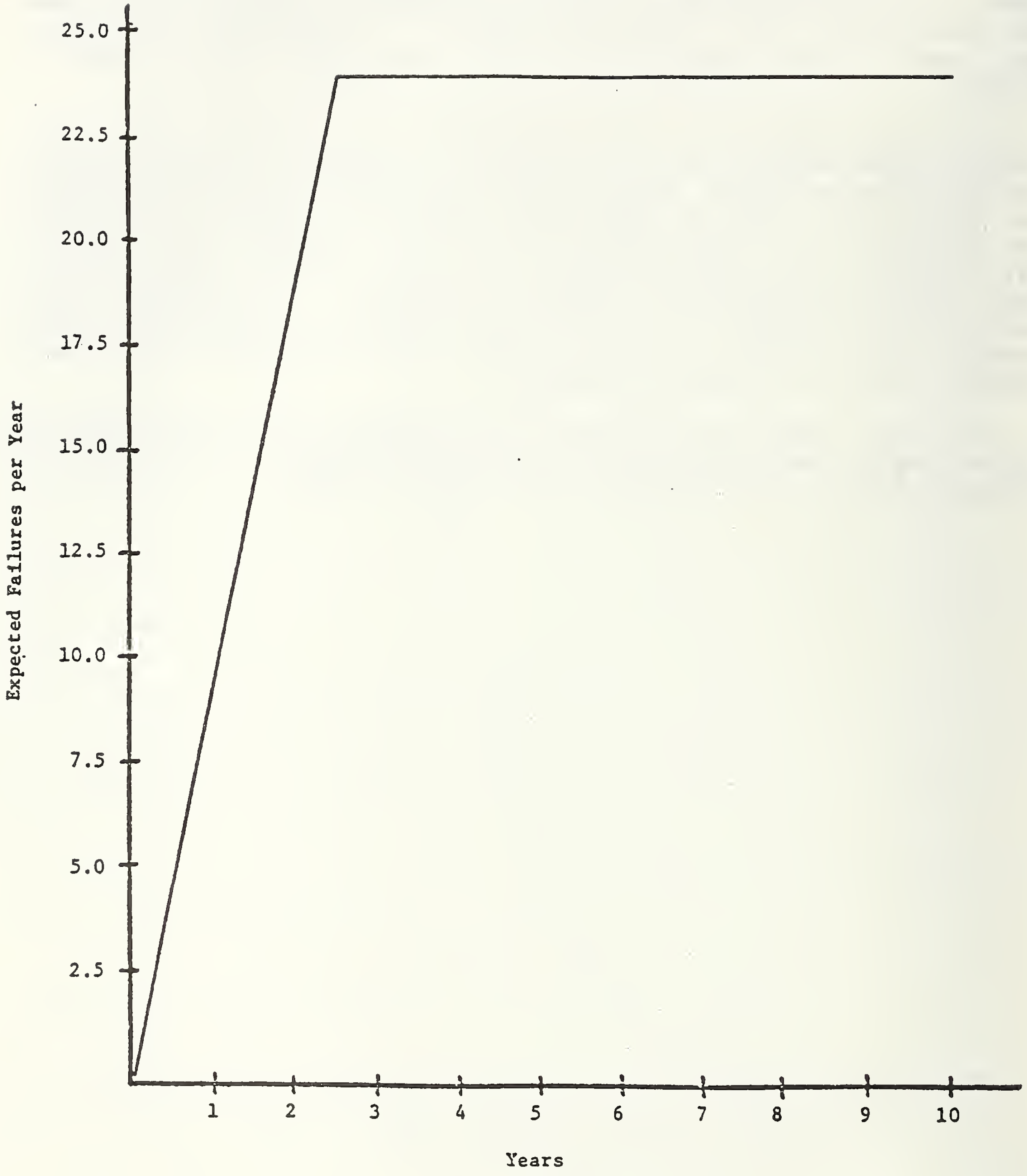


FIGURE 6.4

TOTAL PREDICTED TRANSMISSION FAILURES PER YEAR

DURING THE SERVICE LIFE OF THE BUSES

CHAPTER SEVEN

COMPONENT REPLACEMENT MILEAGE INTERVAL DETERMINATION

Chapter One introduced four component maintenance policies and succeeding chapters have demonstrated how an analysis of component failure mileage patterns can help in determining the best policy for each analyzed component. The four policies were condition-based maintenance, fixed-mileage maintenance, operate-until-failure maintenance, and design-out maintenance. For three of these policies, all except fixed-mileage, it is evident when the component should be replaced: when it shows signs of excess wear or when it fails outright.

However, the optimal replacement interval is not straightforward in the case of fixed-mileage maintenance, FMM. The purpose of this chapter is to outline a simple procedure for determining the optimal fixed-mileage replacement interval. This is the interval that minimizes the total costs of the component to the transit agency during the life cycle of the buses. The technique assumes that it costs more to replace the component after it fails than before it fails. This is either because the failure can cause additional mechanical damage to the bus or because, if the bus breaks down in service or is otherwise disabled, there is inconvenience to both passengers and the transit agency.

Background

There are some parts and components which typically are replaced or overhauled on a fixed mileage or FMM basis. For example, oil, fluids, and filters are commonly changed after a fixed number of miles have passed since they were last changed. These intervals are based on such criteria as manufacturer recommendations, industry practice and transit agency experience. However, there are no well-defined replacement intervals for most other cases where FMM is an appropriate maintenance strategy. Often, the optimal replacement interval is a function of the unique operating environment of each transit system.

Chapter One cited the example of a Southwestern transit agency maintenance manager who overhauled his Detroit Diesel Allison V730 transmissions at 100,000 mile intervals. There were no recommended standards for intervals between transmission overhauls that he could follow, so the maintenance manager selected 100,000 miles by trial-and-error. He tried larger mileage intervals but found that too many transmissions were failing while the buses were in service. Eventually he determined that an interval of 100,000 miles achieved a satisfactory trade-off between overly frequent transmission overhauls and inconvenient in-service failures. The operational benefits of this policy to the transit agency were reviewed in Chapter One.

If the component failure mileage pattern is known, this information can be used to more efficiently select the optimal interval which minimizes costs. Three reasons for using the failure mileage pattern are as follows:

1. A simple procedure, described in this chapter, can determine the optimal replacement interval based on the component failure mileage pattern.

2. A replacement interval can be selected without the need for a time-consuming trial-and-error process.

3. Since the failure pattern is based on past component failures, it takes into account the unique operating environment of the transit agency. For example, the Southwestern transit system mentioned above serves a metropolitan area that is hilly, dry, and dusty. This affects the wear and tear on their transmissions; the 100,000 mile interval they selected may not be applicable to other operating environments.

Information Requirements

Two types of information are required to estimate the optimal mileage interval between replacements. These are replacement cost information and the Weibull distribution failure mileage factors.

Cost Information. The following cost information is needed to determine the best mileage interval between replacements:

1. The approximate parts and labor cost to remove and replace the part or component before it fails.

2. The approximate cost to remove and replace the part or component after it fails. If the failure is likely to cause additional mechanical damage, the cost to repair the damage should also be included. Similarly, if the failure will cause a service disruption, then an estimated cost penalty for the disruption should be included.

The cost of making the repair before the failure must be less than the cost of making the repair after failure. Otherwise, the part or component should not be a candidate for fixed mileage replacement.

The air compressors analyzed in Chapter Four were found to be a good candidate for fixed mileage replacement. If the air compressor fails while in service, it will cause a service disruption. Therefore, it will cost more to replace after the failure because of in-service disruption penalities than it would before it fails. To illustrate, the costs below are used:

Cost to remove and replace an air compressor before failure:

1. Labor cost: 3.5 hours at \$15.00/hr = \$ 52.00
2. Parts cost: Rebuilt air compressor = 280.00

Total: \$332.00

Cost to remove and replace an air compressor after failure:

1. Labor cost: 3.5 hours at \$15.00/hr = \$ 52.00
2. Parts cost: Rebuilt air compressor = 280.00
3. Service disruption penalty = 1,000.00
4. Penalty for unscheduled repair = 100.00

Total: \$1,432.00

Failure Pattern Information. In Chapters Three through Six methods were presented for estimating Weibull distribution factors for various transit bus components. These factors are used to estimate the best replacement interval so that the analysis can take into account the specific failure pattern of the part or component analyzed.

The reason that the failure pattern needs to be taken into account is that replaced operable parts or components will still have some of their useful life remaining regardless of the interval selected. This means that replacements will be more frequent under FMM than under an operate-until-failure maintenance policy. On the other hand, parts or components that are replaced before they fail do not incur the higher cost of replacement after failure. The optimal mileage interval for replacement is a trade-off between the cost of replacing a component with some useful remaining life and the cost of replacement after failure. Making this trade-off requires information on when the part or component is expected to fail, hence the failure pattern needs to be known.

Optimal Replacement Interval Methodology

The following methodology was first published by Glasser in 1967 (1). To find the best replacement interval, two constants must be calculated, R1 and R2.

R1 is the ratio of the costs of replacing the part or component after it has failed divided by the cost of replacement before it has failed.

$$R1 = \frac{\text{The cost of replacing after failure}}{\text{The cost of replacing before failure}}$$

This ratio must be greater than one, otherwise fixed-mileage replacement should not be considered. For the air compressor example:

$$R1 = \frac{\$1{,}432}{\$332} = 4.3$$

R2 is a measure of the relative width of the failure mileage pattern. To get R2 the following formula is used:

$$R2 = \frac{(\bar{X} - D)}{SD}$$

For the air compressor example:

$\bar{X}$ = 127,675 miles, (estimated mean mileage to failure),

SD = 51,501 miles, (standard deviation),

D = 20,315 miles, (Weibull minimum life term).

$$R2 = \frac{(127{,}675 \text{ miles} - 20{,}315 \text{ miles})}{51{,}501 \text{ miles}} = \frac{107{,}360 \text{ miles}}{51{,}501 \text{ miles}}$$

$$R2 = 2.08$$

Once R1 and R2 are determined the chart in Figure 7.1 is used to find "z," which is used in the formula to determine the optimal replacement mileage interval. For the air compressor example, dashed lines are drawn in Figure 7.1 for R1 = 4.3 and R2 = 2.08. The intersection of these two lines is the value of z relative to the marked z contour lines. In this case they intersect between the contours where z equals -1.0 and -0.5. Interpolating between these two contours, the intersection is at about z = -0.8.

The optimal replacement interval is determined by using the following formula:

Optimal replacement interval = $\bar{X}$ + (z x SD)

For the air compressor example:

$$\text{Optimal replacement interval} = 127{,}675 + (-0.8 \times 51{,}501)$$
$$= 86{,}474 \text{ miles}$$

The maintenance manager would round this off to 90,000 miles, which then becomes the mileage interval between air compressor replacements. Any air compressor which has not yet failed after 90,000 miles of service is replaced at this mileage.

Concluding Remarks

The interval between component replacements should be ad-

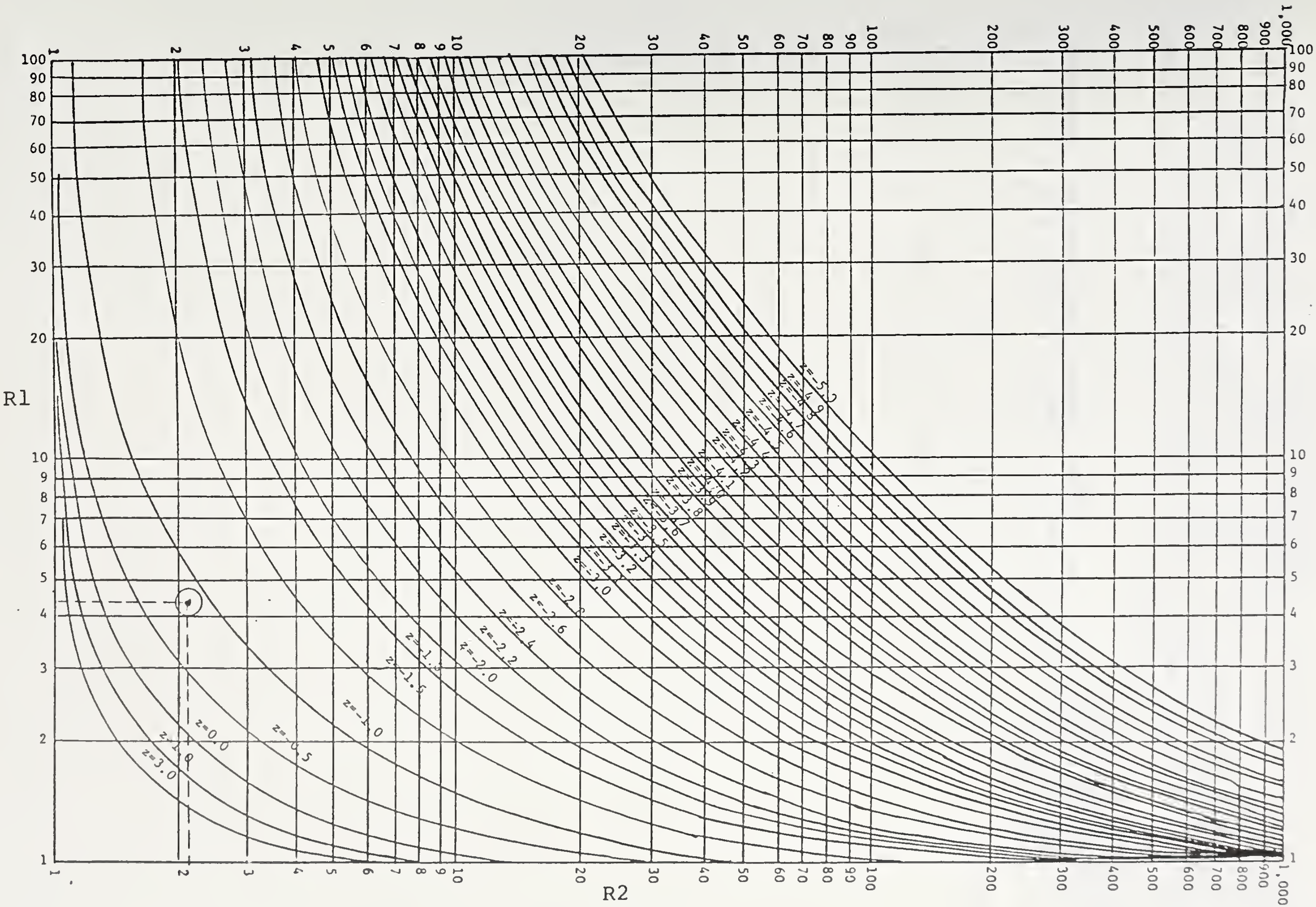


FIGURE 7.1

Z NUMBERS FOR OPTIMAL REPLACEMENT MILEAGE INTERVAL DETERMINATION justed to coincide with bringing the bus in for other maintenance activities and for maintenance shop scheduling convenience. Furthermore, the maintenance manager should adjust the replacement mileage to fit changing conditions. For example, if an improvement is made to the part or component, the interval should be changed based on new operating experience and knowledge of the new failure pattern.

List of References

1. Glasser, Gerald J. "The Age Replacement Problem." Technometrics, Vol. 9, No. 1, February 1967, pp. 83-91.

STUDY QUESTION

Consider the generator failures in Chapter Five, for which a fixed-mileage maintenance policy might be appropriate based on the Weibull distribution failure mileage analysis. The Weibull minimum life term, D, was 31,852 miles. The estimated mean mileage to failure, $\bar{X}$, was 94,486 miles, and the standard deviation, SD, was 36,010 miles. If the cost to replace a generator is $200 before failure and $600 afterwards, then determine the optimal replacement mileage for the generators.

STUDY QUESTION: ANSWER

The cost ratio, R1, is determined as follows:

$$R1 = \frac{\$600}{\$200} = 3.00$$

R2 is found as follows:

$$R2 = \frac{\bar{X} - D}{SD}$$

$$= \frac{94{,}486 \text{ miles} - 31{,}852 \text{ miles}}{36{,}010 \text{ miles}}$$

$$R2 = 1.74$$

From Figure 7.1, z is found to be $z = 0.0$. The optimal replacement mileage interval is:

$$\bar{X} + (z \times SD) = \bar{X} = 94{,}486 \text{ miles.}$$

Since z is about equal to zero, then the optimal interval is the mean mileage to failure. The maintenance manager might round this off to either 90,000 or 100,000 miles. Note, however, that since generator failures are detectable, the preferable maintenance policy is condition-based maintenance.

CHAPTER EIGHT

APPLICATIONS OF LIFE CYCLE COSTING

This chapter is a self-contained discussion of the applications of life cycle costing analysis of buses and their components for making vehicle procurement and replacement decisions. There are two major points demonstrated in this chapter. The first is that life cycle costing does have more applications than just procurement. The second is that sound economic principles, including accounting for the time value of money, are inherent to life cycle costing. Examples will illustrate both points.

The topics covered in this chapter are:

- What is life cycle costing?
- Applications of Life Cycle Costing in the Transit Industry.
- Prerequisites to Effective Life Cycle Cost Vehicle Procurement.
- Review of Economic Principles.
- Information Needed for Vehicle Procurement.
- Selection Among Competing Bidders in Vehicle Procurement.
- When Should Buses be Replaced?
- Purchasing New or Rehabilitated Buses.

What is Life Cycle Costing?

Life cycle costing is an economic evaluation scheme which accounts for the capital, operating, and maintenance costs during the usable life of an investment. In theory, it is both a "common sense" approach to equipment procurement and a well-established evaluation procedure in engineering economics.

In the public sector, life cycle costing has been promoted as an innovative alternative to equipment procurement based on minimum initial capital cost, the "lowest bid". In the Federal government life cycle costing has been used for military procurement by the Department of Defense since the 1960s. It also is used by the General Services Administration for the purchase of such standardized items as typewriters and office supplies.

The U.S. Urban Mass Transportation Administration (UMTA), in response to Congressional dictates, first required life cycle costing for the purchase of transit vehicles in 1982 (Federal Register, Vol. 47, No. 33, February 18, 1982, pp. 7361-7364) and later, in 1983, UMTA made it optional. The 1982 guidelines said that transit agencies should select as "cost drivers" those items which account for 75 percent or more of operating and maintenance costs during the life of the bus. Typical cost drivers include preventive maintenance and major component repairs. UMTA suggested that manufacturers be required to provide estimates of these cost drivers in the procurement process.

Applications of Life Cycle Costing in the Transit Industry

There are six primary uses for life cycle costing in the transit industry, as adapted from Seldon (5). These are discussed below and applications are presented later in the chapter.

1. Long-range planning and budgeting. Gathering the data needed to do life cycle analysis forces the transit agency to clarify and identify the operational and maintenance cost elements of the organization. For example, if the agency is planning a major acquisition of new buses as part of a service expansion, what are the budget implications over the projected life of these buses? Life cycle costing should facilitate the projection of agency budgets over a long period of time.

2. Comparison of competing programs. Life cycle costing can provide some of the information needed for broader policy-making, such as proposals to implement light rail services as an alternative to expanded bus services. Other examples include decisions to purchase different types of buses (e.g., vans, articulated buses, minibuses) or proposals to purchase used or rehabilitated buses.

3. Comparison of maintenance strategies. There are alternatives in maintenance management which are best analyzed in the long range, in keeping with the life cycle costing approach. For example, will increased preventive maintenance extend the life of the buses? Should all maintenance work be done in-house, or would it be more economical to contract for some specialized maintenance work with outsiders?

4. Decisions about the replacement of aging equipment. There are a variety of strategies for determining when to replace aging vehicles and most would benefit from the information needed for life cycle costing. In fact, life cycle costing would enable transit agencies to more effectively monitor the costs associated with retaining their aging buses. This chapter is concluded with an example of bus replacement decision-making where the choice is to keep an old bus or replace it with a new or rehabilitated bus.

5. Control over an ongoing program. The effective management of any program requires adequate information on what aspects contribute to costs. Life cycle costing implies the development of a data base which should help ongoing performance monitoring.

This is of particular significance to transit agencies in the 1980s which are experiencing soaring operating deficits at a time of diminished financial resources.

6. Selection among competing contractors. Finally, life costing, in principle, is the rational economic approach to evaluating alternative bids for equipment, including transit buses. Procurement examples which account for the time value of money are included in this chapter.

Prerequisites to Effective Life Cycle Cost Vehicle Procurement

The successful application of life cycle cost techniques to vehicle procurement requires several things. First, transit agencies must have the ability to identify, measure, and evaluate the cost drivers affecting the operating and maintenance costs of their buses. Second, the manufacturers of buses must be willing and able to provide operating and maintenance cost estimates of the various components included in their bus designs. Third, bus manufacturers and transit agencies must develop and maintain good working relationships and mutual understanding.

Six prerequisites to successful life cycle cost procurement are presented below which should foster the ability of the transit agency to identify its cost drivers and improve its ability to work intelligently with the manufacturers.

1. Standard and uniform guidelines for life cycle cost procurement are necessary, both to ease the task for the transit agency and to encourage the manufacturers to provide the appropriate information. For example, if preventive maintenance is a life cycle cost driver, the manufacturer should specify the mileage intervals and what is to be done at each interval based on transit agency specifications. Thus, each manufacturer provides comparable information to the agency. Similarly, manufacturers estimates of fuel economy can vary widely. The transit agency should specify the fuel test methodology and a representative operating profile, using identical criteria for all manufacturers submitting procurement bids. It should be required that the bus being offered is identical to the bus being tested.

2. Transit operators need adequate records to support life cycle analysis and to monitor the results once the new buses are obtained. In addition, the availability of comprehensive cost and frequency of occurrence records would make it possible to more efficiently manage maintenance activities and procedures. Such records should be computerized to make it easier to analyze data. For example, meaningful cost and frequency of repair predictions can be accomplished with a relatively small number of cases (e.g., ten to twenty buses), but the analysis is best done on a computer. Economic analysis also is easier to do on a computer.

3. Transit agencies should integrate their operating and maintenance records with both long range and annual budget and operations planning. As noted earlier, there are a variety of applications for life cycle costing information in addition to vehicle procurement.

4. Cost and frequency of occurrence records should be collected for individual buses. This information is needed to find out when old buses should be replaced as demonstrated in the last example presented in this chapter. A southwestern transit agency maintenance manager, responsible for a fleet of 100 buses, told the authors that 100 buses were about the limit of his ability to be personally familiar with each vehicle's maintenance history without the help of a good record-keeping system. With computer-based records, summaries easily can be provided for bus fleet and model totals.

5. The time value of money should be considered in life cycle cost procurement, mainly because of the long time period involved (12 years typical bus life) and the magnitude of fuel and maintenance costs which are incurred over time. Uncertainties such as future fuel prices and maintenance expenses are best accounted for by the thoughtful and conservative use of economic principles. Most of the examples in this chapter illustrate how this is done.

6. None of the above prerequisites are easy to implement without adequate staff expertise and the availability of training courses and guidelines. Management information systems and a capable supporting staff should be recognized as fundamental components of transit agency administration. This is discussed in more detail in the chapter entitled, "Information System Development."

7. Top-level management support is needed because life cycle costing departs from traditional procurement practices and it requires more staff time to prepare and evaluate such analyses. Management must be willing to provide the staff and training resources needed to satisfy prerequisite six.

Review of Economic Principles

In this section economic principles, including the time value of money, are reviewed and applied to an example which consists of twenty years of hypothetical bus operating cost records. The computations covered include depreciation, annual life cycle cost, and present worth analysis of total life cycle costs.

Transit Bus Life. For purposes of economic analysis a transit bus is assumed to have three different lives. Life is the period of time that the bus is used by one transit agency.

1. Actual useful life. This is the time period that the bus is actually used. This may or may not be the same as the depreciation or economic life.

2. Depreciation life. This is the anticipated life of the bus. For accounting purposes, to aid in estimating annual costs of ownership, the original capital cost is depreciated annually over this life. Conventional practice in the transit industry is to use a depreciation life of 12 years for new transit buses. Note that after 12 years the bus may still have useful life and be kept in operation by the transit agency.

3. Economic life. This is the length of ownership for which the net annual cost of the bus is a minimum. This will be illustrated in the following example. The economic life is the life at which life cycle costs to the transit agency are minimized. This may or may not correspond to the anticipated depreciation life of 12 years. As will be seen later in this chapter, the concept of economic life is applied to vehicle replacement decisions. An aging bus should be replaced if it can be shown that a replacement will incur lower annual costs on a life cycle basis.

In order to illustrate the application of economic principles to the life cycle costing of a bus, hypothetical operating and maintenance costs are indicated in Table 8.1 for a projected useful bus life of 20 years and a depreciation life of 12 years. As the bus ages the annual miles of service are gradually reduced as indicated in column 2. Annual fuel costs (column 4) are based on projected annual mileage taking into account decreasing fuel efficiency in miles/gallon as the bus ages. Maintenance costs (column 5) are made up but reflect both increased costs over time and the annual impacts of major component overhauls.

Depreciation Cost. Although the Federal government does not permit depreciation accounting of Federally-granted funds, this should not prevent the transit agency from including depreciation in its economic analysis of bus life cycle costs. This is the way the original capital costs of buses are accounted for on an annual basis. If capital costs are not included a vehicle replacement analysis will indicate that new buses should be replaced every year because of low initial maintenance costs. Thus, the following computations relate to economic evaluation, not transit agency finances or budget computations. Their sole purpose is enable better decisions about bus replacements.

There are several commonly used depreciation techniques. The one selected here is the double declining balance depreciation method, which assumes that the annual cost of depreciation is a constant percentage of the book value of the bus at the beginning of each year.

The constant percentage, k, is determined from the following formula:

TABLE 8.1

BUS ANNUAL TOTAL COSTS OVER A 20 YEAR USEFUL LIFE CYCLE

Bus Age (years) (1)	Miles/ Year (2)	Depreciation Cost, $ (3)	Fuel Cost, $ (4)	Maintenance Cost, $ (5)	Total Annual Cost, $ (6)	Annual Life Cycle Cost/Mile, $ (7)
1	47,500	26,666.67	13,350	5,300	45,316.67	0.95
2	47,500	22,222.22	13,350	9,170	44,742.22	0.94
3	47,500	18,518.52	13,350	11,050	42,918.52	0.90
4	47,500	15,432.10	13,350	14,900	43,682.10	0.92
5	47,500	12,860.08	13,350	15,850	42,060.08	0.89
6	42,500	10,716.73	12,700	14,300	37,716.73	0.89
7	42,500	8,930.61	12,700	15,100	36,730.61	0.86
8	42,500	7,442.18	12,700	16,800	36,942.18	0.87
9	42,500	6,201.81	12,700	15,700	34,601.81	0.81
10	42,500	5,168.18	12,700	16,200	34,068.18	0.80
11	37,500	4,306.82	12,000	13,000	29,306.82	0.78
12	37,500	3,589.01	12,000	13,400	28,989.01	0.77
13	37,500	0.00	12,000	16,800	28,800.00	0.77
14	37,500	0.00	12,000	18,100	30,100.00	0.80
15	37,500	0.00	12,000	21,300	33,300.00	0.89
16	32,500	0.00	11,300	22,650	33,950.00	1.04
17	32,500	0.00	11,300	26,100	37,400.00	1.15
18	32,500	0.00	11,300	25,950	37,200.00	1.14
19	32,500	0.00	11,300	28,300	39,600.00	1.22
20	32,500	0.00	11,300	27,650	38,950.00	1.20

$k = 2/N$, where N = the assumed depreciation life

In this example a depreciation life of 12 years is assumed. The value for k is 1/6 in this case.

The depreciation in the first year is computed as follows:

$d_1 = P \times k$, where d_1 = depreciation in the first year
P = original capital cost.

In the example depicted in Table 8.1 the original capital cost of the bus is \$160,000. The depreciation in the first year is $\$160,000/6 = d_1 = \$26,666.67$. This is entered in the first line of column 3 of Table 8.1.

In succeeding years the depreciation is computed from the following formula:

$d_n = (P_{n-1}) \times k$, where: n = bus age in years
d_n = depreciation that year
P_{n-1} = original capital cost <u>minus</u> the sum of the depreciations computed for the previous years, 1 to n - 1.

For example, in the second year:

$$d_2 = (\$160,000 - \$26,666.67)/6 = \$22,222.22$$

In the third year:

$$d_3 = (\$160,000 - \$26,666.67 - \$22,222.22)/6 = \$18,518.52$$

Depreciation costs are continued in a like manner through year 12. After that the bus is considered fully depreciated and a value of zero is entered in column 3 of Table 8.1.

In theory, when using the double declining balance depreciation method the salvage value of the bus after N years is found from the following formula:

$F = P(1 - k)^N$, where: F = salvage value at bus age N

In the example:

$$F = \$160,000(1 - 1/6)^{12} = \$17,945.06.$$

In reality, the salvage value of the bus is its market value at the time, which is whatever the transit agency can sell it for after 12 years.

<u>Annual Life Cycle Cost</u>. The annual life cycle cost for the bus example in Table 8.1 is found by summing together the annual depreciation, fuel cost, and maintenance cost (columns 3, 4 and

5) and entering this value in column 6. Thus, for the first year the annual cost is:

$26,666.67 + $13,350.00 + $5,300 = $45,316.67

The annual life cycle cost per mile is found by dividing the annual cost by the miles accumulated that year. In the first year, this annual life cycle cost per mile is:

$45,316.67 / 47,500 miles = $ 0.95 per mile

This value is entered in column 7, the last column in Table 8.1.

In this example the annual life cycle costs per mile initially tend to decrease each year. This is because depreciation costs decline each year and the miles driven per year declines every five years, reducing to some extent fuel and maintenance expenses. If the economic life of the bus is considered to be the life at the minimum annual expense, the figures in Table 8.1 would conclude that the economic life of these buses is about 11 to 13 years. Note, however, that the buses in Table 8.1 see less use after 13 years than newer buses and probably they aren't very reliable. Neither of these important factors are accounted for in Table 8.1 as costs to the transit agency.

Annual life cycle cost per mile from Table 8.1 is plotted in Figure 8.1. Such plots are typical for aging equipment and help in figuring out when equipment should be replaced. Later in this chapter the question of when to replace an aging bus will be described in more detail.

Present Worth Analysis. In bus procurement the time value of money can be taken into account by discounting the costs incurred during the lifetime of a bus. This is done by computing the present worth of future sums of money. The formula for doing this is called the single payment present worth factor:

$PWF = 1 / (1 - i)^n$, where: PWF = single payment present worth factor

i = discount rate expressed as a decimal (e.g., $i = 0.10$ for 10 percent discount rate)

n = number of years

For example, if the discount rate is 10 percent and n = one year, then PWF = 0.9091. Values for the single payment present worth factor typically are found from tables in engineering economics textbooks or computed directly with business applications pocket calculators.

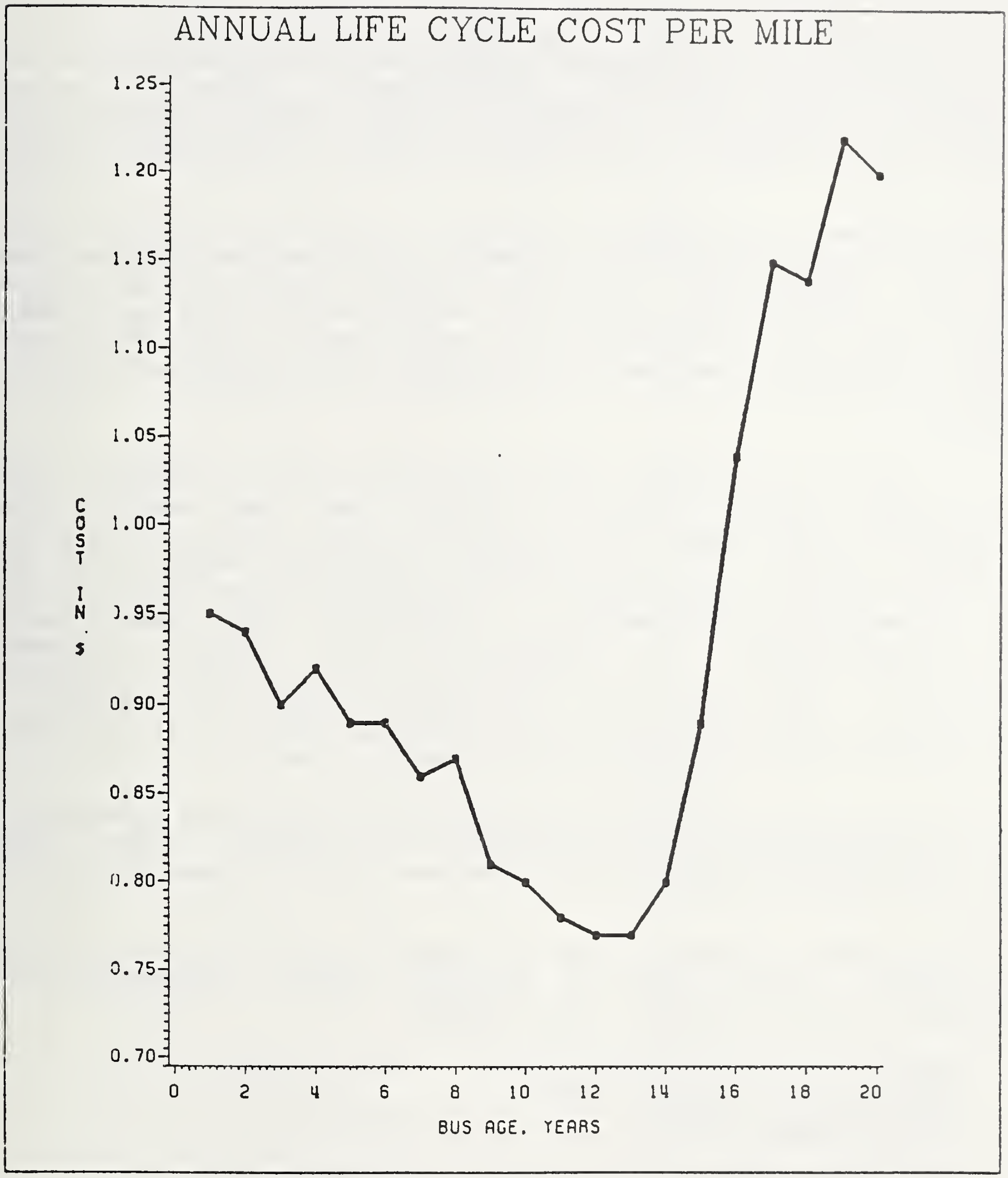


FIGURE 8.1

HYPOTHETICAL BUS ANNUAL LIFE CYCLE COSTS PER MILE

The discount rate, i, reflects the degree of uncertainty in the future as well as prevalent economic interest rates. The examples in this chapter use a discount rate of 10 percent and this rate is suggested for economic studies of public projects unless the transit agency has other preferences or policies. For the convenience of the reader a table of single payment present worth factors for a discount rate of 10 percent is presented in Table 8.2.

The annual fuel and maintenance costs from Table 8.1 are converted to present worths in Table 8.3 using the numbers from Table 8.2. Depreciation costs are deleted from the computations because in this section they are considered strictly as a bookkeeping device for estimating true annual costs of bus ownership. It should be rather obvious to note that the present worth of an initial capital investment of $160,000 to purchase a bus is $160,000.

In Table 8.3 the fuel and maintenance costs (columns 2 and 3) are added together (sums in column 4) and then multiplied by the present worth factor (column 5) for each year of bus life. The result (column 6) is the present worth of those costs for each future year. For example, in the 5th year of bus life the fuel and maintenance costs sum to $29,200. The present worth factor from Table 8.2 for 5 years is 0.6209. Thus, the present worth of the costs in the fifth year are: $29,200 x 0.6209 = $18,130.28. The sum of all the present worths in column 6 is $232,613.34, the present worth at the time the bus is purchased of 20 future years of fuel and maintenance costs.

Note that the present worth factor substantially reduces the value of expenditure which are made in the future. Actual total fuel and maintenance costs per bus over the 20 year period are $594,320 (sum of column 4).

The proper numbers to use in computing life cycle costs are the present worths of the initial capital investment and recurring ownership costs. For example:

Initial purchase price of bus:	$160,000.00
Present worth of fuel and maintenance costs:	+ $232,613.34
Total life cycle cost:	$392,613.34

It is the present worth of all life cycle costs which should be used when comparing one bus model against another in vehicle procurement. This is discussed in more detail in the next section.

Assumptions. In the next section vehicle procurement is based on estimated maintenance and fuel expenses provided by manufacturers. The transit agency can substitute its own figures but there is no guarantee that past operating experiences will be

TABLE 8.2

SINGLE PAYMENT PRESENT WORTH FACTORS FOR i = 10 PERCENT

Bus Age, n	Present Worth Factor*
1	0.9091
2	0.8264
3	0.7513
4	0.6830
5	0.6209
6	0.5645
7	0.5132
8	0.4665
9	0.4241
10	0.3855
11	0.3505
12	0.3186
13	0.2897
14	0.2633
15	0.2394
16	0.2176
17	0.1978
18	0.1799
19	0.1635
20	0.1486

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* Single payment present worth factor

$$= \frac{1}{(1 + i)^n} \quad \text{where } i = 0.10 \text{ or } 10 \text{ percent}$$

TABLE 8.3

PRESENT WORTH OF FUEL AND MAINTENANCE COSTS

Bus Age (years) (1)	Fuel Cost, $ (2)	Maintenance Cost, $ (3)	Total Cost, $ (4)	x	Present Worth Factor (5)	=	Present Worth, $ (6)
1	13,500	5,300	18,650	x	0.9091	=	16,954.72
2	13,500	9,170	22,520	x	0.8264	=	18,610.53
3	13,500	11,050	24,400	x	0.7513	=	18,331.72
4	13,500	14,900	28,250	x	0.6830	=	19,294.75
5	13,500	15,850	29,200	x	0.6209	=	18,130.28
6	12,700	14,300	27,000	x	0.5645	=	15,241.50
7	12,700	15,100	27,800	x	0.5132	=	14,266.96
8	12,700	16,800	29,500	x	0.4665	=	13,761.75
9	12,700	15,700	28,400	x	0.4241	=	12,044.44
10	12,700	16,200	28,900	x	0.3855	=	11,140.95
11	12,000	13,000	25,000	x	0.3505	=	8,762.50
12	12,000	13,400	25,400	x	0.3186	=	8,092.44
13	12,000	16,800	28,800	x	0.2897	=	8,343.36
14	12,000	18,100	30,100	x	0.2633	=	7,925.33
15	12,000	21,300	33,300	x	0.2394	=	7,972.02
16	11,300	22,650	33,950	x	0.2176	=	7,387.52
17	11,300	26,100	37,400	x	0.1978	=	7,397.72
18	11,300	25,950	37,200	x	0.1799	=	6,692.28
19	11,300	28,300	39,600	x	0.1635	=	6,474.60
20	11,300	27,650	38,950	x	0.1486	=	5,787.97
Sum =	$246,750	$347,570	$594,320				$232,613.34

realized with new buses. Other chapters of this report show how the transit agency can make forecasts of component reliability and replacement rates. In a later section of this chapter it will be shown how this information can be used to determine the present worth of specific component repairs for the life of a bus.

The above example also neglected the effects of inflation on costs over time. The reader is advised to consult textbooks on engineering economics for techniques of handling costs which grow by a uniform increment or a constant percentage over time, e.g., maintenance labor rates which increase x percent a year. Note, however, that predicting future price trends is a speculative effort at best. Inflation has typically increased over time but at unpredictable and variable rates. Fuel costs have varied up and down in past decades and likely will continue to do so.

The safest course for the analyst is probably to not get too involved in predicting the unpredictable. The discount rate includes future cost uncertainty. If all costs inflate at about the same rate then there is no compelling reason to predict inflation as long as alternative bus models are economically treated in the same manner. Thus, it is not likely that the analyst should need to forecast inflation rates or cost growth rates. Instead, more attention should be paid to the levels of maintenance likely to be incurred each year throughout the life of the bus using the techniques presented in the other chapters.

Information Needed for Vehicle Procurement.

Obtaining buses with the lowest acquisition costs may not be the least expensive option for the agency in the long run. The total cost of ownership of a bus consists of the following factors:

- Initial acquisition cost,
- Operating costs which include fuel, tires, preventive maintenance, repairs, and major component rebuilding and replacement, and which recur periodically over time,
- Salvage or resale value when the transit agency decides to get rid of the bus at the end of its economic life.

Ownership cost does not include driver wages or transit administration overhead since these costs are generally independent of the types of buses purchased. One notable exception, however, is the articulated bus which offers more passenger seats per driver. It would be appropriate in this case to account for increased driver productivity in life cycle costing.

A comprehensive list of potential life cycle cost factors or "drivers" is presented in Table 8.4. Including all such factors

TABLE 8.4

LIST OF POTENTIAL OPERATING FACTORS FOR LIFE CYCLE COST PROCUREMENT

- BODY
    - Shell
    - External and Applied Panels
    - Finish
    - Skirt Aprons
    - Floors
    - Steps and Stepwells
    - Wheel Housing
    - Passenger Doors
    - Service Compartment Service Doors
    - Operating Components
    - Door Actuators
    - Windshield Wiper/Washer
    - Light Control and Instruments
    - Fare Box
    - Loading System
    - Signals
    - Interior
    - Mirror
    - Passenger Seats
    - Driver Seats
    - Floor Covering
    - Panels and Bulkheads
    - Access Doors
    - Stanchions and Handrails
    - Windows
    - Driver's Windows
    - Side Windows
- CHASSIS
    - Propulsion System
    - Engine
    - Cooling System
    - Transmission
    - Engine Accessories
    - Hydraulic Drive
    - Final Drive
    - Rear Axle
    - Drive Shaft
    - Suspension
    - Springs and Shock Absorbers
    - Front Axle
    - Kneeling Equipment
    - Steering
    - Brake Mechanism
    - Hub and Drums
    - Air System
    - Brake Linings
    - General Chassis
    - Wheels
    - Fuel System
    - Bumper System
    - Frame
    - Electrical System
    - Electrical Components
    - Climate Control
    - Heating
    - Air Conditioning
    - Ventilation
    - Radio & Public Address System
    - Mobile Radio System
    - Public Address System
- ROAD CALLS
- PREVENTIVE MAINTENANCE
    - Oil Change
    - Tuneup
    - Inspections
    - Lubrications
    - Cleaning and Washing
- OPERATING FACTORS
    - Fuel
    - Tires
    - Oil

Source: American Public Transit Association.

in a life cycle analysis, however, is likely to complicate and prolong the analysis without significantly improving the end result or changing the procurement decision.

The following eight major cost drivers generally comprise more than 75 percent of the operating cost of a bus:

1. fuel
2. tires
3. engine oil
4. brakes
5. transmission
6. engine
7. air conditioning
8. preventive maintenance

Life cycle cost procurement should include evaluation of these cost drivers. In addition, other elements such as body corrosion, other body work, and the electrical system, may be used based on past experiences or concerns of the transit agency.

As an example of these cost factors, Table 8.5 provides a breakdown of fuel and maintenance expenses for 22 new buses in their first and third years of operation by the Central Oklahoma Transportation and Parking Authority. Maintenance costs are seen to increase over time, exceeding fuel costs after three years.

By calculating the present worth of costs which would be incurred for each selected cost driver over the entire life of the bus and adding up the results, the total life cycle costs can be determined. This is done for each competing bus in the procurement. From this, the bus with the lowest overall life cycle cost can be determined.

The operating cost data required for the life cycle calculations can be obtained from the competing manufacturers or from transit agency records. The following data typically are requested from the manufacturer according to the American Public Transit Association for the eight major cost drivers noted above:

- the bus acquisition cost
- the estimated lifetime mileage of the bus, typically 500,000 miles
- the estimated fuel consumption of the bus, preferably based on fuel economy tests for the bus which simulate typical bus duty cycles
- the estimated number of miles between transmission overhauls
- the number of labor hours required to remove and reinstall the transmission

TABLE 8.5

OPERATING COST DRIVERS FOR GENERAL MOTORS RTSII MODEL 04 BUSES IN OKLAHOMA CITY*

	Average Operating Cost per Bus	
Cost Driver	First Year	Third Year
Brakes	$1,328	$1,128
Electrical System	404	1,069
Air Conditioning	214	1,028
Preventive Maintenance	651	746
Engine	151	349
Transmission	28	186
Miscellaneous	810	3,062
Subtotal:	$ 3,586	$7,568
Fuel	11,250	7,506
Total Operating Cost:	$14,836	$15,074
Average Mileage:	52,941	35,322
Cost/Mile:	$ 0.28	$ 0.43

* Average costs for 22 General Motors RTSII Model 04 buses acquired by the Central Oklahoma Transportation and Parking Authority in 1981.

- the number of labor hours required to dismantle, overhaul and test the transmission
- the cost of the materials (parts) required to overhaul the transmission
- the estimated number of miles between brake repairs using old drums
- the estimated number of miles between brake repairs installing new drums
- the cost of materials (parts) required to repair brakes using old drums
- the cost of materials (parts) required to repair brakes installing new drums
- the estimated number of hours between air conditioning compressor overhauls
- the estimated number of hours required to rebuild the air conditioning compressor
- the cost of materials (parts) required to rebuild the air conditioning compressor
- the estimated number of miles between air conditioning blower motor overhauls
- the number of labor hours required to remove and reinstall the air conditioning blower motor
- the number of labor hours required to rebuild the air conditioning blower motor
- the estimated number of miles between air conditioning condenser motor overhauls
- the number of labor hours required to remove and reinstall the air conditioning condenser motor
- the number of labor hours required to rebuild the air conditioning condenser motor
- the schedule of preventive maintenance actions in miles
- the number of preventive maintenance actions for the life of the bus
- the number of labor hours required to perform each preventive maintenance action

- the cost of materials (parts) required for each preventive maintenance action.

The following data typically are provided by the transit agency:

- the cost of fuel per gallon
- the hourly labor wage rate for the personnel needed to perform the various repairs and preventive maintenance actions
- the number of years the operator plans to keep the bus
- the number of miles the bus will be operated per year (average utilization rate) and passenger loads (bus weight),

As an example of the operating cost data that can be collected, the information requested from manufacturers by the Central Oklahoma Transportation and Parking Authority (COTPA) in a 1981 procurement is presented in Table 8.6.

Although the information on the major cost drivers is obtained from the manufacturer, it can be helpful for the transit agency to have its own figures. Conscientious transit operators, including COTPA, are alert to and commonly specify specific types and brands of components (e.g., air conditioning units or engines) in their bid specifications because of past maintenance experiences, either within the agency or by comparing experiences with other agencies. Maintaining a data base of cost and frequency of occurrence statistics for the transit agency's own fleet can only facilitate the procurement process. It gives the agency a basis for assessing the validity of manufacturer claims or justifying the specification of specific components. It also enables the agency to account for local climatic and bus duty cycle conditions.

Selection Among Competing Bidders in Vehicle Procurement

The present worth of the life cycle costs for the buses of each competing manufacturer is determined as follows. The costs included in the computation are the acquisition price, up front spare parts and delivery charges, and the eight major cost drivers listed in Table 8.6. The manufacturer has provided the cost information for each cost driver indicated in Table 8.6.

The total present worth of fuel costs during the 12 years of anticipated bus life is computed in Table 8.7 (Work Sheet 17). In this case the bus is expected to accumulate 47,500 miles per year in its first five years of use (column 2), then declining mileages as the bus ages. The reader can readily see that any annual mileage schedule can be accommodated in this work sheet.

TABLE 8.6

LIFE CYCLE COST PROCUREMENT INFORMATION REQUIRED FROM MANUFACTURERS BY CENTRAL OKLAHOMA TRANSPORTATION AND PARKING AUTHORITY

COST FACTOR	INFORMATION REQUIRED [a]
FUEL CONSUMPTION	Fuel economy in miles per gallon based on specified fuel economy test operations.
OIL CONSUMPTION	Consumption (excluding oil changes) in miles per quart
TIRES	Number of tires (brand specified by COTPA) required for 500,000 miles of anticipated bus use
BRAKE RELINING (front and rear)	Parts and labor for life of bus, including expected interval in miles between replacements and overhauls
PREVENTIVE MAINTENANCE	
Oil change & filter	Parts and labor, expected intervals
Engine air filter	Parts and labor, expected intervals
Engine Tune-up	Parts and labor, expected intervals
Transmission	Parts and labor, expected intervals
Air conditioning	Parts and labor, expected intervals
Chassis lubrication	Parts and labor, expected intervals
Differential	Parts and labor, expected intervals
Brake adjustment	Parts and labor, expected intervals
ENGINE REPLACEMENT AND OVERHAUL	Parts and labor, expected intervals
TRANSMISSION REPLACEMENT AND OVERHAUL	Parts and labor, expected intervals
AIR CONDITIONING COMPRESSOR REPLACEMENT AND OVERHAUL	Parts and labor, expected intervals

[a] The manufacturer is required to tabulate all of the above costs and maintenance performance intervals and provide total maintenance costs for the life of the bus using labor, fuel, and oil costs supplied by COTPA as well as miscellaneous maintenance practices information.

TABLE 8.7

WORK SHEET 17

PRESENT WORTH ANALYSIS OF ANNUALLY RECURRING COST DRIVERS

Cost Driver: Annual Fuel Consumption

Bus Age (years) (1)	Mileage per year (2)		Cost per Mile (3)		Total Annual Cost (4)		Present Worth Factor * (5)		Present Worth (6)
1	47,500	x	$ 0.249	=	$ 11,828	x	0.9091	=	$ 10,753
2	47,500	x	$ 0.249	=	$ 11,828	x	0.8264	=	$ 9,775
3	47,500	x	$ 0.249	=	$ 11,828	x	0.7513	=	$ 8,886
4	47,500	x	$ 0.249	=	$ 11,828	x	0.6830	=	$ 8,079
5	47,500	x	$ 0.249	=	$ 11,828	x	0.6209	=	$ 7,344
6	40,000	x	$ 0.249	=	$ 9,960	x	0.5645	=	$ 5,622
7	40,000	x	$ 0.249	=	$ 9,960	x	0.5132	=	$ 5,111
8	40,000	x	$ 0.249	=	$ 9,960	x	0.4665	=	$ 4,646
9	40,000	x	$ 0.249	=	$ 9,960	x	0.4241	=	$ 4,224
10	40,000	x	$ 0.249	=	$ 9,960	x	0.3855	=	$ 3,840
11	35,000	x	$ 0.249	=	$ 8,715	x	0.3505	=	$ 3,055
12	35,000	x	$ 0.249	=	$ 8,715	x	0.3186	=	$ 2,777
							Total Present Worth:		$ 74,112

---

* Single payment present worth factor with discount rate = 10 percent.

Annual fuel costs are calculated as follows. The manufacturer provides the estimated fuel economy, in this case 4.218 miles per gallon based on the duty cycle and bus weights specified by the transit agency. The transit agency provides the cost of fuel, \$1.05 per gallon. Fuel cost per mile is found from the following formula:

$$\text{Fuel cost per mile} = \frac{\text{fuel cost/gallon}}{\text{fuel economy in miles/gallon}}$$

$$= \frac{\$1.05\text{/gallon}}{4.218\text{ miles/gallon}} = \$0.249\text{/mile}$$

This number, entered in column 3 of Table 8.7, is multiplied by the annual miles to get the annual fuel cost. For example, in the first year the annual fuel cost is 47,500 miles x \$0.249/mile = \$11,828, which is entered in column 4 of Table 8.7.

The annual fuel costs are multiplied by the single payment present worth factors found in Table 8.2 for a discount rate of 10 percent (entered in column 5) which results in the present worth of each annual expenditure for fuel. The results are indicated in the last column of Table 8.7.

The present worth of the total fuel costs over the 12 years is \$74,112, which is substantially less than the direct sum of the annual fuel costs, \$126,370. Neglected in these computations are changes in fuel cost over the years, an unpredictable factor, and changes in bus fuel economy as it ages. These uncertainties are among those which are accounted for in the discounting of the costs.

Other cost drivers which are incurred every year are handled in a similar manner. These may include oil, tires, brake repairs, and preventive maintenance. The number of repair events per year is computed for each driver and the costs tabulated on an annual basis for input to the present worth analysis.

Next, consider the present worth of transmission replacements and overhauls, a much less frequent repair. Table 8.8 (Work Sheet 18) provides the manufacturer's estimates of labor hours and cost of materials for transmission overhauls. The manufacturer estimates the mileage between overhauls to be 265,000 miles but the transit agency, based on its own previous maintenance experience with similar transmissions, disputes this figure and uses its own estimate of 100,000 miles between overhauls.

The transit agency provides the labor rate of \$16.50 per hour. The calculations in Table 8.8 estimate that the total cost per transmission overhaul will be \$1,963, and this cost will be

TABLE 8.8

WORK SHEET 18

MAINTENANCE TASK COSTING

Cost Driver: Transmission Overhaul

Maintenance Task	Estimated Labor Hours*	Labor Rate**	Cost of Materials*	Cost per Task
1. Remove and reinstall transmission	5.45	x $ 16.50	+ $ 65	= $ 155
2. Dismantle, overhaul, and test	18.75	x $ 16.50	+ $ 1,500	= $ 1,809
			Total Cost:	$ 1,964

Expected interval in miles: * 100,000

* Supplied by bus manufacturer

** Supplied by transit agency

NOTE: Manufacturer provides an estimate of 265,000 miles between transmission overhauls but transit agency substitutes 100,000 miles based on its past operating experience.

incurred once every 100,000 miles of accumulated bus use.

The present worth of the transmission overhaul costs over the projected lifetime of the bus is calculated in Table 8.9 (Work Sheet 19). This table assumes that the bus will accumulate the same annual mileages as indicated in Table 8.7 through its 12 years of anticipated service. The cumulative miles are indicated in column 2 and total 507,500 miles.

Since the transmissions are predicted to need overhauls at 100,000 mile intervals, overhauls are predicted after 100,000; 200,000; 300,000; 400,000; and 500,000 miles of service. The year in which each mileage occurs is indicated in column 3 of Table 8.9 and the present worth of the overhaul expense for that year is computed. A discount rate of 10 percent was used in Table 8.9, hence the present worth factors are the same as were used in Table 8.3.

The total present worth of the transmission overhauls is the sum of the numbers in the last column of Table 8.9, or $4,460. Note that this is substantially less than the simple sum of the four transmission overhauls expected during the 500,000 mile lifetime of the bus, which is $7,856.

Finally, note that looking at the time frequency of overhauls has resulted in the final failure, at 500,000 miles, being deleted from the analysis. This is because the bus has reached the end of its projected life. At that time, 12 years, evaluation needs to made of what to do with the bus. Should the transmission be overhauled again? This depends on what the transit agency intends to do with the bus: scrap it, sell it, or rehabilitate it. These questions are addressed in the next section.

Present worth analysis clarifies a problem several transit agencies have encountered in conventional life cycle costing, which includes a "frequency interval factor." This factor is the total life mileage of the bus, 500,000 miles, divided by the manufacturer's estimate of the miles between major service items. For example, suppose that the manufacturer estimates that the interval between major engine overhauls is 360,000 miles. The frequency interval factor is computed as follows:

$$\text{factor} = \frac{500{,}000 \text{ miles}}{360{,}000 \text{ miles}} = 1.39$$

This factor is multiplied by the estimated engine overhaul cost to determine the life cycle cost of engine overhauls per bus. Assuming the cost per overhaul to be $5,950.00, the life cycle cost for engine overhauls is:

cost = 1.39 x $5,950.00 = $ 8,271.

Some transit agencies have complained that certain manufac-

TABLE 8.9

WORK SHEET 19

PRESENT WORTH ANALYSIS OF INFREQUENT MAINTENANCE COST DRIVERS

Cost Driver: Transmission Overhaul

Bus Age (years) (1)	Cumulative Miles (2)	Cost Driver Occurrence* (3)		Cost (4)		Present Worth Factor** (5)		Present Worth (6)
1	47,500	no	$		x		= $	
2	95,000	no	$		x		= $	
3	142,500	failure	$	1,964	x	0.7513	= $	1,476
4	190,000	no	$		x		= $	
5	237,500	failure	$	1,964	x	0.6209	= $	1,219
6	277,500	no	$		x		= $	
7	317,500	failure	$	1,964	x	0.5132	= $	1,008
8	357,500	no	$		x		= $	
9	397,500	no	$		x		= $	
10	437,500	failure	$	1,964	x	0.3855	= $	757
11	472,500	no	$		x		= $	
12	507,500	failure***	$		x		= $	
						Total Present Worth:	$	4,460

---

* Transmissions expected to fail every 100,000 miles according to transit agency records. Hence, failures requiring overhaul are predicted at 100,000; 200,000; 300,000; 400,000; and 500,000 miles.

** Single payment present worth factor with discount rate = 10 percent.

*** Failure at end of bus life excluded from calculations.

turers round the frequency interval factor down to the nearest whole number. In the above example, this would be a rounded factor of 1, which would make the life cycle cost $5,950.00 instead of $8,271. This would give this manufacturer a substantial cost advantage over competitors with a similar or even the same engine who did not round the factor.

The rationale for rounding is evident in the present worth tabulation, in that it assumes the component has some remaining useful life at the end of 500,000 miles or fails at the end of the bus life. A present worth tabulation clarifies the assumptions being made and should ensure that the figures for each manufacturer are treated equitably.

Table 8.10 presents a tabulation of the life cycle costs and other factors for bus procurement, in this case assuming four competing manufacturers. The list is headed by the acquisition price and costs of spare parts and delivery, all present costs by definition. This is followed by the present worths of the eight major cost drivers. The fuel and transmission overhaul costs computed in Tables 8.7 and 8.9, $74,112 and $4,460 respectively, are entered under manufacturer A in Table 8.10. All of these costs are summed to determine the total life cycle cost for each manufacturer. Based on these costs alone, the manufacturer with the lowest total life cycle costs would be selected.

Not included in this example is consideration of performance and standardization, also part of the procurement process. Performance indicators for each manufacturer include financial resources of the manufacturer, training and other technical support, availability of service and repair parts through dealers or other organizations, ability to deliver the buses in a timely fashion, and bus features which exceed the minimum specifications of the agency. Performance, since it largely includes qualitative or difficult-to-quantify factors, might be included as a comparative ranking in Table 8.10.

Standardization aspects include the degree of similarity or interchangeability of bus components. Lack of standardization can incur costs associated with the need for additional service training, tools, inventory, and other facilities. Transit agencies typically request a standardization cost estimate from each manufacturer. This cost can be treated as a present cost and so entered in Table 8.10.

The total procurement evaluation, based on all the information summarized in Table 8.10, is not attempted in this chapter as it is to a considerable extent a matter of judgement and specific to the transit agency making the procurement decision. Note, however, that the intent of present worth analysis is to make the life cycle cost procurement process as systematic and clear as possible. This should help eliminate misunderstandings between manufacturers and transit agencies.

TABLE 8.10

COMPARATIVE LIFE CYCLE COST EVALUATION OF COMPETING MANUFACTURERS

Present Worth Life Cycle Cost, $	Manufacturers			
	A	B	C	D
Acquisition Price*	$	$	$	$
Spare Parts and Delivery*				
Fuel Consumption	74,112			
Oil Consumption				
Tires				
Brake Relining				
Preventive Maintenance				
Engine Replacement and Overhaul				
Transmission Replacement and Overhaul	4,460			
Air Conditioner Compressor Replacement and Overhaul				
TOTAL LIFE CYCLE COST				
STANDARDIZATION*				
PERFORMANCE				

* These costs are considered as present costs and are entered as quoted by the manufacturer with no present worth discounting.

When Should Buses Be Replaced?

When a bus has reached the end of its economic life it is time to consider replacing the bus or keeping it in operation for a few more years. In the transit industry it is standard practice to keep a new bus for 12 years or 500,000 miles, which thus becomes the depreciation life of the bus. The economic life of a bus tends to approximate the depreciation life of 12 years as was indicated in the hypothetical data of Table 8.1 and Figure 8.1.

In economic analysis the existing bus is termed the defender. The possible replacement, whether it be a new, used, or rehabilitated bus, is the challenger. These terms are commonly used in this situation because the economic decision technique is based on a comparison of the annual costs of the alternatives for one more year. In other words, should the existing "defender" bus be kept by the transit agency one more year? The answer is no if it can be shown that a replacement (the "challenger") will cost less the next year.

The economic analysis of bus replacements follows these guidelines:

1. All computations are done on an equivalent annual cost basis. First, this is done because the decision is based on annual expenses for the next year only. Second, this permits comparisons of alternatives which have different lives. A new bus typically has a depreciation life of 12 years. Rehabilitated buses typically should last an additional 3 to 10 years depending on how much remanufacturing is done. Who knows how long a used bus, particularly the defender, will last? A present worth analysis comparison is possible but all alternatives should have equal life spans. This becomes difficult (at least, cumbersome) if one alternative has a life of 12 years, another 8 years, and the defender, say, 3 years.

2. The capital value of the defender bus is its present market value, not its book value based on depreciation (note the figures in Table 8.1). All past capital and maintenance investments are neglected in the analysis; they are no longer germane to the analysis. Past investments are unrecoverable now. The value of the defender is whatever the transit agency can sell it for on the open market.

3. There is no point in looking beyond the next year's operating costs for the defender bus. Since it is an old bus these are rather unpredictable anyway. In the coming year the transit agency has a good idea of what duty cycles the bus will perform and likely maintenance expenses. For example, the old bus may be "defending" itself because the maintenance manager knows it has been unreliable in the past and likely will need major engine and transmission overhauls before the year is out. Or perhaps the defender is already sitting out back waiting for overhauls.

4. The defender should be replaced if its annual cost in the coming year is greater than the equivalent annual cost of a replacement bus, new or rehabilitated. This statement is based on the assumption that the defender will continue to deteriorate over time. Thus, it will be even more expensive to operate in the second and succeeding years.

As will be seen in the example in the next section, the economic analysis may not be as simple a calculation as implied by the above statements. The defending bus, because it is old, already may be putting in less miles than a newer bus. It may generate excessive road calls or otherwise be unreliable, thus relegating it to tripper or spare status. A new or rehabilitated replacement should be more reliable and expected to see more active service. Thus, there is more to the replacement decision than just economics. It is hard to quantify "reliability" and hard to take different duty cycles into account. Therefore, the transit agency should look at the economic comparisons as just one element, albeit a useful and important one, in the replacement decision process.

Purchasing New or Rehabilitated Buses.

Consider a defending bus which is 12 years old and has accumulated about 500,000 miles of service. The maintenance manager anticipates that it will need about $23,000 in work in the coming year, including routine maintenance appropriate to its age. Some major component overhauls are anticipated, too. The bus has incurred high maintenance costs in the past and is expected to continue to do so in the future. Recent fuel consumption records indicate that it consumes 3.9 gallons/mile. The bus averages 1,200 miles between road calls. It has been relegated to tripper status and thus will be operated about 25,000 miles in the coming year. Is it time to replace the bus?

The transit manager intends to compare the cost of keeping this defender bus for one more year with the cost of two alternative challengers, a new bus and a rehabilitated bus. The comparison will be made on an equivalent annual cost basis. That is, all investments will be converted to their annual equivalents. Acquisition prices and salvage values are converted to equivalent annual costs by using the capital recovery factor:

$$\underset{\text{(crf-i-n)}}{\text{capital recovery factor}} = \frac{i\ (1+i)^n}{(1+i)^n - 1}$$

where: n = number of years

i = discount rate expressed as a decimal, e.g., 0.10 for 10 percent

Table 8.11 provides capital recovery factors for a discount rate of 10 percent and 12 years of bus life.

Fuel costs and maintenance costs already are provided on an annual basis, hence, they do not need to be converted. If road calls are assumed to cost \$100, which includes tow truck expense and ridership inconvenience and delay, then the annual cost of road calls is:

$$\frac{25{,}000 \text{ miles/year}}{1{,}200 \text{ miles/call}} \times \$100/\text{call} = \$2{,}083.33$$

Acquisition costs and salvage value (the market value of the bus when it is sold) are converted to equivalent annual costs using the following formula:

capital recovery = (P - S) x (crf-i-n) + Si

where: P = present worth of defender or acquisition cost of challenger

S = salvage value (selling price) at end of n years

i = discount rate

n = remaining bus life in years

For example, suppose that the defender bus can be sold as is for \$5,000 today, its present worth. One year from now it will be worth only \$3,000. The capital recovery for the defender bus thus is:

(\$5,000 - \$3,000) x (crf-10%-1) + (\$3,000) x (0.10)

or

\$2,000 x. 1.10000 + \$3,000(0.10)

which is \$2,500.

TABLE 8.11

CAPITAL RECOVERY FACTORS FOR i = 10 PERCENT

Bus Age, n	Capital Recovery Factor*
1	1.10000
2	0.57619
3	0.40211
4	0.31547
5	0.26380
6	0.22961
7	0.20541
8	0.18744
9	0.17364
10	0.16275
11	0.15396
12	0.14676

---

* Capital recovery factor, (crf-i-n)

$$= \frac{i(1+i)^n}{(1+i)^n - 1}$$ where $i = 0.10$, or 10 percent

The total equivalent annual cost of keeping the defender bus one more year is:

Annual fuel cost:	$ 6,730.77	(25,000 miles of service)
Annual maintenance:	23,000.00	(manager's estimate)
Annual road calls:	2,083.33	
Capital recovery:	2,500.00	
Total annual cost:	$34,314.10	
Annual cost/mile:	$ 1.37	

The next step is to compare the defender bus with the equivalent annual costs of new and rehabilitated bus alternatives on a cost per bus mile basis. The computations for the two alternatives are presented in Table 8.12.

Note that when annual costs vary over time they can be converted to a single equivalent annual cost by first computing the present worth of all the annual cost items, and then converting this to one equivalent annual cost by multiplying it by the capital recovery factor for the total years involved. This is illustrated in Table 8.12 for the new bus. Annual fuel costs are projected to decline over time as the new bus ages and sees less service.

For simplicity constant annual maintenance costs were assumed for both new and rehabilitated buses in Table 8.12. If the transit agency has other information which leads to varying annual maintenance costs, the same conversion as was done with fuel costs could be applied to the maintenance costs.

The equivalent annual costs for the three alternatives are summarized below:

Alternative	Equivalent Annual Cost	Cost/Mile
Defender Bus (one more year only)	$34,314.10	$ 1.37
New Bus (12 year life)	51,327.22	1.21
Rehabilitated Bus (6 year life)	47,720.75	1.19

With these figures the answer seems to be to replace the defender with a rehabilitated bus if annual costs per mile are considered. However, some managers might argue that if the defender were to get a rebuilt engine and transmission (needed in the first year)

TABLE 8.12

EQUIVALENT ANNUAL COST COMPARISON OF A NEW BUS VERSUS A REHABILITATED BUS

New Bus (Life = 12 years, 42,292 miles/year)		Rehabilitated Bus (Life = 6 years, 40,000 miles/year)	
Annual Fuel Cost:[1]	$10,873.40	Annual Fuel Cost:	$11,500.00
Annual Maintenance:	16,500.00	Annual Maintenance:	18,000.00
Annual Road Calls:[2]	939.82	Annual Road Calls:[2]	1,000.00
Capital Recovery:[3]	23,014.00	Capital Recovery:[4]	17,220.75
ANNUAL COST:	$51,327.22	ANNUAL COST:	$47,720.75
ANNUAL COST/MILE:	$ 1.21	ANNUAL COST/MILE:	$ 1.19

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[1] This is derived from the fuel schedule in Table 8.7 by multiplying the present worth of the 12 years of fuel costs by the capital recovery factor for 12 years:

Annual fuel cost = $ 74,089.65 x (crf-10%-12)
= $ 74,089.65 x 0.14676
= $ 10,873.40

[2] Each road call is assumed to cost $100 which includes tow truck expenses and ridership inconvenience and delay. New buses are assumed to average 4,500 miles between road calls and rehabilitated buses 4,000 miles. For example, with a new bus:

Annual road calls = $100/call (42,292 miles/year)
/(4,500 miles/call)
$939.82/year

[3] For a new bus assume an acquisition price of $160,000 and a salvage value of $10,000 after a depreciation life of 12 years. Hence:

Capital recovery = ($160,000 - $10,000) x (crf-10%-12)
+ $10,000 x 0.10
= $150,000 x 0.14676 + $10,000 x 0.10
= $ 23,014.00

[4] For a rehabilitated bus assume an acquisition price of $75,000 with no salvage value after a depreciation life of 6 years. Hence:

Capital recovery = $75,000 x (crf-10%-6)
= $75,000 x 0.22961
= $17,220.75

it should be kept for at least several more years, say three. In this case, its equivalent annual costs should cover three years, and the cost/mile might be decreased. Again, note that future maintenance costs for an unreliable defender are relatively unpredictable. Thus, it may be prudent to consider costs for just the coming year.

The numbers also demonstrate the economic appeal of rehabilitated buses. Even though the rehabilitated bus is assumed to incur greater annual fuel and maintenance costs in Table 12, its lower initial acquisition price results in a somewhat lower equivalent annual cost to the transit agency relative to a new bus.

Rehabilitation of transit buses has grown substantially since the late 1970's, in part following national trends of remanufacturing transportation vehicles and other expensive capital investments. Also, it has seemed attractive to rehabilitate the old but reliable and familiar "New Look" buses which long have dominated transit fleets instead of investing in the more expensive advanced design buses (ADB) now being manufactured.

In the late 1970's the first ADB buses were perceived by some transit agencies to have reliability problems and there were long acquisition lead times. A 1983 UMTA report, Economic Comparison of New Buses Versus Rehabilitated Buses (UMTA-IT-06-0219-02-2), found that rehabilitated buses were distinctly less expensive to purchase, they were perhaps just as reliable in terms of road call frequencies as new buses, and they achieved 25 to 35 percent more miles per gallon than new advanced design buses. The UMTA survey found that no transit agency which had purchased rehabilitated buses had done a comprehensive life cycle cost analysis, in large part because of the uncertainty of operating and maintenance data for rehabilitated buses as well as their projected life. Capital costs per rehabilitated bus ranged from $22,000 to $85,000 (1979-1982 figures) with corresponding extensions of 3 to 10 years in bus life depending on the extend of the remanufacturing.

A similar economic analysis could be done for the purchase of used buses. Finally, if articulated buses were an alternative, the analysis would also have to account for differences in bus utilization and driver productivity. One way to do this would be to compute equivalent annual costs on a per seat-mile basis assuming that the extra seats would be justified by passenger loadings. Typically, articulated buses are assigned to routes with high passenger demands such that fewer articulated buses would be needed to provide the same level of service as conventional 40-ft. buses.

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STUDY QUESTIONS

(1) The acquisition cost of a new bus is $135,000. Determine the depreciation for this bus in the third year of its service using the double declining balance depreciation method. Assume a depreciation life for the bus of 12 years.

(2) For the above bus annual fuel costs are estimated at $11,500. In its third year of service maintenance expenses are estimated at $6,000.

   (a) Compute the total annual life cycle cost for this bus in its third year of service.

   (b) What is the present worth of the life cycle cost in part (a), assuming a discount rate of 10 percent?

(3) As part of the bus procurement process Manufacturer A submits a bid which includes an estimated fuel consumption rate of 3.784 miles per gallon. Determine the present worth of the fuel consumed during the projected lifetime of this bus. Assume a life of 12 years and a discount rate of 10 percent. The transit agency projects that the bus will be used 45,000 miles per year in its first six years and then 37,500 miles per year for the remaining six years. The transit agency estimates the fuel cost to be $1.05 per gallon. Use Work Sheet 17 for the computations.

(4) As part of the bus procurement prcess Manufacturer A submits a bid which includes information on the following maintenance cost driver, engine overhauls. There are two tasks to be performed. The manufacturer estimates that it will take 12.55 labor hours to remove and replace the engine, accompanied by $50 worth of materials. The overhaul, itself, will take 48.75 labor hours and $3,800 in materials. The expected interval between engine overhauls, according to Manufacturer A, is 350,000 miles.

Determine the present worth of the expenditures for engine overhauls during the lifetime of the bus. Assume that the bus will be kept in service for 12 years with the cumulative miles indicated in Table 8.9. Assume a discount rate of 10 percent and a labor wage rate of $13.50 per hour. Use Work Sheets 18 and 19 for your computations.

(5) In the last section of the life cycle costing chapter the transit agency considered replacing its existing 12-year-old buses with either new or rehabilitated buses. Suppose the transit agency also has the opportunity to purchase a fleet of used buses for $25,000 each. These used buses are eight years old so the transit agency assumes a remaining life of four years. The used buses have been averaging 30,000 miles of service a year and 3.95 miles per gallon. It is estimated that these buses will cost $20,000 a year to maintain. An annual cost of $1,500 is assumed for road calls. The transit agency estimates that the used buses will have a resale (salvage) value of $5,000 after four more years of use. Assuming that fuel costs $1.05 per gallon and a ten percent discount rate, should the transit agency buy these used buses instead of new or rehabilitated buses?

ANSWERS TO STUDY QUESTIONS

(1) Use the following formula:

$d_n = (P_{n-1}) \times k$, where:

n = bus age in years

d_n = depreciation in year n

P_{n-1} = original capital cost minus the sum of the depreciations computed for the previous years, 1 to n-1.

$k = 2/N$, where
N = depreciation life

In this problem the depreciation life is 12 years, hence, $k = 2/12 = 1/6$. The depreciation in the third year is found from the following computations:

Year, n	P_{n-1}	x k = d_n
1	(135,000 - 0)	x 1/6 = $ 22,500
2	(135,000 - 22,500)	x 1/6 = $ 18,750
3	(135,000 - 22,500 - 18,750)	x 1/6 = $ 15,625

The depreciation in the third year is $ 15,625.

(2) (a) The total annual life cycle cost is the sum of the depreciation, fuel, and maintenance costs. For the third year of service:

$15,625 + $11,500 + $6,000 = $ 33,125.

(b) The present worth of $33,125 is found by multiplying this number by the present worth factor for n = 3 years and a discount rate of 10 percent. From Table 8.2 this factor is 0.7513. Hence:

$33,125 x 0.7513 = $ 24,887.

(3) The computations are indicated in Work Sheet 17 on the next page. The cost of fuel per mile is computed as follows:

$$\frac{\text{fuel cost/gallon}}{\text{fuel economy in miles/gal}} = \frac{\$1.05/\text{gal}}{3.784 \text{ mpg}} = \$0.277 \text{ per mile}$$

This cost factor is entered in column (3) of the work sheet. Note that the work sheet can accomodate changing cost factors over time. For example, the transit agency may wish to account for changes in fuel cost over time, or increase the fuel economy figure as the bus ages. As indicated in Work Sheet 17, the total present worth of the twelve years of fuel consumption is $79,964.

Note that there are only two different annual costs in the work sheet, $12,487 for the first six years and $10,406 in the remaining six years. There is a short-cut method which can be used to simplify the present worth computations in such cases, namely, another compound interest formula called the UNIFORM SERIES PRESENT WORTH FACTOR:

$$\text{uniform series pwf} = \frac{(1 + i)^n - 1}{i(1 + i)^n}$$

The present worth of a series of n equal annual costs is found by multiplying the annual cost by this factor. For example, the present worth of the first six years of equal annual fuel costs, $12,487, is computed as follows:

$$\text{total present worth} = \$12,487 \times \frac{(1 + 0.10)^6 - 1}{0.10(1 + 0.10)^6}$$

$$= \$12,487 \times 4.355$$

$$= \$54,381$$

STUDY QUESTION 3: ANSWERS

WORK SHEET 17

PRESENT WORTH ANALYSIS OF ANNUALLY RECURRING COST DRIVERS

Cost Driver: Annual Fuel Consumption

Bus Age (years) (1)	Mileage per year (2)		Cost per Mile (3)		Total Annual Cost (4)		Present Worth Factor* (5)		Present Worth (6)
1	45,000	x	$ 0.277	=	$ 12,487	x	0.9091	=	$ 11,352
2	45,000	x	$ 0.277	=	$ 12,487	x	0.8264	=	$ 10,319
3	45,000	x	$ 0.277	=	$ 12,487	x	0.7513	=	$ 9,381
4	45,000	x	$ 0.277	=	$ 12,487	x	0.6830	=	$ 8,529
5	45,000	x	$ 0.277	=	$ 12,487	x	0.6209	=	$ 7,753
6	45,000	x	$ 0.277	=	$ 12,487	x	0.5645	=	$ 7,049
7	37,500	x	$ 0.277	=	$ 10,406	x	0.5132	=	$ 5,340
8	37,500	x	$ 0.277	=	$ 10,406	x	0.4665	=	$ 4,854
9	37,500	x	$ 0.277	=	$ 10,406	x	0.4241	=	$ 4,413
10	37,500	x	$ 0.277	=	$ 10,406	x	0.3855	=	$ 4,012
11	37,500	x	$ 0.277	=	$ 10,406	x	0.3505	=	$ 3,647
12	37,500	x	$ 0.277	=	$ 10,406	x	0.3186	=	$ 3,315
					Total Present Worth:				$ 79,964

---

* Single payment present worth factor with discount rate = 10 percent.

Values for the uniform series present worth factor also can be found in tables in any engineering economics textbook. The total present worth for the second series of six annual costs similarly is computed:

total present worth = \$10,406 x 4.355
= \$45,318

However, this is the total present worth at the start of the seventh year of bus life, not today. This figure is easily converted to a total present worth today by multiplying it by the single payment present worth factor for n = 6 years. This value, for a 10 percent discount rate, is found in Table 8.2 to be 0.5645. The total present worth today is:

total present worth <u>today</u> = \$10,406 x 4.355 x 0.5645
= \$45,318 x 0.5645
= \$25,582

Finally, the two present worth totals are summed to obtain the present worth of all 12 years of bus life:

grand total present worth = \$54,381 + 25,582
= <u>\$79,963</u>

This is the same figure as computed in Work Sheet 17.

(4) The information supplied by the manufacturer, along with the labor wage rate provided by the transit agency, is tabulated in Work Sheet 18. The total cost to perform each engine overhaul is found to be $4,677. Based on the cumulative miles indicated in Work Sheet 19, the engine is predicted to need an overhaul after 350,000 miles or in its eighth year of service. Multiplying the overhaul cost by the appropriate present worth factor (from Table 8.2 for 8 years and 10 percent discount rate, the factor is 0.4665) yields the present worth of this first overhaul, which is $2,182.

At the end of 12 years the first engine overhaul is predicted to be still in service. In fact, it will have accumulated only 150,000 miles out of a projected life of 350,000 miles. Should a cost be assigned to the miles accumulated by this overhaul? The suggested answer is no, because the second engine failure is estimated at 700,000 miles, which is beyond the projected life of the bus. Since the expense for a second overhaul does not occur within 12 years, it is not included in the present worth estimate for bus procurement. The value of the engine is part of the salvage value of the bus after 12 years and it may contribute to bus replacement decisions made at that time. Thus, the present worth of engine overhauls for Manufacturer A is the previously determined $2,182, and it is this figure which is entered in Table 8.10, the comparative evaluation of competing manufacturers.

STUDY QUESTION 4: ANSWERS

WORK SHEET 18

MAINTENANCE TASK COSTING

Cost Driver: Engine Overhaul

Maintenance Task	Estimated Labor Hours*	Labor Rate**	Cost of Materials*	Cost per Task
1. Remove and replace	12.55	x $ 13.50	+ $ 50	= $ 219
2. Overhaul	48.75	x $ 13.50	+ $ 3,800	= $ 4,458
			Total Cost:	$ 4,677

Expected interval in miles: * 350,000

---

* Supplied by bus manufacturer

** Supplied by transit agency

STUDY QUESTION 4: ANSWERS

WORK SHEET 19

PRESENT WORTH ANALYSIS OF INFREQUENT MAINTENANCE COST DRIVERS

Cost Driver: Engine Overhaul

Bus Age (years) (1)	Cumulative Miles (2)	Cost Driver Occurrence (3)	Cost (4)		Present Worth Factor* (5)		Present Worth (6)
1	47,500	no	$	x		=	$
2	95,000	no	$	x		=	$
3	142,500	no	$	x		=	$
4	190,000	no	$	x		=	$
5	237,500	no	$	x		=	$
6	277,500	no	$	x		=	$
7	317,500	no	$	x		=	$
8	357,500	failure	$ 4,677	x	0.4665	=	$ 2,182
9	397,500	no	$	x		=	$
10	437,500	no	$	x		=	$
11	472,500	no	$	x		=	$
12	507,500	no	$	x		=	$

Total Present Worth: $ 2,182

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* Single payment present worth factor with discount rate = 10 percent.

(5) As with the other alternatives the equivalent annual costs will be computed for owning the used buses for four years. The first step is converting the purchase price and salvage value to an equivalent annual cost by using the capital recovery factor:

$$\begin{aligned} \text{capital recovery} &= (P - S) \times (crf - i - n) + Si \\ &= (\$25{,}000 - \$5{,}000) \times (crf - 10\% - 4) \\ &\qquad + \$5{,}000 \times 0.10 \\ &= \$20{,}000 \times 0.31547 + \$500 \\ &= \$6{,}809 \end{aligned}$$

The annual fuel cost is computed as follows:

$$\begin{aligned} \text{annual fuel cost} &= \frac{30{,}000 \text{ miles} \times \$1.05/\text{gallon}}{3.95 \text{ miles/gallon}} \\ &= \$7{,}975 \end{aligned}$$

The total equivalent annual cost for each used bus is:

Annual fuel cost:	$ 7,975
Annual maintenance:	20,000
Annual road calls:	1,500
Capital recovery:	6,809
Total annual cost:	$36,284
Annual cost/mile:	$1.21

Based on the annual cost per mile the used buses are superior to the defender buses and comparable in cost to the new or rehabilitated alternatives. Note that the transit agency could use these computations to negotiate the price for the used buses. In other words, the agency could determine a maximum price it would be willing to pay to purchase these used buses rather than new or rehabilitated buses.

CHAPTER NINE

CONCEPTUALIZING AND PLANNING OF MAINTENANCE MANAGEMENT INFORMATION SYSTEMS AND CONSIDERATIONS FOR SYSTEM DESIGN

Introduction

There are a number of maintenance management techniques that have been implemented in practice and/or discussed in the literature. These methods include component failure forecasting to aid in planning labor needs, parts inventory quantities and maintenance budgeting; time standards for mechanics; work load scheduling and planning; maintenance job method analysis; life cycle costing for budget analysis and vehicle replacement decisions; maintenance performance indicators; and other systematic management techniques. Through experimentation it has been found that the application of systematic techniques can have dramatic impacts on a maintenance system's performance. For example, computer simulation has shown that the introduction of work load scheduling and simple maintenance job prioritization rules (based on the expected number of man hours a job will take) can decrease the average number of buses out of service for maintenance work by as much as 20 percent (1).

However, no systematic management technique is implementable without the availability of quality maintenance information. In fact, even rudimentary management procedures are impossible without good information. For example, a common practice for maintenance managers is to monitor fluids consumption rates (fuel, oil, transmission fluid and coolant) and flag vehicles that have rates which are unusually high or low. Flagging a vehicle and inspecting it can identify difficulties before a major failure occurs. However, even the simple flagging of vehicles that experience fluid consumption rates that are out of a tolerance level requires a good information base.

To capitalize on these management techniques, a good information base is essential. Although computerization of an information base is not a mandatory feature of having a good information system, a computer can certainly be a tremendous improvement over a paper system.

One of the most troublesome aspects of computerization is that computer system development is commonly left up to computer experts. However, the development of a computerized information system is too important to the user to be left up to computer experts. The users (the maintenance manager, the parts and inventory manager, etc.) should take the leading role in the system's conceptualization and planning and be deeply involved in the system's design, implementation and maintenance. Unfortunately, the typical computer system development techniques usually require knowledge of computers, data structures, software, computer technical jargon, etc. which may be foreign to non-computer experts (the users).

Fortunately, the conceptualizing and planning of a computer system does not require technical knowledge of computers. The first part of this chapter shows how to lay out the plans for an information system using a tool that works like an information flow road map. The road map is called a "data flow diagram". The diagram requires very little knowledge of computers and allows the users to communicate their desires to computer experts. The data flow diagram is illustrated using the information flows of an actual transit system, the Metropolitan Transit Transit Authority of Wichita, Kansas. The second part of the chapter discusses technical aspects of designing the performance of maintenance information systems, inventory systems, semi-automatic fueling systems, and designing the man-machine interface.

Information System Development

The development of any computerized information system should go through five stages.* They are:

1. Conceptualizing: What are the objectives? What is expected from the information system? This is first determined through management-level brainstorming sessions.

2. Planning: This is the determination of information needs and evaluation methods. Planning should result in a system performance specification.

3. Design: What hardware and software is required to meet performance specifications? How will the system be organized? How will transit agency procedures be changed? What about staff training?

4. Implementation: During this step the new information systems is installed. Transit agency staff become operational in its use and the "bugs" are worked out of the system.

5. Maintenance: This stage covers the life of the system after the system builders are done with their implementation. The first half of this chapter deals only with the first two stages, Conceptualization and Planning. The second half covers technical considerations of Design.

Where Does the Fleet Manager Get Involved?

In candid discussions with some fleet managers that have information systems, they have admitted that "the system doesn't quite fit my needs" or "it doesn't give me the information I

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* These five stages are defined by Mathews (2).

want", or even worse, "the system doesn't require detailed input information, so I don't get the information at the level of detail I need." One of the reasons for these complaints is that there was not enough involvement of the fleet manager at the very beginning of the system development.

During the first two stages of system development (Conceptualizing and Planning), the fleet manager must take a leading role. The reason for this is that the computer experts and system salespersons do not understand the agency's information needs as well as the fleet manager*. In the three remaining stages (Design, Implementation and Maintenance) the computer experts can take a leading role with continuing guidance from the fleet manager.

To further demonstrate why it is important for the fleet manager to be involved at the very beginning, consider the cost of making computer system changes after the system has been installed. Figure 9.1 shows the relative cost for fixing a computer information system at each of the five stages.** For example, an error found during the Conceptualizing stage may have a relative cost of 0.5 (say $100). To correct it later, in the Maintenance stage, this same error may have a relative cost of 15.5 to correct: 31 times more costly ($100 x 31 = $3,100)! Therefore, it makes sense to tie down the fleet manager's needs in the early stages of development. Once a system is implemented it may be too costly to change it to the way it should have been in the first place! In other words, transit agencies can't afford to wait for their staff to "see what they get" before they "know what they want."

The Fleet Manager's Involvement in Conceptualizing

During the Conceptualizing stage of an information system's development, the system's goals and objectives must be developed. At this point, the scope of the system must be decided. For example, a limited system might deal just with data collection at the fueling island, while a broad system may fully integrate all transit management activities (maintenance, parts inventory, service scheduling, accounting, etc.). Another system dimension that must be decided on is the ability to change the system to adapt to the changing needs of the transit agency. Can staff people make these changes? How free are they to analyze and interpret the data generated by the system?

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* The computer expert, in this case, may be a member of the agency's data processing department, a consultant, or a computer software/hardware vendor.

** Figure 9.1 was adapted from relative cost information given in (3).

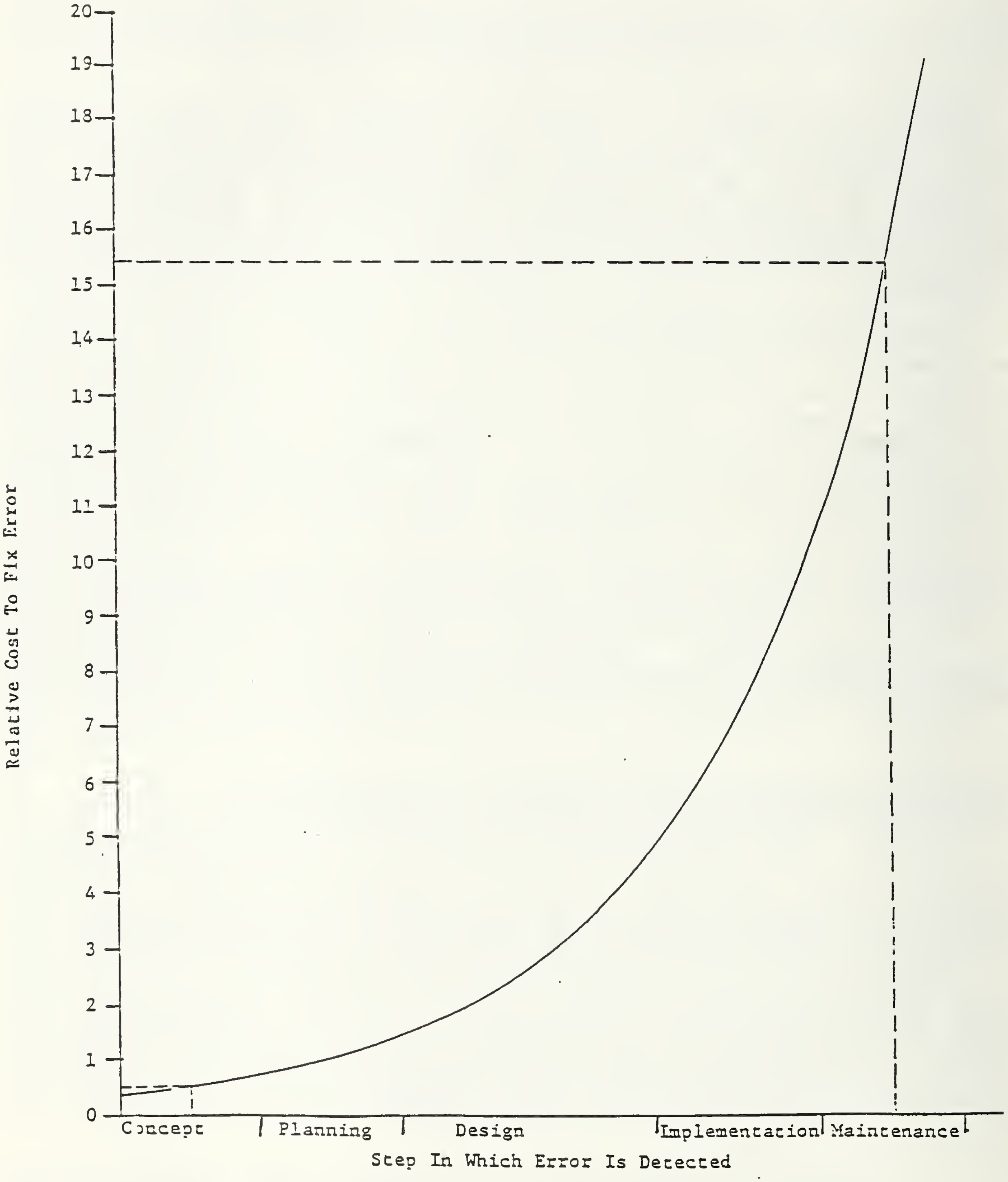

FIGURE 9.1

THE RELATIVE COST TO FIX AN ERROR AT EACH STAGE

The broadness of a computer system and its ability to control more functions is generally described as being integrated. Usually computer experts will argue that broadly integrated systems are preferable to single function systems. For example, computerized inventory systems should hook directly into a computer maintenance information system. If the two systems are integrated, then when a part is assigned to a work order the computer automatically adjusts the inventory records. Similarly, experts will argue that flexibility to customize is important. Generally, the users will grow and change. If the user can customize the system, the system can grow and change with the user. Unfortunately, flexible and integrated systems are more expensive.

The Fleet Manager's Involvement in Planning

As part of Planning, a functional specification must be designed. Because the system is largely for the fleet manager's benefit, the functional specification must be designed to meet the fleet manager's needs and desires. The difficulty in developing the system specification lies in presenting these information needs and desires in a format that can be easily understood.

In the remainder of this section a graphical technique is introduced which can help the fleet manager develop a system specification.* This technique largely involves putting together a picture of the information flows for the transit agency based on information only the fleet manager knows and understands. The technique can be used without a background in computers. The main benefits of the technique are:

1. It helps the fleet manager to better understand and clarify current information flows.

2. It helps in determining what information flows could and should be computerized.

3. It aids in finding new information flows that are made possible with a computer.

4. It creates a picture that can be understood and agreed to by all involved (this is planning!).

5. It shows how the system should function without regard to the type of computer (such as, microcomputer or minicomputer) or other physical requirements.

6. It expresses preferences and system trade-offs.

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* This technique is adapted from (4).

Functional Specification Design Technique

This technique uses diagrams to construct the specification rather than words. To understand why this graphical approach is much easier than a written description of all the specifications, suppose that a specification for a building had to be written rather than charted with blueprints. It would take hundreds of pages of English text to describe the dimensions and locations of each door, window, wall, column, joist, etc. Instead, a plan can provide the same information. The same is true with computerized information systems. One or two data flow diagrams can replace several pages of text. As the old saying goes, "a picture is worth a thousand words".

Data Flow Diagrams. The data flow diagram has only four types of symbols, each representing an activity in the flow of data. To illustrate each one, consider a simple example:

A bus driver reports a mechanical problem on a bus and triggers a chain of events which eventually results in the bus getting fixed. For now, consider only what happens when the shop foreman receives the notice of the problem.

The driver submits a defect card at the end of the shift which notes "soft brakes." The defect card goes to the shop foreman (an "information flow"!) who must decide on a maintenance action. The shop foreman might check the bus's maintenance history to see when the brakes last were inspected or repaired, another information flow. Next, the foreman decides whether the bus should be taken off service until it is repaired ("dead status") or if the bus can make tripper runs ("deferred status") while waiting for maintenance. The foreman changes the status of the bus and writes a work order indicating that the bus's brakes must be checked, two more information flows.

To diagram these information flows, it is first necessary to identify each flow and activity:

1. The defect card is generated by the driver.

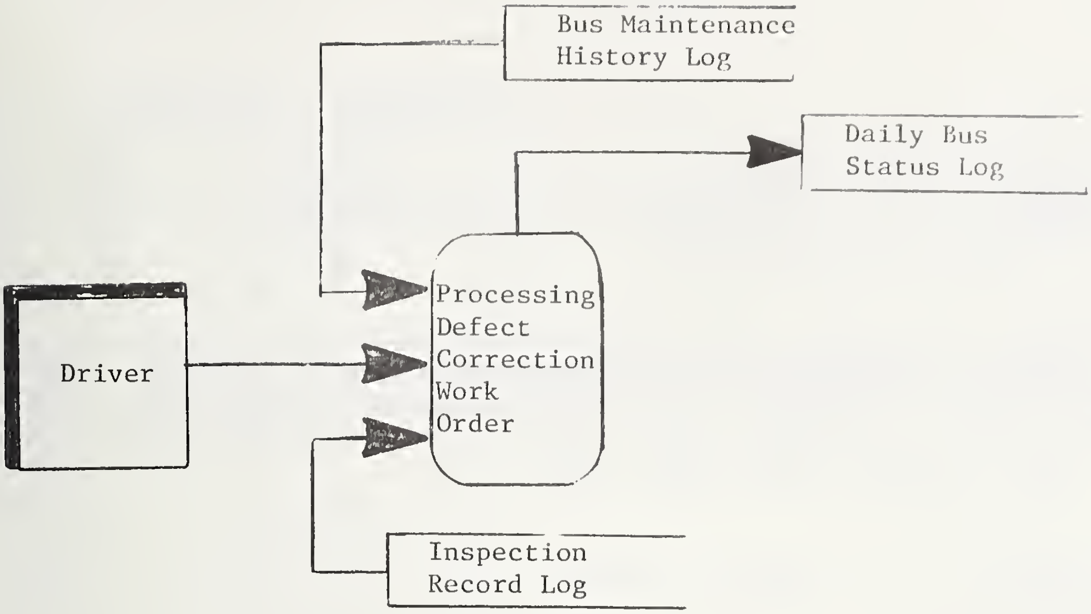
2. The defect card goes (flows) to the shop foreman.

3. The foreman responds by checking the bus maintenance history and inspection log.

4. The foreman then posts a new status for the bus (dead, deferred or active).

5. The foreman submits a work order to the maintenance shop.

The diagram below shows this flow of information.



This diagram uses just four symbols. Their meanings are;

1. Double Square: The double square is an external source or destination of information. In this example the driver is considered to be external to the maintenance system but this was simply a matter of choice. Alternatively, drivers might be considered part of the maintenance system.

Source or destination of data

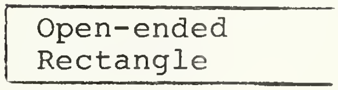
2. Arrow: The arrow represents a data flow. These can be identified in existing paper maintenance systems because they only transmit information that is later recorded somewhere else. For example, the defect card or the work order are messages that are only of temporary value and they only provide data about one bus. Later, the results of the action taken in response to the message are recorded or used somewhere else.

Data Flow (Message)

3. Rounded Rectangle: The rounded rectangle shows that the data is processed. In the example, the shop foreman gets the defect card and starts processing data by figuring out what should be done. While deciding what to do (processing), the foreman may look at other records, in this case bus inspection and maintenance records. The results of the foreman's process is a change in the status of the bus and a maintenance work order.

Process Which Transforms Data

4. Open-ended Rectangle: The open-ended rectangle stands for a data store. A data store is where information is kept. For example, the bus status log keeps data on several buses and even though it changes from one day to the next, it is a long term record of the work flow.



Store of Data

The data flow diagram uses only these four symbols. In the Planning stage it is not necessary to translate these flows, processes, and data records into computer programs or functions. The diagram simply describes the relationship between the various functions of the system. Later, in the Design stage, computer programmers can figure out the details.

With the data flow diagram one need not worry about including procedures to double check for common mistakes which could be made when recording data. For example, on the defect card the driver may enter the wrong bus number. While processing the defect card, a check should be run to see if the driver was actually assigned to the indicated bus. Although these checks are important, they do not need to be considered at the data flow diagram stage. A good computer programmer will know that the program must check for common errors.

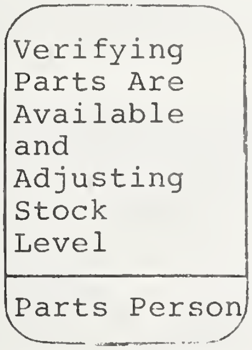
Drawing a Data Flow Diagram. To illustrate the drawing of a data flow diagram, we are going to use an actual, small transit property (60 buses) as an example. The transit property is the Metropolitan Transit Authority of Wichita, Kansas. This example makes no attempt to approach the technical level of a computer programmer or a system analyst.

When drawing-up a data flow diagram, there are five conventions to always remember.

1. Do not cross data flow lines if possible.

2. In order to avoid crossing lines, it is acceptable to draw external entities and data stores twice or more. To indicate that the same external entity or data store appears more than once in the data flow diagram, draw a line in the corner of the data entity symbol and a line across the left end of the data store symbol as shown below.

3. To help identify a process put the title of the individual doing the process at the bottom of the rounded rectangle as shown below:



4. A minimum of three drafts of the data flow diagram should be made. After completing each draft of the diagram, one will find ways to improve it and find data flows that were overlooked.

5. Neatness does not count!

Starting a Data Flow Diagram. Where to start the designing of a data flow diagram largely depends on what is presently being done. Presumably, most properties at least have a paper-based work order system. Therefore, most fleet managers have some kind of record-keeping system to start with.

Many transit systems have excellent paper or paper and computer record-keeping systems. Often these systems have taken years to perfect and they are tailored after the fleet manager's maintenance philosophy. These existing systems provide excellent starting points from which to start. Potential improvements will become apparent as the diagram is developed and additions can be made to later drafts of the data flow diagram.

To provide an example, the paper information system kept by the MTA is analyzed first. To do this, all of the forms used by the MTA maintenance department in its activities are first collected together. Using the paper forms a description of the external entities, the data flows, and the data stores is developed.

External Entities. The external entities are easily defined because they are individuals who start the paper flow but are external to the maintenance system. A driver submitting a defect may be considered an external entity. By submitting a defect card, the driver starts the paper information flow.

Sometimes it is not clear cut whether an entity is external or not. For example, the fueler starts a paper flow by submitting a fueling and fluids consumed report. Whether the fuelers are external or internal is simply a matter of definition. The external entities used for the MTA example are:

Drivers
Dispatcher
Fuelers
Maintenance and services contractors
Parts vendors
Transit system management

Data Flows. In a good paper information system, almost all data flows will be represented by a form or report. For example, the driver's defect card is a form that transmits data. However, even at the best managed transit properties not all data flows are formalized with their own form or report. For example, at the MTA the night fuelers occasionally spot a defect that a driver did not report. If the defect is minor, the fueler will fix the defect. If the defect is major, the fueler will change the status of the bus on the daily work sheet (a status log of the condition of each bus that is waiting for maintenance) and leave a note for the shop foreman. The next day the dispatcher sees that the status of the bus has been changed so it is not assigned to a driver. The shop foreman finds the note and writes a maintenance work order. Although information flowed from the fueler to the shop foreman, the MTA has no specific form or report for this information flow.

Another subtle example of data flow occurs during the requisition of a part. The mechanic asks the parts man for a specific part. This request is a data flow. In the next step the parts man "processes" the verbal request by looking up the part number and its availability. The second data flow is the availability or unavailability (stock-out) of the part. The next process is to get the part if it is available. Thus, the parts example illustrates two data flows that were verbal and had no forms: 1) the parts request and, 2) the availability or unavailability of the part.

To identify the data flows, start by identifying and classifying all of the paper forms. The following is a list and description of all the MTA forms and reports that are considered to act as data flows.

1. Driver's Bad Order (Defect) Card. These are used by the driver to describe mechanical defects on their bus.

2. Notice of an Inspection Due. This is sent from the bookkeeper to the shop foreman and indicates that a bus is approaching the mileage level where another inspection will be required.

3. Notice of Inspection Completed. This is sent from the shop foreman to the bookkeeper and it indicates that the inspection has been completed. The bookkeeper starts accumulating miles until the next inspection.

4. Fueling Sheets. This report is generated by the nightly fueler and contains the fuel and fluids (i.e., oil, coolant and transmission fluid) consumed by each bus. This report is given to the bookkeeper.

5. Bus Line Report. This report is generated by the nightly fueler and identifies the location of each bus after fueling and cleaning. The report is given to the dispatcher.

6. Daily Mileage Report. The daily mileages accumulated by the bus are based on route miles. The dispatcher creates a report of all mileages accumulated by all the buses and the report is given to the bookkeeper.

7. Work Orders. Work orders are the heart of any maintenance information system, paper or computerized. A work order is a written history of each individual maintenance action. From the work order, information is later collected as inputs to summary reports. At the MTA, work orders are used to transmit data in a number of ways. Below are listed each of the distinct ways a work order is used to transmit information. In the data flow diagram each will appear as a separate data flow.

    a. The work order is used to tell maintenance to process a bus inspection.

    b. Maintenance uses the work order to tell the shop foreman that the inspection was completed and that the bus is okay or that further maintenance work is needed.

    c. The work order is used to tell maintenance to correct a bus defect.

    d. The work order is used to tell the shop foreman that a bus's defect was corrected and what was done to correct the defect.

    e. A work order is used to tell a maintenance contractor to perform a service (e.g., rebuild a transmission or dispatch a tow truck to a road call).

    f. The work order is used to show the shop foreman that the maintenance contractor has completed his service.

    g. The work order is used to transmit to the bookkeeper the direct costs (labor and material costs) of an inspection, maintenance task, defect repair, or contracted maintenance work.

8. Purchase Orders. These are used to purchase materials from vendors. Purchase orders provide several types of data flows. They are:

   a. To tell the vendor to deliver material.

   b. The returned purchase order tells the parts man to add the material to the inventory records and to create parts cards.

   c. The purchase order is finally transmitted to the bookkeeper and the bookkeeper processes payment of the vendor.

9. Parts Cards. These are cards attached to each part in the inventory. The card lists the part number, cost, English description, the bus on which the part was used, and the date of its installation. Parts cards have two information flows. They are:

   a. When the part is requisitioned, it goes with the work order while the defect is corrected.

   b. When the work order is returned, after the defect has been corrected, the parts card supplies the part's direct cost information.

10. There are several end-of-the-month reports generated which provide management information. Each report is processed by the bookkeeper. These monthly reports are:

    a. Fluids and fuel used per month and current inventory levels.

    b. The monthly mileage, fuel and oil consumption quantities per bus and per mile.

    c. The total monthly fuel, oil, parts, and maintenance labor cost, miles and cost per mile by the entire fleet, by bus model type, and by bus.

    d. Parts purchased and parts cost by purchase order or contract.

There are several more data flows other than those represented by forms. These are general verbal data flows or flows from a data store to a process. For example, when the shop foreman receives a card from a driver reporting a defect (say, transmission slipping), the foreman will probably look the bus up in the maintenance history. The information found in the history log is a data flow. Such data flows will become obvious when drawing the data flow diagram.

Data Stores. Data stores can be easily identified because they contain information gathered from several individual data flows. For example, the bus maintenance history ledger summarizes the results of numerous work orders. The data stores identified at the MTA are:

1. Daily Work Sheets. This sheet lists the status of buses that currently require maintenance. As buses require maintenance work they are added to the list and when repaired they are taken off the list. The list also defines the status of a bus. For example, a bus with a cracked tail light cover can be used in service but eventually needs to be brought in for repair. Such buses are given tripper status which means that the dispatcher can assign the bus to tripper runs, thus making the bus available for the majority of the day. Buses with more serious defects are assigned dead status, thus stopping the dispatcher from assigning the bus to any run. Buses that are not repaired during that day are transferred to the next day's work sheet.

2. Parts Card File. In this file there is a card for each part. The card lists when parts are received and disbursed, how many are on hand, the vendor and the part cost.

3. Daily and Monthly Miles, Inspection and Fuel. This is an accumulative log of the fuel and fluid each bus has consumed each month and the miles each bus has accumulated since the beginning of the month, and since the last inspection. At the end of the month the miles since the last inspection are carried over to the next month.

4. Bus Maintenance History Ledger. Once a work order has been processed, component and major part replacements are posted to the bus maintenance history ledger. The information on the ledger is the date of the repair, the mileage of the bus and serial number of reusable parts and components.

5. Inspection Record Log. When the shop foreman receives an inspection notice the receipt of the notice is added to the inspection record log. This record is used to determine which type of inspection should be done next (e.g., 3,000, 6,000, 12,000 mile inspections).

6. Annual Cost Ledger. This ledger contains all of the monthly sums of parts, labor, fluid, fuel and contract service costs per bus (direct maintenance and operation costs). The total costs are produced on an annual basis from this ledger.

Drawing the Data Flow Diagram. The next step in preparing to draw the data flow diagram is to list out each of the data flows and determine what process, data store or external entity the data flows link together. Specifically, on both ends of the data flow arrow there must be a process, data store or external entity. The MTA's list is shown in Table 9.1.

TABLE 9.1

LIST OF DATA FLOWS FOR THE MTA

Generates Data Flow	Data Flow	Receives Data Flow
Driver	Driver's bad order card	Shop foreman processing defect correction work orders
Bookkeeper Processing monthly miles and inspection records	Notice of inspection	Shop foreman processing inspection work order
Shop foreman processing inspection completed work orders	Notice of inspection completed	Bookkeeper processing monthly miles and inspection records
Fuelers	Fueling sheets	Bookkeeper processing monthly miles and inspection records
Fuelers	Bus line report	Dispatcher
Dispatcher	Daily mileage report	Bookkeeper processing monthly miles and inspection records
Foreman processing inspection work orders	Work order initiating inspection	Maintenance personnel processing inspections
Maintenance personnel processing inspection	Work order for completion of inspection	Shop foreman processing inspection completion work orders
Shop foreman processing all completed work orders	Work order initiating correction of defect	Maintenance personnel processing defect corrections
Maintenance personnel processing defect corrections	Work order indicating correction completed	Shop foreman processing defect correction work orders
Shop foreman processing contract work orders	Work order indicating required contract services	Maintenance contractor

(continued)

TABLE 9.1. (continued)

Generates Data Flow	Data Flow	Receives Data Flow
Maintenance contractor	Work order indicating completion of contract service	Shop foreman processing completion of contract work orders
Shop foreman all work order completion processes	Work order used to transmit direct costs	Bookkeeper processing monthly cost reports
Parts man processing requisition from vendor	Purchase order to vendor	Vendor
Vendor	Purchase order material received	Parts man processing inventory update
Parts man processing inventory update (entering new parts in card file and creating parts card)	Purchase order for payment	Bookkeeper processing payment and posting to monthly reports
Parts man processing parts requisition	Parts card assignment to defect correction	Maintenance personnel processing defection correction
Maintenance personnel processing defect correction	Parts card with completed work order	Shop foreman processing defect correction work orders
Bookkeeper processing monthly reports	Monthly reports	Management

With this information collected the data flow diagram can be drawn. Find a big sheet of paper, a table, and pencil. Then start with an external entity and start tracing the data flows. The second draft is shown in Figure 9.2 and a final draft (drawn with drafting tools) is shown in Figure 9.3.

Drawing the data flow diagram was simply a matter of connecting the processes and data stores with data flows. Now that the data flow diagram has been drawn, it is wise to have others check it over for accuracy and make the necessary corrections. Spend a few minutes inspecting the diagram and see if it looks like the data flow at your maintenance system.

Designing Functional Specifications for an Information System

Now that the existing system has been laid-out, the last step in the Planning stage can begin. This step consists of deciding what is desired from the computerized information system relative to the data flow diagram. These are three aspects to this step:

1. Determine which existing functions should be computerized. For example, at the MTA the bookkeeper processes an accumulative report of miles, fuel and miles since the last inspection. This would be a relatively simple function to automate and therefore, a strong candidate for computerization. Further, the functions which the bookkeeper performs are relatively standard. Therefore, inexpensive general bookkeeping software is available to process the bookkeeper's records.

2. Determine the information data flows you would like to have available but which are not feasible without computerization. For example, the MTA keeps fairly accurate parts inventory information. From their existing records it would be possible to calculate part usage rates (the quantity of a specific part used per month). This information would be useful in setting stock levels. However, the extensive labor required to calculate part usage rates makes it impracticable. New information flows, like part usage rates, can be added to the data flow diagram by drawing an arrow between the parts card file and the management and labeling it part usage rates.

3. Determine which functions are not currently conducted but would be feasible with computerization. For example, the MTA does not currently schedule the work flow through the maintenance shop. Maintenance work orders are processed at random. Computerization would help in the creation of time standards and thus allow the computer to prioritize maintenance work orders according to the estimated time it takes to complete the work order. These new functions can be represented by new rounded rectangles.

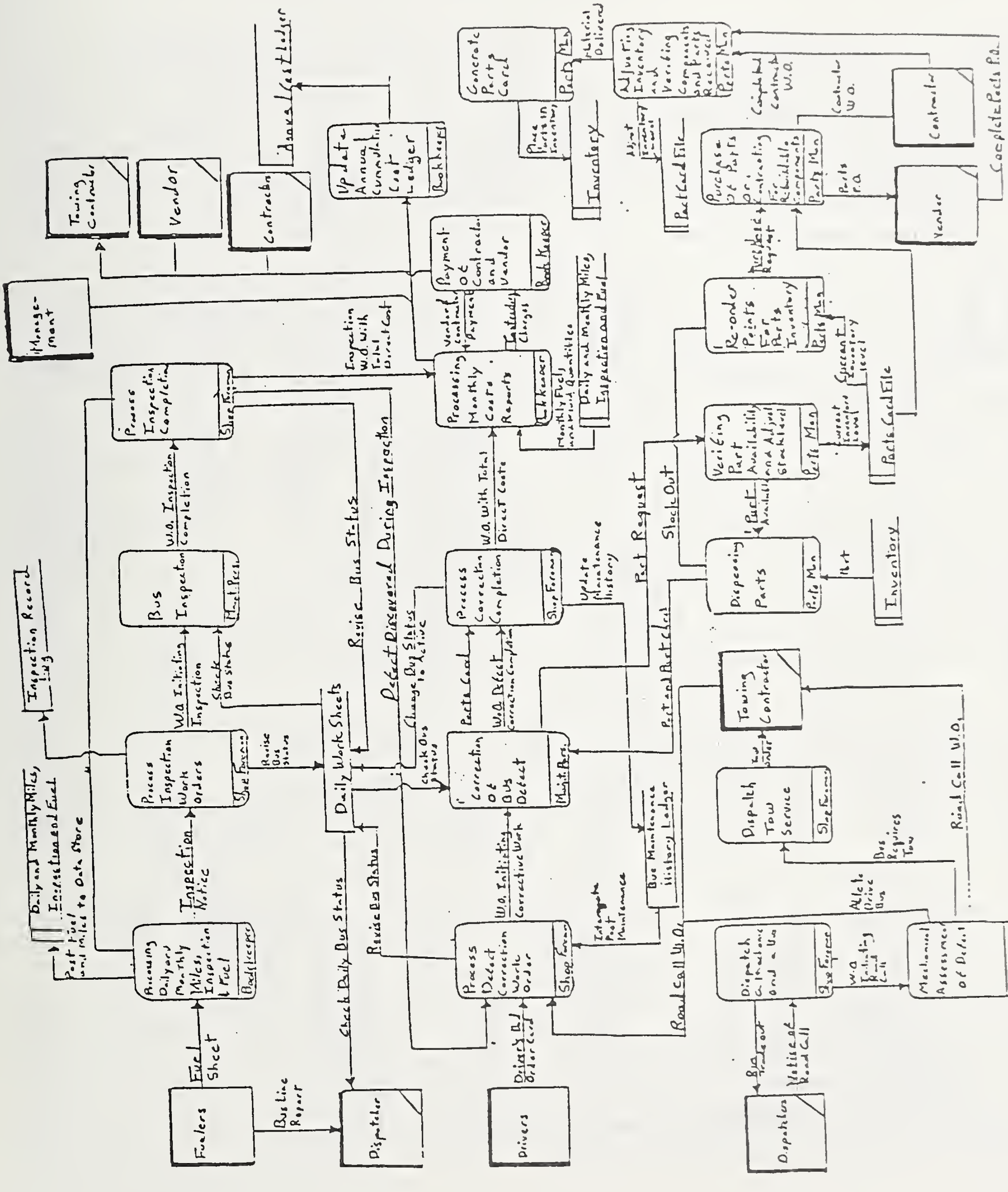


FIGURE 9.2

SECOND DRAFT OF THE DATA FLOW DIAGRAM

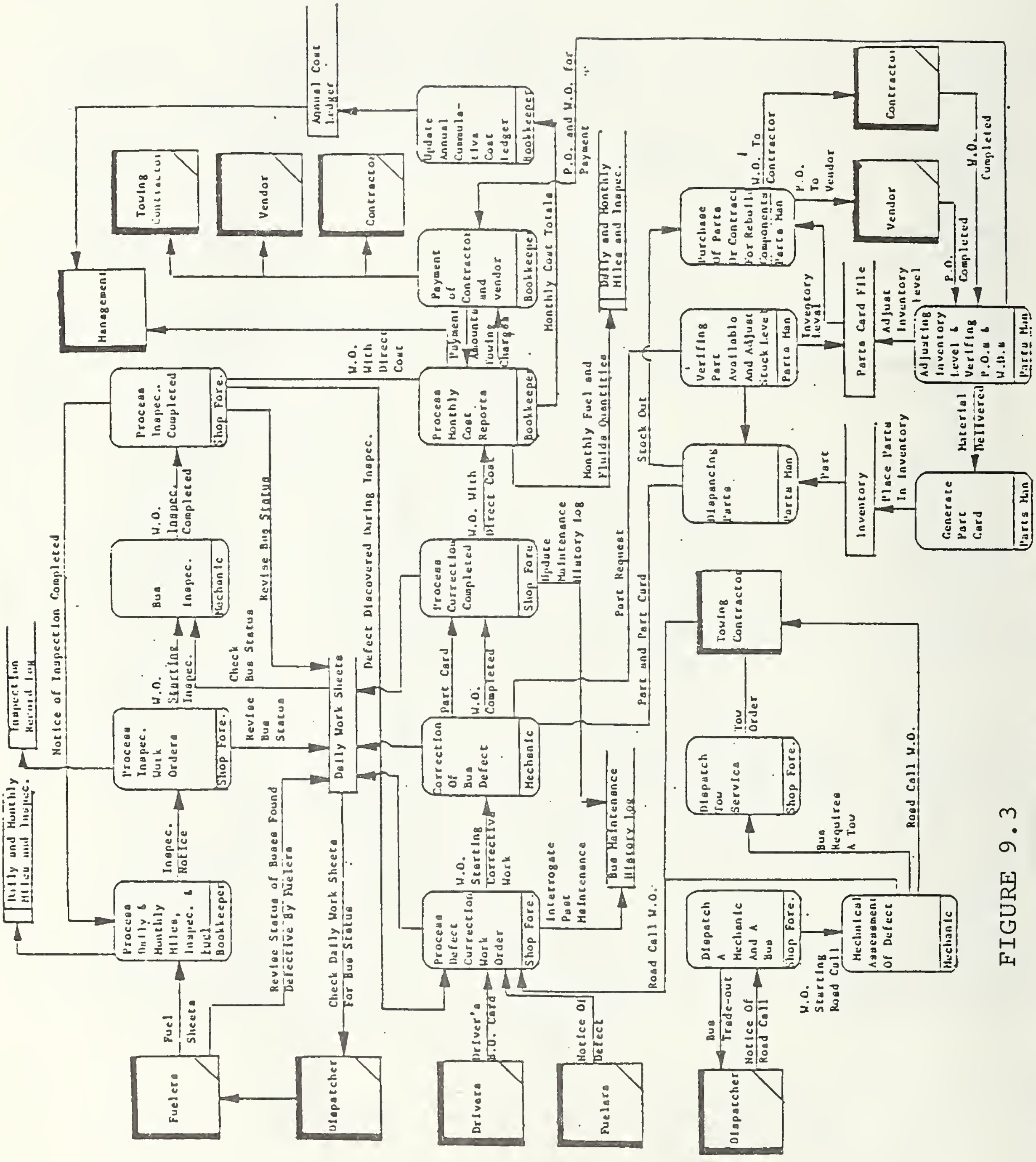


FIGURE 9.3

THIRD DRAFT OF THE DATA FLOW DIAGRAM

The desired new data flows and information functions to be automated can be added to the data flow diagram. After amending the data flow diagram, it is now time to conclude the Planning stage by meeting with computer experts and transit management to make system performance trade-offs.

The MTA example also provides a good example of the trade-offs that can be made at this stage. The MTA intends to examine not only the computer needs of the maintenance department but also planning, accounting, routing and scheduling, paratransit administration and property administration. Therefore, the managers of each department must make a trade-off between their needs and the limited funds that are available for computerization.

A computer expert with knowledge of the cost of computers and computer programs must be an integral part of the trade-off decision making. The expert will estimate the costs of the functions that the individual departments want to have performed by the computer. As an example of the trade-offs that may be considered, the MTA would like to automate inventory and maintenance history record-keeping. Because these functions will require special purpose software, an integrated inventory and maintenance system will be expensive and may use up all the funds available for computerization. This may be unacceptable to the managers of other departments. On the other hand, maintenance information functions currently performed by the bookkeeper could be easily automated by using inexpensive computer programs that can be used by other departments to keep their records. Therefore, the managers must trade-off the levels of available funds with their competing desires for automation.

The data flow diagram shows what you are getting for your money in a visual and non-technical format. Therefore, it is possible for fleet managers, who may have little technical training in computers, to meaningfully participate in the trade-offs made regarding system performance.

Technical Aspects of the System Design

The previous sections are intended to assist in the planning of fleet management information systems at a non-technical level and without consideration of the physical constraints of the computer or the organizational constraints within the transit agency. The following sections examine the technical aspects of system design. Included are discussions of maintenance systems, inventory systems and semi-automatic fueling systems.* Some of the technical aspects of the man-machine interface and system layout are also covered.

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* This information is a summary of the information found in (5) and (6).

The three types of systems (maintenance, inventory and fueling systems) are discussed separately. However, the three systems should be fully integrated. In other words, the computer user should be able to get information from any of three systems and not be aware that they are different systems.

The computer system's ability to meet management needs is largely constrained by the computer programs (software) used by the system. Regardless of the sophistication of the computer equipment (hardware), if the software is unable to prepare the information required by the performance specification, the system will not be doing its job. Therefore, while designing the system, it is important to first consider the performance of the software. The next most important consideration in constraining the abilities of the system is the layout of the system. The layout of the system defines how many computers there are going to be (one or more), how powerful the computer(s) will need to be, how data will be stored, and how data are entered. The last in importance is the selection of the brand of computer equipment (hardware). Once the performance expectations of the software have been determined, the hardware choices are largely constrained to those which are compatible with the performance of the selected software.

Maintenance System Design

At a minimum the maintenance system should include information on the following three areas of fleet status:

1. Vehicle Reliability Indicators. Reliability is the likelihood of a vehicle operating properly at any given time. Common indicators of reliability include the average miles between road calls and the average miles between component failures.

2. Vehicle Maintainability Indicators. Maintainability is a measurement of the time needed to fix a failure and perform prevention maintenance. Direct labor hours devoted to various types of repairs per component failure, by vehicle type and by fleet can be used as indicators of vehicle maintainability.

3. Vehicle Availability Indicators. Availability is the likelihood of a bus being operational at any point in time. Information systems should provide such indicators of vehicle availability as the number of open work orders, the average duration of open work orders, current spare levels, and so forth.

Vehicle reliability, maintainability and availability are rudimentary measures of fleet and vehicle status. Some of the system functional requirements that should be considered during the planning stage include the following paragraphs.

Comparative Analysis. The information system should be able to produce reports of maintenance activities in numerical and possibly graphical form. Statistical information summed across the entire fleet or across a bus model can be compared to individual buses. For example, the comparison of the oil consumption of one bus to the average oil consumption of the other buses of the same model is useful in diagnosing engine problems.

The information system should be able to aid the manager in making comparisons to find buses with exceptional parts or fluids consumption rates (high or low) which indicate a maintenance problem. For common indicators of maintenance problems such as fuel mileage, oil consumption and brake shoe life, the system should automatically flag exceptions.

Information Classification. The system should be able to summarize and report information at every level of breakdown imaginable. The system should allow the user to analyze any reported activity. For example, the user should be able to compute direct labor costs for brake repairs and be able to stratify the average labor hours at the fleet, model and individual bus levels. Other classification levels would include the vehicle system or component, the individual(s) performing the work task, location, and whether the work was preventive or corrective.

Data Input. The data input system should not require the user to input what is routine or obvious. For example, preventive inspections require that certain standard activities take place. It is important that the system records that the inspection took place, the direct labor time that the inspection consumed, and the identity of the inspector. However, it is not important to report each and every activity; the user knows what activities take place during an inspection without the system's help. If a problem is found (e.g., the vehicle needs a brake system overhaul) then the defect and the overhaul should be recorded in the information system. This system of reporting only the unusual is known as "reporting by exception."

Another way of minimizing what is stored is to only store an individual description of what represents a significant activity. Other activities can be accounted for in general categories with known descriptions. Guidelines for determining whether an activity is significant enough to warrant being described are:

1. If an activity requires more than 1 to 2 hours to complete,

2. If the activity requires the efforts of several individuals,

3. If the activity has a relatively high cost in terms of parts used and/or labor,

4. If the activity is related to vehicle safety systems, because of potential liability,

5. If the activity is one of several included in a standard procedure (like the activities included in a preventative inspection), then the performance of the overall procedure is significant but not each individual activity, and

6. If the activity is part of a fleet or model-wide campaign, then the procedure is known and only the fact that the individual bus had the procedure completed is significant.

Information Detail. Information must be scaled by the level of detail to match the informational needs of individuals at various levels of the organization. For example, the general manager may need condensed information in the form of performance indicators. The maintenance manager may only need to see daily summaries of normal activities (i.e., the number of preventive inspections conducted, direct labor hours, etc.) and exceptions that have been flagged. The shop floor supervisor needs access to daily work logs to make schedules and allocate assignments. The mechanic needs access to work histories to determine if a diagnosed problem is a recurring one and how it was taken care of previously. Each individual requires access to the same data store but at different levels of detail.

Inventory System Design

The primary purposes of the inventory system are to help ensure that there are neither too few nor too many parts and components on hand. Too few will increase the downtime of a bus requiring a part or component. Too many increases the dollars tied up in the parts and components inventory. To enable the proper management of inventory quantities, the system should automatically flag parts that need to be reordered, produce inventory dollar values, and report the average usage rate for each part. However, in combination with a maintenance system, the inventory system can provide much more assistance than just helping to control inventory quantities.

The inventory system should interface directly with the maintenance system. The interface enables the inclusion of part and component costs and usage information in maintenance activity information system reports without re-entry of data. Part and component usage statistics can thus be accessed directly from the maintenance system. Further, the inventory system can flag high usage items. For example, suppose an item, like a voltage regulator, exhibits an unusually high usage rate and is flagged. At that time, the manager can decide whether to investigate this problem further.

To permit the inventory system to interface with the maintenance system, the two must recognize the same part and component coding system. The coding system should include the identity of the coach that the part was assigned to, thus allowing failure analysis and allowing the tracking of components. Of course, for the interface system to operate properly, all inventoried items must be coded correctly. Not allowing items to be received without the proper code should be one of the error trapping functions. The coding structure should be simple and recognizable.

The inventory system's primary function is to provide quantity control. However, the information produced by the system should permit the manager to conduct several types of analyses. Three of these are:

1. Vendor Responsiveness Comparisons. The system should keep track of the time elapsed between when the purchase order for parts is issued and when the parts are entered into the system. If a part can be gotten more quickly and with a lower variability in delivery time, then fewer parts need to be held in inventory. Therefore, the shorter and more reliable the delivery time, the smaller the reorder quantity. The smaller the reorder quantity, the smaller the inventory which results in fewer dollars invested in inventory.

2. Vendor Parts Reliability. The maintenance system's frequency of failure information can be stratified by vendor identity to make reliability comparison of parts from different vendors. For example, it would useful to compare the mean miles and variance of miles between brake shoe wear-outs. This comparison would determine which vendor provides shoes that last the longest and are consistent in quality. Differences in the physical reliability of parts from different vendors can be traded-off with purchase price and vendor responsiveness.

3. Component Rebuild Versus Purchase Comparisons. Parts and components can be divided into two types, expendable and repairable. Expendable parts are those which are used only once. Examples of expendable parts include filters, relays, light bulbs and body parts. Repairable parts are those which may be used many times on different buses and after each use the part is repaired, reconditioned or overhauled. Examples of repairable parts include engines, transmissions, and starter motors. The management of repairable parts is much more complex than expendable parts.

   To be able to manage repairable parts requires that the manager be able to track the part through cycles of being refurbished. Tracking of repairable parts permits comparison of failure frequencies and costs of in-house versus off-property repairing of parts.

The tracking of individual repairable items also permits the determination of desirable inventory quantities of repairable items.

Semi-Automatic Fueling System Design

Of the costs associated with operating and maintaining vehicle fleets, fuel generally is the second largest single cost item. Labor costs (driver and mechanic wages) are generally first. Both the size of fuel costs in operating budgets and increasing fuel prices make fuel a likely candidate for cost cutting by agencies facing pressures to economize.

A popular option to aid in the control of fuel costs is the use of semi-automatic fueling systems. Although the capabilities of these systems vary dramatically, at a minimum, through automatic data entry at the fuel pump, the quantity of fuel use by individual vehicles and the dates at which vehicles are fueled is recorded. This allows the fleet manager to track quantities of fuel and other fluids delivered and used.

Very simple software systems will store and report transaction lists. For each time fuel is accessed, the transaction list will generally indicate the vehicle identification code, the date, number of gallons delivered, the fueling location (if there are more than one fueling locations), and the employee's identification code. More sophisticated systems can and should be tied into the maintenance management information system. Sophisticated systems generally offer such options as sending messages to drivers and fuelers, controlling the quantity of fuel delivered to vehicles, and analysis of fuel consumption statistics. More specifically, some of the more sophisticated options include:

1. Validation. Validation of data entered. Data validated includes:

    a. Validation of the current mileage (or hours) entered. For example, the system should not accept a mileage that is less than the one entered in a prior transaction or a mileage that indicates the vehicle has gone an unrealistic distance since the last transaction.

    b. Validation of the vehicle codes and employee authorization codes.

    c. Validation of the fuel products used and the quantities delivered. For example, the system should not allow the delivery of diesel fuel to a vehicle that is listed in the master file as a gasoline engine automobile nor should it allow the delivery of more fuel to a vehicle than the maximum the vehicle could have used given the mileage travelled since the last transaction or more fuel than the capacity of the vehicle's fuel tank.

2. Management Information. The fueling system can directly provide data to the maintenance management information system to assist the fleet manager in better managing the fleet. The information that can be provided includes:

    a. Comparative statistics identifying fuel consumption trends and traits of the fleet, vehicle models and individual vehicles. Vehicle operating costs are highly dependent on fuel costs, thus this information can be used in such high-level management activities as determining vehicle economic replacement intervals and life-cycle costing.

    b. Billing and expense reports.

    c. Exception reports which identify the occurrence of fuel consumption (or consumption of other fluids) which is outside of normal tolerances. Exception reports are an important element in determining a bus's performance and in diagnosing impending mechanical problems.

3. Messages. When a particular driver, fueler or vehicle is identified at the fuel pump, the system may provide messages indicating some special characteristic. For example, the message may tell the fueler that the vehicle is due for preventive maintenance and needs to be positioned in a special location or that the vehicle is part of a special test and the vehicle should not receive normal lubricants.

Fuel Delivery and Systems. The device allowing access to the system is the primary point of control and security. Because of its importance, the type of system used to permit access should be carefully selected to meet the needs of the agency. There are a broad variety of access systems with varying degrees of sophistication and each has good and bad points.

The system chosen for fuel pump control can also constrain the types of data collected at the access point. For example, the primary purpose for key and card systems is to control access to fuel. If the fueling system is intended to collect more than simple transaction information, (i.e., current vehicle mileage, other fluids used, etc.) the key or card system must be augmented with a data entry keypad and a display screen. However, once a data entry keypad is available, it may be possible to control access to the system by typing in authorization codes, thus relieving the need for the key or card controlled system. Therefore, key and card systems may not be appropriate systems if the system performance specifications calls for higher-level information.

The following paragraphs discuss the good and bad points of popular access systems.

Plastic Card Systems. These systems can use plastic cards with hole patterns punched through the cards or with a magnetic strip on the card. Encoded in the holes or on the magnetic strip is the identity of the vehicle. Card systems are inexpensive and functional, however, the integrity of these systems can be easily jeopardized through misuse and abuse of the cards.

The cards are extremely susceptible to misuse. For example, they can be used to open locked doors, as ice scrapers in the winter, and to fuel unauthorized vehicles. Further, they become brittle in extreme heat or cold, thus making it easy for them to become bent or broken. Also, the punch cards can be easily duplicated by punching holes in the same pattern into another card or through a piece of paper.

Individual Keys. In these systems, a key is encoded with the identity of each vehicle. This system is inexpensive and can be efficient if only one driver is given responsibility for a key and always drives the same vehicle. On the other hand, if many individuals drive many different vehicles, a key system can become cumbersome and clumsy. The key can be easily lost or forgotten and, therefore, a backup set of each key is generally maintained.

Key-Like, Memory Devices. These are plastic data keys containing computer memory chips. Information can be read from and written on the chips. Each vehicle is assigned a key and the key's chip contains the vehicle's authorization code. Keys can be coded at the user's site thus making it easy to replace lost keys. Lost keys are automatically disabled which reduces unauthorized fueling.

Keypad Systems. Many systems use a keypad for data entry at the fueling island. Keypads are either the standard raised mechanical key type or touch-sensitive. In keypad systems there are no devices to bend or lose, no chance of reproducing a card, and most people are familiar with similar systems (money card machines at banks and supermarkets). These systems are more expensive to purchase than card and key systems, however, the security and the system integrity are higher.

Bar-Code Readers. Although the technology used in bar-code readers has been used in other applications, its application to semi-automatic fueling systems is recent. The bar-code strips resemble those used on food products in grocery stores. Some bar-codes are mounted on the inside of the fuel door. Others are mounted on the side of the vehicle. These can be read with a hand-held bar-code reader wand or a wall mount reader.

All types of access systems, to some extent, have difficulties with harsh weather and other environmental conditions. For example, moisture from humidity, snow, rain and sleet can tamper with the ability of card and key systems to read accurately. Dirt and oil can block a bar-code, making it impossible to read. To reduce weather problems, enclosures and other protective devices should be provided.

Designing the Man-Machine Interface

The proper planning of the interface of the user and the computer is probably the most complex aspect of the system plan. The behavior of the computer is highly predictable. However, the behavior of the user is not. At the two ends of the system, data input and information retrieval, the computer must interface with the user. At these two points the system must be flexible enough to forgive and adjust to the user mistakes.

Data Entry. Given the state-of-the-art of computer equipment, all data entry should be conducted at an online terminal. With an online terminal system, all data entered should be checked for accuracy while being entered. For example, when entering a bus mileage from a work order report the system should check to see that the mileage is greater than the mileage reported in a previous work order for the same bus and that the mileage is not greater than the mileage normally traveled in the time interval since the last report. Any errors found are immediately identified, thus permitting the user to re-enter the correct information.

The ease with which the user can interact with the system is termed "user friendliness." There are several ways to increase the friendliness of the data entry process. Some examples are:

1. Set up the data entry screen to replicate the data forms (such as, work orders, defect cards, etc.). A screen with specific areas to enter data is known as a mask. The mask enables data entry personnel to follow forms without being concerned with the input screen. To understand the importance of the screen depicting the exact form, suppose a paper form has the employee number before the vehicle number, and the screen has the vehicle number before the employee number. During data entry the user has to transpose the information thus creating the possibility of generating errors during the entry process.

2. Each input number should be checked when it is entered. This means that all data to be input into a mask does not have to be entered before the system performs the validation process. Because the data are re-entered on a "number by number" basis, when an error is flagged the user can choose to either correct the number in error, or recognize that the number is in error, enter the rest of the data called for by the mask and later correct the number in error. Suppose, for example, a vehicle number for a work order is entered and the number is not found in the master file. When the error in the data element is flagged, the user can check to see if the vehicle number is entered correctly from the work

order. If the number on the work order is in error, the user can finish entering the other data off the work order and later investigate the improper number and discover the correct one.

3. Provide error messages which describe the actual error condition, instead of providing an error code number. When an invalid vehicle number is entered the error message should state"Invalid Vehicle Number" or "Vehicle Number Not on Master File", not "error 102......see documentation".

Reporting Information. One of the greatest advantages of a computerized system is that it can process vast quantities of information quickly. However, the production of large quantities of information can also be a detriment. Too much information can bury valuable information in valueless trivia. For example, when interviewed, a fleet manager of a transit system with a computerized information system explained that he did not use the reports because they were too confusing. The reports provided too much information and too often the data were labeled in non-English codes.

To be useful, reports must be selective in providing only the information that the manager needs and the system should provide English titles. Some characteristics that reports should have to increase their usefulness and to increase the efficiency of the manager in interpreting report are:

1. Uniformity. Reports should have uniform formats so that the user can recognize the report's scope and purpose.

2. Information Presentation. Reports should be designed for maximum visual impact and readability. Individuals can more readily interpret graphical presentation of material. Thus, if possible, statistical data should be accompanied by computer plots.

3. Report Accessibility. Terminals, printers and paper copies of reports should all be accessible to the manager.

The report generator of the system should have the ability to process several uniform format reports. These reports should be varied in detail in accordance with the level of the user in the organization. For example, the fleet manager should have a series of summary reports. Typical examples of management level summary reports are:

1. Maintenance Cost by Vehicle System Report (i.e., air-conditioning, engines, chassis, etc.). This report enables the manager to identify high cost (in labor and parts) systems within the fleet.

2. Vehicle Class Performance Report. This report provides average operations and maintenance cost information by vehicle class.

3. In Stock Valuation Report. This provides the manager with current inventory levels and their value.

A more detailed report should be accessed by shop supervisors or when the manager is conducting more indepth analyses. Typical examples of detailed reports are:

1. Bus Case History Reports. On any specific bus this report provides mileage, date, employee, labor and parts cost, and other specific information on each maintenance activity.

2. Component Activity Report. This report provides detailed information on the maintenance activities on a specific component on every bus or every bus within a class.

3. Re-Order Report. This report identifies all stock items that are at or below the minimum established stock level.

System Layout Plan. The system layout plan (configuration) determines where the computer's processor is going to be and what other types of computer equipment (printers, terminals, data storage devices, etc.) are going to be used.* The layout options are many. For example, one option would be to do computing on microcomputers that work by themselves (a stand alone configuration). Another option is to do all computing on a large centralized computer and all user work on the computer through a terminal. Four types of system layouts are described as follows:

1. Local Microcomputer. This is a stand alone system and each user has one computer. An example of this type of system is shown in Figure 9.4. Because there is no connection between computers to share data or programs, the data must be recorded on tapes or disks which can be moved to another machine.

   One of the advantages to this type of system is that it allows each user to work independently. For example, if a microcomputer is used in a fueling system, each site has the flexibility to control its own fuel. Its primary disadvantage is the difficulty in integrating the various functions (inventory, maintenance, and fueling).

2. Multi-User, Central Computer. A centralized processor allows the sharing of programs and data. An example of this type of system is shown in Figure 9.5. A centralized system is generally a powerful enough computer to manage large data sets and provide high-level management information.

   One of the difficulties in having a centralized computer lies in problems encountered in system failures. The entire system may become disabled. A failure of the computer or a failure of the communications lines would result both in dead terminals and the inability to pump fuel.

---

* System layouts were adapted from general system layouts in (7).

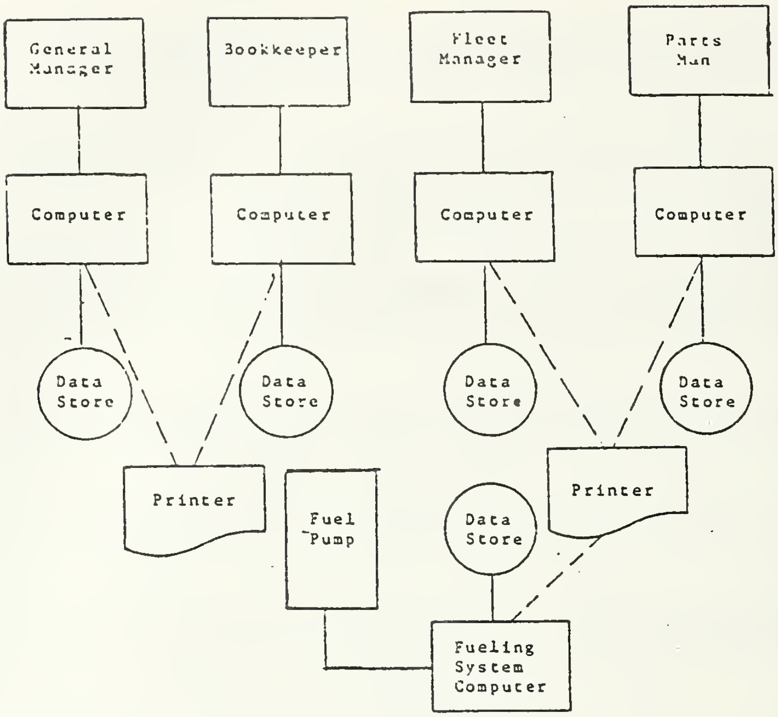

Figure 9.4 Local Microcomputer

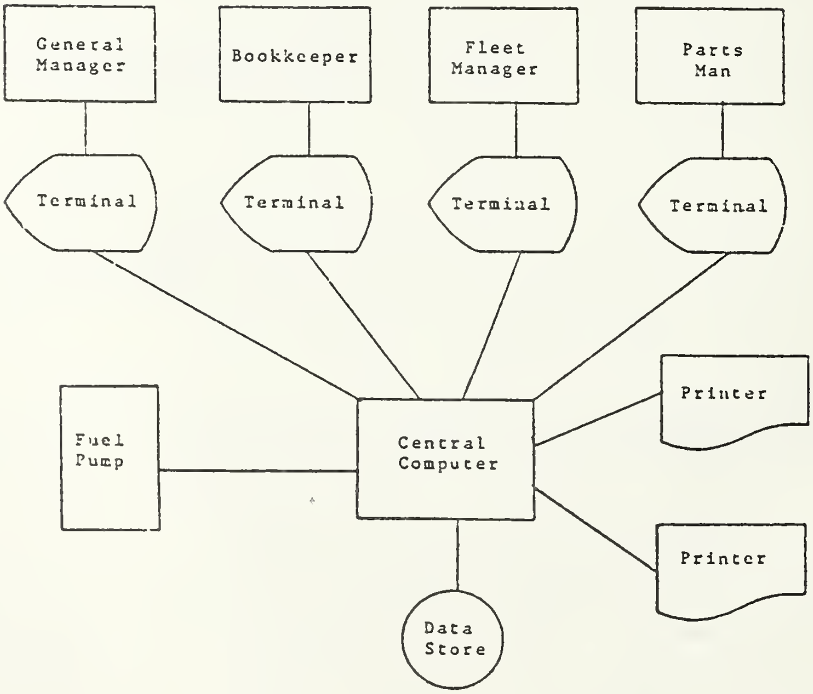

Figure 9.5 Multi-User, Central Computer

3. Network of Microcomputers. This system has all the advantages of the stand-alone system (i.e., independence of users) plus the advantage of being able to share data and computer programs. An example of this type of system is shown in Figure 9.6. A network system also has the advantage over a multi-user, central computer of permitting the number of computers to expand as needs grow. A central computer will have a limit on the number of users that can be attached to the computer.

   A network system's primary disadvantage is that the individual microcomputers are limited in the amount of data that they can individually process efficiently. Therefore, a local area network system may have difficulty summarizing large data sets to provide high level information.

4. Combination Multi-User Computer and Networked Microcomputers. This system has the advantages of all the other three systems. An example of this type of system is shown in Figure 9.7. It allows local computing on microcomputers and additional computing on a central computer. For example, if the stand-alone microcomputer cannot handle a large set of data this activity could be taken over by the central computer with the results transferred back to the microcomputer use.

   A combination, multi-user computer and network of microcomputers increases the reliability of the system (the probability of operating normally over a specific time interval) because of the duplication of computing capabilities. For example, in the case of a fueling system, the central computer could fail while the micro-processor at the pump would continue to control fueling.

Conclusions

The purpose of Chapter Nine is to present information which allows transit agency staff such as fleet managers who may not possess technical training in computers to take the lead in computer system planning. The importance of their involvement lies in the fact that they will have to ultimately live with the system. If the users become more deeply involved with the system's development they will:

1. Better understand their own information data sources, development processes, uses/need, system trade-offs, and how the information flow operates.

2. Receive an information system which will more adequately meet their needs.

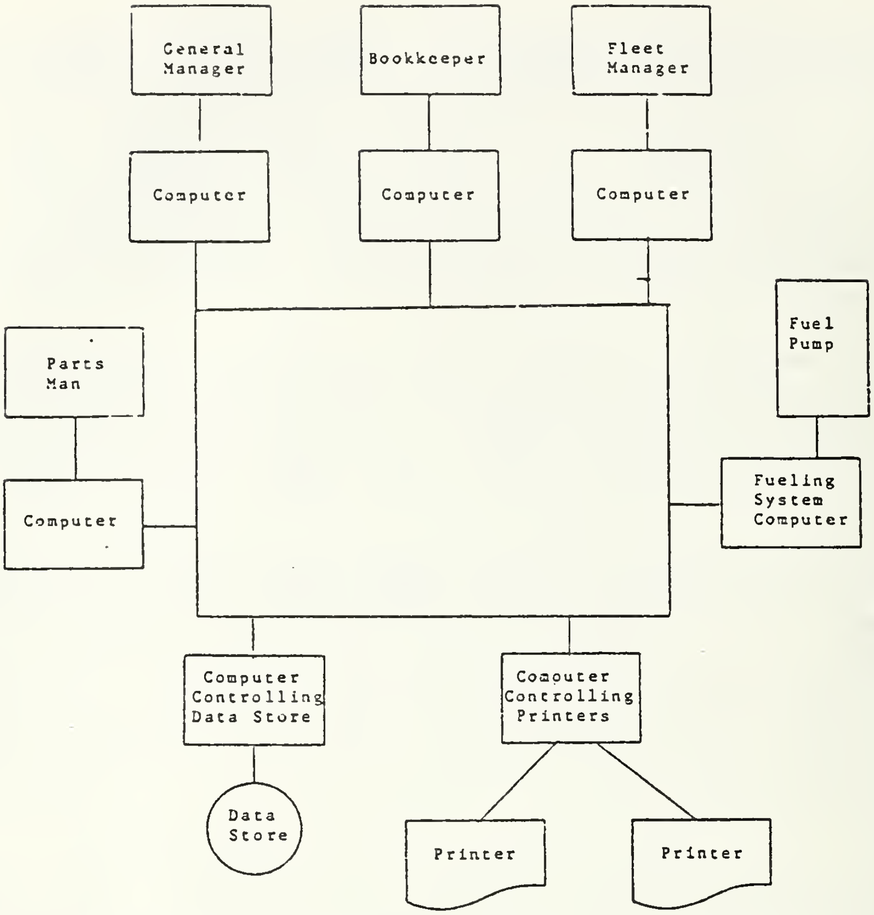

Figure 9.6 Network of Microcomputers

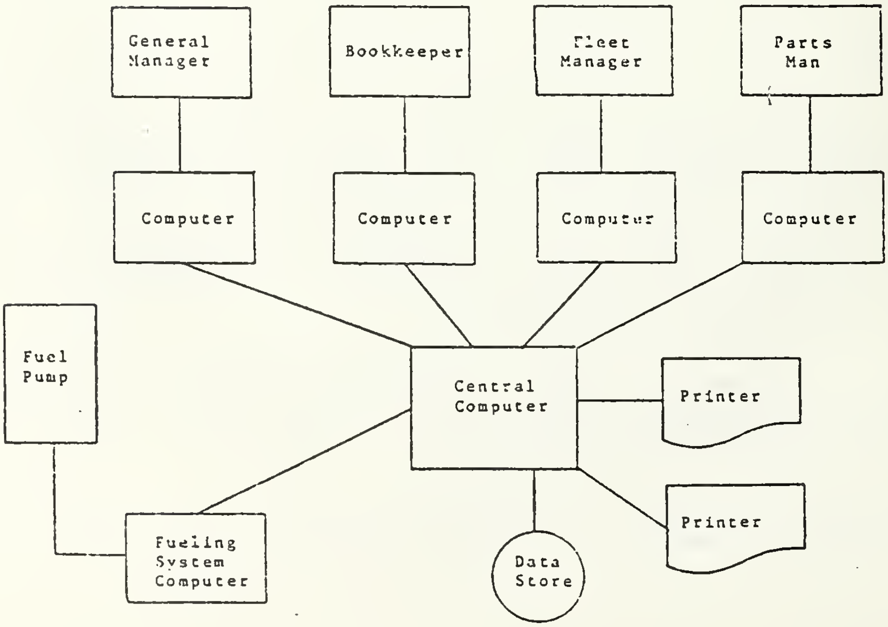

Figure 9.7 Combination of a Multi-User Computer and a Network of Microcomputers

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APPENDIX A

BLANK WORK SHEETS

WORK SHEET 1

TABULATION OF FAILURE MILEAGE FREQUENCIES

Cost Driver____________________ Bus Model____________________

Component Type_________________ Study Dates__________________

Failure Mileage (1)	Failure Mileage (2)	Mileage Class		Tally (5)	Frequency, F (6)
		Lower (3)	Upper (4)		

Number of failure mileages, N________________

Maximum mileage________________ Minimum mileage________________

WORK SHEET 2

COMPUTATION OF MEAN AND STANDARD DEVIATION

Cost Driver__________________ Bus Model__________________

Component Type________________ Study Dates________________

Mileage Class		Class Average X (3)	Freq. F (4)	FX (5)	X^2 (6)	FX^2 (7)
Lower (1)	Upper (2)					
				=		
				=		
				=		
				=		
				=		
				=		
				=		
				=		
				=		
				=		
				=		
				=		
				=		
				=		
				=		
Sums, Σ		N=	ΣFX=		$\Sigma FX^2=$	

Mean $= \bar{X} = \frac{\Sigma FX}{N} =$ __________ / ________ $= \bar{X} =$ __________

Standard Deviation = SD = $\sqrt{\frac{N\Sigma FX^2 - (\Sigma FX)^2}{N(N-1)}}$

$= \sqrt{\frac{\_\_\_ \times \_\_\_\_\_\_ - \_\_\_\_\_\_ \times \_\_\_\_\_\_}{\_\_\_\_\_ \times \_\_\_\_\_}}$ = SD = __________

WORK SHEET 3

SUMMARY SHEET OF FAILURE MILEAGE DISTRIBUTION

Cost Driver____________________ Bus Model____________________

Component Type____________________ Study Dates____________________

Number of cases, N ____________ Mean, $\bar{X}$____________________

Number of failures, NF__________ Std. deviation, SD____________

Maximum mileage______________ $(\bar{X}-D)/SD$____________________

Minimum mileage______________

WEIBULL DISTRIBUTION PARAMETERS:

Minimum life term, D ______________________________

Shape factor, B ________________________________

Characteristic life factor, T ______________________

FAILURE MECHANISM:________________________________

WORK SHEET 4

TABULATION OF FAILURE MILEAGE CUMULATIVE FREQUENCIES

Cost Driver________________________ Bus Model________________________

Component Type________________________ Study Dates________________________

Mileage Class		Frequency, F (3)	Frequency, % (4)	Cumulative Frequency		Percent Surviving (7)
Lower (1)	Upper (2)			Cum F (5)	Cum % (6)	
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
			%		%	%
Sums:		N =	100.0%			

WORK SHEET 5

COMPONENT RELIABILITY ANALYSIS

Cost Driver________________________ Bus Model________________

Component Type_____________________ Study Dates______________

Cumulative Mileage, M (1)	z-score[1] (2)	Table 2.4 Number (3)	Cumulative Percentage[2] (4)	Reliability,[3] R (% Surviving) (5)	Percent Failing in Mileage Interval[4] (6)
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%
			%	%	%

[1] $z = \frac{M - \bar{X}}{SD}$, where M = mileage in column 1, $\bar{X}$ = mean = ________ and SD = standard deviation = ____________

[2] The number in column 3 times 100.

[3] 100 percent less the number in column 4.

[4] Percent failing in mileage interval

= $R_1 - R_2$, where: R_1 = reliability (column 5) at previous cumulative mileage
R_2 = reliability at mileage in column 1

WORK SHEET 6

PREDICTION OF COMPONENT FAILURES OVER TIME

Cost Driver____________________ Bus Model____________________

Component Type____________________ Study Dates____________________

Cumulative Time (1)	Component State (2)	Number of Buses in State (3)	Component Failures Per Time Period (4)	Total Failures Per Time Period (5)
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=
		x	%	=

WORK SHEET 7

FAILURE STATUS OF ORIGINAL COMPONENTS IN WEIBULL FAILURE ANALYSIS

Cost Driver______________________ Bus Model______________

Component Type___________________ Study Dates____________

Bus Number (1)	Component Number (2)	Current Bus Mileage (3)	Original Component Failure Mileage (4)	Status of Unfailed Component*		Comments (7)
				Censor (5)	Susp. (6)	

* CENSORED cases refer to surviving components which have accumulated more miles than any failed component in the data set.

SUSPENDED cases refer to surviving components which have accumulated less miles than at least one of the failed components.

WORK SHEET 8

RANK ORDERING OF SURVIVING AND FAILED COMPONENTS IN WEIBULL FAILURE ANALYSIS

Cost Driver____________________________ Bus Model________________

Component Type__________________________ Study Dates______________

Order Number (1)	Bus Number (2)	Component Number (3)	Failed, Suspended, or Censored (4)	Mileage When Failed, Suspended, or Censored* (5)
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

* Each component is listed in ascending order of the mileage entered in Column 5.

WORK SHEET 9

COMPUTATION OF D, K, AND S FOR WEIBULL FAILURE DATA

Cost Driver__________________________ Bus Model______________________

Component Type________________________ Study Dates____________________

Bus Number (1)	Failure Mileage (2)	Minimum Life, D* (3)	K (4)	$\ln(K_{max})$** (5)	$\ln(K)$ (6)	L*** (7)
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
		−	=		−	=
Sums:			ΣK =			S =

* Minimum life term, D = 0.90 x lowest failure mileage
= 0.90 x __________ = D =__________

** $\ln(K_{max}) = \ln($__________$) =$ _________, where K_{max} is the largest value of K in column 4.

*** If the data set contains <u>suspended</u> failure cases, the L terms must be modified in Work Sheet 10a before computing S.

WORK SHEET 9a

MODIFICATION OF L FACTORS TO COMPUTE S FOR WEIBULL FAILURE DATA WITH SUSPENDED CASES

Cost Driver____________________ Bus Model____________________

Component Type____________________ Study Dates____________________

Bus Number (1)	L* (2)	Adjusted Order No. (3)	Row Number (4)	Ratio (5)	L (6)	Modified L (7)
		/	1	=	x	=
		/	2	=	x	=
		/	3	=	x	=
		/	4	=	x	=
		/	5	=	x	=
		/	6	=	x	=
		/	7	=	x	=
		/	8	=	x	=
		/	9	=	x	=
		/	10	=	x	=
		/	11	=	x	=
		/	12	=	x	=
		/	13	=	x	=
		/	14	=	x	=
		/	15	=	x	=
		/	16	=	x	=
		/	17	=	x	=
		/	18	=	x	=
		/	19	=	x	=
		/	20	=	x	=
Sums:						S =

* From Work Sheet 9, Column 7.

WORK SHEET 10

COMPUTATION OF B AND T FOR WEIBULL FAILURE DATA WHICH CONTAINS ONLY FAILED CASES

Cost Driver____________________ Bus Model____________________

Component Type____________________ Study Dates____________________

INPUT NUMBERS:

N = ____________________
Number of failed components

D = ____________________
Minimum life term
(from Work Sheet 9)

ΣK = ____________________
(from Work Sheet 9)

S = ____________________
(from Work Sheet 9)

SHAPE FACTOR, B:

B = ____________ / ____________ = B = ________________
(Table 3.6 number for N) S

CHARACTERISTIC LIFE FACTOR, T:

K_{ave} = ____________ / ____________ = K_{ave} = ________________
ΣK N

T = ____________ x ____________ = T = ________________
(Table 3.7 number for B) K_{ave}

WORK SHEET 11

COMPONENT RELIABILITY ANALYSIS FOR WEIBULL FAILURE DATA

Cost Driver__________________________ Bus Model________________

Component Type_______________________ Study Dates_______________

Cumulative Mileage, M (1)	Cumulative Probability of Failure[1] (2)	Cumulative Percentage[2] (3)	Reliability,[3] R (% Surviving) (4)	Percent Failing in Mileage Interval[4] (5)	Number Failing in Mileage Interval[5] (6)
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	
		%	%	%	

[1] Cumulative Probability

$$= 1 - e^{-\left[\frac{M - D}{T}\right]^{B}}$$

$$= 1 - e^{-\left[\frac{\underline{\qquad\qquad} - \underline{\qquad\qquad}}{\underline{\qquad\qquad}}\right]^{\underline{\qquad\qquad}}}$$

[2] The number in column 2 times 100.

[3] 100 percent less the number in column 3.

[4] Percent failing in mileage interval

$= R_1 - R_2$, where: R_1 = reliability (column 4) at previous cumulative mileage
R_2 = reliability at mileage in column 1

[5] The total number of failures, N, times the decimal percentage in column 5.

WORK SHEET 12

COMPUTATION OF B AND T FOR WEIBULL FAILURE DATA WHICH CONTAINS CENSORED CASES

Cost Driver________________________	Bus Model________________________
Component Type____________________	Study Dates______________________

INPUT NUMBERS:

A1 = ____________________ (from Table 4.5)	C1 = ____________________ (from Table 4.8)
A2 = ____________________ (from Table 4.6)	C2 = ____________________ (from Table 4.9)
A3 = ____________________ (from Table 4.7)	C3 = ____________________ (from Table 4.10)

N = ____________________ (total number of buses)

S = ____________________ (from Work Sheet 9)

K_{max} = ____________________ (from Work Sheet 9)

SHAPE FACTOR, B:

$$M = \underset{A1}{\underline{\qquad\qquad}} + \underset{A2}{\underline{\qquad\quad}} / \underset{N}{\underline{\qquad}} + \underset{A3}{\underline{\qquad\quad}} / (\underset{N}{\underline{\qquad}})^2 = \underline{\qquad\qquad}$$

$$B = \underset{M}{\underline{\qquad\qquad}} \; x \underset{N}{\underline{\qquad\quad}} \; / \; \underset{S}{\underline{\qquad\qquad}} = B = \underline{\qquad\qquad\quad}$$

CHARACTERISTIC LIFE FACTOR, T:

$$P = \underset{C1}{\underline{\qquad\qquad}} + \underset{C2}{\underline{\qquad\quad}} / \underset{N}{\underline{\qquad}} + \underset{C3}{\underline{\qquad\quad}} / (\underset{N}{\underline{\qquad}})^2 = \underline{\qquad\qquad}$$

$$\ln(T) = \underset{\ln(K_{max})}{\underline{\qquad\qquad}} - \underset{P}{\underline{\qquad\qquad}} \; / \; \underset{B}{\underline{\qquad\quad}} = \underline{\qquad\qquad}$$

T = ________ ("inverse" of ln(T) on a scientific calculator)

WORK SHEET 13

COMPUTING THE ESTIMATED MEAN MILEAGE TO FAILURE AND THE STANDARD DEVIATION FOR WEIBULL FAILURE DATA WITH CENSORED AND SUSPENDED CASES

Cost Driver________________________ Bus Model________________________

Component Type____________________ Study Dates______________________

INPUT NUMBERS:

D = ______________________________
(minimum life term)

B = ______________________________
(Weibull shape factor)

T = ______________________________
(Weibull characteristic life factor)

E1 = ______________________________
(from Table 4.12)

E2 = ______________________________
(from Table 4.13)

ESTIMATED MEAN MILEAGE TO FAILURE, $\bar{X}$:

$$\bar{X} = \underset{T}{\underline{\qquad\qquad}} \;/\; \underset{E1}{\underline{\qquad\qquad}} + \underset{D}{\underline{\qquad\qquad}} = \bar{X} = \underline{\qquad\qquad}$$

ESTIMATED STANDARD DEVIATION, SD:

$$SD = \sqrt{\underset{E2}{\underline{\qquad\qquad}} \times \underset{T}{\underline{(\qquad\qquad)}}^{2}} = SD = \underline{\qquad\qquad}$$

WORK SHEET 14

COMPUTING WEIGHTS AND ADJUSTED ORDER NUMBERS FOR WEIBULL FAILURE ANALYSIS WHEN CASES INCLUDE SUSPENDED CASES

Bus Number (1)	Failed, Suspended, or Censored (2)	Mileage When Failed, Suspended, or Censored (3)	Original Order Number (4)	Reverse Order Number (5)	Weight Calculation (6)	Weight (7)	Adjusted Order Number (8)
							=
							=
							=
							=
							=
							=
							=
							=
							=
							=

WORK SHEET 15

COMPUTATION OF B AND T FOR WEIBULL FAILURE DATA WHICH CONTAINS SUSPENDED CASES

Cost Driver ____________________ Bus Model ____________________

Component Type ____________________ Study Dates ____________________

INPUT NUMBERS:

F1 = ________________ (from Table 5.7)

G1 = ________________ (from Table 5.10)

F2 = ________________ (from Table 5.8)

G2 = ________________ (from Table 5.11)

F3 = ________________ (from Table 5.9)

N = ____________________ (total number of buses)

NS = ____________________________ (total number of suspended cases)

S = __________________ (from Work Sheet 9a)

ratio = ____________________ (from Work Sheet 9a, the ratio in column 5 corresponding to K_{max})

K_{max} = __________________ (from Work Sheet 9)

SHAPE FACTOR, B

$$M = \underset{F1}{\underline{\qquad\qquad}} + \underset{F2}{\underline{\qquad\qquad}} \text{ x } \underset{NS}{\underline{\qquad\quad}} + \underset{F3}{\underline{\qquad\qquad}} \text{ x } (\underset{NS}{\underline{\qquad\quad}})^2$$

$$M = \underline{\qquad\qquad}$$

$$B = (\underset{M}{\underline{\qquad\qquad}} \text{ x } \underset{N}{\underline{\qquad\quad}}) \;/\; \underset{NS}{\underline{\qquad\qquad}} = B = \underline{\qquad\qquad}$$

CHARACTERISTIC LIFE FACTOR, T:

$$P = \underset{G1}{\underline{\qquad\qquad}} - \underset{G2}{\underline{\qquad\qquad}} \text{ x } \underset{N}{\underline{\qquad\quad}} = P = \underline{\qquad\qquad}$$

$$\ln(T) = \ln\ (\underset{K_{max}}{\underline{\qquad\qquad}} / \underset{ratio}{\underline{\qquad\qquad}}) - \underset{P}{\underline{\qquad\qquad}} / \underset{B}{\underline{\qquad\qquad}}$$

ln(T) = __________, T = __________ ("inverse" of ln(T) on a scientific calculator)

WORK SHEET 16

COMPUTATION OF B AND T FOR WEIBULL FAILURE DATA WHICH CONTAINS BOTH CENSORED AND SUSPENDED CASES

Cost Driver______________________ Bus Model________________________

Component Type___________________ Study Dates______________________

INPUT NUMBERS:

The following numbers are obtained from Tables 6.7 to 6.54:

H1 = ____________________ J1 = ____________________

H2 = ____________________ J2 = ____________________

H3 = ____________________ J3 = ____________________

N = ____________________ (total number of buses) S = ____________________ (from Work Sheet 9a)

NS = ____________________ (number of suspended cases) K_{max} = ____________________ (from Work Sheet 9)

NC = ____________________ (number of censored cases) ratio = ____________________ (from Work Sheet 9a, the ratio in column 5 corresponding to K_{max})

(NC/N)x100 = ____________________ (percent of censored cases)

SHAPE FACTOR, B:

$$M = \underset{H1}{\underline{\qquad\qquad}} + \underset{H2}{\underline{\qquad\qquad}} \times \underset{NS}{\underline{\qquad}} + \underset{H3}{\underline{\qquad\qquad}} \times (\underset{NS}{\underline{\qquad}})^2$$

M = ________________

$$B = \underset{M}{\underline{\qquad\qquad}} \times \underset{N}{\underline{\qquad}} / \underset{S}{\underline{\qquad\qquad}} = B = \underline{\qquad\qquad\qquad}$$

CHARACTERISTIC LIFE FACTOR, T:

$$P = \underset{J1}{\underline{\qquad\qquad}} + \underset{J2}{\underline{\qquad\qquad}} \times \underset{NS}{\underline{\qquad}} + \underset{J3}{\underline{\qquad\qquad}} \times (\underset{NS}{\underline{\qquad}})^2$$

P = ________________

$$\ln(T) = \ln\,(\underset{K_{max}}{\underline{\qquad\qquad}} / \underset{ratio}{\underline{\qquad\qquad}}) - \underset{P}{\underline{\qquad\qquad}} / \underset{B}{\underline{\qquad\qquad}}$$

ln(T) = ___________, T = ___________ ("inverse" of ln(T) on a scientific calculator)

WORK SHEET 17

PRESENT WORTH ANALYSIS OF ANNUALLY RECURRING COST DRIVERS

Cost Driver: ________________________________

Bus Age (years) (1)	Mileage per year (2)	Cost per Mile (3)	Total Annual Cost (4)	Present Worth Factor (5)	Present Worth (6)
1	________	x $________	= $________	x ________	= $________
2	________	x $________	= $________	x ________	= $________
3	________	x $________	= $________	x ________	= $________
4	________	x $________	= $________	x ________	= $________
5	________	x $________	= $________	x ________	= $________
6	________	x $________	= $________	x ________	= $________
7	________	x $________	= $________	x ________	= $________
8	________	x $________	= $________	x ________	= $________
9	________	x $________	= $________	x ________	= $________
10	________	x $________	= $________	x ________	= $________
11	________	x $________	= $________	x ________	= $________
12	________	x $________	= $________	x ________	= $________

Total Present Worth: $________

WORK SHEET 18

MAINTENANCE TASK COSTING

Cost Driver:________________________________

Maintenance Task	Estimated Labor Hours*	Labor Rate**	Cost of Materials*	Cost per Task
1.______________ ______________ ______________	____________	x $______	+ $_________	= $__________
2.______________ ______________ ______________	____________	x $______	+ $_________	= $__________
			Total Cost:	$__________

Expected interval in miles: *____________________

* Supplied by bus manufacturer

** Supplied by transit agency

WORK SHEET 19

PRESENT WORTH ANALYSIS OF INFREQUENT MAINTENANCE COST DRIVERS

Cost Driver:____

Bus Age (years) (1)	Cumulative Miles (2)	Cost Driver Occurrence (3)	Cost (4)		Present Worth Factor (5)		Present Worth (6)
1	__________	__________	$________	x	__________	=	$__________
2	__________	__________	$________	x	__________	=	$__________
3	__________	__________	$________	x	__________	=	$__________
4	__________	__________	$________	x	__________	=	$__________
5	__________	__________	$________	x	__________	=	$__________
6	__________	__________	$________	x	__________	=	$__________
7	__________	__________	$________	x	__________	=	$__________
8	__________	__________	$________	x	__________	=	$__________
9	__________	__________	$________	x	__________	=	$__________
10	__________	__________	$________	x	__________	=	$__________
11	__________	__________	$________	x	__________	=	$__________
12	__________	__________	$________	x	__________	=	$__________

Total Present Worth: $__________

APPENDIX B

REFERENCE GUIDE FOR USE OF SCIENTIFIC CALCULATORS

The purpose of this Appendix is to introduce the reader to the use of pocket scientific calculators. If you have not used one of these calculators before, you will find these machines allow the user to do complicated mathematics with the push of a button.

The calculator we are going to demonstrate with is a Texas Instrument TI-35. This calculator costs roughly $22.00 and is sold with an instruction book and separate paper back book that contains many useful formulas for everything from business to games. Other equally acceptable scientific calculators, manufactured by Casio, Radio Shack and Sharp, are listed below. Other brands of calculators that can take logarithms, powers and have statistic keys are acceptable.

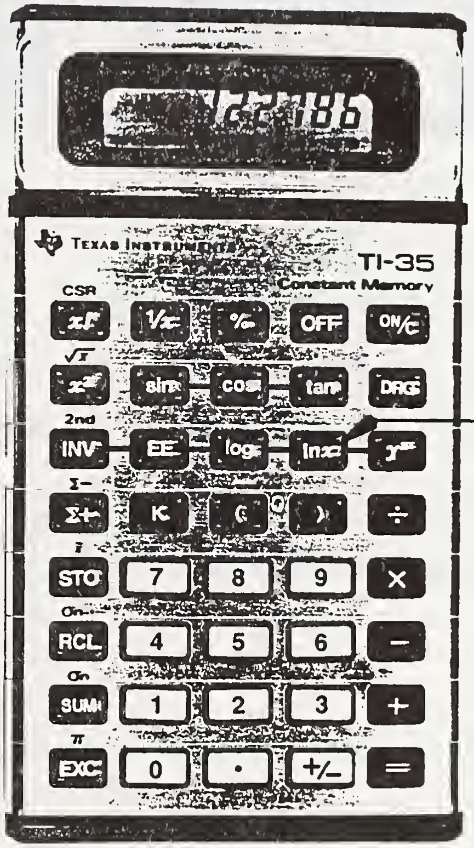
Casio		Radio Shack		Sharp	
Model	Suggested Price	Model	Suggested Price	Model	Suggested Price
FX-82	$19.95	EC-4007	$19.95	EL-509A-7	$19.95
FX-98	$29.95	EC-4006	$21.95	EL-506H-T	$29.95
FX-900/910	$29.95	EC-499	$27.95	EL-510A-T	$24.95
FX-990	$34.95	EC-4004	$34.95	EL-515-T	$34.95
FX-350	$24.95				
FX-550	$34.95				
FX-450	$34.95				

Calculations

Each of the following sections is divided into instructions on each type of calculation. In the body of the guide where a calculation is made, there are references directing you to the place in this Appendix where the calculation is demonstrated.

Taking Natural Logarithms

The calculator gives the natural logarithm of a number by simply entering the number and pressing the $\boxed{\ln x}$ button. For example, suppose we want to convert 122786 to its natural logarithm. First we enter 122786 like it's shown below.



Natural Logarithm Button

We next press $\boxed{\ln x}$ and we get

Two more examples are:

<u>Press</u> Display

387 → $\boxed{\ln_x}$

4821 → $\boxed{\ln_x}$

<u>Caluclating Weibull Cumulative Probabilities</u>

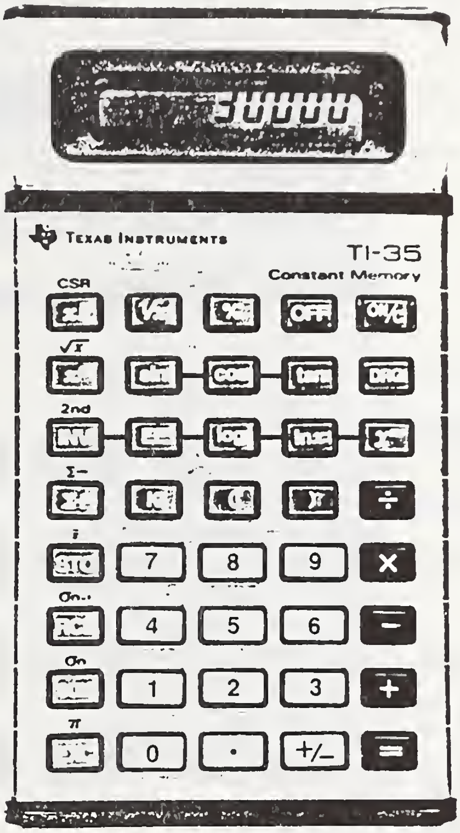
This is a fairly complex calculation and involves four steps. The Weibull cumulative probability formula is:

$$\text{Cumulative Probability} = 1 - e^{\left[-\left(\frac{\text{miles-D}}{T}\right)^{B}\right]}$$

Let D = 3,487, T = 55,107 and B = 1.5187 and suppose we want the cumulative probability at 30,000 miles.

$$? = 1 - e^{\left[-\left(\frac{30{,}000 - 3{,}487}{55.107}\right)^{1.5187}\right]}$$

Step 1: In this step we solve $\frac{30,000 - 3,487}{55107}$. Enter 30,000 as shown below:

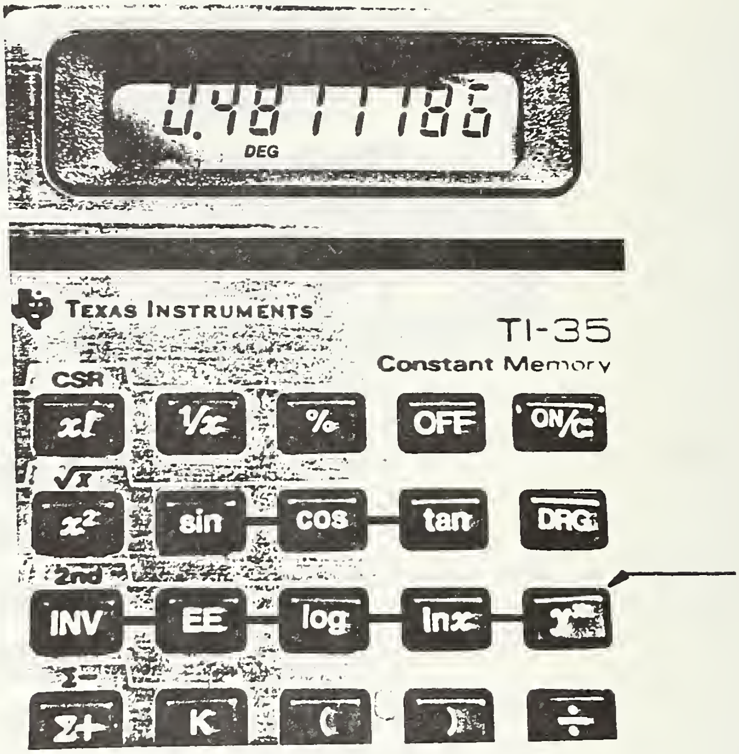


Press

$\boxed{-}$ → 3487 → $\boxed{=}$ → $\boxed{\div}$ → 55107 → $\boxed{=}$

Display

Step 2: In this step we solve $(0.4811186)^{1.587}$. After Step 1 the calculator display should appear as shown below:



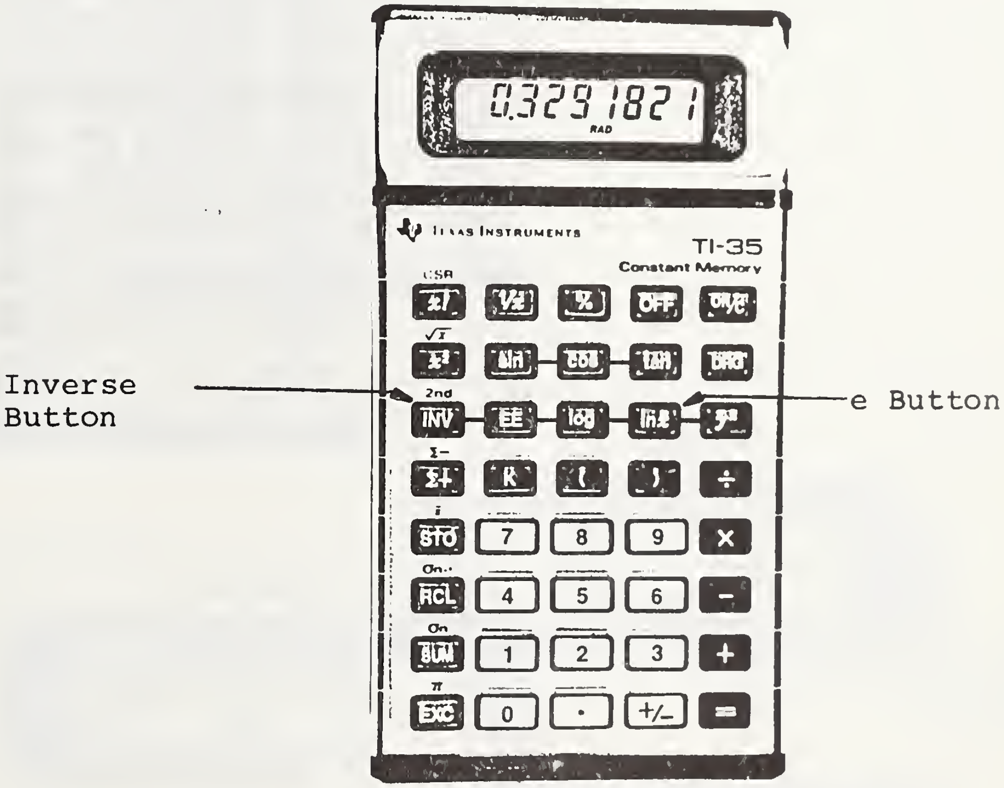
Press

Display

$\boxed{y^x} \longrightarrow 1.5187 \longrightarrow =$

Step 3: In this step we solve $e^{[-0.3291821]}$. To get the e of a number, you press $\boxed{\text{INV}}$ and next press $\boxed{\ln x}$. e is the opposite of the natural logarithm. To see how this work, enter 2 into your calculator and press $\boxed{\ln x}$ and the calculator displays 0.6931472. Next press $\boxed{\text{INV}}$ and then press $\boxed{\ln x}$ and the calculator displays 2 again.

After Step 2, the calculator will appear as shown below.

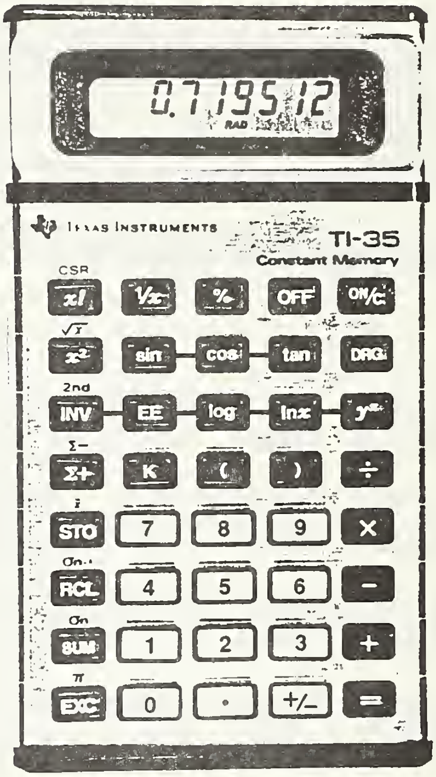


Press

$\boxed{+/-} \rightarrow \boxed{\text{INV}} \rightarrow \boxed{\ln x}$

Display

Step 4: The last step is to solve for 1 - 0.719519. After Step 3, the calculator will appear as shown below.

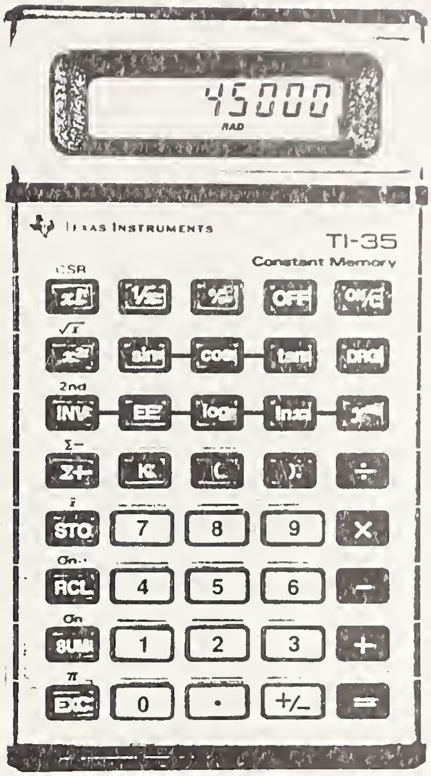


Press

+/– → + → 1 → =

Display

For a second example, suppose we wish to find the cululative probability of 45,000. We start by entering 45000 as shown below:



<u>Press</u>

<u>Display</u>

Step 1 $\boxed{-} \rightarrow 3487 \rightarrow \boxed{=} \rightarrow \boxed{\div} \rightarrow 55107 \rightarrow \boxed{=}$

Step 2 $\boxed{y^x} \rightarrow 1.5187 \rightarrow \boxed{=}$

Step 3 $\boxed{+/-} \rightarrow \boxed{\text{INV}} \rightarrow \boxed{\ln x}$

Step 4 $\boxed{+/-} \rightarrow \boxed{+} \rightarrow 1 \rightarrow \boxed{=}$