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Analysis and Comparison of Separation Measurement Errors in Single Sensor and Multiple Radar Mosaic Display Terminal Environments

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16. Abstract

This paper presents an analysis to estimate and characterize the errors in the measured separation distance between aircraft that are displayed on a radar screen to a controller in a single sensor terminal environment compared to a multiple radar mosaic terminal environment. The error in measured or displayed separation is the difference between the true separation or distance between aircraft in the air and the separation displayed to a controller on a radar screen. In order to eliminate as many variables as possible and to concentrate specifically on the differences between displayed separation errors in the two environments, for the purposes of this analysis, only full operation Mode S secondary beacon surveillance characteristics are considered. A summary of the Mode S secondary radar error sources and characteristics used to model the resultant errors in measured separation between aircraft in single and multi-radar terminal environments is presented. The analysis for average separation errors show that the performance of radars in providing separation services degrades with range. The analysis also shows that when using independent radars in a mosaic display, separation errors will increase, on average, compared to the performance when providing separation with a single radar. The data presented in the section on average separation errors is summarized by plotting the standard deviation of the separation error as a function of range for the single radar case and for the independent mosaic display case. The sections on typical and specific errors in separation measurements illustrate that the separation measurement errors are highly dependent on the geometry of the aircraft and radars. Applying average results to specific geometries can lead to counter intuitive results is illustrated in an example case presented in analysis.

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ABSTRACT

This paper presents an analysis to estimate and characterize the errors in the measured separation distance between aircraft that are displayed on a radar screen to a controller in a single sensor terminal environment compared to a multiple radar mosaic terminal environment. The error in measured or displayed separation is the difference between the true separation or distance between aircraft in the air and the separation displayed to a controller on a radar screen. In order to eliminate as many variables as possible and to concentrate specifically on the differences between displayed separation errors in the two environments, for the purposes of this analysis, only full operation Mode S secondary beacon surveillance characteristics are considered. A summary of the Mode S secondary radar error sources and characteristics used to model the resultant errors in measured separation between aircraft in single and multi-radar terminal environments is presented. The analysis for average separation errors show that the performance of radars in providing separation services degrades with range. The analysis also shows that when using independent radars in a mosaic display, separation errors will increase, on average, compared to the performance when providing separation with a single radar. The data presented in the section on average separation errors is summarized by plotting the standard deviation of the separation error as a function of range for the single radar case and for the independent mosaic display case. The sections on typical and specific errors in separation measurements illustrate that the separation measurement errors are highly dependent on the geometry of the aircraft and radars. Applying average results to specific geometries can lead to counter intuitive results is illustrated in an example case presented in analysis.

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1. INTRODUCTION

The purpose of this analysis is to estimate and characterize the errors in the measured separation distance between aircraft that are displayed on a radar screen to a controller in a single sensor terminal environment compared to a multiple radar mosaic terminal environment. The error in measured or displayed separation is the difference between the true separation or distance between aircraft in the air and the separation displayed to a controller on a radar screen. The true separation is a continuous function of time while the displayed separation is updated at discreet times and is, therefore, a discontinuous function of time. A single sensor terminal environment is one in which the aircraft being separated are tracked by the same radar. In a multiple radar mosaic terminal environment, multiple radars are used to track the aircraft with each radar responsible for the display of aircraft in a defined area. This is accomplished by dividing the terminal area into multiple rectangles known as a radar sort boxes and assigning a single radar (the "preferred sensor") as being primarily responsible for providing position reports for that sort box. The radar sort boxes define the boundaries that determine which radar's position estimate will be displayed for a moving aircraft.

Current separation standards require aircraft less than 1000 feet apart in altitude to be separated by at least 3 nautical miles if both aircraft are within 40 nautical miles of the radar and below Flight Level 180. If these conditions are not met, which is generally the case in en route airspace, the aircraft must be separated by 5 nautical miles. However, when transitioning from terminal to en route control, the 3 nautical mile requirement can be gradually increased to 5 nautical miles as long as the aircraft are diverging or the lead aircraft is faster.

When multiple terminal areas are in close proximity and have coverage from multiple radar sensors, there is a perceived advantage to providing a mosaic display of the entire terminal area to make use of all of the radars to separate aircraft operating at any of the terminal airports. The potential advantages include expanded airspace coverage and use of track reports from radars that are closer to the targets. The potential disadvantages include increased separation error measurements due to uncorrelated errors when aircraft receiving separation services are tracked by different radars. The question is whether or not the surveillance from a mosaic display will provide surveillance equivalent to that required for the 3 nautical mile separation standards to apply.

In practice, large terminal areas have coverage from a multitude of radar sensor types, each with different characteristics, and separation services are provided to aircraft equipped with avionics of varying degrees of sophistication. In order to eliminate as many variables as possible and to concentrate specifically on the differences between displayed separation errors in the two environments, for the purposes of this analysis, only full operation Mode S secondary beacon surveillance characteristics are considered. This is the surveillance used at major terminals and most aircraft receiving separation services at major terminals are Mode S equipped. Terminal radars (primary and Mode S) report position based on the Mode S reply as long as it is available.

This analysis makes use of two previous analyses: First, work done by ARCON [1] analyzing the mosaic display target accuracy in support of Northern California TRACON, and second, an evaluation of fusion trackers by Lincoln Laboratory [2] that contains measured data on sensor performance in the northeast.

2. MODE S RADAR ERRORS

The following is a summary of the Mode S secondary radar error sources and characteristics used to model the resultant errors in measured separation between aircraft in single and multi-radar terminal environments. Work by ARCON [1] shows that coordinate conversion errors and refraction effects are only significant at very long ranges and were not considered in this analysis which examines the effect of mosaic data for terminal Mode S radars. Errors introduced by aircraft not equipped to report altitude were not included, only Mode S equipped aircraft were considered. Propagation anomalies such as atmospheric ducting were not included. The values of the errors used were based on Mode S secondary radar specifications and field data from ARCON [1] for radars in Southern California Tracon and MIT Lincoln Laboratory [2] for radars in the northeast region. The resulting total errors for individual radars in this analysis is in good agreement with the measured registration errors for individual radars in a study conducted by Lockheed Martin and included as an Appendix in the ARCON [1] report.

2.1 RADAR SITE LOCATION BIAS

Data from ARCON [1] and MIT Lincoln Laboratory [2] indicate that the site location surveys for terminal radars deployed in the field will be typically off by as much as 200 feet. This contributes to what is commonly referred to as "registration" errors. This is of little concern when one radar is tracking all aircraft, but if two different radars are used to track airplanes in the same area this will result in an error in the separation measurement between the aircraft. The radar site location bias, or survey error, is modeled by a random sampling from a uniform azimuth distribution of 0 to 360 degrees and a uniform range distribution of 0 to 200 feet. The location biased is constant for a given radar for all aircraft for all scans. Note that this will have no effect on the separation measurement of two aircraft tracked by a single radar. This error can potentially be eliminated by more accurate site surveys of the fielded radars. This error is illustrated in Figure 1 as a uniform distribution in any direction of up to 200 feet.

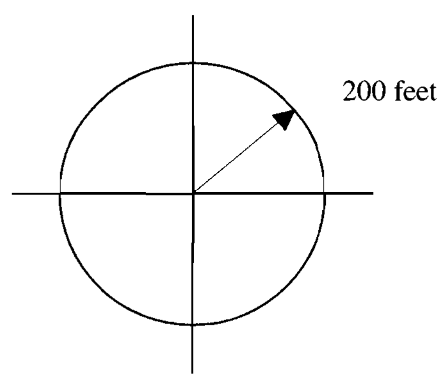

Figure 1. Location Bias

2.2 RANGE ERRORS

There are four sources of error in the Mode S beacon reported range measurement considered in this analysis. A range bias that remains constant for a given radar, range jitter that varies from measurement to measurement, a range error introduced by the aircraft's transponder turnaround time, and the error introduced by the least significant bit encoding of the range measurement for transmission to the ATC display system. These error sources are described below and illustrated in Figure 2. These errors are in good agreement with the sensor error statistics for range errors measured by ARCON [1] in real life conditions using data obtained from Southern California Tracon.

2.2.1 Range Bias

The Mode S specification states that the range error shall not exceed $\pm$ 30 feet bias (including long term drift). This is modeled by sampling from a uniform distribution between plus and minus 30 feet. The bias is considered to be constant for a given radar.

2.2.2 Range Jitter

The Mode S specification states that jitter shall not exceed 25 feet rms. This is modeled by sampling from a normal distribution with a mean of zero and a standard deviation of 25 feet. This is sampled for every scan of every aircraft.

2.2.3 Transponder Range Bias

The Mode S transponder specification sets limits on the response time of the aircraft's transponder. This translates to a $\pm$ 125 feet range bias in the transponder. This is modeled by sampling from a uniform distribution between plus and minus 125 feet, once for each aircraft. The transponder bias is assumed to remain constant for a given aircraft and is independent of the radar. Note that Mode C transponder specifications set limits of $\pm$ 250 feet range bias in the transponder.

2.2.4 Common Digitizer Transmission Error for Range

The Common Digitizer format used to transmit the measured range results in a least significant bit encoding error of 1/64 nmi. This is modeled as a round-off error in the estimated range. The estimated range is first computed using the sampled range errors described above. This number is rounded to the nearest 1/64 n. mi. and that is the range position reported. A nautical mile is based on the international standard adopted by the U.S. of 6076.115 feet.

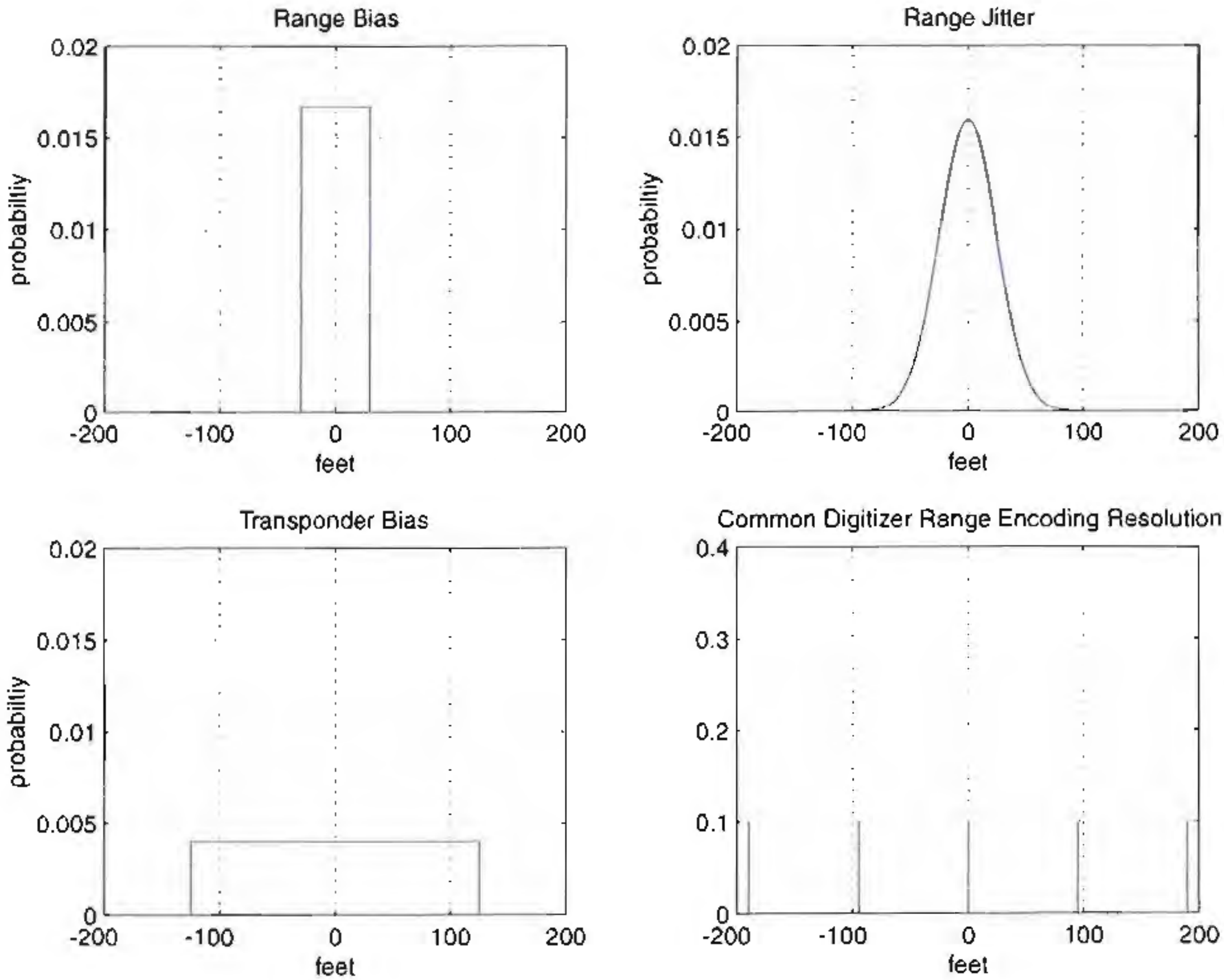

Figure 2. Illustration of the magnitude of the range error sources

2.3 AZIMUTH ERRORS

There are three sources of error in the Mode S beacon reported azimuth measurement considered in this analysis. An azimuth alignment bias which remains constant for a given radar, an azimuth jitter error that varies measurement to measurement, and the error introduced by the least significant bit encoding of the azimuth measurement for transmission. These error sources are described below and illustrated in Figure 3. These errors are in good agreement with the sensor error statistics for range errors measured by ARCON [1] in real life conditions using data obtained from Southern California Tracon. Because azimuth errors are in degrees or radians and their effect on position estimate errors is linear with range, Figure 3 depicts the magnitude of position errors generated by the azimuth errors at a range of 30 nautical miles. Terminal radars are generally used at ranges of up to 60 nautical miles.

2.3.1 Azimuth Alignment Bias

Observations by Lincoln Laboratory staff with experience with fielded radars is that azimuth alignment bias as large as from ± 1.0 to ± 3.0 degrees may exist in terminal radars in the field. Registration algorithms used in a multisensor tracking environment will remove some of this bias but data from ARCON [1] and MIT Lincoln Laboratory [2] both indicated that a residual bias of approximately ± 0.3 degrees will remain. These errors appear to be uniformly distributed. This is of little concern when one radar is tracking all aircraft, but if different radars are used to track airplanes in the same area, this may result in significant errors in estimating the separation between the two aircraft. The ARCON [1] report cites a report by the FAA that investigated azimuth accuracy errors at the ASR-9 / Mode S radar at Denver and concluded that the collective anomalies noted could account for azimuth bias errors of approximately 0.25 to 0.35 degree and that these anomalies were not unique to Denver. For the purposes of this analysis the azimuth alignment bias is assumed to be ± 0.3 degrees. The azimuth alignment bias for a radar is modeled by sampling from a uniform distribution between ± 0.3 degrees. The alignment bias is assumed to be constant over azimuth for a given radar and remains constant measurement to measurement.

2.3.2 Azimuth Jitter Error

The azimuth accuracy required in the Mode S specification is 0.068 degrees, 1 sigma. This is modeled by sampling from a normal distribution with a mean of zero and a standard deviation of 0.068 degrees. This is resampled for every measurement of every aircraft in this analysis.

2.3.3 Common Digitizer Transmission Error for Azimuth

The Common Digitizer format for encoding azimuth measurements results in a least significant bit encoding error of 1/4096 of a scan or a round off to the nearest 0.08789 degrees. The estimated azimuth is first computed as the true azimuth with the azimuth bias and sampled accuracy error (jitter) added or subtracted. The estimated azimuth is then rounded off to the nearest 0.08789 degrees.

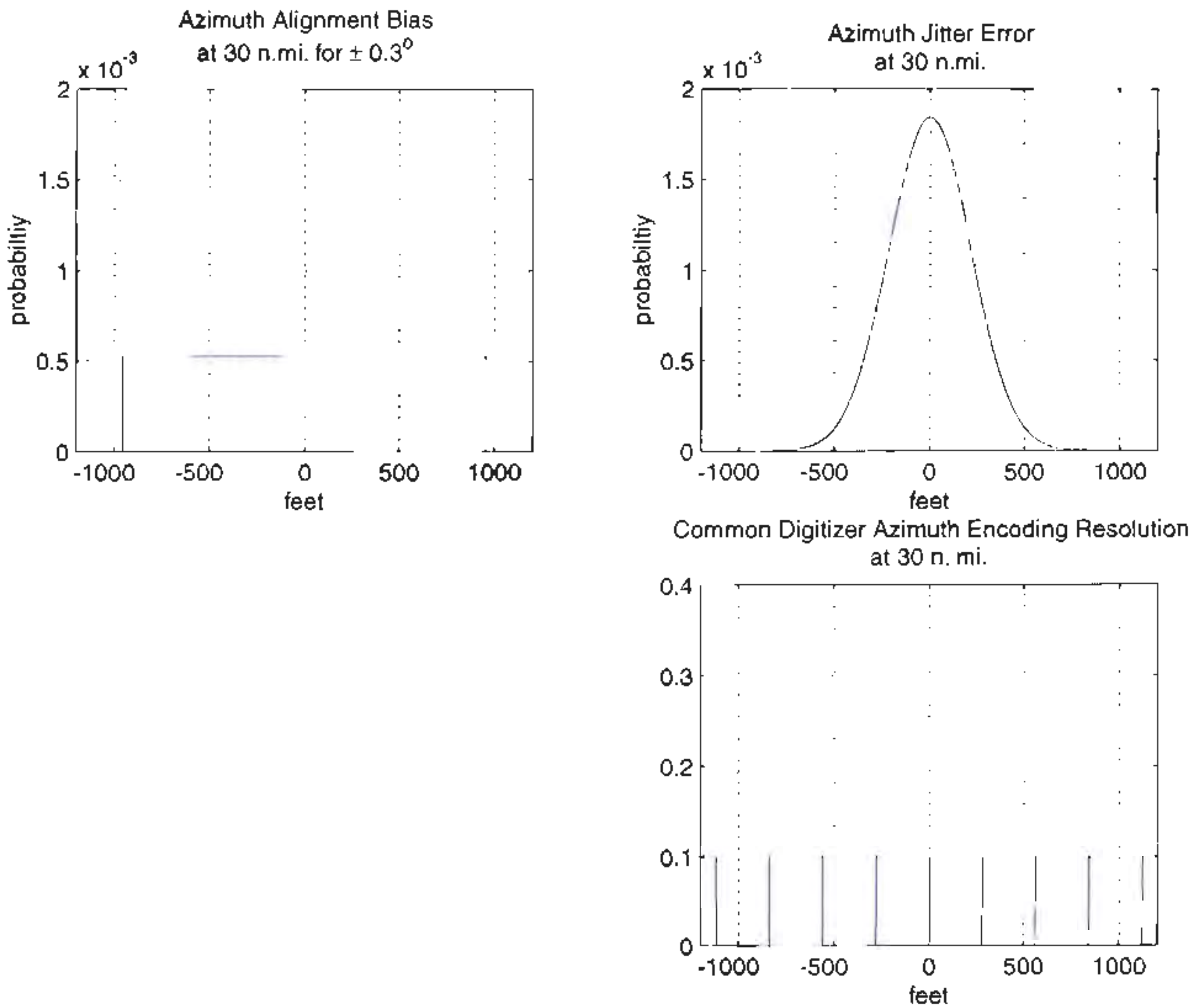

Figure 3. Illustration of the magnitude of the azimuth error sources at a range of 30 nautical miles

2.4 ERRORS IN DISPLAYED SEPARATION DUE TO TIMING

Aircraft position estimates occur as the radar rotates and thus the position estimates of any two aircraft at different azimuth locations will occur at different times. This will result in an error in the measured or displayed separation of the two aircraft. If two aircraft are close to each other and under track by the same radar, then the effect on measured separation will be small. However, if different radars are tracking the two aircraft, this may result in significant errors in the estimated separation of the two aircraft. It is potentially possible to reduce this error by "coasting" tracked aircraft so that the position estimates presented on the radar display are for a common time.

The Advanced Automation System Level Specification and the Standard Terminal Automation Replacement System (STARS) Subsystem Specification both call for Short Range Radar scan times of between 4 and 5 seconds. This results in rotation rates of between 72 and 90 degrees per second. The scan time or update rate is modeled by sampling from a uniform distribution between 4.0 and 5.0 seconds. For a single radar tracking two aircraft, the time difference of the radar "hits" will be the difference in the target azimuths divided by the rotation rate of the radar. However, if different radars are tracking the two aircraft, the time difference of the radar "hits" will be uncorrelated and could differ by as much as half of the maximum update rate or 2.5 seconds.

Note that this does not cause an error in the position estimates, but because the position estimates are taken at different times and the aircraft are in motion, this will result in an error in the apparent separation displayed to the controller. For two aircraft in trail, the effect of this sample time bias will be to reduce the estimate of the separation compared to the true separation if the lead aircraft is sampled first. If the trail aircraft is "hit" first the effect will be to measure a greater than actual separation. The effect on apparent separation is illustrated in Figure 4. It is assumed that any change in ground track angle of an aircraft will be minimal between two radar "hits" and will not have a significant affect on the separation estimate.

The convention used in this analysis to model the errors in displayed separation due to timing is as follows. The "true" separation is taken to be the separation based on the true positions of the two aircraft at the time the "first" aircraft is "hit" by the radar. For two aircraft, the "first" aircraft "hit" is the aircraft that minimizes the time difference between "hits" on the two aircraft. The radar is assumed to rotate clockwise. If one aircraft were northeast of the radar and the second aircraft were east of the radar, then the aircraft northeast of the radar would be considered the "first" aircraft. The "apparent" separation is computed by maintaining the position of the "first" aircraft and moving the "second" aircraft a distance and direction computed from the aircraft's velocity and the time between "hits". The "apparent" separation is the distance between the position of the "first" aircraft when "hit" and the position of the second aircraft when "hit". The "true" separation is based on the position of both aircraft when the "first" aircraft is "hit". The difference in separation is known as the timing error or sample time bias and is added or subtracted as appropriate to the "measured" separation based on modeled aircraft positions reported by the radar.

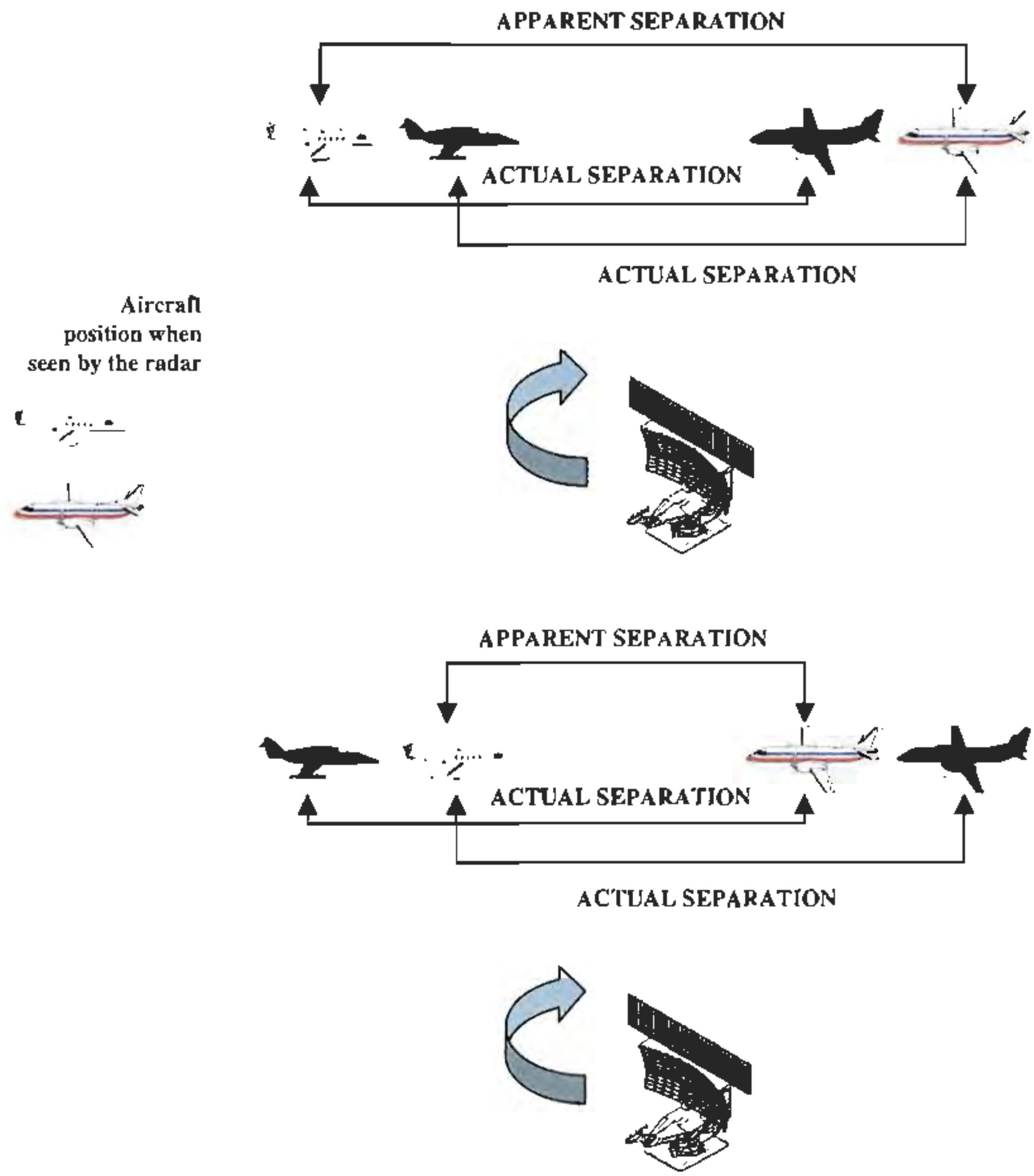

Figure 4. Illustration of the effect of clock bias on separation measurement

2.5 PERFORMANCE METRIC

Air traffic control provides a separation service and, therefore, the appropriate metric for surveillance performance in supporting this service is the error between the actual separation of the aircraft and the separation displayed to a controller on a radar screen. This is the error of interest when comparing the surveillance performance for a single radar versus that under a mosaic display of multiple sensors. This error will not in general be correctly determined by convolving average position errors of radars as a function of range for two reasons. First, some errors in a radar's measurements of aircraft position are correlated when under surveillance by one radar and will have a relatively small effect on measured separation error. Some position errors are uncorrelated when the aircraft are under surveillance by different radars and will have a larger effect on the measured separation error. Second, this ignores the errors due to the timing of the aircraft position estimates, which are a function of the azimuth difference when aircraft are tracked by one radar but uncorrelated when tracked by different radars.

It is important to treat the correlations of positional errors and timing errors correctly when comparing the errors in the displayed separation presented to the controller for the case of one radar tracking both aircraft versus different radars tracking the aircraft being separated.

3. SEPARATION MEASUREMENT ERROR ANALYSIS

Three types of separation performance analysis simulations are identified: 1) average performance, 2) typical performance, and 3) specific performance. The methods of simulation for each of the performance analysis types are described below. Table 1 summarizes the error characteristics for each of these three types of performance analyses.

3.1 AVERAGE SEPARATION ERRORS

The purpose of this analysis is to determine the average observed separation distributions for aircraft that are actually separated by exactly three nautical miles after modeling the errors in position estimates described above. This will be a function of range from the radar and the orientation of the aircraft. In order to obtain the average performance, all of the characteristic radar errors are independently resampled for each measurement. The procedure for a single radar is to randomly orient two aircraft that are separated by three miles with the midpoint of their locations at the specified range from the radar. This is illustrated in Figure 5. In the case of two radars, each tracking one of the two aircraft, the midpoint of the separation of the aircraft is kept at a constant range but moved in a direction orthogonal to the line between the radars to provide a sampling of orientations. Thus a set of average observed distributions would be generated for a given range at a specified Θ. This is also illustrated in Figure 5.

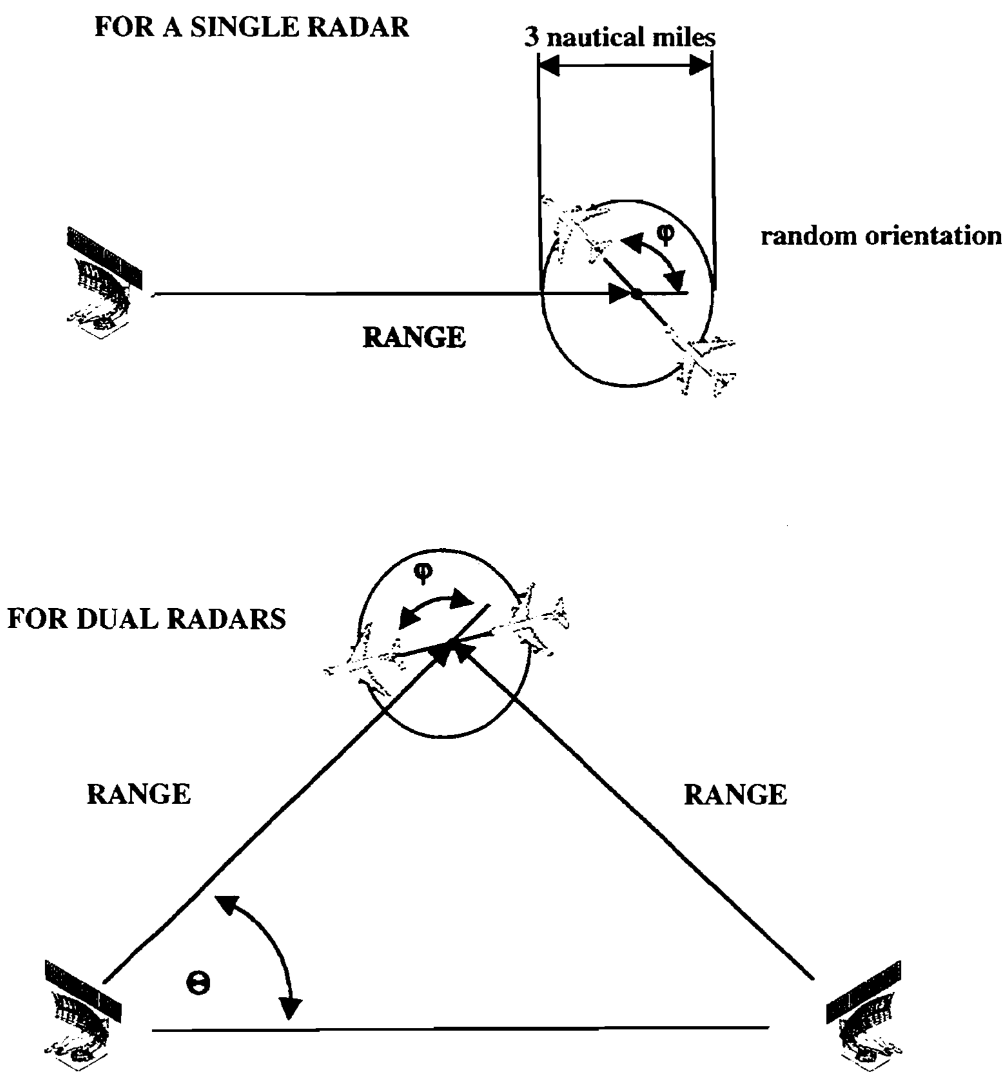

Figure 5. Average performance measurement statistics were generated by placing the midpoint of aircraft three nautical miles in-trail at specified ranges and randomly orienting the aircrafts' paths about φ.

3.2 TYPICAL SEPARATION ERROR CHARACTERISTICS

The purpose of this type of analysis is to generate typically observed separation error characteristics for a specified location of two aircraft and two radars. The aircraft track and ground speed are specified. The radar range bias, azimuth bias and location bias are randomly selected once and held constant for each radar. The transponder range bias is sampled once and held constant for each aircraft. The range jitter and azimuth sample error are resampled and rounded off according to the CD format. The apparent separation error introduced by timing differences is computed once based on a sampled rotation rate of each radar and the aircraft velocities. Estimates and plots of position and separation are provided for each radar tracking both aircraft and both combinations of different radars tracking each aircraft. This is illustrated in Figure 6.

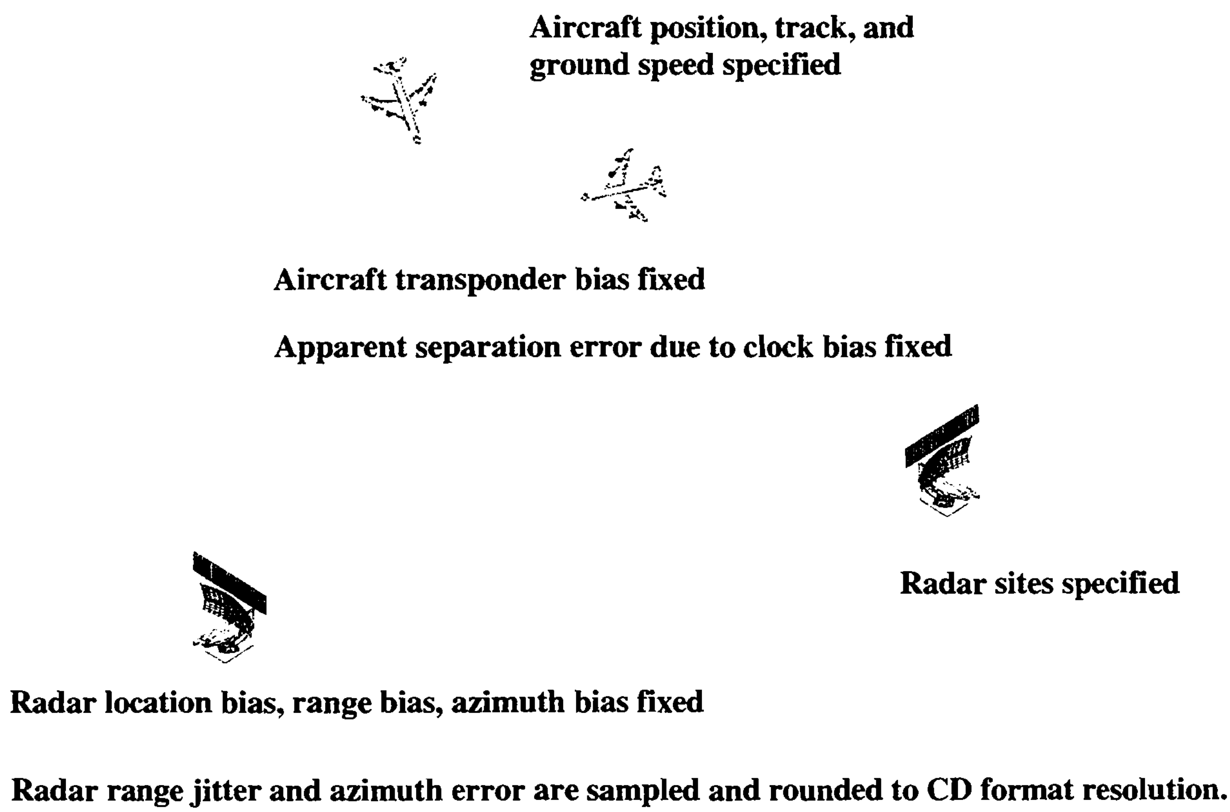

Figure 6. Typical errors in estimating aircraft separation were measured for specified radar and aircraft geometries. Bias errors were sampled once; jitter errors were sampled for each simulation.

3.3 SEPARATION ERRORS FOR SPECIFIC SIMULATION CASES

The purpose of this analysis is to simulate specific "hit" to "hit" performance that would be observed for specified radar locations and specified flight paths of two aircraft. The individual measurements are recorded. The relative orientation and geometry of the aircraft to the radars and will change for each measurement. Radar coverage areas must be specified. This is illustrated in Figure 7.

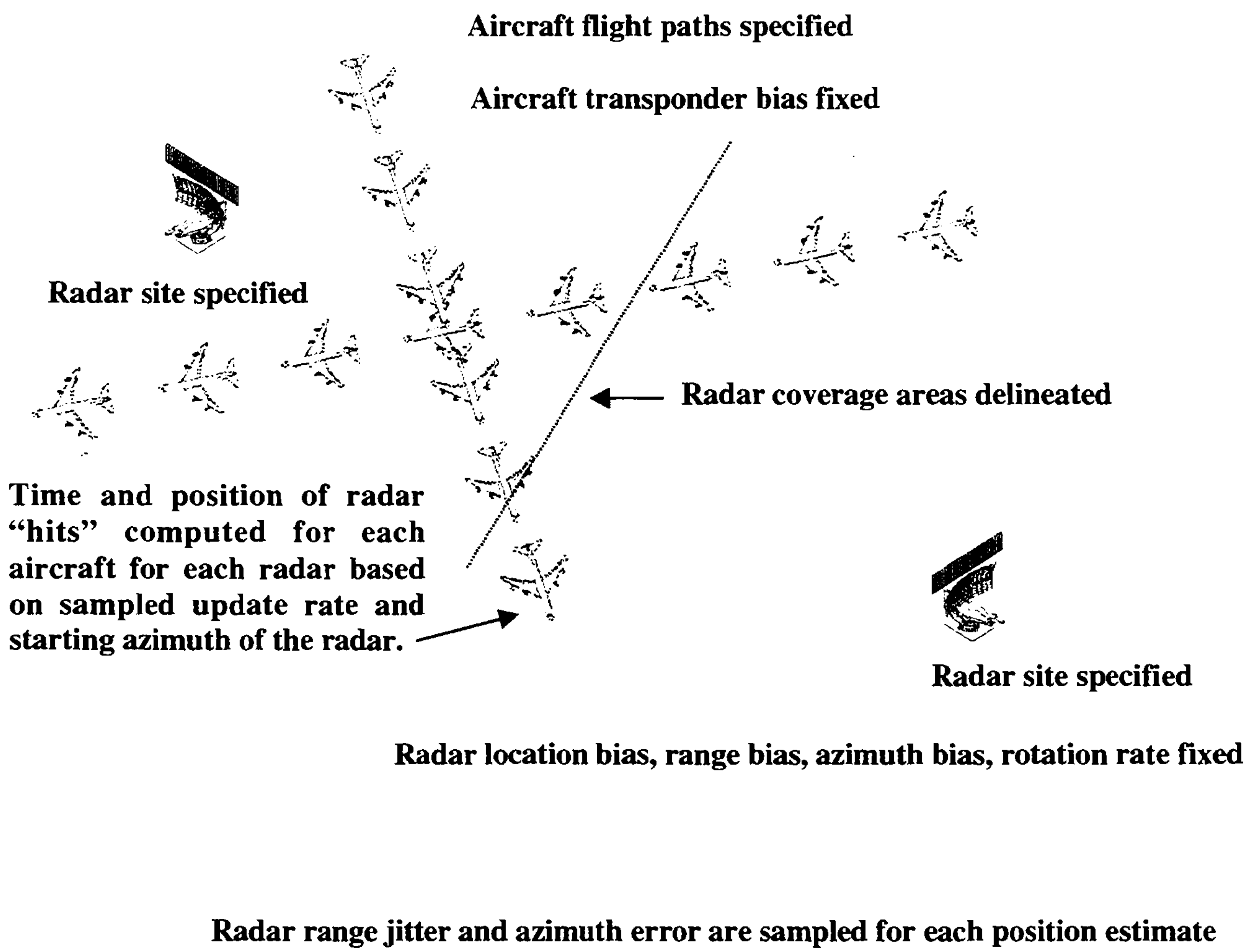

Figure 7. Separation measurement performance for specific cases use specified radar locations and aircraft flight paths to simulate errors for each individual position measurement.

Table 1. Error Characteristics for Types of Simulation

	Average	Typical	Specific
Aircraft Position Track and Ground Speed	Sampled every time. Aircraft are spaced three miles apart with the midpoint of separation at a fixed range from the radar(s). Orientation of aircraft randomly sampled. Ground speed is assumed to be 200 knots.	Specified as input and held constant.	Aircraft tracks and ground speed specified as input. Position updated for every radar "hit."
Radar Site Location Bias	Sampled every time.	Sampled once for each radar and held constant.	Sampled once for each radar and held constant.
Range Bias	Sampled every time.	Sampled once for each radar and held constant.	Sampled once for each radar and held constant.
Range Jitter	Sampled every time.	Sampled every time.	Sampled every time.
Transponder Range Bias	Sampled every time.	Sampled once for each aircraft and held constant. Independent of radar.	Sampled once for each aircraft and held constant. Independent of radar.
Range CD Rounding	Rounded every time.	Rounded every time.	Rounded every time.
Azimuth Alignment Bias	Sampled every time.	Sampled once for each radar and held constant.	Sampled once for each radar and held constant.
Azimuth Jitter	Sampled every time.	Sampled every time.	Sampled every time.
Azimuth CD Rounding	Rounded every time.	Rounded every time.	Rounded every time.
Errors in Separation Due to Timing	Radar update rates sampled every time for each radar. Random starting angles assumed. Timing interval and separation error computed for each sample.	Update rates sampled once for each radar. Apparent separation error due to clock bias computed once and held constant.	Starting angle and rotation rate sampled once and specific positions and times computed for each "hit" of each aircraft. Displayed separation updated with each "hit."

4. SIMULATION PROCEDURES

The following sections present a high level description of the various simulation procedures used in the analysis. Detailed step-by-step psuedo code descriptions and MATLAB scripts are contained as an Appendix in Chapter 8. The following descriptions are based on the concept of multiple "runs" or trials in a Monte Carlo fashion. In fact, the scripts described in the Appendix make use of MATLAB's matrix and vector computation capabilities and actually store the variables for each trial as an element in a vector or matrix, thus there are almost no time-consuming "do-loops" in the scripts themselves. However, for the purposes of understanding the analysis, it may be convenient to think of individual trials or runs.

4.1 AVERAGE ERRORS IN SEPARATION MEASUREMENTS, ONE RADAR

The purpose of this analysis is to estimate the average errors in the separation displayed to a controller using a single radar tracking two aircraft three miles in trail with velocities of 200 knots. These errors will be a function of range. The geometry is illustrated in Figure 5. As described in Chapter 3 and in Table 1, all errors, including site location and bias errors are sampled for each run. The error characteristics are for a terminal radar with operating Mode S.

4.1.1 Input

The range and sample size are input to the simulation. The range is the range of the aircraft from the radar in nautical miles. The two aircraft are assumed to be separated by three nautical miles and the range is to the point midway between the two aircraft. The aircraft will be randomly oriented about a circle whose origin is at the specified range as illustrated in Figure 5. The sample size is the number of cases or trials that will be run.

4.1.2 Output

The outputs are vectors of length sample size containing the position measurement errors for the two aircraft and the separation estimates between the aircraft for each run. The position estimate errors and separation estimates are in nautical miles. A figure with three subplots containing histograms of the separation estimates and aircraft position estimate errors is plotted and the mean and standard deviation of the separation estimates is written on the plot.

4.1.3 Modeling Procedure

The single radar is assumed to be at the center of an x-y coordinate system with coordinates (0,0). For each run, the two aircraft are positioned with the midpoint between the two aircraft at the input range and at a randomly sampled angle φ as illustrated in Figure 5. The midpoint coordinate is (range,0). The true positions of the aircraft are computed based on the randomly sampled φ and specified range. Next the true range and azimuth are computed from the radar to each aircraft. For each run, a site location bias is sampled from a uniform distribution between 0 and 200 feet in a random direction.

The next step is to choose a range bias error, a range jitter error, and a range transponder error for each aircraft by sampling from the error distributions described in Section 2.2 and illustrated in Figure 2. The estimated range for each run is computed by adding the errors to the true range. The reported range is computed by rounding off to the CD reporting format which is the nearest 1/64 nautical mile.

Similarly the azimuth bias and jitter error are chosen by sampling from the error distributions described in Section 2.3 and illustrated in Figure 3. The estimated azimuth is rounded off to the nearest 1/4096 of a scan to compute the reported azimuth.

The reported aircraft x,y positions are computed from the reported range and azimuth and then offset by the radar site location bias. The x,y position errors are computed by comparing the reported x,y positions to the true x,y positions of the aircraft.

The error in displayed separation due to the sample time bias described in Section 2.4 must be computed before an error in separation measurement can be computed. The sample time bias is computed by determining the time between radar hits. The update rate of the radar is randomly sampled between 4 and 5 seconds. The aircraft that is hit first is the one that minimizes the time between aircraft hits and will depend on the randomly sampled geometry. The aircraft that is hit second is moved in the appropriate direction based on a velocity of 200 knots and the time between hits of the two aircraft. The separation estimate is computed from the reported x,y positions of the aircraft and corrected for the time bias.

The reported position errors and separation estimates are returned as output and a figure with three subplots is drawn. The upper subplot shows the distribution of separation estimates as a histogram with the mean and standard deviation printed on the plot. The two lower subplots contain histograms of the position estimate errors for each aircraft. The actual separation of the two aircraft is three nautical miles and the separation estimates should have a mean of three miles. Any deviation is due to the random sampling. The standard deviation of separation estimate errors gives an indication of the inaccuracies in the separation estimates.

4.2 AVERAGE ERRORS IN SEPARATION MEASUREMENTS, TWO RADARS

4.2.1 Input

The range, sample size and angle Θ between the radars and aircraft as shown in Figure 5 are input to the simulation. The range is the range of the aircraft from the radar in nautical miles. The two aircraft are assumed to be separated by three nautical miles and the range is to the point midway between the two aircraft. The aircraft will be randomly oriented about a circle whose origin is at the specified range as shown in the lower illustration in Figure 5. The sample size is the number of cases or trials that will be run.

4.2.2 Output

The outputs vectors and plot are the same as for the single radar simulations.

4.2.3 Modeling Procedure

The modeling procedure for the two radar case is the same as for the one radar case described above except that each aircraft is tracked by the radar nearest that aircraft. All errors for the radars are sampled separately. The error in displayed separation due to the sample time bias is random because the update rates of the two radars are uncorrelated. The procedure used is to first independently sample radar update rates of between 4 and 5 seconds for the two radars and then sample a time bias of between 0 and half the update rate of the slower radar. The aircraft that is hit first is randomly determined and the aircraft hit second is moved the appropriate distance.

4.3 TYPICAL ERRORS IN SEPARATION MEASUREMENT

The purpose of this analysis is to show typical error characteristics for a given geometry of radars and aircraft. The bias errors are sampled once and the jitter errors are resampled for each run.

4.3.1 Input

The x and y positions of the two radars and the two aircraft (nautical miles from a (0,0) reference), the ground track (degrees) and ground speed (knots) of the two aircraft, and the sample size are input.

4.3.2 Output

The outputs are vectors of length sample size containing the position measurements errors and separation measurement errors of the aircraft. There are four position errors vectors returned, corresponding to the cases where radar 1 is tracking aircraft 1, radar 1 is tracking aircraft 2, radar 2 is tracking aircraft 1, and radar 2 is tracking aircraft 2. There are four vectors of separation estimates from the simulation runs. The four vectors correspond to the cases where 1) radar 1 was tracking both aircraft, 2) radar 2 was tracking both aircraft, 3) radar 1 was tracking aircraft 1 and radar 2 was tracking aircraft 2, and 4) radar 1 was tracking aircraft 2 and radar 2 was tracking aircraft 1.

In addition several figures are produced. The first figure shows the relative geometry of the radars and the aircraft. A red leader line showing the path of the aircraft during one minute of flight indicates the aircraft tracks and speeds. There are four additional figures produced corresponding to the four permutations of the two radars tracking the two aircraft. Each of these figures consists of four subplots showing the aircraft positions and the individual reported positions generated by the simulation runs. A histogram of the separation estimates is generated with the mean and standard deviation in separation estimates printed on the graph. The actual separation is also printed on the subplot showing both aircraft positions and position estimates. Each subplot is labeled to make it clear which radar was tracking which aircraft.

4.3.3 Modeling Procedure

The simulation procedure is basically the same as for the average performance simulations except that the radar site location bias, range bias and azimuth alignment bias are sampled only once for each radar and held constant. The update rates for each radar are also sampled once and held constant. The transponder bias is sampled once for each aircraft and held constant. The range jitter and azimuth jitter are sampled every run and added to the other errors and the result rounded off to the CD reporting values to yield the reported range and azimuth estimates. The reported aircraft x,y position and position error are computed as before and include the radar site location bias error. The error in displayed separation due to the sample time bias is computed once and held constant. For the two cases where one radar is tracking both aircraft the sample time bias is directly computed from the sampled update rate and delta azimuth. The time bias is used with the aircraft ground track and velocity to move the second aircraft hit. For the two cases where different radars are tracking the aircraft the delta time between hits is randomly chosen between zero and half the update rate of the slower radar. The sample time bias and resulting apparent motion of the second aircraft hit are computed once and applied to each run.

4.4 SPECIFIC ERRORS IN DISPLAYED SEPARATION

The specific simulation specifies the location of two radars and the flight tracks of two aircraft. The bias errors are sampled once and the aircraft position estimates, including jitter errors are made for each turn of the radar as the aircraft move. The error in displayed separation due to the sample time bias is implicitly included since the times of the position estimates and changes in the displayed separation are computed based on the radar update rate and aircraft motion. The actual separation is a smooth continuous function of time while the estimated separation is updated with each hit of the radar.

4.4.1 Input

The x,y positions of two radars and the x,y initial positions, ground speeds, and ground tracks of the two aircraft are input as well as the run time of the simulation in seconds.

4.4.2 Output

The output is a set of vectors that contain a list of the individual position measurement errors and a set of matrices of the displayed separation measurement errors as a function of time. The matrices of displayed separation measurement errors contain times in column 1 corresponding to an update of either aircraft and the displayed separation error at that time. The displayed errors as a function of time will be linear between entries. The length of the vectors and matrices will depend on the individual radar rotation rates, movement of the aircraft, and run time. Output data for position errors and displayed separation errors is made for all four permutations of two radars tracking two aircraft. Six figures are produced. The figures can be edited by the property editor in MATLAB to change any of the scales. The first figure is a color-coded plot that presents the location of the two radars and the true location of each aircraft when it is hit by each radar. The location of radar 1 and the location of either aircraft when hit by radar 1 are shown in blue and the location and hits of radar 2 are shown in red, therefore a set of red dots and a set of blue dots indicate the hits on an aircraft by both radars. The second figure contains four subplots of the actual separation versus time and the displayed target separation with no position error as a function of time for each of the four permutations of radars tracking aircraft. The difference is due only to the movement of the aircraft between hits by the radar. This figure covers the entire run time. A third figure presents a blow-up of the second figure for a portion of the run time. The scales of any figure can be changed by MATLAB's plot editor. The fourth figure is similar to the first figure but presents the individual radar position estimates, including errors, using the same color-coding. The fifth figure has the same format as the second figure, but now the actual separation versus displayed separation (including the radar position estimate errors) is plotted. The sixth plot is a blow-up of this figure for a portion of the run time. The matrices output containing the displayed separation errors as a function of time can be used to create histograms over any portion of the run time.

4.4.3 Modeling Procedure

The radar update rates are randomly sampled from between 4.0 and 5.0 seconds and the starting point for each radar is randomly selected.

The times when each radar will hit each aircraft are computed by finding the zeros of the expression describing the difference between the Θ of the radar as a function of time and the relative Θ to each aircraft as a function of time. The Θ of the radar as a function of time is computed from the starting point of the radar and the update rate. The relative Θ to each aircraft as a function of time is computed from the aircraft motion, which is specified in the input as a starting position and a ground track and velocity.

Next, the true x,y positions of each aircraft are computed at the times they are hit by either radar. These are plotted along with the radar positions as figure 1.

The next step is to compute and plot the separation that would be displayed to the controller as a function of time if there were no errors in the radar position reports. The displayed separation will change with each update of either aircraft position report. This is accomplished by creating four matrices, one for each permutation of the two radars tracking the two aircraft, (i.e., one matrix would be for radar 1 tracking aircraft 1 and radar 2 tracking aircraft 2, etc.). Each matrix contains the times of the hit of either aircraft by its respective radar and the true x,y positions of both aircraft at the times either aircraft is hit along with the true separation between the aircraft. The displayed separation is constant until the time either aircraft's position is updated with a new hit. This is plotted in figures 2 and 3 as described in the output.

The true ranges and azimuths to each aircraft are computed at the times of the radar hits for the four permutations of radars tracking aircraft.

A radar site location bias, range bias, and azimuth bias are sampled once for each radar and a transponder bias is sampled once for each aircraft. The range jitter and azimuth jitter are sampled for each hit of each radar on each aircraft. The estimated range and azimuth are computed for each hit of each radar on each aircraft by adding the range bias and jitter and transponder bias to the true range and the azimuth bias and azimuth jitter to the true azimuth. The reported values of range and azimuth are computed by rounding to the CD resolution values.

The reported x,y values for each hit of each radar on each aircraft are computed from the reported range and azimuth values and adjusted according to the site location bias. The reported x,y values for each hit of each radar are plotted along with the radar locations as figure 4. The position estimate errors for each hit of each aircraft by each radar are computed and stored to be returned as output.

Matrices similar to those created above for the separation that would be displayed to the controller as a function of time if there were no errors in the radar position reports are created, but the reported x,y position reports are now used in place of the true x,y values. The displayed separation including radar errors as a function of time is plotted along with the true separation for each of the permutations of radars tracking aircraft as figure 5 and 6 as described in the output.

The vectors of position error and matrices of displayed separation error as a function of time are returned as output for the four permutations of the two radars tracking the two aircraft.

5. RESULTS

5.1 AVERAGE ERRORS IN SEPARATION ESTIMATES

The results of the simulations to measure the average errors in separation estimates as a function of range for a single radar tracking two aircraft and for two radars, each tracking one of the aircraft, is presented in sections 5.1.1 and 5.1.2 in Figures 8-23. In these simulations the true separation of the aircraft was 3 nautical miles and the aircraft were randomly oriented such that the mid-point between them was at the range specified. Each simulation was run 50,000 times. Since these are measurements of average performance as a function of range, each type of error, as described in Table 1, was resampled for each run.

The results of each simulation are plotted in Figures 8-23. To facilitate comparisons, the axes were held constant for all plots. The top center plot is a histogram of the estimated separation based on the aircraft positions measured by the radar including errors and the bottom two plots are histograms of the absolute error in position estimates for the two aircraft.

The histograms for estimated separation are unbiased and centered on the actual separation of 3 nautical miles. This is because all of the bias errors were resampled for each run. The reason the histograms are not "smooth" is because the discreet position reports for range and azimuth caused by the Common Digitizer format result in discreet possible separation measurements. Note that the individual position errors for either aircraft are only a function of range whether one radar or separate radars are tracking both aircraft. Note that there is very little quantitative difference in the separation estimate error for a single radar at 40 nmi. (σ = 0.053 nmi.) and at 60 nmi. (σ = 0.077 nmi.). This small difference (0.024 nmi = 150 feet) would suggest that it should be possible to extend the range at which 3 nmi. separation can be used, given appropriate safety analysis. The separation estimates are better when a single radar is tracking the aircraft because bias errors result in correlated position errors. When different radars are tracking the aircraft, the separation estimates are not as good, even though the position estimate quality is the same. This is because the position errors are uncorrelated and because of the larger errors in separation measurements cause by the uncorrelated intersensor timing. Note that for the case of two radars there seems to be an error that remains even as the range goes down to 5 nautical miles. This is due to the intersensor timing error and will remain regardless or range.

The simulations for two radars were all run with Θ = 0° where Θ is the angle between a line connecting the radars and the aircraft position midpoint as described in section 3.1 and illustrated in Figure 5, except that for a range of 30 nautical miles, figures 18-20 show the results for Θ = 0°, 30°, and 45°. As anticipated, the random orientations of the aircraft mean that the results are a function of range but not the angle Θ between the radars.

5.1.1 Average Measurements for a Single Radar

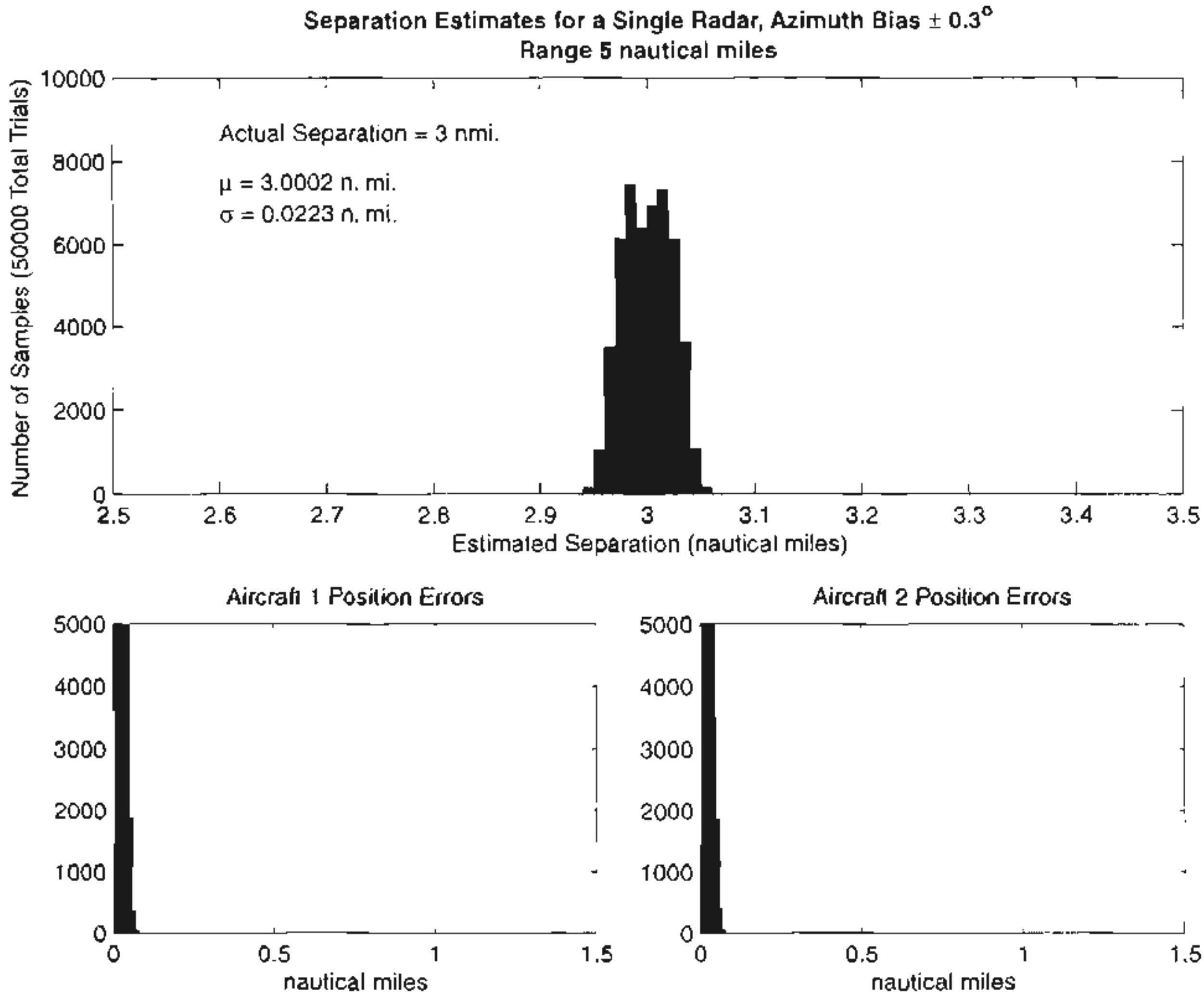

Figure 8. Average separation estimates for a single radar at a 5 nmi range

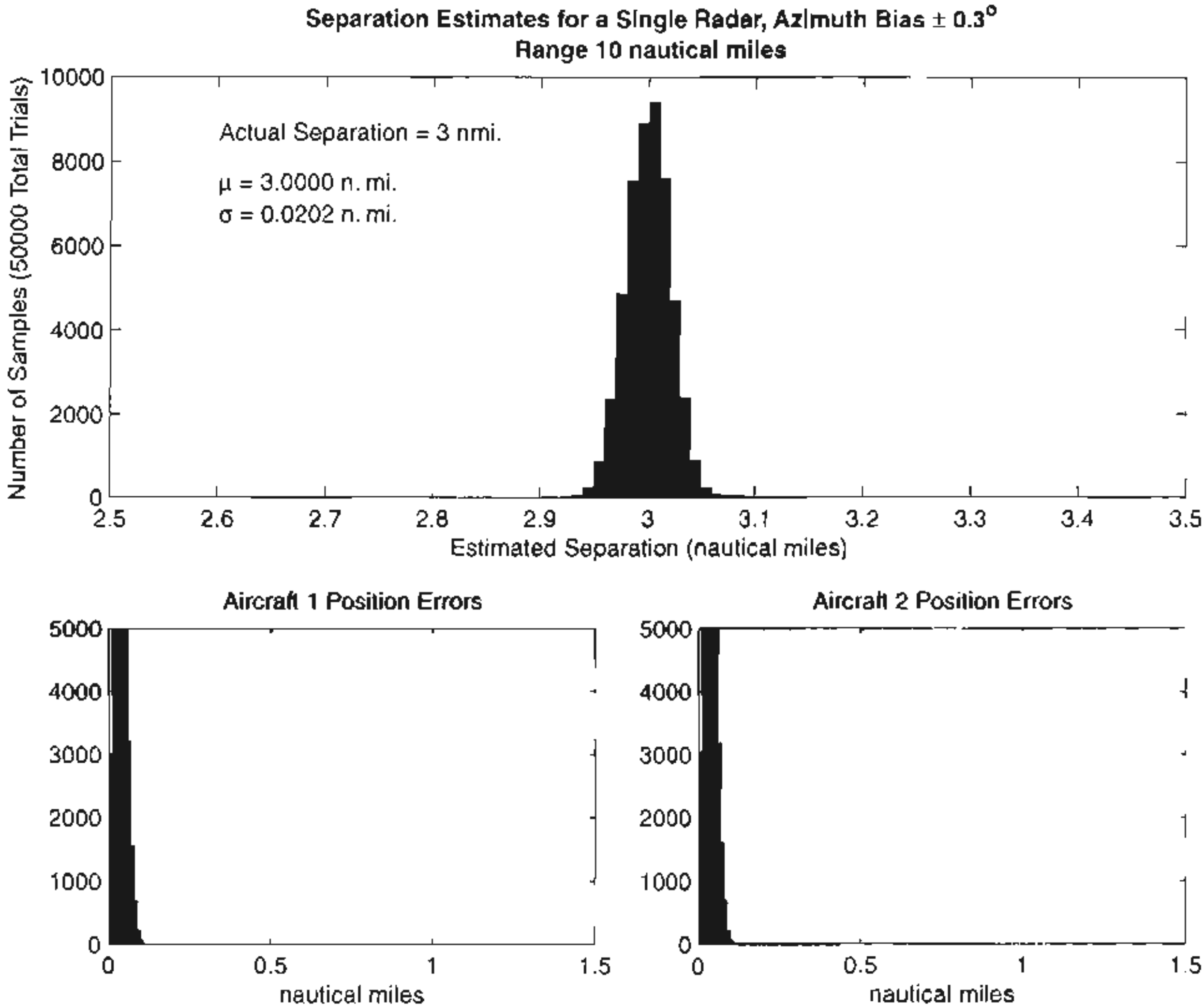

Figure 9. Average separation estimates for a single radar at a 10 nmi range

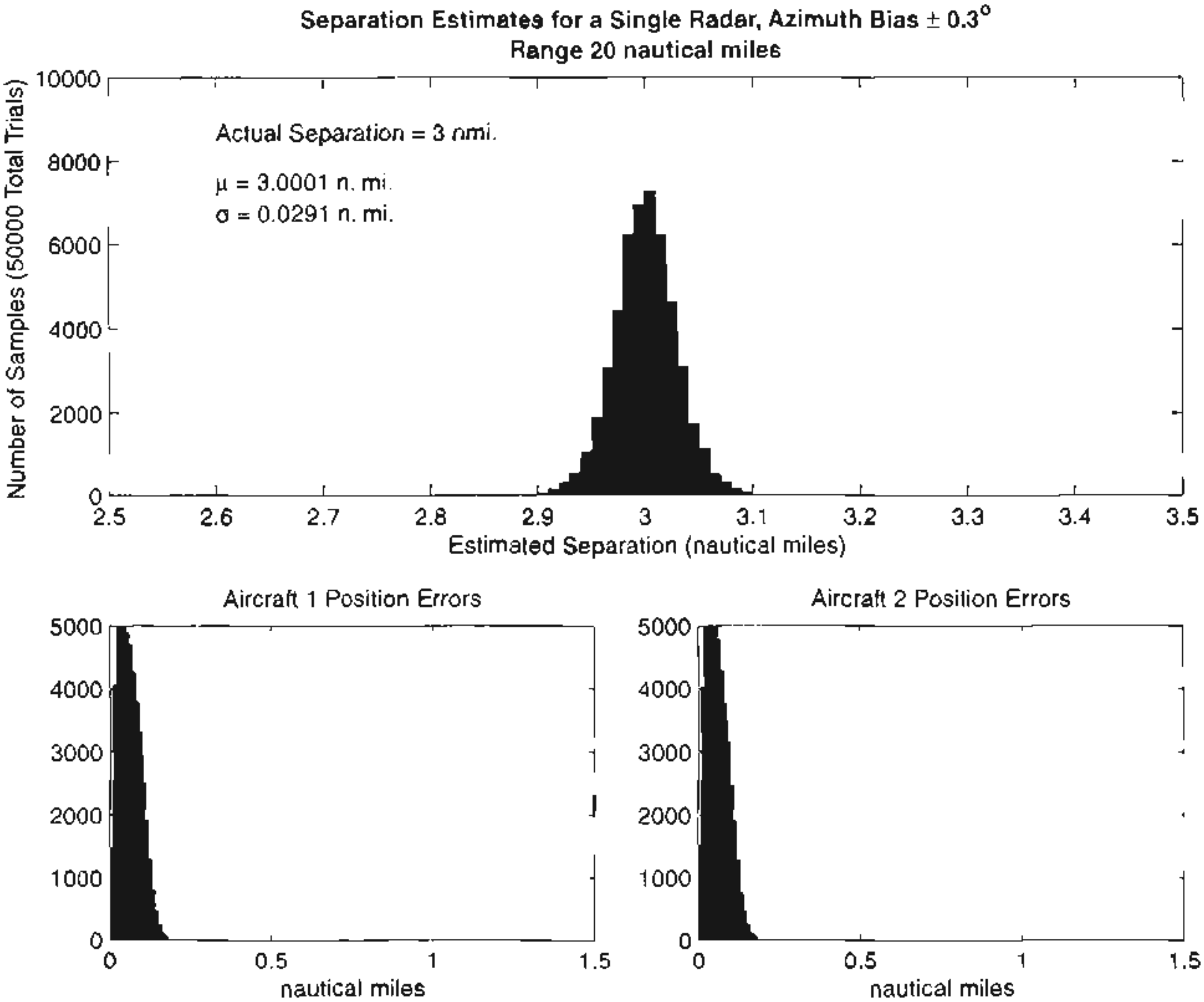

Figure 10. Average separation estimates for a single radar at a 20 nmi range

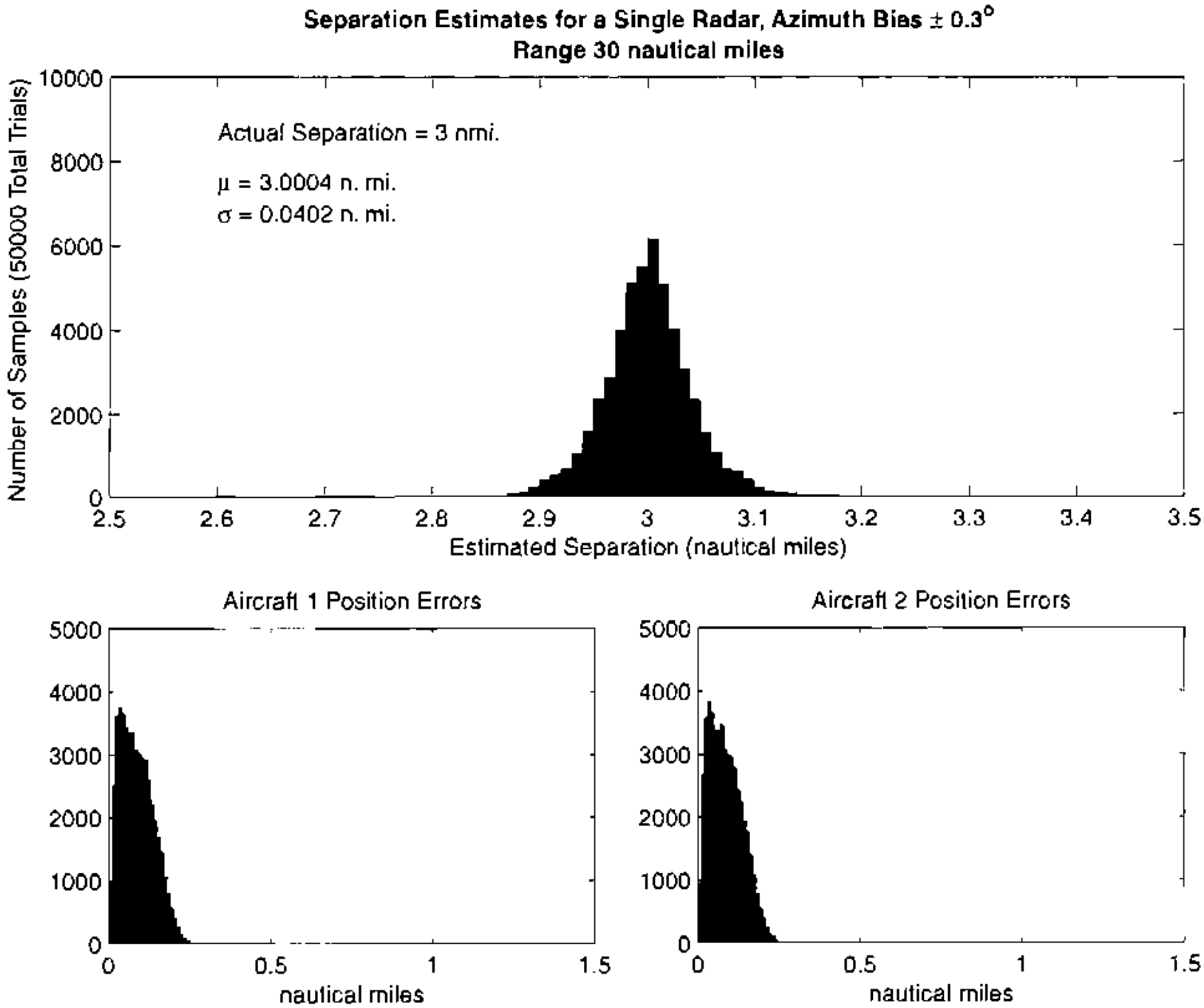

Figure 11. Average separation estimates for a single radar at a 30 nmi range

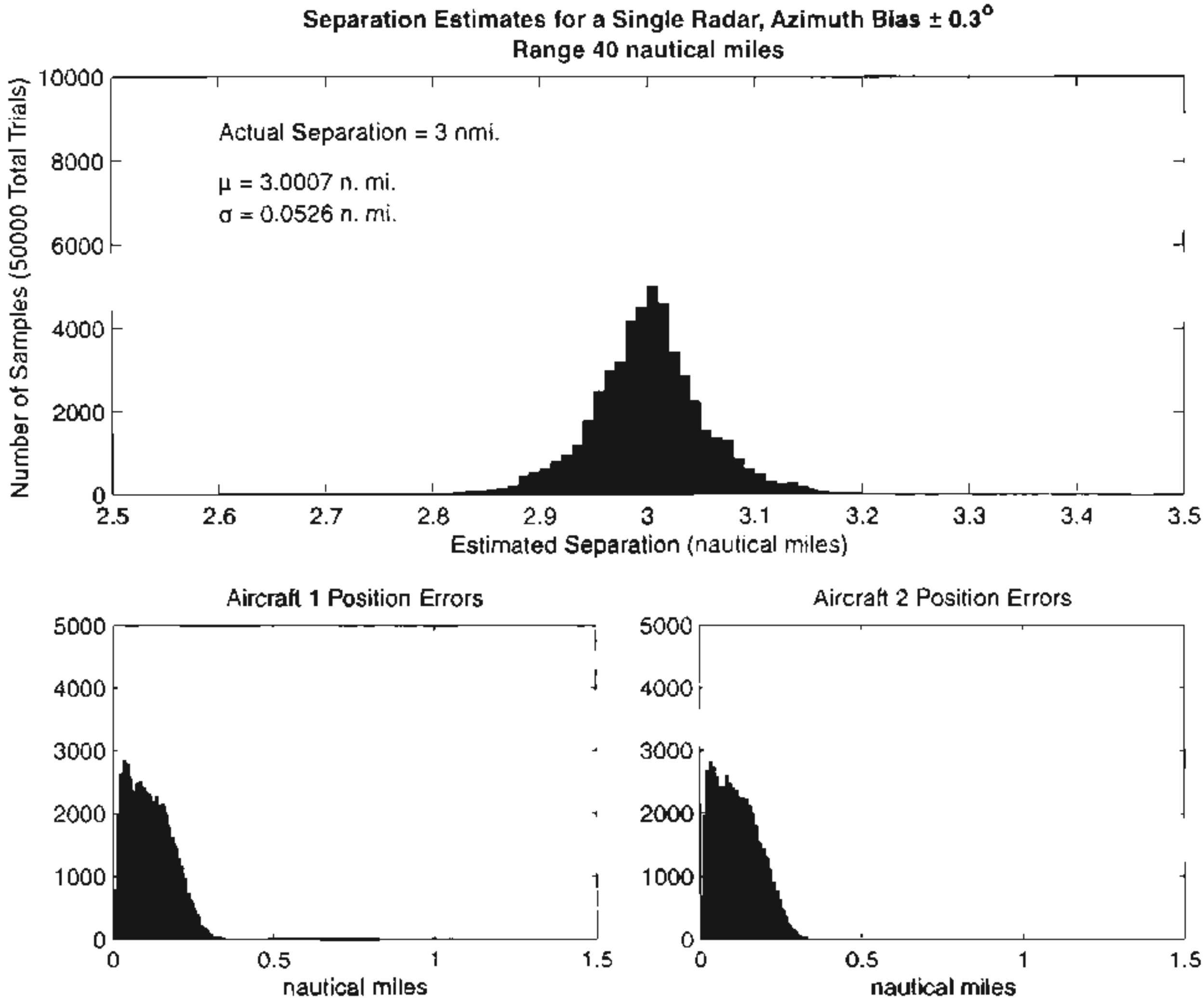

Figure 12. Average separation estimates for a single radar at a 40 nmi range

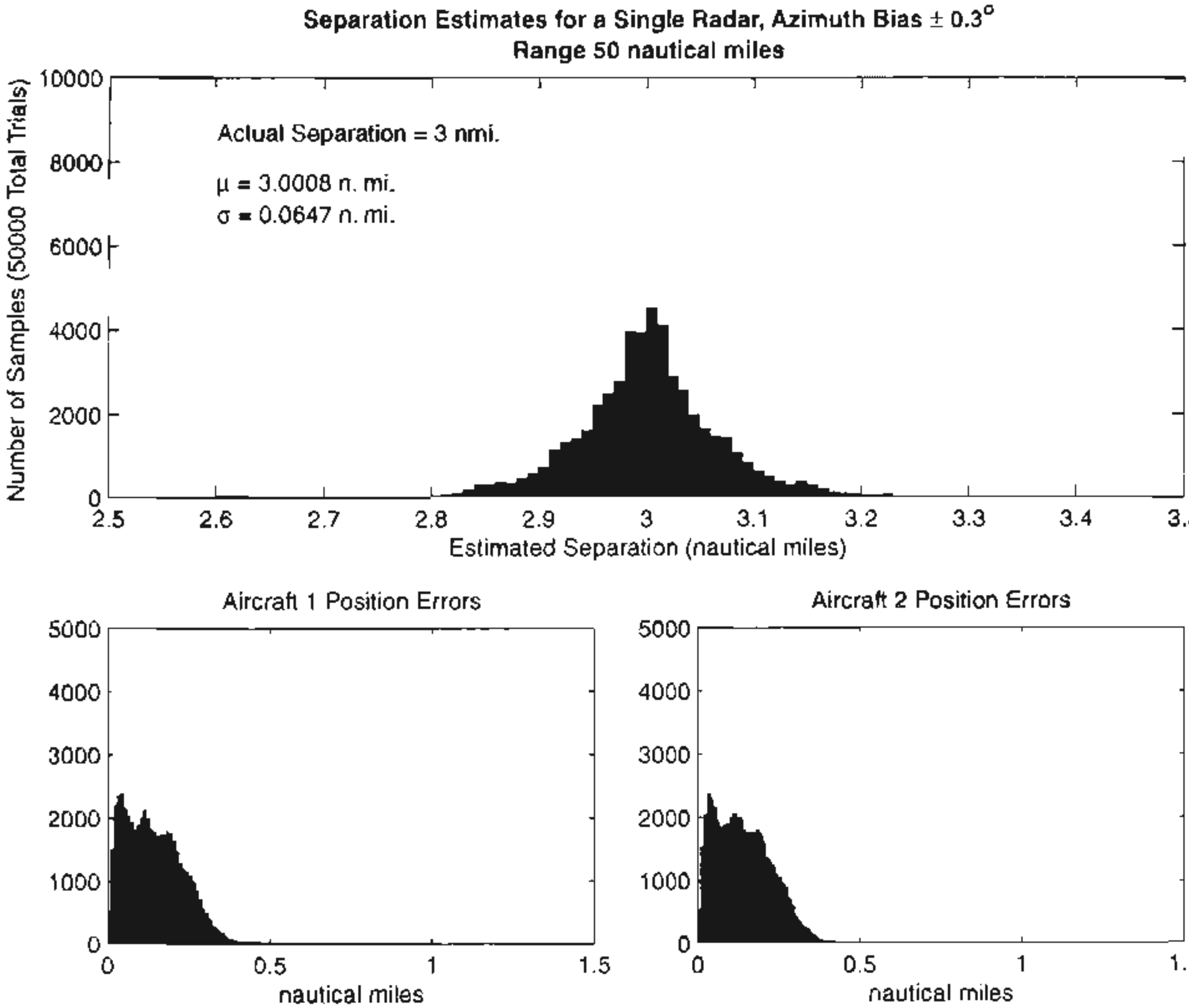

Figure 13. Average separation estimates for a single radar at a 50 nmi range

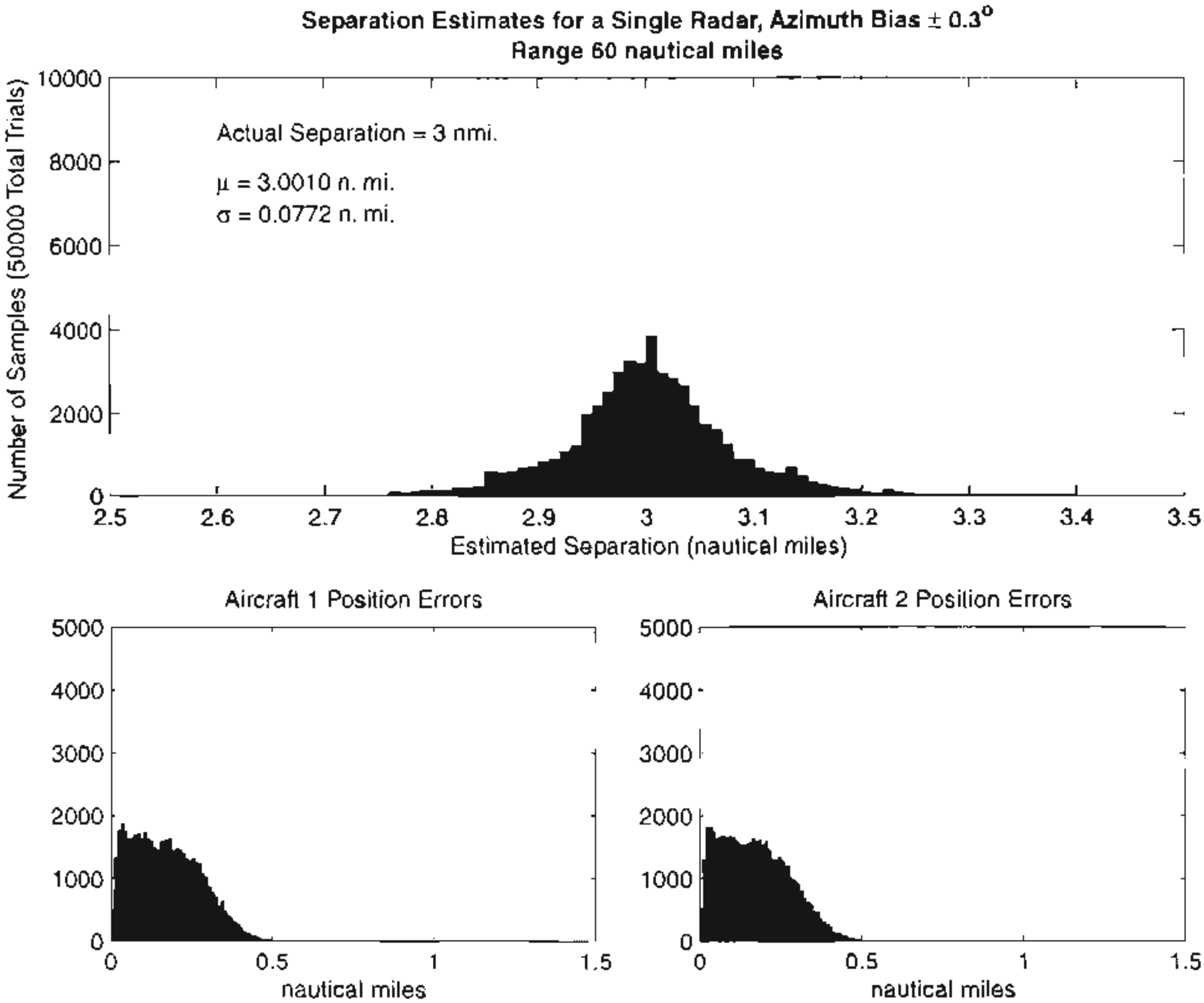

Figure 14. Average separation estimates for a single radar at a 60 nmi range

5.1.2 Average Measurements for Two Radars

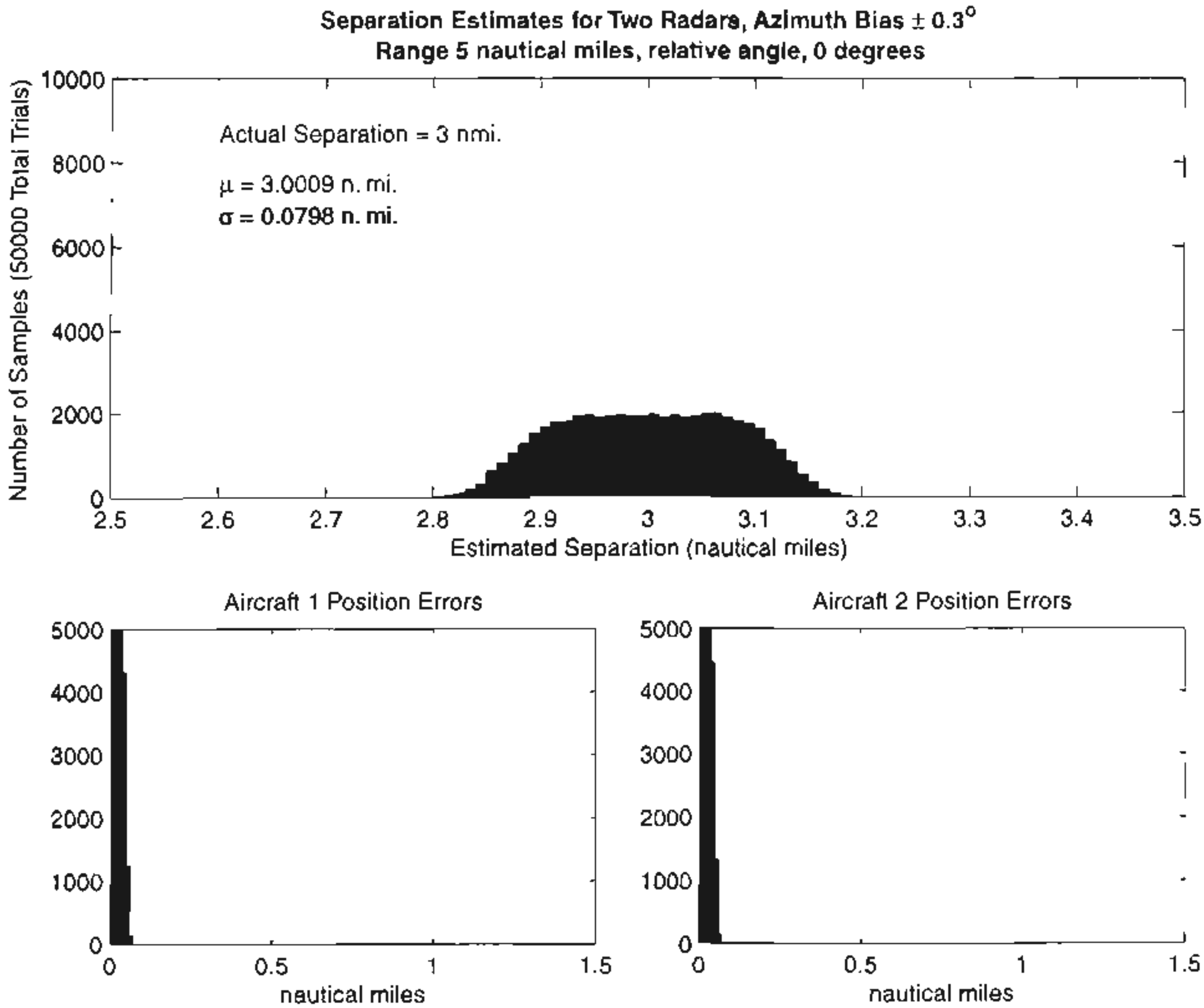

Figure 15. Average separation estimates for independent radars at 5 nmi Range, $\Theta = 0°$

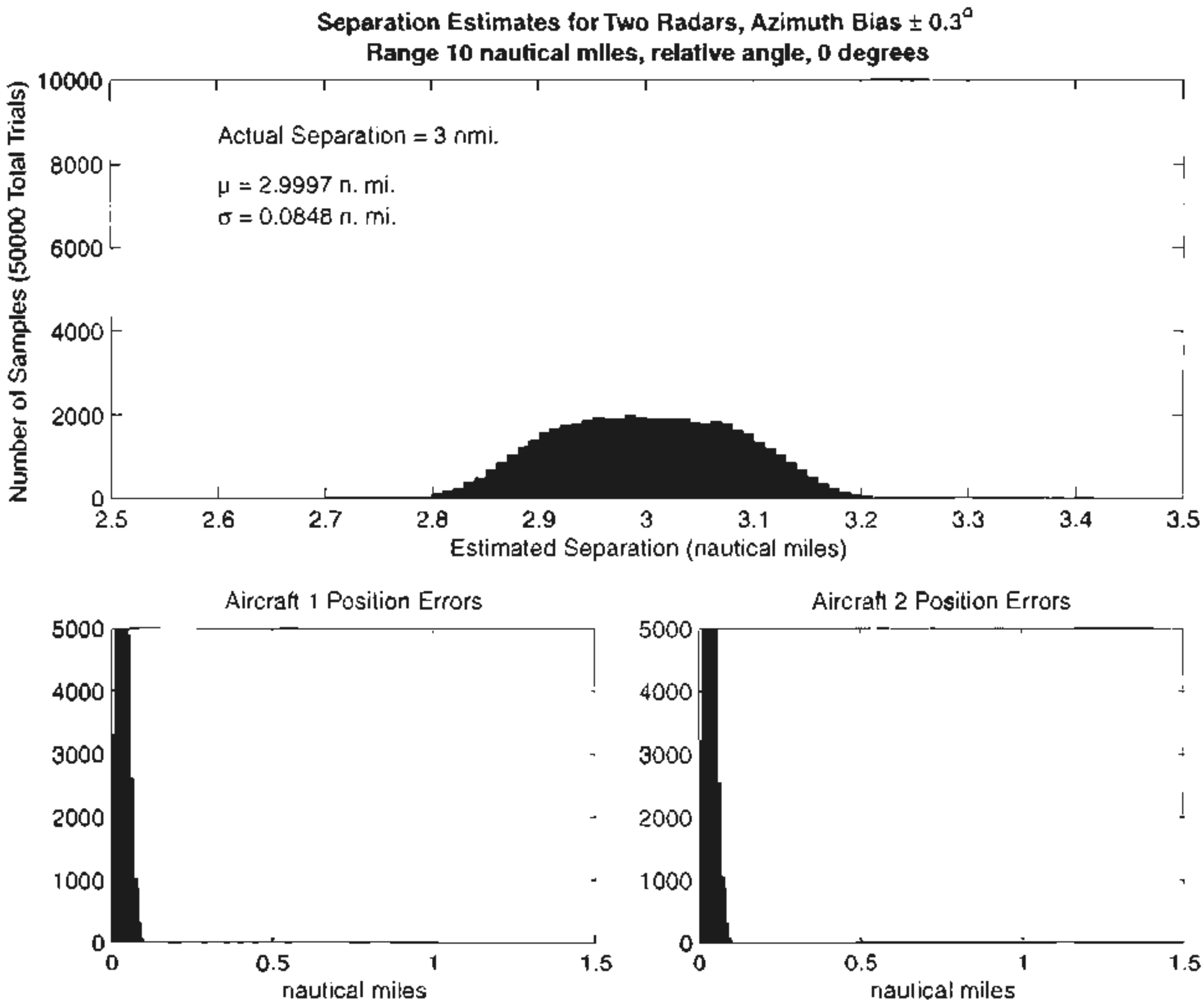

Figure 16. Average separation estimates for independent radars at 10 nmi Range, $\Theta = 0°$

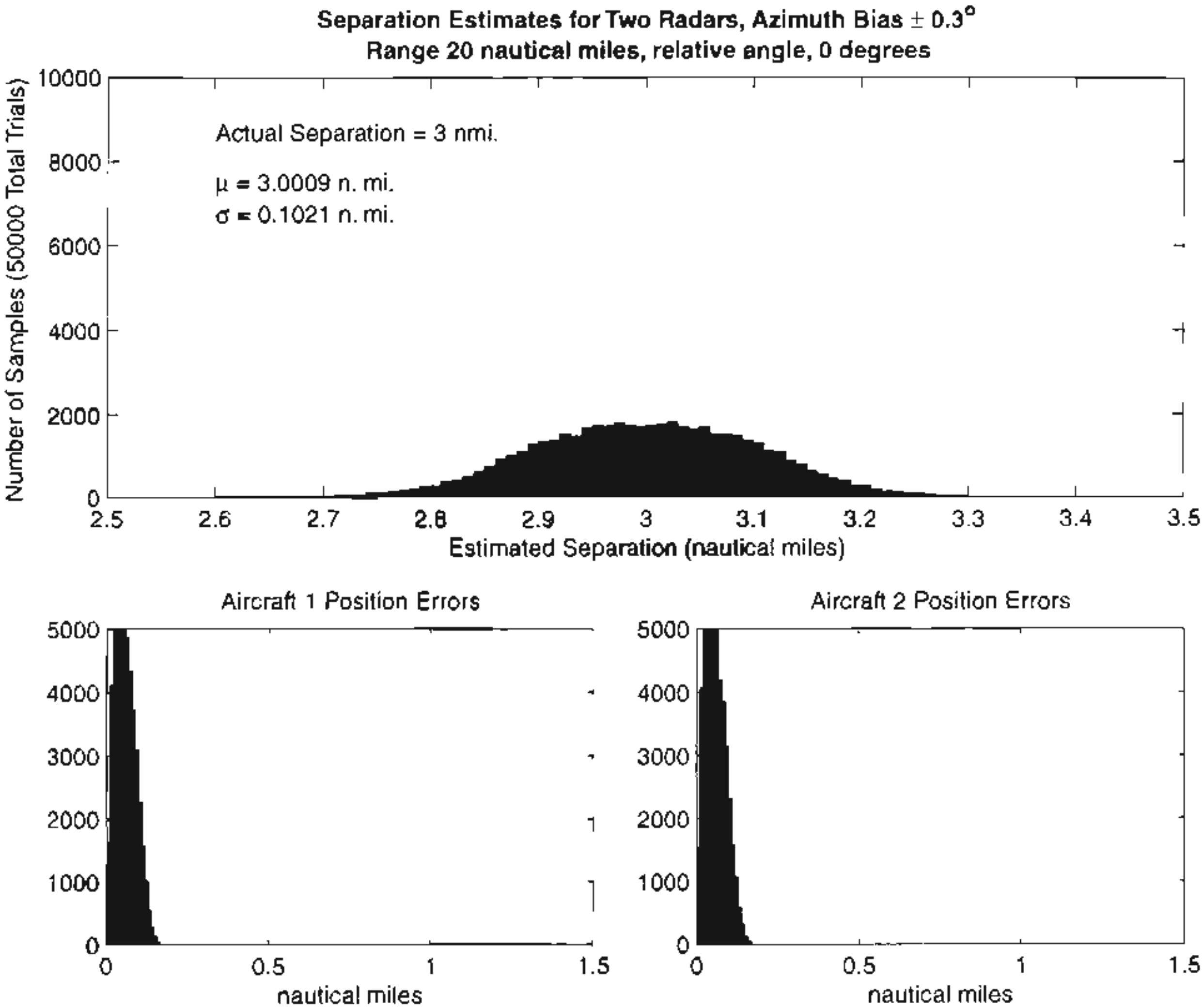

Figure 17. Average separation estimates for independent radars at 20 nmi Range, $\Theta = 0°$

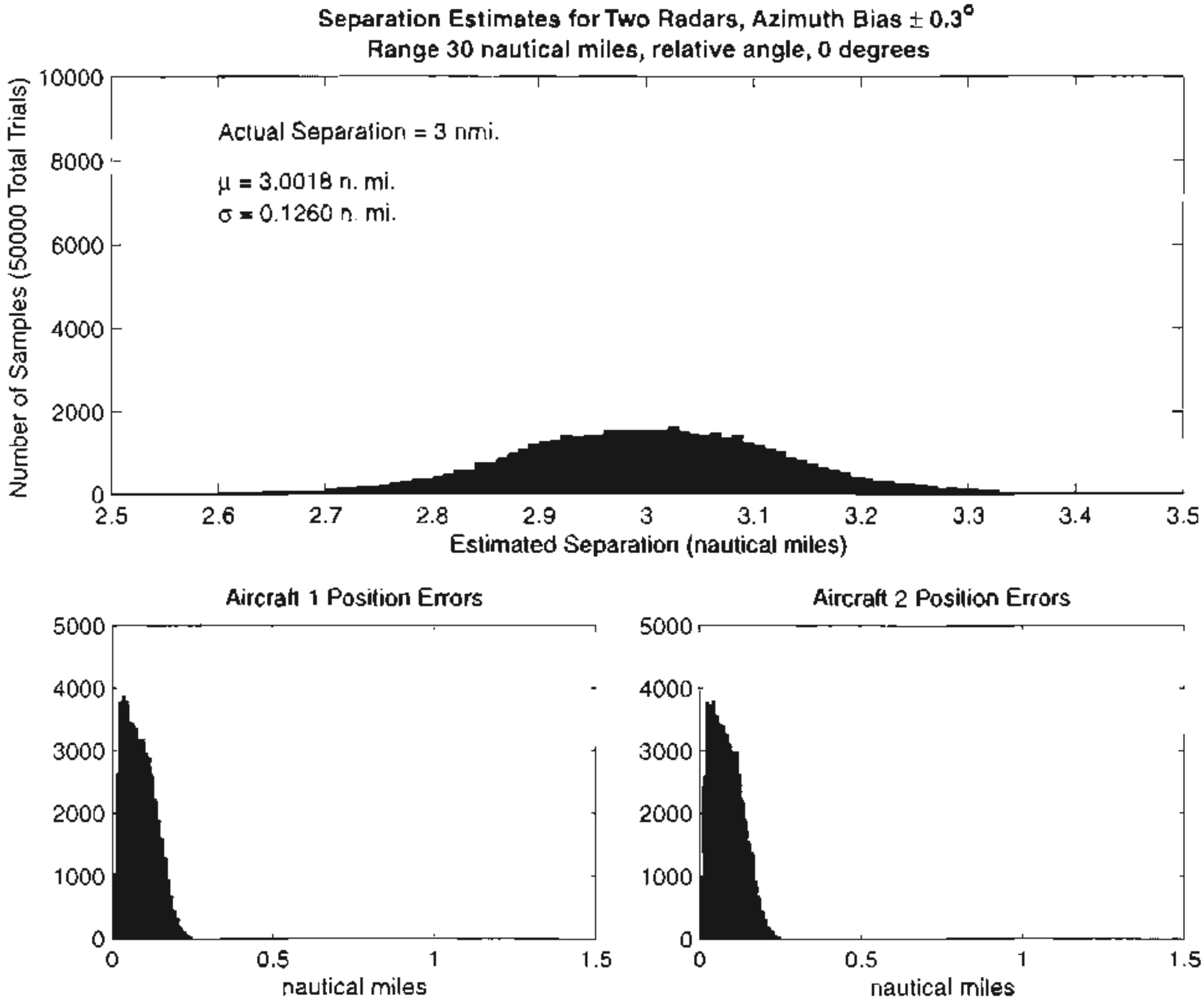

Figure 18. Average separation estimates for independent radars at 30 nmi Range, $\Theta = 0°$

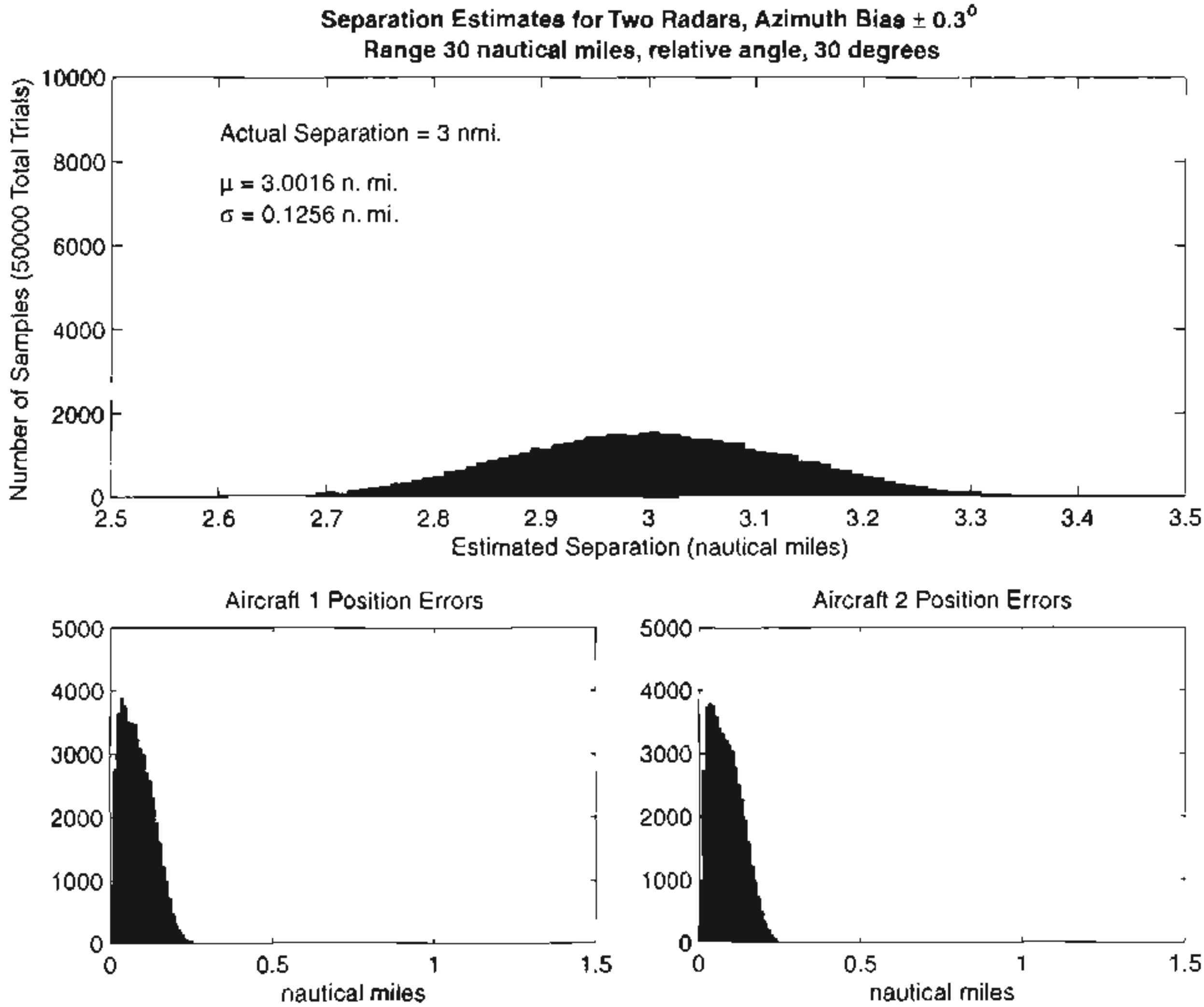

Figure 19. Average separation estimates for independent radars at 30 nmi Range, $\Theta = 30°$

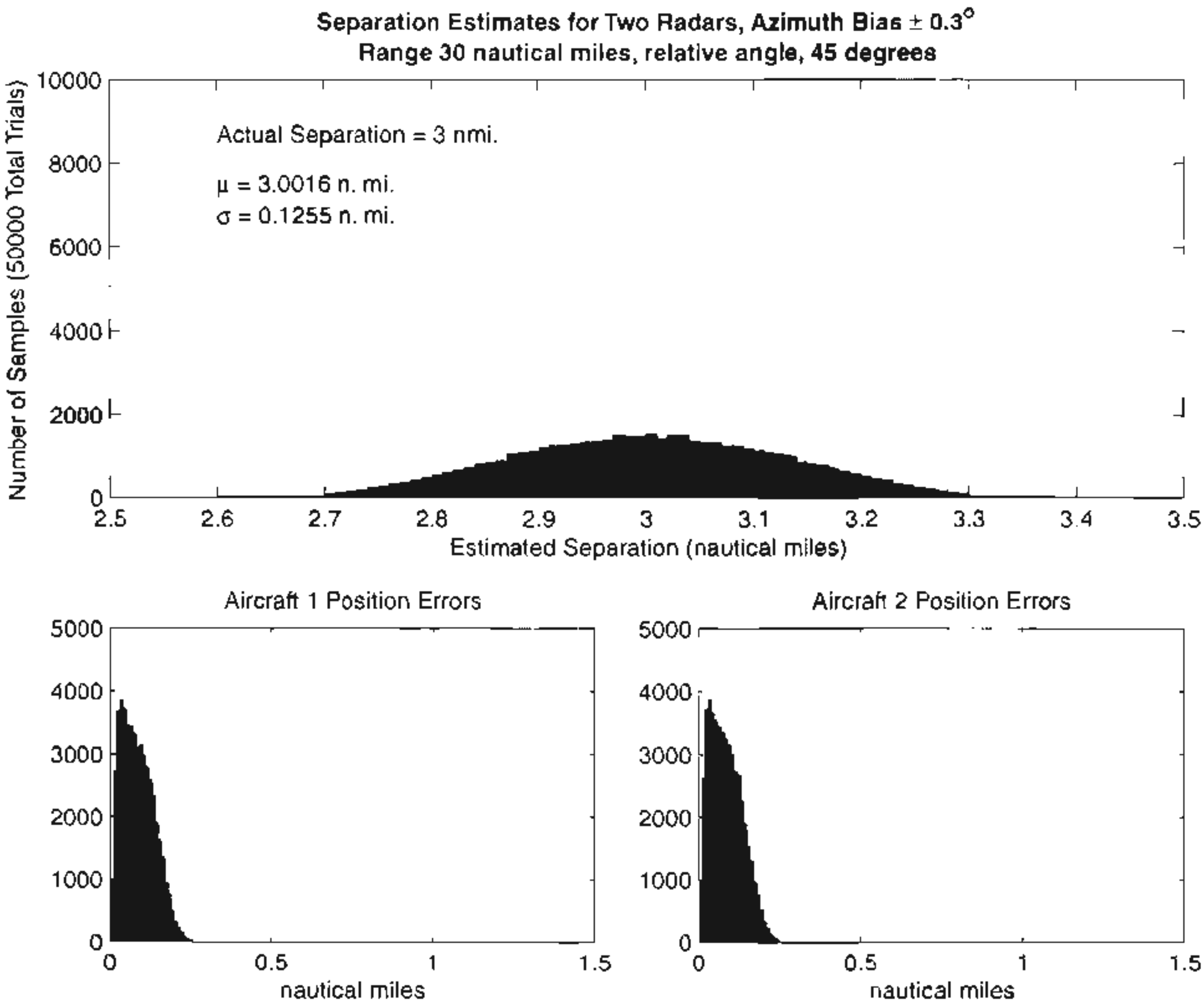

Figure 20. Average separation estimates for independent radars at 30 nmi Range. Θ = 45°

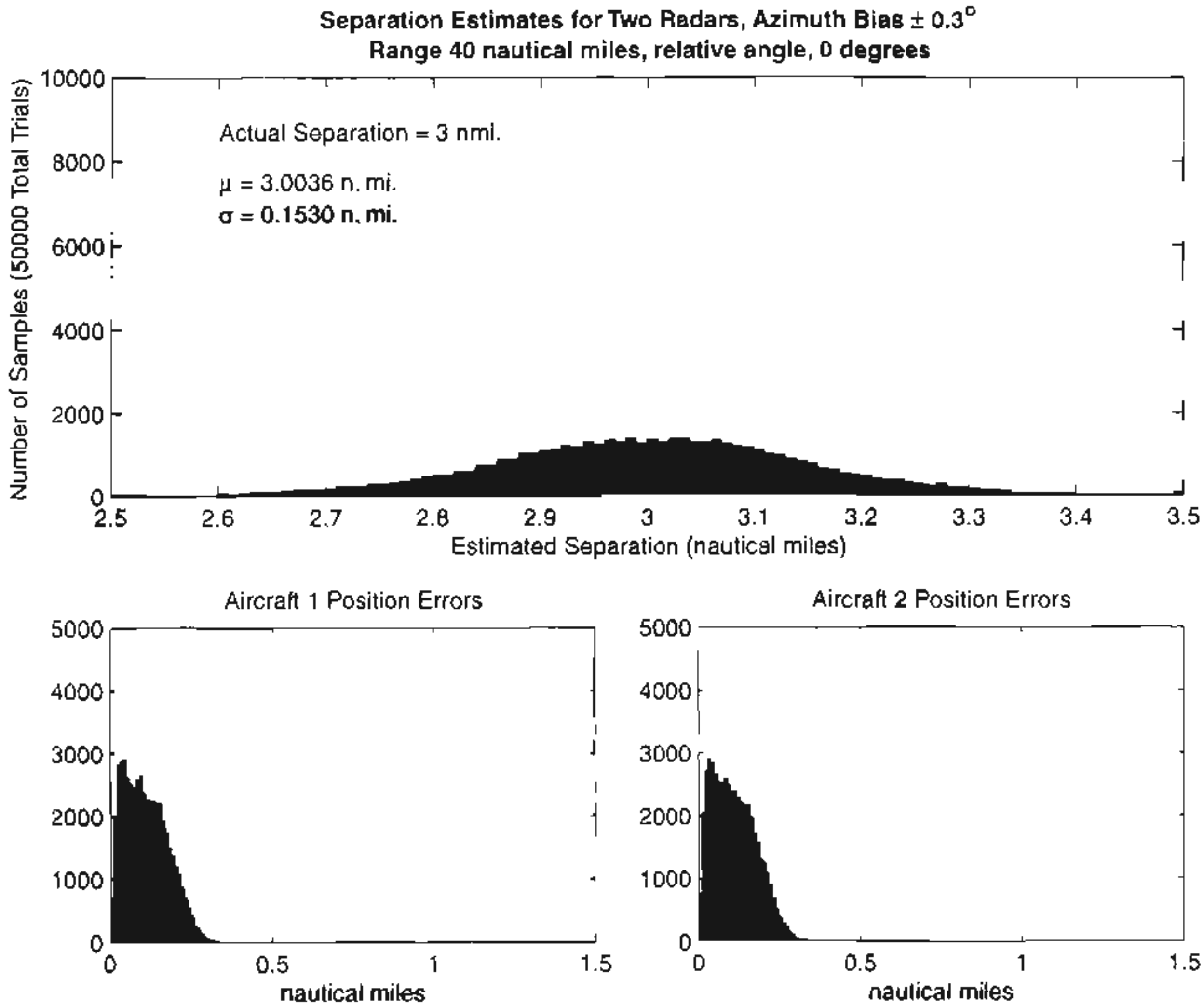

Figure 21. Average separation estimates for independent radars at 40 nmi Range, $\Theta = 0^{\circ}$

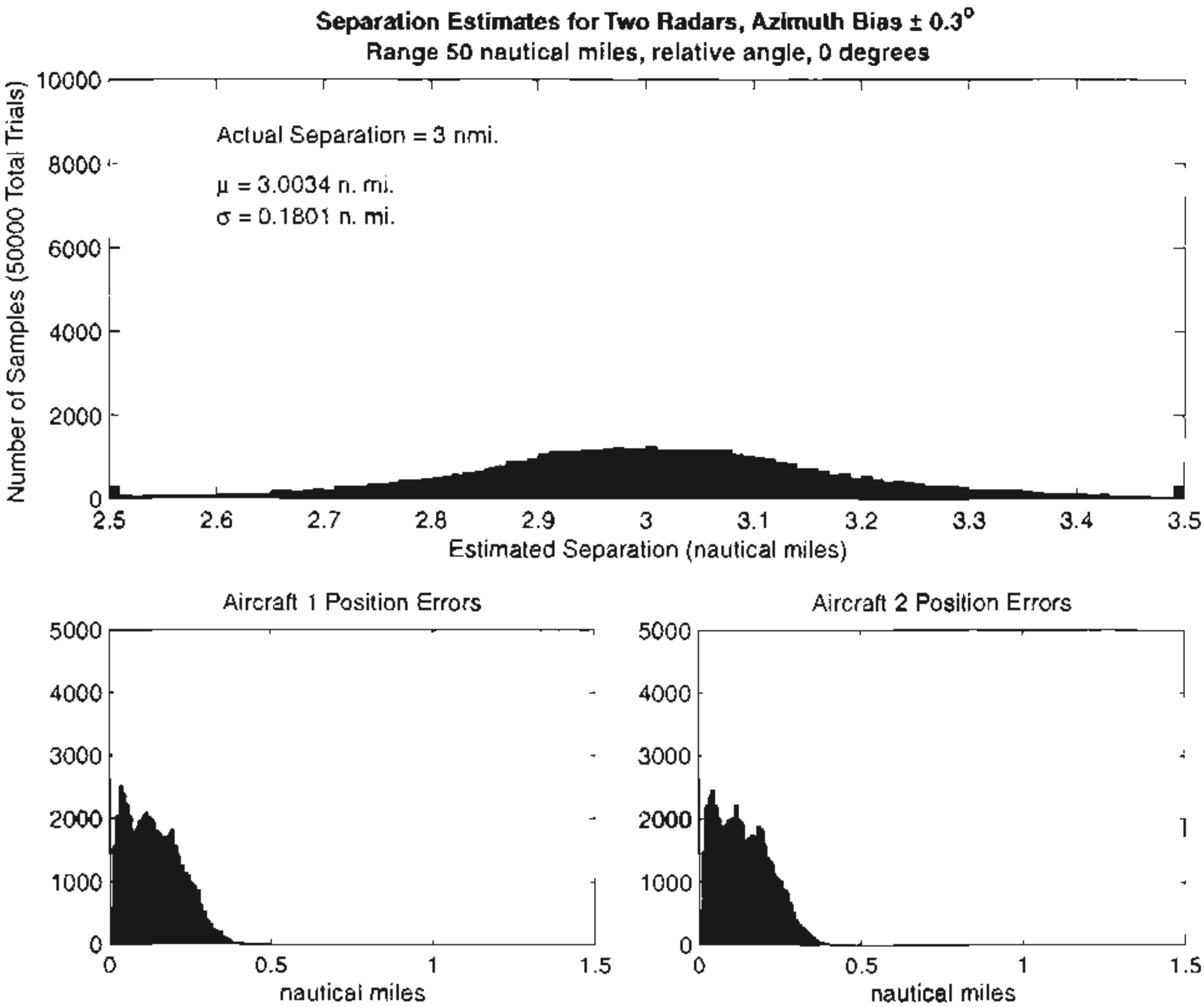

Figure 22. Average separation estimates for independent radars at 50 nmi Range, Θ = 0°

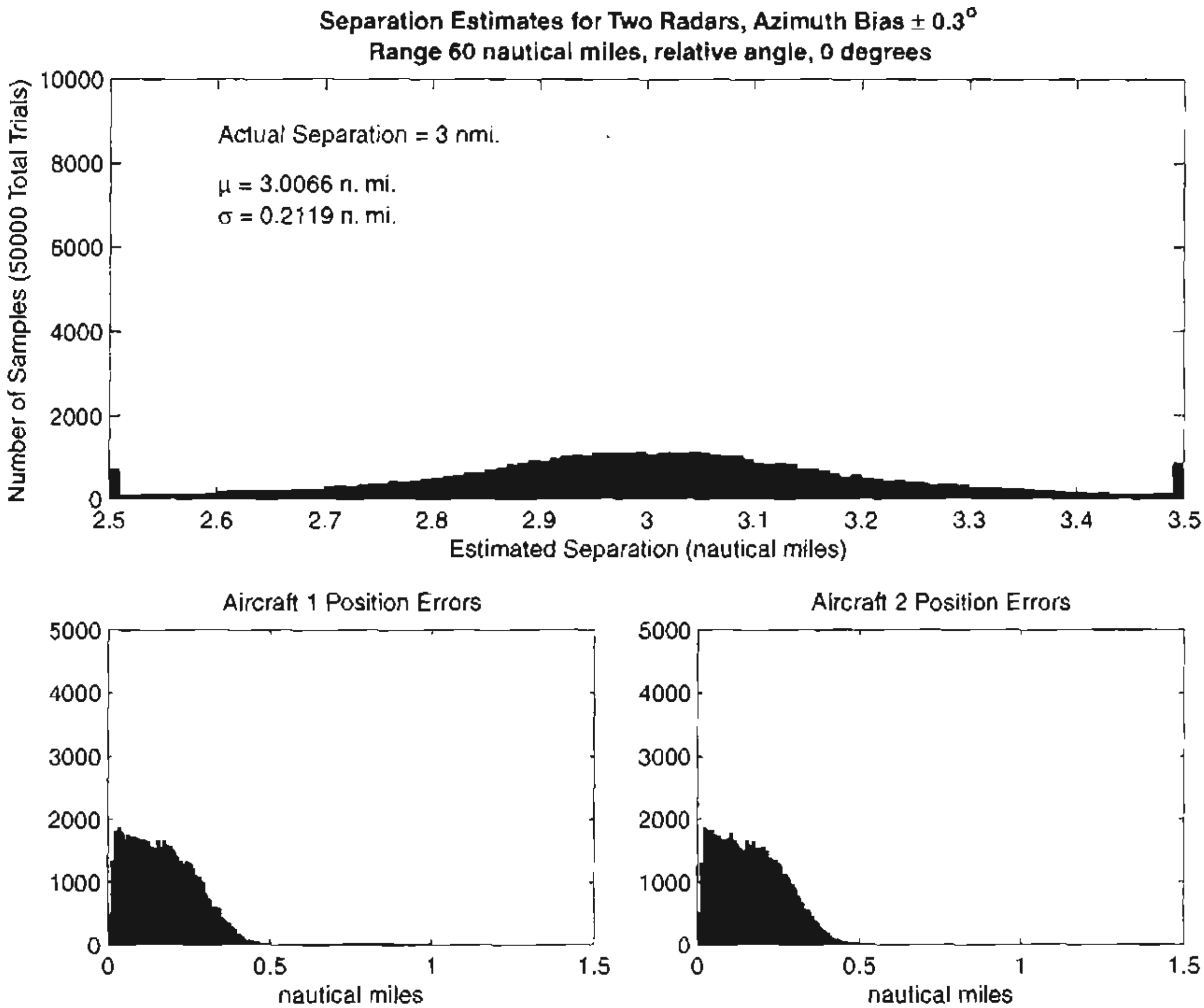

Figure 23. Average separation estimates for independent radars at 60 nmi Range. $\Theta = 0°$

5.2 TYPICAL ERRORS IN SEPARATION MEASUREMENTS

The purpose of this analysis is to gain an understanding of the characteristics of the errors in separation measurements when comparing separation estimates under surveillance with one radar to separation estimates when one radar is tracking one aircraft and another radar is tracking the second aircraft. In this simulation the two radar positions and the two aircraft positions and velocities are specified and held constant. The radar site location and bias errors are sampled once and held constant. The aircraft transponder bias is sampled once for each aircraft and held constant. The radar update rates are sampled once for each radar and the apparent separation error due to the clock bias, or difference in the timing of the position estimates, is computed and held constant. The jitter errors are sampled for each run in the simulation and the position reports are the discreet values allowed by the Common Digitizer formats. Obviously independent runs of this simulation will give different results depending on the biases sampled for that run. The position estimates of the individual aircraft will exhibit biases as will the separation estimates.

Five cases are included here representing typical cases of interest where independent radars may be tracking the two aircraft. In each case, the first figure is a plot showing the geometry of the radar and aircraft positions input for that case. The radars are represented by stars and the aircraft positions by circles. The red lines coming from the circles represent the aircraft heading and speed. The direction of the red line is the heading and the length of the red line represents the distance the aircraft will travel in one minute. The simulation produces four additional plots for each run corresponding to the cases where the first radar tracks both aircraft, the second radar tracks both aircraft, the two combinations of one radar tracking one aircraft and one radar tracking the other aircraft. Not all of the plots produced by the simulation for each of the three cases are presented. In two cases three plots are presented, two plots for one radar tracking both aircraft and a plot for independent radars tracking the two aircraft with the radar nearest the aircraft doing the tracking. In three cases the additional plot for independent radars tracking both aircraft is also presented. The labeling on the plots makes it clear which radar is tracking which aircraft.

Each figures consists of four subplots. The top left subplot shows the actual aircraft position as a circle and the position estimates made by the radar and reported in Common Digitizer format as cyan crosses. The bottom two plots are blowups showing the individual aircraft and position estimates. The red line for one of the aircraft indicates the distance the aircraft traveled after the first aircraft was "hit" before it was "hit." The convention adopted here is that the true positions of the aircraft are the positions specified as input and that the difference between the positions is the true separation. Because the aircraft positions are not sampled at the same time, the red line indicates the point at which the second aircraft would have moved before it's position was estimated by a radar "hit." The first aircraft "hit" will have a red dot indicating no motion. The length of the red line will depend on the relative geometry and speed of the aircraft. The difference in the timing of the "hits" depends on the once sampled update rates and starting angles of the radars.

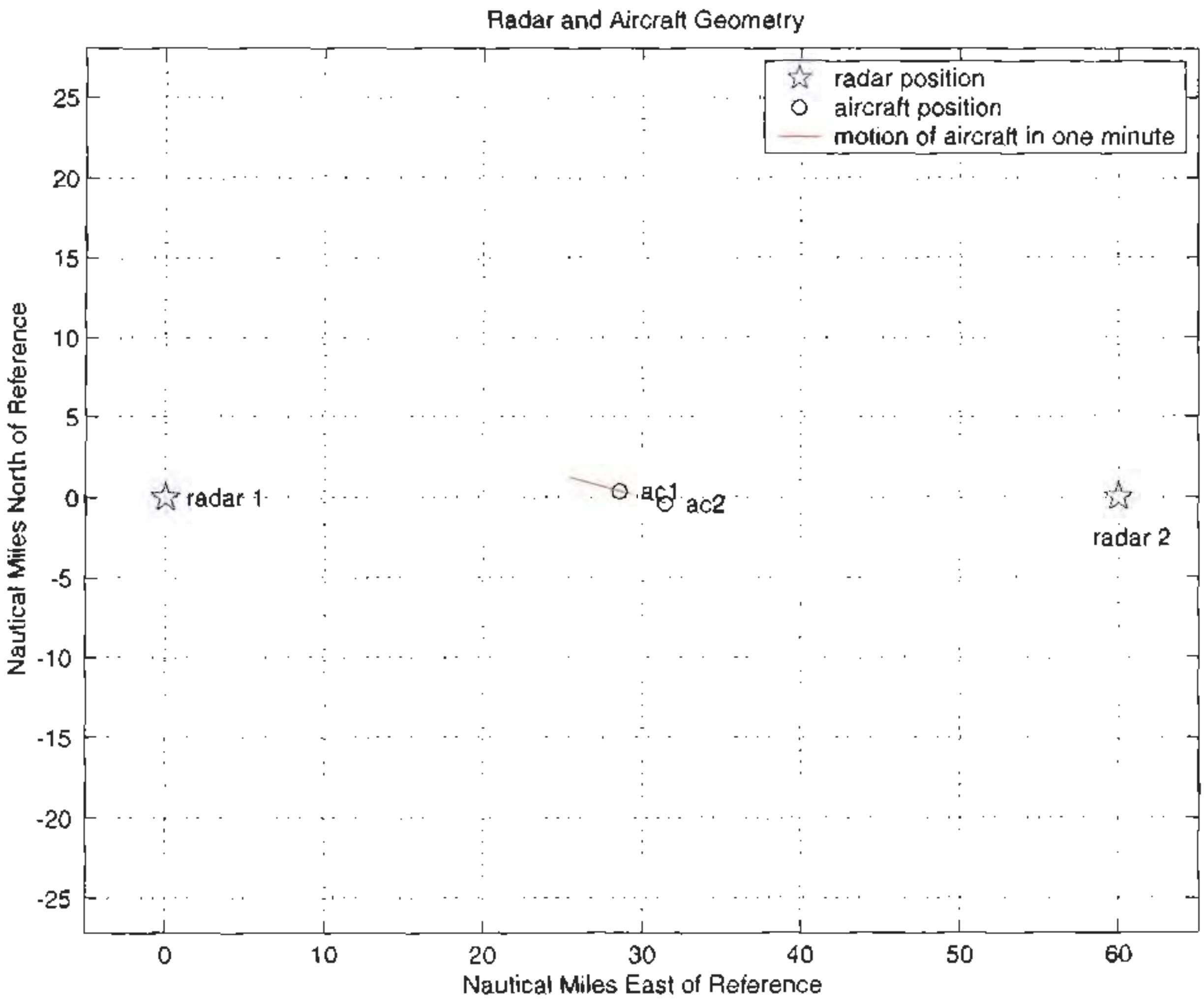

Figure 24. Radar and aircraft geometry for Case 1

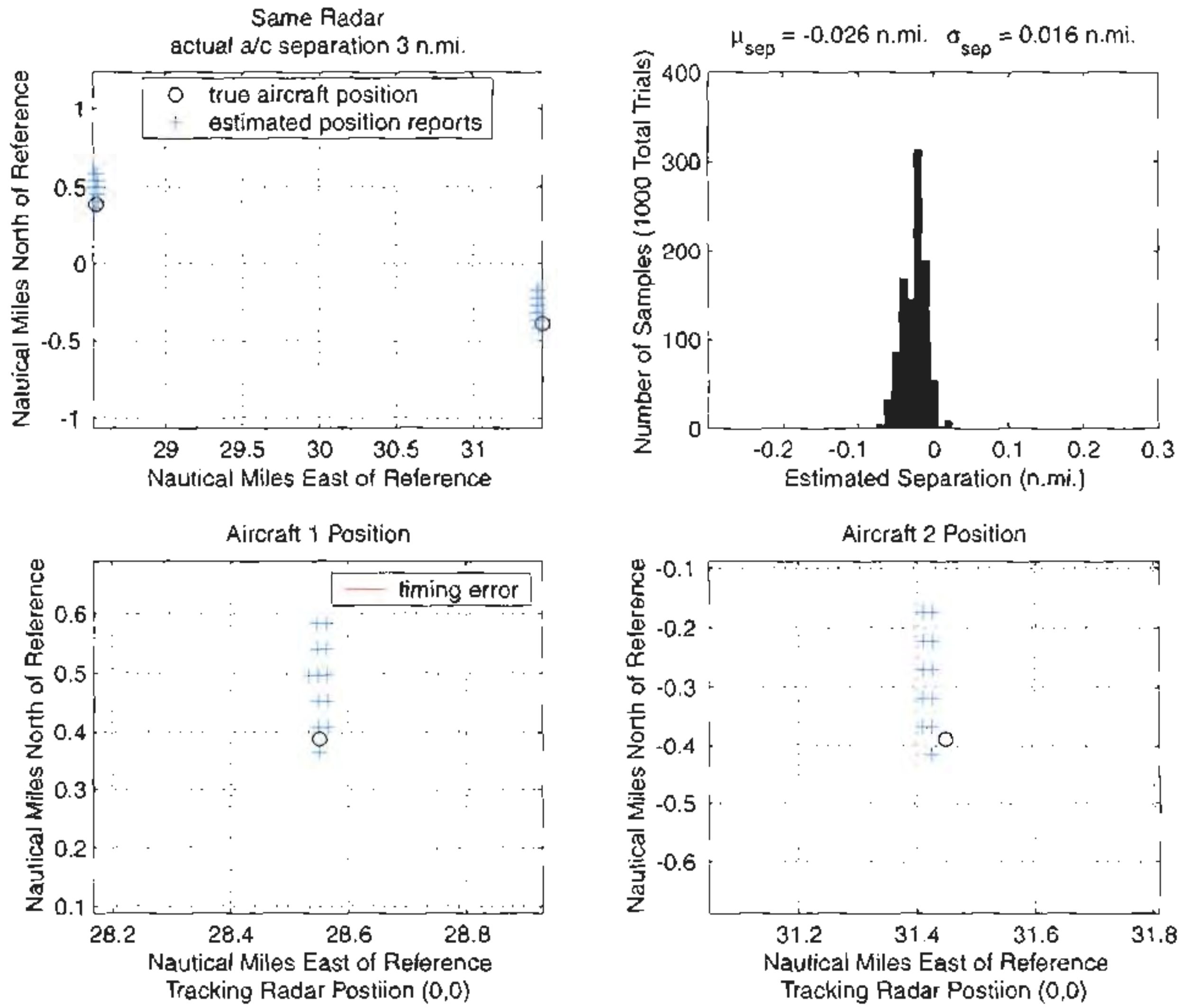

Figure 25. Typical separation estimates for radar 1, Case 1

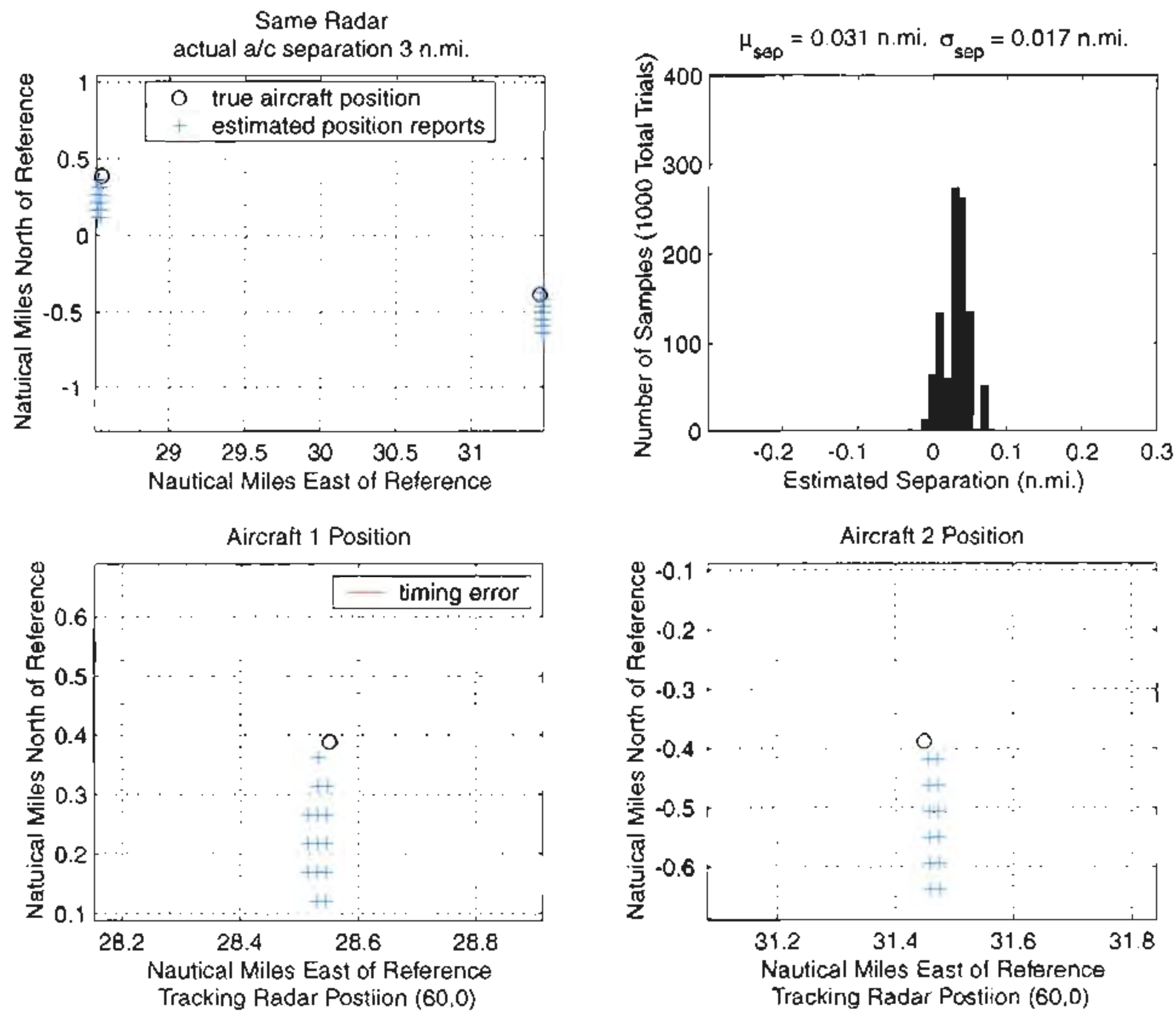

Figure 26. Typical separation estimates for radar 2, Case 1

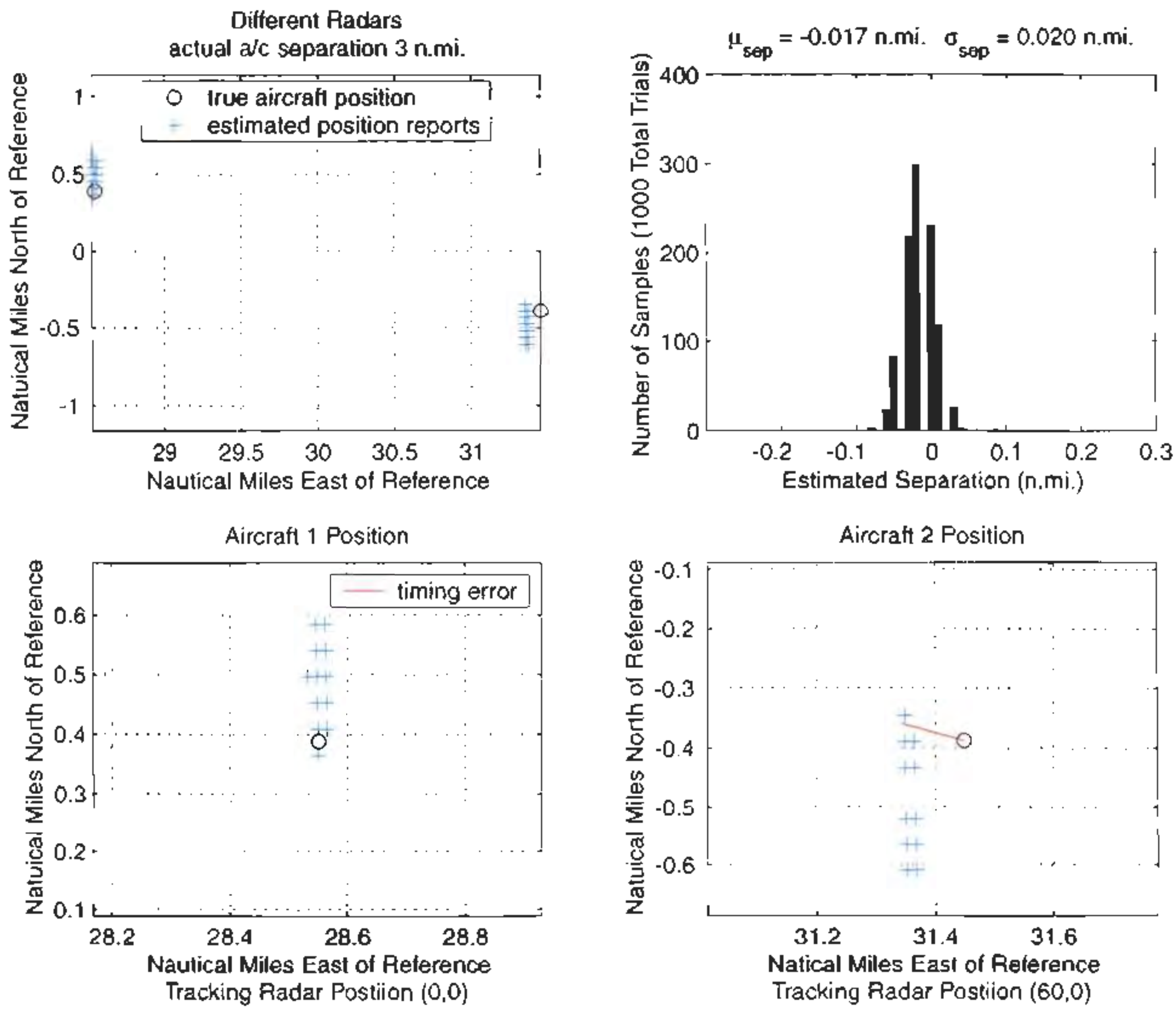

Figure 27. Typical separation estimates independent radars, Case 1

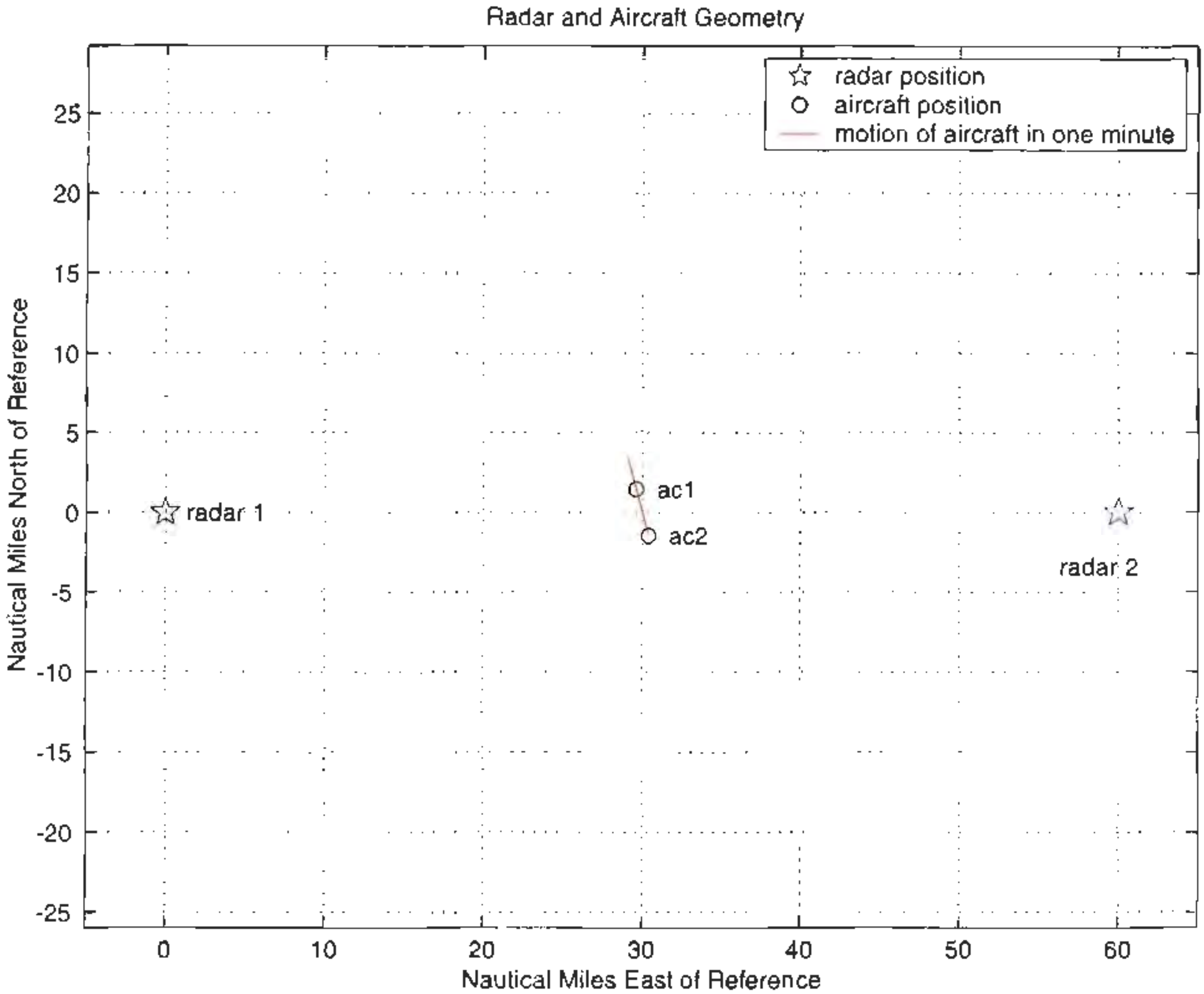

Figure 28. Radar and aircraft geometry for Case 2

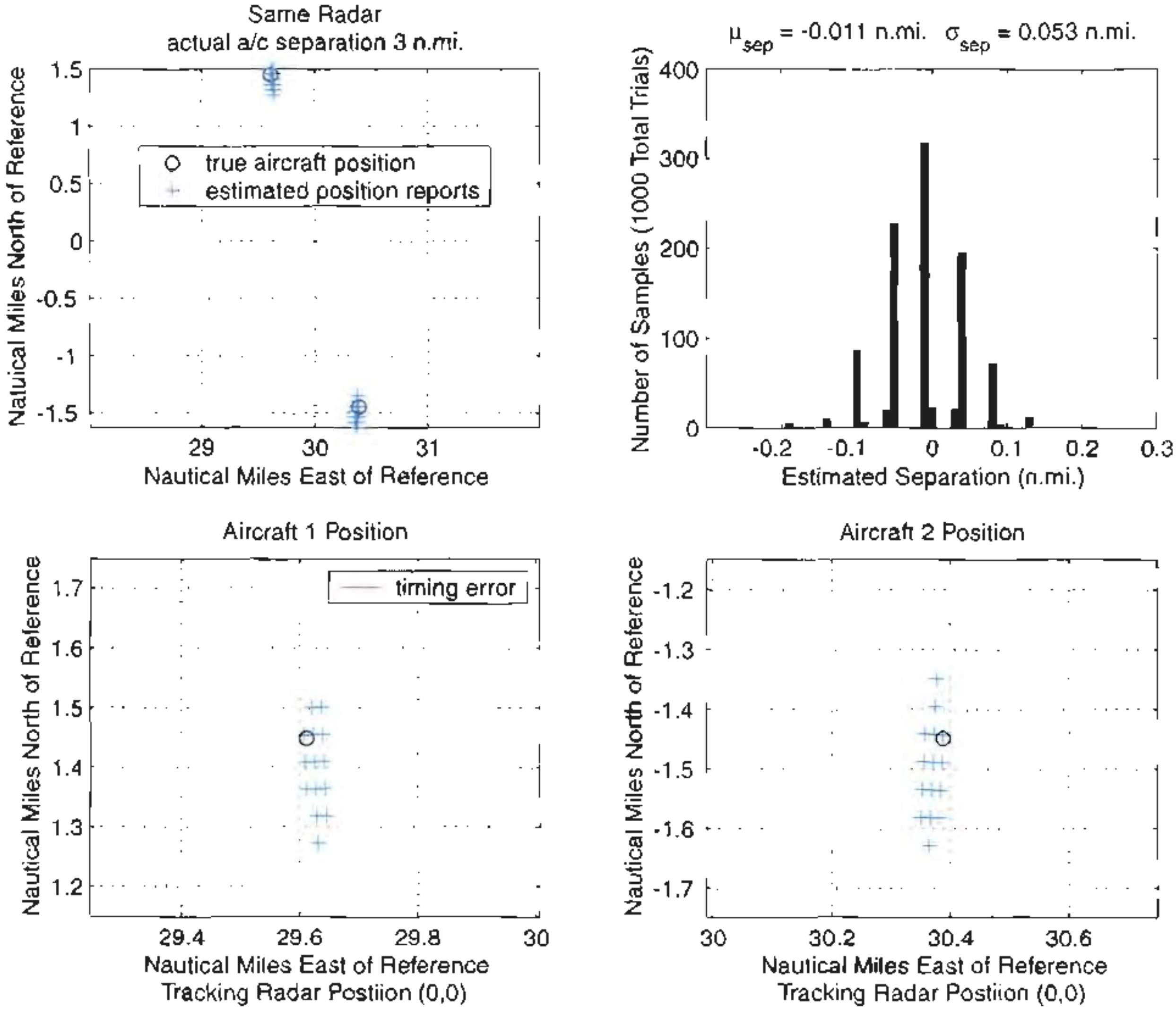

Figure 29. Typical separation estimates for radar 1, Case 2

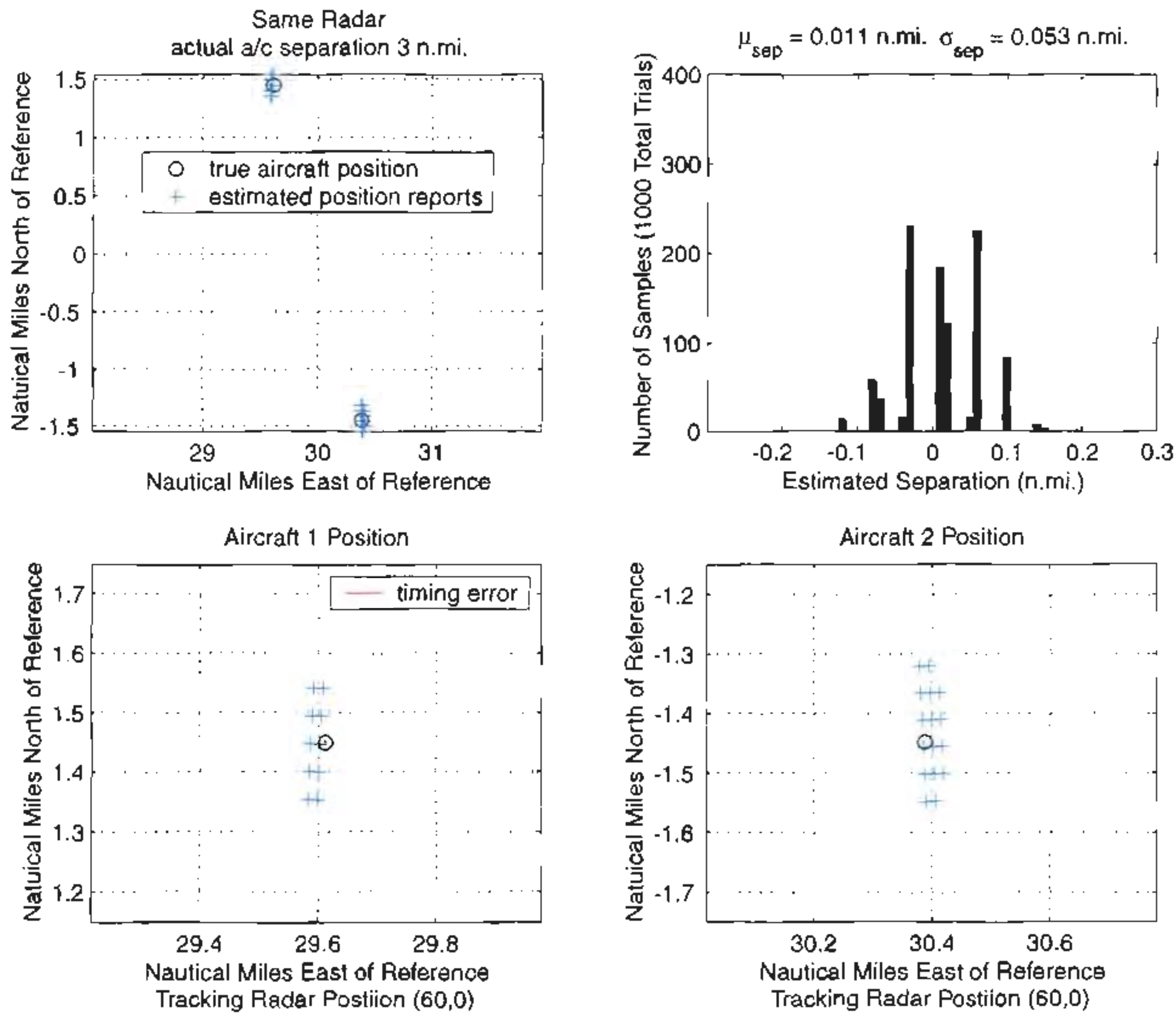

Figure 30. Typical separation estimates for radar 2, Case 2

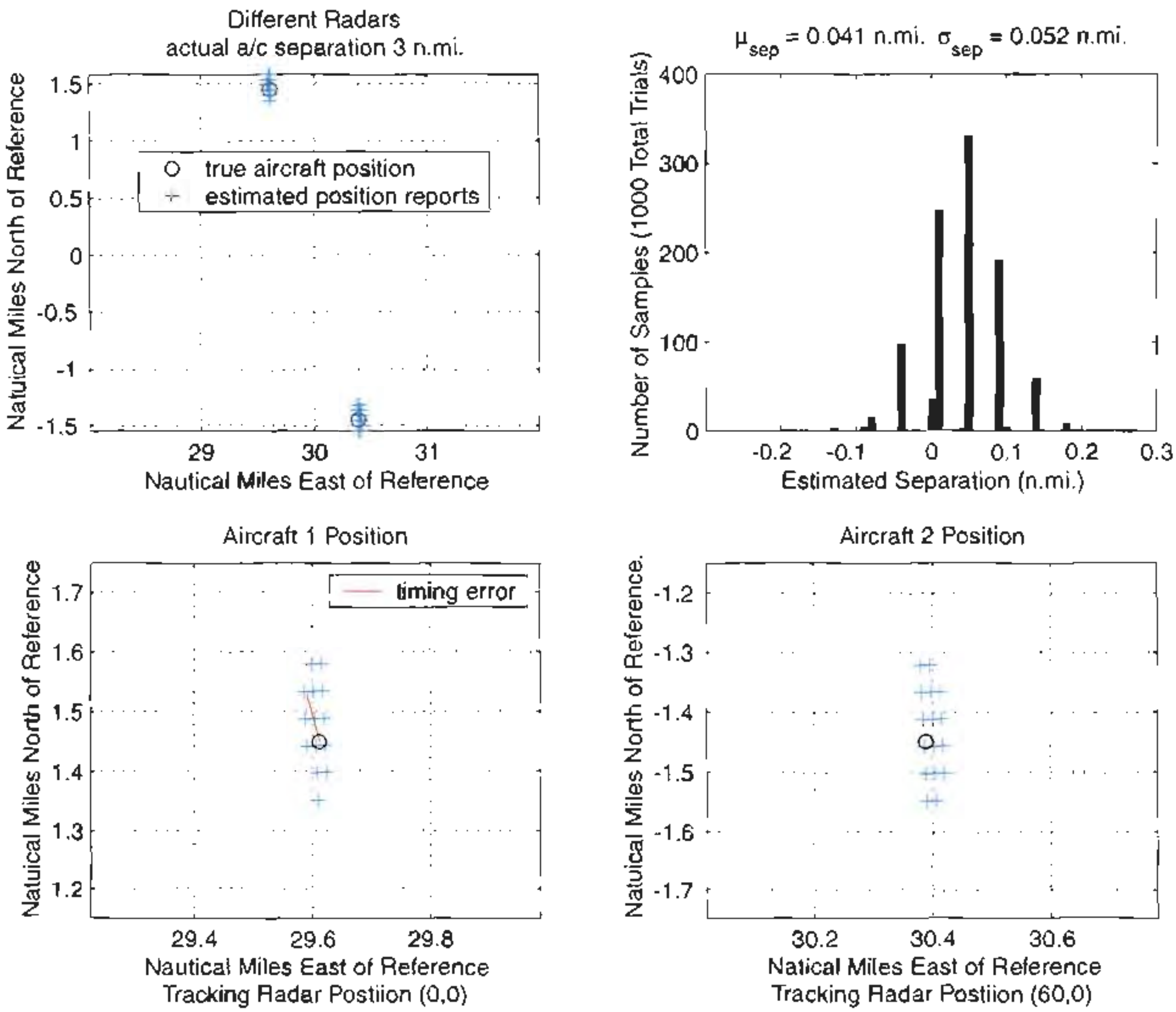

Figure 31. Typical separation estimates independent radars, Case 2

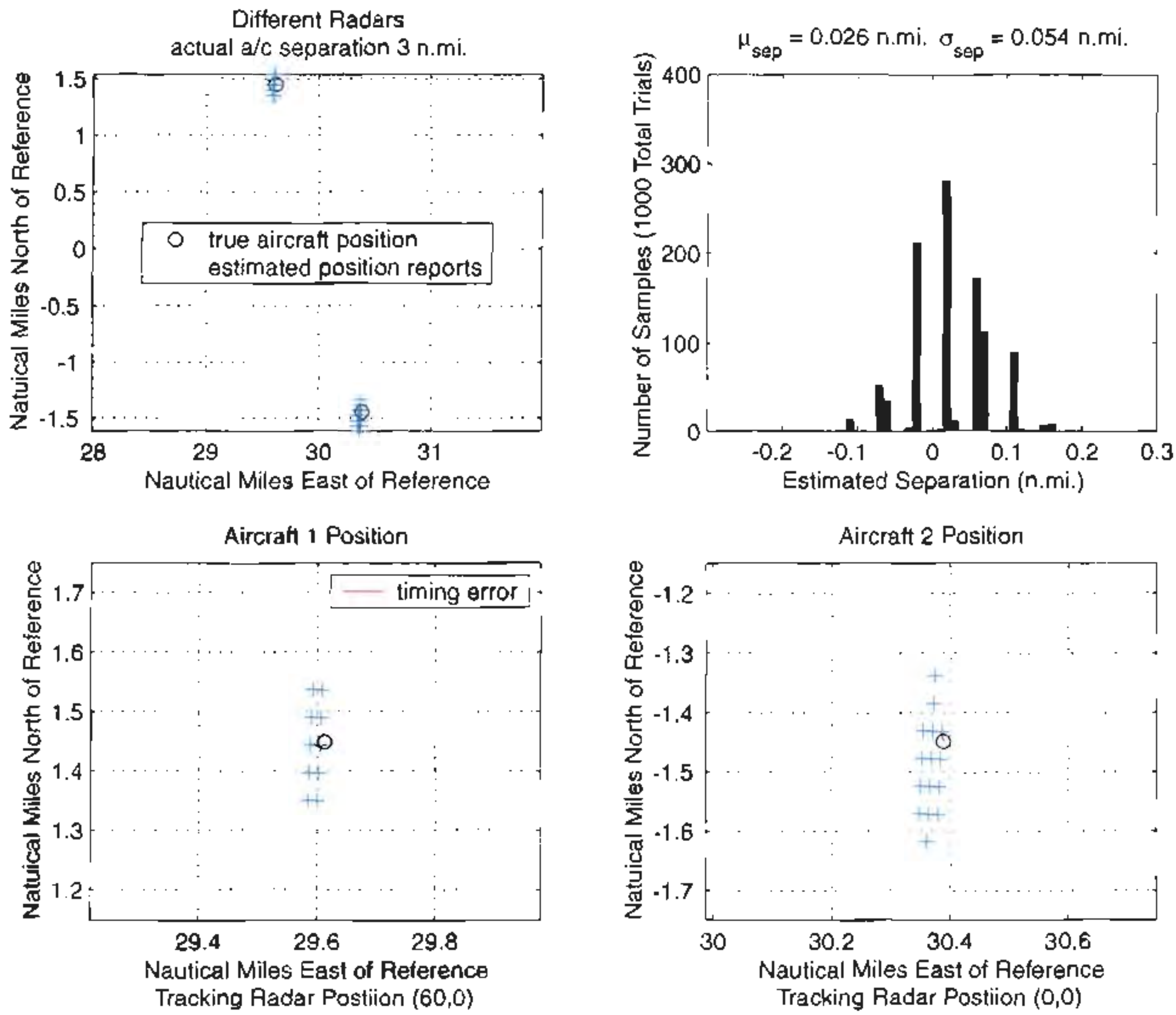

Figure 32. Typical separation estimates additional case of independent radars, Case 2

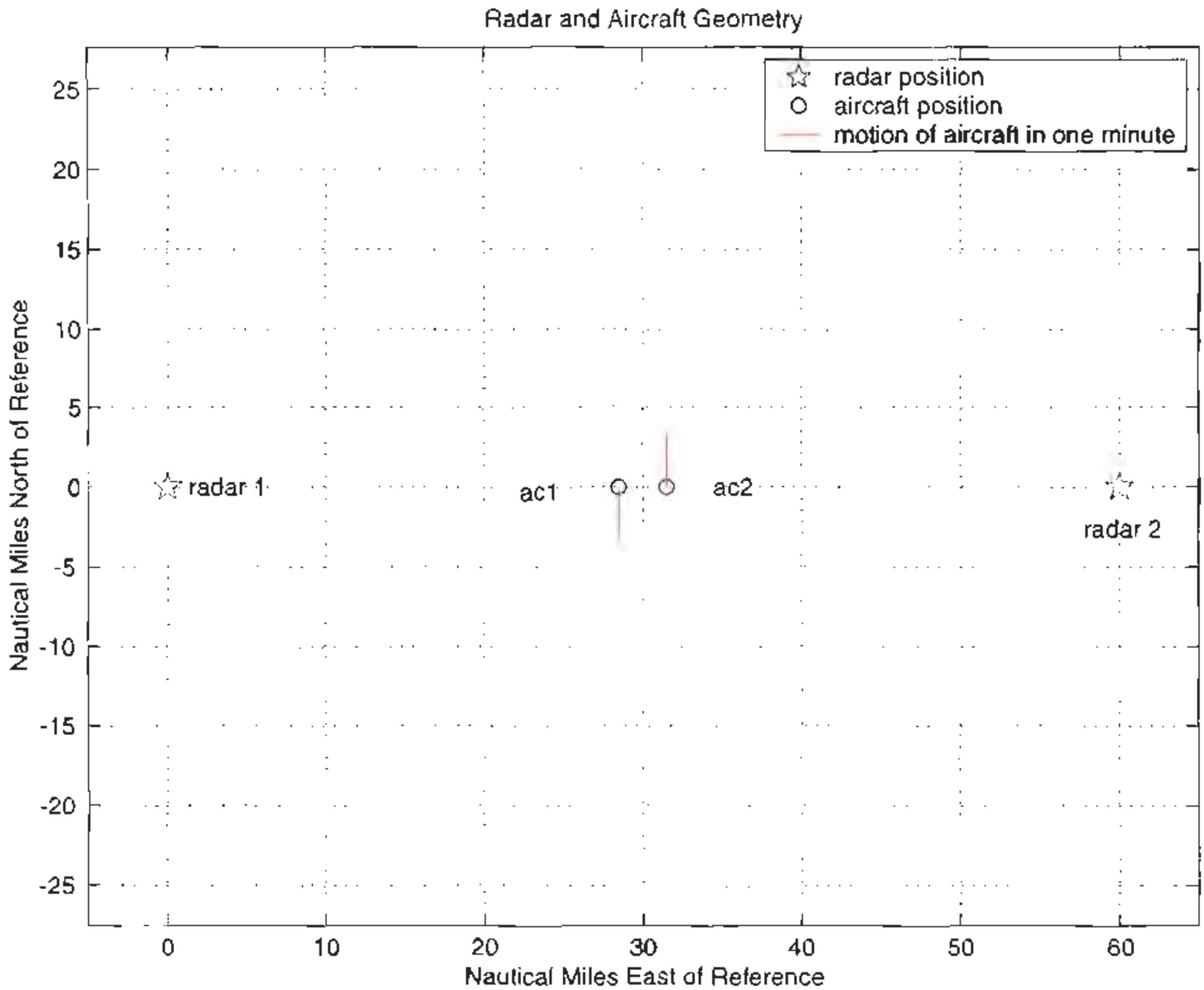

Figure 33. Radar and aircraft geometry for Case 3

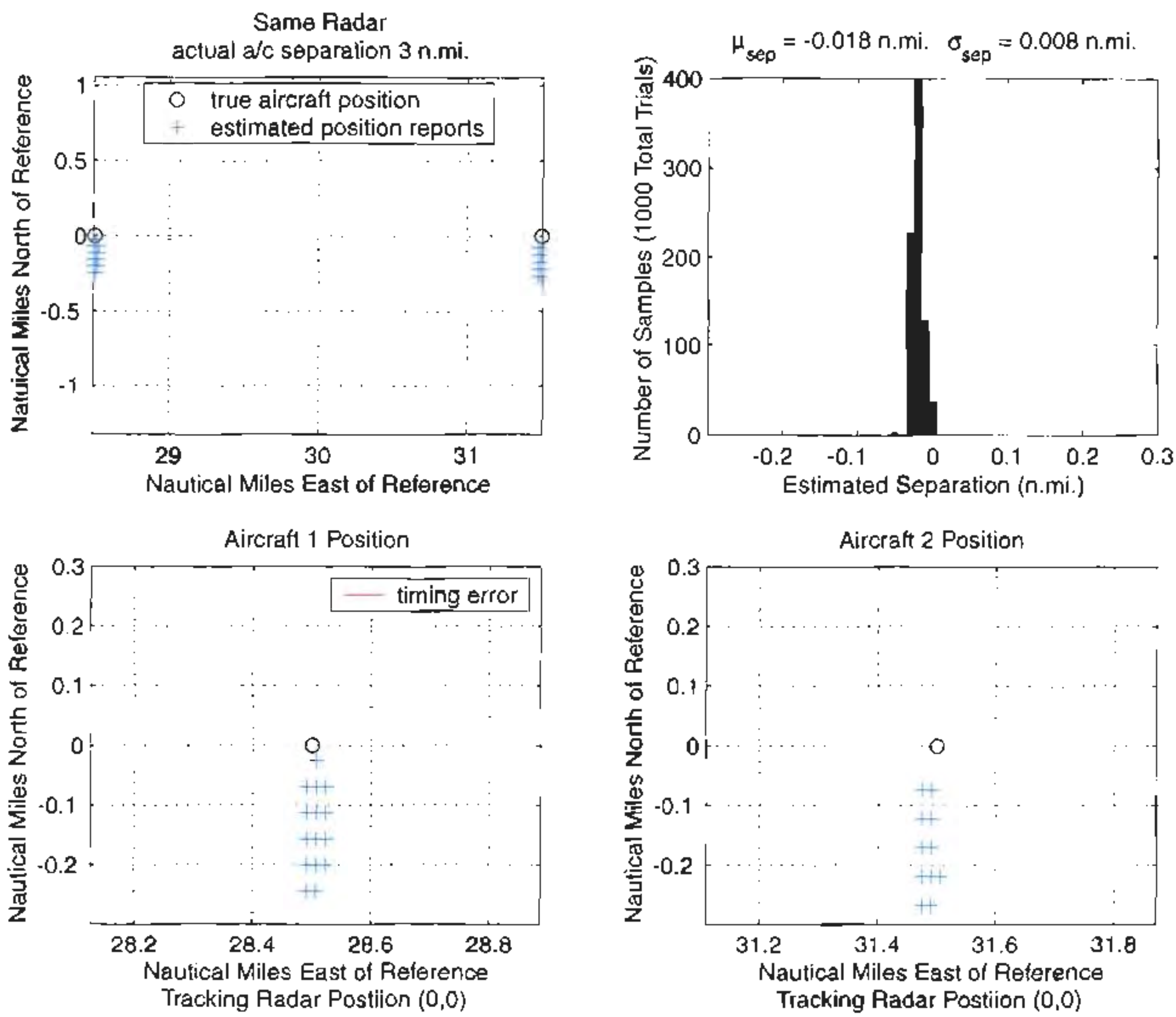

Figure 34. Typical separation estimates for radar 1, Cose 3

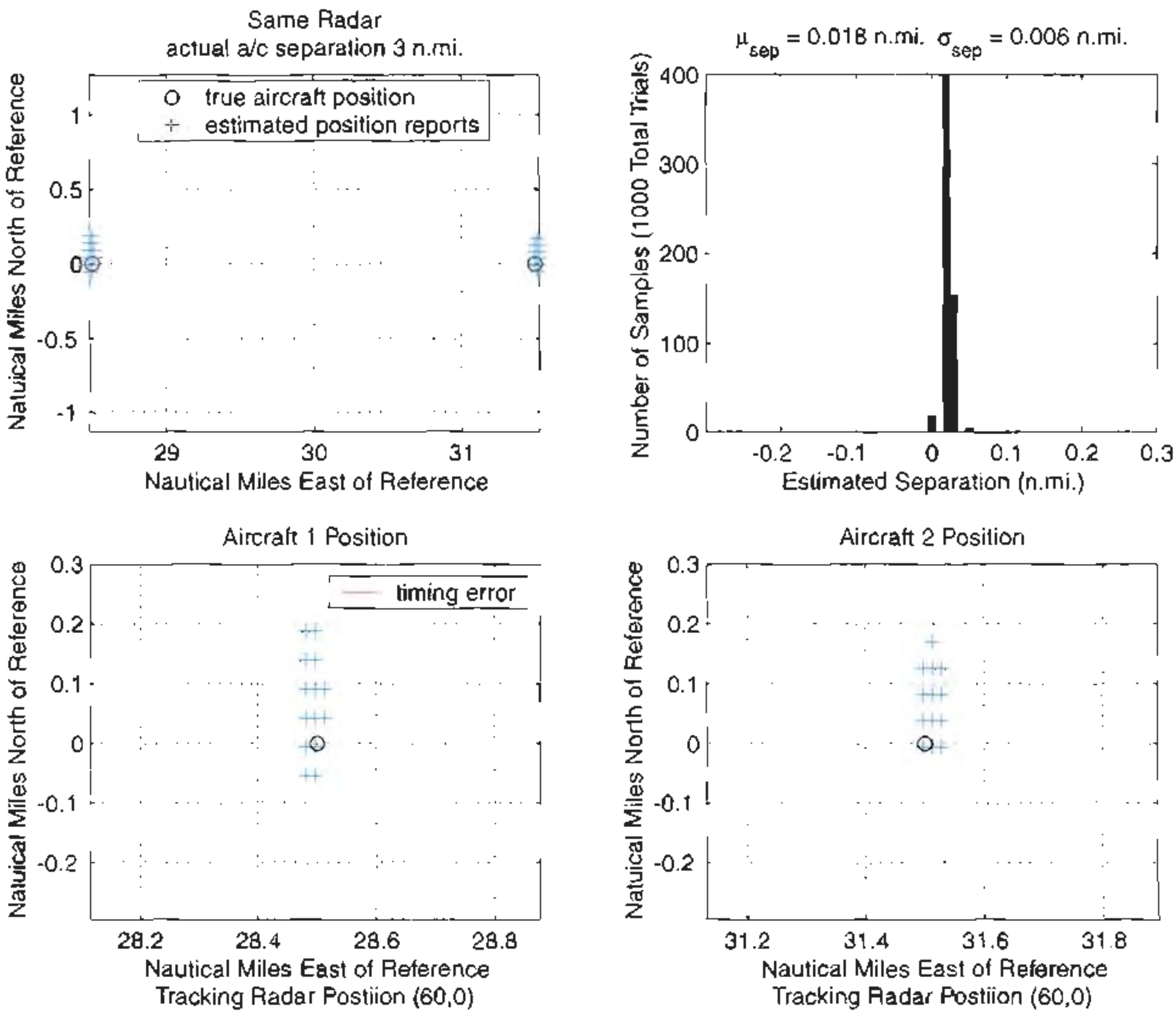

Figure 35. Typical separation estimates for radar 2, Case 3

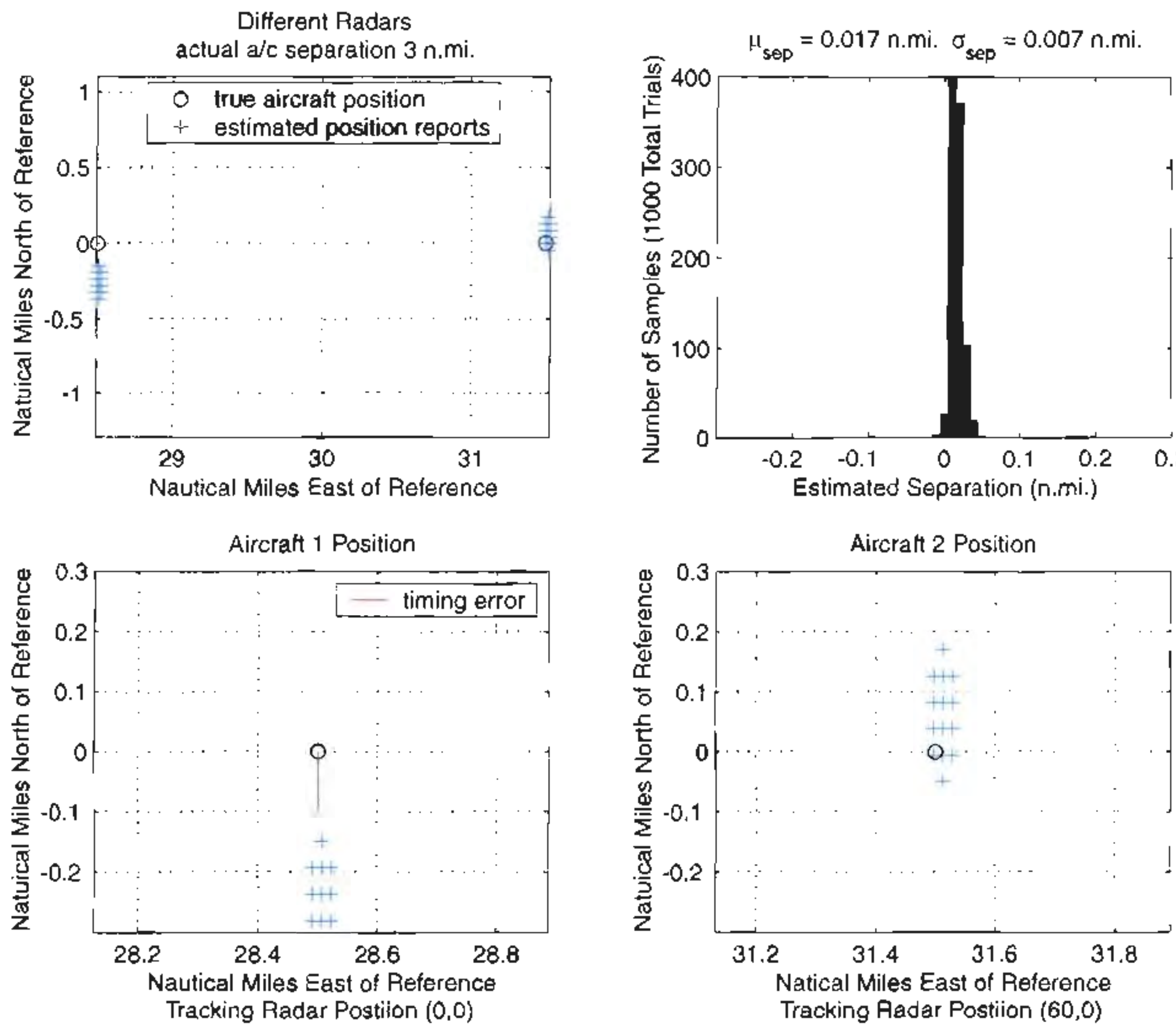

Figure 36. Typical separation estimates independent radars, Case 3

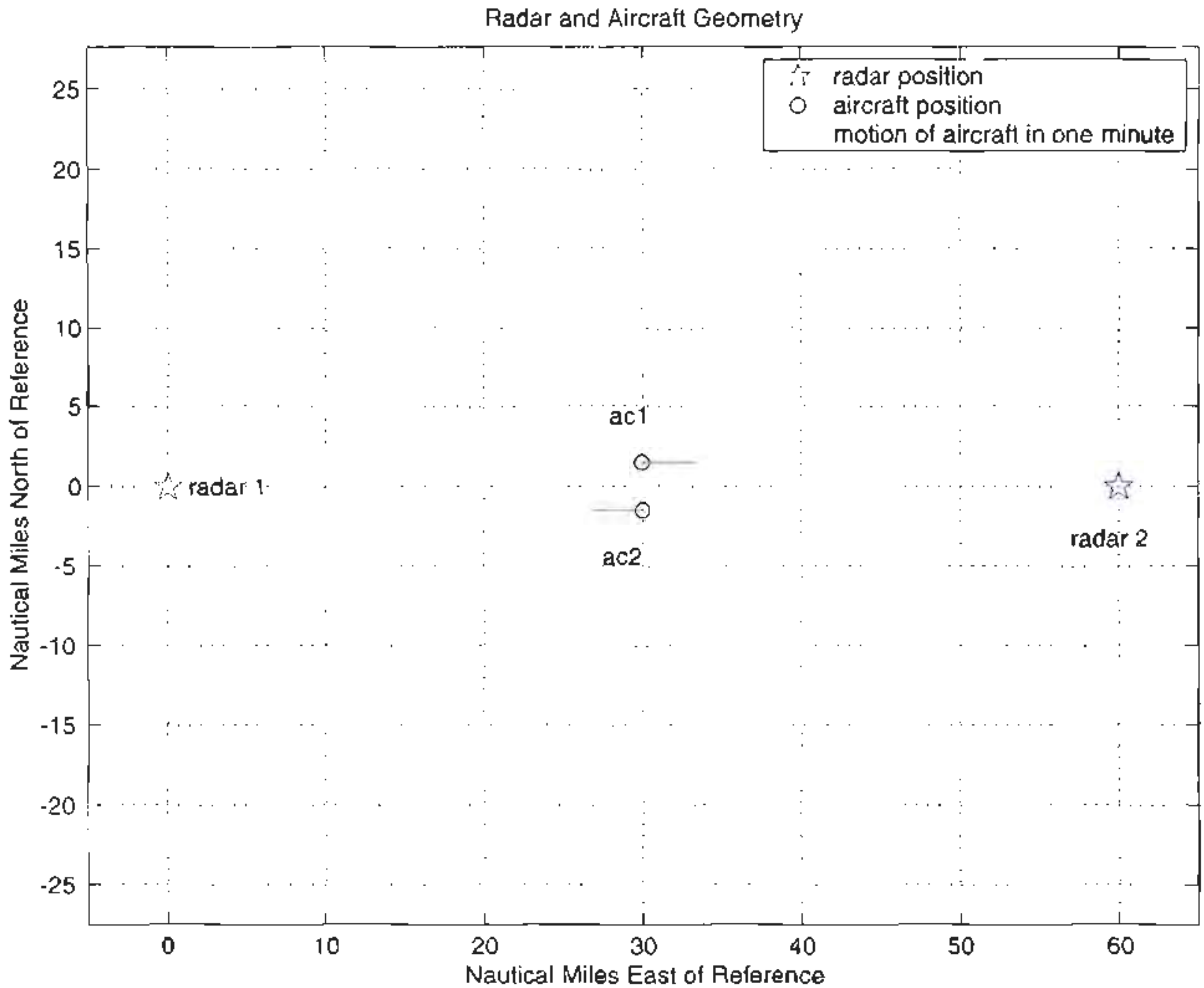

Figure 37. Radar and aircraft geometry for Case 4

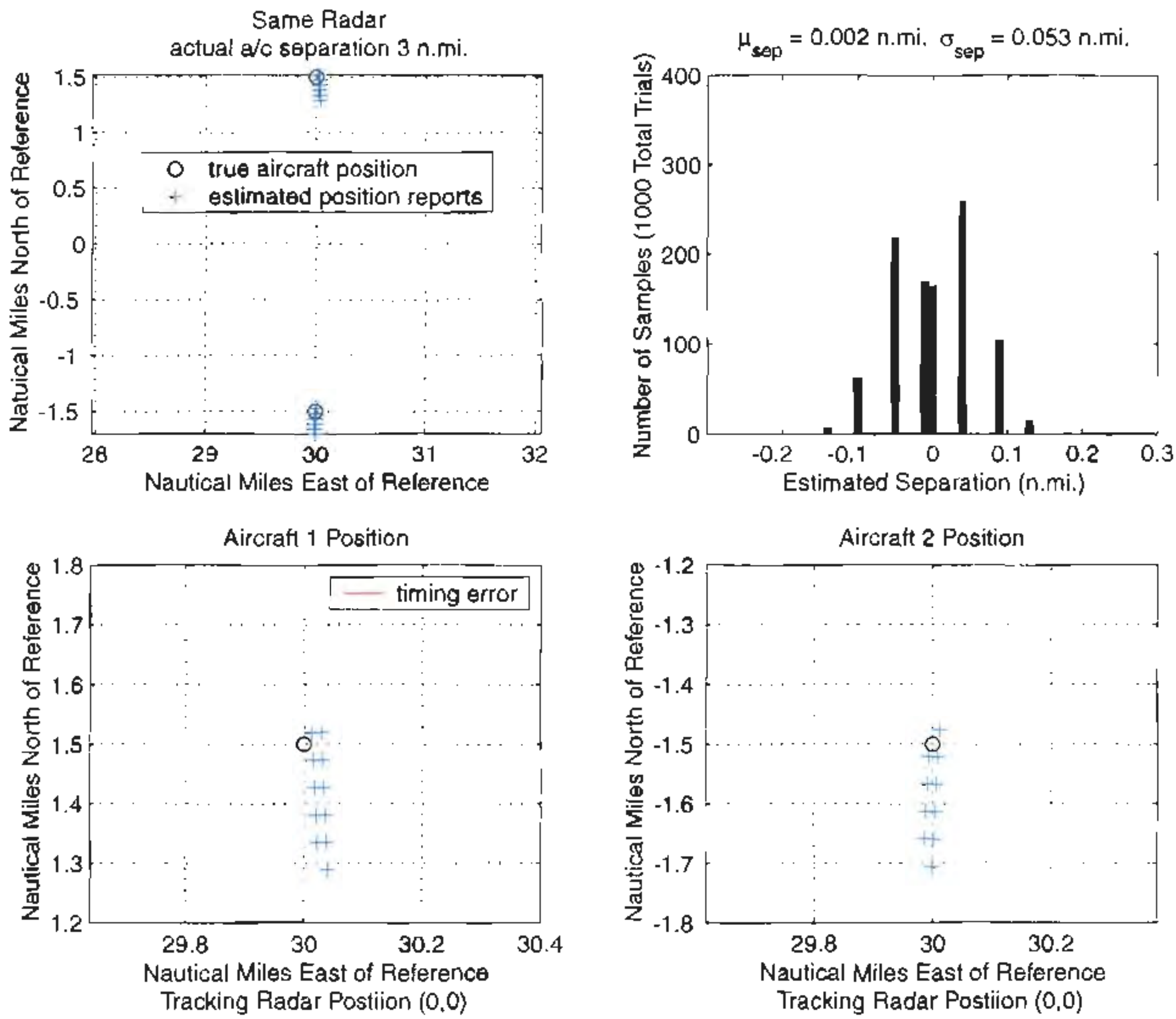

Figure 38. Typical separation estimates for radar 1, Case 4

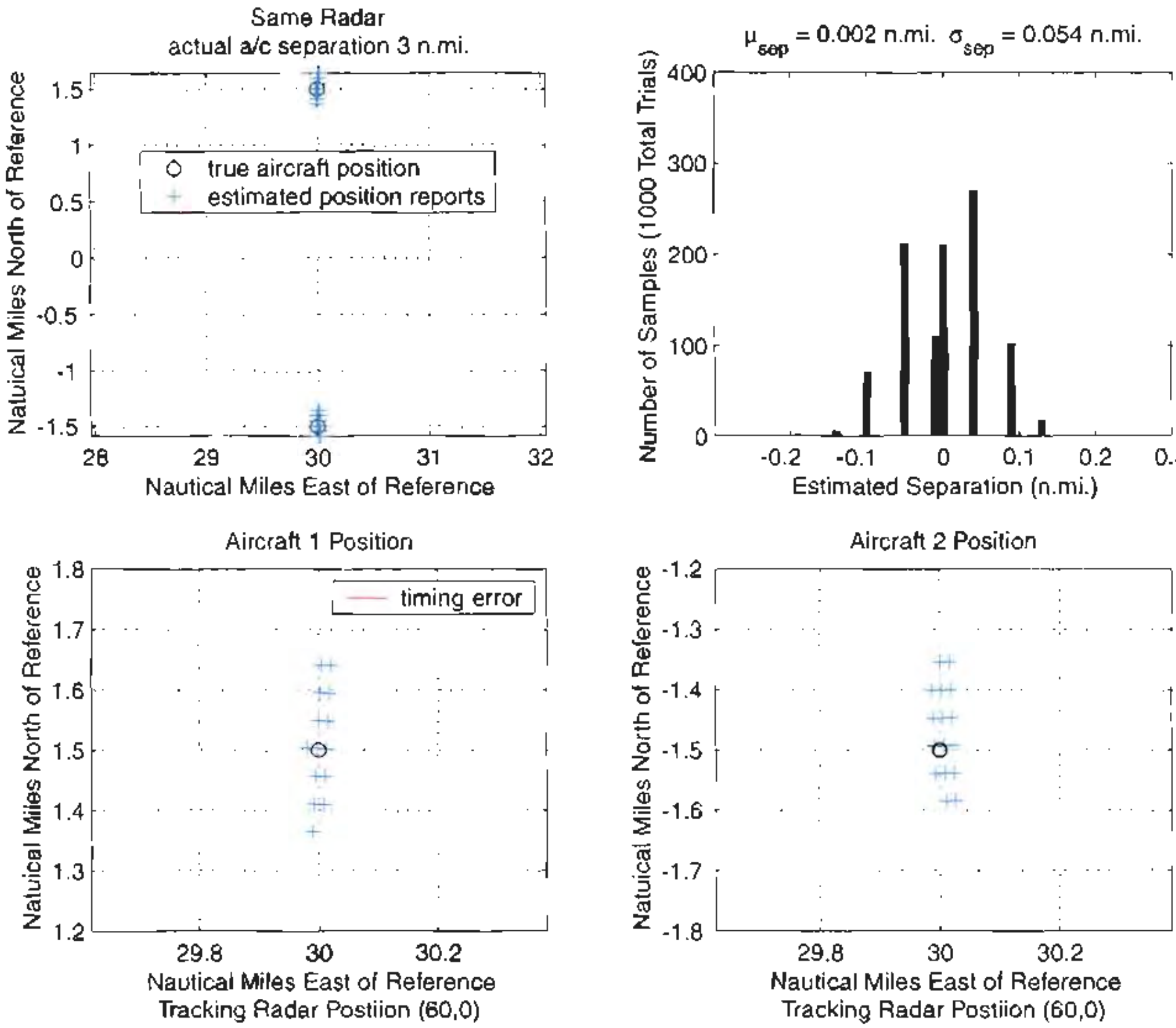

Figure 39. Typical separation estimates for radar 2, Case 4

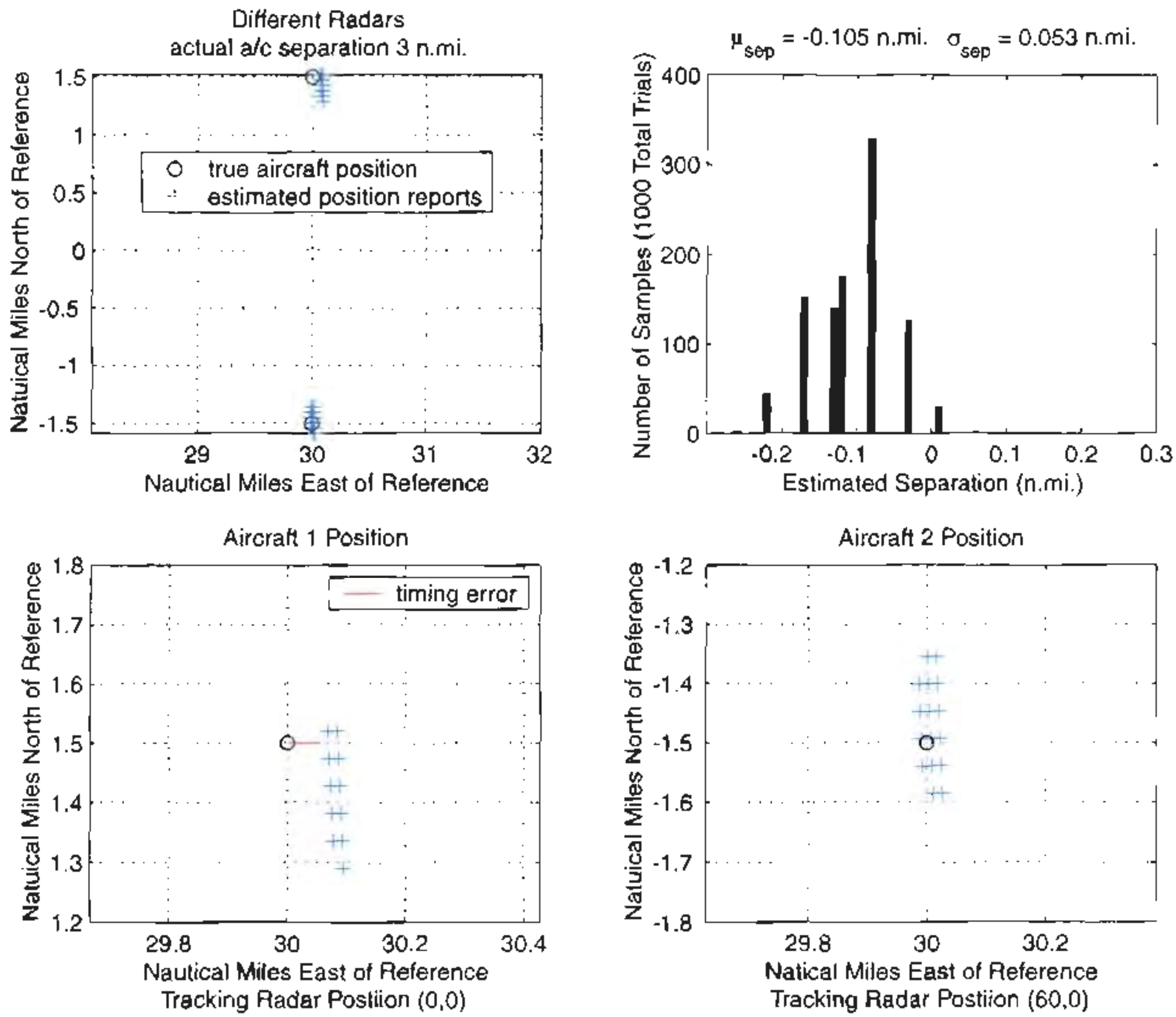

Figure 40. Typical separation estimates independent radars, Case 4

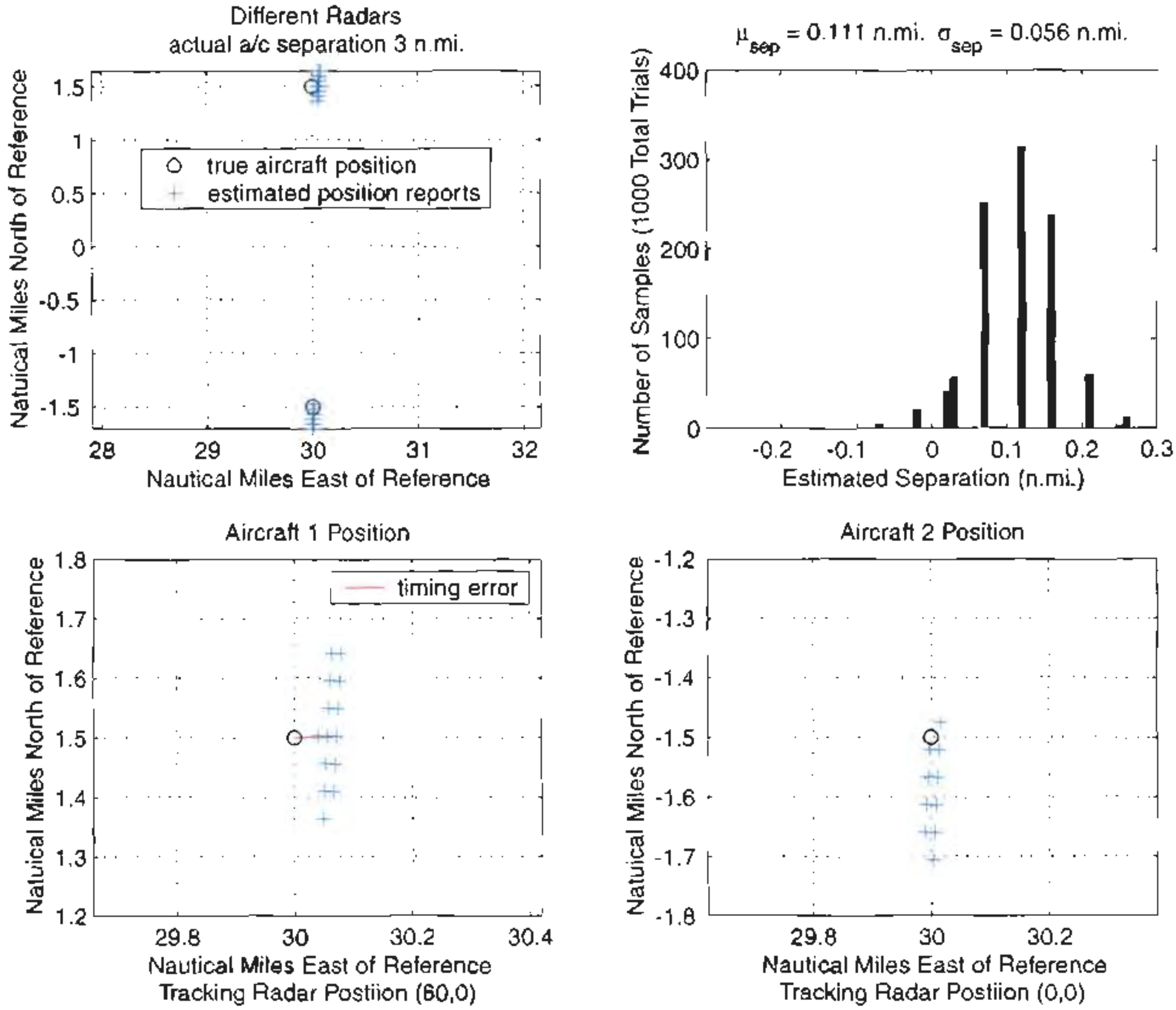

Figure 41. Typical separation estimates additional case of independent radars, Case 4

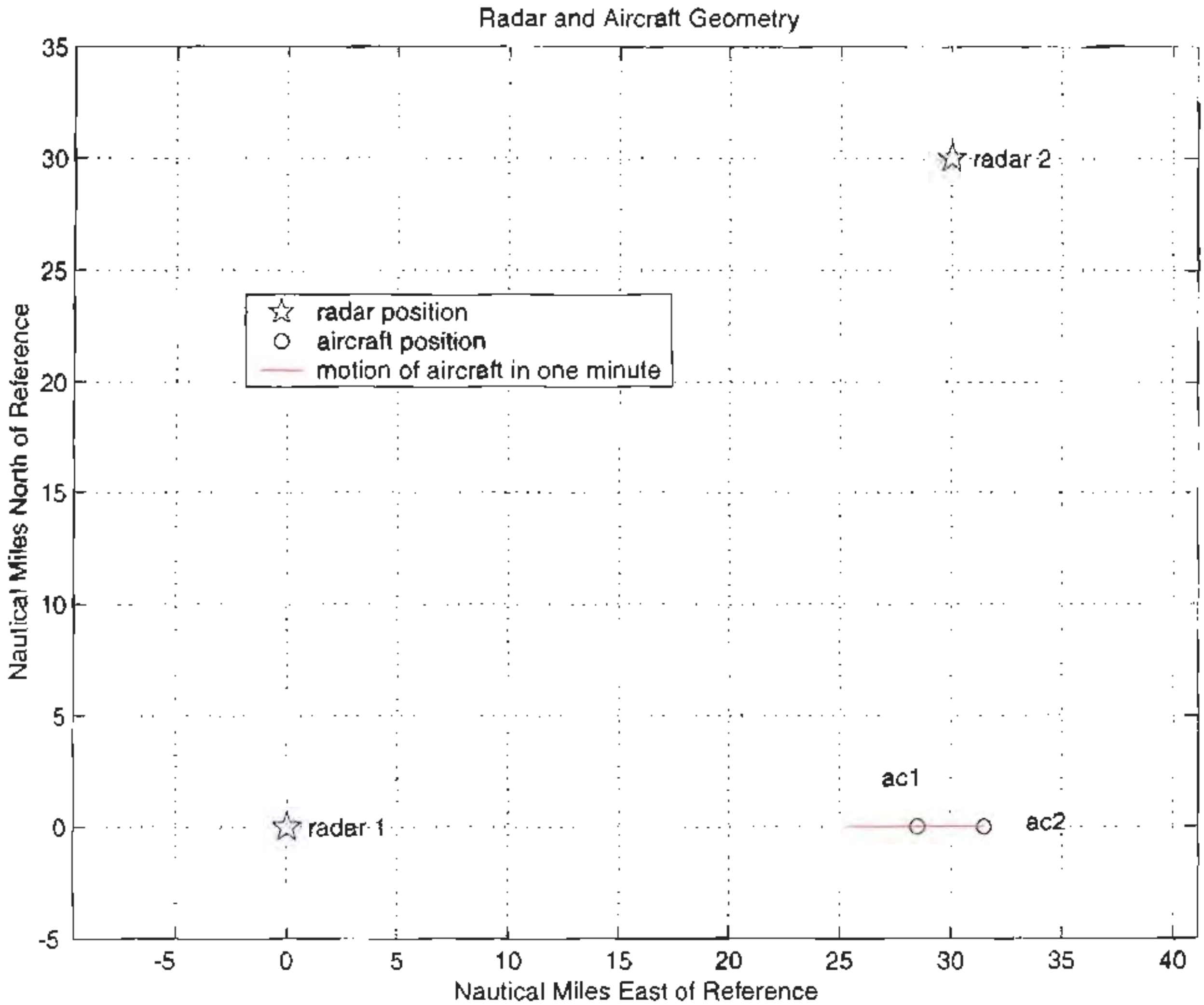

Figure 42. Radar and aircraft geometry for Case 5

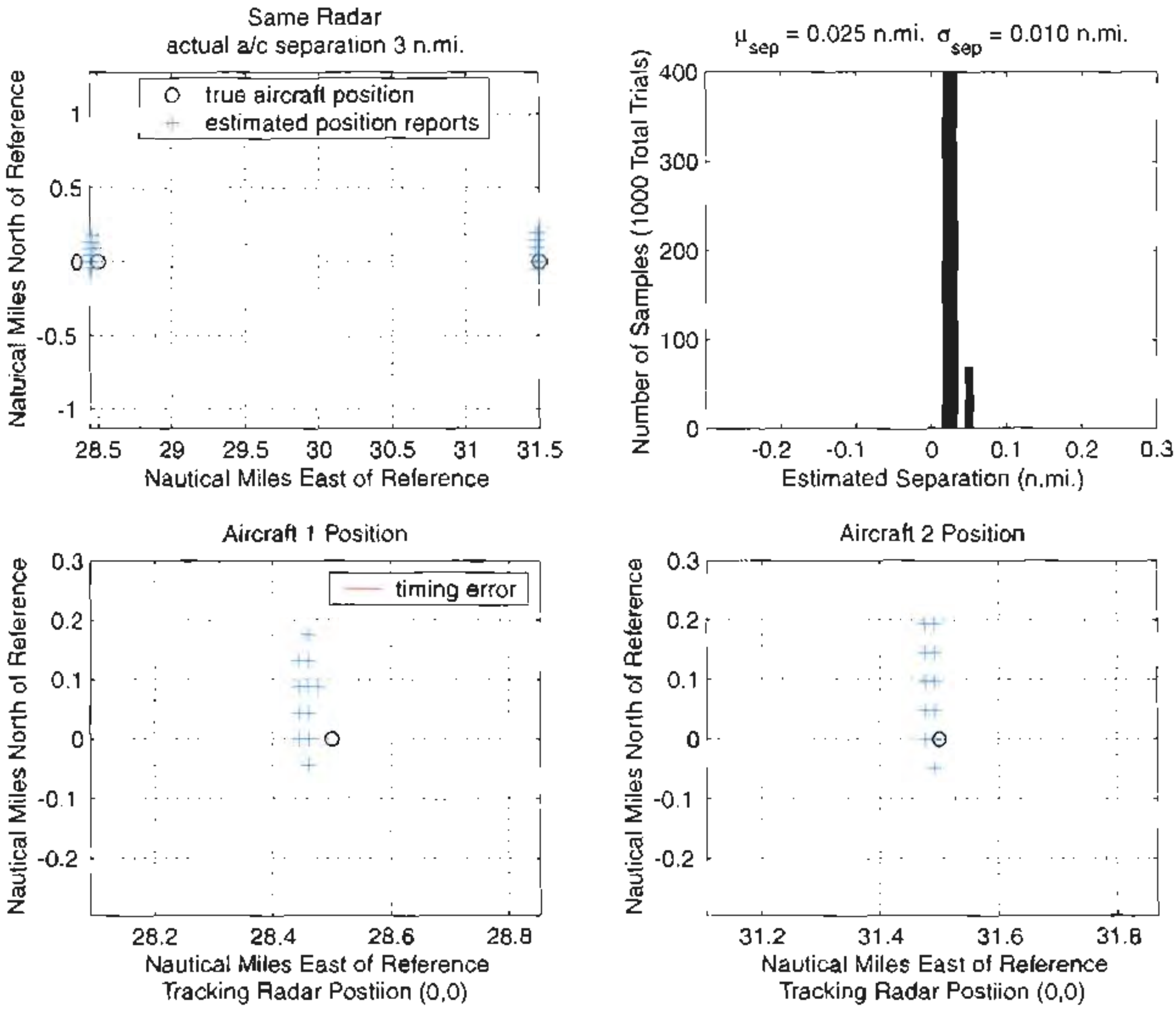

Figure 43. Typical separation estimates for radar 1, Case 5

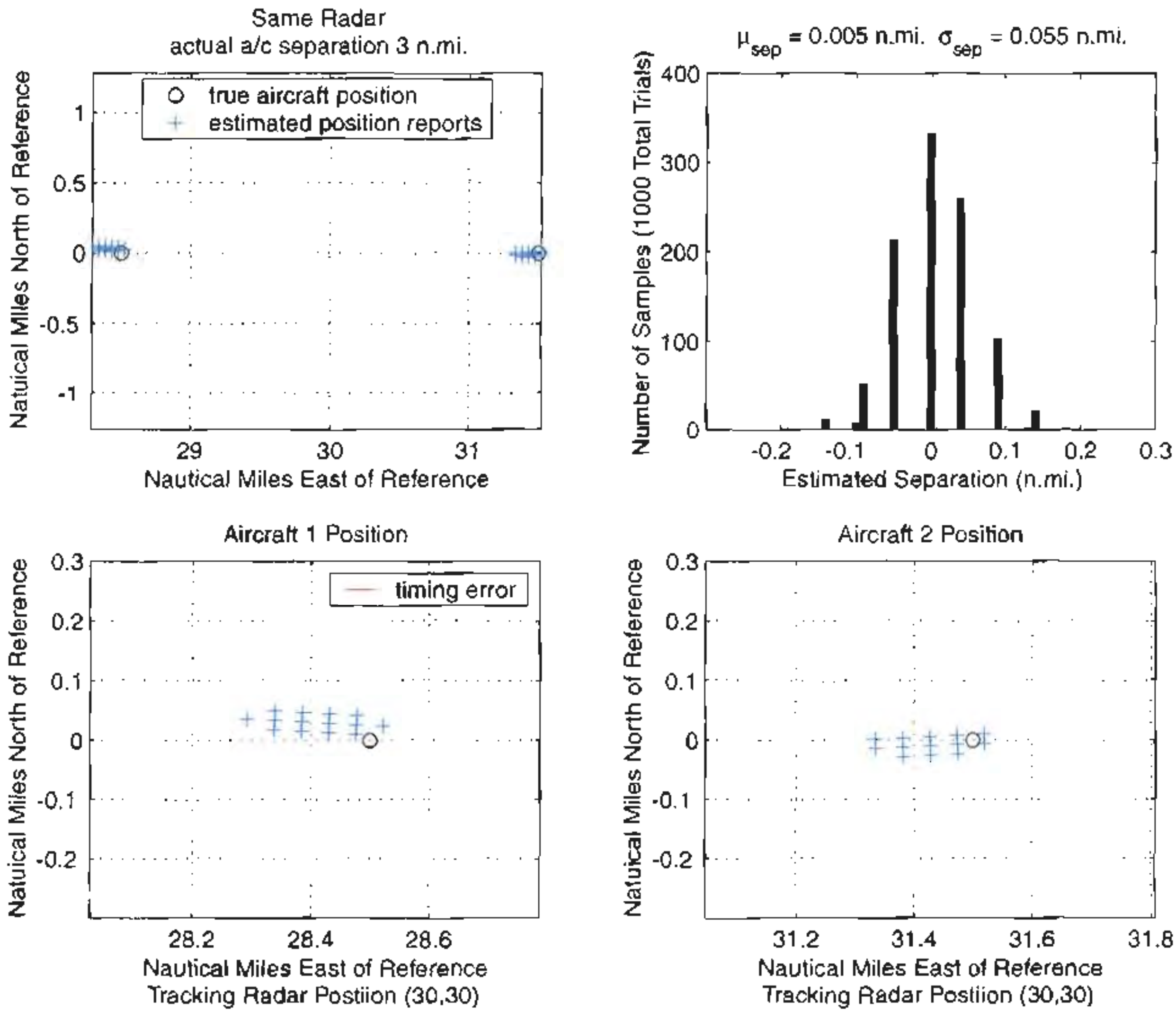

Figure 44. Typical separation estimates for radar 2, Case 5

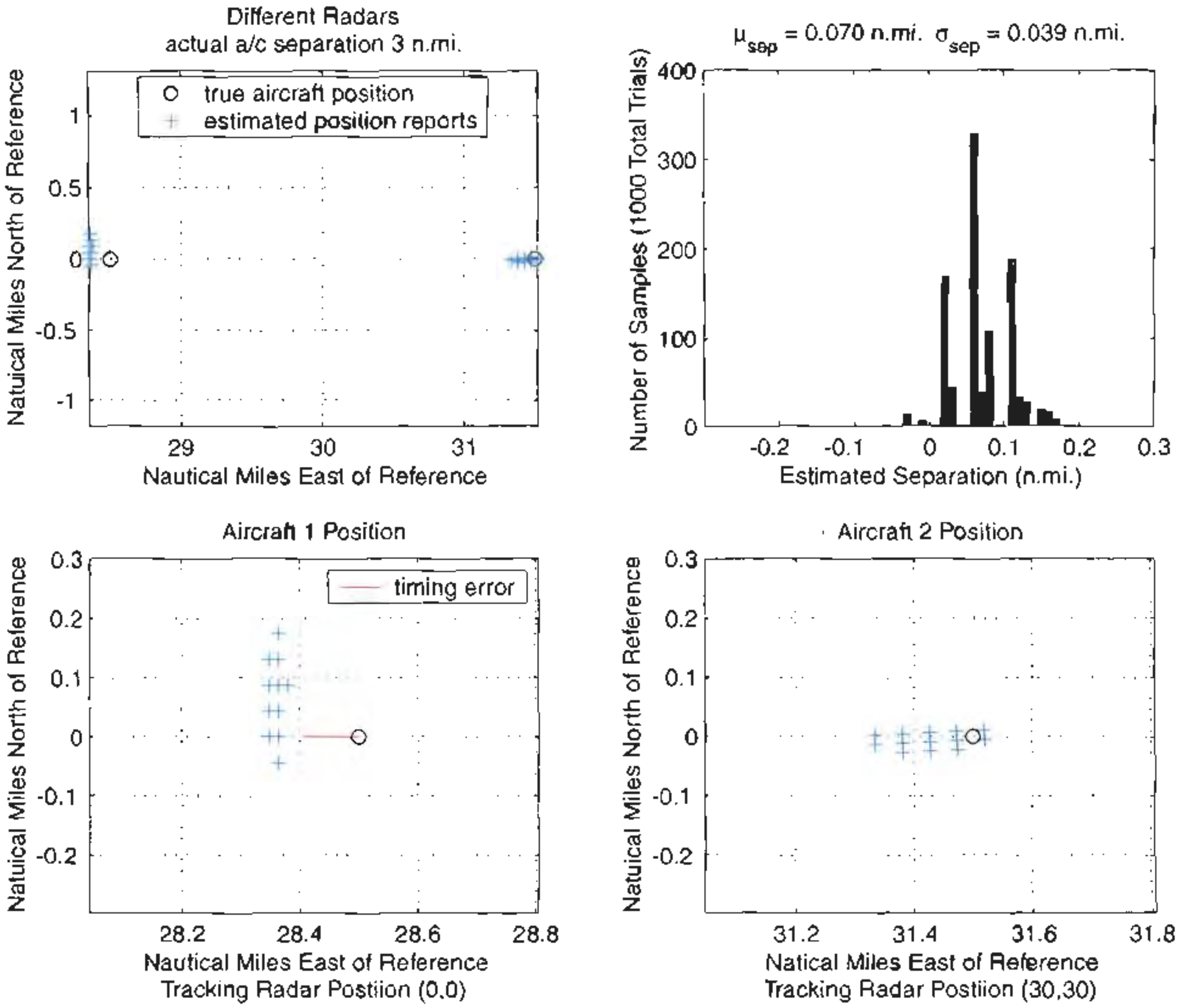

Figure 45. Typical separation estimates independent radars. Case 5

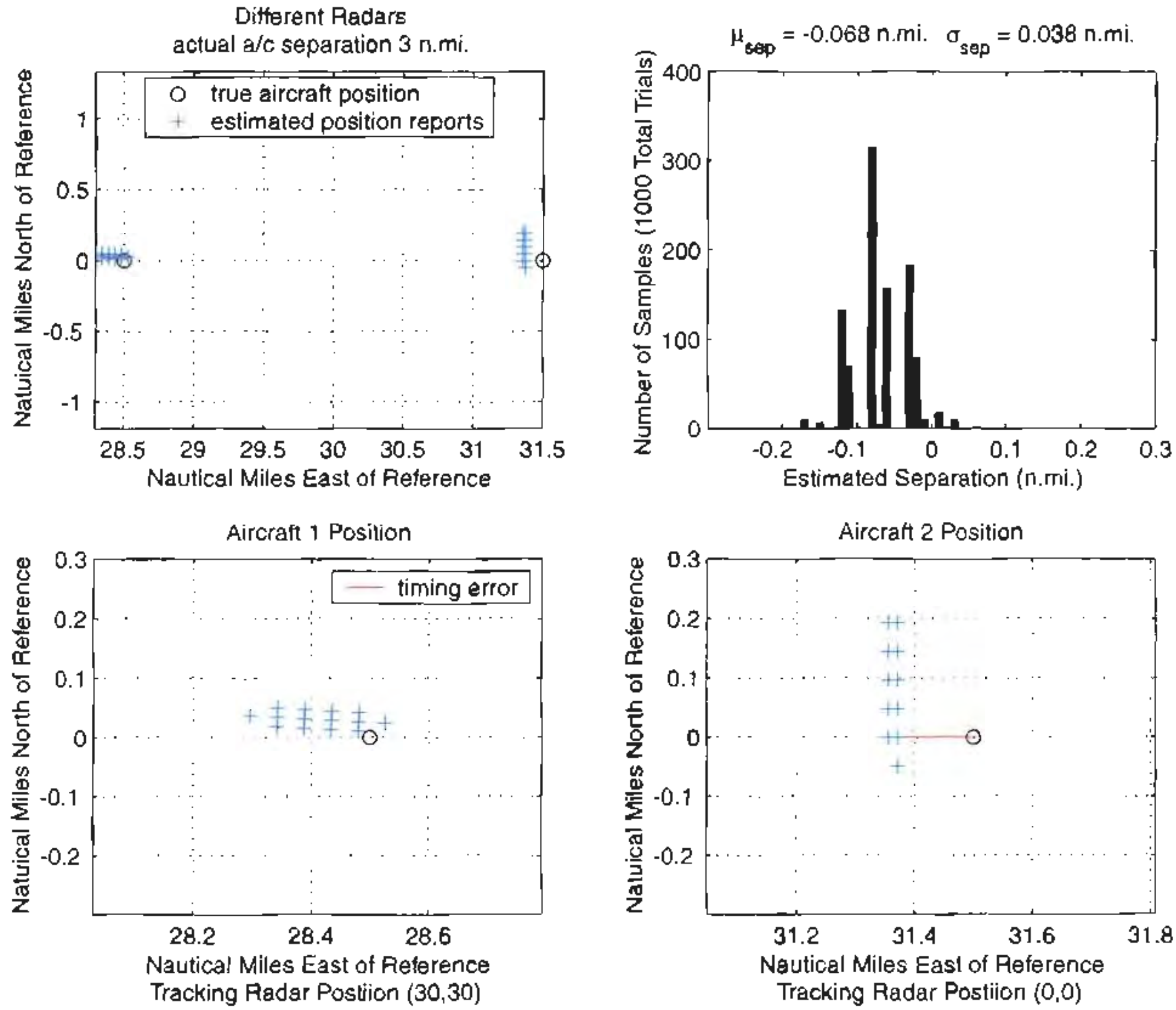

Figure 46. Typical separation estimates additional case of independent radars, Case 5

5.3 SPECIFIC MEASUREMENTS

The purpose of developing a specific analysis capability is to be able to simulate the specific "hit" to "hit" performance that would be observed for specified radar locations and the specified flight paths of two aircraft. The individual measurements are recorded. The relative orientation and geometry of the aircraft to the radars will change for each measurement. The radar bias errors and aircraft transponder errors are sampled once and held constant. The range and azimuth jitter errors vary "hit" to "hit." The radar starting orientation is randomly selected and the update rate is randomly sampled between 4 and 5 seconds and held constant for each radar. The specific times and true positions of the aircraft for each "hit" by each radar are calculated solving the simultaneous equations for the azimuth of the radar and the positions of the aircraft, both continuous functions of time.

The case presented here involves two aircraft traveling towards radar 1 which might simulate arrivals at an airport near radar 1. The lead aircraft, aircraft 1, is traveling at a ground speed of 150 knots and a heading of 217°. The second aircraft has a groundspeed of 185 knots and a heading of 224°. The aircraft start out approximately 11 nautical miles apart and converge to approximately 3 nautical miles apart near radar 1. During the initial part of their paths the aircraft pass very near to radar 2. The runtime is 12 minutes. The flight path and radar geometry is illustrated in Figure 47. A zoom in of a portion of the flight paths is shown in Figure 48 and illustrates the "hit" to "hit" variation. The blue dots are track hits of the two aircraft by radar 1 and the red dots are track hits of the two aircraft by radar 2.

The results of this simulation are presented in Figures 49 through 54. Figures 49 and 50 plot the actual separation of the aircraft and the separation displayed on the radar screen as a function of time. Figure 50 is a zoom in of a portion of Figure 49 to show more clarity. The four sub-plots are the actual and displayed separation for the possible cases of each radar tracking each aircraft. The actual separation is a continuous function of time and is the same in all plots. The displayed separation is discontinuous and jumps as each aircraft's position estimate is updated on the screen. Note that even if there were zero error in the position estimates, the displayed separation estimate would be in error because the aircraft positions were not estimated at the same time. For the case where radar 1 tracks both aircraft the error "spikes" seen are of short duration because the relative azimuth between the aircraft is small as seen from radar 1. The flip-flop in the direction of the error spikes for the cases where radar 1 tracks both aircraft and radar 2 tracks both aircraft represents the point where the relative azimuths of the aircraft presented to the radar change so that the first aircraft "hit" changes. The periodicity of the error spikes seen when different radars are tracking the aircraft result from different update rates sampled for the radar. This results in a periodic change in the relative timing of the radar "hits" on the two aircraft. This is similar to the phenomena for example when observing windshield wipers on a bus driven by independent motors. Periodically they are in sync and out of sync.

Figures 51 through 53 plot the actual separation error (difference between the actual and displayed separations presented above) as a function of time for two cases. The case where radar 1 tracks both aircraft for the entire approach and the case where radar 2 tracks the aircraft until they are within 15 nautical miles of radar 1 at which point radar 1 tracks the aircraft. In this second case there is a transition period where radar 1 will be tracking aircraft 1 and radar 2 will be tracking aircraft 2 which can be seen upper plot of Figure 51. Figures 52 and 53 are zoomed in on portions of Figure 51 during the initial tracking period and during the transition period. This compares the performance of radar 1 tracking both aircraft with the performance of radar 2 tracking the aircraft until they are within 15 nautical miles of radar 1. Figure 54 are histograms of the separation errors in the initial tracking period and in the transition period comparing the performance of having radar 1 track both aircraft with the performance of having radar 2 track the aircraft initially and then transitioning to tracking by radar 1. They were generated by randomly sampling times during the initial period and transition periods and measuring the error at those randomly sampled times. The small bumps in the histograms for radar 1 tracking both aircraft that appear at approximately 2.5 nautical miles correspond to the spikes seen in Figures 52 and 53 which are the display errors during the very short period between the updates of the two aircraft. A human factors study would be required to determine if these errors are significant and should be included. Similarly, the bifurcation in the histogram for the case of radar 2 tracking both aircraft is evident from the error plots in Figure 52.

The conventional logic has been that in cases like this where a terminal radar is located at an airport and providing separation services for arriving (and departing) aircraft, and where the aircraft were far from the terminal radar (over 25 - 30 miles) but happen to be "passing by" a radar that was very near by (5-10 miles), performance would be enhanced by using the "radar of opportunity" while the aircraft were closer to it. This would be accomplished through a mosaic display of that data to the terminal display. The rational is that radar positional accuracy degrades with distance from the radar so the separation service would benefit from the greater accuracy of the nearby radar.

In the simulation presented here using the geometry described above the results show that this is not necessarily the case for the following reasons.

First, while it is true that accuracy in general does degrade in range, range measurements in general do not, and in a geometry like this where aircraft are following in trail generally towards the airport, the separation measurement is mostly a function of the difference in range measurements.

Second, aircraft in this geometry being tracked by the airport's radar generally have small azimuth differences and thus move little between the hits on the two aircraft. But when tracked by the nearby radar, the geometry is such that the azimuth is larger and thus the time between hits on the aircraft is larger. At typical speeds, this motion of the aircraft between hits can be the dominant error between the separation displayed on the screen and the actual separation.

Third, there will necessarily be a period of transition when the aircraft nearest the airport is being tacked by the airport's radar and the other aircraft by the "radar of opportunity." During this period the positional errors are uncorrelated as are the update times so the errors in separation display are greatest.

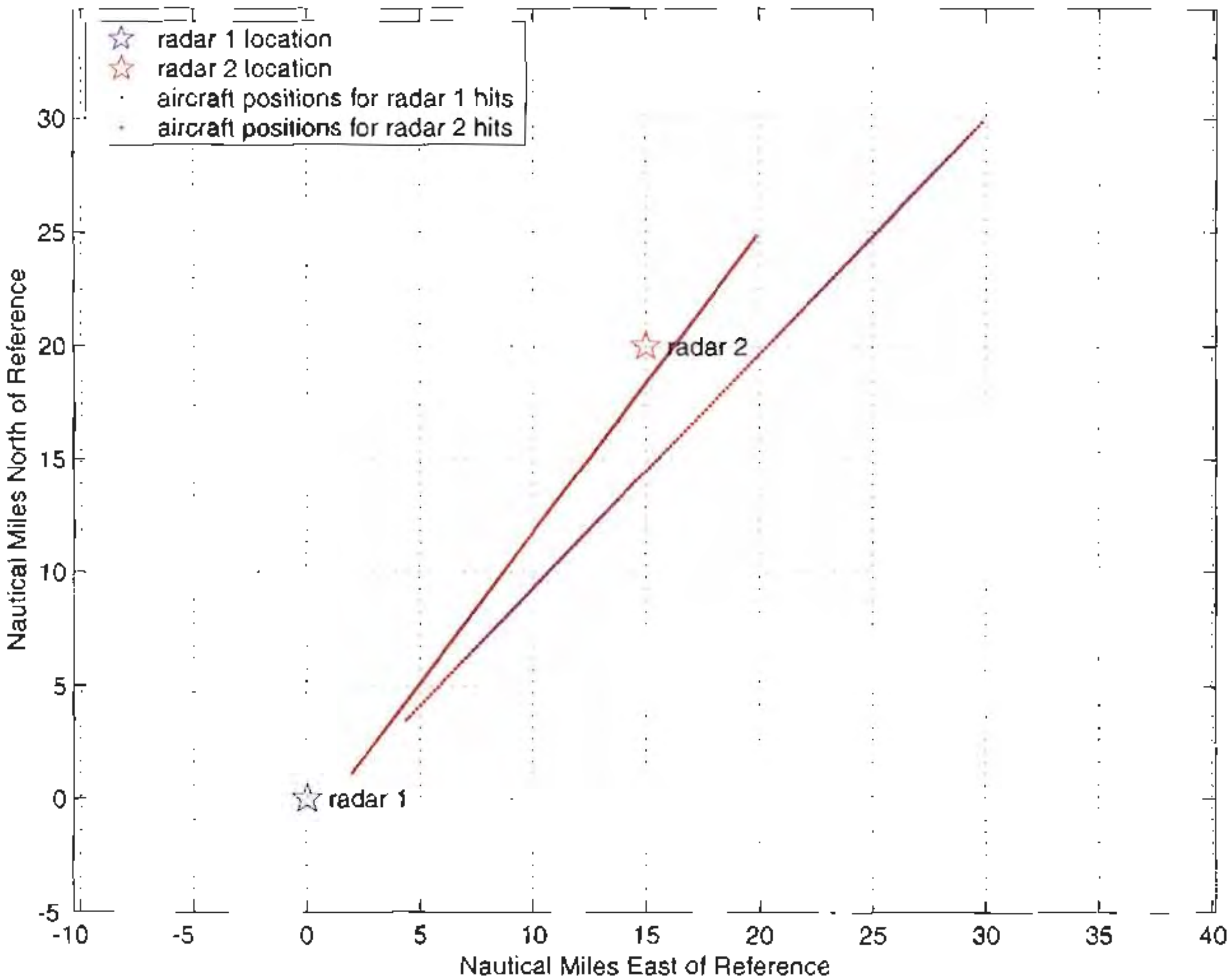

Figure 47. Radar and aircraft tracking geometry for a specific case

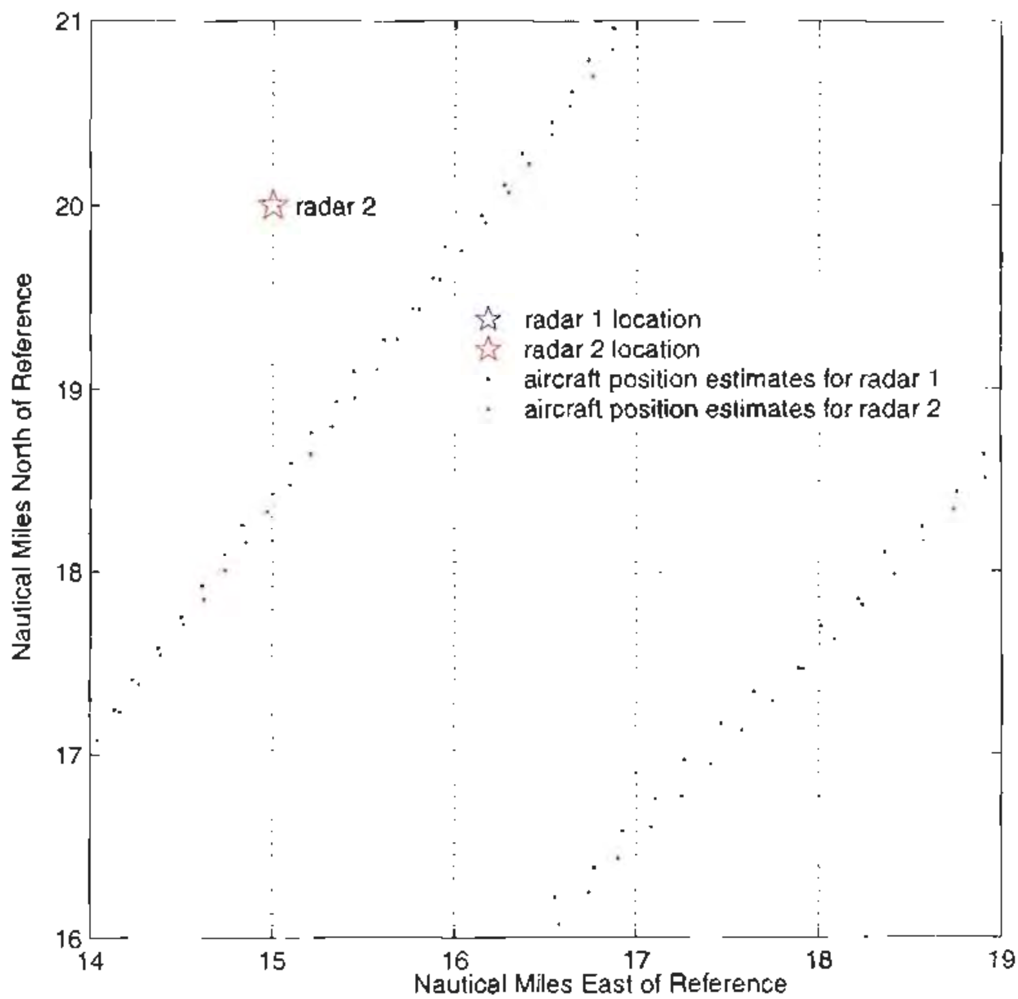

Figure 48. Radar and aircraft tracking geometry for a specific case, zoom-in

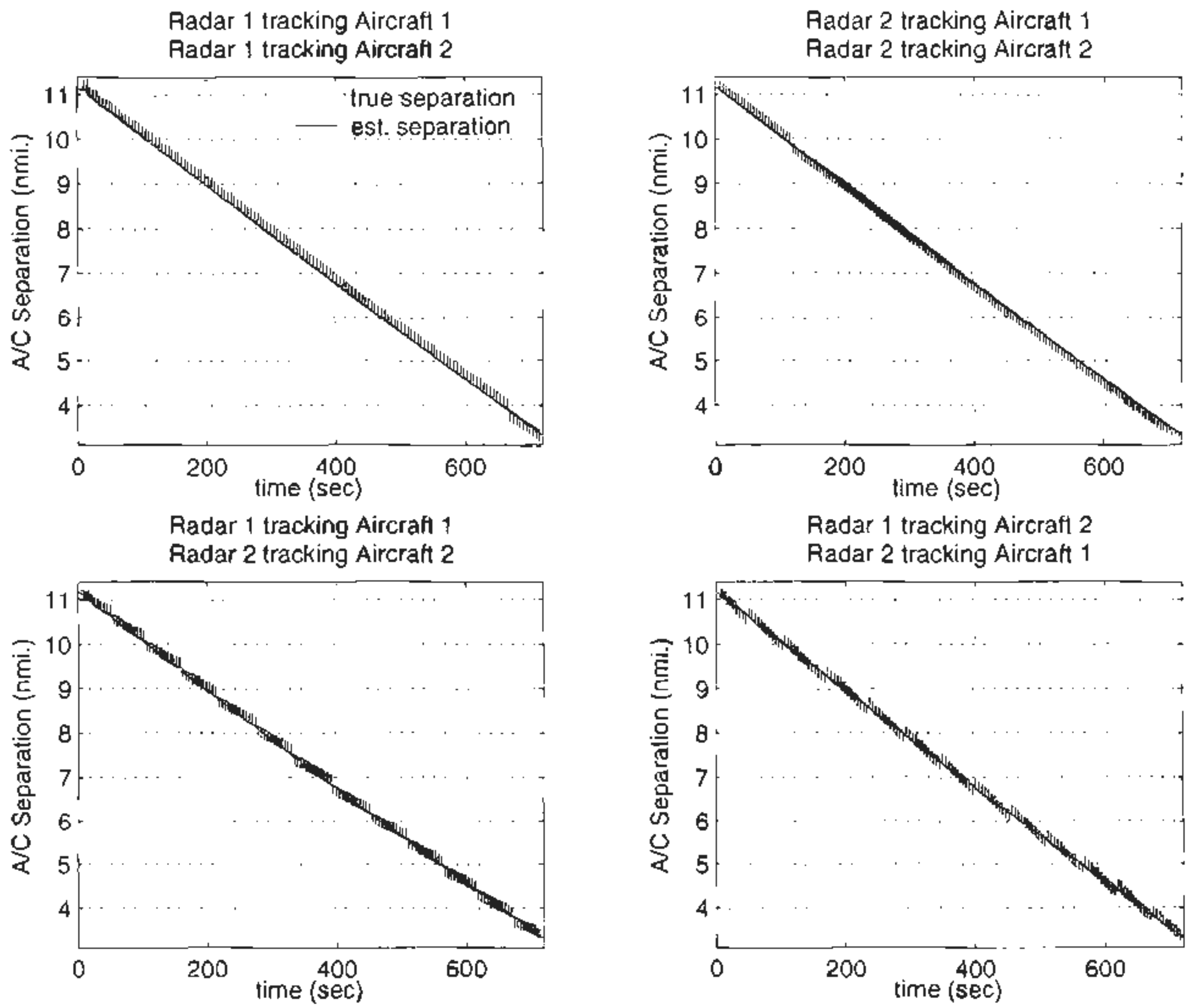

Figure 49. Displayed and actual separation as a function of time for a specific case for each combination of radar tracking aircraft

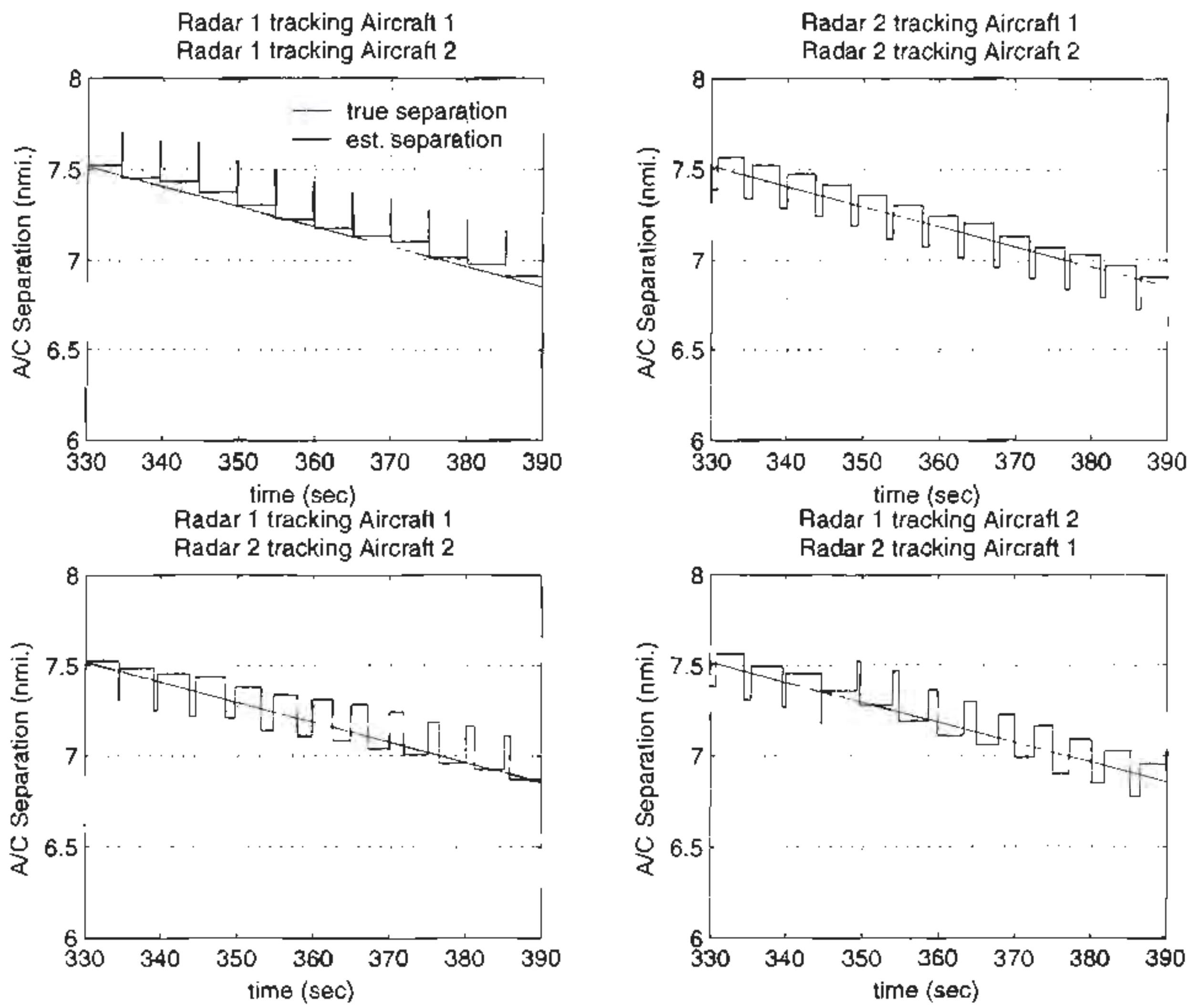

Figure 50. Displayed and actual separation as a function of time for a specific case for each combination of radar tracking aircraft, zoom-in of one minute during the middle of the track

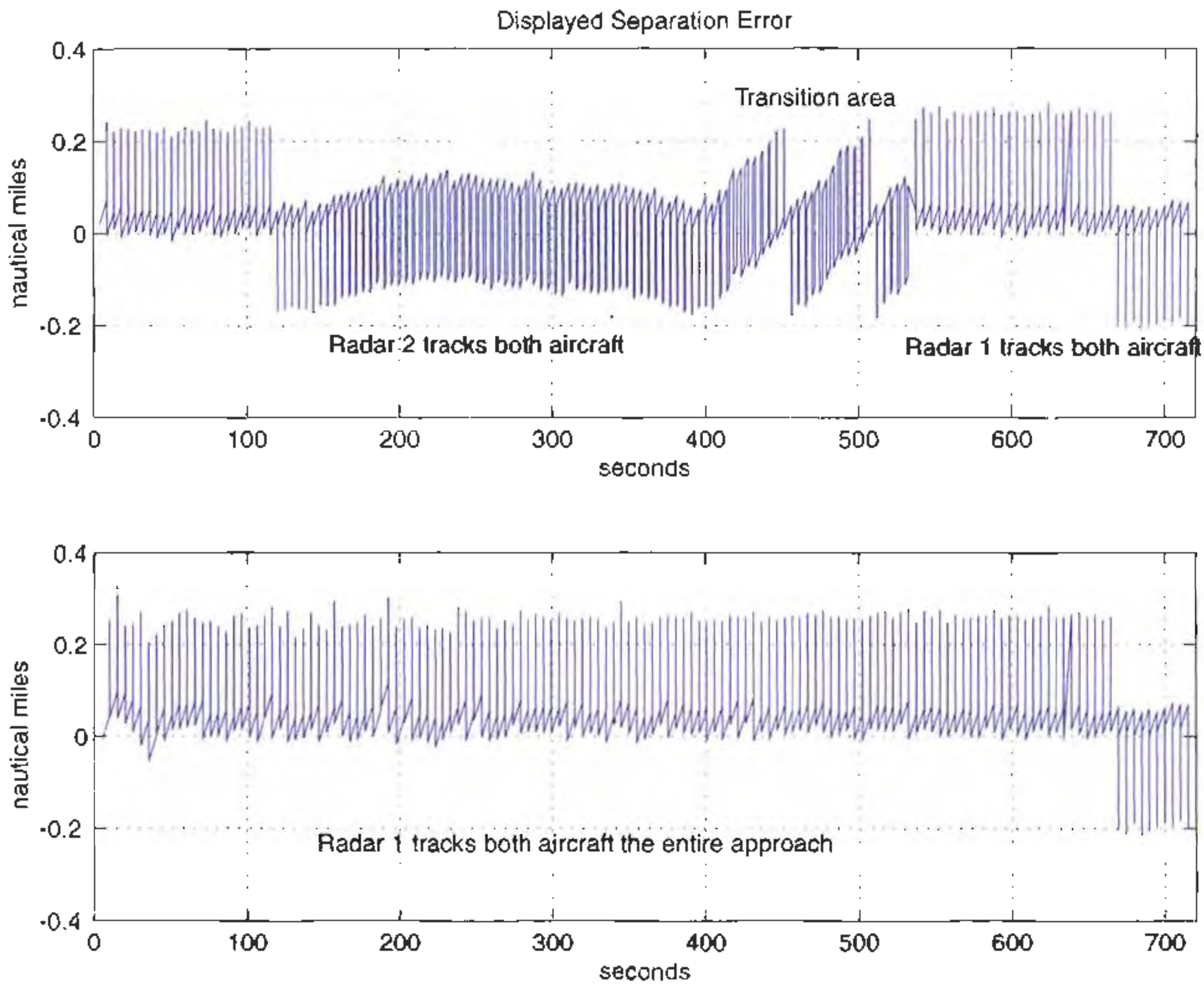

Figure 51. Displayed separation error as a function of time for specific case for radar 1 tracking both aircraft compared with radar 2 tracking until aircraft are within 15 nmi range of radar 1.

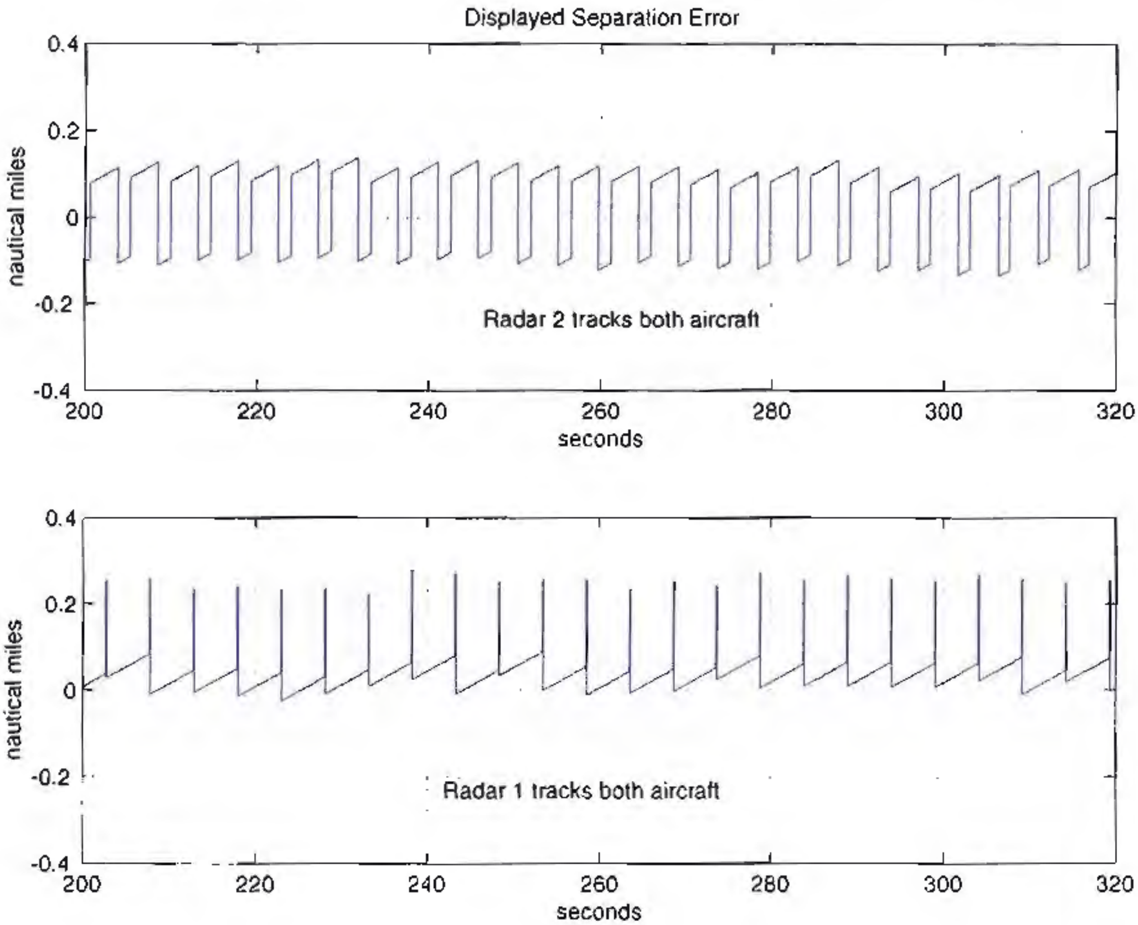

Figure 52. Displayed separation error as a function of time for specific case for radar 1 tracking both aircraft compared with radar 2 tracking until aircraft are within 15 nmi range of radar 1 – two minutes of initial tracking.

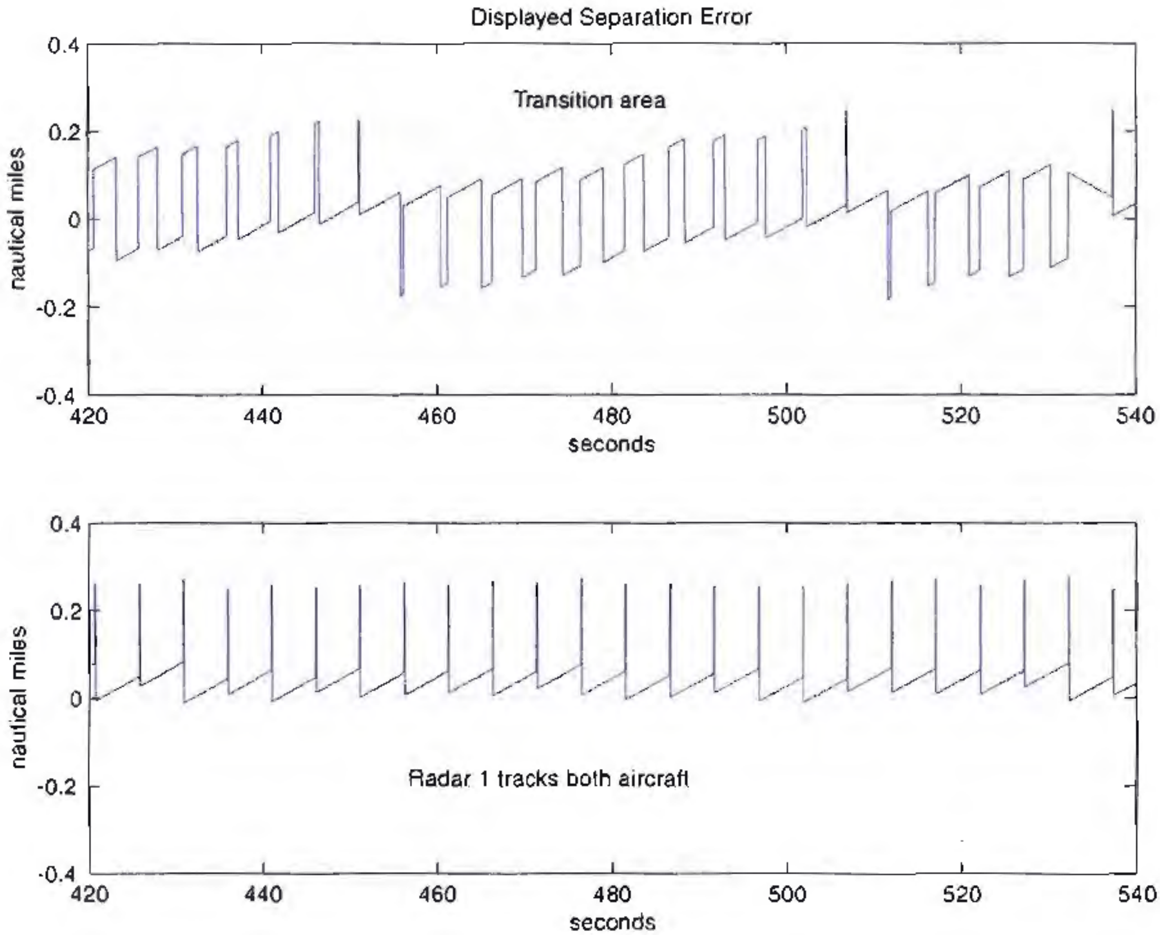

Figure 53. Displayed separation error as a function of time for specific case for radar 1 tracking both aircraft compared with radar 2 tracking until aircraft are within 15 nmi range of radar 1 – transition area

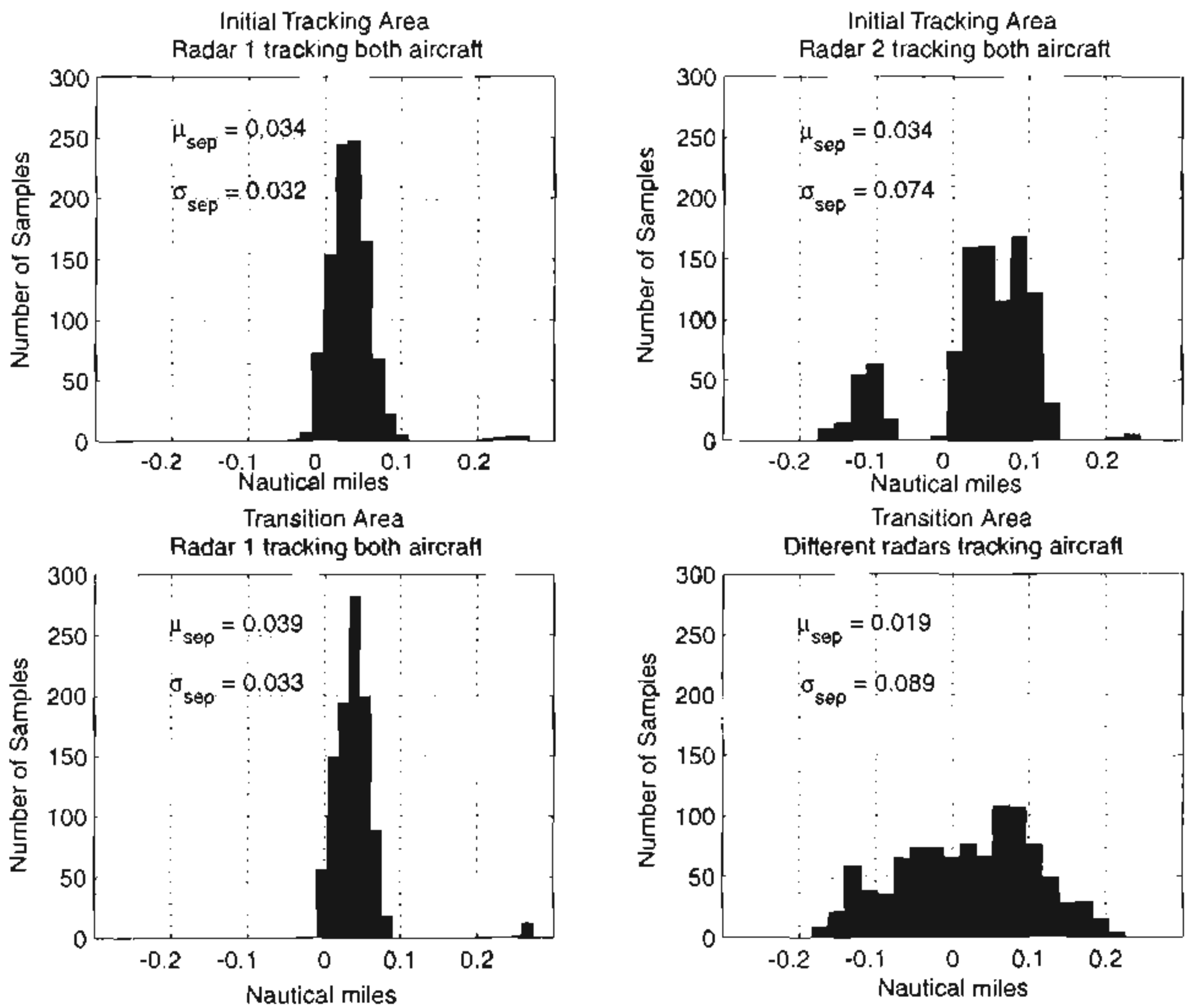

Figure 54. Histograms of displayed separation error in the initial tracking area and in the transition area for the cases where radar 1 tracked both aircraft and the cases where radar 2 tracked both aircraft in the initial tracking area and radar 1 tracked the aircraft within a 15 nmi range.

6. SUMMARY AND CONCLUSIONS

An appropriate metric for measuring the performance of radars in providing separation services is the error in separation presented to the controller. This is not easily deduced directly from the reported positional accuracy of the radar because the error sources have different characteristics. Some errors are correlated and will effect the positional estimates of two aircraft such that the error in separation is reduced. Some errors are direct functions of range and some are not. In addition, because the position estimates are taken at different times, the separation displayed to the controller will have errors due to the relative motion of the aircraft between position estimates.

The approach taken in this analysis is to examine average separation errors, typical separation errors, and specific separation errors, through Monte Carlo simulation based on the known errors in Mode S surveillance. This simulation accounted for both correlated and non-correlated errors, including the important effect of the error introduced when aircraft positions are measured at different times by different radars and presented to a controller on a mosaic display. These errors have not been included in previous analysis (e,g, Ref. [1]).

The analysis for average separation errors show that the performance of radars in providing separation services degrades with range. The analysis also shows that when using independent radars in a mosaic display, errors in estimated separation will increase, on average, compared to the performance when providing separation with one radar. This is primarily because the bias errors and errors due to inter-sensor timing of position estimates are uncorrelated when using independent radars to estimate the separation between two aircraft. The data presented in the section on average separation errors can be summarized by plotting the standard deviation of the separation error as a function of range for the single radar case and for the independent mosaic display case. This is the standard deviation in average measured separation error for two aircraft in trail three nautical miles apart in a random geometry. This summary is presented in Figure 55. Note that the mean of the estimated separations was three miles because all of the errors were sampled for each trial and the bias errors were averaged out in the results.

From Figure 55, it can be seen that the standard deviation of the error in displayed separation for a single radar at a range of 40 nautical miles is 0.053 nautical mile. The standard deviation is 0.077 nautical mile at the maximum typical range of a terminal radar (60 nmi.). This small difference in displayed separation range (0.024 nmi. or 150 feet) is on the order of a wingspan of a transport aircraft and is in stark contrast to the large change in allowed separation standard (3 nmi. vs. 5 nmi. for aircraft at FL180 and below) for the two cases.

From Figure 55, the standard deviation of the separation measurement errors for independent radars with a mosaic display is larger than that for a single radar, approximately three times as great in the 20 to 40 nautical mile range. The increase in the standard deviation of the separation measurement errors at close range is because the separation of the aircraft, three miles, causes an increase in the relative angle between the aircraft on average and thus increases the delta time between hits which results in a greater separation error. This error begins to dominate at very short ranges.

It is also apparent that there is no range for independent radars with mosaic display that will give equivalent displayed separation performance, on average, as that of a single radar at 40 nautical miles. Therefore, it does not appear possible to argue that the single sensor separation standards should apply to mosaic display, purely on a surveillance equivalence basis. However, the relatively low values of the standard deviation of displayed separation error (~ 0.15 – 0.20 nmi.) for mosaic displays at ranges of 40 to 60 nautical miles suggest that further safety analysis be performed to assess the application of 3 nmi. standards for terminal and mosaic displays.

The sections on typical and specific errors in separation measurements illustrate that the separation measurement errors are highly dependent on the geometry of the aircraft and radars. Applying average results to specific geometries can lead to counter intuitive results as was illustrated in the case presented in the specific analysis.

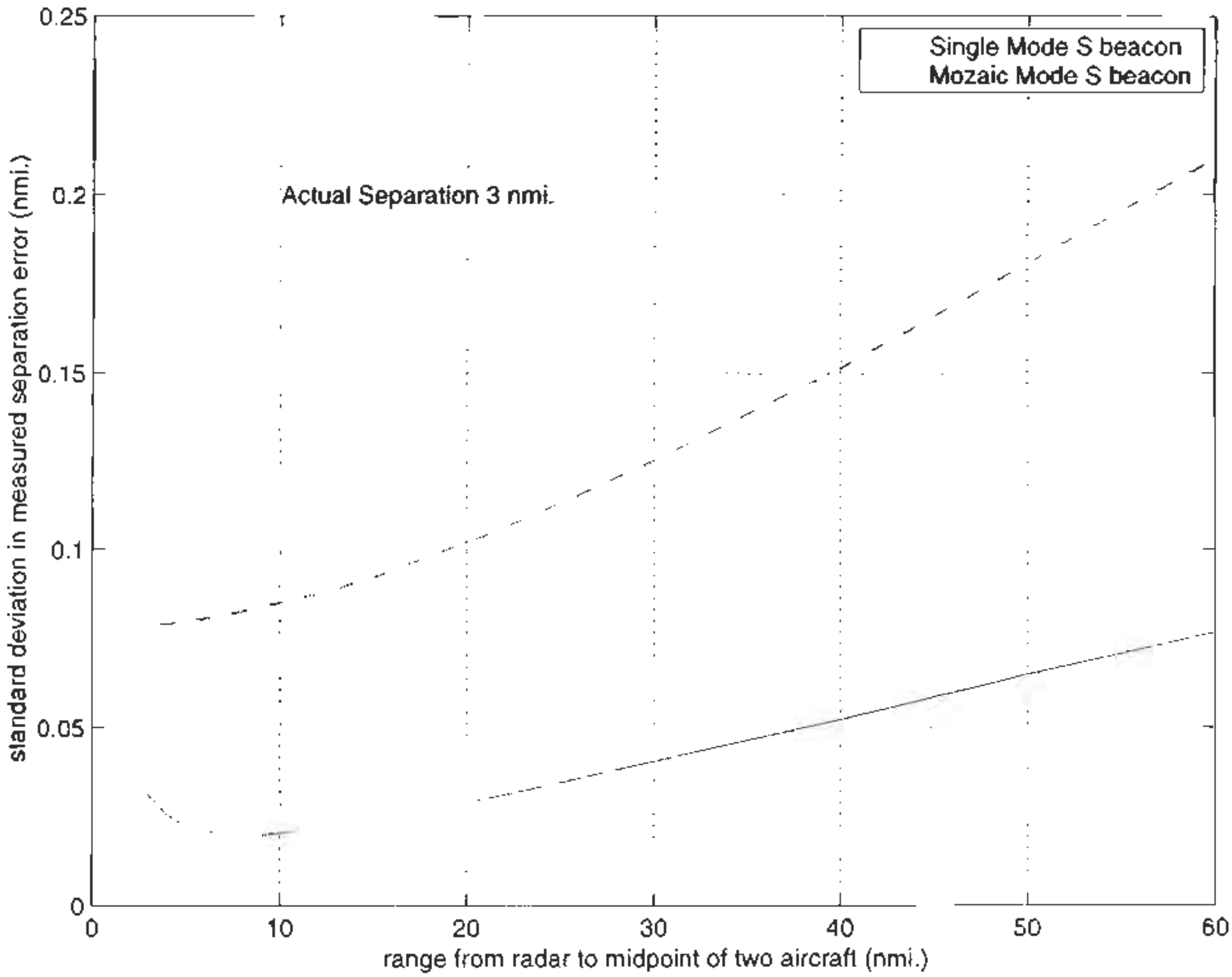

Figure 55. Standard deviation in measured separation error as a function of range for a single radar and for independent radars with a mosaic display. Range is the distance from the radar(s) to a point midway between two aircraft separated by 3 nautical miles as illustrated in Figure 4.

7. REFERENCES

1. *Mosaic Display Target Accuracy Study, Final Report*, for the Federal Aviation Administration, Under Contract DTFA01-97-C-00046, ARCON Corporation, Waltham MA., December 13, 1999.

2. J.L. Gertz, A.D. Kaminsky, *COTS Fusion Tracker Evaluation*, Project Report ATC-302, prepared for the Federal Aviation Administration, MIT Lincoln Laboratory, February 15, 2002.

A. APPENDIX, MATLAB SIMULATION PROCEDURES AND SCRIPTS

The following sections describe the exact procedures used in creating the simulations described in the body of the report. Each type of simulation is described with a step by step psuedo code description followed by the MATLAB script that was used. The step numbers in the psuedo code descriptions match the step numbers in the scripts.

A.1 AVERAGE ERRORS IN SEPARATION MEASUREMENTS, ONE RADAR

A.1.1 Input

The **range** and **samplesize** are input to the simulation.

The **range** is the range of the aircraft from the radar in nautical miles. The two aircraft are assumed to be separated by three nautical miles and the **range** is to the point midway between the two aircraft. The aircraft will be randomly oriented about a circle whose origin is at the specified **range**. The **samplesize** is the number of cases that will be run.

A.1.2 Output

The outputs are vectors of length **samplesize** containing the separation estimates **ES** and position errors of the aircraft **PE1** and **PE2**. A histogram of the samples with the mean and standard deviation is plotted.

A.1.3 Modeling Procedure

1. Place the true radar position at the center of the coordinate system.

$$(\mathbf{xradar}_{\mathbf{true}} , \mathbf{yradar}_{\mathbf{true}}) = (0 , 0)$$

2. Define Common Digitizer format vectors that will be used to round off estimates to the nearest 1/64 nautical mile in range and nearest 1/4096 scan in azimuth (radians).

$$\mathbf{CD}_{\mathbf{range}} = [0 : 1/64 : 100]$$

$$\mathbf{CD}_{\mathbf{azimuth}} = [0 : 2\pi/4096 : 2\pi]$$

3. Define vectors of length **samplesize** of the true positions of the two aircraft. The two aircraft true positions are randomly oriented for each sample measurement with 3.0 miles separation such that the midpoint of the separation is at (**range**, 0). The first aircraft will be positioned at

$$(\mathbf{xac}_{true(ac1)}, \mathbf{yac}_{true(ac1)}) = (\mathbf{range} + 1.5 * \cos(\varphi) , 1.5 * \sin(\varphi))$$

and the second aircraft at

$$(\mathbf{xac}_{true(ac2)}, \mathbf{yac}_{true(ac2)}) = (\mathbf{range} - 1.5 * \cos(\varphi) , -1.5 * \sin(\varphi))$$

where φ is a vector of randomly selected angles between 0 and 360 degrees.

4. Compute vectors of the true range and azimuth from the radar for each aircraft. The azimuth is computed in radians positive for positive $\mathbf{yac}_{true(aci)}$ and negative for negative $\mathbf{yac}_{true(aci)}$

$$\mathbf{range}_{true(aci)} = \text{sqrt} (\mathbf{xac}^2_{true(aci)} + \mathbf{yac}^2_{true(aci)})$$

$$\theta_{true(aci)} = \arctan (\mathbf{yac}_{true(aci)} / \mathbf{xac}_{true(aci)})$$

5. Define a vector of location bias errors by random sampling ϕ from a uniformly distributed azimuth error between [0°, 360°] and $\mathbf{location}_{bias}$ from a uniformly distributed distance error of [0, 200] feet. Note that because this is an average computation the location bias error is resampled for each element in the vector. Determine the surveyed error in radar location.

$$(\mathbf{xradar}_{surveyerror}, \mathbf{yradar}_{surveyerror}) = (\mathbf{location}_{bias} * \cos(\phi) , \mathbf{location}_{bias} * \sin(\phi))$$

6. Define a vector of range bias errors $\mathbf{range}_{bias}$ by random sampling from a uniform distribution between [-30, +30] feet. Note that because this is an average computation the range bias is resampled for each element in the vector. Note also that the same vector $\mathbf{range}_{bias}$ is used for both aircraft.

7. Define two vectors of range jitter errors, one for each aircraft, $\mathbf{range}_{jitter(ac1)}$ and $\mathbf{range}_{jitter(ac2)}$ by random sampling from a normal distribution with $\mu = 0$ feet and $\sigma = 25$ feet.

8. Define two vectors of transponder range bias errors $\mathbf{xponder}_{bias(ac1)}$ and $\mathbf{xponder}_{bias(ac2)}$ by random sampling from a uniform distribution between [-125, +125] feet. Note that because this is an average computation the transponder range bias is resampled for each element of the vector.

9. Compute the estimated range for each aircraft as the true range plus the sum of the range errors by adding the vectors

$$\mathbf{range}_{estimate(aci)} = \mathbf{range}_{true(aci)} + (\mathbf{range}_{bias} + \mathbf{range}_{jitter(aci)} + \mathbf{xponder}_{bias(aci)}) / (6076.115)$$

10. Compute a vector of the reported ranges for each aircraft as the estimated ranges rounded off to the nearest 1/64 n. mi. This is done by linearly interpolating each of the value of $\mathbf{range}_{estimate(aci)}$ using the $\mathbf{CD}_{range}$ vector as both the x and y value for the function being interpolated and using the nearest neighbor option for the interpolation.

$$\mathbf{range}_{\mathbf{reported(ac}i\mathbf{)}} = \text{interp1}(\mathbf{CD}_{\mathbf{range}}, \mathbf{CD}_{\mathbf{range}}, \mathbf{range}_{\mathbf{estimate(ac}i\mathbf{)}}, \text{'nearest'})$$

11. Define a vector of azimuth bias errors $\boldsymbol{\theta}_{\mathbf{bias}}$ by random sampling from a uniform distribution between [-0.3°, +0.3°]. Note that because this is an average computation the azimuth bias is resampled for each element of the vector. Note also that the same vector $\boldsymbol{\theta}_{\mathbf{bias}}$ is used for both aircraft.

12. Define two vectors of azimuth errors, one for each aircraft, $\boldsymbol{\theta}_{\mathbf{error(ac1)}}$ and $\boldsymbol{\theta}_{\mathbf{error(ac2)}}$ by random sampling from a normal distribution with $\mu = 0°$ and $\sigma = 0.068°$.

13. Compute the estimated azimuth for each aircraft as the true azimuth plus the sum of the azimuth bias and azimuth error by adding the vectors.

$$\boldsymbol{\theta}_{\mathbf{estimate(ac}i\mathbf{)}} = \boldsymbol{\theta}_{\mathbf{true(ac}i\mathbf{)}} + (\boldsymbol{\theta}_{\mathbf{bias}} + \boldsymbol{\theta}_{\mathbf{jitter(ac}i\mathbf{)}})$$

14. Compute a vector of the reported azimuths for each aircraft as the estimated azimuths rounded off to the nearest 1/4096 of a scan. This is done by linearly interpolating each of the values of $\boldsymbol{\theta}_{\mathbf{estimate(ac}i\mathbf{)}}$ using the $\mathbf{CD}_{\mathbf{azimuth}}$ vector as both the x and y value of the function being interpolated and using the nearest neighbor option for the interpolation.

$$\boldsymbol{\theta}_{\mathbf{reported(ac}i\mathbf{)}} = \text{interp1}(\mathbf{CD}_{\mathbf{azimuth}}, \mathbf{CD}_{\mathbf{azimuth}}, \boldsymbol{\theta}_{\mathbf{estimate(ac}i\mathbf{)}}, \text{'nearest'})$$

15. Compute vectors of reported aircraft x and y positions. This is based on the reported range and azimuth offset by the location bias.

$$\mathbf{xac}_{\mathbf{reported(ac}i\mathbf{)}} = \mathbf{range}_{\mathbf{reported(ac}i\mathbf{)}} * \cos(\boldsymbol{\theta}_{\mathbf{reported(ac}i\mathbf{)}}) + \mathbf{xradar}_{\mathbf{surveyerror}}$$

$$\mathbf{yac}_{\mathbf{reported(ac}i\mathbf{)}} = \mathbf{range}_{\mathbf{reported(ac}i\mathbf{)}} * \sin(\boldsymbol{\theta}_{\mathbf{reported(ac}i\mathbf{)}}) + \mathbf{yradar}_{\mathbf{surveyerror}}$$

16. Compute vectors of errors in the reported aircraft x and y positions.

$$\mathbf{dxac}_{\mathbf{(ac}i\mathbf{)}} = \mathbf{xac}_{\mathbf{reported(ac}i\mathbf{)}} - \mathbf{xac}_{\mathbf{true(ac}i\mathbf{)}}$$

$$\mathbf{dyac}_{\mathbf{(ac}i\mathbf{)}} = \mathbf{yac}_{\mathbf{reported(ac}i\mathbf{)}} - \mathbf{yac}_{\mathbf{true(ac}i\mathbf{)}}$$

$$\mathbf{PE}\boldsymbol{i} = \text{sqrt} (\mathbf{dxac}^2_{\mathbf{(ac}i\mathbf{)}} + \mathbf{dyac}^2_{\mathbf{(ac}i\mathbf{)}})$$

17. Define a vector of length samplesize of radar rotation rates $\mathbf{radar}_{\mathbf{rotationrate}}$ by random sampling from a uniform distribution between [$2\pi/5.0$, $2\pi/4.0$] radians per second. Note that because this is an average computation the rotation rate is resampled for each element in the vector.

18. Compute the error from timing $\mathbf{error}_{\mathbf{clockbias}}$ on the separation measurements by dividing the difference in azimuth between the two aircraft by the radar rotation rate to determine the time between "hits" and then multiplying the result by the assumed velocity of 200 knots. Aircraft 1

is assumed to be the lead aircraft and aircraft 2 the trail aircraft. According to Figure 4, if the lead aircraft (aircraft 1) is "hit" first, then there is an apparent reduction in measured separation and $\mathbf{error}_{\mathbf{clockbias}}$ should be subtracted from the measured separation. If the trail aircraft (aircraft 2) is "hit" first, then $\mathbf{error}_{\mathbf{clockbias}}$ should be added to the measured separation. $\boldsymbol{\theta}_{\mathbf{true(ac}i)}$ is computed as the arctan of ($\mathbf{yac}_{\mathbf{true(ac}i)}$ / $\mathbf{xac}_{\mathbf{true(ac}i)}$) so a positive $\mathbf{yac}_{\mathbf{true(ac}i)}$ results in a positive value for $\boldsymbol{\theta}_{\mathbf{true(ac}i)}$ and a negative $\mathbf{yac}_{\mathbf{true(ac}i)}$ results in a negative value for $\boldsymbol{\theta}_{\mathbf{true(ac}i)}$. Recall in this model $\mathbf{xac}_{\mathbf{true(ac}i)}$ is always positive. Therefore, if aircraft 1 has a positive value for $\mathbf{yac}_{\mathbf{true(ac1)}}$ and is therefore "hit" first, then the error should be subtracted. If aircraft 2 has a positive value for $\mathbf{yac}_{\mathbf{true(ac1)}}$ and is therefore "hit" first the error should be added. If we compute the clock bias error in nautical miles according to the equation below, the error will be positive in value when aircraft 1 is "hit" first and negative in value if aircraft 2 is "hit" first (because $\boldsymbol{\theta}_{\mathbf{true(ac1)}} - \boldsymbol{\theta}_{\mathbf{true(ac2)}}$ will be negative in value) and thus the value should always be subtracted from the estimated separation.

$$\mathbf{error}_{\mathbf{clockbias}} = ((\boldsymbol{\theta}_{\mathbf{true(ac1)}} - \boldsymbol{\theta}_{\mathbf{true(ac2)}}) / \mathbf{radar}_{\mathbf{rotationrate}}) * (200 / 3600)$$

19. Compute the vector of estimated separation from the reported position reports and the error due to timing.

$$\mathbf{ES} = \mathrm{sqrt} ((\mathbf{xac}_{\mathbf{reported(ac1)}} - \mathbf{xac}_{\mathbf{reported(ac2)}})^2 + (\mathbf{yac}_{\mathbf{reported(ac1)}} - \mathbf{yac}_{\mathbf{reported(ac2)}})^2) - \mathbf{error}_{\mathbf{clockbias}}$$

20. Compute statistics on measurement errors. Compute the mean μ and standard deviations σ of the values in the vectors **PE1, PE2** and **ES**.

Plot the results. Plot histograms of the values in the vectors **PE1, PE2** and **ES** for the range modeled and label the plots with the range and values of μ and σ.

A.2 AVERAGE ERRORS IN SEPARATION MEASUREMENTS, TWO RADARS

A.2.1 Input

The **range, samplesize, and Θ** are input to the simulation.

The **range** is the range of the aircraft from either radar in nautical miles. The relative angle **Θ** is the angle from a line connecting the two radars to the center of separation of the two aircraft as illustrated in Figure 5. The distance between the two radars is adjusted to maintain the specified **range**. The two aircraft are assumed to be separated by three nautical miles and the **range** is to the point midway between the two aircraft. The aircraft will be randomly oriented about a circle whose origin is at the specified range. The **samplesize** is the number of cases run.

A.2.2 Output

The outputs are vectors of length **samplesize** containing the separation estimates **ES** and position errors of the aircraft **PE1** and **PE2**. A histogram of the samples with the mean and standard deviation is plotted.

A.2.3 Modeling Procedure

1. Convert **Θ** to radians and compute the true radar positions placing radar 1 at the center of the coordinate system and the second radar so that they will both have a distance **range** to the center of the separation of the aircraft at an angle **Θ** from the line connecting the radars.

$$\Theta = \Theta * (\pi/180)$$

$$(\ \mathbf{xradar}_{\mathbf{true(radar1)}}, \mathbf{yradar}_{\mathbf{true(radar1)}}\ ) = (\ 0\ ,\ 0\ )$$

$$(\ \mathbf{xradar}_{\mathbf{true(radar2)}}, \mathbf{yradar}_{\mathbf{true(radar2)}}\ ) = (\ 2 * \mathbf{range} * \cos(\Theta)\ ,\ 0\ )$$

2. Define Common Digitizer format vectors that will be used to round off estimates to the nearest 1/64 nautical mile in range and nearest 1/4096 scan in azimuth (radians). Note that the possible values of $\mathbf{CD}_{\mathbf{azimuth}}$ had to be extended from the one radar case. This is because the second radar will see aircraft at the azimuth limits when **Θ** = 0 and additional errors introduced could put the reported values over the conventional ± π limits.

$$\mathbf{CD}_{\mathbf{range}} = [-10/64 : 1/64 : 100]$$

$$\mathbf{CD}_{\mathbf{azimuth}} = [-5\pi/4 : 2\pi/4096 : 5\pi/4]$$

3. Define vectors of length **samplesize** of the true positions of the two aircraft. The two aircraft true positions are randomly oriented for each sample measurement with 3.0 miles separation such that the midpoint of the separation is at (**range***cos(Θ) , **range***sin(Θ)). The first aircraft will be positioned at

$$(\mathbf{xac}_{true(ac1)}, \mathbf{yac}_{true(ac1)}) = (\mathbf{range}*\cos(\Theta) + 1.5*\cos(\varphi), \mathbf{range}*\sin(\Theta) + 1.5*\sin(\varphi))$$

and the second aircraft at

$$(\mathbf{xac}_{true(ac2)}, \mathbf{yac}_{true(ac2)}) = (\mathbf{range}*\cos(\Theta) - 1.5*\cos(\varphi), \mathbf{range}*\sin(\Theta) - 1.5*\sin(\varphi))$$

where φ is a vector of randomly selected angles between 0 and 360 degrees.

4. a. Compute the vector **rtcase** of length **samplesize** that stands for radar tracking case. **rtcase** will contain a 1 if aircraft 1 is nearest radar 1 and (necessarily from the geometry chosen) aircraft 2 is nearest radar 2. **rtcase** will contain a 0 if aircraft 1 is nearest radar 2 and aircraft 2 is nearest radar 1. **rtcaseopp** will contain the opposite values of 0 and 1. The following analysis computes all combinations of the two radars tracking the two aircraft and these vectors are used to produce final statistical results under the assumption that the nearest radar will be tracking the respective aircraft.

$$\mathbf{rtcase} = 1 \text{ when } \mathbf{xac}_{true(ac1)} <= \mathbf{xac}_{true(ac2)}$$

$$\mathbf{rtcase} = 0 \text{ when } \mathbf{xac}_{true(ac1)} > \mathbf{xac}_{true(ac2)}$$

$$\mathbf{rtcaseopp} = 0 \text{ when } \mathbf{xac}_{true(ac1)} <= \mathbf{xac}_{true(ac2)}$$

$$\mathbf{rtcaseopp} = 1 \text{ when } \mathbf{xac}_{true(ac1)} > \mathbf{xac}_{true(ac2)}$$

   b. Compute vectors of the true range and azimuth from each of the radars to each aircraft of the aircraft. The azimuth is computed in radians positive for positive $\mathbf{yac}_{true(aci)}$ and negative for negative $\mathbf{yac}_{true(aci)}$

$$\mathbf{range}_{true(radarj,aci)} = \text{sqrt}\left((\mathbf{xac}_{true(aci)} - \mathbf{xradar}_{true(radarj)})^2 + (\mathbf{yac}_{true(aci)} - \mathbf{yradar}_{true(radarj)})^2 \right)$$

$$\boldsymbol{\theta}_{true(radarj,aci)} = \arctan\left((\mathbf{yac}_{true(aci)} - \mathbf{yradar}_{true(radarj)}) / (\mathbf{xac}_{true(aci)} - \mathbf{xradar}_{true(radarj)}) \right)$$

5. Define vectors of location bias errors for each radar by random sampling ϕ from a uniformly distributed azimuth error between [0°, 360°] and $\mathbf{location}_{bias}$ from a uniformly distributed distance error of [0 , 200] feet. Note that because this is an average computation the location bias error is resampled for each element in the vector. Determine the surveyed error in radar location.

$$(\mathbf{xradar}_{surveyerror(radar j)} , \mathbf{yradar}_{surveyerror(radar j)}) = (\mathbf{location}_{bias j} * \cos(\phi_j) , \mathbf{location}_{bias j} * \sin(\phi_j))$$

6. Define vectors of range bias errors $\mathbf{range}_{bias(radar j)}$ by random sampling from a uniform distribution between [-30 , +30] feet. Note that because this is an average computation the range bias is resampled for each element in the vector. Note also that the same vector $\mathbf{range}_{bias(radar j)}$ for a given radar is used for range computation to both aircraft by that radar.

7. Define four vectors of range jitter errors, one for each radar tracking each aircraft, $\mathbf{range}_{jitter(radar j,ac i)}$ by random sampling from a normal distribution with $\mu = 0$ feet and $\sigma = 25$ feet.

8. Define two vectors of transponder range bias errors $\mathbf{xponder}_{bias(ac i)}$ by random sampling from a uniform distribution between [-125 , +125] feet. Note that because this is an average computation the transponder range bias is resampled for each element of the vector.

9. Compute the estimated range from each radar to each aircraft as the true range plus the sum of the range errors by adding the vectors

$$\mathbf{range}_{estimate(radar j,ac i)} = \mathbf{range}_{true(radar j,ac i)} + (\mathbf{range}_{bias(radar j)} + \mathbf{range}_{jitter(radar j,ac i)} + \mathbf{xponder}_{bias(ac i)}) / (6076.115)$$

10. Compute a vector of the reported ranges for each radar to each aircraft as the estimated ranges rounded off to the nearest 1/64 n. mi. This is done by linearly interpolating each of the value of $\mathbf{range}_{estimate(radar j,ac i)}$ using the $\mathbf{CD}_{range}$ vector as both the x and y value for the function being interpolated and using the nearest neighbor option for the interpolation.

$$\mathbf{range}_{reported(radar j,ac i)} = \text{interp1}(\mathbf{CD}_{range} , \mathbf{CD}_{range} , \mathbf{range}_{estimate(radar j,ac i)} ,\text{'nearest'})$$

11. Define a vector of azimuth bias errors for each radar $\boldsymbol{\theta}_{bias(radar j)}$ by random sampling from a uniform distribution between [-0.3°, +0.3°]. Note that because this is an average computation the azimuth bias is resampled for each element of the vector. Note also that the same vector $\boldsymbol{\theta}_{bias(radar j)}$ is used for both aircraft for a given radar.

12. Define vectors of azimuth errors for each radar tracking each aircraft, $\boldsymbol{\theta}_{error(radar j,ac i)}$ by random sampling from a normal distribution with $\mu = 0°$ and $\sigma = 0.068°$.

13. Compute the estimated azimuth for each radar tracking each aircraft as the true azimuth plus the sum of the azimuth bias and azimuth error by adding the vectors.

$$\boldsymbol{\theta}_{estimate(radar j,ac i)} = \boldsymbol{\theta}_{true(radar j,ac i)} + (\boldsymbol{\theta}_{bias(radar j)} + \boldsymbol{\theta}_{jitter(radar j,ac i)})$$

14. Compute a vector of the reported azimuths for each radar tracking each aircraft as the estimated azimuth values rounded off to the nearest 1/4096 of a scan. This is done by linearly

interpolating each of the values of $\boldsymbol{\theta}_{\mathbf{estimate(radar}j\mathbf{,ac}i\mathbf{)}}$ using the $\mathbf{CD_{azimuth}}$ vector as both the x and y value of the function being interpolated and using the nearest neighbor option for the interpolation.

$$\boldsymbol{\theta}_{\mathbf{reported(radar}j\mathbf{,ac}i\mathbf{)}} = \text{interp1}(\mathbf{CD_{azimuth}}, \mathbf{CD_{azimuth}}, \boldsymbol{\theta}_{\mathbf{estimate(radar}j\mathbf{,ac}i\mathbf{)}}, \text{'nearest'})$$

15. Compute four vectors of reported aircraft x and y positions, one for each radar tracking each aircraft. This is based on the reported range and azimuth offset by the location bias.

$$\mathbf{xac}_{\mathbf{reported(radar}j\mathbf{,ac}i\mathbf{)}} = \mathbf{range}_{\mathbf{reported(radar}j\mathbf{,ac}i\mathbf{)}} * \cos(\boldsymbol{\theta}_{\mathbf{reported(radar}j\mathbf{,ac}i\mathbf{)}}) + \mathbf{xradar}_{\mathbf{surveyerror(radar}j\mathbf{)}}$$

$$\mathbf{yac}_{\mathbf{reported(radar}j\mathbf{,ac}i\mathbf{)}} = \mathbf{range}_{\mathbf{reported(radar}j\mathbf{,ac}i\mathbf{)}} * \sin(\boldsymbol{\theta}_{\mathbf{reported(radar}j\mathbf{,ac}i\mathbf{)}}) + \mathbf{yradar}_{\mathbf{surveyerror(radar}j\mathbf{)}}$$

16. Compute four vectors of errors in the reported aircraft x and y positions.

$$\mathbf{dxac}_{\mathbf{(radar}j\mathbf{,ac}i\mathbf{)}} = \mathbf{xac}_{\mathbf{reported(radar}j\mathbf{,ac}i\mathbf{)}} - \mathbf{xac}_{\mathbf{true(ac}i\mathbf{)}}$$

$$\mathbf{dyac}_{\mathbf{(radar}j\mathbf{,ac}i\mathbf{)}} = \mathbf{yac}_{\mathbf{reported(radar}j\mathbf{,ac}i\mathbf{)}} - \mathbf{yac}_{\mathbf{true(ac}i\mathbf{)}}$$

$$\mathbf{positionerror}_{\mathbf{(radar}j\mathbf{,ac}i\mathbf{)}} = \text{sqrt}\ (\ \mathbf{dxac}^2_{\mathbf{(radar}j\mathbf{,ac}i\mathbf{)}} + \mathbf{dyac}^2_{\mathbf{(radar}j\mathbf{,ac}i\mathbf{)}}\ )$$

17. Compute two vectors of aircraft position errors if the nearest radar is tracking the aircraft. This is accomplished by multiplying , the vectors of 0 and 1 that were defined earlier and define which of the two cases (radar 1 tracking aircraft 1 and radar 2 tracking aircraft 2, OR, radar 1 tracking aircraft 2 and radar 2 tracking aircraft 1) is true for each sample by the computed position errors

$$\mathbf{PE1 = rtcace*positionerror_{(radar1,ac1)} + rtcaseopp*\ positionerror_{(radar2,ac1)}}$$

$$\mathbf{PE2 = rtcace*positionerror_{(radar2,ac2)} + rtcaseopp*\ positionerror_{(radar1,ac2)}}$$

18. Define a two vectors of length samplesize of radar update rates $\mathbf{updaterate}_{\mathbf{radar}j}$ by random sampling from a uniform distribution between [4.0 , 5.0] seconds. Note that because this is an average computation the rotation rate is resampled for each element in the vector.

19. Determine the vector of length samplesize of maximum radar rotation rates.

$$\mathbf{maxupdaterate = max\ [\ updaterate_{radar1}\ ,\ updaterate_{radar2}\ ]}$$

Compute a vector of the clock bias error **clockbias** on the separation measurements by randomly sampling from a uniform distribution between –0.5 and 0.5 and multiplying by the maximum update rate and then multiplying by an assumed aircraft speed of 200 knots. The times of radar "hits" are uncorrelated and can differ by as much as half of the maximum update rate. The effect on apparent separation is randomly increased or decreased by the distance an aircraft travels during that time.

$$\textbf{clockbias} = (200 / 3600) * \textbf{maxupdaterate} * \textbf{random [-0.5 , 0.5]}$$

20. Compute the vector of estimated separation from the reported position reports and the clock bias error by first computing vectors for the two cases **ES1122** (estimated separation when radar 1 tracks aircraft 1 and radar 2 tracks aircraft 2) and ES1221 (estimated separation when radar 1 tracks aircraft 2 and radar 2 tracks aircraft 1) and multiplying by the appropriate radar tracking cases vectors of 0 and 1.

$$\textbf{ES1122} = \mathrm{sqrt}\ ((\textbf{xac}_{\textbf{reported(radar1,ac1)}} - \textbf{xac}_{\textbf{reported(radar2,ac2)}})^2 + (\textbf{yac}_{\textbf{reported(radar1,ac1)}} - \textbf{yac}_{\textbf{reported(radar2,ac2)}})^2) \quad + \textbf{clockbias}$$

$$\textbf{ES1221} = \mathrm{sqrt}\ ((\textbf{xac}_{\textbf{reported(radar1,ac2)}} - \textbf{xac}_{\textbf{reported(radar2,ac1)}})^2 + (\textbf{yac}_{\textbf{reported(radar1,ac2)}} - \textbf{yac}_{\textbf{reported(radar2,ac1)}})^2) \quad + \textbf{clockbias}$$

$$\textbf{ES} = \textbf{rtcase} * \textbf{ES1122} + \textbf{rtcaseopp} * \textbf{ES1221}$$

21. Compute statistics on measurement errors. Compute the mean μ and standard deviations σ of the values in the vectors **PE1,PE2** and **ES**.

Plot the results. Plot histograms of the values in the vectors **PE1,PE2** and **ES** for the range modeled and label the plots with the range and values of μ and σ.

A.3 TYPICAL ERRORS IN SEPARATION MEASUREMENT

A.3.1 Input

The x and y positions of the two radars and the two aircraft (nmi from (0,0) reference), the heading (degrees) and ground speed (knots) of the aircraft, and the sample size are input.

$\mathbf{xradar}_{true(radar1)}$, $\mathbf{yradar}_{true(radar1)}$

$\mathbf{xradar}_{true(radar2)}$, $\mathbf{yradar}_{true(radar2)}$

$\mathbf{xac}_{true(ac1)}$, $\mathbf{yac}_{true(ac1)}$, $\mathbf{head}_{(ac1)}$, $\mathbf{grdspeed}_{(ac1)}$

$\mathbf{xac}_{true(ac2)}$, $\mathbf{yac}_{true(ac2)}$, $\mathbf{head}_{(ac2)}$, $\mathbf{grdspeed}_{(ac2)}$

samplesize

A.3.2 Output

The outputs are vectors of length **samplesize** containing the position measurements errors and separation measurement errors of the aircraft as follows:

PE11 position error of radar 1 tracking aircraft 1

PE12 position error of radar 1 tracking aircraft 2

PE21 position error of radar 2 tracking aircraft 1

PE22 position error of radar 2 tracking aircraft 2

ESE1112 estimated separation with radar 1 tracking aircraft 1 and radar 1 tracking aircraft 2

ESE2122 estimated separation with radar 2 tracking aircraft 1 and radar 2 tracking aircraft 2

ESE1122 estimated separation with radar 1 tracking aircraft 1 and radar 2 tracking aircraft 2

ESE1221 estimated separation with radar 1 tracking aircraft 2 and radar 2 tracking aircraft 1

A.3.3 Modeling Procedure

1. Define Common Digitizer format vectors that will be used to round off estimates to the nearest 1/64 nautical mile in range and nearest 1/4096 scan in azimuth (radians).

$$CD_{range} = [-10/64 : 1/64 : 100]$$

$$CD_{azimuth} = [-5\pi/4 : 2\pi/4096 : 5\pi/4]$$

2. Define the update rates of the radars, **urr1** and **urr2**, by sampling once for each radar from a uniform distribution between 4.0 and 5.0 seconds. Define the maximum update rate **mur** as the larger of the two values.

3. Compute the true range (nmi) and azimuth (± radians, trig. orientation) from each of the radars to each aircraft of the aircraft. The azimuth is computed in radians positive for positive $\mathbf{yac}_{true(aci)}$ and negative for negative $\mathbf{yac}_{true(aci)}$

$$\mathbf{range}_{true(radarj,aci)} = \text{sqrt}\left((\mathbf{xac}_{true(aci)} - \mathbf{xradar}_{true(radarj)})^2 + (\mathbf{yac}_{true(aci)} - \mathbf{yradar}_{true(radarj)})^2 \right)$$

$$\boldsymbol{\theta}_{true(radarj,aci)} = \arctan\left((\mathbf{yac}_{true(aci)} - \mathbf{yradar}_{true(radarj)}) / (\mathbf{xac}_{true(aci)} - \mathbf{xradar}_{true(radarj)}) \right)$$

4. Define a location bias error for each radar by random sampling ϕ from a uniformly distributed azimuth error between [0° , 360°] and $\mathbf{location}_{bias}$ from a uniformly distributed distance error of [0 , 200] feet. Note that because this is a typical computation the location bias error is sampled only once and remains fixed for each radar throughout the remainder of the simulation. Determine the surveyed error in radar location.

$$(\mathbf{xradar}_{surveyerror(radarj)}, \mathbf{yradar}_{surveyerror(radarj)}) = (\mathbf{location}_{biasj} * \cos(\phi_j), \mathbf{location}_{biasj} * \sin(\phi_j))$$

5. Define a range bias error for each radar $\mathbf{range}_{bias(radarj)}$ by random sampling from a uniform distribution between [-30 , +30] feet. Note that because this is an typical computation the range bias is sampled once for each radar and remains fixed.

6. Define four vectors of range jitter errors, one for each radar tracking each aircraft, $\mathbf{range}_{jitter(radarj,aci)}$ by random sampling from a normal distribution with $\mu = 0$ feet and $\sigma = 25$ feet.

7. Define the transponder range bias error for each aircraft $\mathbf{xponder}_{bias(aci)}$ by random sampling from a uniform distribution between [-125 , +125] feet. Note that because this is typical computation the transponder range bias is sampled only once and remains fixed.

8. Compute vectors for the estimated range from each radar to each aircraft as the true range plus the sum of the range errors by adding the errors. Note that $\mathbf{range}_{estimate(radarj,aci)}$, $\mathbf{range}_{true(radarj,aci)}$ and $\mathbf{range}_{jitter(radarj,aci)}$ are vectors of length **samplesize** and $\mathbf{range}_{bias(radarj)}$ and $\mathbf{xponder}_{bias(aci)}$ are scalars. The true range is already in nautical miles but the errors are in feet, so they are converted to nautical miles.

$$\mathbf{range}_{estimate(radar\,j,ac\,i)} = \mathbf{range}_{true(radar\,j,ac\,i)} + (\mathbf{range}_{bias(radar\,j)} + \mathbf{range}_{jitter(radar\,j,ac\,i)} + \mathbf{xponder}_{bias(ac\,i)}) \,/\, (6076.115)$$

9. Compute vectors of the reported ranges for each radar to each aircraft as the estimated ranges rounded off to the nearest 1/64 n. mi. This is done by linearly interpolating each of the value of $\mathbf{range}_{estimate(radar\,j,ac\,i)}$ using the $\mathbf{CD}_{range}$ vector as both the x and y value for the function being interpolated and using the nearest neighbor option for the interpolation.

$$\mathbf{range}_{reported(radar\,j,ac\,i)} = \text{interp1}(\mathbf{CD}_{range}, \mathbf{CD}_{range}, \mathbf{range}_{estimate(radar\,j,ac\,i)}, \text{'nearest'})$$

10. Define a scalar value of azimuth bias error for each radar $\boldsymbol{\theta}_{bias(radar\,j)}$ by random sampling from a uniform distribution between [-0.3°, +0.3°]. Note that because this is a typical computation, the azimuth bias is sampled once for each radar and remains fixed.

11. Define vectors of length **samplesize** of azimuth jitter errors for each radar tracking each aircraft, $\boldsymbol{\theta}_{jitter(radar\,j,ac\,i)}$ by random sampling from a normal distribution with $\mu = 0°$ and $\sigma = 0.068°$.

12. Compute four vectors of length **samplesize** for the estimated azimuth for each radar tracking each aircraft as the true azimuth plus the sum of the azimuth bias and azimuth error. The true azimuth and azimuth bias are scalars and the azimuth errors are vectors of length **samplesize**.

$$\boldsymbol{\theta}_{estimate(radar\,j,ac\,i)} = \boldsymbol{\theta}_{true(radar\,j,ac\,i)} + (\boldsymbol{\theta}_{bias(radar\,j)} + \boldsymbol{\theta}_{jitter(radar\,j,ac\,i)})$$

13. Compute vectors of the reported azimuths for each radar tracking each aircraft as the estimated azimuth values rounded off to the nearest 1/4096 of a scan. This is done by linearly interpolating each of the values of $\boldsymbol{\theta}_{estimate(radar\,j,ac\,i)}$ using the $\mathbf{CD}_{azimuth}$ vector as both the x and y value of the function being interpolated and using the nearest neighbor option for the interpolation.

$$\boldsymbol{\theta}_{reported(radar\,j,ac\,i)} = \text{interp1}(\mathbf{CD}_{azimuth}, \mathbf{CD}_{azimuth}, \boldsymbol{\theta}_{estimate(radar\,j,ac\,i)}, \text{'nearest'})$$

14. Compute four vectors of reported aircraft x and y positions, one for each radar tracking each aircraft. This is based on the reported range and azimuth offset by the location bias.

$$\mathbf{xac}_{reported(radar\,j,ac\,i)} = \mathbf{range}_{reported(radar\,j,ac\,i)} * \cos(\boldsymbol{\theta}_{reported(radar\,j,ac\,i)}) + \mathbf{xradar}_{surveyerror(radar\,j)}$$

$$\mathbf{yac}_{reported(radar\,j,ac\,i)} = \mathbf{range}_{reported(radar\,j,ac\,i)} * \sin(\boldsymbol{\theta}_{reported(radar\,j,ac\,i)}) + \mathbf{yradar}_{surveyerror(radar\,j)}$$

15. Compute four vectors of errors, **PE11, PE12, PE21, PE22**, in the reported aircraft x and y positions. These vectors are returned by the function.

$$\mathbf{dxac}_{(radar\,j,ac\,i)} = \mathbf{xac}_{reported(radar\,j,ac\,i)} - \mathbf{xac}_{true(ac\,i)}$$

$$\mathbf{dyac}_{(\mathbf{radar}j,\mathbf{ac}i)} = \mathbf{yac}_{\mathbf{reported}(\mathbf{radar}j,\mathbf{ac}i)} - \mathbf{yac}_{\mathbf{true}(\mathbf{ac}i)}$$

$$\mathrm{PE}(\mathbf{radar}j,\mathbf{ac}i) = \mathrm{sqrt}\ (\ \mathbf{dxac}^2_{(\mathbf{radar}j,\mathbf{ac}i)} + \mathbf{dyac}^2_{(\mathbf{radar}j,\mathbf{ac}i)}\ )$$

16. Compute the vectors of estimated separation (before adding the error for clock bias) from the reported position reports.

$$\mathbf{ES1112} = \mathrm{sqrt}\ (\ (\mathbf{xac}_{\mathbf{reported}(\mathbf{radar1},\mathbf{ac1})} - \mathbf{xac}_{\mathbf{reported}(\mathbf{radar1},\mathbf{ac2})})^2 + (\mathbf{yac}_{\mathbf{reported}(\mathbf{radar1},\mathbf{ac1})} - \mathbf{yac}_{\mathbf{reported}(\mathbf{radar1},\mathbf{ac2})})^2)$$

$$\mathbf{ES2122} = \mathrm{sqrt}\ (\ (\mathbf{xac}_{\mathbf{reported}(\mathbf{radar2},\mathbf{ac1})} - \mathbf{xac}_{\mathbf{reported}(\mathbf{radar2},\mathbf{ac2})})^2 + (\mathbf{yac}_{\mathbf{reported}(\mathbf{radar2},\mathbf{ac1})} - \mathbf{yac}_{\mathbf{reported}(\mathbf{radar2},\mathbf{ac2})})^2)$$

$$\mathbf{ES1122} = \mathrm{sqrt}\ (\ (\mathbf{xac}_{\mathbf{reported}(\mathbf{radar1},\mathbf{ac1})} - \mathbf{xac}_{\mathbf{reported}(\mathbf{radar2},\mathbf{ac2})})^2 + (\mathbf{yac}_{\mathbf{reported}(\mathbf{radar1},\mathbf{ac1})} - \mathbf{yac}_{\mathbf{reported}(\mathbf{radar2},\mathbf{ac2})})^2)$$

$$\mathbf{ES1221} = \mathrm{sqrt}\ (\ (\mathbf{xac}_{\mathbf{reported}(\mathbf{radar1},\mathbf{ac2})} - \mathbf{xac}_{\mathbf{reported}(\mathbf{radar2},\mathbf{ac1})})^2 + (\mathbf{yac}_{\mathbf{reported}(\mathbf{radar1},\mathbf{ac2})} - \mathbf{yac}_{\mathbf{reported}(\mathbf{radar2},\mathbf{ac1})})^2)$$

17. Compute the timing error for the four combinations of radar tracking aircraft. These will be added to **ES1112**, **ES2122**, **ES1122**, and **ES1221** to calculate the apparent separation. The first step is to calculate which aircraft is "hit" "first" by the radar. Between two aircraft, the aircraft designated as hit first is the one that minimizes the time between the hits of the two aircraft. For the two cases where the same radar is tracking both aircraft it is the aircraft that is hit first while the radar is rotating clockwise. The azimuth notation is trigonometric from $-\pi$ to $+\pi$. For the two cases where different radars are tracking different aircraft, the radars are assumed uncorrelated and the aircraft that is hit first is assumed random and the maximum difference in time between hits will be a randomly sampled value for up to half the maximum update rate. The track monitoring variables, $\mathbf{tm1122}_{\mathbf{ac}i}$, $\mathbf{tm1221}_{\mathbf{ac}i}$, $\mathbf{tm1112}_{\mathbf{ac}i}$, $\mathbf{tm2122}_{\mathbf{ac}i}$, are set equal to 1 if aircraft i is hit second and thus will be the aircraft that apparently moved since the first aircraft was hit. It will be set equal to 0 if aircraft i is the first aircraft hit and will not be moved in the computation of apparent separation. Thus $\mathbf{tm1122}_{\mathbf{ac1}} = 1$ means that for radar 1 tracking aircraft 1 and radar 2 tracking aircraft 2, aircraft 1 was hit second and will be moved. $\mathbf{tm1122}_{\mathbf{ac2}}$ will necessarily be set to 0.

$$s = \mathrm{random}[\ 0\ ,\ 1\ ]$$

$$\text{if } s \le 0.5,\ \mathbf{tm1122}_{\mathbf{ac1}} = 1 \text{ and } \mathbf{tm1122}_{\mathbf{ac2}} = 0$$

$$\text{else } \mathbf{tm1122}_{\mathbf{ac1}} = 0 \text{ and } \mathbf{tm1122}_{\mathbf{ac2}} = 1$$

$$s = \text{random}[\,0\,,\,1\,]$$

if $s \le 0.5$, $\mathbf{tm1221}_{ac1} = 1$ and $\mathbf{tm1221}_{ac2} = 0$

else $\mathbf{tm1221}_{ac1} = 0$ and $\mathbf{tm1221}_{ac2} = 1$

if $(\theta_{true(radar1,ac2)} - \theta_{true(radar1,ac1)}) \ge 0 \;\&\; (\theta_{true(radar1,ac2)} - \theta_{true(radar1,ac1)}) \le \pi$

$\mathbf{tm1112}_{ac1} = 1$ and $\mathbf{tm1112}_{ac2} = 0$

else if $(\theta_{true(radar1,ac2)} - \theta_{true(radar1,ac1)}) \ge 0 \;\&\; (\theta_{true(radar1,ac2)} - \theta_{true(radar1,ac1)}) > \pi$

$\mathbf{tm1112}_{ac1} = 0$ and $\mathbf{tm1112}_{ac2} = 1$

else if $(\theta_{true(radar1,ac1)} - \theta_{true(radar1,ac2)}) > 0 \;\&\; (\theta_{true(radar1,ac1)} - \theta_{true(radar1,ac2)}) \le \pi$

$\mathbf{tm1112}_{ac1} = 0$ and $\mathbf{tm1112}_{ac2} = 1$

else if $(\theta_{true(radar1,ac1)} - \theta_{true(radar1,ac2)}) > 0 \;\&\; (\theta_{true(radar1,ac1)} - \theta_{truc(radar1,ac2)}) > \pi$

$\mathbf{tm1112}_{ac1} = 1$ and $\mathbf{tm1112}_{ac2} = 0$

if $(\theta_{true(radar2,ac2)} - \theta_{true(radar2,ac1)}) \ge 0 \;\&\; (\theta_{true(radar2,ac2)} - \theta_{true(radar2,ac1)}) \le \pi$

$\mathbf{tm2122}_{ac1} = 1$ and $\mathbf{tm2122}_{ac2} = 0$

else if $(\theta_{true(radar2,ac2)} - \theta_{true(radar2,ac1)}) \ge 0 \;\&\; (\theta_{true(radar2,ac2)} - \theta_{true(radar2,ac1)}) > \pi$

$\mathbf{tm2122}_{ac1} = 0$ and $\mathbf{tm2122}_{ac2} = 1$

else if $(\theta_{true(radar2,ac1)} - \theta_{true(radar2,ac2)}) > 0 \;\&\; (\theta_{true(radar2,ac1)} - \theta_{true(radar2,ac2)}) \le \pi$

$\mathbf{tm2122}_{ac1} = 0$ and $\mathbf{tm2122}_{ac2} = 1$

else if $(\theta_{true(radar2,ac1)} - \theta_{true(radar2,ac2)}) > 0 \;\&\; (\theta_{true(radar2,ac1)} - \theta_{true(radar2,ac2)}) > \pi$

$\mathbf{tm2122}_{ac1} = 1$ and $\mathbf{tm2122}_{ac2} = 0$

18. Compute the delta time between the hit of the first aircraft and the hit of the second aircraft. For the cases of different radars tracking different aircraft the radars are assumed uncorrelated

and the delta time is randomly chosen between zero and half the update rate of the slower radar. For the cases of the same radar tracking both aircraft the delta time is explicitly calculated.

$$\mathbf{cb\Delta t1122} = (\mathbf{mur}/2)*\text{random}\,[\,0\,,1\,]$$

$$\mathbf{cb\Delta t1221} = (\mathbf{mur}/2)*\text{random}\,[\,0\,,1\,]$$

$$\text{if } (\theta_{true(radar1,ac2)} - \theta_{true(radar1,ac1)}) \geq 0 \;\; \& \;\; (\theta_{true(radar1,ac2)} - \theta_{true(radar1,ac1)}) \leq \pi$$

$$\mathbf{cb\Delta t1112} = (\mathbf{urr1}/2\pi)*(\theta_{true(radar1,ac2)} - \theta_{true(radar1,ac1)})$$

$$\text{else if } (\theta_{true(radar1,ac2)} - \theta_{true(radar1,ac1)}) \geq 0 \;\; \& \;\; (\theta_{true(radar1,ac2)} - \theta_{true(radar1,ac1)}) > \pi$$

$$\mathbf{cb\Delta t1112} = (\mathbf{urr1}/2\pi)*[\,2\pi - (\theta_{true(radar1,ac2)} - \theta_{true(radar1,ac1)})]$$

$$\text{else if } (\theta_{true(radar1,ac1)} - \theta_{true(radar1,ac2)}) > 0 \;\; \& \;\; (\theta_{true(radar1,ac1)} - \theta_{true(radar1,ac2)}) \leq \pi$$

$$\mathbf{cb\Delta t1112} = (\mathbf{urr1}/2\pi)*(\theta_{true(radar1,ac1)} - \theta_{true(radar1,ac2)})$$

$$\text{else if } (\theta_{true(radar1,ac1)} - \theta_{true(radar1,ac2)}) > 0 \;\; \& \;\; (\theta_{true(radar1,ac1)} - \theta_{true(radar1,ac2)}) > \pi$$

$$\mathbf{cb\Delta t1112} = (\mathbf{urr1}/2\pi)*[2\pi - (\theta_{true(radar1,ac1)} - \theta_{true(radar1,ac2)})]$$

$$\text{if } (\theta_{true(radar2,ac2)} - \theta_{true(radar2,ac1)}) \geq 0 \;\; \& \;\; (\theta_{true(radar2,ac2)} - \theta_{true(radar2,ac1)}) \leq \pi$$

$$\mathbf{cb\Delta t2122} = (\mathbf{urr2}/2\pi)*(\theta_{true(radar2,ac2)} - \theta_{true(radar2,ac1)})$$

$$\text{else if } (\theta_{true(radar2,ac2)} - \theta_{true(radar2,ac1)}) \geq 0 \;\; \& \;\; (\theta_{true(radar2,ac2)} - \theta_{true(radar2,ac1)}) > \pi$$

$$\mathbf{cb\Delta t2122} = (\mathbf{urr2}/2\pi)*[2\pi - (\theta_{true(radar2,ac2)} - \theta_{true(radar2,ac1)})]$$

$$\text{else if } (\theta_{true(radar2,ac1)} - \theta_{true(radar2,ac2)}) > 0 \;\; \& \;\; (\theta_{true(radar2,ac1)} - \theta_{true(radar2,ac2)}) \leq \pi$$

$$\mathbf{cb\Delta t2122} = (\mathbf{urr2}/2\pi)*(\theta_{true(radar2,ac1)} - \theta_{true(radar2,ac2)})$$

$$\text{else if } (\theta_{true(radar2,ac1)} - \theta_{true(radar2,ac2)}) > 0 \;\; \& \;\; (\theta_{true(radar2,ac1)} - \theta_{true(radar2,ac2)}) > \pi$$

$$\mathbf{cb\Delta t2122} = (\mathbf{urr2}/2\pi)*[2\pi - (\theta_{true(radar2,ac1)} - \theta_{true(radar2,ac2)})]$$

19. Compute the apparent x and y position for the aircraft that moved, i.e. the second aircraft hit. The tm variable which has a value of 1 for the aircraft hit second and a value of 0 for the

aircraft hit first is used to determine which aircraft is actually moved. This must be computed for each combination of radar tracking each aircraft because the delta times and which aircraft is hit first depends on the which radars are tracking which airplanes. Note that heading is in degrees measured from north so delta x will be the sin of the heading and delta y the cos of the heading. The delta time is converted to hours because the ground speed is in knots. The x apparent is in nautical miles from the 0,0 reference point.

$$\mathbf{xac1112}_{apparent(aci)} = \mathbf{xac}_{true(aci)} + \mathbf{grdspeed}_{(aci)} * \mathbf{sin}((\pi/180)\mathbf{head}_{(aci)}) * \mathbf{cb\Delta t1112}/3600 * \mathbf{tm1112}_{aci}$$

$$\mathbf{xac2122}_{apparent(aci)} = \mathbf{xac}_{true(aci)} + \mathbf{grdspeed}_{(aci)} * \mathbf{sin}((\pi/180)\mathbf{head}_{(aci)}) * \mathbf{cb\Delta t2122}/3600 * \mathbf{tm2122}_{aci}$$

$$\mathbf{xac1122}_{apparent(aci)} = \mathbf{xac}_{true(aci)} + \mathbf{grdspeed}_{(aci)} * \mathbf{sin}((\pi/180)\mathbf{head}_{(aci)}) * \mathbf{cb\Delta t1122}/3600 * \mathbf{tm1122}_{aci}$$

$$\mathbf{xac1221}_{apparent(aci)} = \mathbf{xac}_{true(aci)} + \mathbf{grdspeed}_{(aci)} * \mathbf{sin}((\pi/180)\mathbf{head}_{(aci)}) * \mathbf{cb\Delta t1221}/3600 * \mathbf{tm1221}_{aci}$$

$$\mathbf{yac1112}_{apparent(aci)} = \mathbf{xac}_{true(aci)} + \mathbf{grdspeed}_{(aci)} * \mathbf{cos}((\pi/180)\mathbf{head}_{(aci)}) * \mathbf{cb\Delta t1112}/3600 * \mathbf{tm1112}_{aci}$$

$$\mathbf{yac2122}_{apparent(aci)} = \mathbf{xac}_{true(aci)} + \mathbf{grdspeed}_{(aci)} * \mathbf{cos}((\pi/180)\mathbf{head}_{(aci)}) * \mathbf{cb\Delta t2122}/3600 * \mathbf{tm2122}_{aci}$$

$$\mathbf{yac1122}_{apparent(aci)} = \mathbf{xac}_{true(aci)} + \mathbf{grdspeed}_{(aci)} * \mathbf{cos}((\pi/180)\mathbf{head}_{(aci)}) * \mathbf{cb\Delta t1122}/3600 * \mathbf{tm1122}_{aci}$$

$$\mathbf{yac1221}_{apparent(aci)} = \mathbf{xac}_{true(aci)} + \mathbf{grdspeed}_{(aci)} * \mathbf{cos}((\pi/180)\mathbf{head}_{(aci)}) * \mathbf{cb\Delta t1221}/3600 * \mathbf{tm1221}_{aci}$$

20. Compute the true separation and the apparent separation. Note for the computation of apparent separation the apparent x and y for both aircraft is used but the tracking monitor variable tm will be 0 for one of the aircraft and so the apparent x,y will be the same as the true x,y.

$$\mathbf{sep}_{true} = \mathrm{sqrt}((\mathbf{xac}_{true(ac1)} - \mathbf{xac}_{true(ac2)})^2 + (\mathbf{yac}_{true(ac1)} - \mathbf{yac}_{true(ac2)})^2)$$

$$\mathbf{sep1112}_{apparent} = \mathrm{sqrt}((\mathbf{xac1112}_{apparent(ac1)} - \mathbf{xac1112}_{apparent(ac2)})^2 + (\mathbf{yac1112}_{apparent(ac1)} - \mathbf{yac1112}_{apparent(ac2)})^2)$$

$$\mathbf{sep2122}_{apparent} = \mathrm{sqrt}((\mathbf{xac2122}_{apparent(ac1)} - \mathbf{xac2122}_{apparent(ac2)})^2 + (\mathbf{yac2122}_{apparent(ac1)} - \mathbf{yac2122}_{apparent(ac2)})^2)$$

$$\textbf{sep1122}_{\text{apparent}} = \text{sqrt}((\textbf{xac1122}_{\text{apparent(ac1)}} - \textbf{xac1122}_{\text{apparent(ac2)}})^2 + (\textbf{yac1122}_{\text{apparent(ac1)}} - \textbf{yac1122}_{\text{apparent(ac2)}})^2)$$

$$\textbf{sep1221}_{\text{apparent}} = \text{sqrt}((\textbf{xac1221}_{\text{apparent(ac1)}} - \textbf{xac1221}_{\text{apparent(ac2)}})^2 + (\textbf{yac1221}_{\text{apparent(ac1)}} - \textbf{yac1221}_{\text{apparent(ac2)}})^2)$$

21. Compute the clock bias as the difference between the apparent separation and the true separation.

$$\textbf{cb1112} = \textbf{sep1112}_{\text{apparent}} - \textbf{sep}_{\text{true}}$$

$$\textbf{cb2122} = \textbf{sep2122}_{\text{apparent}} - \textbf{sep}_{\text{true}}$$

$$\textbf{cb1122} = \textbf{sep1122}_{\text{apparent}} - \textbf{sep}_{\text{true}}$$

$$\textbf{cb1221} = \textbf{sep1221}_{\text{apparent}} - \textbf{sep}_{\text{true}}$$

22. Apply the clock bias to the separation calculated from the reported values of aircraft position.

$$\textbf{ES1112} = \textbf{ES1112} + \textbf{cb1112}$$

$$\textbf{ES2122} = \textbf{ES2122} + \textbf{cb2122}$$

$$\textbf{ES1122} = \textbf{ES1122} + \textbf{cb1122}$$

$$\textbf{ES1221} = \textbf{ES1221} + \textbf{cb1221}$$

23. Compute the separation error for each combination of radar tracking aircraft

$$\textbf{ESE1112} = \textbf{ES1112} - \textbf{sep}_{\text{true}}$$

$$\textbf{ESE2122} = \textbf{ES2122} - \textbf{sep}_{\text{true}}$$

$$\textbf{ESE1122} = \textbf{ES1122} - \textbf{sep}_{\text{true}}$$

$$\textbf{ESE1221} = \textbf{ES1221} - \textbf{sep}_{\text{true}}$$

24. Compute the mean and standard deviations of the position errors and estimated separation measurement errors.

$$\mu_{\text{separation error1112}} = \textbf{mean[ESE1112]} \quad , \quad \sigma_{\text{separation error1112}} = \textbf{std[ESE1112]}$$

$$\mu_{\text{separation error2122}} = \textbf{mean[ESE2122]} \quad , \quad \sigma_{\text{separation error2122}} = \textbf{std[ESE2122]}$$

$$\mu_{\text{separation error1122}} = \textbf{mean[ESE1122]} \quad , \quad \sigma_{\text{separation error1122}} = \textbf{std[ESE1122]}$$

$$\mu_{\text{separation error1221}} = \text{mean[ESE1221]} \quad , \quad \sigma_{\text{separation error1221}} = \text{std[ESE1221]}$$

$$\mu_{\text{position error 11}} = \text{mean[PE11]} \quad , \quad \sigma_{\text{position error 11}} = \text{std[PE11]}$$

$$\mu_{\text{position error 12}} = \text{mean[PE12]} \quad , \quad \sigma_{\text{position error 12}} = \text{std [PE12]}$$

$$\mu_{\text{position error 21}} = \text{mean[PE21]} \quad , \quad \sigma_{\text{position error 21}} = \text{std [PE21]}$$

$$\mu_{\text{position error 22}} = \text{mean[PE22]} \quad , \quad \sigma_{\text{position error 22}} = \text{std [PE22]}$$

25. Before plotting, the reported aircraft positions must be adjusted for clock bias. The presumption is that the positions input for the two aircraft are the true positions of the aircraft at the time the first aircraft is hit. The plots of aircraft reported position must adjust the reported positions reports of the second aircraft hit to adjust for clock bias. The **tm** variable insures that only the reports for the second aircraft hit are moved.

$$\mathbf{xac1112}_{\text{reported}(aci)} = \mathbf{xac}_{\text{reported(radar1},aci)} + \mathbf{grdspeed}_{(aci)} * \mathbf{sin}((\pi/180)\mathbf{head}_{(aci)}) * \mathbf{cb\Delta t1112}/3600 * \mathbf{tm1112}_{aci}$$

$$\mathbf{xac2122}_{\text{reported}(aci)} = \mathbf{xac}_{\text{reported(radar2},aci)} + \mathbf{grdspeed}_{(aci)} * \mathbf{sin}((\pi/180)\mathbf{head}_{(aci)}) * \mathbf{cb\Delta t2122}/3600 * \mathbf{tm2122}_{aci}$$

$$\mathbf{xac1122}_{\text{reported(ac1)}} = \mathbf{xac}_{\text{reported(radar1,ac1)}} + \mathbf{grdspeed}_{(ac1)} * \mathbf{sin}((\pi/180)\mathbf{head}_{(ac1)}) * \mathbf{cb\Delta t1122}/3600 * \mathbf{tm1122}_{ac1}$$

$$\mathbf{xac1122}_{\text{reported(ac2)}} = \mathbf{xac}_{\text{reported(radar2,ac2)}} + \mathbf{grdspeed}_{(ac2)} * \mathbf{sin}((\pi/180)\mathbf{head}_{(ac2)}) * \mathbf{cb\Delta t1122}/3600 * \mathbf{tm1122}_{ac2}$$

$$\mathbf{xac1221}_{\text{reported(ac1)}} = \mathbf{xac}_{\text{reported(radar2,ac1)}} + \mathbf{grdspeed}_{(ac1)} * \mathbf{sin}((\pi/180)\mathbf{head}_{(ac1)}) * \mathbf{cb\Delta t1221}/3600 * \mathbf{tm1221}_{ac1}$$

$$\mathbf{xac1221}_{\text{reported(ac2)}} = \mathbf{xac}_{\text{reported(radar1,ac2)}} + \mathbf{grdspeed}_{(ac2)} * \mathbf{sin}((\pi/180)\mathbf{head}_{(ac2)}) * \mathbf{cb\Delta t1221}/3600 * \mathbf{tm1221}_{ac2}$$

$$\mathbf{yac1112}_{\text{reported}(aci)} = \mathbf{yac}_{\text{reported(radar1},aci)} + \mathbf{grdspeed}_{(aci)} * \mathbf{cos}((\pi/180)\mathbf{head}_{(aci)}) * \mathbf{cb\Delta t1112}/3600 * \mathbf{tm1112}_{aci}$$

$$\mathbf{yac2122}_{\text{reported}(aci)} = \mathbf{yac}_{\text{reported(radar2},aci)} + \mathbf{grdspeed}_{(aci)} * \mathbf{cos}((\pi/180)\mathbf{head}_{(aci)}) * \mathbf{cb\Delta t2122}/3600 * \mathbf{tm2122}_{aci}$$

$$\mathbf{yac1122}_{reported(ac1)} = \mathbf{yac}_{reported(radar1,ac1)} + \mathbf{grdspeed}_{(ac1)} * \cos((\pi/180)\mathbf{head}_{(ac1)}) * \mathbf{cb\Delta t1122}/3600 * \mathbf{tm1122}_{ac1}$$

$$\mathbf{yac1122}_{reported(ac2)} = \mathbf{yac}_{reported(radar2,ac2)} + \mathbf{grdspeed}_{(ac2)} * \cos((\pi/180)\mathbf{head}_{(ac2)}) * \mathbf{cb\Delta t1122}/3600 * \mathbf{tm1122}_{ac2}$$

$$\mathbf{yac1221}_{reported(ac1)} = \mathbf{yac}_{reported(radar2,ac1)} + \mathbf{grdspeed}_{(ac1)} * \cos((\pi/180)\mathbf{head}_{(ac1)}) * \mathbf{cb\Delta t1221}/3600 * \mathbf{tm1221}_{ac1}$$

$$\mathbf{yac1221}_{reported(ac2)} = \mathbf{yac}_{reported(radar1,ac2)} + \mathbf{grdspeed}_{(ac2)} * \cos((\pi/180)\mathbf{head}_{(ac2)}) * \mathbf{cb\Delta t1221}/3600 * \mathbf{tm1221}_{ac2}$$

26. Create four figures containing plots of the position estimates and true position of the aircraft and a histogram of the separation estimation errors, one for each combination of each radar tracking each aircraft. Print out the mean and standard deviations of the errors on the graphs.

A.4 SPECIFIC ERRORS IN DISPLAYED SEPARATION

A.4.1 Input

The x and y positions of two radars and the initial position of two aircraft (nmi from (0,0) reference), the heading (degrees) and ground speed (knots) of the aircraft, and the run time in seconds are input.

$\mathbf{xradar}_{true(radar1)}$, $\mathbf{yradar}_{true(radar1)}$

$\mathbf{xradar}_{true(radar2)}$, $\mathbf{yradar}_{true(radar2)}$

$\mathbf{xac}_{initial(ac1)}$, $\mathbf{yac}_{initial(ac1)}$, $\mathbf{head}_{(ac1)}$, $\mathbf{grdspeed}_{(ac1)}$

$\mathbf{xac}_{initial(ac2)}$, $\mathbf{yac}_{initial(ac2)}$, $\mathbf{head}_{(ac2)}$, $\mathbf{grdspeed}_{(ac2)}$

runtime

A.4.2 Output

The outputs are vectors that containing a list of the position measurements errors and matrices of the displayed separation measurement errors as a function of time. The matrices of displayed separation measurement errors contain times in column 1 corresponding to an update by either radar and the displayed separation error at that time. The displayed errors as a function of time will be linear between entries. The length of the vectors and matrices will depend on the individual radar rotation rates, movement of the aircraft, and runtime. The output vectors and matrices are:

PE11 position error of radar 1 tracking aircraft 1

PE12 position error of radar 1 tracking aircraft 2

PE21 position error of radar 2 tracking aircraft 1

PE22 position error of radar 2 tracking aircraft 2

ESE1112 displayed separation with radar 1 tracking aircraft 1 and radar 1 tracking aircraft 2 vs. t

ESE2122 displayed separation with radar 2 tracking aircraft 1 and radar 2 tracking aircraft 2 vs. t

ESE1122 displayed separation with radar 1 tracking aircraft 1 and radar 2 tracking aircraft 2 vs. t

ESE1221 displayed separation with radar 1 tracking aircraft 2 and radar 2 tracking aircraft 1 vs. t

A.4.3 Modeling Procedure

1. Define Common Digitizer format vectors that will be used to round off estimates to the nearest 1/64 nautical mile in range and nearest 1/4096 scan in azimuth (radians).

$$\mathbf{CD}_{\mathbf{range}} = [-10/64 : 1/64 : 100]$$

$$\mathbf{CD}_{\mathbf{azimuth}} = [-5\pi/4 : 2\pi/4096 : 5\pi/4]$$

2. Define the update rates of the radars, **urr1** and **urr2**, by sampling once for each radar from a uniform distribution between 4.0 and 5.0 seconds.

3. Define the radar start times **tsr1** and **tsr2** by sampling once for each radar between 0 and the update rate of that radar.

4. Compute the samplesizes for the two radar hits as the number of rotations during the runtime. **fix** rounds down to the next lowest integer.

$$\mathbf{samplesize}_i = \mathbf{fix}((\mathbf{runtime\text{-}tsr}_i)/\mathbf{urr}_i)$$

5. Compute vectors of the times of the hits of each radar i on each aircraft j **th*ij***. This is done by finding the zeros of the function describing the difference between the time dependent angle of the radar and the time dependent equation describing the aircraft's angular relationship to the radar. The zeros of the function describing the difference in these angles is found within specified limits corresponding to each single rotation of the radar.

The aircraft x and y positions as a function of time are described by:

$$\mathbf{ac}_j\mathbf{x(t)} = \mathbf{xac}_{\mathbf{initial(ac}j)} + (\mathbf{grdspeed}_{(\mathbf{ac}j)} / 3600) * \mathbf{t} * \sin(\mathbf{head}_{(\mathbf{ac}j)} * (\boldsymbol{\pi/180}))$$

$$\mathbf{ac}_j\mathbf{y(t)} = \mathbf{yac}_{\mathbf{initial(ac}j)} + (\mathbf{grdspeed}_{(\mathbf{ac}j)} / 3600) * \mathbf{t} * \cos(\mathbf{head}_{(\mathbf{ac}j)} * (\boldsymbol{\pi/180}))$$

and the aircraft position angle relative to the radar as:

$$\arctan((\mathbf{ac}_j\mathbf{y(t)} - \mathbf{yradar}_{\mathbf{true(radar}i)}) / (\mathbf{ac}_j\mathbf{x(t)} - \mathbf{xradar}_{\mathbf{true(radar}i)}))$$

The angle of the radar is expressed as a discontinuous function of time **t** between $-\pi$ and $+\pi$ for each rotation i of the radar starting with $i = 1$:

$$\pi - 2\pi((\mathbf{t} - (\mathbf{tsr}_{\mathbf{radar}i} + (i - 1) * \mathbf{urr}_{\mathbf{radar}i})) / \mathbf{urr}_{\mathbf{radar}i})$$

The times of hit for radar i on aircraft j **thij** consists of finding the zeros of the difference of the angles, or solving, for each rotation k of the radar i, the equation:

$$[\,\pi - 2\pi\,(\ (\mathbf{t} - (\mathbf{tsr}_{\mathbf{radar}i} + (k - 1) * \mathbf{urr}_{\mathbf{radar}i}\,))\,/\,\mathbf{urr}_{\mathbf{radar}i}\,)\,] - [\arctan\,((\mathbf{ac}_j\mathbf{y(t)} - \mathbf{yradar}_{\mathbf{true(radar}i)}\,)\,/\,(\mathbf{ac}_j\mathbf{x(t)} - \mathbf{xradar}_{\mathbf{true(radar}i)}\,)\,] = 0$$

6. Next compute the vectors of true x an y positions of the aircraft j when they are hit by radar i

$$\mathbf{xac}ij_{\mathbf{true}} = \mathbf{xac}_{\mathbf{initial(ac}j)} + (\mathbf{grdspeed}_{(\mathbf{ac}j)} / 3600) * \mathbf{th}ij * \sin(\mathbf{head}_{(\mathbf{ac}j)} * (\pi/\mathbf{180}))$$

$$\mathbf{yac}ij_{\mathbf{true}} = \mathbf{yac}_{\mathbf{initial(ac}j)} + (\mathbf{grdspeed}_{(\mathbf{ac}j)} / 3600) * \mathbf{th}ij * \cos(\mathbf{head}_{(\mathbf{ac}j)} * (\pi/\mathbf{180}))$$

7. The geometry of the radars and the true positions of the aircraft when hit by the radars is plotted.

8. The next step is to plot the apparent separation caused by the "clock bias" or fact that the aircraft are hit at different times and are moving between display updates. This is the separation that appears on the radar screen as a function of time. This is done for each of the four combinations of radar tracking aircraft (i.e. radar 1 tracking aircraft 1 and radar 2 tracking aircraft 2.)

8 a. First create combination vectors of the hit times. **tcom1112** is a vector for all of the hit times for radar 1 hitting aircraft 1 and radar 1 hitting aircraft 2. This is created by concatenating and sorting **th11** and **th12**. Create **tcom1112 tcom2122 tcom1122** and **tcom1221**.

8 b. Eliminate any duplicate entries in **tcom1112 tcom2122 tcom1122** and **tcom1221**.

8 c. Replicate each time hit entry and shift the value of one entry **th** - ε and one entry **th** + ε and throw out the first entry in the vector. These vectors, **tcomrep1112**, **tcomrep2122**, **tcomrep1122** and **tcomrep1221** contain a sorted list of hit times of both aircraft except one entry is just before the hit and one entry is just after the hit. This allows linear interpolation holding separation values constant between update times.

8 d. Similarly create replicated time vectors of the individual vectors **th11**, **th12**, **th21**, and **th22** that contain values of one entry **th** - ε and one entry **th** + ε and throw out the first entry in the vector. These vectors are called **th11rep**, **th12rep**, **th21rep**, and **th22rep**.

8 e. Create replicated vectors **xac11truerep**, **xac12truerep**, **xac21truerep**, **xac22truerep**, **yac11truerep**, **yac12truerep**, **yac21truerep**, **yac22truerep** for the true x and y positions of the aircraft when they are hit by each radar and eliminate the final entry.

8 f. Create matrices **DISP1112, DISP2122**, **DISP1122**, **DISP1221** of the displayed positions for each combination of radar tracking aircraft, i.e. **DISP1122** is a matrix for the case of radar 1

tracking aircraft 1 and radar 2 tracking aircraft 2. The first column contains the replicated times for all hits, **tcomrep** for the respective tracking case, i.e. the first column of **DISP1122** contains **tcomrep1122**. The second, third, fourth, and fifth columns are the x and y values of aircraft 1 and 2 respectively at the times in column 1. These are obtained by linearly interpolating the replicated values of x and y versus the replicated times. For example, column 2 of **DISP1122** contains the x values of aircraft 1 at the times in column 1, **tcomrep1122,** obtained by linearly interpolating **th11rep** versus **xac11truerep** for the times in **tcomrep1122**. The sixth and seventh columns contain the Δx and Δy values of the aircraft and the eight column contains the displayed separation (with no radar errors) as a function of the time in column 1. The replication of times $\pm\varepsilon$ allows for the displayed separation to be constant between updates as it would be displayed on a radar screen.

8 g. Create a function for the true separation of the aircraft as a function of time based on their initial positions, and constant headings and groundspeeds.

8 h. Plot the displayed target separation with no radar error, due only to clock bias, versus the true separation as a function of time for all combinations of radar tracking aircraft. The difference between the displayed and true separation is due only to the movement of the aircraft between radar hits.

9. Compute the true ranges and azimuths at the times of radar hit for all combinations of radars tracking aircraft. $\mathbf{range}\boldsymbol{ij}_{\mathbf{true}}$ and $\boldsymbol{\Theta ij}_{\mathbf{true}}$ refer to range and azimuth for radar i hitting aircraft j at the times radar i hit aircraft j.

$$\mathbf{range}_{\mathbf{true(radar}i\mathbf{,ac}j\mathbf{)}} = \mathrm{sqrt}((\mathbf{xac}\boldsymbol{ij}_{\mathbf{true}} - \mathbf{xradar}_{\mathbf{true(radar}i\mathbf{)}})\hat{}2 + (\mathbf{yac}\boldsymbol{ij}_{\mathbf{true}} - \mathbf{yradar}_{\mathbf{true(radar}i\mathbf{)}})\hat{}2)$$

$$\boldsymbol{\Theta}_{\mathbf{true(radar}i\mathbf{,ac}j\mathbf{)}} = \arctan((\mathbf{yac}\boldsymbol{ij}_{\mathbf{true}} - \mathbf{yradar}_{\mathbf{true(radar}i\mathbf{)}})/(\mathbf{xac}\boldsymbol{ij}_{\mathbf{true}} - \mathbf{xradar}_{\mathbf{true(radar}i\mathbf{)}}))$$

10. Define radar site location bias errors $\mathbf{xradar}_{\mathbf{surveyerror(radar}i\mathbf{)}}$, $\mathbf{yradar}_{\mathbf{surveyerror(radar}i\mathbf{)}}$ by sampling once for each radar from a uniform random direction [0-360°] and a uniform random distance [0-200]feet and convert to nautical miles.

11. Define a range bias error for each radar $\mathbf{range}_{\mathbf{bias(radar}i\mathbf{)}}$ by a random sampling from a uniform distribution between [-30 , +30] feet once for each radar.

12. Define four vectors of range jitter errors, one for each radar tracking each aircraft, $\mathbf{range}_{\mathbf{jitter(radar}i\mathbf{,ac}j\mathbf{)}}$ by random sampling from a normal distribution with $\mu = 0$ feet and $\sigma = 25$ feet. Sample once for each radar hit $\mathbf{th}\boldsymbol{ij}$. The lengths of the vectors will be different depending on how many times a radar hit an aircraft.

13. Define the transponder range bias error for each aircraft $\mathbf{xponder}_{\mathbf{bias(ac}j\mathbf{)}}$ by random sampling once for each aircraft from a uniform distribution between [-125 , +125] feet.

14. Compute vectors for the estimated range from each radar to each aircraft for each hit as the true range plus the sum of the range errors by adding the errors.

$$\mathbf{range}_{estimated(radar\mathit{i},ac\mathit{j})} = \mathbf{range}_{true(radar\mathit{i},ac\mathit{j})} + (\mathbf{range}_{bias(radar\mathit{i})} + \mathbf{range}_{jitter(radar\mathit{i},ac\mathit{j})} + \mathbf{xponder}_{bias(ac\mathit{j})}) / (6076.115)$$

15. Compute vectors of the reported ranges for each radar to each aircraft as the estimated ranges rounded off to the nearest 1/64 n. mi. This is done by linearly interpolating each of the value of $\mathbf{range}_{estimated(radar\mathit{i},ac\mathit{j})}$ using the $\mathbf{CD}_{range}$ vector as both the x and y value for the function being interpolated and using the nearest neighbor option for the interpolation.

$$\mathbf{range}_{reported(radar\mathit{i},ac\mathit{j})} = \text{interp1}(\mathbf{CD}_{range}, \mathbf{CD}_{range}, \mathbf{range}_{estimated(radar\mathit{i},ac\mathit{j})}\text{'nearest'})$$

16. Define a value for radar azimuth bias error for each radar $\boldsymbol{\theta}_{bias(radar\mathit{i})}$ by random sampling from a uniform distribution between [-0.3°, +0.3°]. Note that because this is a typical computation, the azimuth bias is sampled once for each radar and remains fixed.

17. Define vectors of the length of **th*ij*** of azimuth jitter errors for each radar tracking each aircraft, $\boldsymbol{\theta}_{jitter(radar\mathit{i},ac\mathit{j})}$ by random sampling from a normal distribution with $\mu = 0°$ and $\sigma = 0.068°$. Convert to radians.

18. Compute four vectors of the length of **th*ij*** for the estimated azimuth for each radar tracking each aircraft as the true azimuth plus the sum of the azimuth bias and azimuth error. The true azimuth and azimuth bias are scalars and the azimuth errors are vectors of the length of **th*ij***.

$$\boldsymbol{\theta}_{estimate(radar\mathit{i},ac\mathit{j})} = \boldsymbol{\theta}_{true(radar\mathit{i},ac\mathit{j})} + (\boldsymbol{\theta}_{bias(radar\mathit{i})} + \boldsymbol{\theta}_{jitter(radar\mathit{i},ac\mathit{j})})$$

19. Compute vectors of the reported azimuths for each radar tracking each aircraft as the estimated azimuth values rounded off to the nearest 1/4096 of a scan. This is done by linearly interpolating each of the values of $\boldsymbol{\theta}_{estimate(radar\mathit{i},ac\mathit{j})}$ using the $\mathbf{CD}_{azimuth}$ vector as both the x and y value of the function being interpolated and using the nearest neighbor option for the interpolation.

$$\boldsymbol{\theta}_{reported(radar\mathit{i},ac\mathit{j})} = \text{interp1}(\mathbf{CD}_{azimuth}, \mathbf{CD}_{azimuth}, \boldsymbol{\theta}_{estimate(radar\mathit{i},ac\mathit{j})}, \text{'nearest'})$$

20. Compute the vectors of reported x y positions for both aircraft for both radars.

$$\mathbf{xac\mathit{ij}}_{reported} = \mathbf{range}_{reported(radar\mathit{i},ac\mathit{j})} * \cos(\boldsymbol{\theta}_{reported(radar\mathit{i},ac\mathit{j})}) + \mathbf{xradar}_{surveyerror(radar\mathit{i})}$$

$$\mathbf{yac\mathit{ij}}_{reported} = \mathbf{range}_{reported(radar\mathit{i},ac\mathit{j})} * \sin(\boldsymbol{\theta}_{reported(radar\mathit{i},ac\mathit{j})}) + \mathbf{yradar}_{surveyerror(radar\mathit{i})}$$

21. Compute and return the vectors of distance errors in reported position of the aircraft.

$$\Delta\mathbf{xac\mathit{ij}} = \mathbf{xac\mathit{ij}}_{reported} - \mathbf{xac\mathit{ij}}_{true}$$

$$\Delta \mathbf{yac}ij = \mathbf{yac}ij_{\mathbf{reported}} - \mathbf{yac}ij_{\mathbf{true}}$$

$$\mathbf{PE}ij = \mathrm{sqrt}(\Delta \mathbf{xac}ij \text{ ^2} + \Delta \mathbf{yac}ij \text{ ^2})$$

22. Plot the geometry of the radars and the displayed positions of the aircraft when hit by the radars.

23. Compute and plot the apparent or displayed separation versus time for position reports that include the errors and the clock bias.

23 a. Create replicated vectors for the reported x and y position reports of the aircraft when they are hit by each radar and eliminate the final entry. **xac11reportedrep, xac12reportedrep, xac21reportedrep, xac22reportedrep, yac11reportedrep, yac12reportedrep, yac21reportedrep, yac22reportedrep**.

23 b. Create matrices **ADISP1112**, **ADISP2122**, **ADISP1122**, **ADISP1221** of the displayed positions for each combination of radar tracking aircraft, i.e. **ADISP1122** is a matrix for the case of radar 1 tracking aircraft 1 and radar 2 tracking aircraft 2. The first column contains the replicated times for all hits, **tcomrep** for the respective tracking case, i.e. the first column of **ADISP1122** contains **tcomrep1122**. The second, third, fourth, and fifth columns are the reported values of x and y for aircraft 1 and 2 respectively at the times in column 1 which are displayed on the radar screen. These are obtained by linearly interpolating the replicated values of reported x and y created in step 23a. versus the replicated times. The replication of times and position reports is necessary to hold the displayed values of x and y constant between updates. For example, column 2 of **ADISP1122** contains the reported (displayed) x values of aircraft 1 at the times in column 1, **tcomrep1122,** obtained by linearly interpolating **th11rep** versus **xac11reportedrep** for the times in **tcomrep1122**. The sixth and seventh columns contain the Δx and Δy values of the aircraft and the eight column contains the displayed separation including radar errors as a function of the time in column 1. The replication of times $\pm\varepsilon$ allows for the displayed separation to be constant between updates as it would be displayed on a radar screen.

23 c. Plot the displayed (reported) target separation as a function of time versus the true separation for all combinations of radar tracking aircraft. The difference between the displayed and true separation is due to both position reporting errors and the movement of the aircraft between hits by the radar.

24. Create and return the matrices of errors in the separation measurements.

ESE1112 displayed separation with radar 1 tracking aircraft 1 and radar 1 tracking aircraft 2 vs. t

ESE2122 displayed separation with radar 2 tracking aircraft 1 and radar 2 tracking aircraft 2 vs. t

ESE1122 displayed separation with radar 1 tracking aircraft 1 and radar 2 tracking aircraft 2 vs. t

ESE1221 displayed separation with radar 1 tracking aircraft 2 and radar 2 tracking aircraft 1 vs. t

24 a. Use temporary matrices **TESE** for computations and return two columns in the form of **ESE**. The first column of **TESE1112, TESE2122, TESE1122,** and **TESE1221** contain the first column of the respective **ADISP** matrix, the replicated times for all radar hits.

24 b. Compute the true values of x and y for aircraft 1 and 2 at the times in column 1 and place them in columns 2 through 5 respectively of **TESE1112**, **TESE2122**, **TESE1122**, and **TESE1221** using the aircraft initial positions, groundspeeds and headings.

24 c. Compute the delta x and delta y values of between the aircraft and put them in columns 6 and 7 respectively of **TESE1112, TESE2122, TESE1122,** and **TESE1221**.

24 d. Compute the actual separation from the delta x and delta y values and place in columns 8 of **TESE1112, TESE2122, TESE1122,** and **TESE1221**.

24 e. Copy the displayed separation from column 8 of **ADSIP1112, ADSIP2122, ADSIP1122, ADSIP1221**, into column 9 of **TESE1112, TESE2122, TESE112,** and **TESE1221**.

24 f. Compute the error between the displayed separation in column 9 and the actual separation in column 8 and place the separation error in column 10 of **TESE1112, TESE2122, TESE112,** and **TESE1221**.

24 g. Return the matrices of estimated separation errors versus time, **ESE1112, ESE2122, ESE1122,** and **ESE1221** by copying the times from column 1 into column 1 and the errors in displayed separation from column 10 into column 2 from **TESE1112, TESE2122, TESE112,** and **TESE1221** respectively. The **ESE** matrices will contain the replicated times in column 1 and the separation errors in column 2. The errors will jump with the discontinuity of the update and will vary linearly between updates. For instance, if aircraft are maintaining a constant separation then the error will be constant until the next update, but if the aircraft are changing separation there will be a linear change in error as a faction of time until the next discontinuity at the next update.