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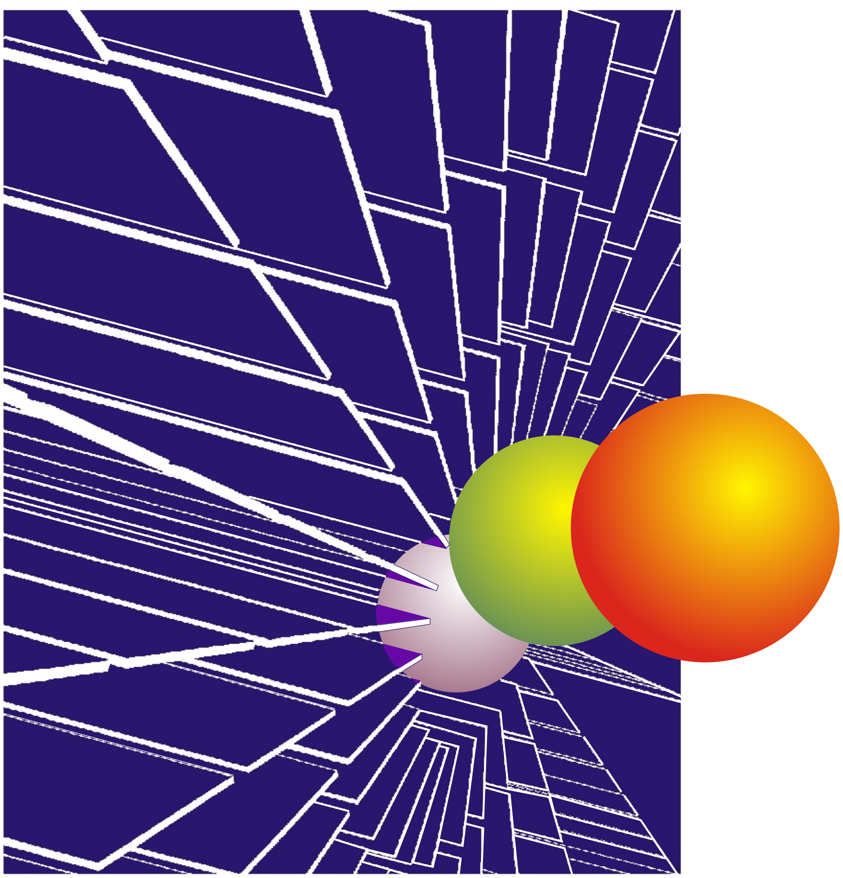
RDT 04-001

Strengthening of Martin Springs Outer Road Bridge, Phelps County

RI 02-007



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16. Abstract
This report presents the use of externally bonded fiber reinforced polymers (FRP) laminates and near surface mounted FRP bars for the flexural strengthening of a concrete bridge. The bridge selected for this project is a three-span simply supported reinforced concrete slab with no transverse steel reinforcement, load posted and located on Martin Spring Outer Road in Phelps County, MO. The original construction combined with the presence of very rigid parapets caused the formation of a wide longitudinal crack which resulted in the slab to behave as two separate elements. The structural behavior was verified implementing the bridge model in a FEM program. The bridge analysis was performed for maximum loads determined in accordance to AASHTO 17th edition. The strengthening scheme was designed in compliance with the ACI 440.2R-02 design guide for externally bonded FRP materials, to avoid further cracking and such that the transverse flexural capacity be higher than the cracking moment. Both FRP strengthening techniques were easily implemented and showed satisfactory performance. An initial load test, to evaluate the structural behavior, was performed prior the strengthening following the AASHTO specifications. The retrofitting of the structure was employed in the summer of 2002, after the major cracks were injected to allow continuity in the cross section. Once the repair work was completed, another load test, identical in procedure to the previous one, was performed to evaluate the efficiency of the strengthening. As a result of this testing, and a report based on this testing by an independent consultant, the load posting of the bridge was removed. A third and last load test was performed in summer 2003, 12 months after the strengthening was finished, to evaluate the long term behavior of the bridge and to investigate whether any type of degradation occurred during the elapsed period. Comparison of the results of the last two load tests showed no significant degradation occurred during the 12 months period. Further, no more cracking was noted in the concrete deck as a result of the strengthening program.

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CENTER FOR INFRASTRUCTURE ENGINEERING STUDIES

STRENGTHENING OF MARTIN SPRINGS
OUTER ROAD BRIDGE, PHELPS COUNTY

by

Nestore Galati
Paolo Casadei
Antonio Nanni

January 30, 2004

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STRENGTHENING OF MARTIN SPRINGS OUTER ROAD BRIDGE, PHELPS COUNTY

PREPARED FOR THE

MISSOURI DEPARTMENT OF TRANSPORTATION

IN COOPERATION WITH THE

U.S. DEPARTMENT OF TRANSPORTATION

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UNIVERSITY OF MISSOURI – ROLLA

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STRENGTHENING OF MARTIN SPRINGS OUTER ROAD BRIDGE, PHELPS COUNTY

EXECUTIVE SUMMARY

This report presents the use of externally bonded fiber reinforced polymers (FRP) laminates and near surface mounted FRP bars for the flexural strengthening of a concrete bridge. The bridge selected for this project is a three-span simply supported reinforced concrete slab with no transverse steel reinforcement, load posted and located on Martin Spring Outer Road in Phelps County, MO. The original construction combined with the presence of very rigid parapets caused the formation of a wide longitudinal crack which resulted in the slab to behave as two separate elements. The structural behavior was verified implementing the bridge model in a FEM program.

The bridge analysis was performed for maximum loads determined in accordance to AASHTO 17th edition. The strengthening scheme was designed in compliance with the ACI 440.2R-02 design guide for externally bonded FRP materials, to avoid further cracking and such that the transverse flexural capacity be higher than the cracking moment. Both FRP strengthening techniques were easily implemented and showed satisfactory performance. An initial load test, to evaluate the structural behavior, was performed prior the strengthening following the AASHTO specifications.

The retrofitting of the structure was employed in the summer of 2002, after the major cracks were injected to allow continuity in the cross section. Once the repair work was completed, another load test, identical in procedure to the previous one, was performed to evaluate the efficiency of the strengthening. As a result, the load posting of the bridge was removed. A third and last load test was performed in summer 2003, 12 months after the strengthening was finished, to evaluate the long term behavior of the bridge and to investigate whether any type of degradation occurred during the elapsed period. Comparison of the results of the last two load tests showed no significant degradation occurred during the 12 months period. Further, no more cracking was noted in the concrete deck as a result of the strengthening program.

ACKNOWLEDGMENTS

The project was made possible with the financial support received from the Missouri Department of Transportation, UMR - University Transportation Center on Advanced Materials. Master Contractors installed the FRP systems. Hughes Brothers Inc. and Master Builders provided the FRP materials.

Phelps County's commissioners and staff provided the opportunity and helped in the implementation.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOTATIONS

C_E	environmental reduction factor
E_c	longitudinal modulus of elasticity of concrete, psi
E_f	longitudinal modulus of elasticity of the longitudinal FRP reinforcement, psi
E_s	longitudinal modulus of elasticity of the steel reinforcement, psi
f'_c	concrete compressive strength, psi
f^*_{fu}	guaranteed tensile strength, ksi
f_{fu}	design tensile strength, ks
f_y	yield stress of the steel shear reinforcement, ksi
I_g	gross moment of inertia of the section, in^4
I	live load impact factor
L	span length, ft
M_{cr}	cracking moment of the section, kip-ft
M_n	ultimate moment capacity, kip-ft
M_u	design moment demand, kip-ft
P_i	load on one wheel of the HS20-44 loading truck, kip
V_c	concrete contribution to the shear capacity, kip
V_f	FRP reinforcement contribution to the shear capacity, kip
β_d	modification factor based on the ratio of the modulus of the FRP reinforcement to that of steel reinforcement
ϕ	strength reduction factor
ϕM_n	design moment capacity, kip-ft
ε^*_{fu}	guaranteed ultimate strain
ε_{fu}	design ultimate strain
ρ_f	reinforcement ratio of the FRP-reinforced section
ω_D	total dead load, lb/ft
ω_u	ultimate values of bending moments and shear forces, lb/ft

1. INTRODUCTION

1.1 Objectives/technical approach

The overall objective of this research project was to demonstrate the feasibility of externally bonding fiber reinforced polymer (FRP) reinforcement for the flexural strengthening of concrete bridge structures.

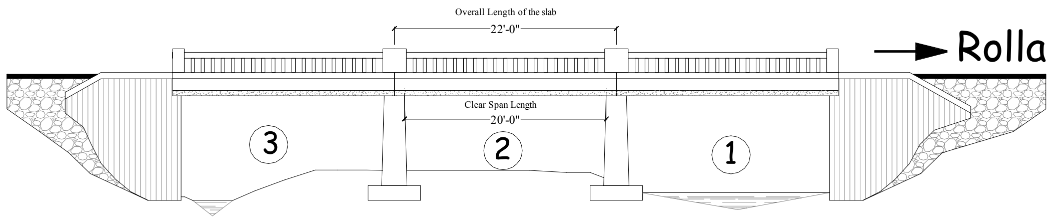
The bridge selected for demonstration of the FRP strengthening technology is located on old Route 66, now Martin Springs Outer Road, in Phelps County, Missouri (see Figure 1-1-a). This bridge was commissioned in 1926 and was originally on a gravel road. In 1951, the last miles of US Route 66 through Phelps County were concrete paved, and in 1972, Route 66 was replaced by I-44. Commissioning of I-44 led to a significant decrease in traffic along Route 66. Load posting of this bridge (a load restriction posting of S-16 trucks over 13 tons (11.79 tons in SI units) at 15 mph (24.14 km/hr) (see Figure 1-1- b), except for single unit trucks H-20 weight limit to 19 tons (17.24 tons in SI units), and all other trucks weight limit 30 tons (27.21 tons in SI units)), was approved around 1985 and had a significant impact on the local economy also when I-44 is closed for accidents or other reasons, heavy unauthorized traffic has to cross this bridge posing safety concerns. It is anticipated that the load posting could be removed as the result of the proposed strengthening scheme.

a) Side-view of the Bridge

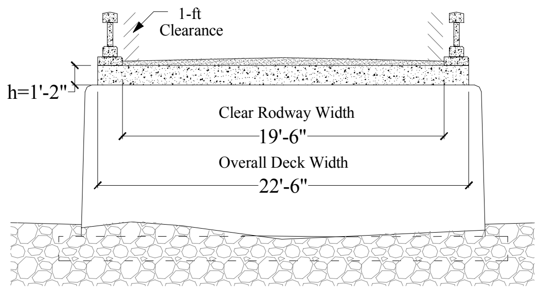
b) Load Posting Prior to Strengthening

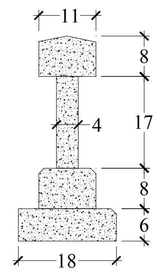
Figure 1-1 – Martin Springs Bridge

This bridge is a three-span simply supported reinforced concrete slab. The total bridge length is 66 ft (20.12 m) and the total width of the deck is 22.5 ft (6.86 m). Figure 1-2 shows a detailed geometry of the bridge. Based on visual and Non Destructive Testing (NDT) evaluation, it was determined that the superstructure is a solid concrete slab 14 in (35.56 cm) thick, running from pier to pier, the longitudinal reinforcement is made of #8 (∅25.4 mm) bars spaced at 5 in (12.7 cm) on centers, and no transverse reinforcing is present. From cores (cylinders 3 in×6 in, 7.62 cm× 15.24 cm) , the average compressive strength of the concrete was measured to be 4100 psi (28.27 MPa); the yield of the steel was also tested on one bar sample, and resulted to be 32 ksi (220.63 MPa).



a) Side View





b) Section View c) Detail of Parapet

Figure 1-2 - Martin Spring Bridge Geometry

This bridge represents an ideal case for the application of FRP composites since its deficiency is due primarily to a lack of transverse reinforcing steel (Stone et al. 2002, Alkhrdaji et al. 1999, Nanni et al. 1997). Upon inspecting the bridge, the area where the FRP was to be installed showed excellent surface integrity. A single crack extends longitudinally through the three spans along the centerline. The crack was over 1in (2.54 cm) wide at some locations (see Figure 1-3). There was no significant cracking at any other location and only minor corrosion of the reinforcement was detected.

Figure 1-3 - Soffit Slab Longitudinal Crack

This demonstration consisted of four major tasks, namely:

1. Design of the required transversal reinforcement;
2. On-site load tests before and after strengthening to demonstrate the effectiveness of the FRP reinforcement;
3. Field construction; and
4. Development of a Finite Element Model to validate the experimental data collected in the field.

It is envisioned that this strengthening technique will lead to a bridge strengthening protocol for consideration by MoDOT for future applications.

1.2 Background & Significance of Work

1.2.1 FRP Composites

Fiber-reinforced polymer (FRP) material systems, composed of fibers embedded in a polymeric matrix, exhibit several properties suitable for their use as structural reinforcement (Iyer and Sen 1991, JSCE Sub-Committee on Continuous Fiber Reinforcement 1992, White 1992, Neale and Labossiere 1992, Nanni 1993, Nanni and Dolan 1993, ACI Committee 440 1996, El-Badry 1996, Nanni 1997, Alkhrdaji et al. 1999, De Lorenzis et al. 2000, Nanni 2001). FRP composites are anisotropic and characterized by excellent tensile strength in the direction of the fibers. They do not exhibit yielding, but instead are elastic up to failure. FRP composites are corrosion resistant, and therefore should perform better than other construction materials in terms of weathering behavior.

1.2.1.1 Externally Bonded Repair for Flexural Strengthening

Structural retrofit work has come to the forefront of industry practice in response to the problem of aging infrastructure and buildings worldwide. This problem, coupled with revisions in structural codes to better accommodate natural phenomena, creates the need for the development of successful structural retrofit technologies. The most important characteristics of repair-type work are: predominance of labor and shut-down costs as opposed to material costs, time and site constraints, long-term durability, difficulty in methodology selection and design, and effectiveness evaluation. An effective method for upgrading reinforced concrete (RC) members (prestressed and non-prestressed) is plate bonding. In Germany and Switzerland during the mid-80's, replacement of steel with FRP plates began to be viewed as a promising improvement in externally bonded repair. The advantages of FRP versus steel for the reinforcement of concrete structures include lower installation costs, improved corrosion resistance, on-site flexibility of use, and small changes in member size after repair. Of all countries, Japan has seen the largest number of field applications using bonded FRP composites (Nanni 1995).

1.2.1.2 Near-Surface Mounted (NSM) FRP for Flexural Strengthening

The use of Near-Surface Mounted (NSM) FRP bars or tapes is emerging as a valid alternative to externally bonded FRP laminates. Embedment of the bars or tapes is achieved by grooving the surface of the member to be strengthened along the desired direction. The groove is filled half way with epoxy paste, the FRP bars/tapes are then placed in the groove and lightly pressed, so forcing the paste to flow around the bar and fill completely between the bar and the sides of the groove. The groove is then filled with more paste and the surface is leveled. The use of NSM FRP technique is an attractive method for increasing the flexural and the shear strength of deficient RC members and, in certain cases, can be more convenient than using FRP laminates (Alkhrdaji et al. 1999, De Lorenzis et al. 2000, Nanni et al. 2001). The NSM FRP technique does not require any surface preparation work and requires minimal installation time compared to FRP laminates. Another advantage is the feasibility of anchoring the bars or tapes into members adjacent to the one to be strengthened. In addition, this technique becomes particularly attractive for strengthening in the negative moment regions, where external reinforcement would be subjected to mechanical and environmental damage and would require protective cover which could interfere with the presence of floor finishes.

2. BRIDGE ANALYSIS

2.1 Load Combinations

For the structural analysis of the bridge the ultimate values of bending moments and shear forces are computed by multiplying their nominal values by the dead and live factors and by the impact factor according to AASHTO (2002) as shown in Eq. (2.1):

$$\omega_u = 1.3\left[\beta_d D + 1.67\left(1+I\right)L\right] \tag{2.1}$$

where D is the dead load, L is the live load, β_d=1.0 as per AASHTO (2002) Table 3.22.1A, and I (maximum 30%) is the live load impact calculated as follows:

$$I = min\left\{\frac{50}{L+125}, 0.3\right\} = min\left\{\frac{50}{22+125}, 0.3\right\} = 0.3 \tag{2.2}$$

and L=22 ft (6.70 m) represents the span length from center to center of supports.

2.2 Design Truck and Design Lanes

Prior to the design of the strengthening, the analysis of the bridge was conducted by considering a HS20-44 truck load (which represents the design truck load as per AASHTO (2002) Section 3.7.4) having geometrical characteristics and weight properties shown in Figure 2-1 and Figure 2-2. The loading conditions required to be checked are laid out in Figure 2-3.

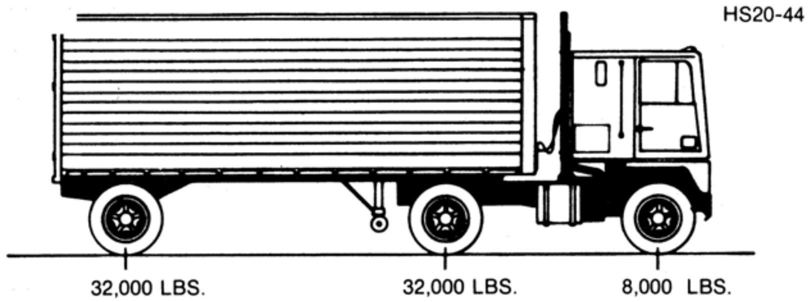


Figure 2-1 - Truck Loading

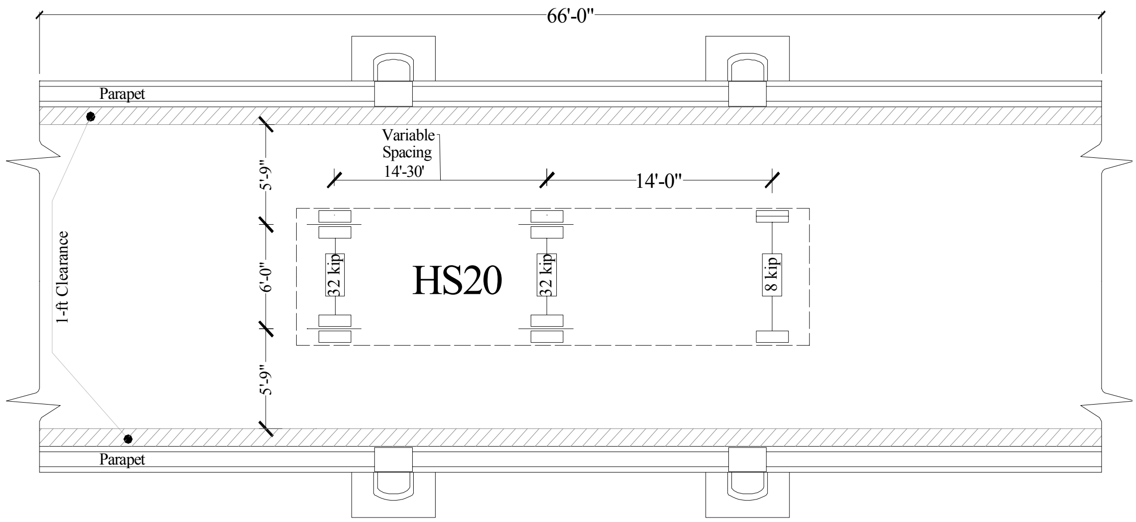
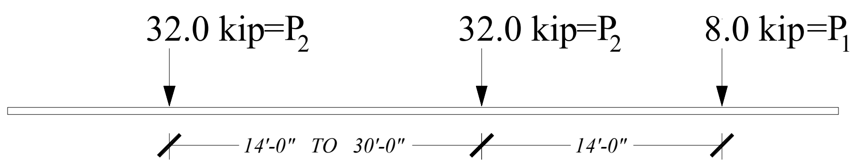
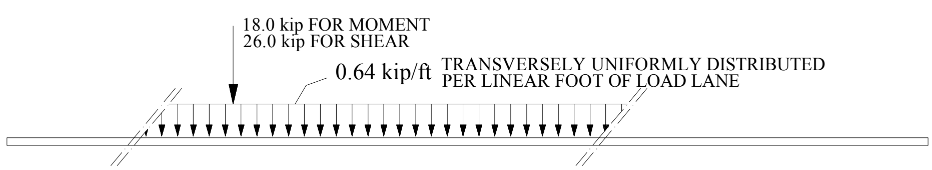


Figure 2-2 - Truck Load and Truck Lanes



a) Design Truck (HS20-44)



b) Design Lane

Figure 2-3 - Loading Conditions

Figure 2-3a represents the HS20-44 design truck already described in Figure 2-2. Given the specific bridge geometry, the worst loading scenario, causing maximum moment at mid span (see Figure 2-4) and shear at the support (see Figure 2-5), is obtained for the minimum spacing of 14.0 ft (4.27 m) between the two rear axles.

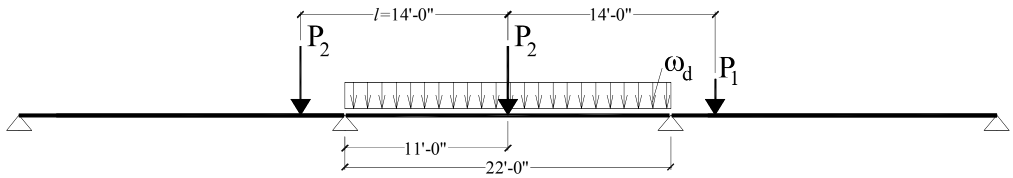


Figure 2-4 - Flexural Design Configuration

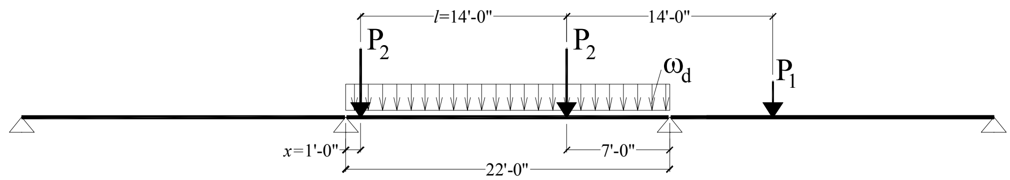


Figure 2-5 - Shear Design Configuration

The design lane loading condition (AASHTO, 2002 Section 3.6) consists of a load of 640 lbs per linear foot (9.35 kN/m), uniformly distributed in the longitudinal direction with a single concentrated load so placed on the span as to produce maximum stress. The concentrated load and uniform load is considered to be uniformly distributed over a 10'-0" (3.05 m) width on a line normal to the center lane of the lane. The intensity of the concentrated load is represented in Figure 2-3b for both bending moments and shear forces. This load shall be placed in such positions within the design lane as to produce the maximum stress in the member.

2.3 Slab Analysis

The deck is considered to be a one-way slab, disregarding the contribution of the parapets. For simplicity, the deck has been studied considering the overall width of the transversal cross section.

The dead load was computed considering the self-weight of the concrete slab plus the permanent weight of the top layer of asphalt. The weight of parapets has been computed according to the geometrical properties of Figure 1-2c and, for simplicity, distributed throughout the width of the slab.

Table 2-1 presents a summary of these values.

Computations for the design lane and the design truck load have been carried out and it has been found that the design truck load is the controlling loading condition.

Table 2-1 -Dead Load (1 k/ft = 14.7 kN/m)

Slab Self-Weight	$\omega_{d1} = (0.15\,k/ft^3)(270/12\,ft)(14/12\,ft) =$	3.94	k/ft
Asphalt Weight	$\omega_{d2} = (0.14\,k/ft^3)(234/12\,ft)(6/12\,ft) =$	1.37	k/ft
Parapet Weight	$\omega_{d3} = \left(0.15k/ft^3\right)\left[\left(326.49/12^2\,ft^2\right)\times 2\right] =$	0.68	k/ft
Total Dead Load	$\omega_D = \omega_{d1} + \omega_{d2} + \omega_{d3} =$	5.99	k/ft

For the flexural analysis, the critical loading condition corresponds to the case when the truck has one of its rear axles at the mid-span of the member (see Figure 2-4). The factored ultimate moment demand is computed for the entire slab in Eq.(2.3):

$$M_u = \frac{1.3\times\omega_D L^2}{8} + \frac{1.3\times1.67\times1.3\times P_2 L}{4} \tag{2.3}$$

$$M_u = \frac{1.3(5.99)(22)^2}{8} + \frac{1.3\times1.67\times1.3\times(32)(22)}{4} = 968\ k-ft\ \ (1312\ kN-m) \tag{2.4}$$

For the shear analysis, the critical loading condition is when one rear axle is closer to one support and the other is 14 ft (4.27 m) away over the span (see Figure 2-5). The factored ultimate shear demand is computed for the entire slab in Eq. (2.5):

$$V_u = \frac{1.3\times\omega_D L}{2} + 1.3\times1.67\times1.3\left(P_2 + P_2 - \frac{P_2(l+x)+P_2 x}{L}\right) \tag{2.5}$$

$$V_u = \frac{1.3(5.99)(22)}{2} + 1.3\times1.67\times1.3\left(32+32-\frac{32(15)+32(1)}{22}\right) = 200.6kip\ \ (892kN) \tag{2.6}$$

The bridge geometry and material properties are reported in

Table **2-2** along with the computed nominal flexural and shear capacities based on conventional RC theory. Since both ϕM_n and ϕV_n are larger than M_u and V_u respectively, no strengthening is needed for load posting removal.

The cracking moment of a unit strip has been computed (see Eq.(2.7)) to design a strengthening scheme able to ensure that $\phi M_{n,transv.}$ is larger than or equal to the cracking moment.

$$M_{cr} = \frac{7.5\sqrt{f_c'}I_g}{h/2} = \frac{7.5\sqrt{4100}(2744)}{7} = 15.7\,k-ft/ft\ \ (21kN-m/m) \tag{2.7}$$

Where I_g represents the gross moment of inertia of the concrete cross section with b = 12 in (30.48 cm) and h = 14 in (35.56 cm).

Table 2-2 - Flexural and Shear Capacity

b in [cm]	h in [cm]	d in [cm]	A_s in^2 [cm^2]	ϕM_n k-ft [kN-m]	ϕV_n kip [kN]	M_u k-ft [kN-m]	V_u kip [kN]
270 [685.8]	14 [35.5]	12.7 [32.4]	42.7 [275.5]	1229 [1666]	370 [1646]	968 [1312]	200.6 [892]

3. BRIDGE STRENTHENING

The objective of the strengthening is to provide the necessary transverse reinforcement so that the load posting can be removed. Since no reinforcement was provided in the transverse direction, minimal strengthening is needed to ensure that the transverse design moment capacity is larger or equal to the cracking moment computed in Eq.(2.7), in order to avoid further crack openings and deterioration of the concrete due to water percolation through the cracks.

Two commercially available carbon FRP systems have been adopted: (1) externally bonded Carbon Fiber Reinforced Polymers (CFRP) laminates installed by manual wet lay-up, and (2) Near-surface mounted CFRP rectangular bars embedded in pre-made grooves and bonded in place with an epoxy-based paste. The main difference between these two techniques belongs to the surface preparation necessary before the application of the strengthening that in turn depends upon the conditions of the concrete substrate on which the laminates and bars are bonded.

Before surface preparation for FRP application, the central crack was repaired in order to re-establish material continuity and assure no water percolation through the crack. For this purpose, the crack was sealed using an epoxy-paste and then injected with a very low viscosity resin as shown in Figure 3-1a-b. Once the crack had been repaired, FRP was applied following the design provisions.

The design of both FRP technologies is carried out according to the principles of ACI 440.2R-02 (ACI 440 in the following). The properties of the FRP composite materials used in the design are summarized in Table 3-1 and

Table 3-2. The reported FRP properties are guaranteed values.

The ϕ factors used to convert nominal values to design capacities are obtained as specified in AASHTO (2002) for the as-built and from ACI 440 for the strengthened members.

Material properties of the FRP reinforcement reported by manufacturers, such as the ultimate tensile strength, typically do not consider long-term exposure to environmental conditions, and should be considered as initial properties. FRP properties to be used in all design equations are given as follows (ACI 440):

$$f_{fu} = C_E f^*_{fu}$$
$$\varepsilon_{fu} = C_E \varepsilon^*_{fu} \quad (3.1)$$

where f_{fu} and ε_{fu} are the FRP design tensile strength and ultimate strain considering the environmental reduction factor (C_E) as given in Table 7.1 (ACI 440), and f^*_{fu} and ε^*_{fu} represent the FRP guaranteed tensile strength and ultimate strain as reported by the manufacturer. The FRP design modulus of elasticity is the average value as reported by the manufacturer. Calculations for both NSM FRP bars and FRP laminates are shown in Appendix I.

a) Crack Sealed Previous to Injection b) Crack Injection under the Bridge

Figure 3-1 – Repair of Central Crack

Table 3-1– Properties of CFRP Laminate Constituent Materials

Material	Ultimate tensile strength f^*_{fu} ksi [MPa]	Ultimate strain ε^*_{fu} in/in [mm/mm]	Tensile modulus E_f ksi [GPa]	Nominal thickness t_f in [mm]
Primer*	2.5 [17.2]	40	104 [0.7]	-
Putty*	2.2 [15.2]	7.0	260 [1.8]	-
Saturant*	8.0 [55.2]	7.0	260 [1.8]	-
High Strength Carbon Fiber**	550 [3790]	0.017	33,000 [228]	0.0065 [0.1651]

* Values provided by the manufacturer (Watson Bowman Acme Corp. (2002))
** Tested as laminate with properties related to fiber area (Yang, X., 2002)

Table 3-2 – Properties of NSM CFRP Constituent Materials

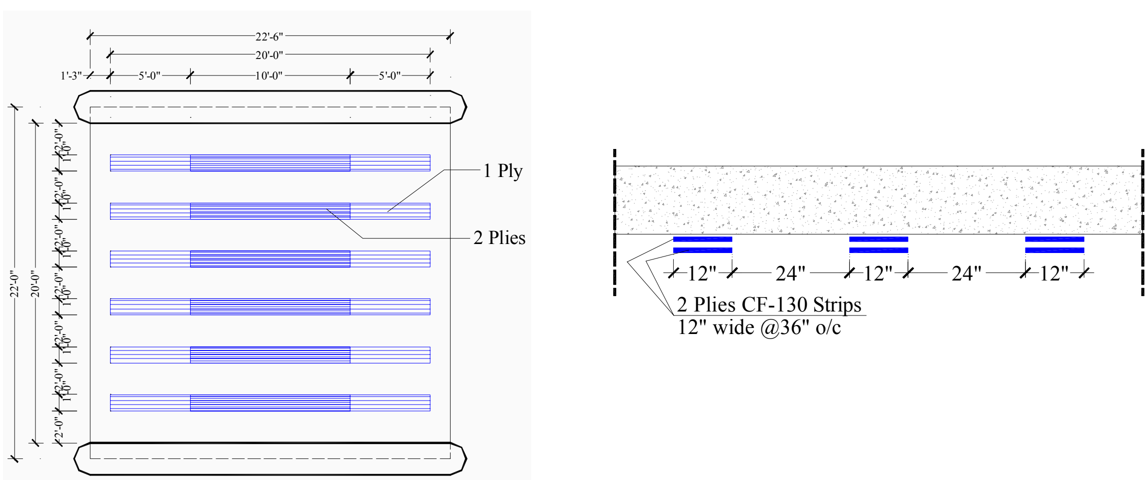
Material	Ultimate tensile strength f^*_{fu} Ksi [MPa]	Ultimate Strain ε^*_{fu} [in/in]	Tensile modulus E_f ksi [GPa]	Cross Sectional Area in^2 [mm^2]	Dimensions in×in [mm×mm]
Concresive 1420 Epoxy*	4 [27.58]	0.1	-	-	-
Aslan 500 Carbon Tape**	300 [2,068]	0.017	19000 [131]	0.05 [32.2]	0.079×0.63 [2×16]

* Values provided by the manufacturer (Watson Bowman Acme Corp. (2002))

** Values provided by the manufacturer and related to cross sectional area (Hughes Brothers, Inc. (2002))

3.1 Externally Bonded CFRP Laminates

The material properties of the laminates that have been used are listed on Table 3-1. The design for externally bonded laminates called for a total of six, 12 in (30.48 cm) wide, single ply CFRP strips overlapping at center span for 10 ft (3.05 m). The strips were evenly spaced over the width of 20 ft (6.09 m) and ran the entire width of the slab, as shown in Figure 3-2. The moment capacity provided with this strengthening scheme is equal to ϕM_n=16.5 k-ft (23 kN-m). The CFRP laminates were applied by a certified contractor in accordance to manufacturer's specification (Watson Bowman Acme Corp.,2002) (see Figure 3-3).



a) Plan View b) Section View

Figure 3-2 – Strengthening with Laminates on Span 1 and 3

a) Surface Preparation with Primer and Putty

b) Application of Saturant

c) Application of CFRP Laminates

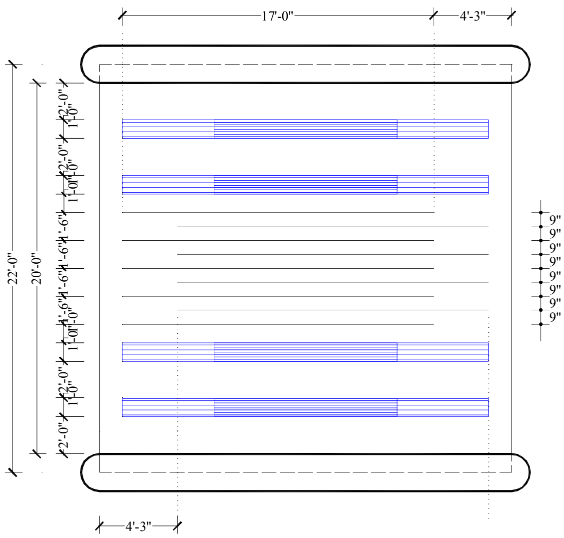
d) Application Completed

Figure 3-3 – Phases of CFRP Laminate Application

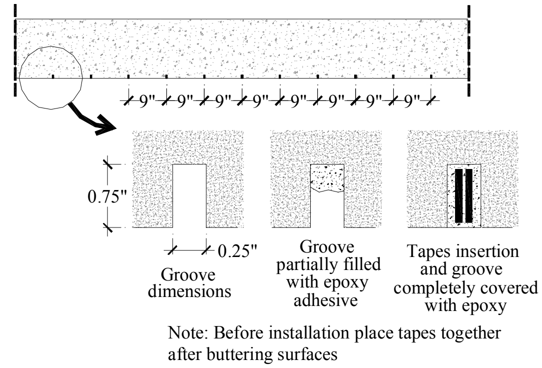
3.2 Near Surface Mounted Rectangular Bars

The material properties of the NSM and epoxy paste that have been used are listed on

Table 3-2. The required number of near-surface mounted reinforcement was determined to be two CRFP tapes per slot on a 9 in (22.86 cm) groove spacing. The bars were embedded in 17 ft (5.18 m) long, ¾ in (19.05 mm) deep, and ¼ in (6.35 mm) wide grooves cut onto the soffit of the bridge deck as shown in Figure 3-4. The moment capacity provided with this strengthening scheme is equal to ϕM_n=15.5 k-ft (21.01 kN-m). NSM bars were applied by a certified contractor following the specifications prescribed by the University of Missouri - Rolla (see Figure 3-5).



a) Plan View



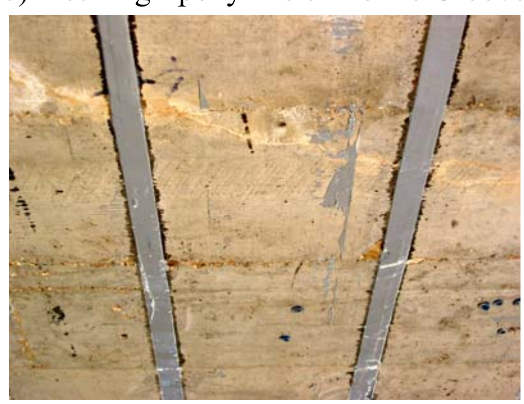
b) Section View

Figure 3-4 – Strengthening with NSM Bars and CFRP Laminate on Span 2

a) Grooves Prepared as per Design Geometry

b) Inserting Epoxy Paste into the Groove

c) Insertion of NSM Bar into the Groove



d) Application Completed

Figure 3-5 – Phases of NSM Bar Application

4. FIELD EVALUATION

Although in-situ bridge load testing is recommended by the AASHTO (2002) Specification as an "effective means of evaluating the structural performance of a bridge," no guidelines currently exist for bridge load test protocols. In each case the load test objectives, load configuration, instrumentation type and placement, and analysis techniques are to be determined by the organization conducting the test.

In order to validate the behavior of the bridge prior and after strengthening, static load tests were performed with a H20 truck (see Figure 4-1). Although H20 and HS20 trucks differ in their geometry, the loading configuration that maximize the stresses and deflections at mid span could still be accomplished (see Figure 4-2).

Displacements in the longitudinal and transversal direction were measured using eight Linear Variable Differential Transducers (LVDTs) and a data acquisition system under a total of three passes, one central and two laterals. For each pass, three stops were executed with the truck having its rear axle centered over the marks on the asphalt (see Figure 4-3). During each stop, the truck stationed for at least two minutes before proceeding to the next location in order to allow stable readings.

Figure 4-1 – Load Test with H20 Truck

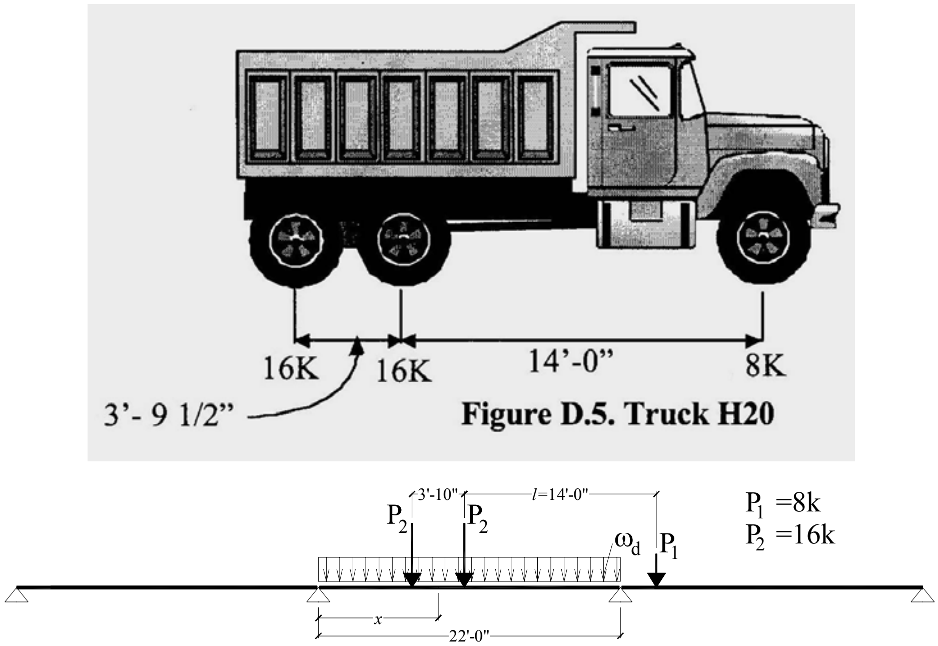


Figure 4-2 – H20 Legal Truck

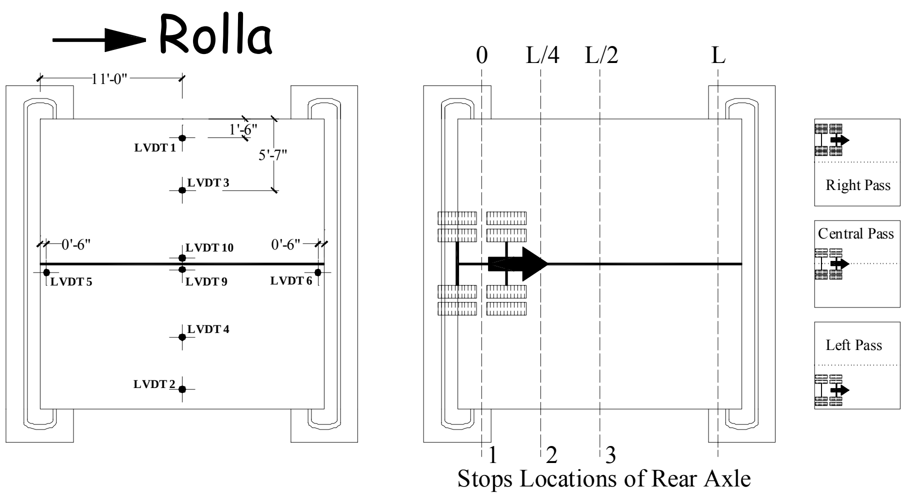


Figure 4-3 – LVDTs Positions and Truck Stops

The instrumentation layout was designed to gain the maximum amount of information about the structure. It was assumed that the bridge acted symmetrically, therefore instrumentation was concentrated on one half of the bridge.

The results of the first load test, relative to the stop No.3, are reported in Figure 4-4. All diagrams show the discontinuity caused by the longitudinal crack. The bridge performed well in terms of overall deflection. In fact, the maximum deflection measured during the load test is below the allowable deflection prescribed by AASHTO, 2002 Section 8.9.3 ($\delta_{max} \leq$ L/800 =0.33in (8.38mm)).

A second load test was performed after the installation of the FRP materials. The monitoring devices were placed at the same locations as the previous load test.

Test results of the second load test, as expected, show a slight improvement in the deflection of the deck in both the longitudinal and transversal direction (see Figure 4-5 and Figure 4-6, respectively).

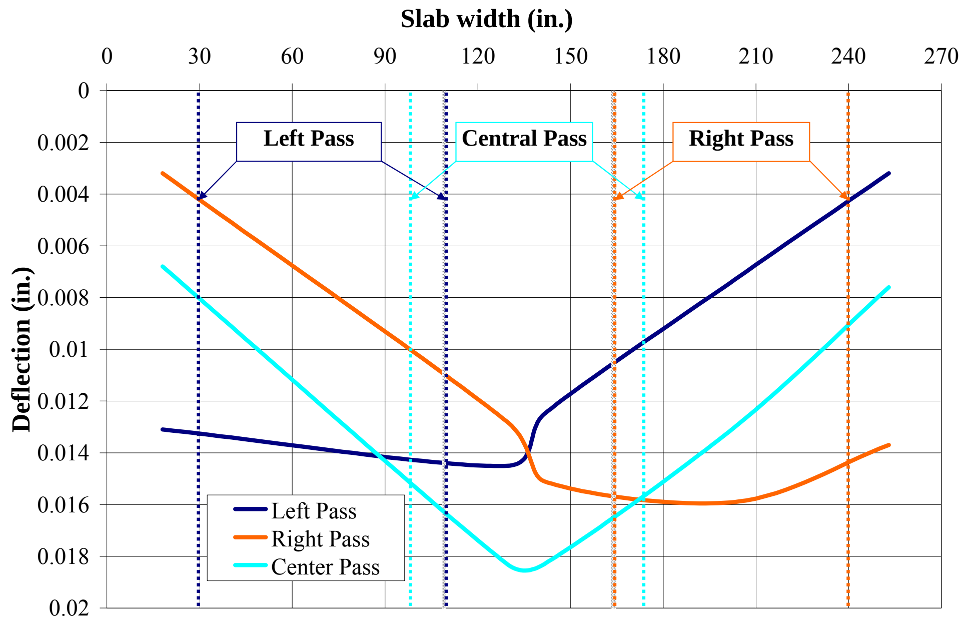


Figure 4-4 – Mid Span Deflection in the Transverse Direction, Stop No.3

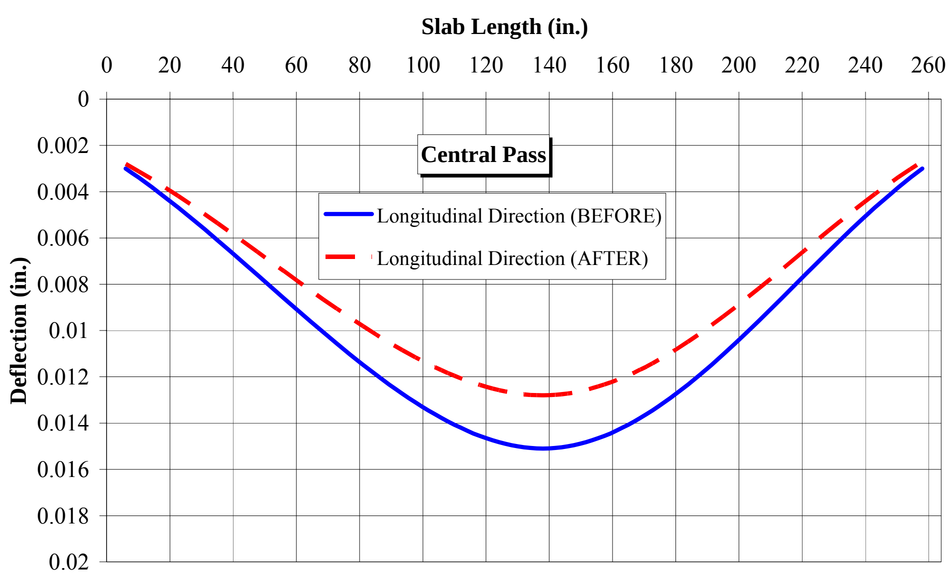


Figure 4-5 – Center Line Deflection in the Longitudinal Direction, Stop No.3

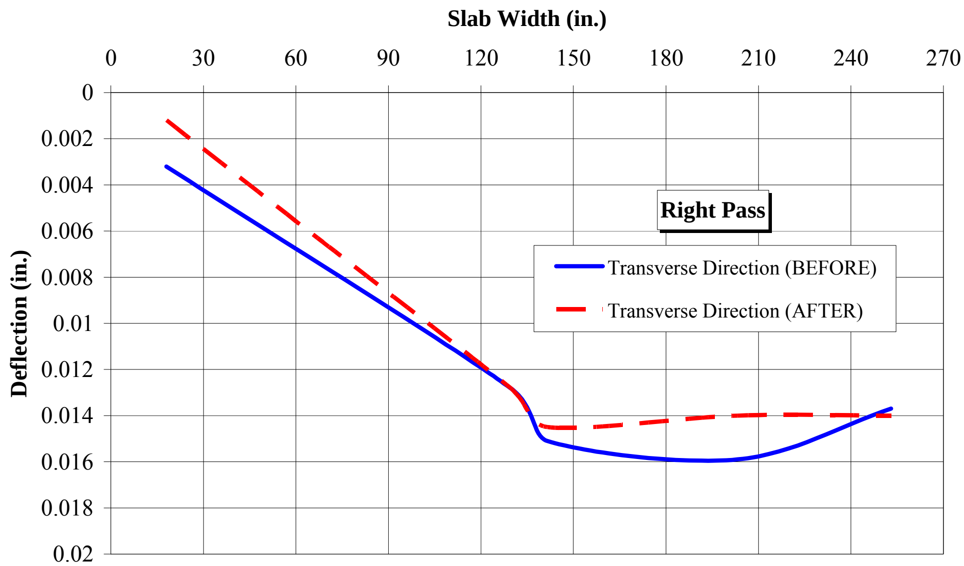


Figure 4-6 – Mid Span Deflection in the Transverse Direction, Stop No.3

5. ADDITIONAL LOAD TEST

As indicated in Figure 5-1, Figure 5-2 and Figure 5-3, the load test was repeated in September 2003 at a distance of one year from strengthening. The same load on the truck was used before and after the strengthening. From the graphs presented herein it is clear that the deflection magnitude has not significantly changed.

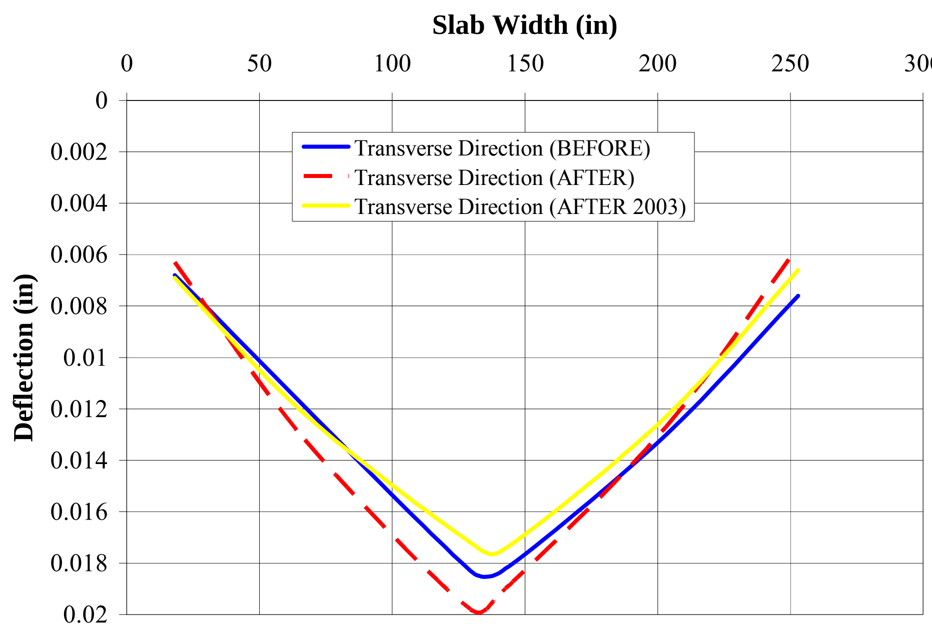


Figure 5-1 –Mid Span Deflection in the Transverse Direction, Central Pass, Stop No.3

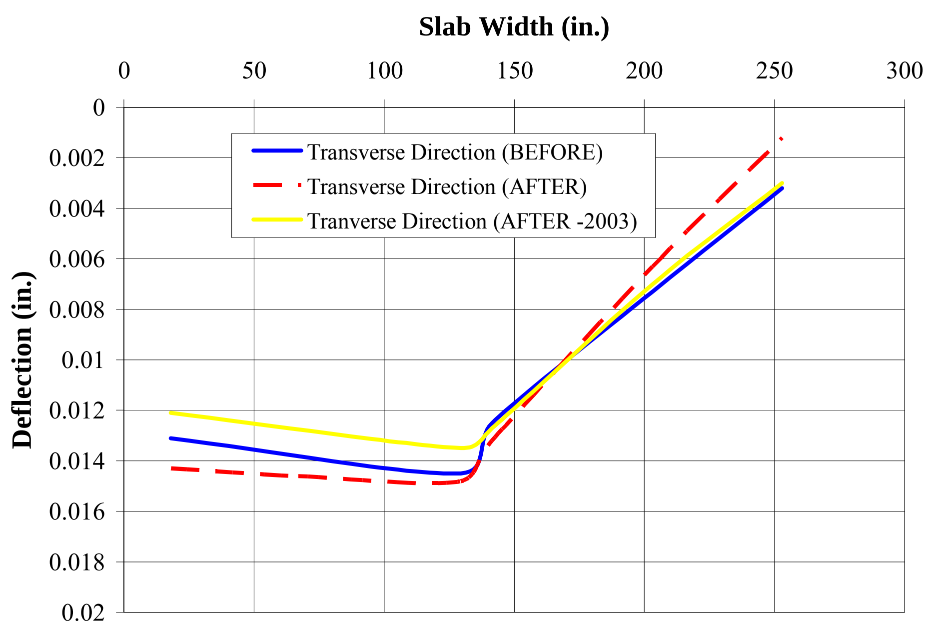


Figure 5-2– Mid Span Deflection in the Transverse Direction, Left Pass, Stop No.3

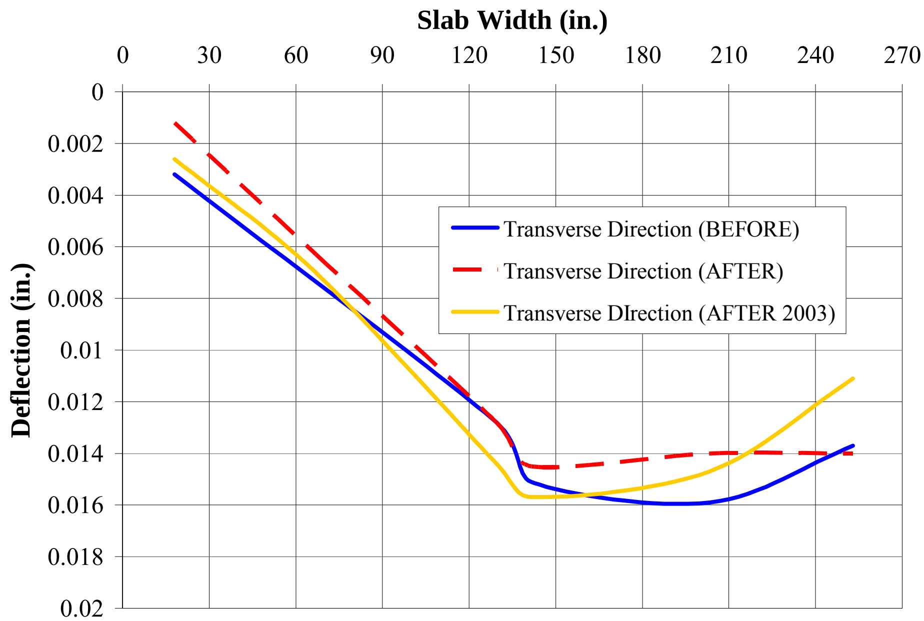


Figure 5-3 – Mid Span Deflection in the Transverse Direction, Right Pass, Stop No.3

6. FEM ANALYSIS

To validate the data obtained from the load tests, a linear elastic FEM analysis was conducted. For this purpose a commercially available finite element program ANSYS 7.0 was used.

The element SOLID65 was chosen to model the concrete. SOLID65 is used for the three-dimensional modeling of solids with or without reinforcing bars. The solid is capable of cracking in tension and crushing in compression. In concrete applications, for example, the solid capability of the element may be used to model the concrete while the rebar capability is available for modeling reinforcement behavior. The element is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. Up to three different rebar specifications may be defined.

SOLID65 is subject to the following assumption and restrictions:

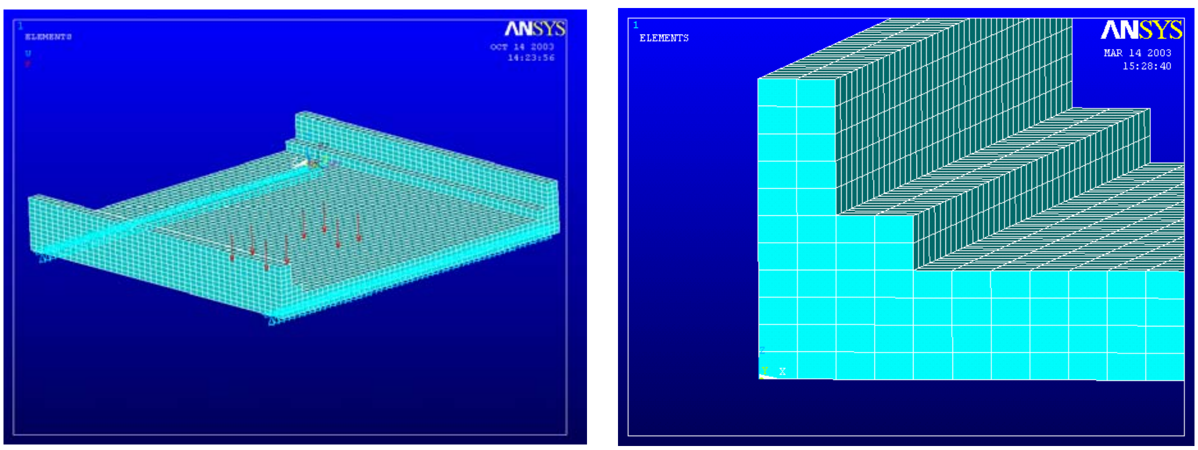
1. Cracking is permitted in three orthogonal directions at each integration point;
2. If cracking occurs at an integration point, the cracking is modeled through an adjustment of material properties which effectively treats the cracking as a "smeared band" of cracks, rather than discrete cracks;
3. The concrete material is assumed to be initially isotropic;
4. Whenever the reinforcement capability of the element is used, the reinforcement is assumed to be "smeared" throughout the element;

5. In addition to cracking and crushing, the concrete may also undergo plasticity, with the Drucker-Prager failure surface being most commonly used. In this case, the plasticity is done before the cracking and crushing checks.

For this project, the material properties of concrete were assumed to be isotropic and linear elastic, since the applied load was relatively low. The modulus of elasticity of the concrete was based on the measured compressive strength of the cores obtained from the slab according to the standard equation ACI 318-02 Section 8.5.1:

$$E_c = 57000\sqrt{f_c^{'}} \approx 3.6\times10^6\, psi\ (24.8\ GPa) \qquad (6.1)$$

Each element was meshed to be 3.5 in×5 in×6 in (8.9 cm×12.7 cm×15.2 cm). In order to take into account the presence of the parapet and curb, an equivalent, less complex shape was chosen. Boundary conditions were simulated as simply supported at both ends (see Figure 6-1).



a) Entire Model   b) Detail of Parapet

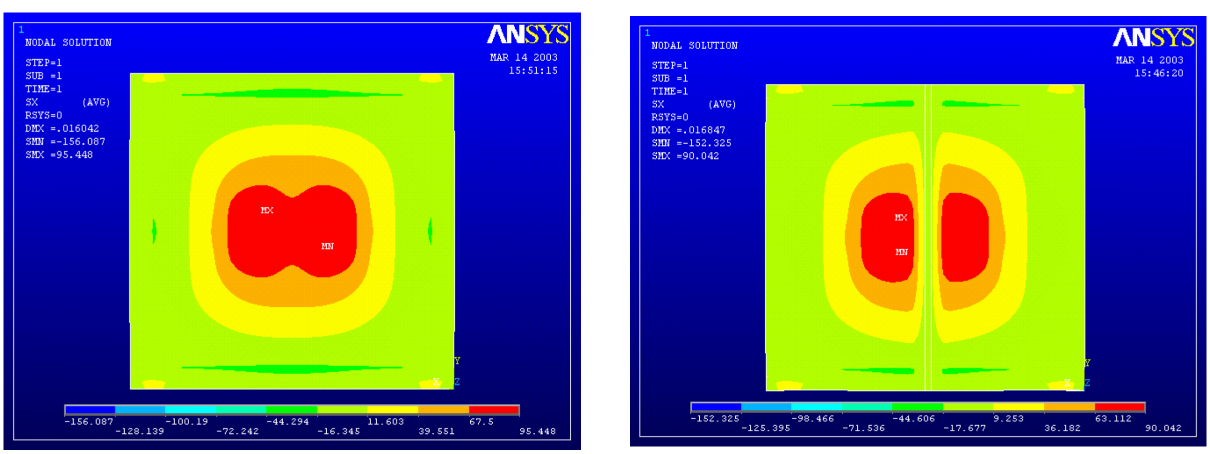
Figure 6-1 – FEM Model Geometry

To take into account the presence of the longitudinal crack, the modulus of elasticity of the central elements was reduced thousandths times with respect to the value expressed in Eq.(6.1). From in-situ inspection, the depth and width of the crack was chosen to be equal to one element dimensions. The load was applied on 8 nodes simulating the truck wheels; each force was equal to 4 kip (17.8 kN) for the H20 truck.

The experimental and analytical results for the central and right passes in the transversal direction are reported in Figure 6-2. The graph shows a good match in deflection between the experimental and analytical results.

Average S_x stresses (stresses in the transversal direction) are plotted in Figure 6-3, for both the un-cracked and cracked models. They show how the presence of the rigid parapets has a significant effect on the overall behavior of the bridge, justifying the presence of peak horizontal stresses along the slab centerline (tensile stresses are positive) which caused the formation of the crack. The strengthening with FRP can overcome these stresses and guarantee a flexural capacity in the transversal direction higher then the cracking moment, blocking new crack's opening.

Figure 6-2 – Comparison of Experimental and Analytical Results in the Transversal Direction



a) S_x in Model Slab with no Crack

b) S_x in Model Slab with Crack

Figure 6-3 – FEM Results of S_x Average Stresses for Axle Position at Stop 3

7. LOAD RATING

Bridge load rating calculations provide a basis for determining the safe load carrying capacity of a bridge. According to the Missouri Department of Transportation (MoDOT), anytime a bridge is built, rehabilitated, or reevaluated for any reason, inventory and operating ratings are required using the Load Factor rating. All bridges should be rated at two load levels, the maximum load level called the Operating Rating and a lower load level called the Inventory Rating. The Operating Rating is the maximum permissible load that should be allowed on the bridge. Exceeding this level could damage the bridge. The Inventory Rating is the load level the bridge can carry on a daily basis without damaging the bridge.

In Missouri, for the Load Factor Method, the Operating Rating is based on the appropriate ultimate capacity using current AASHTO specifications (AASHTO, 1996). The Inventory Rating is taken as 60% of the Operating Rating.

The vehicle used for the live load calculations in the Load Factor Method is the HS20 truck. If the stress levels produced by this vehicle configuration are exceeded, load posting may be required.

The tables below show the Rating Factor and Load Rating for this bridge. The method for determining the rating factor is that outlined by AASHTO in the Manual for Condition Evaluation of Bridges (AASHTO, 1994). Equation (7.1) was used:

$$RF = \frac{C - A_1 D}{A_2 L\left(1 + I\right)} \qquad (7.1)$$

where: RF is the Rating Factor, C is the capacity of the member, D is the dead load effect on the member, L is the live load effect on the member, I is the impact factor to be used with the live load effect, A_1 is the factor for dead loads, and A_2 is the factor for live loads. Since the load factor method is being used, A_1 is taken as 1.3 and A_2 varies depending on the desired rating level. For Inventory rating, A_2 = 2.17, and for Operating Rating, A_2 = 1.3.

To determine the rating (RT) of the bridge Equation (7.1) was used:

$$RT = \left(RF\right)W \qquad (7.1)$$

In the above equation, W is the weight of the nominal truck used to determine the live load effect.

For the Martin Springs Bridge, the Load Rating was calculated for a number of different trucks, HS20, H20, 3S2, and MO5. The different ratings are used for different purposes by the bridge owner. For each of the different loading conditions, the maximum shear and maximum moment were calculated. Impact factors are also taken into account for Load Ratings. This value is 30% for the Martin Springs Bridge. The shear and moment values for the deck are shown below in Table 7-1.

Table 7-1 - Maximum Shear and Moment due to Live Load

Truck	Maximum Shear (kip)	Maximum Moment (k-ft.)	Maximum Shear with Impact (kip)	Maximum Moment with Impact (k-ft.)
HS20	43.16	174.17	56.11	226.42
MO5	30.06	200.83	39.08	261.08
H20	39.68	146.58	51.58	190.56
3S2	30.37	146.83	39.48	190.88

Table 7-2 below gives the results of the Load Rating pertaining to moment and Table 7-3 shows the results for shear. All calculations for the load rating are located in Appendix II.

Table 7-2 - Rating Factor for the Slab (Bending Moment)

Truck	Rating Factor (RF)	Rating (RT) (Tons)	Rating Type
HS20	2.095	75.4	Operating
HS20	1.255	45.2	Inventory
MO5	1.817	65.4	Operating
H20	2.140	42.8	Posting
3S2	2.137	78.3	Posting

* All Units Expressed in English System

Table 7-3 - Rating Factor for the Slab (Shear)

Truck	Rating Factor (RF)	Rating (RT) (Tons)	Rating Type
HS20	3.546	127.7	Operating
HS20	2.124	76.5	Inventory
MO5	3.857	141.3	Operating
H20	4.379	87.6	Posting
3S2	4.334	158.8	Posting

* All Units Expressed in English System

Since the factors RF are greater than 1 then the bridge does not need to be load posted. In addition, from Table 7-2 and Table 7-3 the maximum operating and inventory load can be found as 75T and 45T respectively.

8. REPORT BY INDIPENDENT CONSULTANT

Based on the results provided by UMR, a Bridge Engineering Assistance Program (BEAP) report on the structure was prepared by an independent consultant in the summer 2003. The consultant, based on given information` regarding the condition of the structure, quantity and location of existing steel reinforcement, and on load test results conducted by UMR, rated the structure to demonstrate that the posting could be removed. The strengthening of the bridge in the transversal direction was necessary to the removal of the load posting. In fact, as proved by the load testing prior to strengthening, even though the bridge performed well in terms of overall deflection, all diagrams showed the discontinuity caused by the longitudinal crack. Without such strengthening, the increased loads, resulting from removal of the load posting, could possibly cause an increment of the longitudinal crack width and therefore compromise the serviceability of the structure.

9. CONCLUSIONS

Conclusions based on the retrofitting of the bridge utilizing FRP materials can be summarized as follows:

- FRP systems, either in the form of externally bonded laminates and near surface mounted bars, showed to be a feasible solution for the strengthening of the concrete bridge
- There is great appeal in the short timeline for installation. In addition, the retrofitting of the bridge can be obtained without interrupting the traffic
- As a result of FRP strengthening, load posting of the bridge was removed
- In situ load testing has proven to be useful and convincing
- The FEM analysis has shown good match with experimental results demonstrating the effectiveness of the strengthening technique.

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APPENDIX I

Strengthening of Martin Springs Outer Road Bridge, Phelps County

Project Design of the FRP Laminates Reinforcement

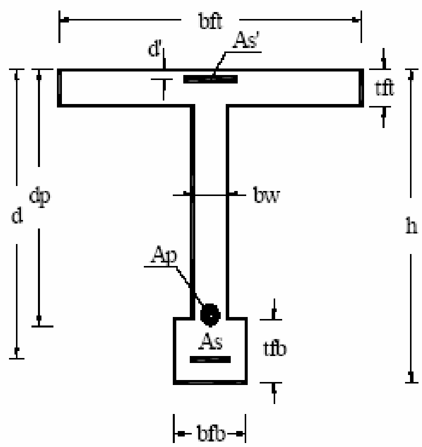
Designed by Nestore Galati

Required Information about the Existing Structure

Select Units

System := 1

Selected system
1 -- US Customary
2 -- SI



Section Dimensions

h := 14	Total section height, [in] or [mm]
bw := 12	Width of web, [in] or [mm]
bft := 12	Width of top flange (zero for rectangular sections), [in] or [mm]
tft := 0	Thickness of top flange (zero for rectangular sections), [in] or [mm]
bfb := 0	Width of bottom flange (zero for rectangular or T sections), [in] or [mm]
tfb := 0	Thickness of bottom flange (zero for rectangular or T sections), [in] or [mm]

Reinforcement Layout

As := 0	Area of mild tension steel, [in^2] or [mm^2]
d := 6.25	Depth to the mild tension steel centroid, [in] or [mm]
As' := 0	Area of mild compression steel, [in^2] or [mm^2]
d' := 0	Depth to the mild compression steel centroid, [in] or [mm]
Ap := 0	Area of prestressing steel, [in^2] or [mm^2]
dp := 23	Depth to the prestressing steel centroid, [in] or [mm]
Bond := 1	Type of tendon installation (Enter 1 for bonded, 0 for unbonded)

Load and Span Information

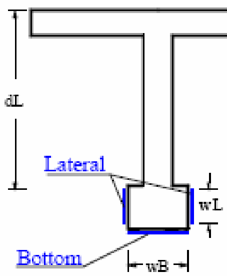
$Mcr := 15.7$	Cracking Moment of the Section, [k-ft] or [kN-m]
$Mu := Mcr$	Factored moment to be resisted by the strengthened element, [k-ft] or [kN-m]
$Ms := 0.5 \cdot Mu$	Service moment to be resisted by the strengthened element, [k-ft] or [kN-m]
$Mip := 0$	Moment in place at the time of FRP installation, [k-ft] or [kN-m]
$Mso := 0$	Original service moment before strenghtening, [k-ft] or [kN-m]
$Ln := 0$	Clear span (Only if unbonded prestressing steel is used), [ft] or [m]
$Lr := 0$	Ratio of loaded spans to total spans (e.g., 0.5 for alternate bay loading)

Material Property Specifications

$f'c := 4100$	Nominal compressive strength of the concrete, [psi] or [MPa]
$\varepsilon_{cu} := 0.003$	Maximum compressive strain for concrete, [in/in] or [mm/mm]
$fy := 32$	Yield strength of the mild steel, [ksi] or [MPa]
$Es := 29000$	Modulus of elasticity of the mild steel, [ksi] or [MPa]
$fpu := 1$	Ultimate strength of the prestressing steel: 1 -- 250 ksi 3 -- 1720 MPa 2 -- 270 ksi 4 -- 1860 MPa
$fpe := 200$	Effective stress in the tendons due to prestress, [ksi] or [MPa]
$fpy := 243$	Yield strength of the prestressing steel, [ksi] or [MPa]
$Ep := 28000$	Modulus of elasticity of the prestressing steel, [ksi] or [MPa]

Required FRP Design Information

FRP Sheets



q := 30 — Number of requested iterations (30 recommended; higher number increase precision and time computation)

Bottom (B)

Producer_B := Fyfe / MAPEI / **MBT** / Mitsubishi

Fiber_B := **Carbon** / Aramid / Gass

Type_B := **MBrace CF130** / MBrace CF530 / MBrace AK60 / MBrace EG900

Exposure_B := Interior Exposure / **Exterior Exposure** / Aggressive Exposure

Lateral (L)

Producer_L := **None** / Fyfe / MAPEI / MBT

Fiber_L := Carbon / Aramid / Gass

Type_L := MBrace CF130 / MBrace CF530 / MBrace AK60 / MBrace EG900

Exposure_L := Interior Exposure / Exterior Exposure / Aggressive Exposure

Bottom	Lateral	
ffu_B = 550	ffu_L = 0	Ultimate tensile strength of the FRP, [ksi] or [MPa]
εfu_B = 0.017	εfu_L = 0	Ultimate rupture strain of the FRP, [in/in] or [mm/mm]
Ef_B = 33000	Ef_L = 0	Tensile modulus of elasticity of the FRP, [ksi] or [GPa]
tf_B = 0.0065	tf_L = 0	Nominal design thickness of one ply of the FRP, [in] or [mm]
Ccr_B = 0.55	Ccr_L = 0	Creep rupture stress limit (Table 9.1 ACI 440F)
Ce_B = 0.85	Ce_L = 0	Reduction factor for environmental exposure (Table 8.1, ACI 440F)

Layout of the FRP Reinforcement *(Skip this section if FRP is NOT present)*

$wB := 6$	Width of FRP Bottom sheets [in] or [mm]
$NB := 2$	Number of FRP Bottom sheets
$wL := 0$	Width of FRP Lateral sheets, [in] or [mm]
$NL := 0$	Number of FRP Lateral sheets
$dL := 0$	Depth to the top fiber of FRP Lateral sheets, [in] or [mm]
$p := 30$	Number of divisions for lateral strenghtening (30 recommended; higher number increase precision and time computation)
$\psi_f := 0.85$	Additional reduction factor for FRP (Eq. (9-2), ACI 440F)

Initial Strain

Detailed Calculation of the Design Moment Capacity

Neutral axis position

- **Before cracking**

Part	Area	y	Area x y
Top Flange (TP)	$(bft - bw)\cdot tft$	$0.5tft$	$0.5(bft - bw)\cdot tft^2$
Web (W)	$bw\cdot h$	$0.5\cdot h$	$0.5bw\cdot h^2$
Bottom Flange (BF)	$(bfb - bw)\cdot tfb$	$h - 0.5tfb$	$(bfb - bw)\cdot tfb\cdot(h - 0.5\cdot tfb)$
Top Steel (TS)	$(n - 1)\cdot As'$	d'	$(n - 1)\cdot As'\cdot d'$
Prestressing Steel (PS)	$(n_p - 1)\cdot Ap$	dp	$(n_p - 1)\cdot Ap\cdot dp$
Bottom Steel (BS)	$(n - 1)\cdot As$	d	$(n - 1)\cdot As\cdot d$

$$c_{b\_cr} := \frac{0.5(bft - bw)\cdot tft^2 + 0.5bw\cdot h^2 + (bfb - bw)\cdot tfb\cdot(h - 0.5\cdot tfb) + (n - 1)\cdot As'\cdot d' \ldots + (n_p - 1)\cdot Ap\cdot dp + (n - 1)\cdot As\cdot d}{[(bft - bw)\cdot tft] + bw\cdot h + (bfb - bw)\cdot tfb + (n - 1)\cdot As' + (n_p - 1)\cdot Ap + (n - 1)\cdot As}$$

- **After cracking**

Guess value for c:
$c := 0.1\cdot d$

1) Neutral axis inside the flange and above the compression steel:

Part	Area	y	Area x y
Top Flange	$bft\cdot c$	$0.5\cdot c$	$0.5\cdot bft\cdot c^2$
Web	– – –	– – –	– – –
Bottom Flange	– – –	– – –	– – –
Top Steel	$n\cdot As'$	$c - d'$	$n\cdot As'\cdot(c - d')$
Prestressing Steel	$n_p\cdot Ap$	$c - dp$	$n_p\cdot Ap\cdot(c - dp)$
Bottom Steel	$n\cdot As$	$c - d$	$n\cdot As\cdot(c - d)$

Given

$$0.5\cdot bft\cdot c^2 + n\cdot As'\cdot(c - d') + n_p\cdot Ap\cdot(c - dp) + n\cdot As\cdot(c - d) = 0$$

$c_1 := \mathrm{Find}(c)$

2) Neutral axis inside the flange and below the compression steel:

Part	Area	y	Area x y
Top Flange	$bft \cdot c$	$0.5 \cdot c$	$0.5 \cdot bft \cdot c^2$
Web	---	---	---
Bottom Flange	---	---	---
Top Steel	$(n-1) \cdot As'$	$c - d'$	$(n-1) \cdot As' \cdot (c - d')$
Prestressing Steel	$n_p \cdot Ap$	$c - dp$	$n_p \cdot Ap \cdot (c - dp)$
Bottom Steel	$n \cdot As$	$c - d$	$n \cdot As \cdot (c - d)$

Given

$$0.5 \cdot bft \cdot c^2 + (n-1) \cdot As' \cdot (c - d') + n_p \cdot Ap \cdot (c - dp) + n \cdot As \cdot (c - d) = 0$$

$$c_2 := \mathrm{Find}(c)$$

3) Neutral axis cuts the web:

Part	Area	y	Area x y
Top Flange	$(bft - bw) \cdot tft$	$c - 0.5 \cdot tft$	$(bft - bw) \cdot tft \cdot (c - 0.5 \cdot tft)$
Web	$bw \cdot c$	$0.5 \cdot c$	$0.5 \cdot bw \cdot c^2$
Bottom Flange	---	---	---
Top Steel	$(n-1) \cdot As'$	$c - d'$	$(n-1) \cdot As' \cdot (c - d')$
Prestressing Steel	$n_p \cdot Ap$	$c - dp$	$n_p \cdot Ap \cdot (c - dp)$
Bottom Steel	$n \cdot As$	$c - d$	$n \cdot As \cdot (c - d)$

Given

$$(bft - bw) \cdot tft \cdot (c - 0.5 \cdot tft) + 0.5 \cdot bw \cdot c^2 + (n-1) \cdot As' \cdot (c - d') + n_p \cdot Ap \cdot (c - dp) + n \cdot As \cdot (c - d) = 0$$

$$c_3 := \mathrm{Find}(c)$$

The neutral axis position after cracking is given by:

$$c_{a\_cr} := \left| \begin{array}{l} c_1 \ \text{ if } \ c_1 < d' \wedge c_1 \neq 0 \\ c_2 \ \text{ if } \ c_2 \leq tft \wedge c_2 \neq 0 \\ c_3 \ \text{ otherwise} \end{array} \right.$$

Moment of Inertia

- **Before cracking**

Part	Area	y	$I_{own\ axis}$	Area x y^2
TF	$(bft - bw)\cdot tft$	$c_{b\_cr} - 0.5tft$	$\frac{(bft - bw)\cdot tft^3}{12}$	$(bft - bw)\cdot tft\cdot(c_{b\_cr} - 0.5tft)^2$
W	$bw\cdot h$	$c_{b\_cr} - 0.5\cdot h$	$\frac{bw\cdot h^3}{12}$	$bw\cdot h\cdot(c_{b\_cr} - 0.5\cdot h)^2$
BF	$(bfb - bw)\cdot tfb$	$c_{b\_cr} - (h - 0.5tfb)$	$\frac{(bfb - bw)\cdot tfb^3}{12}$	$(bfb - bw)\cdot tfb\cdot\left[c_{b\_cr} - (h - 0.5tfb)\right]$
TS	$(n - 1)\cdot As'$	$c_{b\_cr} - d'$	---	$(n - 1)\cdot As'\cdot(c_{b\_cr} - d')^2$
PS	$(n_p - 1)\cdot Ap$	$c_{b\_cr} - dp$	---	$(n_p - 1)\cdot Ap\cdot(c_{b\_cr} - dp)^2$
BS	$(n - 1)\cdot As$	$c_{b\_cr} - d$	---	$(n - 1)\cdot As\cdot(c_{b\_cr} - d)^2$

$$I_g := \frac{(bft - bw)\cdot tft^3}{12} + (bft - bw)\cdot tft\cdot(c_{b\_cr} - 0.5tft)^2 + \frac{bw\cdot h^3}{12} + bw\cdot h\cdot(c_{b\_cr} - 0.5\cdot h)^2 \ldots$$
$$+ \frac{(bfb - bw)\cdot tfb^3}{12} + (bfb - bw)\cdot tfb\cdot\left[c_{b\_cr} - (h - 0.5tfb)\right] \ldots$$
$$+ (n - 1)\cdot As'\cdot(c_{b\_cr} - d')^2 + (n_p - 1)\cdot Ap\cdot(c_{b\_cr} - dp)^2 + (n - 1)\cdot As\cdot(c_{b\_cr} - d)^2$$

- **After cracking**

1) Neutral axis inside the flange and above the compression steel:

Part	Area	y	$I_{own\ axis}$	Area x y^2
TF	$bft\cdot c_1$	$\frac{c_1}{2}$	$\frac{bft\cdot c_1^3}{12}$	$\frac{bft\cdot c_1^3}{4}$
W	---	---	---	---
BF	---	---	---	---
TS	$n\cdot As'$	$c_1 - d'$	---	$n\cdot As'\cdot(c_1 - d')^2$
PS	$n_p\cdot Ap$	$c_1 - dp$	---	$n_p\cdot Ap\cdot(c_1 - dp)^2$
BS	$n\cdot As$	$c_1 - d$	---	$n\cdot As\cdot(c_1 - d)^2$

$$I_{cr\_1} := \frac{bft \cdot c_1^3}{12} + \frac{bft \cdot c_1^3}{4} + n \cdot As' \cdot (c_1 - d')^2 + n_p \cdot Ap \cdot (c_1 - dp)^2 + n \cdot As \cdot (c_1 - d)^2$$

2) Neutral axis inside the flange and below the compression steel:

Part	**Area**	**y**	$I_{own\ axis}$	**Area x y²**
TF	$bft \cdot c_2$	$\frac{c_2}{2}$	$\frac{bft \cdot c_2^3}{12}$	$\frac{bft \cdot c_2^3}{4}$
W	---	---	---	---
BF	---	---	---	---
TS	$(n - 1) \cdot As'$	$c_2 - d'$	---	$(n - 1) \cdot As' \cdot (c_2 - d')^2$
PS	$n_p \cdot Ap$	$c_2 - dp$	---	$n_p \cdot Ap \cdot (c_2 - dp)^2$
BS	$n \cdot As$	$c_2 - d$	---	$n \cdot As \cdot (c_2 - d)^2$

$$I_{cr\_2} := \frac{bft \cdot c_2^3}{12} + \frac{bft \cdot c_2^3}{4} + (n - 1) \cdot As' \cdot (c_2 - d')^2 + n_p \cdot Ap \cdot (c_2 - dp)^2 + n \cdot As \cdot (c_2 - d)^2$$

3) Neutral axis cuts the web:

Part	**Area**	**y**	$I_{own\ axis}$	**Area x y²**
TF	$(bft - bw) \cdot tft$	$c_3 - \frac{tft}{2}$	$\frac{(bft - bw) \cdot tft^3}{12}$	$(bft - bw) \cdot tft \cdot \left(c_3 - \frac{tft}{2}\right)^2$
W	$bw \cdot c_3$	$\frac{c_3}{2}$	$\frac{bw \cdot c_3^3}{12}$	$\frac{bw \cdot c_3^3}{4}$
BF	---	---	---	---
TS	$(n - 1) \cdot As'$	$c_3 - d'$	---	$(n - 1) \cdot As' \cdot (c_3 - d')^2$
PS	$n_p \cdot Ap$	$c_3 - dp$	---	$n_p \cdot Ap \cdot (c_3 - dp)^2$
BS	$n \cdot As$	$c_3 - d$	---	$n \cdot As \cdot (c_3 - d)^2$

$$I_{cr\_3} := \frac{(bft - bw) \cdot tft^3}{12} + (bft - bw) \cdot tft \cdot \left(c_3 - \frac{tft}{2}\right)^2 + \frac{bw \cdot c_3^3}{12} + \frac{bw \cdot c_3^3}{4} \ldots$$
$$+ (n - 1) \cdot As' \cdot (c_3 - d')^2 + n_p \cdot Ap \cdot (c_3 - dp)^2 + n \cdot As \cdot (c_3 - d)^2$$

The moment of inertia of the cracked concrete section is given by:

$$I_{cr} := \begin{cases} I_{cr\_1} & \text{if } c_1 < d' \wedge c_1 \neq 0 \\ I_{cr\_2} & \text{if } c_2 \leq tft \wedge c_2 \neq 0 \\ I_{cr\_3} & \text{otherwise} \end{cases} \qquad I_{cr} = 0$$

Initial Strain in the Concrete ($\mathbf{e_{bi}}$)

Initial strain in the concrete depends either by the applied load during the FRP installation (Mip) and by the prestressing steel if present. Defining r, radius of gyration of the concrete section ($r^2 = I / A_c$), the initial strain can be found by using equation [1].

Cracking moment, area of the concrete cross section, and radius of gyration are written below (the radius of gyration is given for uncracked [r_g] and cracked [r_{cr}] sections):

$$f_d := \frac{M_{ip} \cdot \frac{h}{2}}{I_g}$$

$$M_{cr} := \begin{cases} \dfrac{7.5\sqrt{f'c} \cdot I_g}{h - c_{b\_cr}} & \text{if } Ap = 0 \\ \dfrac{I_{cr}}{h - c_{b\_cr}} \cdot \left(7.5 \cdot \sqrt{f'c} + f_{pe} - f_d\right) & \text{if } Ap \neq 0 \end{cases}$$

$$A_c := (bft - bw) \cdot tft + bw \cdot h + (bfb - bw) \cdot tfb$$

$$r_g := \sqrt{\frac{I_g}{A_c}} \qquad r_{cr} := \sqrt{\frac{I_{cr}}{A_c}}$$

Effective prestress force at the time of FRP installation (P_e), and eccentricity of the prestress force with respect to the neutral axis (cgc, see figure) of the concrete section before (e_g) and after (e_{cr}) cracking are shown below:

$$P_e := Ap \cdot f_{pe}$$

$$e_g := dp - c_{b\_cr}$$

$$e_{cr} := dp - c_{a\_cr}$$

The initial strain in the concrete for uncracked and cracked sections is:

$$\varepsilon_{bi\_g} := \frac{M_{ip}}{I_g \cdot E_c} \cdot \left(h - c_{b\_cr}\right) - \frac{P_e}{A_c \cdot E_c} \cdot \left[1 + \frac{e_g \cdot \left(h - c_{b\_cr}\right)}{r_g^{\,2}}\right]$$

$$\varepsilon_{bi\_cr} := \frac{M_{ip}}{I_{cr} \cdot E_c} \cdot \left(h - c_{a\_cr}\right)$$

$$\varepsilon_{bi} := \begin{cases} \varepsilon_{bi\_g} & \text{if } M_{so} \leq M_{cr} \\ \varepsilon_{bi\_cr} & \text{if } M_{so} > M_{cr} \end{cases} \qquad \textbf{[1]}$$

$\varepsilon_{bi} = 0$

$$c_i := \begin{cases} c_{b\_cr} & \text{if } M_{so} \le M_{cr} \\ c_{a\_cr} & \text{otherwise} \end{cases}$$

The initial strain in the top fiber concrete, in the compression steel, and in the mild tension steel can be written as:

$$\varepsilon_{ci} := \frac{c_i}{h - c_i} \cdot \varepsilon_{bi}$$

$$\varepsilon'_{si} := \frac{c_i - d'}{h - c_i} \cdot \varepsilon_{bi}$$

$$\varepsilon_{si} := \frac{d - c_i}{h - c_i} \cdot \varepsilon_{bi}$$

Initial Strain

CONCRETE CRUSHING (sub c)

Strain

$$\varepsilon'_{sc}(z) := \begin{cases} \dfrac{d' - c(z)}{c(z)} \cdot \varepsilon_{cu} & \text{if } As' \ne 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\varepsilon_{Ljc}(z, j) := \begin{cases} \dfrac{D(j) - c(z)}{c(z)} \cdot \varepsilon_{cu} & \text{if Producer\_L} \ne 1 \\ 0 & \text{otherwise} \end{cases}$$

$\kappa_{m\_L} = 0$

$\varepsilon_{Ljc}(z, j) := \min\left(\varepsilon_{Ljc}(z, j), \kappa_{m\_L} \cdot \varepsilon_{fu\_L}\right)$

$$\Omega_u := \begin{cases} \dfrac{3.0}{\dfrac{L_u}{dp}} \cdot Lr & \text{if Bond} = 0 \\ 1 & \text{if Bond} = 1 \\ 0 & \text{if } Ap = 0 \end{cases}$$

$$\varepsilon_{pc}(z) := \begin{cases} \min\left[\dfrac{P_e}{Ap \cdot E_p} + \dfrac{P_e}{A_c \cdot E_c} \cdot \left(1 + \dfrac{{e_g}^2}{{r_g}^2}\right) + \Omega_u \cdot \dfrac{dp - c(z)}{c(z)} \cdot \varepsilon_{cu}, 0.03\right] & \text{if } Ap \ne 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\varepsilon_{sc}(z) := \begin{cases} \dfrac{d - c(z)}{c(z)} \cdot \varepsilon_{cu} & \text{if } As \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\varepsilon_{rc}(z) := \begin{cases} \dfrac{dr - c(z)}{c(z)} \cdot \varepsilon_{cu} & \text{if } Producer\_N \neq 1 \\ 0 & \text{otherwise} \end{cases}$$

$\kappa_{m\_r} = 0$

$\varepsilon_{rc}(z) := \min\left(\varepsilon_{rc}(z), \kappa_{m\_r} \cdot \varepsilon_{fu\_N}\right)$

$$\varepsilon_{Bc}(z) := \begin{cases} \dfrac{h - c(z)}{c(z)} \cdot \varepsilon_{cu} & \text{if } Producer\_B \neq 1 \\ 0 & \text{otherwise} \end{cases}$$

$\kappa_{m\_B} = 0.9$

$\varepsilon_{Bc}(z) := \min\left(\varepsilon_{Bc}(z), \kappa_{m\_B} \cdot \varepsilon_{fu\_B}\right)$

Stress

$f'_{sc}(z) := \varepsilon'_{sc}(z) \cdot E_s$

$$f_{sc}(z) := \begin{cases} f_y & \text{if } f'_{sc}(z) \geq f_y \\ -f_y & \text{if } f'_{sc}(z) \leq -f_y \\ f'_{sc}(z) & \text{otherwise} \end{cases}$$

$f_{Ljc}(z, j) := \varepsilon_{Ljc}(z, j) \cdot E_{f\_L}$

$$f_{pc}(z) := \begin{cases} \min\left(\varepsilon_{pc}(z) \cdot E_p, 0.94 \cdot f_{py}\right) & \text{if } Bond = 0 \\ \text{otherwise} & \\ \quad \text{if } fpu = 270 & \\ \quad\quad \begin{cases} \varepsilon_{pc}(z) \cdot E_p & \text{if } \varepsilon_{pc}(z) \leq 0.008 \\ f_{pu} - 2000 - \dfrac{75}{\varepsilon_{pc}(z) - 0.0065} & \text{otherwise} \end{cases} & \\ \quad \text{if } fpu = 250 & \\ \quad\quad \begin{cases} \varepsilon_{pc}(z) \cdot E_p & \text{if } \varepsilon_{pc}(z) \leq 0.0076 \\ f_{pu} - 2000 - \dfrac{58}{\varepsilon_{pc}(z) - 0.006} & \text{otherwise} \end{cases} & \end{cases}$$

$$f_{sc}(z) := \varepsilon_{sc}(z) \cdot E_s$$

$$f_{sc}(z) := \begin{cases} f_y & \text{if } f_{sc}(z) \geq f_y \\ -f_y & \text{if } f_{sc}(z) \leq -f_y \\ f_{sc}(z) & \text{otherwise} \end{cases}$$

$$f_{rc}(z) := \varepsilon_{rc}(z) \cdot E_{f\_N}$$

$$f_{Bc}(z) := \varepsilon_{Bc}(z) \cdot E_{f\_B}$$

Force

$$F'_{sc}(z) := \begin{cases} f_{sc}(z) \cdot As' & \text{if } c(z) < d' \\ \left(f_{sc}(z) - 0.85 \cdot fc\right) \cdot As' & \text{otherwise} \end{cases}$$

$$Aj := NL \cdot tf\_L \cdot \frac{wL}{p}$$

$$F_{Ljc}(z, j) := f_{Ljc}(z, j) \cdot Aj$$

$$F_{pc}(z) := Ap \cdot f_{pc}(z)$$

$$F_{sc}(z) := \begin{cases} f_{sc}(z) \cdot As & \text{if } c(z) < d \\ \left(f_{sc}(z) - 0.85fc\right) \cdot As & \text{otherwise} \end{cases}$$

$$A_r := Nr \cdot Ar$$

$$F_{rc}(z) := f_{rc}(z) \cdot A_r$$

$$A_B := wB \cdot NB \cdot tf\_B$$

$$F_{Bc}(z) := f_{Bc}(z) \cdot A_B$$

$$\alpha_{1c} := 0.85$$

$$\beta_1 := \begin{cases} 0.85 & \text{if } fc \leq 4000 \\ 1.05 - 0.05 \cdot \frac{fc}{1000} & \text{if } 4000 < fc < 8000 \\ 0.65 & \text{if } fc \geq 8000 \end{cases}$$

$\beta_1 = 0.845$

$a(z) := \beta_1 \cdot c(z)$

$$C_{cc}(z) := \begin{cases} (\alpha_{1c} \cdot fc \cdot a(z) \cdot bw) & \text{if } bft = 0 \\ \alpha_{1c} \cdot fc \cdot a(z) \cdot bft & \text{if } c(z) \le tft \wedge bft \ne 0 \\ \alpha_{1c} \cdot fc \cdot [a(z) \cdot bft - (a(z) - tft) \cdot (bft - bw)] & \text{if } c(z) > tft \wedge bft \ne 0 \end{cases}$$

Equilibrium Condition:

$z := h$

Given

$z > c_b$

$$C_{cc}(z) - \left(F'_{sc}(z) + \sum_{j=0}^{p-1} F_{Ljc}(z, j) + F_{pc}(z) + F_{sc}(z) + F_{rc}(z) + F_{Bc}(z) \right) = 0$$

$eq_c := \text{Minerr}(z)$

$eq_c = 2.1$

$z := \frac{h}{q}, \frac{2 \cdot h}{q} .. h$

Moment

$$M_c(z) := F'_{sc}(z) \cdot \left(d' - \frac{a(z)}{2} \right) + \sum_{j=0}^{p-1} \psi_f \cdot F_{Ljc}(z, j) \cdot \left(D(j) - \frac{a(z)}{2} \right) + F_{pc}(z) \cdot \left(dp - \frac{a(z)}{2} \right) \dots$$
$$+ F_{sc}(z) \cdot \left(d - \frac{a(z)}{2} \right) + \psi_r \cdot F_{rc}(z) \cdot \left(dr - \frac{a(z)}{2} \right) + \psi_f \cdot F_{Bc}(z) \cdot \left(h - \frac{a(z)}{2} \right)$$

$M_c(eq_c) = 3.665 \times 10^5$

Tension Controlled Failure

Equilibrium Condition

$$\text{Failure\_Mode} := \begin{cases} \text{"Tension Controlled"} & \text{if } c(eq_t) < c_b \\ \text{"Concrete Crushing"} & \text{if } c(eq_c) > c_b \end{cases}$$

$$eq := \begin{cases} eq_t & \text{if } c(eq_t) < c_b \\ eq_c & \text{if } c(eq_c) > c_b \end{cases}$$

Tension failure

Concrete crushing

$$eq = 1.164$$

$$M(z) := \begin{cases} M_t(z) & \text{if } c(eq_t) < c_b \\ M_c(z) & \text{if } c(eq_c) > c_b \end{cases}$$

$$M(z) := \frac{M(z)}{12000}$$

$$Mn := M(eq)$$

$$Mn = 31.465$$

$$\varepsilon_c := \begin{cases} \varepsilon_{ct}(eq) & \text{if } c(eq_t) < c_b \\ \varepsilon_{cu} & \text{if } c(eq_c) > c_b \end{cases}$$

$$\varepsilon_c = 0.0012$$

$$\varepsilon'_s := \begin{cases} 0 & \text{if } As' = 0 \\ \text{otherwise} & \\ \quad \begin{cases} -\varepsilon'_{st}(eq_t) & \text{if } c(eq_t) < c_b \\ -\varepsilon'_{sc}(eq_c) & \text{if } c(eq_c) > c_b \end{cases} \end{cases}$$

$$\varepsilon'_s = 0$$

$$\varepsilon p := \begin{cases} \varepsilon_{pt}(eq_t) & \text{if } c(eq_t) < c_b \\ \varepsilon_{pc}(eq_c) & \text{if } c(eq_c) > c_b \end{cases}$$

$$\varepsilon p = 0$$

$$\varepsilon_p := \varepsilon p - \left[\frac{P_e}{Ap \cdot E_p} + \frac{P_e}{A_c \cdot E_c} \cdot \left(1 + \frac{{e_g}^2}{{r_g}^2} \right) \right]$$

$$\varepsilon_p = 0$$

$$\varepsilon_s := \begin{cases} 0 & \text{if } As = 0 \\ \text{otherwise} & \\ \quad \begin{cases} \varepsilon_{st}(eq_t) & \text{if } c(eq_t) < c_b \\ \varepsilon_{sc}(eq_c) & \text{if } c(eq_c) > c_b \end{cases} \end{cases}$$

$$\varepsilon_s = 0$$

$$\varepsilon_r := \begin{cases} 0 & \text{if } Nr = 0 \\ \text{otherwise} & \\ \quad \begin{cases} \varepsilon_{rt}(eq_t) & \text{if } c(eq_t) < c_b \\ \varepsilon_{rc}(eq_c) & \text{if } c(eq_c) > c_b \end{cases} \end{cases}$$

$$\varepsilon_r := \begin{cases} 0 & \text{if } Producer\_N = 1 \\ \text{otherwise} & \\ \quad \begin{cases} \varepsilon_{rt}(eq_t) & \text{if } c(eq_t) < c_b \\ \varepsilon_{rc}(eq_c) & \text{if } c(eq_c) > c_b \end{cases} \end{cases}$$

$$\varepsilon_r = 0$$

$$\varepsilon r := \varepsilon_r - \varepsilon_{bi}$$

$$\varepsilon_f := \begin{cases} 0 & \text{if } NB = 0 \\ \text{otherwise} & \\ \quad \begin{cases} \varepsilon_{Bt}(eq_t) & \text{if } c(eq_t) < c_b \\ \varepsilon_{Bc}(eq_c) & \text{if } c(eq_c) > c_b \end{cases} \end{cases}$$

$$\varepsilon_f := \begin{cases} 0 & \text{if } Producer\_B = 1 \\ \text{otherwise} & \\ \quad \begin{cases} \varepsilon_{Bt}(eq_t) & \text{if } c(eq_t) < c_b \\ \varepsilon_{Bc}(eq_c) & \text{if } c(eq_c) > c_b \end{cases} \end{cases}$$

$$\varepsilon_f = 0.0128$$

$$\phi := 0.7$$

$$\phi Mn := \phi \cdot Mn$$

Result of the Strengthening Analysis

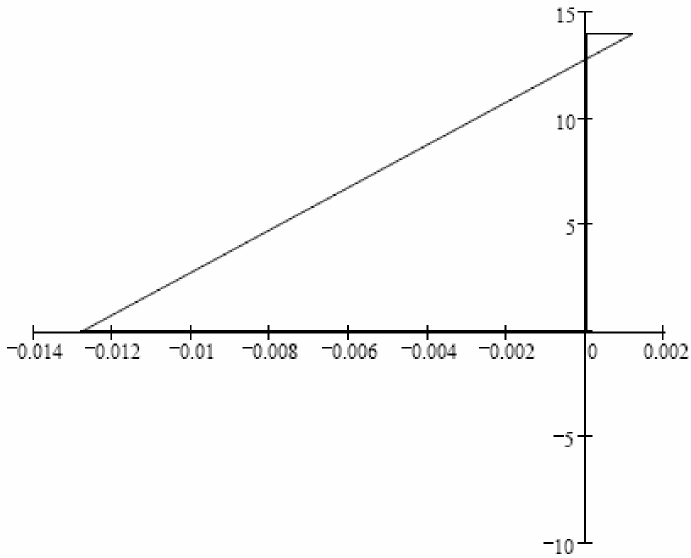
Design Ultimate Moment Capacity

$\phi Mn = 16.52$ $\quad Mu = 15.7$ $\quad$ Design moment capacity vs. moment demand, [k-ft] or [kN-m]

$\kappa_{m\_B} = 0.9$

$\kappa_{m\_L} = 0$

$\kappa_{m\_r} = 0$



Failure_Mode = "Tension Controlled"

$c_b = 2.662$	Depth to the neutral axis for balanced failure, [in] or [mm]
$c = 1.164$	Depth to the neutral axis, [in] or [mm]
$\varepsilon_c = 0.00116$	Maximum strain in the concrete
$\varepsilon'_s = 0$	Strain in the compression steel
$\varepsilon_p = 0$	Strain in the prestressing steel
$\varepsilon_s = 0$	Strain in the tension steel
$\varepsilon r = 0$	Strain at the NSM rod level
$\varepsilon_f = 0.01278$	Strain at the bottom layer of FRP level

Check the Stresses at Service Load Level (Only if FRP is Present)

$f_{cs} = 943$	$F_{cs} = 1845$	Concrete stress at service vs. service stress limit, [psi] or [MPa]
$f'_{ss} = 0$	$F'_{ss} = 24000$	Mild compression steel stress at service vs. service stress limit, [psi] or [MPa]
$f_{ps} = 0$	$F_{ps} = 185000$	Prestressng steel stress at service vs. service stress limit, [psi] or [MPa]
$f_{ss} = 0$	$F_{ss} = 48000$	Mild tension steel stress at service vs. service stress limit, [psi] or [MPa]
$f_{fs} = 88856$	$F_{fs} = 257125$	FRP service stress vs. creep rupture stress limit, [psi] or [MPa]
$f_{rs} = 0$	$F_{rs} = 0$	NSM rod service stress vs. creep rupture stress limit, [psi] or [MPa]

Strengthening of Martin Springs Outer Road Bridge, Phelps County

Project Design of the NSM FRP Bars Reinforcement

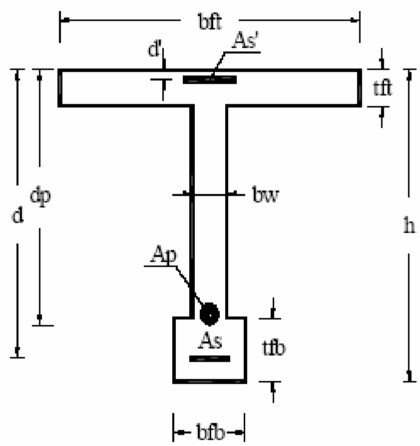
Designed by Nestore Galati

Required Information about the Existing Structure

Select Units

System := 1

Selected system
1 -- US Customary
2 -- SI



Section Dimensions

h := 14	Total section height, [in] or [mm]
bw := 12	Width of web, [in] or [mm]
bft := 12	Width of top flange (zero for rectangular sections), [in] or [mm]
tft := 0	Thickness of top flange (zero for rectangular sections), [in] or [mm]
bfb := 0	Width of bottom flange (zero for rectangular or T sections), [in] or [mm]
tfb := 0	Thickness of bottom flange (zero for rectangular or T sections), [in] or [mm]

Reinforcement Layout

As := 0	Area of mild tension steel, [in^2] or [mm^2]
d := 6.25	Depth to the mild tension steel centroid, [in] or [mm]
As' := 0	Area of mild compression steel, [in^2] or [mm^2]
d' := 0	Depth to the mild compression steel centroid, [in] or [mm]
Ap := 0	Area of prestressing steel, [in^2] or [mm^2]
dp := 23	Depth to the prestressing steel centroid, [in] or [mm]
Bond := 1	Type of tendon installation (Enter 1 for bonded, 0 for unbonded)

Load and Span Information

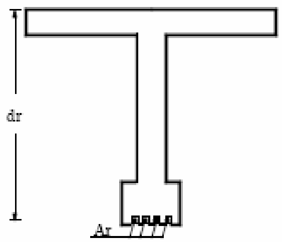
Mcr := 15.7	Cracking Moment of the Section, [k-ft] or [kN-m]
Mu := Mcr	Factored moment to be resisted by the strengthened element, [k-ft] or [kN-m]
Ms := 0.5·Mu	Service moment to be resisted by the strengthened element, [k-ft] or [kN-m]
Mip := 0	Moment in place at the time of FRP installation, [k-ft] or [kN-m]
Mso := 0	Original service moment before strenghtening, [k-ft] or [kN-m]
Ln := 0	Clear span (Only if unbonded prestressing steel is used), [ft] or [m]
Lr := 0	Ratio of loaded spans to total spans (e.g., 0.5 for alternate bay loading)

Material Property Specifications

f'c := 4100	Nominal compressive strength of the concrete, [psi] or [MPa]
ε_{cu} := 0.003	Maximum compressive strain for concrete, [in/in] or [mm/mm]
fy := 32	Yield strength of the mild steel, [ksi] or [MPa]
Es := 29000	Modulus of elasticity of the mild steel, [ksi] or [MPa]
fpu := 1	Ultimate strength of the prestressing steel: 1 -- 250 ksi 3 -- 1720 MPa 2 -- 270 ksi 4 -- 1860 MPa
fpe := 200	Effective stress in the tendons due to prestress, [ksi] or [MPa]
fpy := 243	Yield strength of the prestressing steel, [ksi] or [MPa]
Ep := 28000	Modulus of elasticity of the prestressing steel, [ksi] or [MPa]

Required NSM Design Information

NSM Rods



Producer_N := None / **Hughes**

Material := Aslan 100 GFRP Bar / Aslan 200 CFRP Bar / **Aslan 500 Tape**

Type_N := #2 / #3 / #4 / **Tape**

Exposure_N := Interior Exposure / **Exterior Exposure** / Aggressive Exposure

ffu_N = 200	Ultimate tensile strength of the FRP, [ksi] or [MPa]
εfu_N = 0.01	Ultimate rupture strain of the FRP, [in/in] or [mm/mm]
Ef_N = 20000	Tensile modulus of elasticity of the FRP, [ksi] or [GPa]
Ccr_N = 0.55	Creep rupture stress limit (Table 9.1 ACI 440F)
Ce_N = 0.85	Reduction factor for environmental exposure (Table 8.1, ACI 440F)

Layout of the NSM Reinforcement *(Skip this section if NSM bars are NOT present)*

$Nr := 2 \cdot \frac{12}{9}$	Number of NSM rods
$dr := 13.675$	Depth to the NSM reinforcement centroid, [in] or [mm]
$\psi_r := 0.85$	Additional reduction factor for NSM

Detailed Calculation of the Design Moment Capacity

Neutral axis position

- **Before cracking**

Part	Area	y	Area x y
Top Flange (TP)	$(bft - bw)\cdot tft$	$0.5tft$	$0.5(bft - bw)\cdot tft^2$
Web (W)	$bw\cdot h$	$0.5\cdot h$	$0.5bw\cdot h^2$
Bottom Flange (BF)	$(bfb - bw)\cdot tfb$	$h - 0.5tfb$	$(bfb - bw)\cdot tfb\cdot(h - 0.5\cdot tfb)$
Top Steel (TS)	$(n - 1)\cdot As'$	d'	$(n - 1)\cdot As'\cdot d'$
Prestressing Steel (PS)	$(n_p - 1)\cdot Ap$	dp	$(n_p - 1)\cdot Ap\cdot dp$
Bottom Steel (BS)	$(n - 1)\cdot As$	d	$(n - 1)\cdot As\cdot d$

$$c_{b\_cr} := \frac{0.5(bft - bw)\cdot tft^2 + 0.5bw\cdot h^2 + (bfb - bw)\cdot tfb\cdot(h - 0.5\cdot tfb) + (n-1)\cdot As'\cdot d' \ldots + (n_p - 1)\cdot Ap\cdot dp + (n-1)\cdot As\cdot d}{[(bft - bw)\cdot tft] + bw\cdot h + (bfb - bw)\cdot tfb + (n-1)\cdot As' + (n_p - 1)\cdot Ap + (n-1)\cdot As}$$

- **After cracking**

Guess value for c:
$c := 0.1\cdot d$

1) Neutral axis inside the flange and above the compression steel:

Part	Area	y	Area x y
Top Flange	$bft\cdot c$	$0.5\cdot c$	$0.5\cdot bft\cdot c^2$
Web	– – –	– – –	– – –
Bottom Flange	– – –	– – –	– – –
Top Steel	$n\cdot As'$	$c - d'$	$n\cdot As'\cdot(c - d')$
Prestressing Steel	$n_p\cdot Ap$	$c - dp$	$n_p\cdot Ap\cdot(c - dp)$
Bottom Steel	$n\cdot As$	$c - d$	$n\cdot As\cdot(c - d)$

Given

$$0.5\cdot bft\cdot c^2 + n\cdot As'\cdot(c - d') + n_p\cdot Ap\cdot(c - dp) + n\cdot As\cdot(c - d) = 0$$

$c_1 := \mathrm{Find}(c)$

2) Neutral axis inside the flange and below the compression steel:

Part	Area	y	Area x y
Top Flange	$bft \cdot c$	$0.5 \cdot c$	$0.5 \cdot bft \cdot c^2$
Web	---	---	---
Bottom Flange	---	---	---
Top Steel	$(n-1) \cdot As'$	$c - d'$	$(n-1) \cdot As' \cdot (c - d')$
Prestressing Steel	$n_p \cdot Ap$	$c - dp$	$n_p \cdot Ap \cdot (c - dp)$
Bottom Steel	$n \cdot As$	$c - d$	$n \cdot As \cdot (c - d)$

Given

$$0.5 \cdot bft \cdot c^2 + (n-1) \cdot As' \cdot (c - d') + n_p \cdot Ap \cdot (c - dp) + n \cdot As \cdot (c - d) = 0$$

$$c_2 := \mathrm{Find}(c)$$

3) Neutral axis cuts the web:

Part	Area	y	Area x y
Top Flange	$(bft - bw) \cdot tft$	$c - 0.5 \cdot tft$	$(bft - bw) \cdot tft \cdot (c - 0.5 \cdot tft)$
Web	$bw \cdot c$	$0.5 \cdot c$	$0.5 \cdot bw \cdot c^2$
Bottom Flange	---	---	---
Top Steel	$(n-1) \cdot As'$	$c - d'$	$(n-1) \cdot As' \cdot (c - d')$
Prestressing Steel	$n_p \cdot Ap$	$c - dp$	$n_p \cdot Ap \cdot (c - dp)$
Bottom Steel	$n \cdot As$	$c - d$	$n \cdot As \cdot (c - d)$

Given

$$(bft - bw) \cdot tft \cdot (c - 0.5 \cdot tft) + 0.5 \cdot bw \cdot c^2 + (n-1) \cdot As' \cdot (c - d') + n_p \cdot Ap \cdot (c - dp) + n \cdot As \cdot (c - d) = 0$$

$$c_3 := \mathrm{Find}(c)$$

The neutral axis position after cracking is given by:

$$c_{a\_cr} := \left| \begin{array}{l} c_1 \ \text{ if } \ c_1 < d' \wedge c_1 \neq 0 \\ c_2 \ \text{ if } \ c_2 \leq tft \wedge c_2 \neq 0 \\ c_3 \ \text{ otherwise} \end{array} \right.$$

Moment of Inertia

- **Before cracking**

Part	Area	y	$I_{own\ axis}$	Area x y^2
TF	$(bft - bw)\cdot tft$	$c_{b\_cr} - 0.5tft$	$\frac{(bft - bw)\cdot tft^3}{12}$	$(bft - bw)\cdot tft\cdot(c_{b\_cr} - 0.5tft)^2$
W	$bw\cdot h$	$c_{b\_cr} - 0.5\cdot h$	$\frac{bw\cdot h^3}{12}$	$bw\cdot h\cdot(c_{b\_cr} - 0.5\cdot h)^2$
BF	$(bfb - bw)\cdot tfb$	$c_{b\_cr} - (h - 0.5tfb)$	$\frac{(bfb - bw)\cdot tfb^3}{12}$	$(bfb - bw)\cdot tfb\cdot[c_{b\_cr} - (h - 0.5tfb)]$
TS	$(n - 1)\cdot As'$	$c_{b\_cr} - d'$	---	$(n - 1)\cdot As'\cdot(c_{b\_cr} - d')^2$
PS	$(n_p - 1)\cdot Ap$	$c_{b\_cr} - dp$	---	$(n_p - 1)\cdot Ap\cdot(c_{b\_cr} - dp)^2$
BS	$(n - 1)\cdot As$	$c_{b\_cr} - d$	---	$(n - 1)\cdot As\cdot(c_{b\_cr} - d)^2$

$$I_g := \frac{(bft - bw)\cdot tft^3}{12} + (bft - bw)\cdot tft\cdot(c_{b\_cr} - 0.5tft)^2 + \frac{bw\cdot h^3}{12} + bw\cdot h\cdot(c_{b\_cr} - 0.5\cdot h)^2 \ldots$$
$$+ \frac{(bfb - bw)\cdot tfb^3}{12} + (bfb - bw)\cdot tfb\cdot[c_{b\_cr} - (h - 0.5tfb)] \ldots$$
$$+ (n - 1)\cdot As'\cdot(c_{b\_cr} - d')^2 + (n_p - 1)\cdot Ap\cdot(c_{b\_cr} - dp)^2 + (n - 1)\cdot As\cdot(c_{b\_cr} - d)^2$$

- **After cracking**

1) Neutral axis inside the flange and above the compression steel:

Part	Area	y	$I_{own\ axis}$	Area x y^2
TF	$bft\cdot c_1$	$\frac{c_1}{2}$	$\frac{bft\cdot c_1^3}{12}$	$\frac{bft\cdot c_1^3}{4}$
W	---	---	---	---
BF	---	---	---	---
TS	$n\cdot As'$	$c_1 - d'$	---	$n\cdot As'\cdot(c_1 - d')^2$
PS	$n_p\cdot Ap$	$c_1 - dp$	---	$n_p\cdot Ap\cdot(c_1 - dp)^2$
BS	$n\cdot As$	$c_1 - d$	---	$n\cdot As\cdot(c_1 - d)^2$

$$I_{cr\_1} := \frac{bft \cdot c_1^3}{12} + \frac{bft \cdot c_1^3}{4} + n \cdot As' \cdot (c_1 - d')^2 + n_p \cdot Ap \cdot (c_1 - dp)^2 + n \cdot As \cdot (c_1 - d)^2$$

2) Neutral axis inside the flange and below the compression steel:

Part	**Area**	**y**	$I_{own\ axis}$	**Area x y²**
TF	$bft \cdot c_2$	$\frac{c_2}{2}$	$\frac{bft \cdot c_2^3}{12}$	$\frac{bft \cdot c_2^3}{4}$
W	---	---	---	---
BF	---	---	---	---
TS	$(n-1) \cdot As'$	$c_2 - d'$	---	$(n-1) \cdot As' \cdot (c_2 - d')^2$
PS	$n_p \cdot Ap$	$c_2 - dp$	---	$n_p \cdot Ap \cdot (c_2 - dp)^2$
BS	$n \cdot As$	$c_2 - d$	---	$n \cdot As \cdot (c_2 - d)^2$

$$I_{cr\_2} := \frac{bft \cdot c_2^3}{12} + \frac{bft \cdot c_2^3}{4} + (n-1) \cdot As' \cdot (c_2 - d')^2 + n_p \cdot Ap \cdot (c_2 - dp)^2 + n \cdot As \cdot (c_2 - d)^2$$

3) Neutral axis cuts the web:

Part	**Area**	**y**	$I_{own\ axis}$	**Area x y²**
TF	$(bft - bw) \cdot tft$	$c_3 - \frac{tft}{2}$	$\frac{(bft - bw) \cdot tft^3}{12}$	$(bft - bw) \cdot tft \cdot \left(c_3 - \frac{tft}{2}\right)^2$
W	$bw \cdot c_3$	$\frac{c_3}{2}$	$\frac{bw \cdot c_3^3}{12}$	$\frac{bw \cdot c_3^3}{4}$
BF	---	---	---	---
TS	$(n-1) \cdot As'$	$c_3 - d'$	---	$(n-1) \cdot As' \cdot (c_3 - d')^2$
PS	$n_p \cdot Ap$	$c_3 - dp$	---	$n_p \cdot Ap \cdot (c_3 - dp)^2$
BS	$n \cdot As$	$c_3 - d$	---	$n \cdot As \cdot (c_3 - d)^2$

$$I_{cr\_3} := \frac{(bft - bw) \cdot tft^3}{12} + (bft - bw) \cdot tft \cdot \left(c_3 - \frac{tft}{2}\right)^2 + \frac{bw \cdot c_3^3}{12} + \frac{bw \cdot c_3^3}{4} \cdots$$
$$+ (n-1) \cdot As' \cdot (c_3 - d')^2 + n_p \cdot Ap \cdot (c_3 - dp)^2 + n \cdot As \cdot (c_3 - d)^2$$

The moment of inertia of the cracked concrete section is given by:

$$I_{cr} := \begin{cases} I_{cr\_1} & \text{if } c_1 < d' \wedge c_1 \neq 0 \\ I_{cr\_2} & \text{if } c_2 \le tft \wedge c_2 \neq 0 \\ I_{cr\_3} & \text{otherwise} \end{cases}$$

$I_{cr} = 0$

Initial Strain in the Concrete (e_{bi})

Initial strain in the concrete depends either by the applied load during the FRP installation (Mip) and by the prestressing steel if present. Defining r, radius of gyration of the concrete section ($r^2 = I / A_c$), the initial strain can be found by using equation [1].

Cracking moment, area of the concrete cross section, and radius of gyration are written below (the radius of gyration is given for uncracked [r_g] and cracked [r_{cr}] sections):

$$f_d := \frac{M_{ip} \cdot \frac{h}{2}}{I_g}$$

$$M_{cr} := \begin{cases} \dfrac{7.5\sqrt{f'c} \cdot I_g}{h - c_{b\_cr}} & \text{if } Ap = 0 \\ \dfrac{I_{cr}}{h - c_{b\_cr}} \cdot \left(7.5 \cdot \sqrt{f'c} + f_{pe} - f_d\right) & \text{if } Ap \neq 0 \end{cases}$$

$$A_c := (bft - bw) \cdot tft + bw \cdot h + (bfb - bw) \cdot tfb$$

$$r_g := \sqrt{\frac{I_g}{A_c}} \qquad r_{cr} := \sqrt{\frac{I_{cr}}{A_c}}$$

Effective prestress force at the time of FRP installation (P_e), and eccentricity of the prestress force with respect to the neutral axis (cgc, see figure) of the concrete section before (e_g) and after (e_{cr}) cracking are shown below:

$$P_e := Ap \cdot f_{pe}$$

$$e_g := dp - c_{b\_cr}$$

$$e_{cr} := dp - c_{a\_cr}$$

The initial strain in the concrete for uncracked and cracked sections is:

$$\varepsilon_{bi\_g} := \frac{M_{ip}}{I_g \cdot E_c} \cdot \left(h - c_{b\_cr}\right) - \frac{P_e}{A_c \cdot E_c} \cdot \left[1 + \frac{e_g \cdot \left(h - c_{b\_cr}\right)}{r_g^{\,2}}\right]$$

$$\varepsilon_{bi\_cr} := \frac{M_{ip}}{I_{cr} \cdot E_c} \cdot \left(h - c_{a\_cr}\right)$$

$$\varepsilon_{bi} := \begin{cases} \varepsilon_{bi\_g} & \text{if } M_{so} \le M_{cr} \\ \varepsilon_{bi\_cr} & \text{if } M_{so} > M_{cr} \end{cases} \qquad \textbf{[1]}$$

$\varepsilon_{bi} = 0$

$$c_i := \begin{cases} c_{b\_cr} & \text{if } M_{so} \le M_{cr} \\ c_{a\_cr} & \text{otherwise} \end{cases}$$

The initial strain in the top fiber concrete, in the compression steel, and in the mild tension steel can be written as:

$$\varepsilon_{ci} := \frac{c_i}{h - c_i} \cdot \varepsilon_{bi}$$

$$\varepsilon'_{si} := \frac{c_i - d'}{h - c_i} \cdot \varepsilon_{bi}$$

$$\varepsilon_{si} := \frac{d - c_i}{h - c_i} \cdot \varepsilon_{bi}$$

Initial Strain

CONCRETE CRUSHING (sub c)

Strain

$$\varepsilon'_{sc}(z) := \begin{cases} \dfrac{d' - c(z)}{c(z)} \cdot \varepsilon_{cu} & \text{if } As' \ne 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\varepsilon_{Ljc}(z, j) := \begin{cases} \dfrac{D(j) - c(z)}{c(z)} \cdot \varepsilon_{cu} & \text{if Producer\_L} \ne 1 \\ 0 & \text{otherwise} \end{cases}$$

$\kappa_{m\_L} = 0$

$$\varepsilon_{Ljc}(z, j) := \min\left(\varepsilon_{Ljc}(z, j), \kappa_{m\_L} \cdot \varepsilon_{fu\_L}\right)$$

$$\Omega_u := \begin{cases} \dfrac{3.0}{\dfrac{L_n}{dp}} \cdot Lr & \text{if Bond} = 0 \\ 1 & \text{if Bond} = 1 \\ 0 & \text{if } Ap = 0 \end{cases}$$

$$\varepsilon_{pc}(z) := \begin{cases} \min\left[\dfrac{P_e}{Ap \cdot E_p} + \dfrac{P_e}{A_c \cdot E_c} \cdot \left(1 + \dfrac{e_g^{\,2}}{r_g^{\,2}}\right) + \Omega_u \cdot \dfrac{dp - c(z)}{c(z)} \cdot \varepsilon_{cu}, 0.03\right] & \text{if } Ap \ne 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\varepsilon_{sc}(z) := \begin{cases} \dfrac{d - c(z)}{c(z)} \cdot \varepsilon_{cu} & \text{if } As \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\varepsilon_{rc}(z) := \begin{cases} \dfrac{dr - c(z)}{c(z)} \cdot \varepsilon_{cu} & \text{if Producer\_N} \neq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\kappa_{m\_r} = 0$$

$$\varepsilon_{rc}(z) := \min\left(\varepsilon_{rc}(z), \kappa_{m\_r} \cdot \varepsilon_{fu\_N}\right)$$

$$\varepsilon_{Bc}(z) := \begin{cases} \dfrac{h - c(z)}{c(z)} \cdot \varepsilon_{cu} & \text{if Producer\_B} \neq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\kappa_{m\_B} = 0.9$$

$$\varepsilon_{Bc}(z) := \min\left(\varepsilon_{Bc}(z), \kappa_{m\_B} \cdot \varepsilon_{fu\_B}\right)$$

Stress

$$f'_{sc}(z) := \varepsilon'_{sc}(z) \cdot E_s$$

$$f_{sc}(z) := \begin{cases} f_y & \text{if } f'_{sc}(z) \geq f_y \\ -f_y & \text{if } f'_{sc}(z) \leq -f_y \\ f'_{sc}(z) & \text{otherwise} \end{cases}$$

$$f_{Ljc}(z, j) := \varepsilon_{Ljc}(z, j) \cdot E_{f\_L}$$

$$f_{pc}(z) := \begin{cases} \min\left(\varepsilon_{pc}(z) \cdot E_p, 0.94 \cdot f_{py}\right) & \text{if Bond} = 0 \\ \text{otherwise} & \\ \quad \text{if } fpu = 270 & \\ \quad\quad \begin{cases} \varepsilon_{pc}(z) \cdot E_p & \text{if } \varepsilon_{pc}(z) \leq 0.008 \\ f_{pu} - 2000 - \dfrac{75}{\varepsilon_{pc}(z) - 0.0065} & \text{otherwise} \end{cases} & \\ \quad \text{if } fpu = 250 & \\ \quad\quad \begin{cases} \varepsilon_{pc}(z) \cdot E_p & \text{if } \varepsilon_{pc}(z) \leq 0.0076 \\ f_{pu} - 2000 - \dfrac{58}{\varepsilon_{pc}(z) - 0.006} & \text{otherwise} \end{cases} & \end{cases}$$

$$f_{sc}(z) := \varepsilon_{sc}(z) \cdot E_s$$

$$f_{sc}(z) := \begin{cases} f_y & \text{if } f_{sc}(z) \geq f_y \\ -f_y & \text{if } f_{sc}(z) \leq -f_y \\ f_{sc}(z) & \text{otherwise} \end{cases}$$

$$f_{rc}(z) := \varepsilon_{rc}(z) \cdot E_{f\_N}$$

$$f_{Bc}(z) := \varepsilon_{Bc}(z) \cdot E_{f\_B}$$

Force

$$F'_{sc}(z) := \begin{cases} f'_{sc}(z) \cdot As' & \text{if } c(z) < d' \\ \left(f'_{sc}(z) - 0.85 \cdot fc\right) \cdot As' & \text{otherwise} \end{cases}$$

$$Aj := NL \cdot tf\_L \cdot \frac{wL}{p}$$

$$F_{Ljc}(z, j) := f_{Ljc}(z, j) \cdot Aj$$

$$F_{pc}(z) := Ap \cdot f_{pc}(z)$$

$$F_{sc}(z) := \begin{cases} f_{sc}(z) \cdot As & \text{if } c(z) < d \\ \left(f_{sc}(z) - 0.85fc\right) \cdot As & \text{otherwise} \end{cases}$$

$$A_r := Nr \cdot Ar$$

$$F_{rc}(z) := f_{rc}(z) \cdot A_r$$

$$A_B := wB \cdot NB \cdot tf\_B$$

$$F_{Bc}(z) := f_{Bc}(z) \cdot A_B$$

$$\alpha_{1c} := 0.85$$

$$\beta_1 := \begin{cases} 0.85 & \text{if } fc \leq 4000 \\ 1.05 - 0.05 \cdot \frac{fc}{1000} & \text{if } 4000 < fc < 8000 \\ 0.65 & \text{if } fc \geq 8000 \end{cases}$$

$\beta_1 = 0.845$

$a(z) := \beta_1 \cdot c(z)$

$$C_{cc}(z) := \begin{cases} (\alpha_{1c} \cdot fc \cdot a(z) \cdot bw) & \text{if } bft = 0 \\ \alpha_{1c} \cdot fc \cdot a(z) \cdot bft & \text{if } c(z) \le tft \wedge bft \ne 0 \\ \alpha_{1c} \cdot fc \cdot [\, a(z) \cdot bft - (a(z) - tft) \cdot (bft - bw) \,] & \text{if } c(z) > tft \wedge bft \ne 0 \end{cases}$$

Equilibrium Condition:

$z := h$

Given

$z > c_b$

$$C_{cc}(z) - \left(F'_{sc}(z) + \sum_{j=0}^{p-1} F_{Ljc}(z,j) + F_{pc}(z) + F_{sc}(z) + F_{rc}(z) + F_{Bc}(z) \right) = 0$$

$eq_c := \text{Minerr}(z)$

$eq_c = 2.1$

$z := \frac{h}{q}, \frac{2 \cdot h}{q} .. h$

Moment

$$M_c(z) := F'_{sc}(z) \cdot \left(d' - \frac{a(z)}{2} \right) + \sum_{j=0}^{p-1} \psi_f \cdot F_{Ljc}(z,j) \cdot \left(D(j) - \frac{a(z)}{2} \right) + F_{pc}(z) \cdot \left(dp - \frac{a(z)}{2} \right) \ldots$$
$$+ F_{sc}(z) \cdot \left(d - \frac{a(z)}{2} \right) + \psi_r \cdot F_{rc}(z) \cdot \left(dr - \frac{a(z)}{2} \right) + \psi_f \cdot F_{Bc}(z) \cdot \left(h - \frac{a(z)}{2} \right)$$

$M_c(eq_c) = 3.665 \times 10^5$

Tension Controlled Failure

Equilibrium Condition

$$\text{Failure\_Mode} := \begin{cases} \text{"Tension Controlled"} & \text{if } c(eq_t) < c_b \\ \text{"Concrete Crushing"} & \text{if } c(eq_c) > c_b \end{cases}$$

$$eq := \begin{cases} eq_t & \text{if } c(eq_t) < c_b \\ eq_c & \text{if } c(eq_c) > c_b \end{cases}$$

Tension failure

Concrete crushing

$$eq = 1.164$$

$$M(z) := \begin{cases} M_t(z) & \text{if } c(eq_t) < c_b \\ M_c(z) & \text{if } c(eq_c) > c_b \end{cases}$$

$$M(z) := \frac{M(z)}{12000}$$

$$Mn := M(eq)$$

$$Mn = 31.465$$

$$\varepsilon_c := \begin{cases} \varepsilon_{ct}(eq) & \text{if } c(eq_t) < c_b \\ \varepsilon_{cu} & \text{if } c(eq_c) > c_b \end{cases}$$

$$\varepsilon_c = 0.0012$$

$$\varepsilon'_s := \begin{cases} 0 & \text{if } As' = 0 \\ \text{otherwise} & \\ \quad \begin{cases} -\varepsilon'_{st}(eq_t) & \text{if } c(eq_t) < c_b \\ -\varepsilon'_{sc}(eq_c) & \text{if } c(eq_c) > c_b \end{cases} \end{cases}$$

$$\varepsilon'_s = 0$$

$$\varepsilon p := \begin{cases} \varepsilon_{pt}(eq_t) & \text{if } c(eq_t) < c_b \\ \varepsilon_{pc}(eq_c) & \text{if } c(eq_c) > c_b \end{cases}$$

$$\varepsilon p = 0$$

$$\varepsilon_p := \varepsilon p - \left[\frac{P_e}{Ap \cdot E_p} + \frac{P_e}{A_c \cdot E_c} \cdot \left(1 + \frac{{e_g}^2}{{r_g}^2}\right)\right]$$

$$\varepsilon_p = 0$$

Result of the Strengthening Analysis

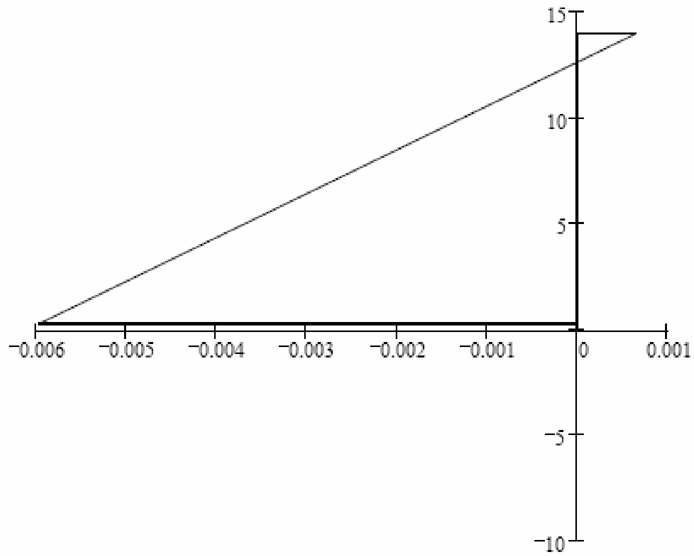
Design Ultimate Moment Capacity

$\phi Mn = 15.48$ $Mu = 15.7$ Design moment capacity vs. moment demand, [k-ft] or [kN-m]

$\kappa_{m\_B} = 0$

$\kappa_{m\_L} = 0$

$\kappa_{m\_r} = 0.7$



Failure_Mode = "Tension Controlled"

$c_b = 4.584$	Depth to the neutral axis for balanced failure, [in] or [mm]
$c = 1.328$	Depth to the neutral axis, [in] or [mm]
$\varepsilon_c = 0.00064$	Maximum strain in the concrete
$\varepsilon'_s = 0$	Strain in the compression steel
$\varepsilon_p = 0$	Strain in the prestressing steel
$\varepsilon_s = 0$	Strain in the tension steel
$\varepsilon r = 0.00595$	Strain at the NSM rod level
$\varepsilon_f = 0$	Strain at the bottom layer of FRP level

Check the Stresses at Service Load Level (Only if FRP is Present)

$f_{cs} = 799$	$F_{cs} = 1845$	Concrete stress at service vs. service stress limit, [psi] or [MPa]
$f_{ss} = 0$	$F'_{ss} = 24000$	Mild compression steel stress at service vs. service stress limit, [psi] or [MPa]
$f_{ps} = 0$	$F_{ps} = 185000$	Prestressng steel stress at service vs. service stress limit, [psi] or [MPa]
$f_{ss} = 0$	$F_{ss} = 48000$	Mild tension steel stress at service vs. service stress limit, [psi] or [MPa]
$f_{fs} = 0$	$F_{fs} = 0$	FRP service stress vs. creep rupture stress limit, [psi] or [MPa]
$f_{rs} = 53613$	$F_{rs} = 140250$	NSM rod service stress vs. creep rupture stress limit, [psi] or [MPa]

APPENDIX II

STRUCTURAL ANALISYS HS20

Inputs

$k_L := 1.00$	--	Coefficient of lateral distribution
$P1 := 8000$	lb	Wheel load *a*
$P2 := 32000$	lb	Wheel load *b*
$P3 := 32000$	lb	Wheel load *c*
$P4 := 0$	lb	Wheel load *d*
$P5 := 0$	lb	Wheel load *e*
$l_1 := 12 \cdot 14$	in	Distance from 1st to 2nd loads
$l_2 := 12 \cdot 14$	in	Distance from 2nd to 3rd loads
$l_3 := 0$	in	Distance from 3rd to 4th loads
$l_4 := 0$	in	Distance from 4th to 5th loads
$L := 12 \cdot 22$	in	Length of Span
$Trucks := 1$	Number of Trucks in train	
$Space := 360$	Space between Trucks in train	
$n := 500$		
$m := 100$		

$$x_{max} := L + (l_1 + l_2 + l_3 + l_4)Trucks + (Trucks - 1)Space$$

$$z := 0, \frac{L}{n} .. L \quad x := 0, \left(0 + \frac{1}{m}\right) .. x_{max}$$

$$P_1(x) := \begin{cases} P1 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$P_2(x) := \begin{cases} P2 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$P_3(x) := \begin{cases} P3 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$P_4(x) := \begin{cases} P4 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$P_5(x) := \begin{cases} P5 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$R1_1(x) := \begin{cases} \dfrac{P_1(x)\cdot(L-x)}{L} & \text{if } (0 < x < L) \\ 0 & \text{otherwise} \end{cases} \qquad R2_1(x) := \begin{cases} \dfrac{P_1(x)\cdot(x)}{L} & \text{if } 0 < x < L \\ 0 & \text{otherwise} \end{cases}$$

$$R1_2(x) := \begin{cases} \dfrac{P_2(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_2(x) := \begin{cases} \dfrac{P_2(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

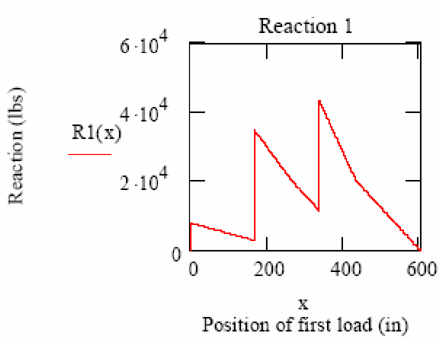
$$R1_3(x) := \begin{cases} \dfrac{P_3(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_3(x) := \begin{cases} \dfrac{P_3(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

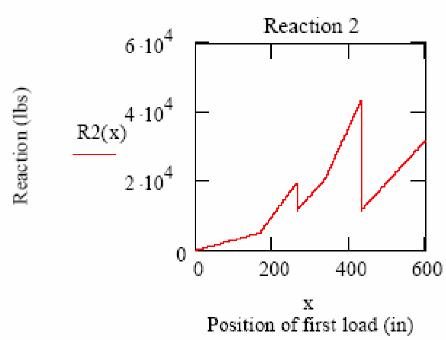
$$R1_4(x) := \begin{cases} \dfrac{P_4(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_4(x) := \begin{cases} \dfrac{P_4(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$R1_5(x) := \begin{cases} \dfrac{P_5(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_5(x) := \begin{cases} \dfrac{P_5(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$R1(x) := \begin{cases} 0 \quad \text{if } x < 0 \\ \text{if } (0) \le L \\ \quad \begin{cases} R1_1(x) & \text{if } 0 \le x \le l_1 \\ \left(R1_1(x) + R1_2(x - l_1)\right) & \text{if } l_1 < x \le (l_1 + l_2) \\ \left(R1_1(x) + R1_2(x - l_1) + R1_3(x - l_1 - l_2)\right) & \text{if } (l_1 + l_2) < x \le (l_1 + l_2 + l_3) \\ \begin{pmatrix} R1_1(x) \ldots \\ +R1_2(x - l_1) \ldots \\ +R1_3(x - l_1 - l_2) \ldots \\ +R1_4(x - l_1 - l_2 - l_3) \end{pmatrix} & \text{if } (l_1 + l_2 + l_3) < x \le (l_1 + l_2 + l_3 + l_4) \\ \begin{pmatrix} R1_1(x) \ldots \\ +R1_2(x - l_1) \ldots \\ +R1_3(x - l_1 - l_2) \ldots \\ +R1_4(x - l_1 - l_2 - l_3) \ldots \\ +R1_5(x - l_1 - l_2 - l_3 - l_4) \end{pmatrix} & \text{if } (l_1 + l_2 + l_3 + l_4) < x \le (L + l_1 + l_2 + l_3 + l_4) \end{cases} \end{cases}$$

$$R2(x) := \begin{cases} 0 & \text{if } x < 0 \\ \text{if } (0) \le L & \\ \quad R2_1(x) & \text{if } 0 \le x \le l_1 \\ \quad \left(R2_1(x) + R2_2(x - l_1)\right) & \text{if } l_1 < x \le (l_1 + l_2) \\ \quad \left(R2_1(x) + R2_2(x - l_1) + R2_3(x - l_1 - l_2)\right) & \text{if } (l_1 + l_2) < x \le (l_1 + l_2 + l_3) \\ \quad \left(R2_1(x) \ldots + R2_2(x - l_1) \ldots + R2_3(x - l_1 - l_2) \ldots + R2_4(x - l_1 - l_2 - l_3)\right) & \text{if } (l_1 + l_2 + l_3) < x \le (l_1 + l_2 + l_3 + l_4) \\ \quad \left(R2_1(x) \ldots + R2_2(x - l_1) \ldots + R2_3(x - l_1 - l_2) \ldots + R2_4(x - l_1 - l_2 - l_3) \ldots + R2_5(x - l_1 - l_2 - l_3 - l_4)\right) & \text{if } (l_1 + l_2 + l_3 + l_4) < x \le (L + l_1 + l_2 + l_3 + l_4) \end{cases}$$





Shear

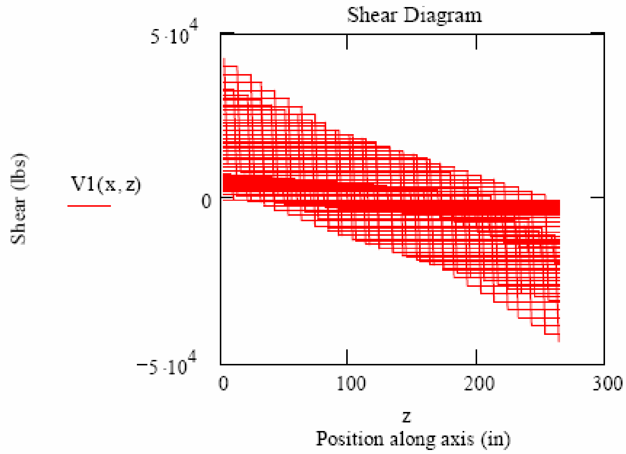
$n := 100$

$m := .1$

$z := 0, \frac{L}{n} .. L$

$x := 0, \left(0 + \frac{1}{m}\right) .. x_{max}$

$$
V1(x,z) := \left|
\begin{array}{l}
\text{if } \left(l_1 + l_2 + l_3 + l_4\right) < x \le \left(L + l_1 + l_2 + l_3 + l_4\right) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } 0 < z \le \left(x - l_1 - l_2 - l_3 - l_4\right) \\
\left(R1(x) - P_5\left(x - l_1 - l_2 - l_3 - l_4\right)\right) \ \text{ if } \left(x - l_1 - l_2 - l_3 - l_4\right) < z \le \left(x - l_1 - l_2 - l_3\right) \\
\left[R1(x) - \left(\begin{array}{l} P_5\left(x - l_1 - l_2 - l_3 - l_4\right) \ldots \\ + P_4\left(x - l_1 - l_2 - l_3\right) \end{array}\right)\right] \ \text{ if } \left(x - l_1 - l_2 - l_3\right) < z \le \left(x - l_1 - l_2\right) \\
\left[R1(x) - \left(\begin{array}{l} P_5\left(x - l_1 - l_2 - l_3 - l_4\right) \ldots \\ + P_4\left(x - l_1 - l_2 - l_3\right) + P_3\left(x - l_1 - l_2\right) \end{array}\right)\right] \ \text{ if } \left(x - l_1 - l_2\right) < z \le \left(x - l_1\right) \\
\left[R1(x) - \left[\begin{array}{l} P_5\left(x - l_1 - l_2 - l_3 - l_4\right) \ldots \\ + \left(P_4\left(x - l_1 - l_2 - l_3\right) + P_3\left(x - l_1 - l_2\right)\right) + P_2\left(x - l_1\right) \end{array}\right]\right] \ \text{ if } \left(x - l_1\right) < z \le (x) \\
\left[R1(x) - \left(\begin{array}{l} P_5\left(x - l_1 - l_2 - l_3 - l_4\right) \ldots \\ + P_4\left(x - l_1 - l_2 - l_3\right) \ldots \\ + P_3\left(x - l_1 - l_2\right) + P_2\left(x - l_1\right) + P_1(x) \end{array}\right)\right] \ \text{ if } (x) < z \le (L)
\end{array}\right. \\
\text{if } \left(l_1 + l_2 + l_3\right) < x \le \left(l_1 + l_2 + l_3 + l_4\right) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } 0 < z \le \left(x - l_1 - l_2 - l_3\right) \\
\left(R1(x) - P_4\left(x - l_1 - l_2 - l_3\right)\right) \ \text{ if } \left(x - l_1 - l_2 - l_3\right) < z \le \left(x - l_1 - l_2\right) \\
\left(R1(x) - P_4\left(x - l_1 - l_2 - l_3\right) - P_3\left(x - l_1 - l_2\right)\right) \ \text{ if } \left(x - l_1 - l_2\right) < z \le \left(x - l_1\right) \\
\left(R1(x) - P_4\left(x - l_1 - l_2 - l_3\right) - P_3\left(x - l_1 - l_2\right) - P_2\left(x - l_1\right)\right) \ \text{ if } \left(x - l_1\right) < z \le (x) \\
\left(R1(x) - P_4\left(x - l_1 - l_2 - l_3\right) - P_3\left(x - l_1 - l_2\right) - P_2\left(x - l_1\right) - P_1(x)\right) \ \text{ if } (x) < z \le (L)
\end{array}\right. \\
\text{if } \left(l_1 + l_2\right) < x \le \left(l_1 + l_2 + l_3\right) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } (0) < z \le \left(x - l_1 - l_2\right) \\
\left(R1(x) - P_3\left(x - l_1 - l_2\right)\right) \ \text{ if } \left(x - l_1 - l_2\right) < z \le \left(x - l_1\right) \\
\left(R1(x) - P_3\left(x - l_1 - l_2\right) - P_2\left(x - l_1\right)\right) \ \text{ if } \left(x - l_1\right) < z \le (x) \\
\left(R1(x) - P_3\left(x - l_1 - l_2\right) - P_2\left(x - l_1\right) - P_1(x)\right) \ \text{ if } (x) < z \le (L)
\end{array}\right. \\
\text{if } \left(l_1\right) < x \le \left(l_1 + l_2\right) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } 0 < z \le \left(x - l_1\right) \\
\left(R1(x) - P_2\left(x - l_1\right)\right) \ \text{ if } \left(x - l_1\right) < z \le (x) \\
\left(R1(x) - P_2\left(x - l_1\right) - P_1(x)\right) \ \text{ if } (x) < z \le (L)
\end{array}\right. \\
\text{if } (0) < x \le \left(l_1\right) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } (0) < z \le (x) \\
\left(R1(x) - P_1(x)\right) \ \text{ if } (x) < z \le (L)
\end{array}\right.
\end{array}\right.
$$



Moment

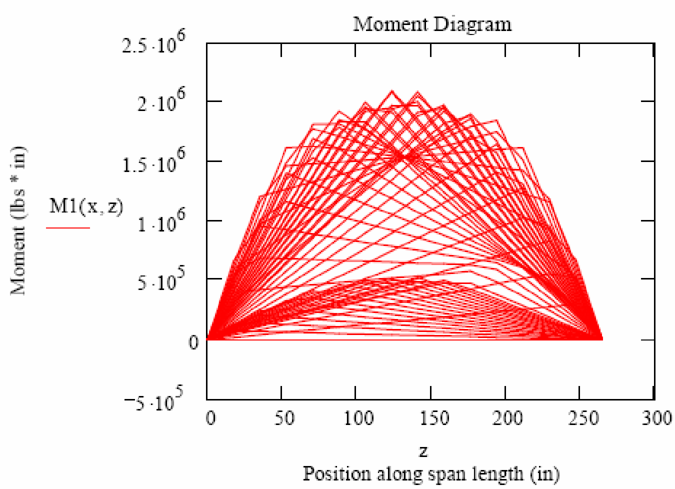
$n := 15$

$m := .1$

$z := 0, \frac{L}{n} .. L$

$x := 0, \left(0 + \frac{1}{m}\right) .. x_{max}$

$M1(x,z) := \int_0^z V1(x,z)\,dz$



STRUCTURAL ANALISYS H20

Inputs

$k_L := 1.00$	--	Coefficient of lateral distribution
$P1 := 8000$	lb	Wheel load *a*
$P2 := 16000$	lb	Wheel load *b*
$P3 := 16000$	lb	Wheel load *c*
$P4 := 0$	lb	Wheel load *d*
$P5 := 0$	lb	Wheel load *e*
$l_1 := 12.1042 \cdot 12$	in	Distance from 1st to 2nd loads
$l_2 := 3.7917 \cdot 12$	in	Distance from 2nd to 3rd loads
$l_3 := 0$	in	Distance from 3rd to 4th loads
$l_4 := 0$	in	Distance from 4th to 5th loads
$L := 12 \cdot 22$	in	Length of Span
$\text{Trucks} := 1$	Number of Trucks in train	
$\text{Space} := 360$	Space between Trucks in train	
$n := 500$		
$m := 100$		

$$x_{max} := L + \left(l_1 + l_2 + l_3 + l_4\right)\text{Trucks} + (\text{Trucks} - 1)\text{Space}$$

$$z := 0, \frac{L}{n} .. L \qquad x := 0, \left(0 + \frac{1}{m}\right) .. x_{max}$$

$$P_1(x) := \begin{vmatrix} P1 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{matrix}$$

$$P_2(x) := \begin{vmatrix} P2 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{matrix}$$

$$P_3(x) := \begin{vmatrix} P3 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{matrix}$$

$$P_4(x) := \begin{vmatrix} P4 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{matrix}$$

$$P_5(x) := \begin{vmatrix} P5 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{matrix}$$

$$R1_1(x) := \begin{cases} \dfrac{P_1(x)\cdot(L-x)}{L} & \text{if } (0 < x < L) \\ 0 & \text{otherwise} \end{cases} \qquad R2_1(x) := \begin{cases} \dfrac{P_1(x)\cdot(x)}{L} & \text{if } 0 < x < L \\ 0 & \text{otherwise} \end{cases}$$

$$R1_2(x) := \begin{cases} \dfrac{P_2(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_2(x) := \begin{cases} \dfrac{P_2(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

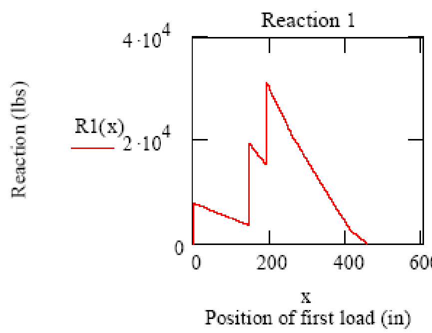
$$R1_3(x) := \begin{cases} \dfrac{P_3(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_3(x) := \begin{cases} \dfrac{P_3(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

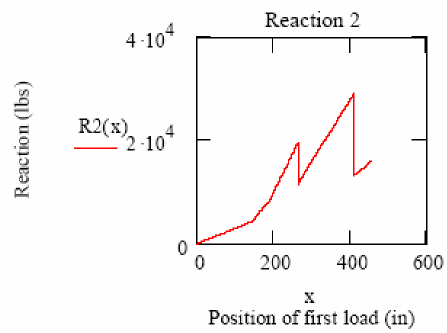
$$R1_4(x) := \begin{cases} \dfrac{P_4(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_4(x) := \begin{cases} \dfrac{P_4(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$R1_5(x) := \begin{cases} \dfrac{P_5(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_5(x) := \begin{cases} \dfrac{P_5(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$R1(x) := \begin{array}{|l} 0 \quad \text{if } x < 0 \\ \text{if } (0) \le L \\ \quad \begin{array}{|l} R1_1(x) \quad \text{if } 0 \le x \le l_1 \\ \left(R1_1(x) + R1_2(x - l_1)\right) \quad \text{if } l_1 < x \le (l_1 + l_2) \\ \left(R1_1(x) + R1_2(x - l_1) + R1_3(x - l_1 - l_2)\right) \quad \text{if } (l_1 + l_2) < x \le (l_1 + l_2 + l_3) \\ \begin{pmatrix} R1_1(x) \ldots \\ +R1_2(x - l_1) \ldots \\ +R1_3(x - l_1 - l_2) \ldots \\ +R1_4(x - l_1 - l_2 - l_3) \end{pmatrix} \quad \text{if } (l_1 + l_2 + l_3) < x \le (l_1 + l_2 + l_3 + l_4) \\ \begin{pmatrix} R1_1(x) \ldots \\ +R1_2(x - l_1) \ldots \\ +R1_3(x - l_1 - l_2) \ldots \\ +R1_4(x - l_1 - l_2 - l_3) \ldots \\ +R1_5(x - l_1 - l_2 - l_3 - l_4) \end{pmatrix} \quad \text{if } (l_1 + l_2 + l_3 + l_4) < x \le (L + l_1 + l_2 + l_3 + l_4) \end{array} \end{array}$$

$$R2(x) := \begin{array}{|l} 0 \ \text{ if } \ x < 0 \\ \text{if } \ (0) \le L \\ \quad \begin{array}{|l} R2_1(x) \ \text{ if } \ 0 \le x \le l_1 \\ \left(R2_1(x) + R2_2(x - l_1)\right) \ \text{ if } \ l_1 < x \le (l_1 + l_2) \\ \left(R2_1(x) + R2_2(x - l_1) + R2_3(x - l_1 - l_2)\right) \ \text{ if } \ (l_1 + l_2) < x \le (l_1 + l_2 + l_3) \\ \begin{pmatrix} R2_1(x) \ldots \\ + R2_2(x - l_1) \ldots \\ + R2_3(x - l_1 - l_2) \ldots \\ + R2_4(x - l_1 - l_2 - l_3) \end{pmatrix} \ \text{ if } \ (l_1 + l_2 + l_3) < x \le (l_1 + l_2 + l_3 + l_4) \\ \begin{pmatrix} R2_1(x) \ldots \\ + R2_2(x - l_1) \ldots \\ + R2_3(x - l_1 - l_2) \ldots \\ + R2_4(x - l_1 - l_2 - l_3) \ldots \\ + R2_5(x - l_1 - l_2 - l_3 - l_4) \end{pmatrix} \ \text{ if } \ (l_1 + l_2 + l_3 + l_4) < x \le (L + l_1 + l_2 + l_3 + l_4) \end{array} \end{array}$$





Shear

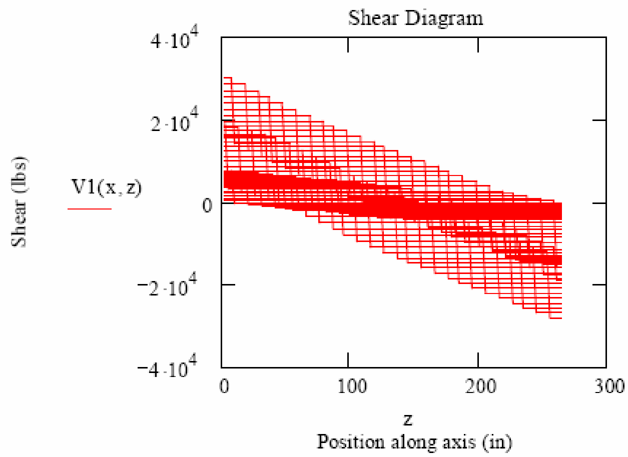
$n := 100$

$m := .1$

$z := 0, \frac{L}{n} .. L$

$x := 0, \left(0 + \frac{1}{m}\right) .. x_{max}$

$$
V1(x,z) := \left|
\begin{array}{l}
\text{if } (l_1 + l_2 + l_3 + l_4) < x \le (L + l_1 + l_2 + l_3 + l_4) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } 0 < z \le (x - l_1 - l_2 - l_3 - l_4) \\
(R1(x) - P_5(x - l_1 - l_2 - l_3 - l_4)) \ \text{ if } (x - l_1 - l_2 - l_3 - l_4) < z \le (x - l_1 - l_2 - l_3) \\
\left[R1(x) - \left(P_5(x - l_1 - l_2 - l_3 - l_4) \ldots + P_4(x - l_1 - l_2 - l_3)\right)\right] \ \text{ if } (x - l_1 - l_2 - l_3) < z \le (x - l_1 - l_2) \\
\left[R1(x) - \left(P_5(x - l_1 - l_2 - l_3 - l_4) \ldots + P_4(x - l_1 - l_2 - l_3) + P_3(x - l_1 - l_2)\right)\right] \ \text{ if } (x - l_1 - l_2) < z \le (x - l_1) \\
\left[R1(x) - \left[P_5(x - l_1 - l_2 - l_3 - l_4) \ldots + (P_4(x - l_1 - l_2 - l_3) + P_3(x - l_1 - l_2)) + P_2(x - l_1)\right]\right] \ \text{ if } (x - l_1) < z \le (x) \\
\left[R1(x) - \left(P_5(x - l_1 - l_2 - l_3 - l_4) \ldots + P_4(x - l_1 - l_2 - l_3) \ldots + P_3(x - l_1 - l_2) + P_2(x - l_1) + P_1(x)\right)\right] \ \text{ if } (x) < z \le (L)
\end{array}
\right. \\
\text{if } (l_1 + l_2 + l_3) < x \le (l_1 + l_2 + l_3 + l_4) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } 0 < z \le (x - l_1 - l_2 - l_3) \\
(R1(x) - P_4(x - l_1 - l_2 - l_3)) \ \text{ if } (x - l_1 - l_2 - l_3) < z \le (x - l_1 - l_2) \\
(R1(x) - P_4(x - l_1 - l_2 - l_3) - P_3(x - l_1 - l_2)) \ \text{ if } (x - l_1 - l_2) < z \le (x - l_1) \\
(R1(x) - P_4(x - l_1 - l_2 - l_3) - P_3(x - l_1 - l_2) - P_2(x - l_1)) \ \text{ if } (x - l_1) < z \le (x) \\
(R1(x) - P_4(x - l_1 - l_2 - l_3) - P_3(x - l_1 - l_2) - P_2(x - l_1) - P_1(x)) \ \text{ if } (x) < z \le (L)
\end{array}
\right. \\
\text{if } (l_1 + l_2) < x \le (l_1 + l_2 + l_3) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } (0) < z \le (x - l_1 - l_2) \\
(R1(x) - P_3(x - l_1 - l_2)) \ \text{ if } (x - l_1 - l_2) < z \le (x - l_1) \\
(R1(x) - P_3(x - l_1 - l_2) - P_2(x - l_1)) \ \text{ if } (x - l_1) < z \le (x) \\
(R1(x) - P_3(x - l_1 - l_2) - P_2(x - l_1) - P_1(x)) \ \text{ if } (x) < z \le (L)
\end{array}
\right. \\
\text{if } (l_1) < x \le (l_1 + l_2) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } 0 < z \le (x - l_1) \\
(R1(x) - P_2(x - l_1)) \ \text{ if } (x - l_1) < z \le (x) \\
(R1(x) - P_2(x - l_1) - P_1(x)) \ \text{ if } (x) < z \le (L)
\end{array}
\right. \\
\text{if } (0) < x \le (l_1) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } (0) < z \le (x) \\
(R1(x) - P_1(x)) \ \text{ if } (x) < z \le (L)
\end{array}
\right.
\end{array}
\right.
$$



Moment

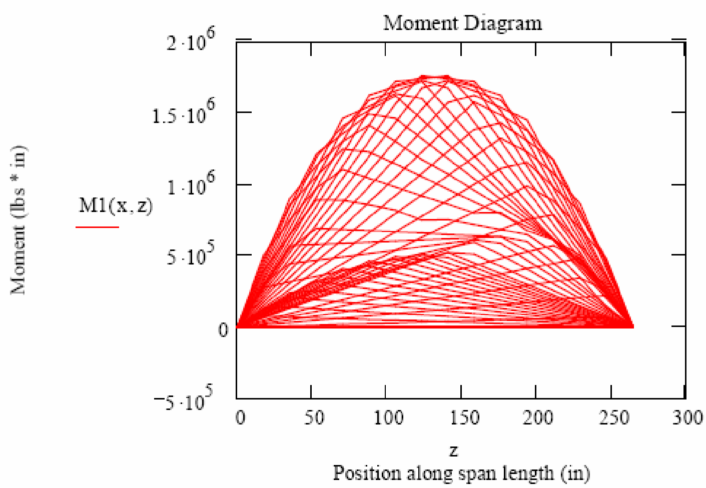
$n := 15$

$m := .1$

$z := 0, \frac{L}{n} .. L$

$x := 0, \left(0 + \frac{1}{m}\right) .. x_{max}$

$$M1(x,z) := \int_0^z V1(x,z)\,dz$$



STRUCTURAL ANALISYS MO5

Inputs

$k_L := 1.00$	--	Coefficient of lateral distribution
$P1 := 9280$	lb	Wheel load *a*
$P2 := 16000$	lb	Wheel load *b*
$P3 := 16000$	lb	Wheel load *c*
$P4 := 16000$	lb	Wheel load *d*
$P5 := 16000$	lb	Wheel load *e*
$l_1 := 12.1042 \cdot 12$	in	Distance from 1st to 2nd loads
$l_2 := 23.4167 \cdot 12$	in	Distance from 2nd to 3rd loads
$l_3 := 3.7917 \cdot 12$	in	Distance from 3rd to 4th loads
$l_4 := 3.7917 \cdot 12$	in	Distance from 4th to 5th loads
$L := 12 \cdot 22$	in	Length of Span
$Trucks := 1$	Number of Trucks in train	
$Space := 360$	Space between Trucks in train	
$n := 500$		
$m := 100$		

$$x_{max} := L + \left(l_1 + l_2 + l_3 + l_4\right) Trucks + (Trucks - 1) Space$$

$$z := 0, \frac{L}{n} .. L \quad x := 0, \left(0 + \frac{1}{m}\right) .. x_{max}$$

$$P_1(x) := \begin{cases} P1 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$P_2(x) := \begin{cases} P2 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$P_3(x) := \begin{cases} P3 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$P_4(x) := \begin{cases} P4 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$P_5(x) := \begin{cases} P5 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$R1_1(x) := \begin{cases} \dfrac{P_1(x)\cdot(L-x)}{L} & \text{if } (0 < x < L) \\ 0 & \text{otherwise} \end{cases} \qquad R2_1(x) := \begin{cases} \dfrac{P_1(x)\cdot(x)}{L} & \text{if } 0 < x < L \\ 0 & \text{otherwise} \end{cases}$$

$$R1_2(x) := \begin{cases} \dfrac{P_2(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_2(x) := \begin{cases} \dfrac{P_2(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

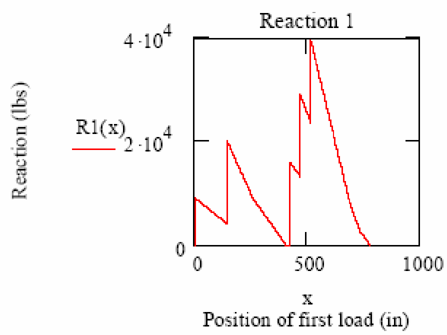
$$R1_3(x) := \begin{cases} \dfrac{P_3(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_3(x) := \begin{cases} \dfrac{P_3(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

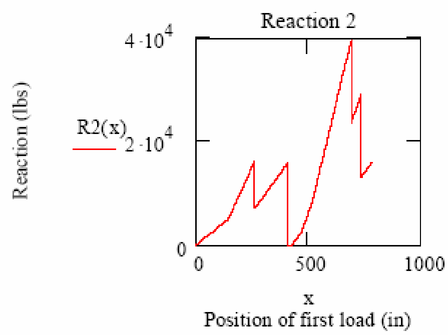
$$R1_4(x) := \begin{cases} \dfrac{P_4(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_4(x) := \begin{cases} \dfrac{P_4(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$R1_5(x) := \begin{cases} \dfrac{P_5(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_5(x) := \begin{cases} \dfrac{P_5(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$R1(x) := \begin{cases} 0 \quad \text{if } x < 0 \\ \text{if } (0) \le L \\ \quad \begin{cases} R1_1(x) & \text{if } 0 \le x \le l_1 \\ \left(R1_1(x) + R1_2(x - l_1)\right) & \text{if } l_1 < x \le (l_1 + l_2) \\ \left(R1_1(x) + R1_2(x - l_1) + R1_3(x - l_1 - l_2)\right) & \text{if } (l_1 + l_2) < x \le (l_1 + l_2 + l_3) \\ \begin{pmatrix} R1_1(x) \ldots \\ +R1_2(x - l_1) \ldots \\ +R1_3(x - l_1 - l_2) \ldots \\ +R1_4(x - l_1 - l_2 - l_3) \end{pmatrix} & \text{if } (l_1 + l_2 + l_3) < x \le (l_1 + l_2 + l_3 + l_4) \\ \begin{pmatrix} R1_1(x) \ldots \\ +R1_2(x - l_1) \ldots \\ +R1_3(x - l_1 - l_2) \ldots \\ +R1_4(x - l_1 - l_2 - l_3) \ldots \\ +R1_5(x - l_1 - l_2 - l_3 - l_4) \end{pmatrix} & \text{if } (l_1 + l_2 + l_3 + l_4) < x \le (L + l_1 + l_2 + l_3 + l_4) \end{cases} \end{cases}$$

$$
R2(x) := \begin{array}{|l} 0 \ \text{ if } \ x < 0 \\ \text{if } \ (0) \le L \\ \quad \begin{array}{|l} R2_1(x) \ \text{ if } \ 0 \le x \le l_1 \\ \left(R2_1(x) + R2_2(x - l_1)\right) \ \text{ if } \ l_1 < x \le (l_1 + l_2) \\ \left(R2_1(x) + R2_2(x - l_1) + R2_3(x - l_1 - l_2)\right) \ \text{ if } \ (l_1 + l_2) < x \le (l_1 + l_2 + l_3) \\ \begin{pmatrix} R2_1(x) \ ... \\ + R2_2(x - l_1) \ ... \\ + R2_3(x - l_1 - l_2) \ ... \\ + R2_4(x - l_1 - l_2 - l_3) \end{pmatrix} \ \text{ if } \ (l_1 + l_2 + l_3) < x \le (l_1 + l_2 + l_3 + l_4) \\ \begin{pmatrix} R2_1(x) \ ... \\ + R2_2(x - l_1) \ ... \\ + R2_3(x - l_1 - l_2) \ ... \\ + R2_4(x - l_1 - l_2 - l_3) \ ... \\ + R2_5(x - l_1 - l_2 - l_3 - l_4) \end{pmatrix} \ \text{ if } \ (l_1 + l_2 + l_3 + l_4) < x \le (L + l_1 + l_2 + l_3 + l_4) \end{array} \end{array}
$$



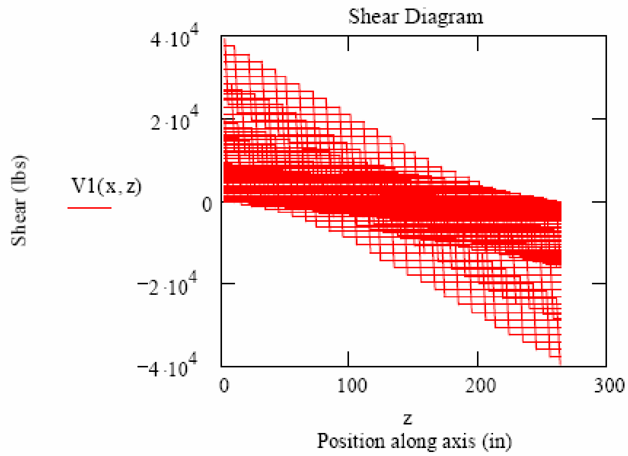


Shear

$n := 100$ $\qquad$ $m := .1$

$z := 0, \frac{L}{n} .. L$ $\qquad$ $x := 0, \left(0 + \frac{1}{m}\right) .. x_{max}$

$$
\mathrm{V1}(x,z) := \left|
\begin{array}{l}
\text{if } (l_1 + l_2 + l_3 + l_4) < x \le (L + l_1 + l_2 + l_3 + l_4) \\
\quad \left|
\begin{array}{l}
\mathrm{R1}(x) \ \text{ if } 0 < z \le (x - l_1 - l_2 - l_3 - l_4) \\
\left(\mathrm{R1}(x) - P_5(x - l_1 - l_2 - l_3 - l_4)\right) \ \text{ if } (x - l_1 - l_2 - l_3 - l_4) < z \le (x - l_1 - l_2 - l_3) \\
\left[\mathrm{R1}(x) - \left(\begin{array}{l} P_5(x - l_1 - l_2 - l_3 - l_4) \ldots \\ + P_4(x - l_1 - l_2 - l_3) \end{array}\right)\right] \ \text{ if } (x - l_1 - l_2 - l_3) < z \le (x - l_1 - l_2) \\
\left[\mathrm{R1}(x) - \left(\begin{array}{l} P_5(x - l_1 - l_2 - l_3 - l_4) \ldots \\ + P_4(x - l_1 - l_2 - l_3) + P_3(x - l_1 - l_2) \end{array}\right)\right] \ \text{ if } (x - l_1 - l_2) < z \le (x - l_1) \\
\left[\mathrm{R1}(x) - \left[\begin{array}{l} P_5(x - l_1 - l_2 - l_3 - l_4) \ldots \\ + \left(P_4(x - l_1 - l_2 - l_3) + P_3(x - l_1 - l_2)\right) + P_2(x - l_1) \end{array}\right]\right] \ \text{ if } (x - l_1) < z \le (x) \\
\left[\mathrm{R1}(x) - \left(\begin{array}{l} P_5(x - l_1 - l_2 - l_3 - l_4) \ldots \\ + P_4(x - l_1 - l_2 - l_3) \ldots \\ + P_3(x - l_1 - l_2) + P_2(x - l_1) + P_1(x) \end{array}\right)\right] \ \text{ if } (x) < z \le (L)
\end{array}\right. \\
\text{if } (l_1 + l_2 + l_3) < x \le (l_1 + l_2 + l_3 + l_4) \\
\quad \left|
\begin{array}{l}
\mathrm{R1}(x) \ \text{ if } 0 < z \le (x - l_1 - l_2 - l_3) \\
\left(\mathrm{R1}(x) - P_4(x - l_1 - l_2 - l_3)\right) \ \text{ if } (x - l_1 - l_2 - l_3) < z \le (x - l_1 - l_2) \\
\left(\mathrm{R1}(x) - P_4(x - l_1 - l_2 - l_3) - P_3(x - l_1 - l_2)\right) \ \text{ if } (x - l_1 - l_2) < z \le (x - l_1) \\
\left(\mathrm{R1}(x) - P_4(x - l_1 - l_2 - l_3) - P_3(x - l_1 - l_2) - P_2(x - l_1)\right) \ \text{ if } (x - l_1) < z \le (x) \\
\left(\mathrm{R1}(x) - P_4(x - l_1 - l_2 - l_3) - P_3(x - l_1 - l_2) - P_2(x - l_1) - P_1(x)\right) \ \text{ if } (x) < z \le (L)
\end{array}\right. \\
\text{if } (l_1 + l_2) < x \le (l_1 + l_2 + l_3) \\
\quad \left|
\begin{array}{l}
\mathrm{R1}(x) \ \text{ if } (0) < z \le (x - l_1 - l_2) \\
\left(\mathrm{R1}(x) - P_3(x - l_1 - l_2)\right) \ \text{ if } (x - l_1 - l_2) < z \le (x - l_1) \\
\left(\mathrm{R1}(x) - P_3(x - l_1 - l_2) - P_2(x - l_1)\right) \ \text{ if } (x - l_1) < z \le (x) \\
\left(\mathrm{R1}(x) - P_3(x - l_1 - l_2) - P_2(x - l_1) - P_1(x)\right) \ \text{ if } (x) < z \le (L)
\end{array}\right. \\
\text{if } (l_1) < x \le (l_1 + l_2) \\
\quad \left|
\begin{array}{l}
\mathrm{R1}(x) \ \text{ if } 0 < z \le (x - l_1) \\
\left(\mathrm{R1}(x) - P_2(x - l_1)\right) \ \text{ if } (x - l_1) < z \le (x) \\
\left(\mathrm{R1}(x) - P_2(x - l_1) - P_1(x)\right) \ \text{ if } (x) < z \le (L)
\end{array}\right. \\
\text{if } (0) < x \le (l_1) \\
\quad \left|
\begin{array}{l}
\mathrm{R1}(x) \ \text{ if } (0) < z \le (x) \\
\left(\mathrm{R1}(x) - P_1(x)\right) \ \text{ if } (x) < z \le (L)
\end{array}\right.
\end{array}\right.
$$



Moment

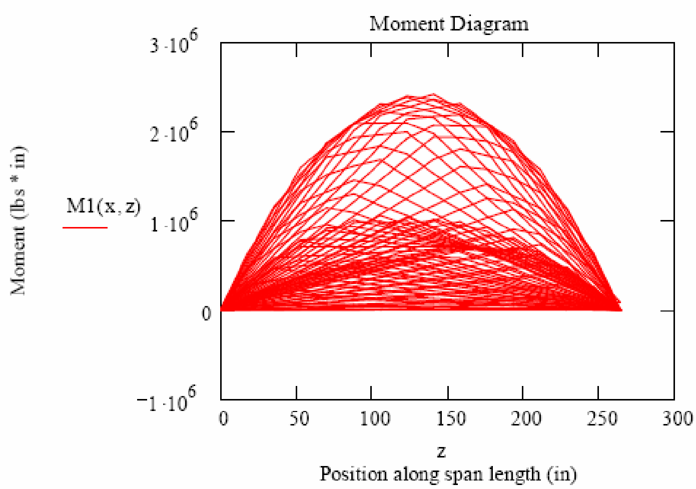
$n := 15$

$z := 0, \frac{L}{n} .. L$

$m := .1$

$x := 0, \left(0 + \frac{1}{m}\right) .. x_{max}$

$$M1(x,z) := \int_0^z V1(x,z)\,dz$$



STRUCTURAL ANALISYS 3S2

Inputs

$k_L := 1.00$	--	Coefficient of lateral distribution
$P1 := 9280$	lb	Wheel load *a*
$P2 := 16000$	lb	Wheel load *b*
$P3 := 16000$	lb	Wheel load *c*
$P4 := 16000$	lb	Wheel load *d*
$P5 := 16000$	lb	Wheel load *e*
$l_1 := 12.1042 \cdot 12$	in	Distance from 1st to 2nd loads
$l_2 := 3.7917 \cdot 12$	in	Distance from 2nd to 3rd loads
$l_3 := 23.4167 \cdot 12$	in	Distance from 3rd to 4th loads
$l_4 := 3.7917 \cdot 12$	in	Distance from 4th to 5th loads
$L := 12 \cdot 22$	in	Length of Span
$Trucks := 1$	Number of Trucks in train	
$Space := 360$	Space between Trucks in train	
$n := 500$		
$m := 100$		

$$x_{max} := L + (l_1 + l_2 + l_3 + l_4)Trucks + (Trucks - 1)Space$$

$$z := 0, \frac{L}{n} .. L \quad x := 0, \left(0 + \frac{1}{m}\right) .. x_{max}$$

$$P_1(x) := \begin{cases} P1 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$P_2(x) := \begin{cases} P2 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$P_3(x) := \begin{cases} P3 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$P_4(x) := \begin{cases} P4 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$P_5(x) := \begin{cases} P5 & \text{if } 0 \le x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$R1_1(x) := \begin{cases} \dfrac{P_1(x)\cdot(L-x)}{L} & \text{if } (0 < x < L) \\ 0 & \text{otherwise} \end{cases} \qquad R2_1(x) := \begin{cases} \dfrac{P_1(x)\cdot(x)}{L} & \text{if } 0 < x < L \\ 0 & \text{otherwise} \end{cases}$$

$$R1_2(x) := \begin{cases} \dfrac{P_2(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_2(x) := \begin{cases} \dfrac{P_2(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

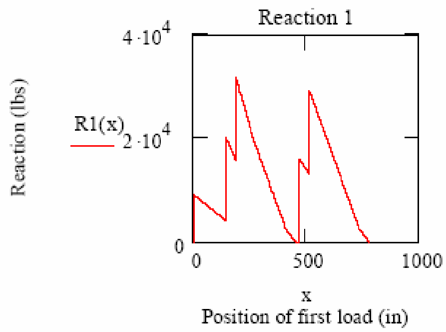
$$R1_3(x) := \begin{cases} \dfrac{P_3(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_3(x) := \begin{cases} \dfrac{P_3(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

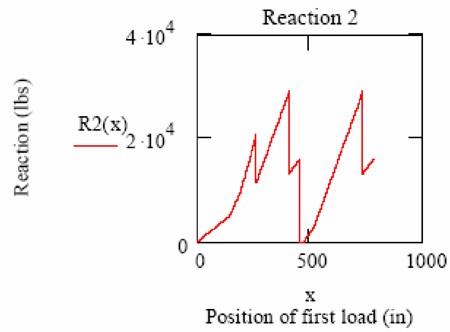
$$R1_4(x) := \begin{cases} \dfrac{P_4(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_4(x) := \begin{cases} \dfrac{P_4(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$R1_5(x) := \begin{cases} \dfrac{P_5(x)\cdot(L-x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases} \qquad R2_5(x) := \begin{cases} \dfrac{P_5(x)\cdot(x)}{L} & \text{if } 0 < x \le L \\ 0 & \text{otherwise} \end{cases}$$

$$R1(x) := \begin{cases} 0 & \text{if } x < 0 \\ \text{if } (0) \le L & \\ \quad \begin{cases} R1_1(x) & \text{if } 0 \le x \le l_1 \\ \left(R1_1(x) + R1_2(x - l_1)\right) & \text{if } l_1 < x \le (l_1 + l_2) \\ \left(R1_1(x) + R1_2(x - l_1) + R1_3(x - l_1 - l_2)\right) & \text{if } (l_1 + l_2) < x \le (l_1 + l_2 + l_3) \\ \begin{pmatrix} R1_1(x) \ldots \\ +R1_2(x - l_1) \ldots \\ +R1_3(x - l_1 - l_2) \ldots \\ +R1_4(x - l_1 - l_2 - l_3) \end{pmatrix} & \text{if } (l_1 + l_2 + l_3) < x \le (l_1 + l_2 + l_3 + l_4) \\ \begin{pmatrix} R1_1(x) \ldots \\ +R1_2(x - l_1) \ldots \\ +R1_3(x - l_1 - l_2) \ldots \\ +R1_4(x - l_1 - l_2 - l_3) \ldots \\ +R1_5(x - l_1 - l_2 - l_3 - l_4) \end{pmatrix} & \text{if } (l_1 + l_2 + l_3 + l_4) < x \le (L + l_1 + l_2 + l_3 + l_4) \end{cases} \end{cases}$$

$$R2(x) := \left| \begin{array}{l} 0 \quad \text{if } x < 0 \\ \text{if } (0) \le L \\ \quad \left| \begin{array}{l} R2_1(x) \quad \text{if } 0 \le x \le l_1 \\ \left(R2_1(x) + R2_2(x - l_1)\right) \quad \text{if } l_1 < x \le (l_1 + l_2) \\ \left(R2_1(x) + R2_2(x - l_1) + R2_3(x - l_1 - l_2)\right) \quad \text{if } (l_1 + l_2) < x \le (l_1 + l_2 + l_3) \\ \left(\begin{array}{l} R2_1(x) \ldots \\ + R2_2(x - l_1) \ldots \\ + R2_3(x - l_1 - l_2) \ldots \\ + R2_4(x - l_1 - l_2 - l_3) \end{array}\right) \quad \text{if } (l_1 + l_2 + l_3) < x \le (l_1 + l_2 + l_3 + l_4) \\ \left(\begin{array}{l} R2_1(x) \ldots \\ + R2_2(x - l_1) \ldots \\ + R2_3(x - l_1 - l_2) \ldots \\ + R2_4(x - l_1 - l_2 - l_3) \ldots \\ + R2_5(x - l_1 - l_2 - l_3 - l_4) \end{array}\right) \quad \text{if } (l_1 + l_2 + l_3 + l_4) < x \le (L + l_1 + l_2 + l_3 + l_4) \end{array} \right. \end{array} \right.$$





Shear

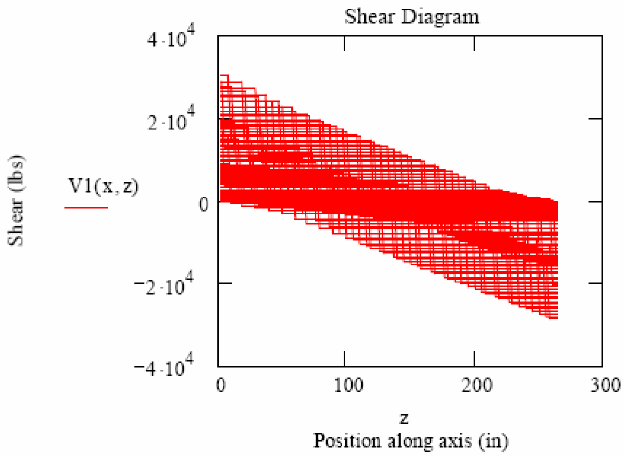
$n := 100$

$m := .1$

$z := 0, \frac{L}{n} .. L$

$x := 0, \left(0 + \frac{1}{m}\right) .. x_{max}$

$$
V1(x,z) := \left|
\begin{array}{l}
\text{if } \left(l_1 + l_2 + l_3 + l_4\right) < x \le \left(L + l_1 + l_2 + l_3 + l_4\right) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } 0 < z \le \left(x - l_1 - l_2 - l_3 - l_4\right) \\
\left(R1(x) - P_5\left(x - l_1 - l_2 - l_3 - l_4\right)\right) \ \text{ if } \left(x - l_1 - l_2 - l_3 - l_4\right) < z \le \left(x - l_1 - l_2 - l_3\right) \\
\left[R1(x) - \left(\begin{array}{l} P_5\left(x - l_1 - l_2 - l_3 - l_4\right) \ldots \\ + P_4\left(x - l_1 - l_2 - l_3\right) \end{array}\right)\right] \ \text{ if } \left(x - l_1 - l_2 - l_3\right) < z \le \left(x - l_1 - l_2\right) \\
\left[R1(x) - \left(\begin{array}{l} P_5\left(x - l_1 - l_2 - l_3 - l_4\right) \ldots \\ + P_4\left(x - l_1 - l_2 - l_3\right) + P_3\left(x - l_1 - l_2\right) \end{array}\right)\right] \ \text{ if } \left(x - l_1 - l_2\right) < z \le \left(x - l_1\right) \\
\left[R1(x) - \left[\begin{array}{l} P_5\left(x - l_1 - l_2 - l_3 - l_4\right) \ldots \\ + \left(P_4\left(x - l_1 - l_2 - l_3\right) + P_3\left(x - l_1 - l_2\right)\right) + P_2\left(x - l_1\right) \end{array}\right]\right] \ \text{ if } \left(x - l_1\right) < z \le (x) \\
\left[R1(x) - \left(\begin{array}{l} P_5\left(x - l_1 - l_2 - l_3 - l_4\right) \ldots \\ + P_4\left(x - l_1 - l_2 - l_3\right) \ldots \\ + P_3\left(x - l_1 - l_2\right) + P_2\left(x - l_1\right) + P_1(x) \end{array}\right)\right] \ \text{ if } (x) < z \le (L)
\end{array}\right. \\
\text{if } \left(l_1 + l_2 + l_3\right) < x \le \left(l_1 + l_2 + l_3 + l_4\right) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } 0 < z \le \left(x - l_1 - l_2 - l_3\right) \\
\left(R1(x) - P_4\left(x - l_1 - l_2 - l_3\right)\right) \ \text{ if } \left(x - l_1 - l_2 - l_3\right) < z \le \left(x - l_1 - l_2\right) \\
\left(R1(x) - P_4\left(x - l_1 - l_2 - l_3\right) - P_3\left(x - l_1 - l_2\right)\right) \ \text{ if } \left(x - l_1 - l_2\right) < z \le \left(x - l_1\right) \\
\left(R1(x) - P_4\left(x - l_1 - l_2 - l_3\right) - P_3\left(x - l_1 - l_2\right) - P_2\left(x - l_1\right)\right) \ \text{ if } \left(x - l_1\right) < z \le (x) \\
\left(R1(x) - P_4\left(x - l_1 - l_2 - l_3\right) - P_3\left(x - l_1 - l_2\right) - P_2\left(x - l_1\right) - P_1(x)\right) \ \text{ if } (x) < z \le (L)
\end{array}\right. \\
\text{if } \left(l_1 + l_2\right) < x \le \left(l_1 + l_2 + l_3\right) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } (0) < z \le \left(x - l_1 - l_2\right) \\
\left(R1(x) - P_3\left(x - l_1 - l_2\right)\right) \ \text{ if } \left(x - l_1 - l_2\right) < z \le \left(x - l_1\right) \\
\left(R1(x) - P_3\left(x - l_1 - l_2\right) - P_2\left(x - l_1\right)\right) \ \text{ if } \left(x - l_1\right) < z \le (x) \\
\left(R1(x) - P_3\left(x - l_1 - l_2\right) - P_2\left(x - l_1\right) - P_1(x)\right) \ \text{ if } (x) < z \le (L)
\end{array}\right. \\
\text{if } \left(l_1\right) < x \le \left(l_1 + l_2\right) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } 0 < z \le \left(x - l_1\right) \\
\left(R1(x) - P_2\left(x - l_1\right)\right) \ \text{ if } \left(x - l_1\right) < z \le (x) \\
\left(R1(x) - P_2\left(x - l_1\right) - P_1(x)\right) \ \text{ if } (x) < z \le (L)
\end{array}\right. \\
\text{if } (0) < x \le \left(l_1\right) \\
\quad \left|
\begin{array}{l}
R1(x) \ \text{ if } (0) < z \le (x) \\
\left(R1(x) - P_1(x)\right) \ \text{ if } (x) < z \le (L)
\end{array}\right.
\end{array}\right.
$$



Moment

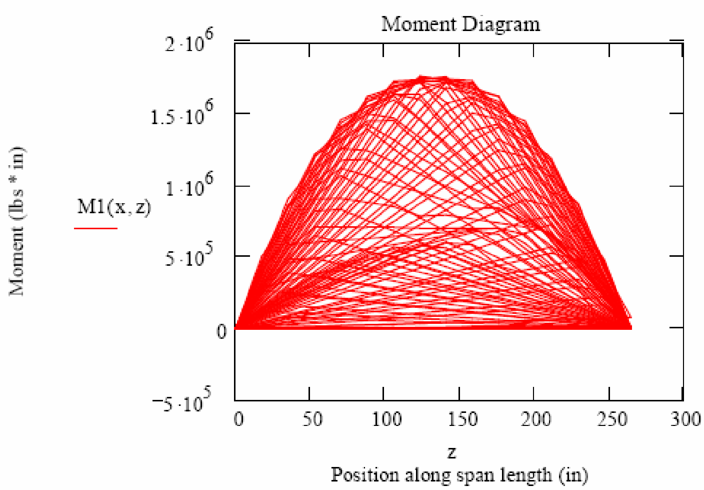
$n := 15$

$m := .1$

$z := 0, \frac{L}{n} .. L$

$x := 0, \left(0 + \frac{1}{m}\right) .. x_{max}$

$$M1(x,z) := \int_0^z V1(x,z)\,dz$$



Load Rating for Bridge Structures

Bridge:	Martin Springs	Performed By:	Nestore Galati
Bridge #:		Date:	10/10/2003
City:	Rolla	Member Rated:	Slab
County:	Phelps		
State:	Missouri		

Loading

M_{DL} =	**471.113**	k-ft	(Moment due to Dead Load)
V_{DL} =	**85.657**	kip	(Shear due to Dead Load)

	HS20	MO5	H20	3S2		
M_{LL} =	**226.42**	**261.08**	**190.56**	**190.88**	k-ft	(Unfactored Moment from Live Load incl Impact)
V_{LL} =	**56.11**	**51.58**	**39.08**	**39.48**	kip	(Unfactored Shear from Live Load incl Impact)

Capacity

ΦM_N =	1229	k-ft	(Factored Moment Capacity)
ΦV_N =	370	kip	(Factored Shear Capacity)

Load Rating (Moment)			
Rating Type	Truck	Rating Factor	Rating (Tons)
Operating	**HS20**	**2.095**	**75.4**
Inventory	**HS20**	**1.255**	**45.2**
Operating	MO5	1.817	65.4
Posting	H20 Lgl	2.140	42.8
Posting	3S2	2.137	78.3

Load Rating (Shear)			
Rating Type	Truck	Rating Factor	Rating (Tons)
Operating	HS20	3.546	127.7
Inventory	HS20	2.124	76.5
Operating	MO5	3.857	141.3
Posting	H20 Lgl	4.379	87.6
Posting	3S2	4.334	158.8