

# **Intermodal Logistic System Network Design with Expedited Transportation Services**

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## ABSTRACT

A comprehensive network design planning framework is critical to the capacity, efficiency and reliability of a global supply chain system. Most studies on logistics network planning are based on simple and deterministic assumptions of day-to-day supply chain operations. However, the performance of a network logistic system is in fact largely affected by uncertainties in demand rate and transportation lead times. This study proposes a mathematical model for the design of a two-echelon supply chain where a set of suppliers serve a set of terminal facilities that receive uncertain customer demands. This model integrates a number of system decisions in both the planning and operational stages, including facility location, multi-level service assignments, multi-modal transportation configuration, and inventory management, to minimize the expected system cost under uncertainties from both suppliers and demands. We also consider probabilistic supplier disruptions that may halt product supply from certain suppliers from time to time. We developed a customized solution approach based on Lagrangian relaxation that can solve these models efficiently and accurately. Numerical examples are conducted to test the proposed model and draw managerial insights into how the key parameters affect the optimal system design. Finally, a user friendly web-based interface is developed for biofuel supply chain network design with graphic interactions.



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## TABLE OF CONTENTS

ABSTRACT.....	III
ACKNOWLEDGMENTS .....	V
TABLE OF CONTENTS.....	VII
INTRODUCTION .....	1
OBJECTIVE .....	5
SCOPE .....	6
METHODOLOGY .....	7
Network Model Formulation .....	8
Solution Algorithm .....	11
Lagrangian Relaxation .....	11
Feasible Solution.....	13
Updating Lagrangian Multipliers.....	16
DISCUSSION OF RESULTS .....	17
CONCLUSIONS.....	25
RECOMMENDATIONS .....	26
REFERENCES .....	27
APPENDIX.....	30





## INTRODUCTION

A significant portion of Logistics investment and operations cost are attributed to suboptimal logistics network planning and unreliable day-to-day operations. In the planning stage, excessive installations and unbalanced spatial distribution of service facilities obviously incur a waste of investment and resources, while underinvestment in facility infrastructures will hinder following transportation operations and inventory management throughout the operational horizon. In day-to-day operations, a series of interdependent decisions on several operational components, including inventory level, shipment amount and delivery mode, will be dynamically made at each terminal facility so as to satisfy stochastic customer demand in time. Supply chain operations are susceptible to various uncertainties such as facility disruptions, transportation delays, and customer demand fluctuations.

For decades, numerous efforts have been made to address planning-level logistics network design. Early studies stem from facility location problems, which can be traced back to about a century ago (Weber, 1957). Traditional location models, e.g., the p-median and uncapacitated facility location models, simply assume that the long-term operational costs can be captured by one-period transportation operations from service facilities to customer locations. See Daskin (1995) and Drezner (2002) for a review on these developments. As the global competition increased in the late 1970s, researchers tried to use location models to address strategical planning issues in logistics systems where complex transportation and inventory management operations are involved (M. Melo, Nickel, & Saldanha-Da-Gama, 2009). Facility location models were adapted to incorporate shipment costs for multi-period dynamic operations (M. T. Melo, Nickel, & Da Gama, 2006) and multi-commodity flows (Klose & Drexler, 2005). These models are also extended to multi-layer (or multi-echelon) supply chain distribution system design (Şahin & Süral, 2007).

In addition to all the studies on facility location problems, on the operational side, freight lead time uncertainties and customer demand fluctuations have been well recognized as major challenges to inventory management and customer service quality. Recently, a number of studies investigated how operational disruptions impact inventory management (Berk & Arreola-Risa, 1994; Chopra, Reinhardt, & Mohan, 2007; Dada & Petruzzi, 2007; Gupta, 1996; Parlar & Berkin, 1991; Parlar & Perry, 1995; Parlar, 1997; L Qi, Shen, & Snyder, 2009; Ross, Rong, & Snyder, 2008; Sheffi, 2001; Tomlin, 2006). Most of these studies are however limited to relatively simple supply chain structures (e.g., a single facility or a series supply chain) and therefore cannot provide a network perspective for large-scale logistics systems planning. However, for a realistic supply chain system that faces both facility disruptions and operational uncertainties simultaneously, it is imperative to have a system

design method that is not only robust against facility disruption risks but immune to operational uncertainties. Uncertainties from both customer demand generation and supply service reliability have been recognized and investigated (Cui, Ouyang, & Shen, 2010; Daskin, 1983; Li & Ouyang, 2010; Snyder & Daskin, 2005)

As evidenced in recent catastrophic events (e.g., West Coast Lockdown (Gibson, Defee, & Ishfaq, 2015), Szechuan Earthquake (Chan, 2008), Fukushima nuclear leak (Holt, Campbell, & Nikitin, 2012), Hurricane Sandy (Blake, Kimberlain, Berg, Cangialosi, & Beven II, 2013)), supply chain facilities are vulnerable to various natural and anthropogenic disruption risks such as floods, earthquakes, power outages, and labor actions. Recently, there have been many studies on reliable facility location design with the aim of increasing the expected performance of a supply chain system across various facility disruption scenarios. To ensure customer service levels after a disruption happens, one way is to hold a high inventory of commodities at the downstream terminals (or retailer stores), which however incurs excessive inventory holding cost. Or expedited transportation can be used to largely reduce the delivery time to avoid accumulation of unmet demand, which however may dramatically increase transportation cost due to expensive expedited services. When expedited transportation is available, the needed safety inventory can be significantly reduced. Although an expedited shipment usually costs way more than the regular service, it can much improve the service quality even if only used in emergent occasions (e.g., when the safety inventory is about to deplete). This series of uncertainties throughout these interdependent planning and operational stages, if not properly managed, may seriously damage system performance and deteriorate customer satisfaction.

In reality, however, uncertainties exist almost ubiquitously throughout all components in a supply chain. Studies in 1980s (Batta, Dolan, & Krishnamurthy, 1989; Daskin, 1982, 1983; ReVelle & Hogan, 1989) pointed out the need for facility redundancy under stochastic demand. Later studies (Lee, Padmanabhan, & Whang, 1997; Ouyang & Daganzo, 2006; Ouyang & Li, 2010) further recognized that demand uncertainties cause serious challenges to inventory management when transportation takes long and uncertain lead times. To address this problem, facility location design has been integrated into inventory management to balance the tradeoff between spatial inventory distribution and transportation (Chen, Li, & Ouyang, 2011; Daskin, Coullard, & Shen, 2002; Lian Qi, Shen, & Snyder, 2010; Shen, Coullard, & Daskin, 2003; Shen & Qi, 2007; Shu, Teo, & Shen, 2005; Snyder, Daskin, & Teo, 2007). Many developments simply treated operational components as separable linear terms (Carlsson & Rönnqvist, 2005; Cordeau, Pasin, & Solomon, 2006; Eskigun, Uzsoy, Preckel, & Beaujon, 2005; Sadjady & Davoudpour, 2012; Wilhelm, Liang, Rao, Warriar, & Zhu, 2005), which however fail to capture the interdependence among inventory management and transportation mode choices

in an uncertain environment. However, none of these studies consider expedited shipment options and unexpected facility disruptions and thus they are not suitable for incorporating transportation mode configuration decisions in system network planning. This study will bridge these gaps by proposing an integrated logistics planning framework that combines supply selection, transportation mode configurations, and plant inventory management decisions all together.



## OBJECTIVE

This study proposes an integrated logistics network design framework that incorporates inventory management, expediting transportation options and facility disruption risks. We will particularly focus on a supply chain system (e.g., a distributed manufacture system) where a set of terminal facilities (e.g., plants) order products (or parts) from a set of external suppliers. This system will be responsible to the initial fixed investment to select suppliers and setup the service relationships in the planning stage, and inventory holding costs at the terminal facilities and both regular and expedited transportation costs from suppliers to facilities in the operational stage. Note that the effect of facility disruptions and that from demand and transportation uncertainties are highly coupled. For example, disruptions of facilities will reduce candidate suppliers to customer terminals, which may in consequence increase transportation uncertainties and cumulate more unmet demand. Mathematical models will be created to determine the optimal supplier location selection, supplier to plant assignment, expedited shipment configuration, and inventory stock level at each plant that collectively minimize the total expected system cost over the entire planning horizon. These models are non-linear integer programming problems and solving them is very challenging and substantial modeling efforts are needed to develop a comprehensive yet computationally-tractable model to solve this problem. We propose a customized solution approach based on Lagrangian relaxation that decompose the problem into a set of relatively easy sub-problems. We conduct a set of case studies to show that the proposed approach can efficiently solve problem instances of different scales. We draw from case study results several managerial insights on the interdependence among planning decisions, transportation configurations and inventory management strategies, and we also discuss the effects of key parameters on the optimal system design results.

## **SCOPE**

This study is presenting an integrated methodological framework that takes advantage of optional expedited transportation services and addresses decision components in both planning and operational stages simultaneously. This framework bridges the gap between planning models of network logistics systems and operational models of multimodal transportation configuration and inventory management decisions considering the facility disruption risks. It enables logistics planners to ponder all these involved critical decisions in an integrated manner and design a system that performs more reliably and runs at a lower cost compared to traditional results. As demonstrated in the numerical examples, our proposed model framework can efficiently and accurately solve an integrated logistics system design problem, and the optimal design solution can balance all cost components (including initial investment, regular and expedited transportation cost, and inventory management cost) and thus yields a minimum expected net cost. In addition, we showed interesting managerial insights into the optimal system design, such as relative importance and savings from integrating the expedited shipment option under different problem settings.

## METHODOLOGY

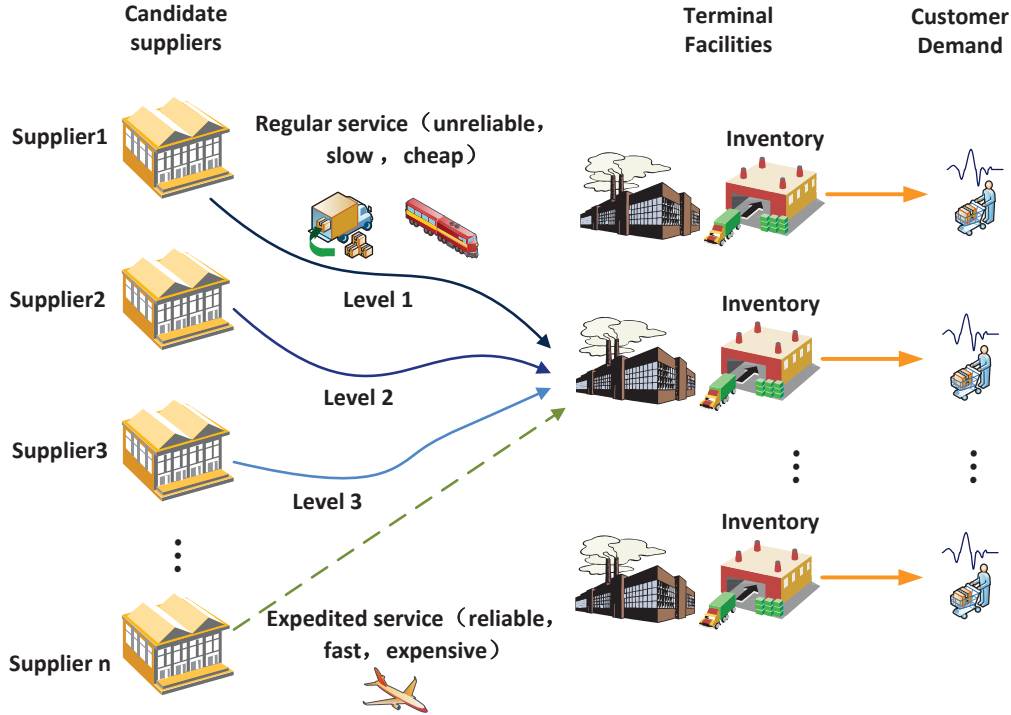
In this section, we formulate the full network design problem into a non-linear integer programming model or its variants and propose a customized solution approach for solving these network models. For the convenience of the readers, the mathematical notation is summarized in Table 1.

**Table 1** Notation List.

$d_j$	Demand rate at the terminal $j$
$e_{ij}$	Unit expedited shipment cost from supplier $i$ to terminal $j$
$f_i$	Fixed cost to install supplier $i$
$h_j$	Unit inventory holding cost at facility $j$
$q$	Supplier disruption probability for the regular service
$r_{ij}$	Unit regular shipment cost from supplier $i$ to terminal $j$
$t_{ij}$	Expected regular shipment lead time from supplier $i$ to terminal $j$
$L$	Maximum assignment level
$P_{ij}(S_j)$	Stock-out probability at terminal $j$ with base stock $S_j$ and regular supplier $i$
$S_j$	Base-stock position at terminal $j$
$\bar{S}_j$	Maximum allowable base-stock position at terminal $j$
$X_i$	Whether supplier $i$ is installed for service
$Y_{ijl}$	Whether supplier $i$ provides regular service to terminal $j$ at assignment level $l$
$Z_{ij}$	Whether supplier $i$ provides expedited service to terminal $j$
<b>I</b>	Set of candidate suppliers, indexed by $i$
<b>J</b>	Set of terminal facilities, indexed by $j$
<b>L</b> = $\{1, 2, \dots, L\}$	Set of assignment levels, indexed by $l$
<b>S<sub>j</sub></b> = $\{1, 2, \dots, \bar{S}_j\}$	Set of candidate base-stock positions

## Network Model Formulation

Figure 1 illustrates the studied supply chain system, which includes set of terminal facilities denoted by  $\mathbf{J}$  and a set of candidate suppliers denoted by  $\mathbf{I}$ . Each terminal  $j \in \mathbf{J}$  receives discrete demand for a certain commodity from a fixed pool of customers over time. We assume that at each terminal  $j$ , demand units arrive randomly with an expected rate of  $d_j$ .



**Figure 1** Illustration of the studied supply chain system.

To feed the arriving demand, we assume that each terminal  $j$  initially keeps a base-stock position  $S_j \in \mathbf{S}_j := \{0, 1, 2, \dots, \bar{S}_j\}$  where  $\bar{S}_j$  is a given capacity of the inventory at  $j$ , and the cost of holding one unit base stock per unit time is  $h_j \geq 0$ . This yields the system inventory cost as follows

$$C^H := \sum_{j \in \mathbf{J}} h_j S_j \quad (1)$$

Whenever receiving a demand unit, terminal  $j$  will first check its on-hand inventory and take one unit from this inventory, if any, to feed this demand unit. Meanwhile in order to maintain the base-stock inventory position, terminal  $j$  can place an order right away from a supplier from  $\mathbf{I}$ . This study considers possible supplier disruptions and assumes that each supplier can be disrupted independently any time at an identical probability  $q$ . To mitigate the impact from uncertain disruptions, a terminal is assigned to  $L > 1$  suppliers at different priority levels for regular shipments. Every time, this terminal scans through these assigned suppliers



from level 1 through level  $L$  and places the order to the first functioning supplier. For the notation convenience, we define level set  $\mathbf{L} := \{1, 2, \dots, L\}$ . In this way, the probability for a terminal to be served by its level- $l$  supplier is  $(1 - q)q^{l-1}, \forall l \in \mathbf{L}$ . The assignments are specified by binary variables  $[Y_{ijl}]_{i \in \mathbf{I}, j \in \mathbf{J}, l \in \mathbf{L}}$  such that  $Y_{ijl} = 1$  if supplier  $i$  is assigned to terminal  $j$  at level  $l$  or  $Y_{ijl} = 0$  otherwise. Let  $r_{ij}$  denote the cost to ship a unit commodity from supplier  $i$  to terminal  $j$ , and then the total expected regular shipment cost is

$$C^R := \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{l \in \mathbf{L}} d_j r_{ij} (1 - q) q^{l-1} Y_{ijl}. \quad (2)$$

We assume that the studied supply chain system has to maintain very high service quality such that customer demand has to be served right after it arrives. Despite being an economic option, a regular shipment is usually slow and unreliable. We assume that a regular shipment from supplier  $i$  to terminal  $j$  takes a random lead time with an expected value of  $t_{ij}$ . Since a longer shipment time is usually associated with a higher shipment cost for the same mode of transportation, we assume that  $r_{ij} \geq r_{i'j} \Leftrightarrow t_{ij} \geq t_{i'j}, \forall i \neq i' \in \mathbf{I}, j \in \mathbf{J}$ . In case that the regular shipments cannot arrive in time to meet the outstanding demand, the on-hand inventory at terminal  $j$  may be depleted, particularly when the realized demand rate is high. In this case, terminal  $j$  has to activate expedited transportation that always delivers shipments in a negligible lead time. We assume that every supplier provides an emergent expedited service that is independent from the regular service and never disrupts. When the on-hand inventory is depleted, a terminal uses the expedited service from a selected supplier, which however costs much more than regular transportation. Let  $e_{ij}$  denote the cost to obtain an expedited shipment from supplier  $i$  to terminal  $j$ , which shall satisfy  $e_{ij} \gg r_{i'j}, \forall i' \in \mathbf{I}$ . In order to quantify the expected expedited transportation cost, we will first quantify the probability for a terminal to activate the expedited service. Conditioning on that supplier  $i$  is the active regular service provider to terminal  $j$ , the probability for terminal  $j$  to use the expedited service can be represented as a function of initial inventory  $S_j$  based on a truncated Poisson distribution (Li, 2013),

$$P_{ij}(S_j) = \frac{(d_j t_{ij})^{S_j} / S_j!}{\sum_{s=0}^{S_j} (d_j t_{ij})^s / s!}. \quad (3)$$

Note that once terminal  $j$  places an expedited order from supplier  $i'$ , then no regular order is placed to the incumbent regular supplier  $i$ , and thus the actual additional cost due to this expedited order is  $e_{i'j} - r_{ij}$ . Define variables  $[Z_{ij}]_{i \in \mathbf{I}, j \in \mathbf{J}}$  to denote the expedited service assignments such that  $Z_{ij} = 1$  if terminal  $j$ 's expedited service provider is supplier  $i$  or  $Z_{ij} =$

0 otherwise. Then the total expected additional cost due to expedited shipments (or the marginal expedited cost) can be formulated as

$$C^M := \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}} \sum_{l \in \mathbf{L}} \sum_{i' \in \mathbf{I}} d_j (e_{i'j} - r_{ij}) (1-q) q^{l-1} P_{ij}(S_j) Z_{i'j} Y_{ijl}. \quad (4)$$

Another risk that the regular service is subject to is that all its suppliers may be disrupted simultaneously at probability  $q^L$ . If this happens, the regular service to this terminal becomes inactive, and we assume that it is now only served by emergency shipments from the previously assigned expedited supplier. The emergency cost structure stays the same as the previously defined expedited cost structure since they come from the same sources. Thus the expected system emergency cost is formulated as

$$C^E := \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}} d_j e_{ij} q^L Z_{ij}. \quad (5)$$

Finally, in this supply chain system, if candidate supplier  $i$  is used by one or more terminals for either regular or expedited service, a fixed installation cost  $f_i$  (prorated per unit time) is incurred. Define binary variables  $[X_i]_{i \in \mathbf{I}}$  to denote the supplier location decisions such that  $X_i = 1$  if candidate supplier  $i$  is installed or  $X_i = 0$  otherwise. This results in the system fixed installation cost as follows,

$$C^F := \sum_{i \in \mathbf{I}} f_i X_i. \quad (6)$$

The system design includes integrated decisions of supplier location  $[X_i]$ , regular service assignments  $[Y_{ijl}]$ , expedited service assignments  $[Z_{ij}]$ , and initial inventory positions  $[S_j]$  that collectively minimize the total system cost composed of (1), (2), (4), (5) and (6). Note that these cost components shall generally exhibit the following tradeoffs. Increasing supplier installations shall raise one-time fixed cost (6) but reduce day-to-day operational costs(2), (4) and (5). The higher inventory positions  $[S_j]$  we set, which though increase inventory cost (1), the less frequent expedited shipments are needed according to probability function (3), and thus the less extra expedited transportation cost (4) is consumed. In order to quantitatively solve the detailed system design, the follow integer programming model is formulated.

$$\begin{aligned} \min C := & \sum_{i \in \mathbf{I}} f_i X_i + \sum_{j \in \mathbf{J}} h_j S_j \\ & + \sum_{j \in \mathbf{J}} d_j \left\{ \sum_{i \in \mathbf{I}} \sum_{l \in \mathbf{L}} \left[ r_{ij} + \sum_{i' \in \mathbf{I}} (e_{i'j} - r_{ij}) Z_{i'j} P_{ij}(S_j) \right] (1-q) q^{l-1} Y_{ijl} + q^L \sum_{i \in \mathbf{I}} e_{ij} Z_{ij} \right\}, \end{aligned} \quad (7)$$

$$\text{s.t.} \quad \sum_{l \in \mathbf{L}} Y_{ijl} - X_i \leq 0, \quad \forall i \in \mathbf{I}, \quad j \in \mathbf{J}, \quad (8)$$

$$Z_{ij} - X_i \leq 0, \quad \forall i \in \mathbf{I}, j \in \mathbf{J}, \quad (9)$$

$$\sum_{i \in \mathbf{I}} Y_{ijl} = 1, \quad \forall j \in \mathbf{J}, l \in \mathbf{L}, \quad (10)$$

$$\sum_{i \in \mathbf{I}} Z_{ij} = 1, \quad \forall j \in \mathbf{J}, \quad (11)$$

$$S_j \in \mathbf{S}_j, \quad \forall j \in \mathbf{J}, \quad (12)$$

$$Y_{ijl} \in \{0,1\}, \quad \forall i \in \mathbf{I}, j \in \mathbf{J}, l \in \mathbf{L}, \quad (13)$$

$$Z_{ij} \in \{0,1\}, \quad \forall i \in \mathbf{I}, j \in \mathbf{J}, \quad (14)$$

$$X_i \in \{0,1\}, \quad \forall i \in \mathbf{I}. \quad (15)$$

Objective aims to minimize the summation of all cost components (2), (4), (5) and (6) across the entire system. Constraints (8) and (9) indicate that a supplier need to be installed first prior to its usage. Constraint (8) also ensures that if one of the suppliers is selected to provide the regular service to a terminal, it can only serve this terminal at one assignment level. Constraint (10) requires that one terminal has one and only one regular supplier at each level. Constraint (11) postulates that each terminal is assigned to one and only one expedited supplier. Constraints (12) - (15) are the corresponding integer and variable constraints for all variables.

### Solution Algorithm

This section develops a customized solution approach that can efficiently obtain a near-optimum solution to the problem (7) – (15), which is an NP-hard problem and extremely difficult to solve. We propose a Lagrangian relaxation algorithm that decomposes this problem into a number of relatively easy sub-problems that can be solved to obtain a lower bound to the original optimal objective (7). Also, the relaxed solution is utilized to construct a feasible solution that yields an upper bound to the objective. Finally, a sub-gradient search approach that iteratively updates both relaxed and feasible solutions to reduce the optimality gap between the upper bound (the best feasible solution) and the lower bound (the best relaxed solution) of the true optimum to an acceptable tolerance (or zero in ideal cases) is presented in this section.

### Lagrangian Relaxation

The Lagrangian relaxation algorithm basically relaxes constraints (8) and (9), and add them to objective (8a) with a set of Lagrangian multipliers  $\lambda := \{\lambda_{ij} \geq 0\}_{i \in \mathbf{I}, j \in \mathbf{J}}$  and  $\mu := \{\mu_{ij} \geq 0\}_{i \in \mathbf{I}, j \in \mathbf{J}}$ . We further add the following constraints

$$\sum_{l \in \mathbf{L}} Y_{ijl} \leq 1, \quad \forall i \in \mathbf{I}, j \in \mathbf{J}, \quad (16)$$

which are redundant to constraints (8) and are only used to improve the relaxed problem solution. Therefore, the relaxed problem can be formulated as follows:

$$\Delta(\boldsymbol{\lambda}, \boldsymbol{\mu}) := \min_{\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{S}} \sum_{i \in \mathbf{I}} \left[ f_i - \sum_{j \in \mathbf{J}} (\lambda_{ij} + \mu_{ij}) \right] X_i + \sum_{j \in \mathbf{J}} \left\{ \sum_{i \in \mathbf{I}} \sum_{l \in \mathbf{L}} \left( \sum_{i' \in \mathbf{I}} \alpha_{i'jl} Z_{i'j} + \beta_{ijl} \right) Y_{ijl} + h_j S_j \right\}, \quad (17)$$

subject to constraints (9)-(16), where

$$\alpha_{i'jl} = d_j \left[ (e_{i'j} - r_{ij}) P_{ij}(S_j) (1-q) q^{l-1} + \frac{e_{ij} q^L}{L} \right] + \frac{\mu_{i'j}}{L}, \quad (18)$$

and,

$$\beta_{ijl} = d_j r_{ij} (1-q) q^{l-1} + \lambda_{ij}. \quad (19)$$

In the above relaxed problem, the variables  $\mathbf{X}$  are separated from the other variables. This allows us to decompose the relaxed problem into two sets of sub-problems. The first set only includes one sub-problem involving variables  $\mathbf{X}$ :

$$\Gamma(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \min_{\mathbf{X}} \sum_{i \in \mathbf{I}} \left[ f_i - \sum_{j \in \mathbf{J}} (\lambda_{ij} + \mu_{ij}) \right] X_i, \quad (20)$$

subject to binary constraint (15). Sub-problem (20) could be simply solved by setting  $X_i = 1$  if  $f_i - \sum_{j \in \mathbf{J}} (\lambda_{ij} + \mu_{ij}) \leq 0$  or  $X_i = 0$  otherwise, which only takes a time complexity of  $O(|\mathbf{I}||\mathbf{J}|)$ . The second set contains  $|\mathbf{J}|$  sub-problems, each associated with a terminal  $j \in \mathbf{J}$ , as follows:

$$\Phi_j(\boldsymbol{\lambda}, \boldsymbol{\mu}) := \min_{\{Y_{ijl}, Z_{ij}\}_{i \in \mathbf{I}, l \in \mathbf{L}}, S_j} \sum_{i \in \mathbf{I}} \sum_{l \in \mathbf{L}} \left( \sum_{i' \in \mathbf{I}} \alpha_{i'jl} Z_{i'j} + \beta_{ijl} \right) Y_{ijl} + h_j S_j, \forall j \in \mathbf{J}, \quad (21)$$

subject to (10)-(14), (16) (where  $\alpha_{i'jl}$  and  $\beta_{ijl}$  were formulated in (18) and (19)). We reformulate sub-problem (21) as a combinatorial problem to facilitate the solution algorithm. Define set  $\mathbf{K} = \{(i_1, i_2, \dots, i_L) | i_1 \neq i_2 \neq \dots \neq i_L \in \mathbf{I}\}$ , where each  $(i_1, i_2, \dots, i_L)$  specifies a strategy to assign the regular suppliers to terminal  $j$  at all  $L$  levels sequentially; i.e., supplier  $i_l$  is assigned to terminal  $j$  at level  $l, \forall l \in \mathbf{L}$ . For short we denote vector  $(i_1, i_2, \dots, i_L)$  with alias  $k$ . Then sub-problems (21) can be rewritten as:

$$\Phi_j(\boldsymbol{\lambda}, \boldsymbol{\mu}) := \min_{i' \in \mathbf{I}, k \in \mathbf{K}, S_j \in \mathbf{S}_j} C_{ki'j}(S_j) := A_{ki'j}(S_j) + h_j S_j + B_{ki'j}, \forall j \in \mathbf{J}, \quad (22)$$

where

$$A_{ki'j} = d_j \sum_{l \in \mathbf{L}} \left[ (e_{i'j} - r_{i'j}) (1-q) q^{l-1} P_{i'j}(S_j) \right],$$

$$B_{ki'j} = \sum_{l \in \mathbf{L}} \left[ d_j r_{ij} (1-q) q^{l-1} + \lambda_{ij} \right] + \frac{d_j e_{ij} q^L + \mu_{ij}}{L}. \quad (23)$$

For given  $k$  and  $i'$ ,  $\min_{S_j \in \mathcal{S}_j} C_{ki'j}(S_j)$  can be solved with a bisection search method (BS) described in Appendix A. With this, problem (17) can be solved by a customized enumeration algorithm (EA) the does an exhaustive search through  $k \in \mathbf{K}, i' \in \mathbf{I}$  for every  $j \in \mathbf{J}$ , as follows:

**Step EA1:** For each terminal  $j \in \mathbf{J}$ , we iterate through  $(k, i') \in (\mathbf{K}, \mathbf{I})$  that specifies terminal  $j$ 's assignment strategy of both regular and expedited suppliers, and call the BS algorithm to solve  $S_j^* := \arg \min_{S_j \in \mathcal{S}_j} C_{ki'j}(S_j)$ .

**Step EA2:** Find  $(k^* = (i_1^*, i_2^*, \dots, i_L^*), i'^*) := \operatorname{argmin}_{k \in \mathbf{K}, i' \in \mathbf{I}} C_{ki'j}(S_j^*)$ ;

**Step EA3:** Return the optimal assignment strategy  $(k^*, i'^*)$  and inventory position  $S_j^*$ ;

**Step EA4:** Repeat EA1-3 for every supplier  $j \in \mathbf{J}$  to get the optimal solution to  $\mathbf{X}, \mathbf{Y}$  and  $\mathbf{Z}$  as follows:

$$Y_{ijl} = \begin{cases} 1 & \text{if } i = i_l^*; \\ 0 & \text{otherwise,} \end{cases} \quad Z_{ij} = \begin{cases} 1 & \text{if } i = i'^*; \\ 0 & \text{otherwise,} \end{cases} \quad X_i = \max_{j \in \mathbf{J}, l \in \mathbf{L}} \{Y_{ijl}, Z_{ij}\}, \quad \forall j \in \mathbf{J}, i \in \mathbf{I}, l \in \mathbf{L}. \quad (22)$$

Note that in the worst case, the time complexity of the EA algorithm for solving sub-problems (21) is  $O(|\mathbf{J}| |\mathbf{I}|^{L+1} \ln(\bar{S}_j))$ . By solving sub-problems (20) and (21), the object value of relaxed problem (17) for one set of given  $\lambda$  and  $\mu$  is equal to:

$$\Delta(\lambda, \mu) = \Gamma(\lambda, \mu) + \sum_{j \in \mathbf{J}} \Phi_j(\lambda, \mu), \quad (23)$$

which is a lower bound for the optimal value of problem (7)-(15) based on the duality property of Lagrangian relaxation (Geoffrion, 1974). Note the time complexity of sub-problem (20) is of a lower order. Therefore, sub-problem (21) dominates the total time complexity of the relaxed problem (23).

### Feasible Solution

If the solution obtained by solving relaxed sub-problems is found to be feasible to the primal problem (7)-(15) and yield an identical objective value, then it will be also the optimal solution to the primal problem. Otherwise, which is the most likely case, we will use certain heuristics to construct a feasible solution. One intuitive way is to keep  $\mathbf{Y}, \mathbf{Z}, \mathbf{S}$  values and adjust the  $\mathbf{X}$  values as:

$$X_i = \max_{j \in \mathbf{J}, l \in \mathbf{L}} \{Y_{ijl}, Z_{ij}\}, \quad \forall i \in \mathbf{I}, \quad (24)$$

which could be solved very efficiently, i.e., in a time on the order of  $O(|\mathbf{I}||\mathbf{J}|L)$ . However, as  $\mathbf{Y}, \mathbf{Z}$  values in the relaxed solution are usually much scattered, this feasible solution likely yields an excessive number of suppliers, leading to an unnecessarily high total cost. A better feasible solution is to fix  $\mathbf{X}$  and adjust the other variables accordingly. Define  $\bar{\mathbf{I}} := \{i | X_i = 1, \forall i \in \mathbf{I}\}$  to be the set of installed suppliers in the relaxed solution. Then, by setting  $\lambda_{ij} = \mu_{ij} = 0$  and replacing  $\mathbf{I}$  with  $\bar{\mathbf{I}}$  in sub-problems(21), other feasible variable values can be determined by solving sub-problems (24)-(29):

$$\bar{\Phi}_j(\boldsymbol{\lambda}, \boldsymbol{\mu}) := \min_{\{Y_{ij}, Z_{ij}\}_{i \in \bar{\mathbf{I}}, l \in \mathbf{L}}, S_j} \sum_{i \in \bar{\mathbf{I}}} \sum_{l \in \mathbf{L}} \left( \sum_{i' \in \bar{\mathbf{I}}} \bar{\alpha}_{i'ij} Z_{i'j} + \bar{\beta}_{ijl} \right) Y_{ijl} + h_j S_j, \quad \forall j \in \mathbf{J}, \quad (24)$$

$$\text{Subject to } \sum_{l \in \mathbf{L}} Y_{ijl} \leq 1, \quad \forall i \in \bar{\mathbf{I}}, \quad j \in \mathbf{J}, \quad (25)$$

$$\sum_{i \in \bar{\mathbf{I}}} Y_{ijl} = 1, \quad \forall j \in \mathbf{J}, \quad l \in \mathbf{L}, \quad (26)$$

$$\sum_{i \in \bar{\mathbf{I}}} Z_{ij} = 1, \quad \forall j \in \mathbf{J}, \quad (27)$$

where

$$\bar{\alpha}_{i'ij} = d_j \left[ (e_{i'j} - r_{ij}) P_{ij}(S_j) (1-q) q^{l-1} + \frac{e_{ij} q^L}{L} \right], \quad (28)$$

$$\bar{\beta}_{ijl} = d_j r_{ij} (1-q) q^{l-1}. \quad (29)$$

Then similar to the transformation from (21) to (22), we also define set  $\bar{\mathbf{K}} = \{(i_1, i_2, \dots, i_L) | i_1 \neq i_2 \neq \dots \neq i_L \in \bar{\mathbf{I}}\}$  as all the strategies to assign regular suppliers from  $\bar{\mathbf{I}}$  to terminal  $j$  at all  $L$  levels, and we also use alias  $k$  to represent vector  $(i_1, i_2, \dots, i_L)$  for short. Then the transformed sub-problems are formulated as

$$\bar{\Phi}_j(\boldsymbol{\lambda}, \boldsymbol{\mu}) := \min_{i' \in \bar{\mathbf{I}}, k \in \bar{\mathbf{K}}, S_j \in \mathbf{S}_j} \bar{C}_{ki'j}(S_j) := \bar{A}_{kj} + \bar{B}_{ki'j} + h_j S_j, \quad \forall j \in \mathbf{J} \quad (30)$$

where

$$\bar{A}_{kj} = d_j \sum_{l \in \mathbf{L}} r_{i_l j} (1-q) q^{l-1} (1 - P_{i_l j}(S_j)), \quad (31)$$

$$\bar{B}_{ki'j} = e_{i'j} \left( \frac{d_j q^L}{L} + d_j \sum_{l \in \mathbf{L}} (1-q) q^{l-1} P_{i_l j}(S_j) \right). \quad (32)$$

The exact optimal solution to each sub-problem (24)-(27) with any  $j \in \mathbf{J}$  can be solved as follows. First denote  $\bar{i}'_j^* := \operatorname{argmin}_{i' \in \bar{\mathbf{I}}} e_{i'j}$ . Then we denote with vector  $\bar{k}_j^* = (\bar{i}_1^{j*}, \bar{i}_2^{j*}, \dots, \bar{i}_L^{j*}) \in \bar{\mathbf{K}}$  the first  $L$  regular service suppliers sorted by the shipment cost to terminal  $j$ , i.e.,  $r_{\bar{i}_l^{j*} j} \leq r_{\bar{i}_{l+1}^{j*} j} \leq r_{i_l j}, \forall l < m \in \mathbf{L}, i \notin \bar{k}_j^*$ . Finally, define  $\bar{S}_j^* := \min_{S_j \in \mathbf{S}_j} \bar{C}_{\bar{k}_j^* \bar{i}'_j^*}(S_j)$ , which can be again efficiently solved with the BS algorithm in Appendix A. The following

proposition proves that  $(\bar{i}_j^*, \bar{k}_j^*, \bar{S}_j^*)$  is the optimal solution to sub-problem (24)-(27) with respect to terminal  $j$ .

**Proposition1.**  $(\bar{i}_j^*, \bar{k}_j^*, \bar{S}_j^*) = \min_{i' \in \bar{\mathbf{I}}, k \in \bar{\mathbf{K}}, S_j \in \mathbf{S}_j} \bar{C}_{ki'j}(S_j), \forall j \in \mathbf{J}$ .

**Proof.** First, it can be seen from the structure of sub-problem (24)-(27) that as  $i'$  varies while the other variables are fixed,  $\bar{C}_{ki'j}(S_j)$  increases with  $e_{i'j}$ . Therefore the optimal solution to  $i'$  is  $\bar{i}_j^*$ .

Let  $\hat{S}_j$  denote the optimal value of  $S_j$ , then the optimal solution to  $k$  is  $\hat{k}_j := (\hat{i}_1^j, \hat{i}_2^j, \dots, \hat{i}_L^j) := \min_{k \in \bar{\mathbf{K}}} \bar{C}_{k\bar{i}_j^*j}(\hat{S}_j)$ . We will prove  $\hat{k}_j = \bar{k}_j^*$  by contradiction. If there exists  $l \in \mathbf{L}$  such that  $r_{\hat{i}_l^j} > r_{\hat{i}_{l+1}^j}$ . We construct a new feasible solution  $\tilde{k}_j := (\hat{i}_1^j, \dots, \hat{i}_{l+1}^j, \hat{i}_l^j, \dots, \hat{i}_L^j)$  by swapping the levels of  $\hat{i}_l^j$  and  $\hat{i}_{l+1}^j$  in  $\hat{k}_j$ , and then we compare the difference between the two costs with respect to  $\hat{k}_j$  and  $\tilde{k}_j$ , respectively,

$$\begin{aligned} \bar{C}_{\tilde{k}_j\bar{i}_j^*j}(\hat{S}_j) - \bar{C}_{\hat{k}_j\bar{i}_j^*j}(\hat{S}_j) &= (1-q)^2 q^{l-1} d_j \left[ (r_{\hat{i}_l^j} - r_{\hat{i}_{l+1}^j}) + \right. \\ &\quad \left. e_{\bar{i}_j^*j} \left( P_{\hat{i}_l^j}(\hat{S}_j) - P_{\hat{i}_{l+1}^j}(\hat{S}_j) \right) - \left( r_{\hat{i}_l^j} P_{\hat{i}_l^j}(\hat{S}_j) - r_{\hat{i}_{l+1}^j} P_{\hat{i}_l^j}(\hat{S}_j) \right) \right] > \\ &\quad (1-q)^2 q^{l-1} d_j \left[ (r_{\hat{i}_l^j} - r_{\hat{i}_{l+1}^j}) + \left( e_{\bar{i}_j^*j} - r_{\hat{i}_l^j} \right) \left( P_{\hat{i}_l^j}(\hat{S}_j) - P_{\hat{i}_{l+1}^j}(\hat{S}_j) \right) \right] \end{aligned}$$

Note that  $r_{\hat{i}_l^j} - r_{\hat{i}_{l+1}^j} > 0$ , and  $P_{\hat{i}_l^j}(\hat{S}_j) - P_{\hat{i}_{l+1}^j}(\hat{S}_j) > 0$  due to the assumption that  $r_{ij} \geq r_{i'j} \Leftrightarrow t_{ij} \geq t_{i'j}, \forall i \neq i' \in \mathbf{I}, j \in \mathbf{J}$ . Then we obtain  $\bar{C}_{\tilde{k}_j\bar{i}_j^*j}(\hat{S}_j) - \bar{C}_{\hat{k}_j\bar{i}_j^*j}(\hat{S}_j) \geq 0$ , which is contradictive to the premise that  $\hat{k}_j$  is the optimal solution. Therefore, we prove  $\hat{i}_1^j \leq \hat{i}_2^j \leq \dots \leq \hat{i}_L^j$ . If there exists a  $i \in \bar{\mathbf{I}} \setminus \hat{k}_j$  and some  $l \in \mathbf{L}$  having  $r_{i'j} > r_{ij}$ , replacing  $\hat{i}_l^j$  with  $i$  in  $\hat{k}_j$  will further reduce cost  $\bar{C}_{\hat{k}_j\bar{i}_j^*j}(\hat{S}_j)$  with a similar argument, which is a contradiction, too. This proves that  $\hat{k}_j = \bar{k}_j^*$ . Finally,  $\hat{S}_j = \bar{S}_j^*$  obviously holds since  $\hat{k}_j = \bar{k}_j^*$ . This completes the proof.  $\square$

By solving problem (24)-(27) for all  $j \in \mathbf{J}$ , a feasible solution to the primal problem can be obtained as follows:

$$Y_{ijl} = \begin{cases} 1 & \text{if } i = \bar{i}_j^*; \\ 0 & \text{otherwise,} \end{cases} \quad Z_{ij} = \begin{cases} 1 & \text{if } i = \bar{i}_j^*; \\ 0 & \text{otherwise,} \end{cases} \quad S_j = \bar{S}_j^*, \quad \forall j \in \mathbf{J}, i \in \mathbf{I}, l \in \mathbf{L}. \quad (33)$$

This algorithm is fast (only taking a solution time of  $\mathcal{O}\left(|\mathbf{J}|\max\{|\mathbf{I}|\ln(|\mathbf{I}|), L\ln(\bar{S}_j)\}\right)$ ). Plugging these feasible solution values into primal objective function (7), we obtain an upper bound to the optimal objective value as well

### **Updating Lagrangian Multipliers**

If the upper bound objective based on (19) is equal to the lower bound (11), then we know this bound is the optimal objective value, and the corresponding feasible solution is an optimal solution. Otherwise, we will iteratively update the multipliers  $\lambda$  and  $\mu$  based on the difference between the current relaxed and feasible solutions so as to obtain better solutions and find the tightest lower bound. A subgradient algorithm is used to complete this iterative procedure as described in Appendix B.



## DISCUSSION OF RESULTS

In this section, a series of numerical examples are presented to test the proposed model and provide useful managerial insights based on the datasets provided in Daskin (1995), i.e., a 49-site network involving 48 continental state capital cities and Washington D.C. The numerical algorithms are coded with MATLAB and implemented on a PC with 3.40 GHz CPU and 8 GB RAM. The LR parameters are set as  $\tau = 1$ ,  $\bar{\tau} = 10^{-3}$ ,  $K = 5$ ,  $\theta = 1.005$ , and  $\bar{K} = 60$ . We assume that each site has both a candidate supplier and a terminal facility, and the parameters are generated as follows. We set  $h_j = h$ ,  $r_{ij} = c_r \delta_{ij}$ , and  $t_{ij} = c_l \delta_{ij}$ ,  $\forall i \in \mathbf{I}, j \in \mathbf{J}$ , where  $h, c_r$  and  $c_l$  are constant coefficients and  $\delta_{ij}$  is the great-circle distance between sites  $i$  and  $j$ . Each  $e_{ij}$  is set to be an independent realization of a uniformly distributed random variable in interval  $[1, 1 + c_e] \cdot \max_{a_{i' \in \mathbf{I}}} r_{i'j}$ , where  $c_e \geq 0$  is a constant scalar. In addition, we assume that each  $f_i$  is the product of the corresponding city population and a scalar  $c_f$ , and each  $d_j$  is the product of the corresponding state population and a scalar  $c_d$ . We set  $L = 3$  for all the cases.

Firstly we test model (7)-(15). Table 2 summarizes the results of 4 instances on the 49-site network by varying failure probability  $q$ , where we set  $h = 100$ ,  $c_r = 0.01$ ,  $c_e = 1$ ,  $c_f = 0.02$ ,  $c_d = 10^{-5}$ , and  $c_l = 10^{-4}$ . The optimal gap between the final feasible objective value and the best relaxed objective is denoted by  $G$ , the solution time is denoted by  $T$ . The optimal system total inventory and the optimal number of selected suppliers are denoted by  $S$  and  $N$ , respectively. Moreover, define

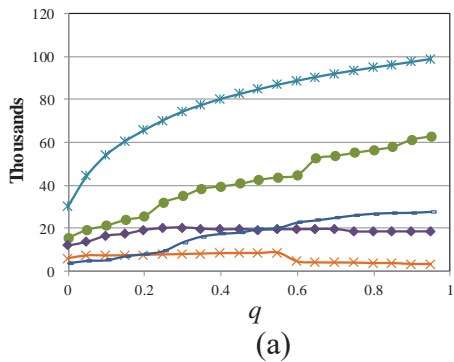
$$P^E := \frac{\sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{l \in \mathbf{L}} d_j (1-q) q^{l-1} P_{ij}(S_j) Y_{ijl}^*}{\sum_{j \in \mathbf{J}} d_j}$$

as the percentage of demand served by the expedited shipments, where  $Y_{ijl}^*$  is the best solution to  $Y_{ijl}$ . Inventory cost, regular shipment cost, marginal expedited shipment, emergency cost, supplier set-up cost, and total system cost are denoted by  $C^H$ ,  $C^R$ ,  $C^M$ ,  $C^E$ ,  $C^F$ , and  $C$ , respectively, as defined in equations (1), (2), (4)-(7).

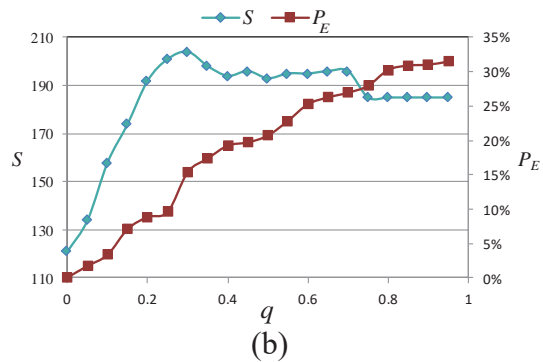
**Table 2** Numerical results for the 49-site network ( $c_f=0.02$ ,  $c_d=10^{-5}$ ,  $c_l=10^{-4}$ ).

#	q	T	G(%)	N	C <sup>F</sup>	S	C <sup>H</sup>	C <sup>R</sup>	C <sup>M</sup>	P <sup>E</sup> (%)	C
1	0.1	3154	0.18	12	21198	165	16500	7323.2	4862.3	3.73	49883.5
2	0.3	3367	0.22	16	34858	204	20400	7932.3	13357	14.9	76547.3
3	0.5	3571	0.41	21	42724	193	19300	8419.9	19478	20.6	89921.9
4	0.7	3653	0.47	24	53379	195	19500	3192.1	27354	26.1	94833.2

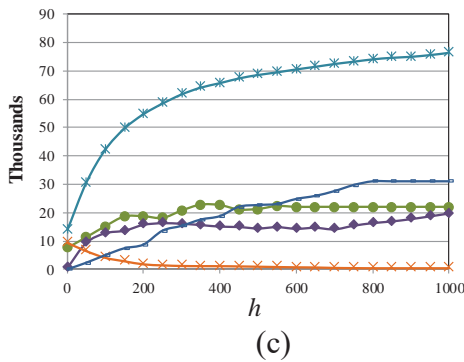
We note in Table 2 that all the instances are solved with  $G < 1\%$  in one hour. This indicates that our approach can solve problem instances of a realistic size to a near-optimum solution with a reasonable solution efficiency. When  $q$  increases,  $C^F$ ,  $C^M$ ,  $P^E$  and  $C$  significantly increase, while  $C^H$  and  $C^R$  seem to increase first and then drop. This indicates that as  $q$  rises, all the cost components will increase at first, leading to a sharp growth of the total cost. Nevertheless, when  $q$  keeps increasing, regular service is unreliable and keeping a higher inventory is no longer an appealing solution. Instead, a higher percentage of expedited shipments are needed to keep the service quality. Meanwhile more suppliers are installed to shorten the shipping distance and offset the increasing expedited shipment cost.



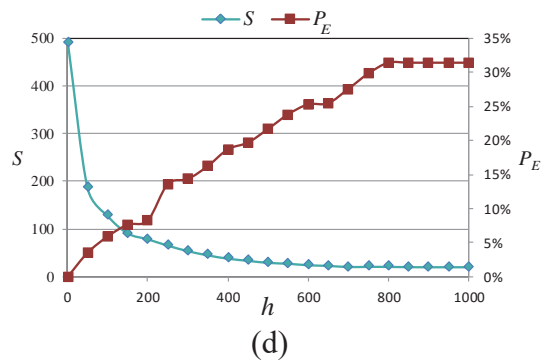
(a)



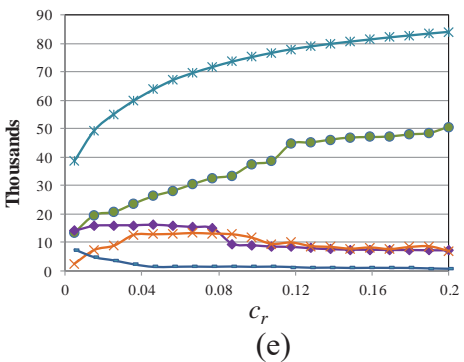
(b)



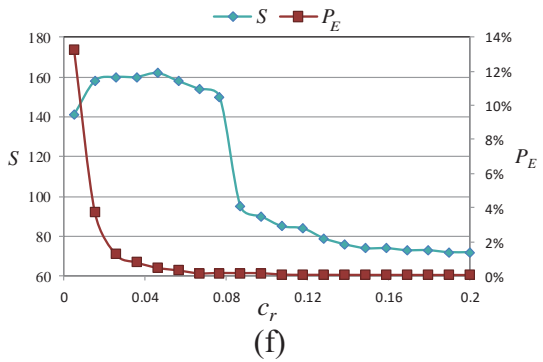
(c)



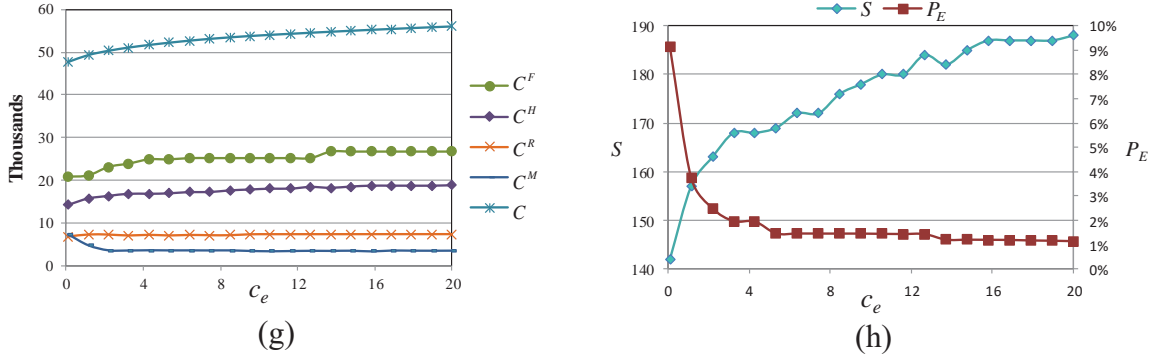
(d)



(e)



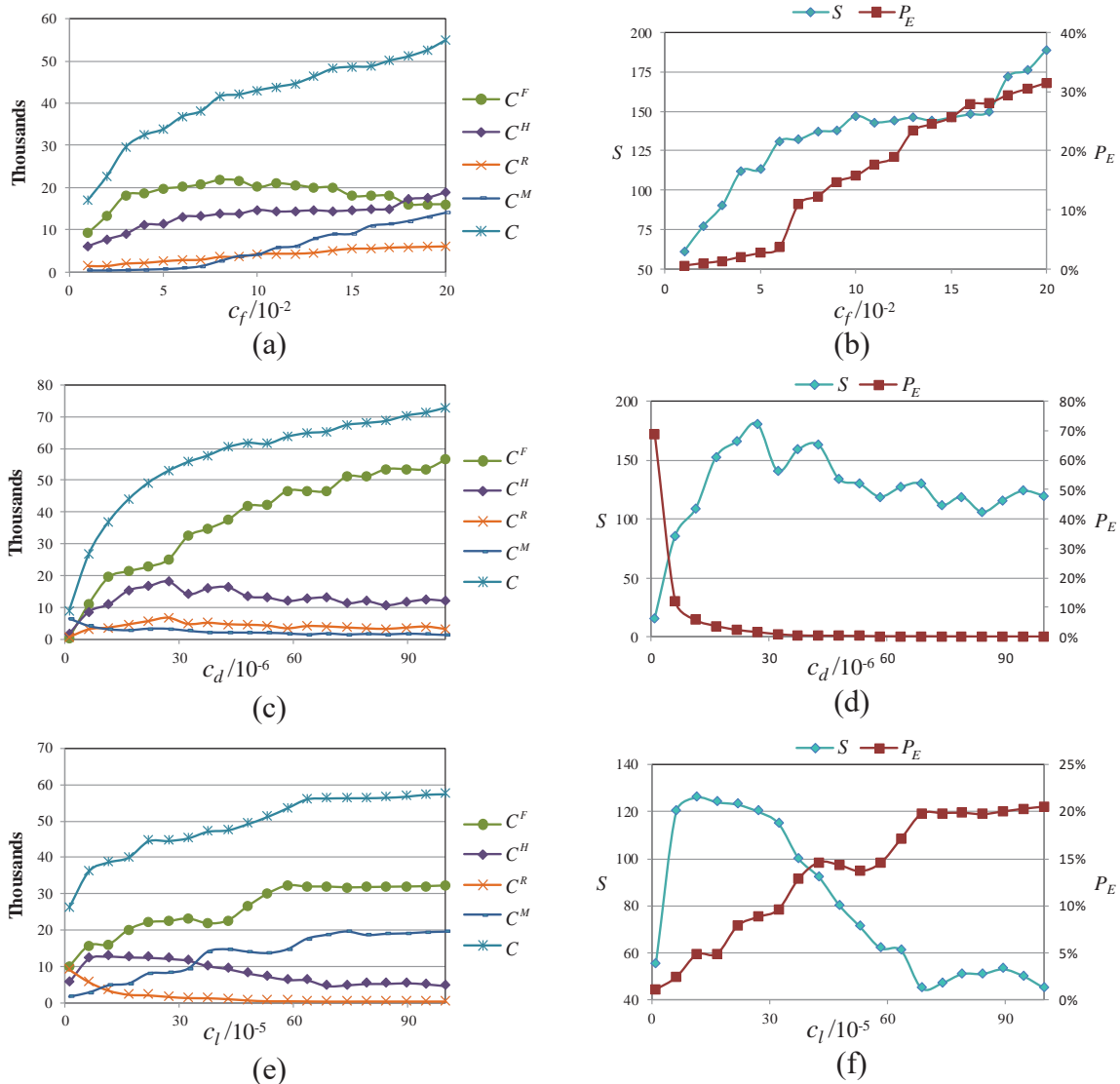
(f)



**Figure 2** Sensitivity analysis on parameters  $q$  ((a) and (b)),  $h$  ((c) and (d)),  $c_r$  ((e) and (f) ) and  $c_e$  ((g) and (h)).

Figure 2 shows four sets of more detailed sensitivity results, where we can see all the cost components, the inventory position  $S$  and the expedited service percentage change over key parameters  $q$ ,  $h$ ,  $c_r$  and  $c_e$ . The default parameters are set as  $q = 0.1$ ,  $h = 100$ ,  $c_r = 0.01$ ,  $c_e = 1$ ,  $c_f = 0.02$ ,  $c_d = 10^{-5}$ , and  $c_l = 10^{-4}$ , and only one parameter value varies in each experiment. In Figure 2 (a), as  $q$  grows from 0 to 1,  $C^F$  and  $C^M$  generally increase, while  $C^H$  increases slightly first and then drops, and  $C^R$  is originally stationary and then drops. Also, the total cost  $C$  has a sharp increase from around 30000 to 80000, then followed by a constant and slower increasing rate as  $q$  becomes larger. Figure 2 (b) shows that  $P^E$  rapidly increases with the growth of  $q$ , while  $S$  rises at first and then drops slowly. It's probably because when  $q$  increases, the regular service from upstream suppliers become increasingly unreliable, and thus the probability of accessing backup suppliers and expedited services grows. Then more suppliers are selected and higher inventory positions are needed to offset the growth of the shipment costs. Furthermore, as  $q$  keeps rising, selecting more suppliers gradually becomes the only cure and higher inventory positions are not as helpful. Meanwhile, expedited shipments gradually take over regular shipments and become the dominating shipment mode. In Figure 2 (c), when  $h$  grows from 1 to 1000,  $C^F$ ,  $C^H$  and  $C^M$  generally increase,  $C^R$  continually drop to almost zero, and  $C$  increases strictly first followed by a slower growth. Figure 2 (d) shows that the increase of  $h$  rapidly brings down  $S$  to a slowing down trend in the tail, while  $P^E$  generally increase. This implies that installing more suppliers does not help much when  $h$  is large, while using more expedited shipments seems more effective in offsetting the inventory cost growth. We can see in Figure 2 (e) and (f) that both  $C$  and  $C^F$  increase with the growth of  $c_r$  from 0.005 to 0.2, while  $P^E$  and  $C^M$  keep decreasing to almost zero.  $C^H$  and  $C^R$  grow slowly at first and then drop, which seems to be consistent with the variation of  $S$  in Figure 2(f). This is probably because as  $c_r$  grows, the regular shipment cost increases, and thus a higher inventory is needed to offset the growth of expedited shipment cost. The higher inventory leads to a continuous drop of the expedited shipments and a slight increase of the

regular shipment cost initially. Nevertheless, as  $c_r$  continues to grow (the shipment cost correspondingly increases), building more facilities becomes a better solution to offset the shipment cost growth, which finally brings down the total inventory. In Figure 2(g) and (h), as  $c_e$  increases,  $S$  increases significantly and  $P^E$  drops sharply, but the total cost and all its components do not change too much. This indicates that expedited shipments actually cause little increase in overall cost under the optimal inventory management and transportation configuration strategy, and thus it is an appealing strategy to combine both regular and expedited shipments to reduce the system cost and increase the system reliability.



**Figure 3** Sensitivity analysis on parameters  $c_f$  ((a) and (b)),  $c_d$  ((c) and (d)), and  $c_l$  ((e) and (f)).

Besides, we also tested how the results vary with the magnitudes of supplier installation cost (in terms of  $c_f$ ), customer demand rates (in terms of  $c_d$ ) and lead times (in terms of  $c_l$ ), as

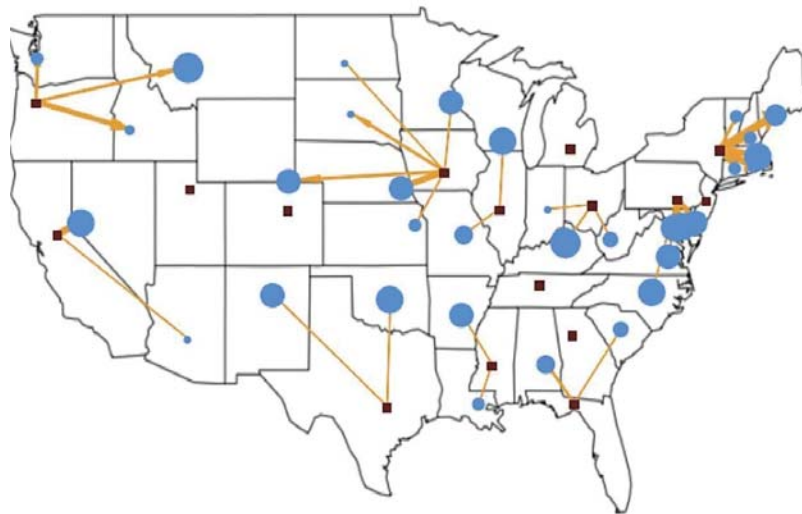
shown in Figure 3. It can be seen in Figure 3 (a) and (b) that, when  $c_f$  initially grows, all cost components and the total cost generally increase. As  $c_f$  keeps increasing,  $C^F$  increases first and then turns down and  $C^R$  flattens out. This is probably because that, the growth of supplier installation cost likely decreases the number of suppliers, which consequentially raises the shipment distance, cost and leading time. Nevertheless, when  $c_f$  continues to increase, the number of suppliers is so small that increasing the inventory position alone is not enough to keep the service quality and thus using more expedited services seems necessary. Figure 3 (c) shows that the increase of  $c_d$  initially raises all cost components except  $C^R$ . Then  $C^F$  keeps increasing but  $C^H$  and  $C^R$  decrease slightly with a slowing down trend in the tails. We also see in Figure 3 (d) that  $S$  increases quickly initially and then flattens out, while  $P^E$  significantly decreases to almost zero. It is probably because that expedited services are more suitable for the cases with low demands when the suppliers are scattered and high inventory positions are unnecessary. Nevertheless, as demands increase, regular shipments will become the main shipment mode instead. Figure 3 (e) and Figure 3 (f) show that as  $c_l$  grows,  $C^F$ ,  $C^M$  and  $P_E$  increase while  $C^R$  drop, and  $C^H$  and  $S$  increases at the beginning then drops. This shows that growth of regular shipment delay will cause the decreasing of inventory positions and consequently increasing the expedited service seems to become a better solution to improve the service quality.

We also test how the variations of  $q$  affect the optimal suppliers' layouts and terminal facilities' assignments. Again we set  $h = 100$ ,  $c_r = 0.01$ ,  $c_e = 1$ ,  $c_f = 0.02$ ,  $c_d = 10^{-5}$ , and  $c_l = 10^{-4}$  and each sub-figure in Figure 4 shows the optimal layout for a different  $q$  value among 0, 0.3 and 0.6. In each sub-figure, the squares denote the selected supplier locations and the circles represent the terminal facilities with their area sizes proportional to the base-stock positions. The arrows show how the selected suppliers are assigned to each facility with each arrow's width proportional to the percentage of the corresponding expedited shipments and different colors denoting different levels, i.e., yellow for the first level, green for the second level and pink for the third level.

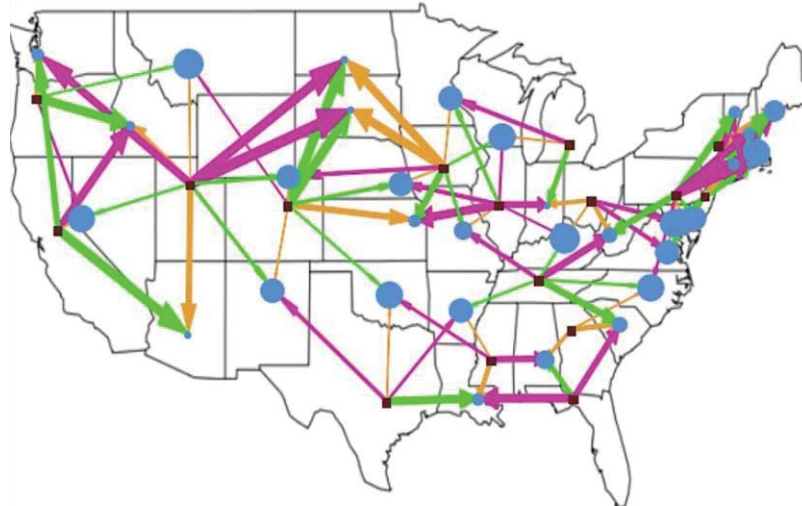
In Figure 4 (a), when the facility disruption risks are ignored ( $q = 0$ ), the problem would be similar to the integrated model proposed by Li (2013), in which all suppliers are assumed to be reliable and is considered as the benchmark case in our problem. By comparing Figure 4 (a) and (b), we note that as failure probability  $q$  increases from 0 to 0.3, 5 more supplier installations are selected and more frequent expedited shipments are needed, in particular for the facilities that are far away from their suppliers. This implies that when primary supplier becomes unreliable and backup facilities are needed, a proper solution is selecting more suppliers to reduce the overall shipment cost. Generally, the expedition

percentage increases with the assignment level, and facilities served by farther suppliers hold higher inventory positions.

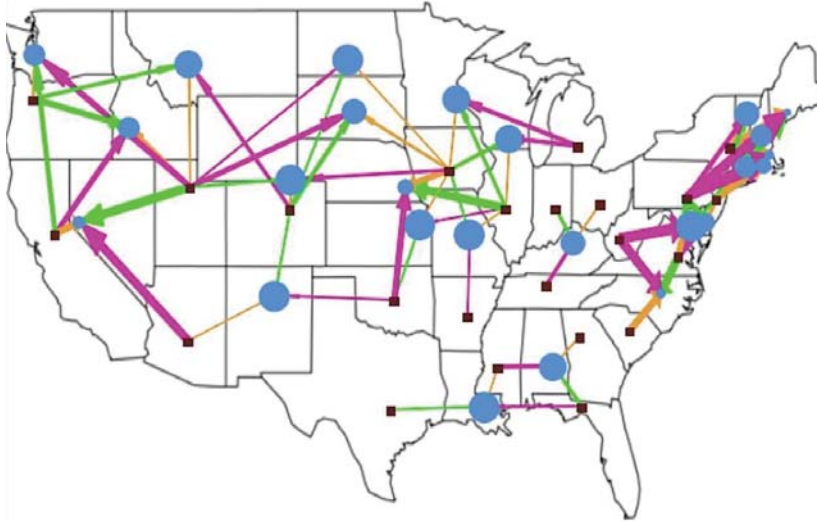
As  $q$  further rises to 0.6, we can see that many more suppliers are installed and the inventory positions of facilities increase generally, but the expedition percentage generally drops. This is probably because that more unreliable suppliers lead to a higher probability of activating services from higher-level suppliers and all the supplier's simultaneous disruption, such that more suppliers are needed to be installed to maintain essential supplying service. Interestingly, we can see that 7 more suppliers are installed and most of them (5 suppliers) are located in the northeastern areas with higher population and more facilities. Therefore, under the optimal planning, facilities with more demand may be met first to reduce the shipment costs as much as possible.



(a)  $q=0\%$



(b)  $q=30\%$



(c)  $q=60\%$

**Figure 4** Optimal network layouts for different  $q$ .





## CONCLUSIONS

This paper proposes a reliable logistics network design framework that integrates one-time location selection decisions and long-term operational strategies of shipment expedition and inventory management in an uncertain environment. We formulated a nonlinear mixed integer model to investigate this problem. The major contributions of the paper to the literature are that (i) the expedited transportation decisions have been integrated into the logistics network design framework, (ii) the possibility of supplier failures are considered in the design of integrated logistics system and (iii) a customized solution approach has been developed to efficiently solve this integrated logistics system design problem. Since the proposed model is difficult to solve (nonlinear integer programming problem), a customized solution approach is created based on Lagrangian relaxation. This solution approach is able to solve this model efficiently and accurately, as evidenced in a set of numerical experiments. These experiments also revealed a number of managerial insights on how the parameter values affect the optimal design results. For example, we found that the optimal network layout and the related cost components can vary significantly with different failure probabilities. We also found that when the upstream suppliers become unreliable, expedited shipments will be used more frequently in relation to regular shipments. The increased expedited shipments can ensure the reliability of the integrated logistics system without an excessive increase of the operating costs. The optimal network layout also shows that when the supplier failure probability increases, areas with more customer demands tend to have a high priority to receive services, despite the higher transportation costs. For this project, we created a user friendly web-based interface for biofuel logistics network design available at <http://biofuel.msstate.edu/>.

## RECOMMENDATIONS

This study established an open-ended design framework that can be extended in several directions. First, it will be interesting to examine the effect of more general demand and lead time distributions other than an independent Poisson distribution. Second, this study assumes that the expedited shipment is “non-fallible”, which may be not realistic for some applications where the suppliers may suffer serious disasters causing both services failed. Third, it might be worth considering positive lead times even for expedited shipments for some applications where the expedited delivery time is still noticeable. Finally, this study is set as a two-echelon system where the locations and demand of all the terminal facility are considered to be in the basic conditions and independent of the network design results. However, in some other applications, a more general structure is needed for terminal facilities distribution planning. Extending the current two-echelon network to a more general structure will be an interesting research topic.

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## APPENDIX

### A Bisectioning algorithm to solve (22)

**Step BS0:** For a given set of  $k \in \mathbf{K}, i' \in \mathbf{I}, j \in \mathbf{J}$ , initialize two search bounds of as  $S_L := 0$  and  $S_U := \bar{S}_j$ , and the difference slope of  $C_{ki'j}(S_j)$  defined in equation (22) with respect to  $S_L$  as:

$$G_L := h_j + d_j \sum_{l \in \mathbf{L}} (e_{ij} - r_{ij}) (1-q) q^{l-1} (P_{ij}(0) - P_{ij}(1))$$

, and that with respect to  $S_U$  as:

$$G_U := h_j + d_j \sum_{l \in \mathbf{L}} (e_{ij} - r_{ij}) (1-q) q^{l-1} (P_{ij}(\bar{S}_j) - P_{ij}(\bar{S}_j - 1)).$$

**Step BS1:** If  $G_L, G_U \geq 0$ , set optimal  $S^* := S_L$ . Or if  $G_L, G_U < 0$ , set optimal  $S^* := S_U$ . Or if  $S_U - S_L \leq 1$ , set  $S^* := S_L$  if  $C_{ki'j}(S_L) \leq C_{ki'j}(S_U)$  or  $S^* := S_U$  otherwise. If  $S^*$  is found, go to Step BS3.

**Step BS2:** Set the middle point  $S_M := \lfloor (S_L + S_U) / 2 \rfloor$ . Calculate the slope at  $S_M$  as:

$$G_M := h_j + d_j \sum_{l \in \mathbf{L}} (e_{ij} - r_{ij}) (1-q) q^{l-1} (P_{ij}(0) - P_{ij}(1)) \text{ if } S_M = 0 \text{ or}$$

$$G_M := h_j + d_j \sum_{l \in \mathbf{L}} (e_{ij} - r_{ij}) (1-q) q^{l-1} (P_{ij}(S_M) - P_{ij}(S_M - 1)) \text{ otherwise. If } S_M > 0, \text{ set}$$

$S_U = S_M$  and  $G_U = G_M$ ; otherwise, set  $S_L = S_M$  and  $G_L = G_M$ . Go to Step BS1.

**Step BS3:** Return  $S_j^*$  and  $C_{ki'j}(S_j^*)$  as the optimal solution and the optimal objective value to problem (22), respectively.

### B Subgradient algorithm to update Lagrangian multipliers

**Step SG0:** Set initial multipliers  $\lambda_{ij}^0 = \mu_{ij}^0 = 0, \forall i \in \mathbf{I}, j \in \mathbf{J}$ . Set an auxiliary scalar  $0 < \tau \leq 2$  and an iteration index  $k := 0$ . Set the best known upper bound objective  $C := +\infty$ .

**Step SG1:** Solve relaxed problem  $\Delta(\lambda^k, \mu^k)$  with the solution approach proposed in Section 4.1, and  $\{X_i^k\}, \{Y_{ij}^k\}, \{Z_{ij}^k\}, \{S_j^k\}$  denote its optimal solution. If the objective value of  $\Delta(\lambda^k, \mu^k)$  does not improve in  $K$  consecutive iterations (where  $K$  is a predefined number, e.g., 5), we update  $\tau = \tau / \theta$ , where  $\theta$  is a contraction ratio greater than 1.

**Step SG2:** Adapt  $\{X_i^k\}, \{Y_{ij}^k\}, \{Z_{ij}^k\}, \{S_j^k\}$  to a set of feasible solution with the algorithm described in Section 4.2. Set  $c$  equal to this feasible objective if  $c$  is greater than it.

**Step SG3:** Calculate the step size as follows

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<sup>1</sup> In the denominator of this formula, we use the absolute value instead of the squared Euclidean norm, because we found it helps improve the solution efficiency.

$$t_k := \frac{\tau(C - \Delta(\boldsymbol{\lambda}^k, \boldsymbol{\mu}^k))}{\sum_{i \in \mathbf{I}, j \in \mathbf{J}} \left( \left( \sum_{l \in \mathbf{L}} Y_{ijl}^k - X_i^k \right)^+ + (Z_{ij}^k - X_i^k)^+ \right)}.$$

Then update multipliers as follows

$$\lambda_{ij}^{k+1} = \left[ \lambda_{ij}^k + t_k \left( \sum_{l \in \mathbf{L}} Y_{ijl}^k - X_i^k \right) \right]^+, \mu_{ij}^{k+1} = \left[ \mu_{ij}^k + t_k (Z_{ij}^k - X_i^k) \right]^+, \forall i \in \mathbf{I}, j \in \mathbf{J}.$$

**Step SG4:** Terminate this algorithm if (i) optimality gap  $\frac{C - \Delta(\boldsymbol{\lambda}^k, \boldsymbol{\mu}^k)}{C} \leq \varepsilon$  where  $\varepsilon$  is a pre-specified error tolerance, (ii)  $\tau$  is smaller than a minimum value  $\bar{\tau}$ , or (iii)  $k$  exceeds a maximum iteration number  $\bar{K}$ ; return the best feasible solution as the near-optimum solution. Otherwise  $k = k + 1$ , and go to Step SG1.