AN AIRLIFT HUB-AND-SPOKE LOCATION-ROUTING MODEL WITH TIME WINDOWS: CASE STUDY OF THE CONUS-TO-KOREA AIRLIFT PROBLEM

THESIS

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AFIT/GOR/ENS/98M

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THESIS

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Acknowledgments

This thesis offers a combined location-routing model for strategic airlift aircraft in a hub-and-spoke configuration. It is the result of eight months of exhaustive research, and the development of linear programming code. As a result, I've personally learned a lot about mathematical programming and the Air Mobility Command's (AMC) airlift system, and it is my sincere hope this model will be useful to both AMC and follow-on researchers.

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David W. Cox

Table of Contents

List of Figures

List of Tables

AFIT/GOR/ENS/98M-07

Abstract

Traditionally, the United States Air Force's Air Mobility Command (AMC) has used the concept of direct delivery to airlift cargo and passengers from a point of embarkation to a point of debarkation. While this method of one onload and one offload for each cargo load makes cargo tracking, or in-transit visibility (ITV), easy, and makes deliveries quickly, it appears direct delivery has significant disadvantages in many airlift scenarios. This study develops an alternative military airlift method utilizing concepts from the hub-and-spoke configuration used in the commercial airline industry. The goals of this project are two-fold: to develop an alternative hub-and-spoke combined location-routing mixed integer programming prototype model, and then to use this model to determine what advantages a hub-and-spoke system offers, and in which scenarios it is best-suited, when compared to the direct delivery method.

Three types of bases are incorporated into the model: supply bases (hubs for the line-haul aircraft), transshipment bases (hubs for the local-delivery aircraft), and destination (demand) bases. The model features the following elements: time windows, cargo tracking capability, multiple frequency servicing, aircraft basing assignments and routing, and the selection of the optimal number of local-delivery aircraft to be used. The model incorporates ideas from the following works: the hierarchical model of Perl and Daskin (1983), time windows features of Chan (1991), combining subtour-breaking and range constraints (Kulkarni and Bhave, 1985) and multiple servicing frequency via the

clustering co-location method for binary variables (Baker, 1991). Additionally, an original approach for cargo tracking is developed and incorporated. As a case study, the CONUS-to-Korea transoceanic airlift problem is used to test the model.

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Chapter 1 Background and Problem Statement

1.1 Background and Motivation for This Thesis

In 1995, Major Tom White, an analyst with the Air Mobility Command (AMC) Studies and Analysis Flight, hypothesized that there may be situations where a hub-and-spoke airlift network, similar to those in use today in the civilian airline sector, may be more efficient than the traditional direct delivery method favored for decades by AMC. In a nutshell, direct delivery seeks to use one onload (at the point of embarkation, or POE) and one offload (at the point of debarkation, or POD) for each piece of cargo. This method makes the tracking of cargo easy. The system envisioned by White employed transshipment bases between the POE and POD where cargo could be transloaded (transferred) from larger cargo aircraft with longer ranges to smaller, more efficient aircraft for in-theater delivery. He called this system the Selective Transload as Force Multiplier for Aircraft, or STAFMA. Numerous simulation runs conducted using the Korean theater of operations as a testbed indicated that, on average, the STAFMA hub-and-spoke system resulted in an increase in cargo throughput (total tonnage delivered) compared to direct delivery. In one run, the increase was more than 15 percent greater than via direct delivery. AMC has expressed interest in a deterministic model of a hub-and-spoke system to further explore this intermediary transshipment depot idea.

In that same year, the Projection Forces Division of the Office of the Secretary of Defense (PA&E) commissioned the RAND Corporation to determine the in-theater roles for the C-17 Globemaster III airlifter, which is due to completely replace the C-141 Starlifter by the year 2006. (20:viii) The results of the RAND study "suggest that the Air Force should plan for a substantial level of C-17 operations in-theater during regional contingencies". (26:xii) Interestingly enough, White used the C-17 aircraft in an intra-theater role in his STAFMA simulation in much the manner RAND recommends it be used.

This thesis fulfills AMC's desire for a deterministic, theater hub-and-spoke location-and-routing model while incorporating ideas gleaned from the RAND study. The model features many desirable elements inherent to the airlift world: time windows, multiple frequency servicing, aircraft basing and routing, cargo tracking, and selecting the optimal number of aircraft to be used in theater. The intent is to provide Air Mobility Command with a prototype hub-and-spoke location-and-routing model. The mathematical modeling formulation is written in CPLEX coding, and model runs were performed on a Sun SPARCstation 10, using version 3.0 of the CPLEX solver, at the Air Force Institute of Technology.

As a result of this study, a hub-and-spoke model now exists to help determine what advantages, and in which types of airlift situations, the hub-and-spoke network provides when compared to direct delivery. This prototype model provides a solid baseline model which can be enlarged in scale and fine-tuned for comparison with AMC's Airlift Flow Model (direct delivery via simulation) and the Naval Postgraduate/RAND Mobility Optimizer (NRMO) model (direct delivery deterministic linear programming model).

This model is unique in several respects in the air mobility literature. First, it is believed to be the only model to incorporate the idea of transloading cargo to an intermediary transshipment node between the cargo's origin and destination points. Since this transshipment action makes cargo tracking much more difficult than cargo tracking during direct delivery, a method for performing this crucial task was developed for this model.

Another difference lies in the model's incorporation of strategic <u>and</u> tactical airlift concepts. AMC historically has focused its modeling efforts on the strategic airlift problem. This approach made lots of sense when the U. S. military's main focus was to counter the Soviet threat. However, the breakup of the Soviet monolith has resulted in more attention being paid to smaller, regional conflicts, low intensity conflicts, operations other than war, etc.

> This is in response ... to the "new world order", wherein the strategic confrontation between the East and the West is now replaced by regional conflicts which can flare up at a moment's notice. Strategic mobility requirements are now over shadowed by tactical transportation demands. (13:33)

Current models, in their focus on the strategic aspect of airlift, typically aggregate individual bases in a particular region, or an entire country, into "supernodes", for simplicity, and to make the calculations more tractable. This makes these models less equipped to answer such questions as which individual airfields should be used to base individual aircraft, or which origin-destination pairs are most advantageous.

The model developed here seeks to strike a more even balance between the tactical and strategic aspects of airlift. It retains the capabilities of strategic airlift models, but utilizes individual aircraft and bases, which are critical features to possess if a detailed analysis of a more tactical airlift scenario is desired. Thus, this model offers great flexibility for a variety of military airlift scenarios.

There are several benefits in formulating hub-and-spoke as a deterministic linear integer program versus a simulation. Mathematical programming (MP) models directly provide the optimal answer to a problem. Analysts don't have to set up and interpret multiple runs of an MP model, as they must with current simulations. Additionally,

> MP models determine the optimal solution by sequentially examining the entire solution space. Therefore… they consider all possible combinations of different types and quantities of transport assets, all cargoes, and all time periods. Simulation models produce good solutions according to the quality of the embedded decision rules. However, it is unlikely that their solutions are even locally optimum let optimum over the entire range of possibilities (34:7)

Perhaps the greatest benefit an MP model provides is sensitivity analysis information. For example, dual variables from model runs indicate which constraints in the primal problem are the "most-constraining", dual prices indicate the amount by which the objective function would improve given a unit of increase in a constraint's right-hand side, and reduced costs are the amount by which the objective function coefficient of a variable would need to improve before that variable would entire the basis. Admittedly, sensitivity information is tricky with integer-valued variables, and the model herein is a mixed integer program (MIP). Nevertheless, some insights can be gained relative to sensitivity.

Furthermore, because this model does use integer-valued variables, the output (unlike NRMO) is straightforward and definitive. The model tells us exactly which aircraft are utilized, which routes they fly, how much cargo each aircraft carries, etc.

1.2 Background for the Case Study Scenario

The border between North and South Korea (the fabled 38th parallel) is arguably the most heavily defended and potentially most explosive border on the earth. Although a cease-fire has been in place for nearly 50 years, the two Koreas have never signed a truce agreement, and are technically still at war with one another. And despite the floods and famine which have somewhat tempered the threatening rhetoric from the North in the past two years, the peninsula remains a virtual powder keg of tension which could ignite quickly at any time. The United States has pledged to defend its South Korean ally from any aggression from North Korea, and currently has 37,000 troops stationed south of the border (41:11). Should any hostilities ever appear imminent, these troops would need to be re-supplied with equipment, food, ammunition, and augmented by additional troops from the United States. Since "air mobility delivers the bulk of the initial time-critical forces and supplies", airlift into South Korea would be critically important. (20:1-9)

As the Air Force's service component of the unified United States Transportation Command (USTRANSCOM), the Air Mobility Command (AMC) is tasked to provide the airlift required in any U.S. military response to hostilities anywhere on the globe. As a result, AMC would be responsible to provide the airlift to Korea, or anyplace else, as deemed necessary by the National Command Authorities. (20:2-5)

AMC has historically used a concept known as *direct delivery* in hauling cargo around the globe. This "paradigm" seeks to carry cargo from the port of embarkation (POE) to the port of debarkation (POD) utilizing the minimum number of onloads and offloads of cargo possible. This method makes "in-transit visibility" (ITV), or the tracking of cargo, as simple as possible. (20:1-19) Once cargo is onloaded to an airlifter traveling all the way from the POE to the POD, knowing exactly where the cargo is at all times is a trivial point - it can only be on the aircraft! (Note: Don't mistake the term *direct delivery* to mean "fort-to-foxhole", where cargo is delivered directly to the end-users; i.e. the Army troops in the field. Direct delivery means that cargo is airlifted directly from the supply source to an airfield in the destination country - ideally as close as possible to the troops in the field. But from the airfield the cargo must then be loaded onto trucks, rail, etc. and delivered to the foxhole. In this sense, every *direct delivery* made by AMC can be thought of as a hub-and-spoke delivery where the PODs are the hubs and the end-users (the "foxholes") are the spokes).

The traditional approach to an airlift operation into Korea via direct delivery would be if C-5s and C-17s flew the POE-to-POD routes from the CONUS to main bases in South Korea, where C-130s, perhaps some other C-17s, ground, and rail transport would distribute the cargo to the forward operating bases (FOBs) in more austere or isolated spots throughout the country. (Note: Currently, AMC still possesses approximately 150 C-141s in the active and reserve fleets. However, this aircraft is being retired, with the year 2006 being the target date for the complete retirement of the C-141 and replacement by 120 C-17s. (20:viii) In this thesis, as in White's STAFMA simulation, I'm considering the C-17, and not the C-141, for a military Hub-and-spoke system).

There are several possible problems with this direct delivery approach into Korea, however. The first problem relates to a constraint known as Maximum Aircraft on Ground, or MOG. (20:10 of Acronyms) The acronym MOG is a somewhat esoteric, and therefore, an often misunderstood, term.

> Although this term literally refers to the maximum number of aircraft which can be accommodated on the airfield (usually the parking MOG), it is often specialized to refer to the working MOG (maximum number of aircraft which can be simultaneously "worked" by maintenance, aerial port, and others), the fuel MOG (maximum number which can be simultaneously refueled) or other constraining factors. (19:6)

For example, the AMC ground time planning factors for the C-17 is 2 hours and 15 minutes, which is often abbreviated as, simply, 2 + 15. (19:18) Consider a base which we've determined has the physical space to simultaneously park 10 C-17s, but due to limitations in refueling capability, material handling equipment (MHE), power carts, transient alert personnel, etc., the base can only service 5 C-17s from arrival to departure in 2 + 15 ground time. Then the working MOG of this base is 5, not 10!

MOG constraints are among the most limiting factors to throughput in the real-world airlift system, and must be taken into account. As Chan points out,

> An important bottleneck of transportation systems is often found in the terminal environment. An example in the airlift world is the airfield where cargo is loaded or unloaded from the aircraft and a number of services may need to be performed on the aircraft. (14:25)

However, determining the MOG value of any base is difficult. It varies with the type(s) of aircraft being considered, and can even change from day to day due to manning levels, weather, operating hours, etc. Analysts from the AMC Studies and Analysis Flight recommend using a MOG for most Japanese bases of 10 (for <u>only</u> C-17s) or 5 (for <u>only</u> C-5s). (44:1) In the simple examples which follow, I arbitrarily assumed a MOG of 10 at each base in the model. (These models don't involve the use of more than 6 aircraft, so this choice had no effect on the solution and were not a limiting factor in the examples which follow). Section 4.1 will explain in detail how the MOG values used in the final model formulation were selected.

Two other problems in the CONUS-to-Korea airlift scenario are a direct result of the peculiar physical realities of the Korean theater. The distance between Korea and the CONUS (in the most direct routing possible, McChord to Osan; see Table 5 in Chapter 4) is greater than 4500 nautical miles. This distance is beyond a C-17's maximum unrefueled range, even if carrying no cargo whatsoever, and at the outer limit of an unrefueled C-5 carrying a modest load of only 46 tons after fuel reserves are accounted for. (44:1) This means that either the entire fleet of airlifters would require at least one inflight aerial refueling, or stops would have to be made enroute to Korea.

The distance between CONUS and Korea also has negative ramifications where crew duty day (CDD) is concerned. As we'll see in Chapter 4, C-5 crews flying all the way to Korea will literally run out of crew day within 2-4 hours of landing. With a planned ground offload time of 3.25 hours, this likely means the crew will have to remain overnight (RON) in country, and the plane will have to be flown out of Korea by a fresh crew, or it will take up valuable limited ramp space and aggravate the bottleneck at the POD.

Furthermore, with all types of aircraft flying into Korea and aircraft already in country, it's quite possible that the availability of fuel for airlift aircraft could be limited. If this is the case, the need for fuel could become a major constraint for any airlift operations to the peninsula, making direct delivery from the CONUS an extremely difficult prospect. Any cargo aircraft landing in Korea would need to have enough fuel to depart and recover to a refueling base or reach an aerial refueling track, or it would be stuck on the ground in Korea.

As mentioned in the previous section, White's STAFMA concept came about as a result of some serious reflection on this same CONUS-to-Korea airlift problem. Its underlying premise is to develop a hub-and-spoke system by selecting hub bases in an intermediate location (White selected Japan, which appears to be the most realistic option due to its physical location) where cargo could be transloaded from longer-range C-5 aircraft to C-17 aircraft, which are much more "MOG-efficient", and more capable of landing at short-field, austere locations.

Assumptions made in the STAFMA study include the following:

A. Under direct delivery, C-5s and C-17s would perform trunk-deliveries from Tinker AFB (an aggregated "supernode" representing all the POEs in the CONUS), with refueling stops at Elmendorf AFB, Alaska, enroute to Kimhae AB, Korea (the aggregated "supernode" POD). Return flights to the CONUS would stop at Yokota AB, Japan, and Travis AFB, for refueling enroute to Tinker AFB.

B. The maximum payloads for the C-5s and C-17s are 62 tons and 19 tons, respectively

C. The logistics departure reliability of the C-5 and C-17 are 76% and 96%, respectively

D. Under the STAFMA system, C-5s would be based at Tinker AFB (the aggregate POE), and make refueling stops at Elmendorf AFB enroute to Yokota AB (the aggregate transload base). There, cargo would be transloaded onto C-17s, which would be based at Yokota. The C-5s would return to Tinker with an enroute stop at Travis AFB. Meanwhile, C-17s from Yokota would make deliveries to Kimhae AB (the aggregate POD) and return to Yokota.

Simulation runs at AMC Studies and Analysis Flight indicate that hub-and-spoke offers the following advantages over direct delivery, in a Korean airlift scenario:

1. Increased payloads are possible using a transload concept versus Direct Delivery. In all simulation runs, cargo throughput (measured in tons per day) using STAFMA was greater than or equal to the throughput using Direct Delivery. White attributed this increase in throughput to the larger C-5 and C-17 payloads possible due to the shorter distances flown by the aircraft.
2. Transloading cargo from C-5s onto C-17s at the transshipment bases allows spare C-5 aircrews to stage out of a non-hostile environment for return flights to the CONUS. (Note: we can extend this line of thinking to include Civil Reserve Air Fleet (CRAF) carriers, who may balk at the prospect of flying into destinations in a possible threat area)
3. No fuel will now be needed in Korea by AMC aircraft
4. By using C-17s (our most MOG-efficient airlifter) to perform our deliveries into Korea, we minimize the MOG problem at the PODs

1.3 Problem Statement

Ultimately, we want to determine if, and when, the hub-and-spoke concept presents advantages over the current direct delivery system. The goals of this study are as follows:

1. Develop a prototype hub-and-spoke airlift model
2. Use the model to compare hub-and-spoke with direct delivery for the CONUS-to-Korea problem using the following metrics:

A. Inflight refueling support requirements

B. Maximum payloads available

C. Minimizing total cargo closure times

D. Maximizing aircrew productivity given CDD limitations

E. Minimizing bottlenecks in the system

F. Ability to easily track cargo from origin to destination

G. Meeting time window deadlines for cargo delivery

3. Generalize the results of this comparison for any given airlift problem by offering "rules of thumb" for determining which airlift method is better-suited for a given scenario

4. Encourage further research on the hub-and-spoke concept as an alternative to direct delivery

Since no existing deterministic models incorporate the hub-and-spoke concept, the first step towards our goal is to build one, essentially from scratch. We can borrow some features from other location-routing models, but overcoming the difficulties posed by varying model assumptions, and then linking these features together in one single model, is difficult. Furthermore, we'll need to develop an original method for cargo tracking to suit our model, since the aircraft making the final deliveries to the PODs are different from the aircraft which originally picked up the cargo at the POEs. The trick is to build a model complex enough to yield results which are realistic and useful, but simple enough to be understandable and tractable.

Chapter 2 Literature Review

2.1 Introduction

This chapter is a general overview of the literature available which is closely related to the hub-and-spoke problem. The vehicle routing problem (VRP), the facility location problem (FLP), and the combined location-routing problem (LRP) are discussed. General terminology and principles are explained, and the advantages and disadvantages of the different types of models are highlighted. Additionally, an explanation of complexity is included to highlight the difficulties inherent to solving a hub-and-spoke combined location-routing model.

2.2 Vehicle Routing Problems

The basic problem upon which all vehicle routing problems is based is the famous traveling salesmen problem (TSP). The TSP is as follows: given a salesman (or vehicle), a finite set of N nodes (destinations), and distances between these nodes, find the tour which begins at a node (you may select any node as the starting point), visits all the other (N - 1) nodes exactly once, and returns to the origin node, in the shortest cumulative distance. (23:101) This problem may not sound terribly difficult, but solution by complete enumeration becomes extremely time-consuming, as Reeves points out:

> As the starting point is arbitrary, there are clearly (N - 1)! possible solutions (or (N - 1)!/2 if the distance between every pair of cities is the same regardless of the direction

> of travel). Suppose we have a computer that can list all possible solutions of a 20 city problem in 1 hour. Then, using the above formula, it would clearly take 20 hours to solve a 21-city problem, and 17.5 days to solve a 22-city problem; a 25-city problem would take nearly 6 centuries. (39:7).

If we include more than one salesman (vehicle) and simply stipulate that, among all salesmen, all the nodes will be visited exactly once, in the shortest total distance, we now have a "multiple salesmen TSP".

Notice the similarity between the multiple salesmen TSP and the vehicle routing problem (VRP), as defined by LaPorte, Louveaux and Mercure:

> The classical vehicle routing problem (VRP) consists of optimally designing vehicle routes from one or several depots to a set of customers in such a way that:
> (i) All vehicles start and end their journey at the same depot
> (ii) All customers are served once by exactly one vehicle, but a vehicle route may include several customers
> (iii) Some side constraints on the routes are satisfied
> (iv) The sum of vehicle utilization costs and of routing costs is minimized (31:71)

Assumptions made in classical VRPs include:

A. The demands of cargo at each destination are fixed

B. The starting/ending locations (i.e. the hubs) are fixed (31:72)

Careful observation reveals that the VRP has simply added to the TSP the further dimension requiring that <u>demands</u> be delivered to each node (customer/destination). Thus, *the classical VRP is in reality a multiple TSP in which delivery requirements are placed upon the various destination points*. <u>The location of the origin(s) is known and fixed beforehand</u>, as well as the locations and demand requirements of the destination nodes. The VRP becomes a problem in selecting the optimal routes (arcs) from the origin(s) to the destinations, ensuring all destinations are visited exactly once, while meeting the destination delivery requirements. Costs, in the form of distance, time or money, are placed on each arc and the optimal solution is the one which minimizes the total cost while meeting all constraints. Additionally, it is not uncommon to incorporate vehicle "range" constraints (time or distance limitations), and capacity constraints, into VRPs.

There is an obvious problem in applying the standard VRP format to the routing of aircraft (military and civilian). The assumption that all destinations must be visited exactly once (no more, no less) is unrealistic. Military necessity certainly allows for multiple visits to any given node (destination base) and, if no supplies are required at a location on any given day, there is no sense in ensuring an aircraft flies into that base on that day. Therefore, modifications to the classical VRP formulation probably will need to be made when applying elements of this problem to U. S. Air Force mobility airlift scenarios.

2.3 Facility Location Problems

Notice above that in the VRP the location of the origin(s) is known and fixed beforehand. This is rather presumptuous. What if we want to determine the optimal facility (origin) location(s) to provide goods (cargo) to customers? This problem is the closely related facility location problem. In its generic form, this is a spatial resource allocation problem. The literature contains a host of warehouse location problems for the business world, dealing with finding the optimal locations for depots. The objective function of the general location problem minimizes costs (money, time or distance) subject to supply-demand relationships between the nodes.

In <u>Facility Location and Land Use - Multi-Criteria Analysis of Spatial-Temporal Information</u>, Chan points out "in many locational problems the cost associated with placing a facility at a certain site depends not only on the distances from other facilities and the demands, but also on the interaction with other facilities". (11:4-28) He provides a good explanation of a specific type of location problem known as the quadratic assignment problem (QAP). Here, we are given a set of fixed nodes (bases) and our goal is to determine which units (i.e. aircraft squadrons, wings, etc.) to locate where so as to minimize the movement of supplies from the hubs to the destinations (11:4-28).

Personal experience with QAPs has shown they are not viable solution methods for problems with more than four or five nodes (particularly if using a "non-industrial strength" linear programming solver such as the student version of LINDO), as *they grow <u>extremely</u> quickly in size* as the number of nodes under consideration increases. O'Kelly makes this point also, in his article "A quadratic integer program for the location of interacting hub facilities". He states that "a full enumeration (of all possible connection allocations between nodes) would involve the solution of a large number of quadratic assignment problems, which is by no means an easy computational task". (37:403)

Despite the wealth of information regarding warehouse location models, research on hub location models is far less common in the literature. Campbell makes this assertion: "Recent surveys of facility location research testify to the breadth of problems considered. One area that has so far received limited attention is hub location problems". (10:387) There are several ways to view aircraft hub location problems, as Campbell points out:

> Hub location problems may be classified by the way in which demand points are assigned, or allocated, to hubs. One possibility is single allocation, in which each demand point is allocated to a single hub (i.e. each demand point can send and receive via only a single hub). A second possibility is multiple allocation, in which a demand point may send and receive via more than one hub. (10:388)

What limited aircraft hub location literature does exist has focused on the civilian airline hub-and-spoke system. Unfortunately, a major assumption in the civilian airline industry is that travelers will want to move among all the various destinations in various directions. In the airline world, virtually every airport acts as an origin for some travelers and a destination for others. But in a military re-supply scenario, there is a definite distinction between the supply bases and destination bases. Cargo and passengers travel from supply nodes to demand (destination) nodes. There is generally a very distinctive direction of aircraft flow from supply bases to demand bases, instead of the two-way travel amongst virtually every hub/spoke and hub/hub pair of airports in the civilian airline industry. This difference makes existing civilian models difficult to adapt to a military airlift problem.

2.4 Combined Location-Routing Problems

Given the need to: A. Find the optimal locations for hubs, and then

B. Find the optimal routes from the hubs to the destinations, one's initial thought naturally is to first select hubs based upon a modified hub (facility) location model, and then use a vehicle routing model to find the optimal routing structure to deliver supplies. This sequential method will probably work for situations that consider straight line distance between the hub and destination. However, Balakrishnan, Ward and Wong point out that if an aircraft will visit several destinations in one flight, the "customer service cost" will depend on all of the customers serviced along the same vehicle route (6:37). This "actual route" cost can select different hub locations. Hence, "*the sequential solution of a classical facility location and a vehicle routing model can therefore lead to a* ***suboptimal*** *design for the distribution system*". (6:37) (italics and underlining added for emphasis)

The distinct possibility of a suboptimal solution is an important shortcoming of the sequential approach. But since location and routing decisions are closely related, analysts have begun seeking optimal approaches to find both the best locations and routings for vehicles simultaneously. Models striving for this dual goal are known as combined facility location/vehicle routing problems (or location-routing problems - LRPs - for short). LaPorte, Louveaux and Mercure define this more complex problem: "Location-routing problems (LRPs) are VRPs in which the set of depots is not known a priori. Instead, depot sites with given operating costs must be determined from a candidate set, simultaneously with the optimal delivery routes". (31:72) But there is "no free lunch" here. Balakrishnan et al. assert that, when dealing with LRPs, "such integrated models are complex and their design poses challenges in combining the short-term operational considerations of vehicle routing with the medium/long-term strategic issues of facility location". (6:35)

The general formulation of LRP models, as explained in "Integrated Facility Location and Vehicle Routing Models: Recent Work and Future Prospects", by Balakrishnan, Ward, and Wong, is as follows: (6:38)

Objective: The objective is to minimize hub operations costs plus routing costs

Subject to:

Every destination must be serviced

Standard traveling salesman constraints apply:

- the in-degree to destinations = out-degree from destinations
- subtour breaking constraints are used to eliminate illegal subtours that do not touch every depot

Forced/linking constraints

- No routes are allowed to use a hub unless aircraft are based there

Route Restrictions

- Constraint on total number of aircraft available
- Vehicle capacity constraints (weight, space limitations)
- Constraints on the number of planes available or tours that can originate from each hub
- Range (time or distance) limitations

Depot (Hub) Restrictions

- Upper and lower bounds exist on the number of hubs that can be established
- Fuel and parking availability
- Restrictions on throughput of each depot

Another type of location-routing formulation is given by Chan and is called the "multi-facility/multi-route/multi-criteria/nested (m/m/m/n)" model. It "includes multi-objective functions consisting of minimum path chains (in addition to tours) and maximal coverage, and most importantly, a nested location-routing formulation wherein both vehicle and commodity flows are analyzed". (11:9-50) The formulation is ideal for modeling civilian hub-and-spoke systems where each airport has customers wishing to travel to numerous other airports in the system. In that environment, each airport has both a supply and a demand, an assumption that is contrary to our scenario. While the m/m/m/n model could "handle" the route structure in our scenario, it would probably require some modifications to make it more conducive to the scenario we have here, where we'll have pure supply nodes, transshipment nodes with neither demand nor supply, and pure demand nodes.

The truth is, every real-world situation is unique. No "off-the-shelf" software is likely to have been built using assumptions and requirements which duplicate those of our particular scenario. The best one can usually hope for is to find a model which is designed for situations similar to the problem they wish to solve, and "tweak" it by adding or deleting features from the model. If no suitable such models exist, the analyst must build a model from scratch. As we'll see in the next sections of this chapter, we'll incorporate features from numerous existing models, and also incorporate some original ideas and methods of our own to develop a model well-suited to solve our specific problem.

2.5 Complexity Theory and Problem Classification

Before we build our model, it's helpful to understand a little bit about complexity theory and problem classification. Entire books have been written on these topics, so we'll just touch briefly on some key concepts.

The Operations Research (OR) community has come to regard a class of optimization problems as "easy" if we can develop an algorithm to solve <u>every instance</u> of the problem class in polynomial time (i.e. by an algorithm that requires a number of operations that is polynomial in the size of the input data for the problem). In reality, despite decades of research, many network and combinatorial optimization problems <u>including the TSP</u> have never been shown to be easy because no one has been able to develop an efficient algorithm to solve them. This has led the OR community to consider these problems inherently "hard" in the sense that <u>no efficient</u> algorithm could ever solve these problems. The term "NP-Completeness" has come to mean a class of optimization problems which share some generic difficulty which is beyond the capabilities of polynomial-time algorithms (1:788-789):

> The theory of NP-Completeness helps us to classify a given problem into two broad classes: (1) easy problems that can be solved by polynomial-time algorithms, and (2) hard problems that are not likely to be solved in polynomial time and for which all known algorithms require exponential running time. (1:790)

The theory also requires that problems be stated so that we can answer them with a "yes" or "no". This yes-no version of a problem is called the "recognition problem", and helps us place problems into different classes.

Problem classes include P, NP, NP-Complete and NP-Hard, where P refers to polynomial time and NP refers to non-deterministic polynomial. In a nutshell, each recognition problem belongs to one of the following four classes:

1. Class P, if some polynomial-time algorithm solves it (examples include shortest path, maximum flow, minimum cost, and assignment and matching problems)
2. Class NP, if for every "yes" instance there is a polynomial length verification that this instance is a "yes" instance
3. Class NP-Hard, if we are able to show that all problems in NP polynomially to some problem "Q", but we are unable to argue that "Q" $\in$ NP. Then, "Q" doesn't qualify to be called NP-Complete, although it is as hard as any problem in NP. (11:36 of Appendix)

4. Class NP-Complete, if: A. The recognition problem itself is in Class NP, and

B. All other problems in Class NP polynomially transform to the recognition problem (1:790-796)

(Problems that are both NP-Hard and members of Class NP are called NP-Complete) (11:36 of Appendix)

In simpler English, "NP-Hard algorithms tend to be exponential, requiring, in the worst case, a number of computations proportional to 2^N. Thus, every time a node is added, the problem size could double". (34:8). The explanations of the various categories above also suggest that NP-Complete problems are the "hardest" of all problems to solve. (39:9) Alas, the TSP belongs to Class NP-Complete, which makes it an incredibly time-consuming problem to solve (1:797) "Finding the tour among the 50 state capitals in the United States, for instance, could require many billions of years, with the fastest computer available". (11:35 of Appendix) And since the combined-location routing problem is even more complex than the TSP, it also is in the NP-Complete class. This tells us that solving such a problem to optimality (i.e. without the use of heuristics to achieve a "near optimal" answer) will take lots of time, as we'll soon see.

Chapter 3 Methodology

3.1 Introduction

This chapter is divided into two parts. The first part (sections 3.2-3.9) explains the concepts and features needed to build an LRP suitable for use in a military airlift system. This part utilizes a building block approach to show how elements such as aircraft routing, cargo tracking, time windows, hierarchical (multi-stage) basing, multiple frequency of visits, and aircraft basing assignments can be added, piece by piece, to a "generic" LRP, resulting in a hub-and-spoke airlift network model. In the second part (sections 3.10-3.11) we'll put all of these elements together to build a comprehensive hub-and-spoke combined location-routing model. We'll then use this model to examine a notional example of a CONUS-to-Korea airlift problem.

3.2 A Starting Point: The Hierarchical Model

Now that we've become somewhat familiar with the terminology and concepts of LRPs, we'll begin building a prototype model. Keep in mind that building a combined location-routing model is a slow, arduous process. As with nearly any large, complex mixed integer program, the best approach to build the model is by stages. This step-by-step approach enables the analyst to add features to the model incrementally, and enables him to ensure the model not only provides information correctly, but also provides the correct (desired) information.

Our major assumption for using a hub-and-spoke format is that large, long-range cargo aircraft will haul cargo from the supply points over the longest leg distances to intermediate (transshipment) depots. Here, the cargo will be transloaded to aircraft which are able to land at smaller, more austere fields, have better short-range performance characteristics, and are more MOG-efficient. These shorter range aircraft will carry the cargo to the final destination points, as close as possible to the foxhole.

We'll re-state the underlying premise for this case study: AMC's largest, most long-range aircraft (C-5s) will be based at supply nodes in the continental United States (CONUS), and fly out-and-back missions to the depots, located between the CONUS supply bases and the destinations in Korea. With one inflight refueling (A/R) enroute assumed at the halfway point to Korea (approximately 2360 NM after takeoff), a C-5 could carry a maximum theoretical payload of about 112 tons. (Note: The best places geographically for KC-10 and KC-135 aircraft to be based to support a refueling track midway between the CONUS and Korea are Elmendorf and Eielsen AFBs in Alaska, since the halfway point lies several hundred miles northwest of Adak, Alaska, above the Aleutian islands. The next best location in terms of proximity to the refueling track would be Hickam AFB, Hawaii.) This amounts to 77 percent of its maximum cargo weight capacity of 145.5 tons. Since airlift aircraft nearly always will reach full capacity volume-wise well before they exceed their cargo weight capacity, it's fair to say that this 112 ton maximum weight restriction will not prohibit the C-5 from carrying a full load of cargo. Therefore, we maximize the use of the C-5 as a cargo hauler if we fly approximately 4400-4800 total NM with one A/R midway.

Where is the best location for our candidate depots (transshipment points)? An ideal location geographically, due to its distance from the CONUS ($\approx$4500 NM), and its close proximity to Korea, is Japan (see Figure 1). It is here that we'll select our candidate C-17 hubs. (Note: This all assumes, of course, that Japan is willing and able to allow the United States to use its airfields in this manner. The political sensitivities of this assumption are well beyond the scope of this thesis. For our purposes, we make this assumption due the geographical advantages, and to illustrate the model).

It is in Japan, we'll thus assume, that the C-5s will transload their cargo onto a more tactically efficient aircraft with the capability of flying into smaller fields close to the U. S. Army's tactical assembly areas (TAAs). AMC's newest airlifter, the C-17, is ideally suited for this role, for several reasons. First, it was designed to be utilized for airlift missions into austere airfields. Additionally, the range/payload curve (we'll examine this more in Chapter 4) shows that the C-17 will maximize it's payload carrying capabilities on such missions. Thirdly, since the C-5 and C-17 are the only two aircraft in the U. S. inventory with the ability to carry <u>all</u> cargo types, including outsize cargo, we can confidently assume <u>all</u> cargo loads are possible for use with this model. With these points in mind, we'll incorporate C-17s into our model to fly from their depots in Japan to one or more destinations in Korea, offload the cargo, then return to Japan.

We therefore want to assign C-5s to CONUS supply bases (nodes) and base the C-17s at the transshipment nodes in Japan. Additionally, we want to select the optimal routes for both aircraft to fly to meet cargo demands in Korea. (see Figure 2)

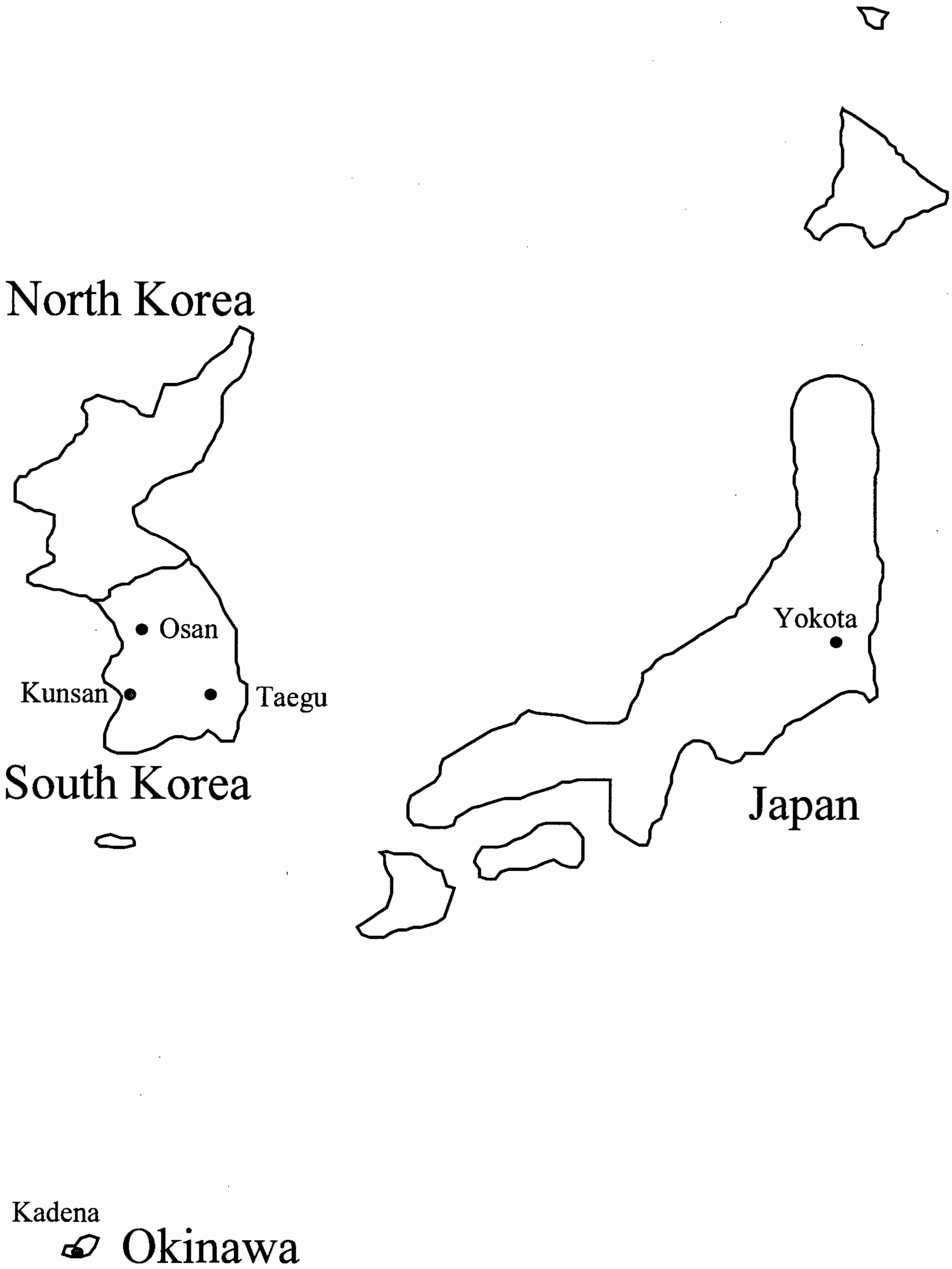


Figure 1 - Bases of Interest in Korea and Japan

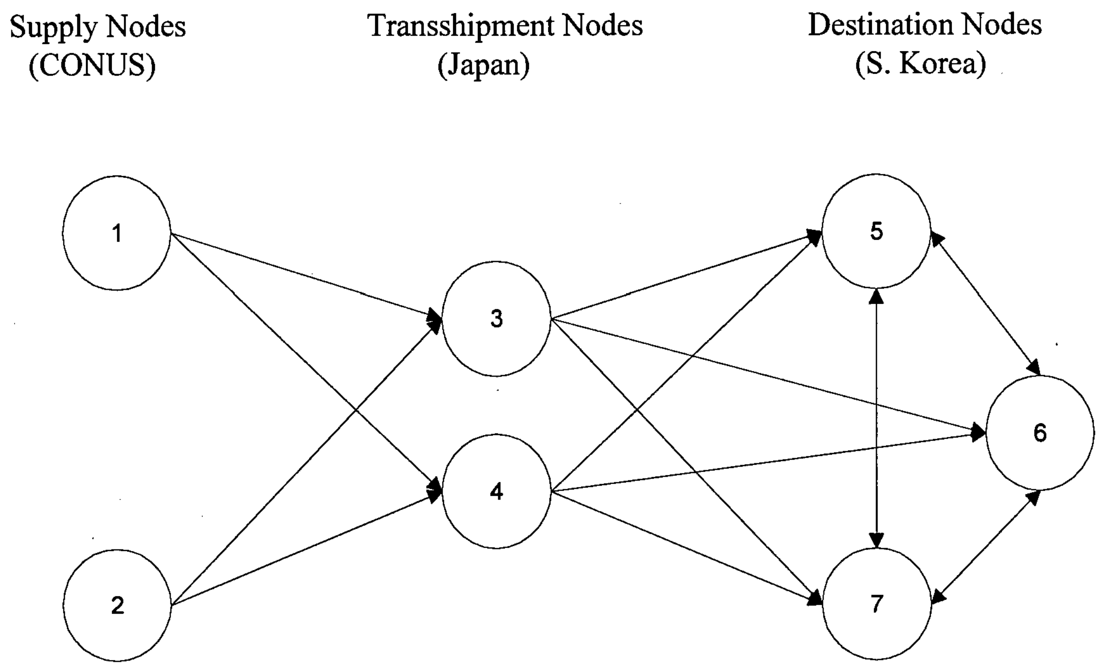


Figure 2 - Three Types of Nodes

We now have three different types of bases (nodes):

1. C-5 hubs in the CONUS (supply nodes)
2. C-17 hubs in Japan (transshipment or depot nodes)
3. Demand points in Korea (destination nodes)

Take note that the most basic (and common) LRPs have only two types of nodes: supply nodes and demand nodes. So our starting point already is more complex than a basic LRP! We've added the additional dimension of transshipment nodes, which have no supply and no demand. In 1983 Perl and Daskin formulated a similar model consisting of these same three types of nodes, and coined it a "hierarchical" model. As Chan describes, this model selects the depot locations, the number of aircraft assigned to each depot, and the optimal routing of each aircraft. "The problem is hierarchical in the sense that goods are shipped to two levels of facilities. First, goods are shipped from supply nodes to the depots, then the goods are shipped from the depots to the demand nodes". (11:9-36) We can use Perl and Daskin's formulation as our starting point, and improve upon it.

Perl and Daskin's original formulation minimizes the total cost associated with building depots and the delivery costs for transporting goods from supply points to depots and then depots to demand bases. (38:118) We are not as interested in minimizing costs as we are in minimizing time. Thus, we need to make modifications to their proposed hierarchical model to reflect our goal of delivering cargo on time.

Let's define the following variables and set notations to be used:

X^h_{ij} is a binary variable equal to 1 if aircraft with tail number h flies the arc from i to j, and equal to 0 otherwise

X_{ij} is a binary variable equal to 1 if demand node j is supplied by a plane based at depot i, and equal to 0 otherwise

Z_{0j} is a real-valued variable, equal to the amount of cargo delivered from all supply nodes to transshipment (depot) node j

Y_i is a binary variable equal to 1 if any aircraft are assigned to depot i, and 0 otherwise

T^h_j is the time that aircraft h spends at node j

H is the set of all aircraft

I is the set of all nodes

S is the set of all supply nodes

D is the set of all depots (transshipment) nodes

F is the set of all destinations

M_i is the set of all nodes with directed arcs terminating at node I

Additionally, the following parameters are incorporated into the model:

d_{ij} is the distance (in time units) between nodes i and j

f_j is the demand at depot j

U_h is the crew duty day (beginning at initial takeoff time) for the crew flying aircraft "h"

U_c is the crew duty day (beginning at initial takeoff time) for the crew flying aircraft "c"

P_j is the capacity at depot j

V_h is the vehicle capacity of aircraft "h"

V_c is the vehicle capacity of aircraft "c"

h is an individual aircraft tail number for a depot-based aircraft

c is an individual aircraft tail number for a supply-based aircraft

Unless otherwise specified, in equations (1) - (21) the following superscripts and subscripts refer to the following types of nodes and aircraft:

a and *m* both refer to the supply nodes in set *S*

b and *n* both refer to the depot nodes in set *D*

j, g and *l* all refer to the destination nodes in set *F*

i and *k* refer to the depot and destination nodes in either set *D* or *F*

Our objective function is to minimize:

(1) $$\sum_{b}\sum_{j}\sum_{h} d_{bj} X^{h}{}_{bj} + \sum_{g}\sum_{l}\sum_{h} d_{gl} X^{h}{}_{gl} + \sum_{h}\sum_{k} T^{h}{}_{k} + \sum_{a}\sum_{b}\sum_{c} d_{ab} X^{c}{}_{ab} + \sum_{c}\sum_{b} T^{c}{}_{b},$$

where g ≠ 1

(Note: We only are concerned with times for flights/ground delays in the direction of the supply nodes <u>to</u> the demand nodes, <u>not</u> round trip times)

We need to ensure each demand node point j is visited at least once (optional, if this requirement is unnecessary), so we have:

(2) $$\sum_{h}\sum_{i} X^{h}{}_{ij} \geq 1, \ \forall \text{ j, where i} \neq \text{j}$$

We must ensure the vehicle capacities are not exceeded:

(3) $$\sum\sum\sum f_{j} X^{h}{}_{ij} \leq V_{h}, \ \forall \text{h, where i} \neq \text{j}$$

The next constraint limits the maximum length of a tour, including stop-over times (for onloading/offloading), to ensure it does not exceed crew duty day. Equation (4a) is a range constraint for depot-based aircraft, and equation (4b) is for supply-based aircraft:

(4a) $$\sum_{i} T^{h}{}_{i} + \sum_{b}\sum_{j} d_{bj} X^{h}{}_{bj} + \sum_{j}\sum_{b} d_{jb} X^{h}{}_{jb} \leq U_{h}, \forall h$$

(4b) $$\sum_{a} T^{c}{}_{a} + \sum_{a}\sum_{b} d_{ab} X^{c}{}_{ab} + \sum_{b}\sum_{a} d_{ba} X^{c}{}_{ba} \leq U_{c}, \forall c$$

Each delivery tour must be connected to one of the depots in a subtour-breaking constraint, as follows:

(5) $\sum_{g}\sum_{j} X^{h}{}_{gj} \leq |F| - 1$, $\forall$ subsets of F containing two or more nodes, where g $\neq$ j, and $\forall$ h

The number of times we enter any node equals the number of times we exit that node (route continuity):

(6a) $\sum_{b} X^{h}{}_{bj} - \sum_{b} X^{h}{}_{jb} = 0, \forall j, h$ (for depot-based aircraft)

(6b) $\sum_{a} X^{c}{}_{ab} - \sum_{a} X^{c}{}_{ba} = 0, \forall b, c$ (for supply-based aircraft)

The amount of cargo sent from the supply nodes to the depots equals the number of supplies sent from the depots to the demand nodes:

(7) $Z_{0b} - \sum_{j} f_j X_{jb} = 0, \forall b$

We must ensure capacity at the depots is not exceeded:

(8) $Z_{0b} - P_b Y_b \leq 0, \forall b$

We next must "link" the $X^{h}{}_{ij}$ (allocation) and X_{ij} (routing) variables by stipulating that a demand node can be served only if a tour connects it to a depot:

(9a) $\sum_{i} X^{h}{}_{ji} + \sum_{k} X^{h}{}_{kb} - X_{jb} \leq 1, \forall h, j, b,$ where j ≠ i, and k ≠ b (for depot-based aircraft)

(9b) $\sum_{a} X^{c}{}_{ba} + \sum_{b} X^{c}{}_{ba} - X_{ba} \leq 1, \forall a, b, c$ (for supply-based aircraft)

Constraints (1) through (9b) above form the "hierarchical" model, which is the basis upon which we'll add new features in our goal of a hub-and-spoke system for military airlift.

Capabilities

The hierarchical model has the following capabilities:

1. It optimally assigns individual aircraft to supply nodes or depots
2. The model's solution enables us to track the cargo from depot to destination

Assumptions

Assumptions inherent to the hierarchical model:

1. Each aircraft has a capacity (weight, volume, etc.)
2. Each depot likewise has a capacity (weight, volume of supplies it can store, or maximum number of aircraft which can transition through the depot)
3. All cargo destined to a demand node originates from a supply node (i.e. the supply and demand at the depots is zero)

Limitations

1. We cannot specify individual cargo pieces by type. This pre-supposes all cargo pieces are homogeneous (i.e. same contents, same volume). We're only able to specify the weight requirement at each demand site

2. We cannot yet track cargo from supply node to destination
3. Similarly, we cannot place time window constraints on our itineraries without further modifications.

The following sections will address these shortcomings.

3.3 Time Windows

In theory, if we impose no constraints on when we require cargo to be delivered, one single aircraft could eventually fulfill our entire airlift needs. (It just may take years and years for a customer to eventually receive his cargo!) In reality, critical cargo only meets a need if it is delivered by a certain time/date. (For example, if an army unit will run out of ammunition 7 days from now if they're not re-supplied before then, it is useless if they don't receive this ammunition until 30 days from now). Only by stipulating that goods must be delivered not later than a certain time can we be sure our airlift truly serves our customers usefully.

Similarly, many types of cargo cannot be delivered too early, or they are useless as well. For example, it does no good to transport Humvees or tanks to a demand node before drivers have arrived at the demand nodes to offload these vehicles. The most pressing immediate need, then, is to ensure our cargo is delivered within a certain time window.

The contingency plans for all AMC airlift scenarios use a document known as the Time-Phased Force Deployment Data (TPFDD) document specifying origins, destinations, types of cargo, and delivery timelines for the given scenario. (20:14 of Acronyms) In any TPFDD, "Each cargo has two dates: The available to load date (ALD) (which) indicates when the cargo is at a port and ready to be placed on a transportation asset (and) the required delivery date (RDD) (that) specifies when the cargo must be at the theater port of entry." (34:11) In essence, we have a "not earlier than" (NET) time and a "not later than" (NLT) time defining when our cargo must be delivered. (The NLT and RDD times are identical, and for all practical purposes we can assume that the ALD date and the NET times are one and the same). These two times, associated with each individual piece of cargo listed on the TPFDD, will form the lower and upper bounds of our time window. (Note: In actuality the TPFDD specifies cargo ALDs and RDDs in terms of days, not hours. In an effort to make this model more universally pertinent, I chose to use hours, since in normal day-to-day airlift operations takeoffs and landings are restricted on an hourly basis. For example, Yokota, Kadena and Osan have quiet hours in effect each evening and during holidays. Thus, our model is amenable to the restrictions and realities of everyday airlift operations, and therefore is just as useful for everyday operations as it is for larger military buildups using airlift operations. Furthermore, since the distances between bases are measured in hours of flying time in the model, our units of measurement are the same)

Methods for incorporating time constraints into network models are fairly widespread in the literature. One of the most flexible methods was developed by Chan, and combines dual and primal variables. This method ingeniously inserts "odometer" variables, which track the elapsed time used by aircraft flying into each node. The user can thereby incorporate NET and NLT times for when an aircraft must visit a given demand node. Chan's formulation also incorporates dwell times, whereby an aircraft can delay at a node, if necessary, for the time window at the succeeding node to open. (11:8-26) We can use these concepts and build on them to incorporate time windows into our problem.

To our hierarchical model (equations (1) -(9b)) we need to add the following variables:

D^i is the clock time at which an aircraft arrives at node i, where $i \in I$

G_h is the minimum ground time necessary to offload cargo at a given node

We now must add the following constraints to our hierarchical model to introduce our time window requirements:

(10a) $\sum_b \sum_j X^h{}_{bj} \leq 1, \forall h$ (for depot-based aircraft)

(10b) $\sum_a \sum_b X^c{}_{ab} \leq 1, \forall c$ (for supply-based aircraft)

Equation (10a) ensures that each depot-based aircraft used travels from a depot to a demand node, and equation (10b) ensures that each supply-based aircraft used travels from a supply node to a depot.

(11a) $(\sum_i X^h{}_{ij}) - (1/G_h)(T^h{}_j) \leq 0,\ \forall\ h, j,$ where $i \neq j$ (for depot-based aircraft)

(11b) $(\sum_a X^c{}_{ab}) - (1/G_c)(T^c{}_b) \leq 0,\ \forall\ c, b$ (for supply-based aircraft)

Equations (11a) and (11b) ensure that the offload/ground dwell time at node j is at least G_h time units (for depot-based aircraft) or G_c time units (for supply-based aircraft). For example, if we require at least 2 hours of ground time to offload cargo from aircraft "h", then G_h equals 2 hours, so $1/G_h$ equals .5 hours. This means the last term in equation (11a) above would be $.5T^h_j$, which ensures T^h_j is at least 2 hours.

The real magic of the time window characteristic lies in the following four formulas, which combine four variables (D^j, D^i, D^a, and D^b) that arise from the dual formulation into the primal formulation's constraint set :

(12a) $D^j \geq (D^i + T^h{}_i + D^h{}_{ij}) - (1 - X^h{}_{ij})U_h$, $\forall$ *h, i, j,* where i ≠ j (for depot-based aircraft)

(12b) $D^b \geq (D^a + T^c{}_a + D^c{}_{ab}) - (1 - X^c{}_{ab})U_c$, $\forall$ *c, a, b* (for supply-based aircraft)

and

(13a) $D^j \leq (D^i + T^h{}_i + D^h{}_{ij}) + (1 - X^h{}_{ij})U_h$, $\forall$ *h, i, j,* where i ≠ j (for depot-based aircraft)

(13b) $D^b \leq (D^a + T^c{}_a + D^c{}_{ab}) + (1 - X^c{}_{ab})U_c$, $\forall$ *c, a, b* (for supply-based aircraft)

Altogether, equations (12a), (12b), (13a) and (13b) calculate the arrival times, D^b, to the depots, and D^j, to the destination bases. Notice (in equations (12a) and (13a)) that when $X^h_{ij} = 1$, they determine D^j in terms of the arrival time D^i at node i preceding node j on a tour, the dwell time T^h_i at node i, and the travel time D^h_{ij} between nodes i and j. When $X^c_{ab} = 1$, equations (12b) and (13b) similarly calculate D^b via D^a, T^c_a, and D^c_{ab}. (11:8-26 to 8-27)

We first stipulate that the clock begins at time zero at all the supply nodes. It turns out that to accurately measure the odometer reading at each node visited, it's necessary to initialize the D^a values (where $a \in S$) to the minimum offload time required at the depot stops. By doing this, the D^b values (where $b \in D$, the depots) tell us the transload completion time at depot b after the cargo has been transloaded from the C-5 to the C-17, which in turn is the earliest time the C-17s may depart from depot b.

The only remaining goal in this "hierarchical-plus-time-windows" model is to specify the NET and NLT times (bounds) at the destination nodes. If, for example, cargo must be delivered to node 5 no earlier than 4 time units (hours, days, minutes, etc.) after the clock begins, and not later than 8 time units, we'd add the following two constraints: $D^5 \geq 4$, and $D^5 \leq 8$.

As with any model, this "hierarchical-plus-time-windows" formulation (eqs 1-13) has its strengths and weaknesses.

Capabilities

The "hierarchical-plus-time-windows" model has these capabilities:

1. Selects optimal locations for supply bases and transshipment depots

2. Assigns the optimal mix of aircraft (from the total "h" that we specify) to each supply and depot base
3. Determines the optimal routes for each of these aircraft to satisfy all demands and meet all time window requirements

Assumptions

1. Aircraft assigned to supply nodes and transshipment depots will return to their origin base at the end of each round-trip delivery tour
2. The objective is to minimize the sum of all the times prior to/until the cargo is delivered to the destination nodes
3. We must specify a crew duty day time limit for each aircraft tail number and a maximum cargo capacity for each aircraft and depot
4. We specify how many of each type of aircraft are available for the model to assign and route in the model

Limitations

1. If we make our time windows impossible to meet, the model will be infeasible, instead of telling us how close we can get to the "next best " feasible solution
2. Thus far, the model assumes that the requirements at any one depot is not greater than the cargo capability of one aircraft. Our model is thus <u>not</u> yet built to handle a situation where we need to visit a destination more than once. We

need to incorporate the capability for a destination to be visited multiple times, since this will almost certainly be the case in any real-world, large-scale airlift scenario.

3. Although we track aircraft, we haven't yet achieved the goal of tracking cargo from supply source to destination.

3.4 Example of the "Hierarchical-Plus-Time-Windows" Model

Let's look at an example to see how this "hierarchical-plus-time-windows" model we've built works. This is useful not only to help visualize how all the equations (1)-(13b) are written out in mixed integer programming form, but the solution will help the reader understand the capabilities, assumption and limitations listed above more easily. Additionally, it will make our model more understandable as we add more features in the succeeding sections.

Refer to Figure 3. We have two supply bases (nodes 6 and 7), two transshipment bases (nodes 1 and 2) and three demand bases (nodes 3, 4 and 5). Distances (in hours) are given for each possible arc which can be flown. Demands at nodes 3, 4 and 5 are given as 15,000, 10,000 and 12,000 pounds of cargo, respectively. We'll assume that we have two aircraft to be based at nodes 6 and/or 7, each with a capacity of hauling 35,000 pounds to depot 1 or 2. We also have two aircraft, each with a cargo capacity of 30,000 pounds, to be based at nodes 1 and/or 2. These aircraft will deliver cargo to meet the demands at nodes 3, 4 and 5. In addition to determining the optimal basing for each aircraft, we want

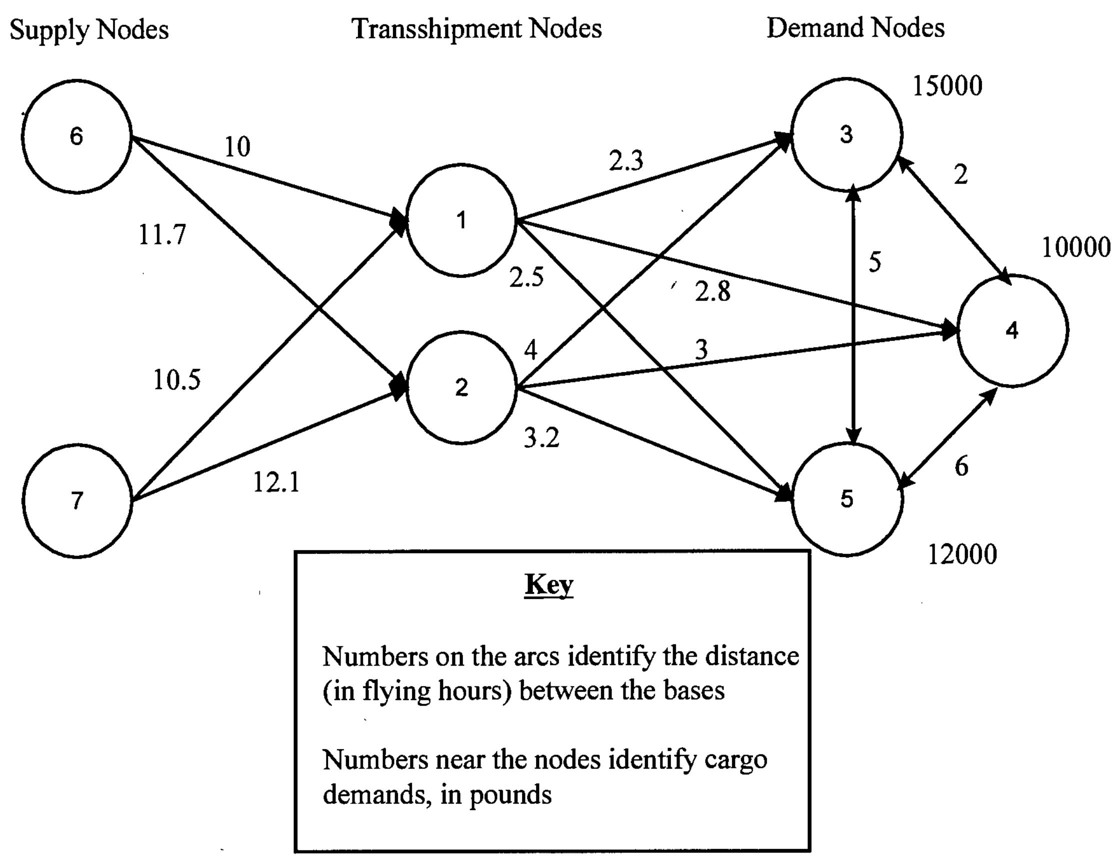


Figure 3 - "Hierarchical-Plus-Time-Windows" Model

to know which route each aircraft will fly. Let's assume we cannot fly any of the supply-based aircraft or depot-based aircraft for more than 16 hours. Suppose we specify our NET and NLT times (in total elapsed hours, beginning at zero hours) at our demand nodes as follows:

Node	NET	NLT
3	16	20
4	17.5	21.5
5	19	25

Additionally, we'll assume the minimum ground/offload times for the supply-based aircraft is 2 hours, and the depot-based aircraft need a minimum of .5 hours to offload. The capacity of the depots is 50,000 pounds apiece.

The formulation of this problem is listed as Appendix 1A, and the solution is given in Appendix 1B. The optimal solution is to base both supply aircraft at node 6 and both depot aircraft at node 1. A total of 37,000 pounds of cargo is hauled by the supply-based aircraft to node 1, from which both depart node 1 at clock time 4.5, and arrive at node 1 at time 14.5. After two hours offload time, the two depot-based aircraft depart node 1 at time 16.5 hours. One aircraft departs with 25,000 pounds of cargo and flies to nodes 3 and 4, then returns to node 1. It arrives at node 3 at time 18.8, requires .5 hours to offload 15,000 pounds, and departs at time 19.3. It then arrives at node 4 at time 21.3 and spends .5 hours offloading its remaining 10,000 pounds of cargo, then returns to node 1. The other depot-based aircraft departs node 1 at time 16.5, with 12,000 pounds of cargo. It flies to demand node 5, arriving at time 19.0, spends .5 hours offloading, then returns to node 1.

This model, simple as it might be, points out the strengths and weaknesses of our formulation thus far. Most importantly, it demonstrates how our binary-valued X^h_{ij} and X_{ij} variables limit our model to single-frequency visits. In the example just given, had our demand at node 3 been 40,000, *our model would not have visited it at all, because this demand cannot be met by one aircraft* (recall that our capacity for a depot-based aircraft is 30,000 pounds), and no provisions are made for more than one aircraft visiting any single demand node. Additionally, until now, we've assumed all cargo is exactly the same, and we've simply ensured a certain amount of weight of this cargo has reached each destination. To make our model more useful, we'll need to find a way to specify the CONUS origin and the Korean destination for each cargo piece to be delivered.

We also can see how large this model is growing in size. We're individually tracking each aircraft tail number, and each tail number originating from a supply node generates [2 X *(# of depots)* X *(# of supply nodes)*] X^h_{ij} variables, and each tail number based at a depot generates [(# *of depots* + # *of demand nodes*) X (# *of depots* + # *of demand nodes - 1*]) X^h_{ij} variables! Our model has yet to incorporate multiple-frequency capability and cargo tracking features, but we already have 56 X^h_{ij} binary variables and 95 variables altogether, while using only 4 aircraft and 7 nodes!

3.5 Multiple Frequency of Service

Let's now address the problem of frequency of service. As was the case with time-windows, numerous methods exist that allow for destinations to be visited more than once. The trick is to find a method which can be adapted to fit our current model. This is no small task. Since we've defined our X^h_{ij} and X_{ij} variables to be binary, any methods for multiple frequency servicing which utilize general integer variables to count frequencies flown would not be easily adaptable, and would require extensive reformulation of the current model.

Perhaps the most simple method for incorporating multiple-frequency visits found in the literature, which is also consistent with our notation to date, is offered by Baker (5:30-31). This method "splits" demand locations into multiple nodes which are colocated, but which have no arcs connecting them to one another. The sum of demands at all the "split" nodes combined equals the original demand at the location in question. Since each "split node" requires only one visit, we retain the binary variable characteristic of our arcs.

Additionally, we'll combine our range limitation constraints (equation 4) with our subtour-breaking constraints (equation 5). (This change is trivial for our current model, but in larger models with more than three demand nodes, equation (5) gets extremely tedious to formulate for each two-some, three-some, etc. of demand nodes). The way equations (4) and (5) are combined is given by Kulkarni and Bhave, and explained by Chan. (11:9-93)

(14) $UX_{jg} + D^j - D^g \leq U - d_{jg}, \forall j, g,$ where $j \neq g$

In equation (14) above, D^g and D^j are real-valued variables associated with each node, which tell us how much range has been accumulated, beginning at "time 0", upon reaching nodes g and j, respectively. The D^g and D^j variables can be thought of as "odometer' readings, which record the time (or distance) elapsed. These variables are one and the same with the D^i and D^j variables present in equations (12a) and (13a) in the previous section. Notice that when X_{jg} takes the value 1, equation (14) becomes

(15) $D^j + d_{jg} \leq D^g, \forall j, g,$ where $j \neq g$

which means that the range (time) elapsed at node g must be increased by at least the flying distance (time) between g and node j, the node immediately prior to g. And when X_{jg} takes the value 0, the right-hand side of equation (14) becomes large relative to the left-hand side, and the constraint is non-binding. (11:9-93)

Our goal is to replace equations (4) and (5) with equation (14). We must add two more constraints before doing this, however, because Kulkarni and Bhave's method does not account for distances flown during the first flight leg (arc) immediately after departing a depot, nor the tour's final leg returning to the depot. Therefore, we need to ensure the "odometer" variables account for these legs.

The first equation we'll add accounts for the range used upon departing a depot. The equation is as follows:

(16) $$d_{bj}X_{bj} \leq D^j, \forall b$$

Similarly, to ensure we account for the final tour leg returning to the depot:

(17) $$d_{bj}X_{bj} + D^j \leq U, \forall b$$

One problem remains, however. Notice that Kulkarni and Bhave's notation, X_{bj}, is different from our tail-number-specific notation, $X^h{}_{bj}$. Therefore, equations (14), (16) and (17) are not currently suitable to our existing model. This is easily remedied, though, by substituting $X^h{}_{bj}$ for X_{bj} in these equations. One important point to be aware of is that *our formulation allows us to assign a different odometer variable to each split node*, which is much more flexible and realistic than Baker's formulation. For instance, if node 8 has a demand of 10,000, and it is split into a grouping of five nodes (say, 8A, 8B, 8C, 8D and 8E), we will replace the subscript 8 ($\in F$) with 8A, 8B, 8C, 8D and 8E ($\in F$), and we'll have odometer variables D^{8A}, D^{8B}, D^{8C}, D^{8D}, and D^{8E} for each of the five splits of node 8. *This allows us, if desired, to specify different time windows for each aircraft visiting a given destination!* This is a powerful and useful feature, as we'll now demonstrate.

In the case of node 8 above, for example, suppose cargo being delivered to node 8 can arrive NET hour 18 and NLT hour 23. To prevent five aircraft from converging upon node 8 at the same time, we can assign five individual time windows; one for each split node. For example, we can assign the time windows as follows:

Node	NET	NLT
8A	18	19
8B	19	20
8C	20	21
8D	21	22
8E	22	23

This ensures no more than two aircraft could ever arrive at node 8 over any given 60-minute period. (When the important real-world constraint known as MOG is discussed in depth in Chapter 4, we'll see why this capability can be very useful).

By contrast, Baker's formulation would require us to have only one odometer variable, D^8, effectively forcing all five aircraft visiting node 8 to arrive at the same time. The reason for this limitation lies in his formulation, which was built for a courier service scenario in which the number of aircraft visiting a base in a given time window was very small. Thus, this formulation did not force all visits to a split node to occur via different tours. (5:31) But our model, while more complex, is also more flexible. Other constraints we've already incorporated prevent any tour from servicing any grouping of nodes more than once. *Thus, we can have multiple visits to the same node (via single visits to node splits) which occur at different times*, which is a very desirable feature, and adds realism to our model.

In summary, we've added the ability to provide multiple visits to any demand node via "node splits" without forsaking our binary X_{ij}^{h} binary variable arc structure. Additionally, we've eliminated the need for equations (4) and (5), which could prove to be excessively complex as the number of nodes increases, and replaced them with:

(18) $U_h X^h{}_{gj} + D^g - D^j \leq U_h - d_{gj}, \forall h, g, j,$ where g ≠ j

(19) $d_{bj} X^h{}_{bj} \leq D^j, \forall h, b, j$

(20) $d_{bj} X_{jb} + D^j \leq U\ , \forall b, j$

Lastly, to ensure each type of aircraft doesn't exceed its maximum crew duty day (CDD), we add equations (21a) and (21b):

(21a) $D^j - D^b \leq U_h, \forall j, b, h$ (for depot-based aircraft)

(21b) $D^b - D^a \leq U_c, \forall b, a, c$ (for supply-based aircraft)

3.6 How Demand Assignments to Split Nodes Can Affect the Solution

To see how our "hierarchical-plus-time-windows-plus-multiple-frequency" model (let's call equations 1-3, 6-13, 18-21 the HTWMF model) works, let's quickly re-visit our previous example from Figure 3. Let's change the demand at node 3 from its original value of 15,000 pounds to a new value of 31,000 pounds. Additionally, we'll change the capacity of each depot (transshipment node) from 50,000 pounds to 60,000 pounds. Since our depot-based aircraft have a capacity of only 30,000 pounds, the new 31,000 pound demand at node 3 cannot be met by a single aircraft. Therefore, we'll split node 3 into nodes 3A and 3B, and arbitrarily (for now) assign 3A and 3B with demands of 11,000 pounds and 20,000 pounds, respectively. Refer to Figure 4 below.

The formulation of our model with this new feature included is given in Appendix 2A. The solution (as detailed in Appendix 2B) assigns both supply aircraft to node 6, from which they deliver a combined payload of 53,000 pounds to node 1. Both depot-based aircraft are assigned to node 1. The first depot-based aircraft flies to node 3B to deliver 20,000 pounds, then delivers 10,000 pounds to node 4, and returns to node 1. Our second depot-based plane hauls 11,000 pounds to node 3A, then 12,000 pounds to node 5, and returns to node 1.

As always, our model has strengths and weaknesses. On the positive side, we've added the ability to service demand nodes with more than one aircraft - an absolute necessity for any realistic airlift model. One limitation of which we need to be aware, however, is that *the choice of demands for the split nodes affects the solution.* The

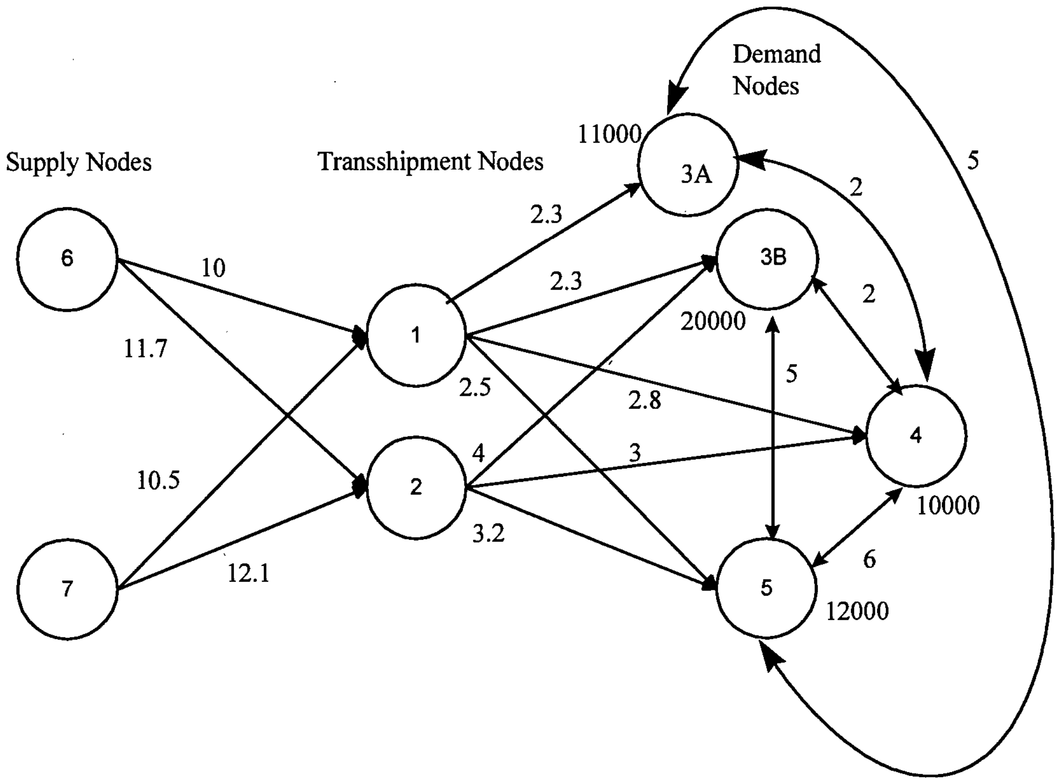


Figure 4 - Splitting Node 3

assumptions made above in Figure 4 demonstrate this point well. By selecting demands at nodes 3A and 3B of 11,000 and 20,000, we *artificially* ensured both depot-based aircraft carried cargo to two demand nodes apiece.

We can show that, had we assigned our 31,000 pound demand at node 3 into demands of 30,000 pounds and 1,000 pounds to split nodes 3A and 3B, respectively, our problem would have a different solution. Refer to Figure 5.

Since the depot-based aircraft capacity is 30,000 pounds, one can easily visualize how one feasible solution would be to have one fully-loaded aircraft haul 30,000 pounds from node 1 to 3A and return empty to node 1, while the second aircraft would depart

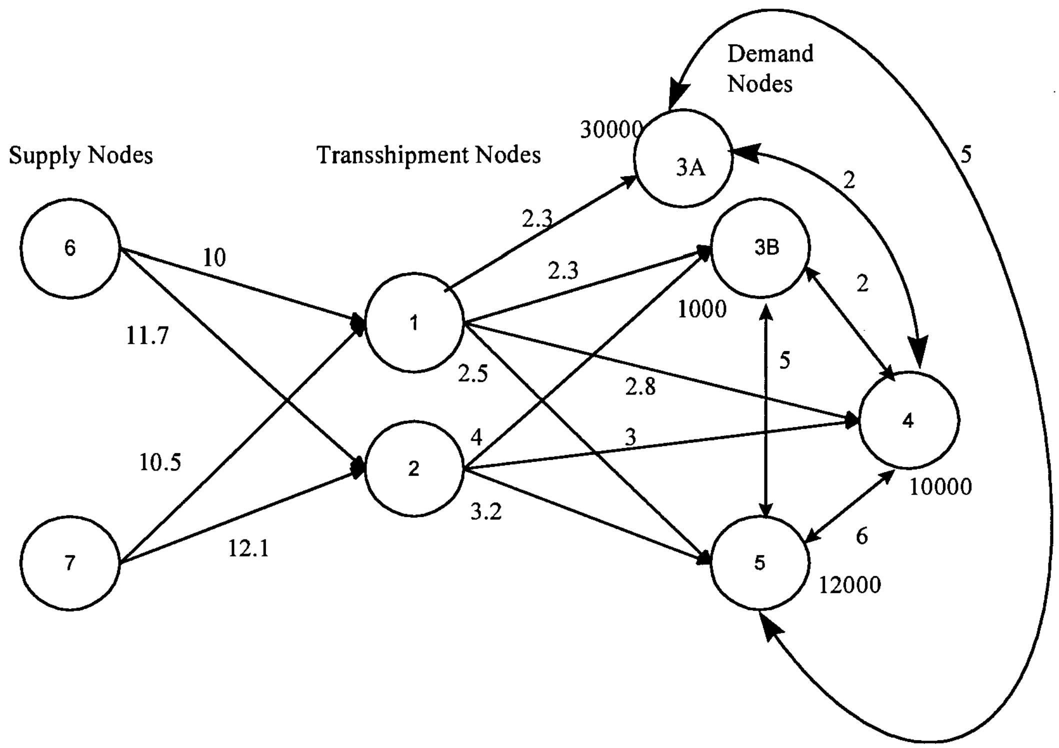


Figure 5 - Changing Node 3 Demands

node 1 and visit nodes 3B, 4 and 5 in succession, meeting their demands of 1,000, 10,000 and 12,000 pounds. This, in fact, is the optimal solution. The formulation and solution of the model using the 30,000 and 1,000 pound demands at split nodes 3A and 3B are presented in Appendix 3A and Appendix 3B, respectively.

Contrast the objective function value of Appendix 2A (Figure 4 Problem) to Appendix 3A (Figure 5 Problem) Which is better? Since our objective function to this point minimizes the sum of elapsed times incurred in meeting our demands, Appendix 2A has a "better" (smaller) objective function value (43.6 hours for Figure 4 versus 44.6 hours for Figure 5), so it would appear the demand assignments in Figure 4 would be preferable to those of Figure 5. *But Figure 5 clearly makes more logical sense*. In a real-world scenario it is undesirable to have an aircrew deliver only part of its cargo to one destination and then fly to another destination to offload the remaining cargo when it all could have been offloaded at the first destination.

If you're not convinced yet, consider one more example. Refer to Figure 6 on the following page. Suppose we decide to split our 31,000 pound demand at node 3 evenly between split nodes 3A and 3B, so each has a demand of 15,500 pounds. Also, suppose node 4 has a demand of 15,000 pounds and node 5 has a demand of 1,000 pounds. Our total demand is thus 15,500 + 15,500 + 15,000 + 1,000 = 47,000 pounds. Since our depot-based aircraft each has a capacity of 30,000 pounds, it seems logical to assume we'd only need two aircraft (two tours) to meet our demands. But upon close examination, *we have a problem due to the way we split node 3*. There is no way to meet all the demands with two aircraft! We need *three* aircraft to meet the demands, and each would travel nearly half empty (one could deliver to node 3A, one to node 3B, and the third to nodes 4 and 5)!

Had we assigned the demands at nodes 3A and 3B as 30,000 and 1,000 pounds, respectively, two aircraft would suffice easily. One aircraft would deliver a full load to node 3A, and the second aircraft would easily meet the demands of 1,000, 15,000 and 1,000 pounds at nodes 3B, 4 and 5, respectively, with 13,000 pounds of capacity to spare!

These examples show how the demand assignments to split nodes impacts our optimal solution. The logic in carrying loads which are as full as possible suggests a methodology for how we should assign our split node demands.

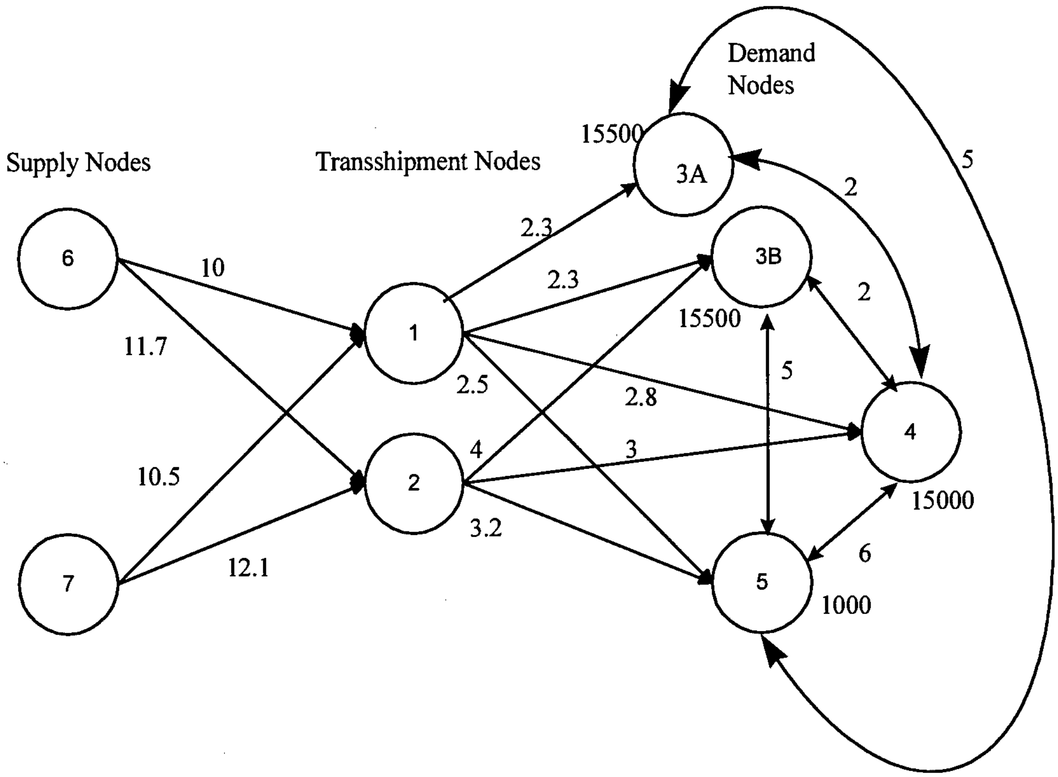


Figure 6 - Changing Node 3 Demands Again

3.7 A Methodology For Assigning Demands To Split Nodes

Take the total demand required at each node. Divide this by the capacity of the aircraft which may deliver goods to that node. The integer portion of this quotient is the number of splits which will have a demand equal to the capacity of the aircraft, and the fraction left over is the demand of the last split. For example, with an aircraft capacity of 30,000 pounds and a total demand at node 3 of 31,000 pounds, we did the correct assignment in Figure 5 by assigning a 30,000 pound demand to node 3A and a 1,000 pound demand to node 3B. This method makes more sense (than the way we split node 3's demands in Figure 4 or Figure 6) from both a transportation and military standpoint;

it is more desirable to minimize the number of stops each C-17 must make than to force them to make numerous enroute stops with less than full payloads.

(Note: A more formalized optimal partitioning method is presented in Chapter V, Section D, of "A Multiple-Depot, Multiple-Vehicle, Location-Routing Problem With Stochastically-Processed Demands", by Chan, Carter and Burnes (see reference 12 in the bibliography).)

3.8 Cargo Tracking

We've now reached the point where the problem of tracking our cargo must be addressed. This promises to greatly add complexity to our model, but it is absolutely necessary to provide any realistic usefulness as a military transportation network model.

It is to this end we'll now introduce a key variable to our existing HTWMF model; the X^h_{ijsd} variable. This is similar to our X^h_{ij} aircraft tracking variable, with the additional "s" and "d" subscripts. The "s" refers to the CONUS supply node from which our cargo originates via a C-5, and the "d" refers to the depot in Japan from which our C-17 departs and delivers the cargo to "j", our destination in Korea. Thus, X^h_{ijsd} is a cargo tracking variable which tells us which C-17 (tail number "h") meets the demands of Korean destination "j" for cargo from CONUS supply node "s". The "d" depot identifier subscript is necessary for ensuring the total tonnage of cargo delivered to Korea from a Japan depot is exactly equal to the total tonnage of cargo sent to that depot from the CONUS.

This creation of X^h_{ijsd} not only adds many more variables to the model; the number of constraints increases dramatically, as well. We'll need to "link" the X^h_{ijsd} cargo tracking variables to their X^h_{ij} aircraft tracking brethren.

Two similar constraints are used to link the C-17's X^h_{ijsd} cargo tracking variables with the X^h_{ij} C-17 aircraft routing variables. In order to ensure we don't visit a destination with no cargo to offload there, we can build a constraint set to make sure each X^h_{ij} routing variable is equal to zero unless a corresponding X^h_{ijsd} cargo tracking variable is equal to one. Likewise, we cannot offload cargo from any aircraft to a destination unless that aircraft actually visits that destination. So we have to include constraints that ensure no X^h_{ijsd} cargo tracking variable equals one unless its corresponding X^h_{ij} routing variable is also equal to one. While these constraints may seem obvious, failure to include them could result in the model believing that cargo demands are being met, when in fact no aircraft are visiting these "satisfied" nodes!

3.9 Determining the Optimal Number of C-17s

The final feature of our model that merits an explanation concerns the desire to determine the optimal number of C-17s to be based at the depots. In the first generations of this model I actually calculated the number of C-17s required to meet the destination demands, and incorporated this many C-17s in the model. (similar to equation 10). The model then determined which depots to base each of the C-17s at, which routes they'd fly, and what cargo they'd carry. The drawback is obvious: it required me to figure out how many C-17s were needed beforehand. While this may not be difficult for a small problem, in larger problems this could become quite time-consuming. More importantly, the whole methodology was artificial. Why not let the model do the work for me? This not only saved time - it is much more realistic and makes the model more powerful and user-friendly.

How, then, do we let the model determine the optimal number of aircraft for us? The methodology used here is as follows:

1. Add up the total weight of all demands going to the destination bases
2. Divide this number by the C-17 capacity
3. Round up (if necessary) to the nearest integer
4. This gives us the minimum number of C-17s that are necessary to physically haul enough cargo to meet destination demands
5. Sum up the number of split nodes. This gives us the maximum number of C-17s that could be required to meet the destination demands, since each split must be visited exactly once - no more, no less.

In the model, let "h", the number of C-17s to be considered, equal the maximum number possibly needed from step 5 above. We can then insert a constraint set that tells the model to use no fewer than the minimum number we calculated in step 4 above. Since our objective function is a minimization, we can "penalize" the model for selecting more C-17s than those required to meet the demands at the destinations. We can do this by adding a "penalty factor" equal to a C-17 crew duty day (16 hours) for each C-17 selected to be used by the model. It's clear that now the model will minimize the number of C-17 aircraft utilized.

3.10 Putting It All Together - The Hub-and-Spoke Model

We're now ready to construct our model, which we'll refer to as the "Hub-and-Spoke" model. This model incorporates the elements of hierarchy, time windows, multiple frequency servicing, cargo and aircraft tracking, assigning aircraft to depots, and determining the optimal number of depot-based aircraft needed to meet destination demands. Specifically, the required inputs and associated outputs for our Hub -and-Spoke model are as follows:

Given:

1. An assignment of C-5s to CONUS bases (pre-assigned by the model user)
2. A set of candidate C-17 hubs (depots) in Japan
3. Destination bases in Korea, each having a specific demand for cargo from the CONUS bases
4. The minimum number of C-17s needed to meet the demands (see steps 1-4 of Section 3.9)
5. A candidate number (maximum that might be needed to meet all Korean base demands) of C-17s, as determined by the user (see step 5 of Section 3.9)
6. Time window requirements to be met for each split node

The Hub-and Spoke model will:

1. Select the optimal C-5 CONUS-to-Japan routes
2. Select which Japan depot(s) should be used, and assign each C-17 aircraft to a depot
3. Determine the optimal C-17 routes to/from Korea
4. Meet all time window requirements, if this is possible, while delivering cargo in the fastest possible time
5. Meet all Korean destination demands
6. Track cargo (i.e. ensure demands for cargo from specified CONUS supply bases are delivered in the proper amounts to each Korean destination)

The model uses the following variables, which we'll define:

Xh_{ijsd} (cargo tracking) is a binary variable equal to 1 if the C-17 with tail number h flies the arc from node i to node j carrying cargo from CONUS node s which was transshipped through Japan depot d, and equals 0 otherwise

Xh_{ij} (aircraft tracking) is a binary variable equal to 1 if the C-17 aircraft with tail number h flies the arc from i to j, and equal to 0 otherwise

Cc_{ij} (aircraft tracking) is a binary variable equal to 1 if the C-5 aircraft with tail number c flies the arc from i to j, and equal to 0 otherwise

C_{ij} is a binary variable equal to 1 if depot i is supplied by a C-5 based at supply node j, and equal to 0 otherwise (i.e. the C-5 allocation variable)

X_{ij} is a binary variable equal to 1 if demand node i is supplied by a C-17 based at depot j, and equal to 0 otherwise (i.e. the C-17 allocation variable)

Z_{ij} is a real-valued variable, equal to the amount of cargo delivered from supply node i to transshipment (depot) node j

T_{hj} is the ground delay dwell time that C-17 aircraft h spends at node j

B_{hj} is the ground delay dwell time (including transload of cargo time) that C-5 aircraft h spends at node j

$S^c{}_j$ (or S_{cj}) is the departure time (the number of hours after the 32-hour clock starts at time 0.0) of C-5 aircraft c departing from CONUS node j

D_i has a slightly different meaning depending on whether node i is a CONUS supply node, a Japanese depot or Korean destination. When $i \in D$ (i.e. a depot in Japan), D_i is the transload completion time of C-5 cargo onto C-17s in Japan. This is the earliest departure time from Japan for any C-17 based at depot i. When $i \in F$ (i.e. a destination in Korea), D_i is the arrival time (before offloading cargo) for the C-17 delivering cargo to destination i. As mentioned in Section 3.3, when $i \in S$ (i.e. a CONUS supply node), D_i is given a value equal to the <u>minimum</u> offload time required at the depot stops.

The following sets are used in the formulation:

H_c is the set of all C-5 aircraft

H_x is the set of all C-17 aircraft

I is the set of all nodes

S is the set of all CONUS supply nodes

D is the set of all Japan-based depots (transshipment) nodes

F is the set of all Korea-based destinations

M_i is the set of all nodes with directed arcs terminating at node i

Likewise, the following parameters are needed in the model:

d_{ij} is the distance (in time units) between nodes i and j

$f_{j,a}$ is the cargo demand (in tons) at destination j from CONUS supply node a

U_h is the crew duty day (beginning at initial takeoff time) for the aircrew flying aircraft h

V_h is the vehicle capacity of aircraft h

h is an individual C-17 aircraft tail number

c is an individual C-5 aircraft tail number

G_h is the minimum ground time necessary to offload cargo from a C-17 at a destination

G_c is the minimum ground time necessary to transload cargo from a C-5 at a depot

Unless otherwise specified, in equations (22) - (54) the following superscripts and subscripts refer to the following types of nodes and aircraft:

a and *m* both refer to the supply nodes in set S

b and *n* both refer to the depot nodes in set D

j, *g* and *l* all refer to the destination nodes in set F

i and *k* refer to the depot and destination nodes in either set D or F

The objective function, which we will MINIMIZE, is as follows:

$$(22)\quad \sum_h\sum_b\sum_j d_{bj}X^h{}_{bj} + \sum_h\sum_g\sum_l d_{gl}X^h{}_{gl} + \sum_a\sum_b\sum_c d_{ab}C^c{}_{ab} + \sum_h\sum_i T_{hi}$$
$$+ \sum_j\sum_b X_{jb} + \sum_b\sum_a C_{ba} + 16\sum_b\sum_j\sum_h X^h{}_{bj}$$

Notice that the $\sum_j\sum_b X_{jb}$ and $\sum_b\sum_a C_{ba}$ terms are not needed for minimizing the time to deliver goods - they are included only to ensure any such binary variable not specifically forced to have a value of one will take on a value of zero, so our solution set will be correct.

Our first constraint ensures our C-17 crew duty day (CDD) is not more than 16 hours:

$$(23)\quad D_j - D_b \leq 16, \forall j, b$$

Likewise, we make sure our C-5 augmented CDD is not greater than 16 hours:

$$(24)\quad D_b - D_a \leq 16, \forall b, a$$

Constraint (25) is a range ("odometer variable") computation for the first legs flown by the C-17s (from the Japan depots, "b", to the Korean bases, "j"). It ensures D_j is no earlier than the time to fly directly from Japan node b to Korean node j:

$$(25)\quad d_{bj}X^h{}_{bj} - D_j \leq 0, \forall b, j, h$$

Similarly, the range computation for the final leg flown by C-17 (returning to the Japan depots) is:

$$(26) \quad d_{jb}X^{h}{}_{jb} + D_{j} \leq 32, \forall b, j, h$$

The "32" on the right hand side refers to the sum of the C-5 CDD (16 hours) and the C-17 CDD (16 hours). In essence, it tells us that the elapsed time from the start of our time period until a C-17 returns from its final delivery cannot exceed 32 hours.

Our next two constraints, (27) and (28), link the ground dwell times (T_{hi}), the time window arrival times (D_i and D_j), and the 16 hour C-17 CDD. Together, they calculate D_j, the arrival time at Korean destination node j:

$$(27) \quad 16X^{h}{}_{ij} - T_{hi} - D_{i} + D_{j} \leq 16 + d_{ij} \quad \text{and}$$

$$(28) \quad 16X^{h}{}_{ij} + T_{hi} + D_{i} - D_{j} \leq 16 - d_{ij}, \forall h, i, j$$

Constraint (29) ensures that the minimum number of C-17s required to fulfill demands in Korea depart from the depot(s) in Japan:

$$(29) \quad \sum_{h}\sum_{b}\sum_{j} X^{h}{}_{bj} \geq (\sum_{j}\sum_{a} f_{j,a}) / V_{h} \text{ (rounded up to the nearest integer)}$$

Similarly, equation (30) ensures that this same number of C-17s returns to their Japanese base(s) of origin:

(30) $\sum_h \sum_b \sum_j X^h{}_{jb} \geq (\sum_j \sum_a f_{j,a}) / V_h$ (rounded up to the nearest integer)

Equation (31) ensures the ground dwell time for aircraft h at node j is not less than G_h hours, the minimum offload time for C-17 aircraft "h":

(31) $\sum_i X^h{}_{ij} - (1/G_h)(T_{hj}) \leq 0, \forall h, j$, where i $\neq$ j

(Note: For a C-17, the planned ground offload time is 2+15, or 2.25 hours = G_h. Therefore, $1/G_h \cong .44444444$) (19:18)

The following constraint is the same as equation (6). It is a route continuity constraint stipulating that the number of times we enter any Korean destination node "j" equals the number of times we exit that node:

(32) $\sum_i X^h{}_{ij} - \sum_i X^h{}_{ji} = 0, \forall h, j$, where i $\neq$ j

Equation (33) ensures each split demand node "j" is visited exactly once (similar to equation (2)):

(33) $\sum_h \sum_i X^h{}_{ij} = 1, \forall j$ where i $\neq$ j

We need to make sure our vehicle weight capacities are not exceeded, yielding:

(34) $\sum_i \sum_j \sum_a \sum_b f_{j,a} X^h{}_{ijab} \leq V_h, \forall h$, where i $\neq$ j

Equation (35) is similar to (32), but it states that the number of times we depart from a Japanese depot "b" equals the number of times we return to that depot (i.e. if C-17 "h" departs from a Japanese node, it must return there):

$$(35) \quad \sum_j X^h{}_{jb} - \sum_j X^h{}_{bj} = 0, \forall h, j, b, \text{ where i} \neq \text{j}$$

Equation (36) ensures demand node "j" will have its cargo requirements from CONUS node "a" fulfilled by only one aircraft:

$$(36) \quad \sum_h \sum_i \sum_b X^h{}_{ijab} = 1, \forall j, a, \text{ where i} \neq \text{j}$$

Note: *Even if there is <u>no</u> demand at a destination node for cargo from a given CONUS node*, we still will have one C-17 visit this destination to fulfill all the demands from <u>other</u> CONUS nodes, so *the right hand side (RHS) of this constraint is <u>always</u> 1.*

Equation (37) "links" the C-17 routing (X^h_{ij}) and allocation (X_{ij}) variables by stipulating that a demand node "j" can only be served if a tour connects it to a depot "b":

$$(37) \quad \sum_i X^h{}_{ji} + \sum_i X^h{}_{ib} - X_{jb} \leq 1, \forall h, j, b, \text{ where i} \neq \text{j}$$

If a C-17 leaves Korean destination "j" and enters Japan depot "b", equation (37) above forces X_{jb} to equal 1. Notice that our objective function , equation (22), contains the term $\sum_j \sum_b X_{jb}$. Unless equation (37) forces any X_{jb} to equal 1, by minimizing the objective function we'll ensure X_{jb} takes a value of 0!

Equation (38) ensures each X^h_{ij} C-17 aircraft tracking variable is equal to zero unless a corresponding X^h_{ijsd} cargo tracking variable is equal to one (i.e. if any aircraft visits node j, then it must be delivering cargo from at least one CONUS supply node to node j):

(38) $$\sum_a \sum_b X^h_{ijab} - X^h_{ij} \geq 0, \forall h,i,j, \text{ where i} \neq \text{j}$$

Equation (39) ensures any X^h_{ijsd} cargo tracking variable cannot be equal to 1 (i.e. it must be equal to 0) unless its corresponding aircraft tracking variable X^h_{ij} is equal to 1:

(39) $$X^h_{ij} - X^h_{ijab} \geq 0, \forall h,i,j, \text{ where i} \neq \text{j}$$

Equation (40) states that if any C-17 tail number "h", based at Japan depot "b", visits demand node "k", it exits node "k" (i.e. this is a continuity constraint for X^h_{ijsd} cargo tracking variables):

(40) $$\sum_i \sum_a (X^h_{ijab} - X^h_{jiab}) = 0, \forall h,j,b \text{ where i} \neq \text{j}$$

Equation (41) sets the initial condition that a C-17 cannot be hauling cargo from depot "b" if that C-17 is flying into/from a depot other than "b":

(41) $X^{h}{}_{ikab} = 0, \forall h, i, k, a, b,$ whenever i $\neq$ b or k $\neq$ b, and we stipulate in all cases that i $\neq$ k

Equation (42) is similar to equation (7). It calculates Z_{ab}, the amount of cargo delivered from CONUS node "a" to Japan depot "b". It is equal to the total amount of cargo hauled from depot "b" and delivered to satisfy the destination node demands:

(42) $Z_{ab} - (\sum_h \sum_i \sum_j f_{j,a} X^{h}{}_{ijab}) = 0, \forall a, b,$ where i $\neq$ j

Constraint (43) ensures the total cargo demands at the depots does not exceed the combined capacity of all C-5s hauling cargo to the depots:

(43) $(\sum_c V_c C^{c}{}_{ab}) - Z_{ab} \geq 0, \forall a, b$

Constraint (44) ensures that whenever C-5 tail number "c" is based at a CONUS supply node "a", then it will depart from node "a" to a Japanese depot "b". Similarly, equation (45) ensures that if C-5 tail number "c" is based at a CONUS supply node "a", then any aircraft tracking variables which indicate that C-5 tail number "c" departed from a CONUS node other than "a" will have a value of 0. Let's define these constraints as follows:

$\forall\ a \in S,\ \exists\ b_a \subseteq D$ and $b^*{}_a \subseteq D,$ where $b_a \cap b^*{}_a = \varnothing$ and $b_a \cup b^*{}_a = D,$

such that:

(44) $\sum_{b_a} C^c{}_{ab_a} = 1, \forall c, a,$

and

(45) $\sum_{b^*_a} C^c{}_{ab^*_a} = 0, \forall c, a$

Equation (46) ensures that the offload time (the B_{ij} ground dwell time variable) of each C-5 at its Japan depot transload site takes at least G_c hours (where G_c is the pre-specified minimum offload time for C-5 with tail number "c"):

(46) $C^c{}_{ab} - (1/G_c)B_{cb} \leq 0, \forall c, a, b$

(Note: For a C-5, the planned ground offload time is 3 +15, or 3.25 hours. So in this case, $G_c = 3.25$, so $1/G_c \cong .30769$) (19:18)

Equation (47) makes sure that each C-5 returns to its CONUS base of origin from the Japan depot at which it transloads its cargo:

(47) $C^c{}_{ab} - C^c{}_{ba} = 0, \forall c, a, b$

In Section 1.2 the concept of MOG was explained. We need to ensure that the maximum number of C-5s and C-17s combined in any 32-hour period (the period each "run" of this model examines) is not greater than the maximum number possible due to the MOG value for both C-5s and C-17s together at each Japanese depot. (Note: In this model's context, we're defining the planned time to calculate MOG as the <u>entire 32-hour</u>

period for one model run, not the usual method of using the average aircraft offload time. This allows the MOG values used to be consistent with our model's formulation. We can thus use the term "MOG32" to mean the maximum number of aircraft which can transition through a given base in a 32-hour period). From this assumption comes equation (48), which stipulates that the sum of all C-5s entering and departing depot "b", plus the sum of all C-17s entering and departing depot "b", is less than or equal to the MOG32 value for depot "b". (Note: In Section 4.2 we'll analyze the methodology for determining the MOG32 values). We have:

$$(48) \quad \sum_{c}\sum_{a} C^{c}{}_{ab} + \sum_{c}\sum_{a} C^{c}{}_{ba} + \sum_{h}\sum_{j} X^{h}{}_{bj} + \sum_{h}\sum_{j} X^{h}{}_{jb} \leq \text{(MOG32 value of depot "b")}, \ \forall b$$

Likewise, we'll have a MOG32 constraint limiting the number of C-17s which can visit and depart from each Korea destination in the 32-hour period. This constraint will take the form:

$$(49) \quad \sum_{i}\sum_{h \in H_x} X^{h}{}_{ij} + \sum_{i}\sum_{h \in H_x} X^{h}{}_{ji} \leq \text{(MOG32 value of destination "j")}, \ \forall j, \text{ where } i \neq j$$

Equation (50) is the C-5 equivalent version of equation (37), which is used to calculate the value of the C-17 allocation variables (X_{ij}). Equation (50) links the C-5 allocation and routing variables to determine the value of the C_{ij} allocation variables. The first term, $\sum_{m} C^{h}{}_{am}$, is the number of ways for C-5 "h" to depart CONUS supply node "a", and the second term, $\sum_{n} C^{h}{}_{nb}$, is the number of ways in which C-5 "h" can visit

Japan depot “”b”. Any C_{ab} allocation variable must equal 1 if one of the C^h_{am} and one of the C^h_{nb} variables are both equal to 1 (i.e. CONUS node “a” supplies depot “b” whenever a C-5 departs from node “a” and visits “b”):

$$(50) \quad \sum_m C^c{}_{bm} + \sum_n C^c{}_{na} - C_{ba} \leq 1, \forall c,a,b$$

Equations (51) and (52) are similar to eqs (27) and (28). They link the S_{ca} CONUS departure times (referenced to time 0.0 hours), the “D_j” time window arrival times (odometer variables) and the C-5 16 hour CDD. Together, they calculate D_b, the transload completion times of cargo from C-5 aircraft h to a C-17 at Japan depot b:

$$(51) \quad 16C^c{}_{ab} + S_{ca} + D_a - D_b \leq 16 - d_{ab}, \forall h,a,b$$

$$(52) \quad 16C^c{}_{ab} - S_{ca} - D_a + D_b \leq 16 + d_{ab}, \forall h,a,b$$

We’ll now formulate a numerical example using these equations

3.11 A Numerical Example - Case Study

Recall the methodology for determining the demands for split destination nodes as explained in Section 3.6. In order to determine the coefficients for the X^h_{ijsd} variables (i.e. the demands) for each “split” destination, we’ll take the total demand at a given destination from each CONUS supply base and divide by the C-17’s capacity. We’ll now use realistic values for our leg distances and payload capacities. As we’ll explain in Chapter 4, the maximum possible payload for a C-17 flying from Japan to Korea, making as many as 4 visits (no more than this are possible due to crew duty day limits), and returning to Japan, is 86 tons. (Note: As previously mentioned, we'd likely reach our volume capacity prior to this weight capacity, but for simplicity we'll assume our C-17 payload limit is 86 tons) We'll assign a "split" node for every time we can assign a demand of exactly 86 to a split. When we can no longer divide demands from any CONUS base by 86 tons, we'll combine the "leftovers" into one or more split nodes.

For example, suppose node 3 in Korea requires 250 tons from CONUS node 6 and 230 tons from CONUS node 7. Let's first form split nodes 3A and 3B, each with 86-ton demands from CONUS node 6 (with 78 tons left over), and then form nodes 3C and 3D with 86-tons demands each from node 7 (with 58 tons remaining). Next, we'll combine the leftovers (136 total tons) and form nodes 3E and 3F. Node 3E will have a 78-ton demand from node 6 and an 8-ton demand from node 7 (this totals to 86 tons), and node 3F will have demands of 0 tons from node 6 and 50 tons from node 7.

This node-splitting convention makes three realistic assumptions:

1. To the maximum extent possible, C-17s will carry full cargo loads
2. To minimize crew day lengths and possible exposure to enemy forces, C-17s will fly as few legs as possible (at the same time, the minimum number of C-17s required to meet the demands will be used)
3. To minimize "sorting" of cargo (by CONUS site) at the transshipment depots (and thereby make tracking cargo as easy as possible), C-17s will carry as much cargo as possible from one CONUS supply node

The following example illustrates this methodology, bringing together all the concepts discussed thus far.

Given:

- CONUS supply nodes 6 and 7 (representing McChord and Travis AFBs, respectively)
- Japan-based transshipment nodes 1 and 2 (representing Yokota and Kadena ABs, respectively)
- Korean destination nodes 3, 4 and 5 (representing Osan, Taegu and Kunsan ABs, respectively)
- Osan AB requires 57 tons from node McChord and 49 tons from Travis
- Taegu requires 49 tons from node McChord and 69 tons from Travis
- Kunsan requires 66 tons from McChord and 54 tons from Travis

Refer to Figure 7 for a graphic of this scenario.

We'll first determine our "split" destination nodes. Node 3 (Osan) can be split into nodes 3A (with a 57-ton requirement from McChord and a 29-ton requirement from Travis) and 3B (with a demand of 0 tons from McChord and a 20-ton requirement from Travis). Node 4 (Taegu) will be split into nodes 4A and 4B, with demands of 17 and 32 tons from McChord, respectively, and demands of 69 tons and 0 tons from Travis, respectively. Lastly, we'll split node 5 (Kunsan) into nodes 5A and 5B. Node 5A will have requirements from McChord and Travis of 66 tons and 20 tons, and node 5B will have demands 0 tons and 34 tons from McChord and Travis.

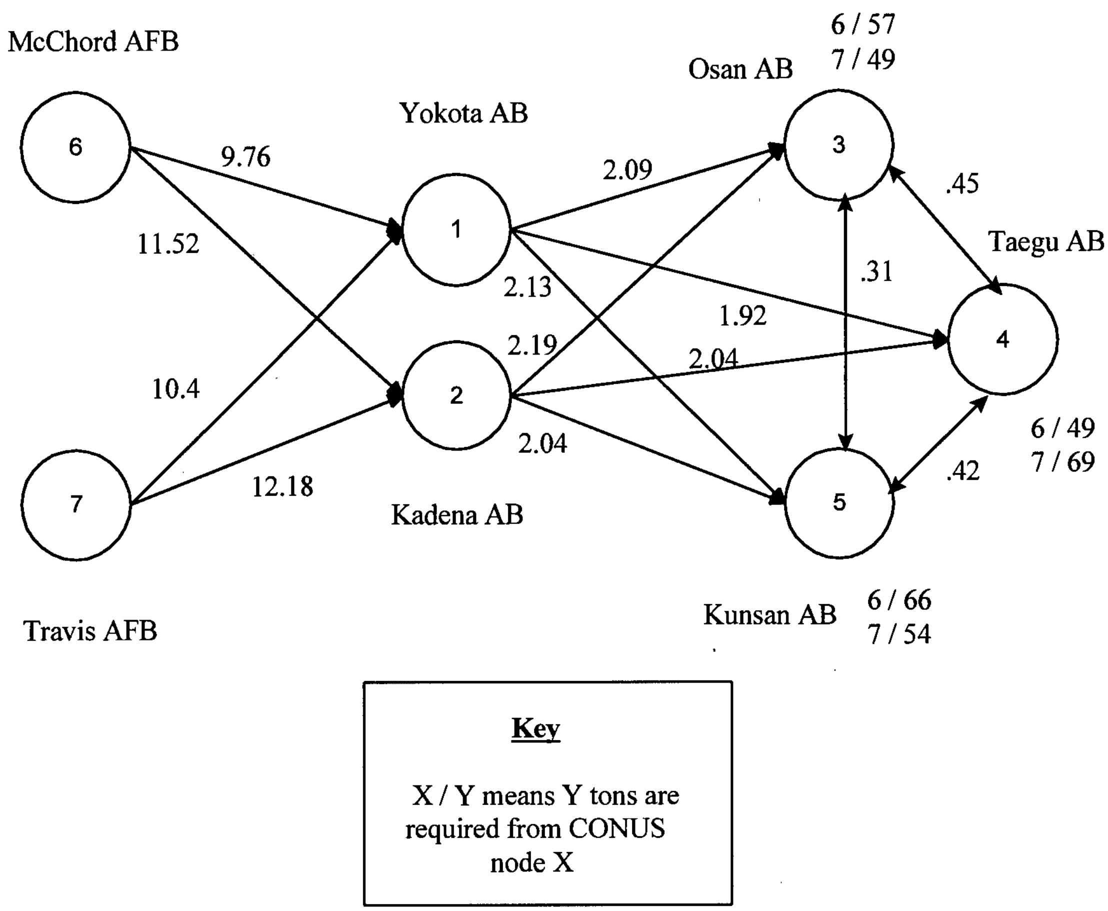


Figure 7 - Case Study Model Before Splitting Nodes

We also need to determine how many C-5s are needed. We'll assume each C-5 performs one A/R midway between the CONUS and Japan, and thus has a capacity of 114 tons (in Chapter 4 we'll explain how to determine payloads). Since McChord must provide 57 + 49 + 66 = 172 tons, we need to assign two C-5s to this CONUS base .

Similarly, we'll need two C-5s at Travis, since the total demand originating from there is 49 + 69 + 54 = 172 tons. Figure 8 depicts how our Hub-and-Spoke network now appears after performing our node splits, and with our 4 C-5s pre-assigned to our CONUS bases.

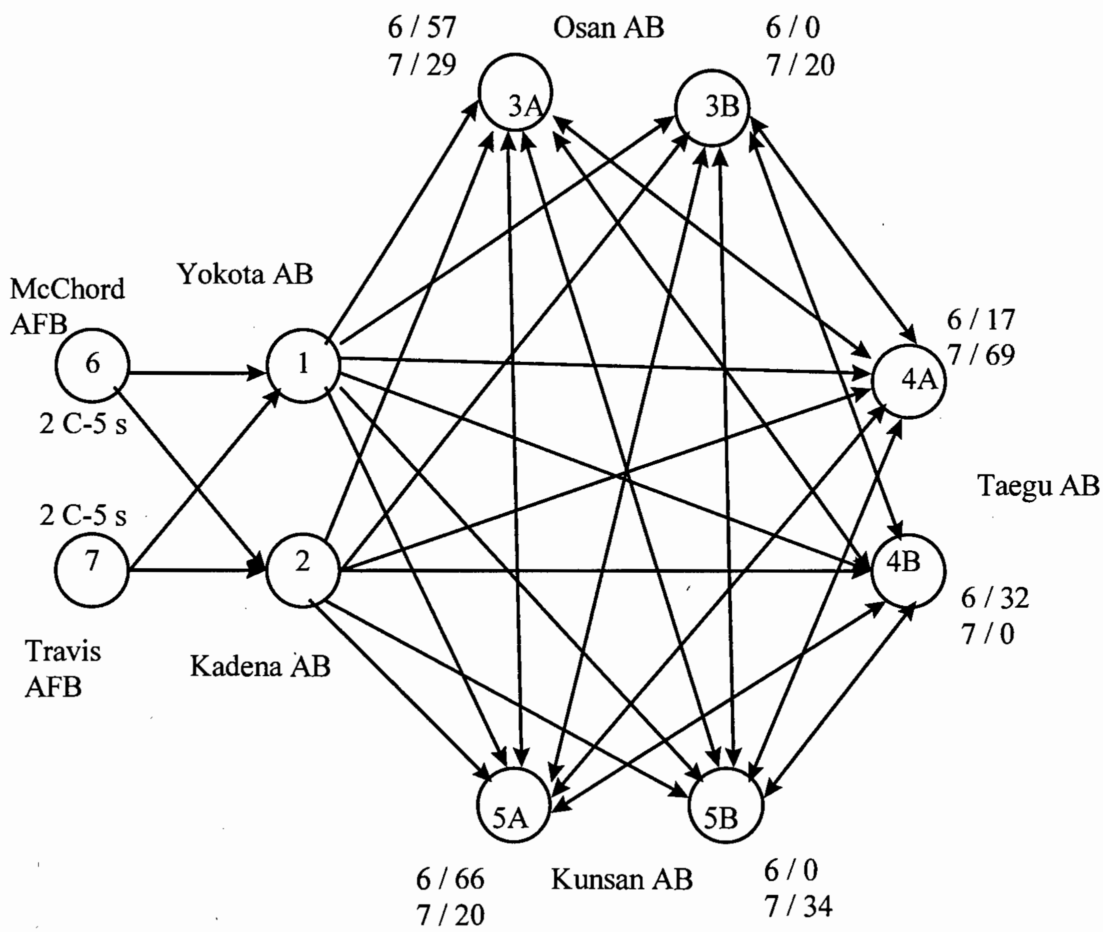


Figure 8 - Case Study Model After Splitting Nodes

Next, we need to determine how many C-17s are needed. If we incorporated as many C-17s as we have split destination nodes, we'd guarantee coverage of all demand nodes. Therefore, this is the <u>most</u> C-17s we'd ever need. And since one C-17 may be able to deliver to more than one "leftover" split node (which has a demand less than the C-17s capacity) we could possibly fulfill our demands with fewer C-17s. The way to let the model and solver select the optimal number of C-17s is to incorporate constraints allowing all C-17s to be used (in our example, six C-17s, since we have six split nodes), but add a penalty to the objective function forcing the model to use the minimum number of C-17s necessary. Since the crew duty day of a C-17 crew is 16 hours, and we're minimizing total hours flown, if we add a penalty of 16 hours for each C-17 used, we'll effectively ensure the model will select the minimum number of C-17s required to meet the demands.

We'll use the same time window requirements as in the previous examples:

	Node	NET	NLT
(Osan AB)	3	16	20
(Taegu AB)	4	17.5	21.5
(Kunsan AB)	5	19	25

In the AMC airlift system there are generally five types of cargo capable of being hauled by aircraft (20:1-11, 1-12):

A). *Bulk cargo* is typically loaded on 463L pallets or containers, and is transportable by common cargo aircraft

B). *Oversize cargo* exceeds the usable pallet dimensions, is not greater than 1090 inches in length, 117 inches in width, and 105 inches in height, and is transportable by the C-5, C-17, C-141, C-130 and KC-10 (ex. Humvee)

C). *Outsize cargo* exceeds the dimensions of oversize, *and is only transportable* *via the C-5 or C-17* (ex. M1A1 Abrams tank)

D). *Rolling stock* can be driven directly on or off the aircraft

E). *Special cargo* requires special preparation or handling (ex. nuclear weapons)

If we were to specify each ton of cargo by type, we'd add many more subscripts, variables, constraints, and complexity to our model. Therefore, in the interest of simplicity, we'll classify our cargo only by weight and CONUS origin. (The fact that we've selected the C-5 and C-17 makes this decision less of a contentious issue, as these two aircraft are the only two capable of carrying all possible cargo types, so our model doesn't run the risk of, say, a C-130 or Civil Reserve Air Fleet (CRAF) civilian carrier trying to haul an M1A2 Abrams tank, for example). The bottom line is this: It's important to realize that in this formulation cargo is homogeneous in the sense that all cargo is assumed to have the same weight and volume per item. At the same time, it is not homogeneous in that we specify the CONUS origin for the cargo weight needing to be delivered to each Korea base. Perhaps the easiest way to understand this concept is with an analogy, of sorts. Think of cargo as being individual soldiers, each with the same height, weight and volume. Although all soldiers in this hypothetical are physically identical, they come from different locations in the U.S. Further, each stateside soldier is to be deployed to a pre-determined Korean base. Our model does the same for cargo. Each piece of cargo has a specific CONUS departure point and destination in Korea, and our model determines the optimal routing of the C-5s and the optimal basing and routing of the C-17s to ensure each ton of cargo reaches it's intended destination.

In this manner, we are aggregating the CONUS portion of the model, but disaggregating the rest. Unlike other airlift models such as NRMO and THRUPUT II, our model incorporates individual aircraft routes and individual bases. Admittedly, this level of detail adds to the complexity of this model. However, it also adds to its fidelity and usefulness from the strategic and tactical aspects. And here is where the beauty of the model appears. It doesn't simply send the entire army from one CONUS supernode to one Korean supernode. It determines how much of a given cargo load at any CONUS base should be apportioned, sent to the depots via C-5, and then determines which individual C-17s and routes to fly from Japan to Korea to deliver the individual pounds of cargo to each individual specified Korean destination base.

Our time window requirements at our destination bases are the same as in our previous examples:

Node	NET	NLT
3	16	20
4	17.5	21.5
5	19	25

The formulation and solution to the scenario depicted in Figure 7 is presented in Appendix 4A and Appendix 4B, respectively. The problem was solved via the CPLEX linear solver, version 3.0, at the Air Force Institute of Technology. It took 27 hours and 20 minutes to solve! (Recall the discussion on computational complexity in Section 2.5. This formulation in Appendix 4A contains 2686 individual constraints and 1581 decision variables, of which 1488 are binary integers. It is this large number of integer variables which cause the long solution times.)

A graphical depiction of the solution is shown in Figure 9, on the following page. As depicted, C-5 aircraft #1 and #3 depart from McChord and Travis, respectively, and haul 83 tons and 89 tons, respectively, to Kadena. Similarly, C-5 aircraft #2 and # 4 depart from McChord and Travis, respectively, and haul 89 tons and 83 tons, respectively, to Yokota. From Japan, 4 of the 6 possible C-17s are used to deliver cargo to Korea, as follows:

C-17 #1 and C-17 #2 are not needed to ensure demands are met

C-17 #3 delivers 57 tons from McChord and 29 tons from Travis to Osan

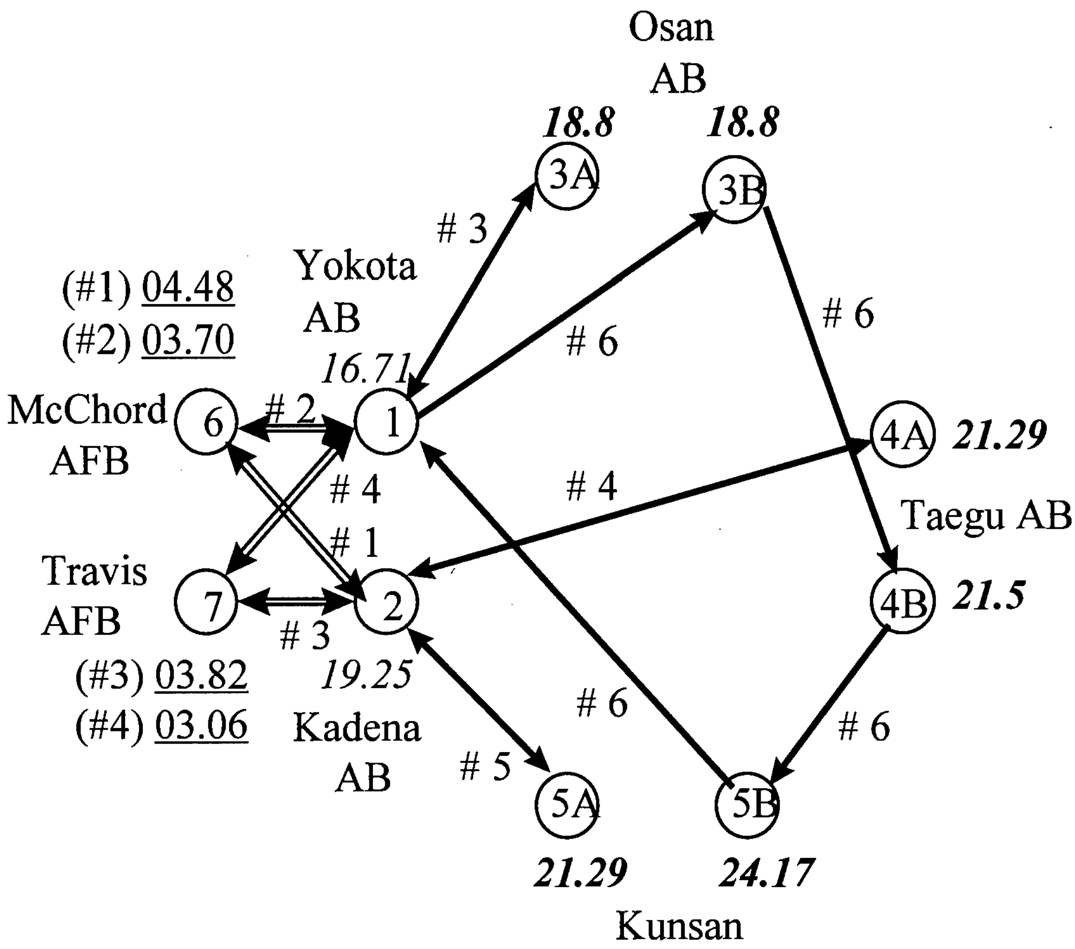
C-17 #4 delivers 17 tons from McChord and 69 tons from Travis to Taegu

C-17 #5 delivers 66 tons from McChord and 20 tons from Travis to Kunsan

C-17 #6 first delivers 20 tons from Travis to Osan, then hauls 32 tons from McChord to Taegu, then carries, and finally hauls 34 tons from Travis to Kunsan

As we can see, all cargo demands are satisfied, cargo is delivered from the pre-specified CONUS bases, all time windows are met, the optimal number of C-17s to be used is given, C-17 depot basing assignments are given, and optimal routing for the C-5s and the C-17s is determined.

Close examination of Figure 9 reveals that <u>none</u> of the C-17 arrival times coincides with the NET times specified by our time windows. In fact, C-17 #4 visits Taegu (node 4B) at 21.5 hours, the NLT deadline for arrival to Taegu!



KEY

(# h) <u>XX.XX</u> = C-5 Dept Time

XX.XX = Transload Completion Time

XX.XX = C-17 Arrival Time

TIME WINDOWS

Node	NET	NLT
Osan AB	16	20
Taegu AB	17.5	21.5
Kunsan AB	19	25

Figure 9 - Solution to the Case Study Scenario

This raises an interesting question. What would the optimal solution to our scenario be if decided that it was more desirable to arrive in Korea as close to the NET times as possible (this implies that earlier arrivals are better, versus the original objective function's view that, as long as we meet the time window constraints, it doesn't matter when in the time window we actually arrive).

To solve this new problem, we simply re-formulate the objective function of our model. The new objective function simply minimizes the sum of the arrival times into Korea, as well as minimizing the number of C-17s used

$$(53) \quad \text{MIN} \sum_j D_j + 16\sum_b \sum_j \sum_h X^h{}_{bj} + \sum_j \sum_b X_{jb} + \sum_b \sum_a C_{ba}$$

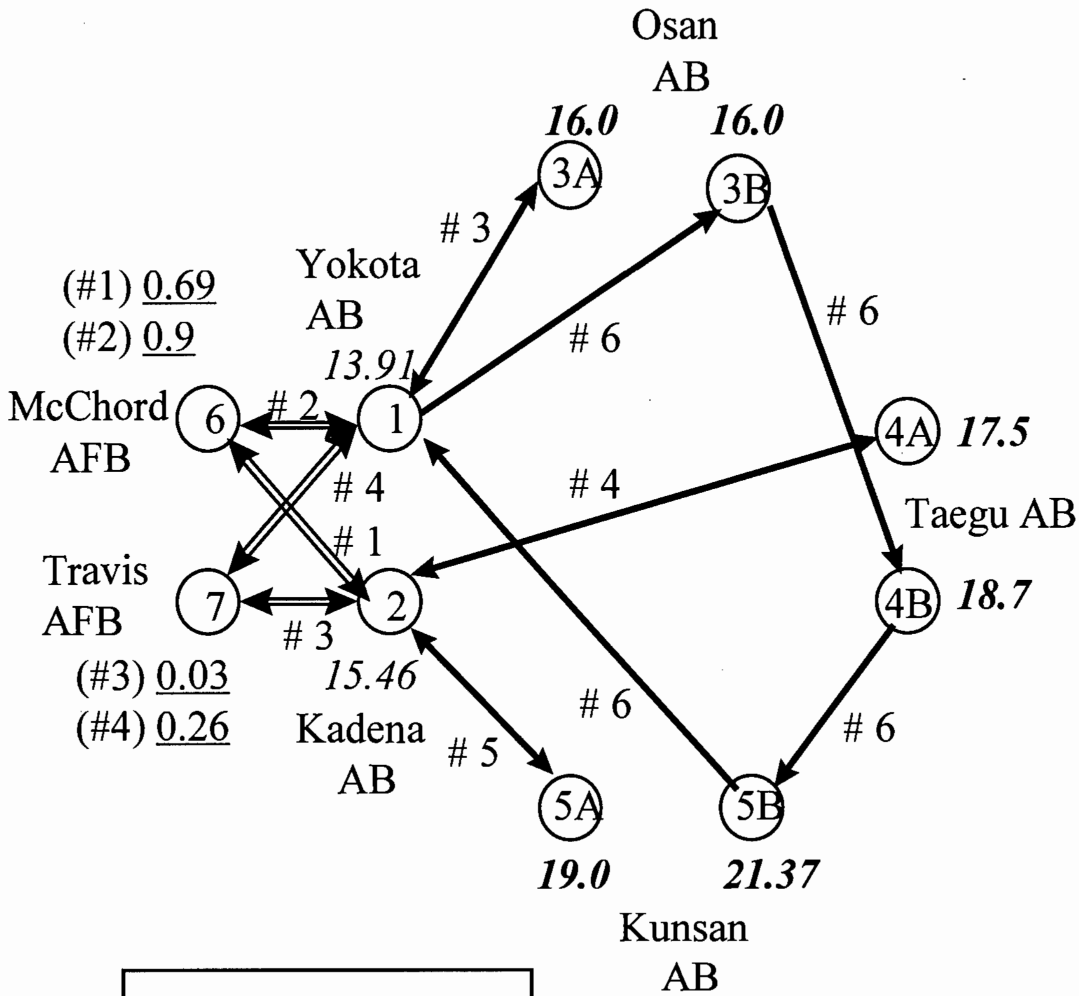
(Note: Recall from Section 3.10 that the $\sum_j \sum_b X_{jb}$ and $\sum_b \sum_a C_{ba}$ terms are not needed for minimizing the time to deliver goods - they are included only to ensure any such binary variable not specifically forced to have a value of one will take on a value of zero)

The new objective function (equation (53)), and optimal solution to the model with this new objective, are provided in Appendix 5A and 5B, respectively. (Note: There is no need to provide the constraints of the model, as they are exactly the same as the model given in Appendix 4A.) Surprisingly, this model is much more time consuming to solve than the original formulation. It required 94 hours to solve on the Sun SPARCstation 10 using the CPLEX Version 3.0 solver!

A graphical depiction of the solution is given in Figure 10. Notice that the routing and cargo deliveries of all aircraft are exactly the same as in Figure 9. The most noticeable differences are the C-5 CONUS departure times, transload completion times at the Japanese bases, and the C-17 arrival times in Korea. Since our objective was to arrive at the Korean destinations as early as possible, four of the six Korean arrival times exactly match the NET times!

This appears to be a "better" solution than the one depicted in Figure 9. Aside form the obvious differences in departure, transload, and arrival times, the other difference (which is not at all obvious) between the solutions shown Figures 9 and 10 is this: Figure 9's objective function minimizes ground dwell times, so none of the crews "burn CDD" sitting on the ground waiting for a time window downstream to open up. In the solution to Figure 10's problem, C-17 aircraft # 5 actually has a 1.5 hour delay at Kadena AB, after its transload is completed, before it can depart for Kunsan AB. If it doesn't delay, it will arrive at Kunsan 1.5 hours before the time window allows. This is wasteful in that 1.5 hours of its 16-hour CDD are spent doing nothing.

The whole point of comparing Figures 9 and 10 is to show that there is no "correct' objective function for this problem. It all depends on the goals of the user. If we want to maximize useful CDD and simply meet the time windows, Figure 9 is the optimal solution. However, if arriving earlier in the time window is considered better than arriving later in the time window, and delays are not a concern, then Figure 10 is the optimal solution.



KEY

(# h) <u>XX.XX</u> = C-5 Dept Time

XX.XX = Transload Completion Time

XX.XX = C-17 Arrival Time

TIME WINDOWS

Node	NET	NLT
Osan AB	16	20
Taegu AB	17.5	21.5
Kunsan AB	19	25

Figure 10 - Solution After Changing the Objective Function

There are numerous other ways we could define the objective, and the objective may actually change over time or by the current situation. The beauty of a linear model is we can make quick changes to the objective function (as well as the constraints) to suit our needs.

Chapter 4 Data and Analysis

4.1 Introduction

In Sections 4.2 though 4.4 we'll discuss how the values for crucial parameters in the model were determined. Then in Section 4.5 we'll do a comparison of Hub-and-Spoke versus Direct Delivery, using seven different metrics, to see in which types of airlift scenarios each method has advantages and disadvantages. From a head-to-head comparison, we can suggest general guidelines for which delivery method is better for a given situation. One major goal in this study , aside from building a Hub-and-Spoke model to analyze the CONUS-to-Korea airlift scenario, is to be able to utilize the model to make more general conclusions about the use of Hub-and-Spoke for any airlift operation, anywhere in the world. Finally, Section 4.6 supports the hypothesis that the C-17 is better-suited in certain airlift scenarios, including the CONUS-to-Korea problem, for local (theater) deliveries versus trunk (strategic) deliveries.

4.2 Explanation of MOG32 Values Used

We mentioned back in Section 1.2 that a ballpark value for MOG at Japanese bases is 10 for <u>only</u> C-17s and 5 for <u>only</u> C-5s. It's reasonable to assume, then that the MOG value at a Japanese base if both C-17s and C-5s will transit through is a value between 5 and 10, which we'll assume is 6. Keep in mind that this MOG value is the average number of aircraft to transit through a base in a planned time period, equal in duration to the average ground time (i. e. minimum cargo offload time). We know that the average ground times are 2 + 15 for a C-17 and 3 + 15 for a C-5, so we'll assume the average ground time for both C-17s and C-5s at a Japanese base is 2 + 45. Since one run of model is NGT 32 hours, (and $32 \div 2.75 \cong 11.6$) We could conceivably have 11.6 periods where 6 aircraft on average, per period, could transit through Japan. Therefore, a total of 70 (since $11.6 \times 6 \cong 70$) C-17s and C-5s combined could transit through each Japanese depot. We can thus write our depot MOG32 constraint (eq 54) as:

$$(54) \qquad \sum_{h \in H_c} \sum_{a} C^h{}_{ab} + \sum_{h \in H_x} X^h{}_{bj} \leq 70 \text{ (= MOG32 value of depot ``b'')} \ , \forall b \in D$$

(Note: Recall that the planned ground time in this context refers to the entire 32 hour length of one model run) Admittedly, this is simplistic and assumes maximal flow with no possibility of delays, but this does set an upper limit on the number of aircraft to transition through each Japan base in one run of our model.

Likewise, there will be a MOG constraint for each Korean destination base. For the purposes of our final model run problem we made the simplifying assumption that all Korean destinations have a MOG of 6 regarding C-17s, meaning each Korean base can service at most 6 C-17s in each 2+15 hour planned offload period. Again, with one model run covering a 32-hour period, (and $32 \div 2.25 \cong 14.2$) we could conceivably have 14.2 periods where 6 C-17s could transit through each Korean destination. This means 85 (since 14.2 x $6 \cong 85$) C-17s could transit through each Korean destination in 32 hours. We can thus write (eq 55) as:

(55) $\sum_{i}\sum_{h \in H_x} X^h_{ij} \leq 85$ (= MOG32 value of destination "j"), $\forall j \in F, i \in D$ and F, but where $i \neq j$, and i and j are not splits in the same node set

One inaccuracy to this method is the fact that, theoretically, our model could allow as many as 85 aircraft to arrive at a given Korean destination in a very short time period; possibly all at the exact same time! This is where the fact that our model allows us to assign individual time windows to each destination node is extremely useful. In Section 3.5 we gave an example of how different time windows could be devised for each split node to ensure aircraft arrivals to any given destination are spread out. This method can be used as a "supplement" to equation (55). While adding individual time constraints to individual splits increases the complexity of the model, it also makes it much more realistic.

4.3 Distance Calculations

It's obvious that a very important factor in our model is the distance, d_{ij}, expressed in hours, between bases "i" and "j". To calculate these d_{ij} values we need to know two things: the nautical mileage distance between i and j, and the speed of the aircraft between i and j. One may reasonably ask who we don't simply express d_{ij} as the nautical mile distance between i and j and save ourselves a step. There are two very important reasons we don't do this.

The first reason is pragmatic. One goal of our model is to meet window constraints, which are expressed in hours, so in order to talk "apples and apples" versus “apples and oranges” we need to standardize our metric by expressing distance in time units.

A less obvious reason for converting our mileage distances to time distances is due to the fact that flying time (in hours) is not directly proportional to distance (in nautical miles). Consider the fact that every aircraft always must takeoff and land, regardless of the flight distance, and the slower speeds at which it performs these phases of flight has more of an adverse impact on the overall flight time of a shorter flight than it does on a longer flight. (For example, suppose it takes an aircraft 20 minutes total time to takeoff, climb and accelerate to cruise speed, descend and land. If we cruise at 5 miles per minute, then this will make a theoretical 5 mile flight a 21 minute odyssey; 20 minutes for the ascent/descent and 1 minute for cruise. On the other hand, a 500 mile flight will take 120 minutes; 20 minutes for the ascent/descent and 100 minutes of cruise. Clearly, we see that if we simply took the 5 mile flight’s enroute time of 21 minutes and multiplied by 100 to estimate the flight time of the 500 mile trip, our 100 X 21 = 2100 minutes calculation would be grossly inaccurate). The point is, distance and time are not directly proportional over short distances in the flying arena due to the relatively large “fixed costs” of takeoff, climb, descent, and landing. Shorter flights will have a lower average speed since the ratio of cruise time to total flight time is much smaller than for a longer distance flight. Thus, time is a more accurate measure of cost than distance.

We need a way to factor the takeoff, climb, descent and landing times into our measurements to be accurate. To accurately convert these base-to-base mileage distances to time, I’ve consulted Air Force Pamphlet 10-1403, Air Mobility Planning Factors. This official planning document, used by the Air Mobility Command Studies and Analysis Flight for airlift analysis and modeling, offers a table of aircraft block speeds. Aircraft block speed is defined as "the average true airspeed over a specified distance, including takeoff, climb, cruise, descent, approach, landing, and taxi to block-in (i.e. parking)". (19:3) This value will take into account the fact that cruise speeds will be higher on average for greater distance flights.

Air Force Pamphlet 10-1403, Air Mobility Planning Factors, lists the following data for C-17 and C-5 block speeds: (19:17)

Table 1 - Aircraft Block Speeds (NM/hour True Airspeed)

Type	500 NM	1000 NM	1500 NM	2000 NM	2500 NM	3000 NM	3500 NM	4000 NM	5000 NM
C-17	243	348	386	402	410	415	421	430	----
C-5	242	347	385	401	409	414	420	429	429

If we divide the nautical mileage by the block speed, we get a block time (in hours):

Table 2 - Aircraft Block Times (Hours)

Type	500 NM	1000 NM	1500 NM	2000 NM	2500 NM	3000 NM	3500 NM	4000 NM	5000 NM
C-17	2.0576	2.87356	3.88601	4.97512	6.09756	7.22892	8.31354	9.30233	----
C-5	2.0661	2.88184	3.8961	4.98753	6.11247	7.24638	8.33333	9.32401	11.655

Now let's take the distances and block hours and perform a linear regression for both the C-17 and the C-5. This will result in an equation to accurately determine the block time for a given distance, d_{ij}, extremely quickly. Our regressions for the C-17 and C-5 follow, as Tables 3 and 4, respectively.

Table 3 - C-17 Regression for Block Speed Formula

C-17 Block Hours

DIST (NM)	TIME (HRs)
500	2.057613169
1000	2.873563218
1500	3.886010363
2000	4.975124378
2500	6.097560976
3000	7.228915663
3500	8.313539192
4000	9.302325581

Regression Statistics	
Multiple R	0.999318341
R Square	0.998637147
Adjusted R Square	0.998410005
Standard Error	0.103631715
Observations	8

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	47.21666385	47.21666385	4396.529	7.91442E-10
Residual	6	0.064437194	0.010739532		
Total	7	47.28110104			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.820544822	0.080749182	10.16164875	5.29E-05	0.622958547
DIST	0.002120572	3.19814E-05	66.3063259	7.91E-10	0.002042316

	Upper 95%	*Lower 95.0%*	*Upper 95.0%*
	1.018131096	0.622958547	1.018131096
	0.002198828	0.002042316	0.002198828

RESIDUAL OUTPUT | PROBABILITY OUTPUT

Observation	*Predicted TIME*	*Residuals*	*Standard Residuals*	*Percentile*	*TIME*
1	1.880830765	0.176782404	1.70587164	6.25	2.057613169
2	2.941116709	-0.06755349	-0.651861163	18.75	2.873563218
3	4.001402652	-0.11539229	-1.113484318	31.25	3.886010363
4	5.061688596	-0.086564218	-0.835306235	43.75	4.975124378
5	6.121974539	-0.024413564	-0.235580041	56.25	6.097560976
6	7.182260483	0.04665518	0.450201754	68.75	7.228915663
7	8.242546426	0.070992766	0.685048647	81.25	8.313539192
8	9.30283237	-0.000506788	-0.004890283	93.75	9.302325581

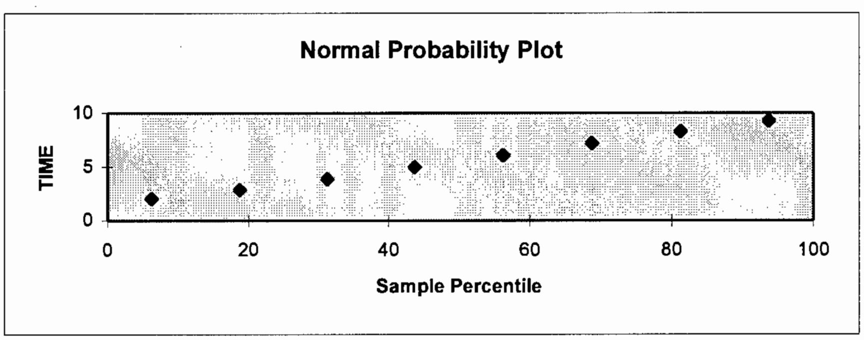


Table 4 - C-5 Regression for Block Speed Formula

C-5 Block Hours

DIST (NM)	TIME (HRs)
500	2.066115702
1000	2.88184438
1500	3.896103896
2000	4.987531172
2500	6.112469438
3000	7.246376812
3500	8.333333333
4000	9.324009324
5000	11.65501166

Regression Statistics	
Multiple R	0.999446362
R Square	0.99889303
Adjusted R Square	0.998734891
Standard Error	0.112471822
Observations	9

ANOVA

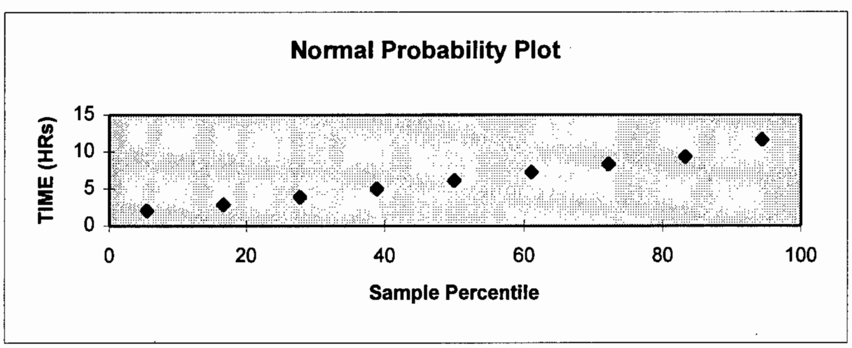
	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	79.904008	79.904008	6316.567	1.314E-11
Residual	7	0.088549376	0.012649911		
Total	8	79.99255737			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.773497938	0.078756186	9.82142448	2.41E-05	0.587269283	0.959726593	0.587269283	0.959726593
DIST (NM)	0.00215397	2.71019E-05	79.47683254	1.31E-11	0.002089885	0.002218056	0.002089885	0.002218056

RESIDUAL OUTPUT

PROBABILITY OUTPUT

Observation	*Predicted TIME*	*Residuals*	*Standard Residuals*	*Percentile*	*TIME (HRs)*
1	1.850483031	0.215632672	1.917215062	5.555556	2.066115702
2	2.927468124	-0.045623743	-0.405645986	16.66667	2.88184438
3	4.004453216	-0.10834932	-0.963346358	27.77778	3.896103896
4	5.081438309	-0.093907137	-0.834939235	38.88889	4.987531172
5	6.158423402	-0.045953965	-0.408582022	50	6.112469438
6	7.235408495	0.010968317	0.097520573	61.11111	7.246376812
7	8.312393588	0.020939745	0.186177702	72.22222	8.333333333
8	9.389378681	-0.065369357	-0.581206523	83.33333	9.324009324
9	11.54334887	0.111662788	0.992806787	94.44444	11.65501166



The regression equation to determine the C-17 block time is given on Table 3 in the "coefficients" section as:

(56) $.002121X + .8205448 = Y$, where X is the distance (d_{ij}) in nautical miles, and Y is the block time, in hours

Similarly, to determine the C-5 block time, we again look at the "coefficients" block of the regression output as given on Table 4 to get:

(57) $.002154X + .7734979 = Y$, where X is the distance (d_{ij}) in nautical miles, and Y is the block time, in hours

How do we determine the nautical mileage between bases "i" and "j"? The simplest way is to determine the direct point-to-point great circle distance "as the crow flies". AMC Studies and Analysis has an Excel spreadsheet called "distcalc.xls" which does this quickly, and was used to generate Table 5, on the following page, for selected CONUS, Japanese, and Korean bases.

Recognize that in reality aircraft usually cannot fly direct great circle routes due to special use airspace (warning and prohibited areas), noise abatement areas, national areas of identification (ADIZ boundaries), etc. To avoid these areas and for the benefit of ground-based controllers, aircrews normally file flight plans according to established jet routes. These "highways of the sky" have been established that, by design, result in slightly greater flying distances between two bases than the great circle distance.

Table 5 - Great Circle Nautical Mileage Between Selected Bases

Distmtrx		KTCM	KSUU	RJTY	RODN	RJFF	RJCO	RJBB	RJSM	RJN	RJAA	RJOO	ROAH	RJTT	RJFU	RKTN	RKPK	RKSO	RKJK	RKSS
McChord	KTCM	0	534	4172	4989	4571	3804	4342	3900	4279	4136	4349	5002	4166	4618	4526	4549	4525	4591	4511
Travis	KSUU		0	4478	5295	4903	4148	4654	4228	4592	4438	4666	5307	4469	4949	4878	4895	4889	4951	4878
Yokota	RJTY			0	819	458	452	181	312	122	50	200	831	24	497	520	509	600	619	614
Kadena	RODN				0	455	1206	648	1095	711	858	642	12	827	410	574	533	645	576	674
Fukuoka	RJFF					0	769	282	675	336	507	258	467	475	47	164	122	270	235	299
Sapporo	RJCO						0	575	145	517	444	572	1218	460	816	730	748	751	807	745
Kansai	RJBB							0	451	63	228	33	660	195	318	369	350	464	467	483
Misawa	RJSM								0	390	301	453	1107	318	721	664	674	704	751	704
Nagoya	RJNN									0	172	78	723	140	375	405	391	493	505	510
Narita	RJAA										0	249	870	32	545	570	559	650	669	663
Osaka	RJOO											0	654	217	297	338	320	431	437	450
Naha	ROAH												0	839	421	584	544	655	585	683
Tokyo Intl	RJTT													0	513	541	529	623	640	637
Nagasaki	RJFU														0	188	144	288	242	318
Taegu	RKTN															0	44	107	100	135
Kimhae	RKPK																0	148	122	176
Osan	RKSO																	0	74	30
Kunsan	RKJK																		0	99
Kimpo	RKSS																			0

However, for flights between bases more than a few hundred miles apart via "straight line", the difference is negligible. Our use of the distances in Table 5 are therefore completely acceptable.

Now, if we take our mileage distance from Table 5 and call this number "X", we then plug this "X" value into the corresponding regression equation (eqs (56) and (57)) to get "Y", the block time "distance" d_{ij} (in hours) between bases "i" and "j". Thus, equations (56) and (57) give us a rapid way to calculate block times between bases.

There is one important caveat to be aware of, however. Since distance and time are not proportional linearly for short distances, as we've pointed out, using equations (56) and (57) for "short" distances will lead to underestimating block times. What, then, is considered a "short" distance?

AMC analysts recommend that distances less than 500 NM are to be considered "short". Unfortunately, all Korean bases are less than 500 NM apart, There also are several instances of distances between Japan bases and Korean bases being less than 500 NM. AMC analysts recommend using a blockspeed of 240 knots (nautical miles/hour) for distances below 500 NM for all aircraft, including the C-17 and C-5. (One sole exception is the C-130, which use a blockspeed of 180 nautical miles/hour). (18:22)

While 240 knots is a pretty accurate estimate, we can be more even more accurate for shorter distances. For example, consider a flight from Kadena AB to Naha International Airport. While the straight line distance between the two fields is only 12 NM, from personnel experience I know that 12/240 = .05 hours = 3 minutes is completely unrealistic as an estimate of the time from taxi out at one base through landing and taxiing in at the other airport. Any crewmember will tell you that the minimum time needed to taxi out, takeoff, then fly any approach, land, and taxi back in to that very same base (i.e. theoretically, a straight whose distance is zero!) requires approximately .3 hours! With this in mind, it's fair to place a lower limit of .3 hours on our d_{ij} table for flights less than 72 NM (72/240 = .3).

We can now convert all the mileage distances to block times via the following rule:

1. If the mileage distance $\geq$ 500 NM, use equation (56) for the C-17 and equation (57) for the C-5
2. If the mileage distance < 500 NM but > 72 NM, divide the mileage distance by 240 to find the block time (in hours) for the C-17 and C-5
3. If the distance $\leq$ 72 NM, use .3 hours as the block time for the C-17 and C-5

Using these rules, we can equivalently express Table 5 in the more useful form of block time (i.e. $d_{ij,}$) for the C-17 and the C-5, in Tables 6 and 7, respectively. The values in Tables 6 and 7 are used for the dij entries in equations (22) - (52) of our Hub-and-Spoke model.

Table 6 - C-17 d_{ij} Block Time Values, in Hours, for Selected Bases

C-17 BlockTime		KTCM	KSUU	RJTY	RODN	RJFF	RJCO	RJBB	RJSM	RJNN	RJAA	RJOO	ROAH	RJTT	RJFU	RKTN	RKPK	RKSO	RKJK	RKSS
McChord	KTCM	0	1.95	9.67	11.4	10.5	8.89	10	9.091	9.89	9.59	10	11.43	9.65	10.6	10.4	10.47	10.4	10.6	10.4
Travis	KSUU		0	10.3	12.05	11.2	9.62	10.7	9.786	10.6	10.2	10.7	12.07	10.3	11.3	11.2	11.2	11.2	11.3	11.2
Yokota	RJTY			0	2.557	1.91	1.88	0.75	1.3	0.51	0.21	0.83	2.583	0.1	2.07	1.92	1.9	2.09	2.13	2.12
Kadena	RODN				0	1.9	3.38	2.19	3.143	2.33	2.64	2.18	0.05	2.57	1.71	2.04	1.951	2.19	2.04	2.25
Fukuoka	RJFF					0	2.45	1.18	2.252	1.4	1.9	1.08	1.946	1.98	0.2	0.68	0.508	1.13	0.98	1.25
Sapporo	RJCO						0	2.04	0.604	1.92	1.85	2.03	3.403	1.92	2.55	2.37	2.407	2.41	2.53	2.4
Kansai	RJBB							0	1.879	0.26	0.95	0.14	2.22	0.81	1.33	1.54	1.458	1.93	1.95	2.01
Misawa	RJSM								0	1.63	1.25	1.89	3.168	1.33	2.35	2.23	2.25	2.31	2.41	2.31
Nagoya	RJNN									0	0.72	0.33	2.354	0.58	1.56	1.69	1.629	2.05	1.89	1.9
Narita	RJAA										0	1.04	2.665	0.13	1.98	2.03	2.006	2.2	2.24	2.23
Osaka	RJOO											0	2.207	0.9	1.24	1.41	1.333	1.8	1.82	1.88
Naha	ROAH												0	2.6	1.75	2.06	1.974	2.21	2.06	2.27
Tokyo Intl	RJTT													0	1.91	1.97	1.942	2.14	2.18	2.17
Nagasaki	RJFU														0	0.78	0.6	1.2	1.01	1.33
Taegu	RKTN															0	0.183	0.45	0.42	0.56
Kimhae	RKPK																0	0.62	0.51	0.73
Osan	RKSO																	0	0.31	0.13
Kunsan	RKJK																		0	0.41
Kimpo	RKSS																			0

Table 7 - C-5 d_{ij} Block Time Values, in Hours, for Selected Bases

C-5 Block Time		KTCM	KSUU	RJTY	RODN	RJFF	RJCO	RJBB	RJSM	RJNN	RJAA	RJOO	ROAH	RJTT	RJFU	RKTN	RKPK	RKSO	RKJK	RKSS
McChord	KTCM	0	1.92	9.76	11.52	10.6	8.97	10.1	9.174	9.99	9.68	10.1	11.55	9.75	10.7	10.5	10.57	10.5	10.7	10.5
Travis	KSUU		0	10.4	12.18	11.3	9.71	10.8	9.881	10.7	10.3	10.8	12.2	10.4	11.4	11.3	11.32	11.3	11.4	11.3
Yokota	RJTY			0	2.538	1.91	1.88	0.75	1.3	0.51	0.21	0.83	2.563	0.1	2.07	1.89	1.87	2.07	2.11	2.1
Kadena	RODN				0	1.9	3.37	2.17	3.132	2.3	2.62	2.16	0.05	2.55	1.71	2.01	1.922	2.16	2.01	2.23
Fukuoka	RJFF					0	2.43	1.18	2.227	1.4	1.87	1.08	1.946	1.98	0.2	0.68	0.508	1.13	0.98	1.25
Sapporo	RJCO						0	2.01	0.604	1.89	1.85	2.01	3.397	1.92	2.53	2.35	2.385	2.39	2.51	2.38
Kansai	RJBB							0	1.879	0.26	0.95	0.14	2.195	0.81	1.33	1.54	1.458	1.93	1.95	2.01
Misawa	RJSM								0	1.63	1.25	1.89	3.158	1.33	2.33	2.2	2.225	2.29	2.39	2.29
Nagoya	RJNN									0	0.72	0.33	2.331	0.58	1.56	1.69	1.629	2.05	1.86	1.87
Narita	RJAA										0	1.04	2.647	0.13	1.95	2	1.978	2.17	2.21	2.2
Osaka	RJOO											0	2.182	0.9	1.24	1.41	1.333	1.8	1.82	1.88
Naha	ROAH												0	2.58	1.75	2.03	1.945	2.18	2.03	2.24
Tokyo Intl	RJTT													0	1.88	1.94	1.913	2.12	2.15	2.15
Nagasaki	RJFU														0	0.78	0.6	1.2	1.01	1.33
Taegu	RKTN															0	0.183	0.45	0.42	0.56
Kimhae	RKPK																0	0.62	0.51	0.73
Osan	RKSO																	0	0.31	0.13
Kunsan	RKJK																		0	0.41
Kimpo	RKSS																			0

Since this model can be used for any scenario anywhere in the world, the three possibility rule presented above can be used to build similar block value tables simply by utilizing the flying distances between each desired base.

4.4 Payload Calculations

Admittedly, determining the payload of any aircraft type is an inexact science. Everything from distances, winds, volume, weight, and altitudes flown can profoundly affect an aircraft's maximum payload. We'll make the simplifying assumption that volume is not a problem, and each aircraft modeled will have a weight capacity. (In reality, most airlift aircraft will fill up with respect to volume before they reach their weight-carrying capacity, so this assumption is not entirely indicative of reality, but it is more than sufficient for the purposes at hand). Our distance table (Table 5) above will be indispensable in helping us calculate our payloads.

Every cargo aircraft has a payload/range curve which tells us what cargo weight capacity it has if it flies a given distance. We can estimate this curve fairly accurately using piecewise linear approximation. Analysts at AMC often use the values in Table 8 as the extreme points for their payload/range curve approximations for the C-5 and C-17.

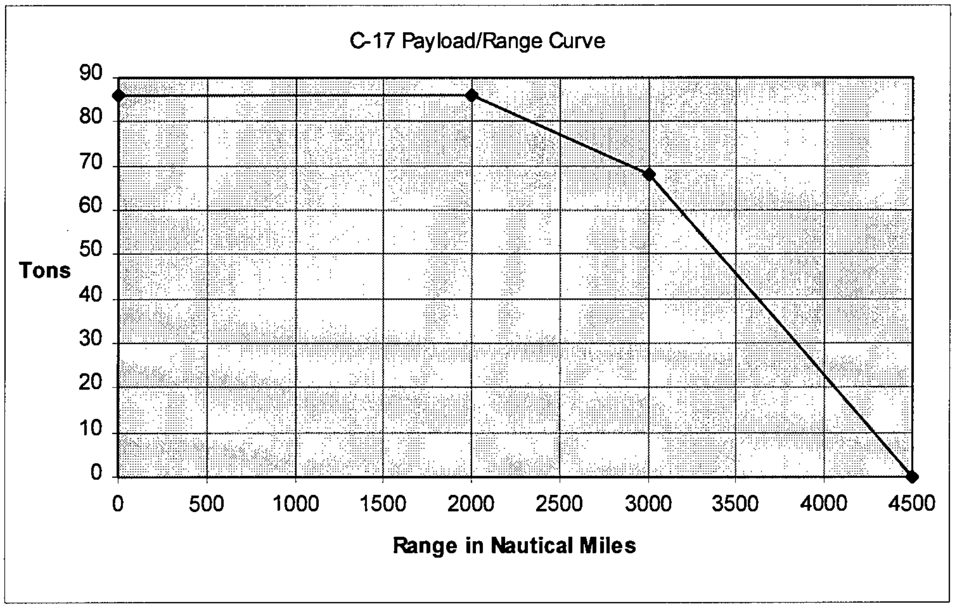
If we plot these points and "connect the dots" we get the payload/range curves for the C-17 and C-5 (Tables 9 and 10, respectively) on the following pages. We could use the tables simply by entering on the horizontal axis with the distance, moving vertically upwards until touching the curve, then moving horizontally to the left to the vertical axis and reading off the payload. This is imprecise, however, and fortunately

Table 8 - Extreme Points For Approximating
the C-5/C-17 Payload/Range Curves

C-5 Range (NM)	C-5 Tons		C-17 Range (NM)	C-17 Tons
0	145.5		0	86.1
1000	145.5		2000	86.1
5500	34.5		3000	68.2
6500	0		4500	0

there is an easier way, which is also precisely accurate. Since each curve consists of three straight line segments, we can simply calculate the equation for each of these line segments, which can then tell us exactly what our maximum payload is for a given distance.

Table 9 - C-17 Payload/Range Curve



After calculating the equation for each line segment on the C-17 curve, we have the following formulas:

(58) If C-17 distance > 0 NM but ≤ 2000 NM :

Tonnage = 86.1

(59) If C-17 distance > 2000 NM but ≤ 3000 NM:

Tonnage = (-.0179) X (Range) + 121.9

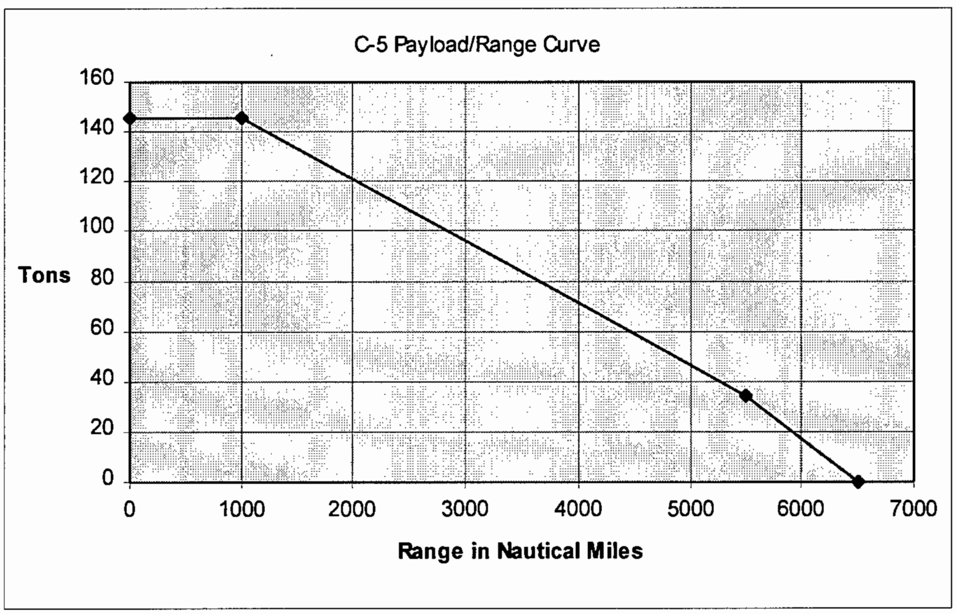
(60) If C-17 distance > 3000 NM but ≤ 4500 NM :

Tonnage = (-.04547) X (Range) + 204.6.9

(61) If C-17 distance > 4500 NM :

Tonnage = 0

Table 10 - C-5 Payload/Range Curve



Likewise, for the C-5 we have the following formulas:

(62) If C-5 distance > 0 NM but ≤ 1000 NM :

$$\text{Tonnage} = 145.5$$

(63) If C-5 distance > 1000 NM but ≤ 5500 NM :

$$\text{Tonnage} = (-.02467) \text{ X } (\text{Distance}) + 170.1667$$

(64) If C-5 distance > 5500 NM but ≤ 6500 NM:

$$\text{Tonnage} = (-.0345) \text{ X } (\text{Distance}) + 224.25$$

(65) If C-5 distance > 6500 NM:

$$\text{Tonnage} = 0$$

In our case study we assumed each C-5 performed one enroute refueling halfway between the CONUS and Japan. This means the refuelings took place approximately 2360 NM after takeoff. Using equation (63) we can calculate that the maximum payload is approximately 114 tons. Similarly, using the average distance between Japanese and Korean bases, 536 NM, with equation (58), we have a maximum C-17 payload of 86 tons. These payload values were used in our case study model.

This all assumes, of course, that the halfway point is a logical, and possible, A/R location for the tanker. In this case study it is. For the model to be used in a more generic format, the midpoint may not be accessible range-wise to tankers for providing any sizeable fuel offload. In Section 4.6 we'll see how we can determine the A/R support requirements for a mobility aircraft in <u>any</u> scenario, and examine in detail the CONUS-to-Korea refueling requirements for C-17s using the Direct Delivery method.

4.5 Comparison Between Hub-and-Spoke and Direct Delivery

In this section we'll make quantitative comparisons between Hub-and-Spoke and Direct Delivery in our CONUS-Korea scenario. (Comparisons are also made, and conclusions drawn, for the generic airlift situation anywhere in the world). The following metrics will be used to compare the two methods:

1). The need for inflight refueling support

2). Maximum payloads available

3). Crew Duty Day (CDD) time restrictions

4). MOG limitations

5). Ease of cargo tracking

6). Ability to meet time windows

7). Minimum number of airlifters needed

Each of these metrics in considered in turn, and conclusions drawn from head-to-head comparisons between Hub-and-Spoke and Direct Delivery are given. Keep in mind that Hub-and-Spoke implicitly assumes that a nation is able and willing to grant the United States access to its airports and/or air bases. We've stated up front in this analysis already that we're assuming bases in Japan are available as C-17 depot locations. As more and more bases outside the U. S. are closing, the availability of transshipment hubs is becoming more scarce.

***** Conclusions pertaining to *our particular case study* are given in *italic* font *****

***** More universal conclusions which pertain to **any given airlift scenario** are given in **bold** font *****

Three possible aircraft combinations were considered:

A. Only C-17s are used

B. Only C-5s are used

C. Both C-17s and C-5s are used

Obviously, possibilities A and B above make the transload of cargo from a C-5 to a C-17 meaningless, so in these instances Hub-and-Spoke simply means utilizing the transshipment bases as stopover/ground refueling/crew staging bases.

The following terms and information are used:

A/R refers to inflight refueling via a KC-10 or KC-135 tanker

Average distance from CONUS to Japan (based on average of all bases on Table 5) (including Naha and Kadena on Okinawa) = 4519 NM

= 10.40 hours via C-17 or 10.50 hours via C-5 (not including Okinawan airfields) = 4393 NM

= 10.14 hours via C-17 or 10.24 hours via C-5

Average distance from CONUS to Korea = 4719 NM

= 10.80 hours for C-17 or 10.94 hours via C-5

Average distance from Japan to Korean bases = 536 NM

= 1.96 hours via C-17 or 1.93 hours via C-5

Average distance between Korean bases = 148 NM

=.615 hours via C-17 or C-5

We've assumed each **Japan** depot has a **MOG** of:

10, if only C-17s visit

5, if only C-5s visit

6, if both C-17s and C-5s visit

Likewise, we've assumed each **Korean** destination has a **MOG** of:

6, if only C-17s visit

3, if only C-5s visit

4, if both C-17s and C-5s visit

CDD for a basic C-5 or C-17 crew (i.e. not augmented with spare crewmembers) is 16 hours. Recall that CDD refers to Crew Duty Day (the maximum number of hours an aircrew can perform flight duties)

Metric 1). The need for inflight refueling support

Consideration	Hub-and-Spoke	Direct Delivery
Do C-17s require inflight refueling?	No	Yes
KC-135/KC-10 support requirements for C-17s?	None	1 or 2 refuelings per C-17
Do C-5s require inflight refueling?	Yes, for useful payload	Yes, for useful payload

In our scenario, Hub-and-Spoke eliminates the need for C-17 A/R support, so it is more desirable than Direct Delivery. The distances flown by C-5s in Hub-and-Spoke are only 200 NM less than they'd be with Direct Delivery, so the Hub-and-Spoke benefits are negligible for the C-5. (Note: In Section 4.6 we'll examine this A/R support requirement for C-17s in depth).

In general, since Hub-and-Spoke reduces the distances each individual aircraft must fly, less inflight refueling support will be needed, freeing up tanker aircraft to refuel other receivers. This makes Hub-and-Spoke preferable to Direct Delivery where inflight refueling needs are concerned.

Metric 2). Maximum payloads available

Consideration	Hub-and-Spoke	Direct Delivery
C-17 max payload?	86.1 tons	N/A if no refueling With 1 refueling at midway point to Korea (2360 NM) = 79.66 tons With 2 refuelings (at the 1573 and 3146 NM points) = 86.1 tons
C-5 max payload?	With no refueling = 58.7 tons With 1 A/R enroute = 114.4 tons With 2 A/Rs enroute = 133 tons	With no refueling = 53.7 tons With 1 A/R enroute = 112 tons With 2 A/Rs enroute = 131.4 tons

Hub-and-Spoke has a clear advantage in terms of C-17 payload capability, since the shorter distances involved means less fuel is required on board so more cargo can be carried. Likewise, Hub-and-Spoke has a small advantage in allowing a slightly greater C-5 payload. This advantage is lessened as the number of inflight refuelings is increased.

In general, the shorter flying distances inherent with Hub-and-Spoke mean less fuel is required to be carried (without air refueling), allowing increased payloads among all airlifters.

Metric 3). Crew Duty Day (CDD) time restrictions

Consideration	Hub-and-Spoke	Direct Delivery
Maximum number of offloads in Korea possible, per C-17, with a 16 hour CDD?	4 (3.91 hours to fly to/from Korea, .615 hours of enroute time between bases, and 2.25 hours offload per stop) = 14.76 hrs	2 (10.83 hours one-way time to Korea, with the same .615 and 2.25 hour restrictions) = 15.95 hrs but this forces C-17 and crew to RON in Korea
Is C-5 CDD (16 hour) a problem for a flight from the CONUS?	No (10.51 hours between CONUS and Japan, and 3.25 hours to offload) = 13.76hrs CDD, and allows crews to stage out of Japan	No (10.94 hours between CONUS and Korea, and 3.25 hours to offload) = 14.19 hrs CDD but forces crews to stage out of Korea

CDD can limit the number of visits each aircraft can make to destination bases, as well as constrain where the aircrew (and aircraft) must remain overnight (RON). In our case scenario, the great majority of each C-17's CDD is spent in Korea making deliveries, whereas the cruise time spent crossing the Pacific Ocean during Direct Delivery cuts their possible visits in half. This makes Hub-and-Spoke clearly a better method for the C-17. Since the distance flown by each C-5 in our case study is only lessened by 200 NM with Hub-and-Spoke, neither method appears more beneficial in terms of C-5 CDD.

We must also mention that staging out of Japan, out of harms way, is preferable to staging out of Korea, where bases likely will become high-threat areas. For these two *reasons, in our case study scenario, the CDD constraints hinder Direct Delivery and clearly favor Hub-and-Spoke.*

Common sense tells us that since Hub-and-Spoke shortens the leg distances (assuming the transshipment base(s) are selected somewhere between the origins and destinations) it should never be a disadvantage where CDD limits are concerned, and most often will be more advantageous than Direct Delivery.

Metric 4). MOG Limitations

Consideration	Hub-and-Spoke	Direct Delivery
MOG situation: How many C-5s and/or C-17s can be used in one 32-hour period <u>if C-5s and C-17s are used?</u>	Japan depots can accommodate a combined total of 70 C-5s and C-17s, and Korean bases could accommodate 85 C-17s (see Section 4.2, eqs (53) and (54)	Korean destinations can accommodate a combined total of only 47 C-5s and C-17s (Our MOG for both C-5s and C-17s at a Korean base is 4 every 2.75 hours, so we get $(32 \div 2.75) \times 4 \cong 47$
How many C-5s and/or C-17s can be used in one 32-hour period <u>if only C-5s are used?</u>	Japan depots can service 49 C-5s. Recall that a Japan depot has a MOG of 5 for C-5s: ($[32 \div 3.25] \times 5 \cong 49$). However, we'd be limited to 30 C-5s into Korea	Korean destinations can accommodate 30 C-5s. Since the C-5 MOG for a Korean base is 3: $(32 \div 3.25) \times 3 \cong 30$
How many C-5s and/or C-17s can be used in one 32-hour period <u>if only C-17s are used?</u>	Japan depots can service 142 C-17s. Recall that a Japan depot has a C-17 MOG of 10: ($[32 \div 2.25] \times 10 \cong 142$). However, we'd be limited to 85 C-17s into Korea.	Korean destinations can accommodate 85 C-17s. Since the C-17 MOG for a Korean base is 6, we get: $(32 \div 2.25) \times 6 \cong 85$

As far as MOG limitations go, if we used *<u>only</u>* *C-17s* *<u>or</u>* *C-5s in our scenario we'd be limited to the same amount of aircraft used in each 32-hour period due to the MOG limitations at the Korean destinations, so at first glance neither method seems to be advantageous. Keep in mind, though, that if* *<u>only</u>* *C-17s* *<u>or</u>* *C-5s were used in an airlift to Korea, no cargo transload is required, but Japanese bases could act as valuable refueling points and/or crew staging bases away from harm's way. Therefore, Hub-and-Spoke has these potential advantages.*

It's much more likely that, in a real-world airlift, military necessity would require that <u>both</u> the C-17 <u>and</u> C-5 be used, as we've done in our model run. And in this situation, Hub-and-Spoke allows for more aircraft being entered into the system, if desired.

Important Notes:

1). The values of MOG used in any given scenario are the primary factors which determine the maximum number of aircraft allowable into the airlift system. Had the MOG values of the Korean and Japanese bases been reversed, Hub-and-Spoke would allow $(32 \div 2.75) \times 4 \cong 47$ aircraft (combined total of C-5s and C-17s) into Japan, and $(32 \div 2.25) \times 6 \cong 85$ C-17s into Korea, whereas Direct Delivery would allow $(32 \div 2.75) \times 6 \cong 70$ C-5s and C-17s together into Korea. In this instance, the limitation on aircraft into Japan clearly makes Direct Delivery a more accommodating delivery method.

2). Also keep in mind that our MOG32 values refer to the maximum number of aircraft that could transition through <u>a given base</u> in 32 hours. If the number of transshipment bases available is larger than the number of destinations, then it's quite possible, even if the destination bases have larger MOG32 values, more total aircraft could transition through the transshipment bases! The bottom line is that neither Hub-and-Spoke nor Direct Delivery is always a better delivery method where MOG is concerned.

With these points stated, we can offer this **rule of thumb**:

A) If our transshipment bases have larger MOG values than our destination bases, (and if there are at least as many transshipment bases as destination bases), Hub-and-Spoke appears to be more advantageous

B) If our destination bases have larger MOG values than our transshipment bases, (and if there are as many, or more, destination bases than transshipment bases), the bottleneck will occur at the transshipment bases, so Direct Delivery appears to be the better delivery method

---

Metric 5 - Ease of Cargo Tracking

Consideration	Hub-and-Spoke	Direct Delivery
Ease of Cargo Tracking	Possibly difficult due to transload at the depots	Easy

Direct Delivery undoubtedly is superior in terms of ease of cargo tracking.

Metric 6 - Time Windows

Consideration	Hub-and-Spoke	Direct Delivery
Time Windows (Are they met?)	Yes	Yes

Our problems have had very artificial time windows, given in terms of 4-6 hour blocks, and not beginning any earlier than 16 hours into the start of the clock. Since the CDDs of the C-5 and C-17 are both 16 hours, these time windows would artificially prohibit any Direct Delivery scenario from meeting time windows, since all crews start at clock time 0 and run out of CDD exactly at the time when deliveries can begin. Furthermore, in real-world TPFDDs it is much more likely to have time windows expressed in days, not hours. Therefore, it's not easy to use the prototype model developed here, with the numbers used, to compare Hub-and-Spoke to Direct Delivery.

What we can say with confidence, though, is that in real-world situations it is difficult to state that one delivery method is better at meeting time window constraints. On the one hand, Hub-and-Spoke requires that cargo be transloaded at the depots, creating a delay of several hours (approximately 3.25 hours in a C-5 to C-17 transload) are used up before final deliveries can be made to the destinations. Direct Delivery makes this delay unnecessary, so it is a better cargo delivery method in this respect.

However, since Hub-and-Spoke normally allows much shorter distances to be flown by individual aircrews, so a smaller portion of each crew's duty day is spent in the air cruising, more time can be devoted to offloading cargo and visiting multiple destinations. The shorter leg distances resulting from Hub-and-Spoke suggest that in this respect it is superior to Direct Delivery.

In the CONUS-to-Korea problem, both methods appear equally good at ensuring time windows can be met. **As a general rule** it appears that:

A) Direct Delivery may make more sense if meeting a NLT time is critical, since it eliminates the delays associated with transload of cargo.

B) If time windows are not prohibitively small, Hub-and-Spoke appears more advantageous, since generally, more offloads can made at the destinations by the C-17s, due to the CDD and payload advantages offered by Hub-and-Spoke.

Metric 7 - Minimum Number of Aircraft Needed

Consideration	Hub-and-Spoke	Direct Delivery
How many aircraft are needed (in the sample scenario) to meet all the destination demands with no A/R available?	6 C-5s and 4 C-17s	8 C-5s (C-17s don't have the range to make any deliveries from the CONUS without A/R enroute
How many aircraft are needed (in the sample scenario) to meet all the destination demands with 1 A/R available?	(With 1 A/R available for each C-5; the C-17s don't require A/R) 4 C-5s and 4 C-17s	(With 1 A/R available for each C-5 and C-17) 6 C-5s (limited by CDD) or 6 C-17s (limited by payload)
How many aircraft are needed (in the sample scenario) to meet all the destination demands with 2 A/Rs available?	(With 2 A/Rs available for each C-5; the C-17s don't require A/R) 4 C-5s and 4 C-17s	(With 2 A/Rs available for each C-5 and C-17) 6 C-5s (limited by CDD) or 4 C-17s (payload is increased due to A/Rs)

These results concerning the minimum number of aircraft required to meet the example problem cargo requirements, with A/R availability varying, are extremely interesting and point out some important facts:

1) **The biggest limitation on the C-5 in a CONUS-to-Korea Direct Delivery airlift is the planned offload time at each stop (3.25 hours) compared to the C-17 (2.25 hours). Due to the long flying times to get to Korea from the CONUS, only one offload can be performed, due to CDD limitations, in the C-5. By contrast, the C-17 can make 2 offloads at different Korean destinations (just barely - the C-17 CDD is between 15-16 hours with 2 offloads!). This points out the importance of the C-17's faster offload ability versus the C-5s. Augmenting the C-5 aircrew (adding additional spare crewmembers) changes the CDD limitation from 16 hours to 24 hours, and would allow the C-5 to make multiple stops in Korea.**
2) **The C-17 has nowhere near the range of the C-5, so any Direct Delivery airlift to Korea using C-17s requires at least one A/R. Even without A/R, the C-5 still can haul a modest cargo load.**
3) **Hub-and-Spoke using Japanese bases as C-17 hubs allows the C-17s to take advantage of their quick turn times <u>and</u> eliminates the need for A/R, so full payloads can be carried. Additionally, since 10-12 hours of a crew's duty day aren't wasted flying between the CONUS and Japan, C-17s can utilize their 16 hour crew duty day spending almost all that time making deliveries.**

4) **Provided A/R is available, the C-17's better use of CDD may or may not offset its smaller payload when compared to the C-5. Neither aircraft is better in every Direct Delivery scenario. If sheer payload is the top priority the C-5 can theoretically carry as much as 1.68 times the cargo of the C-17 (145 tons versus 86 tons). In the CONUS-to-Korea problem, it may be more important to get the cargo offloaded more expeditiously, or visit more than one destination. In these situations, the C-17 is better-suited for the task.**

4.6 Is the C-17 Better-Suited for Hub-and-Spoke or Direct Delivery?

The C-17 is truly the only airlifter in the AMC inventory capable of being used in either a strategic, long-range role or a shorter-range, tactical role. In fact, this ability to "wear two hats" was one of the main selling points of the C-17 to Congress. However, the stated purpose for the C-17 has focused on its Direct Delivery role. Indeed, the Air Mobility Master Plan explicitly states that "the C-17 brings to life the concept of direct delivery - the air movement of cargo and/or personnel from an airlift point of embarkation to a location as close as practical to the customer's final destination". (20:5-22)

Recall, as mentioned in Section 1.1, that in 1995 the RAND Corporation was asked to conduct a study, called the C-17 Tactical Utility Analysis, to examine possible roles for the C-17 as an in-theater airlifter. (26:iii) Since RAND's study was conducted under the premise of a Direct Delivery airlift from the CONUS, the term "in-theater airlift" refers to flights made delivering cargo from the PODs to forward operating bases (FOBs), all of which are located in the same destination country. One of the two scenarios RAND analyzed was a deployment from the CONUS to Korea (the other was a deployment to the Middle East). RAND modeled the use of C-130s and C-17s for POD-to-FOB cargo hauling, and concluded that:

> There is robust role for...C-17s in theater during major regional contingencies...The C-17 was increasingly favored over the C-130 as beddown-base capacity became more constrained. The C-17 makes better use of parking space per ton of cargo delivered...The rapid on- and off-load capability, fast en route speeds, and large cargo capacity of the C-17 make the in-theater mission a preferred role. (26:37-39)

If we believe RAND's analysis is correct, it certainly seems reasonable to conclude that using at least some C-17s in the Japan-to-Korea role, as we've done, makes sense. But any analysis requires more than intuition; we need to back up our claims with numbers.

One way we can make a comparison between the C-17's tactical short-range use in Hub-and-Spoke versus it's strategic, long-range role during Direct Delivery, is by evaluating the A/R support requirements and maximum payload each aircraft can deliver every 16 hours (one aircrew's maximum crew duty day limit). We've seen that the Hub-and-Spoke system in the Korean case study allows each C-17 to carry its full payload of 86 tons, with no A/R requirements, and make as many as 4 offloads within 16 hours. What kind of payload can the C-17 deliver in the CONUS-to-Korea problem using Direct

Delivery, given that it receives the required A/R support, and exactly how much A/R support is needed?

Let's assume the C-17 takes off with as much fuel as it can carry (payload permitting), and refuels to its full weight capacity (again, payload permitting). The midpoint of the route is the location at which the maximum possible payload from the CONUS to this point equals the maximum payload the aircraft can carry from this point to Korea (disregarding fluctuations in winds). Thus, the midpoint of the CONUS-to-Korea direct routes is the ideal A/R point.

We'll find the midpoint between Korea and the CONUS bases for the Direct Delivery routes. (Since the Korean bases are so close to each other, for all practical purposes we can select a single Korean base, say Taegu, to find these midpoints, with negligible loss of accuracy . However, since McChord and Travis are hundreds of miles apart, we'll consider both of these CONUS bases individually).

Using a globe and checking it with the AMC-provided "distcalc" spreadsheet, for the McChord-to-Korea route, the midpoint lies approximately at coordinates:

N 55' 00" E 176' 30",

and for the Travis-to-Korea route the midpoint is found at 100 NM northwest of Adak, Alaska, at coordinates:

N 53' 00" W 178' 00".

A pictorial representation of the two Direct Delivery routes is shown in Figure 11:

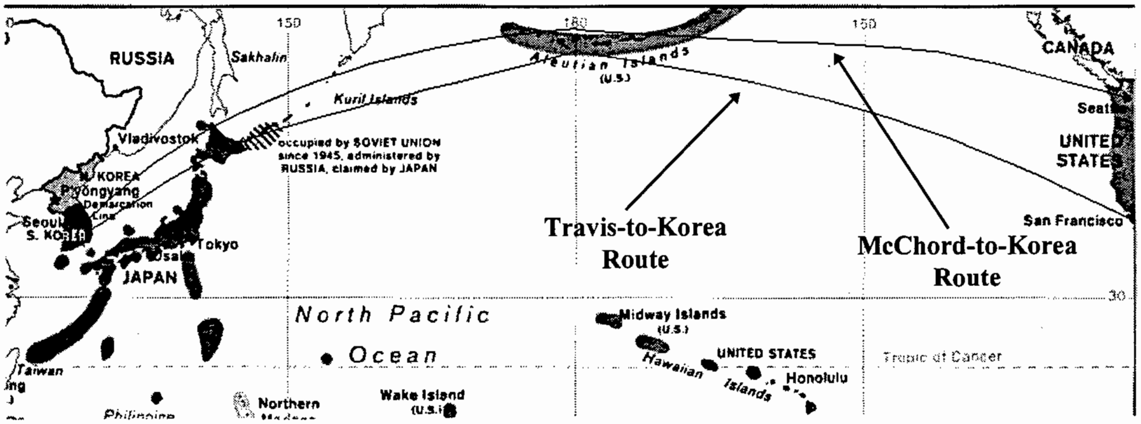


Figure 11 - Direct Delivery Routes

The three most logical (and possible) places to base our tankers are Elmendorf and Eielson AFBs, Alaska, and Hickam AFB, Hawaii. From a strictly locational proximity viewpoint (i.e. discounting fuel stocks, ramp space, maintenance availability, etc.), the base closest to our two direct route midpoints is the best choice. This is due the fact that tankers can reach the refueling track quickly and have more fuel available to offload to the receiver aircraft. The distance between the three candidate tanker bases and the two optimal A/R locations (midpoints) is given in Tables 11 and 12 as follows:

Table 11 - Distance to the McChord-to-Korea A/R Midpoint

Base	**Distance to** McChord-to-Korea Direct Route **Midpoint**
Elmendorf AFB, AK	1126 NM
Eielson AFB, AK	1216 NM
Hickam AFB, HI	2341 NM

Table 12 - Distance to the Travis-to-Korea A/R Midpoint

Base	**Distance to** Travis-to-Korea Direct Route **Midpoint**
Elmendorf AFB, AK	1039 NM
Eielson AFB, AK	1164 NM
Hickam AFB, HI	2128 NM

We can see that, proximity-wise, Elmendorf is the best location for either route.

Next, we need to determine the amount of fuel available for offload to each C-17, and determine the maximum cargo payload each C-17 could carry to Korea. For this data we refer to Table 10 in AFPAM 10-1403, Air Mobility Planning Factors. It lists the tanker offload capabilities of the KC-135E, KC-135R/T, and KC-10 refuelers, based upon their mission radius. The data assumes the tankers depart with their maximum fuel weights, and spend one hour refueling at the A/R track refueling. The data is given in Table 13, as follows:

Table 13 - Tanker Offload Capabilities

Aircraft	Takeoff Gross Weight	Takeoff Fuel Load	**Max Offload Available (lbs)**			
			Mission Radius			
	lbs	lbs	500 nm	1000 nm	1500 nm	2500 nm
KC-135E	275,000	160,000	101,200	78,600	55,800	10,500
KC-135R/T	301,700	180,000	122,200	99,400	76,400	30,700
KC-10	587,000	327,000	233,500	195,200	156,000	78,700

We can perform linear regression on the data in the table to find formulas to calculate the maximum offload of any of the tanker aircraft for any given mission radius. This is done in Tables 14-16, which follow.

Table 14 - Linear Regression for KC-135E Offload Capabilities

KC-135E OFFLOAD CAPABILTIES

RADIUS	OFFLOAD
500	101200
1000	78600
1500	55800
2500	10500

Regression Statistics	
Multiple R	0.999999175
R Square	0.99999835
Adjusted R Square	0.999997525
Standard Error	60.94494002
Observations	4

ANOVA

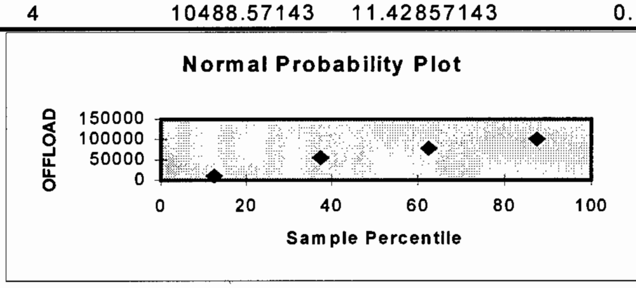
	df	*SS*	*MS*	*F*
Regression	1	4501980071	4501980071	1212072
Residual	2	7428.571429	3714.285714	
Total	3	4501987500		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	123902.8571	64.33331571	1925.951675	2.7E-07
RADIUS	-45.36571429	0.0412063	-1100.941214	8.25E-07

Significance F	*Lower 95%*	*Upper 95%*
8.25033E-07	123626.053	124179.6613
	-45.54301081	-45.18841776

RESIDUAL OUTPUT

Observation	*Predicted OFFLOAD*	*Residuals*	*Standard Residuals*
1	101220	-20	-0.328165062
2	78537.14286	62.85714286	1.031375908
3	55854.28571	-54.28571429	-0.890733739
4	10488.57143	11.42857143	0.187522892



For the KC-135E tanker, the maximum offload available, in pounds, is:

(66) (-45.366) X (mission radius) + 123,903 ≅ Maximum Offload Available

Table 15 - Linear Regression for KC-135R/T Offload Capabilities

KC-135R/T OFFLOAD CAPABILITIES

RADIUS	OFFLOAD
500	122200
1000	99400
1500	76400
2500	30700

Regression Statistics	
Multiple R	0.999999189
R Square	0.999998379
Adjusted R Square	0.999997568
Standard Error	60.94494002
Observations	4

ANOVA

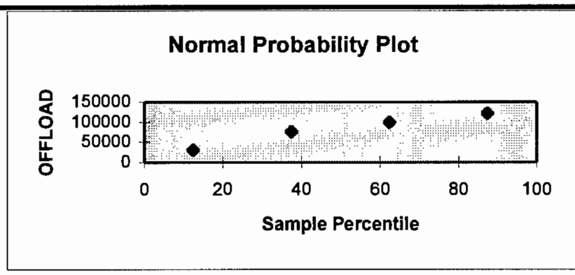
	df	*SS*	*MS*	*F*
Regression	1	4581720071	4581720071	1233540
Residual	2	7428.571429	3714.285714	
Total	3	4581727500		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	145102.8571	64.33331571	2255.485444	1.966E-07
RADIUS	-45.76571429	0.0412063	-1110.648468	8.107E-07

Significance F	*Lower 95%*	*Upper 95%*
8.10674E-07	144826.053	145379.6613
	-45.94301081	-45.58841776

RESIDUAL OUTPUT

Observation	*Predicted OFFLOAD*	*Residuals*	*Standard Residuals*
1	122220	-20	-0.328165062
2	99337.14286	62.85714286	1.031375908
3	76454.28571	-54.28571429	-0.890733739
4	30688.57143	11.42857143	0.187522892



For the KC-135R/T tanker, the maximum offload available, in pounds, is:

(67) (-45.766) X (mission radius) + 145,103 $\cong$ Maximum Offload Available

Table 16 - Linear Regression for KC-10 Offload Capabilities

KC-10 OFFLOAD CAPABILTIES

RADIUS	OFFLOAD
500	233500
1000	195200
1500	156000
2500	78700

Regression Statistics	
Multiple R	0.999994843
R Square	0.999989686
Adjusted R Squa	0.999984528
Standard Error	260.2196874
Observations	4

ANOVA

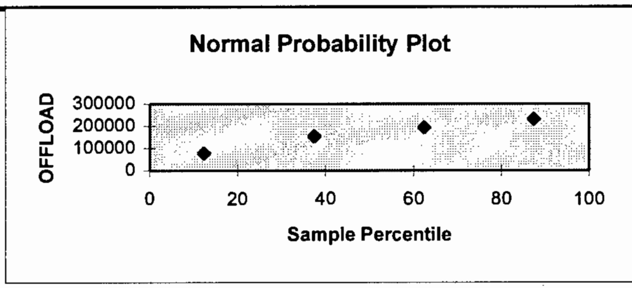
	df	*SS*	*MS*	*F*
Regression	1	13129954571	13129954571	193902.3
Residual	2	135428.5714	67714.28571	
Total	3	13130090000		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	272377.1429	274.6872062	991.5902043	1.02E-06
RADIUS	-77.47428571	0.175940621	-440.3433691	5.16E-06

Significance F	*Lower 95%*	*Upper 95%*
5.1572E-06	271195.2584	273559.0273
	-78.23129763	-76.7172738

RESIDUAL OUTPUT

Observation	*Predicted OFFLOAD*	*Residuals*	*Standard Residuals*
1	233640	-140	-0.538006949
2	194902.8571	297.1428571	1.141892299
3	156165.7143	-165.7142857	-0.636824551
4	78691.42857	8.571428571	0.032939201



For the KC-10 tanker, the maximum offload available, in pounds, is:

(68) (-77.474) X (mission radius) + 272,377 $\cong$ Maximum Offload Available

Now that we can determine the amount of fuel any tanker has to offer a receiver, we need to find out how much fuel each C-17 is likely to need by the time it reaches the A/R point, so that we can determine how many tanker aircraft may be needed. Once again we refer to AFPAM 10-1403, Air Mobility Planning Factors. The following formula is given to determine the offload required per receiver:

(69) Offload required = (Dist / TAS X Fuel Flow) - Total Fuel + Dest Reserve,

where: Dist = the total distance flown by the receiver from takeoff to landing

TAS = Average true airspeed (for mobility aircraft, this is Blockspeed)

Fuel Flow = Fuel burn rate in pounds/hour

Total Fuel = Total fuel on board at takeoff

Dest Reserve = Required fuel at destination

For the C-17 traveling from the CONUS to Korea, we can use the following approximate values for these variables:

Dist = 4526 (for McChord to Taegu), or 4878 (for Travis to Taegu)

TAS = 430

Fuel Flow ≅ 21,440 (from AFPAM 10-1403, Table 9) (19:22)

Total Fuel ≅ 133,180 (Assuming an 79.66 ton cargo load; recall that this is the maximum payload with only one A/R enroute, vs 86.1 tons with Hub-and-Spoke)

Dest Reserve ≅ 22,000

Crunching these numbers in equation (69) yields an approximate fuel onload requirement for each McChord-based C-17 of 114,488 pounds. Similarly, each Travis-based C-17 requires about 132,039 pounds.

Now we can go back to equations (66) - (68) and calculate the maximum offload of our three tanker types from Elmendorf, Eielson, and Hickam AFBs to see how many tankers are required to meet each C-17s fuel requirements. Table 17 gives the maximum fuel offload values for the tankers from these three bases:

Table 17 - Maximum Offloads (in pounds) Each Tanker Can Provide

	A/R on McChord to Korea Route			A/R on Travis to Korea Route		
Tanker Type	Elmendorf	Eielson	Hickam	Elmendorf	Eielson	Hickam
KC-135E	72,821	68,738	17,701	76,768	71,097	27,364
KC-135R/T	93,570	89,452	37,965	97,552	91,831	47,713
KC-10	185,141	178,169	91,010	191,882	182,197	107,512

Using the data in Table 17 we can now formulate Table 18 which tells us how many of each type of tanker is required to meet each C-17's refueling requirements during Direct Delivery.

Table 18 offers convincing evidence that, if A/R resources (aircraft, crews, fuel stocks, maintenance, etc.) are not plentiful, then Direct Delivery from CONUS to Korea via C-17 is a difficult proposition. Not only does it require anywhere from 1 - 7 tankers per C-17, the payload possible is nearly 6.5 tons less (per C-17) than with

Table 18 - Minimum Number of Tankers Needed per C-17

	A/R on McChord to Korea Route			A/R on Travis to Korea Route		
Tanker Type	Elmendorf	Eielson	Hickam	Elmendorf	Eielson	Hickam
KC-135E	2	2	7	2	2	5
KC-135R/T	2	2	4	2	2	3
KC-10	1	1	2	1	1	2

Hub-and-Spoke when we use Japan as the C-17 hub. Additionally, 10 - 12 hours of the crew duty day is virtually wasted during long trans-Pacific cruising portions of each flight, so fewer offloads can be made in Korea. Finally, Direct Delivery does not provide a safe haven staging base for spare crews, maintenance personnel, etc. as does Hub-and-Spoke, which minimizes the aircraft's, aircrew's, and support personnel's exposure to possible hostile activity.

Based upon the results in the tables above, in the CONUS-to-Korea airlift problem, strong evidence exists suggesting that C-17s are better-suited for the Hub-and-Spoke system, assuming Japan is available as the transshipment (hub) location.

Chapter 5 Conclusions and Recommendations

5.1 Significance of Results

The vast majority of time spent on this project was in the development of the model itself, as presented in Section 3.10. It is believed to be the first (and currently, the only) deterministic model utilizing the hierarchical structure enabling the analysis of a Hub-and-Spoke airlift system. Its existence is important for the following reasons.

First, the model can easily be enlarged to incorporate more bases, aircraft, and cargo. An equation generator such as GAMS (General Algebraic Modeling System) or MPL (Mathematical Programming Language) could be used, along with mathematical equations (22) - (52) in Chapter 3 of this study, to enlarge the problem's scope to any size desirable. The model can then be applied to any scenario globally, whether it be the Middle East, Europe, Asia, etc. All that is required is a change in the coefficients and/or parameters in the model.

Second, the model utilizes and tracks individual aircraft, flying from/to individual air bases, and is based on an hourly (32-hour) timeline. These features make the model amenable to small-scale, everyday operations (when quiet hours, restricted flying times, etc. are a part of reality) as well as larger-scale airlifts. By using individual bases instead of aggregate "supernodes", while retaining a transoceanic, long-range scope, the model provides a nice balance between strategic airlift and tactical airlift.

Third, the analysis concurs with White's STAFMA hypothesis that there are airlift scenarios where Hub-and-Spoke is better-suited for the task than Direct Delivery. White's main conclusion focused on the increase in payloads possible in the CONUS-to-Korea problem (as much as 15% in some simulation runs) by transloading cargo. This study not only agrees that increased payloads are possible via Hub-and-Spoke, it demonstrates that:

A). More visits to destinations are possible per aircraft due to more effective use of CDD (i. e. Instead of wasting 10-12 hours of each crew day crossing the Pacific, C-17 crews spend only 4 hours flying to/from Korea, which gives them 12 hours of crew duty day to make deliveries. This may actually allow them to make two round trips from Japan per crew, which effectively doubles the payload each aircrew can deliver)

B). Significantly less tanker A/R support is required with Hub-and-Spoke (C-17s can easily visit multiple destinations in Korea and return to Japan with no need whatsoever for refueling enroute) This frees up a large number of tankers for other air refueling missions

C). Less congestion occurs at the PODs due to the C-17's "MOG efficiency" compared to the C-5

D). Hubs act as safe haven staging bases for aircrews and AMC aircraft, away from possible hostile activity

E). The need for ground refueling at the congested PODs is eliminated

Fourth, the model supports the conclusions reached by the 1995 RAND C-17 Tactical Utility Analysis, which found that at least a portion of the C-17 fleet should be used for shorter-range, tactical airlift mission in theater. This takes advantage of the C-17's quick-turn capabilities, fast cruising speeds, and MOG efficiency. The findings of

Section 4.6 show the huge differences in A/R support required when using the C-17 in a Direct Delivery fashion versus in a Hub-and-Spoke structure for any airlift to Korea. This conclusion is particularly important, since it conflicts with Air Force's stated purpose of utilizing the C-17 as a direct delivery airlifter.

The disadvantages of Hub-and-Spoke are few:

A). Cargo tracking is more difficult, due to the transloading process, than with Direct Delivery

B). Cargo may be 3-4 hours slower in delivery time to the PODs, compared to Direct Delivery, due to the transload time at the C-17 hub location

C). Requires the agreement of a host nation to act as a transshipment location

However, this study has shown that the advantages of Hub-and-Spoke are many. It is my hope that this study encourages further research on the suitability of Hub-and-Spoke for the Air Force, and the role of the C-17 for theater delivery as well as strategic delivery.

5.2 Recommendations for Further Study

As noted in Section 2.5, finding the optimal solution to a problem in Class NP-Complete is a computationally difficult, time-consuming proposition. All the various runs using the Hub-and-Spoke developed in this study took from 18 hours to an unbelievable 94 hours to solve, using the CPLEX linear solver, version 3.0, on a Sun SPARCstation 10 computer at the Air Force Institute of Technology. This is a significant amount of time, considering the small size of nodes and aircraft modeled (10 nodes, 10 aircraft). This strongly suggests a need to find ways to cut solution times for future model runs. Possible approaches include the following:

1). Search for ways to pre-process the model. For example, one may be able to:

   A). Eliminate any redundant constraints

   B). Solve the model's LP relaxation, look for any integer-valued variables, fix them to these optimal values, then re-run the model

   C). Find ways to combine constraints to reduce the overall number in the formulation

2). Attempt a decomposition method, such as Dantzig-Wolfe or Bender's partitioning method in an attempt to "divide and conquer" the larger problem by taking advantage of its special structure and breaking it into subproblems

3). Apply a heuristic, which is "a technique which seeks good (i.e. near-optimal) solutions at a reasonable computational cost without being able to guarantee either feasibility or optimality, or even in many cases to state how close to optimality a particular feasible solution is". (39:6)

One advantage of the heuristic option is that this study already has four different formulations fully written in integer LP form, with the optimal solutions given, in Appendices 1-4. Thus, any heuristic employed already has four separate yardsticks by which it could measure its accuracy.

Regarding future improvements to this model, several thoughts immediately come to mind. First, due to the way cargo tracking variables were incorporated, and the way we pre-specified the way cargo demands and their CONUS origins were assigned to the split nodes, the majority of the model's variables are binary. While this gives the user the capability of tracking individual aircraft, it also adds significantly to the number of variables and constraints present. If a way can be found to eliminate some of these binary variables and use ordinary integers instead, the model should become more tractable to handle. This idea seems worthy of future exploration.

Second, although the model is extremely flexible in that individual time windows can be built in for every split node, it is inflexible as currently presented because if any single time window in the entire model cannot be met, the problem is infeasible and provides no answer whatsoever. Other airlift models, notably the NRMO model developed jointly by the Naval Postgraduate School and RAND Corporation, offers a feature known as "soft" time windows. This allows cargo to be delivered outside of the time window (early or late), with a penalty assessed for such deliveries. This is more realistic, because the TPFDDs that specify delivery times are overly optimistic, and may even set time windows which are impossible to meet. A soft time window allows for this possibility, so the model can still find the "best" answer. Incorporating a soft time window into this model should not be too difficult, and make it more useful and realistic.

Third, finding a way to incorporate A/R into the model would allow to user to determine the required number of tankers needed, fuel offloads required, optimal tanker bases, and maximum cargo payloads for airlift aircraft with no need for laborious manual calculations as I've done in Section 4.6. In fact, this would empower the model greatly by enabling it to model both Direct Delivery and Hub-and-Spoke! If the number of available tankers is used as a MOG constraint, and the A/R track is used as the hub location, this model would in essence become two models in one. Readers are referred to the detailed explanation in Section II of Chapter 9, entitled "Single-Facility/Single-Route/Multi-Criteria Problem", in Chan's forthcoming text (reference 11 in the bibliography). In a nutshell, the user can integrate the maximum-covering location problem and the shortest-path routing problem. This procedure can be used to determine exactly where the optimal A/R track should be located, given the cargo airlifter's great circle path from takeoff to landing and the tanker departure base. (11:9-5 to 9-9)

Fourth, as mentioned in Section 3.11, AMC categorizes its cargo into five types: bulk, oversize, outsize, rolling stock, and special cargo. Additionally, cargo aircraft nearly always will be limited by the volume (not weight) they can carry. This model specifies cargo solely by weight and CONUS base of origin, not cargo type or volume. Future modifications could add "cargo type" identifiers to each cargo load, and volume constraints to each aircraft. The drawback is that while this would increase the model's fidelity, it would also greatly increase the number of variables and complexity, which in turn would increase the solver solution times.

5.3 Conclusion

The United States Air Force for decades has airlifted cargo all around the world using the concept of Direct Delivery. While this airlift method most certainly is expeditious and makes in-transit visibility of cargo a routine affair, the system appears to have notable disadvantages in some airlift scenarios. Two of the most significant limitations on the Direct Delivery system relate to origin and destination bottlenecks, and the tremendous A/R support required for transoceanic flights.

This study considers an alternative system using a Hub-and-Spoke hierarchical structure to reduce A/R support requirements and minimize MOG problems at destinations. As no such model previously existed in the air mobility literature, a prototype deterministic mixed-integer program is developed herein. This model is an extremely flexible and useful tool for evaluating a possible alternative to the current Direct Delivery airlift method which AMC has used for decades. The C-17 Globemaster III possesses features such as outsize cargo capability, quick-turn times, and the smallest MOG value of any existing airlifter, which make it particularly well-suited for a Hub-and-Spoke structure.

As the U. S. military services continue to be downsized, while at the same time receive more and more taskings worldwide, our mobility system is being strained more and more. With the advent of new systems, such as the C-17, a new airlift structure which offers numerous advantages over Direct Delivery deserves serious consideration.

Consider also that "commercial air carriers account for 93 percent of (US Transportation Command's) long-range passenger capability and 32 percent ot its long-range cargo capability". (22:44) During Operations Desert Shield and Desert Storm, the Civil Reserve Air Fleet (CRAF) flew more than 5000 missions for AMC (known then as the Military Airlift Command, or MAC). In fact, "more than 60 percent of the troops and 25 percent of the cargo airlifted into or out of the theater went by airliners". (17:xi) Evidence also points toward increasing reliance upon CRAF as a force multiplier in contingencies. The possible future employment of CRAF necessitates a close examination of the Hub-and-Spoke concept, particularly since Hub-and-Spoke provides safe haven transshipment bases. This advantage is not insignificant in light of civilian carrier concerns of carrying cargo and passengers to possible hostile areas, as occurred during Desert Shield/Desert Storm. Indeed, a post-Gulf War study by the RAND Corporation concludes that commercial air carriers "are much less likely to volunteer valuable assets without appropriate liability protection". (17:70)

In conclusion, it is my sincere hope that, by providing evidence that a Hub-and-Spoke airlift system deserves serious consideration for implementation in certain military airlift scenarios, this study will encourage further research on the concept as an alternative to Direct Delivery.

Appendix 1A - Math formulation for Figure 3

CPLEX 3.0 Formulation for the "Hierarchical-Plus-Time-Windows" example problem associated with Figure 3, in which cargo demands at nodes 3, 4 and 5 are 15000, 10000 and 12000 pounds, respectively.

NOTE: We've added two things to the objective function which are not standard to equation (1). From experience, if a Y_i or X_{ij} variable is not specifically assigned a value of 0 or 1, the solver will arbitrarily select either value. To avoid confusion when examining our solution, we want the solver to assign a value 0 unless the variable requires a value of 1. Therefore, to equation (1) we've added the terms $\sum_b Y_b$ and $\sum_b \sum_a X_{ba}$. This results in the terms X31, X32, X41, X42, X51, X52, X16, X17, X26, X27, Y1 and Y2 being added to our objective function.

The solution to this formulation is given in Appendix 1B.

```
MINIMIZE

2.3X113+2.8X114+2.5X115+4X123+3X124+3.2X125+2X134+5X135+2X143+5X153+
6X145+6X154+2.3X213+2.8X214+2.5X215+4X223+3X224+3.2X225+2X234+5X235+
2X243+5X253+6X245+6X254+T11+T12+T21+T22+T13+T23+T14+T24+T15+T25+X
31+X32+X41+X42+X51+X52+10X361+10X461+11.7X362+11.7X462+10.5X371+10.5
X471+12.1X372+12.1X472+T31+T32+T41+T42+X16+X17+X26+X27+Y1+Y2
ST
16X113+T11+D1-D3<=13.7
16X113-T11-D1+D3<=18.3
16X123+T12+D2-D3<=12
16X123-T12-D2+D3<=20
16X143+T14+D4-D3<=14
16X143-T14-D4+D3<=18
16X153+T15+D5-D3<=11
16X153-T15-D5+D3<=21
16X213+T21+D1-D3<=13.7
16X213-T21-D1+D3<=18.3
16X223+T22+D2-D3<=12
16X223-T22-D2+D3<=20
16X243+T24+D4-D3<=14
16X243-T24-D4+D3<=18
16X253+T25+D5-D3<=11
16X253-T25-D5+D3<=21
```

16X114+T11+D1-D4<=13.2
16X114-T11-D1+D4<=18.8
16X124+T12+D2-D4<=13
16X124-T12-D2+D4<=19
16X134+T13+D3-D4<=14
16X134-T13-D3+D4<=18
16X154+T15+D5-D4<=10
16X154-T15-D5+D4<=22
16X214+T21+D1-D4<=13.2
16X214-T21-D1+D4<=18.8
16X224+T22+D2-D4<=13
16X224-T22-D2+D4<=19
16X234+T23+D3-D4<=14
16X234-T23-D3+D4<=18
16X254+T25+D5-D4<=10
16X254-T25-D5+D4<=22
16X115+T11+D1-D5<=13.5
16X115-T11-D1+D5<=18.5
16X125+T12+D2-D5<=12.8
16X125-T12-D2+D5<=19.2
16X135+T13+D3-D5<=11
16X135-T13-D3+D5<=21
16X145+T14+D4-D5<=10
16X145-T14-D4+D5<=22
16X215+T21+D1-D5<=13.5
16X215-T21-D1+D5<=18.5
16X225+T22+D2-D5<=12.8
16X225-T22-D2+D5<=19.2
16X235+T23+D3-D5<=11
16X235-T23-D3+D5<=21
16X245+T24+D4-D5<=10
16X245-T24-D4+D5<=22
X113+X114+X115+X123+X124+X125=1
X213+X214+X215+X223+X224+X225=1
X131+X141+X151+X132+X142+X152=1
X231+X241+X251+X232+X242+X252=1
-X113-X123-X143-X153+2T13>=0
-X213-X223-X243-X253+2T23>=0
-X114-X124-X134-X154+2T14>=0
-X214-X224-X234-X254+2T24>=0
-X115-X125-X135-X145+2T15>=0
-X215-X225-X235-X245+2T25>=0
X113-X131+X123-X132-X134-X135+X143+X153=0

X114-X141+X124-X142+X134-X143-X145+X154=0
X115-X151+X125-X152+X135-X153+X145-X154=0
-X135-X153>=-1
-X235-X253>=-1
-X134-X143>=-1
-X234-X243>=-1
-X145-X154>=-1
-X245-X254>=-1
-X134-X135-X143-X153-X145-X154>=-2
-X234-X235-X243-X253-X245-X254>=-2
-15000X113-10000X114-12000X115-15000X123-10000X124-12000X125-
10000X134-12000X135-15000X143-15000X153-12000X145-10000X154>=-30000
-15000X213-10000X214-12000X215-15000X223-10000X224-12000X225-
10000X234-12000X235-15000X243-15000X253-12000X245-10000X254>=-30000
X213+X223-X231-X232-X234-X235+X243+X253=0
X214-X241+X224-X242+X234-X243-X245+X254=0
X215-X251+X225-X252+X235-X253+X245-X254=0
X113+X123+X143+X153+X213+X223+X243+X253>=1
X114+X124+X134+X154+X214+X224+X234+X254>=1
X115+X125+X135+X145+X215+X225+X235+X245>=1
2.3X113+2.8X114+2.5X115+4X123+3X124+3.2X125+2.3X131+4X132+2X134
+5X135+2.8X141+3X142+2X143+6X145+2.5X151+3.2X152+5X153+6X154
+T11+T12+T13+T14+T15<=16
2.3X213+2.8X214+2.5X215+4X223+3X224+3.2X225+2.3X231+4X232+2X234
+5X235+2.8X241+3X242+2X243+6X245+2.5X251+3.2X252+5X253+6X254
+T21+T22+T23+T24+T25<=16
X131+X141+X151-X113-X114-X115=0
X231+X241+X251-X213-X214-X215=0
X132+X142+X152-X123-X124-X125=0
X232+X242+X252-X223-X224-X225=0
Z01-15000X31-10000X41-12000X51=0
Z02-15000X32-10000X42-12000X52=0
Z01-50000Y1<=0
Z02-50000Y2<=0
X131+X132+X134+X135+X121+X131+X141+X151-X31<=1
X231+X232+X234+X235+X221+X231+X241+X251-X31<=1
X131+X132+X134+X135+X112+X132+X142+X152-X32<=1
X231+X232+X234+X235+X212+X232+X242+X252-X32<=1
X141+X142+X143+X145+X121+X131+X141+X151-X41<=1
X241+X242+X243+X245+X221+X231+X241+X251-X41<=1
X141+X142+X143+X145+X112+X132+X142+X152-X42<=1
X241+X242+X243+X245+X212+X232+X242+X252-X42<=1
X151+X152+X153+X154+X121+X131+X141+X151-X51<=1

X251+X252+X253+X254+X221+X231+X241+X251-X51<=1
X151+X152+X153+X154+X112+X132+X142+X152-X52<=1
X251+X252+X253+X254+X212+X232+X242+X252-X52<=1
X361+X362+X371+X372=1
X461+X462+X471+X472=1
X316+X326+X317+X327=1
X416+X426+X417+X427=1
X361+X371-.5T31<=0
X461+X471-.5T41<=0
X362+X372-.5T32<=0
X462+X472-.5T42<=0
X361+X371-X316-X317=0
X362+X372-X326-X327=0
X461+X471-X416-X417=0
X462+X472-X426-X427=0
Z01-35000X361-35000X461-35000X371-35000X471<=0
Z02-35000X362-35000X462-35000X372-35000X472<=0
X361+X371+X461+X471<=10
X362+X372+X462+X472<=10
X316+X317+X316+X326-X16<=1
X416+X417+X416+X426-X16<=1
X316+X317+X317+X327-X17<=1
X416+X417+X417+X427-X17<=1
X326+X327+X316+X326-X26<=1
X426+X427+X416+X426-X26<=1
X326+X327+X317+X327-X27<=1
X426+X427+X417+X427-X27<=1
X361+X362-X316-X326=0
X371+X372-X317-X327=0
X461+X462-X416-X426=0
X471+X472-X417-X427=0
16X361+T36+D6-D1<=6
16X361-T36-D6+D1<=26
16X371+T37+D7-D1<=5.5
16X371-T37-D7+D1<=26.5
16X461+T46+D6-D1<=6
16X461-T46-D6+D1<=26
16X471+T47+D7-D1<=5.5
16X471-T47-D7+D1<=26.5
16X362+T36+D6-D2<=4.3
16X362-T36-D6+D2<=27.7
16X372+T37+D7-D2<=3.9
16X372-T37-D7+D2<=28.1

16X462+T46+D6-D2<=4.3
16X462-T46-D6+D2<=27.7
16X472+T47+D7-D2<=3.9
16X472-T47-D7+D2<=28.1
BOUNDS
D3>=16
D3<=20
D4>=17.5
D4<=21.5
D5>=19
D5<=25
D6<=2
D7<=2
D6>=2
D7>=2
X112<=0
X113<=1
X114<=1
X115<=1
X121<=0
X131<=1
X141<=1
X151<=1
X123<=1
X124<=1
X125<=1
X132<=1
X142<=1
X152<=1
X134<=1
X135<=1
X143<=1
X153<=1
X145<=1
X154<=1
X212<=0
X213<=1
X214<=1
X215<=1
X221<=0
X231<=1
X241<=1
X251<=1

X223<=1
X224<=1
X225<=1
X232<=1
X242<=1
X252<=1
X234<=1
X235<=1
X243<=1
X253<=1
X245<=1
X254<=1
Y1<=1
Y2<=1
X31<=1
X41<=1
X51<=1
X32<=1
X42<=1
X52<=1
X316<=1
X317<=1
X326<=1
X327<=1
X361<=1
X371<=1
X362<=1
X372<=1
X416<=1
X417<=1
X426<=1
X427<=1
X461<=1
X471<=1
X462<=1
X472<=1
X16<=1
X17<=1
X26<=1
X27<=1
INTEGERS
X112
X113

X114
X115
X121
X131
X141
X151
X123
X124
X125
X132
X142
X152
X134
X135
X143
X153
X145
X154
X212
X213
X214
X215
X221
X223
X241
X231
X251
X224
X225
X232
X242
X252
X234
X235
X243
X253
X245
X254
Y1
Y2
X31
X41
X51

X32
X42
X52
X316
X317
X326
X327
X361
X371
X362
X372
X416
X417
X426
X427
X461
X471
X462
X472
X16
X17
X26
X27
END

Appendix 1B - Solution for Figure 3

CPLEX 3.0 Solution to the "Hierarchical-Plus-Time-Windows" example problem associated with Figure 3, in which cargo demands at nodes 3, 4 and 5 are 15000, 10000 and 12000 pounds, respectively.

The formulation of this problem is presented in Appendix 1A.

Integer Optimal Solution: Objective = 3.7300000000e+01
Solution Time = 0.55 sec. Iterations = 216 Nodes = 25

Variable Name	Solution Value
X113	1.000000
X134	1.000000
X215	1.000000
T13	0.500000
T14	0.500000
T25	0.500000
X31	1.000000
X41	1.000000
X51	1.000000
X361	1.000000
X461	1.000000
T31	2.000000
T41	2.000000
X16	1.000000
Y1	1.000000
D1	16.500000
D3	18.800000
D2	2.300000
D4	21.300000
D5	19.000000
X141	1.000000
X251	1.000000
Z01	37000.000000
X316	1.000000
X416	1.000000
T36	4.500000
D6	2.000000
T37	4.200000
D7	2.000000
T46	4.500000
T47	4.200000

All other variables in the range 1-95 are zero.

Appendix 2A - Math formulation for Figure 4

CPLEX 3.0 Formulation for the "Hierarchical-Plus-Time-Windows-Plus-Multiple-Frequency" example problem associated with Figure 4, in which cargo demands at nodes 3A, 3B, 4 and 5 are 11000, 20000, 10000 and 12000 pounds, respectively.

The solution to this formulation is given in Appendix 2B.

MINIMIZE

2.3X113A+2.3X113B+2.8X114+2.5X115+4X123A+4X123B+3X124+3.2X125+2X13A
4+2X13B4+5X13A5+
5X13B5+2X143A+2X143B+5X153A+5X153B+6X145+6X154+2.3X213A+2.3X213B+
2.8X214+2.5X215+
4X223A+4X223B+3X224+3.2X225+2X23A4+2X23B4+5X23A5+5X23B5+
2X243A+2X243B+5X253A+5X253B+6X245+6X254+T11+T12+T21+T22+T13A+T13
B+T23A+T23B+T14+
T24+T15+T25+X3A1+X3B1+X3A2+X3B2+X41+X42+X51+X52+10X361+10X461+1
1.7X362+11.7X462+
10.5X371+10.5X471+12.1X372+12.1X472+T31+T32+T41+T42+X16+X17+X26+X27+
Y1+Y2
ST
D3A - D1 <= 16
D3B - D1 <= 16
D4 - D1 <= 16
D5 - D1 <= 16
D3A - D2 <= 16
D3B - D2 <= 16
D4 - D2 <= 16
D5 - D2 <= 16
D1 - D6 <= 16
D1 - D7 <= 16
D2 - D6 <= 16
D2 - D7 <= 16

16X13A4+D3A-D4<=14
16X143A+D4-D3A<=14
16X13B4+D3B-D4<=14
16X143B+D4-D3B<=14
16X13A5+D3A-D5<=11
16X153A+D5-D3A<=11
16X13B5+D3B-D5<=11

16X153B+D5-D3B<=11
16X145+D4-D5<=10
16X154+D5-D4<=10

2.3X113A-D3A<=0
2.3X113B-D3B<=0
2.3X213A-D3A<=0
2.3X213B-D3B<=0
2.8X114-D4<=0
2.8X214-D4<=0
2.5X115-D5<=0
2.5X215-D5<=0
4X123A-D3A<=0
4X123B-D3B<=0
4X223A-D3A<=0
4X223B-D3B<=0
3X124-D4<=0
3X224-D4<=0
3.2X125-D5<=0
3.2X225-D5<=0

2.3X13A1+D3A<=32
2.3X13B1+D3B<=32
2.3X23A1+D3A<=32
2.3X23B1+D3B<=32
4X13A2+D3A<=32
4X13B2+D3B<=32
4X23A2+D3A<=32
4X23B2+D3B<=32
2.8X141+D4<=32
2.8X241+D4<=32
3X142+D4<=32
3X242+D4<=32
2.5X151+D5<=32
2.5X251+D5<=32
3.2X152+D5<=32
3.2X252+D5<=32

16X113A+T11+D1-D3A<=13.7
16X113A-T11-D1+D3A<=18.3
16X123A+T12+D2-D3A<=12
16X123A-T12-D2+D3A<=20
16X143A+T14+D4-D3A<=14

16X143A-T14-D4+D3A<=18
16X153A+T15+D5-D3A<=11
16X153A-T15-D5+D3A<=21
16X213A+T21+D1-D3A<=13.7
16X213A-T21-D1+D3A<=18.3
16X223A+T22+D2-D3A<=12
16X223A-T22-D2+D3A<=20
16X243A+T24+D4-D3A<=14
16X243A-T24-D4+D3A<=18
16X253A+T25+D5-D3A<=11
16X253A-T25-D5+D3A<=21

16X113B+T11+D1-D3B<=13.7
16X113B-T11-D1+D3B<=18.3
16X123B+T12+D2-D3B<=12
16X123B-T12-D2+D3B<=20
16X143B+T14+D4-D3B<=14
16X143B-T14-D4+D3B<=18
16X153B+T15+D5-D3B<=11
16X153B-T15-D5+D3B<=21
16X213B+T21+D1-D3B<=13.7
16X213B-T21-D1+D3B<=18.3
16X223B+T22+D2-D3B<=12
16X223B-T22-D2+D3B<=20
16X243B+T24+D4-D3B<=14
16X243B-T24-D4+D3B<=18
16X253B+T25+D5-D3B<11
16X253B-T25-D5+D3B<=21

16X114+T11+D1-D4<=13.2
16X114-T11-D1+D4<=18.8
16X124+T12+D2-D4<=13
16X124-T12-D2+D4<=19

16X13A4+T13A+D3A-D4<=14
16X13A4-T13A-D3A+D4<=18

16X13B4+T13B+D3B-D4<=14
16X13B4-T13B-D3B+D4<=18

16X154+T15+D5-D4<=10
16X154-T15-D5+D4<=22
16X214+T21+D1-D4<=13.2

16X214-T21-D1+D4<=18.8
16X224+T22+D2-D4<=13
16X224-T22-D2+D4<=19

16X23A4+T23A+D3A-D4<=14
16X23A4-T23A-D3A+D4<=18

16X23B4+T23B+D3B-D4<=14
16X23B4-T23B-D3B+D4<=18

16X254+T25+D5-D4<=10
16X254-T25-D5+D4<=22
16X115+T11+D1-D5<=13.5
16X115-T11-D1+D5<=18.5
16X125+T12+D2-D5<=12.8
16X125-T12-D2+D5<=19.2

16X13A5+T13A+D3A-D5<=11
16X13A5-T13A-D3A+D5<=21

16X13B5+T13B+D3B-D5<=11
16X13B5-T13B-D3B+D5<=21

16X145+T14+D4-D5<=10
16X145-T14-D4+D5<=22
16X215+T21+D1-D5<=13.5
16X215-T21-D1+D5<=18.5
16X225+T22+D2-D5<=12.8
16X225-T22-D2+D5<=19.2

16X23A5+T23A+D3A-D5<=11
16X23A5-T23A-D3A+D5<=21

16X23B5+T23B+D3B-D5<=11
16X23B5-T23B-D3B+D5<=21

16X245+T24+D4-D5<=10
16X245-T24-D4+D5<=22

X113A+X113B+X114+X115+X123A+X123B+X124+X125=1
X213A+X213B+X214+X215+X223A+X223B+X224+X225=1
X13A1+X13B1+X141+X151+X13A2+X13B2+X142+X152=1
X23A1+X23B1+X241+X251+X23A2+X23B2+X242+X252=1

-X113A-X123A-X143A-X153A+2T13A>=0
-X213A-X223A-X243A-X253A+2T23A>=0

-X113B-X123B-X143B-X153B+2T13B>=0
-X213B-X223B-X243B-X253B+2T23B>=0

-X114-X124-X13A4-X13B4-X154+2T14>=0
-X214-X224-X23A4-X23B4-X254+2T24>=0
-X115-X125-X13A5-X13B5-X145+2T15>=0
-X215-X225-X23A5-X23B5-X245+2T25>=0

X113A-X13A1+X123A-X13A2-X13A4-X13A5+X143A+X153A=0
X113B-X13B1+X123B-X13B2-X13B4-X13B5+X143B+X153B=0
X114-X141+X124-X142+X13A4-X143A+X13B4-X143B-X145+X154=0
X115-X151+X125-X152+X13A5-X153A+X13B5-X153B+X145-X154=0

-11000X113A-20000X113B-10000X114-12000X115-11000X123A-
20000X123B-10000X124-12000X125-10000X13A4-10000X13B4-
12000X13A5-12000X13B5-11000X143A-20000X143B-11000X153A-
20000X153B-12000X145-10000X154>=-30000

-11000X213A-20000X213B-10000X214-12000X215-11000X223A-
20000X223B-10000X224-12000X225-10000X23A4-10000X23B4-
12000X23A5-12000X23B5-11000X243A-20000X243B-11000X253A-
20000X253B-12000X245-10000X254>=-30000

X213A-X23A1+X223A-X23A2-X23A4-X23A5+X243A+X253A=0
X213B-X23B1+X223B-X23B2-X23B4-X23B5+X243B+X253B=0
X214-X241+X224-X242+X23A4-X243A+X23B4-X243B-X245+X254=0
X215-X251+X225-X252+X23A5-X253A+X23B5-X253B+X245-X254=0

X113A+X123A+X143A+X153A+X213A+X223A+X243A+X253A>=1
X113B+X123B+X143B+X153B+X213B+X223B+X243B+X253B>=1
X114+X124+X13A4+X13B4+X154+X214+X224+X23A4+X23B4+X254>=1
X115+X125+X13A5+X13B5+X145+X215+X225+X23A5+X23B5+X245>=1

X13A1+X13B1+X141+X151-X113A-X113B-X114-X115=0
X23A1+X23B1+X241+X251-X213A-X213B-X214-X215=0
X13A2+X13B2+X142+X152-X123A-X123B-X124-X125=0
X23A2+X23B2+X242+X252-X223A-X223B-X224-X225=0

Z01-11000X3A1-20000X3B1-10000X41-12000X51=0

Z02-11000X3A2-20000X3B2-10000X42-12000X52=0

Z01-60000Y1<=0
Z02-60000Y2<=0

X13A1+X13A2+X13A4+X13A5+X121+X13A1+X13B1+X141+X151-X3A1<=1
X13B1+X13B2+X13B4+X13B5+X121+X13A1+X13B1+X141+X151-X3B1<=1
X23A1+X23A2+X23A4+X23A5+X221+X23A1+X23B1+X241+X251-X3A1<=1
X23B1+X23B2+X23B4+X23B5+X221+X23A1+X23B1+X241+X251-X3B1<=1
X13A1+X13A2+X13A4+X13A5+X112+X13A2+X13B2+X142+X152-X3A2<=1
X13B1+X13B2+X13B4+X13B5+X112+X13A2+X13B2+X142+X152-X3B2<=1
X141+X142+X143A+X143B+X145+X121+X13A1+X13B1+X141+X151-X41<=1
X241+X242+X243A+X243B+X245+X221+X23A1+X23B1+X241+X251-X41<=1
X141+X142+X143A+X143B+X145+X112+X13A2+X13B2+X142+X152-X42<=1
X241+X242+X243A+X243B+X245+X212+X23A2+X23B2+X242+X252-X42<=1
X151+X152+X153A+X153B+X154+X121+X13A1+X13B1+X141+X151-X51<=1
X251+X252+X253A+X253B+X254+X221+X23A1+X23B1+X241+X251-X51<=1
X151+X152+X153A+X153B+X154+X112+X13A2+X13B2+X142+X152-X52<=1
X251+X252+X253A+X253B+X254+X212+X23A2+X23B2+X242+X252-X52<=1

X361+X362+X371+X372=1
X461+X462+X471+X472=1
X316+X326+X317+X327=1
X416+X426+X417+X427=1
X361+X371-.5T31<=0
X461+X471-.5T41<=0
X362+X372-.5T32<=0
X462+X472-.5T42<=0
X361+X371-X316-X317=0
X362+X372-X326-X327=0
X461+X471-X416-X417=0
X462+X472-X426-X427=0
Z01-35000X361-35000X461-35000X371-35000X471<=0
Z02-35000X362-35000X462-35000X372-35000X472<=0
X361+X371+X461+X471<=10
X362+X372+X462+X472<=10
X316+X317+X316+X326-X16<=1
X416+X417+X416+X426-X16<=1
X316+X317+X317+X327-X17<=1
X416+X417+X417+X427-X17<=1
X326+X327+X316+X326-X26<=1
X426+X427+X416+X426-X26<=1
X326+X327+X317+X327-X27<=1

X426+X427+X417+X427-X27<=1
X361+X362-X316-X326=0
X371+X372-X317-X327=0
X461+X462-X416-X426=0
X471+X472-X417-X427=0
16X361+T36+D6-D1<=6
16X361-T36-D6+D1<=26
16X371+T37+D7-D1<=5.5
16X371-T37-D7+D1<=26.5
16X461+T46+D6-D1<=6
16X461-T46-D6+D1<=26
16X471+T47+D7-D1<=5.5
16X471-T47-D7+D1<=26.5
16X362+T36+D6-D2<=4.3
16X362-T36-D6+D2<=27.7
16X372+T37+D7-D2<=3.9
16X372-T37-D7+D2<=28.1
16X462+T46+D6-D2<=4.3
16X462-T46-D6+D2<=27.7
16X472+T47+D7-D2<=3.9
16X472-T47-D7+D2<=28.1

BOUNDS
D6=2
D7=2

D3A>=16
D3B>=16
D3A<=20
D3B<=20

D4>=17.5
D4<=21.5
D5>=19
D5<=25

INTEGERS
X112
X113A
X113B
X114
X115

X121
X13A1
X13B1
X141
X151
X123A
X123B
X124
X125
X13A2
X13B2
X142
X152
X13A4
X13B4
X13A5
X13B5
X143A
X143B
X153A
X153B
X145
X154
X212
X213A
X213B
X214
X215
X221
X223A
X223B
X241
X23A1
X23B1
X251
X224
X225
X23A2
X23B2
X242
X252
X23A4
X23B4

X23A5
X23B5
X243A
X243B
X253A
X253B
X245
X254
Y1
Y2
X3A1
X3B1
X41
X51
X3A2
X3B2
X42
X52
X316
X317
X326
X327
X361
X371
X362
X372
X16
X17
X26
X27
END

Appendix 2B - Solution for Figure 4

Solution to the "Hierarchical-Plus-Time-Windows-Plus-Multiple-Frequency" example problem associated with Figure 4, in which cargo demands at nodes 3A, 3B, 4 and 5 are 11000, 20000, 10000 and 12000 pounds, respectively.

The formulation of this problem is presented in Appendix 2A.

Integer Optimal Solution: Objective = 4.3600000000e+01
Solution Time = 4.82 sec. Iterations = 1625 Nodes = 292

Variable Name	Solution Value
X113A	1.000000
X13A5	1.000000
X213B	1.000000
X23B4	1.000000
T13A	0.500000
T23B	0.500000
T24	0.500000
T15	0.500000
X3A1	1.000000
X3B1	1.000000
X41	1.000000
X51	1.000000
X361	1.000000
X461	1.000000
T31	2.000000
T41	2.000000
X16	1.000000
Y1	1.000000
D3A	17.500000
D1	15.200000
D3B	17.500000
D4	20.000000
D5	23.000000
D2	18.000000
D6	2.000000
D7	2.000000
X241	1.000000
X151	1.000000
Z01	53000.000000
X316	1.000000
X416	1.000000

T36	3.200000
T46	3.200000

All other variables in the range 1-116 are zero.

Appendix 3A - Math formulation for Figure 5

CPLEX 3.0 Formulation for the "Hierarchical-Plus-Time-Windows-Plus-Multiple-Frequency" example problem associated with Figure 5, in which cargo demands at nodes 3A, 3B, 4 and 5 are 30000, 1000, 10000 and 12000 pounds, respectively.

The solution to this formulation is given in Appendix 3B.

MINIMIZE

2.3X113A+2.3X113B+2.8X114+2.5X115+4X123A+4X123B+3X124+3.2X125+2X13A
4+2X13B4+5X13A5+
5X13B5+2X143A+2X143B+5X153A+5X153B+6X145+6X154+2.3X213A+2.3X213B+
2.8X214+2.5X215+
4X223A+4X223B+3X224+3.2X225+2X23A4+2X23B4+5X23A5+5X23B5+
2X243A+2X243B+5X253A+5X253B+6X245+6X254+T11+T12+T21+T22+T13A+T13
B+T23A+T23B+T14+
T24+T15+T25+X3A1+X3B1+X3A2+X3B2+X41+X42+X51+X52+10X361+10X461+1
1.7X362+11.7X462+
10.5X371+10.5X471+12.1X372+12.1X472+T31+T32+T41+T42+X16+X17+X26+X27+
Y1+Y2
ST
D3A - D1 <= 16
D3B - D1 <= 16
D4 - D1 <= 16
D5 - D1 <= 16
D3A - D2 <= 16
D3B - D2 <= 16
D4 - D2 <= 16
D5 - D2 <= 16
D1 - D6 <= 16
D1 - D7 <= 16
D2 - D6 <= 16
D2 - D7 <= 16

16X13A4+D3A-D4<=14
16X143A+D4-D3A<=14
16X13B4+D3B-D4<=14
16X143B+D4-D3B<=14
16X13A5+D3A-D5<=11
16X153A+D5-D3A<=11
16X13B5+D3B-D5<=11

16X153B+D5-D3B<=11
16X145+D4-D5<=10
16X154+D5-D4<=10

2.3X113A-D3A<=0
2.3X113B-D3B<=0
2.3X213A-D3A<=0
2.3X213B-D3B<=0
2.8X114-D4<=0
2.8X214-D4<=0
2.5X115-D5<=0
2.5X215-D5<=0
4X123A-D3A<=0
4X123B-D3B<=0
4X223A-D3A<=0
4X223B-D3B<=0
3X124-D4<=0
3X224-D4<=0
3.2X125-D5<=0
3.2X225-D5<=0

2.3X13A1+D3A<=32
2.3X13B1+D3B<=32
2.3X23A1+D3A<=32
2.3X23B1+D3B<=32
4X13A2+D3A<=32
4X13B2+D3B<=32
4X23A2+D3A<=32
4X23B2+D3B<=32
2.8X141+D4<=32
2.8X241+D4<=32
3X142+D4<=32
3X242+D4<=32
2.5X151+D5<=32
2.5X251+D5<=32
3.2X152+D5<=32
3.2X252+D5<=32

16X113A+T11+D1-D3A<=13.7
16X113A-T11-D1+D3A<=18.3
16X123A+T12+D2-D3A<=12
16X123A-T12-D2+D3A<=20
16X143A+T14+D4-D3A<=14

16X143A-T14-D4+D3A<=18
16X153A+T15+D5-D3A<=11
16X153A-T15-D5+D3A<=21
16X213A+T21+D1-D3A<=13.7
16X213A-T21-D1+D3A<=18.3
16X223A+T22+D2-D3A<=12
16X223A-T22-D2+D3A<=20
16X243A+T24+D4-D3A<=14
16X243A-T24-D4+D3A<=18
16X253A+T25+D5-D3A<=11
16X253A-T25-D5+D3A<=21

16X113B+T11+D1-D3B<=13.7
16X113B-T11-D1+D3B<=18.3
16X123B+T12+D2-D3B<=12
16X123B-T12-D2+D3B<=20
16X143B+T14+D4-D3B<=14
16X143B-T14-D4+D3B<=18
16X153B+T15+D5-D3B<=11
16X153B-T15-D5+D3B<=21
16X213B+T21+D1-D3B<=13.7
16X213B-T21-D1+D3B<=18.3
16X223B+T22+D2-D3B<=12
16X223B-T22-D2+D3B<=20
16X243B+T24+D4-D3B<=14
16X243B-T24-D4+D3B<=18
16X253B+T25+D5-D3B<11
16X253B-T25-D5+D3B<=21

16X114+T11+D1-D4<=13.2
16X114-T11-D1+D4<=18.8
16X124+T12+D2-D4<=13
16X124-T12-D2+D4<=19

16X13A4+T13A+D3A-D4<=14
16X13A4-T13A-D3A+D4<=18

16X13B4+T13B+D3B-D4<=14
16X13B4-T13B-D3B+D4<=18

16X154+T15+D5-D4<=10
16X154-T15-D5+D4<=22
16X214+T21+D1-D4<=13.2

16X214-T21-D1+D4<=18.8
16X224+T22+D2-D4<=13
16X224-T22-D2+D4<=19

16X23A4+T23A+D3A-D4<=14
16X23A4-T23A-D3A+D4<=18

16X23B4+T23B+D3B-D4<=14
16X23B4-T23B-D3B+D4<=18

16X254+T25+D5-D4<=10
16X254-T25-D5+D4<=22
16X115+T11+D1-D5<=13.5
16X115-T11-D1+D5<=18.5
16X125+T12+D2-D5<=12.8
16X125-T12-D2+D5<=19.2

16X13A5+T13A+D3A-D5<=11
16X13A5-T13A-D3A+D5<=21

16X13B5+T13B+D3B-D5<=11
16X13B5-T13B-D3B+D5<=21

16X145+T14+D4-D5<=10
16X145-T14-D4+D5<=22
16X215+T21+D1-D5<=13.5
16X215-T21-D1+D5<=18.5
16X225+T22+D2-D5<=12.8
16X225-T22-D2+D5<=19.2

16X23A5+T23A+D3A-D5<=11
16X23A5-T23A-D3A+D5<=21

16X23B5+T23B+D3B-D5<=11
16X23B5-T23B-D3B+D5<=21

16X245+T24+D4-D5<=10
16X245-T24-D4+D5<=22

X113A+X113B+X114+X115+X123A+X123B+X124+X125=1
X213A+X213B+X214+X215+X223A+X223B+X224+X225=1
X13A1+X13B1+X141+X151+X13A2+X13B2+X142+X152=1
X23A1+X23B1+X241+X251+X23A2+X23B2+X242+X252=1

-X113A-X123A-X143A-X153A+2T13A>=0
-X213A-X223A-X243A-X253A+2T23A>=0

-X113B-X123B-X143B-X153B+2T13B>=0
-X213B-X223B-X243B-X253B+2T23B>=0

-X114-X124-X13A4-X13B4-X154+2T14>=0
-X214-X224-X23A4-X23B4-X254+2T24>=0
-X115-X125-X13A5-X13B5-X145+2T15>=0
-X215-X225-X23A5-X23B5-X245+2T25>=0

X113A-X13A1+X123A-X13A2-X13A4-X13A5+X143A+X153A=0
X113B-X13B1+X123B-X13B2-X13B4-X13B5+X143B+X153B=0
X114-X141+X124-X142+X13A4-X143A+X13B4-X143B-X145+X154=0
X115-X151+X125-X152+X13A5-X153A+X13B5-X153B+X145-X154=0

-30000X113A-1000X113B-10000X114-12000X115-30000X123A-1000X123B-10000X124-12000X125-10000X13A4-10000X13B4-12000X13A5-12000X13B5-30000X143A-1000X143B-30000X153A-1000X153B-12000X145-10000X154>=-30000

-30000X213A-1000X213B-10000X214-12000X215-30000X223A-1000X223B-10000X224-12000X225-10000X23A4-10000X23B4-12000X23A5-12000X23B5-30000X243A-1000X243B-30000X253A-1000X253B-12000X245-10000X254>=-30000

X213A-X23A1+X223A-X23A2-X23A4-X23A5+X243A+X253A=0
X213B-X23B1+X223B-X23B2-X23B4-X23B5+X243B+X253B=0
X214-X241+X224-X242+X23A4-X243A+X23B4-X243B-X245+X254=0
X215-X251+X225-X252+X23A5-X253A+X23B5-X253B+X245-X254=0

X113A+X123A+X143A+X153A+X213A+X223A+X243A+X253A>=1
X113B+X123B+X143B+X153B+X213B+X223B+X243B+X253B>=1
X114+X124+X13A4+X13B4+X154+X214+X224+X23A4+X23B4+X254>=1
X115+X125+X13A5+X13B5+X145+X215+X225+X23A5+X23B5+X245>=1

X13A1+X13B1+X141+X151-X113A-X113B-X114-X115=0
X23A1+X23B1+X241+X251-X213A-X213B-X214-X215=0
X13A2+X13B2+X142+X152-X123A-X123B-X124-X125=0
X23A2+X23B2+X242+X252-X223A-X223B-X224-X225=0

Z01-30000X3A1-1000X3B1-10000X41-12000X51=0

Z02-30000X3A2-1000X3B2-10000X42-12000X52=0

Z01-60000Y1<=0
Z02-60000Y2<=0

X13A1+X13A2+X13A4+X13A5+X121+X13A1+X13B1+X141+X151-X3A1<=1
X13B1+X13B2+X13B4+X13B5+X121+X13A1+X13B1+X141+X151-X3B1<=1
X23A1+X23A2+X23A4+X23A5+X221+X23A1+X23B1+X241+X251-X3A1<=1
X23B1+X23B2+X23B4+X23B5+X221+X23A1+X23B1+X241+X251-X3B1<=1
X13A1+X13A2+X13A4+X13A5+X112+X13A2+X13B2+X142+X152-X3A2<=1
X13B1+X13B2+X13B4+X13B5+X112+X13A2+X13B2+X142+X152-X3B2<=1
X141+X142+X143A+X143B+X145+X121+X13A1+X13B1+X141+X151-X41<=1
X241+X242+X243A+X243B+X245+X221+X23A1+X23B1+X241+X251-X41<=1
X141+X142+X143A+X143B+X145+X112+X13A2+X13B2+X142+X152-X42<=1
X241+X242+X243A+X243B+X245+X212+X23A2+X23B2+X242+X252-X42<=1
X151+X152+X153A+X153B+X154+X121+X13A1+X13B1+X141+X151-X51<=1
X251+X252+X253A+X253B+X254+X221+X23A1+X23B1+X241+X251-X51<=1
X151+X152+X153A+X153B+X154+X112+X13A2+X13B2+X142+X152-X52<=1
X251+X252+X253A+X253B+X254+X212+X23A2+X23B2+X242+X252-X52<=1

X361+X362+X371+X372=1
X461+X462+X471+X472=1
X316+X326+X317+X327=1
X416+X426+X417+X427=1
X361+X371-.5T31<=0
X461+X471-.5T41<=0
X362+X372-.5T32<=0
X462+X472-.5T42<=0
X361+X371-X316-X317=0
X362+X372-X326-X327=0
X461+X471-X416-X417=0
X462+X472-X426-X427=0
Z01-35000X361-35000X461-35000X371-35000X471<=0
Z02-35000X362-35000X462-35000X372-35000X472<=0
X361+X371+X461+X471<=10
X362+X372+X462+X472<=10
X316+X317+X316+X326-X16<=1
X416+X417+X416+X426-X16<=1
X316+X317+X317+X327-X17<=1
X416+X417+X417+X427-X17<=1
X326+X327+X316+X326-X26<=1
X426+X427+X416+X426-X26<=1
X326+X327+X317+X327-X27<=1

X426+X427+X417+X427-X27<=1
X361+X362-X316-X326=0
X371+X372-X317-X327=0
X461+X462-X416-X426=0
X471+X472-X417-X427=0
16X361+T36+D6-D1<=6
16X361-T36-D6+D1<=26
16X371+T37+D7-D1<=5.5
16X371-T37-D7+D1<=26.5
16X461+T46+D6-D1<=6
16X461-T46-D6+D1<=26
16X471+T47+D7-D1<=5.5
16X471-T47-D7+D1<=26.5
16X362+T36+D6-D2<=4.3
16X362-T36-D6+D2<=27.7
16X372+T37+D7-D2<=3.9
16X372-T37-D7+D2<=28.1
16X462+T46+D6-D2<=4.3
16X462-T46-D6+D2<=27.7
16X472+T47+D7-D2<=3.9
16X472-T47-D7+D2<=28.1

BOUNDS
D6=2
D7=2

D3A>=16
D3B>=16
D3A<=20
D3B<=20

D4>=17.5
D4<=21.5
D5>=19
D5<=25

INTEGERS
X112
X113A
X113B
X114
X115

X121
X13A1
X13B1
X141
X151
X123A
X123B
X124
X125
X13A2
X13B2
X142
X152
X13A4
X13B4
X13A5
X13B5
X143A
X143B
X153A
X153B
X145
X154
X212
X213A
X213B
X214
X215
X221
X223A
X223B
X241
X23A1
X23B1
X251
X224
X225
X23A2
X23B2
X242
X252
X23A4
X23B4

X23A5
X23B5
X243A
X243B
X253A
X253B
X245
X254
Y1
Y2
X3A1
X3B1
X41
X51
X3A2
X3B2
X42
X52
X316
X317
X326
X327
X361
X371
X362
X372
X16
X17
X26
X27
END

Appendix 3B - Solution for Figure 5

Solution to the "Hierarchical-Plus-Time-Windows-Plus-Multiple-Frequency" example problem associated with Figure 5, in which cargo demands at nodes 3A, 3B, 4 and 5 are 30000, 1000, 10000 and 12000 pounds, respectively. (The formulation of this problem is presented in Appendix 3A)

Integer Optimal Solution: Objective = 4.4600000000e+01
Solution Time = 1.55 sec. Iterations = 667 Nodes = 66

Variable Name	Solution Value
X113A	1.000000
X213B	1.000000
X23B4	1.000000
X245	1.000000
T13A	0.500000
T23B	0.500000
T24	0.500000
T25	0.500000
X3A1	1.000000
X3B1	1.000000
X41	1.000000
X51	1.000000
X361	1.000000
X461	1.000000
T31	2.000000
T41	2.000000
X16	1.000000
Y1	1.000000
D3A	16.000000
D1	13.700000
D3B	16.000000
D4	18.500000
D5	25.000000
D2	9.000000
D6	2.000000
D7	2.000000
X13A1	1.000000
X251	1.000000
Z01	53000.000000
X316	1.000000
X416	1.000000

T36	1.700000
T46	1.700000
	All other variables in the range 51-116 are zero.

Appendix 4A - Math formulation for Figure 7

CPLEX 3.0 Formulation for the "Hub-and-Spoke" case study scenario problem associated with Figure 7, in which cargo demands at nodes 3A, 3B, 4A, 4B, 5A and 5B are specified by weights and CONUS origin.

The solution to this case study formulation is given in Appendix 4B.

Via Equation (22):

MINIMIZE

2.09X113A+2.09X113B+1.92X114A+1.92X114B+2.13X115A+2.13X115B+2.19X123
A+2.19X123B+
2.04X124A+2.04X124B+2.04X125A+2.04X125B+.45X13A4A+.45X13A4B+.31X13A5
A+.31X13A5B+
.45X13B4A+.45X13B4B+.31X13B5A+.31X13B5B+.45X14A3A+.45X14A3B+.42X14
A5A+.42X14A5B+
.45X14B3A+.45X14B3B+.42X14B5A+.42X14B5B+.31X15A3A+.31X15A3B+.42X15
A4A+.42X15A4B+
.31X15B3A+.31X15B3B+.42X15B4A+.42X15B4B+

2.09X213A+2.09X213B+1.92X214A+1.92X214B+2.13X215A+2.13X215B+2.19X223
A+2.19X223B+
2.04X224A+2.04X224B+2.04X225A+2.04X225B+.45X23A4A+.45X23A4B+.31X23A5
A+.31X23A5B+
.45X23B4A+.45X23B4B+.31X23B5A+.31X23B5B+.45X24A3A+.45X24A3B+.42X24
A5A+.42X24A5B+
.45X24B3A+.45X24B3B+.42X24B5A+.42X24B5B+.31X25A3A+.31X25A3B+.42X25
A4A+.42X25A4B+
.31X25B3A+.31X25B3B+.42X25B4A+.42X25B4B+

2.09X313A+2.09X313B+1.92X314A+1.92X314B+2.13X315A+2.13X315B+2.19X323
A+2.19X323B+
2.04X324A+2.04X324B+2.04X325A+2.04X325B+.45X33A4A+.45X33A4B+.31X33A5
A+.31X33A5B+
.45X33B4A+.45X33B4B+.31X33B5A+.31X33B5B+.45X34A3A+.45X34A3B+.42X34
A5A+.42X34A5B+
.45X34B3A+.45X34B3B+.42X34B5A+.42X34B5B+.31X35A3A+.31X35A3B+.42X35
A4A+.42X35A4B+
.31X35B3A+.31X35B3B+.42X35B4A+.42X35B4B+

2.09X413A+2.09X413B+1.92X414A+1.92X414B+2.13X415A+2.13X415B+2.19X423
A+2.19X423B+
2.04X424A+2.04X424B+2.04X425A+2.04X425B+.45X43A4A+.45X43A4B+.31X43A5
A+.31X43A5B+
.45X43B4A+.45X43B4B+.31X43B5A+.31X43B5B+.45X44A3A+.45X44A3B+.42X44
A5A+.42X44A5B+
.45X44B3A+.45X44B3B+.42X44B5A+.42X44B5B+.31X45A3A+.31X45A3B+.42X45
A4A+.42X45A4B+
.31X45B3A+.31X45B3B+.42X45B4A+.42X45B4B+

2.09X513A+2.09X513B+1.92X514A+1.92X514B+2.13X515A+2.13X515B+2.19X523
A+2.19X523B+
2.04X524A+2.04X524B+2.04X525A+2.04X525B+.45X53A4A+.45X53A4B+.31X53A5
A+.31X53A5B+
.45X53B4A+.45X53B4B+.31X53B5A+.31X53B5B+.45X54A3A+.45X54A3B+.42X54
A5A+.42X54A5B+
.45X54B3A+.452X54B3B+.42X54B5A+.42X54B5B+.31X55A3A+.31X55A3B+.42X55
A4A+.42X55A4B+
.31X55B3A+.31X55B3B+.42X55B4A+.42X55B4B+

2.09X613A+2.09X613B+1.92X614A+1.92X614B+2.13X615A+2.13X615B+2.19X623
A+2.19X623B+
2.04X624A+2.04X624B+2.04X625A+2.04X625B+.45X63A4A+.45X63A4B+.31X63A5
A+.31X63A5B+
.45X63B4A+.45X63B4B+.31X63B5A+.31X63B5B+.45X64A3A+.45X64A3B+.42X64
A5A+.42X64A5B+
.45X64B3A+.45X64B3B+.42X64B5A+.42X64B5B+.31X65A3A+.31X65A3B+.42X65
A4A+.42X65A4B+
.31X65B3A+.31X65B3B+.42X65B4A+.42X65B4B+

16X113A+16X113B+16X114A+16X114B+16X115A+16X115B+
16X123A+16X123B+16X124A+16X124B+16X125A+16X125B+

16X213A+16X213B+16X214A+16X214B+16X215A+16X215B+
16X223A+16X223B+16X224A+16X224B+16X225A+16X225B+

16X313A+16X313B+16X314A+16X314B+16X315A+16X315B+
16X323A+16X323B+16X324A+16X324B+16X325A+16X325B+

16X413A+16X413B+16X414A+16X414B+16X415A+16X415B+
16X423A+16X423B+16X424A+16X424B+16X425A+16X425B+

16X513A+16X513B+16X514A+16X514B+16X515A+16X515B+

16X523A+16X523B+16X524A+16X524B+16X525A+16X525B+

16X613A+16X613B+16X614A+16X614B+16X615A+16X615B+
16X623A+16X623B+16X624A+16X624B+16X625A+16X625B+

T11+T12+T13A+T13B+T14A+T14B+T15A+T15B+
T21+T22+T23A+T23B+T24A+T24B+T25A+T25B+
T31+T32+T33A+T33B+T34A+T34B+T35A+T35B+
T41+T42+T43A+T43B+T44A+T44B+T45A+T45B+
T51+T52+T53A+T53B+T54A+T54B+T55A+T55B+
T61+T62+T63A+T63B+T64A+T64B+T65A+T65B+

X3A1+X3A2+X3B1+X3B2+X4A1+X4A2+X4B1+X4B2+X5A1+X5A2+X5B1+X5B2+

9.76C161+11.52C162+9.76C261+11.52C262+10.4C371+12.18C372+10.4C471+
12.18C472+C16+C17+C26+C27

ST

Via Equation (23):

D3A - D1 <= 16
D3B - D1 <= 16
D4A- D1 <= 16
D4B- D1 <= 16
D5A - D1 <= 16
D5B- D1 <= 16
D3A - D2 <= 16
D3B - D2 <= 16
D4A - D2 <= 16
D4B - D2 <= 16
D5A - D2 <= 16
D5B - D2 <= 16

Via Equation (24):

D1 - D6 <= 16
D1 - D7 <= 16
D2 - D6 <= 16
D2 - D7 <= 16

Via Equation (25):

2.09X113A-D3A<=0
2.09X113B-D3B<=0
2.09X213A-D3A<=0
2.09X213B-D3B<=0
2.09X313A-D3A<=0
2.09X313B-D3B<=0
2.09X413A-D3A<=0
2.09X413B-D3B<=0
2.09X513A-D3A<=0
2.09X513B-D3B<=0
2.09X613A-D3A<=0
2.09X613B-D3B<=0

1.92X114A-D4A<=0
1.92X114B-D4B<=0
1.92X214A-D4A<=0
1.92X214B-D4B<=0
1.92X314A-D4A<=0
1.92X314B-D4B<=0
1.92X414A-D4A<=0
1.92X414B-D4B<=0
1.92X514A-D4A<=0
1.92X514B-D4B<=0
1.92X614A-D4A<=0
1.92X614B-D4B<=0

2.13X115A-D5A<=0
2.13X115B-D5B<=0
2.13X215A-D5A<=0
2.13X215B-D5B<=0
2.13X315A-D5A<=0
2.13X315B-D5B<=0
2.13X415A-D5A<=0
2.13X415B-D5B<=0
2.13X515A-D5A<=0
2.13X515B-D5B<=0
2.13X615A-D5A<=0
2.13X615B-D5B<=0

2.19X123A-D3A<=0
2.19X123B-D3B<=0

2.19X223A-D3A<=0
2.19X223B-D3B<=0
2.19X323A-D3A<=0
2.19X323B-D3B<=0
2.19X423A-D3A<=0
2.19X423B-D3B<=0
2.19X523A-D3A<=0
2.19X523B-D3B<=0
2.19X623A-D3A<=0
2.19X623B-D3B<=0

2.04X124A-D4A<=0
2.04X124B-D4B<=0
2.04X224A-D4A<=0
2.04X224B-D4B<=0
2.04X324A-D4A<=0
2.04X324B-D4B<=0
2.04X424A-D4A<=0
2.04X424B-D4B<=0
2.04X524A-D4A<=0
2.04X524B-D4B<=0
2.04X624A-D4A<=0
2.04X624B-D4B<=0

2.04X125A-D5A<=0
2.04X125B-D5B<=0
2.04X225A-D5A<=0
2.04X225B-D5B<=0
2.04X325A-D5A<=0
2.04X325B-D5B<=0
2.04X425A-D5A<=0
2.04X425B-D5B<=0
2.04X525A-D5A<=0
2.04X525B-D5B<=0
2.04X625A-D5A<=0
2.04X625B-D5B<=0

Via Equation (26):

2.09X13A1+D3A<=32
2.09X13B1+D3B<=32
2.09X23A1+D3A<=32
2.09X23B1+D3B<=32

2.09X33A1+D3A<=32
2.09X33B1+D3B<=32
2.09X43A1+D3A<=32
2.09X43B1+D3B<=32
2.09X53A1+D3A<=32
2.09X53B1+D3B<=32
2.09X63A1+D3A<=32
2.09X63B1+D3B<=32

2.19X13A2+D3A<=32
2.19X13B2+D3B<=32
2.19X23A2+D3A<=32
2.19X23B2+D3B<=32
2.19X33A2+D3A<=32
2.19X33B2+D3B<=32
2.19X43A2+D3A<=32
2.19X43B2+D3B<=32
2.19X53A2+D3A<=32
2.19X53B2+D3B<=32
2.19X63A2+D3A<=32
2.19X63B2+D3B<=32

1.92X14A1+D4A<=32
1.92X14B1+D4B<=32
1.92X24A1+D4A<=32
1.92X24B1+D4B<=32
1.92X34A1+D4A<=32
1.92X34B1+D4B<=32
1.92X44A1+D4A<=32
1.92X44B1+D4B<=32
1.92X54A1+D4A<=32
1.92X54B1+D4B<=32
1.92X64A1+D4A<=32
1.92X64B1+D4B<=32

2.04X14A2+D4A<=32
2.04X14B2+D4B<=32
2.04X24A2+D4A<=32
2.04X24B2+D4B<=32
2.04X34A2+D4A<=32
2.04X34B2+D4B<=32
2.04X44A2+D4A<=32
2.04X44B2+D4B<=32

2.04X54A2+D4A<=32
2.04X54B2+D4B<=32
2.04X64A2+D4A<=32
2.04X64B2+D4B<=32

2.13X15A1+D5A<=32
2.13X15B1+D5B<=32
2.13X25A1+D5A<=32
2.13X25B1+D5B<=32
2.13X35A1+D5A<=32
2.13X35B1+D5B<=32
2.13X45A1+D5A<=32
2.13X45B1+D5B<=32
2.13X55A1+D5A<=32
2.13X55B1+D5B<=32
2.13X65A1+D5A<=32
2.13X65B1+D5B<=32

2.04X15A2+D5A<=32
2.04X15B2+D5B<=32
2.04X25A2+D5A<=32
2.04X25B2+D5B<=32
2.04X35A2+D5A<=32
2.04X35B2+D5B<=32
2.04X45A2+D5A<=32
2.04X45B2+D5B<=32
2.04X55A2+D5A<=32
2.04X55B2+D5B<=32
2.04X65A2+D5A<=32
2.04X65B2+D5B<=32

Via Equations (27) and (28):

16X113A+T11+D1-D3A<=13.91
16X113A-T11-D1+D3A<=18.09
16X123A+T12+D2-D3A<=13.81
16X123A-T12-D2+D3A<=18.19
16X14A3A+T14A+D4A-D3A<=15.55
16X14A3A-T14A-D4A+D3A<=16.45
16X14B3A+T14B+D4B-D3A<=15.55
16X14B3A-T14B-D4B+D3A<=16.45
16X15A3A+T15A+D5A-D3A<=15.69
16X15A3A-T15A-D5A+D3A<=16.31

16X15B3A+T15B+D5B-D3A<=15.69
16X15B3A-T15B-D5B+D3A<=16.31

16X213A+T21+D1-D3A<=13.91
16X213A-T21-D1+D3A<=18.09
16X223A+T22+D2-D3A<=13.81
16X223A-T22-D2+D3A<=18.19
16X24A3A+T24A+D4A-D3A<=15.55
16X24A3A-T24A-D4A+D3A<=16.45
16X24B3A+T24B+D4B-D3A<=15.55
16X24B3A-T24B-D4B+D3A<=16.45
16X25A3A+T25A+D5A-D3A<=15.69
16X25A3A-T25A-D5A+D3A<=16.31
16X25B3A+T25B+D5B-D3A<=15.69
16X25B3A-T25B-D5B+D3A<=16.31

16X313A+T31+D1-D3A<=13.91
16X313A-T31-D1+D3A<=18.09
16X323A+T32+D2-D3A<=13.81
16X323A-T32-D2+D3A<=18.19
16X34A3A+T34A+D4A-D3A<=15.55
16X34A3A-T34A-D4A+D3A<=16.45
16X34B3A+T34B+D4B-D3A<=15.55
16X34B3A-T34B-D4B+D3A<=16.45
16X35A3A+T35A+D5A-D3A<=15.69
16X35A3A-T35A-D5A+D3A<=16.31
16X35B3A+T35B+D5B-D3A<=15.69
16X35B3A-T35B-D5B+D3A<=16.31

16X413A+T41+D1-D3A<=13.91
16X413A-T41-D1+D3A<=18.09
16X423A+T42+D2-D3A<=13.81
16X423A-T42-D2+D3A<=18.19
16X44A3A+T44A+D4A-D3A<=15.55
16X44A3A-T44A-D4A+D3A<=16.45
16X44B3A+T44B+D4B-D3A<=15.55
16X44B3A-T44B-D4B+D3A<=16.45
16X45A3A+T45A+D5A-D3A<=15.69
16X45A3A-T45A-D5A+D3A<=16.31
16X45B3A+T45B+D5B-D3A<=15.69
16X45B3A-T45B-D5B+D3A<=16.31

16X513A+T51+D1-D3A<=13.91

16X513A-T51-D1+D3A<=18.09
16X523A+T52+D2-D3A<=13.81
16X523A-T52-D2+D3A<=18.19
16X54A3A+T54A+D4A-D3A<=15.55
16X54A3A-T54A-D4A+D3A<=16.45
16X54B3A+T54B+D4B-D3A<=15.55
16X54B3A-T54B-D4B+D3A<=16.45
16X55A3A+T55A+D5A-D3A<=15.69
16X55A3A-T55A-D5A+D3A<=16.31
16X55B3A+T55B+D5B-D3A<=15.69
16X55B3A-T55B-D5B+D3A<=16.31

16X613A+T61+D1-D3A<=13.91
16X613A-T61-D1+D3A<=18.09
16X623A+T62+D2-D3A<=13.81
16X623A-T62-D2+D3A<=18.19
16X64A3A+T64A+D4A-D3A<=15.55
16X64A3A-T64A-D4A+D3A<=16.45
16X64B3A+T64B+D4B-D3A<=15.55
16X64B3A-T64B-D4B+D3A<=16.45
16X65A3A+T65A+D5A-D3A<=15.69
16X65A3A-T65A-D5A+D3A<=16.31
16X65B3A+T65B+D5B-D3A<=15.69
16X65B3A-T65B-D5B+D3A<=16.31

16X113B+T11+D1-D3B<=13.91
16X113B+T11-D1+D3B<=18.09
16X123B+T12+D2-D3B<=13.81
16X123B-T12-D2+D3B<=18.19
16X14A3B+T14A+D4A-D3B<=15.55
16X14A3B-T14A-D4A+D3B<=16.45
16X14B3B+T14B+D4B-D3B<=15.55
16X14B3B-T14B-D4B+D3B<=16.45
16X15A3B+T15A+D5A-D3B<=15.69
16X15A3B-T15A-D5A+D3B<=16.31
16X15B3B+T15B+D5B-D3B<=15.69
16X15B3B-T15B-D5B+D3B<=16.31

16X213B+T21+D1-D3B<=13.91
16X213B+T21-D1+D3B<=18.09
16X223B+T22+D2-D3B<=13.81
16X223B-T22-D2+D3B<=18.19
16X24A3B+T24A+D4A-D3B<=15.55

16X24A3B-T24A-D4A+D3B<=16.45
16X24B3B+T24B+D4B-D3B<=15.55
16X24B3B-T24B-D4B+D3B<=16.45
16X25A3B+T25A+D5A-D3B<=15.69
16X25A3B-T25A-D5A+D3B<=16.31
16X25B3B+T25B+D5B-D3B<=15.69
16X25B3B-T25B-D5B+D3B<=16.31

16X313B+T31+D1-D3B<=13.91
16X313B+T31-D1+D3B<=18.09
16X323B+T32+D2-D3B<=13.81
16X323B-T32-D2+D3B<=18.19
16X34A3B+T34A+D4A-D3B<=15.55
16X34A3B-T34A-D4A+D3B<=16.45
16X34B3B+T34B+D4B-D3B<=15.55
16X34B3B-T34B-D4B+D3B<=16.45
16X35A3B+T35A+D5A-D3B<=15.69
16X35A3B-T35A-D5A+D3B<=16.31
16X35B3B+T35B+D5B-D3B<=15.69
16X35B3B-T35B-D5B+D3B<=16.31

16X413B+T41+D1-D3B<=13.91
16X413B+T41-D1+D3B<=18.09
16X423B+T42+D2-D3B<=13.81
16X423B-T42-D2+D3B<=18.19
16X44A3B+T44A+D4A-D3B<=15.55
16X44A3B-T44A-D4A+D3B<=16.45
16X44B3B+T44B+D4B-D3B<=15.55
16X44B3B-T44B-D4B+D3B<=16.45
16X45A3B+T45A+D5A-D3B<=15.69
16X45A3B-T45A-D5A+D3B<=16.31
16X45B3B+T45B+D5B-D3B<=15.69
16X45B3B-T45B-D5B+D3B<=16.31

16X513B+T51+D1-D3B<=13.91
16X513B+T51-D1+D3B<=18.09
16X523B+T52+D2-D3B<=13.81
16X523B-T52-D2+D3B<=18.19
16X54A3B+T54A+D4A-D3B<=15.55
16X54A3B-T54A-D4A+D3B<=16.45
16X54B3B+T54B+D4B-D3B<=15.55
16X54B3B-T54B-D4B+D3B<=16.45
16X55A3B+T55A+D5A-D3B<=15.69

16X55A3B-T55A-D5A+D3B<=16.31
16X55B3B+T55B+D5B-D3B<=15.69
16X55B3B-T55B-D5B+D3B<=16.31

16X613B+T61+D1-D3B<=13.91
16X613B+T61-D1+D3B<=18.09
16X623B+T62+D2-D3B<=13.81
16X623B-T62-D2+D3B<=18.19
16X64A3B+T64A+D4A-D3B<=15.55
16X64A3B-T64A-D4A+D3B<=16.45
16X64B3B+T64B+D4B-D3B<=15.55
16X64B3B-T64B-D4B+D3B<=16.45
16X65A3B+T65A+D5A-D3B<=15.69
16X65A3B-T65A-D5A+D3B<=16.31
16X65B3B+T65B+D5B-D3B<=15.69
16X65B3B-T65B-D5B+D3B<=16.31

16X114A+T11+D1-D4A<=14.08
16X114A-T11-D1+D4A<=15.92
16X124A+T12+D2-D4A<=13.96
16X124A-T12-D2+D4A<=18.04
16X13A4A+T13A+D3A-D4A<=15.55
16X13A4A-T13A-D3A+D4A<=16.45
16X13B4A+T13B+D3B-D4A<=15.55
16X13B4A-T13B-D3B+D4A<=16.45
16X15A4A+T15A+D5A-D4A<=15.58
16X15A4A-T15A-D5A+D4A<=16.42
16X15B4A+T15B+D5B-D4A<=15.58
16X15B4A-T15B-D5B+D4A<=16.42

16X214A+T21+D1-D4A<=14.08
16X214A-T21-D1+D4A<=15.92
16X224A+T22+D2-D4A<=13.96
16X224A-T22-D2+D4A<=18.04
16X23A4A+T23A+D3A-D4A<=15.55
16X23A4A-T23A-D3A+D4A<=16.45
16X23B4A+T23B+D3B-D4A<=15.55
16X23B4A-T23B-D3B+D4A<=16.45
16X25A4A+T25A+D5A-D4A<=15.58
16X25A4A-T25A-D5A+D4A<=16.42
16X25B4A+T25B+D5B-D4A<=15.58
16X25B4A-T25B-D5B+D4A<=16.42

16X314A+T31+D1-D4A<=14.08
16X314A-T31-D1+D4A<=15.92
16X324A+T32+D2-D4A<=13.96
16X324A-T32-D2+D4A<=18.04
16X33A4A+T33A+D3A-D4A<=15.55
16X33A4A-T33A-D3A+D4A<=16.45
16X33B4A+T33B+D3B-D4A<=15.55
16X33B4A-T33B-D3B+D4A<=16.45
16X35A4A+T35A+D5A-D4A<=15.58
16X35A4A-T35A-D5A+D4A<=16.42
16X35B4A+T35B+D5B-D4A<=15.58
16X35B4A-T35B-D5B+D4A<=16.42

16X414A+T41+D1-D4A<=14.08
16X414A-T41-D1+D4A<=15.92
16X424A+T42+D2-D4A<=13.96
16X424A-T42-D2+D4A<=18.04
16X43A4A+T43A+D3A-D4A<=15.55
16X43A4A-T43A-D3A+D4A<=16.45
16X43B4A+T43B+D3B-D4A<=15.55
16X43B4A-T43B-D3B+D4A<=16.45
16X45A4A+T45A+D5A-D4A<=15.58
16X45A4A-T45A-D5A+D4A<=16.42
16X45B4A+T45B+D5B-D4A<=15.58
16X45B4A-T45B-D5B+D4A<=16.42

16X514A+T51+D1-D4A<=14.08
16X514A-T51-D1+D4A<=15.92
16X524A+T52+D2-D4A<=13.96
16X524A-T52-D2+D4A<=18.04
16X53A4A+T53A+D3A-D4A<=15.55
16X53A4A-T53A-D3A+D4A<=16.45
16X53B4A+T53B+D3B-D4A<=15.55
16X53B4A-T53B-D3B+D4A<=16.45
16X55A4A+T55A+D5A-D4A<=15.58
16X55A4A-T55A-D5A+D4A<=16.42
16X55B4A+T55B+D5B-D4A<=15.58
16X55B4A-T55B-D5B+D4A<=16.42

16X614A+T61+D1-D4A<=14.08
16X614A-T61-D1+D4A<=15.92
16X624A+T62+D2-D4A<=13.96
16X624A-T62-D2+D4A<=18.04

16X63A4A+T63A+D3A-D4A<=15.55
16X63A4A-T63A-D3A+D4A<=16.45
16X63B4A+T63B+D3B-D4A<=15.55
16X63B4A-T63B-D3B+D4A<=16.45
16X65A4A+T65A+D5A-D4A<=15.58
16X65A4A-T65A-D5A+D4A<=16.42
16X65B4A+T65B+D5B-D4A<=15.58
16X65B4A-T65B-D5B+D4A<=16.42

16X114B+T11+D1-D4B<=14.08
16X114B-T11-D1+D4B<=15.92
16X124B+T12+D2-D4B<=13.96
16X124B-T12-D2+D4B<=18.04
16X13A4B+T13A+D3A-D4B<=15.55
16X13A4B-T13A-D3A+D4B<=16.45
16X13B4B+T13B+D3B-D4B<=15.55
16X13B4B-T13B-D3B+D4B<=16.45
16X15A4B+T15A+D5A-D4B<=15.58
16X15A4B-T15A-D5A+D4B<=16.42
16X15B4B+T15B+D5B-D4B<=15.58
16X15B4B-T15B-D5B+D4B<=16.42

16X214B+T21+D1-D4B<=14.08
16X214B-T21-D1+D4B<=15.92
16X224B+T22+D2-D4B<=13.96
16X224B-T22-D2+D4B<=18.04
16X23A4B+T23A+D3A-D4B<=15.55
16X23A4B-T23A-D3A+D4B<=16.45
16X23B4B+T23B+D3B-D4B<=15.55
16X23B4B-T23B-D3B+D4B<=16.45
16X25A4B+T25A+D5A-D4B<=15.58
16X25A4B-T25A-D5A+D4B<=16.42
16X25B4B+T25B+D5B-D4B<=15.58
16X25B4B-T25B-D5B+D4B<=16.42

16X314B+T31+D1-D4B<=14.08
16X314B-T31-D1+D4B<=15.92
16X324B+T32+D2-D4B<=13.96
16X324B-T32-D2+D4B<=18.04
16X33A4B+T33A+D3A-D4B<=15.55
16X33A4B-T33A-D3A+D4B<=16.45
16X33B4B+T33B+D3B-D4B<=15.55
16X33B4B-T33B-D3B+D4B<=16.45

16X35A4B+T35A+D5A-D4B<=15.58
16X35A4B-T35A-D5A+D4B<=16.42
16X35B4B+T35B+D5B-D4B<=15.58
16X35B4B-T35B-D5B+D4B<=16.42

16X414B+T41+D1-D4B<=14.08
16X414B-T41-D1+D4B<=15.92
16X424B+T42+D2-D4B<=13.96
16X424B-T42-D2+D4B<=18.04
16X43A4B+T43A+D3A-D4B<=15.55
16X43A4B-T43A-D3A+D4B<=16.45
16X43B4B+T43B+D3B-D4B<=15.55
16X43B4B-T43B-D3B+D4B<=16.45
16X45A4B+T45A+D5A-D4B<=15.58
16X45A4B-T45A-D5A+D4B<=16.42
16X45B4B+T45B+D5B-D4B<=15.58
16X45B4B-T45B-D5B+D4B<=16.42

16X514B+T51+D1-D4B<=14.08
16X514B-T51-D1+D4B<=15.92
16X524B+T52+D2-D4B<=13.96
16X524B-T52-D2+D4B<=18.04
16X53A4B+T53A+D3A-D4B<=15.55
16X53A4B-T53A-D3A+D4B<=16.45
16X53B4B+T53B+D3B-D4B<=15.55
16X53B4B-T53B-D3B+D4B<=16.45
16X55A4B+T55A+D5A-D4B<=15.58
16X55A4B-T55A-D5A+D4B<=16.42
16X55B4B+T55B+D5B-D4B<=15.58
16X55B4B-T55B-D5B+D4B<=16.42

16X614B+T61+D1-D4B<=14.08
16X614B-T61-D1+D4B<=15.92
16X624B+T62+D2-D4B<=13.96
16X624B-T62-D2+D4B<=18.04
16X63A4B+T63A+D3A-D4B<=15.55
16X63A4B-T63A-D3A+D4B<=16.45
16X63B4B+T63B+D3B-D4B<=15.55
16X63B4B-T63B-D3B+D4B<=16.45
16X65A4B+T65A+D5A-D4B<=15.58
16X65A4B-T65A-D5A+D4B<=16.42
16X65B4B+T65B+D5B-D4B<=15.58
16X65B4B-T65B-D5B+D4B<=16.42

16X115A+T11+D1-D5A<=13.87
16X115A-T11-D1+D5A<=18.13
16X125A+T12+D2-D5A<=13.96
16X125A-T12-D2+D5A<=18.04
16X13A5A+T13A+D3A-D5A<=15.69
16X13A5A-T13A-D3A+D5A<=16.31
16X13B5A+T13B+D3B-D5A<=15.69
16X13B5A-T13B-D3B+D5A<=16.31
16X14A5A+T14A+D4A-D5A<=15.58
16X14A5A-T14A-D4A+D5A<=16.42
16X14B5A+T14B+D4B-D5A<=15.58
16X14B5A-T14B-D4B+D5A<=16.42

16X215A+T21+D1-D5A<=13.87
16X215A-T21-D1+D5A<=18.13
16X225A+T22+D2-D5A<=13.96
16X225A-T22-D2+D5A<=18.04
16X23A5A+T23A+D3A-D5A<=15.69
16X23A5A-T23A-D3A+D5A<=16.31
16X23B5A+T23B+D3B-D5A<=15.69
16X23B5A-T23B-D3B+D5A<=16.31
16X24A5A+T24A+D4A-D5A<=15.58
16X24A5A-T24A-D4A+D5A<=16.42
16X24B5A+T24B+D4B-D5A<=15.58
16X24B5A-T24B-D4B+D5A<=16.42

16X315A+T31+D1-D5A<=13.87
16X315A-T31-D1+D5A<=18.13
16X325A+T32+D2-D5A<=13.96
16X325A-T32-D2+D5A<=18.04
16X33A5A+T33A+D3A-D5A<=15.69
16X33A5A-T33A-D3A+D5A<=16.31
16X33B5A+T33B+D3B-D5A<=15.69
16X33B5A-T33B-D3B+D5A<=16.31
16X34A5A+T34A+D4A-D5A<=15.58
16X34A5A-T34A-D4A+D5A<=16.42
16X34B5A+T34B+D4B-D5A<=15.58
16X34B5A-T34B-D4B+D5A<=16.42

16X415A+T41+D1-D5A<=13.87
16X415A-T41-D1+D5A<=18.13
16X425A+T42+D2-D5A<=13.96

16X425A-T42-D2+D5A<=18.04
16X43A5A+T43A+D3A-D5A<=15.69
16X43A5A-T43A-D3A+D5A<=16.31
16X43B5A+T43B+D3B-D5A<=15.69
16X43B5A-T43B-D3B+D5A<=16.31
16X44A5A+T44A+D4A-D5A<=15.58
16X44A5A-T44A-D4A+D5A<=16.42
16X44B5A+T44B+D4B-D5A<=15.58
16X44B5A-T44B-D4B+D5A<=16.42

16X515A+T51+D1-D5A<=13.87
16X515A-T51-D1+D5A<=18.13
16X525A+T52+D2-D5A<=13.96
16X525A-T52-D2+D5A<=18.04
16X53A5A+T53A+D3A-D5A<=15.69
16X53A5A-T53A-D3A+D5A<=16.31
16X53B5A+T53B+D3B-D5A<=15.69
16X53B5A-T53B-D3B+D5A<=16.31
16X54A5A+T54A+D4A-D5A<=15.58
16X54A5A-T54A-D4A+D5A<=16.42
16X54B5A+T54B+D4B-D5A<=15.58
16X54B5A-T54B-D4B+D5A<=16.42

16X615A+T61+D1-D5A<=13.87
16X615A-T61-D1+D5A<=18.13
16X625A+T62+D2-D5A<=13.96
16X625A-T62-D2+D5A<=18.04
16X63A5A+T63A+D3A-D5A<=15.69
16X63A5A-T63A-D3A+D5A<=16.31
16X63B5A+T63B+D3B-D5A<=15.69
16X63B5A-T63B-D3B+D5A<=16.31
16X64A5A+T64A+D4A-D5A<=15.58
16X64A5A-T64A-D4A+D5A<=16.42
16X64B5A+T64B+D4B-D5A<=15.58
16X64B5A-T64B-D4B+D5A<=16.42

16X115B+T11+D1-D5B<=13.87
16X115B-T11-D1+D5B<=18.13
16X125B+T12+D2-D5B<=13.96
16X125B-T12-D2+D5B<=18.04
16X13A5B+T13A+D3A-D5B<=15.69
16X13A5B-T13A-D3A+D5B<=16.31
16X13B5B+T13B+D3B-D5B<=15.69

16X13B5B-T13B-D3B+D5B<=16.31
16X14A5B+T14A+D4A-D5B<=15.58
16X14A5B-T14A-D4A+D5B<=16.42
16X14B5B+T14B+D4B-D5B<=15.58
16X14B5B-T14B-D4B+D5B<=16.42

16X215B+T21+D1-D5B<=13.87
16X215B-T21-D1+D5B<=18.13
16X225B+T22+D2-D5B<=13.96
16X225B-T22-D2+D5B<=18.04
16X23A5B+T23A+D3A-D5B<=15.69
16X23A5B-T23A-D3A+D5B<=16.31
16X23B5B+T23B+D3B-D5B<=15.69
16X23B5B-T23B-D3B+D5B<=16.31
16X24A5B+T24A+D4A-D5B<=15.58
16X24A5B-T24A-D4A+D5B<=16.42
16X24B5B+T24B+D4B-D5B<=15.58
16X24B5B-T24B-D4B+D5B<=16.42

16X315B+T31+D1-D5B<=13.87
16X315B-T31-D1+D5B<=18.13
16X325B+T32+D2-D5B<=13.96
16X325B-T32-D2+D5B<=18.04
16X33A5B+T33A+D3A-D5B<=15.69
16X33A5B-T33A-D3A+D5B<=16.31
16X33B5B+T33B+D3B-D5B<=15.69
16X33B5B-T33B-D3B+D5B<=16.31
16X34A5B+T34A+D4A-D5B<=15.58
16X34A5B-T34A-D4A+D5B<=16.42
16X34B5B+T34B+D4B-D5B<=15.58
16X34B5B-T34B-D4B+D5B<=16.42

16X415B+T41+D1-D5B<=13.87
16X415B-T41-D1+D5B<=18.13
16X425B+T42+D2-D5B<=13.96
16X425B-T42-D2+D5B<=18.04
16X43A5B+T43A+D3A-D5B<=15.69
16X43A5B-T43A-D3A+D5B<=16.31
16X43B5B+T43B+D3B-D5B<=15.69
16X43B5B-T43B-D3B+D5B<=16.31
16X44A5B+T44A+D4A-D5B<=15.58
16X44A5B-T44A-D4A+D5B<=16.42
16X44B5B+T44B+D4B-D5B<=15.58

16X44B5B-T44B-D4B+D5B<=16.42

16X515B+T51+D1-D5B<=13.87
16X515B-T51-D1+D5B<=18.13
16X525B+T52+D2-D5B<=13.96
16X525B-T52-D2+D5B<=18.04
16X53A5B+T53A+D3A-D5B<=15.69
16X53A5B-T53A-D3A+D5B<=16.31
16X53B5B+T53B+D3B-D5B<=15.69
16X53B5B-T53B-D3B+D5B<=16.31
16X54A5B+T54A+D4A-D5B<=15.58
16X54A5B-T54A-D4A+D5B<=16.42
16X54B5B+T54B+D4B-D5B<=15.58
16X54B5B-T54B-D4B+D5B<=16.42

16X615B+T61+D1-D5B<=13.87
16X615B-T61-D1+D5B<=18.13
16X625B+T62+D2-D5B<=13.96
16X625B-T62-D2+D5B<=18.04
16X63A5B+T63A+D3A-D5B<=15.69
16X63A5B-T63A-D3A+D5B<=16.31
16X63B5B+T63B+D3B-D5B<=15.69
16X63B5B-T63B-D3B+D5B<=16.31
16X64A5B+T64A+D4A-D5B<=15.58
16X64A5B-T64A-D4A+D5B<=16.42
16X64B5B+T64B+D4B-D5B<=15.58
16X64B5B-T64B-D4B+D5B<=16.42

Via Equation (29):

X113A+X113B+X114A+X114B+X115A+X115B+
X123A+X123B+X124A+X124B+X125A+X125B+
X213A+X213B+X214A+X214B+X215A+X215B+
X223A+X223B+X224A+X224B+X225A+X225B+
X313A+X313B+X314A+X314B+X315A+X315B+
X323A+X323B+X324A+X324B+X325A+X325B+
X413A+X413B+X414A+X414B+X415A+X415B+
X423A+X423B+X424A+X424B+X425A+X425B+
X513A+X513B+X514A+X514B+X515A+X515B+
X523A+X523B+X524A+X524B+X525A+X525B+
X613A+X613B+X614A+X614B+X615A+X615B+
X623A+X623B+X624A+X624B+X625A+X625B>=4

Via Equation (30):

X13A1+X13B1+X14A1+X14B1+X15A1+X15B1+
X13A2+X13B2+X14A2+X14B2+X15A2+X15B2+
X23A1+X23B1+X24A1+X24B1+X25A1+X25B1+
X23A2+X23B2+X24A2+X24B2+X25A2+X25B2+
X33A1+X33B1+X34A1+X34B1+X35A1+X35B1+
X33A2+X33B2+X34A2+X34B2+X35A2+X35B2+
X43A1+X43B1+X44A1+X44B1+X45A1+X45B1+
X43A2+X43B2+X44A2+X44B2+X45A2+X45B2+
X53A1+X53B1+X54A1+X54B1+X55A1+X55B1+
X53A2+X53B2+X54A2+X54B2+X55A2+X55B2+
X63A1+X63B1+X64A1+X64B1+X65A1+X65B1+
X63A2+X63B2+X64A2+X64B2+X65A2+X65B2>=4

Via Equation (31):

X113A+X123A+X14A3A+X14B3A+X15A3A+X15B3A-.44444444T13A<=0
X213A+X223A+X24A3A+X24B3A+X25A3A+X25B3A-.44444444T23A<=0
X313A+X323A+X34A3A+X34B3A+X35A3A+X35B3A-.44444444T33A<=0
X413A+X423A+X44A3A+X44B3A+X45A3A+X45B3A-.44444444T43A<=0
X513A+X523A+X54A3A+X54B3A+X55A3A+X55B3A-.44444444T53A<=0
X613A+X623A+X64A3A+X64B3A+X65A3A+X65B3A-.44444444T63A<=0

X113B+X123B+X14A3B+X14B3B+X15A3B+X15B3B-.44444444T13B<=0
X213B+X223B+X24A3B+X24B3B+X25A3B+X25B3B-.44444444T23B<=0
X313B+X323B+X34A3B+X34B3B+X35A3B+X35B3B-.44444444T33B<=0
X413B+X423B+X44A3B+X44B3B+X45A3B+X45B3B-.44444444T43B<=0
X513B+X523B+X54A3B+X54B3B+X55A3B+X55B3B-.44444444T53B<=0
X613B+X623B+X64A3B+X64B3B+X65A3B+X65B3B-.44444444T63B<=0

X114A+X124A+X13A4A+X13B4A+X15A4A+X15B4A-.44444444T14A<=0
X214A+X224A+X23A4A+X23B4A+X25A4A+X25B4A-.44444444T24A<=0
X314A+X324A+X33A4A+X33B4A+X35A4A+X35B4A-.44444444T34A<=0
X414A+X424A+X43A4A+X43B4A+X45A4A+X45B4A-.44444444T44A<=0
X514A+X524A+X53A4A+X53B4A+X55A4A+X55B4A-.44444444T54A<=0
X614A+X624A+X63A4A+X63B4A+X65A4A+X65B4A-.44444444T64A<=0

X114B+X124B+X13A4B+X13B4B+X15A4B+X15B4B-.44444444T14B<=0
X214B+X224B+X23A4B+X23B4B+X25A4B+X25B4B-.44444444T24B<=0
X314B+X324B+X33A4B+X33B4B+X35A4B+X35B4B-.44444444T34B<=0
X414B+X424B+X43A4B+X43B4B+X45A4B+X45B4B-.44444444T44B<=0
X514B+X524B+X53A4B+X53B4B+X55A4B+X55B4B-.44444444T54B<=0

X614B+X624B+X63A4B+X63B4B+X65A4B+X65B4B-.44444444T64B<=0

X115A+X125A+X13A5A+X13B5A+X14A5A+X14B5A-.44444444T15A<=0
X215A+X225A+X23A5A+X23B5A+X24A5A+X24B5A-.44444444T25A<=0
X315A+X325A+X33A5A+X33B5A+X34A5A+X34B5A-.44444444T35A<=0
X415A+X425A+X43A5A+X43B5A+X44A5A+X44B5A-.44444444T45A<=0
X515A+X525A+X53A5A+X53B5A+X54A5A+X54B5A-.44444444T55A<=0
X615A+X625A+X63A5A+X63B5A+X64A5A+X64B5A-.44444444T65A<=0

X115B+X125B+X13A5B+X13B5B+X14A5B+X14B5B-.44444444T15B<=0
X215B+X225B+X23A5B+X23B5B+X24A5B+X24B5B-.44444444T25B<=0
B315B+X325B+X33A5B+X33B5B+X34A5B+X34B5B-.44444444T35B<=0
X415B+X425B+X43A5B+X43B5B+X44A5B+X44B5B-.44444444T45B<=0
X515B+X525B+X53A5B+X53B5B+X54A5B+X54B5B-.44444444T55B<=0
X615B+X625B+X63A5B+X63B5B+X64A5B+X64B5B-.44444444T65B<=0

Via Equation (32):

X113A-X13A1+X123A-X13A2+X14A3A-X13A4A+X14B3A-X13A4B+X15A3A-X13A5A+X15B3A-X13A5B=0
X213A-X23A1+X223A-X23A2+X24A3A-X23A4A+X24B3A-X23A4B+X25A3A-X23A5A+X25B3A-X23A5B=0
X313A-X33A1+X323A-X33A2+X34A3A-X33A4A+X34B3A-X33A4B+X35A3A-X33A5A+X35B3A-X33A5B=0
X413A-X43A1+X423A-X43A2+X44A3A-X43A4A+X44B3A-X43A4B+X45A3A-X43A5A+X45B3A-X43A5B=0
X513A-X53A1+X523A-X53A2+X54A3A-X53A4A+X54B3A-X53A4B+X55A3A-X53A5A+X55B3A-X53A5B=0
X613A-X63A1+X623A-X63A2+X64A3A-X63A4A+X64B3A-X63A4B+X65A3A-X63A5A+X65B3A-X63A5B=0

X113B-X13B1+X123B-X13B2+X14A3B-X13B4A+X14B3B-X13B4B+X15A3B-X13B5A+X15B3B-X13B5B=0
X213B-X23B1+X223B-X23B2+X24A3B-X23B4A+X24B3B-X23B4B+X25A3B-X23B5A+X25B3B-X23B5B=0
X313B-X33B1+X323B-X33B2+X34A3B-X33B4A+X34B3B-X33B4B+X35A3B-X33B5A+X35B3B-X33B5B=0
X413B-X43B1+X423B-X43B2+X44A3B-X43B4A+X44B3B-X43B4B+X45A3B-X43B5A+X45B3B-X43B5B=0
X513B-X53B1+X523B-X53B2+X54A3B-X53B4A+X54B3B-X53B4B+X55A3B-X53B5A+X55B3B-X53B5B=0
X613B-X63B1+X623B-X63B2+X64A3B-X63B4A+X64B3B-X63B4B+X65A3B-X63B5A+X65B3B-X63B5B=0

X114A-X14A1+X124A-X14A2+X13A4A-X14A3A+X13B4A-X14A3B+X15A4A-X14A5A+X15B4A-X14A5B=0
X214A-X24A1+X224A-X24A2+X23A4A-X24A3A+X23B4A-X24A3B+X25A4A-X24A5A+X25B4A-X24A5B=0
X314A-X34A1+X324A-X34A2+X33A4A-X34A3A+X33B4A-X34A3B+X35A4A-X34A5A+X35B4A-X34A5B=0
X414A-X44A1+X424A-X44A2+X43A4A-X44A3A+X43B4A-X44A3B+X45A4A-X44A5A+X45B4A-X44A5B=0
X514A-X54A1+X524A-X54A2+X53A4A-X54A3A+X53B4A-X54A3B+X55A4A-X54A5A+X55B4A-X54A5B=0
X614A-X64A1+X624A-X64A2+X63A4A-X64A3A+X63B4A-X64A3B+X65A4A-X64A5A+X65B4A-X64A5B=0

X114B-X14B1+X124B-X14B2+X13A4B-X14B3A+X13B4B-X14B3B+X15A4B-X14B5A+X15B4B-X14B5B=0
X214B-X24B1+X224B-X24B2+X23A4B-X24B3A+X23B4B-X24B3B+X25A4B-X24B5A+X25B4B-X24B5B=0
X314B-X34B1+X324B-X34B2+X33A4B-X34B3A+X33B4B-X34B3B+X35A4B-X34B5A+X35B4B-X34B5B=0
X414B-X44B1+X424B-X44B2+X43A4B-X44B3A+X43B4B-X44B3B+X45A4B-X44B5A+X45B4B-X44B5B=0
X514B-X54B1+X524B-X54B2+X53A4B-X54B3A+X53B4B-X54B3B+X55A4B-X54B5A+X55B4B-X54B5B=0
X614B-X64B1+X624B-X64B2+X63A4B-X64B3A+X63B4B-X64B3B+X65A4B-X64B5A+X65B4B-X64B5B=0

X115A-X15A1+X125A-X15A2+X13A5A-X15A3A+X13B5A-X15A3B+X14A5A-X15A4A+X14B5A-X15A4B=0
X215A-X25A1+X225A-X25A2+X23A5A-X25A3A+X23B5A-X25A3B+X24A5A-X25A4A+X24B5A-X25A4B=0
X315A-X35A1+X325A-X35A2+X33A5A-X35A3A+X33B5A-X35A3B+X34A5A-X35A4A+X34B5A-X35A4B=0
X415A-X45A1+X425A-X45A2+X43A5A-X45A3A+X43B5A-X45A3B+X44A5A-X45A4A+X44B5A-X45A4B=0
X515A-X55A1+X525A-X55A2+X53A5A-X55A3A+X53B5A-X55A3B+X54A5A-X55A4A+X54B5A-X55A4B=0
X615A-X65A1+X625A-X65A2+X63A5A-X65A3A+X63B5A-X65A3B+X64A5A-X65A4A+X64B5A-X65A4B=0

X115B-X15B1+X125B-X15B2+X13A5B-X15B3A+X13B5B-X15B3B+X14A5B-X15B4A+X14B5B-X15B4B=0

X215B-X25B1+X225B-X25B2+X23A5B-X25B3A+X23B5B-X25B3B+X24A5B-X25B4A+X24B5B-X25B4B=0
X315B-X35B1+X325B-X35B2+X33A5B-X35B3A+X33B5B-X35B3B+X34A5B-X35B4A+X34B5B-X35B4B=0
X415B-X45B1+X425B-X45B2+X43A5B-X45B3A+X43B5B-X45B3B+X44A5B-X45B4A+X44B5B-X45B4B=0
X515B-X55B1+X525B-X55B2+X53A5B-X55B3A+X53B5B-X55B3B+X54A5B-X55B4A+X54B5B-X55B4B=0
X615B-X65B1+X625B-X65B2+X63A5B-X65B3A+X63B5B-X65B3B+X64A5B-X65B4A+X64B5B-X65B4B=0

Via Equation (33):

X113A+X123A+X14A3A+X14B3A+X15A3A+X15B3A+X213A+X223A+X24A3A+X24B3A+X25A3A+X25B3A+
X313A+X323A+X34A3A+X34B3A+X35A3A+X35B3A+X413A+X423A+X44A3A+X44B3A+X45A3A+X45B3A+
X513A+X523A+X54A3A+X54B3A+X55A3A+X55B3A+X613A+X623A+X64A3A+X64B3A+X65A3A+X65B3A=1

X113B+X123B+X14A3B+X14B3B+X15A3B+X15B3B+X213B+X223B+X24A3B+X24B3B+X25A3B+X25B3B+
X313B+X323B+X34A3B+X34B3B+X35A3B+X35B3B+X413B+X423B+X44A3B+X44B3B+X45A3B+X45B3B+
X513B+X523B+X54A3B+X54B3B+X55A3B+X55B3B+X613B+X623B+X64A3B+X64B3B+X65A3B+X65B3B=1

X114A+X124A+X13A4A+X13B4A+X15A4A+X15B4A+X214A+X224A+X23A4A+X23B4A+X25A4A+X25B4A+
X314A+X324A+X33A4A+X33B4A+X35A4A+X35B4A+X414A+X424A+X43A4A+X43B4A+X45A4A+X45B4A+
X514A+X524A+X53A4A+X53B4A+X55A4A+X55B4A+X614A+X624A+X63A4A+X63B4A+X65A4A+X65B4A=1

X114B+X124B+X13A4B+X13B4B+X15A4B+X15B4B+X214B+X224B+X23A4B+X23B4B+X25A4B+X25B4B+
X314B+X324B+X33A4B+X33B4B+X35A4B+X35B4B+X414B+X424B+X43A4B+X43B4B+X45A4B+X45B4B+
X514B+X524B+X53A4B+X53B4B+X55A4B+X55B4B+X614B+X624B+X63A4B+X63B4B+X65A4B+X65B4B=1

X115A+X125A+X13A5A+X13B5A+X14A5A+X14B5A+X215A+X225A+X23A5A+X23B5A+X24A5A+X24B5A+

X315A+X325A+X33A5A+X33B5A+X34A5A+X34B5A+X415A+X425A+X43A5A+X43B5A+X44A5A+X44B5A+
X515A+X525A+X53A5A+X53B5A+X54A5A+X54B5A+X615A+X625A+X63A5A+X63B5A+X64A5A+X64B5A=1

X115B+X125B+X13A5B+X13B5B+X14A5B+X14B5B+X215B+X225B+X23A5B+X23B5B+X24A5B+X24B5B+
X315B+X325B+X33A5B+X33B5B+X34A5B+X34B5B+X415B+X425B+X43A5B+X43B5B+X44A5B+X44B5B+
X515B+X525B+X53A5B+X53B5B+X54A5B+X54B5B+X615B+X625B+X63A5B+X63B5B+X64A5B+X64B5B=1

Via Equation (34):

57X113A61+29X113A71+0X113B61+20X113B71+17X114A61+32X114B61+
69X114A71+0X114B71+66X115A61+20X115A71+0X115B61+34X115B71+
57X123A61+29X123A71+20X123B71+17X124A61+32X124B61+69X124A71+
66X125A61+20X125A71+34X125B71+17X13A4A61+69X13A4A71+
32X13A4B61+66X13A5A61+20X13A5A71+34X13A5B71+17X13B4A61+
69X13B4A71+32X13B4B61+66X13B5A61+20X13B5A71+34X13B5B71+
57X14A3A61+29X14A3A71+20X14A3B71+66X14A5A61+20X14A5A71+
34X14A5B71+57X14B3A61+29X14B3A71+20X14B3B71+66X14B5A61+
20X14B5A71+34X14B5B71+57X15A3A61+29X15A3A71+20X15A3B71+
17X15A4A61+69X15A4A71+32X15A4B61+57X15B3A61+29X15B3A71+
20X15B3B71+17X15B4A61+69X15B4A71+32X15B4B61+
57X113A62+29X113A72+0X113B62+20X113B72+17X114A62+32X114B62+
69X114A72+0X114B72+66X115A62+20X115A72+0X115B62+34X115B72+
57X123A62+29X123A72+20X123B72+17X124A62+32X124B62+69X124A72+
66X125A62+20X125A72+34X125B72+17X13A4A62+69X13A4A72+
32X13A4B62+66X13A5A62+20X13A5A72+34X13A5B72+17X13B4A62+
69X13B4A72+32X13B4B62+66X13B5A62+20X13B5A72+34X13B5B72+
57X14A3A62+29X14A3A72+20X14A3B72+66X14A5A62+20X14A5A72+
34X14A5B72+57X14B3A62+29X14B3A72+20X14B3B72+66X14B5A62+
20X14B5A72+34X14B5B72+57X15A3A62+29X15A3A72+20X15A3B72+
17X15A4A62+69X15A4A72+32X15A4B62+57X15B3A62+29X15B3A72+
20X15B3B72+17X15B4A62+69X15B4A72+32X15B4B62<=86

57X213A61+29X213A71+0X213B61+20X213B71+17X214A61+32X214B61+
69X214A71+0X214B71+66X215A61+20X215A71+0X215B61+34X215B71+
57X223A61+29X223A71+20X223B71+17X224A61+32X224B61+69X224A71+
66X225A61+20X225A71+34X225B71+17X23A4A61+69X23A4A71+
32X23A4B61+66X23A5A61+20X23A5A71+34X23A5B71+17X23B4A61+
69X23B4A71+32X23B4B61+66X23B5A61+20X23B5A71+34X23B5B71+

57X24A3A61+29X24A3A71+20X24A3B71+66X24A5A61+20X24A5A71+
34X24A5B71+57X24B3A61+29X24B3A71+20X24B3B71+66X24B5A61+
20X24B5A71+34X24B5B71+57X25A3A61+29X25A3A71+20X25A3B71+
17X25A4A61+69X25A4A71+32X25A4B61+57X25B3A61+29X25B3A71+
20X25B3B71+17X25B4A61+69X25B4A71+32X25B4B61+
57X213A62+29X213A72+0X213B62+20X213B72+17X214A62+32X214B62+
69X214A72+0X214B72+66X215A62+20X215A72+0X215B62+34X215B72+
57X223A62+29X223A72+20X223B72+17X224A62+32X224B62+69X224A72+
66X225A62+20X225A72+34X225B72+17X23A4A62+69X23A4A72+
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34X24A5B72+57X24B3A62+29X24B3A72+20X24B3B72+66X24B5A62+
20X24B5A72+34X24B5B72+57X25A3A62+29X25A3A72+20X25A3B72+
17X25A4A62+69X25A4A72+32X25A4B62+57X25B3A62+29X25B3A72+
20X25B3B72+17X25B4A62+69X25B4A72+32X25B4B62<=86

57X313A61+29X313A71+0X313B61+20X313B71+17X314A61+32X314B61+
69X314A71+0X314B71+66X315A61+20X315A71+0X315B61+34X315B71+
57X323A61+29X323A71+20X323B71+17X324A61+32X324B61+69X324A71+
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17X35A4A61+69X35A4A71+32X35A4B61+57X35B3A61+29X35B3A71+
20X35B3B71+17X35B4A61+69X35B4A71+32X35B4B61+
57X313A62+29X313A72+0X313B62+20X313B72+17X314A62+32X314B62+
69X314A72+0X314B72+66X315A62+20X315A72+0X315B62+34X315B72+
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32X33A4B62+66X33A5A62+20X33A5A72+34X33A5B72+17X33B4A62+
69X33B4A72+32X33B4B62+66X33B5A62+20X33B5A72+34X33B5B72+
57X34A3A62+29X34A3A72+20X34A3B72+66X34A5A62+20X34A5A72+
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17X35A4A62+69X35A4A72+32X35A4B62+57X35B3A62+29X35B3A72+
20X35B3B72+17X35B4A62+69X35B4A72+32X35B4B62<=86

57X413A61+29X413A71+0X413B61+20X413B71+17X414A61+32X414B61+
69X414A71+0X414B71+66X415A61+20X415A71+0X415B61+34X415B71+
57X423A61+29X423A71+20X423B71+17X424A61+32X424B61+69X424A71+

66X425A61+20X425A71+34X425B71+17X43A4A61+69X43A4A71+
32X43A4B61+66X43A5A61+20X43A5A71+34X43A5B71+17X43B4A61+
69X43B4A71+32X43B4B61+66X43B5A61+20X43B5A71+34X43B5B71+
57X44A3A61+29X44A3A71+20X44A3B71+66X44A5A61+20X44A5A71+
34X44A5B71+57X44B3A61+29X44B3A71+20X44B3B71+66X44B5A61+
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17X45A4A61+69X45A4A71+32X45A4B61+57X45B3A61+29X45B3A71+
20X45B3B71+17X45B4A61+69X45B4A71+32X45B4B61+
57X413A62+29X413A72+0X413B62+20X413B72+17X414A62+32X414B62+
69X414A72+0X414B72+66X415A62+20X415A72+0X415B62+34X415B72+
57X423A62+29X423A72+20X423B72+17X424A62+32X424B62+69X424A72+
66X425A62+20X425A72+34X425B72+17X43A4A62+69X43A4A72+
32X43A4B62+66X43A5A62+20X43A5A72+34X43A5B72+17X43B4A62+
69X43B4A72+32X43B4B62+66X43B5A62+20X43B5A72+34X43B5B72+
57X44A3A62+29X44A3A72+20X44A3B72+66X44A5A62+20X44A5A72+
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20X44B5A72+34X44B5B72+57X45A3A62+29X45A3A72+20X45A3B72+
17X45A4A62+69X45A4A72+32X45A4B62+57X45B3A62+29X45B3A72+
20X45B3B72+17X45B4A62+69X45B4A72+32X45B4B62<=86

57X513A61+29X513A71+0X513B61+20X513B71+17X514A61+32X514B61+
69X514A71+0X514B71+66X515A61+20X515A71+0X515B61+34X515B71+
57X523A61+29X523A71+20X523B71+17X524A61+32X524B61+69X524A71+
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17X55A4A62+69X55A4A72+32X55A4B62+57X55B3A62+29X55B3A72+
20X55B3B72+17X55B4A62+69X55B4A72+32X55B4B62<=86

57X613A61+29X613A71+0X613B61+20X613B71+17X614A61+32X614B61+
69X614A71+0X614B71+66X615A61+20X615A71+0X615B61+34X615B71+
57X623A61+29X623A71+20X623B71+17X624A61+32X624B61+69X624A71+
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34X64A5B71+57X64B3A61+29X64B3A71+20X64B3B71+66X64B5A61+
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17X65A4A61+69X65A4A71+32X65A4B61+57X65B3A61+29X65B3A71+
20X65B3B71+17X65B4A61+69X65B4A71+32X65B4B61+
57X613A62+29X613A72+0X613B62+20X613B72+17X614A62+32X614B62+
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20X64B5A72+34X64B5B72+57X65A3A62+29X65A3A72+20X65A3B72+
17X65A4A62+69X65A4A72+32X65A4B62+57X65B3A62+29X65B3A72+
20X65B3B72+17X65B4A62+69X65B4A72+32X65B4B62<=86

Via Equation (35):

X13A1+X13B1+X14A1+X14B1+X15A1+X15B1-X113A-X113B-X114A-X114B-
X115A-X115B=0
X23A1+X23B1+X24A1+X24B1+X25A1+X25B1-X213A-X213B-X214A-X214B-
X215A-X215B=0
X33A1+X33B1+X34A1+X34B1+X35A1+X35B1-X313A-X313B-X314A-X314B-
X315A-X315B=0
X43A1+X43B1+X44A1+X44B1+X45A1+X45B1-X413A-X413B-X414A-X414B-
X415A-X415B=0
X53A1+X53B1+X54A1+X54B1+X55A1+X55B1-X513A-X513B-X514A-X514B-
X515A-X515B=0
X63A1+X63B1+X64A1+X64B1+X65A1+X65B1-X613A-X613B-X614A-X614B-
X615A-X615B=0

X13A2+X13B2+X14A2+X14B2+X15A2+X15B2-X123A-X123B-X124A-X124B-
X125A-X125B=0
X23A2+X23B2+X24A2+X24B2+X25A2+X25B2-X223A-X223B-X224A-X224B-
X225A-X225B=0

X33A2+X33B2+X34A2+X34B2+X35A2+X35B2-X323A-X323B-X324A-X324B-X325A-X325B=0
X43A2+X43B2+X44A2+X44B2+X45A2+X45B2-X423A-X423B-X424A-X424B-X425A-X425B=0
X53A2+X53B2+X54A2+X54B2+X55A2+X55B2-X523A-X523B-X524A-X524B-X525A-X525B=0
X63A2+X63B2+X64A2+X64B2+X65A2+X65B2-X623A-X623B-X624A-X624B-X625A-X625B=0

Via Equation (36):

X113A61+X213A61+X313A61+X413A61+X513A61+X613A61+
X123A61+X223A61+X323A61+X423A61+X523A61+X623A61+
X14A3A61+X24A3A61+X34A3A61+X44A3A61+X54A3A61+X64A3A61+
X14B3A61+X24B3A61+X34B3A61+X44B3A61+X54B3A61+X64B3A61+
X15A3A61+X25A3A61+X35A3A61+X45A3A61+X55A3A61+X65A3A61+
X15B3A61+X25B3A61+X35B3A61+X45B3A61+X55B3A61+X65B3A61+
X113A62+X213A62+X313A62+X413A62+X513A62+X613A62+
X123A62+X223A62+X323A62+X423A62+X523A62+X623A62+
X14A3A62+X24A3A62+X34A3A62+X44A3A62+X54A3A62+X64A3A62+
X14B3A62+X24B3A62+X34B3A62+X44B3A62+X54B3A62+X64B3A62+
X15A3A62+X25A3A62+X35A3A62+X45A3A62+X55A3A62+X65A3A62+
X15B3A62+X25B3A62+X35B3A62+X45B3A62+X55B3A62+X65B3A62=1

X113A71+X213A71+X313A71+X413A71+X513A71+X613A71+
X123A71+X223A71+X323A71+X423A71+X523A71+X623A71+
X14A3A71+X24A3A71+X34A3A71+X44A3A71+X54A3A71+X64A3A71+
X14B3A71+X24B3A71+X34B3A71+X44B3A71+X54B3A71+X64B3A71+
X15A3A71+X25A3A71+X35A3A71+X45A3A71+X55A3A71+X65A3A71+
X15B3A71+X25B3A71+X35B3A71+X45B3A71+X55B3A71+X65B3A71+
X113A72+X213A72+X313A72+X413A72+X513A72+X613A72+
X123A72+X223A72+X323A72+X423A72+X523A72+X623A72+
X14A3A72+X24A3A72+X34A3A72+X44A3A72+X54A3A72+X64A3A72+
X14B3A72+X24B3A72+X34B3A72+X44B3A72+X54B3A72+X64B3A72+
X15A3A72+X25A3A72+X35A3A72+X45A3A72+X55A3A72+X65A3A72+
X15B3A72+X25B3A72+X35B3A72+X45B3A72+X55B3A72+X65B3A72=1

X113B61+X213B61+X313B61+X413B61+X513B61+X613B61+
X123B61+X223B61+X323B61+X423B61+X523B61+X623B61+
X14A3B61+X24A3B61+X34A3B61+X44A3B61+X54A3B61+X64A3B61+
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X15A3B61+X25A3B61+X35A3B61+X45A3B61+X55A3B61+X65A3B61+
X15B3B61+X25B3B61+X35B3B61+X45B3B61+X55B3B61+X65B3B61+

X113B62+X213B62+X313B62+X413B62+X513B62+X613B62+
X123B62+X223B62+X323B62+X423B62+X523B62+X623B62+
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X15A3B62+X25A3B62+X35A3B62+X45A3B62+X55A3B62+X65A3B62+
X15B3B62+X25B3B62+X35B3B62+X45B3B62+X55B3B62+X65B3B62=1

X113B71+X213B71+X313B71+X413B71+X513B71+X613B71+
X123B71+X223B71+X323B71+X423B71+X523B71+X623B71+
X14A3B71+X24A3B71+X34A3B71+X44A3B71+X54A3B71+X64A3B71+
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X15A3B72+X25A3B72+X35A3B72+X45A3B72+X55A3B72+X65A3B72+
X15B3B72+X25B3B72+X35B3B72+X45B3B72+X55B3B72+X65B3B72=1

X114A61+X214A61+X314A61+X414A61+X514A61+X614A61+
X124A61+X224A61+X324A61+X424A61+X524A61+X624A61+
X13A4A61+X23A4A61+X33A4A61+X43A4A61+X53A4A61+X63A4A61+
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X15A4A61+X25A4A61+X35A4A61+X45A4A61+X55A4A61+X65A4A61+
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X15A4A62+X25A4A62+X35A4A62+X45A4A62+X55A4A62+X65A4A62+
X15B4A62+X25B4A62+X35B4A62+X45B4A62+X55B4A62+X65B4A62=1

X114A71+X214A71+X314A71+X414A71+X514A71+X614A71+
X124A71+X224A71+X324A71+X424A71+X524A71+X624A71+
X13A4A71+X23A4A71+X33A4A71+X43A4A71+X53A4A71+X63A4A71+
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X15A4A72+X25A4A72+X35A4A72+X45A4A72+X55A4A72+X65A4A72+
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Via Equation (37):

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2+X54B2+X55A2+X55B2-X5B2<=1
X65B1+X65B2+X65B3A+X65B3B+X65B4A+X65B4B+X612+X63A2+X63B2+X64A
2+X64B2+X65A2+X65B2-X5B2<=1

Via Equation (38):

X113A61+X113A71+X113A62+X113A72-X113A>=0
X113B61+X113B71+X113B62+X113B72-X113B>=0
X114A61+X114A71+X114A62+X114A72-X114A>=0
X114B61+X114B71+X114B62+X114B72-X114B>=0
X115A61+X115A71+X115A62+X115A72-X115A>=0
X115B61+X115B71+X115B62+X115B72-X115B>=0
X123A61+X123A71+X123A62+X123A72-X123A>=0
X123B61+X123B71+X123B62+X123B72-X123B>=0
X124A61+X124A71+X124A62+X124A72-X124A>=0
X124B61+X124B71+X124B62+X124B72-X124B>=0
X125A61+X125A71+X125A62+X125A72-X125A>=0
X125B61+X125B71+X125B62+X125B72-X125B>=0
X13A161+X13A171+X13A162+X13A172-X13A1>=0
X13A261+X13A271+X13A262+X13A272-X13A2>=0
X13A4A61+X13A4A71+X13A4A62+X13A4A72-X13A4A>=0

X13A4B61+X13A4B71+X13A4B62+X13A4B72-X13A4B>=0
X13A5A61+X13A5A71+X13A5A62+X13A5A72-X13A5A>=0
X13A5B61+X13A5B71+X13A5B62+X13A5B72-X13A5B>=0
X13B161+X13B171+X13B162+X13B172-X13B1>=0
X13B261+X13B271+X13B262+X13B272-X13B2>=0
X13B4A61+X13B4A71+X13B4A62+X13B4A72-X13B4A>=0
X13B4B61+X13B4B71+X13B4B62+X13B4B72-X13B4B>=0
X13B5A61+X13B5A71+X13B5A62+X13B5A72-X13B5A>=0
X13B5B61+X13B5B71+X13B5B62+X13B5B72-X13B5B>=0
X14A161+X14A171+X14A162+X14A172-X14A1>=0
X14A261+X14A271+X14A262+X14A272-X14A2>=0
X14A3A61+X14A3A71+X14A3A62+X14A3A72-X14A3A>=0
X14A3B61+X14A3B71+X14A3B62+X14A3B72-X14A3B>=0
X14A5A61+X14A5A71+X14A5A62+X14A5A72-X14A5A>=0
X14A5B61+X14A5B71+X14A5B62+X14A5B72-X14A5B>=0
X14B161+X14B171+X14B162+X14B172-X14B1>=0
X14B261+X14B271+X14B262+X14B272-X14B2>=0
X14B3A61+X14B3A71+X14B3A62+X14B3A72-X14B3A>=0
X14B3B61+X14B3B71+X14B3B62+X14B3B72-X14B3B>=0
X14B5A61+X14B5A71+X14B5A62+X14B5A72-X14B5A>=0
X14B5B61+X14B5B71+X14B5B62+X14B5B72-X14B5B>=0
X15A161+X15A171+X15A162+X15A172-X15A1>=0
X15A261+X15A271+X15A262+X15A272-X15A2>=0
X15A3A61+X15A3A71+X15A3A62+X15A3A72-X15A3A>=0
X15A3B61+X15A3B71+X15A3B62+X15A3B72-X15A3B>=0
X15A4A61+X15A4A71+X15A4A62+X15A4A72-X15A4A>=0
X15A4B61+X15A4B71+X15A4B62+X15A4B72-X15A4B>=0
X15B161+X15B171+X15B162+X15B172-X15B1>=0
X15B261+X15B271+X15B262+X15B272-X15B2>=0
X15B3A61+X15B3A71+X15B3A62+X15B3A72-X15B3A>=0
X15B3B61+X15B3B71+X15B3B62+X15B3B72-X15B3B>=0
X15B4A61+X15B4A71+X15B4A62+X15B4A72-X15B4A>=0
X15B4B61+X15B4B71+X15B4B62+X15B4B72-X15B4B>=0

X213A61+X213A71+X213A62+X213A72-X213A>=0
X213B61+X213B71+X213B62+X213B72-X213B>=0
X214A61+X214A71+X214A62+X214A72-X214A>=0
X214B61+X214B71+X214B62+X214B72-X214B>=0
X215A61+X215A71+X215A62+X215A72-X215A>=0
X215B61+X215B71+X215B62+X215B72-X215B>=0
X223A61+X223A71+X223A62+X223A72-X223A>=0
X223B61+X223B71+X223B62+X223B72-X223B>=0
X224A61+X224A71+X224A62+X224A72-X224A>=0

X224B61+X224B71+X224B62+X224B72-X224B>=0
X225A61+X225A71+X225A62+X225A72-X225A>=0
X225B61+X225B71+X225B62+X225B72-X225B>=0
X23A161+X23A171+X23A162+X23A172-X23A1>=0
X23A261+X23A271+X23A262+X23A272-X23A2>=0
X23A4A61+X23A4A71+X23A4A62+X23A4A72-X23A4A>=0
X23A4B61+X23A4B71+X23A4B62+X23A4B72-X23A4B>=0
X23A5A61+X23A5A71+X23A5A62+X23A5A72-X23A5A>=0
X23A5B61+X23A5B71+X23A5B62+X23A5B72-X23A5B>=0
X23B161+X23B171+X23B162+X23B172-X23B1>=0
X23B261+X23B271+X23B262+X23B272-X23B2>=0
X23B4A61+X23B4A71+X23B4A62+X23B4A72-X23B4A>=0
X23B4B61+X23B4B71+X23B4B62+X23B4B72-X23B4B>=0
X23B5A61+X23B5A71+X23B5A62+X23B5A72-X23B5A>=0
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X24A161+X24A171+X24A162+X24A172-X24A1>=0
X24A261+X24A271+X24A262+X24A272-X24A2>=0
X24A3A61+X24A3A71+X24A3A62+X24A3A72-X24A3A>=0
X24A3B61+X24A3B71+X24A3B62+X24A3B72-X24A3B>=0
X24A5A61+X24A5A71+X24A5A62+X24A5A72-X24A5A>=0
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X24B161+X24B171+X24B162+X24B172-X24B1>=0
X24B261+X24B271+X24B262+X24B272-X24B2>=0
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X24B5A61+X24B5A71+X24B5A62+X24B5A72-X24B5A>=0
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X25A161+X25A171+X25A162+X25A172-X25A1>=0
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X25A3A61+X25A3A71+X25A3A62+X25A3A72-X25A3A>=0
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X25A4A61+X25A4A71+X25A4A62+X25A4A72-X25A4A>=0
X25A4B61+X25A4B71+X25A4B62+X25A4B72-X25A4B>=0
X25B161+X25B171+X25B162+X25B172-X25B1>=0
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X25B4A61+X25B4A71+X25B4A62+X25B4A72-X25B4A>=0
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X313A61+X313A71+X313A62+X313A72-X313A>=0
X313B61+X313B71+X313B62+X313B72-X313B>=0
X314A61+X314A71+X314A62+X314A72-X314A>=0

X314B61+X314B71+X314B62+X314B72-X314B>=0
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X315B61+X315B71+X315B62+X315B72-X315B>=0
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X54A5A61+X54A5A71+X54A5A62+X54A5A72-X54A5A>=0
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X54B3A61+X54B3A71+X54B3A62+X54B3A72-X54B3A>=0
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X54B5A61+X54B5A71+X54B5A62+X54B5A72-X54B5A>=0
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X55B261+X55B271+X55B262+X55B272-X55B2>=0
X55B3A61+X55B3A71+X55B3A62+X55B3A72-X55B3A>=0
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X55B4A61+X55B4A71+X55B4A62+X55B4A72-X55B4A>=0
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X624A61+X624A71+X624A62+X624A72-X624A>=0
X624B61+X624B71+X624B62+X624B72-X624B>=0
X625A61+X625A71+X625A62+X625A72-X625A>=0
X625B61+X625B71+X625B62+X625B72-X625B>=0
X63A161+X63A171+X63A162+X63A172-X63A1>=0
X63A261+X63A271+X63A262+X63A272-X63A2>=0
X63A4A61+X63A4A71+X63A4A62+X63A4A72-X63A4A>=0
X63A4B61+X63A4B71+X63A4B62+X63A4B72-X63A4B>=0
X63A5A61+X63A5A71+X63A5A62+X63A5A72-X63A5A>=0
X63A5B61+X63A5B71+X63A5B62+X63A5B72-X63A5B>=0
X63B161+X63B171+X63B162+X63B172-X63B1>=0
X63B261+X63B271+X63B262+X63B272-X63B2>=0
X63B4A61+X63B4A71+X63B4A62+X63B4A72-X63B4A>=0
X63B4B61+X63B4B71+X63B4B62+X63B4B72-X63B4B>=0
X63B5A61+X63B5A71+X63B5A62+X63B5A72-X63B5A>=0
X63B5B61+X63B5B71+X63B5B62+X63B5B72-X63B5B>=0
X64A161+X64A171+X64A162+X64A172-X64A1>=0
X64A261+X64A271+X64A262+X64A272-X64A2>=0
X64A3A61+X64A3A71+X64A3A62+X64A3A72-X64A3A>=0
X64A3B61+X64A3B71+X64A3B62+X64A3B72-X64A3B>=0
X64A5A61+X64A5A71+X64A5A62+X64A5A72-X64A5A>=0

X64A5B61+X64A5B71+X64A5B62+X64A5B72-X64A5B>=0
X64B161+X64B171+X64B162+X64B172-X64B1>=0
X64B261+X64B271+X64B262+X64B272-X64B2>=0
X64B3A61+X64B3A71+X64B3A62+X64B3A72-X64B3A>=0
X64B3B61+X64B3B71+X64B3B62+X64B3B72-X64B3B>=0
X64B5A61+X64B5A71+X64B5A62+X64B5A72-X64B5A>=0
X64B5B61+X64B5B71+X64B5B62+X64B5B72-X64B5B>=0
X65A161+X65A171+X65A162+X65A172-X65A1>=0
X65A261+X65A271+X65A262+X65A272-X65A2>=0
X65A3A61+X65A3A71+X65A3A62+X65A3A72-X65A3A>=0
X65A3B61+X65A3B71+X65A3B62+X65A3B72-X65A3B>=0
X65A4A61+X65A4A71+X65A4A62+X65A4A72-X65A4A>=0
X65A4B61+X65A4B71+X65A4B62+X65A4B72-X65A4B>=0
X65B161+X65B171+X65B162+X65B172-X65B1>=0
X65B261+X65B271+X65B262+X65B272-X65B2>=0
X65B3A61+X65B3A71+X65B3A62+X65B3A72-X65B3A>=0
X65B3B61+X65B3B71+X65B3B62+X65B3B72-X65B3B>=0
X65B4A61+X65B4A71+X65B4A62+X65B4A72-X65B4A>=0
X65B4B61+X65B4B71+X65B4B62+X65B4B72-X65B4B>=0

Via Equation (39):

X113A-X113A61>=0
X113B-X113B61>=0
X114A-X114A61>=0
X114B-X114B61>=0
X115A-X115A61>=0
X115B-X115B61>=0
X123A-X123A61>=0
X123B-X123B61>=0
X124A-X124A61>=0
X124B-X124B61>=0
X125A-X125A61>=0
X125B-X125B61>=0
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X13B4A-X13B4A61>=0

X13B4B-X13B4B61>=0
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X13B5B-X13B5B61>=0
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X15B3B-X15B3B61>=0
X15B4A-X15B4A61>=0
X15B4B-X15B4B61>=0

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X114B-X114B62>=0
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X115B-X115B62>=0
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X124A-X124A62>=0
X124B-X124B62>=0
X125A-X125A62>=0
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X15B3B-X15B3B62>=0
X15B4A-X15B4A62>=0
X15B4B-X15B4B62>=0

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X214B-X214B61>=0
X215A-X215A61>=0
X215B-X215B61>=0
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X223B-X223B61>=0
X224A-X224A61>=0

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X215B-X215B62>=0
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X223B-X223B62>=0
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X224B-X224B62>=0
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X55B3A-X55B3A72>=0
X55B3B-X55B3B72>=0
X55B4A-X55B4A72>=0
X55B4B-X55B4B72>=0

X613A-X613A71>=0
X613B-X613B71>=0
X614A-X614A71>=0
X614B-X614B71>=0
X615A-X615A71>=0
X615B-X615B71>=0
X623A-X623A71>=0
X623B-X623B71>=0
X624A-X624A71>=0
X624B-X624B71>=0
X625A-X625A71>=0
X625B-X625B71>=0
X63A1-X63A171>=0
X63A2-X63A271>=0
X63A4A-X63A4A71>=0
X63A4B-X63A4B71>=0
X63A5A-X63A5A71>=0

X63A5B-X63A5B71>=0
X63B1-X63B171>=0
X63B2-X63B271>=0
X63B4A-X63B4A71>=0
X63B4B-X63B4B71>=0
X63B5A-X63B5A71>=0
X63B5B-X63B5B71>=0
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X64A3B-X64A3B71>=0
X64A5A-X64A5A71>=0
X64A5B-X64A5B71>=0
X64B1-X64B171>=0
X64B2-X64B271>=0
X64B3A-X64B3A71>=0
X64B3B-X64B3B71>=0
X64B5A-X64B5A71>=0
X64B5B-X64B5B71>=0
X65A1-X65A171>=0
X65A2-X65A271>=0
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X65A3B-X65A3B71>=0
X65A4A-X65A4A71>=0
X65A4B-X65A4B71>=0
X65B1-X65B171>=0
X65B2-X65B271>=0
X65B3A-X65B3A71>=0
X65B3B-X65B3B71>=0
X65B4A-X65B4A71>=0
X65B4B-X65B4B71>=0

X613A-X613A72>=0
X613B-X613B72>=0
X614A-X614A72>=0
X614B-X614B72>=0
X615A-X615A72>=0
X615B-X615B72>=0
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X623B-X623B72>=0
X624A-X624A72>=0
X624B-X624B72>=0
X625A-X625A72>=0

X625B-X625B72>=0
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X63A2-X63A272>=0
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X63A4B-X63A4B72>=0
X63A5A-X63A5A72>=0
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Via Equation (40):

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X13A171+X123A71-X13A271+X14A3A71-X13A4A71+X14B3A71-
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X33A171+X323A71-X33A271+X34A3A71-X33A4A71+X34B3A71-
X33A4B71+X35A3A71-X33A5A71+X35B3A71-X33A5B71=0
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X43A171+X423A71-X43A271+X44A3A71-X43A4A71+X44B3A71-
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X63A171+X623A71-X63A271+X64A3A71-X63A4A71+X64B3A71-
X63A4B71+X65A3A71-X63A5A71+X65B3A71-X63A5B71=0

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X13B4B61+X15A3B61-
X13B5A61+X15B3B61-X13B5B61+ X113B71-X13B171+X123B71-
X13B271+X14A3B71-X13B4A71+X14B3B71-X13B4B71+X15A3B71-
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X43B4B71+X45A3B71-X43B5A71+X45B3B71-X43B5B71=0
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X13B4B62+X15A3B62-
X13B5A62+X15B3B62-X13B5B62+ X113B72-X13B172+X123B72-
X13B272+X14A3B72-X13B4A72+X14B3B72-X13B4B72+X15A3B72-
X13B5A72+X15B3B72-X13B5B72=0
X213B62-X23B162+X223B62-X23B262+X24A3B62-X23B4A62+X24B3B62-
X23B4B62+X25A3B62-X23B5A62+X25B3B62-X23B5B62+ X213B72-
X23B172+X223B72-X23B272+X24A3B72-X23B4A72+X24B3B72-
X23B4B72+X25A3B72-X23B5A72+X25B3B72-X23B5B72=0
X313B62-X33B162+X323B62-X33B262+X34A3B62-X33B4A62+X34B3B62-
X33B4B62+X35A3B62-X33B5A62+X35B3B62-X33B5B62+X313B72-
X33B172+X323B72-X33B272+X34A3B72-X33B4A72+X34B3B72-
X33B4B72+X35A3B72-X33B5A72+X35B3B72-X33B5B72=0
X413B62-X43B162+X423B62-X43B262+X44A3B62-X43B4A62+X44B3B62-
X43B4B62+X45A3B62-X43B5A62+X45B3B62-X43B5B62+X413B72-
X43B172+X423B72-X43B272+X44A3B72-X43B4A72+X44B3B72-
X43B4B72+X45A3B72-X43B5A72+X45B3B72-X43B5B72=0
X513B62-X53B162+X523B62-X53B262+X54A3B62-X53B4A62+X54B3B62-
X53B4B62+X55A3B62-
X53B5A62+X55B3B62-X53B5B62+ X513B72-X53B172+X523B72-
X53B272+X54A3B72-X53B4A72+X54B3B72-X53B4B72+X55A3B72-
X53B5A72+X55B3B72-X53B5B72=0
X613B62-X63B162+X623B62-X63B262+X64A3B62-X63B4A62+X64B3B62-
X63B4B62+X65A3B62-
X63B5A62+X65B3B62-X63B5B62+ X613B72-X63B172+X623B72-
X63B272+X64A3B72-X63B4A72+X64B3B72-X63B4B72+X65A3B72-
X63B5A72+X65B3B72-X63B5B72=0

X114A62-X14A162+X124A62-X14A262+X13A4A62-X14A3A62+X13B4A62-
X14A3B62+X15A4A62-X14A5A62+X15B4A62-X14A5B62+X114A72-
X14A172+X124A72-X14A272+X13A4A72-X14A3A72+X13B4A72-
X14A3B72+X15A4A72-X14A5A72+X15B4A72-X14A5B72=0
X214A62-X24A162+X224A62-X24A262+X23A4A62-X24A3A62+X23B4A62-
X24A3B62+X25A4A62-X24A5A62+X25B4A62-X24A5B62+X214A72-
X24A172+X224A72-X24A272+X23A4A72-X24A3A72+X23B4A72-
X24A3B72+X25A4A72-X24A5A72+X25B4A72-X24A5B72=0

X314A62-X34A162+X324A62-X34A262+X33A4A62-X34A3A62+X33B4A62-
X34A3B62+X35A4A62-X34A5A62+X35B4A62-X34A5B62+X314A72-
X34A172+X324A72-X34A272+X33A4A72-X34A3A72+X33B4A72-
X34A3B72+X35A4A72-X34A5A72+X35B4A72-X34A5B72=0
X414A62-X44A162+X424A62-X44A262+X43A4A62-X44A3A62+X43B4A62-
X44A3B62+X45A4A62-X44A5A62+X45B4A62-X44A5B62+X414A72-
X44A172+X424A72-X44A272+X43A4A72-X44A3A72+X43B4A72-
X44A3B72+X45A4A72-X44A5A72+X45B4A72-X44A5B72=0
X514A62-X54A162+X524A62-X54A262+X53A4A62-X54A3A62+X53B4A62-
X54A3B62+X55A4A62-X54A5A62+X55B4A62-X54A5B62+X514A72-
X54A172+X524A72-X54A272+X53A4A72-X54A3A72+X53B4A72-
X54A3B72+X55A4A72-X54A5A72+X55B4A72-X54A5B72=0
X614A62-X64A162+X624A62-X64A262+X63A4A62-X64A3A62+X63B4A62-
X64A3B62+X65A4A62-X64A5A62+X65B4A62-X64A5B62+X614A72-
X64A172+X624A72-X64A272+X63A4A72-X64A3A72+X63B4A72-
X64A3B72+X65A4A72-X64A5A72+X65B4A72-X64A5B72=0

X114B62-X14B162+X124B62-X14B262+X13A4B62-X14B3A62+X13B4B62-
X14B3B62+X15A4B62-X14B5A62+X15B4B62-X14B5B62+X114B72-
X14B172+X124B72-X14B272+X13A4B72-X14B3A72+X13B4B72-
X14B3B72+X15A4B72-X14B5A72+X15B4B72-X14B5B72=0
X214B62-X24B162+X224B62-X24B262+X23A4B62-X24B3A62+X23B4B62-
X24B3B62+X25A4B62-X24B5A62+X25B4B62-X24B5B62+X214B72-
X24B172+X224B72-X24B272+X23A4B72-X24B3A72+X23B4B72-
X24B3B72+X25A4B72-X24B5A72+X25B4B72-X24B5B72=0
X314B62-X34B162+X324B62-X34B262+X33A4B62-X34B3A62+X33B4B62-
X34B3B62+X35A4B62-X34B5A62+X35B4B62-X34B5B62+X314B72-
X34B172+X324B72-X34B272+X33A4B72-X34B3A72+X33B4B72-
X34B3B72+X35A4B72-X34B5A72+X35B4B72-X34B5B72=0
X414B62-X44B162+X424B62-X44B262+X43A4B62-X44B3A62+X43B4B62-
X44B3B62+X45A4B62-X44B5A62+X45B4B62-X44B5B62+X414B72-
X44B172+X424B72-X44B272+X43A4B72-X44B3A72+X43B4B72-
X44B3B72+X45A4B72-X44B5A72+X45B4B72-X44B5B72=0
X514B62-X54B162+X524B62-X54B262+X53A4B62-X54B3A62+X53B4B62-
X54B3B62+X55A4B62-X54B5A62+X55B4B62-X54B5B62+X514B72-
X54B172+X524B72-X54B272+X53A4B72-X54B3A72+X53B4B72-
X54B3B72+X55A4B72-X54B5A72+X55B4B72-X54B5B72=0
X614B62-X64B162+X624B62-X64B262+X63A4B62-X64B3A62+X63B4B62-
X64B3B62+X65A4B62-X64B5A62+X65B4B62-X64B5B62+X614B72-
X64B172+X624B72-X64B272+X63A4B72-X64B3A72+X63B4B72-
X64B3B72+X65A4B72-X64B5A72+X65B4B72-X64B5B72=0

X115A62-X15A162+X125A62-X15A262+X13A5A62-X15A3A62+X13B5A62-X15A3B62+X14A5A62-X15A4A62+X14B5A62-X15A4B62+X115A72-X15A172+X125A72-X15A272+X13A5A72-X15A3A72+X13B5A72-X15A3B72+X14A5A72-X15A4A72+X14B5A72-X15A4B72=0
X215A62-X25A162+X225A62-X25A262+X23A5A62-X25A3A62+X23B5A62-X25A3B62+X24A5A62-X25A4A62+X24B5A62-X25A4B62+X215A72-X25A172+X225A72-X25A272+X23A5A72-X25A3A72+X23B5A72-X25A3B72+X24A5A72-X25A4A72+X24B5A72-X25A4B72=0
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X615A62-X65A162+X625A62-X65A262+X63A5A62-X65A3A62+X63B5A62-X65A3B62+X64A5A62-X65A4A62+X64B5A62-X65A4B62+X615A72-X65A172+X625A72-X65A272+X63A5A72-X65A3A72+X63B5A72-X65A3B72+X64A5A72-X65A4A72+X64B5A72-X65A4B72=0

X115A62-X15A162+X125A62-X15A262+X13A5A62-X15A3A62+X13B5A62-X15A3B62+X14A5A62-X15A4A62+X14B5A62-X15A4B62+X115A72-X15A172+X125A72-X15A272+X13A5A72-X15A3A72+X13B5A72-X15A3B72+X14A5A72-X15A4A72+X14B5A72-X15A4B72=0
X215B62-X25B162+X225B62-X25B262+X23A5B62-X25B3A62+X23B5B62-X25B3B62+X24A5B62-X25B4A62+X24B5B62-X25B4B62+X215B72-X25B172+X225B72-X25B272+X23A5B72-X25B3A72+X23B5B72-X25B3B72+X24A5B72-X25B4A72+X24B5B72-X25B4B72=0
X315B62-X35B162+X325B62-X35B262+X33A5B62-X35B3A62+X33B5B62-X35B3B62+X34A5B62-X35B4A62+X34B5B62-X35B4B62+X315B72-X35B172+X325B72-X35B272+X33A5B72-X35B3A72+X33B5B72-X35B3B72+X34A5B72-X35B4A72+X34B5B72-X35B4B72=0
X415B62-X45B162+X425B62-X45B262+X43A5B62-X45B3A62+X43B5B62-X45B3B62+X44A5B62-X45B4A62+X44B5B62-X45B4B62+X415B72-X45B172+X425B72-X45B272+X43A5B72-X45B3A72+X43B5B72-X45B3B72+X44A5B72-X45B4A72+X44B5B72-X45B4B72=0
X515A62-X55A162+X525A62-X55A262+X53A5A62-X55A3A62+X53B5A62-X55A3B62+X54A5A62-X55A4A62+X54B5A62-X55A4B62+X515A72-

X55A172+X525A72-X55A272+X53A5A72-X55A3A72+X53B5A72-
X55A3B72+X54A5A72-X55A4A72+X54B5A72-X55A4B72=0
X615A62-X65A162+X625A62-X65A262+X63A5A62-X65A3A62+X63B5A62-
X65A3B62+X64A5A62-X65A4A62+X64B5A62-X65A4B62+X615A72-
X65A172+X625A72-X65A272+X63A5A72-X65A3A72+X63B5A72-
X65A3B72+X64A5A72-X65A4A72+X64B5A72-X65A4B72=0

Via Equation (41):

X123A61=0
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X125B61=0
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X13B261=0
X14A261=0
X14B261=0
X15A261=0
X15B261=0

X223A61=0
X223B61=0
X224A61=0
X224B61=0
X225A61=0
X225B61=0
X23A261=0
X23B261=0
X24A261=0
X24B261=0
X25A261=0
X25B261=0

X323A61=0
X323B61=0
X324A61=0
X324B61=0
X325A61=0
X325B61=0
X33A261=0
X33B261=0

X34A261=0
X34B261=0
X35A261=0
X35B261=0

X423A61=0
X423B61=0
X424A61=0
X424B61=0
X425A61=0
X425B61=0
X43A261=0
X43B261=0
X44A261=0
X44B261=0
X45A261=0
X45B261=0

X523A61=0
X523B61=0
X524A61=0
X524B61=0
X525A61=0
X525B61=0
X53A261=0
X53B261=0
X54A261=0
X54B261=0
X55A261=0
X55B261=0

X623A61=0
X623B61=0
X624A61=0
X624B61=0
X625A61=0
X625B61=0
X63A261=0
X63B261=0
X64A261=0
X64B261=0
X65A261=0
X65B261=0

X123A71=0
X123B71=0
X124A71=0
X124B71=0
X125A71=0
X125B71=0
X13A271=0
X13B271=0
X14A271=0
X14B271=0
X15A271=0
X15B271=0

X223A71=0
X223B71=0
X224A71=0
X224B71=0
X225A71=0
X225B71=0
X23A271=0
X23B271=0
X24A271=0
X24B271=0
X25A271=0
X25B271=0

X323A71=0
X323B71=0
X324A71=0
X324B71=0
X325A71=0
X325B71=0
X33A271=0
X33B271=0
X34A271=0
X34B271=0
X35A271=0
X35B271=0

X423A71=0
X423B71=0
X424A71=0

X424B71=0
X425A71=0
X425B71=0
X43A271=0
X43B271=0
X44A271=0
X44B271=0
X45A271=0
X45B271=0

X523A71=0
X523B71=0
X524A71=0
X524B71=0
X525A71=0
X525B71=0
X53A271=0
X53B271=0
X54A271=0
X54B271=0
X55A271=0
X55B271=0

X623A71=0
X623B71=0
X624A71=0
X624B71=0
X625A71=0
X625B71=0
X63A271=0
X63B271=0
X64A271=0
X64B271=0
X65A271=0
X65B271=0

X113A62=0
X113B62=0
X114A62=0
X114B62=0
X115A62=0
X115B62=0
X13A162=0

X13B162=0
X14A162=0
X14B162=0
X15A162=0
X15B162=0

X213A62=0
X213B62=0
X214A62=0
X214B62=0
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X215B62=0
X23A162=0
X23B162=0
X24A162=0
X24B162=0
X25A162=0
X25B162=0

X313A62=0
X313B62=0
X314A62=0
X314B62=0
X315A62=0
X315B62=0
X33A162=0
X33B162=0
X34A162=0
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X415B62=0
X43A162=0
X43B162=0
X44A162=0
X44B162=0
X45A162=0

X45B162=0

X513A62=0
X513B62=0
X514A62=0
X514B62=0
X515A62=0
X515B62=0
X53A162=0
X53B162=0
X54A162=0
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X615B62=0
X63A162=0
X63B162=0
X64A162=0
X64B162=0
X65A162=0
X65B162=0

X113A72=0
X113B72=0
X114A72=0
X114B72=0
X115A72=0
X115B72=0
X13A172=0
X13B172=0
X14A172=0
X14B172=0
X15A172=0
X15B172=0

X213A72=0
X213B72=0

X214A72=0
X214B72=0
X215A72=0
X215B72=0
X23A172=0
X23B172=0
X24A172=0
X24B172=0
X25A172=0
X25B172=0

X313A72=0
X313B72=0
X314A72=0
X314B72=0
X315A72=0
X315B72=0
X33A172=0
X33B172=0
X34A172=0
X34B172=0
X35A172=0
X35B172=0

X413A72=0
X413B72=0
X414A72=0
X414B72=0
X415A72=0
X415B72=0
X43A172=0
X43B172=0
X44A172=0
X44B172=0
X45A172=0
X45B172=0

X513A72=0
X513B72=0
X514A72=0
X514B72=0
X515A72=0
X515B72=0

X53A172=0
X53B172=0
X54A172=0
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X613B72=0
X614A72=0
X614B72=0
X615A72=0
X615B72=0
X63A172=0
X63B172=0
X64A172=0
X64B172=0
X65A172=0
X65B172=0

Via Equation (42):

Z61-
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57X313A61-57X323A61-57X34A3A61-57X34B3A61-57X35A3A61-57X35B3A61-
57X413A61-57X423A61-57X44A3A61-57X44B3A61-57X45A3A61-57X45B3A61-
57X513A61-57X523A61-57X54A3A61-57X54B3A61-57X55A3A61-57X55B3A61-
57X613A61-57X623A61-57X64A3A61-57X64B3A61-57X65A3A61-57X65B3A61-
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32X114B61-32X124B61-32X13A4B61-32X13B4B61-32X15A4B61-32X15B4B61-
32X214B61-32X224B61-32X23A4B61-32X23B4B61-32X25A4B61-32X25B4B61-

32X314B61-32X324B61-32X33A4B61-32X33B4B61-32X35A4B61-32X35B4B61-
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Via Equation (43):

114C161+114C261+114C361+114C461-Z61>=0
114C162+114C262+114C362+114C462-Z62>=0
114C171+114C271+114C371+114C471-Z71>=0
114C172+114C272+114C372+114C472-Z72>=0

Via Equation (44):

C161+C162=1
C261+C262=1
C371+C372=1
C471+C472=1
C116+C126=1
C216+C226=1
C317+C327=1
C417+C427=1

Via Equation (45):

C171+C172=0
C271+C272=0
C361+C362=0
C461+C462=0
C117+C127=0
C217+C227=0
C316+C326=0
C416+C426=0

Via Equation (46):

C161-.307692307B11<=0
C162-.307692307B12<=0
C171-.307692307B11<=0
C172-.307692307B12<=0
C261-.307692307B21<=0
C262-.307692307B22<=0
C271-.307692307B21<=0
C272-.307692307B22<=0

C361-.307692307B31<=0
C362-.307692307B32<=0
C371-.307692307B31<=0
C372-.307692307B32<=0
C461-.307692307B41<=0
C462-.307692307B42<=0
C471-.307692307B41<=0
C472-.307692307B42<=0

Via Equation (47):

C161-C116=0
C162-C126=0
C171-C117=0
C172-C127=0
C261-C216=0
C262-C226=0
C271-C217=0
C272-C227=0
C361-C316=0
C362-C326=0
C371-C317=0
C372-C327=0
C461-C416=0
C462-C426=0
C471-C417=0
C472-C427=0

Via Equation (48):

C161+C171+C261+C271+C361+C371+C461+C471+C116+C117+C216+C217+C316+
C317+C416+C417+
X113A+X113B+X114A+X114B+X115A+X115B+
X213A+X213B+X214A+X214B+X215A+X215B+
X313A+X313B+X314A+X314B+X315A+X315B+
X413A+X413B+X414A+X414B+X415A+X415B+
X513A+X513B+X514A+X514B+X515A+X515B+
X613A+X613B+X614A+X614B+X615A+X615B+
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X43A1+X43B1+X44A1+X44B1+X45A1+X45B1+
X53A1+X53B1+X54A1+X54B1+X55A1+X55B1+

X63A1+X63B1+X64A1+X64B1+X65A1+X65B1<=70

C162+C172+C262+C272+C362+C372+C462+C472+C126+C127+C226+C227+C326+
C327+C426+C427+
X123A+X123B+X124A+X124B+X125A+X125B+
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X613A+X63A1+X623A+X63A2+X64A3A+X63A4A+X64B3A+X63A4B+X65A3A+X
63A5A+X65B3A+X63A5B
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3B5A+X15B3B+X13B5B+
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X314A+X34A1+X324A+X34A2+X33A4A+X34A3A+X33B4A+X34A3B+X35A4A+X
34A5A+X35B4A+X34A5B+
X414A+X44A1+X424A+X44A2+X43A4A+X44A3A+X43B4A+X44A3B+X45A4A+X
44A5A+X45B4A+X44A5B+
X514A+X54A1+X524A+X54A2+X53A4A+X54A3A+X53B4A+X54A3B+X55A4A+X
54A5A+X55B4A+X54A5B+
X614A+X64A1+X624A+X64A2+X63A4A+X64A3A+X63B4A+X64A3B+X65A4A+X
64A5A+X65B4A+X64A5B
<=85

X114B+X14B1+X124B+X14B2+X13A4B+X14B3A+X13B4B+X14B3B+X15A4B+X1
4B5A+X15B4B+X14B5B+
X214B+X24B1+X224B+X24B2+X23A4B+X24B3A+X23B4B+X24B3B+X25A4B+X2
4B5A+X25B4B+X24B5B+
X314B+X34B1+X324B+X34B2+X33A4B+X34B3A+X33B4B+X34B3B+X35A4B+X3
4B5A+X35B4B+X34B5B+
X414B+X44B1+X424B+X44B2+X43A4B+X44B3A+X43B4B+X44B3B+X45A4B+X4
4B5A+X45B4B+X44B5B+
X514B+X54B1+X524B+X54B2+X53A4B+X54B3A+X53B4B+X54B3B+X55A4B+X5
4B5A+X55B4B+X54B5B+
X614B+X64B1+X624B+X64B2+X63A4B+X64B3A+X63B4B+X64B3B+X65A4B+X6
4B5A+X65B4B+X64B5B
<=85

X115A+X15A1+X125A+X15A2+X13A5A+X15A3A+X13B5A+X15A3B+X14A5A+X
15A4A+X14B5A+X15A4B+
X215A+X25A1+X225A+X25A2+X23A5A+X25A3A+X23B5A+X25A3B+X24A5A+X
25A4A+X24B5A+X25A4B+
X315A+X35A1+X325A+X35A2+X33A5A+X35A3A+X33B5A+X35A3B+X34A5A+X
35A4A+X34B5A+X35A4B+
X415A+X45A1+X425A+X45A2+X43A5A+X45A3A+X43B5A+X45A3B+X44A5A+X
45A4A+X44B5A+X45A4B+
X515A+X55A1+X525A+X55A2+X53A5A+X55A3A+X53B5A+X55A3B+X54A5A+X
55A4A+X54B5A+X55A4B+
X615A+X65A1+X625A+X65A2+X63A5A+X65A3A+X63B5A+X65A3B+X64A5A+X
65A4A+X64B5A+X65A4B
<=85

X115B+X15B1+X125B+X15B2+X13A5B+X15B3A+X13B5B+X15B3B+X14A5B+X1
5B4A+X14B5B+X15B4B+
X215B+X25B1+X225B+X25B2+X23A5B+X25B3A+X23B5B+X25B3B+X24A5B+X2
5B4A+X24B5B+X25B4B+
X315B+X35B1+X325B+X35B2+X33A5B+X35B3A+X33B5B+X35B3B+X34A5B+X3
5B4A+X34B5B+X35B4B+
X415B+X45B1+X425B+X45B2+X43A5B+X45B3A+X43B5B+X45B3B+X44A5B+X4
5B4A+X44B5B+X45B4B+
X515B+X55B1+X525B+X55B2+X53A5B+X55B3A+X53B5B+X55B3B+X54A5B+X5
5B4A+X54B5B+X55B4B+
X615B+X65B1+X625B+X65B2+X63A5B+X65B3A+X63B5B+X65B3B+X64A5B+X6
5B4A+X64B5B+X65B4B
<=85

Vià Equation (50):

C116+C117+C116+C126-C16<=1
C216+C217+C216+C226-C16<=1
C316+C317+C316+C326-C16<=1
C416+C417+C416+C426-C16<=1
C116+C117+C117+C217-C17<=1
C216+C217+C217+C227-C17<=1
C316+C317+C317+C327-C17<=1
C416+C417+C417+C427-C17<=1
C126+C127+C116+C126-C26<=1
C226+C227+C216+C226-C26<=1
C326+C327+C316+C326-C26<=1
C426+C427+C416+C426-C26<=1
C126+C127+C117+C127-C27<=1
C226+C227+C217+C227-C27<=1
C326+C327+C317+C327-C27<=1
C426+C427+C417+C427-C27<=1

Via Equations (51) and (52):

16C161+S16+D6-D1<=6.24
16C161-S16-D6+D1<=25.76
16C171+S17+D7-D1<=5.6
16C171-S17-D7+D1<=26.4
16C261+S26+D6-D1<=6.24
16C261-S26-D6+D1<=25.76
16C271+S27+D7-D1<=5.6

16C271-S27-D7+D1<=26.4
16C162+S16+D6-D2<=4.48
16C162-S16-D6+D2<=27.52
16C172+S17+D7-D2<=3.82
16C172-S17-D7+D2<=28.18
16C262+S26+D6-D2<=4.48
16C262-S26-D6+D2<=27.52
16C272+S27+D7-D2<=3.82
16C272-S27-D7+D2<=28.18
16C361+S36+D6-D1<=6.24
16C361-S36-D6+D1<=25.76
16C371+S37+D7-D1<=5.6
16C371-S37-D7+D1<=26.4
16C461+S46+D6-D1<=6.24
16C461-S46-D6+D1<=25.76
16C471+S47+D7-D1<=5.6
16C471-S47-D7+D1<=26.4
16C362+S36+D6-D2<=4.48
16C362-S36-D6+D2<=27.52
16C372+S37+D7-D2<=3.82
16C372-S37-D7+D2<=28.18
16C462+S46+D6-D2<=4.48
16C462-S46-D6+D2<=27.52
16C472+S47+D7-D2<=3.82
16C472-S47-D7+D2<=28.18

BOUNDS

The first twelve bounds set the NET and NLT time windows for the split destination nodes

D3A>=16
D3A<=20
D4A>=17.5
D4A<=21.5
D5A>=19
D5A<=25
D3B>=16
D3B<=20
D4B>=17.5
D4B<=21.5
D5B>=19
D5B<=25

The following four bounds pre-specify that the C-5 crews "arrive" to their aircraft at least 3.25 hours after the clock starts. This accounts for the 3.25 minimum transload time at the depots, and is necessary for ensuring the D_i odometer variables (when "i" represents a depot) correctly calculate D_i as the transload completion time at depot "i".

D6<=3.25
D7<=3.25
D6>=3.25
D7>=3.25

INTEGERS

(Note: In CPLEX, the declaration INTEGERS means binary variables)

X113A
X113B
X114A
X114B
X115A
X115B
X123A
X123B
X124A
X124B
X125A
X125B
X13A1
X13A2
X13A4A
X13A4B
X13A5A
X13A5B
X13B1
X13B2
X13B4A
X13B4B
X13B5A
X13B5B
X14A1
X14A2
X14A3A

X14A3B
X14A5A
X14A5B
X14B1
X14B2
X14B3A
X14B3B
X14B5A
X14B5B
X15A1
X15A2
X15A3A
X15A3B
X15A4A
X15A4B
X15B1
X15B2
X15B3A
X15B3B
X15B4A
X15B4B

X213A
X213B
X214A
X214B
X215A
X215B
X223A
X223B
X224A
X224B
X225A
X225B
X23A1
X23A2
X23A4A
X23A4B
X23A5A
X23A5B
X23B1
X23B2
X23B4A

X23B4B
X23B5A
X23B5B
X24A1
X24A2
X24A3A
X24A3B
X24A5A
X24A5B
X24B1
X24B2
X24B3A
X24B3B
X24B5A
X24B5B
X25A1
X25A2
X25A3A
X25A3B
X25A4A
X25A4B
X25B1
X25B2
X25B3A
X25B3B
X25B4A
X25B4B

X313A
X313B
X314A
X314B
X315A
X315B
X323A
X323B
X324A
X324B
X325A
X325B
X33A1
X33A2
X33A4A

X33A4B
X33A5A
X33A5B
X33B1
X33B2
X33B4A
X33B4B
X33B5A
X33B5B
X34A1
X34A2
X34A3A
X34A3B
X34A5A
X34A5B
X34B1
X34B2
X34B3A
X34B3B
X34B5A
X34B5B
X35A1
X35A2
X35A3A
X35A3B
X35A4A
X35A4B
X35B1
X35B2
X35B3A
X35B3B
X35B4A
X35B4B

X413A
X413B
X414A
X414B
X415A
X415B
X423A
X423B
X424A

X424B
X425A
X425B
X43A1
X43A2
X43A4A
X43A4B
X43A5A
X43A5B
X43B1
X43B2
X43B4A
X43B4B
X43B5A
X43B5B
X44A1
X44A2
X44A3A
X44A3B
X44A5A
X44A5B
X44B1
X44B2
X44B3A
X44B3B
X44B5A
X44B5B
X45A1
X45A2
X45A3A
X45A3B
X45A4A
X45A4B
X45B1
X45B2
X45B3A
X45B3B
X45B4A
X45B4B

X513A
X513B
X514A

X514B
X515A
X515B
X523A
X523B
X524A
X524B
X525A
X525B
X53A1
X53A2
X53A4A
X53A4B
X53A5A
X53A5B
X53B1
X53B2
X53B4A
X53B4B
X53B5A
X53B5B

X54A1
X54A2
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X54A3B
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X55A3B
X55A4A
X55A4B
X55B1
X55B2
X55B3A

X55B3B
X55B4A
X55B4B

X613A
X613B
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X614B
X615A
X615B
X623A
X623B
X624A
X624B
X625A
X625B
X63A1
X63A2
X63A4A
X63A4B
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X63A5B
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X64A3B
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X64A5B
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X64B3A
X64B3B
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X65A1
X65A2
X65A3A

X65A3B
X65A4A
X65A4B
X65B1
X65B2
X65B3A
X65B3B
X65B4A
X65B4B

X3A1
X3A2
X3B1
X3B2
X4A1
X4A2
X4B1
X4B2
X5A1
X5A2
X5B1
X5B2

X113A61
X113B61
X114A61
X114B61
X115A61
X115B61
X123A61
X123B61
X124A61
X124B61
X125A61
X125B61
X13A161
X13A261
X13A4A61
X13A4B61
X13A5A61
X13A5B61
X13B161
X13B261

X13B4A61
X13B4B61
X13B5A61
X13B5B61
X14A161
X14A261
X14A3A61
X14A3B61
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X14A5B61
X14B161
X14B261
X14B3A61
X14B3B61
X14B5A61
X14B5B61
X15A161
X15A261
X15A3A61
X15A3B61
X15A4A61
X15A4B61
X15B161
X15B261
X15B3A61
X15B3B61
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X15B4B61

X213A61
X213B61
X214A61
X214B61
X215A61
X215B61
X223A61
X223B61
X224A61
X224B61
X225A61
X225B61
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X23A261

X23A4A61
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X23A5A61
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X23B4B61
X23B5A61
X23B5B61
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X324B61
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X324B72
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X35A3B72
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X35A4B72
X35B172
X35B272
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X35B3B72
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X35B4B72

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X413B72
X414A72
X414B72
X415A72
X415B72
X423A72
X423B72
X424A72

X424B72
X425A72
X425B72
X43A172
X43A272
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X44A5B72
X44B172
X44B272
X44B3A72
X44B3B72
X44B5A72
X44B5B72
X45A172
X45A272
X45A3A72
X45A3B72
X45A4A72
X45A4B72
X45B172
X45B272
X45B3A72
X45B3B72
X45B4A72
X45B4B72

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X513B72
X514A72

X514B72
X515A72
X515B72
X523A72
X523B72
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X524B72
X525A72
X525B72
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X54B172
X54B272
X54B3A72
X54B3B72
X54B5A72
X54B5B72
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X55A3B72
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X55A4B72
X55B172
X55B272
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X55B3B72

X55B4A72
X55B4B72

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X624B72
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X625B72
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X65B4B71

C116

C126
C117
C127
C161
C162
C171
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C216
C226
C217
C227
C261
C262
C271
C272
C316
C326
C317
C327
C361
C362
C371
C372
C416
C426
C417
C427
C461
C462
C471
C472

C16
C17
C26
C27
END

Appendix 4B - Solution for Figure 7

Solution to the "Hub-and-Spoke" case study scenario problem associated with Figure 7 and depicted graphically in Figure 9, in which cargo demands at nodes 3A, 3B, 4A, 4B, 5A and 5B are specified by weights and CONUS origin.

The formulation of this problem is presented in Appendix 4A

Node limit, integer feasible: Objective = 1.4049000013e+02
Solution Time = 69874.03 sec. Iterations = 2624989 Nodes = 20000

Variable Name	Solution Value
X313A	1.000000
X424A	1.000000
X525A	1.000000
X613B	1.000000
X63B4B	1.000000
X64B5B	1.000000
T33A	2.250000
T44A	2.250000
T55A	2.250000
T63B	2.250000
T64B	2.250000
T65B	2.250000
X3A1	1.000000
X3B1	1.000000
X4A2	1.000000
X4B1	1.000000
X5A2	1.000000
X5B1	1.000000
C162	1.000000
C261	1.000000
C372	1.000000
C471	1.000000
C16	1.000000
C17	1.000000
C26	1.000000
C27	1.000000
D3A	18.800000
D1	16.710000
D3B	18.800000
D4A	21.290000

D4B	21.500000
D5A	21.290000
D5B	24.170000
D2	19.250000
D6	3.250000
D7	3.250000
X33A1	1.000000
X44A2	1.000000
X65B1	1.000000
X55A2	1.000000
X313A61	1.000000
X313A71	1.000000
X424A62	1.000000
X424A72	1.000000
X525A62	1.000000
X525A72	1.000000
X613B61	1.000000
X613B71	1.000000
X63B4B61	1.000000
X64B5B71	1.000000
X63B4B71	1.000000
X64B5B61	1.000000
X33A161	1.000000
X33A171	1.000000
X44A262	1.000000
X44A272	1.000000
X55A262	1.000000
X55A272	1.000000
X65B161	1.000000
X65B171	1.000000
Z61	89.000000
Z62	83.000000
Z71	83.000000
Z72	89.000000
C126	1.000000
C216	1.000000
C327	1.000000
C417	1.000000
B12	3.250000
B21	3.250000
B32	3.250000
B41	3.250000
S16	4.480000

S26	3.700000
S37	3.820000
S47	3.060000

All other variables in the range 1401-1581 are zero.

Appendix 5A - New objective function for Figure 7 problem

The new objective function of equation (53) (minimizing the sum of the Korean arrival times, along with minimizing the number of C-17s used) used for the "Hub-and-Spoke" case study scenario problem associated with Figure 7.

MINIMIZE

D3A+D3B+D4A+D4B+D5A+D5B+

16X113A+16X113B+16X114A+16X114B+16X115A+16X115B+
16X123A+16X123B+16X124A+16X124B+16X125A+16X125B+

16X213A+16X213B+16X214A+16X214B+16X215A+16X215B+
16X223A+16X223B+16X224A+16X224B+16X225A+16X225B+

16X313A+16X313B+16X314A+16X314B+16X315A+16X315B+
16X323A+16X323B+16X324A+16X324B+16X325A+16X325B+

16X413A+16X413B+16X414A+16X414B+16X415A+16X415B+
16X423A+16X423B+16X424A+16X424B+16X425A+16X425B+

16X513A+16X513B+16X514A+16X514B+16X515A+16X515B+
16X523A+16X523B+16X524A+16X524B+16X525A+16X525B+

16X613A+16X613B+16X614A+16X614B+16X615A+16X615B+
16X623A+16X623B+16X624A+16X624B+16X625A+16X625B+

X3A1+X3A2+X3B1+X3B2+X4A1+X4A2+X4B1+X4B2+X5A1+X5A2+X5B1+X5B2+

C16+C17+C26+C27

Appendix 5B - Solution to Figure 7 problem with new objective function

Solution to the "Hub-and-Spoke" case study scenario problem associated with Figure 7. and depicted graphically in Figure 10, <u>after</u> changing the objective function from equation (22) to equation (53) (minimizing the sum of the Korean arrival times).

Node limit, integer feasible: Objective = 1.8257000007e+02
Solution Time = 338454.85 sec. Iterations = 11351791 Nodes = 20000

CPLEX> Display values of which variable(s): Variable Name Solution Value

D3A	16.000000
D3B	16.000000
D4A	17.500000
D4B	18.700000
D5A	19.000000
D5B	21.370000
X613B	1.000000
X525A	1.000000
X424A	1.000000
X313A	1.000000
X3A1	1.000000
X3B1	1.000000
X4A2	1.000000
X4B1	1.000000
X5A2	1.000000
X5B1	1.000000
C16	1.000000
C17	1.000000
C26	1.000000
C27	1.000000
D1	13.910000
D2	15.460000
D6	3.250000
D7	3.250000
X33A1	1.000000
X44A2	1.000000
X65B1	1.000000
X55A2	1.000000
T12	14.350000
T21	16.000000

T22	14.350000
T62	14.350000
T64B	2.250000
T65B	2.250000
T51	16.000000
T52	1.500000
T55A	2.250000
T44A	2.250000
T63B	2.250000
T33A	2.250000
X63B4B	1.000000
X64B5B	1.000000
X613B61	1.000000
X613B71	1.000000
X63B4B61	1.000000
X64B5B71	1.000000
X525A62	1.000000
X525A72	1.000000
X424A62	1.000000
X424A72	1.000000
X313A61	1.000000
X313A71	1.000000
X63B4B71	1.000000
X64B5B61	1.000000
X65B161	1.000000
X65B171	1.000000
X55A262	1.000000
X55A272	1.000000
X44A262	1.000000
X44A272	1.000000
X33A161	1.000000
X33A171	1.000000
Z61	89.000000
Z62	83.000000
Z71	83.000000
Z72	89.000000
C261	1.000000
C162	1.000000
C471	1.000000
C372	1.000000
C216	1.000000
C126	1.000000
C327	1.000000

C417	1.000000
B21	3.250000
B12	3.250000
B32	3.250000
B41	3.250000
S26	0.900000
S16	0.690000
S37	0.030000
S47	0.260000

All other variables in the range 1-1581 are zero.

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Vita

Major David W. Cox was born on 6 November, 1962, in Worcester, Massachusetts. He graduated from Saint John's High School in 1980, and enrolled at Holy Cross College on an Air Force ROTC scholarship that same year. He graduated with a Bachelor of Arts Degree in Mathematics in 1984 and entered Undergraduate Navigator Training at Mather AFB, CA. There, he earned his navigator wings in May of 1985 with a KC-135 assignment to Grand Forks AFB, ND. He earned a Master of Aeronautical Science degree from Embry-Riddle Aeronautical University, a private pilot's license, and represented the 319th Bomb Wing in Strategic Air Command's Navigation and Bombing Competition in 1988. He was selected to attend Undergraduate Pilot Training at Reese AFB, Texas, where he earned his pilot wings in August, 1989, and received an HC-130 assignment to Eglin AFB, FL.

He flew numerous combat missions during Operation Desert Storm, earning two Air Medals, and was reassigned to Kadena AB, Japan, in 1993. There, he became an instructor pilot, flight commander, chief pilot, and then chief of current operations for the 353rd Special Operations Group. In 1996 he entered the Graduate School of Engineering at the Air Force Institute of Technology. His follow-on assignment is to Headquarters, Air Mobility Command, Scott AFB, Illinois.

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13. ABSTRACT *(Maximum 200 words)*
Traditionally, the United States Air Force's Air Mobility Command (AMC) has used the concept of direct delivery to airlift cargo and passengers from a point of embarkation to a point of debarkation. This study develops an alternative hub-and-spoke combined location-routing integer linear programming prototype model, and uses this model to determine what advantages a hub-and-spoke system offers, and in which scenarios it is better-suited than the direct delivery method. Additionally, the analysis suggests that the C-17 may be better-suited, in certain airlift situations, for theater airlift versus strategic (direct delivery) airlift. The model features the following elements: time windows, cargo tracking capability, multiple frequency servicing, aircraft basing assignments and routing, and the selection of the optimal number of local-delivery aircraft to be used. The model is an extension on the following works: the hierarchical model of Perl and Daskin (1983), time windows features of Chan (1991), combining subtour-breaking and range constraints of Kulkarni and Bhave (1985), and multiple servicing frequency via the clustering co-location method for binary variables of Baker (1991). Additionally, an original approach for cargo tracking is developed and incorporated. A notional CONUS-to-Korea transoceanic airlift problem is used to demonstrate the numerous features and power of the model.

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