Reinforcement-Learning-Based Cooperative Adaptive Cruise Control of Buses in the Lincoln Tunnel Corridor with Time-Varying Topology

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Abstract—The exclusive bus lane (XBL) is one of the most popular bus transit systems in US. The Lincoln Tunnel utilizes an XBL through the tunnel in the AM peak period. This paper proposes a novel data-driven cooperative adaptive cruise control (CACC) algorithm that aims to minimize a cost function for connected and autonomous buses along the XBL. Different from existing model-based CACC algorithms, the proposed approach employs the idea of reinforcement learning (RL), which does not rely on accurate knowledge of bus dynamics. Considering a time-varying topology where each autonomous vehicle can only receive information from preceding vehicles that are within its communication range, a distributed controller is learned real-time by online headway, velocity, and acceleration data collected from system trajectories. The convergence of the proposed algorithm and the stability of the closed-loop system are rigorously analyzed. The effectiveness of the proposed approach is demonstrated using a well-calibrated Paramics microscopic traffic simulation model of the XBL corridor. Simulation results show that the travel times in the autonomous version of the XBL are close to the present day travel times even when the bus volume is increased by 30%.

Index Terms—Reinforcement learning, connected and autonomous vehicles, cooperative adaptive cruise control, time-varying topology

I. INTRODUCTION

The Lincoln Tunnel is an 80-year-old, 1.5-mile-long tunnel that connects Weehawken, New Jersey and Manhattan, New York. It is one of the busiest crossings in the United States, carrying approximately 40 million vehicles in 2016 [1]. It also serves as one of the major bus transit corridors that directly connects to the Port Authority Bus Terminal (PABT) in midtown Manhattan, New York. The Lincoln Tunnel Exclusive Bus Lane, known as the XBL, is a 2.5-mile contra-flow bus lane traveling along New Jersey Route 495, leading from the New Jersey Turnpike to the Lincoln Tunnel [2]. The XBL acts as a one-lane isolated contraflow bus lane that operates only between 6-10AM every day and is separated from the oncoming traffic by cylindrical traffic delineators [3]. The XBL caters to an average of 1850 buses daily during peak hours (22% of peak-hour vehicles) and carries approximately 89% of peak-hour customers in the Lincoln Tunnel; see [2], [4].

While the XBL is one of the most successful and productive bus rapid transit systems in the United States, the practical capacity of the XBL has often been exceeded during morning peak hours, resulting in considerable delays for the buses [5]. Fig. 1 shows an example of the congestion along the XBL. The Port Authority of New York and New Jersey (PANYNJ) predicts that the demand of daily buses using PABT will increase by 15% by 2040. This increasing demand urges the agencies to explore supply/demand strategies that can “increase or manage capacity along the Lincoln Tunnel Corridor - either by improving corridor operations or PABT facility operations [4]”. A prior study completed in September 2016, entitled “Trans-Hudson Commuting Capacity Study”, evaluates various strategies to meet the projected 2040 trans-Hudson commuter demand, taking into account the conceptual planning of PABT replacement [4]. This prior study suggests opportunities to develop new technologies, especially connected and autonomous vehicle technologies [6], to improve the operational efficiency of the Lincoln Tunnel Corridor by dispatching in real time. The use of platooning buses via adaptive cruise control (ACC) or cooperative adaptive cruise control (CACC) technology could significantly increase the throughput of the existing XBL without any huge investment.

ACC is an enhancement of the classical cruise control algorithms that can reduce driver workload and fatigue. It automatically regulates the dynamics of individual vehicles by measuring the headway and velocity via on-board sensors, such as radars, lasers and video cameras. It is an example of application of real-time control techniques on traffic systems [7]–[9]. Ozbay et al [10], [11] study the feasibility of automating the exclusive bus lane (XBL) in the Lincoln tunnel corridor using three different adaptive cruise controllers developed in Paramics simulation network, namely P controller, PI controller, and PID controller. These elementary linear controllers are achieved by adjusting the speed of the bus based on its speed and spacing measurements with respect to the bus ahead. The performance of each controller and the travel times are examined, and the results are compared with the simulation model that describes the traffic scenarios in...
2005 with approximately 1,700 buses on the XBL during the peak period and a peak hour volume of 730 buses [3], [10]. The results have shown that all three controllers performed much better than the human driver in the simulation model. In addition, all the three controllers are able to stop the bus quickly and safely in the simulations in case of emergency. A qualitative cost analysis indicates that automation of the bus lane is beneficial.

CACC is an extended version of ACC. In comparison with the sensor-based ACC, the CACC technology has a longer communication range that enables it to alert drivers about potential collisions. This is more beneficial in some blind intersections. Additionally, CACC usually considers a longitudinal platoon of vehicles, and well-formed platoon with smaller time headway manipulated by CACC can further reduce aerodynamic drag, which leads to fuel saving.

In the last decade, several vehicular platoon control algorithms have been developed through model-based control theory. For instance, an optimal connected cruise control algorithm has been developed for a platoon of mixed human-driven and autonomous vehicles [12]. Considering the state and input constraints, the model predictive control (MPC) has been employed to design CACC algorithms for vehicular platoons [13], [14]. The robust MPC strategy has been utilized for CACC design in the presence of model uncertainties [15]. The effect of delay on the platoon performance has also been studied [12], [16], [17]. In our previous work [18], a data-driven adaptive optimal control approach has been proposed for a platoon of human-operated and autonomous vehicles in the presence of input delay.

The effectiveness of CACC on safety, traffic flow, and the environment has also been tested in different traffic scenarios with human-driven and autonomous vehicles [19]–[23]. Shladover et al. have compared the impact of CACC and ACC on freeway traffic flow [21]. They claim that ACC marginally changes the lane capacity and that, while CACC can significantly improve the capacity. Moreover, the capacity increases as the penetration of autonomous vehicles increases. This is consistent with the conclusion drawn in [20]. The authors of [20] study a freeway lane drop as a shockwave induced by a disturbance limits the traffic capacity. They disclose the fact that a high CACC-penetration rate (> 60%) can improve traffic flow while mitigating the serious shockwave effect, especially in the condition of high traffic volume. This is attributed to the increased average speed, dramatic reduction of the number of shock waves, and low speed variance. The influence of CACC on both traffic congestion and environment is investigated in [22]. Specifically, they explore the possibility of integrating both the CACC and intelligent traffic signals. The benefits of implementing the integrated algorithm are expected to reduce the traffic delay by up to 91% and reduce fuel consumption by up to 75%.

Although CACC has been extensively studied by many researchers, there are still some open issues when one implements this technology in connected buses on XBL. First, the existing CACC methods are usually model-based. The first step in the model-based control method is often building a mathematical model for the plant in question. However, system modeling often involves modeling errors. A controller designed based on an inaccurate model may destabilize the vehicle system, which threatens the safety of the autonomous buses and their surrounding traffic. Robust control can stabilize the vehicle system if there are errors or uncertainties in the model, but this compromises the system’s transient response. Second, in order to ameliorate the transient performance and to reduce the fuel usage of the autonomous buses, one hopes to design an optimal controller through minimizing some predefined cost functions. Nevertheless, a common feature of most optimal control methods is that they rely on the accurate knowledge of the system model. Last but not least, in the framework of connected vehicles and CACC, each vehicle is supposed to receive information from preceding vehicles (active vehicles) who stay within the communication range. Notice that the number of active vehicles usually varies since the relative distances between buses change with time. This renders the communication topology as time-varying, which is usually a challenge for the adaptive/optimal control design.

In order to overcome these technical obstacles, this paper proposes a novel data-driven distributed control strategy for connected and autonomous buses on XBL. To be more specific, the proposed control strategy employs the idea of reinforcement learning (RL) [24]–[32]. RL is an active research branch of artificial intelligence, which is viewed as a practically sound data-driven optimal control approach as well. The main feature of RL is that it is able to approximate the optimal control strategy and the corresponding cost function in an iterative fashion, without accurate knowledge of the vehicle dynamics. This feature helps overcome two main drawbacks of the traditional dynamic programming, i.e., the curse of dimensionality [33] and curse of modeling [34].

The contributions of this paper are listed as follows.

1) A novel data-driven CACC method for connected and autonomous buses is proposed through RL. This is different from most existing model-based CACC methods [12]–[14], [16], [17] in that the former essentially relies on the collected online headway, velocity, and acceleration data. The proposed RL algorithm can be used to learn the optimal controllers with a satisfactory transient response in the absence of prior knowledge of vehicle dynamics.

Fig. 1. Bus in the XBL (Taken in June, 2017)
2) This paper distinguishes itself from our previous work [35], [36] through combining the ideas of RL and distributed control [37]. Compared with centralized control, distributed control strategies do not rely on the assumption that each vehicle can communicate with a central location and share information by a fully connected network, which reduces communication cost. Also, it is widely accepted that centralized control is fragile as it requires the information of all the agents in the network. In this sense, the distributed control strategy is more robust to communication failure or information loss, as it uses only the information of neighboring agents within the communication range.

3) To the best of our knowledge, the time-varying topology, or more accurately the state-dependent connectivity [38], is considered for the CACC design for the first time. The connectivity of vehicles depends on the relative distance and communication range, making this approach more reflective of real-world conditions. The data-driven controller design is constructive, while the closed-loop system stability and optimality analyses are rigorously completed in this scenario.

The remainder is organized as follows. In Section II, we develop a Paramics micro-traffic simulation model, which is calibrated by real traffic data. In Section III, the data-driven CACC is designed along with stability and optimality analyses. The traffic simulation results are included in Section IV. Concluding remarks and future work are contained in Section V.

Notations. Throughout this paper, $\mathbb{C}^-$ stands for the open left-half complex plane. $|\cdot|$ represents the Euclidean norm for vectors and the induced norm for matrices, $\otimes$ indicates the Kronecker product operator and $\text{vec}(A) = [a_1^T, a_2^T, \cdots, a_m^T]^T$, where $a_i \in \mathbb{R}^n$ are the columns of $A \in \mathbb{R}^{n \times m}$. For a symmetric matrix $P \in \mathbb{R}^{n \times m}$, $\text{vec}(P) = [p_{11}, 2p_{12}, \cdots, 2p_{1m}, p_{22}, 2p_{23}, \cdots, 2p_{m-1, m}, p_{mm}]^T \in \mathbb{R}^{\frac{1}{2}m(m+1)}$. For an arbitrary column vector $v \in \mathbb{R}^m$, $|v|_P$ stands for $v^T P v$, and $\text{vec}(v) = [v_1^T, v_2^T, \cdots, v_1^T, v_2^T, v_3^T, \cdots, v_{m-1}^T, v_m^T]^T \in \mathbb{R}^{\frac{1}{2}m(m+1)}$. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and the minimum eigenvalue of a symmetric matrix $P$, respectively.

II. MICROSCOPIC TRAFFIC SIMULATION MODEL OF XBL

Paramics is a micro-traffic simulation software that is widely used for the study of traffic operations. The software allows users to setup vehicles, traffic signals, ramps, and so forth. Fig. 2 shows the network simulation model that is built in Paramics. The Paramics model simulates the one-lane bus-only XBL from 6AM to 10AM. The one-lane structure restricts any merge, split or lane changing behaviors for buses. There are no interactions between buses and general traffic on the XBL.

First, we develop the skeleton network by placing nodes and connecting these nodes using links. Other stages include assigning priorities at junctions, defining vehicle types, setting restrictions and demands. After that, we calibrate the traffic network such that the simulation results match traffic data extracted from the Lincoln Tunnel. The Origin-Destination (O-D) matrices are estimated for 15-minute intervals based on the traffic volumes obtained from New Jersey Department of Transportation and New York City Department of Transportation, and the toll plaza demand data [39]. The current number of buses utilizing XBL is obtained from PANYNJ’s official website with 1850 buses during peak period [2]. Taking advantage of buses equipped with GPS devices, the travel time and headway information of buses are extracted via New Jersey Transit (NJT) “MyBus Now” platform [40], a real-time service information system that provides estimated vehicle arrival times and map locations for NJT buses. A web scraper that behaves as a “virtual sensor” [41] is developed to retrieve real-time bus schedules and delays for the corresponding bus lines from “MyBus Now” [42]. More details about the virtual sensor methodology can be found in [41]. One month of bus data (Apr. to May, 2017) are processed and cleaned. The model is then calibrated by modifying certain features such as reaction times and safe headways so that it replicates the existing traffic conditions. Ten simulation runs are performed with different random seeds so that the stochasticity associated with the microsimulation model can be minimized. Geoffrey E. Heavers (GEH) statistic is used to compare the field bus volumes with simulation bus volumes. Ten simulation results with different random seeds show that 91% observed GEHs were less than 5. The difference between field travel times and model travel times remains less than 10% on all time intervals except two occasions when it increases to 11%–13%. Moreover, the mean of model travel time $\mu_M$ is selected to compare with the mean of field travel time $\mu_F$ with two-sample $t$-test. It shows that the hypothesis $\mu_M = \mu_F$ is accepted at 5% significant level. These results indicate that the calibrated network is consistent with that of the field traffic conditions. More information on issues related to large scale traffic calibration in general can be found in the previous study [43]. Note that the travel times collected through NJT MyBus Now application are for buses traveling from the teardrop on the New Jersey side (entry point of XBL, refer to Fig. 2) to PABT at Manhattan. The same entering and exit points are used to collect travel time from the micro-simulation model.
III. CONTROLLER DESIGN

In this Section, we first present the formulation of the CACC problem. Then, we propose a novel data-driven distributed controller design algorithm. The stability and optimality of the closed-loop system are rigorously analyzed.

Consider a platoon of $n$ autonomous buses. Let $h_i$ be the headway of the $i$th vehicle in the platoon, i.e., the bumper-to-bumper distance between the $i$th vehicle and the $(i-1)$th vehicle. Time headway $[44]$ and constant spacing $[45]$ are two main spacing policies in the CACC. This paper adopts the former policy since there are different speed limits in each snap of the road. The desired headway is chosen by $h_i^* = \tau v_i(t) + h_0$, where $\tau$ is the time headway and $h_0$ is named the standstill headway. Define $\Delta h_i(t) = h_i(t) - h_i^*$ and $\Delta v_i(t) = v_i - v_i(t)$. By $[46]$, the dynamics of the $i$th vehicle can be described by

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + D_i x_{i-1}(t)$$  \hspace{1cm} (1)

where $u_i$ represents the desired acceleration of vehicle $i$. For $j = i-1, i$, $x_k = [\Delta h_k, \Delta v_k, a_k]^T$ includes the headway and velocity errors, and the acceleration of vehicle $k$. The system matrices are

$$A_i = \begin{bmatrix} 0 & 1 & -\tau \\ 0 & 0 & -1 \\ 0 & 0 & -T_i \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, D_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

with $G_i$ the system gain and $T_i$ the system time constant.

Assume there is a fictitious vehicle running before the leading vehicle in a constant velocity. In this setting, the state $x_1$ of the leader in the platoon also satisfies the equation (1) with $x_0 \equiv 0$. Given the system (1), define a time-varying digraph $\mathcal{G}(t) = \{\mathcal{V}, \mathcal{E}(t)\}$, $\mathcal{V} = \{1, \cdots, n\}$ is the node set. $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$ refers to the edge set. If the distance between vehicle $i$ and $k$ is smaller than the minimum between the communication range of vehicle $k$ and $i$ at time $t$, then the edge $(k, i) \in \mathcal{E}(t)$. Denote $\mathcal{N}_i(t)$ the set of all the nodes $k$ such that $(k, i) \in \mathcal{E}(t)$. For instance, the set $\mathcal{N}_i(t) = \{i-1, i-2\}$ at time $t$ since there are only two preceding vehicles staying in the communication range of vehicle $i$ in Fig. 3. Interestingly, this time-varying topology depends on the system state, which is named state-dependent connectivity $[38]$. Moreover, the communication topology is unidirectional in this paper, which means each bus is only able to receive its preceding vehicle in the communication range.

One of control goals in this paper is to let the headway and velocity errors and acceleration of each bus asymptotically converge to zero, i.e., $\lim_{t \to \infty} x_i(t) = 0$ for $i = 1, 2, \cdots, n$. In order to improve the transient response of the system, one can design an optimal controller through minimizing the following cost function

$$J = \int_0^\infty (x^T Q x + u^T R u) d\tau$$  \hspace{1cm} (2)

where $Q = \text{blockdiag}(Q_1, Q_2, \cdots, Q_n)$, $R = \text{blockdiag}(R_1, R_2, \cdots, R_n)$, $x = [x^T_1, x^T_2, \cdots, x^T_n]^T$, and $u = [u_1, u_2, \cdots, u_n]^T$. By linear optimal control theory $[47]$, there exists an optimal controller is $u = -K^* x$ such that the cost function (2) achieves its minimum $J^* = x^* P^* x$. However, this optimal controller is centralized, which cannot be implemented in the vehicular platoon with state-dependent connectivity condition. Taking this scenario into consideration, if all the system matrices are available, then one can design the following model-based suboptimal distributed controller for each autonomous bus:

$$u_i(t) = -K_i^* \zeta_i(t)$$  \hspace{1cm} (3)

where

$$\zeta_i(t) = \frac{1}{|\mathcal{N}_i(t)|} \sum_{k \in \mathcal{N}_i(t)} (x_i(t) - x_k(t))$$  \hspace{1cm} (4)

In (4), $|\mathcal{N}_i(t)|$ denotes the cardinality of the set $\mathcal{N}_i(t)$. If $\mathcal{N}_i(t) = \emptyset$, we split the platoon by letting vehicle $i$ be a new leader operating in a constant speed. It is merged to the previous platoon until time $T_i$ when $\mathcal{N}_i(T_i) \neq \emptyset$.

Choosing weight matrices such that $Q_i, R_i > 0$, the desired control gain for the $i$th vehicle can be computed by

$$K_i^* = R_i^{-1} B_i^T P_i^*$$  \hspace{1cm} (5)

where matrix $P_i^*$ indicates the solution to the following algebraic Riccati equation

$$A_i^T P_i^* + P_i^* A_i + Q_i - P_i^* B_i R_i^{-1} B_i^T P_i^* = 0.$$  \hspace{1cm} (6)

However, the model-based control approaches (3) are hard to implement since identifying the system dynamics accurately is usually a challenging task. Also, it is almost impossible to know the system matrices of all the vehicles considering different types and conditions of vehicles on the road. We will propose a data-driven control approach to learn the distributed controller (3) without knowledge of system matrices $A_i$ and $B_i$.

A. Data-Driven CACC Design

To begin with, we rewrite the system (1) as

$$\dot{x}_i = (A_i - B_i K_{ij}) x_i + B_i (K_{ij} x_i + u_i) + D_i x_{i-1}$$  \hspace{1cm} (7)

where, for $j = 1, 2, \cdots$, the control gain $K_{ij}$ is a stabilizing control gain.

Let $P_{ij}$ be the unique solution to the following Lyapunov equation

$$(A_i - B_i K_{ij})^T P_{ij} + P_{ij} (A_i - B_i K_{ij})^T + Q_i + K_{ij}^T R_i K_{ij} = 0,$$  \hspace{1cm} (8)
and $K_{i,j+1}$ be updated by
\begin{equation}
K_{i,j+1} = R_i^{-1}B_i^TP_{i,j},
\end{equation}

Along the solutions to (7), by equations (8)-(9), it follows that
\begin{equation}
x_i(t + \delta t)P_{ij}x_i(t + \delta t) - x_i(t)P_{ij}x_i(t) = t
\end{equation}
\begin{equation}
+ 2\left\{u_i + K_{ij}x_i\right\}^T (Q_i + K_{ij}^TR_iK_{ij}) x_i d\tau
+ 2\left\{u_i + K_{ij}x_i\right\}^T R_iK_{i,j+1}x_i d\tau
+ 2\left\{u_i + K_{ij}x_i\right\}^T [I \otimes K_{ij}^T_Ri] x_i d\tau
+ 2\left\{u_i + K_{ij}x_i\right\}^T D_i^TP_{i,j}x_i d\tau.
\end{equation}

By Kronecker product representation, we obtain
\begin{equation}
x_i(t)P_{ij}x_i = \left(x_i^T \otimes x_i^T\right) \text{vec} (Q_i + K_{ij}^TR_iK_{ij}),
\end{equation}
\begin{equation}
x_i(t)D_i^TP_{i,j} = \left(x_i^T \otimes x_i^T\right) \text{vec} (D_i^TP_{i,j}),
\end{equation}
\begin{equation}
\left\{u_i + K_{ij}x_i\right\}^T R_iK_{i,j+1}x_i = \left[\left(x_i^T \otimes x_i^T\right)(I \otimes K_{ij}^T_Ri) + \left(x_i^T \otimes x_i^T\right)(I \otimes R_i)\right] \text{vec} (K_{ij+1}x_i - K_{ij}x_i)\nonumber
\end{equation}
\begin{equation}
\left\{u_i + K_{ij}x_i\right\}^T [I \otimes K_{ij}^T_Ri] x_i d\tau + 2\left\{u_i + K_{ij}x_i\right\}^T D_i^TP_{i,j}x_i d\tau.
\end{equation}

Moreover, for any two vectors $a \in \mathbb{R}^n_a, b \in \mathbb{R}^n_b$ and a sufficiently large number $s > 0$, define
\begin{equation}
\delta_a = [\text{vec}(a(t_1)) - \text{vec}(a(t_0)), \ldots, \text{vec}(a(t_s)) - \text{vec}(a(t_{s-1}))]^T \in \mathbb{R}^{n_a(n_a + 1)},
\end{equation}
\begin{equation}
\Gamma_{a,b} = \begin{bmatrix}
t_1 & t_2 & \ldots & t_s \\
a \otimes b d\tau & a \otimes b d\tau & \ldots & a \otimes b d\tau
\end{bmatrix}^T \in \mathbb{R}^{s \times n_an_b},
\end{equation}
\begin{equation}
(10) \text{ implies the following linear equation}
\end{equation}
\begin{equation}
\Psi_i^{(j)} \begin{bmatrix} \text{vec}(P_{ij}) \\ \text{vec}(K_{ij+1}) \\ \text{vec}(D_i^TP_{ij}) \end{bmatrix} = \Phi_i^{(j)}
\end{equation}
\begin{equation}
\text{where}
\end{equation}
\begin{equation}
\Psi_i^{(j)} = [\delta_x, -2\Gamma_{x_i,x_i} (I \otimes K_{ij}^T_Ri) - 2\Gamma_{x_i,u_i} (I \otimes R_i),
-2\Gamma_{x_i,x_i-1} \in \mathbb{R}^{s \times 18},
\Phi_i^{(j)} = - \Gamma_{x_i,x_i} \text{vec} (Q_i + K_{ij}^T_RiK_{ij}) \in \mathbb{R}^s.
\end{equation}

The full column rank of $\Psi_i^{(j)}$ is guaranteed under some mild conditions [48], [49] similar to persistent excitation, which may be satisfied by adding some exploration noise into the control input. In this way, the solution to (11) can be uniquely obtained via
\begin{equation}
\begin{bmatrix} \text{vec}(P_{ij}) \\ \text{vec}(K_{ij+1}) \\ \text{vec}(D_i^TP_{ij}) \end{bmatrix} = \left(\Psi_i^{(j)} \Psi_i^{(j)}\right)^{-1} \Psi_i^{(j)} \Phi_i^{(j)}
\end{equation}

Now, we are ready to propose our data-driven CACC algorithm, Algorithm 1 to approximate the control gain $K_i^*$ in the distributed CACC (13).

\textbf{Algorithm 1 Data-Driven CACC Algorithm}
\begin{enumerate}
\item $i \leftarrow 1$
\item \textbf{repeat}
\item Apply an initial control policy $u_i = -K_{i0}x_i + \epsilon_i$ with exploration noise $\epsilon_i$ and $K_{i0}$ a stabilizing control gain.
\item $j \leftarrow 0$
\item \textbf{repeat}
\item Solve $P_{ij}$ and $K_{i,j+1}$ from (12) via online input-state data.
\item $j \leftarrow j + 1$
\item \textbf{until} $|P_{ij} - P_{i,j-1}| < \epsilon_i$ with $\epsilon_i$ a small positive constant.
\item $j^* \leftarrow j$
\item \textbf{Obtain} the following suboptimal controller
\item $u_i^* = -K_{i,j^*} \xi_i$
\item \textbf{until} $i = n + 1$
\end{enumerate}

The convergence of the proposed Algorithm 1 is analyzed in the following.

\textbf{Theorem 1.} For $i = 1, 2, \ldots, n$, the sequences $\{P_{ij}\}_{j=0}^\infty$ and $\{K_{ij}\}_{j=1}^\infty$ computed through Algorithm 1 converge to $P_i^*$ and $K_i^*$, respectively.

\textbf{Proof.} For all $i = 1, 2, \ldots, n$, it is checkable that matrices $P_{ij}$ and $K_{ij}$ that are uniquely determined by (12) solve equations (8)-(9). By reference [50], as the iteration $j \to \infty$, matrices $P_{ij}$ and $K_{ij}$ solving from (8)-(9) converge to matrices $P_i^*$ and $K_i^*$, respectively. This immediately implies the convergence of sequences $\{P_{ij}\}_{j=0}^\infty$ and $\{K_{ij}\}_{j=1}^\infty$. The proof is thus completed.

\textbf{Remark 1.} Note that Algorithm 1 employs the idea from the policy iteration method in reinforcement learning; see [25]. The step 6 includes both policy evaluation and policy improvement.

\section{B. Stability and Optimality Analysis}

We will analyze the stability of the closed-loop system in the following theorem.

\textbf{Theorem 2.} The origin of the system (1) in closed-loop with (13) learned by data-driven control Algorithm 1 is an exponentially stable equilibrium.

\textbf{Proof.} For the vehicle $i$, the closed-loop system can be described in the following form
\begin{equation}
\dot{x}_i = (A_i - B_iK_{i,j^*}^*)x_i + \sum_{k \in N_i(t)} \frac{1}{|N_i(t)|} B_iK_{i,j^*}^*x_k + D_i x_{i-1}
\end{equation}
where, given a small enough constant $\epsilon_i$, the matrix $A_i - B_iK_{i,j^*}^*$ is always a Hurwitz matrix.
For the leading vehicle, we can always find $\beta_1, \lambda_1 > 0$ such that

$$|x_1(t)| = |e^{(A_1 - B_1 K_{i,j}^*)^T} x_1(0)| \leq \beta_1 e^{\lambda_1 t} |x_1(0)|$$

due to the fact that $x_0 \equiv 0$.

For the second vehicle, we can always find $\beta_2, \lambda_2 > 0$ such that

$$|x_2(t)| = |e^{(A_2 - B_2 K_{i,j}^*)^T} x_2(0)| + |B_2 K_{i,j}^* + D_2|$$

$$|x_1(t)| e^{(A_2 - B_2 K_{i,j}^*)^T} x_1(0) d\tau|$$

$$\leq \beta_2 e^{\lambda_2 t} \left| \begin{bmatrix} x_1^T(0) \\ x_2^T(0) \end{bmatrix} \right|^T.$$

Similarly, we can observe that, for $i = 2, 3, \cdots, n$, there always exist $\beta_i > 0, \lambda_i > 0$ such that

$$|x_i(t)| \leq \beta_i e^{\lambda_i t} \left| \begin{bmatrix} x_i^T(0) \\ x_{i+1}^T(0) \end{bmatrix} \right|^T$$

which directly implies that the closed-loop system is exponentially stable. The proof is completed.

The following theorem compares the minimum cost $J^* = x^T(0) P^* x(0)$ with the cost $J^\dagger$ associated with the distributed controller (13).

**Theorem 3.** There exists a constant $\mu > 0$ such that $J^\dagger \leq \mu J^*.$

**Proof.** Write the closed-loop platooning system in a compact form

$$\dot{x} = A_i(t) x$$  \hspace{1cm} (14)

where $x = \begin{bmatrix} x_1^T \\ x_2^T \\ \cdots \\ x_n^T \end{bmatrix}.$

The exponential stability of this system has been shown in Theorem 2. Therefore, the state transition matrix $\Phi(\tau, t)$ satisfies $|\Phi(\tau, t)| \leq e^{\lambda(\tau - t)}, \forall \tau \geq t \geq 0.$ Let $P^\dagger(t) = \int_t^\infty \Phi^T(t, \tau) P^\dagger(\tau) \Phi(t, \tau) d\tau.$ It is checkable that there exists some $c_1, c_2 > 0$ such that $c_1 |\phi|^2 \leq \phi^T P^\dagger(\tau) \phi \leq c_2 |\phi|^2,$ which implies that $P^\dagger(\tau)$ is positive definite and upper bounded by some $sup_{\tau \geq 0} |P^\dagger(\tau)| < c_3.$ The definition of $P^\dagger(\tau)$ shows that it is symmetric and continuously differentiable. By the fact that $\frac{d\phi}{dt} = -\phi^T A_i(t) \phi, \Phi(\tau, \tau) = I,$ and Leibniz integral rule, we have

$$\dot{P}^\dagger(t) = \int_t^\infty \frac{\partial}{\partial \tau} \Phi^T(t, \tau) \Phi(\tau, t) d\tau - \Phi^T(\tau, \tau) \Phi(\tau, \tau)$$

$$= - P^\dagger(t) A_i(t) - A_i^T(t) P^\dagger(t) - I.\hspace{1cm} (15)$$

Along the solutions of (14), from (15), we have

$$\int_0^\infty |x|^2 |d\tau| \leq x^T(0) P^\dagger(0) x(0) \leq c_2 |x(0)|^2.$$

Then, the cost $J^\dagger$ is bounded above by

$$J^\dagger \leq \lambda_M(Q) + n |R| \left( \max_i |K_{i,j}^*| \right)^2 \int_0^\infty |x|^2 |d\tau|$$

$$\leq \frac{c_2 \left[ \lambda_M(Q) + n |R| \left( \max_i |K_{i,j}^*| \right)^2 \right]}{\lambda_m(P^\dagger)} J^*$$

$$:= \mu J^*.$$

The proof is thus completed.

**Fig. 4.** Paramics simulation architecture

**Fig. 5.** Snapshot of buses on XBL in closed-loop with data-driven CACC controller, simulation in Paramics

**IV. MICROSCOPIC TRAFFIC SIMULATION RESULT AND ANALYSIS**

The proposed data-driven CACC Algorithm 1 is implemented in the micro-traffic simulation. Similar to reference [35], the data-driven controller is programmed via application programming interface (API) provided by Paramics. The API allows users to override the existing car following and other driver behavior characteristics [51]. There are four kinds of interfaces, named getting a value from (QPG), setting a value (QPS), overriding (QPO), extended (QPX). When we finish the simulation, we collect simulation data to compare the control performance of the proposed data-driven control algorithm with that of the manual control. Fig. 4 shows how the control Algorithm 1 is implemented in Paramics simulation.

Note that the details of Paramics internal models are not exactly known by us. That is exactly why we use Paramics to validate the proposed data-driven idea. Fig. 5 is a snapshot of buses on XBL (with green shape) by using data-driven control Algorithm 1. These buses operate with roughly the same headway.

Notice that Paramics uses its own micro-traffic model with a number of heuristic decision making rules for lane changing and exiting to mimic actual human decision making in practice. Hence, we let the default Paramics control method output the manual control performance. In order to minimize the stochasticity of the simulation model, five simulation runs are performed with different random seeds.

For the purpose of simulation, the online position, velocity,
and acceleration data of buses are collected to learn the value matrix $P_{ij}$ and control gain $K_{ij}$ with the vehicle index $i$ and the iteration $j$. In order to test the convergence of Algorithm 1, a platoon of 4 buses is taken as an example. The comparison of learned value matrix $P_{ij}$ and corresponding optimal value matrix $P^*_i$ (solving from ARE (6)) of the $i$th bus is depicted in Fig. 8, while the comparison of learned control gain $K_{ij}$ and the optimal gain $K^*_i$ is given in Fig. 9. It shows that the stopping criterion for all the subsystems is satisfied in less than 15 iterations.

In this experiment, the communication range is set as $r_i = 300m$. The desired time headway is $\tau = 1.25s$. We test the effect of the platoon size on the travel time of autonomous buses. The result is illustrated in Tab. I with respect to different platoon size from $p = 3$ to $p = 7$. It is shown in the last row that the average travel time is ordered by $\bar{T}_1 < \bar{T}_4 < \bar{T}_3 < \bar{T}_5 < \bar{T}_6$. It is suggested that the platoon size is set as $p = 4$ instead of a larger platoon size $p = 7$ since the former is able to reduce the computational load with only a slight increase of travel time, i.e., $\bar{T}_4 - \bar{T}_7 = 0.016$ min.

Fig. 6 shows histograms of velocity and spacing errors of the buses on the XBL lane under manual control mode, while Fig. 7 shows histograms under data-driven CACC Algorithm 1. The velocity error distribution of both cases are bell-shaped. The spacing errors of both cases tend to generate distributions skewed to the left. The standard variations of velocity and headway errors under manual control mode are 1.8493 and 153.98, respectively. The standard variations of velocity and headway errors under CACC algorithm are 0.9799 and 127.23, both of which are smaller than that of manual control mode. The standard deviations of velocity and headway have been used as surrogate safety measurements (SSM) to analyze the traffic safety; see [52] and references therein. Hence, it implies that the CACC is able to potentially reduce the crash risk.
The proposed data-driven method performs better than the PID control method used in [10] from both theoretical and experimental perspectives. Theoretically, the proposed method can ensure the stability of the connected bus systems in the autonomous XBL is increased to observe the increase in travel time data (field data) collected from the Lincoln Tunnel. It is depicted in Fig. 10 that the data-driven CACC algorithm does not produce as much benefit as before. During this hour, the travel time becomes close to the present case when the bus volume is increased by 20%. The maximum travel time when the bus volume is increased by 20% (16.1933 minutes) is comparable with the maximum field travel time (16.0977 minutes) from 9 AM to 10 AM.

Tab. II illustrates the corresponding average travel time for different volumes of buses under the proposed approach. The average travel time of field data is 13.0073 minutes, which is close to that of the data-driven CACC algorithm with 30% increase in bus volume, i.e. 13.0805 minutes. For the purpose of comparison, we use PID control algorithm in [10]. It shows in Fig. 10 that the bus volume was increased by 20% under PID control. The proposed data-driven CACC method is able to further improve the traffic conditions for the whole corridor.

Last but not least, we select a platoon of four buses. Let the leader begin to decelerate from a steady-state velocity 30m/s to another steady-state 25m/s at t = 50s. Fig. 11 depicts the response of the whole platoon, showing that all the vehicles can smoothly track the new desired velocity without overshoot.

V. CONCLUSIONS

In this paper, a data-driven cooperative adaptive cruise control (CACC) method is proposed for controlling a fleet of autonomous buses on the exclusive bus lane of the Lincoln Tunnel corridor by means of reinforcement learning and optimal control theories. The proposed control method is able to increase the traffic throughput and save the travel time by reducing the impact of the human drivers and by removing the assumption on the exact knowledge of the vehicle dynamics. The proposed data-driven method performs better than the PID control method used in [10] from both theoretical and experimental perspectives. Theoretically, the proposed method can ensure the stability of the connected bus systems in the

### TABLE I

<table>
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<tr>
<th>Time Period</th>
<th>$T_3$</th>
<th>$T_4$</th>
<th>$T_5$</th>
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<tr>
<td>7 : 45 – 8 : 00</td>
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<td>8 : 00 – 8 : 15</td>
<td>11.6953</td>
<td>11.6953</td>
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<td>8 : 15 – 8 : 30</td>
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<td>11.6193</td>
<td>11.6193</td>
<td>11.6193</td>
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</tr>
<tr>
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<td>13.0570</td>
<td>13.0570</td>
<td>13.0570</td>
<td>13.0570</td>
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<td>9 : 45 – 10 : 00</td>
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<td>12.4740</td>
<td>12.4740</td>
<td>12.4740</td>
<td>12.4740</td>
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<tr>
<td>Average $T_p$</td>
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<td>11.3992</td>
<td>11.4358</td>
<td>11.5309</td>
<td>11.3827</td>
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### TABLE II

<table>
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<tr>
<th>Case</th>
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<td>Field</td>
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<td>Data-Driven CACC</td>
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<tr>
<td>15% increase in Vol. Data-Driven CACC</td>
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</tr>
<tr>
<td>20% increase in Vol. Data-Driven CACC</td>
<td>12.5711</td>
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<tr>
<td>30% increase in Vol. Data-Driven CACC</td>
<td>13.0805</td>
</tr>
</tbody>
</table>

**Fig. 10.** Travel time for different bus volumes

**Fig. 11.** Velocities of the vehicular platoon

We collect the travel times every 15 minutes using the proposed data-driven CACC algorithm through taking $p = 4$. For the purpose of comparison, we use the real XBL travel time data (field data) collected from the Lincoln Tunnel. It is depicted in Fig. 10 that the data-driven algorithm is able to save travel time for buses on the XBL. The volume of buses in the autonomous XBL is increased to observe the increase in travel time. It is checkable that the travel time is overall shorter than the present case (field) while the bus volume is increased by 30% from 6:15 AM to 9 AM. For the period 9-10 AM, since the current field bus volume is already at capacity, the proposed CACC algorithm does not produce as much benefit as before. During this hour, the travel time becomes close to the present case when the bus volume is increased by 20%. The maximum travel time when the bus volume is increased by 20% (16.1933 minutes) is comparable with the maximum field travel time (16.0977 minutes) from 9 AM to 10 AM.
presence of unknown vehicle dynamics. Note that stability is a critically important factor in ensuring the safety of vehicles. Moreover, the proposed method yields an optimal controller with respect to a predefined cost. A key strategy of the proposed distributed controller scheme is the use of a vehicle-to-vehicle communication network such that each autonomous vehicle can communicate with multiple vehicles in the communication range. These observations show that the proposed method can improve transient performance for a closed-loop, connected bus system when compared with existing vehicle control methods. This contributes to smoother traffic flow and shorter travel times. The Paramics microscopic traffic simulation results show that, under the proposed control method, the travel times of buses in the autonomous exclusive bus lanes are close to the present day travel times when the bus demand is increased by 30%. Compared with the earlier PID method in [10], we further increase the traffic flow by 10%.

Future work includes developing advanced CACC methods for a connected bus system when nonlinear models are used, for mixed human-driven and autonomous vehicle environments, and for a vehicular network with changing topology. Future work also includes collecting more field data using automated image processing techniques and developing a novel application programming interface to improve the performance of the micro-simulation model.

REFERENCES


